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# Stokes drag on axially symmetric body in micro polar fluid

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## Abstract

In a recent paper, Srivastava et al. (2016) have tested the proposed formula based on DS-conjecture Datta and Srivastava, (1999) of Stokes drag on axially symmetric bodies placed under micropolar fluid to improve the drag value under Oseen's limit. In the present work, proof of the proposed drag formula is given for both axial and transverse Stokes flow of micropolar fluid under certain body geometry constraints mainly of continuously turning tangent on body curve in meridional plane as assumed in DS-conjecture. The general expression of drag immediately reduces to the value of drag in classical Newtonian fluid as micro polarity parameter k approaches to zero. The proposed expressions of Stokes drag are applied to various axially symmetric bodies like sphere, spheroid, deformed sphere, cycloidal body, Cassini body, Hypocycloidal body and egg-shaped body and results are in agreement with some known values available in the literature in the limiting cases.

**Keywords:** Stokes drag; axially symmetric body; meridional plane; micropolar fluid; deformed sphere; cycloidal body; Cassini body; egg-shaped body

**MSC 2010 No.:** 76D05, 76D07

## 1. Introduction

The theory of micro polar fluid was first introduced by Eringen (1966). The micro polar fluid differs with classical Newtonian fluid by only microstructure properties viz; micro-rotation and micro-inertia. Complex fluids like polymeric suspensions, animal blood, liquid crystals, lubricants, colloidal suspensions, bubbly fluids, granular fluids are few examples of micro polar fluids. In such fluid, solitary particle can rotate independently from the rotation and motion of fluid in complete form. The creep movement of small particles in a fluid is common in bio-engineering, chemical engineering and naval engineering. It should be noted that the important quantity in solving all problems of fluid dynamics is the drag force rather than a detailed description of the flow field. To deal with the Stokes drag on axially symmetric body placed under the micro polar fluid or Stokes flow past an axially symmetric body in micro polar fluid is always a typical task though the nature of Stokes equations are linear.

Ariman et al. (1973) provided a complete review over micro polar continuum fluid mechanics. Ramkissoon and Majumdar (1976) gave a formula for Stokes drag on axially symmetric body placed under micropolar fluid in terms of stream function which immediately reduces to the formula of drag provided by Payne and Pell (1960) on removing the micro polar effect from expression of drag. In their work, Datta and Rathore (1983) dealt a problem of slow uniform flow of a micropolar fluid past a prolate spheroid. In this paper, they proved that the drag for the polar case can be deduced from the non-polar by taking k=0, where k is micro polarity parameter, simply through multiplication by the factor 1 + μ/k with k into the expression of drag or replacing k by μ + k. Avoiding the selection of coordinate systems for chosen body, Datta and Srivastava (1999) provided a simple Stokes drag formula for axially symmetric body placed in classical Newtonian fluid which was purely based on geometry of axi-symmetric body with the condition of continuously turning tangent on body curve in meridional plane. Combining this fact with DS-conjecture (1999) along with Brenner's formula (1961), Srivastava et al. (2016) provided the Oseen's correction to Stokes drag on axially symmetric particle in micro polar fluid. In this paper, the proposed drag formula is tested for sphere, spheroid and deformed sphere. There exists some related work on Stokes flow past axially symmetric bodies of micro polar fluid mainly by Ramkissoon and O'Neill (1983), Iyengar and Charya (1993), Hayakawa (2000), Palaniappan and Ramkissoon (2005), Hoffmann et al. (2007), Shu and Lee (2008), Sherief et al. (2010), Deo and Shukla (2012) etc.

Now, in the present paper, author attempted to provide complete proof of drag formula for axially symmetric body placed in Stokes flow of micro polar fluid. The method exploits the well-known drag formula for a sphere and the step used in its derivation [page 122, Happel and Brenner (1964)]. Using this method, first a simple formula is obtained for evaluating the drag force on axially symmetric body, with continuously turning tangent, placed in a uniform stream along the axis of symmetry, and then the method is extended to the transverse flow situation. In this process, we consider viscosity coefficient μ + k instead of μ for micro polar fluid where k is the small micro polarity parameter.

## 2. Method

### 2.1. Axial Flow

Let us consider the axially symmetric body of characteristic length L placed along its axis(x-axis, say) in a uniform stream $U$ of micropolar viscous fluid of density $\rho_1$ , kinematic viscosity ν, viscosity coefficient $\mu$ and micro polar parameter $k$. When Reynolds number $UL/\nu$ is small, the steady motion is governed by Stokes equations (Ramkissoon and Majumdar, 1976),

$$-(\mu+k)\nabla\times\nabla\times u+k\ \nabla\times\omega-\nabla p\ +\ \rho_1 F=0\,, \tag{1}$$

$$(\alpha_1+\alpha_2+\alpha_3)\nabla(\nabla.\omega)-\alpha_3\nabla\times\omega+k(\nabla\times u)-2k\omega=0, \tag{2}$$

$$\nabla.u=0. \tag{3}$$

subject to the no-slip and no spin boundary condition. In the above equations, $\boldsymbol{u}$ is the velocity vector, $\boldsymbol{\omega}$ the micro-rotation or spin vector, $\mu$ the viscosity coefficient, p the pressure, '$k$' the coupling constant or micro polarity of the fluid, **F** the external force per unit mass and $\alpha_1$, $\alpha_2$, $\alpha_3$ are characteristics constants of the particular fluid under consideration. It should be noted here that the general expression for Stokes drag on small axially symmetric particle placed under slow incompressible viscous micro polar fluid with small micro polarity experiencing no external body forces is proposed with the restriction of no-slip and no spin boundary conditions.

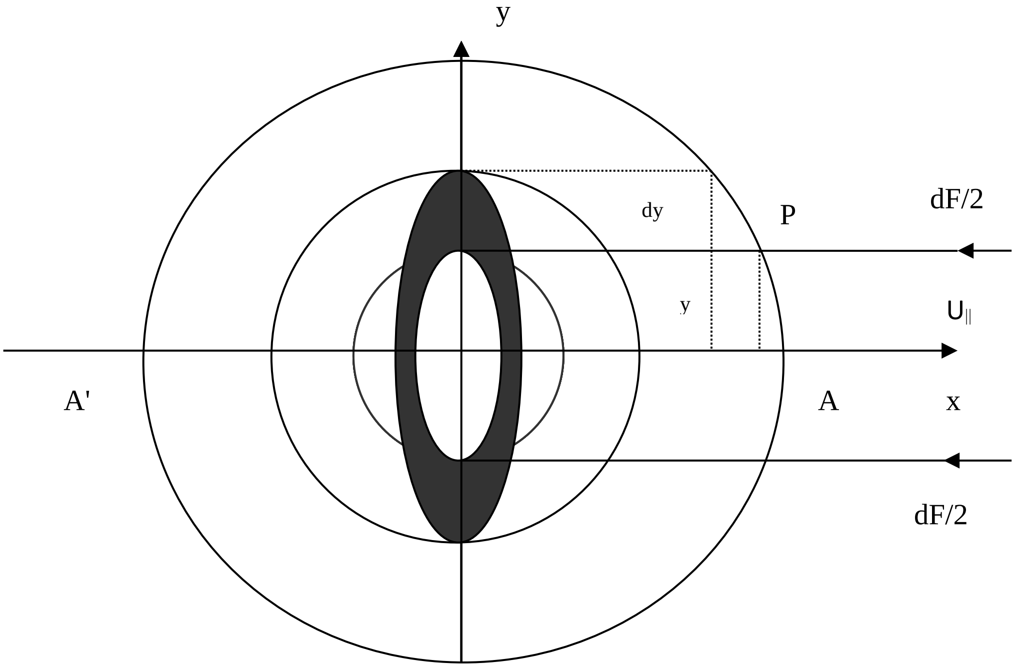


**Figure 1(a).** **Elemental force system on the sphere**

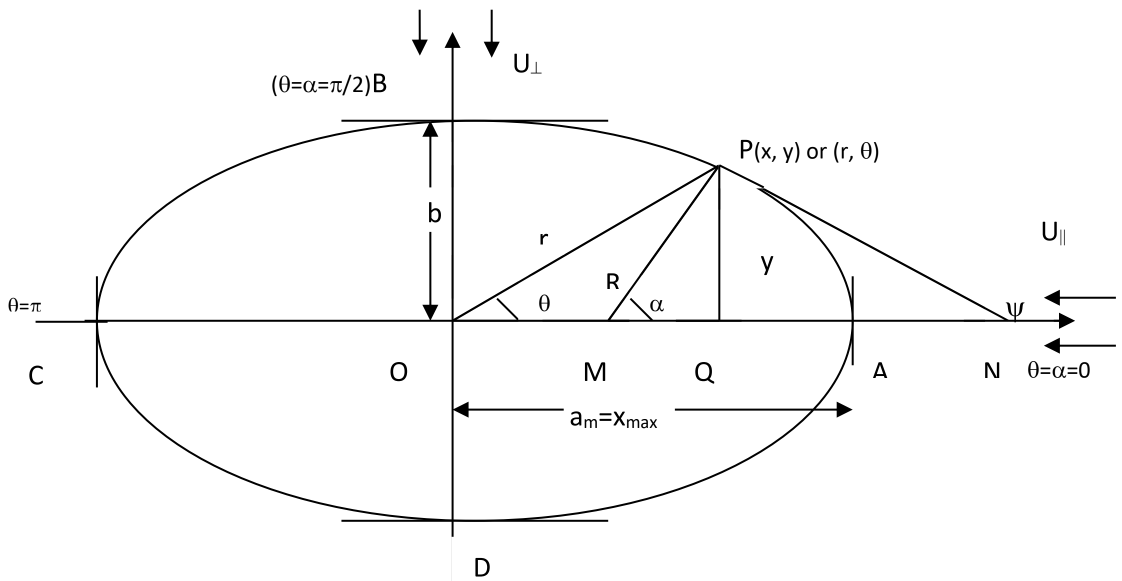


**Figure 1(b).** Geometry of axially symmetric body

For the case of a sphere of radius $R$, the solution is easily obtained and on evaluating the stress, the drag force $F$ comes out as in Happel and Brenner (1964, p. 122)

$$F = \frac{9}{2}\pi\ (\mu + k)\ U \int_0^{\pi} R\ \sin^3\alpha\ d\alpha = \lambda R, \tag{4}$$

where

$$\lambda\ = 6\pi\ (\mu + k)U. \tag{5}$$

This shows that the drag force increases linearly with the radius of the sphere. In other words, the difference between drag force on two spheres of radii $y$ and $y + dy$ is given by

$$dF = \lambda dy. \tag{6}$$

A sphere of radius '$b$' is obtained by rotating the curve $x = b\,cos\,t, y = b\,sin\,t (0 \le t \le \pi)$ about the x-axis and the force $F = \lambda b$ is obtained from (6) as $\int_0^b \lambda\,dy$ exhibiting that the force system $dF$ may be considered as lying in the xy plane. The element force $dF$ may be decomposed into two parts $(1/2)\,dF$, each acting over the upper half and lower half; $(1/2)\,dF$ on the upper half acts at a height y(say) above the x-axis. The total force $F/2$ on the upper half, may be considered as made up of these differential forces $dF/2$ acting over elements corresponding to a system of half spheres of radii increasing from 0 to b and spread over from A to A′(Figure 1(a)). The moment of this force system (taken to be in the *xy*-plane) about O, provides

$$h\left(\frac{F}{2}\right) = M = \frac{1}{2}\int_0^b y\,dF = \frac{1}{2}\,\lambda \int_0^b y\,dy = \frac{1}{4}\lambda b^2,$$

or

$$F = \frac{1}{2}\frac{\lambda b^2}{h}\ , \tag{7}$$

where 'h' is the height of centroid of the force system. In the case of a sphere of radius '$b$' we have $F = \lambda b$, and so we get from (7), $h = b/2$, as it should be. Next, we can express (4) also as

$$F = \int_{\alpha=0}^{\pi} df, \tag{8}$$

where

$$df = \frac{3}{4}\lambda R\ \sin^3\alpha\ d\alpha. \tag{9}$$

Is the elemental force on a circular ring element at $P$ Figure 1(a). as in Happel and Brenner (1964, equation (4-17.23), p. 122). For the purpose of calculating $F/2$, the force on upper half, $(1/2)df$ may be taken to be acting at height $\eta$ (say), above x-axis, given by

$$h = \frac{\int \eta \left(\frac{df}{2}\right)}{\int \frac{df}{2}} = \frac{\int_0^\pi \eta \left(\frac{3}{4}\lambda R \sin^3 \alpha \, d\alpha\right)}{\int_0^\pi \frac{3}{4}\lambda R \sin^3 \alpha \, d\alpha} = \frac{3}{4}\int_0^\pi \eta \sin^3 \alpha \, d\alpha,$$

taking $\eta = R/2$, the result is seen to correspond to the value $h = b/2$ confirmed earlier. Thus, we have

$$h = \frac{3}{8}\int_0^\pi R \sin^3 \alpha \, d\alpha. \tag{10}$$

It is proposed that the formula (10) holds good for an axially symmetric body also, when R is interpreted as the normal distance PM between the point P on the body and the point of intersection M of the normal at P with axis of symmetry and α as its slope in Figure 1(b). On inserting the value of h from (10) in (7), we finally obtained the expression of drag on axially symmetric body in axial flow

$$F_{||} = \frac{1}{2}\frac{\lambda b^2}{h_{||}} = \frac{4}{3}\frac{\lambda b^2}{\int_0^\pi R \ sin^3 \alpha \, d\alpha}, where\ \lambda = 6\pi(\mu + k)\ U_{||},$$

$$= \frac{8\pi(\mu+k) b^2 U_{||}}{\int_0^\pi R \ sin^3 \alpha \, d\alpha}, \tag{11}$$

where the suffix '$||$' has been introduced to assert that the force is in the axial direction. While using (11), it should be kept in mind that '$b$' denotes intercept between the meridian curve and the axis of the normal perpendicular to the axis i.e., $b = R \ at \ \alpha = \pi/2$.

Sometimes it will be convenient to work in Cartesian co-ordinates. Therefore, referring to the Figure 1(b), for the profile geometry, we have

$$y = R \sin\alpha \ , \quad \tan\alpha = -\left(\frac{dy}{dx}\right)^{-1} = -\frac{dx}{dy} = -x'. \tag{12}$$

Using above transformation, we may express (10) as

$$h_{||} = -\frac{3}{4}\int_0^a \frac{yy''}{\left(1+y'^2\right)^2} dx \ , \tag{13}$$

where $2a_m$ represents the axial length of the body and dashes represents derivatives with respect to $x$. In the sequel, it will be found simpler to work with y as the independent variable. Thus, $h_x$ assumes the form

$$\boldsymbol{h}_{||} = -\frac{3}{4}\int_0^b \frac{yx'^2x''}{(1+x'^2)^2} \boldsymbol{dy}, \tag{14}$$

where dashes represents derivatives with respect to y.

### 2.2. Transverse Flow

We set up a polar coordinate system $(R, \beta, \gamma)$ with $\beta$ as the polar angle with y-axis and $\gamma$ the azimuthal angle in *zx* plane. Since y-axis is not the axis of symmetry for the body we cannot make use of circular ring elemental force (3/4) $\lambda R \sin^3 \beta \, d\beta$ corresponding to (9). But we can easily write down the elemental force on the element $R^2 \sin\beta \, d\beta \, d\gamma$ as

$$\delta f = \frac{3\lambda R}{8\pi} \sin^3 \beta \, d\beta \, d\gamma.$$

Transforming the above to the polar coordinate $(R, \theta, \varphi)$ with the *x*-axis as the polar axis, we have

$$\delta f = \frac{3\lambda R}{8\pi} \left(1 - \sin^3 \alpha \cos^2 \phi\right) \sin\alpha \, d\alpha \, d\phi,$$

as the force on the element $R^2 \sin \alpha \, d\alpha \, d\varphi$. On integrating over φ from 0 to $2\pi$, we get

$$df_{\perp} = \frac{3\lambda R}{8} (2 - \sin^3 \alpha) \sin\alpha \, d\alpha, \tag{15}$$

where the suffix '⊥' has been placed to designate the force due to the external flow along the y-axis, the transverse direction.

Integrating $df_{\perp}$ over the surface of the sphere, we get

$$F_{\perp} = \frac{3\lambda}{8} \int_0^{\pi} \left(2 \sin\alpha - \sin^3 \alpha\right) d\alpha = \lambda R. \tag{16}$$

Agreeing with the correct value. This suggests we can take the force $df_{\perp}$ as given by (15) as the element force on the circular ring element at *P*. Although the force $F_{\perp}$ is along the *y* direction, we have reduced it to elemental forces on a system of spheres cantered on the *x*-axis. Since $F_{\perp}$ and $df_{\perp}$ themselves are scalar quantities, on comparing (15) and (9), we can use the analysis as in the axial flow case with '*h*' replaced by

$$h_{\perp} = \frac{3}{16} \int_0^{\pi} R\left(2 \sin\alpha - \sin^3 \alpha\right) d\alpha. \tag{17}$$

Thus, we get from (2.5)

$$\begin{aligned} F_{\perp} &= \frac{1}{2} \frac{\lambda b^2}{h_{\perp}}, \text{ where } \lambda = 6\pi(\mu + k) \, U_{\perp}, \\ &= \frac{16\pi(\mu + k) b^2 U_{\perp}}{\int_0^{\pi} R \left(2\sin\alpha - \sin^3 \alpha\right) d\alpha}. \end{aligned} \tag{18}$$

We have taken up the class of those axially symmetric bodies which possesses continuously turning tangent, placed in a uniform stream U along the axis of symmetry (which is x-axis), as well as constant radius ‘b’ of maximum circular cross-section at the middle of the body.

In the same manner as we did in axial flow, equation (17) may also be written in Cartesian form as (in both cases having x and y treated as independent)

$$h_{\perp} = -\frac{3}{8}\int_0^a \frac{yy''\left[1+2\left(y'\right)^2\right]}{\left[1+\left(y'\right)^2\right]^2}dx, \tag{19}$$

and

$$h_{\perp} = -\frac{3}{8}\int_0^b \frac{yx''\left[2+\left(x'\right)^2\right]}{\left[1+\left(x'\right)^2\right]^2}dy. \tag{20}$$

In (19) and (20), the dashes represents derivative with respect to *x* and *y*, respectively.

This axi-symmetric body is obtained by the revolution of meridional plane curve (depicted in Figure 1(b)) about axis of symmetry which obeys the following limitations:

i. Tangents at the points *A*, on the x-axis, must be vertical,
ii. Tangents at the points *B*, on the y-axis, must be horizontal,
iii. The semi-transverse axis length ‘*b*’ must be fixed.

The point *P* on the curve may be represented by the Cartesian coordinates $(x,y)$ or polar coordinates $(r,\theta)$ respectively , *PN* and *PM* are the length of tangent and normal at the point *P*. The symbol *R* stands for the intercepting length of normal between the point on the curve and point on axis of symmetry and symbol $\alpha$ is the slope of normal *PM* which can be vary from 0 to $\pi$.

The proposed drag formulae is, of, course, subject to restrictions on the geometry of the meridional body profile $y(x)$ of continuously turning tangent implying that $y'(x)$ is continuous together with $y''(x) \neq 0$, thereby avoiding corners or sharp edges or other kind of nodes and straight line portions, $y = ax + b$ , $x_1 \leq x \leq x_2$. If such type of cases arises in the body, the contribution of drag corresponding to those parts will be zero and true drag value experienced by the body may not be achieved. Also, it should be noted here that the method holds good for convex axially symmetric bodies which possesses fore-aft symmetry about the equatorial axis perpendicular to the axis of symmetry (polar axis). Apart from this argument, It is interesting to note here that the proposed conjecture is applicable also to those axi-symmetric bodies which fulfills the condition of continuously turning tangent but does not possesses fore-aft symmetry like egg shaped body (Datta and Srivastava, 1999). This conjecture is much simpler to evaluate the numerical values of drag than other existing numerical methods like Boundary Element Method (BEM), Finite Element Analysis (FEA) etc. as it can be applied to a large set of convex axi-symmetric bodies possessing fore-aft symmetry about maximal radius situated in the middle of the body for which analytical solution is not available or impossible to evaluate. The presence of height of C.G. of force system in the denominator of the expressions of drag (11) and (18) is the conjecture for the proposed formula of drag.

## 3. Flow past a sphere

Let us consider the parametric equation of sphere having radius '$a$' as

$$x = a\cos t,\;\; y = a\sin t\;,\;\; 0 \le t \le \pi. \tag{21}$$

After careful calculation, the expressions of axial and transverse Stokes drag on this sphere placed in micropolar fluid, by using (11) and (18) with fact $b = a,$ comes out to be

$$F_x = F_y = 6\,\pi\,\mu\,\left(1+\frac{k}{\mu}\right)\,U\,a\,, \tag{22}$$

where '$k$' is small micro polarity coefficient. This expression matches with the expression of drag on sphere of radius 'a' placed under micro polar fluid given by Ramkissoon and Majumdar (1976), Datta and Rathore (1984), Shu and Lee (2008) which immediately reduces to classical Stokes drag, 6πμUa, on sphere having radius '$a$', as $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in both axial and transverse cases as

$$C_{F_x} = C_{F_y} = \frac{6\pi\mu(1+\dfrac{k}{\mu})Ua}{6\pi\mu Ua} = 1+\frac{k}{\mu},\quad \frac{k}{\mu} \ge 0. \tag{23}$$

## 3. Flow past spheroid

### 3.1. Prolate spheroid

Suppose a prolate spheroid is generated by the rotation of an ellipse about the x-axis. The parametric equation of the ellipse may be taken as

$$x = a\cos t,\;\; y = a\sin t\;,\;\; 0 \le t \le \pi. \tag{24}$$

After careful calculation, the expressions of axial Stokes drag on this prolate spheroid placed in micro polar fluid, by using equation 11, comes out to be

$$F_x = 16\pi\mu\left(1+\frac{k}{\mu}\right)Uae^3\left[-2e+\left(1+e^2\right)\log\left(\frac{1+e}{1-e}\right)\right]^{-1}, \tag{25}$$

$$= 6\pi\mu\left(1+\frac{k}{\mu}\right)Ua\left[1-\frac{2}{5}e^2-\frac{17}{175}e^4-\cdots\right]. \tag{26}$$

This expression matches with axial Stokes drag calculated by Datta and Rathore (1984) with the help of singularity distribution method which further reduces to the classical expression of drag on prolate spheroid by Datta and Srivastava (1999), as micro polarity coefficient tends to zero, i.e., $k \to 0$. On normalizing with drag on sphere having radius 'a' placed under Newtonian fluid i.e. $6\pi\mu U a$, we can have the expressions of drag coefficient in axial flow as

$$C_{F_x} = \frac{F_x}{6\pi\mu Ua} = \frac{8}{3}\left(1+\frac{k}{\mu}\right) e^3\left[-2e+\left(1+e^2\right)\log\left(\frac{1+e}{1-e}\right)\right]^{-1},$$
$$= \left(1+\frac{k}{\mu}\right)\left[1-\frac{2}{5}e^2-\frac{17}{175}e^4-\cdots\right]. \tag{27}$$

The expressions of transverse Stokes drag on this prolate spheroid placed in micro polar fluid, by using equation 18, comes out to be

$$F_y = 32\pi\mu\left(1+\frac{k}{\mu}\right)Uae^3\left[2e+\left(3e^2-1\right)\log\left(\frac{1+e}{1-e}\right)\right]^{-1}, \tag{28}$$

$$= 6\pi\mu\left(1+\frac{k}{\mu}\right)Ua\left[1-\frac{3}{10}e^2-\frac{57}{700}e^4-\cdots\right]. \tag{29}$$

This expression matches with transverse Stokes drag calculated by Datta and Rathore (1984) with the help of singularity distribution method which further reduces to the classical expression of drag on prolate spheroid by Datta and Srivastava (1999), as micro polarity coefficient tends to zero, i.e., $k \to 0$. On normalizing with drag on sphere having radius 'a' placed under Newtonian fluid i.e. $6\pi\mu U a$, we can have the expressions of drag coefficient in transverse flow as

$$C_{F_y} = \frac{F_y}{6\pi\mu Ua} = \frac{16}{3}\left(1+\frac{k}{\mu}\right)e^3\left[2e+\left(3e^2-1\right)\log\left(\frac{1+e}{1-e}\right)\right]^{-1},$$
$$= \left(1+\frac{k}{\mu}\right)\left[1-\frac{3}{10}e^2-\frac{57}{700}e^4-\cdots\right]. \tag{30}$$

### 3.2. Oblate spheroid

Suppose an oblate spheroid is generated by the rotation of an ellipse about the x-axis. The parametric equation of the ellipse may be taken as

$$x = a\cos t,\quad y = a\sin t\ ,\quad 0 \le t \le \pi. \tag{31}$$

After careful calculation, the expressions of axial Stokes drag on this prolate spheroid placed in micro polar fluid, by using equation 11, comes out to be

$$F_x = 8\,\pi\,\mu\left(1+\frac{k}{\mu}\right)U\,a\,e^3\left[e\sqrt{1-e^2}-\left(1-2e^2\right)\sin^{-1}e\right]^{-1}, \tag{32}$$

$$= 6\,\pi\,\mu\left(1+\frac{k}{\mu}\right)U\,a\left[1-\frac{1}{10}e^2-\frac{31}{1400}e^4+\frac{9}{1400}e^6-\cdots\right]. \tag{33}$$

This expression matches with axial Stokes drag calculated by Datta and Rathore (1984) with the help of singularity distribution method which further reduces to the classical expression of drag on prolate spheroid by Datta and Srivastava (1999), as micro polarity coefficient tends to zero, i.e., $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in axial flow as

$$\begin{aligned} C_{F_x} = \frac{F_x}{6\pi\mu Ua} &= \frac{4}{3}\left(1+\frac{k}{\mu}\right)e^3\left[e\sqrt{1-e^2}-\left(1-2e^2\right)\sin^{-1}e\right]^{-1}, \\ &= \left(1+\frac{k}{\mu}\right)\left[1-\frac{1}{10}e^2-\frac{31}{1400}e^4+\frac{9}{1400}e^6-\cdots\right]. \end{aligned} \tag{34}$$

The expressions of transverse Stokes drag on this oblate spheroid placed in micro polar fluid, by using equation 18, comes out to be

$$\mathrm{F_y} = 16\,\pi\,\mu\left(1+\frac{\mathrm{k}}{\mu}\right)\mathrm{U\,a\,e^3}\left[-\mathrm{e}\sqrt{1-\mathrm{e}^2}+\left(1+2\mathrm{e}^2\right)\sin^{-1}\mathrm{e}\right]^{-1}, \tag{35}$$

$$= 6\,\pi\,\mu\left(1+\frac{\mathrm{k}}{\mu}\right)\mathrm{U\,a}\left[1-\frac{1}{5}\mathrm{e}^2-\frac{79}{1400}\mathrm{e}^4-\cdots\right]. \tag{36}$$

This expression matches with transverse Stokes drag calculated by Datta and Rathore (1984) with the help of singularity distribution method which further reduces to the classical expression of drag on prolate spheroid by Datta and Srivastava (1999), as micro polarity coefficient tends to zero, i.e., $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in transverse flow as

$$\begin{aligned} C_{F_y} = \frac{F_y}{6\pi\mu Ua} &= \frac{16}{3}\left(1+\frac{k}{\mu}\right)e^3\left[-e\sqrt{1-e^2}+\left(1+2e^2\right)\sin^{-1}e\right]^{-1}, \\ &= \left(1+\frac{k}{\mu}\right)\left[1-\frac{1}{5}e^2-\frac{79}{1400}e^4-\cdots\right]. \end{aligned} \tag{37}$$

## 5. Flow past deformed sphere

We consider the polar equation of deformed sphere as

$$r = a\left[1+\varepsilon\sum_{k=0}^{\infty}d_k P_k\left(\mu\right)\right],\ \mu = \cos\theta, \tag{38}$$

where '$\varepsilon$' is deformation parameter and $(r, \theta)$ are polar coordinates.

After careful calculation, the Stokes drag experienced by this deformed sphere placed in axial and transverse uniform stream is given by use of equation (11) and (18) up to first order of '$\varepsilon$', as

$$F_x = 6\ \pi\ \mu\left(1+\frac{k}{\mu}\right)\ Ua\left[1+\varepsilon\left(d_0-\frac{1}{5}d_2\right)\right], \tag{39}$$

$$F_y = 6\ \pi\ \mu\left(1+\frac{k}{\mu}\right)\ Ua\left[1+\varepsilon\left(d_0+\frac{1}{10}d_2\right)\right]. \tag{40}$$

These expressions reduces to those for deformed sphere placed under Newtonian fluid given in paper [Datta and Srivastava, 1999] in the limiting case as $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in both axial and transverse flow cases as

$$C_{F_x} = \frac{F_x}{6\pi\mu Ua} = \left(1+\frac{k}{\mu}\right)\left[1+\varepsilon\left(d_0-\frac{1}{5}d_2\right)\right], \tag{41}$$

$$C_{F_y} = \frac{F_y}{6\pi\mu Ua} = \left(1+\frac{k}{\mu}\right)\left[1+\varepsilon\left(d_0+\frac{1}{10}d_2\right)\right]. \tag{42}$$

## 4. Cycloidal body of revolution

**Case 1.** We consider the equation of cycloidal body of revolution as

$$x = a(t+\sin t),\ \ y = a(1+\cos t),\ \ 0 \le t \le \pi. \tag{43}$$

The Stokes drag on this axially symmetric body of revolution is given by using equation 11 and 18, as

$$F_x = \frac{128}{3}\mu(1+\frac{k}{\mu})\ U\ a\ , \tag{44}$$

$$F_y = \frac{256}{5}\mu(1+\frac{k}{\mu})\ U\ a\ . \tag{45}$$

These expressions reduces to those for cycloidal body placed under Newtonian fluid given in **Datta and Srivastava** (1999) in the limiting case as $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in both axial and transverse flow cases as

$$C_{F_x} = \frac{F_x}{6\pi\mu Ua} = 2.265(1+\frac{k}{\mu}), \tag{46}$$

$$C_{F_y} = \frac{F_y}{6\pi\mu Ua} = 2.7176(1+\frac{k}{\mu}). \tag{47}$$

**Case 2**. Consider the body generated by the rotation about *x*-axis of the curve composed of arcs of two cycloidal parts represented parametrically by

$$x = a\left(1+\cos t\right),\ \ y = a\left(t+\sin t\right)\ ,\ 0 \le t \le \pi, \tag{48}$$

$$x = -a\left(1+\cos t\right),\ \ y = a\left(t+\sin t\right),\ \ 0 \le t \le \pi\ . \tag{49}$$

The Stokes drag on this axially symmetric body of revolution is given by equation 11 and 18, as

$$F_x = \frac{96\pi^3}{3\pi^2+16}\mu(1+\frac{k}{\mu})\ U\ a, \tag{50}$$

$$F_y = \frac{192\pi^3}{9\pi^2+32}\mu(1+\frac{k}{\mu})\ U\ a. \tag{51}$$

These expressions reduces to those for cycloidal body placed under Newtonian fluid given in Datta and Srivastava (1999) in the limiting case as $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in both axial and transverse flow cases as

$$C_{F_x} = 3.461118(1+\frac{k}{\mu}), \tag{52}$$

and

$$C_{F_y} = 2.61319(1+\frac{k}{\mu}). \tag{53}$$

## 5. Flow past Cassini body of revolution

We consider the Cassini body of revolution obtained by revolving the curve

$$y^2 = \frac{2}{3}\left(1+3x^2\right)^{1/2} - x^2 - \frac{1}{3}\ ,\ \ 0 \le x \le 1\ , \tag{54}$$

about x-axis. The Stokes drag on this axially symmetric Cassini body of revolution, on taking $a = 1, b = 0.577$, by using equation 11 and 18, as

$$F_x \cong 0.8\lambda \cong 4.8\ \pi\mu(1+\frac{k}{\mu})U, \tag{55}$$

$$F_y \cong 0.82\lambda \cong 4.92\ \pi\mu(1+\frac{k}{\mu})U. \tag{56}$$

These expressions reduces to those for Cassini body of revolution placed under Newtonian fluid given in Srivastava (2001) in the limiting case as $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in both axial and transverse flow cases as

$$C_{F_x} = 0.8\left(1 + \frac{k}{\mu}\right), \tag{57}$$

$$C_{F_y} = 0.82\left(1 + \frac{k}{\mu}\right). \tag{58}$$

## 6. Hypocycloidal body of revolution

We consider the hypocycloidal body of revolution obtained by revolving the curve

$$y^2 = -3x^2 + \sqrt{\left(1 + 8x^4\right)}\ , \quad 0 \le x \le 1\ . \tag{59}$$

about x-axis. The Stokes drag on this axially symmetric hypocycloidal body of revolution is given by using equation 11 and 18, with $a = 1$, as

$$F_x \cong 1.044\ \lambda \cong 6.264\ \pi\ \mu(1 + \frac{k}{\mu})U\ , \tag{60}$$

$$F_y \cong 1.32\ \lambda \cong 7.92\ \pi\ \mu(1 + \frac{k}{\mu})U\ . \tag{61}$$

These expressions reduces to those for Hypocycloidal body of revolution placed under Newtonian fluid given in Srivastava (2001) in the limiting case as $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in both axial and transverse flow cases as

$$C_{F_x} = 1.044\left(1 + \frac{k}{\mu}\right), \tag{62}$$

$$C_{F_y} = 1.32\left(1 + \frac{k}{\mu}\right). \tag{63}$$

## 7. Egg-shaped body

We consider the drag for an egg-shaped body in which the right portion is in the shape of $a$ half prolate spheroid given parametrically by

$$x = a\cos t\,,\ y = a\sin t\ ,\ 0 \le t \le \frac{\pi}{2}, \tag{64}$$

and left portion is a hemisphere given by

$$x = b\cos t,\ \ y = b\sin t\ ,\ \ \frac{\pi}{2} \le t \le \pi. \tag{65}$$

The Stokes drag on this egg-shaped body of revolution is given by using equation 11 and 18, as

$$F_x = 8\pi\ \mu\ Ua\sqrt{1-e^2}\left[\frac{2}{3} + \frac{\sqrt{1-e^2}}{4e^3}\left\{-2e + \left(1+e^2\right)\log\frac{1+e}{1-e}\right\}\right]^{-1}, \tag{66}$$

$$F_y = 8\ \pi\ \mu\ U\ a\sqrt{1-e^2}\left[\frac{4}{3} + \frac{\sqrt{1-e^2}}{4e^3}\left\{2e + \left(3e^2-1\right)\log\frac{1+e}{1-e}\right\}\right]^{-1}. \tag{67}$$

This expression reduces to those for egg-shaped body of revolution placed under Newtonian fluid given in Datta and Srivastava (1999) in the limiting case as $k \to 0$. On normalizing with drag on sphere having radius '$a$' placed under Newtonian fluid, i.e., $6\pi\mu Ua$, we can have the expressions of drag coefficient in both axial and transverse flow cases as

$$C_{F_x} = \frac{F_x}{6\ \pi\ \mu U\ a} = \frac{4}{3}\ \left(1+\frac{k}{\mu}\right)\sqrt{1-e^2}\left[\frac{2}{3} + \frac{\sqrt{1-e^2}}{4e^3}\left\{-2e + \left(1+e^2\right)\log\frac{1+e}{1-e}\right\}\right]^{-1}, \tag{68}$$

$$C_{F_y} = \frac{F_y}{6\ \pi\ \mu\ U\ a} = \frac{8}{3}\ \left(1+\frac{k}{\mu}\right)\sqrt{1-e^2}\left[\frac{4}{3} + \frac{\sqrt{1-e^2}}{4e^3}\left\{2e + \left(3e^2-1\right)\log\frac{1+e}{1-e}\right\}\right]^{-1}. \tag{69}$$

## 8. Numerical Tables

**Table 1.** Variation of drag coefficient for spherical body (equation 23) with respect to $k/\mu$, the ratio of micro polar parameter and viscosity coefficient of fluid

| | k/μ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $C_{F_x}$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
| $C_{F_y}$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |

**Table 2(i).** Variation of drag coefficient for prolate spheroidal body (equation 27 & 30) with respect to $k/\mu$ and eccentricity $e$, the ratio of micro polar parameter and viscosity coefficient of fluid

| k/μ | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | | 0.2 | | 0.4 | | 0.6 | | 0.8 | | 1.0 | |
| e | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ |
| 0.0 | 1.00 | 1.00 | 1.20 | 1.20 | 1.40 | 1.40 | 1.60 | 1.60 | 1.80 | 1.80 | 2.00 | 2.00 |
| 0.2 | 0.98 | 0.99 | 1.18 | 1.18 | 1.38 | 1.38 | 1.57 | 1.58 | 1.77 | 1.78 | 1.97 | 1.97 |
| 0.4 | 0.93 | 0.95 | 1.12 | 1.14 | 1.31 | 1.33 | 1.49 | 1.52 | 1.68 | 1.71 | 1.87 | 1.90 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.6 | 0.84 | 0.88 | 1.01 | 1.06 | 1.18 | 1.23 | 1.34 | 1.41 | 1.52 | 1.59 | 1.69 | 1.76 |
| 0.8 | 0.70 | 0.77 | 0.84 | 0.92 | 0.98 | 1.08 | 1.13 | 1.24 | 1.27 | 1.39 | 1.40 | 1.54 |
| 1.0 | 0.50 | 0.62 | 0.60 | 0.74 | 0.70 | 0.87 | 0.80 | 0.98 | 0.90 | 1.11 | 1.00 | 1.23 |

**Table 2(ii).** Variation of drag coefficient for oblate spheroidal body (equation 34 and 37) with respect to $k/\mu$ and eccentricity e, the ratio of micro polar parameter and viscosity coefficient of fluid

| | 0.0 | | 0.2 | | 0.4 | | 0.6 | | 0.8 | | 1.0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $e$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ |
| 0.0 | 1.00 | 1.00 | 1.20 | 1.20 | 1.40 | 1.40 | 1.60 | 1.60 | 1.80 | 1.80 | 2.00 | 2.00 |
| 0.2 | 0.99 | 0.99 | 1.19 | 1.19 | 1.39 | 1.39 | 1.59 | 1.59 | 1.79 | 1.78 | 1.99 | 1.98 |
| 0.4 | 0.98 | 0.96 | 1.18 | 1.16 | 1.37 | 1.35 | 1.57 | 1.55 | 1.77 | 1.74 | 1.97 | 1.93 |
| 0.6 | 0.96 | 0.92 | 1.15 | 1.10 | 1.34 | 1.30 | 1.54 | 1.47 | 1.73 | 1.65 | 1.92 | 1.84 |
| 0.8 | 0.92 | 0.85 | 1.11 | 1.01 | 1.23 | 1.18 | 1.48 | 1.36 | 1.67 | 1.53 | 1.85 | 1.69 |
| 1.0 | 0.88 | 0.74 | 1.05 | 0.89 | 1.23 | 1.04 | 1.40 | 1.19 | 1.58 | 1.34 | 1.75 | 1.48 |

**Table 3.** Variation of drag coefficient for deformed sphere (equation 41 and 42) with respect to $k/\mu$ and deformation parameter $\varepsilon$, the ratio of micro polar parameter and viscosity coefficient of fluid ($d0 = d2 = 0.5$)

| | 0.0 | | 0.2 | | 0.4 | | 0.6 | | 0.8 | | 1.0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ε | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ |
| 0.0 | 1.00 | 1.00 | 1.20 | 1.20 | 1.40 | 1.40 | 1.60 | 1.60 | 1.80 | 1.80 | 2.00 | 2.00 |
| 0.2 | 1.08 | 1.11 | 1.30 | 1.33 | 1.51 | 1.55 | 1.73 | 1.78 | 1.94 | 1.99 | 2.16 | 2.22 |
| 0.4 | 1.16 | 1.22 | 1.32 | 1.46 | 1.62 | 1.70 | 1.78 | 1.95 | 2.08 | 2.19 | 2.32 | 2.44 |
| 0.6 | 1.24 | 1.33 | 1.48 | 1.59 | 1.73 | 1.86 | 1.98 | 2.13 | 2.23 | 2.39 | 2.48 | 2.66 |
| 0.8 | 0.70 | 0.77 | 0.84 | 0.92 | 0.98 | 1.08 | 1.12 | 1.23 | 1.27 | 1.39 | 1.40 | 1.54 |
| 1.0 | 1.40 | 1.55 | 1.68 | 1.86 | 1.96 | 2.17 | 2.24 | 2.48 | 2.52 | 2.79 | 2.80 | 3.10 |

**Table 4(i).** Variation of drag coefficient for Cycloidal body (case-1, equation 46 and 47) with respect to $k/\mu$, the ratio of micro polar parameter and viscosity coefficient of fluid

| | $k/\mu$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $C_{F_x}$ | 2.265 | 2.492 | 2.718 | 2.944 | 3.171 | 3.307 | 3.624 | 3.8505 | 4.077 | 4.3035 | 4.530 |
| $C_{F_y}$ | 2.718 | 2.989 | 3.262 | 3.533 | 3.805 | 4.077 | 4.349 | 4.621 | 4.892 | 5.164 | 5.436 |

**Table 4(ii).** Variation of drag coefficient for Cycloidal body (case-2, equation 52 & 53) with respect to $k/\mu$, the ratio of micro polar parameter and viscosity coefficient of fluid

| | $k/\mu$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $C_{F_x}$ | 3.461 | 3.807 | 4.1532 | 4.499 | 4.845 | 5.191 | 5.537 | 5.884 | 6.230 | 6.576 | 6.922 |
| $C_{F_y}$ | 2.613 | 2.874 | 3.136 | 3.397 | 3.658 | 3.919 | 4.181 | 4.442 | 4.704 | 4.965 | 5.226 |

**Table 5.** Variation of drag coefficient for Cassini body (equation 57 & 58) with respect to $k/\mu$, the ratio of micro polar parameter and viscosity coefficient of fluid

| | $k/\mu$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $C_{F_x}$ | 0.8 | 0.88 | 0.96 | 1.04 | 1.12 | 1.2 | 1.28 | 1.36 | 1.44 | 1.52 | 1.6 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_{F_y}$ | 0.82 | 0.982 | 0.984 | 1.066 | 1.148 | 1.23 | 1.312 | 1.394 | 1.476 | 1.558 | 1.64 |

**Table 6.** Variation of drag coefficient for Hypocycloidal body (equation 62 & 63) with respect to $k/\mu$, the ratio of micro polar parameter and viscosity coefficient of fluid

| | $k/\mu$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $C_{F_x}$ | 1.044 | 1.148 | 1.252 | 1.357 | 1.462 | 1.566 | 1.670 | 1.775 | 1.879 | 1.984 | 2.088 |
| $C_{F_y}$ | 1.320 | 1.452 | 1.584 | 1.716 | 1.848 | 1.980 | 2.112 | 2.244 | 2.376 | 2.508 | 2.640 |

**Table 7.** Variation of drag coefficient for egg-shaped body (equation 68 & 69) with respect to $k/\mu$ and eccentricity '$e$'

| | 0.0 | | 0.2 | | 0.4 | | 0.6 | | 0.8 | | 1.0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $e$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ | $C_{F_x}$ | $C_{F_y}$ |
| 0.0 | 1.00 | 1.00 | 1.20 | 1.20 | 1.40 | 1.40 | 1.60 | 1.60 | 1.80 | 1.80 | 2.00 | 2.00 |
| 0.2 | 0.98 | 0.99 | 1.18 | 1.18 | 1.38 | 1.38 | 1.57 | 1.58 | 1.77 | 1.78 | 1.96 | 1.97 |
| 0.4 | 0.93 | 0.94 | 1.11 | 1.12 | 1.29 | 1.31 | 1.48 | 1.5 | 1.67 | 1.68 | 1.85 | 1.87 |
| 0.6 | 0.82 | 0.84 | 0.99 | 1.00 | 1.15 | 1.17 | 1.32 | 1.34 | 1.48 | 1.51 | 1.65 | 1.67 |
| 0.8 | 0.67 | 0.66 | 0.79 | 0.80 | 0.93 | 0.93 | 1.07 | 1.06 | 1.19 | 1.20 | 1.33 | 1.33 |
| 1.0 | 0.44 | 0.39 | 0.52 | 0.46 | 0.61 | 0.54 | 0.70 | 0.62 | 0.79 | 0.70 | 0.87 | 0.77 |

## 9. Numerical Analysis

- According to table-1, the drag coefficients for spherical body increases with respect to the increasing values of k/μ for both cases of axial and transverse flow of micro-polar fluid. For k/μ=1, the drag values are doubled to the drag values for k/μ=0, the case of Newtonian fluid.
- According to table-2(i), the drag coefficients for prolate spheroid decreases with respect to the increasing values of eccentricity 'e' for specific ratio k/μ and for k/μ=1,the values of drag coefficients are more than double to that at k/μ=0 for Newtonian fluid. Similar variation pattern to drag may be explained from table-2(ii) for oblate spheroid.
- According to table-3, the drag coefficients for deformed sphere increases with respect to the increasing values of deformation parameter 'ε' for specific ratio k/μ and for k/μ=1, the values of drag doubled to that at k/μ=0 for Newtonian fluid. Further, for specific deformation parameter 'ε', the drag coefficients increases with respect to increasing values of k/μ.
- According to table-4(i), the drag coefficients for cycloidal body(case-1, equation 43) increases with respect to the increasing values of k/μ and k/μ=1, the values are doubled to that at k/μ=0 for Newtonian fluid. Similar variation pattern to drag reflects from table-4(ii) for cycloidal body(case-2, equation 48 & 49).
- According to table-5 and 6, the drag coefficients for cassini body of revolution and hypocycloidal body of revolution increases with respect to increasing values of k/μ and k/μ =1, the values are doubled to that at k/μ =0 for Newtonian fluid.
- According to table-7, the drag coefficients for egg-shaped body decreases with respect to the increasing values of eccentricity 'e' for specific ratio k/μ and for k/μ =1, the

values are almost doubled to that at $k/\mu = 0$ for Newtonian fluid. While for specific eccentricity, the drag value increases with respect to increasing value of $k/\mu$ .

- All the variations, in every class of bodies, are depicted in figures 2-8.

## 10. Conclusion

In the authors previous published paper [IJAAMM, (2016)], the drag formula was conjectured and Oseen's drag were evaluated with the help of Brenner's formula (1961) on axially symmetric body with continuously turning tangent. The novelty of the manuscript belongs to the proof of previously conjectured drag formula. It is clear from the proposed proof of drag on axially symmetric bodies placed under micro-polar fluid geometric parameters related to the body and micro-rotation parameter of micro-polar fluid contribute significantly in the analytical expression of drag on axially symmetric body. This statement directly leads us to the conclusion that we may find the analytic expression of drag on axially symmetric body by knowing some geometrical parameters along with micro-rotation parameter for micro-polar fluid without solving the cumbersome differential equations governing the physical phenomenon.

The general expression of Stokes drag on axially symmetric particle placed under axial and transverse flow of micro polar fluid is advanced as an extension from Newtonian fluid. The proposed expressions of Stokes drag are applied to various axially symmetric bodies like sphere, spheroid, deformed sphere, cycloidal body, Cassini body, Hypocycloidal body and egg-shaped body and results are in agreement with some known values available in the literature in the limiting cases. It is found, as expected, that the drag values in the present cases are larger to that in case of Newtonian fluid for the same bodies. It is interesting here to note that the proposed result is based on some geometric parameters of body based on DS-conjecture avoiding complex and tedious calculations of coordinate based differential equations of motion. This method may further be advanced to solve many more problems of micropolar fluid-particle interaction in variety of flows. The proposed proof of drag on axially symmetric body placed under micro-polar fluid which was conjectured in previous paper (2016) of author is the main contribution in the present manuscript. This study, in my views, may open the door for the further study of those properties that differs the Newtonian fluid with Micro-polar fluid. Author is working on these problems which may appear in future work.

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## APENDIX

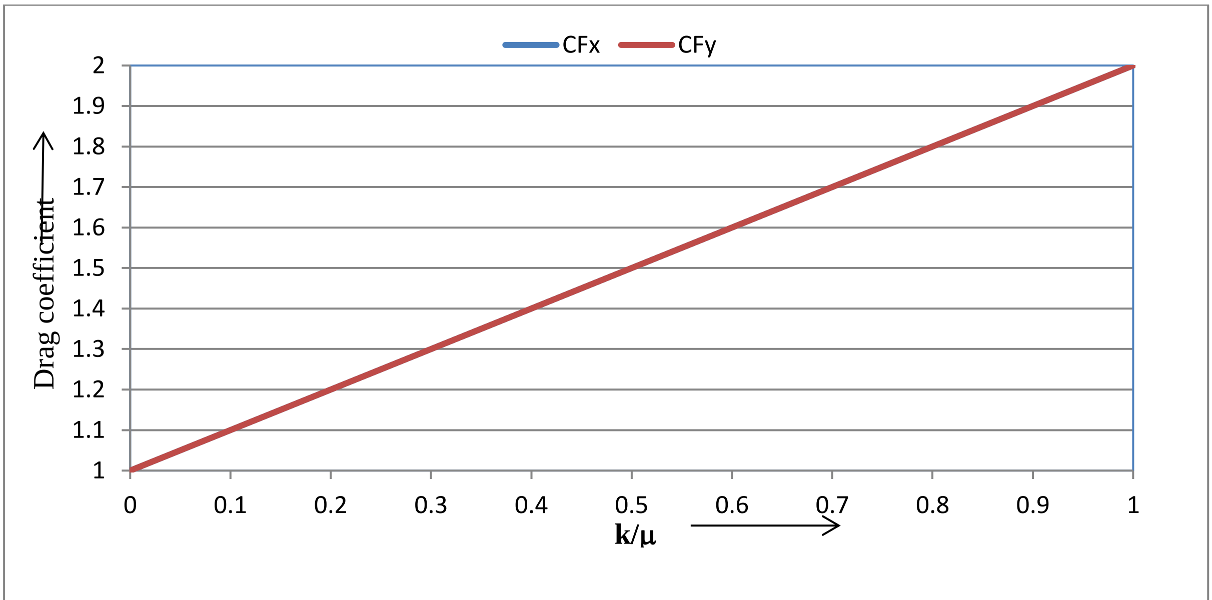


**Figure 2.** Variation of drag coefficient for Sphere with respect to k/μ

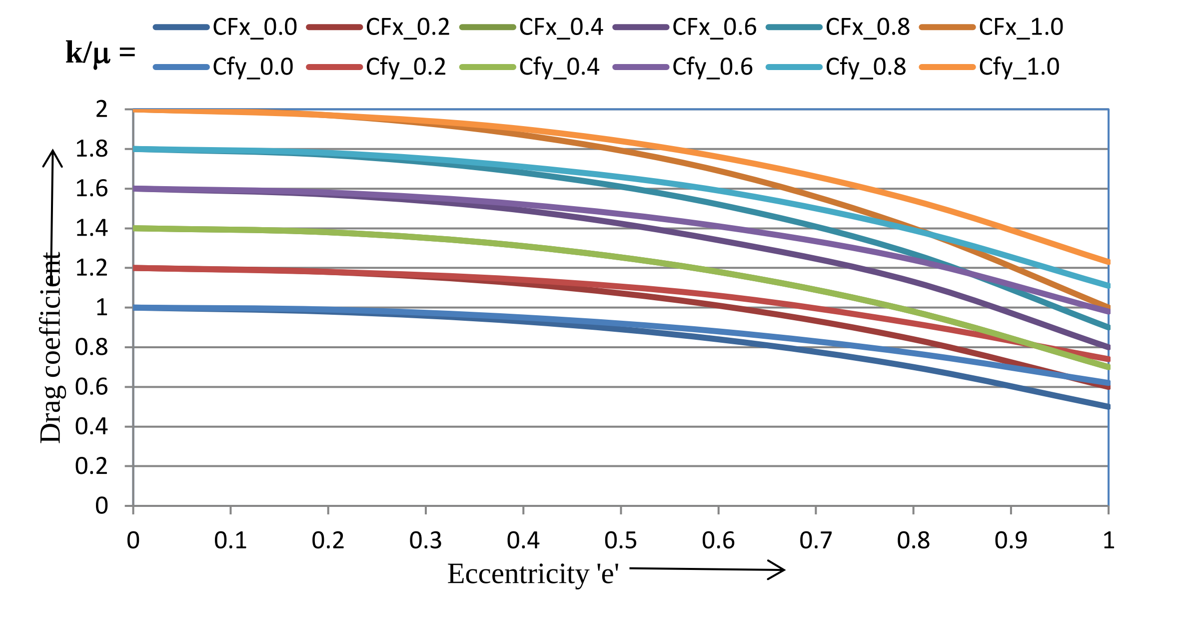


**Figure 3(i).** Variation of drag coefficient for Prolate spheroidal with respect to k/μ and eccentricity 'e' (based on data of table 2(i))

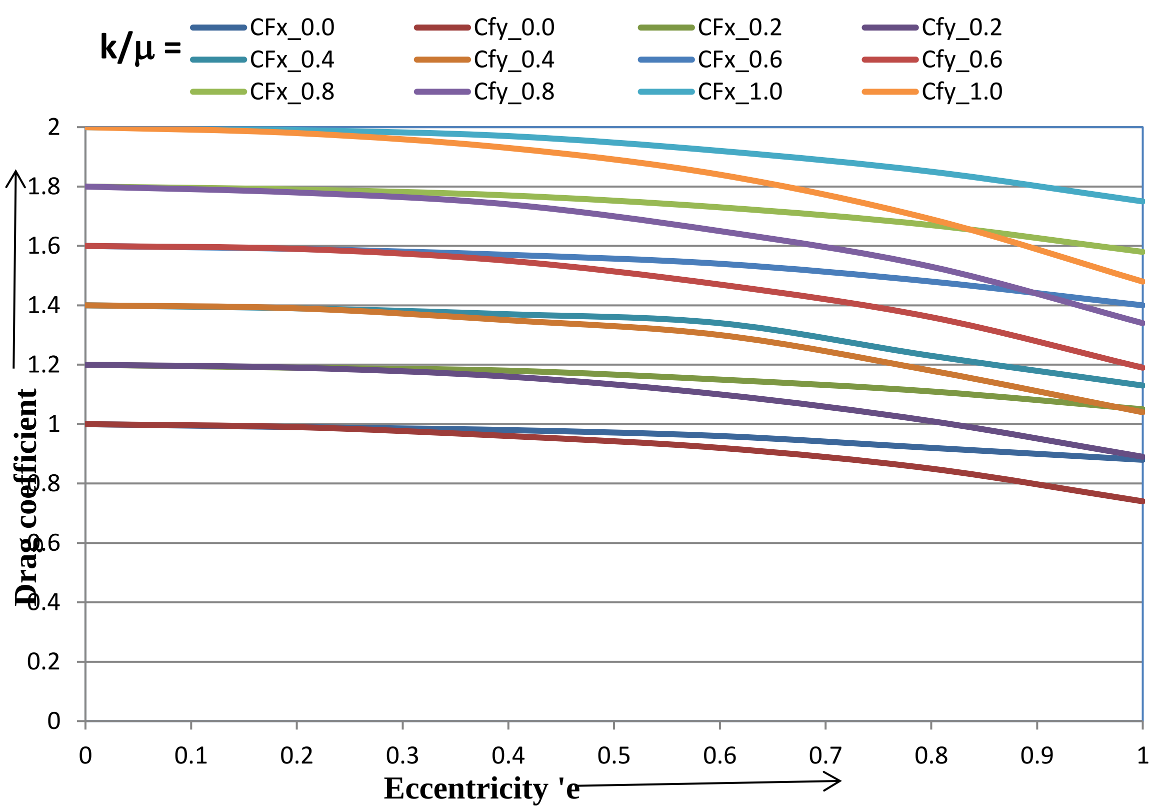


**Figure 3(ii).** Variation of drag coefficient for Oblate spheroidal with respect to k/μ and eccentricity 'e' (based on data of table 2(ii))

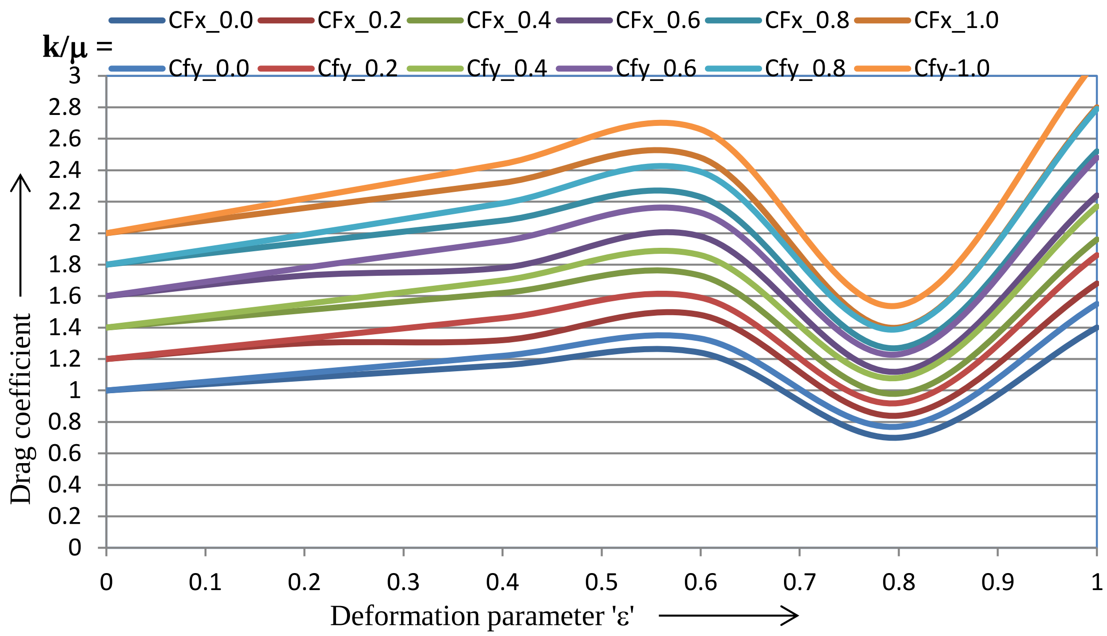


**Figure 4.** Variation of drag coefficient for deformed spherewith respect to k/μ and deformation parameter ε

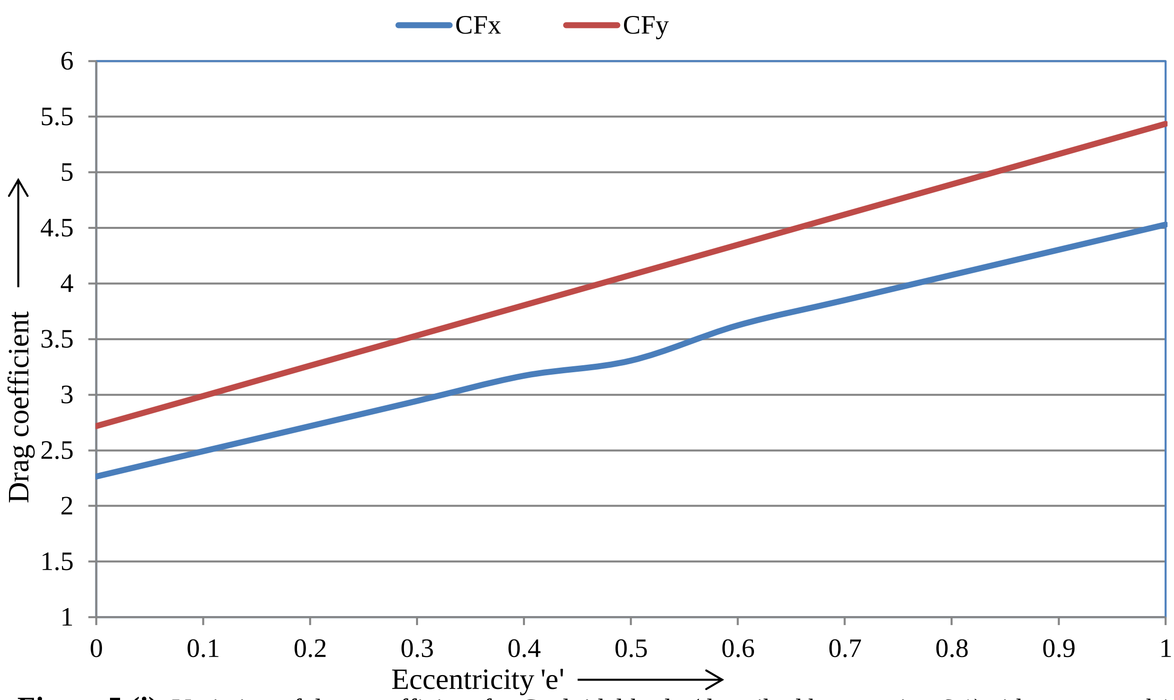


**Figure 5 (i).** Variation of drag coefficient for Cycloidal body (described by equation 6.1)with respect to k/μ

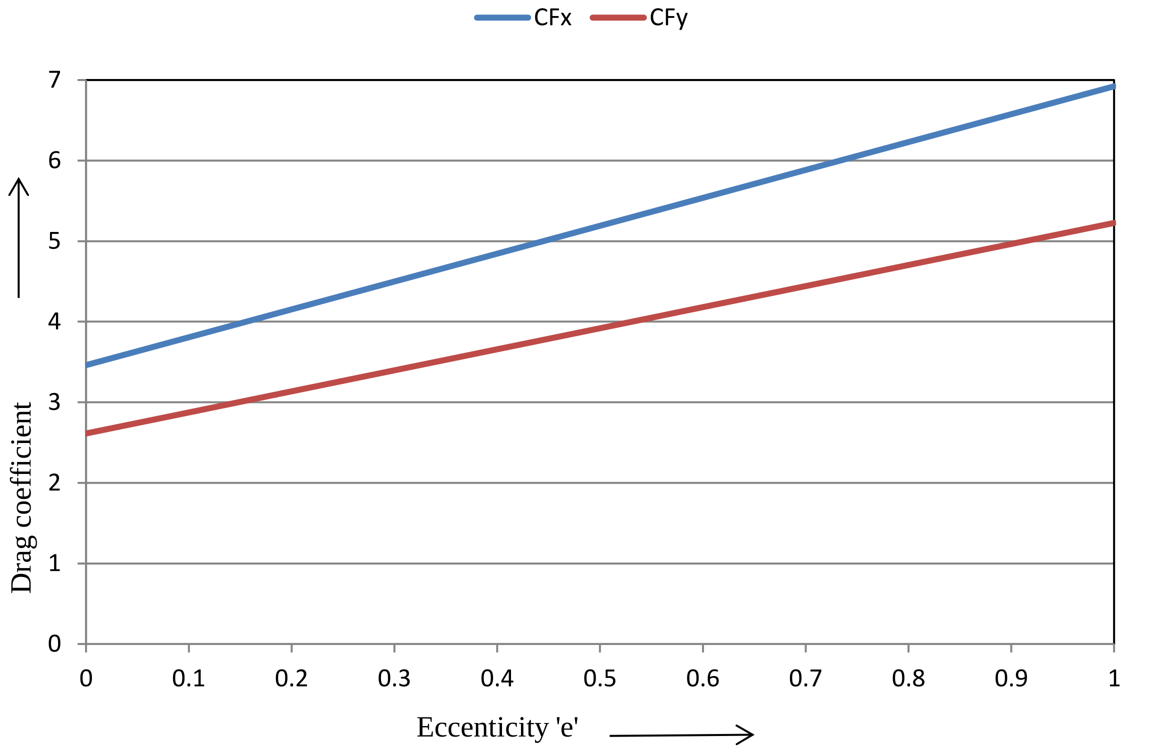


**Figure 5 (ii)** Variation of drag coefficient for Cycloidal body(described by equation 6.5) with respect to k/μ

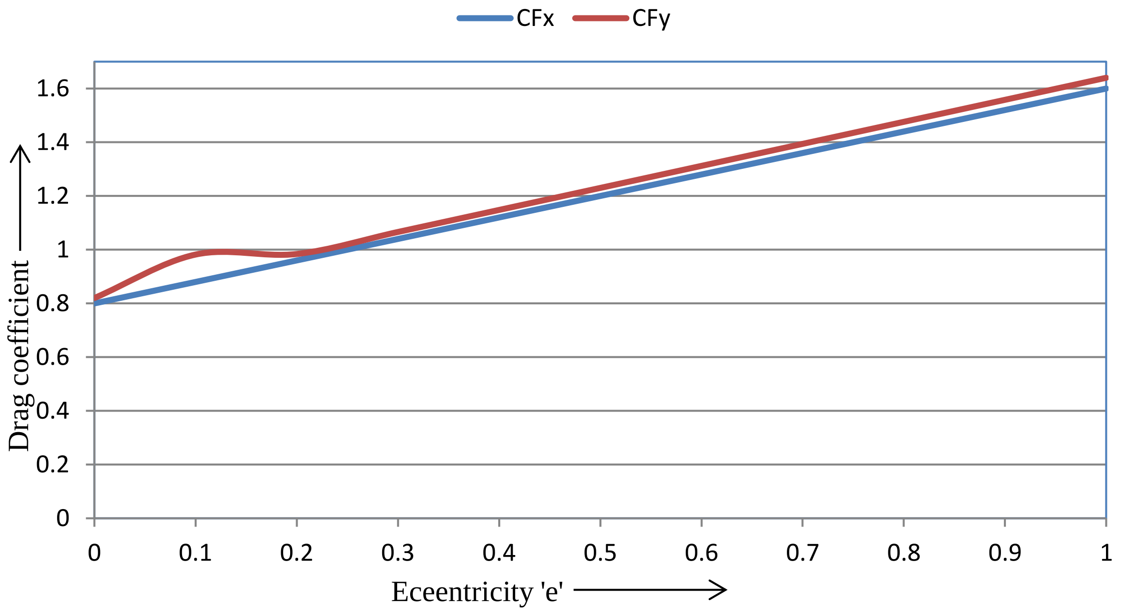


**Figure 6.** Variation of drag coefficient for Cassini body with respect to k/μ

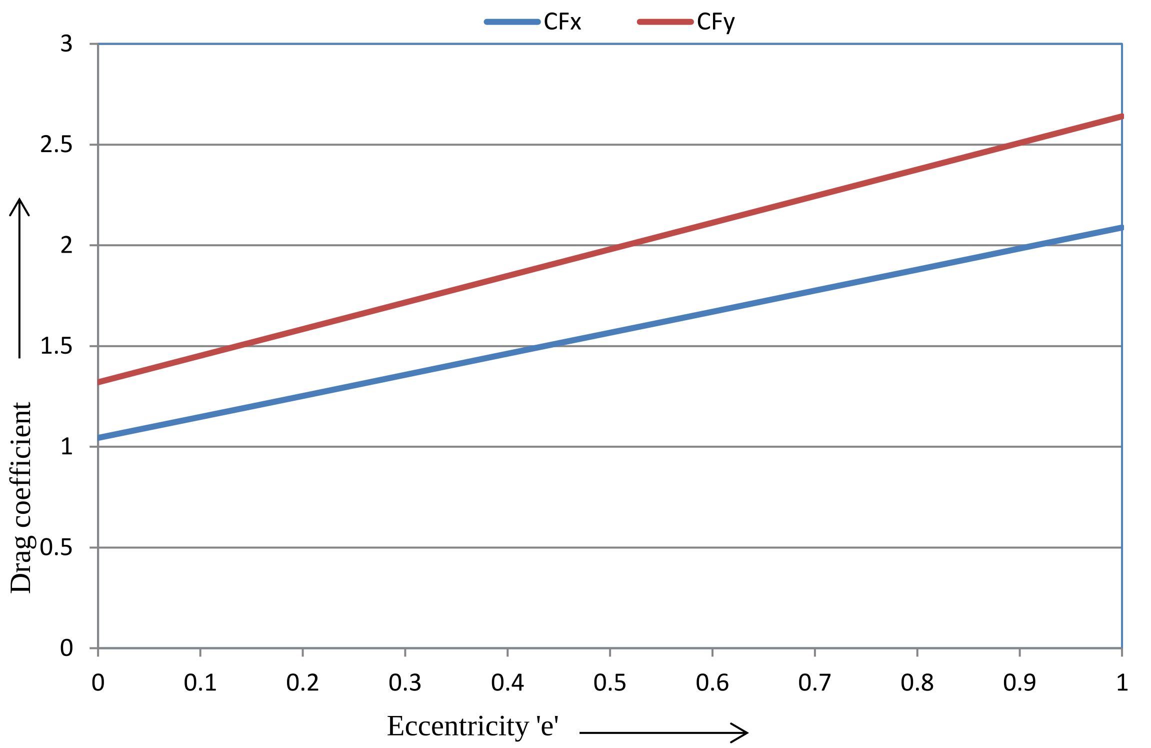


**Figure 7.** Variation of drag coefficient for Hypocycloidal body with respect to k/μ

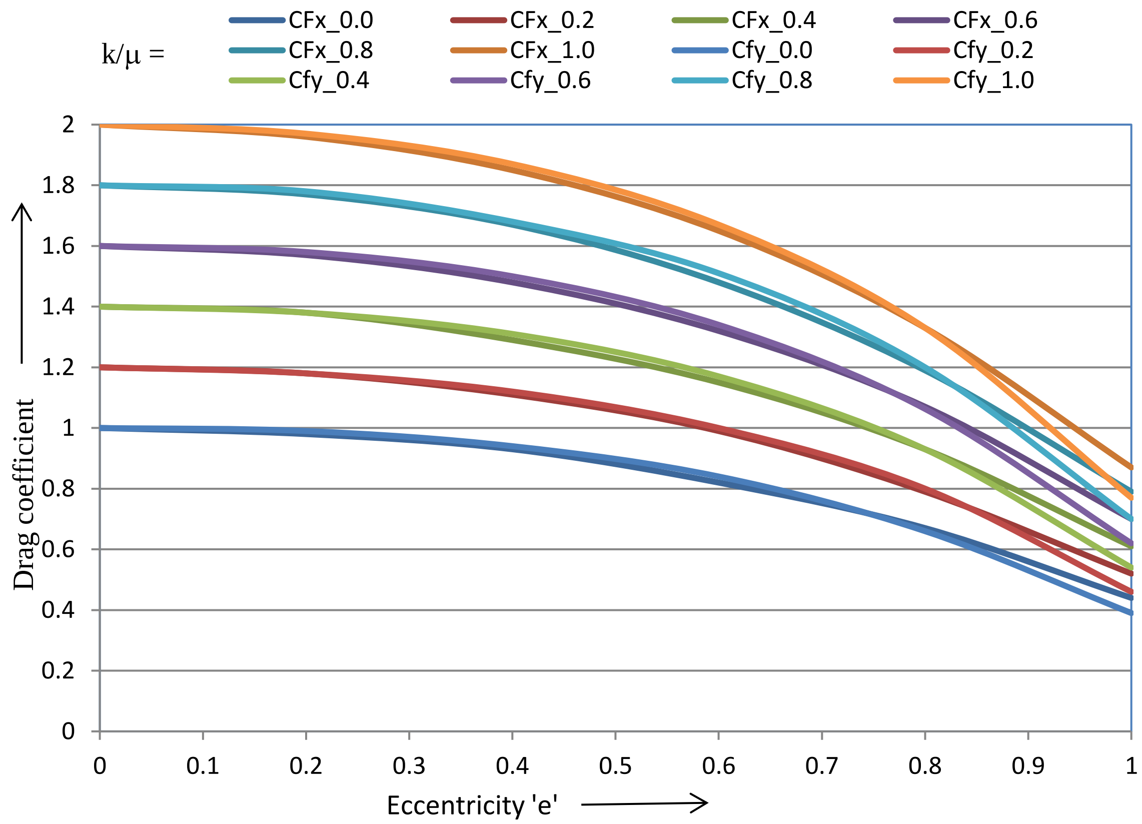


**Figure 8.** Variation of drag coefficient for Egg-shaped body with respect to k/μ and eccentricity 'e'