Applications and Applied Mathematics: An International Journal (AAM)

Volume 15 | Issue 1 | Article 1

6-2020

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Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 15, Issue 1 (June 2020), pp. 1 – 12

Performance Analysis of an $M/M/1$ Queue with N-policy Interrupted Closedown Preventive Maintenance Balking and Feedback

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Received: September 2, 2019; Accepted: April 27, 2020

Abstract

This paper investigates the transient and stationary behavior of a $M/M/1$ queueing model with N-policy, interrupted closedown, balking, feedback and preventive maintenance. The server stays dormant (off state) until N customers accumulate in the queue and then starts an exhaustive service (on state). After the service, each customer may either leave the system or get immediate feedback. When the system becomes empty, the server resumes closedown. If any arrival occurs before the completion of closedown time, the closedown work of the server is interrupted and starts the busy period in an exhaustive manner. If no arrival occurs during the closedown time, the server commences preventive maintenance work. When this period ends, the server moves to the idle state and waits N accumulate for service. When the N^{th} one enters the queue, the server starts the service. The customers may either join the queue or balk when the size of the system is less than N and the server is in off state. The transient and stationary system size probabilities of the proposed model are derived by the method of generating function. Some system performance indices are computed and the numerical simulations are also presented.

Keywords: N-policy queue; Interrupted closedown; Preventive maintenance; Balking; Feedback

MSC 2010 No.: 60K25, 90B22, 68M20

1. Introduction

To closedown the system when it becomes empty plays a key role in various real life situations as they support economically to minimize the expense of an organization. Only few works are investigated in literature related to Markovian queueing models with closedown times and preventive maintenance. As far as the production systems concern, preventive maintenance of machines is very important in the aspect of reducing the wastage of time due to frequent failures (refer Kumar et al. (2015)).

In many practical situations, the service will not be started whenever an arrival occurs. The server fix a threshold value (say, 'N') to start the service and waits idle till 'N' to reach (refer Jain et al. (2016)). During this time, new arrivals are getting discouraged and may decide not to join (*balk*) the queue (refer Haight (1957)). When a customer is dissatisfied with the quality of service that he received, the necessity of immediate *feedback* is unavoidable (refer Takacs (1963)).

The time dependent distribution of the queue length in a single server queueing model was derived by Parthasarathy (1987). Kumar et al. (1993) obtained the probabilities of system size under transient state for a Markovian queueing model with balking using generating function method. Ke et al. (2010) studied a N-policy queue where the customers arrive in batches and the server can avail a maximum of J vacations. They obtained the probability generating function of queue size at an arbitrary epoch for the steady state case.

Kumar et al. (2015) considered a Markovian queueing model where the server undergoes closedown and then maintenance whenever the system becomes empty. They derived the transient system size probabilities and some system measures such as asymptotic behavior of various system state probabilities, average system size, average workload, etc. Jain et al. (2016) studied a multi component machine system with N-policy where the repairman starts the repair when there were N number of the repairable items accumulated in the system. They solved the system state equations by Range-Kutta method. Haight (1957) introduced the balking behavior of customers in queueing models.

Takacs (1963) introduced the queue with feedback customers where each customer either immediately joins the queue for another service or leaves the system, after the service completion. He obtained the steady state queue size distribution and distribution function of sojourn time of a customer in the system. Azhagappan and Deepa (2019) studied a queueing model with single vacation, feedback nature of customers, interrupted closedown time and control of admission during vacation. They obtained the transient system size probabilities and performance measures such as time-dependent mean as well as variance for that model.

Al-Seedy et al. (2009) derived the system size probabilities under transient state for an $M/M/c$ queueing model with balking and reneging using the method of generating function. Kumar and Sharma (2014) studied a finite capacity multi server queueing model with feedback, balking, reneging and retention of reneged customers. They obtained the steady state system size probabilities for that model. Mohanty et al. (1993) analyzed the transient behavior of a finite birth-death process with an application. Montazer-Haghighi et al. (1986) investigated a multiserver Markovian queueing system with balking and reneging. Haghighi and Mishev (2014) studied potential applications of various queueing models in industry and business. Schwarz et al. (2016) performed a survey on time-dependent queueing systems. The *novelty* of this research work is in the usage of the rare parameter "interrupted closedowm" along with the parameters "N-policy" and "preventive maintenance". None of the basic literatures provide research work related to N-policy together with interrupted closedown time and preventive maintenance. This induces us to carry out this research work.

The *motivation* of this research work is in the following points: This paper analyzes the most significant time-dependent system size probabilities for the proposed model. The system measures such as mean, variance and empty state probability are derived for the transient case. This model has potential *applications* in the fields related to the manufacturing systems, service systems, etc. The *probability generating function method* is a common tool to find the steady state as well as transient system size probabilities for many queueing models. This is because of its simplest approach without any complications.

The remaining sections are as follows. The $M/M/1$ queueing model with N-policy, interrupted closedown, balking, feedback and preventive maintenance is described and the transient probabilities are derived in Section 2. The system performance measures such as time-dependent mean, variance, probabilities of closedown, maintenance and empty state are also obtained in Section 3. The stationary system size probabilities are deduced from their transient counterparts in Section 4. Numerical simulations of the proposed model are presented in Section 5. The conclusion and future scope are provided in Section 6.

2. Model Description

We consider a single server Markovian queueing model with N-policy, interrupted closedown, balking, feedback and preventive maintenance. Customers arrive at the rate of α which is exponentially distributed. The arriving customers may either join the queue with a probability θ or balk with a probability $1-\theta$ when the size of the system is less than N and the server is in off state. It is assumed that $\alpha_0 = \theta\alpha$. The server stays dormant until N customers accumulate in the queue and then starts an exhaustive service where the service time follows exponential distribution with the rate of β. After the service, each customer may either leave the system with a probability σ or get immediate feedback with a probability $1-\sigma$.

When the system becomes empty, the server resumes closedown work exponentially at the rate γ. If any customer arrives before the completion of closedown time, the closedown of the server is interrupted and starts a busy period exhaustively. On the other hand, if no customer arrives during closedown time, the server commences preventive maintenance work which follows exponential distribution with parameter ω. At the maintenance completion moment, it moves to the idle state and waits N accumulate for service. When the N^{th} one enters the queue, the server commences the service. The service of customers is based on first come first service. Assume that inter-arrival, ser-

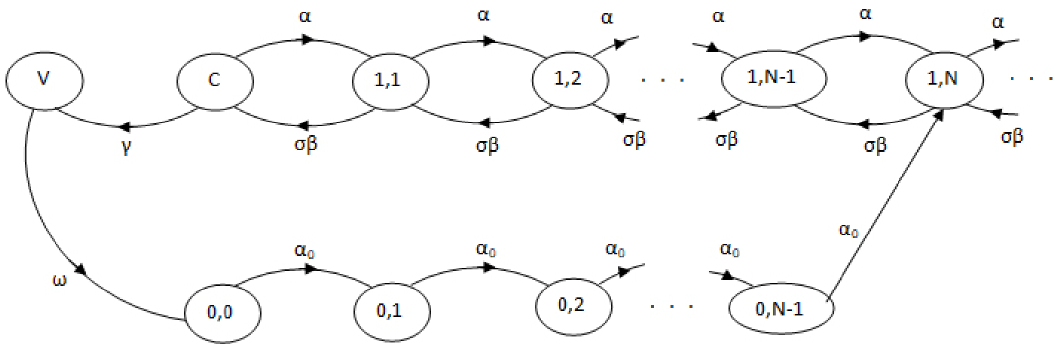

Figure 1. Transition Diagram

vice, closedown and maintenance times are all independent. Figure 1 shows the transition diagram of the present model.

Let $\{\Omega(t), t \geq 0\}$ and $\zeta(t)$ are respectively the size of the system and the server state at time t. Let

$$\zeta(t) = \begin{cases} 0, & \text{off state of the server,} \\ 1, & \text{on state of the server.} \end{cases}$$

Then $\{\zeta(t), \Omega(t),\ t \geq 0\}$ is a Markov process with state space $S = \{V\} \cup \{C\} \cup \{0, j : 0 \leq j \leq N-1\} \cup \{1, j : j \geq 1\}$. Let

$$\begin{aligned} \pi_{1,n}(t) =& P\left\{\Omega(t) = n, \zeta(t) = 1\right\},\ n \geq 1, \\ \pi_{0,n}(t) =& P\left\{\Omega(t) = n, \zeta(t) = 0\right\},\ 0 \leq n \leq N-1, \\ \pi_C(t) =& P\left\{\text{closedown state}\right\}, \\ \pi_V(t) =& P\left\{\text{maintenance state}\right\}. \end{aligned}$$

Then the transient system state equations are

$$\pi'_V(t) = -\omega\pi_V(t) + \gamma\pi_C(t), \tag{1}$$

$$\pi'_C(t) = -(\gamma + \alpha)\pi_C(t) + \sigma\beta\pi_{1,1}(t), \tag{2}$$

$$\pi'_{1,1}(t) = -(\alpha + \sigma\beta)\pi_{1,1}(t) + \sigma\beta\pi_{1,2}(t) + \alpha\pi_C(t), \tag{3}$$

$$\pi'_{1,n}(t) = -(\alpha + \sigma\beta)\pi_{1,n}(t) + \alpha\pi_{1,n-1}(t) + \sigma\beta\pi_{1,n+1}(t),\ n \geq 2,\ n \neq N, \tag{4}$$

$$\pi'_{1,N}(t) = -(\alpha + \sigma\beta)\pi_{1,N}(t) + \alpha\pi_{1,N-1}(t) + \sigma\beta\pi_{1,N+1}(t) + \alpha_0\pi_{0,N-1}(t), \tag{5}$$

$$\pi'_{0,0}(t) = -\alpha_0\pi_{0,0}(t) + \omega\pi_V(t), \tag{6}$$

$$\pi'_{0,n}(t) = -\alpha_0\pi_{0,n}(t) + \alpha_0\pi_{0,n-1}(t),\ 1 \leq n \leq N-1, \tag{7}$$

with $\pi_{0,0}(0) = 1$.

2.1. Transient probabilities

The transient probabilities are derived for the proposed model in this section.

Expression for $\pi_{1,n}(t)$

Define the generating function as

$$G(z,t)=\sum_{n=1}^{\infty}\pi_{1,n}(t)z^{n}. \tag{8}$$

Applying (8) in Equations (3), (4) and (5), we obtain

$$\frac{\partial G(z,t)}{\partial t}=\left[-(\alpha+\sigma\beta)+\frac{\sigma\beta}{z}+\alpha z\right]G(z,t)-\sigma\beta\pi_{1,1}(t)+\alpha_0 z^{N}\pi_{0,N-1}(t)+\alpha\pi_C(t)z.$$

Solving the above partial differential equation, we obtain

$$\begin{aligned}G(z,t)=\alpha_0\int_0^t &\pi_{0,N-1}(u)z^{N}e^{-(\alpha+\sigma\beta)(t-u)}e^{-\left(\alpha z+\frac{\sigma\beta}{z}\right)(t-u)}du\\ -\sigma\beta\int_0^t &\pi_{1,1}(u)e^{-(\alpha+\sigma\beta)(t-u)}e^{-(\alpha z+\frac{\sigma\beta}{z})(t-u)}du\\ +\alpha\int_0^t &\pi_C(u)ze^{-(\alpha+\sigma\beta)(t-u)}e^{-(\alpha z+\frac{\sigma\beta}{z})(t-u)}du.\end{aligned} \tag{9}$$

Let us assume that $h=2\sqrt{\alpha\sigma\beta},\ r=\sqrt{\frac{\alpha}{\sigma\beta}}$. Then

$$e^{-(\alpha z+\frac{\sigma\beta}{z})t}=\sum_{n=-\infty}^{\infty}(rz)^{n}I_n(ht), \tag{10}$$

where $I_n(t)$ is the modified Bessel function of the first kind of order n.

Using (10) in (9) and comparing the coefficients of z^n, for $n\geq 1$, we get

$$\begin{aligned}\pi_{1,n}(t)=&\alpha_0\int_0^t \pi_{0,N-1}(u)e^{-(\alpha+\sigma\beta)(t-u)}r^{n-N}I_{n-N}(h(t-u))du\\ &-\sigma\beta\int_0^t \pi_{1,1}(u)e^{-(\alpha+\sigma\beta)(t-u)}r^{n}I_n(h(t-u))du\\ &+\alpha\int_0^t \pi_C(u)e^{-(\alpha+\sigma\beta)(t-u)}r^{n-1}I_{n-1}(h(t-u))du.\end{aligned} \tag{11}$$

The above expression holds for $n = -1, -2, -3, \ldots$. Using $I_{-n}(y) = I_n(y)$, for $n \geq 1$, we have

$$\begin{aligned} 0 =& \alpha_0 \int_0^t \pi_{0,N-1}(u) e^{-(\alpha+\sigma\beta)(t-u)} r^{-n-N} I_{n+N}(h(t-u)) du \\ & - \sigma\beta \int_0^t \pi_{1,1}(u) e^{-(\alpha+\sigma\beta)(t-u)} r^{-n} I_n(h(t-u)) du \\ & + \alpha \int_0^t \pi_C(u) e^{-(\alpha+\sigma\beta)(t-u)} r^{-n-1} I_{n+1}(h(t-u)) du. \end{aligned} \tag{12}$$

From (11) and (12), for $n \geq 1$, we obtain

$$\begin{aligned} \pi_{1,n}(t) =& \alpha_0 \int_0^t \pi_{0,N-1}(u) e^{-(\alpha+\sigma\beta)(t-u)} r^{n-N} [I_{n-N}(h(t-u)) - I_{n+N}(h(t-u))] du \\ & + \alpha \int_0^t \pi_C(u) e^{-(\alpha+\sigma\beta)(t-u)} r^{n-1} \left[I_{n-1}(h(t-u)) - I_{n+1}(h(t-u))\right] du. \end{aligned} \tag{13}$$

Thus, $\pi_{1,n}(t)$, for $n \geq 1$ are obtained in terms of $\pi_{0,N-1}(t)$ and $\pi_C(t)$.

Expression for $\pi_{0,n}(t)$ and $\pi_{0,0}(t)$

Taking Laplace transform of (7), we get

$$\hat{\pi}_{0,n}(s) = \left(\frac{\alpha_0}{s+\alpha_0}\right)^n \hat{\pi}_{0,0}(s),\ 1 \leq n \leq N-1. \tag{14}$$

Laplace inversion of (14) yields,

$$\pi_{0,n}(t) = \alpha_0^n e^{-\alpha_0 t} \frac{t^{n-1}}{(n-1)!} * \pi_{0,0}(t),\ 1 \leq n \leq N-1. \tag{15}$$

Laplace transform of (6) gives,

$$\hat{\pi}_{0,0}(s) = \left(\frac{1}{s+\alpha_0}\right)\left[1 + \omega \hat{\pi}_V(s)\right]. \tag{16}$$

Laplace inversion of (16) leads to

$$\pi_{0,0}(t) = e^{-\alpha_0 t} + \omega e^{-\alpha_0 t} * \pi_V(t). \tag{17}$$

Thus, $\pi_{0,n}(t)$, for $1 \leq n \leq N-1$, are obtained in terms of $\pi_{0,0}(t)$ whereas $\pi_{0,0}(t)$ is expressed as a function of $\pi_V(t)$.

Expression for $\pi_C(t)$ and $\pi_V(t)$

Taking Laplace transform of (2), we get

$$\hat{\pi}_C(s) = \frac{\sigma\beta}{(s+\gamma+\alpha)}\hat{\pi}_{1,1}(s). \tag{18}$$

On inverse Laplace transform of (18), we obtain

$$\pi_C(t) = \sigma\beta e^{-(\gamma+\alpha)t} * \pi_{1,1}(t). \tag{19}$$

Taking Laplace transform of (1) and using (18), we get

$$\hat{\pi}_V(s) = \left(\frac{\gamma}{s+\omega}\right)\left(\frac{\sigma\beta}{s+\gamma+\alpha}\right)\hat{\pi}_{1,1}(s). \tag{20}$$

On taking inverse Laplace transform of (20), we obtain

$$\pi_V(t) = \gamma\sigma\beta e^{-\omega t} * e^{-(\gamma+\alpha)t} * \pi_{1,1}(t). \tag{21}$$

Thus, $\pi_C(t)$ and $\pi_V(t)$ are all expressed in terms of $\pi_{1,1}(t)$.

Expression for $\pi_{1,1}(t)$

Substituting $n = 1$ in (13) and using (14), we get after some algebra,

$$\hat{\pi}_{1,1}(s) = \hat{A}(s)\sum_{k=0}^{\infty}(\hat{B}(s))^k, \tag{22}$$

where

$$\hat{B}(s) = \frac{\omega\gamma\sigma\beta\hat{A}(s)}{(s+\omega)(s+\gamma+\alpha)} + \frac{r\sigma\beta}{(s+\gamma+\alpha)},$$

$$\hat{A}(s) = \frac{\alpha_0^N r^{2-N}}{\alpha}\left(\frac{p-\sqrt{p^2-h^2}}{h}\right)\frac{1}{(s+\alpha_0)^N}$$

and $p = s + \alpha + \sigma\beta$.

On Laplace inversion of (22), we get

$$\pi_{1,1}(t) = A(t) * \sum_{k=0}^{\infty}(B(t))^{*k}, \tag{23}$$

where

$$B(t) = \omega\gamma\sigma\beta e^{-\omega t} * e^{-(\gamma+\alpha)t} * A(t) + r\sigma\beta e^{-(\gamma+\alpha)t},$$

$$A(t) = \alpha_0^N r^{1-N} e^{-(\alpha+\sigma\beta)t}[I_{N-1}(ht) - I_{N+1}(ht)] * e^{-\alpha_0 t}\frac{t^{N-1}}{(N-1)!}.$$

Thus, the expression for $\pi_{1,1}(t)$ is computed in an explicit manner. The expressions given in (13), (15), (17), (19), (21) and (23) together represent the transient system size probabilities for the proposed model.

2.2. Special case

When $N = 1, \sigma = 1, \theta = 1$, the equations (13), (19) and (21) become

$$\begin{aligned}\pi_{1,n}(t) =& r^{n-1}e^{-(\alpha+\beta)t}\left[I_{n-1}(ht) - I_{n+1}(ht)\right]\\ &+ \gamma\beta r^{n-1}e^{-(\alpha+\beta)t}\left[I_{n-1}(ht) - I_{n+1}(ht)\right] * e^{-\gamma t} * \pi_{1,1}(t),\\ \pi_C(t) =& \beta e^{-\gamma t} * \pi_{1,1}(t),\\ \pi_V(t) =& \gamma\beta e^{-\omega t} * e^{-\gamma t} * \pi_{1,1}(t),\end{aligned}$$

which on simplification coincide with (2.13), (2.26) and (2.25) respectively in Kumar et al. (2015) if $\gamma = \xi$ and $\omega = \eta$.

3. System measures

The system measures like the expected system size, variance, etc. are computed for the transient case in this section.

(I) The average $m(t)$ of $\{\Omega(t)\}$, at time t is

$$m(t) = \alpha_0 \sum_{n=1}^{N-1} \int_0^t \pi_{0,n-1}(u)du + \sum_{n\geq 1}(\alpha - \sigma\beta) \int_0^t \pi_{1,n}(u)du + \alpha \int_0^t \pi_C(u)du.$$

(II) The variance, $v(t)$, of $\{\Omega(t)\}$ at time t is

$$v(t) = q(t) - (m(t))^2,$$

where

$$q(t) = \alpha_0 \sum_{n=1}^{N-1}(2n-1) \int_0^t \pi_{0,n-1}(u)du + \sum_{n\geq 1}\left[2(\alpha - \sigma\beta)n + (\alpha + \sigma\beta)\right] \int_0^t \pi_{1,n}(u)du + \alpha \int_0^t \pi_C(u)du.$$

(III) The probability of closedown, $\pi_C(t)$ is

$$\pi_C(t) = \sigma\beta e^{-(\gamma+\alpha)t} * A(t) * \sum_{k=0}^{\infty}(B(t))^{*k}.$$

(IV) The probability of maintenance, $\pi_V(t)$ is

$$\pi_V(t) = \gamma\sigma\beta e^{-\omega t} * e^{-(\gamma+\alpha)t} * A(t) * \sum_{k=0}^{\infty}(B(t))^{*k}.$$

(V) The empty state probability, $\pi_{0,0}(t)$ is

$$\pi_{0,0}(t) = e^{-\alpha_0 t} + \omega\gamma\sigma\beta e^{-\alpha_0 t} * e^{-\omega t} * e^{-(\gamma+\alpha)t} * A(t) * \sum_{k=0}^{\infty}(B(t))^{*k}.$$

4. Stationary probabilities

The steady state system size probabilities are obtained from the time-dependent counterparts in this section.

$$\pi_{1,n} = \lim_{s\to 0} s\hat{\pi}_{1,n}(s) = r^n \pi_C \left(\frac{p_1 - \sqrt{p_1^2 - h^2}}{h}\right)^n + \alpha_0 \pi_{0,N-1} r^{n-N}$$
$$\times \frac{1}{2\sqrt{p_1^2 - h^2}} \left\{ \left(\frac{p_1 - \sqrt{p_1^2 - h^2}}{h}\right)^{n-N} - \left(\frac{p_1 - \sqrt{p_1^2 - h^2}}{h}\right)^{n+N} \right\},\ n \geq 2,$$
$$\pi_{0,n} = \lim_{s\to 0} s\hat{\pi}_{0,n}(s) = \pi_{0,0},\ 1 \leq n \leq N-1,$$
$$\pi_{0,0} = \lim_{s\to 0} s\hat{\pi}_{0,0}(s) = \frac{\omega}{\alpha_0}\pi_V,$$
$$\pi_V = \lim_{s\to 0} s\hat{\pi}_V(s) = \frac{\gamma}{\omega}\pi_C,$$
$$\pi_C = \lim_{s\to 0} s\hat{\pi}_C(s) = \frac{\sigma\beta}{\gamma+\alpha}\pi_{1,1},$$
$$\pi_{1,1} = \lim_{s\to 0} s\hat{\pi}_{1,1}(s) = \frac{r^{2-N}}{\alpha}\left(\frac{p_1 - \sqrt{p_1^2 - h^2}}{h}\right)^N$$
$$\times \left[1 - \frac{r\sigma\beta}{\gamma+\alpha} - \frac{\gamma\sigma\beta r^{2-N}}{\alpha(\gamma+\alpha)}\left(\frac{p_1 - \sqrt{p_1^2 - h^2}}{h}\right)^N\right]^{-1},$$

where $p_1 = \alpha + \sigma\beta$.

5. Numerical illustration

The numerical simulation is carried over for the model under consideration using MATLAB software in this section.

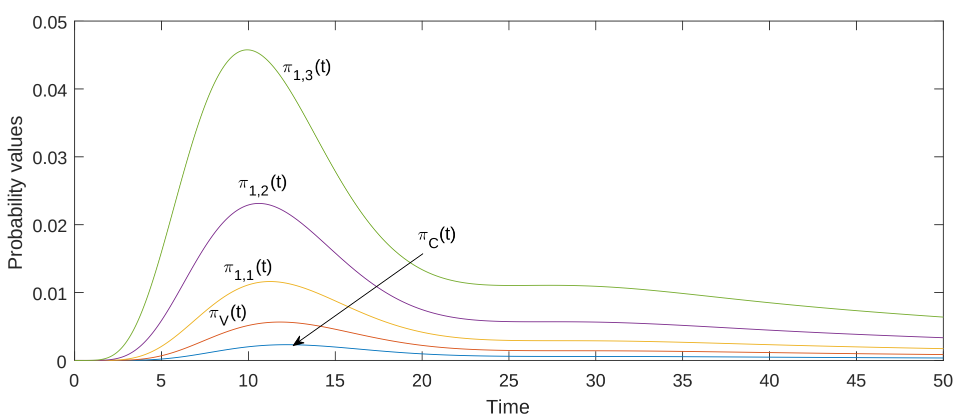

Figure 2. Transient probabilities for the on state of the server

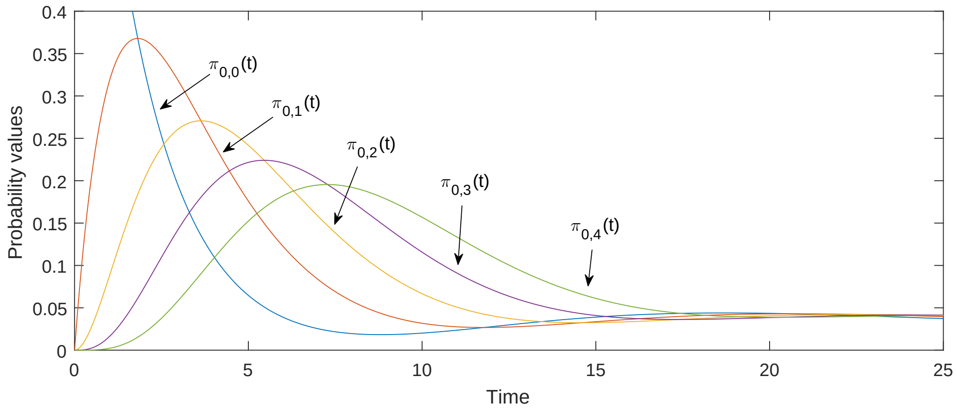

Figure 3. Transient probabilities for the off state of the server

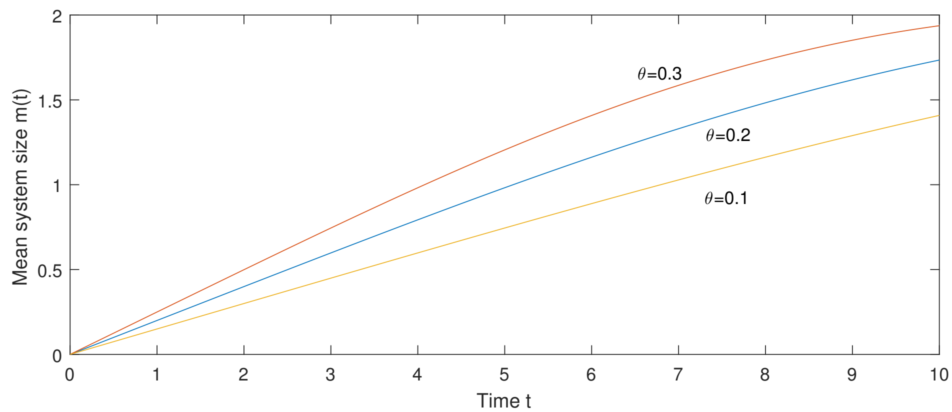

Figure 4. Mean system size Vs θ values

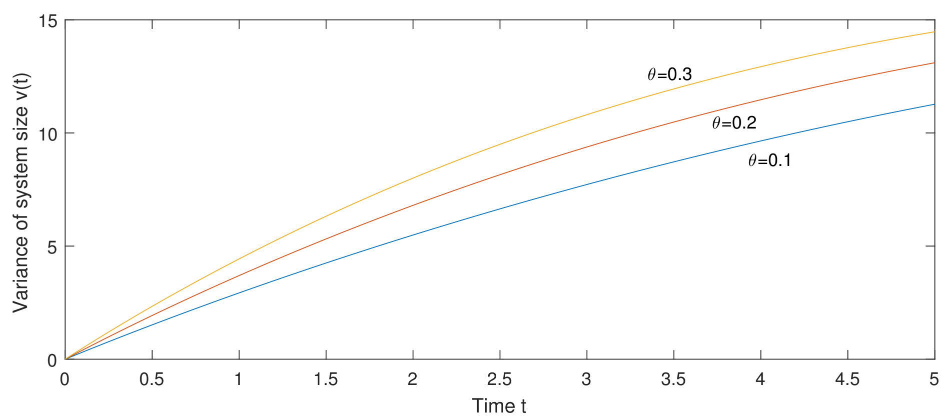

Figure 5. Variance of system size Vs θ values

Figures 2 and 3 give the probability curves corresponding to the on and off state of the server respectively for $\theta = 0.5, \alpha = 0.75, \sigma = 0.5, \beta = 1, \gamma = 0.5, \omega = 0.2$ and $N = 5$. All the probability curves, except $\pi_{0,0}(t)$, increase initially and reach steady state in the long run. Figures 4 and 5 present that the mean and variance of number of customers in the system increase whenever the θ increases.

6. Conclusion and future scope

The analysis of a single server Markovian queueing model with N-policy, interrupted closedown, balking, feedback and preventive maintenance is carried out. Using the method of generating function, the system size probabilities are derived under transient state. Various performance measures like time-dependent mean, variance, probabilities of closedown, maintenance and empty state are also obtained. Numerical illustrations are provided to validate the analytical results. In future, this queueing model may be extended into a multi-server queueing model with N-policy, interrupted closedown, balking, feedback and preventive maintenance.

Acknowledgment:

The authors wish to thank the anonymous referees and Professor Aliakbar Montazer Haghighi for their careful review and valuable suggestions that led to considerable improvement in the presentation of this paper.

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