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Abstract

The dynamical equations of motion for a thin shell wormhole from regular black holes that are supported by Van der Waals (VDW) quintessence equation of state (EoS) are constructed, through cut and -paste technique. The linearized stability of regular wormhole is derived. The presences of unstable and stable static solutions with different value of some parameters are analyzed.

Keywords: Cosmology; General relativity; Singularity; VDW Quintessence; Regular black holes; Astrophysics

MSC 2010 No.: 83C75; 83C57; 83C10; 83F05

1. Introduction

In the general relativity framework, black hole (BH) solutions of Schwarzschild, Reissner-Nordstrom and Kerr-Newman include curvature singularity beyond their event horizons. The extensive understanding of BH requires singularity free solutions. The regular or nonsingular black hole is BHs possessing regular centers. Bardeen (1968) proposed a theoretical method for constructing the preceding regular BH. This type of regular BH has both event and Cauchy horizons, but without a singularity. Later, many regular BH solutions were based on Bardeen's proposal (Ayon-Beato and Garcia (1998), Borde (1994), Bronnikov (2001)). Hayward (2006) analyzed the static regular black holes.

The accelerated expansion of the universe can be explained by the existence of dark energy. There exist a several models represent the dark energy, namely, Cosmological constant, Sahni and Starobinsky (2000), Van der Waals (VDW) Quintessence, Caldwell et al. (1998), Dissipative matter fluid, Sen et al. (2001) , Chaplygin gas, Bento et al. (2002), Phantom energy, Stefanicic (2005), Tracker field, Zlatev et al. (1999), K-essence, malguarti et al. (2003), tachyon matter, Das and Kar (2005). Further, Bronnikov and Fabris (2006) studied regular black holes and phantom wormhole.

Recently, Halilsoy et al. (2014) discussed the regular Hayward black hole. Sharif and Iftikhar (2015) analyzed the scalar thin shell dynamics for a class of regular BHs. Sharif and Mumtaz (2017) investigated the stability of thin shell wormholes from regular ABG black hole. Eid (2019) studied the stability of a regular black holes thin shell wormhole in Reissner-Nordstrom - De Sitter space-time.

Herein, the stability of regular thin shell wormholes (RTSW) supported by VDW quintessence equation of state (EoS) is investigated. The paper is organized in the following format. In Section 2 the dynamics of regular wormhole from black hole is discussed. The linearized stability analysis is given in Section 3. Briefly discussion is given in Section 4. Finally, a general conclusion is providing in Section 5.

2. Dynamics of wormhole from regular black holes

Beato and Garcia (1999) derived a new regular exact black hole solution which comes from the action using nonlinear electrodynamics coupled to general relativity, this action is given by:

$$I = \frac{1}{16\pi} \int \sqrt{-g} \ [R - L(F)] d^4x, \tag{1}$$

where R is the scalar of curvature, $g = det|g_{\mu\nu}|$ and $L(F)$ is the non-linear electrodynamics Lagrangian, $F_{\mu\nu}$ is the Maxwell field tensor, F is the contracted Maxwell scalar ($F = F_{\mu}^{\nu}$). The Beato-Garcia metric, Beato and Garcia (1999), derived from action (1) given by:

$$ds^2 = -H(r)dt^2 + H^{-1}(r)dr^2 + r^2(d\theta^2 + sin^2\theta \ \ d\phi^2), \tag{2}$$

where

$$H(r) = 1 - \frac{2m}{r} + \frac{2m}{r} tanh\left(\frac{q^2}{2mr}\right), \tag{3}$$

where m is the gravitational mass, q is the charge of the black hole. This black hole has two event horizons r_- and r_+ whenever $q \leq 1.05 \ m$. Let the parametric equation of the shell be $r = R(\tau)$, and $R(\tau)$ described the time evolution of the shell. The induced metric on the hypersurface Σ is written as:

$$ds^2 = -d\tau^2 + R^2(\tau)(d\theta^2 + sin^2\theta \ \ d\phi^2), \tag{4}$$

where τ is the proper time on the shell. Applied Darmois – Israel formalism, Israel (1966), to the matter at Σ, the extrinsic curvature is defined by:

$$K_{ij}^{\pm} = -n_{\gamma}^{\pm}\left(\frac{\partial^2 x^{\gamma}}{\partial\xi^i \partial\xi^j} + \Gamma_{\alpha\beta}^{\gamma}\frac{\partial x^{\alpha}}{\partial\xi^i}\frac{\partial x^{\beta}}{\partial\xi^j}\right)\vdots_{\Sigma}, \tag{5}$$

where $n_{\gamma}^{\pm}$ are the unit normal 4-vector. The Lanczos equations are given by:

$$t_{ij} = \frac{-1}{8\pi}\left([K_{ij}] - [K]\ g_{ij}\right), \tag{6}$$

where $[K]$ is the trace of $[K_{ij}] = K_{ij}^{+} - K_{ij}^{-}$ and t_{ij} is the surface stress-energy tensor on the hypersurface Σ, $t_j^i = diag(-\sigma, p_{\theta}, p_{\varphi})$, where p and σ are the pressure and the surface energy density, Sen et al. (2001). The Lanczos equations becomes:

$$\sigma = \frac{-1}{4\pi}[K_{\theta}^{\theta}], \tag{7}$$

$$p = \frac{1}{8\pi}\left([K_{\tau}^{\tau}] + [K_{\theta}^{\theta}]\right). \tag{8}$$

These equations become:

$$\sigma = \frac{-1}{2\pi R}\sqrt{\dot{R}^2 + H(R)}\ , \tag{9}$$

$$p = \frac{2R\ddot{R} + 2\dot{R}^2 + 2H(R) + RH'(R)}{8\pi R\ \sqrt{\dot{R}^2 + H(R)}}, \tag{10}$$

where dot and prime mean derivatives with respect to τ and R, respectively. The van der Waals (VDW) equation of state, Capozziello et al. (2002), is given by:

$$p = \frac{\gamma\sigma}{1-\beta\sigma} - \alpha\sigma^2, \tag{11}$$

where α, β and γ are parameters of the VDW fluid. In the limiting case $(\alpha, \beta) \to 0$, one recovers the dark energy with Chaplygin gas EoS ($p = \gamma\sigma\ , \gamma < -1/3$). Insert equations (9) and (10) into equation (11), the dynamical equation becomes:

$$\pi R\left(2R\ddot{R} + 2\dot{R}^2 + 2H + RH'\right)\left(2\pi R + \beta\sqrt{\dot{R}^2 + H}\right) + 2\left(\dot{R}^2 + H\right) \times \left\{4\gamma\pi^2 R^2 + 2\pi R\alpha\sqrt{\dot{R}^2 + H} + \alpha\beta\left(\dot{R}^2 + H\right)\right\} = 0. \tag{12}$$

It is convenient to define the parameter space of the problem using α, β, γ, q and m as free parameters.

3. Linearized stability analyses

The dynamical equation (12) for the static solution (where $\ddot{R} = \dot{R} = 0$), becomes:

$$\pi R(2H + RH')\left(2\pi R + \beta\sqrt{H}\right) + 2H\left\{4\gamma\pi^2 R^2 + 2\pi R\alpha\sqrt{H} + \alpha\beta H\right\} = 0. \tag{13}$$

The surface energy density and pressure are given in the static case by:

$$\sigma = \frac{-1}{2\pi R}\sqrt{H}\ ,\ \ p = \frac{2H+RH'}{8\pi R\sqrt{H}}. \tag{14}$$

The conservation equation with equations (6, 14) can be defined as:

$$\frac{d}{d\tau}A\sigma + p\frac{dA}{d\tau} = 0\ , \tag{15}$$

where $A = 4\pi R^2$ is the area of the wormhole throat. This equation describes the continuity equation and can be written in the form:

$$\frac{d\sigma}{d\tau} = -\frac{2}{R}(\sigma + p)\frac{dR}{d\tau}\ . \tag{16}$$

And will take the following form:

$$R\sigma' = \ -2(\sigma + p). \tag{17}$$

From equation (9), the dynamical equation of motion of the thin shell wormhole, becomes:

$$\dot{R}^2 + \ V(R) = 0\ \ , \tag{18}$$

where $V(R)$ is known as the effective potential function given by:

$$V(R) = H(R) - 4\pi^2 R^2\sigma^2, \tag{19}$$

Differentiating this equation:

$$V'(R) = H'(R) + 8\pi^2 R\sigma(\sigma + 2p). \tag{20}$$

Taking the first derivative with respect to R of equation (11) and using equation (17) to get:

$$\sigma' + 2p' = \ \sigma'\left\{1 + \frac{2}{1-\beta\sigma}\ [\gamma - 2\alpha\sigma + \beta(p + 3\alpha\sigma^2)]\right\}. \tag{21}$$

The Taylor series expansion of $V(R)$ up to second order around $R_\circ$, is given by:

$$V(R) = \sum_{n=0}^{2} b_n(R - R_\circ)^n\ \ , \qquad b_n = \frac{V^n(R_\circ)}{n!}\ . \tag{22}$$

The second derivative of $V(R)$ is given by:

$$V''(R) = H'' - 8\pi^2(\sigma + 2p)^2 - 16\pi^2\sigma(\sigma + p)(1 + 2\chi^2), \tag{23}$$

where $\chi^2 = p'/\sigma'$ is the square of sound velocity. The stability of static solutions at $R = R_\circ$ requires $V(R_\circ) = 0$ and $V'(R_\circ) = 0$, while $V''(R_\circ)$ becomes:

$$V''(R) = H'' + \frac{H'^2}{2H}\left[-1 + \frac{\beta\sqrt{H}}{2\pi R\left(1+\frac{\beta\sqrt{H}}{2\pi R}\right)}\right] + \frac{H'}{R}\left[1 + \frac{4\gamma\pi^2R^2+4\pi R\alpha\sqrt{H}+3\alpha\beta H}{2\pi^2R^2\left(1+\frac{\beta\sqrt{H}}{2\pi R}\right)}\right]$$
$$-\frac{2H}{R^2}\left[1 + \frac{4\gamma\pi^2R^2+4\pi R\alpha\sqrt{H}+3\alpha\beta H+\pi R\beta\sqrt{H}}{2\pi^2R^2\left(1+\frac{\beta\sqrt{H}}{2\pi R}\right)}\right]. \tag{24}$$

The surface energy density and pressure (14) can be written in the form:

$$\sigma_\circ = \frac{-1}{2\pi R_\circ}\sqrt{1 - \frac{2m}{R_\circ} + \frac{2m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)}\ , \tag{25}$$

$$p_\circ = \frac{1}{4\pi R_\circ}\ \frac{1-\frac{2m}{R_\circ}+\frac{2m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)-\frac{q^2}{{R_\circ}^2}\ sech^2\left(\frac{q^2}{2mR_\circ}\right)}{\sqrt{1-\frac{2m}{R_\circ}+\frac{2m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)}}\ . \tag{26}$$

Using equations (13) and (24), the dynamical equation and the second derivative of the effective potential, become:

$$4\gamma(\pi R_\circ)^2\left[1 - \frac{2m}{R_\circ} + \frac{2m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)\right] + 2(\pi R_\circ)^2\left(1 - \frac{q^2}{{R_\circ}^2}\right) + 2\alpha\beta\left(1 - \frac{2m}{R_\circ}\right)$$
$$\times\left[1 - \frac{2m}{R_\circ} + \frac{4m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)\right] + \left(2(\pi q)^2 + 2\left(\frac{2m}{R_\circ}\right)^2\alpha\beta\right)tanh^2\left(\frac{q^2}{2mR_\circ}\right)$$
$$+\sqrt{1 - \frac{2m}{R_\circ} + \frac{2m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)}\ \left\{2\pi\beta R_\circ\left[1 - \frac{m}{R_\circ} + \frac{m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)\right] +\ \Gamma\right\} = 0, \tag{27}$$

where

$$\Gamma = 4\pi\alpha R_\circ\left[1 - \frac{2m}{R_\circ} + \frac{2m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)\right] - \pi\beta\frac{q^2}{R_\circ}\ \ sech^2\left(\frac{q^2}{2mR_\circ}\right),$$

and

$$V''(R_\circ) = \frac{4m}{{R_\circ}^3}\left[tanh\left(\frac{q^2}{2mR_\circ}\right) - 1\right] + \frac{4q^2}{{R_\circ}^4}\left[1 - \frac{q^2}{4mR_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)\right]sech^2\left(\frac{q^2}{2mR_\circ}\right)$$
$$-\frac{(\pi R_\circ)^2}{\Phi L^2}\Theta^2 + \frac{\Theta}{\Phi R_\circ}(\Phi+\Psi) - \frac{2L^2}{\Phi {R_\circ}^2}(\Phi+\Psi+2\beta L R_\circ), \tag{28}$$

where

$$\Phi = 2(\pi R_\circ)^2 + \pi\beta L R_\circ, \quad \Theta = \frac{2m}{{R_\circ}^2} - \frac{2m}{{R_\circ}^2}tanh\left(\frac{q^2}{2mR_\circ}\right) - \frac{q^2}{{R_\circ}^3}sech^2\left(\frac{q^2}{2mR_\circ}\right),$$

$$\Psi = 4\gamma(\pi R_\circ)^2 + 4\pi\alpha L R_\circ + 3\alpha\beta L^2\ \ ,\quad L = \sqrt{1 - \frac{2m}{R_\circ} + \frac{2m}{R_\circ}tanh\left(\frac{q^2}{2mR_\circ}\right)}\ ,$$

So that, $\dot{R}^2 = -\frac{1}{2}V''(R_\circ)(R - R_\circ)^2 + O[(R - R_\circ)^3]$. The regular TSW is stable under radial perturbations if $V''(R_\circ) > 0$, and unstable when $V''(R_\circ) < 0$. From equation (23), the square of sound velocity χ^2 is given by:

$$\chi^2 < \frac{R_\circ^2\left(2H'' - H^{-1}H'^2\right)}{4\left(2H - R_\circ H'\right)} - \frac{1}{2} \ , \tag{29}$$

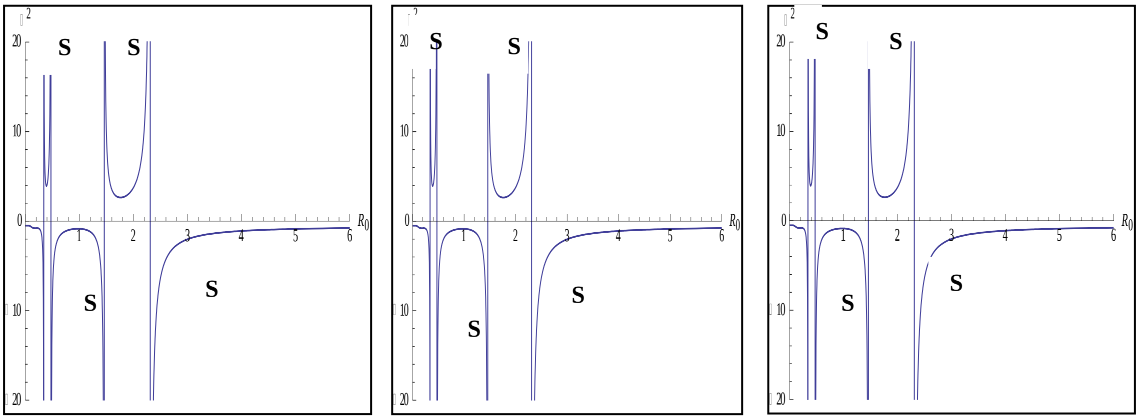
for $V''(R_\circ) > 0$ and

$$\chi^2 = -\frac{1}{2} + \frac{R_\circ^2}{2(2L^2 - \Theta R_\circ)}\left\{-\frac{\Theta^2}{2L^2} + \frac{4m}{R_\circ^3}\ \left[-1 + tanh\left(\frac{q^2}{2mR_\circ}\right)\right] + \Upsilon\right\}, \tag{30}$$

where

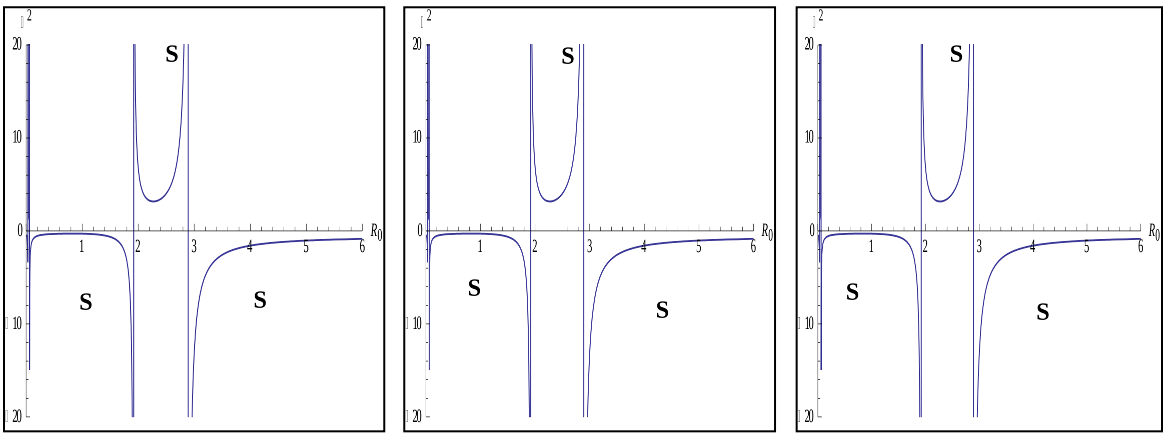
$$\Upsilon = \frac{4q^2}{R_\circ^4}\ \left[1 - \frac{q^2}{4mR_\circ} tanh\left(\frac{q^2}{2mR_\circ}\right)\right] sech^2\left(\frac{q^2}{2mR_\circ}\right),$$

for $V''(R_\circ) = 0$. The variation of χ^2 versus $R_\circ$ is plotted in figures (1- 3) with different values of q, γ, β, α and m as free parameters.



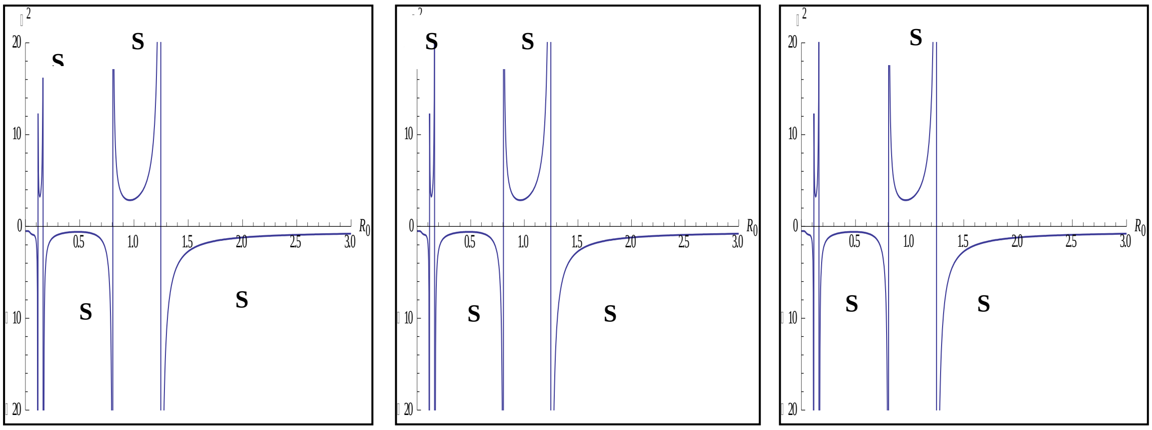
(a) (b) (c)

Figure 1. Stability regions RTSW corresponding to $\alpha = 1, \beta = 1, q = 0.9$ and $m = 1$ with different values of $\boldsymbol{\gamma}.\boldsymbol{\gamma} = \mathbf{1}$, (b) $\boldsymbol{\gamma} = \mathbf{0.4}$, (c) $\boldsymbol{\gamma} = \mathbf{-0.4}$ **;** where S denoted to the stability regions



(a) (b) (c)

Figure 2. Stability regions RTSW corresponding to $\alpha = 1, \beta = 1, q = 0.4$ and $m = 1$ with different values of $\boldsymbol{\gamma}$. $\boldsymbol{\gamma} = \mathbf{1}$, (b) $\boldsymbol{\gamma} = \mathbf{0.4}$, (c) $\boldsymbol{\gamma} = \mathbf{-0.4}$; where S denoted to the stability regions



(a) (b) (c)

Figure 3. Stability regions RTSW corresponding to $\alpha = 1, \beta = 1, q = 0.4$ and $m = 0.5$ with different values of $\boldsymbol{\gamma}$. $\boldsymbol{\gamma = 1}$, (b) $\boldsymbol{\gamma = 0.4}$, (c) $\boldsymbol{\gamma = -0.4}$; where S denoted to the stability regions

4. Discussion

The stability regions have been plotted in the form of parameter χ^2versus $R_\circ$. Figures 1-3 show the stability regions with different values of free parameters in Van der Waals (VDW) quintessence EoS. Therefore, the stability region happens when the charge |q| is slightly smaller than the mass. This result is similar to the result of Eid (2019).

5. Conclusions

The TSW dynamics from regular BH with Van der Waals (VDW) quintessence EoS is derived, using the cut and paste technique. Such kind of EoS can describe the cosmic expansion of the universe without the presence of exotic fluid. Also, Van der Waals (VDW) quintessence fluids can reduce the usage of exotic matter.

The stability analysis of RTSW has been carried out about the static equilibrium solution. From the stability conditions (29-30) the regular TSW is stable if $V''(R_\circ) > 0$, while for $V''(R_\circ) < 0$, the static solution is unstable. The output of RTSW can be either stable or unstable, depending on the mass m, the parameters $q, \gamma, \beta, \alpha$and the initial position $R_\circ$ of the dynamical shell.

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