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Logarithmic Mean Labeling of Some Ladder Related Graphs

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Abstract

In general, the logarithmic mean of two positive integers need not be an integer. Hence, the logarithmic mean is to be an integer; we use either flooring or ceiling function. The logarithmic mean labeling of graphs have been defined in which the edge labels may be assigned by either flooring function or ceiling function. In this, we establish the logarithmic mean labeling on graphs by considering the edge labels obtained only from the flooring function. A logarithmic mean labeling of a graph G with q edges is an injective function from the vertex set of G to 1, 2, 3,..., q+1 such that the edge labels obtained from the flooring function of logarithmic mean of the vertex labels of the end vertices of each edge are all distinct, and the set of edge labels is 1, 2, 3,..., q. A graph is said to be a logarithmic mean graph if it admits a logarithmic mean labeling. In this paper, we study the logarithmic meanness of some ladder related graphs.

Keywords: Graph; Graph labeling; Vertex labeling; Edge labeling; Logarithmic mean labeling; Logarithmic mean graph; Ladder graph; Corona of ladder graph

MSC 2010 No.: 05C25; 05C78

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology, the readers are referred to the book of Harary (1972). For a detailed survey on graph labeling, we refer the book of Gallian (2019).

The study of graceful graphs and graceful labeling methods was first introduced by Rosa (1967). The concept of mean labeling was first introduced and developed by Somasundaram and Ponraj (2003). Further, it was studied by Vasuki et al. (2009, 2010, 2011). Vaidya and Lekha Bijukumar (2010) discussed the mean labeling of some graph operations. The concept of F-Geometric mean labeling was first introduced and developed by Durai Baskar and Arockiaraj (2015, 2017). Amutha (2006) discussed the existence of certain types of graph labelings of step graphs.

Motivated by the works of so many authors in the area of graph labeling, we introduced a new type of labeling called logarithmic mean labeling. In this paper, we have discussed the logarithmic meanness of some ladder related graphs.

2. Preliminaries

In this section, some basic definitions which are to be used in the subsequent sections have been discussed.

Definition 2.1.

A graph G consists of a pair (V, E) where V is a non-empty finite set of vertices and E is a set of unordered pair of elements of V, called edges of G.

Definition 2.2.

A walk of a graph G is an alternating sequence of vertices and edges $v_0, e_1, v_1, \ldots, v_{n-1}, e_n, v_n$ beginning and ending with vertices, in which each edge is incident with the two vertices immediately preceding and following it. This walk joins v_0 and v_n, and it is called a $v_0 - v_n$ walk. A walk is called a path if all the vertices and edges are distinct and a path on n vertices is denoted by P_n.

Definition 2.3.

The degree of a vertex v in a graph G, denoted by $deg\ v$, is the number of edges incident with v. A vertex v in G is called an end vertex (or pendant vertex) if $deg\ v = 1$. An edge incident to a pendant vertex is called a pendant edge.

Definition 2.4.

$G \odot S_m$ is the graph obtained from G by attaching m pendant vertices at each vertex of G.

Definition 2.5.

Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the Cartesian product $G_1 \times G_2$ has $p_1 p_2$ vertices which are $\{(u, v) : u \in G_1, v \in G_2\}$. The edges are obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. The ladder graph L_n is a graph obtained from the Cartesian product of P_2 and P_n.

Definition 2.6.

The triangular ladder $TL_n, n \geq 2$ is a graph obtained by completing the ladder L_n by the edges $u_i v_{i+1}$ for $1 \leq i \leq n-1$, where L_n is the graph $P_2 \times P_n$.

Definition 2.7.

The slanting ladder SL_n is a graph that consists of two copies of P_n having vertex set $\{u_i : 1 \leq i \leq n\} \bigcup \{v_i : 1 \leq i \leq n\}$ and edge set is formed by adjoining u_{i+1} and v_i for all $1 \leq i \leq n-1$.

Definition 2.8.

Let P_n be a path on n vertices denoted by $u_{1,1}, u_{1,2}, u_{1,3}, \ldots, u_{1,n}$ and with $n-1$ edges denoted by $e_1, e_2, \ldots, e_{n-1}$ where e_i is the edge joining the vertices $u_{1,i}$ and $u_{1,i+1}$. On each edge e_i, erect a ladder with $n-(i-1)$ steps including the edge e_i, for $i = 1, 2, 3, \ldots, n-1$. The graph thus obtained is called a one sided step graph and it is denoted by ST_n.

Definition 2.9.

Let P_{2n} be a path on $2n$ vertices $u_{1,1}, u_{1,2}, u_{1,3}, \ldots, u_{1,2n}$ and with $2n-1$ edges $e_1, e_2, \ldots, e_{2n-1}$ where e_i is the edge joining the vertices $u_{1,i}$ and $u_{1,i+1}$. On each edge e_i, we erect a ladder with '$i+1$' steps including the edge e_i, for $i = 1, 2, 3, \ldots, n$ and on each e_i erect a ladder with $2n+1-i$ steps including e_i, for $i = n+1, n+2, \ldots, 2n-1$. The graph thus obtained is called a double sided step graph and it is denoted by $2ST_{2n}$.

Definition 2.10.

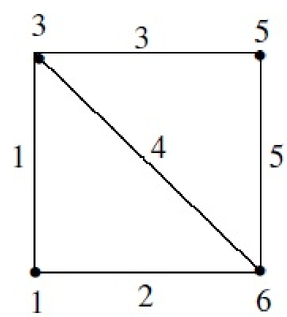
A function f is called a logarithmic mean labeling of a graph $G(V, E)$ if $f : V(G) \to \{1, 2, 3, \ldots, q+1\}$ is injective and the induced function $f^* : E(G) \to \{1, 2, 3, \ldots, q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{f(v) - f(u)}{\ln f(v) - \ln f(u)} \right\rfloor, \text{ for all } uv \in E(G),$$

is bijective. A graph that admits a logarithmic mean labeling is called a logarithmic mean graph.

Example 2.11.

A logarithmic mean graph of a graph $K_4 - e$ is shown below.



In this paper, we have discussed the logarithmic mean labeling of the ladder graphs L_n for $n \geq 2$, $L_n \odot S_m$ for $n \geq 2$ and $m \leq 2$, TL_n for $n \geq 2$, $TL_n \odot S_m$ for $n \geq 2$ and $m \leq 2$, SL_n for $n \geq 2$, $SL_n \odot S_m$ for $n \geq 2$ and $m \leq 2$, step graph ST_n and double sided step graph $2ST_{2n}$.

3. Main Results

Theorem 3.1.

The ladder graph L_n is a logarithmic mean graph, for $n \geq 2$.

Proof:

Let $u_1, u_2, u_3, \ldots, u_n$ and $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the ladder $L_n = P_n \times P_2$. Then, L_n has $2n$ vertices and $3n-2$ edges.

Define $f : V(L_n) \to \{1, 2, 3, \ldots, 3n-1\}$ as follows:

$$f(u_i) = 3i - 1, \text{ for } 1 \leq i \leq n,$$
$$f(v_i) = 3i - 2, \text{ for } 1 \leq i \leq n.$$

The induced edge labeling is as follows:

$$f^*(u_i u_{i+1}) = 3i, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(v_i v_{i+1}) = 3i - 1, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(u_i v_i) = 3i - 2, \text{ for } 1 \leq i \leq n.$$

Hence, f is a logarithmic mean labeling of the ladder L_n. Thus, the ladder L_n is a logarithmic mean graph, for $n \geq 2$. ■

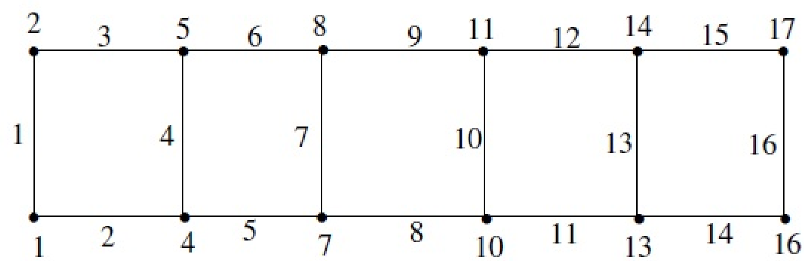


Figure 1. A logarithmic mean labeling of the ladder L_6

Theorem 3.2.

The graph $L_n \odot S_m$ is a logarithmic mean graph for $n \geq 2$ and $m \leq 2$.

Proof:

Let $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ be the vertices of L_n. Let $w_1^{(i)}, w_2^{(i)}, ..., w_m^{(i)} and\ x_1^{(i)}, x_2^{(i)}, ..., x_m^{(i)}$ be the pendent vertices attached at each vertex u_i and v_i of the ladder L_n, for $1 \leq i \leq n$. Then, the edge set is $E(L_n \circ S_m) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_iw_j^{(i)} : 1 \leq i \leq n\ , 1 \leq j \leq m\} \cup \{v_ix_j^{(i)} : 1 \leq i \leq n\ , 1 \leq j \leq m\}$.

Case (i) $m = 1$.
Define $f : V(L_n \odot S_1) \to \{1, 2, 3, \ldots, 5n-1\}$ as follows:

$$\begin{aligned}
f(u_1) &= 3,\\
f(u_i) &= 5i-3,\ \text{for } 2 \leq i \leq n,\\
f(v_1) &= 4,\\
f(v_i) &= 5i-2,\ \text{for } 2 \leq i \leq n,\\
f(w_1^{(i)}) &= 5i-4,\ \text{for } 1 \leq i \leq n,\\
f(x_1^{(1)}) &= 2,\\
f(x_1^{(i)}) &= 5i-1,\ \text{for } 2 \leq i \leq n.
\end{aligned}$$

Then the induced edge labeling is obtained as follows.

$$\begin{aligned}
f^*(u_iu_{i+1}) &= 5i-1,\ \text{for } 1 \leq i \leq n-1,\\
f^*(v_iv_{i+1}) &= 5i,\ \text{for } 1 \leq i \leq n-1,\\
f^*(u_1v_1) &= 3,\\
f^*(u_iv_i) &= 5i-3,\ \text{for } 2 \leq i \leq n,\\
f^*(u_iw_1^{(i)}) &= 5i-4,\ \text{for } 1 \leq i \leq n,\\
f^*(v_1x_1^{(1)}) &= 2,\\
f^*(v_ix_1^{(i)}) &= 5i-2,\ \text{for } 2 \leq i \leq n.
\end{aligned}$$

Case (ii) $m = 2$.
Define $f : V(L_n \odot S_2) \to \{1, 2, 3, \ldots, 7n-1\}$ as follows.

$$f(u_i) = \begin{cases} 3, & i = 1,\\ 7i-2, & 2 \leq i \leq n \text{ and } i \text{ is even},\\ 7i-5, & 2 \leq i \leq n \text{ and } i \text{ is odd}\ , \end{cases}$$

$$f(v_i) = \begin{cases} 5, & i = 1,\\ 7i-4, & 2 \leq i \leq n \text{ and } i \text{ is even},\\ 7i-1, & 2 \leq i \leq n \text{ and } i \text{ is odd}\ , \end{cases}$$

$$f(w_1^{(i)}) = \begin{cases} 1, & i = 1,\\ 7i-3, & 2 \leq i \leq n \text{ and } i \text{ is even},\\ 7i-6, & 2 \leq i \leq n \text{ and } i \text{ is odd}\ , \end{cases}$$

$$f(w_2^{(i)}) = \begin{cases} 2, & i = 1,\\ 7i-1, & 2 \leq i \leq n \text{ and } i \text{ is even},\\ 7i-4, & 2 \leq i \leq n \text{ and } i \text{ is odd}\ , \end{cases}$$

$$f(x_1^{(i)}) = \begin{cases} 4i, & 1 \leq i \leq 2, \\ 7i-6, & 3 \leq i \leq n \text{ and } i \text{ is even,} \\ 7i-3, & 3 \leq i \leq n \text{ and } i \text{ is odd ,} \end{cases}$$

$$f(x_2^{(i)}) = \begin{cases} 6, & i = 1, \\ 7i-5, & 2 \leq i \leq n \text{ and } i \text{ is even,} \\ 7i-2, & 2 \leq i \leq n \text{ and } i \text{ is odd .} \end{cases}$$

Then, the induced edge labeling is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 6, & i = 1, \\ 7i-1, & 2 \leq i \leq n-1 \ , \end{cases}$$

$$f^*(v_iv_{i+1}) = 7i, \text{ for } 1 \leq i \leq n-1 \ ,$$

$$f^*(u_iv_i) = 7i-4, \text{ for } 1 \leq i \leq n \ ,$$

$$f^*(u_iw_1^{(i)}) = \begin{cases} 7i-3, & 1 \leq i \leq n \text{ and } i \text{ is even,} \\ 7i-6, & 1 \leq i \leq n \text{ and } i \text{ is odd ,} \end{cases}$$

$$f^*(u_iw_2^{(i)}) = \begin{cases} 7i-2, & 1 \leq i \leq n \text{ and } i \text{ is even,} \\ 7i-5, & 1 \leq i \leq n \text{ and } i \text{ is odd ,} \end{cases}$$

$$f^*(v_ix_1^{(i)}) = \begin{cases} 7i-6, & 1 \leq i \leq n \text{ and } i \text{ is even,} \\ 7i-3, & 1 \leq i \leq n \text{ and } i \text{ is odd ,} \end{cases}$$

$$f^*(v_ix_2^{(i)}) = \begin{cases} 7i-5, & 1 \leq i \leq n \text{ and } i \text{ is even,} \\ 7i-2, & 1 \leq i \leq n \text{ and } i \text{ is odd .} \end{cases}$$

Hence, f is a logarithmic mean labeling of the graph $L_n \odot S_m$. Thus, the graph $L_n \odot S_m$ is a logarithmic mean graph for $n \geq 2$ and $m \leq 2$. ■

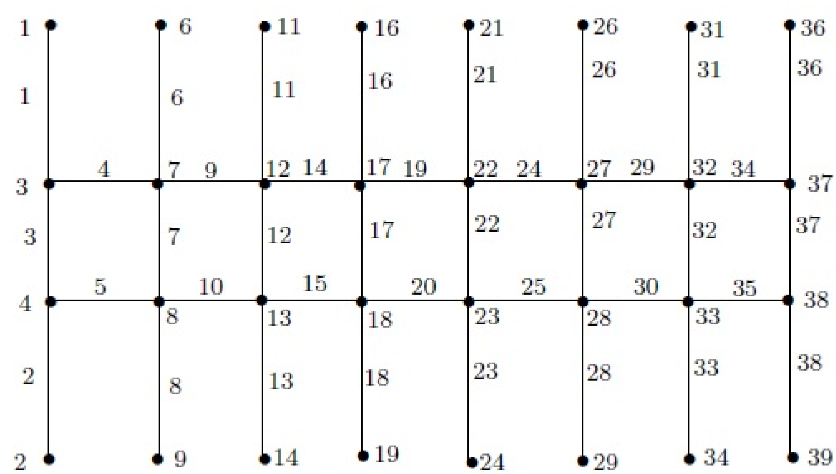


Figure 2. A logarithmic mean labeling of $L_8 \odot S_1$

Theorem 3.3.

The triangular Ladder TL_n is a logarithmic mean graph for $n \geq 2$.

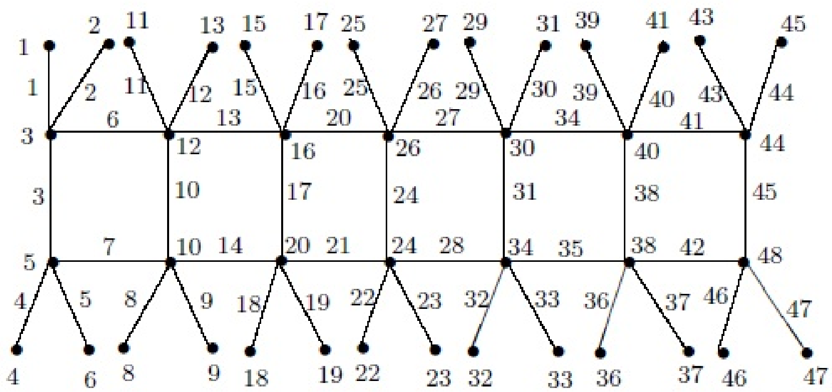


Figure 3. A logarithmic mean labeling of $L_7 \odot S_2$

Proof:

Let $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ be the vertex set of TL_n and $\{u_i u_{i+1}; 1 \le i \le n-1\} \cup \{v_i v_{i+1}; 1 \le i \le n-1\} \cup \{u_i v_i; 1 \le i \le n\} \cup \{u_i v_{i+1}; 1 \le i \le n-1\}$ be the edge set of TL_n. Then, TL_n has $2n$ vertices and $4n-3$ edges.

Define $f : V(TL_n) \to \{1, 2, 3, \dots, 4n-2\}$ as follows.

$$
\begin{aligned}
f(u_i) &= 4i-1, \text{ for } 1 \le i \le n-1,\\
f(u_n) &= 4n-2,\\
f(v_i) &= 4i-3, \text{ for } 1 \le i \le n.
\end{aligned}
$$

The induced edge labeling is as follows.

$$
\begin{aligned}
f^*(u_i u_{i+1}) &= 4i, \text{ for } 1 \le i \le n-1,\\
f^*(u_i v_i) &= 4i-3, \text{ for } 1 \le i \le n,\\
f^*(u_i v_{i+1}) &= 4i-1, \text{ for } 1 \le i \le n-1,\\
f^*(v_i v_{i+1}) &= 4i-2, \text{ for } 1 \le i \le n-1.
\end{aligned}
$$

Hence, f is a logarithmic mean labeling of TL_n. Thus, the Triangular Ladder TL_n is a logarithmic mean graph, for $n \ge 2$. ■

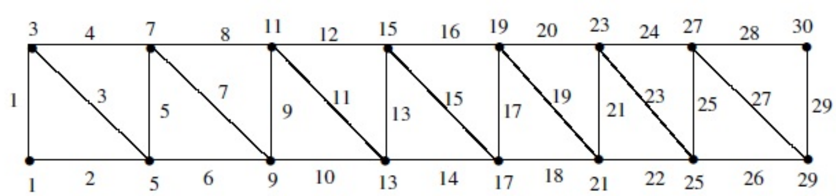


Figure 4. A logarithmic mean labeling of TL_8

Theorem 3.4.

The graph $TL_n \odot S_m$ is a logarithmic mean graph for $n \ge 2$ and $m \le 2$.

Proof:

Let $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ be the vertices of TL_n. Let $w_1^{(i)}, w_2^{(i)}, ..., w_m^{(i)}$ and $x_1^{(i)}, x_2^{(i)}, ..., x_m^{(i)}$ be the pendent vertices attached at each vertex u_i and v_i of the ladder L_n, for $1 \leq i \leq n$.

Then, the edge set is $E(TL_n \circ S_m) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_iw_j^{(i)} : 1 \leq i \leq n\ , 1 \leq j \leq m\} \cup \{v_ix_j^{(i)} : 1 \leq i \leq n\ , 1 \leq j \leq m\} \cup \{u_iv_{i+1} : 1 \leq i \leq n-1\}$.

Case (i) $m = 1$.
Define $f : V(TL_n \odot S_1) \to \{1, 2, 3, \ldots, 6n-2\}$ as follows.

$$\begin{aligned} f(u_i) &= 6i-2, \text{ for } 1 \leq i \leq n, \\ f(v_i) &= 6i-4, \text{ for } 1 \leq i \leq n, \\ f(w_1^{(i)}) &= 6i-3, \text{ for } 1 \leq i \leq n, \\ f(x_1^{(i)}) &= 6i-5, \text{ for } 1 \leq i \leq n. \end{aligned}$$

Then, the induced edge labeling is obtained as follows:

$$\begin{aligned} f^*(u_iu_{i+1}) &= 6i, \text{ for } 1 \leq i \leq n-1, \\ f^*(v_iv_{i+1}) &= 6i-2, \text{ for } 1 \leq i \leq n-1, \\ f^*(u_iv_{i+1}) &= 6i-1, \text{ for } 1 \leq i \leq n-1, \\ f^*(u_iv_i) &= 6i-4, \text{ for } 1 \leq i \leq n, \\ f^*(u_iw_1^{(i)}) &= 6i-3, \text{ for } 1 \leq i \leq n, \\ f^*(v_ix_1^{(i)}) &= 6i-5, \text{ for } 1 \leq i \leq n. \end{aligned}$$

Case (ii) $m = 2$.
Define $f : V(TL_n \odot S_2) \to \{1, 2, 3, \ldots, 8n-2\}$ as follows.

$$\begin{aligned} f(u_1) &= 5, \\ f(u_i) &= 8i-5, \text{ for } 2 \leq i \leq n, \\ f(v_1) &= 3, \\ f(v_i) &= 8i-3, \text{ for } 2 \leq i \leq n, \\ f(w_1^{(1)}) &= 4, \\ f(w_1^{(i)}) &= 8i-7, \text{ for } 2 \leq i \leq n, \\ f(w_2^{(1)}) &= 6, \\ f(w_2^{(i)}) &= 8i-6, \text{ for } 2 \leq i \leq n, \\ f(x_1^{(1)}) &= 1, \\ f(x_1^{(i)}) &= 8i-4, \text{ for } 2 \leq i \leq n, \end{aligned}$$

$$\begin{aligned} f(x_2^{(1)}) &= 2, \\ f(x_2^{(i)}) &= 8i-2, \text{ for } 2 \leq i \leq n. \end{aligned}$$

Then, the induced edge labeling is obtained as follows:

$$\begin{aligned} f^*(u_1u_2) &= 7, \\ f^*(u_iu_{i+1}) &= 8i-2, \text{ for } 2 \leq i \leq n-1, \\ f^*(v_1v_2) &= 6, \\ f^*(v_iv_{i+1}) &= 8i, \text{ for } 2 \leq i \leq n-1, \\ f^*(u_iv_i) &= 8i-5, \text{ for } 1 \leq i \leq n, \\ f^*(u_1v_2) &= 8, \\ f^*(u_iv_{i+1}) &= 8i-1, \text{ for } 2 \leq i \leq n-1, \end{aligned}$$

$$\begin{aligned} f^*(u_1w_1^{(1)}) &= 4, \\ f^*(u_iw_1^{(i)}) &= 8i-7, \text{ for } 2 \leq i \leq n, \end{aligned}$$

$$\begin{aligned} f^*(u_1w_2^{(1)}) &= 5, \\ f^*(u_iw_2^{(i)}) &= 8i-6, \text{ for } 2 \leq i \leq n, \end{aligned}$$

$$\begin{aligned} f^*(v_1x_1^{(1)}) &= 1, \\ f^*(v_ix_1^{(i)}) &= 8i-4, \text{ for } 2 \leq i \leq n, \end{aligned}$$

$$\begin{aligned} f^*(v_1x_2^{(1)}) &= 2, \\ f^*(v_ix_2^{(i)}) &= 8i-3, \text{ for } 2 \leq i \leq n. \end{aligned}$$

Hence, f is a logarithmic mean labeling of the graph $TL_n \odot S_m$. Thus, the graph $TL_n \odot S_m$ is a logarithmic mean graph for $n \geq 2$ and $m \leq 2$. ■

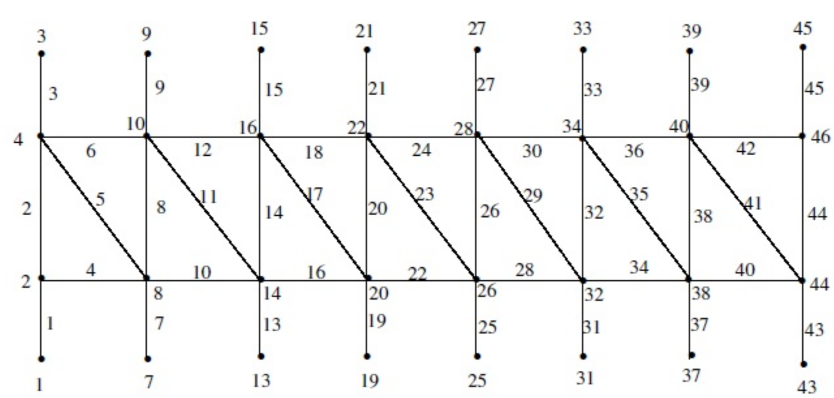


Figure 5. A logarithmic Mean labeling of $TL_8 \circ S_1$

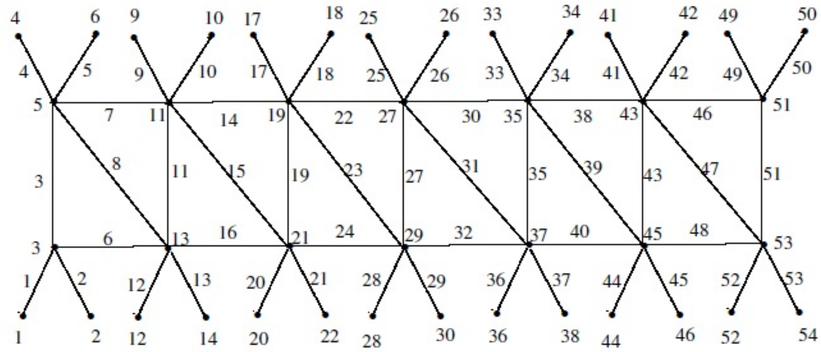


Figure 6. A logarithmic Mean labeling of $TL_7 \circ S_2$

Theorem 3.5.

The graph SL_n is a logarithmic mean graph for $n \geq 2$.

Proof:

Let the vertex set of SL_n be $\{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ and the edge set of SL_n be $\{u_iu_{i+1}; 1 \leq i \leq n-1\} \cup \{v_iv_{i+1}; 1 \leq i \leq n-1\} \cup \{v_iu_{i+1}; 1 \leq i \leq n-1\}$. Then, SL_n has $2n$ vertices and $3n-3$ edges.

Define $f : V(SL_n) \to \{1, 2, 3, \ldots, 3n-2\}$ as follows.

$$\begin{aligned} f(u_1) &= 1, \\ f(u_i) &= 3i-4, \text{ for } 2 \leq i \leq n, \\ f(v_i) &= 3i, \text{ for } 1 \leq i \leq n-1, \\ f(v_n) &= 3n-2. \end{aligned}$$

Then the induced edge labeling is obtained as follows.

$$\begin{aligned} f^*(u_1u_2) &= 1, \\ f^*(u_iu_{i+1}) &= 3i-3, \text{ for } 2 \leq i \leq n-1, \\ f^*(v_iv_{i+1}) &= 3i+1, \text{ for } 1 \leq i \leq n-2, \\ f^*(v_{n-1}v_n) &= 3n-3, \\ f^*(v_iu_{i+1}) &= 3i-1, \text{ for } 1 \leq i \leq n-1. \end{aligned}$$

Hence, f is a logarithmic mean labeling of SL_n. Thus, the slanting ladder SL_n is a logarithmic mean graph for $n \geq 2$. ■

Theorem 3.6.

The graph $SL_n \odot S_m$ is a logarithmic mean graph for $n \geq 2$ and $m \leq 2$.

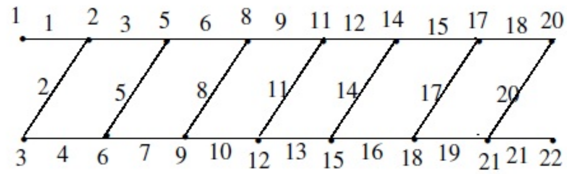


Figure 7. A logarithmic Mean labeling of SL_8

Proof:

Let $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ be the vertices of SL_n. Let $w_1^{(i)}, w_2^{(i)}, ..., w_m^{(i)}$ and $x_1^{(i)}, x_2^{(i)}, ..., x_m^{(i)}$ be the pendent vertices attached at each vertex u_i and v_i of the ladder L_n, for $1 \leq i \leq n$. Then, the edge set is $E(SL_n \circ S_m) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{v_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iw_j^{(i)} : 1 \leq i \leq n\ , 1 \leq j \leq m\} \cup \{v_ix_j^{(i)} : 1 \leq i \leq n\ , 1 \leq j \leq m\}$.

Case (i) $m = 1$ and $n \geq 3$.
Define $f : V(SL_n \odot S_1) \to \{1, 2, 3, \ldots, 5n-2\}$ as follows.

$$f(u_1) = 2,$$
$$f(u_i) = 5i - 6,\ \text{for } 2 \leq i \leq n,$$

$$f(v_i) = 5i,\ \text{for } 1 \leq i \leq n-1,$$
$$f(v_n) = 5n - 2,$$

$$f(w_1^{(1)}) = 1,$$
$$f(w_1^{(i)}) = 5i - 7,\ \text{for } 2 \leq i \leq n,$$

$$f(x_1^{(1)}) = 7,$$
$$f(x_1^{(i)}) = 5i + 1,\ \text{for } 2 \leq i \leq n-1,$$
$$f(x_1^{(n)}) = 5n - 3.$$

Then the induced edge labeling is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 2, & i = 1, \\ 5i - 4, & 2 \leq i \leq n-1, \end{cases}$$
$$f^*(v_iv_{i+1}) = 5i + 2,\ \text{for } 1 \leq i \leq n-2,$$
$$f^*(v_{n-1}v_n) = 5n - 4,$$
$$f^*(v_iu_{i+1}) = 5i - 1,\ \text{for } 1 \leq i \leq n-1,$$
$$f^*(u_1w_1^{(1)}) = 1,$$
$$f^*(u_iw_1^{(i)}) = 5i - 7,\ \text{for } 2 \leq i \leq n,$$
$$f^*(v_ix_1^{(i)}) = 5i,\ \text{for } 1 \leq i \leq n-1,$$
$$f^*(v_nx_1^{(n)}) = 5n - 3.$$

Case (ii) $m = 2$ and $n \geq 3$.

Define $f : V(SL_n \odot S_2) \to \{1, 2, 3, \ldots, 7n-2\}$ as follows:

$$f(u_i) = \begin{cases} 2i+1, & 1 \leq i \leq 2, \\ 7i-6, & 3 \leq i \leq n-1 \text{ and } i \text{ is even}, \\ 7i-9, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd}, \end{cases}$$

$$f(u_n) = \begin{cases} 7n-10, & n \text{ is even}, \\ 7n-9, & n \text{ is odd}, \end{cases}$$

$$f(v_i) = \begin{cases} 8, & i = 1, \\ 7i+2, & 2 \leq i \leq n-3 \text{ and } i \text{ is even}, \\ 7i-1, & 2 \leq i \leq n-3 \text{ and } i \text{ is odd}, \end{cases}$$

$$f(v_{n-2}) = \begin{cases} 7n-13, & n \text{ is even}, \\ 7n-15, & n \text{ is odd}, \end{cases}$$

$$f(v_{n-1}) = 7n-5,$$

$$f(v_n) = 7n-3,$$

$$f(w_1^{(i)}) = \begin{cases} 1, & i = 1, \\ 6i-8, & 2 \leq i \leq 3, \\ 7i-7, & 4 \leq i \leq n-1 \text{ and } i \text{ is even}, \\ 7i-10, & 4 \leq i \leq n-1 \text{ and } i \text{ is odd}, \end{cases}$$

$$f(w_1^{(n)}) = \begin{cases} 7n-11, & n \text{ is even}, \\ 7n-10, & n \text{ is odd}, \end{cases}$$

$$f(w_2^{(i)}) = \begin{cases} 4i-2, & 1 \leq i \leq 2, \\ 7i-5, & 3 \leq i \leq n-1 \text{ and } i \text{ is even}, \\ 7i-8, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd}, \end{cases}$$

$$f(w_2^{(n)}) = \begin{cases} 7n-7, & n \text{ is even}, \\ 7n-8, & n \text{ is odd}, \end{cases}$$

$$f(x_1^{(i)}) = \begin{cases} 9, & i = 1, \\ 7i, & 2 \leq i \leq n-3 \text{ and } i \text{ is even}, \\ 7i-3, & 2 \leq i \leq n-3 \text{ and } i \text{ is odd}, \end{cases}$$

$$f(x_1^{(n-2)}) = \begin{cases} 7n-12, & n \text{ is even}, \\ 7n-17, & n \text{ is odd}, \end{cases}$$

$$f(x_1^{(n-1)}) = \begin{cases} 7n-8, & n \text{ is even}, \\ 7n-7, & n \text{ is odd}, \end{cases}$$

$$f(x_1^{(n)}) = 7n-4,$$

$$f(x_2^{(i)}) = \begin{cases} 11, & i = 1, \\ 7i+1, & 2 \leq i \leq n-3 \text{ and } i \text{ is even}, \\ 7i-2, & 2 \leq i \leq n-3 \text{ and } i \text{ is odd}, \end{cases}$$

$$f(x_2^{(n-2)}) = \begin{cases} 7n-9, & n \text{ is even,} \\ 7n-16, & n \text{ is odd ,} \end{cases}$$
$$f(x_2^{(n-1)}) = 7n-6,$$
$$f(x_2^{(n)}) = 7n-2.$$

Then, the induced edge labeling is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 4i-1, & 1 \le i \le 2, \\ 7i-5, & 3 \le i \le n-2 \text{ ,} \end{cases}$$
$$f^*(u_{n-1}u_n) = \begin{cases} 7n-14, & n \text{ is even,} \\ 7n-12, & n \text{ is odd ,} \end{cases}$$

$$f^*(v_iv_{i+1}) = \begin{cases} 11, & i=1, \\ 7i+3, & 2 \le i \le n-3 \text{ ,} \end{cases}$$
$$f^*(v_{n-2}v_{n-1}) = \begin{cases} 7n-10, & n \text{ is even,} \\ 7n-11, & n \text{ is odd ,} \end{cases}$$

$$f^*(v_{n-1}v_n) = 7n-5,$$
$$f^*(v_iu_{i+1}) = 7i-1, \text{ for } 1 \le i \le n-1 \text{ ,}$$
$$f^*(u_iw_1^{(i)}) = \begin{cases} 1, & i=1, \\ 6i-8, & 2 \le i \le 3, \\ 7i-7, & 4 \le i \le n-1 \text{ and } i \text{ is even,} \\ 7i-10, & 4 \le i \le n-1 \text{ and } i \text{ is odd ,} \end{cases}$$

$$f^*(u_nw_1^{(n)}) = \begin{cases} 7n-11, & n \text{ is even,} \\ 7n-10, & n \text{ is odd ,} \end{cases}$$
$$f^*(u_iw_2^{(i)}) = \begin{cases} 3i-1, & 1 \le i \le 2, \\ 7i-6, & 3 \le i \le n-1 \text{ and } i \text{ is even,} \\ 7i-9, & 3 \le i \le n-1 \text{ and } i \text{ is odd ,} \end{cases}$$

$$f^*(u_nw_2^{(n)}) = 7n-9,$$
$$f^*(v_ix_1^{(i)}) = \begin{cases} 8, & i=1, \\ 7i, & 2 \le i \le n-3 \text{ and } i \text{ is even,} \\ 7i-3, & 2 \le i \le n-3 \text{ and } i \text{ is odd ,} \end{cases}$$

$$f^*(v_{n-2}x_1^{(n-2)}) = \begin{cases} 7n-13, & n \text{ is even,} \\ 7n-17, & n \text{ is odd ,} \end{cases}$$
$$f^*(v_{n-1}x_1^{(n-1)}) = 7n-7,$$
$$f^*(v_nx_1^{(n)}) = 7n-4,$$

$$f^*(v_i x_2^{(i)}) = \begin{cases} 9, & i = 1, \\ 7i+1, & 2 \le i \le n-3 \text{ and } i \text{ is even}, \\ 7i-2, & 2 \le i \le n-3 \text{ and } i \text{ is odd}, \end{cases}$$

$$f^*(v_{n-2} x_2^{(n-2)}) = \begin{cases} 7n-12, & n \text{ is even}, \\ 7n-16, & n \text{ is odd}, \end{cases}$$

$$f^*(v_{n-1} x_2^{(n-1)}) = 7n-6,$$

$$f^*(v_n x_2^{(n)}) = 7n-3.$$

Case (iii) $m = 1, 2$ and $n = 2$.

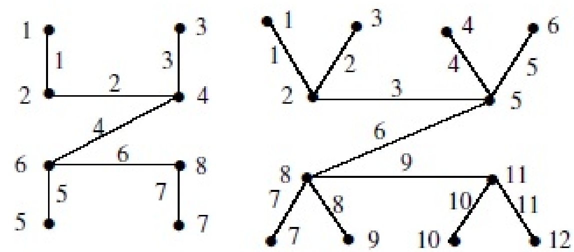


Figure 8. A logarithmic mean labeling of $SL_2 \odot S_1$ and $SL_2 \odot S_2$

Hence, f is a logarithmic mean labeling of the graph $SL_n \odot S_m$. Thus, the graph $SL_n \odot S_m$ is a logarithmic mean graph for $n \ge 2$ and $m \le 2$. ■

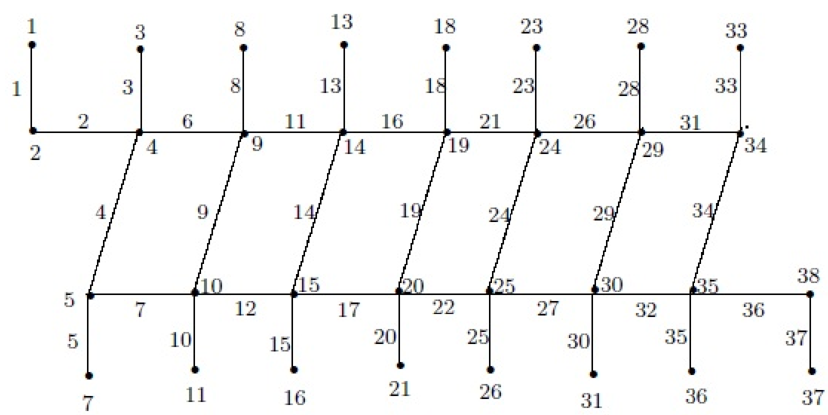


Figure 9. A logarithmic Mean labeling of $SL_8 \circ S_1$

Theorem 3.7.

The graph ST_n is a logarithmic mean graph for $n \ge 2$.

Proof:

Let $u_{1,1}, u_{1,2}, u_{1,3}, \ldots, u_{1,n}, u_{2,1}, u_{2,2}, u_{2,3}, \ldots, u_{2,n}, u_{3,1}, u_{3,2}, u_{3,3}, \ldots, u_{3,n-1}, u_{4,1}, u_{4,2}, u_{4,3}, \ldots, u_{4,n-2}, \ldots, u_{n,1}, u_{n,2}$ be the vertices of the step graph ST_n.

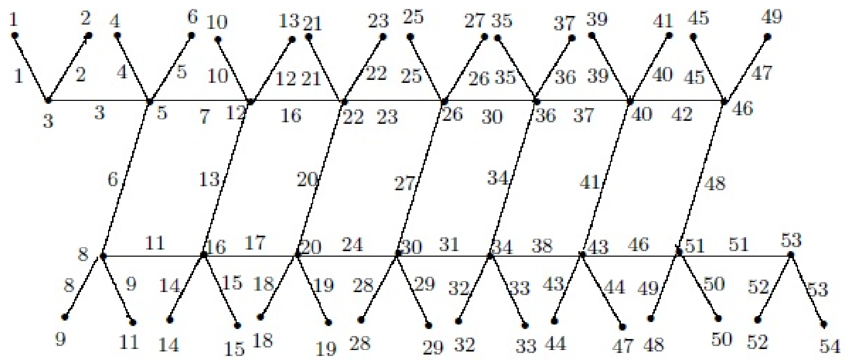


Figure 10. A logarithmic Mean labeling of $SL_8 \circ S_2$

In $u_{i,j}, i$ denotes the row (counted from bottom to top) and j denotes the column (counted from left to right) in which the vertex occurs.

Define $f : V(ST_n) \to \{1, 2, 3, \ldots, n^2 + n - 1\}$ as follows.

$$f(u_{i,j}) = (n+1-i)^2 + j - 1, \text{ for } 2 \le i \le n \text{ and } 1 \le j \le n+2-i\ ,$$
$$f(u_{1,j}) = n^2 + j - 1, \text{ for } 1 \le j \le n.$$

The induced edge labeling is as follows.

$$f^*(u_{i,j}u_{i,j+1}) = (n+1-i)^2 + j - 1, \text{ for } 2 \le i \le n \text{ and } 1 \le j \le n+1-i,$$
$$f^*(u_{1,j}u_{1,j+1}) = n^2 + j - 1, \text{ for } 1 \le j \le n-1,$$
$$f^*(u_{i,j}u_{i+1,j}) = (n+1-i)(n-i) + j - 1, \text{ for } 1 \le i \le n-1 \text{ and } 1 \le j \le n+1-i.$$

Hence, f is a logarithmic mean labeling of ST_n. Thus the graph ST_n is a logarithmic mean graph, for $n \ge 2$. ■

Theorem 3.8.

The graph $2ST_{2n}$ is a logarithmic mean graph, for $n \ge 2$.

Proof:

Let $u_{1,1}, u_{1,2}, u_{1,3}, \ldots, u_{1,n}, u_{2,1}, u_{2,2}, u_{2,3}, \ldots, u_{2,2n}, u_{3,1}, u_{3,2}, u_{3,3}, \ldots, u_{3,2n-2}, u_{4,1}, u_{4,2}, u_{4,3}, \ldots, u_{4,2n-4}, \ldots, u_{n+1,1}, u_{n+1,2}$ be the vertices of the double sided step graph $2ST_{2n}$.

In $u_{i,j}, i$ denotes the row (counted from bottom to top) and j denotes the column (counted from left to right) in which the vertex occurs.

Define $f : V(2ST_{2n}) \to \{1, 2, 3, \ldots, 2n^2 + 3n\}$ as follows,

$$f(u_{1,j}) = \begin{cases} 2n^2 + n + 1 + 2(j-1), & 1 \le j \le n, \\ 2n^2 + 3n - 2(j-n-1), & n+1 \le j \le 2n, \end{cases}$$

for $2 \le i \le n$ and $2 \le j \le n+2-i$,

$$f(u_{i,j}) = 2(n+1-i)^2 + (n+2-i) + 2(j-2),$$

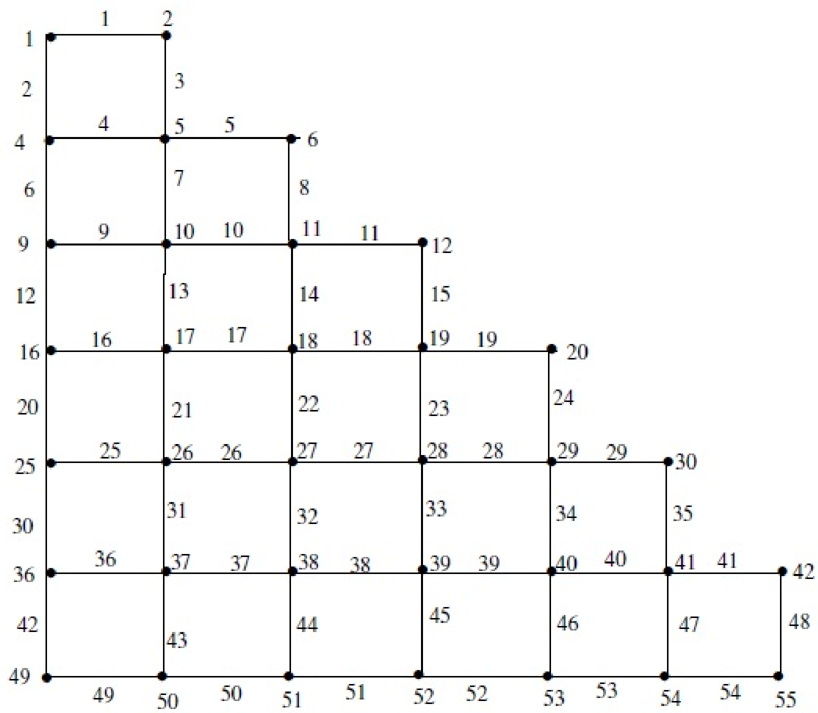


Figure 11. A logarithmic mean labeling of ST_7

for $2 \leq i \leq n$ and $n+3-i \leq j \leq 2n+3-2i$,

$$\begin{aligned}
f(u_{i,j}) &= 2(n+1-i)^2 + 3(n+1-i) - 2(i+j-n-3),\\
f(u_{2,1}) &= 2n^2 + n - 2,\\
f(u_{i,1}) &= 2(n+2-i)^2 + n - i, \text{ for } 3 \leq i \leq n+1\ ,\\
f(u_{i,2n+4-2i}) &= 2(n+2-i)^2 + n + 1 - i, \text{ for } 2 \leq i \leq n+1.
\end{aligned}$$

The induced edge labeling is as follows,

$$f^*(u_{1,j}u_{1,j+1}) = \begin{cases} 2n^2+n+1+2(j-1), 1 \leq j \leq n, \\ 2n^2+3n-2(j-n), \quad n+1 \leq j \leq 2n-1, \end{cases}$$

for $2 \leq i \leq n-1$ and $2 \leq j \leq n+2-i$,

$$f^*(u_{i,j}u_{i,j+1}) = 2(n+1-i)^2 + (n+2-i) + 2(j-2),$$

for $2 \leq i \leq n-1$ and $n+3-i \leq j \leq 2n+2-2i$,

$$f^*(u_{i,j}u_{i,j+1}) = 2(n+1-i)^2 + 3(n+1-i) - 2(i+j-n-2),$$

$$f^*(u_{i,2n+3-2i}u_{i+1,2n+2-2i}) = 2(n+1-i)^2 + (n+1-i), \text{ for } 2 \leq i \leq n,$$

$$f^*(u_{n,2}u_{n,3}) = 4,$$

$$f^*(u_{n+1,1}u_{n+1,2}) = 1,$$

$$f^*(u_{1,1}u_{2,1}) = 2n^2 + n - 1,$$

$$f^*(u_{1,2n}u_{2,2n}) = 2n^2 + n,$$

$$f^*(u_{i,2}u_{i+1,1}) = 2(n+1-i)^2 + n - i, \text{ for } 2 \leq i \leq n,$$

$$f^*(u_{1,j}u_{2,j}) = \begin{cases} 2n^2 - n + 1 + 2(j-2), & 2 \leq j \leq n, \\ 2n^2 + n - 2 - 2(j-n-1), & n+1 \leq j \leq 2n-1, \end{cases}$$

for $2 \leq i \leq n-1$ and $3 \leq j \leq n+2-i$,

$$f^*(u_{i,j}u_{i+1,j-1}) = 2(n+1-i)^2 - (n+2-i) + 2(j-2),$$

for $2 \leq i \leq n-1$ and $n+3-i \leq j \leq 2n+2-2i$,

$$f^*(u_{i,j}u_{i+1,j-1}) = 2(n+1-i)^2 + (n+3-i) - 2(i+j-n-1),$$

$$f^*(u_{i,1}u_{i,2}) = 2(n+1-i)^2 + 3(n+1-i), \text{ for } 2 \leq i \leq n\ ,$$

$$f^*(u_{i,2n+3-2i}u_{i,2n+4-2i}) = 2(n+1-i)^2 + 3(n+1-i) + 1, \text{ for } 2 \leq i \leq n.$$

Hence, f is a logarithmic mean labeling of $2ST_{2n}$. Thus, the graph $2ST_{2n}$ is a logarithmic mean graph, for $n \geq 2$. ■

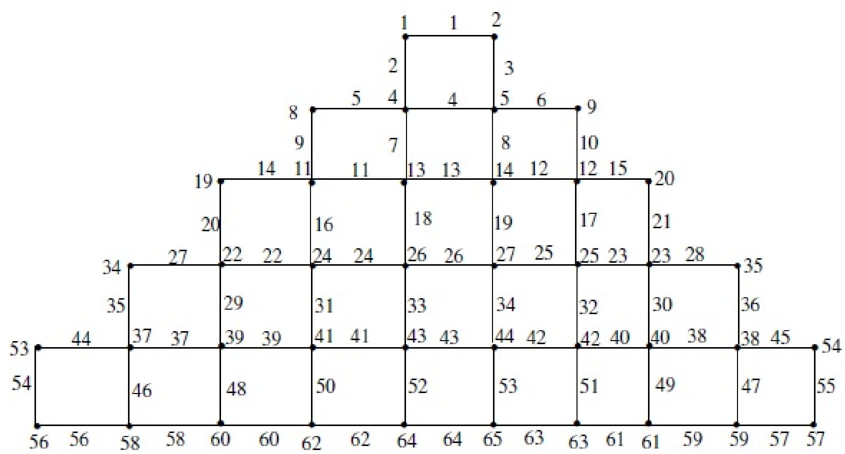


Figure 12. A logarithmic mean labeling of $2ST_{10}$

4. Conclusion

In this paper, I have analyzed the Logarithmic mean labeling of some ladder related graphs such as ladder graph, corona of ladder, triangular ladder graph, corona of triangular ladder, slanting ladder graph, corona of slanting ladder, one sided step graph and two sided step graphs. Herewith I am going to propose the following open problems to the readers for further research work in this.

Open Problem 1.
Find a sub graph of a graph in which the graph is not a Logarithmic mean graph.

Open Problem 2.
Find a necessary condition for a graph to be a Logarithmic mean graph.

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