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The Effects of Variability in Demand and Time Parameters for Multi-Item, Multi-Echelon, Multi-Indenture Reparable Inventory Systems

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THE EFFECTS OF VARIABILITY IN DEMAND AND TIME PARAMETERS FOR MULTI-ITEM, MULTI-ECHELON, MULTI-INDENTURE REPARABLE INVENTORY SYSTEMS

THESIS

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Abstract
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Roberto Carlos Borges de Abreu

Table of Contents

Page

List of Tables

Page

List of Figures

Abstract

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THE EFFECTS OF VARIABILITY IN DEMAND AND TIME PARAMETERS FOR MULTI-ITEM, MULTI-ECHELON, MULTI-INDENTURE REPARABLE INVENTORY SYSTEMS

I. Introduction

Chapter Overview

The main purpose of this chapter is to describe the problem that will be analyzed throughout the research. The research and investigative questions related to the topic will also be presented. The scope of the research, its limitations, needs and other main issues will be addressed as well.

The research is focused on introducing a new approach for reparable items management, which seeks to take into account the effects of variability in Demand, Time-to-Repair (TTR) and Ordering & Ship Time (OST) for expected backorder (EBO) calculations. By analyzing the weakness of some assumptions of the existing models, the present research seeks to propose a new mathematical model (to be called P-METRIC) for EBO calculation, which is based on relaxed assumptions about demand, TTR and OST.

Background

Reparable Inventory System (RIS) will be the generic name used to describe the activities related to the management of reparable items. Reparable items, as opposed to consumable items, are going to be repaired instead of disposed of and replaced by a new item. The high costs usually associated with these kinds of items place them in the upper end of the spectrum of importance when the matter is material management.

The complexity involving the administration of reparable items begins with the stochastic characteristics of its demand but it is not limited to it. Dealing with multiple items, locations, maintenance levels, and item-hierarchies increases the number of variables that a manager has to analyze when managing a RIS.

Several models have been developed over the last 40 years to cope with these problems. The Multi-Echelon Technique for Recoverable Item Control (METRIC), developed by the RAND Corporation for the United States Air Force (USAF) in 1968, forms the basis of many other models. The METRIC model considers two echelons of repair and supply, bases and depot, and takes care of first-indenture items. METRIC's objective function seeks to optimize a system by reducing EBOs at the bases, subjected to constraints usually expressed in terms of cost. The optimization process is guaranteed by a marginal analysis technique, which calculates the Benefit Cost Analysis (BCA) for each additional spare item to be increased in the system. The fundamental basis of METRIC is Palm's Theorem, which states:

> If demand for an item is a Poisson process with annual mean m and if the repair time for each failed unit is independently and identically distributed according to any distribution with mean T years, then the steady-state probability distribution for the number of units in repair has a Poisson distribution with mean m*T. (Sherbrooke, 1992:21)

METRIC's scientific basis, together with its simplicity, makes it a useful model, still used today.

In December of 1973, John A. Muckstadt wrote an article describing the relationship between an assembly and its subassemblies with respect to a RIS. “The model, called MOD-METRIC, an extension of METRIC, permits the explicit consideration of a hierarchical parts structure” (Muckstadt, 1973:472). MOD-METRIC’s objective function, like METRIC, seeks to minimize the EBO for Line Replaceable Units (LRUs) at the bases, subject to cost constraints. The optimization process is also guaranteed by the marginal analysis technique. MOD-METRIC, however, takes in account the effects of having Shop Replaceable Units (SRUs) on the LRU’s repair process. MOD-METRIC’s goal is to calculate the best mix of LRUs and SRUs.

In March of 1985, Sherbrooke described a new model, called VARI-METRIC, which is claimed to be an improvement to EBO calculations. It is called a second-order model because it incorporates two parameters, mean and variance for the number of items in the pipeline, for EBO calculations (Sherbrooke, 1985:318). Based on results gathered from simulation, VARI-METRIC assumes that the number of units from a base that are in re-supply or repair at any point of time can be approximated by a negative binomial distribution, with variance that is never less than the mean (Sherbrooke, 1985:313). As a result of these assumptions, VARI-METRIC “produces an estimate for backorders that exceeds that of METRIC in all cases except when stock levels are zero (when the two models agree)” (Sherbrooke, 1985:318).

Preliminary research using a simulation model described in Chapter III will show that if Demand has distributions different from Poisson, Palm’s theorem cannot be evoked. Additionally, TTR and OST may have considerable variance, affecting the calculation of the expected number of items in backorder situation. Those assumptions of METRIC may result in inaccurate EBO calculations, potentially leading decision makers to take wrong decisions about what to buy as well as where to locate the items under analysis.

This research proposes a new approach that takes into account the stochastic characteristics of Demand, TTR and OST. It will model item demand according to actual or theoretical time-between demand distribution (approximated using statistical tools), which may be different from exponential distribution that results in Poisson demand as assumed by the existing models. TTR and TTR will also be modeled according to a best-fit distribution approximated using statistical tools.

Problem Statement and Contribution of Research

There is concern about the effects of variability in Demand, TTR and OST on EBO calculation for RISs that do not meet the assumptions of Poisson demand. For those systems, variability on the parameters could significantly affect the EBO calculation, as it will be demonstrated later in the Chapter 4. Current models disregard variability assuming Demand as Poisson distributed in order to evoke Palm's theorem (Sherbrooke, 1992:46). This approach assumes a risk of overestimating aircraft availability by underestimating backorders at the system (Sherbrooke, 1985:311, 312).

The proposed method seeks to improve EBO calculation by using a more realistic distribution for Demand, TTR and OST, and accounting for possible effects that may have on EBO prediction.

Research Question

Variability in Demand, TTR, and OST can possibly affect the accuracy of EBO predictions for systems where Demand does not follow Poisson distribution. Existing EBO models are based on assumptions that cannot always be met in the real world. This may adversely affect the suitability of EBO predictions. How could a mathematical analytical model account for variability in Demand, TTR, and OST with respect to EBO calculations in a more accurate way?

Investigative Questions

To help answer the research question, this research must answer the following investigative questions:

1. What is the best form for a mathematical model for EBO calculation that accounts for the stochastic aspects of the demand, time-to-repair and ordering-ship-time that may exist in reparable inventory systems?

2. Do the stochastic aspects of the demand and time parameters affect the EBO calculation in the proposed method?

3. Does the proposed method return different EBO numbers compared to the existing models? How significant is the difference?

4. Which model would provide the most accurate (close to the real world) back order numbers, the proposed method or the existing models?

5. Is the new model time/resource efficient compared to the existing models?

Methodology and Expected Results

This study intends to be a quantitative research aiming to test the suitability of Simulation working together with already existing METRIC theory for accounting EBO for Reparable Inventory Systems.

To answer the first investigative question, the research will demonstrate the mathematical rationality of the proposed method, describing its fundamentals, and logics. There will be described the simulation portion of the method, as well as the analytical portion with its formulas. In describing the proposed method, it is also intended to discuss the fifth investigative question, the possible problems that one may find when implementing the proposed approach such as software needs, time consuming and so forth.

Looking for answering the second and third investigative questions, an experimental design consisted of ten conceptual examples will explore different distributions for Demand and time parameters (TTR and OST) in order to verifying the effects of those variability on the EBO calculation. To assert about the significance of possible differences, this research will test the results of the existing and proposed methods against samples collected from simulation, checking which model, if any, would be inside of a 95 % half-width confidence interval (CI) of the "true" mean value gathered from simulation. Additionally, for each stock level tested, there will be informed which model (exiting or proposed) is closer to the mean simulation value. Finally, the summation of the squared difference between exiting models and simulation, and proposed method and simulation will summarize the range of stock levels tested for each experiment in order to verify which of the models (existing or proposed) deviates more from the mean simulation values.

The fourth investigative question must be answered considering a given system. Therefore, the research will gather information about demand, time parameters, back orders and stock level policy for 20 reparable items from the T-27 Tucano program, at

Lagoa Santa Depot (PAMALS), Brazil. The PAMALS serves the two most important T-27 Tucano's operators, Academia *da Força Aérea* (AFA) and *Comando Aéreo de Treinamento* (CATRE) that together maintain approximately 90 % of the T-27 Tucano's fleet. For the sake of simplicity during the early model development, the research will consider only these two operators, being the system composed of the depot – PAMALS, and the bases – AFA and CATRE. That information will be used for calculating EBO of both existing and proposed models, allowing assert about deviation from the reality.

Scope and Limitations

The main purpose of this study is to demonstrate the effects of the variance in Demand, TTR, and OST on the system's calculated EBO. That will be pursued by working with theoretical examples, and a real world data sampled from a set of 20 reparable items of the T-27 Tucano program, an advanced-training, light-attack aircraft deployed by the Brazilian Air Force and supported by the PAMALS. The conceptual examples will be confined to the study of some hypothetical reparable items, which have Demand approximated by distributions different from the Poisson distribution, used by the existing models. TTR and OST will be assumed as having their own probability distributions. Comparisons will be made for a range of stock-level at base, given a stock level at depot. The set of 20 reparable items of the T-27 Tucano program will be chosen by the Subdivision of Planning (TPLJ), at PAMALS. TPLJ's personnel will gather information about equipment failures over an elected period of 100 days of aircraft operation in the last year (2001). There will be no differentiation from corrective or preventive maintenance. The comparison between the methods for the set of 20 reparable items of the T-27 Tucano will be made only for the stock-level informed by TPLJ. Additionally, it is relevant to say that this study is going to be the first effort toward the utilization of a mathematical model for reparable item management at PAMALS.

Therefore, it is out of the scope of this study either to extend calculations across all reparable items of the T-27 Tucano program or to explore all-possible different theoretical examples.

The following is a list of limitations and risk factors that can represent potential misinterpretations of the results of this research:

1. Appropriateness of the Mathematical Models. Since METRIC models were developed specifically for application to aircraft RIS, any application outside of this area would be suspect. Failures may exhibit different behavior for electronic and/or mechanic equipments outside of aircraft. All models in this study are applied to an aircraft RIS.

2. Sample of Items. It is assumed that a sample of approximately 20 of the most important reparable items of the T-27 Tucano program that have historical data about backorders will provide the research with the information necessary to draw conclusions about the effects of variability on the system. This number needs to be extended if one wants to run the model for a real world situation.

3. Steady-State Behavior. The existing and proposed models assume steady-state behavior for all parameters. Consequently, it is assumed stationary processes for all parameters in the system. Sometimes, this assumption is not true. For example, many items exhibit different Mean Time Between Failures (MTBF) as they pass through a repair process over a long period of time.

4. Time Between Demands. Samples of the time between demands are going to be analyzed through the Arena Input Analyzer in order to verify the best theoretical distribution that fits with the sample values. The selection of the distribution will be done based on the least square error gathered from the Arena Input Analyzer. However, theoretical distributions are still an approximation of the reality, and the “real world” demand behaviors will be simulated using the best-fit theoretical distribution.

5. Back Order Data. Back order reports not always represent the real situation of a system. Since that information is used as a metric of the efficiency, it is expected that sometimes people simply withhold information in order to show the situation better than it really is.

As a result of the scope and limitations, the generalizability of the findings may both decrease and/or be subject to other interpretations. These limitations will be addressed again in Chapter V, when the results will be discussed.

Summary

In Chapter 1, the main purpose of this study was described. The background section discussed the nature of the EBO estimation. The problem statement, the research and investigative questions, the methodology, and scope and limitations were also explained and briefly discussed.

In the next chapter, the literature review will cover the main topics related to this research. In following chapters, the methodology to be used will be described, and then, in the last two chapters, the results obtained from the study and the suggested recommendations will be presented and discussed.

II. Literature Review

Chapter Overview

The first chapter discussed introductory issues. General aspects and background of the problem, the problem statement, research and investigative questions, the methodology, and scope and limitations were described.

This chapter will review relevant literature related to the subject of this research in order to present fundamentals necessary for the research hypothesis. From the management science field, the reasons for having inventories, its risks and benefits, dealing with critical items, and forecasting independent demand for reparable items will be discussed. A review of the nature of random failures and the wear-out process will be presented, extending the discussion to issues related to maintenance policy and activities. The Base Stockage, MEDTIC, and MOD-METRIC models are going to be presented and described in more details. Finally the relationship of the EBO theory with system availability will be discussed.

Inventory Management

Inventory management plays an important role in almost all types of business. Since inventory usually requires a lot of investment, such as materiel, buildings, equipment, and personnel, the ability to manage inventory correctly may represent the ability to be profitable or running a loss. Regardless of this importance, "it is evident that most firms do not fully understand the complexities of inventory management" (Silver, 1998:5). Referring to his experience with consulting project for inventory management issues with local firms, Silver says: "Over the years, we have seen that in more than 90 percent of the cases, improved inventory or production management would lead to cost savings of at least 20 percent, without sacrificing customer service" (Silver, 1998:5).

For public services, inventory management is also of great importance. "Imagine a hospital stocking out of blood, or the air force stocking out of a mission-critical part when the enemy is attacking" (Silver, 1998:3). For the Brazilian Air Force, which runs with short budgets for aircraft spare-part purchases, the importance is even greater. With scarce resources, managers do not have much room for mistakes and each dollar spent impacts directly on metrics like aircraft availability.

Setting the proper level of inventory is a decision that managers usually have to face in business. Stocking out of items may represent risk, such as lost sales and lost customers, but having excess inventory drains profit and ties up capital, making it hard to survive in the competitive marketplace. Managers should balance pros and cons when deciding about the level of items in inventory. According to Silver, several factors can influence the decision to stock or not stock an item, including:

1. The system cost (file maintenance, forecasting, etc.) per unit of stocking an item.

2. The unit variable cost of the item both when it is bought for stock and when it is purchased to meet each demand transaction (A more favorable price may be achieved by the regular larger buys associated with stocking. In addition, a premium per unit may be necessary if the non-stocking purchases are made from a competitor).

3. The cost of a temporary backorder associated with each demand when the item is not stocked.

4. The fixed setup cost associated with replenishment in each context (An account should be taken of possible coordination with other items, because setup costs may be reduced).

5. The carrying charge (including the effects of obsolescence), which, together with the unit variable cost, determines the cost of carrying each unit of inventory per unit time.

6. The frequency and magnitude of demand transactions.

7. The replenishment lead-time. (Silver, 1998:372,373)

Critical Items

Managing inventories may involve dealing with large amount of information. To run the three aircraft programs under its responsibility, for example, the PAMALS have about 30,000 different part numbers in stock, with approximately 5,000 with regular consumption, being used at least once a month. Consequently, a first distinction one may need to do when managing such kind of activity refers to the degree of importance of each item in the system or, in other words, the amount of attention managers should pay for the item administration.

To identify critical items, those that will receive higher priority in the allocation of management time and financial resources, managers usually use the ABC analysis. This analysis involves ranking items into three priority ratings: A (most important), B (intermediate in importance), and C (least important) (Silver, 1998:34). As a result of the ABC analysis, one may find that few items (approximately 20 percent) account for the major part (usually 80 percent) of the dollars tied up. These items are called class A items. Class B items usually account for approximately 30 percent of the items, but representing 15 percent of dollar amount. Class C items would account for the rest of the items, approximately 50 percent, but representing just five percent of the dollars tied up.

Silver also says “An ABC classification need not to be done on the basis of the Distribution by Value curve alone. Managers may shift some SKUs (*Stock-Keeping Unit*) among categories for a number of reasons” (Silver, 1998:35). Items that are crucial to the aircraft operation, for example, those that are stocking-out and resulting in an Aircraft-On-Ground (AOG) situation, would be ranked in the class A, regardless of the amount of dollars they tie up.

In the Brazilian Air Force, reparable items require close management because of several reasons; two reasons of importance are that they are usually expensive items and often critical for aircraft operability. The procurement process also requires more time, since most of those items are imported from other countries. Additionally, their repair processes involve the integration of other fields of logistics, such as transportation and production planning and scheduling, requiring closer management. Finally, the application of ABC analysis usually ends up in a curve, like the Pareto chart represented on the Figure 1 below (Silver, 1998:33).

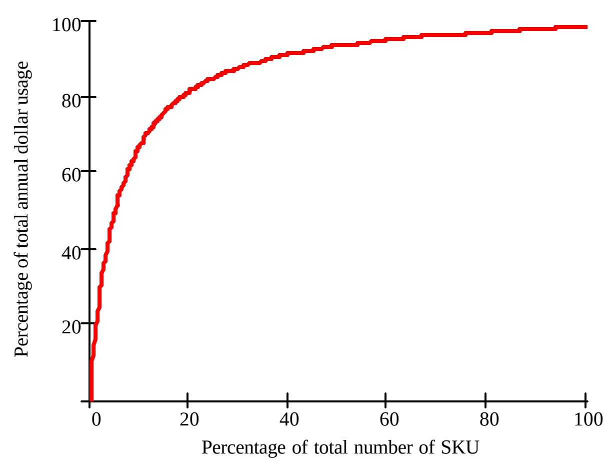


Figure 1. Distribution by Value of SKUs

Demand Forecasting

For reparable inventory system, the terms demand and failure are used most of the time interchangeably (Sherbrooke, 1992:1). In fact, a demand for an item can occur when it either fails or goes to a preventive revision. A primary concern for inventory planning involves demand prediction. Inventory models require a prediction of demand for the forecasted period. Even the best inventory model will not work if the information about the demand is inaccurate (Sherbrooke, 1992:58). This accurate demand information is relevant for both reparable and consumable items.

The complexity of a RIS requires the use of techniques for demand prediction that go beyond just expert opinion. Opinion can provide good insight, but it isn't enough. When it comes to making decisions about the next item to buy and where to locate it, in an environment full of randomness, an analytical approach is indispensable (Sherbrooke, 1992:57).

The primary distinction that must be made about demand refers to the type of demand, that is, whether demand is dependent or independent. By definition, a supply item has dependent demand whenever its requirement is directly related to a need of a higher-level item (Colin, 1973:6). In opposition, an item shows independent demand pattern when it is not possible to directly correlate its demand to a next higher assembly component (Orlicky, 1975:22). Dependent demand exhibits a pattern very different from that of independent demand and must be managed with different techniques (Silver, 1998:594-595). Dependent demand is easier to cope with. Methods like Material Requirement Planning (MRP) and its derivative models can be efficiently used to answer the two basic questions related to “what do we need, and when” in order to fulfill the future needs (Silver, 1998:597).

Planning a RIS requires the use of independent demand techniques. First indenture items (end-items or LRUs) are usually considered independent demand items. Their demands are influenced by market conditions (for the case of an aircraft fleet, the number of flight hours, the number of take-offs and landings, etc). The demands are not directly related to inventory decisions for any other item held in stock, depending on the condition of the item itself in most of the cases (Orlicky, 1975:22). The big difficulty in managing independent of demand is that it is influenced by unpredictable external factors.

Another important characteristic of independent demand is the Time Between Demand (TBD). When modeled as exponentially distributed, for example, that attribute defines the demand distribution as Poisson. Existing EBO models assume TBD as exponential distributed (Sherbrooke, 1992:48,106). That fundamental assumption provides support for further assumptions, like the simple Poisson Process (METRIC and MOD-METRIC) or Poisson Process with a changing mean (VARI-METRIC), for the estimation of the number of items in the pipeline. Those models are based on the already referred to Palm’s Theorem (Sherbrooke, 1992:100).

Failure Rate and Wear-Out Processes

It is now instructive to analyze sources and causes of supply demand. Demand for an item starts when, as a result of maintenance procedures, a spare part is withdrawn from the system in order to effect a repair. Such maintenance procedures can be the consequence of corrective and/or preventive actions. A good understanding of the failure process may provide managers with helpful information to build up better inventory planning.

Some items, like engine parts, tires, batteries, and landing gear parts are more likely to have demand rates that increase with the item service life. The demand for those items is ruled by an underlying wear-out process, and cannot be modeled randomly (Sherbrooke, 1992:83). Sherbrooke suggests that for those items the time between demands does not decrease uniformly like the exponential distribution. Instead, they have a peak value to the right of the origin as in theoretical distributions like gamma, Weibull, or log normal (Sherbrooke, 1992:83). Such different behavior in the failure creation may generate different pattern for demand, potentially affecting the number of items in the pipeline.

Maintenance Policy, Repair Time and Maintenance Activities

Starting with the assumption that all equipment is subjected to failure, a primary concern when planning a RIS should be the maintenance policy. The maintenance policy involves defining who and where (which organization) will perform maintenance activities, and also where to locate equipment and spare parts to support maintenance activities. The decisions about maintenance are important since they directly impact the system performance.

According to Blanchard, in defining maintenance policy, three levels of maintenance are usually considered: organizational, intermediate and depot/producer levels (Blanchard, 1990:42). Organizational maintenance is performed by consumers at the operational site. It may include visual inspection, operational checkout, minor servicing, external adjustment and removal and replacement of some components. Intermediate maintenance is performed by organizations usually located close to the operational sites. It can also be provided by mobile or semi-mobile installations. Its activities may include major servicing, major equipment repair and modifications, complicated adjustments, and limited calibration. Depot/producer maintenance represents the highest type of maintenance-performed tasks that go above and beyond the capability of the organizational and intermediate level. Sometimes, it is provided by the manufacturer or its representative. (Blanchard, 1990:42-43).

The Brazilian Air Force has instituted two levels of maintenance for the aircraft program supported by the PAMALS. Under this concept, the intermediate level would perform both organizational and intermediate level maintenance. However, analyzing the maintenance practices and routines for PAMALS and aircraft operators, it is clear that the local maintenance squadrons (*Esquadrão de Suprimento e Manutenção*, ESM), which are supposed to perform organizational and intermediate levels, actually perform only the conceptual organizational maintenance level routines plus part of the intermediate level, such as substitution of defective items and a few minor-repairs. In fact, almost all end-items are repaired at the PAMALS.

Repair time is the period piece of equipment takes to undergo the maintenance activities. Together with the failure rate (demand), repair time forms the basic parameters of all maintainability prediction (Green, 1991:78).

Maintenance activity comprises the actions taken to keep system in its state of functioning. According to Blanchard, it consists of acts of diagnosing, repairing, or preventing the system of failures (Blanchard, 1990:393). It can be classified into two major categories: corrective maintenance and preventive maintenance. Corrective maintenance refers to the unscheduled actions taken to restore a defective item to a specified level of performance. Preventive maintenance are the scheduled actions taken to keep a system working at specified level of performance. It comprises activities related to systematic inspection, detection, and prevention of impeding failures (Blanchard, 1990:393).

Back Orders and Expected Back Orders

At this point, much has been said about EBO, but without detailed explanation. To understand EBO, it is first necessary to explain what a backorder is. Sherbrooke provides a simple explanation of a backorder, saying "when a malfunction is diagnosed on an aircraft, the malfunctioning item is removed from the aircraft to the base supply. If a spare is available, it is issued and installed on the aircraft. Otherwise, a backorder is established" (Sherbrooke, 1992:6). Therefore, a backorder results from a combination of demand (failures) and supply availability. It could also be defined as the number of items in shortage for a specific site over a specified period of time. EBO, however, is not an actual backorder but it is related to it. EBO is an expected value for the number of backorders. Because the actual backorder number is unknown ahead of time, it can be modeled as a random variable, and could be described by using statistical techniques. EBO can be defined as a prediction of backorder using its probability distribution.

Another way of looking at EBO can be provided by the following example: suppose one wants to calculate EBO for an item over a ten days period. After research at the field, the following data for past backorders is determined (Fulk, 1999:1, numbers intentionally changed)

Table 1. Back Orders

Backorder Events	Day									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1º back order		x	x	x	x	x				
2º back order		x	x	x						
3º back order				x	x	x	x			
4º back order									x	x

The chart above represents the occurrence of backorders over a ten day period. For each event (backorder), the initial and final date was marked with the letter x. Note that in this conceptual example, four backorders have occurred. In the first and eighth days there are no backorders and, because of the different time-length of each backorder, some days one may find up to three backorders (fourth day). Using this example, there are different ways of representing the EBOs. One way to calculate EBO for the period is to take the weighted average of the number of backorder over the ten-day period. The expected backorder for this period can be calculated as shown in the following formula:

$$\text{EBO} = \sum_{d=1}^{10} \frac{x}{n} \tag{2.1}$$

Where: d = the specific day, ranging from day one to day ten.
x = the variable number of backorder for each day.
n = constant of the total of days, which is equal to ten.

Solving the formula above means solving the following equation (Fulk, 1999:34):

$$\text{EBO} = \frac{0}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{2}{10} + \frac{2}{10} + \frac{1}{10} + \frac{0}{10} + \frac{1}{10} + \frac{1}{10} = 1.4$$

The expected backorder for the period is 1.4. Thus, computing EBO takes into account not only the number of backorders, but their durations as well. The EBO represents the number of backorders for the discrete distribution of the daily backorders.

Since EBO is not the same thing as a backorder, why would one want to reduce EBO, instead of reducing actual backorders? While the number of backorders measures one dimension of supply insufficiency, EBO accounts for both the number and duration of the unmet need for an item. Trying to reduce backorder directly, by improving aspects of supply, such as service level and or average fill rate, may lead to resource misallocation. Results gathered from simulation and also a real world field test done by RAND Corporation for the USAF, at George AFB from 1965 to 1966, shows that the use of a EBO as a metric, compared to service level, can both reduce cost and increase aircraft availability at the same time (Sherbrooke, 1992:10).

Thus, reducing EBO one may find that not only backorders, but also the backorder's time-length will be reduced. Consequently, as will be shown shortly, the aircraft availability can also be expected to increase.

Marginal Analysis

While the concept of marginal analysis, also called greedy heuristic, has been used for many years, the earliest published reference about that came from O. Gross, in a paper called *A class of Discrete-Type Minimization Problems*, RAND Corporation, in 1956. The application of this technique for reparable inventory management aims to produce an optimal curve for EBO, which considers not only the EBO reduction produced by the addition of one item in the system, but also the benefit-cost-rate (BCR) analysis related with that increment (Sherbrooke, 1992:29). Marginal analysis would allow optimizing EBO reduction considering different items, with different EBO curves, and with different prices, increasing the system effectiveness per dollar obtained when an additional item is selected for stockage (Sherbrooke, 1992:29). This way, marginal analysis plays an important role when it comes to systems subjected to constrained budget.

The formulation of the marginal analysis as it refers to reparable inventory system is presented by Sherbrooke as the following formula (Sherbrooke, 1992:30):

$$[EBO(s-1) - EBO(s)]/c \qquad (2.2)$$

Where: EBO = expected backorder as a function of s.
s = stock level being analyzed.
s-1 = stock level immediately before s.
c = cost of the item.

Item Approach vs. System Approach

Traditional inventory models seek to balance factors like holding inventory, ordering, and stockout cost when deciding about the stockage of a reparable item. Those models use the item approach and the decisions on the number of spare units of stock to buy of an item are made without considering other items in the system (Sherbrooke, 1992:3). Given an aircraft backorder is usually considered a hole in the aircraft that potentially affects aircraft availability, minimizing backorders of some items without considering the total system may lead to inappropriate resource investment (Sherbrooke, 1992:3). Besides, the management may be also interested on the system performance. At a certain point, for example, the increment on the availability of an item may bring no benefit to the system availability as a whole, representing only cost. The system approach takes care of those issues. In deciding about augmenting an additional item in a system, the system approach looks for an optimal point in the availability curve that represents the best utilization of the resources available. The points on the curve represent both the maximum availability that can be achieved and the minimum required cost to achieve that availability (Sherbrooke, 1992:39). Points above the availability curve would be unfeasible solution, given the actual paradigms. Points below the availability curve would represent non-optimal situations.

Reparable Inventory Models

Since the 1960's years, several mathematical inventory models were developed to facilitate the management of reparable inventory systems. Basically, those models seeks for calculating expected backorders in order to support better decisions of what-to-buy and where-to-locate those items. Three of them are going to be discussed in this section: the Base Stockage Model, the Multi-Echelon Technique for Recoverable Item Control (METRIC), and the Multi-Item, Multi-Echelon, Multi-Indenture System Model (MOD-METRIC). Due to scope issues, this research does not compare the proposed method results with those of VARI-METRIC and MOD-METRIC. However, a simulation model that allows the computation of backorders for assembly and subassemblies of a Multi-Indenture, Single-Site situation is provided in the Appendix B.

Base Stockage Model

In 1965, RAND Corporation developed a model that introduced the optimizing techniques described on the topics above, the marginal analysis and the system approach.

The Base Stockage model scenario consists of: when a item failure, a demand for a serviceable item goes to the warehouse and, if stock level is greater than or equal to one, the demand is fulfilled. Otherwise, a backorder is issued and the warehouse waits for the next available that comes from the base repair shop. The model can be pictured as following (LOGM 628 – 3-32 - Reparable Inventory Class):

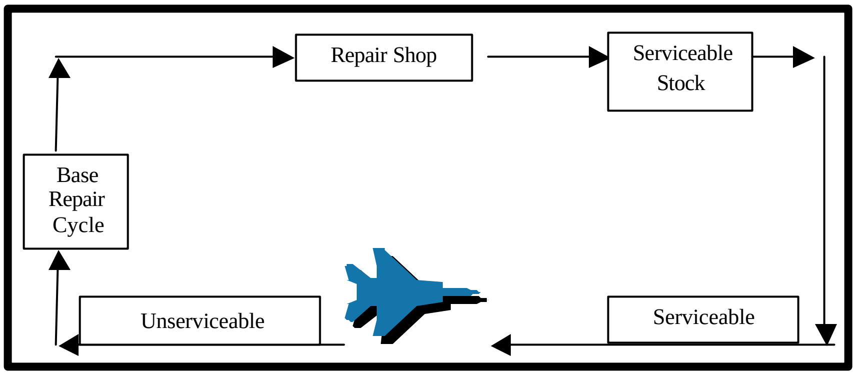


Figure 2. Base Stockage Model Scenario

The expected back order as a function of the stock level is stated as the following function (Sherbrooke, 1992:25):

$$\mathrm{EBO}(s) = \sum_{x=s+1}^{\infty} (x-s)\cdot \mathrm{Prob}(X = x) \tag{2.3}$$

Where: EBO(s) = expected backorder as a function of s.
s = stock level.
x = random variable pipeline.
Prob(X=x) = probability of a random variable x to assume a value X.

The formula asserts that expected backorder is the probability-weighted sum (expected value) of the occurrences when the number of items in the pipeline (demands) exceeds the stock level (supply).

The model assumes infinite repair channels and the demands for items coming from an infinite population. Additionally, the demands for items are assumed independent of the items' repair time, and vice-versa. This way, the expected number of items in the repair channel (pipeline) would be a corollary of a fundamental law from queueing theory, called Little's law, as shown in the subsequent formula (Sherbrooke, 1992:28)

$$\text{Pipeline} = m * T \tag{2.4}$$

Where: m = mean number of items demanded for a period of time.
T = mean time to repair (same unit of time of Demand).

Since demand is assumed Poisson distributed, the number of item in the pipeline should also follow the same distribution (consequence of Palm's theorem, already discussed). Thus, the pipeline is defined as shown in the next formula (Sherbrooke, 1992:20):

$$\text{Prob}(x) = \frac{(m \cdot T)^x e^{-m \cdot T}}{x!} \tag{2.5}$$

Where: Prob(x) defines the probability function of the pipeline values.
x = random variable for the pipeline, assumed Poisson distributed.
m = mean number of items demanded for a period of time.
T = mean time to repair, for any distribution.
e = natural logarithm base, which is approximated to 2.71828.

The objective of this model, called the Base Stockage model, is to minimize the expected number of backorders (EBO) for a give location, constrained by a dollar amount budget. The Base Stockage model dealt with multiple items, but single echelon, single location and single indenture. The mathematical statement of the model is described as following (Sherbrooke, 1992:34):

$$\min_{(s1,s2)} EBO_1(s_1) + EBO_2(s_2) \qquad (2.6)$$

Where: min = minimizing objective function.
$EBO_1(s_1)$ = item 1expected number of backorders as a function of s_1.
s_1 = item 1 stock level.
$EBO_2(s_2)$ = item 2 expected number of backorders as a function of s_2.
s_2 = item 2 stock level.

Only two items compose the system above, item 1 and 2. The statement can be extended for systems with more items. Observe that the objective function looks for a minimization of the total expected backorder, and not for individual items. Additionally, the objective function is constrained by following mathematical statement (Sherbrooke, 1992:34):

$$c_1 s_1 + c_2 s_2 <= C \qquad (2.7)$$

Where: c_1 = cost of item 1.
s_1 = stock level of item 1.
c_2 = cost of item 2.
s_2 = stock level of item 2.
C = total cost represented by a given budget.

Even being considered a step-forward in reparable item modeling at that time, the Base Stockage model was never adopted by the Air Force. Its concepts, however, were used as a foundation of other models (Sherbrooke, 1992:45).

METRIC

The Multi-Echelon Technique for Recoverable Item Control (METRIC) model is considered an improvement of the Base Stockage model. METRIC follows the same logic as the Base Stockage model but expand it to allow modeling the depot portion of the pipeline. METRIC's optimization process is global, that is, it takes into account the entire supply system. The model can both compute requirements and redistribute stock more appropriately when compared with the Base Stockage model (Kutzke and Turner, 1982:26).

The METRIC's scenario consists of: when items failure at base level, a requisition for a serviceable item goes to the base warehouse and, if the stock level for the item is greater than or equal to one, the demand is fulfilled. Otherwise, a base backorder is issued. At the same time, an unserviceable item goes to the base repair shop and there is a constant probability (r) of that item being repaired locally, and (1-r) probability of the item goes to the next echelon (depot) to be repaired. If the item is repaired locally, the base warehouse waits for its repair (or for any other that is already being repaired). Otherwise, the unserviceable item goes to the depot pipeline. In this case, the base supply service issues a requisition of one serviceable item to the depot. In the depot's portion of the pipeline, the requisition for a serviceable item goes to the warehouse and, if the stock level for the item is greater than or equal to one, the demand is fulfilled. Otherwise, a depot backorder is issued and waits for the next serviceable available from the depot repair shop in order to attend in a first-in-first-out (FIFO) rule the demand from the bases. In the cases a base orders an item from the depot, it will take a time for the base to receive the item. This is the ordering & ship time (OST) and it is another factor that affects the item pipeline.

The following figure pictures the METRIC model (LOGM628 6-4 – Reparable Inventory Class)

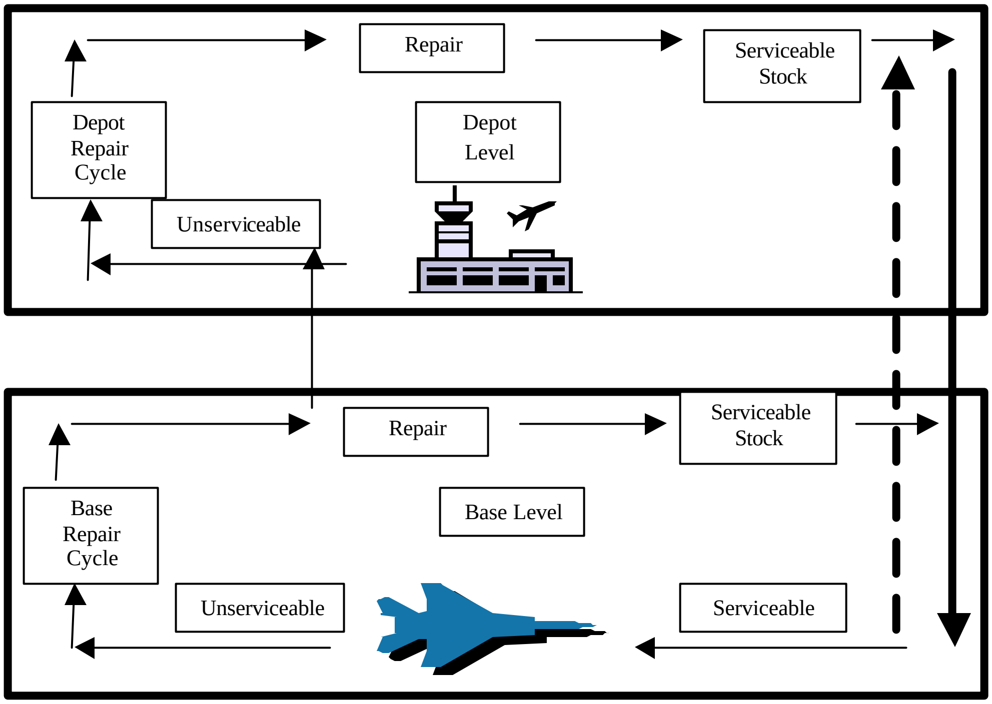


Figure 3. METRIC Model Scenario

The dashed arrow in the figure above represents a base requisition for a serviceable item from the base to the depot. The parallel arrow (bold) represents items being attended from the depot.

The objective of METRIC is similar to the Base Stockage model: METRIC seeks to minimize expected backorders over the specified items subject to the investment constraint. Depot backorders are a factor only as they affect base backorders (Muckstadt, 1973:473)

METRIC Assumptions

These are the main assumptions of METRIC (Sherbrooke, 1992:46):

- The decision as to whether a base repairs an item does not depend on stock level or workload. A fraction of repairs is going to be repaired at base, at a constant probability (r), and (1-r) probability is going to be repaired at depot. The time to repair includes any waiting time, such as supply waiting time or queue-times in the repair process. Additionally, the repair process follows the one-by-one rule, no batching for repair.

- METRIC assumes stationary process for demand, and time parameters. Additionally, demands are described by a logarithmic Poisson process, a member of the compound Poisson family.

- The basic METRIC assumes no lateral supply from other bases; the depot is the only organization allowed to re-supply the base.

- The (S-1,S) inventory policy is appropriate for every item at every echelon.

- No condemnation.

- Serviceable and unserviceable items are equally important for the system.

- The length of time required to repair an item is independent of the number of demands.

The assumptions above have been made for analytic modeling convenience and may not be realistic in most of time. As a result, the accuracy of the model can be harmed. For example, the assumption about Poisson process for demands sometimes is not true. Sherbrooke suggests that some items presents time between demands that do not decrease uniformly like the exponential distribution. Instead, they have a peak value to the right of the origin as in theoretical distributions like Gamma, Weibull, or Lognormal (Sherbrooke, 1992:83). Different distributions for time between demands may lead to different EBO results, calling for a model that could take that into account.

MOD-METRIC

The Multi-Item, Multi-Echelon, Multi-Indenture Model (MOD-METRIC) was introduced by Muckstadt in early 1970's. He observed that METRIC focus on reparable items as a whole tended to concentrate more heavily on inexpensive sub-components because it was able to decrease the backorder level more in buying these items (Kutzke, 1982:28). As Muckstadt points out, "in METRIC a backorder of a module and a backorder for an engine are assumed to be equally undesirable; however, these backorders affect the system in different ways" (Muckstadt, 1973:475). MOD-METRIC, in turn, could describe the logistics relationship between an assembly and its subassemblies, and to compute the spare stock levels for both assembly and subassemblies. MOD-METRIC extends the METRIC's concept to include hierarchical and indenture parts structure, allowing two levels of parts to be considered, an assembly and its subassemblies.

Using the engine problem described in the Muckstadt's paper, the mathematical statement of the problem is (Muckstadt, 1973:477):

$$\min \sum_{i=1}^{M} \sum_{x_i = s_i+1}^{\infty} \left(s_i - x_i\right) \cdot p\left(x_i \cdot \text{given} \cdot \lambda_{ij} \cdot T_{ij}\right) \tag{2.8}$$

Where: min = minimization objective function.
i = any base in the system.
M = number of bases.
s_i = stock level of spare engines at base i.
x_i = number of engines in backorder at a base i.
λ_{ij} = the average number of removals of module j at base i.
T_{ij} = the average re-supply time for module j at base i.

Important to highlight that MOD-METRIC objective is to minimize backorders for assembly items. By doing so, MOD-METRIC considers the expected backorders of subassembly items at the extent that they affect the average re-supply time (T_{ij}) of the assembly item.

Besides, the objective function above is subject to (Muckstadt, 1973:477):

$$\sum_{i=1}^{M} \left(c_E \cdot s_i + \sum_{i=1}^{N} c_j \cdot s_{ij} \right) + \sum_{j=1}^{N} c_j \cdot s_{0j} + c_E \cdot s_0 \leq C \tag{2.9}$$

Where: c_E = unit cost of an engine.

j = any module (subassembly).
c_j = unit cost of module j.
s_{ij} = stock level of module j at base i.
N = number of modules.
s_{0j} = stock level of module j at the depot.
s_0 = stock level of spare engine at depot.
C = dollar budget limit.

The assumptions of MOD-METRIC are the same of METRIC, except for indenture issues and the fundamental difference in the objective function already discussed.

EBO and System Availability

Weapon systems, including military aircraft, should preferably be in a ready condition when an operational demand occurs. Thus, the availability of a system must be a concern for logisticians. The concept of availability, however, can be viewed from several aspects. Generally, availability can be measured by the ratio of system uptime by the system uptime plus downtime, like in the following formula (Green, 1991:71):

$$\text{Availability} = \frac{\text{uptime}}{\text{uptime} + \text{downtime}} \qquad (2.10)$$

Other ways of looking at availability are also described by Green. Inherent Availability (Ai) represents the probability that a system will perform as intended over a given period of time, under an ideal support environment. The support would include all the necessary resources, such as spare and repair parts, test equipment, trained personnel, and etc (Green, 1991:71). Additionally, Inherent Availability does not consider preventive maintenance actions, logistics supply time, and administrative downtime. Inherent Availability is calculated as following (Green, 1991:72):

$$A_i = \frac{MTBF}{MTBF + Mct} \qquad (2.11)$$

Where: MTBF = mean time between failure (see 2.7 ahead).
Mct = represents the statistical mean for all corrective maintenance actions.

The Mct is also referred to as Mean Time To Repair (MTTR), or simply Time To Repair (TTR), as it is used in this thesis.

Additionally, MTBF can be represented by (Green, 1991:72):

$$MTBF = \frac{1}{\lambda} \qquad (2.12)$$

Where: ? = mean demand over a period of time.

Achieved Availability (Aa) represents the probability that a system will perform as intended, under specified conditions, at a given point of time (Green, 1991:72); excluding both logistics supply time and administrative downtime, Aa differs from Ai in that preventive maintenance time is included. It is defined as (Green, 1991:72):

$$A_a = \frac{MTBM}{MTBM + M} \qquad (2.13)$$

Where: MTBM = mean time between maintenance.
M = Mean Maintenance Time.

Important to say that both MTBM and M account for scheduled and unscheduled maintenance.

Additionally, MTBM can be defined by (Green, 1991:72)

$$MTBM = \frac{1}{(MTBMs)^{-1} + (MTBMu)^{-1}} \qquad (2.14)$$

Where: MTBMs = mean time between scheduled maintenance.
MTBMu = mean time between unscheduled maintenance.

Operational Availability (Ao) represents the probability that a system will perform as intended under stated conditions in the operating environment (Green, 1991:73). Operational Availability accounts for both logistics supply time and administrative downtime, as shown in the following formula (Green, 1991:73):

$$A_o = \frac{MTBM + ready\_time}{(MTBM + ready\_time) + MDT} \qquad (2.15)$$

Where: MTBM = mean time between maintenance.
Ready Time = time when the system is ready for use but not being utilized.
MDT = time the system is not in condition to perform its intended function.

MDT includes not only the repair time, but also administrative downtime, waiting-time in queue, and logistics supply time.

Additionally, in defining system availability, Sherbrooke suggests to splitting it into two categories: Maintenance Availability (Aa), the same concept and formulation of Achieved Availability already discussed; and Supply Availability (As), which he defines as shown in the next formula (Sherbrooke, 1992:36):

$$A_s = \frac{MTBM}{MTBM + MSD} \qquad (2.16)$$

Where: MSD = mean supply delay.

MSD refers to the Mean Supply Delay, calculated considering delays originated from both administrative and shortage supply time. As a result, system availability would be the product of Maintenance Availability and Supply Availability (Sherbrooke, 1992:37). It is assumed that Supply Availability is independent of the maintenance policy.

Looking at all formulas above, it is easy to understand that system availability is influenced by events that cause the downtime (backorder) and also by the duration of the event (backorder length). Thus, a better way to account for the system availability should consider both backorder and backorder length. The EBO formulation, which considers the pipeline of the items in demand for its calculation, would better link system availability to item's shortage.

Sherbrooke noted that it would be possible to extend the EBO results of METRIC in order to obtain measures of aircraft availability (Sherbrooke, 1992:38). He defined aircraft availability for a fleet of aircraft as the probability a weapon system is not missing a single reparable, which is shown in the following formula (Sherbrooke, 1992:38):

$$A = 100 \cdot \prod_{i=1}^{I} \left(1 - \frac{EBO_i(s_i)}{N \cdot Z_i}\right)^{Z_j} \qquad (2.17)$$

Where: A = Aircraft availability.
i = each individual item in the system.
I = total number of different items in the system.
N = number of aircrafts in the system.
Z = quantity per aircraft.
s_i = stock level of a specific item.
$EBO_i(s_i)$ = expected backorders for any individual item as a function of stock level.

Additionally, EBO for any individual item is constrained by N^*Z_i for every item in the system.

The logic of this formulation asserts that there exist N*Z locations for a specific item in the system, and the probability of a hole in any of those locations is the ratio

EBO(s)/N*Z for each item. It follows that an aircraft is going to be available only if there is no hole for any of the Z occurrences of each specific item, or for any other.

The Availability is expressed as a fleet percentage, explaining the multiple 100. It assumes independence of failures, equal importance for all items to the availability of the system, serial systems, and no cannibalization. Therefore, as Sherbrooke concludes, the minimization of total backorders could achieve maximization of a weapon system's availability in a fleet (Sherbrooke, 1992:39).

Summary

Chapter II reviewed previous findings that apply to this research effort. First, important issues related to inventory management were discussed, highlighting critical item management, and details about reparable items in the FAB. A discussion about demand forecasting, failure rate and wear-out process was presented along with topics associated with maintenance policy, repair time, and maintenance activities. Additionally, a description of the fundamentals of the existing models (Based Stockage model, METRIC model, and MOD-METRIC) was presented and discussed. Finally, the relationship between expected backorder and system availability was discussed highlighting the importance of studying backorders for the operational field.

In the next chapter, the methodology to be used in assessing the efficacy of an improved METRIC model will be presented and described.

III. Methodology

Chapter Overview

The first two chapters described the problem that motivated this research and the issues related to planning and managing RIS. Chapter III will describe the methodology used in this research. The experimental design will be presented, describing the ten conceptual experiments as well as the T-27 case study. A simulation model in Arena will be introduced as the basis of the proposed method. Then, the mathematical analytical portion of the proposed method will be described and discussed. Finally, considerations about the data collected from the PAMALS will be presented.

The Investigative Questions.

In describing the experiments to be performed in the next sections, this research intends to get information for answering the investigative questions stated earlier in Chapter I. Therefore, it is opportune to remind those questions at this time:

Investigative Question 1. What is the best form for a mathematical model for EBO calculation that accounts for the stochastic aspects of the demand, time-to-repair and ordering-ship-time that may exist in reparable inventory systems?

Investigative Question 2. Do the stochastic aspects of the demand and time parameters affect the EBO calculation in the proposed method?

Investigative Question 3. Does the proposed method return different EBO numbers compared to the existing models? How significant is the difference?

Investigative Question 4. Which model would provide the most accurate (close to the real world) back orders numbers, the proposed method or the existing models?

Investigative Question 5. Is the new model time/resource efficient compared to the existing models?

Describing the Experiments

The experiments consist of two main parts: conceptual theoretical experiments, and the T-27 Tucano case study. The conceptual experiments aim to verify whether different distributions for demand and time parameters influence the EBO calculation, the significance of the difference (if any), and the sensitivity of the proposed method to those differences. Doing so, it is intended to answer both second and third investigative questions. The T-27 Tucano case study aims to verify the suitability of the proposed method for real world situations. This way, it is intended to answer the fourth investigative questions. Additionally, in describing the proposed method it is intended to answer the first and fifth investigative questions.

Conceptual Theoretical Experiment Design

The conceptual examples consist of giving ten different treatments to the Demand and time parameters in order to verify their effects on the EBO results. The first six treatments test different factors and levels for Demand with all other parameters (TTR and OST) kept the same. The last four treatments test different factors and levels for time parameters with Demand kept the same. In the experiments, different factors mean different probability distributions for Demand and time parameters; and different levels refer to the degrees of variance of the factors, which are defined as low variance (LV), or high variance (HV) for all ten treatments. Additionally, since TTR and OST are assumed delays in the pipeline channel, they are tested together, as they were only one parameter. A list of all factors and levels being tested is presented in Appendix E.

The ten treatments consist of the following steps:

1. Selecting Factors and Levels for Demand and Time Parameters. Candidate distributions for Demand (in fact, time between demands), TTR and OST should be recognized by the related literature as a suitable one. That means this research does not want to test just for testing purpose. It is intended to test possible candidates for "real world" situations. However, the intention is not to test the full range of possible distributions. Appendix E provides a list of all factors and levels being tested.

2. Setting the Simulation in Arena. Factors and levels are set up in an Arena simulation model (ahead described) developed for this research and presented in the appendixes A, B, and C. For each treatment, a depot stock level is previously defined as well as a range of base stock levels to be tested. That information is also presented in the Appendix E. Finally, the Arena simulation is set up to return a sample of 30 independent identically distribute replications, each one consisting of 5,000 weeks of running with 1,000 weeks of warm-up time.

3. Gathering Data from the Arena Reports. Data about EBO and the 95 % half-width confidence interval (CI) is collected from each running. The EBO from simulation is then considered the "true" EBO value from simulation.

4. Stating the EBO Simulation Values. The EBO from simulation is stated as a step-function, for future comparisons with both existing and proposed model (ahead defined).

After collecting the data from Arena reports, a comparison between EBO from existing models and simulation, as well as proposed model and simulation is performed for assessing which model (if any) is inside of the 95 % half-width CI of the "true" mean value from simulation for each base stock level tested. Additionally, the models (existing and proposed) are tested against the simulation values in order to verify which model generates the best approximation for each stock level tested. The best approximation consists of the algebraic difference (absolute value) between existing methods and simulation, and proposed method and simulation. Aiming to summarize the results of each one of the 10 experiments, the summation of the squared differences for both existing and proposed models are provided allowing assessing the closeness of each model as a whole.

Finally, all ten treatments refer to a theoretical first-indenture, multi-echelon model, with no condemnation, no lateral re-supply, and no cannibalization

Chapter IV presents a table for each treatment. Additionally, a resume of the first six treatments (testing demand parameter) and last four treatments (testing time parameters) are also presented.

The T-27 Tucano Case Study

The T-27 Tucano case study aims to check the suitability of the proposed method with EBO information collected from the field. Twenty reparable items of the T-27 Tucano program are previously selected by the Subdivision of Planning (TPLJ) at the PAMALS. For scope issues, the system is considered as composed by one depot (PAMALS) and two bases (AFA and CATRE). The experiment consists of the following steps:

1. Setting a Time Frame for Data Collection. The data necessary for EBO calculation is collected from a continuous period of one hundred days of normal operation, from August 13th to November 20th of 2001.

2. Getting the Data for METRIC EBO Calculation. The data includes information about Demand at base level; time to repair (TTR) at base and depot levels, ordering and ship time (OST) at system level, and stock level at bases and depot for all items listed in Appendix D.

3. Additional Information for the Proposed Method EBO Calculation. A sample of time between demands is also collected, since it is a requirement for Arena simulation.

4. Getting Information About EBO Numbers**.** In order to allow comparing the EBO numbers of both proposed and existing methods with those from the real world, information about backorder occurrence and backorder duration are collected for each one of the 20 reparable items, during the same period of one hundred days.

The data collected from the field is then used for calculating METRIC EBO. Additionally, TBD and TTR samples are worked in the Arena Input Analyzer in order to check the best distribution for them. The goodness-of-fit (GOF) test is discussed ahead. That is accomplished for each one of the 20 reparable items, and each location in the system.

Considering the stock level (both depot and bases) gathered from the PAMALS, the proposed method calculates the EBO value according to a methodology to be described in the next sections. Then, for each of the two bases (AFA and CATRE), and each item, a comparison between both existing and proposed models against the EBO information gathered from the PAMALS is performed. That comparison consists of measuring the algebraic difference between existing model's EBO number and the EBO information from the field, as well as proposed method's EBO number and the EBO information from the field. Finally, the summation of the squared difference between the existing models and the EBO information from the field, and the proposed method and EBO information from the field, for each item, summarizes the performance of the models (existing and proposed) for each base.

A Simulation Model in Arena

Assuming a simulation model can better capture the complexities existent in a RIS in regarding to EBO calculation, this research first proposes setting up a model in Arena to accomplish that calculation. The model should be able to calculate the expected backorder (EBO) for each level (base and depot), for first and second indenture items, given a stock level (s). Besides, in order to allow comparing the significance between both existing and proposed method, the model in Arena should return information about the 95 % half-width CI after each replication.

Only the model used in the experimental design (first indenture, multi echelon model) is going to be described here. The simulation models for single base (FISS) and for multi indenture single site (MISS) are pictured in the Appendixes. A and B.

First Indenture Multi Echelon Model Description

Since the first indenture multi echelon model in Arena was made to represent the basic METRIC model scenario described in the literature review, the following description follows a similar rationality. As a result of maintenance actions (corrective or preventive), demands for reparable items are created at the operational level, with batch size equals to one, according to a specified distribution expressed in terms of time between demands (TBD). For each demand created, two actions are taken: a requisition for a serviceable item goes to the base warehouse and the unserviceable item goes to the base repair shop. At the base warehouse, if the stock level for the item is greater than or equal to one, the demand is fulfilled, and the variable stock level at base are assigned, reducing it in minus 1. Otherwise, a base backorder is issued. At the base repair shop, the unserviceable item has a constant probability (r) of being repaired locally, and (1-r) probability of the item goes to the next echelon (depot) to be repaired. If the item is repaired locally, the base warehouse waits for its repair (or for any other that is already being repaired). Otherwise, the unserviceable item goes to the depot pipeline. In this case, the base supply service issues a requisition of one serviceable item to the depot. In the depot's portion of the pipeline, the requisition for a serviceable item goes to the warehouse and, if the stock level for the item is greater than or equal to one, the demand is fulfilled. Otherwise, a depot backorder is issued and waits for the next serviceable available from the depot repair shop in order to attend in a first-in-first-out (FIFO) rule the demand from the bases. In the cases a base orders an item from the depot, it will take a time for the base to receive the item. This is the ordering & ship time (OST) and it is another factor that affects the item pipeline. Note that there are only two types of entities passing through the model: demand for items, and demand for service. Additionally, global variables (variables that belong to all the system) were created to represent backorders and inventory level throughout the systems. The variables inventory level should be given "a priori" of a replication. The variables backorder are measured after each replication of the model. Statistical functions were inserted to collect and analyze the results of each replication. This is the FIME model pictured in Appendix C.

Assumptions of the Simulation Model

The following is a list of assumptions that apply to the simulation models (FISS, MISS, and FIME) above described:

a. Independent Demand. All models assume infinite source of population. That means demand generation is not related or linked to other factors, such as number of aircraft eventually available, number of items already in repair process, or fly hours. This assumption might not be true for "real world" circumstances; however, modeling such relationship would require tremendous field research and programming effort. One of the consequences of such assumption might be, if more items were not repaired (or have their repair time delayed for any reason) the actual demand would tend to decrease, *ceteris paribus*.

b. Repair Process. The decision as to whether a defective item is repaired on base or depot is based on a constant probability value, expressed as a percentage. That means whenever a base has a capability to repair an item, the repair process will be performed at base level, regardless of external factors such as maintenance workload, spare part availability and etc. This is usually true to repairable systems, at least in the Brazilian Air Force.

c. Repair Time. A corollary of the Repair Process assumption links it to the time to repair (TTR). Since workload is not an issue on the described models (observe that the repair times are described as simple delays), capacity is assumed infinite, and consequently one should not expect delays in the repair process caused by waiting-in-queue time. Since capacity throughout systems may not be infinite, as a result, the "real" time to repair would be greater than the one previously assumed. This issue will be addressed later on Chapter V, where this research will recommend ways of fixing (or minimizing) the effects of that assumption.

d. Inventory Policy. Once established the initial stock level values (s), the replenishment point will be s – 1. That means for each item demanded in the system, a replenishment action (which may include repair process with or without spare part application) is going to be taken. In other words, this is the inventory policy (s – 1, s).

e. Condemnations. It is assumed no condemnation as a result of the repair process. In other words, whenever a defective item passes through a repair process the result will be a “ready-for-use” item. Sometimes this is not true. Chances exist that the repair process becomes anti-economic due to several reasons, such as repair cost, unfeasibility of repairing, and etc. Thus, this research will address later on Chapter V ways of dealing with condemnations in the proposed method.

f. Lateral Re-supply. No lateral re-supply is assumed in the proposed method. This assumption may work negatively on the system’s metric. That means, if lateral re-supply takes place the predicted expected backorder would be greater than it really is. Since lateral re-supply is most of the time avoided by the Brazilian Air Force (due to cost of transportation issues), potential lateral re-supply will not be in the scope of this research.

g. Cannibalization. It is assumed no cannibalization in the described models. Cannibalization may result in labor hour misallocations. It should be avoided, but it does happen in the “real world”. Ignoring cannibalization, however, may result in similar situation as in the lateral re-supply assumption. Besides, rules for cannibalization are usually hard to be defined. Thus, it will not be in the scope of this research.

Those assumptions are the same of the basic METRIC assumptions (Sherbrooke, 1992:46). This was intentionally done in order to measure only the effects of the variability of demand and time parameters for the expected backorder calculation.

Following METRIC's assumptions allowed also the verification process that is going to be discussed in the next topic.

Arena Simulation Model Verification

Model's verification process was performed in two stages: attempting to run the model for finding out and correcting errors; and checking if the models were performing as designed.

The first stage, also called "debugging", was performed right after model's development. At this stage, problems like module's data entrance, and module's misunderstanding were solved consulting an Arena textbook (Simulation In Arena) and getting advice from AFIT's students more skilled on Arena (Captain Todd Bertullis).

The second stage verification process referred to verifying if the model would behave as it was supposed to do. That task was performed basically by running the models for beforehand known examples got from Sherbrooke's textbook (Optimal Inventory Modeling Planning), as well as solved-in-class exercises got from handouts of the course Reparable Inventory Management (LOGM 628). In all cases the model provided with EBO answers deviated less than 0.01 from the beforehand known answers. Additionally, changes were made in the EBO parameters of those examples found in the textbook, such as "increasing and decreasing repair time", and "increasing and decreasing demand rate". Then, "what-if" analysis was carried out in order to check model's reaction to the changes. No major problem was found, except for some cases where demand rate was increased to a level not supported by the student version of Arena 3.0, which has a limit of 150 entities in the model at the same time.

Arena Simulation Model Validation

The validation process, which is the task of ensuring that the model behaves the same as the real system (Sadowski, 1998:444), was performed basically by interviewing people of the Subdivision of Planning (TPLJ), Captain Vladimir and Lieutenant Marcio, about the correctness of the assumptions, logic of the paths, and the closeness to the reality of the models. An electronic copy of each model pasted in Word® format document was sent to the TPLJ personnel via electronic-mail, together with a detailed description of the function of each module used in the models. Additionally, a brief explanation about "how it works" of each model was also provided.

According to their opinion, the models could fairly represent reparable inventory systems managed by the PAMALS. However, they pointed out the following problems that could threaten the models' validity:

1. Cannibalization Issues. Captain Vladimir has pointed out cannibalization practice takes place throughout the system but it couldn't be accessed since in most of the times the process happens informally. He said: "frequently, people in charge of aircraft maintenance, trying to expedite maintenance actions, simply exchange spare parts between aircrafts without an accurate control". Additionally, "many of the reparable items managed by the PAMALS have no serial number recorded on it. They are controlled as a total, not individually. That makes it difficult to control." However, since cannibalization usually benefits the system in terms of reducing backorders, such practice is not prohibited. Bottom line, Captain Vladimir said it was not possible to define clearly rules for cannibalization modeling. Thus, cannibalization practices were considered out of the models' scope.

2. Inventory Policy. The inventory policy for reparable items at the PAMALS follows the standard (s – 1, s) for the majority of the items. However, Captain Vladimir stressed out that sometimes operators ask and the TPLJ authorizes additional leveling. He couldn't provide major details of how that happens.

3. Back Order Control. Currently, the PAMALS operates a software called M2421, which is supposed to manage inventory information for both reparable and consumable items, linking the PAMALS and operators. That software was initiated there in the first semester of 2000. However, due to budgetary issues, problems such as follow-up consulting, and the personnel training program has not been a priority. Consequently, Lieutenant Marcio said, “even after working out the basic data collected to this research, he couldn’t guarantee more than 90 % of accuracy for back order information, since the reports of the M2421 does not provide automatically information about backorder length ”.

Possible effects coming from problems above listed will be addressed later on Chapter V.

Defining the Proposed Method for EBO Calculation

The proposed method, called P-METRIC, is a mix of simulation and mathematical analytical model for EBO calculation. Its simulation portion refers to a model developed in Arena environment, which is designed to calculated, among other information, expected backorder numbers for multi echelon, multi indenture, and multi location reparable item systems. Due to didactic reasons, the simulation portion was split into three different models, first indenture single site (FISS) model, multi indenture single site (MISS) model, and first indenture multi echelon (FIME) model, which are presented in the Appendix A to C. The mathematical analytical portion uses the same formulation of METRIC, described in the Chapter II, however, instead of use Poisson distribution, the P-METRIC proposes the use of Gamma distribution to express the number of items in pipeline. Reasons for that are also provided in the coming topics.

Describing the Critical Point (CP)

From the observation of several replications of the simulation models previously discussed, it was noted that the EBO curve of the simulation model has a common behavior when compared with its related mathematical model. Explaining: take for example the first indenture single site model (FISS), Appendix A, which is a representation of the Base Stockage model. Running the model for stock level values ranging from zero to S, one may find that for stock level equal to zero, both existing and simulation models agree on the EBO results (or the difference is not significant, since simulation return a mean and a confidence interval). As long as stock level (S) increases, the difference also increases. Because the EBO numbers of both METRIC and simulation model tend to zero, there will be a point where the difference is maximal. After this point, the difference tends to decrease and for big values of S the models tend to agree in EBO equal to (or approximately) zero. The point of maximal difference hereafter is called critical point (CP).

Similar behavior is observed for multi-echelon situation with just one difference: in the multi-echelon situation, when stock level (S_i) at base equals to zero, EBO from simulation will agree with EBO from METRIC only if the stock level at depot (S_0) also equals to zero. Otherwise, they will disagree and, as it will be demonstrated, the difference can be significant. How to calculate the CP as well as its importance to the proposed method is going to be discussed shortly.

The Importance of the Critical Point for the Proposed Method

The behavior above described indicates that, if the EBO formula described in the Chapter II could be worked mathematically, it would be possible to shift the EBO curve, in order to force it to pass through two points: the initial point, for S = 0, and the critical point, which S is still unknown. The EBO curve resultant of this change should tend toward zero for large values of S, like the EBO of METRIC and simulation do. This way, a modified EBO curve would potentially approximate to the "true" values of simulation. Note that, in the single site situation, for S = 0, EBO value equals to the pipeline mean, which is a corollary of the Little's law (refer to the Base Stockage model described in the Chapter II). The problem would be to find out the CP, that is, the stock level (S) and its respective EBO value in the point of maximal difference. Additionally, for multi-echelon situation, if the stock level at depot (S_0) is greater than or equal to 1, two replications of the simulation model are necessary: the first replication finds out the EBO for $S_i = 0$ (given a value for S_0); the second replication finds out the EBO for S_i = CP (given the same value for S_0). However, there is still a question: what is the S value of the critical point? The next topic discusses this issue.

How to Calculate s Value and EBO at the Critical Point

This research proposes two alternative ways of calculating the S value and its respective EBO number. Considering the single site situation, in the first way, one should start running the simulation model for S = 0, then increasing by one S-value, run the model again and again until to reach the point where the difference start to shrink. The CP would be the point immediately before the difference start decreasing. Note that S is a discrete variable; therefore no fractional values should be assigned to it when simulating. Depending on the characteristics of the simulation model, this could be tremendously time consuming. Another way of looking for the CP was a "non-expected" finding that came up after running the simulation model and comparing the EBO results with those of METRIC model, for several different systems. The rule is quite simple: the critical point is defined as the point where S equals to the initial value of the pipeline mean, or better approximates to it, since pipeline mean is not necessarily an integer value. In other words, the S-value of the critical point is equal (or approximated) to the value of the pipeline for S = 0. With that information, instead of running the simulation model several times to find out the point of maximal difference, one may run the model only for that value of S, and then get the EBO value from the replication. Comparing it with EBO from METRIC, one may find this is the point of maximal difference between METRIC and the simulation model. Extending the concept to multi-echelon situation, if the stock level at depot (S_0) equal to zero, the rule is the same of the single-site situation. Otherwise, there is a need of running the simulation model for stock level at base equals to zero ($S_i = 0$), and stock level at depot equals to N ($S_0 = N$) being N an integer number greater than or equal to 1 that represents the given stock at depot. The EBO from this replication will be used to approximate to the S_i value of the critical point. Then, a second running is used to find out the EBO at the critical point.

Unfortunately, this research cannot provide with a mathematical proof of that, however, pre-experimental replications of the simulation models has corroborated with that.

How to Use the CP Information in the Proposed Method

Assuming that the pipeline value can be Gamma distributed (this is a proposition of the P-METRIC model and it is going to be justified shortly). The following formula represents the probability distribution of a variable Gamma distributed (Devore, 1999:172):

$$f(x) = \begin{cases} \dfrac{1}{\beta^{\alpha}\cdot\Gamma(\alpha)}\cdot x^{\alpha-1}\cdot e^{\frac{-x}{\beta}} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \qquad (3.1)$$

Where: x = Gamma variable.
$f(x)$ = pdf of the Gamma variable.
α and β = parameters of location.
e = natural logarithm base.
Γ = multiplication signal.

The α and β, according to Prof Daniel Reynolds' statistics classes, are parameters of location that ultimately define the mean and variance of the Gamma variable. Recalling the EBO formula of METRIC discussed in the Chapter II, now this research proposes the use of Gamma distribution instead of Poisson distribution for describing the pipeline values. The choice for Gamma distribution is due to the ability of that distribution of assuming different shapes (Devore, 1999:173). Adjusting the two parameters of Gamma (α and β) would allow, for example, changing the shape of the curve according to inputs gathered from the critical point. That would not be possible for Poisson distribution, given that Poisson has just one parameter. Re-writing the EBO formula presented in the Chapter II, the proposed EBO is expressed as:

$$\mathrm{EBO}(s) := \sum_{x=s+1}^{\infty} (x-s)f(x) \qquad (3.2)$$

Where: s = stock level.
x = pipeline variable.
$f(x)$ = pdf of the pipeline variable, assumed Gamma distributed.

Note that the proposed method formula is the same of the METRIC formulation presented in the Chapter II except that the pipeline function is now assumed Gamma distributed, instead of Poisson.

A system of equations is set in order to solve the system of equations below for α and β. The example used refers to a single site situation.

$$\mathrm{EBO}(S = 0) = \sum_{x = 0+1}^{\infty} (x - 0)\cdot\frac{1}{\beta^{\alpha}\cdot\Gamma(\alpha)}\cdot x^{\alpha-1}\cdot e^{\frac{-x}{\beta}} \tag{3.3}$$

$$\mathrm{EBO}(S = CP) = \sum_{x = CP+1}^{\infty} (x - CP)\cdot\frac{1}{\beta^{\alpha}\cdot\Gamma(\alpha)}\cdot x^{\alpha-1}\cdot e^{\frac{-x}{\beta}} \tag{3.4}$$

Where: S = stock level.
EBO (S=0) = mean pipeline value.
x = pipeline variable.
CP = integer approximation of the pipeline mean value.
α, β, Γ incognita already explained.

Although the system of equations above may seem to be very complex of solving, one may find this is not true when mathematical software, like MathCad (used in this research), are available. Note that there are two incognita variables (x is an index variable), α and β (parameters of Gamma distribution), and two known EBO values. The first one, EBO (S=0), is a corollary of Little's law. The second one, EBO (S = CP) is got from the simulatio n running. Next step is to plug the α and β values in the EBO formula and the proposed method EBO function is ready to generate EBO values, given a stock level S.

A comparison of the results of the proposed method, METRIC and simulation model for EBO calculation will be carried out later on Chapter IV.

Considerations About Data Collection and Analysis

The data used in the array of conceptual examples was chosen intending to explore didactic issues. In selecting the data, this research had two main objectives: first, to show that different distributions for demand and time parameters in RISs may lead to different EBO values when compared to existing models; second, to demonstrate it is possible (at this point, a level of hypothesis) to model complex RIS with assumptions about demand and time parameters different from those of METRIC models. Therefore, the data used in those examples are fictitious, didactic data. The methodology of selecting the fictitious data can be said as part of the verification process of the proposed (P-METRIC) method.

T-27 Data Collection

The data used in the T-27 Tucano case study was sampled from a set of initially 25 first indenture reparable items of the T-27 Tucano aircraft program. Preliminary analysis reduced that number to 20 items due to one or more problems related to the profiles described below. A list of the 20 elected items is provided in Appendix D.

Together with the PAMALS, the two major T-27 Tucano's operator, AFA and CATRE, were chosen to participate of this research. The data was collected from a continuous period of one hundred days of normal operation, from August 13th to

November 20th of 2001. No differentiation between corrective and preventive maintenance was done. The information collected referred to parameters of interest for EBO calculation. It also included a sample of time between demands (TBD) at base level, time to repair (TTR) at base and depot levels, ordering and ship time (OST) at system level, and stock level at bases and depot for all items listed in Appendix D. Additionally, in order to allow comparing the EBO numbers of both proposed and existing methods with those from the real world, information about backorder occurrence and backorder duration was collected for each one of the 20 reparable items, during the same period of one hundred days.

The items were selected based on the following initial profiles:

1. Demand Information. The selected items should have good information about demand in the bases and depot levels. The "goodness" of the information refers to aspects both qualitative and quantitative. Items with too few demands in the research period were avoided (quantitative aspect), as well as items whose demands were not reliable for a variety of reasons (qualitative aspect).

2. Time Parameters. Items with chronic repair problems, such as abnormal delay in repair due to equipment breakdowns, were ruled out of the list.

Data Analysis

The data collected was treated statistically in order to fit in both proposed and existing methods. In analyzing the data, the Arena Input Analyzer was used in order to select the best theoretical distribution to the parameters used in the proposed and existing methods.

In preparing the data to get into the METRIC model, the following steps were taken:

1. The mean demand was calculated for each base (AFA and CATRE) according to information gathered from the PAMALS.

2. The same procedure was taken for the Depot TTR. Important to highlight that, according to information from Capt Vladimir, TPLJ at the PAMALS, none of the selected item can be repaired at base. In fact, as he said, the base repair capability could be ignored due two reasons: first, the chance of repairing at base are very small; second, the flight line maintenance only request an serviceable item from the local supply service if the unserviceable item should go to the depot. This situation will be commented later in Chapter V.

3. Since no information about ordering and ship time was provided from the PAMALS, OST is assumed, with the agreement of the PAMALS, as being four days for AFA, and six days for CATRE. This will be also commented in Chapter V.

In preparing the data to get into the proposed method (P-METRIC), the procedures were similar to the METRIC above, except that the same data was submitted to a goodness-of-fit (GOF) test in order to select the best theoretical distribution. The treatment also included a request for measures of variability (standard deviation) of the data.

The Arena Input Analyzer's Goodness of Fit (GOF) Test. The Arena Input Analyzer provided with a "built-in" Kolmogorov-Smirnov GOF test. It automatically selects the theoretical distribution for the data. It also provides with mean, standard deviation of the data and the p-value of the test. The P-value (observed significance level) refers to the smallest level of significance at which the null hypothesis (to be defined ahead) would be rejected (Devore, 1999:341). The two-sided Kolmogorov-Smirnov GOF tests were performed as following (Conover, 1980:346):

1. Data: The data consist of a random sample of values, of size n, associated with a still unknown distribution function, denoted by F(x).

2. Hypothesis: Let G(x) be a completely specified hypothesized distribution function. Thus, the hypothesis are:

   Ho: F(x) = G(x) Null hypothesis

   Ha: F(x) # G(x) Alternative hypothesis

3. Decision Rule: The Arena Input Analyzer automatically returns to the "best-fitted" theoretical distribution. However, to ensure that the theoretical really represents the real data, this research followed a rule suggested by Sadowski. He says:

   If the p-values for one or more distributions are fairly high (e.g. 0.10 or greater), then you can use a theoretical distribution and have a fair degree of confidence that you're getting a good representation of the data (unless your sample size is quite small, in which case the discriminatory power of goodness-of-fit tests is quite weak). If the p-values are low, you may want to use an empirical distribution to better capture the characteristics of the data. (Sadowski, 1998:137)

Summary

In this chapter, the methodology used to model the proposed method was presented and discussed. The ten conceptual experimenting designs as well as the T-27 Tucano case study was described as the tool for verification and validation of the proposed method. Then, the simulation portion of the proposed method was introduced, followed by the mathematical analytical approach. Finally, considerations about data collection for the T-27 Tucano case study were presented and discussed.

Chapter IV will compare the results of both experimental design and the T-27 Tucano case study, in order to answering the investigative questions described earlier.

IV. Results

Chapter Overview

Chapter IV presents the results of each one of the 10 conceptual experiments summarized in the Appendix E. The results of the T-27 Tucano case study is also presented. Then, those results are interpreted and used to help answering the investigative questions.

Results of the Conceptual Experiments

The results of the ten conceptual experiments are presented in the next tables. In the experiments one to six, different treatments for demand variability and demand distribution were applied, but keeping the same mean. All other parameters (Base TTR, Depot TTR, and OST) were kept the same for all six experiments. In the experiments seven to ten, the time parameters were tested all together. All of them refer to first indenture multi echelon (FIME) model. EBO values are calculated for a range of stock levels (s) at base from zero to a minimum of eight, given a depot stock level, using METRIC, P-METRIC and Simulation. For each base stock level, the absolute algebraic differences between METRIC and Simulation (M-S), and P-METRIC (P-S) are presented. The difference between METRIC and P-METRIC is also calculated. A 95% half-width confidence interval (CI) gathered from the replications of the Simulation in Arena is provided followed by the information of which model (METRIC and/or P-METRIC), if any, is inside of the confidence interval. The last column states for each stock level which model (METRIC or P-METRIC) is closer to the simulation values. More details are provided before each table.

Parameters used for METRIC EBO computation:

Demand rate at base = 2.1 items per week.
Probability of repair at base = 0.6.
Probability of repair at depot = 1 – 0.6 = 0.4.
Base repair time = 1.2 weeks.
Depot repair time = 3.5 weeks.
Ordering and ship time = 1.2 weeks.
Depot stock level = 3.
Base stock level ranging from zero to eight.

Additionally, the following information is used in the P-METRIC EBO calculation:

Mean time between demands (TBD) = 0.476 (1/2.1) week, normally distributed.
TBD standard deviation = 0.05.
Mean base time to repair (BTTR) = 1.2 week, lognormal distributed.
BTTR standard deviation = 1.2.
Mean depot time to repair (DTTR) = 3.5 weeks, lognormal distributed.
DTTR standard deviation = 3.
Mean ordering & ship time (OST) = 1.2 weeks gamma distributed, α = 1.2 and β = 1.
The following table shows the result of the Experiment 01.

Table 2. Results of the Experiment 01

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	3.1579	3.0589	3.0591	0.09882	0.0002	0.0990	0.01718	OUTSIDE	INSIDE	P-METRIC
1	2.2004	2.0568	2.0699	0.13054	0.0131	0.1436	0.01695	OUTSIDE	INSIDE	P-METRIC
2	1.3772	1.1527	1.1798	0.19741	0.0271	0.2245	0.01503	OUTSIDE	OUTSIDE	P-METRIC
3	0.7660	0.5438	0.54389	0.22207	0.0001	0.2222	0.01049	OUTSIDE	INSIDE	P-METRIC
4	0.3779	0.2241	0.20284	0.17502	0.0212	0.1538	0.00566	OUTSIDE	OUTSIDE	P-METRIC
5	0.1659	0.0833	0.06257	0.10336	0.0207	0.0826	0.00245	OUTSIDE	OUTSIDE	P-METRIC
6	0.0653	0.0286	0.01616	0.0491	0.0125	0.0367	9.47E-04	OUTSIDE	OUTSIDE	P-METRIC
7	0.0232	0.0092	0.00344	0.01971	0.0058	0.0140	3.73E-04	OUTSIDE	OUTSIDE	P-METRIC
8	0.0075	0.0028	0.00068	0.00678	0.0022	0.0047	1.84E-04	OUTSIDE	OUTSIDE	P-METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of the Experiment 01. The absolute difference between the models followed by the

95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % half-width CI, and which of them is closer to the simulation mean value. The summation of the squared difference between METRIC and simulation for the range of s values tested is 0.10017. The summation of the squared difference between P-METRIC and simulation is 0.0020. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following graphs picture the EBO curves for METRIC, P-METRIC and Simulation model. It refers to the Experiment 01.

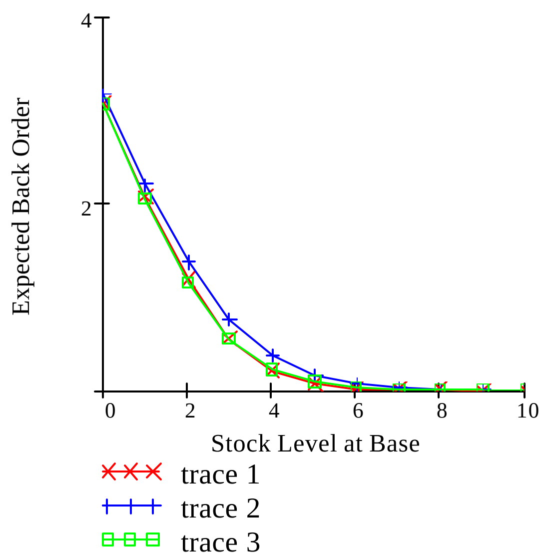


Figure 4. EBO Curves for Experiment 01

The graph above, Figure 04, represents the EBO curve at base level considering METRIC, P-METRIC, and the simulation models (step function created in MathCad in order to compare the models). The x-coordinate (abscissa) axis represents stock level at base, while y-coordinate (ordinate) axis represents EBO values. The following terminology applies to the graph interpretation:

Stock level at base = Range of values used for comparing the models.

Trace 1 = EBO values of simulation expressed as a function of stock level at base.

Trace 2 = EBO values of METRIC expressed as a function of stock level at base.

Trace 3 = EBO values of P-METRIC expressed as a function of stock level at base. A plot of the absolute difference (M-S, and P-S) for the Experiment 01 is provided in the following graph. The x-coordinate (abscissa) axis represents stock levels, while y-coordinate (ordinate) axis represents the absolute difference between EBO values of Simulation and METRIC (M-S), and between EBO values of Simulation and P-METRIC (P-S).

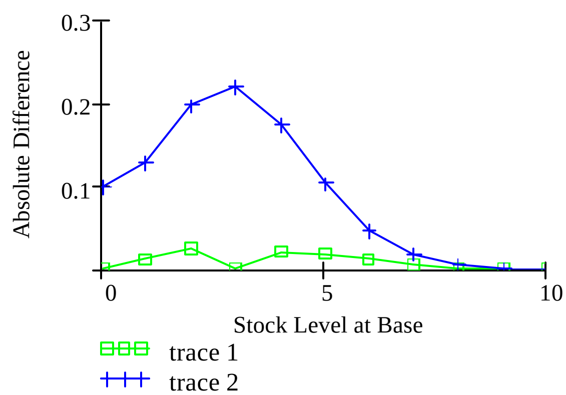


Figure 5. Absolute Dif. Between METRIC/P-METRIC and Simulation for Exp. 01

The graph on figure 05 refers to the absolute difference between both models (METRIC and P-METRIC) and the simulation model, where:

Trace 1 = Difference between EBO values of Simulation and P-METRIC (P-S).

Trace 2 = Difference between EBO values of Simulation and METRIC (M-S).

The following table refers to the Experiment 02. The parameters for METRIC EBO calculation are the same of the Experiment 01. Additionally, the parameters for P-METRIC EBO calculation are the same of the Experiment 01 except for:

TBD standard deviation = 0.135.

Base stock level ranging from zero to eight.

Table 3. Results of Experiment 02

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	3.1579	3.0712	3.0711	0.08682	0.0000	0.0867	0.01576	OUTSIDE	INSIDE	P-METRIC
1	2.2004	2.0687	2.084	0.11644	0.0154	0.1317	0.01563	OUTSIDE	INSIDE	P-METRIC
2	1.3772	1.1745	1.2022	0.17501	0.0277	0.2027	0.01423	OUTSIDE	OUTSIDE	P-METRIC
3	0.766	0.5708	0.57077	0.19519	0.0000	0.1952	0.01106	OUTSIDE	INSIDE	P-METRIC
4	0.3779	0.2458	0.22369	0.15417	0.0221	0.1321	0.00675	OUTSIDE	OUTSIDE	P-METRIC
5	0.1659	0.0966	0.07363	0.0923	0.0230	0.0693	0.00339	OUTSIDE	OUTSIDE	P-METRIC
6	0.0653	0.0354	0.02061	0.04465	0.0148	0.0299	1.57E-03	OUTSIDE	OUTSIDE	P-METRIC
7	0.0231	0.0123	0.0049	0.01825	0.0074	0.0108	6.88E-04	OUTSIDE	OUTSIDE	P-METRIC
8	0.0075	0.0041	0.00102	0.00644	0.0030	0.0034	2.73E-04	OUTSIDE	OUTSIDE	P-METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 02. The absolute difference between the models followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % half-width CI, and which of them are closer to the simulation mean value. The summation of the squared difference between METRIC and simulation for the range of values tested is 0.0788426. The summation of the squared difference between P-METRIC and simulation is 0.0023. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table refers to Experiment 03. The parameters for METRIC EBO calculation are the same of the previous experiments. Additionally, the following information is used in the P-METRIC EBO calculation:

Mean time between demands (TBD)= 0.476 week, gamma distributed, parameters α = 0.476 and β = 1.
Mean base time to repair (BTTR) = 1.2 week, lognormal distributed.
BTTR standard deviation = 1.2.
Mean depot time to repair (DTTR) = 3.5 weeks, lognormal distributed.
DTTR standard deviation = 3.
Mean ordering & ship time (OST) = 1.2 weeks gamma distributed, parameters α = 1.2 and β = 1.
Base stock level ranging from zero to eight.

Table 4. Results of Experiment 03

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	3.15792	3.2329	3.2329	0.07498	0.0000	-0.0750	0.01754	OUTSIDE	INSIDE	P-METRIC
1	2.20044	2.28583	2.3358	0.13536	0.0500	-0.0854	0.01667	OUTSIDE	OUTSIDE	P-METRIC
2	1.37721	1.55722	1.5975	0.22029	0.0403	-0.1800	0.01491	OUTSIDE	OUTSIDE	P-METRIC
3	0.76596	1.0366	1.0366	0.27064	0.0000	-0.2706	0.01217	OUTSIDE	INSIDE	P-METRIC
4	0.37786	0.67906	0.63998	0.26212	0.0391	-0.3012	0.00949	OUTSIDE	OUTSIDE	P-METRIC
5	0.16593	0.4396	0.37713	0.2112	0.0625	-0.2737	0.00718	OUTSIDE	OUTSIDE	P-METRIC
6	0.06526	0.28199	0.21171	0.14645	0.0703	-0.2167	5.32E-03	OUTSIDE	OUTSIDE	P-METRIC
7	0.02315	0.17958	0.11392	0.09077	0.0657	-0.1564	4.06E-03	OUTSIDE	OUTSIDE	P-METRIC
8	0.00746	0.11368	0.0589	0.05144	0.0548	-0.1062	3.14E-03	OUTSIDE	OUTSIDE	METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 03. The absolute difference between the models followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % half-width CI, and which of them are closer to the simulation mean value. The summation of the squared difference between METRIC and simulation for the range of values tested is 0.212495. The summation of the squared difference between P-METRIC and simulation is 0.0218. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table refers to Experiment 04. The parameters for METRIC EBO calculation are the same of the previous experiments. Parameters for P-METRIC EBO calculation are the same of the Experiment 03 except for:

Mean TBD = 0.476 week (same of the previous experiment) gamma distributed, parameters α = 0.0476 and β = 10.
Base stock level ranging from zero to 10.

Table 5. Results of Experiment 04

S	EBO			ABS DIFFERENCE		M -P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	3.15792	3.72407	3.7241	0.56618	0.0000	-0.5662	0.10362	OUTSIDE	INSIDE	P-METRIC
1	2.20044	3.0291	3.1058	0.90536	0.0767	-0.8287	0.09109	OUTSIDE	INSIDE	P-METRIC
2	1.37721	2.51169	2.589	1.21179	0.0773	-1.1345	0.07941	OUTSIDE	INSIDE	P-METRIC
3	0.76596	2.10331	2.1487	1.38274	0.0454	-1.3374	0.06869	OUTSIDE	INSIDE	P-METRIC
4	0.37786	1.77268	1.7727	1.39484	0.0000	-1.3948	0.05891	OUTSIDE	INSIDE	P-METRIC
5	0.16593	1.50099	1.4541	1.28817	0.0469	-1.3351	0.05038	OUTSIDE	INSIDE	P-METRIC
6	0.06526	1.2755	1.1848	1.11954	0.0907	-1.2102	4.26E-02	OUTSIDE	OUTSIDE	P-METRIC
7	0.02315	1.08701	0.959	0.93588	0.1280	-1.0639	3.57E-02	OUTSIDE	OUTSIDE	P-METRIC
8	0.00746	0.92857	0.7709	0.76344	0.1577	-0.9211	2.98E-02	OUTSIDE	OUTSIDE	P-METRIC
9	0.0022	0.79481	0.6159	0.61373	0.17888	-0.7926	0.02482	OUTSIDE	OUTSIDE	P-METRIC
10	0.0006	0.68148	0.4889	0.48827	0.19261	-0.6809	0.0206	OUTSIDE	OUTSIDE	P-METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 04. The absolute difference between the models, followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % half-width CI, and which of them are closer to the simulation mean value. The summation of the squared difference between METRIC and simulation for the range of values tested is 9.186618. The summation of the squared difference between P-METRIC and simulation is 0.13468. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table refers to Experiment 05. The parameters for METRIC EBO calculation are the same of the previous experiments. Additionally, the following information is used in the P-METRIC EBO calculation:

Mean time between demands (TBD)= 0.476 week lognormal distributed.
TBD standard deviation = 0.5.
Mean base time to repair (BTTR) = 1.2 week, lognormal distributed.
BTTR standard deviation = 1.2.
Mean depot time to repair (DTTR) = 3.5 weeks, lognormal distributed.
DTTR standard deviation = 3.
Mean ordering & ship time (OST) = 1.2 weeks gamma distributed, parameters α = 1.2 and β = 1.
Base stock level ranging from zero to eight.

Table 6. Results of Experiment 05

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	**P-METRIC**	**SIM**	**M-S**	**P-S**			**METRIC**	**P-METRIC**	
0	3.15792	3.1573	3.1573	0.00062	0.0000	0.0006	0.02421	INSIDE	INSIDE	P-METRIC
1	2.20044	2.17103	2.2193	0.01886	0.0483	0.0294	0.02348	INSIDE	OUTSIDE	METRIC
2	1.37721	1.37653	1.4215	0.04429	0.0450	0.0007	0.02135	OUTSIDE	OUTSIDE	METRIC
3	0.76596	0.82483	0.82483	0.05887	0.0000	-0.0589	0.01742	OUTSIDE	INSIDE	P-METRIC
4	0.37786	0.47458	0.43382	0.05596	0.0408	-0.0967	0.01221	OUTSIDE	OUTSIDE	P-METRIC
5	0.16593	0.26491	0.20783	0.0419	0.0571	-0.0990	0.00772	OUTSIDE	OUTSIDE	METRIC
6	0.06526	0.14445	0.0911	0.02584	0.0534	-0.0792	4.65E-03	OUTSIDE	OUTSIDE	METRIC
7	0.02315	0.07732	0.03679	0.01364	0.0405	-0.0542	2.78E-03	OUTSIDE	OUTSIDE	METRIC
8	0.00746	0.04077	0.01382	0.00636	0.0270	-0.0333	1.65E-03	OUTSIDE	OUTSIDE	METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 05. The absolute difference between the models followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % half-width CI, and which of them are closer to the simulation mean value. The summation of the squared difference between METRIC and simulation for the range of values tested is 0.008099. The summation of the squared difference between P-METRIC and simulation is 0.01497. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table refers to Experiment 06. The parameters for METRIC EBO calculation are the same of the previous experiments. Parameters for P-METRIC EBO calculation are the same of the Experiment 05 except for:

TBD standard deviation = 2.5.
Base stock level ranging from zero to eight.

Table 7. Results of Experiment 06

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	3.15792	3.5934	3.5934	0.43548	0.0000	-0.4355	0.07044	OUTSIDE	INSIDE	P-METRIC
1	2.20044	2.79652	2.897	0.69656	0.1005	-0.5961	0.06019	OUTSIDE	OUTSIDE	P-METRIC
2	1.37721	2.20213	2.3055	0.92829	0.1034	-0.8249	0.05159	OUTSIDE	OUTSIDE	P-METRIC
3	0.76596	1.74509	1.8051	1.03914	0.0600	-0.9791	0.04381	OUTSIDE	OUTSIDE	P-METRIC
4	0.37786	1.3887	1.3887	1.01084	0.0000	-1.0108	0.03689	OUTSIDE	INSIDE	P-METRIC
5	0.16593	1.10847	1.0498	0.88387	0.0587	-0.9425	0.03066	OUTSIDE	OUTSIDE	P-METRIC
6	0.06526	0.88687	0.7797	0.71444	0.1072	-0.8216	0.02517	OUTSIDE	OUTSIDE	P-METRIC
7	0.02315	0.71091	0.56813	0.54498	0.1428	-0.6878	0.02039	OUTSIDE	OUTSIDE	P-METRIC
8	0.00746	0.57075	0.40631	0.39885	0.1644	-0.5633	0.01633	OUTSIDE	OUTSIDE	P-METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 06. The absolute difference between the models, followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % half-width CI, and which of them are closer to the simulation mean. The summation of the squared difference between METRIC and simulation for the range of values tested is 4.174465. The summation of the squared difference between P-METRIC and simulation is 0.0867. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table summarizes the results of the experiments 1 to 6. The objective is to verify the effects of different treatments on Demand distribution and variance for EBO calculation. Three types of time between demands distributions are tested. Two levels of variance are assigned. The percentage of the times each model is inside of the 95 % half-width CI is calculated dividing the number of times it appears inside of the confidence interval for each treatment. The sum of the squared difference between METRIC/P-METRIC and the simulation model is also presented.

Table 8. Effects of Variability in Demand

TREATMENT	FACTOR TESTED	LEVEL OF VARIANCE	% INSIDE INTERVAL TEST		SUM SQUARED DIF	
	DEM. DIST		METRIC	P-METRIC	METRIC	P-METRIC
1	NORMAL	LV	0	14.28	0.10017	0.002
2	NORMAL	HV	0	14.28	0.07884	0.0023
3	GAMMA	LV	0	0	0.212495	0.0218
4	GAMMA	HV	0	44.44	9.1866	0.13468
5	LOGNORMAL	LV	0	0	0.008099	0.01497
6	LOGNORMAL	HV	0	0	4.174465	0.0867
TOTAL PERCENTAGE/SUM SQ DIF			0	13.63	13.7605	0.26245

Table 10 shows that, for the six treatments of Demand, METRIC is inside the 95 % half-width CI in 0 % of the times. P-METRIC is inside the 95 % half-width CI in 13.63 % of the times, already excluded the two anchor points of P-METRIC approximation. Additionally, the total sum of the squared difference between simulation and METRIC is 13.7605. The total sum of the difference between simulation and P-METRIC is 0.26245. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The next four tables (7 to 10) refer to different treatments applied to TTR and OST. Since these parameters are designed as delays in the simulation model, it is assumed that the variability may affect the EBO calculation similarly, so the treatments are applied on both together. More details are provided before each table.

The following table refers to Experiment 07.

Parameters used for METRIC EBO computation:

Demand rate at base = 4 items per week.
Probability of repair at base = 0.6.
Probability of repair at depot = 1 – 0.6 = 0.4.
Base repair time = 1.2 weeks.
Depot repair time = 3.5 weeks.
Ordering and ship time = 1.2 weeks.
Depot stock level = 6.
Base stock level ranging from zero to 13.

Additionally, the following information is used in the P-METRIC EBO calculation:

Mean time between demands (TBD)= 0.25 week lognormal distributed.
TBD standard deviation = 1.5.
Mean base time to repair (BTTR) = 1.2 week, lognormal distributed.
BTTR standard deviation = 1.2.
Mean depot time to repair (DTTR) = 3.5 weeks, lognormal distributed.
DTTR standard deviation = 3.
Mean ordering & ship time (OST) = 1.2 weeks gamma distributed, parameters α = 1.2 and β = 1.

Base stock level ranging from zero to 13.

Table 9. Results of Experiment 07

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	5.5551	5.5748	5.5748	0.0197	0.0000	-0.0197	0.09606	INSIDE	INSIDE	P-METRIC
1	4.559	4.7128	4.8393	0.2803	0.1265	-0.1538	0.08596	OUTSIDE	OUTSIDE	P-METRIC
2	3.5843	4.0084	4.1856	0.6013	0.1772	-0.4241	0.07644	OUTSIDE	OUTSIDE	P-METRIC
3	2.6694	3.4212	3.5938	0.9244	0.1726	-0.7518	0.0676	OUTSIDE	OUTSIDE	P-METRIC
4	1.8649	2.927	3.0589	1.194	0.1319	-1.0621	0.05943	OUTSIDE	OUTSIDE	P-METRIC
5	1.2139	2.5088	2.5792	1.3653	0.0704	-1.2949	0.05176	OUTSIDE	OUTSIDE	P-METRIC
6	0.7334	2.1533	2.1533	1.4199	0.0000	-1.4199	4.48E-02	OUTSIDE	INSIDE	P-METRIC
7	0.4107	1.8504	1.7799	1.3692	0.0705	-1.4397	3.86E-02	OUTSIDE	OUTSIDE	P-METRIC
8	0.2134	1.5916	1.4565	1.2431	0.1351	-1.3782	3.32E-02	OUTSIDE	OUTSIDE	P-METRIC
9	0.103	1.3702	1.1796	1.0766	0.1906	-1.2672	2.84E-02	OUTSIDE	OUTSIDE	P-METRIC
10	0.0463	1.1804	0.94562	0.8993	0.2348	-1.1341	0.02422	OUTSIDE	OUTSIDE	P-METRIC
11	0.0194	1.0175	0.75026	0.7308	0.2673	-0.9981	0.02054	OUTSIDE	OUTSIDE	P-METRIC
12	7.70E-03	0.8776	0.58905	0.5814	0.2886	-0.8699	0.01727	OUTSIDE	OUTSIDE	P-METRIC
13	2.80E-03	0.7573	0.45818	0.4554	0.2992	-0.7545	0.01436	OUTSIDE	OUTSIDE	P-METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 07. The absolute difference between the models, followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % CI, and which of them are closer to the simulation mean. The summation of the squared difference between METRIC and simulation for the range of values tested is 11.0516. The summation of the squared difference between P-METRIC and simulation is 0.45849. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table refers to the Experiment 08. The parameters for METRIC EBO calculation are the same of the previous experiments. Parameters for P-METRIC EBO calculation are the same as Experiment 07 except for:

Base TTR standard deviation = 5.1.
Depot TTR standard deviation = 12.
Mean ordering & ship time (OST) = 1.2 weeks gamma distributed, parameters $\alpha = 0.12$ and $\beta = 10$.

Base stock level ranging from zero to eight.

Table 10. Results of Experiment 08

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	5.5551	5.9126	5.9127	0.3576	0.0001	-0.3575	0.11314	OUTSIDE	INSIDE	P-METRIC
1	4.559	4.9436	5.0063	0.4473	0.0627	-0.3846	0.10644	OUTSIDE	INSIDE	P-METRIC
2	3.5843	4.0886	4.1887	0.6044	0.1001	-0.5043	0.09695	OUTSIDE	OUTSIDE	P-METRIC
3	2.6694	3.3564	3.4559	0.7865	0.0995	-0.6870	0.08702	OUTSIDE	OUTSIDE	P-METRIC
4	1.8649	2.7402	2.8179	0.953	0.0777	-0.8753	0.0779	OUTSIDE	INSIDE	P-METRIC
5	1.2139	2.2273	2.2688	1.0549	0.0415	-1.0134	0.06922	OUTSIDE	INSIDE	P-METRIC
6	0.7334	1.8041	1.8042	1.0708	0.0001	-1.0707	6.04E-02	OUTSIDE	INSIDE	P-METRIC
7	0.4107	1.457	1.4167	1.006	0.0403	-1.0463	5.19E-02	OUTSIDE	INSIDE	P-METRIC
8	0.2134	1.1738	1.0987	0.8853	0.0751	-0.9604	4.38E-02	OUTSIDE	OUTSIDE	P-METRIC
9	0.103	0.9436	0.84131	0.7383	0.1023	-0.8406	0.03655	OUTSIDE	OUTSIDE	P-METRIC
10	0.0463	0.7572	0.63569	0.5894	0.1215	-0.7109	0.03018	OUTSIDE	OUTSIDE	P-METRIC
11	0.0194	0.6066	0.47333	0.4539	0.1333	-0.5872	0.02475	OUTSIDE	OUTSIDE	P-METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 08. The absolute difference between the models, followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % CI, and which of them are closer to the simulation mean. The summation of the squared difference between METRIC and simulation for the range of values tested is 6.0992. The summation of the squared difference between P-METRIC and simulation is 0.08187. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table refers to the Experiment 09. Parameters used for METRIC EBO computation:

Demand rate at base = 4 items per week.
Probability of repair at base = 0.6.
Probability of repair at depot = 1 – 0.6 = 0.4.
Base repair time = 1.2 weeks.
Depot repair time = 2.3 weeks.

Ordering and ship time = 1.2 weeks.
Depot stock level = 6.
Base stock level ranging from zero to 15.

Additionally, the following information is used in the P-METRIC EBO calculation:

Mean time between demands (TBD) = 0.25 (1/4) week, lognormal distributed.
TBD standard deviation = 1.5.
Mean base time to repair (BTTR) = 1.2 week, normally distributed.
BTTR standard deviation = 0.12.
Mean depot time to repair (DTTR) = 2.3 weeks, normally distributed.
DTTR standard deviation = 0.2.
Mean ordering & ship time (OST) = 1.2 weeks lognormal distributed.
OST standard deviation = 1.2.

Table 11. Results of Experiment 09

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	4.9344	5.5748	5.5748	0.6404	0.0000	-0.6404	0.09606	OUTSIDE	INSIDE	P-METRIC
1	3.9416	4.7128	4.8393	0.8977	0.1265	-0.7712	0.08596	OUTSIDE	OUTSIDE	P-METRIC
2	2.9843	4.0084	4.1856	1.2013	0.1772	-1.0241	0.07644	OUTSIDE	OUTSIDE	P-METRIC
3	2.1146	3.4212	3.5938	1.4792	0.1726	-1.3066	0.0676	OUTSIDE	OUTSIDE	P-METRIC
4	1.389	2.927	3.0589	1.6699	0.1319	-1.5380	0.05943	OUTSIDE	OUTSIDE	P-METRIC
5	0.841	2.5088	2.5792	1.7382	0.0704	-1.6678	0.05176	OUTSIDE	OUTSIDE	P-METRIC
6	0.4685	2.1533	2.1533	1.6848	0.0000	-1.6848	0.04476	OUTSIDE	INSIDE	P-METRIC
7	0.2402	1.8504	1.7799	1.5397	0.0705	-1.6102	0.0386	OUTSIDE	OUTSIDE	P-METRIC
8	0.1136	1.5916	1.4565	1.3429	0.1351	-1.4780	0.0332	OUTSIDE	OUTSIDE	P-METRIC
9	0.0497	1.3702	1.1796	1.1299	0.1906	-1.3205	0.0284	OUTSIDE	OUTSIDE	P-METRIC
10	0.0202	1.1804	0.94562	0.9254	0.2348	-1.1602	0.02422	OUTSIDE	OUTSIDE	P-METRIC
11	0.0076	1.0175	0.75026	0.7426	0.2673	-1.0099	0.02054	OUTSIDE	OUTSIDE	P-METRIC
12	0.0027	0.8776	0.58905	0.5864	0.2886	-0.8749	0.01727	OUTSIDE	OUTSIDE	P-METRIC
13	0.0009	0.7573	0.45814	0.4572	0.2992	-0.7564	0.01436	OUTSIDE	OUTSIDE	P-METRIC
14	0.0003	0.6538	0.35289	0.3526	0.3009	-0.6535	0.01169	OUTSIDE	OUTSIDE	P-METRIC
15	8.23E-05	0.5647	0.26899	0.2689	0.2957	-0.5646	0.0094	OUTSIDE	OUTSIDE	METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 09. The absolute difference between the models, followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % CI, and which of them are closer to the simulation mean. The summation of the squared difference between METRIC and simulation for the range of values tested is 17.85501. The summation of the squared difference between P-METRIC and simulation is 0.63647. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table refers to the Experiment 10. The parameters for METRIC EBO calculation are the same of the Experiment 09. Parameters for P-METRIC EBO calculation are the same of the Experiment 09 except for:

Base TTR standard deviation = 0.35.
Depot TTR standard deviation = 0.7.
OST standard deviation = 4.
Base stock level ranging from zero to 15.

Table 12. Results of the Experiment 10

S	EBO			ABS DIFFERENCE		M - P	95% CI	INTERVAL TEST		CLOSE
	METRIC	P-METRIC	SIM	M-S	P-S			METRIC	P-METRIC	
0	4.9344	5.5887	5.5889	0.6545	0.0002	-0.6543	0.0905	OUTSIDE	INSIDE	P-METRIC
1	3.9416	4.6747	4.7305	0.7889	0.0558	-0.7331	0.08314	OUTSIDE	INSIDE	P-METRIC
2	2.9843	3.9099	4.0019	1.0176	0.0920	-0.9256	0.0748	OUTSIDE	OUTSIDE	P-METRIC
3	2.1146	3.27	3.3697	1.2551	0.0997	-1.1554	0.06651	OUTSIDE	OUTSIDE	P-METRIC
4	1.389	2.7348	2.8161	1.4271	0.0813	-1.3458	0.05857	OUTSIDE	OUTSIDE	P-METRIC
5	0.841	2.2871	2.3323	1.4913	0.0452	-1.4461	0.05134	OUTSIDE	INSIDE	P-METRIC
6	0.4685	1.9127	1.9128	1.4443	0.0001	-1.4442	0.04466	OUTSIDE	INSIDE	P-METRIC
7	0.2402	1.5995	1.5533	1.3131	0.0462	-1.3593	0.03872	OUTSIDE	OUTSIDE	P-METRIC
8	0.1136	1.3376	1.2492	1.1356	0.0884	-1.2240	0.03333	OUTSIDE	OUTSIDE	P-METRIC
9	0.0497	1.1185	0.99512	0.9454	0.1234	-1.0688	0.02849	OUTSIDE	OUTSIDE	P-METRIC
10	0.0202	0.9354	0.78531	0.7651	0.1501	-0.9152	0.02433	OUTSIDE	OUTSIDE	P-METRIC
11	0.0076	0.7822	0.6138	0.6062	0.1684	-0.7746	0.02066	OUTSIDE	OUTSIDE	P-METRIC
12	0.0027	0.6541	0.47514	0.4724	0.1789	-0.6514	0.01746	OUTSIDE	OUTSIDE	P-METRIC
13	0.0009	0.5469	0.36409	0.3632	0.1829	-0.5460	0.01468	OUTSIDE	OUTSIDE	P-METRIC
14	0.0003	0.4574	0.27593	0.2757	0.1814	-0.4571	0.01233	OUTSIDE	OUTSIDE	P-METRIC
15	8.23E-05	0.3824	0.20699	0.2069	0.1755	-0.3823	0.01034	OUTSIDE	OUTSIDE	P-METRIC

The table above shows the EBO values for METRIC, P-METRIC, and simulation model of Experiment 10. The absolute difference between the models, followed by the 95 % half-width CI is presented. It also shows whether or not the models (METRIC and P-METRIC) are inside of the 95 % CI, and which of them are closer to the simulation mean. The summation of the squared difference between METRIC and simulation for the range of values tested is 12.82809. The summation of the squared difference between P-METRIC and simulation is 0.2354. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

The following table summarizes the results of experiments 7 to 10. The objective is to verify the effects of different treatments on time parameters distribution and variance for EBO calculation. The considerations about the 95 % half-width CI, as well as the summation of the squared different are similar to the experiments one to six.

Table 13. Effects of Variability in Time Parameters

TREATMENT	FACTOR TESTED TTR/OST	LEVEL OF VARIANCE	% INSIDE INTERVAL TEST		SUM SQUARED DIF	
			METRIC	P-METRIC	METRIC	P-METRIC
7	LOGN/GAMMA	LV	0	0	11.0516	0.45849
8	LOGN/GAMMA	HV	0	40.00	6.0992	0.08187
9	NORMAL/LOGN	LV	0	0	17.85501	0.63647
10	NORMAL/LOGN	HV	0	14.28	12.82809	0.2354
TOTAL PERCENTAGE/SUM SQ DIF			0	13.04	47.8339	1.41223

Table 13 shows that, for the four treatments on time parameters above, METRIC is inside the 95 % half-width CI in 0 % of the times. P-METRIC is inside the 95 % half-width CI in 13.04 % of the times. Additionally, the total sum of the squared difference between simulation and METRIC is 47.8339. The total sum of the difference between simulation and P-METRIC is 1.41223. The anchor points where P-METRIC EBO curve is adjusted were kept out of this analysis.

Analysis of the Results of the Conceptual Experiments

The results of the conceptual experiments has demonstrated that the proposed approach can do a better job in calculating EBO for systems that do not match the assumptions of METRIC models. That does not mean that METRIC approach is not accurate. In fact, when the assumptions about demand (Poisson) are valid, METRIC has demonstrated to be a strong, accurate model. However, this study aimed to test the cases when the assumptions were not valid. Thus, considering the situations when the METRIC assumptions are not valid, the proposed method seems to be more accurate than METRIC.

By analyzing the results of the 10 conceptual experiments, this research intends to answer the investigative questions number 1, 2, 3, and 5. Investigative question number 4 will be answered based on the analysis of the T-27 Tucano case study.

Investigative Question 1. What is the best form for a mathematical model for EBO calculation that accounts for the stochastic aspects of the demand, time-to-repair and ordering-ship-time that may exist in reparable inventory systems? The use of simulation combined with mathematical model can result in more accurate number for EBO calculation if compared to customary mathematical approaches. The EBO results from the proposed method, even being most of the times outside of the 95 % half-width CI, have demonstrated to be closer to the mean of simulation, compared with existing models. On the other hand, the combined approach is more complex, requiring dealing with simulation and more professional mathematical software since this approach is both driven by simulation and works with non-tabled distribution. Thus, a trade-off between analysis considering needs for accuracy versus the cost in implement such approach should be evaluated.

Investigative Question 2. Do the stochastic aspects of demand and time parameters affect EBO calculation in the proposed method? Answer. In general variability in Demand, TTR, and OST do have an effect. The conceptual experiments 01 to 06 were designed to test the stochastic aspects of demand. The experiments 07 to 10 were designed to test the stochastic aspect of time parameters, TTR and OST as a whole. Factors being tested as well as the levels (low/high variability) are presented in Appendix E. The intention was to test some (not all) possibilities of Demand and TTR/OST, which have been recognized by the related literature.

Stochastic Aspects of Demand. Experiments 01 to 06 allow verifying the effects of two different aspects of demand: firstly, the effects of having the same distribution but with different variance; secondly, the effects of having different demand distributions. Time parameters were kept the same for all first-six experiments. It means same mean (expected value), same variance, and same distribution. To test the effects of having different variances with same demand distribution, it is necessary to compare each pair of experiment (01 and 02; 03 and 04; and 05 and 06). To test the effect of having different demand distribution, it is necessary to compare results throughout the first six experiments.

Effects of Having Different Variances (LV, and HV) for the Same Distribution. Comparing the results of the experiment 01 (time between demands normally distributed with LV) with experiment 02 (time between demands normally distributed with HV) this research concludes there is no significant difference between EBO values of Experiment 01 and Experiment 02. EBO values in the Experiment 02 (HV) are in average 0.02 bigger than Experiment 01 (LV). A possible explanation for this is that it might happen due to the fact that the only one factor being tested was the demand, which was normally distributed. Given the normal distribution is a symmetric distribution (Devore, 1999:154), one can expected that the effects of the variance are balanced. Comparing Experiment 03 (time between demands Gamma distributed with LV) with Experiment 04 (time between demands Gamma distributed with HV) the differences on the EBO values are considerable. For some stock levels (2, 3, and 4), the difference reaches more than 1.1, representing 100% more in favor of the distribution with high variance (Experiment 04). No explanation was found for that; it is proposed that the negative effect of variance acknowledged by several statistic books could be at work (Benson, 1994:702). Similar behavior was observed when comparing experiments 05 (time between demands lognormal distributed with LV) and 06 (time between demands lognormal distributed with HV). However, the differences were smaller than on the previous experiment (comparing 03 to 04).

Effects of Having Different Demand Distributions. Looking at the EBO results from experiments 01 to 06, one may conclude that different demand distributions can lead to different EBO values. Additionally, such difference can be positive or negative when compared to the existing models. In Experiment 01 and 02, where time between demands is normally distributed, EBO values are smaller than METRIC. In the other cases, EBO values are bigger than METRIC.

Stochastic Aspects of Time Parameters (TTR and OST). Since TTR and OST were modeled as delays in the system, it is expected that the effects of the variability on both would similar effects. Similar experiments were conducted to test the effect of having different variances for the same distribution. That was accomplished in the experiments 07, 08, 09, and 10 (see Appendix E for more details). Comparing experiments 07 and 08, the EBO values got bigger when the variance in time parameters got bigger. However, this behavior was not confirmed on the experiments 09 and 10, where the differences can be considered insignificant (around 0.01 in favor of bigger variance, comparing the results of EBO from simulation). There was, again, the normal distribution presented for time to repair in both depot and base. It is out of the scope of this research to explain the reasons of that. Different distributions for demand and time parameters, as well as different variances, potentially lead to different EBO values.

Investigative Question 3**.** Does the proposed method return different EBO numbers compared to the existing models? How significant is the difference? In general, the proposed model yields different EBO predictions. Unlike the METRIC models, the proposed approach has demonstrated to be sensitive to different demand and time parameter distributions, and also to the variability on those parameters. When comparing both models with the 95 % half-width CI of the EBO from simulation, the results are in favor of the proposed method: around 13 % inside the CI, against 0 % METRIC. Again, this research tested only situations where the assumptions of METRIC were not verified.

Investigative Question 5. Is the new model time/resource efficient compared to the existing models? No doubt that the proposed method requires more time and resource to be implemented if compared to the existing models. However, assuming P-METRIC can do a better job in calculating backorders, by relaxing several assumptions that existing models cannot deal with, a corollary of that may be better decisions about resource allocation for reparable items. The following is a list of issues to consider in order to evaluate an implementation of the proposed P-METRIC:

1. Simulation Software. Simulation is the base of the proposed method. P-METRIC requires the use of a dynamic simulation package that works with discrete models. This research used a student version of Arena®, Arena 3.0, that comes with the book Simulation with Arena (Sadowski) that costs less than a hundred dollar. However, a professional version can cost more than U$ 20,000. Thus, it is really important to decide whether or not to get into an investment like that.

2. Mathematic Software. The proposed method requires the use of mathematic software tools capable of solving more complex system of equations. Besides, the distribution used to model the pipeline in the proposed method (Gamma distribution) is not completely tabled like Poisson is. Thus, software like MathCad (used in this research), Mat lab, or similar are highly recommended and a professional version may cost no more than U$ 800.

3. Human Resources. Having simulation software does not mean having models. Modeling system is a task that requires human resources. This research does not have information about cost of labor hours for professional of system modeling and it may vary from place to place.

4. Time Consuming. The proposed method requires more time to work the input data (GOF tests) and also to running the model in the previously discussed points in order to build up the EBO curve. However, given the state-of-the-art of several software for statistics, the time to prepare data to get into the proposed method can be assumed almost the same of the existing models. Furthermore, the goal of the proposed method is to use the information from simulation in order to build a backorder without needing of running the model for each level of decision. This way, once the model had been developed, templates can be used to save time of programming.

Therefore, considering that reparable items are very expensive items, investing on the minimal requirements for implementing the proposed method can be thought as a good cost-benefit trade-off analysis.

The next section presents the results of the EBO calculation for the 20 reparable items of the T-27 Tucano.

Results of the T-27 Tucano Case Study

The T-27 Tucano case study was an attempt to validate the proposed model, checking its accuracy compared with data from the "real world." The results of the T-27 Tucano Case Study are presented in the following tables. The data used for calculating EBO for both METRIC and proposed method are presented in the Appendix D.

Table 14. Results of the T-27 Tucano Case Study (AFA)

ITEM	DEPOT STOCK	AFA STOCK	EBO			DIFFERENCE		SQ DIFFERENCE	
			METRIC	P-METRIC	FIELD	M-S	P-S	METRIC	P-METRIC
1	0	7	0	0	0.418	-0.418	-0.418	0.174724	0.174724
2	0	3	0.0077	0.01492	0.37	-0.3623	-0.35508	0.1312613	0.1260818
3	0	4	1.62678	1.86268	2.05	-0.42322	-0.18732	0.1791152	0.0350888
4	0	2	0.60132	0.44367	0.58	0.02132	-0.13633	0.0004545	0.0185859
5	8	2	0.00008	9.35E-05	0.1	-0.09992	-0.09991	0.009984	0.0099813
6	2	2	0.00026	1.87E-04	0.102	-0.10174	-0.10181	0.010351	0.0103659
7	15	12	0	0	0	0	0	0	0
8	0	2	1.72656	1.62048	0.846	0.88056	0.77448	0.7753859	0.5998193
9	0	4	0.148388	0.13877	0.505	-0.35661	-0.36623	0.1271721	0.1341244
10	3	2	0.0637	0.72257	0.48	-0.4163	0.24257	0.1733057	0.0588402
11	0	2	0.02313	0.02888	0.18	-0.15687	-0.15112	0.0246082	0.0228373
12	0	0	1.39986	1.40599	1.253	0.14686	0.15299	0.0215679	0.0234059
13	30	22	0	1.15E-09	0.28	-0.28	-0.28	0.0784	0.0784
14	0	1	0.04187	0.03713	0.332	-0.29013	-0.29487	0.0841754	0.0869483
15	0	1	2.3415	2.32069	0.9	1.4415	1.42069	2.0779223	2.0183601
16	2	9	0.01139	1.37E-03	0	0.01139	0.001375	0.0001297	1.89E-06
17	0	1	5.01893	4.97294	3.77	1.24893	1.20294	1.5598261	1.4470646
18	1	7	0	4.79E-06	0	0	4.79E-06	0	2.296E-11
19	0	2	0.05091	0.03216	0.74	-0.68909	-0.70784	0.474845	0.5010375
20	0	1	0.34511	0.37471	0.3501	-0.00499	0.02461	2.49E-05	0.0006057

The table above presents the EBO calculated by using METRIC and P-METRIC for the AFA. The stock levels for each one of the items as well as the average backorder were informed by the PAMALS. The EBO information got from the field referred just to the mean, thus no considerations have been made about confidence interval. The differences between METRIC EBO and EBO from the field, as well as proposed method EBO and EBO from the field are presented, and the summation of the squared difference (considering all 20 items together) for METRIC is 5.9032. For P-METRIC is 5.3462.

Table 15. Results of the T-27 Tucano Case Study (CATRE)

ITEM	DEPOT	CATRE	EBO			DIFFERENCE		SQ DIFFERENCE	
	STOCK	STOCK	METRIC	P-METRIC	FIELD	M-S	P-S	METRIC	P-METRIC
1	0	7	0	0.00005	0	0	0.00005	0	2.5E-09
2	0	6	0.00595	0.01059	0.376	-0.37005	-0.36541	0.136937	0.1335245
3	0	7	1.66658	2.09489	2.449	-0.78242	-0.35411	0.6121811	0.1253939
4	0	3	1.38719	1.23738	1.35	0.03719	-0.11262	0.0013831	0.0126833
5	8	11	0	1.90E-08	0	0	1.9E-08	0	3.622E-16
6	2	2	0.00061	4.48E-04	0	0.00061	0.000448	3.721E-07	2.008E-07
7	15	12	0	0	0	0	0	0	0
8	0	5	0.45928	0.39673	0.463	-0.00372	-0.06627	1.384E-05	0.0043917
9	0	10	0.00027	1.48E-07	0	0.00027	1.48E-07	7.29E-08	2.191E-14
10	3	3	0.07609	0.29018	0.52	-0.44391	-0.22982	0.1970561	0.0528172
11	0	5	0.00004	4.97E-04	0.17	-0.16996	-0.1695	0.0288864	0.0287311
12	0	1	1.91702	1.87013	1.938	-0.02098	-0.06787	0.0004402	0.0046063
13	30	54	0	0	0.204	-0.204	-0.204	0.041616	0.041616
14	0	5	0.4596	0.73351	1.887	-1.4274	-1.15349	2.0374708	1.3305392
15	0	2	2.17694	2.08179	2.96	-0.78306	-0.87821	0.613183	0.7712528
16	2	10	0.03132	3.13E-03	0.561	-0.52968	-0.55787	0.2805609	0.3112203
17	0	1	1.53812	1.4571	0.854	0.68412	0.6031	0.4680202	0.3637296
18	1	3	0.20737	0.43893	0.87	-0.66263	-0.43107	0.4390785	0.1858213
19	0	4	0.03409	0.05062	0.338	-0.30391	-0.28738	0.0923613	0.0825873
20	0	2	1.09004	1.46656	1.266	-0.17596	0.20056	0.0309619	0.0402243

The table above presents the EBO calculated by using METRIC and P-METRIC for the CATRE. The stock levels for each one of the items as well as the average backorder were informed by the PAMALS. The EBO information got from the field referred just to the mean, thus no considerations have been made about confidence interval. The differences between METRIC EBO and EBO from the field, as well as proposed method EBO and EBO from the field are presented, and the summation of the squared difference (considering all 20 items together) for METRIC is 4.9801. For P-METRIC is 3.4891.

The following table summarizes the results of the experiments T-27 Tucano case study. The objective is to verify which of the models (METRIC or P-METRIC) provides more accurate (close to real world) back order numbers.

Table 16. T-27 Tucano Case Study Summary

LOCATION	SUM SQ DIFFERENCE		% CLOSENESS		% NO DIFF
	METRIC	P-METRIC	METRIC	P-METRIC	
AFA	5.9032	5.3462	40	45	15
CATRE	4.9801	3.4891	35	55	10
TOTAL	10.8833	8.8353	37.5	50	12.5

The table above shows the resume of the T-27 Tucano case study. The summation of the squared difference of all 20 items is presented for each base, fro both METRIC and P-METRIC. Additionally, the table shows the percentage of the closeness in which each of the models has been close to the EBO from field.

Analysis of the Results of the T-27 Tucano case study

The analysis of the results of the T-27 Tucano case study is now used to help answer the fourth investigative question presented next.

<u>Investigative Question 4</u>. Which model would provide the most accurate (close to the real world) back order numbers, the proposed method or the existing models? The results of the T-27 Tucano case study show that both models are not too far from the "real world" EBO numbers. Considering that both models sometimes underestimate and

sometimes overestimate backorders, the overall result on the back order estimation could be even lower. The EBO estimation of METRIC and P-METRIC are close to the EBO from the field and close to each other. The same can be said about the percentage of the times the models are close to the EBO from field.

The analysis of both conceptual experiments and the T-27 Tucano case study is used to answer the research question.

Research Question. How could a mathematical analytical model account for variability in Demand, TTR, and OST with respect to EBO calculations in a more accurate way? After analyzing the data from both conceptual examples and the T-27 Tucano case study this research concludes that the discussion belongs more to the theoretical field than to the "real world" situation. The improvement in the EBO calculation in terms of the closeness to the EBO from simulation in the conceptual cases was diminished due to fact of both models being outside of the 95 % half-width CI in most of the time. Moreover, the analysis of the T-27 Tucano case study has shown that both METRIC and P-METRIC are close to the EBO from the field, therefore, even in the conceptual experiments the proposed method has demonstrated being more accurate, the discussion really does not matter. That may happen due to compensatory issues such as condemnation, cannibalization, lateral re-supply and other factors not tested in this research that, acting together, results in EBO numbers very close to the METRIC and also to the proposed P-METRIC.

Therefore, the use of simulation combined with mathematical model can result in more accurate number for EBO calculation, but this may not be an important issue for the "real world" reparable inventory systems.

Summary

Chapter IV presented the results of the experimental design used to verify and validate the proposed method. The results of the ten conceptual experiments were presented and analyzed, followed by the T-27 Tucano case study. The analysis of both conceptual experiments and T-27 Tucano case study were used to answering the investigative question and the research question.

Chapter V will present the conclusions of this study and the researcher recommendations.

V. Conclusions and Recommendations

Chapter Overview

This chapter presents the conclusions of this study and the recommendations of the researcher regarding to the matter of EBO prediction.

Conclusions

The results of this study suggest that few improvements were achieved in attempting to model reparable inventory systems using the proposed method. Analyzing the results from both conceptual examples and the T-27 Tucano case study, this research concludes that, except in the conceptual examples, where the proposed method showed a more accurate, even being outside of the 95 % half-width CI most of the time, the proposed method EBO results are very close to the existing METRIC models.

Recommendations

Based on the results collected from the conceptual examples and the T-27 Tucano case study, this research suggests the following recommendations for mangers that work with reparable items:

1. Be Simple. Many issues in the "academic world" are very complex, however, in the real world, thanks God, compensatory forces may simplify them. Therefore, instead of spending effort in attempting to understand the individual complexity of the factors, just look at the overall results. The application of this learning in the management of reparable items may result in the utilization of not only the existing models, with their restrictive assumptions, but also other methods, such as the heuristic approach, which is even more simple than the mathematical model, and it works.

2. Right Tool, Right Job. METRIC has demonstrated to be a good tool for predicting backorders for the regular reparable items. However, in the cases where the complexity happen in scale affecting considerably the EBO prediction, and when that is a matter that should be considered due to amount of money involved or other critical issues, the use of the simulation can bring better results in terms of EBO prediction accuracy. Besides the combined simulation-mathematical approach, this research offers three simulation models for Arena environmental for predicting EBO in Appendix A, B, and C.

Further Studies

From the academic point of view, there are still some fields to be explored in the prediction of backorders. This research recommends the following:

1. Capacity Issues in the Reparable Items Supply Chain. Existing models have ignored this issue in modeling reparable inventory systems. Capacity issues usually create waiting in queue time that potentially affects the pipeline of the item. Simulation can identify this time and include it as a factor to be considered.

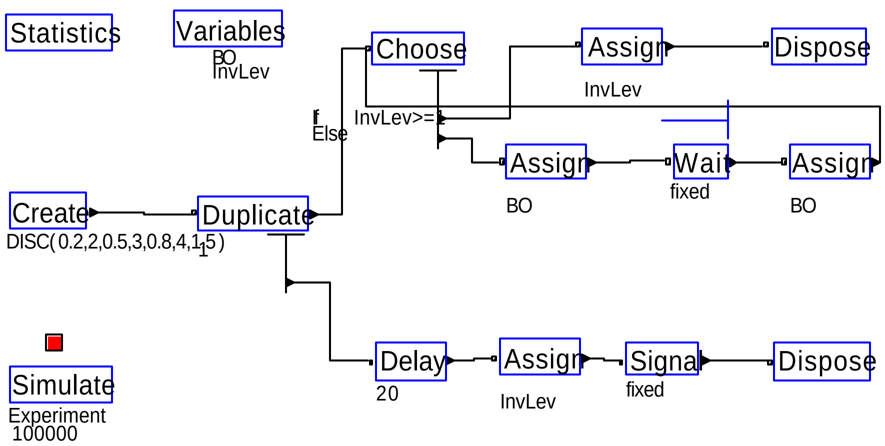
2. Convolution of Different Distributions. A fact observed for this research that could not be addressed due to scope issues was that it seems that the Gamma distribution used in the proposed method seems to work better for specific situations, which can be the resultant of specific distributions or simply the result of the variance of them. Besides, other distributions than Gamma can be tested for modeling the pipeline of reparable items.

From the practical point of view, a good simulation model can take care of the EBO prediction problem, when the complexities regarding to demand, TTR and OST, as well as strategic issues require such control.

Summary of the Research

This research was an attempting of evaluating the effects of the variability in Demand and time parameters (TTR and OST) in EBO prediction for systems that do not match the assumptions of the existing mathematical models. So far, two main approaches had dealt with this issue: mathematical approach, represented by METRIC models that predict backorders based on restrictive assumptions about demand and time parameters, simplifying the problem; simulation approach, that can virtually relax all assumptions, but may become very complex. The proposed method suggests the use of simulation combined with mathematical analytical model to better predict backorder for reparable inventory systems. The results from an experimental design showed some improvements on the EBO prediction for systems that do not match the METRIC assumptions, however, most of the times the proposed method EBO prediction was found outside of the 95 % half-width CI established as a parameter of comparison. Moreover, the results of a field research that collect data from the T-27 Tucano, an advanced-training, light-attack aircraft deployed by the BRAF have demonstrated few improvements in the EBO prediction when compared with the existing models. From the point of view of the researcher, the major contribution of this research was the innovative approach given to the problem.

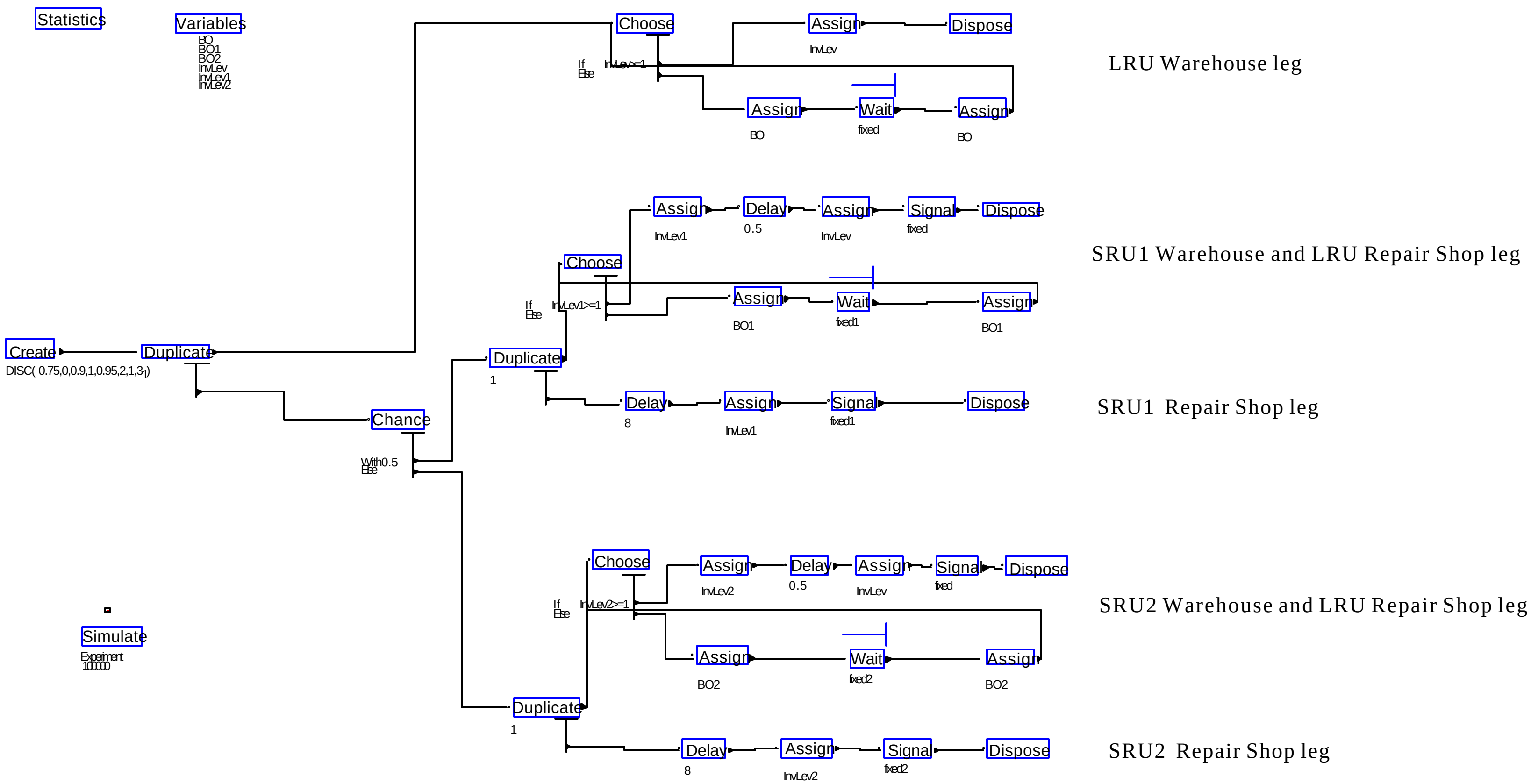
Appendix A. First Indenture Single Site (FISS) Model



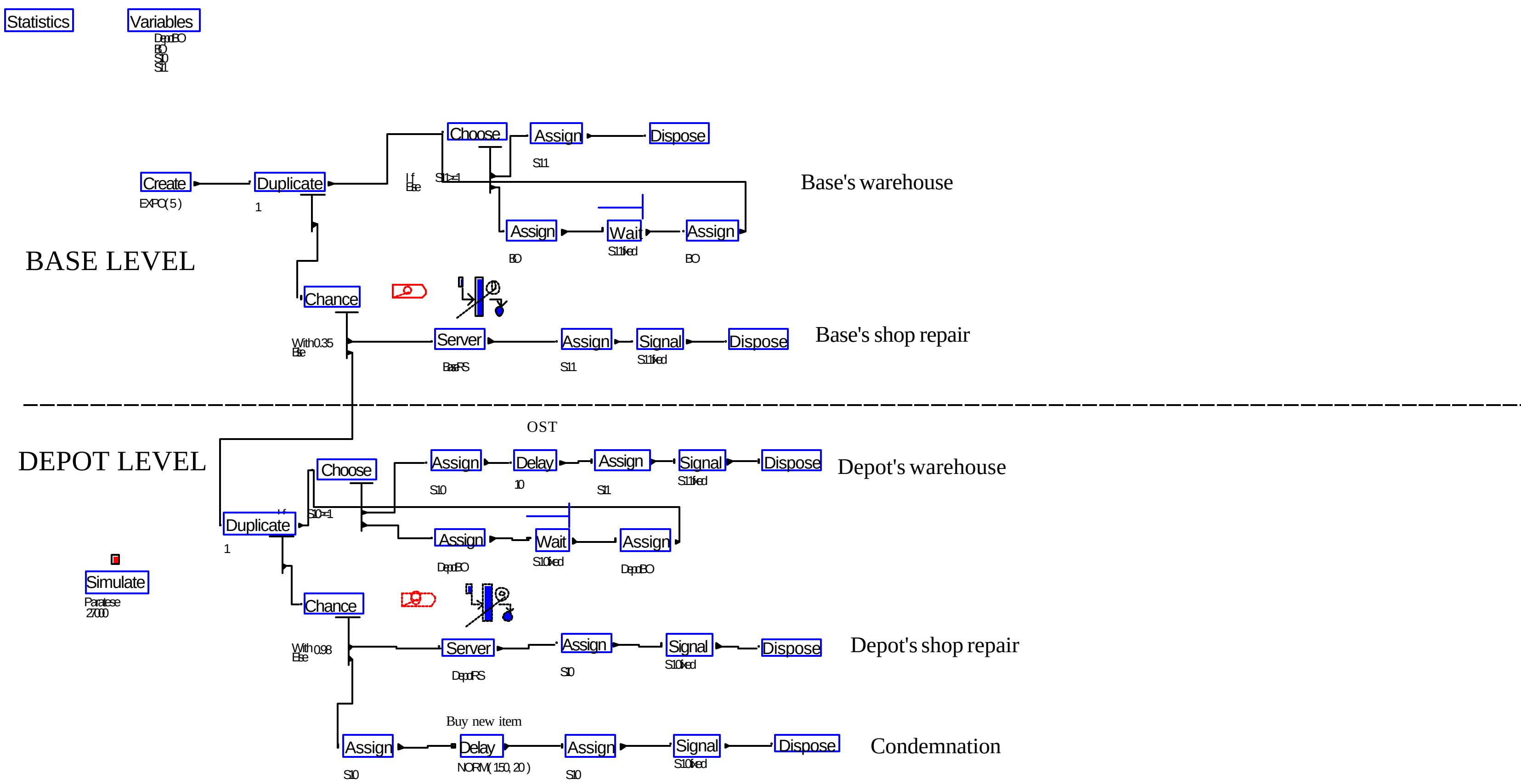
Warehouse leg

Shop Repair leg

Appendix B. Multi Indenture Single Site (MISS) Model



Appendix C. First Indenture Multi Echelon (FIME) Model



Appendix D. List of Items of the T-27 Tucano Case Study

Nomenclature	STOCK LEVEL			DEMAND		DEPOT
	AFA	CATRE	PAMALS	D-AFA	D-CATRE	TTR
CYLINDER ASSEMBLER	7	7	0	0.03	0.04	10.38
ATUADOR LINEAR	3	6	0	0.08	0.18	5.12
BOMBA COMBUSTÍVEL	4	7	0	0.19	0.27	23.645
LIGHT,RECOGNIT	2	3	0	0.09	0.16	19.32
CONJUNTO FREIO	2	11	8	0.02	0.09	16.907
BERÇO DO MOTOR	2	2	2	0.02	0.02	22.24
COMPRESSOR FREON	12	12	15	0.02	0.03	25.018
MOT JAN AR COND.	2	5	0	0.15	0.16	19.798
POWER SUPPLY	4	10	0	0.15	0.16	12.024
CONTACTOR MANÔM.	2	3	3	0.12	0.16	11.55
PAINEL MULT ALARM.	2	5	0	0.02	0.02	24.38
FONTE LUZ CALDA	0	1	0	0.07	0.13	15.998
CILYNDER OXIG	22	54	30	0.22	0.17	24.693
CONJ RODA TPP	1	5	0	0.02	0.24	11.202
CUBO RODA NARI	1	2	0	0.17	0.19	15.44
ELETRIC MOTOR	9	10	2	0.19	0.23	21.582
PROPELLER	1	1	0	0.21	0.08	24.65
CONJ GARRAFA	7	3	1	0.08	0.18	8.331
METER,ELECTRIC	2	4	0	0.1	0.17	3.609
VALVE,BLEEDER,	1	2	0	0.11	0.26	4.76

Other Information:
Demand Rate = unit per day
Probability of Repair at Base = 0 %
Probability of Repair at Depot = 100%
OST AFA = 4 days
OST CATRE = 6 days

Appendix E. Experimental Design

Conceptual Experimenting Design																
Exp	s-dep	s-base 0 to	Factor Tested	Level	Demand/TBD			Base TTR			Depot TTR			OST		
					Dist	Mean	StDev	Dist	Mean	StDev	Dist	Mean	StDev	Dist	Mean	StDev
1	3	8	Demand	LV	Normal	0.476	0.05	LogN	1.2	1.2	LogN	3.5	3	Gamma	1.2	1
2	3	8	Demand	HV	Normal	0.476	0.135	LogN	1.2	1.2	LogN	3.5	3	Gamma	1.2	1
3	3	8	Deamnd	LV	Gamma	0.476	1	LogN	1.2	1.2	LogN	3.5	3	Gamma	1.2	1
4	3	10	Deamnd	HV	Gamma	0.0476	10	LogN	1.2	1.2	LogN	3.5	3	Gamma	1.2	1
5	3	8	Deamnd	LV	LogN	0.476	0.5	LogN	1.2	1.2	LogN	3.5	3	Gamma	1.2	1
6	3	8	Deamnd	HV	LogN	0.476	2.5	LogN	1.2	1.2	LogN	3.5	3	Gamma	1.2	1
7	6	13	TTR/OST	LV	LogN	0.25	1.5	LogN	1.2	1.2	LogN	3.5	3	Gamma	1.2	1
8	6	11	TTR/OST	HV	LogN	0.25	1.5	LogN	1.2	5.1	LogN	3.5	12	Gamma	0.12	10
9	6	15	TTR/OST	LV	LogN	0.25	1.5	Normal	1.2	0.12	Normal	2.3	0.2	LogN	1.2	1.2
10	6	15	TTR/OST	HV	LogN	0.25	1.5	Normal	1.2	0.35	Normal	2.3	0.7	LogN	1.2	4

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Vita

Captain Roberto Carlos Borges de Abreu was born on 31 July 1965 in Bom Jesus do Itabapoana, Brazil. He graduated from the Academia da Força Aérea (AFA) in 1989 with a Bachelor of Science degree in the Science of Management as an Air Force Logistics Officer. Upon graduation from AFA, he was assigned as a supply officer in the Base Aérea de Santa Maria, Rio Grande do Sul (RS). There, Capt Abreu worked as a supply officer for four years and as an acquisition officer for two years. During this time, he did a four-year course in the Universidade Federal de Santa Maria, Santa Maria, RS, graduating with a bachelor in the Science of Accounting.

In January of 1996, Capt Abreu was assigned to the Parque de Material Aeronáutico in Lagoa Santa (PAMALS) as a program manager of the T-25 Universal, a basic-training aircraft of the Brazilian Air Force. During this time, he entered in the five-year course in the Law Faculty of the Universidade Federal de Minas Gerais, Belo Horizonte, Minas Gerais, having completed two years.

In May of 1999, Captain Abreu entered the Instituto de Logística da Aeronáutica at Base Aérea de São Paulo, São Paulo, and graduated in November 1998 in the CELOG, a six-month course of logistics. He was selected to enter the Air Force Institute of Technology at Wright-Patterson AFB, Ohio in 2000 and graduated in 2002 with a Masters degree in Logistics Management. He was subsequently assigned to the Instituto de Logística da Aeronáutica at Base Aérea de São Paulo, São Paulo.

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14. ABSTRACT
This research sought to describe an alternative way for calculating expected back order (EBO) for reparable inventory systems. The high costs associated with reparable items management, together with its importance for system's availability, make the assessment of back orders of great importance in supporting decisions of "what-to-buy" and "where-to-locate" those items. Starting at the point that existing models (METRIC, MOD-METRIC, and VARIMETRIC) rely on some assumptions that often cannot be met in real life, the proposed method (called P-METRIC), which is a mix of simulation and mathematical analytical model, relaxes assumptions about Demand, Time to Repair (TTR), and Ordering & Ship Time (OST) distributions looking for potential differences that may cause on the EBO calculation. The study consists of 10 conceptual examples where the parameters of Demand, TTR, and OST vary according to probability distributions recognized by the related literature. It also presents a case study of 20 reparable items of the T-27 Tucano, an advanced-training, light-attack deployed by the Brazilian Air Force. EBO numbers of the existing and proposed models are compared with results gathered from simulation (conceptual examples) and a field research (T-27 Tucano) in order to allow conclusions about the accuracy and suitability of the proposed method.

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