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Characterizing the Quantum Phase Transition using a Flat Band in Circuit QED Lattices

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Abstract: We show the superradiant phase transition (SPT) can control the existence of flat band in an extended Dicke-Hubbard lattice [1].
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A flat band is a non-dispersive energy band with a zero group velocity. Such a band supports a macroscopic number of degenerate localized states. The Dicke model, which describes collective light-matter interactions, has been predicted to exhibit an intriguing superradiant phase transition (SPT) from the normal phase to the superradiant phase at a critical atom-field coupling λ_c in the thermodynamic limit. Nevertheless, the SPT in an extended Dicke-Hubbard lattice is lacking.

In this paper, we propose a circuit QED lattice in which two-level systems are doped in sublattice A, while sublattice B consists of an atomless cavity [see Fig. 1(a)]. We describe the system with the Hamiltonian

$$H = \sum_{n_A} \omega_A a_{n_A}^\dagger a_{n_A} + \Omega J_{n_A}^z + \frac{\lambda}{\sqrt{N}}(a_{n_A}^\dagger + a_{n_A})(J_{n_A}^+ + J_{n_A}) + \sum_{n_B} \omega_B a_{n_B}^\dagger a_{n_B} - \zeta \sum_{\langle n_A, m_B\rangle} (a_{n_A}^\dagger + a_{n_A})(a_{m_B}^\dagger + a_{m_B}),$$

where the collective coupling constant λ describes the strength of the N spins coupled to the coplanar waveguide

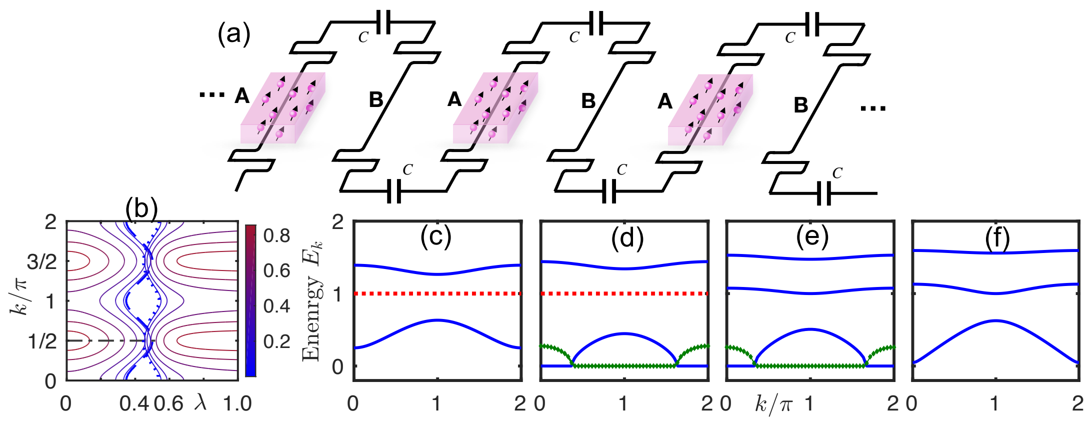

Fig.1 (a) Quasi-one dimensional (Quasi-1D) extended Dicke-Hubbard lattice consists of two-level systems (e.g., an ensemble of NV center spins) in every other coplanar waveguide (CPW) resonator. (b) Contour plot of energy $E(k)$ as a function of spin-field coupling λ and wavenumber k in normal phase (left) and superradiant phase (right), respectively. The dashed (dotted) curve is a zero energy contour in normal phase (superradiant phase), which define the boundaries between regions real and imaginary values of $E(k)$. The region between these two boundaries is called critical region. (c-f) Energy structures of quasi-1D circuit QED lattice. The solid and dotted curves show the real part of the energy, and the diamond curves present the imaginary part. (c) is in normal phase and spin-field coupling is $\lambda = 0.3$. (d,e) are in the unstable region with $\lambda = 0.4, 0.505$, respectively. (f) is in superradiant phase with $\lambda = 0.542$. In the normal phase, there is one flat band (red-dotted) occurs. But in the superradiant phase, this flat band disappears and all bands are dispersive. In (d,e) one eigenvalue becomes imaginary at some certain k which denotes at this case the system is unstable. Here we considered the resonant condition $\omega_A = \omega_B = \Omega = \omega = 1$ and $\zeta = 0.18$.

resonator mode. Here a_{n_A} is annihilation operator for the resonator mode in site n_A with frequency ω_A, and the spins are described by the collective operators $J_{n_A}^z = 1/2 \sum_N \sigma_{n_A}^z$, and $J_{n_A}^\pm = \sum_N \sigma_{n_A}^\pm$,where $\sigma_{n_A}^{z,\pm}$ are Pauli matrices.

To describe the phase transition in the extended Dicke-Hubbard lattice, we extend the method of Ref. [2] to the multi-cavity system and Fourier transform Hamiltonian into k space, then the coefficients are collected into the 6×6 matrix. Diagonalizing the dynamic matrix, in Fig. 1(b) we plot the contour of the real part of the lowest energy spectra versus spin-field coupling λ and wavenumber k in the normal phase and the superradiant phase. Unlike the standard Dicke model which has a single critical point, there is a critical region separating the normal phase and supperradiant phase. As coupling λ increases, the system turns from normal phase to the critical region, and then finally reaches the superradiant phase.

In Figs.1(c-f), we present the dispersion relation of $E(k)$ in the first Brillouin zone for different values of spin-field coupling λ. The dispersion relation of the lattice yields three bands, including a flat band in the normal phase located at $E(k) = \omega$. With the increasing of coupling λ, the lowest eigenvalue becomes completely imaginary for some certain k, which indicates that the system becomes unstable. Despite this, the flat band persists. Physically, a flat band arises due to the destructive interference between two paths. This destructive condition is sensitive to the perturbations of the resonant condition. Only if the eigenfrequencies of spins and cavity mode B are at resonance, i.e., $\Omega = \omega_B$, then the destructive interference condition would be satisfied and flat band would occur. Once the spin-field coupling λ is large enough to surpass the unstable region, the system enters into superradiant phase. Here, the flat band turns to a dispersive one.

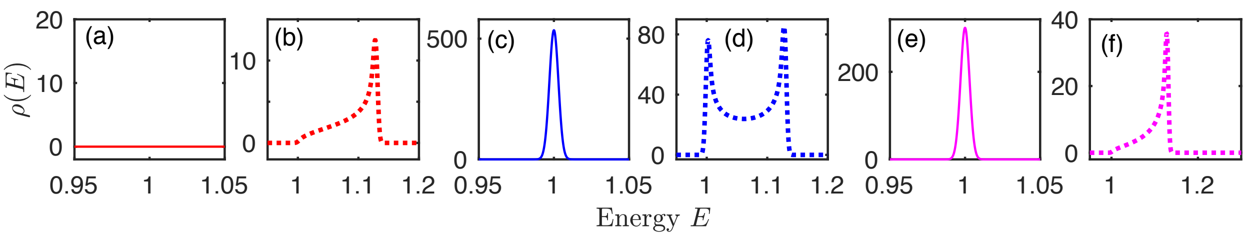

Fig.2 (a-f) The local density of states (LDOS) of three different eigenmodes in quasi-1D circuit QED lattice for spin-field coupling $\lambda = 0.3$ in the normal phase (solid curves) and $\lambda = 0.542$ in the superradiant phase (dashed curves). The red, blue and pink curves, show the LDOS of cavity mode A, cavity mode B and spins, respectively.

To demonstrate the localization induced by the flat band, in Figs. 2(a-f) we plot the local density of states (LDOS) of three different eigenmodes with respect to energy E in the normal phase (solid curves) and superradiant phase (dashed curves), respectively. As shown in Fig. 2(a), in the normal phase, the LDOS of cavity A at $E = 1$ is zero but for cavity mode B and the spin mode, its LDOSs have regular Gaussian-like peaks [see Figs. 2(c) and 2(e)]. In such a regime, both cavity B and the spins are localized at $E = 1$, while cavity A remains completely dark in that the destructive interference between two paths offsets the net flow of particles to cavity mode A. Nevertheless, in the superradiant phase, the effective frequencies of spins and cavity A are off-resonance, hence the destructive interference is destroyed, causing the disappearance of flat-band localization.

To summarize, we have investigated flat-band physics associated with a SPT in hybrid circuit-QED architecture. We found that the critical point that occurs in a single Dicke model becomes a critical region in a lattice geometry that is periodically modulated by wavenumber k. The flat band, and the localization that it engenders, is found exclusively in the low-coupling normal phase of the system. The importance of our work is twofold; on the one hand, SPT can control optical transmission properties of the lattice. Conversely, it also offers a flexible and intuitive method to distinguish the normal phase and superradiant phase via experimentally detectable energy spectrum.

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