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Reliability of wood plastic composites and improving lower percentile estimation via induced percentile censoring

Kevin Andrew Crookston
University of Tennessee

To the Graduate Council:

I am submitting herewith a thesis written by Kevin Andrew Crookston entitled "Reliability of wood plastic composites and improving lower percentile estimation via induced percentile censoring." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Statistics.

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We have read this thesis and recommend its acceptance:

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Dr. Russell Zaretzki

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Reliability of Wood Plastic Composites and Improving Lower Percentile Estimation via Induced Percentile Censoring

Masters Thesis
Presented for the
Master of Science Degree
The University of Tennessee, Knoxville

Kevin Andrew Crookston
May 2009

Dedication

I dedicate this thesis to the Tennessee Forest Products Center at the University of Tennessee, Knoxville. Further dedication is granted to Dr. Frank Guess, Dr. Timothy Young, and Dr. Ramón León for their suggestions, insight, guidance, and encouragement on the research and write-up. Dedication of this work also goes to my father who taught me the value of education, and my mother who taught me perseverance. Most of all, I dedicate this thesis to my wife, Becky, and children William and Katie for their great sacrifices so I could finish the research.

Acknowledgments

Thanks go to Dr. Adrian Bowman from the University of Glasgow who provided insight through several correspondences into the methods of Kaplan-Meier curve comparisons. Dennis Boos from North Carolina State University shared Fortran Code to efficiently make these comparisons. Susan Smith from the University of Tennessee aided in FORTRAN code compilation and execution. Special thanks go to Dr. Frank Guess, Dr. Timothy Young, Dr. Ramón León, and Dr. Russell Zaretzki for their help on the research and review. This research was partially supported by The University of Tennessee Agricultural Experiment Station, Forest Products Center, McIntire-Stennis TEN00MS-89 and USDA CSREES Special Wood Utilization Research Grant R11-2216-100.

Abstract

Wood plastic composites (WPC) are a combination of wood fiber and thermoplastics to form a water resistant substitute for wood in construction. As a manufactured product, practitioners are interested in understanding the reliability of WPC. This thesis explores the reliability of WPC by analyzing data from a WPC manufacturer, and explores a new application of induced percentile right censoring via simulation to improve lower percentile estimation. The research demonstrates significant improvements in the mean squared errors and bias from this percentile right censoring.

Estimates of the reliability of WPC are studied for two industrial extrusion lines at the same facility. A parametric analysis of the extrusion lines reveals only small differences in averages. However, a non-parametric method is presented that reveals differences between Kaplan-Meier survival curves for the modulus of elasticity (MOE) and modulus of rupture (MOR) strength metrics of the WPC industrial data. Although the differences between the two extrusion lines are most prevalent in the middle of the distributions, the consistency between the two lines for the smaller left tail percentiles holds greater interest for safety and liability. Bootstrapping is performed to estimate confidence intervals on the differences between the two lines for the first, fifth, and tenth percentiles of the MOE and the MOR. A statistical difference is found for the MOR at the tenth percentile.

In many applications, one aging behavior is not sufficient for understanding the entire life time. A simulation is conducted to generate data with a bathtub hazard function. The simulation uses induced right censoring to improve the estimates of the lower tail percentiles. A wide range of possible percentile right censoring yields

significant improvements. The smallest mean squared error and bias are achieved when the percentile censoring approaches the point at which aging behavior first shifts. Techniques for finding this optimal point are discussed.

Application of the induced percentile right censoring and the methods used to analyze the WPC data may benefit statisticians, wood scientists, and practitioners by improving the statistical tools for understanding product quality and variability.

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Chapter 1. Introduction

Wood plastic composites (WPC) are a combination of wood flour and thermoplastics (Figure 1-1) that form a wood-like material used in the construction of outdoor decking, railing, furniture, and some automobile parts (Clemons 2002). Perhac (1997) notes, "A few of the positive attributes [of WPC] are the use of recycled materials, low maintenance requirements, high moisture resistance, decay and insect resistance, low splintering, and good machinability." As a manufactured product, practitioners are interested in understanding the reliability of WPC. One issue related to the use of WPC in the market place is the lack of long-term field data related to durability and reliability. As Morrell et al. (2006) noted, there is a continuing need to develop realistic methods for assessing the many aspects of WPC durability, and these methods will continue to evolve as material scientists refine these composites to improve properties. Statistical reliability methods will be essential for improving the understanding of the long-term performance and robustness of WPC.

Figure 1-1: Wood Flour and Thermoplastics

This thesis focuses on statistical methods for studying the reliability of WPC provided by one manufacturer. Although the results are unique to the conditions of the sample, the methods used are general and can be used by others. Background information about WPC and the methods used in this thesis are in Chapter 2.

The data for this analysis are destructive static bending test metrics. The test data are Modulus of Elasticity (MOE) and Modulus of Rupture (MOR), both metrics that govern performance. These metrics are measured in units of Mega Pascals (MPa). MOE is estimated using a standard test method that estimates the flexural properties of WPC and is derived following the ASTM D6109-05 standard method (ASTM 2008). MOR refers to the failure stress (maximum stress) obtained when applying a load to a structural member in flexure. MOR for WPC is derived following the ASTM D7031-04 standards (ASTM 2008).

The WPC manufacturer who supplied the data has two production-size extrusion lines: Line A and Line B. Samples were collected from each line every day from January 1, 2005 to May 2, 2005, and MOE and MOR metrics were obtained from each line. The two extrusion lines are compared in Chapter 3 by analyzing the significance of differences between center and spread of the failure metrics. The metrics' measurements are also used to build probability density functions. Using Log Likelihood estimation (Meeker and Escobar 1998; Tobias and Trindade 1995; Bain and Engelhardt 1992) and Akaike's Information Criterion (Akaike 1974; Bozdogan 2000), the most appropriate distributions are selected for each metric for the two lines. Further, non-parametric Kaplan-Meier survival curves (Kaplan and Meier 1958) are used for comparing intervals of measurements to estimate the reliability of WPC for each line.

In Chapter 4, the selected distributions from Chapter 3 are used for parametric bootstrapping for the 1st, 5th, and 10th percentiles for the MOE and MOR of each line (Efron and Tibshirani 1997; Meeker and Escobar 1998). Confidence intervals are created for each percentile from the bootstraps, visual comparisons are made, and 2-sample parametric bootstrap hypothesis tests show where differences between the two lines occur for these three percentiles.

In many applications, one aging behavior is not sufficient for understanding the entire life time. A simulation is presented in Chapter 5 studying the effect of induced right censoring to improve left tail percentile estimates when the distribution is subject to more than one aging behavior. The research demonstrates significant improvements in the mean squared errors and bias from this percentile right censoring.

Chapter 6 provides the conclusions and present ideas for future research. The mathematical development and MATLAB codes for this thesis are in the appendix.

Chapter 2. Literature Review

2-1. <u>Wood Plastic Composites</u>

Wood plastic composites have been used in Europe since the early 1900's and have been popular in the United States since the 1970's (Balatinecz and Woodhams 1993). Today there are many manufacturers of WPC that produce decking for the European and North American markets. Perhac (2007) explains, "Wood-plastic composites (WPC) are gaining market share in the building industry as a result of chromated copper arsenate (CCA) pressure-treated wood being removed from the market, perceived durability advantages over traditional wood products, and forest conservation concerns."

Wood/natural fiber-plastic composites are a unique development in the wood products industry in that they are an emerging renewable material class based on performance, process, and product design innovation (Smith and Wolcott 2006). Wood fiber used for WPC is commonly in the form of wood flour (fine particles), and typically

Figure 2-1: Example of WPC Decking

makes up 50 percent of the WPC. Either recycled or virgin plastic materials can be used to produce WPC. Some of the thermoplastic resins include low and high density polyethylene, polypropylene, and polyvinylchloride (PVC). In general, polyethylene based WPC are more thermally stable and ductile in nature. Polypropylene based WPCs have higher stiffness and tend to be more brittle.

One issue related to the use of WPC in the market place is the lack of long-term field data related to durability and reliability. As Morrell et al. (2006) noted, there is a continuing need to develop realistic methods for assessing the many aspects of WPC durability, and these methods will continue to evolve as material scientists refine these composites to improve properties. Statistical reliability methods will be essential for improving the understanding of the long-term performance and robustness of WPC.

Important measures of strength for WPC are expressed in terms of the bending tangent Modulus of Elasticity (MOE) and the bending Modulus of Rupture (MOR). Other measures of strength are the tensile strength tangent modulus of elasticity, also denoted MOE, and the tensile strength modulus of rupture, also denoted MOR. The strength metrics analyzed for this thesis are from bending tests. Perhac (2007) comments, "The [bending] modulus of rupture (MOR) is defined as the maximum stress that can be applied to a beam in pure bending before permanent deformation occurs. The [bending] tangent modulus of elasticity (MOE) is defined as the rate of change of strain as a function of stress and is measured as the slope of the straight line portion of a stress-strain diagram taken at any point." Other helpful references for these metrics are Hartsuiker and Welleman (2007) and Hodgkinson (2000).

2-2. Reliability

The term “reliability” was first coined by Samuel T. Coleridge in 1816 when he used it as a description of his friend Robert Southey (Saleh and Marais 2006). The use of reliability for the purpose of engineering was made possible by the development of probability by Blaise Pascal and Pierre de Fermat in 1654, and the need for mass production beginning during the American civil war. Saleh an Marais (2006) state that the “catalyst that accelerated the coming of this new discipline was the (unreliability of the) vacuum tube.” For more on the history of reliability, see Saleh and Marais (2006) and Denson (1998).

Reliability tools are used to improve quality and reduce cost of manufactured products. Statisticians use statistical process control, probability distributions, regression models, maximum likelihood estimation, acceleration models, censoring, bootstrapping, and Kaplan-Meier estimation among other tools to understand the characteristics of a population of manufactured products based on a sample, and make improvements by reducing variation, error, and defects. Tobias and Trindade (1995) state, “One of the most useful skills a reliability specialist can develop is the ability to convert a mass (mess?) of data into a form suitable for meaningful analysis. Raw numbers by themselves are not useful; what is needed is a distillation of the data into information.”

Several authors discuss the study, methods, and tools of reliability with varying degrees of practical and/or theoretical depth. Some authors are Tobias and Trindade (1995); Meeker and Escobar (1998); Kaplan and Meier (1958); Balakrishnan, Kannan, and Nagaraja (2004); Dovich (1990); O’Conner (2002); and Kenett and Zacks (1998). Former graduate research assistants who studied reliability as it pertains to the science of wood composite materials were Perhac (2007), Wang (2007), Chen (2005) and Edwards (2004).

2-3. Bootstrapping

Bootstrapping is one of the methods used in this thesis to study the reliability of WPC. The term bootstrapping comes from the idea of "pulling yourself up by your bootstraps," signifying the idea of self-sustaining without external help. Bootstrapping in statistics is a computer-intensive resampling method used to estimate properties of a statistic that are difficult to calculate using analytical methods (Efron and Tibshirani, 1997).

The procedure for bootstrapping uses a sample as a pseudo population from which samples are drawn with replacement (Chernick 1999). The size of the samples equal the size of the pseudo population, and the number of samples can vary depending on the speed of the computer performing the algorithm. One-thousand samples is a good starting point, and can be increased depending on the importance of the statistic being measured. As a default, the Splida add-in to S-Plus used in this thesis resamples 2000 times and can do so relatively quickly.

A statistic (such as a percentile) is measured from each sample. By the end of the algorithm, there is a distribution of several bootstrap statistics. The mean, standard error, and bias can be measured from the distribution, allowing us to calculate confidence intervals and perform hypothesis tests (Martinez 2002, Lunneborg 2000).

There are several bootstrapping methods depending on the normality of the data, size of the original sample, and knowledge of an underlying distribution. Procedures for non-parametric, parametric, and percentile bootstrap methods are discussed in Meeker and Escobar (1998) and Cheernick (1999). Efron (2003), Efron and Tibshirani (1993), and Davison and Hinkley (1997) discuss bootstrap methodology, theory, and applications. DiCiccio and Efron (1996) present several types of bootstrap confidence intervals including standard, percentile, and bootstrap-t. Polansky (2000) indicates that bootstrap confidence intervals constructed by percentile methods have an upper bound on the coverage probability that can be relatively low.

2-4. Censoring

In statistics, censoring occurs when the value of an observation is only partly known. The concept of censoring was first used by Daniel Bernoulli in his 1766 analysis of smallpox and the use of vaccination (http://www.absoluteastronomy.com/topics/Daniel_Bernoulli 2009, Blower 2004). For a practitioners' guide to the theory and methods of progressive censoring in applied statistics, life-testing, and reliability, see Balakrishnan and Aggarwala (2000). Dalgaard (2008) shows how to use R software to estimate parameters when data is subject to censoring. Other authors are Meeker and Escobar (1998), Tobias and Trindade (1995), and Sun (2006).

Life tests are subject to three kinds of censoring: right, left, and interval. The censored values provide less information than the exact failure times, though are useful in that they indicate the proportion of the sample that can survive beyond (right), before (left), or between (interval) the censored times.

When units are placed on test and the time to failure is measured, there is a possibility that not all units will fail by the end of the test. These units are said to be Type I right censored. Right censoring can also take place when the test is terminated after a predetermined number of units have failed and are called Type II right censored.

Tobias and Trindade (1995) explain the practical benefits of each type. In likelihood estimation, the contribution to the likelihood of a right censored observation is shown in equation 2-1 where t_i is the time of the i^{th} censoring (Meeker and Escobar 1998).

$$L_i(p) = \int_{t_i}^{\infty} f(t)dt = F(\infty) - F(t_i) = 1 - F(t_i) \quad \textbf{(2-1)}$$

Left censoring, on the other hand, occurs if a unit fails before the first inspection. Data that are left censored provide information on the proportion of the sample that fail before time t_i. The contribution to the likelihood is shown in equation 2-2.

$$L_i(p) = \int_{0}^{t_i} f(t)dt = F(t_i) - F(0) = F(t_i) \quad \textbf{(2-2)}$$

Another type of censoring is interval censoring. This is the result of not knowing the exact time of failure of a unit, but knowing that the unit failed between time t_{i-1} and t_i. For interval censoring, the contribution to the likelihood is shown in equation 2-3.

$$L_i(p) = \int_{t_{i-1}}^{t_i} f(t)dt = F(t_i) - F(t_{i-1}) \quad \textbf{(2-3)}$$

All three types of censoring can be used in the same data set. The total likelihood for *n* independent observations for this mixed censoring situation is given in equation 2-4, where $n = \sum_{j=1}^{m+1}(d_j + r_j + l_j)$ and C is a constant depending on the sampling inspection scheme but not on the parameters p (Meeker and Escobar 1998). For more information on the constant term C, see Meeker and Escobar (1998). Programs such as R, Splida, JMP, and SAS can calculate the likelihood of these multi-censored situations.

(2-4)

$$L(p\colon DATA) = C\prod_{i=1}^{n} L_i(p; DATA)$$

$$= C\prod_{i=1}^{m+1}[F(t_i)]^{l_i}\,[F(t_i) - F(t_{i-1})]^{d_i}\,[1 - F(t_i)]^{r_i}$$

Chapter 3. Comparison of Two Wood Plastic Composite Extrusion Processes using Statistical Reliability Analysis

Chapter 3 was submitted to Wood and Fiber Science, April 2008. Co-others for the article are Dr. Frank M. Guess, Dr. Timothy M. Young, and Dr. David Harper. Following is a modification of the article to comply with the format requirements of the thesis.

Comparison of Two Wood Plastic Composite Extrusion Processes using Statistical Reliability Analysis

Estimates of the reliability of wood plastic composites (WPC) are explored for two industrial extrusion lines located at the same facility. A parametric analysis of the extrusion lines reveal only small differences, however a non-parametric method is presented that reveals the statistical differences between Kaplan-Meir survival curves for the modulus of elasticity (MOE) and modulus of rupture (MOR) of WPC industrial data. Distribution fitting as related to selection of the proper statistical methods is also discussed with relevance to estimating the reliability of WPC. The ability to detect statistical differences in the product reliability of WPC between extrusion processes may benefit WPC producers in improving long-term product quality. These methods may also benefit wood scientists by improving the statistical understanding of product quality during experimentation.

3-1. Descriptive Statistics

Figure 3-1 represents side-by-side box and whisker plots for the MOE and MOR from Lines A and B. The points extending beyond the whiskers are *potential* outliers, but only the minimum value in Line B for the MOR seems to stand noticeably low in comparison with the other values from its distribution. In addition to the visual differences between the two lines, formal hypothesis tests indicate significant differences between the means, but not significant differences between the variances. The descriptive statistics and p-values are summarized in Table 3-1.

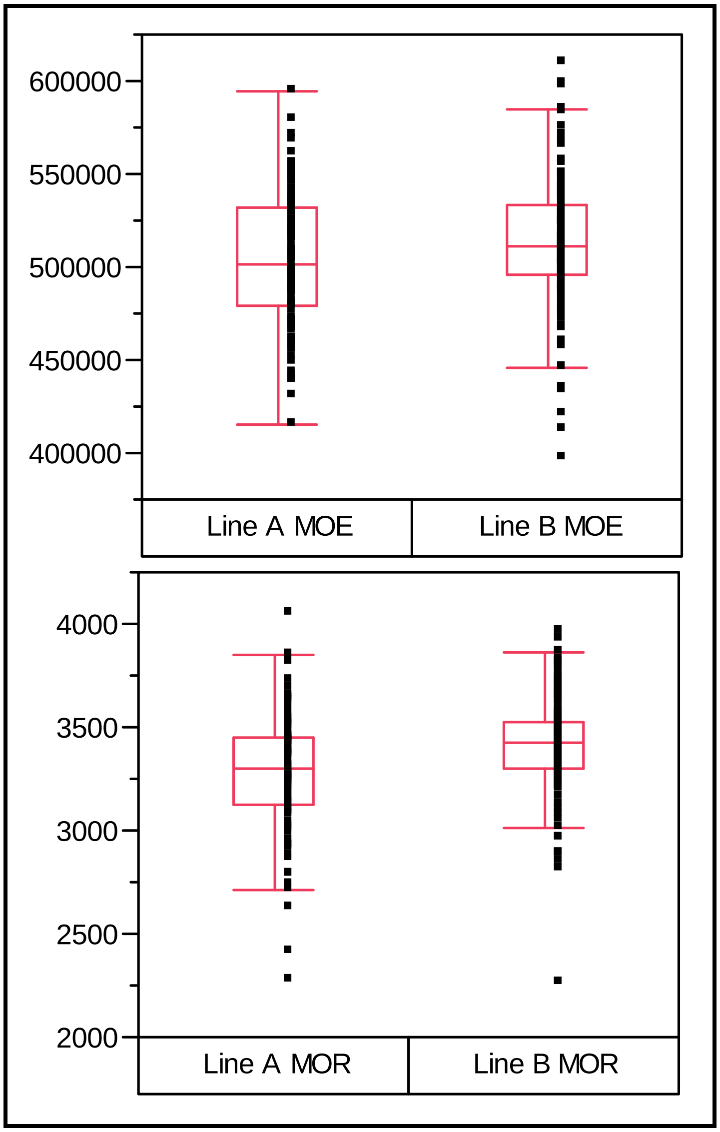

Figure 3-1: Box plots for MOE and MOR for Lines A and B.

Table 3-1: Descriptive statistics and p-values for MOE and MOR for Lines A and B.

	MOE			MOR		
Statistics	Line A	Line B	P-values	Line A	Line B	P-values
Mean	503478.55	512905.14	0.03971	3279.27	3414.23	0.000058
Median	500957.00	511522.00		3294.00	3420.70	
Std Dev	35329.88	35873.55		277.63	235.55	
Variance	1248200301	1286911454	Bartlett's 0.86686 Levene's 0.87733	77076	55482	Bartlett's 0.071773 Levene's 0.229816
CV	7.02%	7.00%		8.47%	6.90%	
IQR	53174.98	38268.74		324.85	225.07	
Skewness	-0.0034	-0.2404		-0.6188	-1.0767	
Kurtosis	-0.4557	1.2618		1.4770	4.3751	

There is evidence that the mean MOE from Line A is statistically different than the mean MOE from Line B. A two-sample, two-tailed t-test comparing the difference between the means returns a p-value of 0.039711. There is also evidence of a statistical difference between the mean MOR between the two lines. The p-value for the two-sample two-tailed t-test comparing the difference in means is 0.000058. At a significance level of $\alpha = 0.05$ we conclude initially from these descriptive statistics that the WPC from Line A are, on average, different than the WPC from Line B. These hypothesis tests merely tell us that differences in the means exist, but not if the differences are important. A practitioner can determine if these differences are large enough to be of concern.

Unlike the means, the variations in MOE and MOR between Lines A and B are not statistically different. The variance for the MOE is greater for Line B than for Line A, but the p-values for comparing the variances are large (0.8669 for Bartlett's test and 0.8773 for Levene's test). As for the MOR, the variance for Line A is greater than for Line B. When comparing the variances, it may be more appropriate to use Levene's test given the potential outlier found in Line B. The p-value for this test is 0.2298. We conclude that these differences are attributed to random chance and not to actual difference in variation.

A measure of the ratio of the standard deviation to the mean is the Coefficient of Variation (CV). The CV for the MOEs for Lines A and B are, respectively, 7.02 percent and 6.99 percent. For the MORs the CV for Lines A and B are, respectively, 8.47 percent and 6.90 percent.

The Inner Quartile Range (IQR) is the range of the middle 50 percent of the data and provides insight to the concentration of the distribution about the median. The box plots in Figure 3-1 show that there is more concentration of the IQR about the median for the MOE (38269) and MOR (225) from Line B than for the MOE (53175) and MOR (325) from Line A.

The four distributions have negative skewness values close to zero indicating slight left skewness. The MOR from Line B is the most severely skewed due to the one early failure.

The kurtosis measures how peeked or flat a distribution is. The MOE from Line A is mound shaped and has a kurtosis of -0.46. The MOR from Line B is concentrated close to the mean and has a large positive kurtosis of 4.38.

The aforementioned descriptive statistics provide the practitioner with an initial assessment of the data quality and product quality for each production line. These metrics of quality can help practitioners start to objectively quantify process variation and be the initial basis of directing resources towards continuous improvement efforts. However, additional statistical methods are warranted to assess the process reliability of the product. The product reliability is reflective of the process reliability, and to improve product reliability would require further investigation into the process.

3-2. Parametric Comparisons of Distributions

3-2-1. Information Criterion and Distribution Selection

Descriptive statistics have provided insight for each production line on the location, variability, and shape of the distributions generated for each line's mechanical properties. Hypothesis tests suggest that the locations of the distributions with respect to the MOE and MOR differ, but the variabilities are similar. In this section, we will fit the data from each of the four groups to the Normal, Log Normal, Largest Extreme Value (LEV), Logistic, Loglogistic, Weibull, and Fréchet distributions in an attempt to determine the underlying distributions of the data sets. Although none of these distributions may fit the data exactly, we will use maximum log likelihood estimation, Akaike's Information Criterion (AIC), and probability plots to find the most useful models (Bozdogan 2000).

Log likelihood estimation is one information criterion for determining the best distribution for a set of data (Bain and Engelhardt 1992; DeGroot and Schervish 2001; Hogg, Craig, and McKean 2004; Rohatgi and Saleh 2000). Precedence is given to the distribution with the largest log likelihood value. This value is calculated by finding the maximum of the log likelihood of the probability density functions. S-PLUS and SPLIDA software are used to estimate these values (Insightful 2008).

Table 3-2: Log Likelihood and AIC for MOE and MOR Line A.

MOE Line A			MOR Line A		
Model	**Log Likelihood**	**AIC**	**Model**	**Log Likelihood**	**AIC**
Normal	-1450	2904	Logistic	-856.4	1716.8
Lognormal	-1451	2906	Weibull	-858.5	1721.0
Loglogistic	-1453	2910	Loglogistic	-858.8	1721.6
Logistic	-1453	2910	Normal	-859.0	1722.0
Weibull	-1455	2914	Lognormal	-863.4	1730.8
LEV	-1458	2920	LEV	-883.0	1770.0
Fréchet	-1461	2926	Fréchet	-895.1	1794.2

Table 3-3: Log Likelihood and AIC for MOE and MOR Line B.

MOE Line B			MOR Line B				
Model	**Log Likelihood**	**AIC**	**Model**	**Log Likelihood (complete)**	**AIC**	**Log Likelihood (excluding day 17)**	**AIC**
Logistic	-1449	2902	Logistic	-831	1667	-817	1637
Loglogistic	-1450	2904	Loglogistic	-834	1672	-818	1639
Normal	-1452	2908	Normal	-839	1682	-819	1642
Lognormal	-1454	2912	Lognormal	-845	1694	-821	1645
Weibull	-1458	2920	Weibull	-836	1676	-824	1651
LEV	-1470	2944	LEV	-875	1753	-836	1676
Fréchet	-1478	2960	Fréchet	-880	1764	-842	1688

A second method used to rank the distributions for the data is AIC (Akaike 1974; Burnham and Anderson 2002; Bozdogan 2000). The AIC is a function of the log likelihood and the number of parameters in the distribution under consideration. This value is calculated using Equation 3-1 where $LogL(\hat{\sigma})$ is the maximum log likelihood value and k is the number of estimated parameters in the density function. The distribution with the smallest AIC is preferred. The advantage of using AIC over the maximum log likelihood is its ability to distinguish simple distributions that have similar maximum log likelihood values from complex distributions.

$$\boldsymbol{AIC = -2LogL(\hat{\sigma}) + 2k} \quad \textbf{(3-1)}$$

Tables 3-2 and 3-3 summarize the information criterion for the MOE and MOR for the two lines. The MOE from Line A is best modeled with a normal distribution, and the MOR from Line A, MOE from Line B, and MOR from Line B are best modeled with the logistic distribution.

Probability plots are a visual method of distribution selection and help strengthen our conclusions from the information criterion. These plots allow us to see where the data fit the distributions under consideration. Figures 3-2, 3-3, 3-4, and 3-5 are the sets of probability plots for the four data sets.

When plotting the data for the MOR from Line B, the failure occurring on day 17 (January 17, 2005) was not consistent with the other MOR data. Since there is no evidence of an error, day 17 cannot be discarded. The data was fit with and without the observation to determine if it made a difference in the selected distribution (Figures 3-5 and 3-6). The Logistic distribution is selected as the best fit with and without day 17.

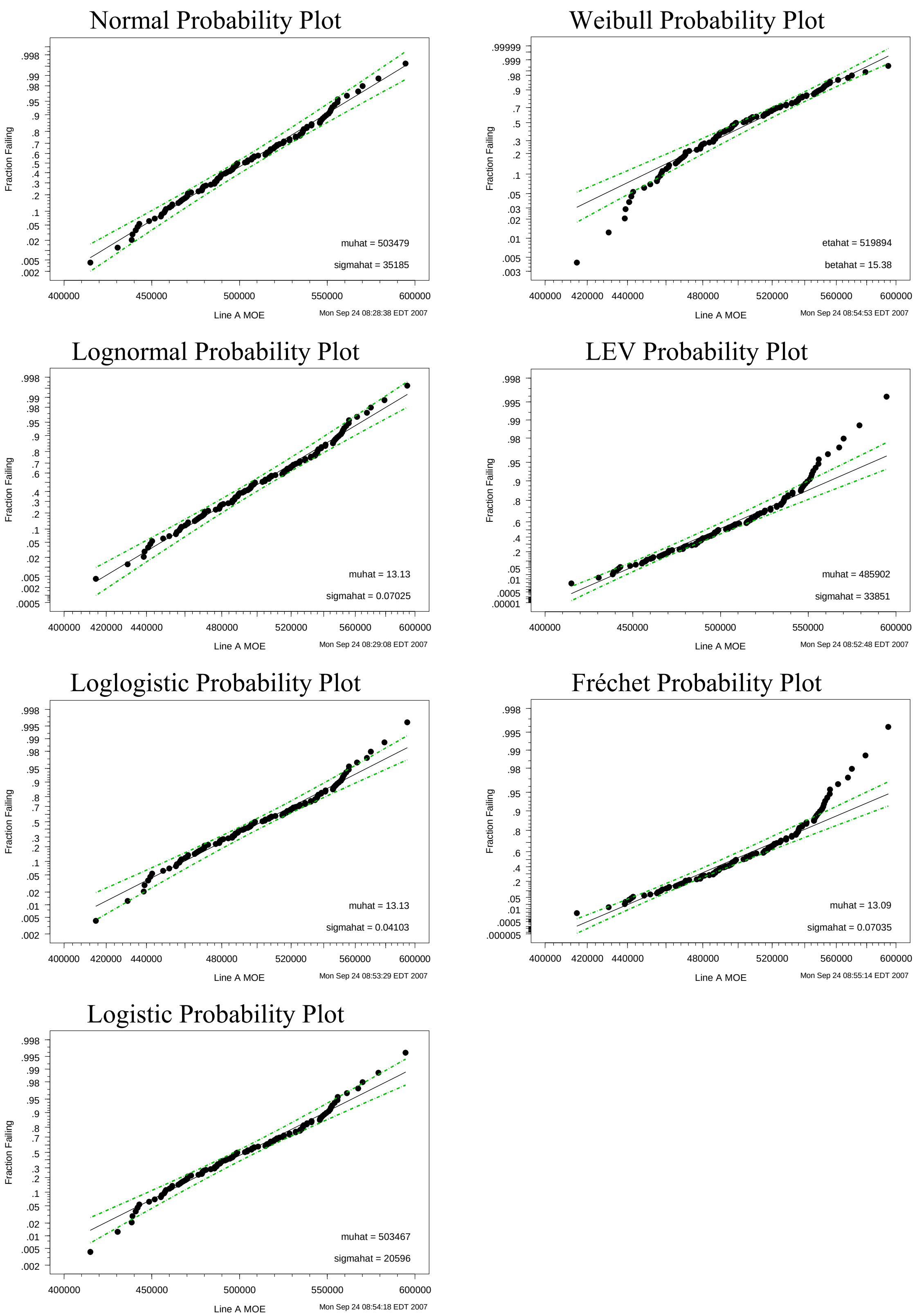

Figure 3-2: Probability plots for several distributions on the Modulus of Elasticity for wood plastic composites from Line A.

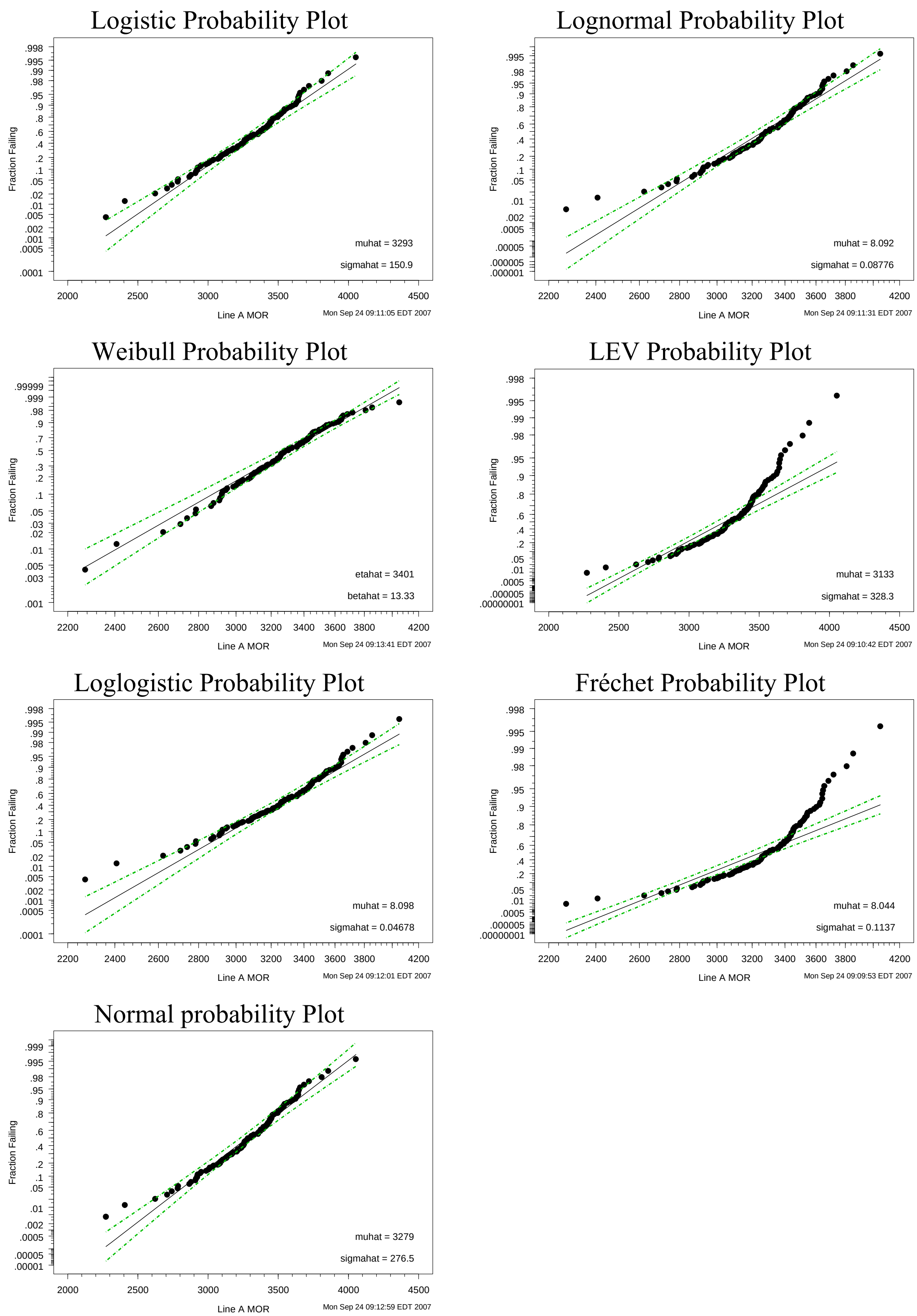

Figure 3-3: Probability plots for several distributions on the Modulus of Rupture for wood plastic composites from Line A.

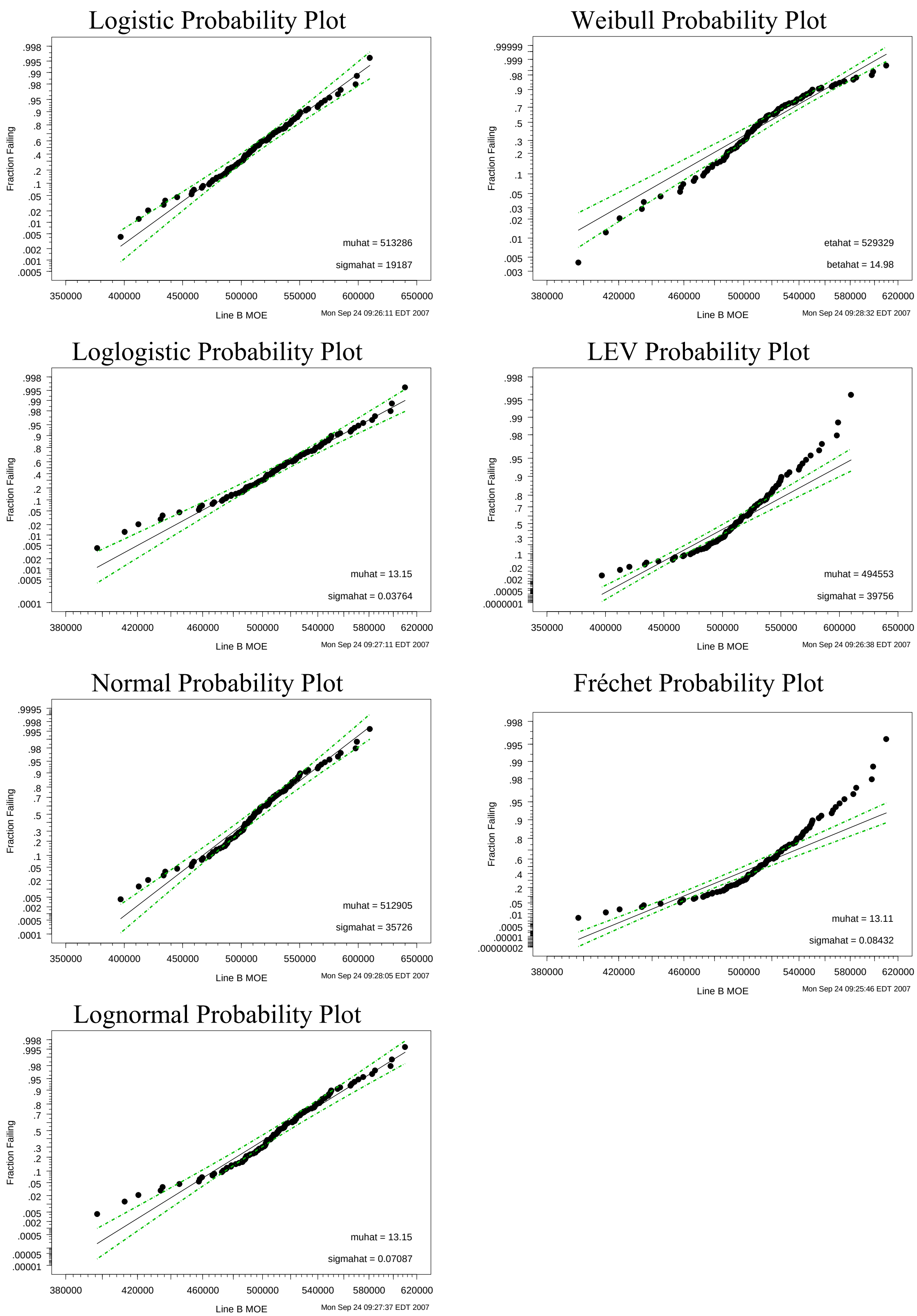

Figure 3-4: Probability plots for several distributions on the Modulus of Elasticity for wood plastic composites from Line B.

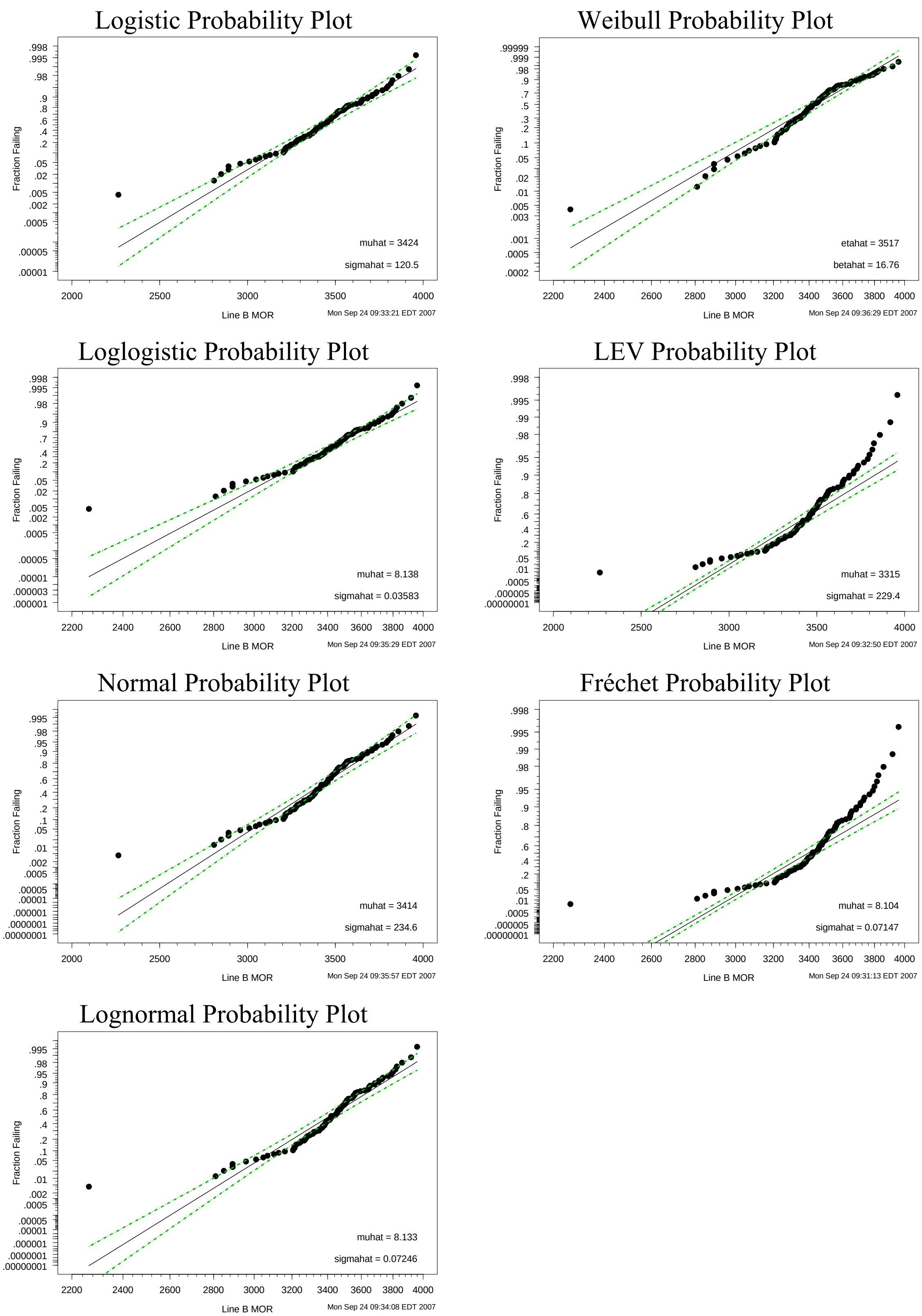

Figure 3-5: Probability plots for several distributions on the Modulus of Rupture for wood plastic composites from Line B with outlier included.

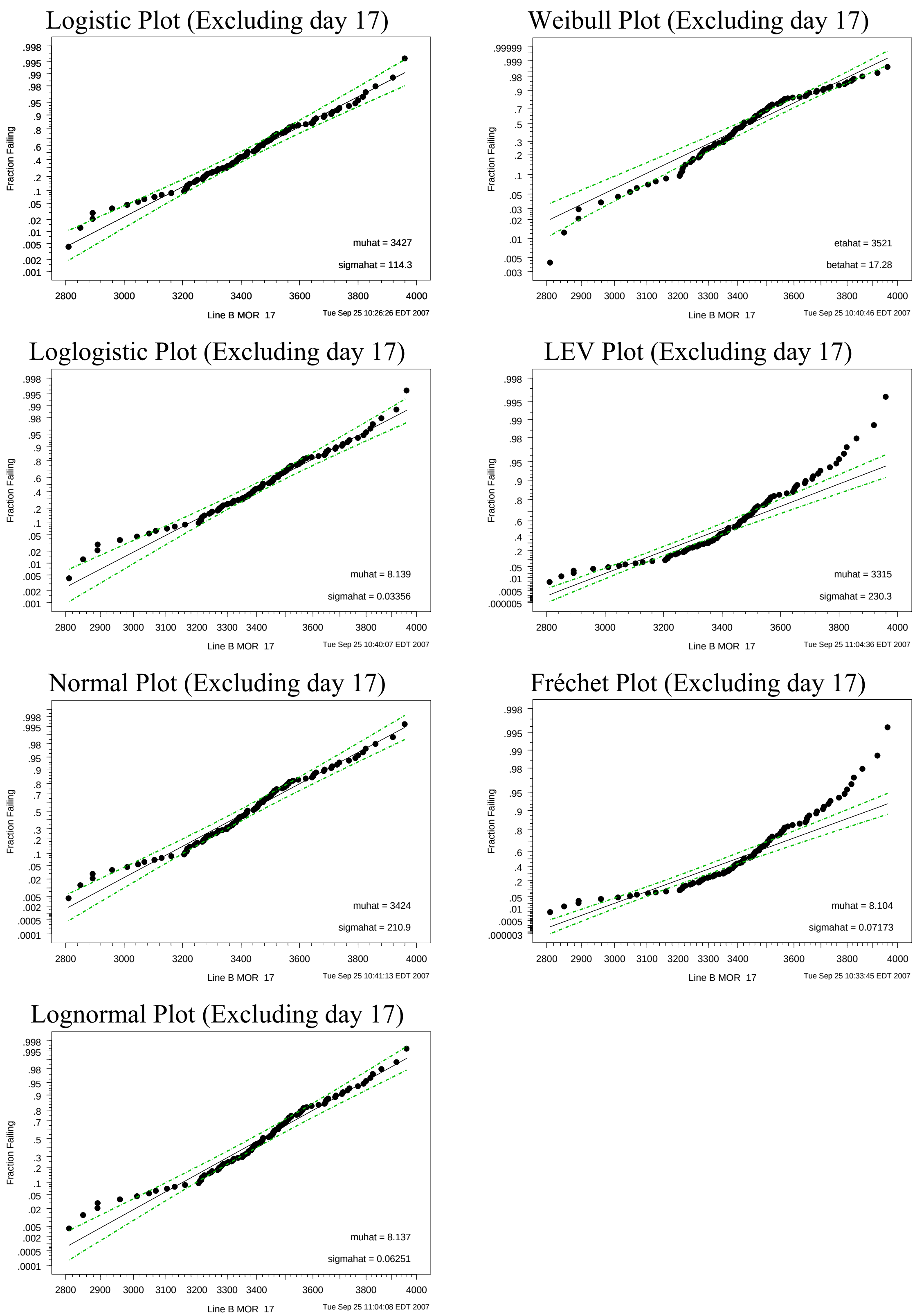

Figure 3-6: Probability plots for several distributions on the Modulus of Rupture for wood plastic composites from Line B with outlier excluded.

The implication of an outlier, as the one described here, is the possibility of miss-specifying the distribution. If the early failure on day 17 is due to a special cause (a cause that is out of the ordinary) it is proper to discard the data point. For this reason, it is important for a practitioner to observe the process and note any possible special causes. On the other hand, if the early failure is due to a common cause (a source of variation that is typical in the process) then the data point is part of the distribution and should be included to get a proper estimate of the parameters.

The implications for the practitioner for identifying the correct distribution of the data are essential for making valid process decisions. Accurate confidence intervals, minimization of serious Type I errors, and strength of conclusions are all dependent on identifying the correct distribution or probability density function. Minimizing the risk of a Type I error is paramount and can prevent invalid conclusions made by analyzing statistics based on the wrong distribution. Accurate confidence intervals and statistical-based decision making are critical for minimizing risk, properly allocating capital, optimizing resource use, minimizing costs, and improving product quality.

3-2-2. Distributions and Parameter Estimation

From the information criteria and probability plots, the normal distribution is assumed for the MOE from Line A. The remaining three distributions are estimated assuming the logistic distribution. Parameter estimates for the four distributions are in Table 3-4.

Proper fitting of distributions and estimating parameters for these distributions are essential for correct applications of statistical methods in science and business. In

Table 3-4: Parameter estimates for the selected Distributions.

Sample	Distribution	Location $\hat{\mu}$ MPa	Shape $\hat{\sigma}$ MPa
Line A MOE	Normal	503,479	35,185
Line A MOR	Logistic	3293	150.9
Line B MOE	Logistic	513,286	19,187
Line B MOR	Logistic	3424	120.5

scientific endeavor, parameters of the distributions are necessary for developing valid confidence intervals and conclusions. As seen in the probability plots, the confidence intervals for each of the selected distributions capture more of the data – particularly in the tails – than the unselected distributions. The parameters of the distributions provide the practitioner with objective insight in assessing the statistical differences between Lines A and B, which may direct the manufacturers towards additional root cause analysis, costs savings, and product quality improvement.

3-3. Non-parametric Reliability Comparisons of Failure Data

In the preceding section, the use of descriptive statistics and the fitting of data to distributions for parameter estimation were highlighted as an essential first step for practitioners in improving the understanding of WPC extrusion processes. In this section, we expand the analysis by using non-parametric reliability methods to assess the reliability of manufactured WPC. Non-parametric reliability methods assume no underlying distribution for the data and can be very helpful when parametric statistical methods are not available for a specific distribution that fits the data.

Reliability methods are important to manufacturers when assessing the quality, durability, and reliability of manufactured product. Product reliability methods applied to experimental data are a common rubric (Rosowsky and Ellingwood 1992; Taylor, Bender, Kline, and Kline, 1992; Ellingwood 1997; Rosowsky, Line, and Line, 2005; Van de Lindt and Rosowsky 2005; Van de Lindt, Huart, and Rosowsky 2005).

The reliability/survival function captures the probability that the system will survive beyond a specified time (or pressure). A common nonparametric analysis is Kaplan-Meier estimation (Kaplan and Meier, 1958; Tobias and Trindade 1995). The Kaplan-Meier estimator (origin of Product Limit Estimator) estimates the survival function from life-time (or pressure to failure) data (Kaplan and Meier 1958).

The Kaplan-Meier statistic is evaluated using equation 3-2 where p_i is the pressure at the i^{th} failure when the failures are ranked in ascending order, n_i is the number of WPC tests that have not failed prior to p_i, and d_i is the number of failures at p_i. This is a general formula used for interval, censored, and read-out data.

$$\hat{S}(p) = \prod_{p_i < p} \frac{n_i - d_i}{n_i} \qquad \textbf{(3-2)}$$

Greenwoods formula (Equation 3-3) is used to calculate the variance of the Kaplan-Meier statistic (Greenwood 1926). When the sample size is large the Kaplan-Meier curve approaches the true distribution of the population.

$$\hat{V}\left(\hat{S}(p)\right) = \hat{S}(p)^2 \sum_{p_i < p} \frac{d_i}{n_i(n_i - d_i)} . \qquad \textbf{(3-3)}$$

Statistical comparison of Kaplan-Meier survival curves indicate that the MOE and MOR differ by production line (Figure 3-7). The WPC production from Line A has a higher probability of failure at lower stress relative to the WPC production from Line B. For example, there is approximately a 0.50 probability that the MOE for the WPC produced from Line A will exceed 50,000 MPa. In contrast there is approximately a 0.70

Kaplan-Meier Curves for Proportion of WPC Surviving Pressure to Failure

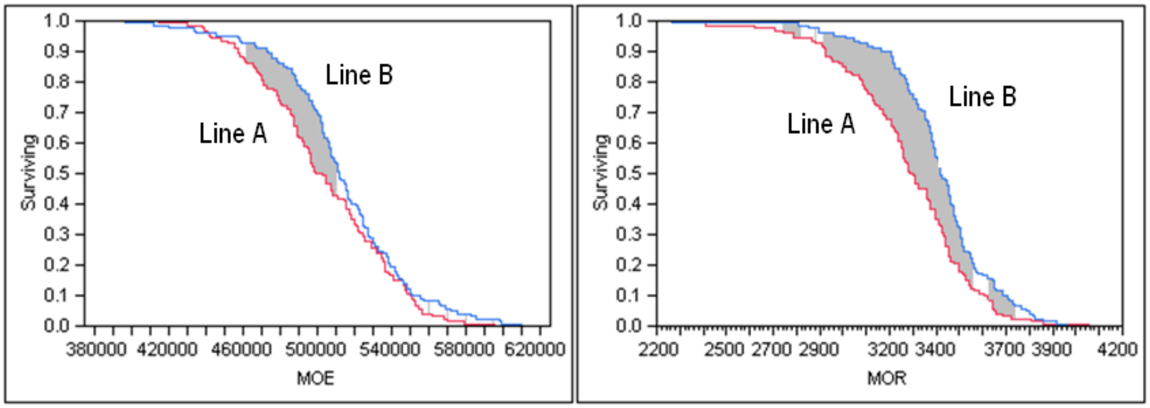

Figure 3-7: Kaplan-Meier Comparison of Lines A and B

probability that the MOE for the WPC produced from Line B will exceed 50,000 MPa. The differences in the survival curves are more pronounced for the MOR when comparing the two production lines.

To enhance the visual comparison of survival curves we use a statistical method to compare these curves for intervals of similarity (Dinse, Boos, and Piegorsch 1993) using Wald's test (Bowman and Young 1996). Code for this test is available for download at www.spcforwood.com. The shaded regions in Figure 3-7 represent the intervals of pressure between which the Kaplan-Meier curve for Line B is statistically greater than the curve for Line A at an $\alpha = 0.05$.

Although the means for Lines A and B are different, the understanding of the reliability of the WPC processes is improved, i.e., intervals of the Kaplan-Meier curves that are statistically dissimilar. The two extrusion lines produce the same WPC product with regard to the MOE below 461,690 MPa and above 510,100 MPa. This accounts for the lower 10.2 percent and the upper 48.0 percent of the MOE data. This may be an undesirable outcome for the WPC manufacturer, i.e., to produce different product reliability for MOE in the range of 461,690 and 510,100 MPa.

With regard to the MOR, the two extrusion lines are similar below 2,742 MPa and above 3,729 MPa. This accounts for the lower two percent and the upper 4.9 percent of the data with a few minor exceptions near the tails. The Kaplan-Meier survival curves as applied to these failure metrics indicate that the majority of WPC produced from Line B is stronger than Line A. This is of concern to the manufacturer if the desired outcome is to produce similar WPC products from both extrusion lines. This may also have important warranty implications for the manufacturer.

3-4. <u>Conclusion</u>

Descriptive statistics are used in this chapter to assess the data quality and statistical differences in the MOE and MOR strength metrics produced from the two WPC production processes. There is statistical evidence that mean and median MOE from Line B are greater than the mean and median MOE from Line A. The mean and median MOR from Line B are also greater than the mean and median MOR from Line A. There was no statistical evidence that the variances from the two extrusion lines were different. The lower strength of the WPC from Line A may be of concern to the manufacturer if the desired outcome from the two production processes is similar strength.

For the MOE from Line A, the normal distribution is the best fit to the data. For MOR from Line A the Logistic distribution provides the best fit to the data. The Logistic distribution is the best fit for both the MOE and MOR from Line B. The distribution fits and parameterization are important for the scientist and practitioner when using parametric statistics methods to objectively quantify product quality. Incorrect assumptions about distributions influence parameter estimates and ultimately affect the validity of conclusions.

The analysis indicated that the reliability of WPC manufactured for each production line is different. Non-parametric Kaplan-Meier survival curves of the MOR indicates that this metric is dissimilar almost 93 percent of the time, i.e., Line B produces a consistently higher MOR. The MOE for both lines is dissimilar approximately 51 percent of the time, i.e., Line B produces a higher MOE between the mid-level percentiles. This is of concern to the manufacturer if the desired outcome is to produce similar WPC quality from both extrusion lines.

Chapter 4. Parametric Comparison of Small Percentiles of Wood Plastic Composites via Bootstrapping

Chapter 4 is an analysis of the lower tails of the failure distributions for the WPC introduced in Chapter 3, and is written for the practitioner. Section 4-1 gives background information on WPC and the data under analysis, followed by the summary and conclusions in section 4-2. For more details on the analysis and the hypothesis tests, section 4-3 is the methods.

Lower Tail Distribution Comparison of Wood Plastic Composites via Bootstrapping

4-1. Background Information

Wood plastic composites are made from a combination of wood flower and plastic to create a durable substitute for lumber in the construction business. Metrics for measuring durability are the Modulus of Elasticity (MOE) and Modulus of Rupture (MOR). These metrics are measured in units of Mega Pascal (MPa). MOE is estimated using a standard test method that estimates the flexural properties of WPC and is derived following the ASTM D6109-05 standard method (ASTM 2008). MOR refers to the failure stress (maximum stress) obtained when applying a load to a structural member in flexure. MOR for wood plastic composites is derived following the ASTM D7031-04 standards (ASTM 2008).

The data used in this study comes from a WPC manufacturer with two production size extrusion lines. For simplicity, these lines are referred to as Line A and Line B. Samples were collected from each line over the period of January 1 to May 2, 2005. The sampling order is preserved, but the method is unknown. MOE and MOR metrics were obtained from each line (Figure 4-1).

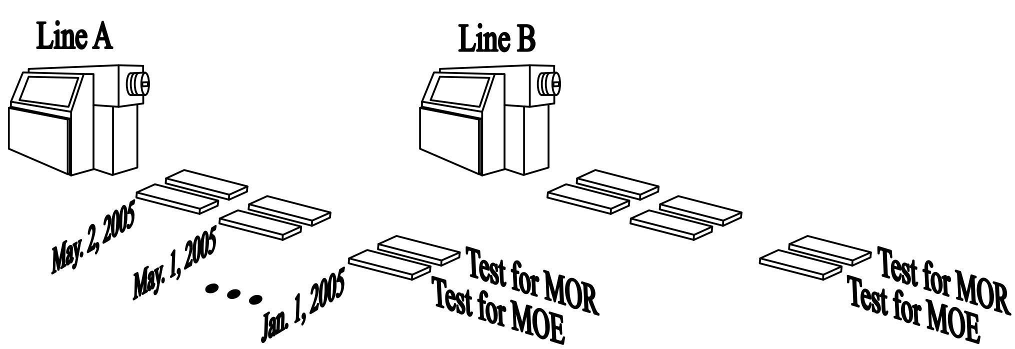

Figure 4-1: Wood Plastic Composite Extrusion Lines

Table 4-1: 95% Bonferroni family-wise Bootstrap Confidence Intervals for MOE and MOR for Lines A and B

MOE 98.31% Individual Bootstrap CI for Differences		
Percentile	Lower	Upper
1st	-42420	21427
5th	-33366	33358
10th	-10059	34499

MOR 98.31% Individual Bootstrap CI for Differences		
Percentile	Lower	Upper
1st	-451	620
5th	-57	505
10th	17	431

4-2. Summary and Conclusions

Table 4-1 has 95% Bonferroni confidence intervals (98.31% individual confidence intervals) for the metrics from Line B minus the metrics from Line A for the 1st, 5th, and 10th percentiles. Since all of the intervals for the MOE contain zero, there is not sufficient evidence to prove that there is a difference in the MOE between the two lines at the three percentiles tested. This is reassuring for practitioners since consistency between lines is desirable. However, since the 10th percentile confidence interval for the MOR is greater than zero, there is sufficient evidence to conclude that there is a difference in the MOR between the two lines in the left tail of the distributions, with Line B producing the stronger WPC. If this statistical difference is of practical importance, further investigation into the cause of this difference is suggested.

4-3. Methods

Figure 4-2a indicates that there are two shifts in the MOE for Line A occurring around January 24 and March 22. Since the sampling method is not known, the first

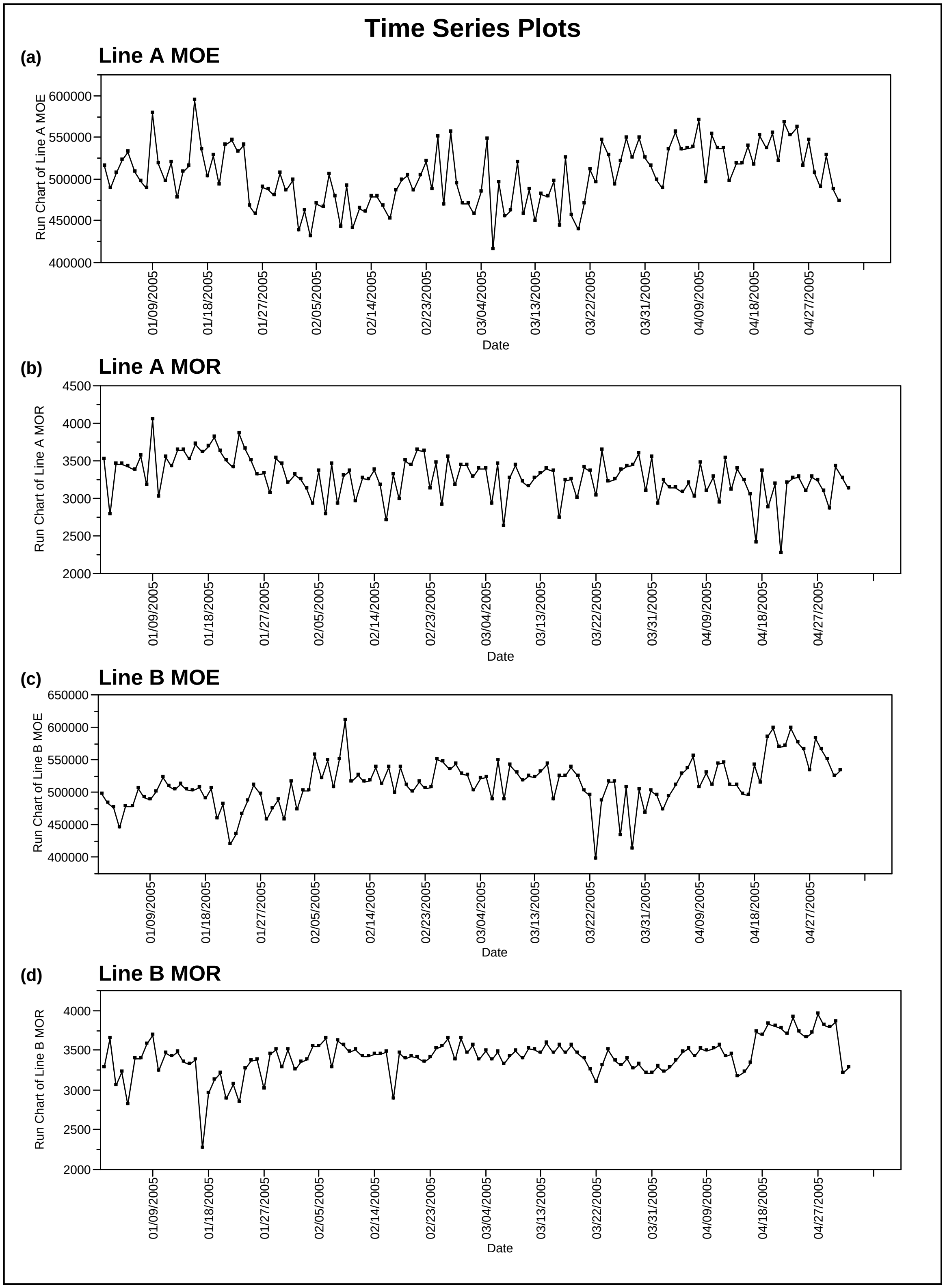

Figure 4-2: Time series for the MOE and MOR for Line A and Line B from January 1 to May 2, 2005.

Table 4-2: Parameter Estimates and 95% Confidence Intervals for selected distributions for the MLE and MOR from Line A and Line B.

Sample	Distribution	Location $\hat{\mu}$ MPa (95%CI)	Shape $\hat{\sigma}$ MPa (95%CI)
Line A MOE	Normal	503,479 (497235,509722)	35,185(31036, 39888)
Line B MOE	Logistic	513,286 (507451, 519120)	19,187 (16508, 22301)
Line A MOR	Logistic	3293 (3247.1, 3339.8)	150.9 (130.1, 175.1)
Line B MOR	Logistic	3424 (3388.0, 3460.8)	120.5 (103.6, 140.2)

recommendation is to keep track of changes in the process. Secondly, a statistically sound sampling method should be used to eliminate bias. If known changes occur, this sampling method should block according to those changes. Assuming that the data provided are representative of the process over time, the following conclusions are made.

Previous research indicates that the MOE for Line A is most appropriately modeled by a normal distribution, and the MOR from Line A, MOE from line B, and MOR from line B are most appropriately modeled by the logistic distribution. These distributions were determined by comparing the log likelihoods and Akaika's Information Criterion from several distributions. Parameter estimates and 95% confidence intervals for each of these distributions are in Table 4-2. See Appendix A-1 for Splida output.

Practitioners are interested in estimating the MOE and MOR for the lower percentiles (i.e. 1%, 5%, and 10%) as an assurance that WPC are meeting reliability requirements. A further interest is a comparison of the reliability between the two lines for these lower percentiles. This chapter focuses on bootstrap confidence intervals for lower tail percentiles assuming the distributions aforementioned. Furthermore, the null hypothesis stating that there is no difference in the MOE and MOR at these percentiles between the two lines will be tested via bootstrapping.

4-3-1. Modulus of Elasticity Bootstrap Confidence Intervals for Percentiles

Assuming the normal distribution for the MOE of WPC extruded from Line A, Table 4-3a has the maximum likelihood estimates and corresponding 95% bootstrap confidence intervals for the 1^{st}, 5^{th}, and 10^{th} percentiles. Assuming the logistic distribution for the MOE of WPC extruded from Line B, Table 4-3b has the maximum likelihood estimates and corresponding 95% bootstrap confidence intervals for the same percentiles. Figure 4-3 is a graphical comparison of these intervals. SPLIDA output for the bootstrap estimates is in Appendix A-2.

4-3-2. Hypothesis Test comparing Lines A and B for the MOE

It appears from Figure 4-3 that Lines A and B are equivalent at the three percentiles. To test this hypothesis, a bootstrap is taken of the percentile differences from each line. Bootstrap histograms from JMP output for this test are in Appendix A-3, and JMP script for conducting this test is in Appendix A-4. A formal test follows.

H_o: There is no difference in the MOE at the 1^{st}, 5^{th}, and 10^{th} percentiles between Lines A and B.

H_a: There is a difference in the MOE at the 1^{st}, 5^{th}, or 10^{th} percentiles between Lines A and B.

By the Bonferroni method, a family-wise confidence level of 95% requires testing each of the three percentiles at a confidence level of $(0.95)^{\frac{1}{3}} \times 100\% = 98.31\%$.

Table 4-4 has the confidence intervals for the three percentiles. Since all of the intervals contain zero, there is not sufficient evidence to prove that there is a difference in the MOE between the two lines at the three percentiles tested. This is reassuring for practitioners since consistency between lines is desirable.

Table 4-3: Maximum Likelihood Estimates of Lower Tail Percentiles and corresponding Bootstrap Confidence Intervals for the MOE

a

Line A MOE 95% Bootstrap CI			
Percentile	Estimate	Lower	Upper
1st	421626	411118	433201
5th	445605	437057	454840
10th	458387	450846	466670

b

Line B MOE 95% Bootstrap CI			
Percentile	Estimate	Lower	Upper
1st	425118	409177	440932
5th	456790	446076	467882
10th	471127	462009	480402

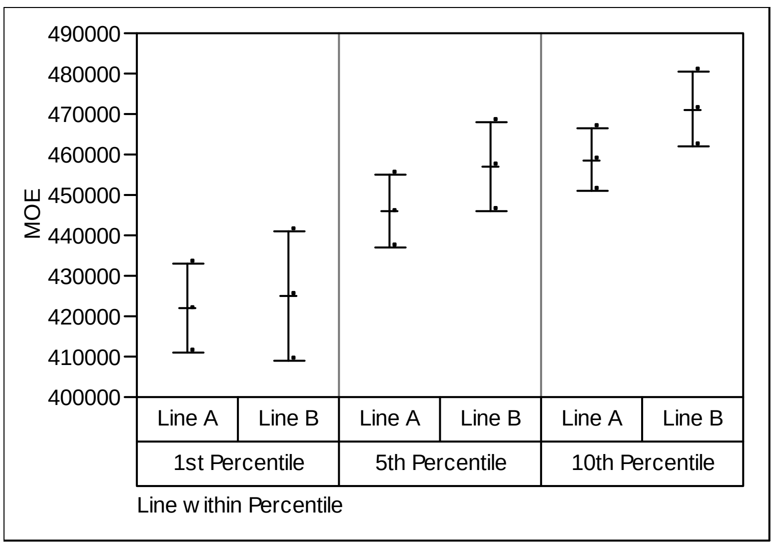

Figure 4-3: Line A and Line B comparison by Percentile for Maximum Likelihood Estimates of Lower Tail Percentiles and corresponding Bootstrap Confidence Intervals for the MOE

Table 4-4: 95% Family-wise Confidence Hypothesis Test for the MOE Differences at lower end Percentiles for Lines A and B

MOE 98.31% Bootstrap CI for Differences		
Percentile	Lower	Upper
1^{st}	-42420	21427
5^{th}	-33366	33358
10^{th}	-10059	34499

4-3-3. Modulus of Rupture Bootstrap Confidence Intervals for Percentiles

The MOR for both Lines A and B are assuming the logistic distribution. For the WPC extruded from Line A, Table 4a has the maximum likelihood estimates and corresponding 95% bootstrap confidence intervals for the 1^{st}, 5^{th}, and 10^{th} percentiles. For the WPC extruded from Line B, Table 4-5b has the maximum likelihood estimates and corresponding 95% bootstrap confidence intervals for the same percentiles. Figure 4-4 is a graphical comparison of these intervals. SPLIDA output for the bootstrap estimates is in Appendix A-2.

4-3-4. Hypothesis Test comparing Lines A and B for the MOR

It appears from Figure 4-4 that Lines A and B are not equivalent at all three percentiles. JMP output for this test is in Appendix A-3. A formal test follows.

Ho: There is no difference in the MOR at the 1st, 5th, and 10th percentiles between Lines A and B.

Ha: There is a difference in the MOR at the 1st, 5th, or 10th percentiles between Lines A and B.

Family-wise Bonferroni confidence level = 95%

Table 4-6 has the confidence intervals for the three percentiles. Since the 10^{th} percentile confidence interval does not contain zero, there is sufficient evidence to reject the null hypothesis and conclude that there is a difference in the MOR between the two lines in the left tail of the distributions. If this statistical difference is of practical importance, further investigation into the cause of this difference is suggested.

Table 4-5: Maximum Likelihood Estimates of Lower Tail Percentiles and corresponding Bootstrap Confidence Intervals for the MOR

a

Line A MOR 95% Bootstrap CI			
Percentile	Estimate	Lower	Upper
1st	2600	2472.6	2722.3
5th	2849.1	2755.1	2937.6
10th	2961.9	2882.4	3037.1

b

Line B MOR 95% Bootstrap CI			
Percentile	Estimate	Lower	Upper
1st	2870.8	2753.4	2979.5
5th	3069.7	2986.1	3145.2
10th	3159.7	3091.9	3220.7

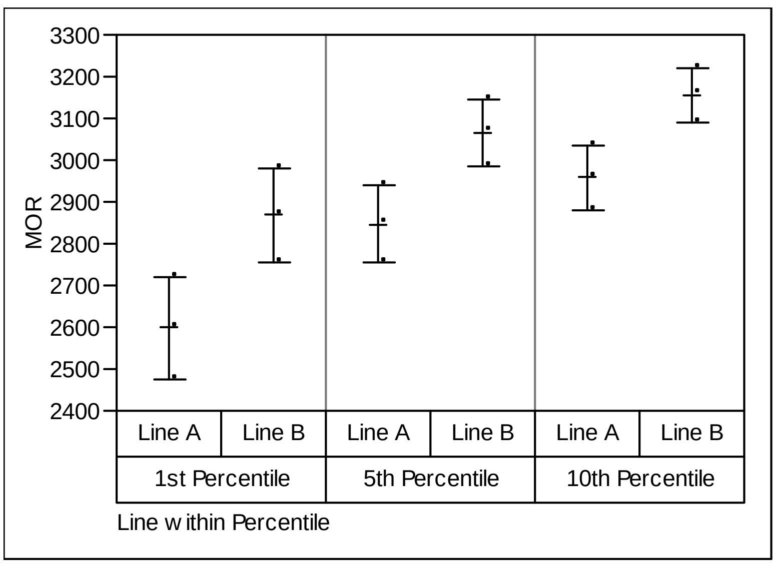

Figure 4-4: Line A and Line B comparison by Percentile for Maximum Likelihood Estimates of Lower Tail Percentiles and corresponding Bootstrap Confidence Intervals for the MOR

Table 4-6: 95% Family-wise Confidence Hypothesis Test for the MOR Differences at lower end Percentiles for Lines A and B

MOR 98.31% Bootstrap CI for Differences		
Percentile	Lower	Upper
1^{st}	-451	620
5^{th}	-57	505
10^{th}	17	431

Chapter 5. Improving Estimates of Lower Percentiles by Percentile Censoring: Theoretical Models & Simulation Results

Results of a simulation studying the effect of right censoring for a two-part Weibull distribution as a means of improving left tail estimation are presented in this chapter. Section 5-1 gives an overview of reliability and the traditional use of data censoring. Section 5-2 introduces the well known bathtub hazard curve. These two concepts are brought together in section 5-3, and the simulation is outlined. Conclusions of the simulation are given in Sections 5-4, and practical applications in Section 5-5.

Improving Estimates of Lower Percentiles by Percentile Censoring: Theoretical Models & Simulation Results

5-1. Traditional Censoring

The study of product reliability allows practitioners to understand products' life expectancies. Reliability can relate to the length of time, level of stress, or frequency of use until product failure. For simplicity, this chapter will refer to any one of these measurements until failure as duration of time. For customer satisfaction, or even liability reasons, practitioners want to know when they can typically expect the first failures to occur.

A reliability study is conducted by placing a sample of products on test and measuring the time to failure for each unit. Meeker and Escobar (1998) give details for accelerating these tests and forecasting results to real time. These authors also discuss methods for choosing the most appropriate Cumulative Distribution Function (CDF) from theoretical and mathematical perspectives. The results from such studies are used to make predictions on the reliability of the populations from which the samples were drawn.

In many cases not all units fail by the end of a test period and the surviving units are said to be right censored. Right censoring can also take place when the test is terminated after a predetermined number of units have failed. These are referred to as Type I and Type II censoring, respectively. Tobias and Trindade (1995) explain the practical benefits of each type. The censored values provide less information than the exact failure times, though are useful in that they indicate the proportion of the sample that can survive beyond the censored time.

5-2. Bathtub Hazard Function

One derivation of the CDF is the hazard function. The hazard is the instantaneous rate of failure at a given point in time (Equation 5-1).

$$\boldsymbol{h(t) = f(t)/S(t)} \quad \textbf{(5-1)}$$

The numerator $f(t)$ is the probability density function, and the denominator $S(t)$ is the survival function (see Appendix A-5 for more information on the relationship and derivation of these functions as they relate to this simulation.) In many applications, the hazard function consists of three intervals. The first interval decreases rapidly and is known as an infant mortality rate. As the week components fail and are removed from the sample, the rate of failures decreases. Remaining components will fail at a constant rate due to random causes, and this interval is known as the usable life. Components that are still in use by the end of the usable life will begin to wear out, and the rate of failures will increase. This type of hazard function is well known as the bathtub curve.

5-3. Simulation

The simulation considers a set of data derived from the Weibull distribution with the characteristics of this bathtub curve. In practice the parameters for each interval of the distribution are unknown. The point in time when the infant mortality ends and the usable life begins is also not known. An undesirable fit occurs when the *full* set of data is modeled with the one distribution, resulting in error of the lower percentile estimates. To correct for this error, this simulation ranks the data and censors a percentage from the right tail *after* all the data have been generated. All failure times in this right tail are set equal to the censored time and are labeled as right censored observations. The data is again fit to the Weibull distribution and the 1st, 5th, and 10th percentiles are estimated. With each iteration of the simulation, the amount of censoring increments by 10 percent until 90 percent of the data is right censored. Precision and accuracy for the estimated percentiles are calculated by a *scaled* version of the Root Mean Squared Error (RMSE) and Bias – namely, the $sRMSE = (RMSE/t_p)\%$ and $sBias = (Bias/t_p)\%$ where t_p is the time at the estimated percentile.

For this simulation a sample of size $n = 200$ is generated with the following characteristics. In the interval $0 \leq t \leq 100$, the shape parameter $\beta = 0.5$, and for $t > 100$ the shape parameter $\beta = 1.2$. These parameters allow for the decreasing hazard rate and a slightly increasing hazard rate characteristic of the bathtub hazard curve. The scale parameter $\eta = 400$ for all values of t. This simulation is generated 1000 times. The MATLAB code is in Appendix A-6.

5-4. <u>Results</u>

Two distributions are displayed in Figure 5.1. The blue curve represents the Weibull distribution with parameters $\beta = 0.5$ and $\eta = 400$ for $0 \leq t \leq 1000$. The red curve represents the two-part Weibull distribution described in section 5-3. Although the two-part Weibull curve is representative of the data, the deviation beginning at $t = 100$ obscures the estimates in the left tail of the curve when all of the data is fit to a Weibull curve with one shape parameter. In practice, the time when this change begins is not

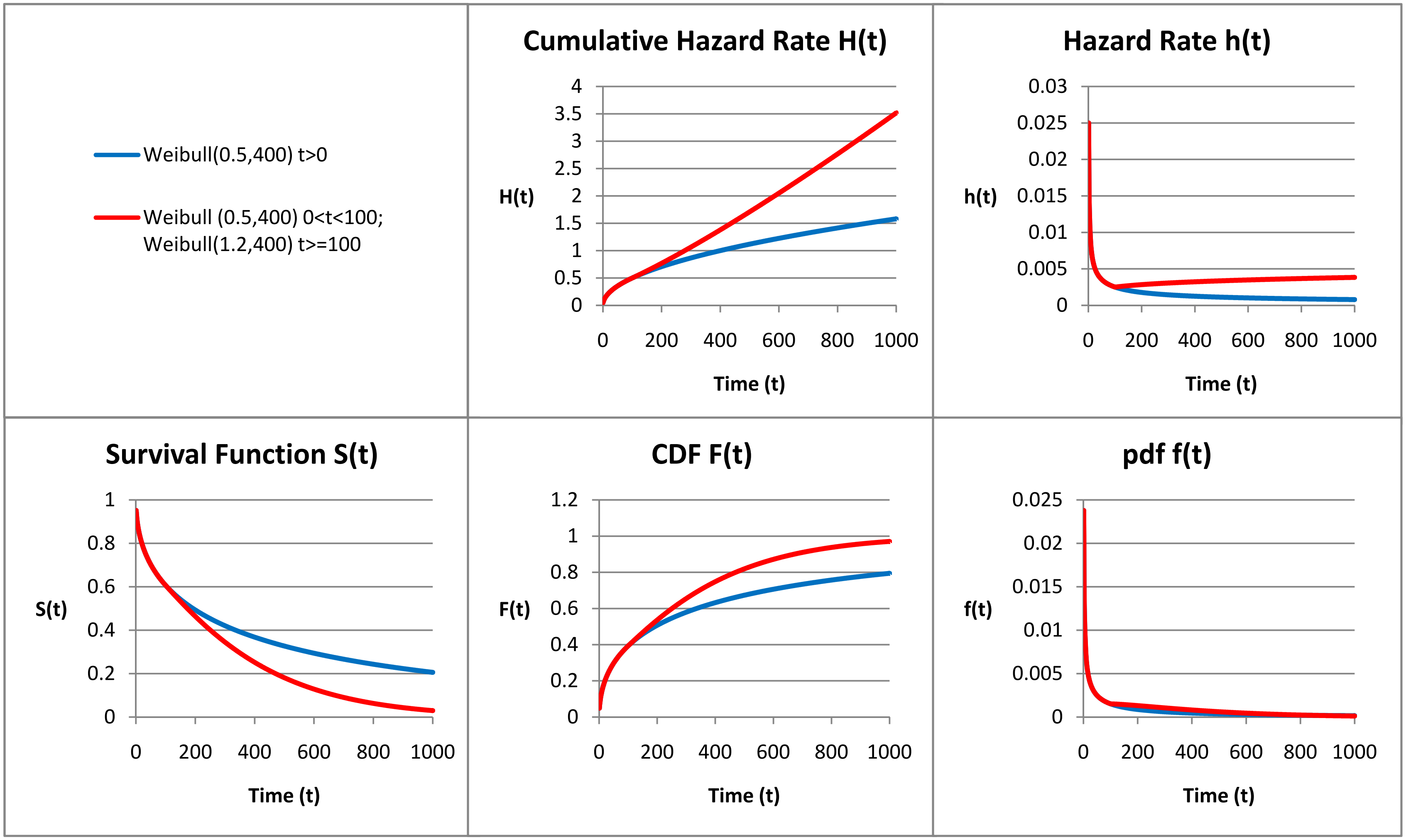

Figure 5-1: Standard Weibull Distribution and Bathtub Hazard Modification of Weibull Distribution

known, and the objective is to find what proportion of the data needs to be censored from the right of the distribution so that the estimated distribution from the data fits the left tail, thus allowing better estimation of the lower percentiles. For the present, the time this deviation begins is known for the purpose of the simulation. Section 5-5 describes how to estimate this time for practical use.

The red curve in the CDF of Figure 5-2a is the theoretical distribution from which the data is generated. Each black curve represents a Weibull CDF with parameters estimated from the generated data. In this instance the full data set is used to estimate the parameters. Figures 5-2b and 5-2c show the sRMSE and sBias for the 1st, 5th, and 10th percentiles. The three histograms in Figure 5-2d, 5-2e and 5-2f represent the distribution of all 1000 time estimates for the three percentiles, and the red bars indicate the true times for these percentiles.

The full output and figures for the simulation are in Appendix A-7. Figure A-7b shows what happens when the top 10 percent of the data is right censored. (The horizontal bar in the first graph represents where the censoring occurs.) The sRMSE and sBias drop nearly 25 percent from when no censoring took place. This improvement in precision and accuracy continue until about 50 percent of the data is censored, and then begins to worsen again at about 80 percent (see Figures 5-3, 5-4, and 5-5).

These results are encouraging because the CDF at time $t = 100$ corresponds to the proportion of $F(100) = 1 - \exp\left[-\left(\frac{100}{400}\right)^{0.5}\right] = 0.39347$. When about 60 percent of the data are right censored, the remaining 40% allow for the best estimation of the lower percentiles.

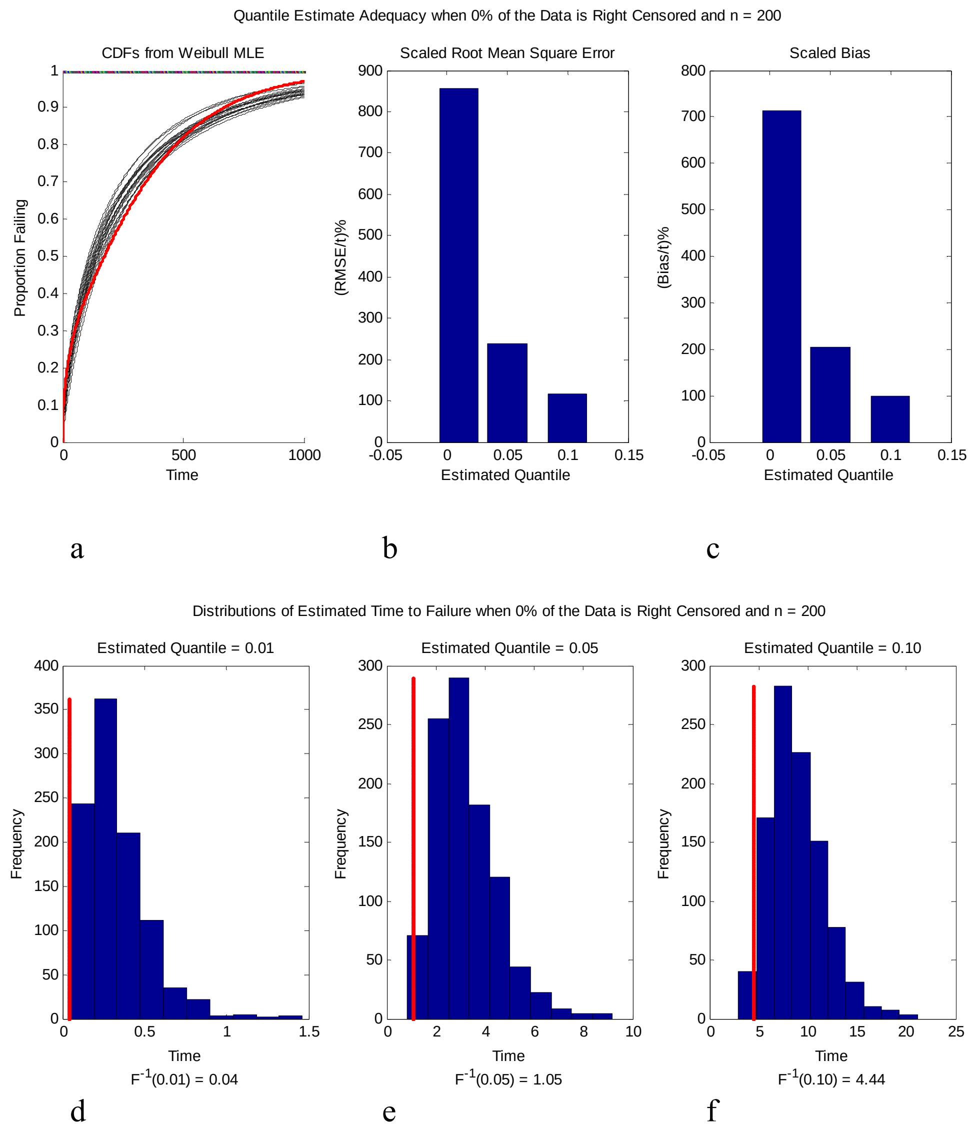

Figure 5-2: Simulation Results for Lower Percentile Estimates with no Censoring

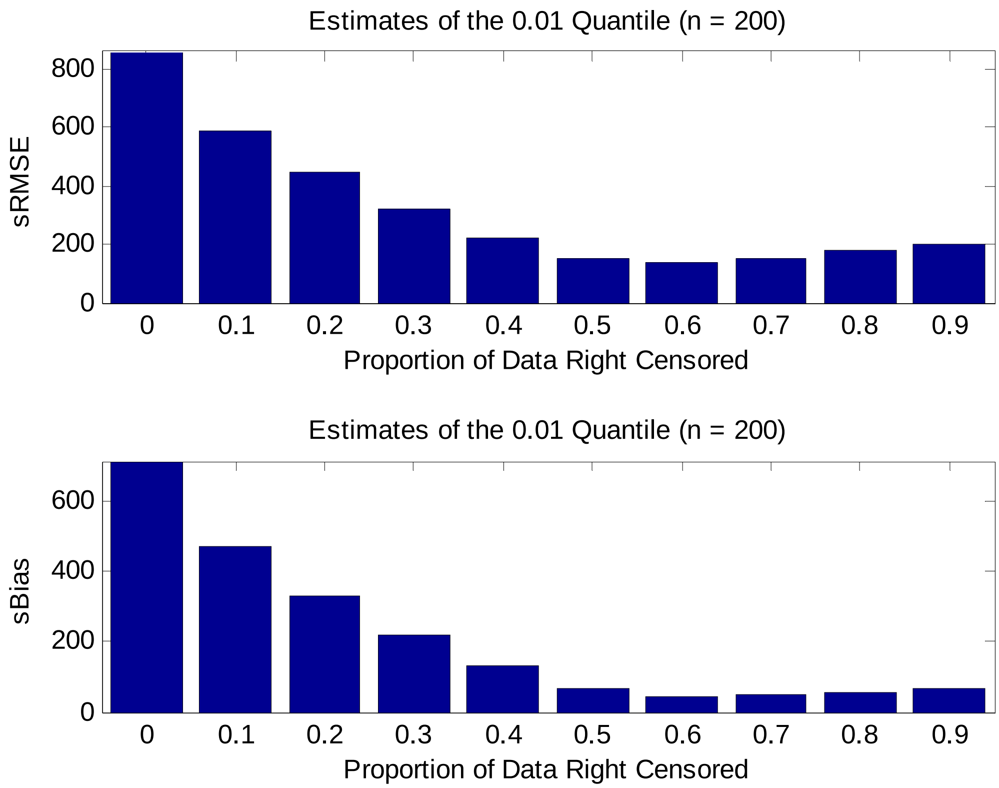

Figure 5-3: Effect of Censoring on the sRMSE and sBias of the 0.01 Quantile for the Weibull Distribution with Two Aging Behaviors

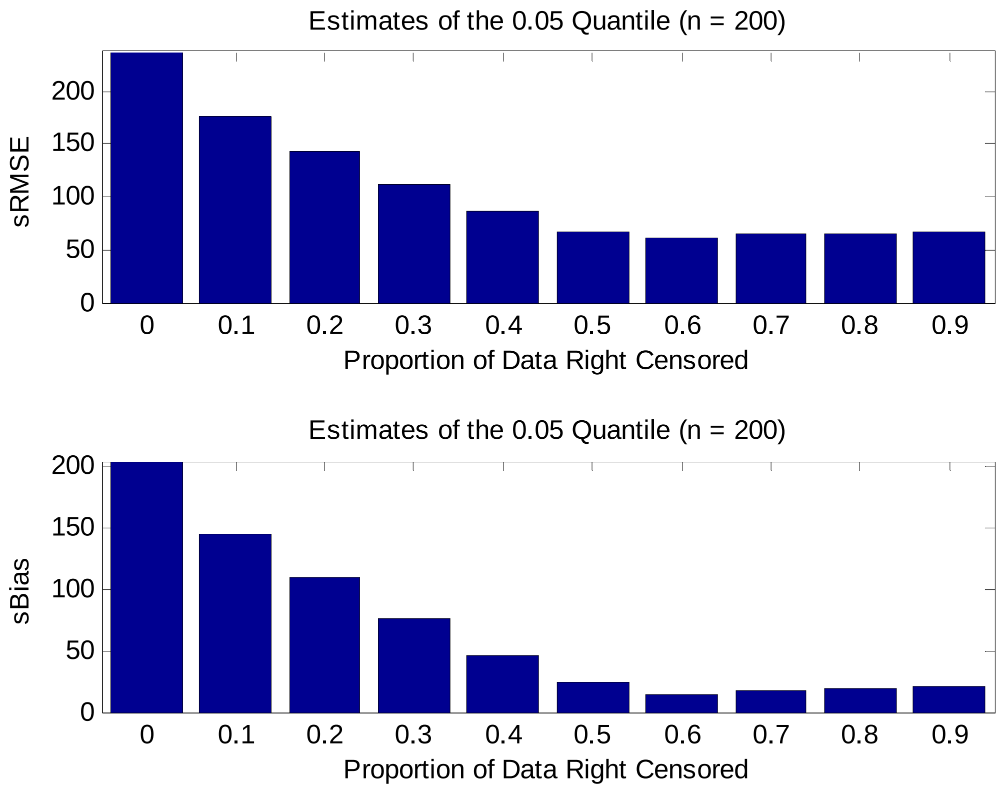

Figure 5-4: Effect of Censoring on the sRMSE and sBias of the 0.05 Quantile for the Weibull Distribution with Two Aging Behaviors

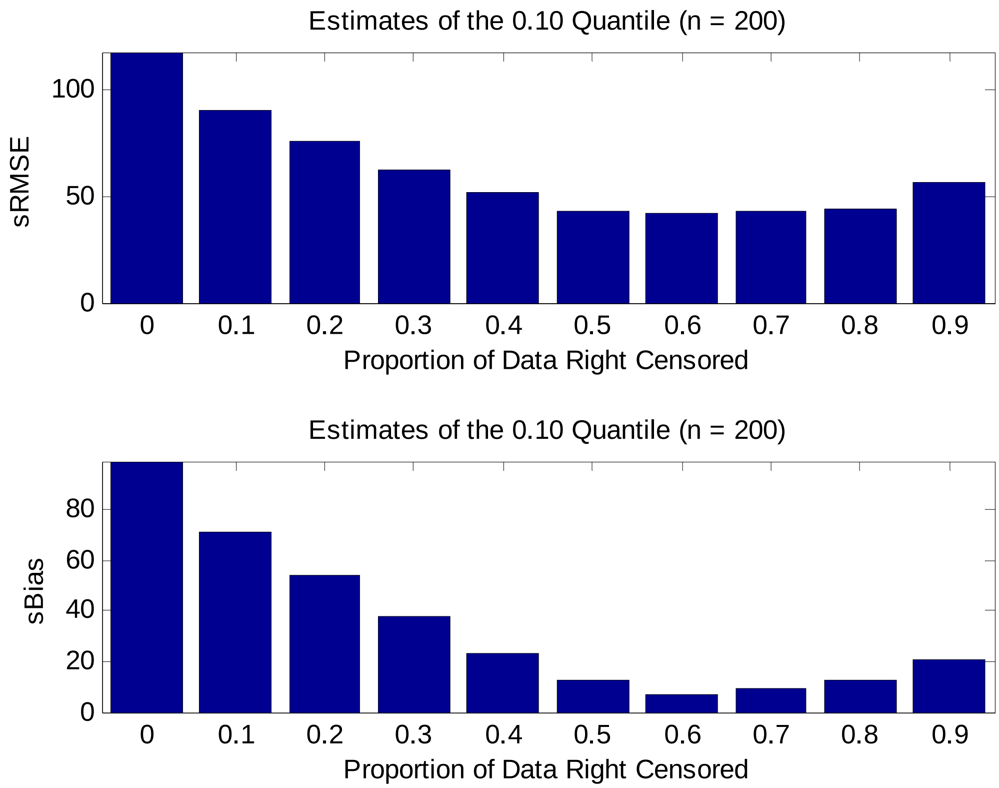

Figure 5-5: Effect of Censoring on the sRMSE and sBias of the 0.10 Quantile for the Weibull Distribution with Two Aging Behaviors

5-5. Practical Application

In practice the time the parameters change in the distribution may not be known, and thus the necessary proportion to censor is not known. A probability plot of the data can bring this information to light. Figure 5-6 is a Weibull probability plot for a sample of 200 values simulated in this program (Appendix A-8). The sharp bend at about 40 percent is an indication of where to censor the data. The rest of the output from this program is in Appendix A-8. In Figures A-8b to A-8j, the data are censored by an additional 10 percent and fit to the Weibull probability plot. Each figure comes closer to fitting a straight line as the censoring approaches $F(100) = 39\%$ – the true value of t where the aging behavior begins to shift.

If there is not a bend in the probability plot, signifying that there is only one aging behavior, then censoring causes harm rather than good. Figure 5-7 is a Weibull probability plot with 200 random and independently distributed values from the Weibull distribution with shape parameter β=0.5 and scale parameter η=400. Figures 5-8, 5-9, and 5-10 show that censoring increases the sMRSE and sBias for 1000 samples of size 200.

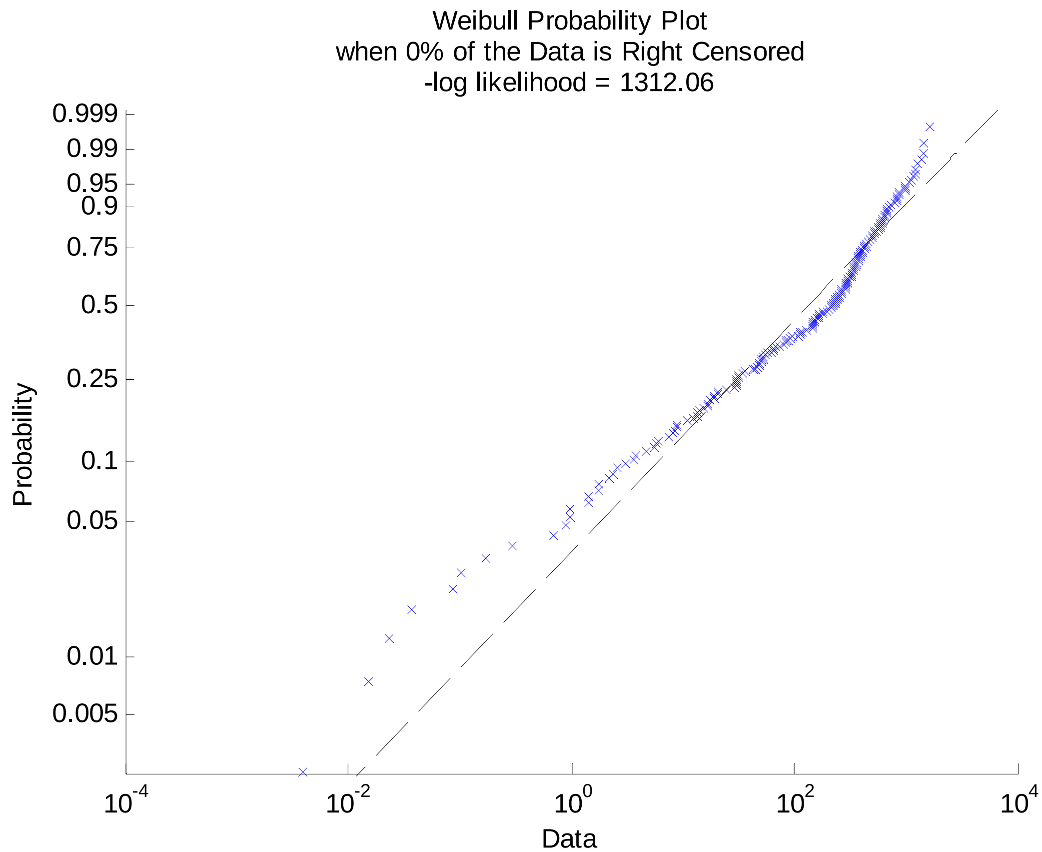

Figure 5-6: Weibull Probability Plot for Weibull with two aging behaviors and with no Censoring

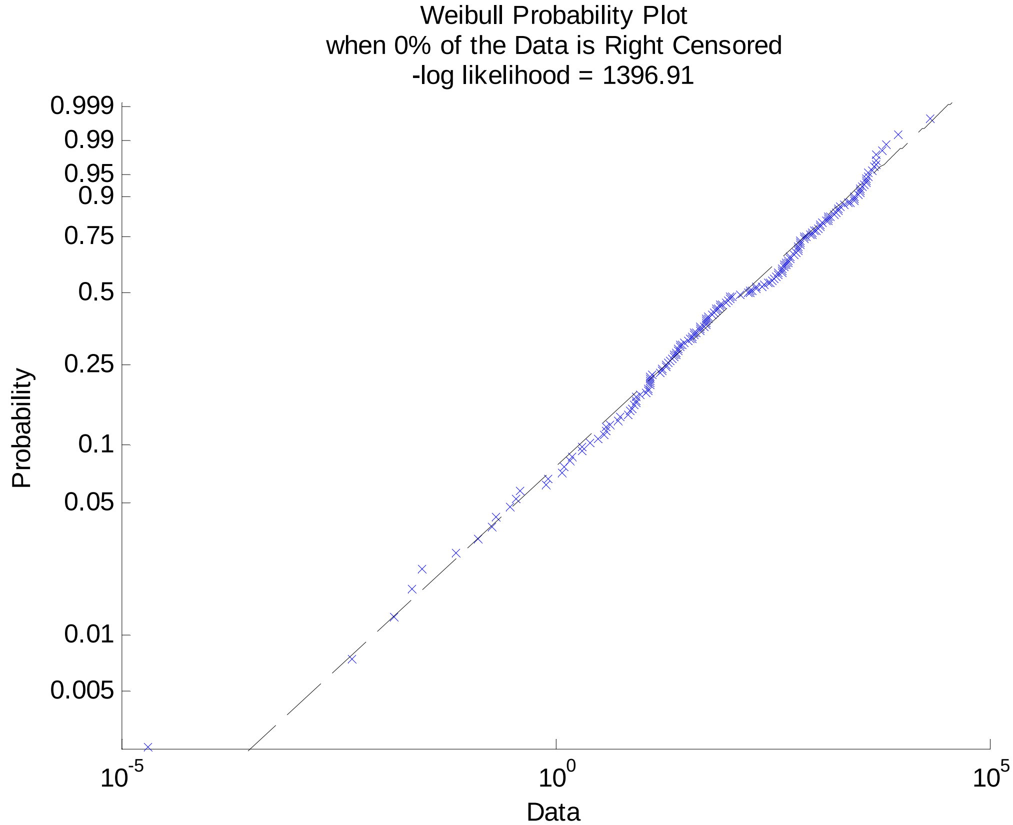

Figure 5-7: Weibull Probability Plot for Weibull with one aging behavior and with no Censoring

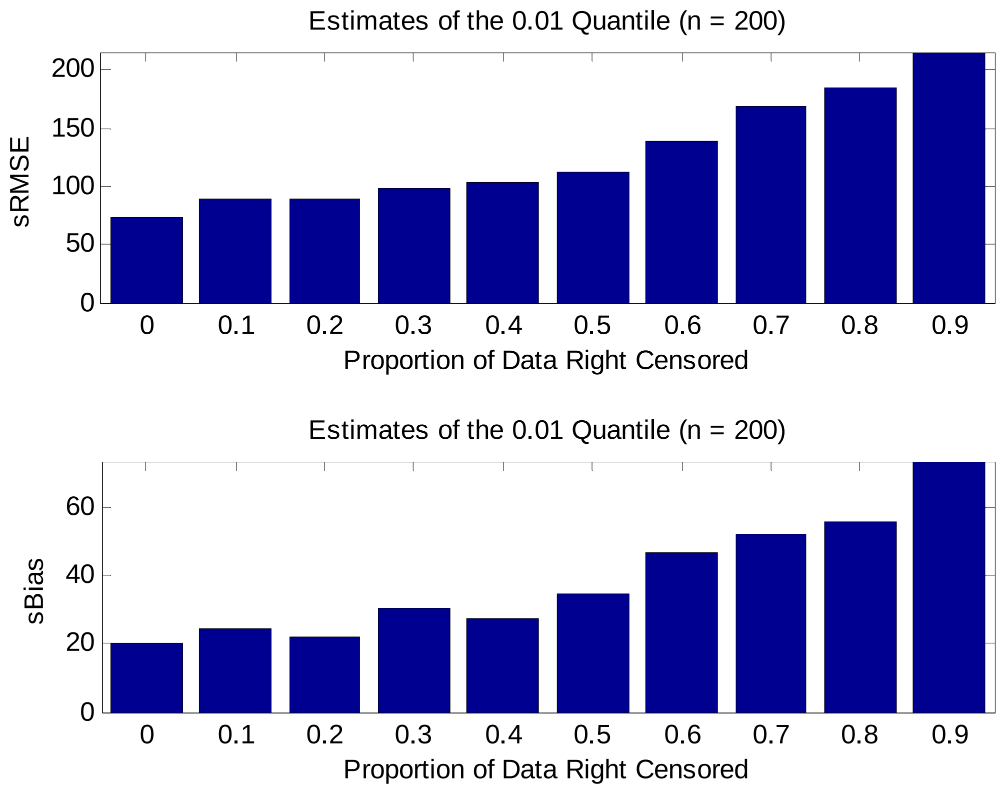

Figure 5-8: Effect of Censoring on the sRMSE and sBias of the 0.01 Quantile for the Weibull Distribution with One Aging Behavior

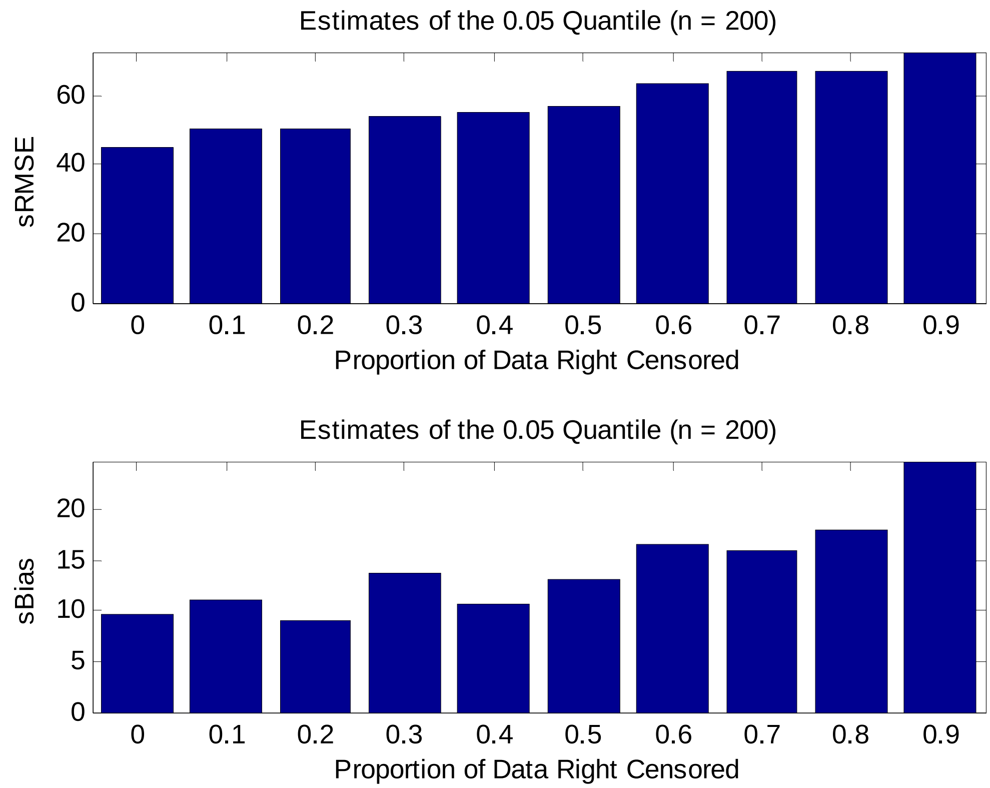

Figure 5-9: Effect of Censoring on the sRMSE and sBias of the 0.05 Quantile for the Weibull Distribution with One Aging Behavior

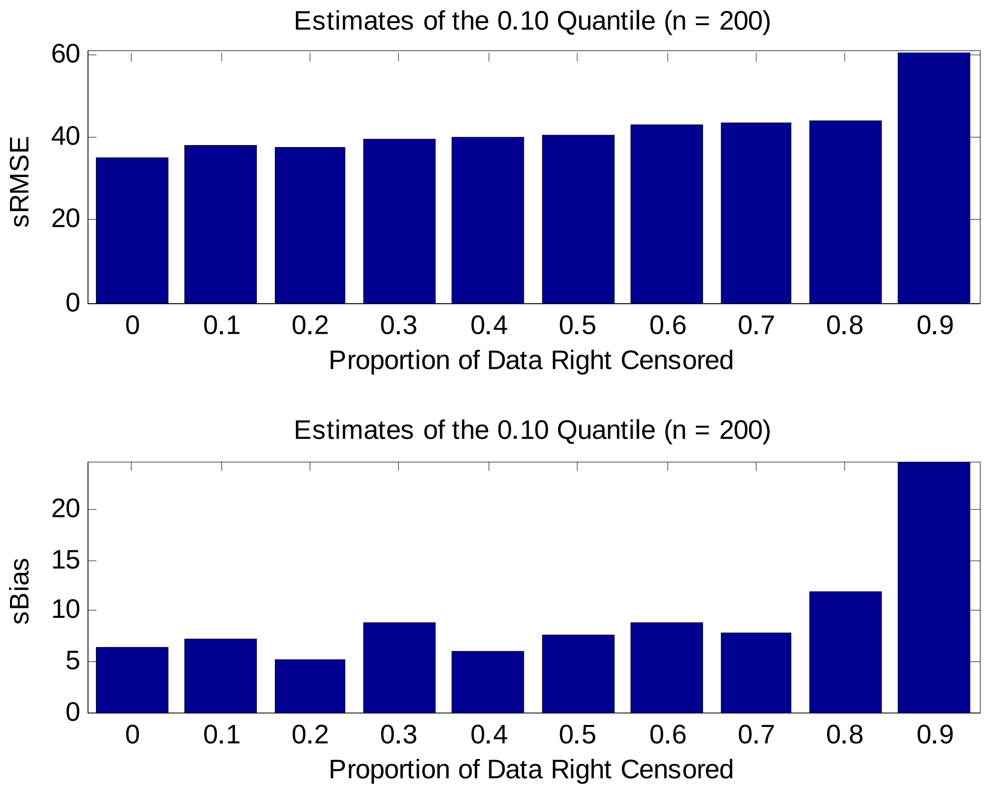

Figure 5-10: Effect of Censoring on the sRMSE and sBias of the 0.10 Quantile for the Weibull Distribution with One Aging Behavior

Chapter 6. Concluding Remarks and Future Research

Good statistical analysis begins with a well planned method for collecting data, and the best data collecting plan begins with the design of an experiment, if possible (Montgomery 2005). The failure data for the two extrusion lines explored in Chapters 3 and 4 are lacking this information. Those who are interested in following the methods in these chapters will do well to first design an experiment and document the procedure.

Assuming that sound statistical practices for data collection were observed, the results from Chapter 3 indicate that on average Line B produces stronger WPC for both the MOE and MOR. Kaplan-Meier comparisons showed that most of this difference is in the middle of the distributions. Chapter 4 used parametric bootstrapping to show that Line B also produces stronger WPC for the MOR at the 10^{th} percentile.

Future research can be done using alternative bootstrapping methods. Several methods for bootstrapping were introduced in Chapter 2, and Chapter 4 implemented the 2-sample, parametric bootstrap. As a follow-up, non-parametric bootstrapping can be used and compared with the results presented in Chapter 4.

The focus of Chapters 4 and 5 has been on the left tails of the failure distributions. The same studies can be made for the right tails. Bootstrapping can be used to test the hypothesis that there is a difference between the two distributions for the MOE and MOR for the 90^{th}, 95^{th}, and 99^{th} percentiles. Such a study – if done with the proper experimentation and interpretation – can aid statisticians and practitioners in learning what factors contribute to stronger WPC.

Simulation results in Chapter 5 provided conclusive evidence that left tail estimates for failure distributions with bathtub hazard rates can be improved with induced percentile right censoring. Section 5-5 gives a method for determining the optimal percentile to induce censoring to minimize the sRMSE and sBias. The Weibull probability plot is a good visual starting point for finding this optimal percentile, but can be followed up with a mathematical approach. An approach comparing the negative log likelihood with the proportion of censoring yielded poor results. Figure 6-1 shows that the negative log likelihood continues to decrease in a linear fashion and gives no help in finding where to censor the data. A possible next step is to fix a penalty to the negative log likelihood for excessive censoring (similar to how the AIC fixes a penalty to the log likelihood for the number of parameters in a model) and maximize the function to find the optimal percentage of right censoring. Another possibility is to calculate a loss function for the optimal percentile and simulate the results.

As previously mentioned, an analysis of the right tails is of equal interest as the left tails. The simulation used in Chapter 5 is limited to right censoring (and therefore left tail estimation) because the MATLAB function `wblfit` is not capable of left or interval censoring. An attempt was made to modify `wblfit.m` to make other censoring methods possible. This attempt was also made by writing a JMP macro using a loss function and the Newton-Rapsin algorithm.

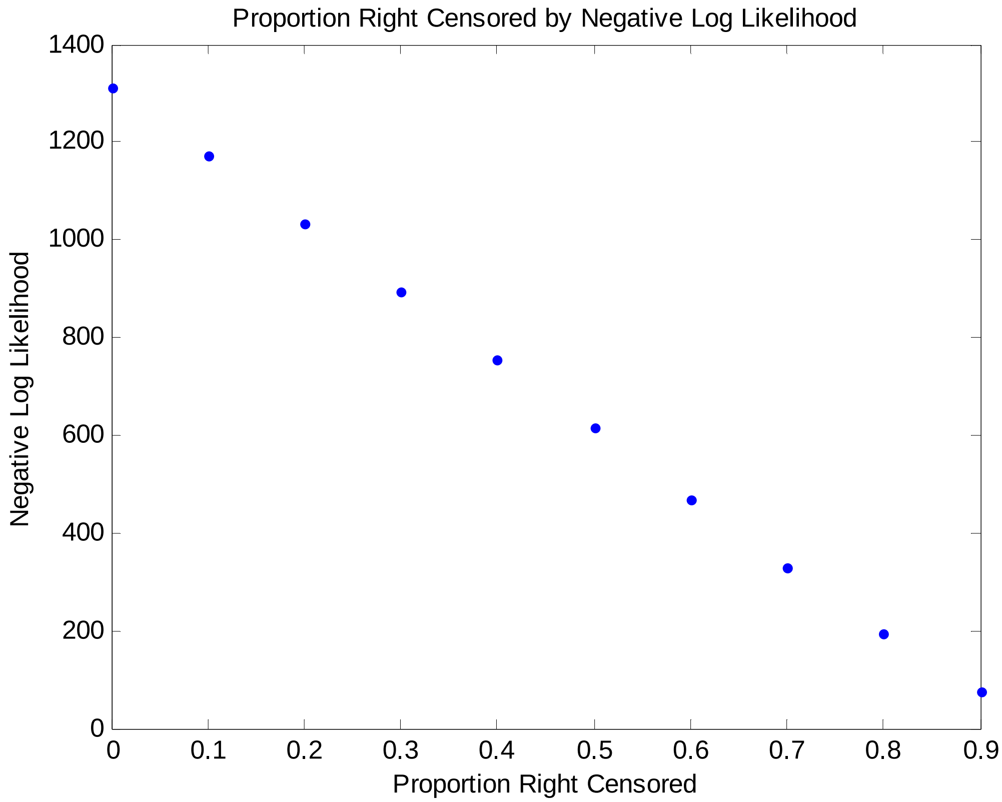

Figure 6-1: Relationship between Proportion Right Censored and the Negative Log Likelihood

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Appendices

A-1. Splida Output for parameter MLE and 95% Confidence Intervals

```
MOEA data

Maximum likelihood estimation results:

Response units: MOE A

Normal Distribution

Log likelihood at maximum point: -1450

  Parameter              Approx Conf. Interval
         MLE Std.Err. 95% Lower 95% Upper
   mu 503479     3185    497235    509722
sigma  35185     2252     31036     39888


The ML estimate of mean time to failure (MTTF) for the
MOEA data is 503479.
An approximate 95% confidence interval is [497235,   509722].
```

```
 MOEB data

Maximum likelihood estimation results:

Response units: MOE B

Logistic Distribution

Log likelihood at maximum point: -1449

  Parameter              Approx Conf. Interval
         MLE Std.Err. 95% Lower 95% Upper
   mu 513286     2977    507451    519120
sigma  19187     1472     16508     22301


The ML estimate of mean time to failure (MTTF) for the
MOEB data is 513286.
An approximate 95% confidence interval is [507451,   519120].
```

MORA data

Maximum likelihood estimation results:

Response units: LINE A MOR

Logistic Distribution

Log likelihood at maximum point: -856.4

```
  Parameter              Approx Conf. Interval
         MLE Std.Err. 95% Lower 95% Upper
   mu 3293.5    23.64    3247.1    3339.8
sigma  150.9    11.44     130.1     175.1
```

The ML estimate of mean time to failure (MTTF) for the
MORA data is 3293.
An approximate 95% confidence interval is [3247,   3340].

MORB data

Maximum likelihood estimation results:

Response units: LINE B MOR

Logistic Distribution

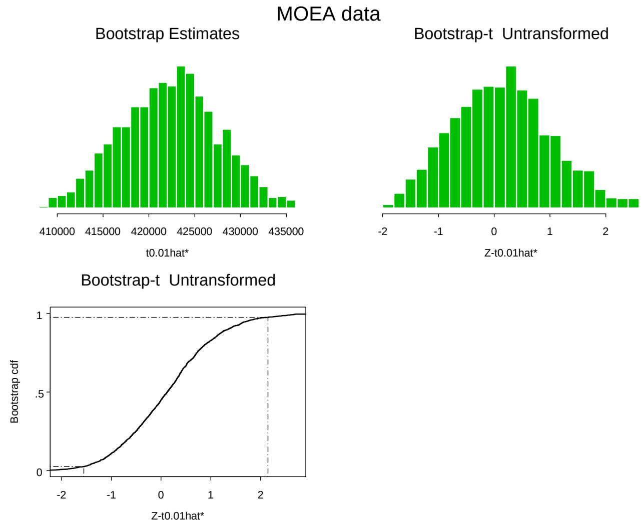
Log likelihood at maximum point: -831.4

```
  Parameter              Approx Conf. Interval
         MLE Std.Err. 95% Lower 95% Upper
   mu 3424.4   18.565    3388.0    3460.8
sigma  120.5    9.301     103.6     140.2
```

The ML estimate of mean time to failure (MTTF) for the
MORB data is 3424.
An approximate 95% confidence interval is [3388,   3461].

A-2. Splida Output for Parametric Bootstrap Confidence Intervals for Lower Percentiles

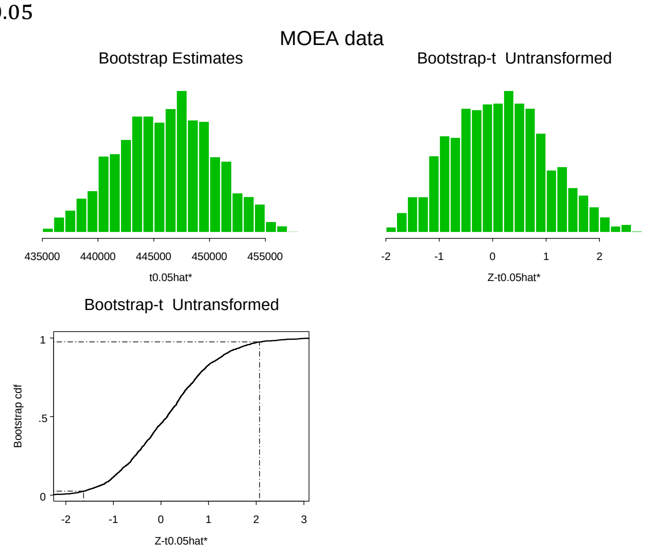
1) Parametric bootstrap CI for lower percentiles (SPLITA)
   a) MOE Line A
      i) $t_{0.01}$




```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.01 is [411118,    433201] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.01 is [408448,    431143] .
```

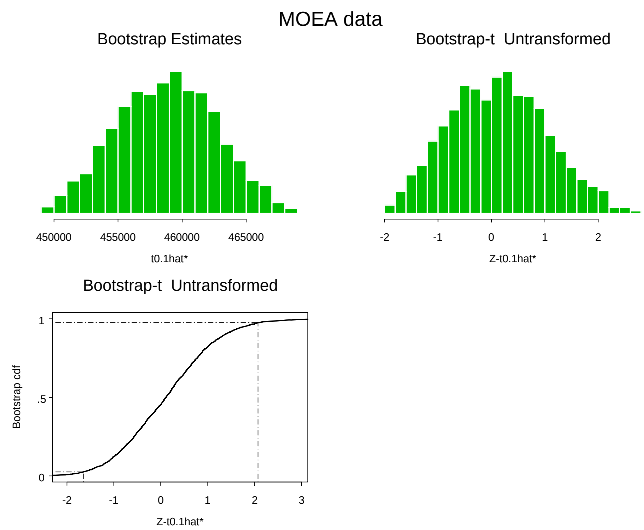
      ii) $t_{0.05}$



```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.05 is [437057,    454840] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.05 is [435480,    453537] .
```

iii) $t_{0.10}$

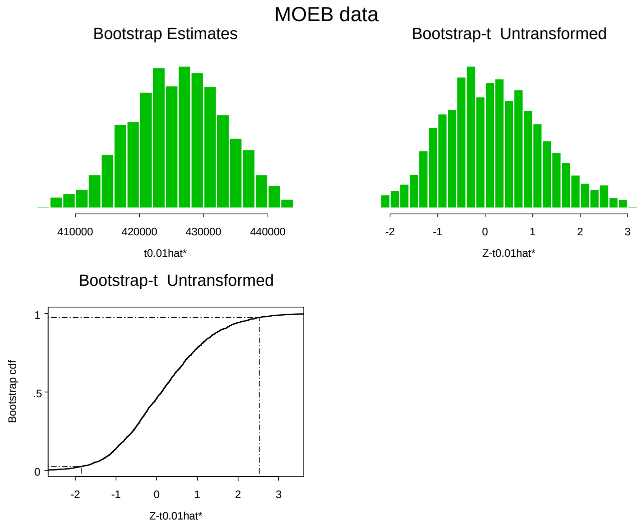



```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.1 is [450846,    466670] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.1 is [449489,    465498] .
```

b) MOE Line B

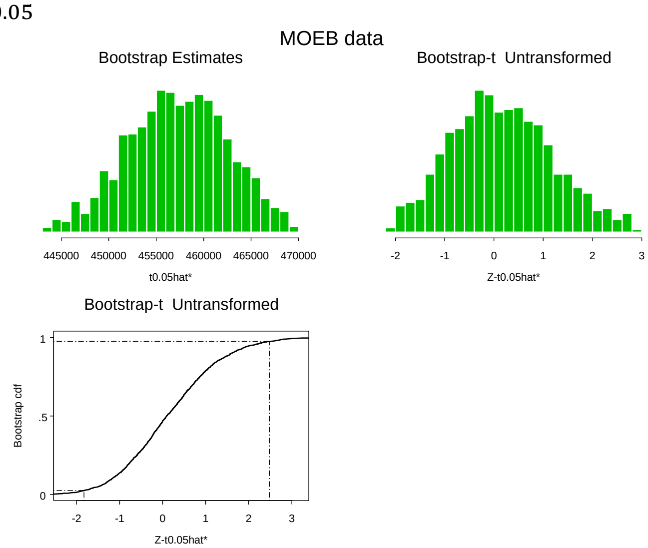
i) $t_{0.01}$




```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.01 is [409177,    440932] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.01 is [406476,    438711] .
```

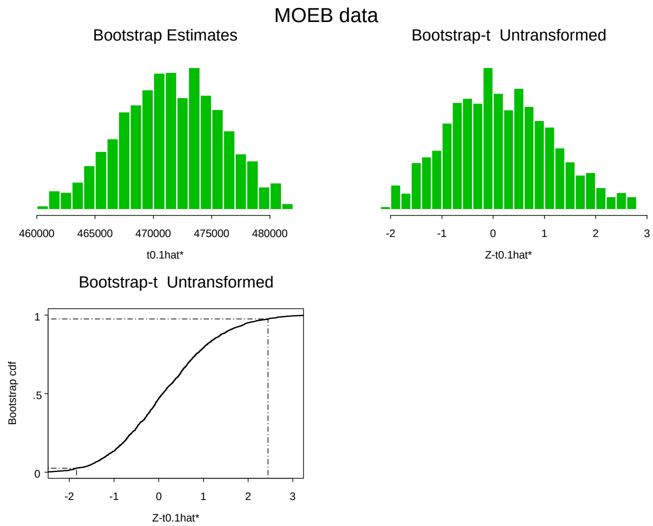
ii) $t_{0.05}$




```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.05 is [446076,    467882] .
```

```
Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.05 is [443754,    466383] .
```

iii) $t_{0.10}$

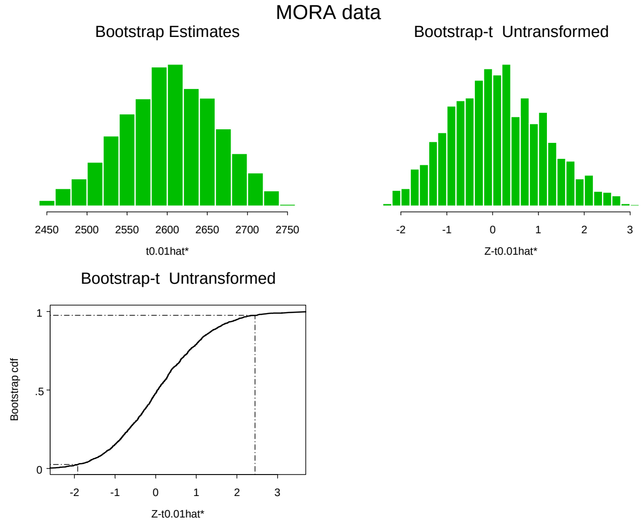



```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.1 is [462009,    480402] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.1 is [460395,    479217] .
```

c) MOR Line A

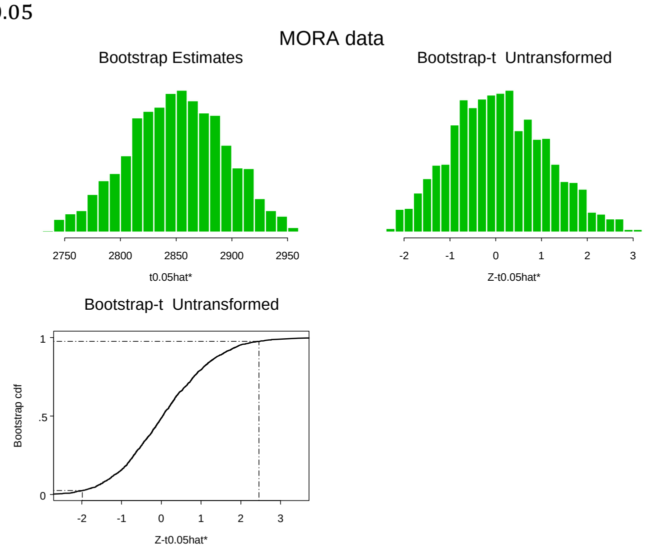
i) $t_{0.01}$




```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.01 is [2472.6,    2722.3] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.01 is [2457,    2712] .
```

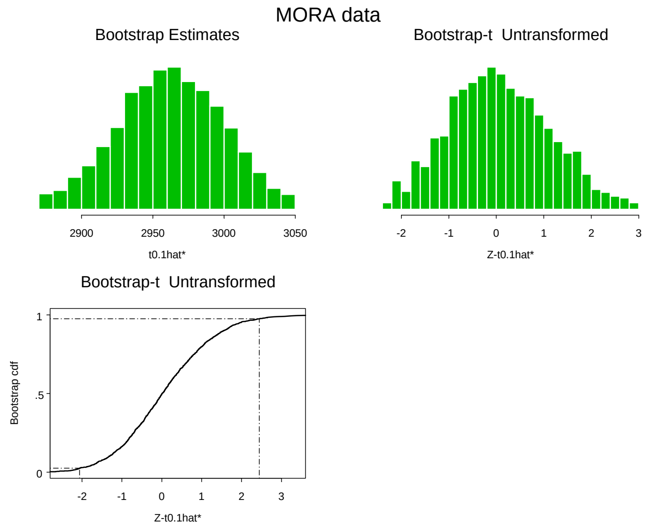
ii) $t_{0.05}$




```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.05 is [2755.1,    2937.6] .
```

```
Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.05 is [2746.3,    2932.1] .
```

iii) $t_{0.10}$

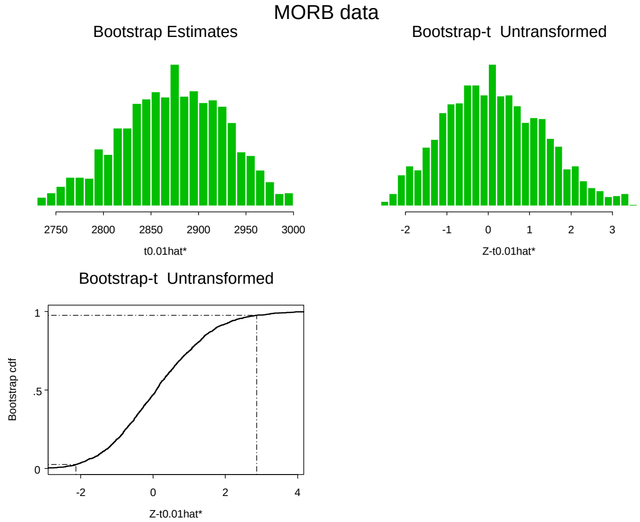



```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.1 is [2882.4,    3037.1] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.1 is [2876.1,    3034.2] .
```

d) MOR Line B

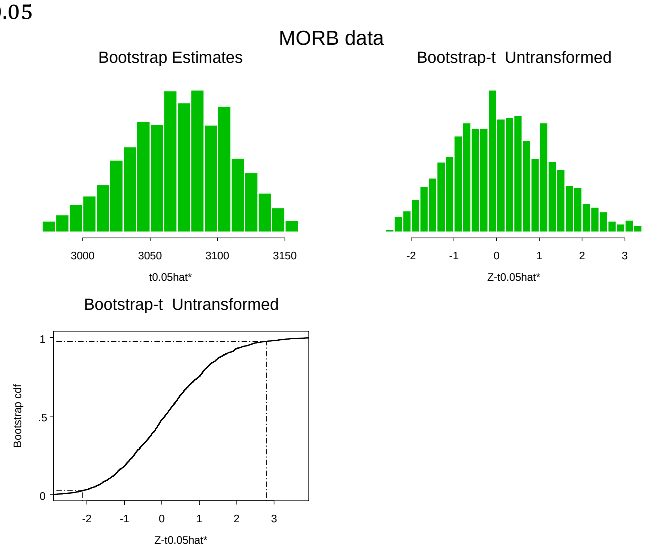
i) $t_{0.01}$




```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.01 is [2753.4,    2979.5] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.01 is [2736.7,    2970.6] .
```

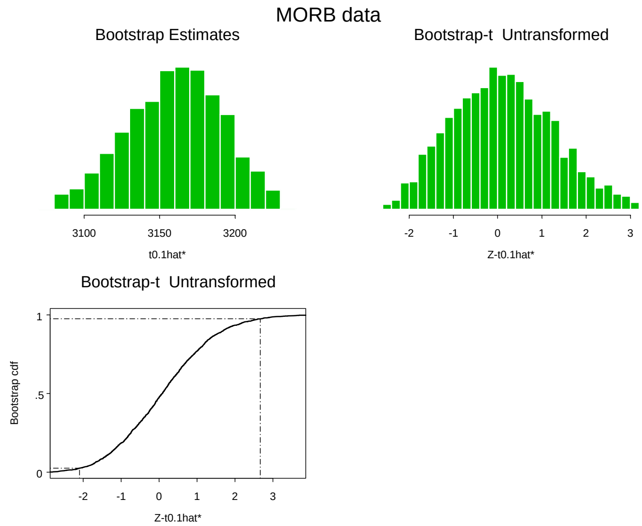
ii) $t_{0.05}$




```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.05 is [2986.1,    3145.2] .
```

```
Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.05 is [2976.8,    3139.9] .
```

iii) $t_{0.10}$



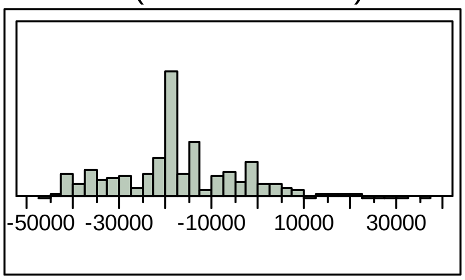

```
Using the percentile method,
an approximate 95 percent confidence interval
for t0.1 is [3091.9,    3220.7] .


Using the boott.notran method,
an approximate 95 percent confidence interval
for t0.1 is [3085.6,    3217.9] .
```

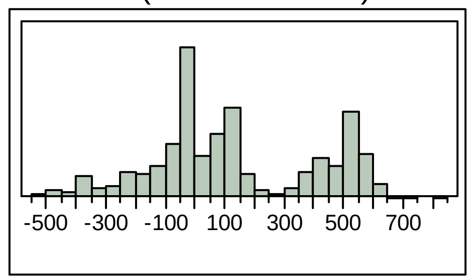
A-3. <u>Bootstrap Histograms of Line Differences in Lower Percentiles</u>

JMP output for the bootstrap distributions of MOE and MOR differences between Line A and Line B. Each distribution consists of 2000 bootstraps.

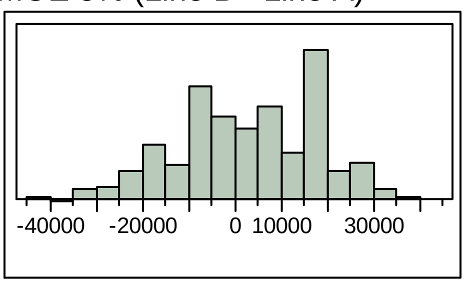
MOE 1% (Line B - Line A)



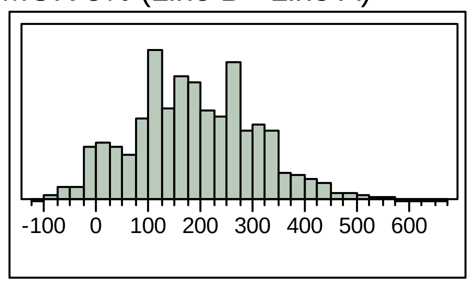
MOR 1% (Line B - Line A)



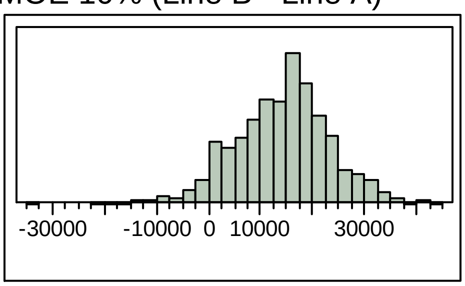
MOE 5% (Line B - Line A)



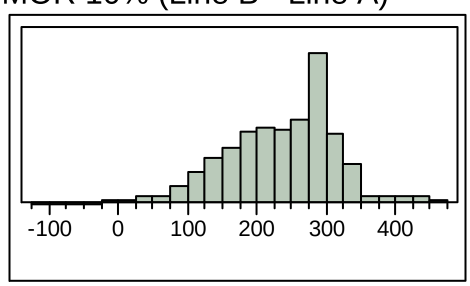
MOR 5% (Line B - Line A)



MOE 10% (Line B - Line A)



MOR 10% (Line B - Line A)



A-4. JMP Script to create Bootstrap Histograms of Line Differences in Lower Percentiles

JMP Script to perform bootstrapping on differences of percentiles. This script was modified from a script provided by Dr. Leon to compare bootstrapped means (Leon 2008).

```
Clear Globals();

dlg = Column Dialog(
      yCol = Col List( "Variable",
            Data Type( Numeric ),
            Min Col(1),
            Max Col(2)
      ),

      Line Up( 2,
            "Number of Samples",     k = Edit Number( 2000 ),
            "Quantile to Bootstrap",q = Edit Number(.10)
      ),

      V List(
            "Demonstration",
            demo = Radio Buttons( "Test Single Quantile", "Compare Two
Quantiles" )
      ),

      "", "Select columns for resampling demonstration"
);


If( dlg["Button"] == -1, Throw( "User cancelled" ) );
Remove From( dlg ); Eval List( dlg );

If( demo == 2 & N Items( yCol ) != 2,
      Dialog(
            "Only one column provided to compare two means", "",
            Button( "OK" )
      );
      Throw( "Only one column provided to compare two means" );
);

dt = Current Data Table();
nn = N Row();

// create containers for resampling results.
m = J( k, 1, . );

// get 'population'.
y = dt << Get As Matrix( yCol );

// collect sample statistics.
Choose( demo,
      // test one quantile.
```

```
      y1 = y[0,1];            // just use the first column.
      For( s = 1, s <= k, s++,
            m[s] = Quantile(q, y1[ J( nn, 1, Random Integer( nn ) ) ]
);
      ),
      // compare two quantiles.
      y1 = y[0,1];
      y2 = y[0,2];            // just use the first column.
      For( s = 1, s <= k, s++,
            m[s] = Quantile(q, y2[ J( nn, 1, Random Integer( nn ) ) ] )
-
                  Quantile(q, y1[ J( nn, 1, Random Integer( nn ) ) ] );
      );
);

// save and show the statistics.
rdt = New Table( "Resample Results",
      New Column( "Sample", Values( 1::k ) ),
      New Column( "Quantile", Values( m ) ),
);


If( demo == 2,
      Column( 2 ) << Set Name( "Difference" );
);

dist = rdt << Distribution( Y( Column( 2 ) ),
Moments(0) );
dist << Set Title( "Bootstrap Statistics" );
```

A-5. Development of CDF and pdf for Chapter 5 Simulation

Equation A-1 is the hazard function for the Weibull distribution. We begin by defining the bathtub hazard function (Equation A-2) for the Chapter 5 simulation by using the Weibull hazard as a starting point.

$$h_{Weibull}(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \quad for\ t > 0, \beta > 0, \eta > 0 \tag{A-1}$$

$$h(t) = \left\{\begin{matrix} \frac{\beta_1}{\eta_1}\left(\frac{t}{\eta_1}\right)^{\beta_1-1} & \text{for } 0 < t < a,\ 0 < \beta_1 < 1 \\ \frac{\beta_2}{\eta_2}\left(\frac{t}{\eta_2}\right)^{\beta_2-1} + c & \text{for } a \le t < \infty,\ \beta_2 \ge 1 \end{matrix}\right\} \tag{A-2}$$

The value c is a constant that allows continuity at $t = a$ and is derived in Equation A-4.

$$\frac{\beta_1}{\eta_1}\left(\frac{a}{\eta_1}\right)^{\beta_1-1} = \frac{\beta_2}{\eta_2}\left(\frac{a}{\eta_2}\right)^{\beta_2-1} + c \tag{A-3}$$

$$c = \frac{\beta_1}{\eta_1}\left(\frac{a}{\eta_1}\right)^{\beta_1-1} - \frac{\beta_2}{\eta_2}\left(\frac{a}{\eta_2}\right)^{\beta_2-1} \tag{A-4}$$

Using the hazard found in Equation A-2 we calculate the cumulative hazard function (Equation A-6) using the definition in Equation A-5.

$$H(t) = \begin{cases} \int_0^t h(u_1)\, du & \text{for } 0 < t < a,\ 0 < \beta_1 < 1 \\ \int_0^a h(u_1)\, du + \int_a^t h(u_2)\, du & \text{for } a \le t < \infty,\ \beta_2 \ge 1 \end{cases} \tag{A-5}$$

where $u_1 = \frac{\beta_1}{\eta_1}\left(\frac{t}{\eta_1}\right)^{\beta_1 - 1}$ and $u_2 = \frac{\beta_2}{\eta_2}\left(\frac{t}{\eta_2}\right)^{\beta_2 - 1} + c$

$$H(t) = \begin{cases} \left(\frac{t}{\eta_1}\right)^{\beta_1} & \text{for } 0 < t < a,\ 0 < \beta_1 < 1 \\ \left(\frac{a}{\eta_1}\right)^{\beta_1} + \left(\frac{t^{\beta_2} - a^{\beta_2}}{\eta_2^{\beta_2}}\right) + c(t-a) & \text{for } a \le t < \infty,\ \beta_2 \ge 1 \end{cases} \tag{A-6}$$

The cumulative hazard function leads to the Survival function (Equation A-8), which in turn leads to the CDF (Equation A-10) and then the pdf (Equation A-12).

$$S(t) = \exp\left(-H(t)\right) \tag{A-7}$$

$$S(t) = \begin{cases} \exp\left[-\left(\frac{t}{\eta_1}\right)^{\beta_1}\right] & \text{for } 0 < t < a,\ 0 < \beta_1 < 1 \\ \exp\left[-\left(\frac{a}{\eta_1}\right)^{\beta_1} - \left(\frac{t^{\beta_2} - a^{\beta_2}}{\eta_2^{\beta_2}}\right) - c(t-a)\right] & \text{for } a \le t < \infty,\ \beta_2 \ge 1 \end{cases} \tag{A-8}$$

$$F(t) = 1 - S(t) \tag{A-9}$$

$$F(t) = \begin{cases} 1 - \exp\left[-\left(\frac{t}{\eta_1}\right)^{\beta_1}\right] & \text{for } 0 < t < a,\ 0 < \beta_1 < 1 \\ 1 - \exp\left[-\left(\frac{a}{\eta_1}\right)^{\beta_1} - \left(\frac{t^{\beta_2} - a^{\beta_2}}{\eta_2^{\beta_2}}\right) - c(t-a)\right] & \text{for } a \le t < \infty,\ \beta_2 \ge 1 \end{cases} \tag{A-10}$$

$$f(t) = h(t)S(t) \quad \textbf{(A-11)}$$

$$f(t) = \begin{Bmatrix} \frac{\beta_1}{\eta_1}\left(\frac{t}{\eta_1}\right)^{\beta_1-1} \times \exp\left[-\left(\frac{t}{\eta_1}\right)^{\beta_1}\right] & \text{for } 0 < t < a,\ 0 < \beta_1 < 1 \\ \left[\frac{\beta_2}{\eta_2}\left(\frac{t}{\eta_2}\right)^{\beta_2-1} + c\right] \times \exp\left[-\left(\frac{a}{\eta_1}\right)^{\beta_1} - \left(\frac{t^{\beta_2}-a^{\beta_2}}{\eta_2^{\beta_2}}\right) - c(t-a)\right] & \text{for } a \le t < \infty,\ \beta_2 \ge 1 \end{Bmatrix} \quad \textbf{(A-12)}$$

A-6. MATLAB code for Chapter 5 Simulation

```
%**********************************************************************
% Kevin Crookston
% March 2009
%
% The purpose of this program is to simulate random numbers from the
% Weibull distribution with different shape parameters for different
% intervals of time, then to use parametric parameter estimation and
% right censoring to fit the lower quantiles of the data set.
%**********************************************************************

clc
clear all

% The following are parameters that may be changed before running the
% program ____________________________________________________________

a    = 100;% time at which parameters change
beta1= .5; % 0 < beta1 < 1, decreasing hazard from 0<=t<a
beta2= 1.2;% 1 <= beta2 < 2, constant or slightly increasing hazard for
t>=a
eta1 = 400;% scale parameter for 0<=t<a
eta2 = 400;% scale parameter for t>=a
n    = 200;% sample size
s    = 500;% number of simulated samples of size n

%quantiles at which the estimates of the CDF will be made (0 < esmt <1)
esmt=[0.01 0.05 0.10];

%proportion of data that are right censored (0 <= censd < 1)
censd= [0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9];
%
______________________________________________________________________
censdquantile=1-censd;
Fa = 1-exp(-((a/eta1)^beta1)); % Pr(t<a)

%**********************************************************************

% Section 1
% Plots of theoretical distribution

% time t
t1=0:a/100:a;
t2=a+a/100:a/100:a*10;
t=[t1,t2];

% hazard h(t)
h1=(beta1/eta1)*(t1./eta1).^(beta1-1);
const1 = (beta1/eta1)*(a/eta1)^(beta1-1)-(beta2/eta2)*(a/eta2)^(beta2-
1);
```

```
h2=(beta2/eta2)*(t2./eta2).^(beta2-1)+const1;
h=[h1,h2];

% Cumulative Hazard H(t)
H1=(t1./eta1).^beta1;
H2=(a/eta1)^beta1+((t2.^beta2-a^beta2)./eta2^beta2)+const1*(t2-a);
H=[H1,H2];

% Cumulative Survival S(t), CDF F(t), pdf f(t)
S=exp(-H);
F=1-S;
f=h.*S;

figure(1)
subplot(2,3,2), plot(t,H,'LineWidth',2),title('Cumulative Hazard
H(t)'),...
 xlabel('t'),ylabel('H(t)');
subplot(2,3,3), plot(t,h,'LineWidth',2),title('Hazard h(t)')
,...
 xlabel('t'),ylabel('h(t)');
subplot(2,3,4), plot(t,S,'LineWidth',2),title('Survival S(t)')
,...
 xlabel('t'),ylabel('S(t)');
subplot(2,3,5), plot(t,F,'LineWidth',2),title('CDF F(t)')
,...
 xlabel('t'),ylabel('F(t)');
subplot(2,3,6), plot(t,f,'LineWidth',2),title('pdf f(t)')
,...
 xlabel('t'),ylabel('f(t)');
%**********************************************************************

% Section 2
% Simulates s Weibull distributions of size n from the CDF defined in
% Section 1, estimates eta and beta for each distribution assuming
% censoring at a predetermined quantile, and plots the estimated
% distribution over the CDF defined in Section 1.

format short g;
disp(sprintf('\nINPUTS'))
disp(sprintf('Wbl(eta,beta):'))
disp(sprintf('(%.1f,%.1f) for 0 <= t <  %.1f', eta1, beta1, a))
disp(sprintf('(%.1f,%.1f) for      t >= %.1f', eta2, beta2, a))
disp(sprintf('F(%.1f)=%.5f', a, Fa))
disp(sprintf('Number of samples: %.0f',s))
disp(sprintf('Observations per sample: %.0f',n))

options = optimset('Display','off'); %Turns off Display for fsolve

for k=1:length(censd)
    for i=1:s
        pr=rand(n,1);
        for j = 1:n
            if pr(j)<Fa
                wbl(j)=eta1*(-log(1-pr(j)))^(1/beta1);
```

```
            else
                const2 = eta2^beta2*((a/eta1)^(beta1)...
                         -(a/eta2)^(beta2)-const1*a+log(1-pr(j)));
                wbl(j) =
fsolve(@(t)(t^beta2+eta2^beta2*const1*t+const2)...
                         ,a,options);
            end
        end

        % Estimate parameters when censd% of data is right censored
        wbl=sort(wbl);
        c=prctile(wbl,censdquantile(k)*100);
        wblc=[wbl(1:floor(n*censdquantile(k))),c];
        censored=[zeros(1,floor(n*censdquantile(k))),1];
        frequ=[ones(1,floor(n*censdquantile(k))),n-length(wblc)+1];
        parmhat=wblfit(wblc,0.05,censored,frequ);

        etahat(i)=parmhat(1);
        betahat(i)=parmhat(2);

        figure(k+1)
        subplot(1,3,1),
        hold on,
        if i <= 20
            plot(t,wblcdf(t,etahat(i),betahat(i)),'k'),
            plot(t,F,'r','LineWidth',2),
            plot(t,censdquantile(k)),
            hold off,
            title({'Estimated Weibull CDFs from Random Samples';...
                sprintf('when %.0f%% of the Data is Right Censored',...
                100*censd(k))}),
            xlabel('Time'), ylabel('Proportion Failing')
        end
    end

%***********************************************************************
****

% Section 3
% This section estimates the time (t) for selected quantiles (esmt)
% from the n estimated CDFs in Section 2 and calculates a Mean Squared
% Error and Bias for each as compared to the actual Time (T).  Scaled
% RMSE and Bias are plotted against the quantile.

    % Time (T) at each value of esmt from the Theoretical distribution
    % in Section 1
    for i=1:length(esmt)
        if esmt(i) < Fa
            T(i) = eta1 * (-log(1 - esmt(i))) ^ (1 / beta1);
        else
            const2 = eta2^beta2*((a/eta1)^(beta1)...
                     -(a/eta2)^(beta2)-const1*a+log(1-esmt(i)));
            T(i) = fsolve(@(t)(t^beta2+eta2^beta2*const1*t+const2)...
                   ,a,options);
```

```
        end
    end

    % Time (t) at each value of esmt from the generated distributions
    % in Section 2 and corresponding MSE and Bias
    for i=1:length(esmt)
        ti(:,k,i)=wblinv(esmt(i),etahat',betahat');
        MSE(k,i)=sum((ti(:,k,i)-T(i)).^2)/s;
        B(k,i)=mean(ti(:,k,i))-T(i);
    end

    %The root MSE and bias are scaled to the estimated quantiles
    sRMSE(k,:) = sqrt(MSE(k,1:length(T)))./T.*100;
    sBias(k,:)    = B(k,1:length(T))./T.*100;

    subplot(1,3,2), bar(esmt, sRMSE(k,1:length(esmt))),
        title('Scaled Root Mean Square Error'),
        xlabel('Estimated Quantile'), ylabel('(RMSE/t)%')
    subplot(1,3,3), bar(esmt, sBias(k,1:length(esmt))),
        title('Scaled Bias'),
        xlabel('Estimated Quantile'), ylabel('(Bias/t)%')

    disp(sprintf('\n%.0f%% Right Censored',100*censd(k)))
    disp(sprintf('OUTPUTS'))
    disp('       Quantile    Time          MSE          RMSE
(RMSE/t)%    Bias         (Bias/t)%')
    disp('-----------------------------------------------------------
---------------------------------')
    disp([esmt',
T',(MSE(k,1:length(T)))',sqrt((MSE(k,1:length(T)))')...
        , (sRMSE(k,1:length(T)))', (B(k,1:length(T)))'...
        , (sBias(k,1:length(T)))'])
end


for k=1:length(censd)
    figure
    h=0;
    for i=1:length(esmt)
        h=h+1;
        N=hist(ti(:,k,i));
        subplot(1,length(esmt),h), hist(ti(:,k,i)),hold on,
        plot(T(i),[0:max(N)/1000:max(N)],'r'),hold off,
        title('Estimated time to Failure'),
        xlabel({sprintf('%.0f%% Right Censored',100*censd(k));...
                sprintf('F^-^1(%.2f) = %.2f',esmt(i),T(i))})
        ylabel(sprintf('Frequency'));
    end
end

%*********************************************************************

% Section 4
% This section plots the scaled RMSE and the scaled Bias by the
```

```
% censoring quantiles for each estimated quantile.
for l=1:length(esmt)
    figure
    subplot(2,1,1), bar(censd,sRMSE(1:length(censd),l)),
        title(sprintf('Estimates of the %.2f Quantile',esmt(l))),
        xlabel('Proportion of Data Right Censored'), ylabel('sRMSE'),
        axis([min(censd)-0.05 max(censd)+0.05 0
max(sRMSE(1:length(censd),l))])
    subplot(2,1,2), bar(censd,sBias(1:length(censd),l)),
        title(sprintf('Estimates of the %.2f Quantile',esmt(l))),
        xlabel('Proportion of Data Right Censored'), ylabel('sBias'),
        axis([min(censd)-0.05 max(censd)+0.05 0
max(sBias(1:length(censd),l))])
end
```

A-7. <u>Output for Chapter 5 Simulation</u>

INPUTS
Wbl(eta,beta):
(400.0,0.5) for 0 <= t < 100.0
(400.0,1.2) for t >= 100.0
F(100.0)=0.39347
Number of samples: 1000
Observations per sample: 200

0% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.11971	0.346	856.35	0.28833	713.61
0.05	1.0524	6.2504	2.5001	237.56	2.1563	204.89
0.1	4.4403	27.372	5.2318	117.83	4.414	99.406

10% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.056064	0.23678	586.03	0.18927	468.44
0.05	1.0524	3.4728	1.8635	177.08	1.5289	145.28
0.1	4.4403	16.167	4.0208	90.552	3.1515	70.974

20% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.032413	0.18003	445.59	0.13459	333.11
0.05	1.0524	2.2737	1.5079	143.28	1.1515	109.42
0.1	4.4403	11.322	3.3648	75.779	2.3964	53.969

30% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.016755	0.12944	320.37	0.087758	217.2
0.05	1.0524	1.3768	1.1734	111.49	0.79684	75.717
0.1	4.4403	7.581	2.7534	62.008	1.6686	37.577

40% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.0079914	0.089395	221.25	0.052015	128.74
0.05	1.0524	0.82338	0.9074	86.222	0.4911	46.665
0.1	4.4403	5.2692	2.2955	51.696	1.0198	22.967

50% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.0037432	0.061181	151.43	0.027397	67.807
0.05	1.0524	0.49139	0.70099	66.609	0.26347	25.035
0.1	4.4403	3.7129	1.9269	43.395	0.55554	12.511

60% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.0030808	0.055505	137.37	0.017047	42.192
0.05	1.0524	0.42086	0.64873	61.643	0.14812	14.075
0.1	4.4403	3.4312	1.8524	41.717	0.31426	7.0774

70% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.0037802	0.061483	152.17	0.020628	51.056
0.05	1.0524	0.46798	0.68409	65.003	0.178	16.914
0.1	4.4403	3.6699	1.9157	43.143	0.40172	9.0472

80% Right Censored
OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.0051461	0.071736	177.55	0.023319	57.716
0.05	1.0524	0.48113	0.69364	65.91	0.19714	18.732
0.1	4.4403	3.7819	1.9447	43.796	0.5444	12.26

90% Right Censored

OUTPUTS

Quantile	Time	MSE	RMSE	(RMSE/t)%	Bias	(Bias/t)%
0.01	0.040404	0.0065914	0.081188	200.94	0.026463	65.497
0.05	1.0524	0.49483	0.70344	66.841	0.22267	21.159
0.1	4.4403	6.2159	2.4932	56.148	0.93285	21.008

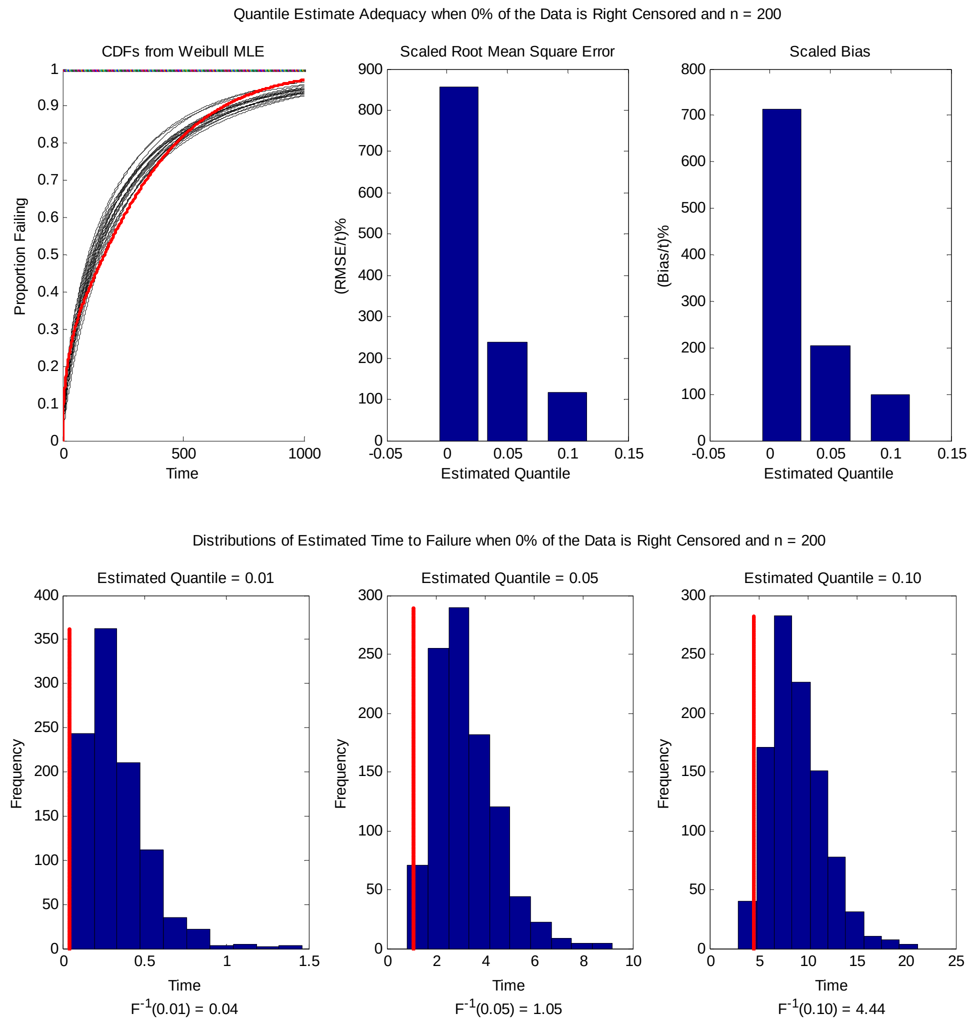
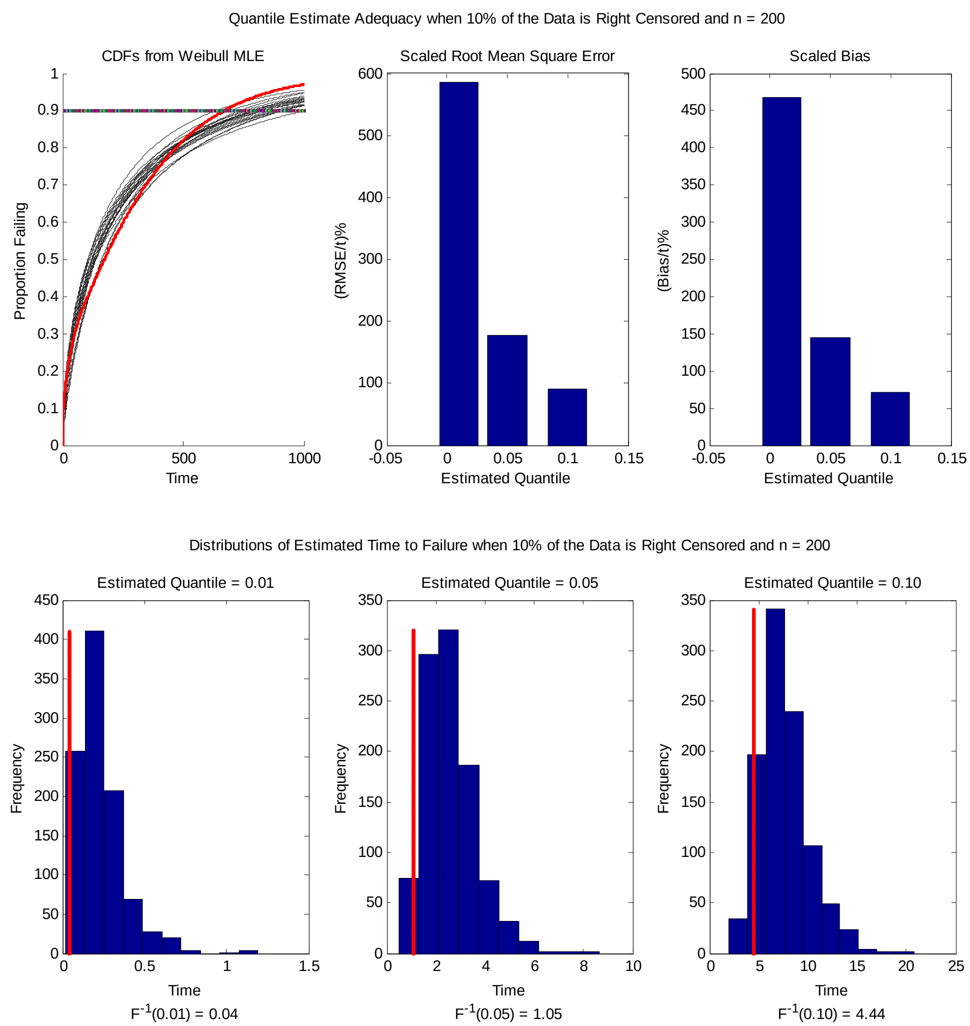

Quantile Estimate Adequacy when 0% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile
Distributions of Estimated Time to Failure when 0% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Estimated Quantile = 0.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.01) = 0.04
F-1(0.05) = 1.05
F-1(0.10) = 4.44

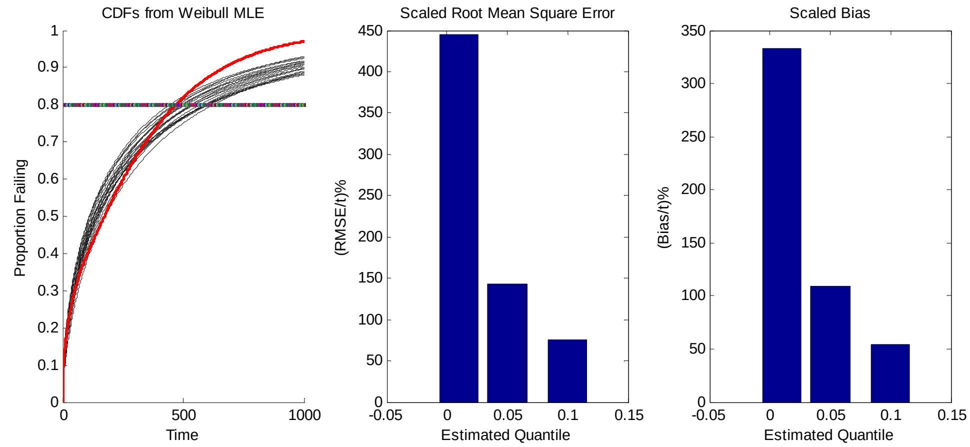
Figure A-7a



Quantile Estimate Adequacy when 10% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile
Distributions of Estimated Time to Failure when 10% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Estimated Quantile = 0.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.01) = 0.04
F-1(0.05) = 1.05
F-1(0.10) = 4.44

Figure A-7b

Quantile Estimate Adequacy when 20% of the Data is Right Censored and n = 200



Distributions of Estimated Time to Failure when 20% of the Data is Right Censored and n = 200

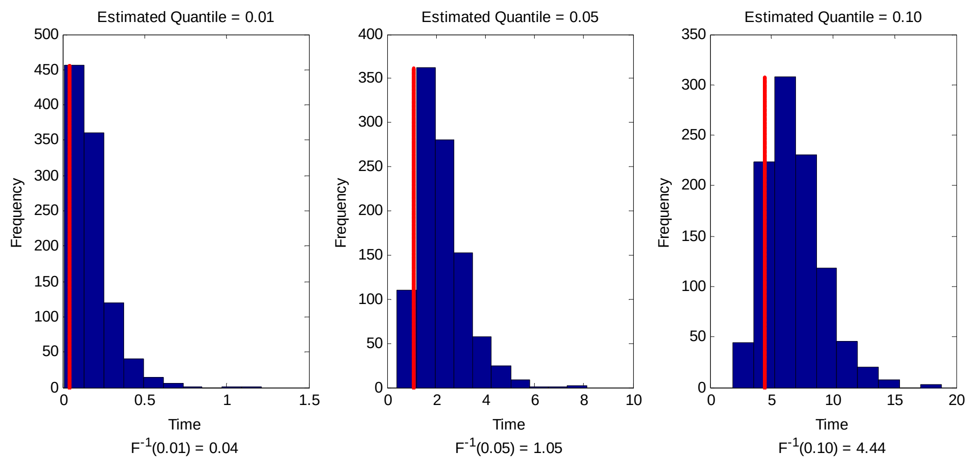


Figure A-7c

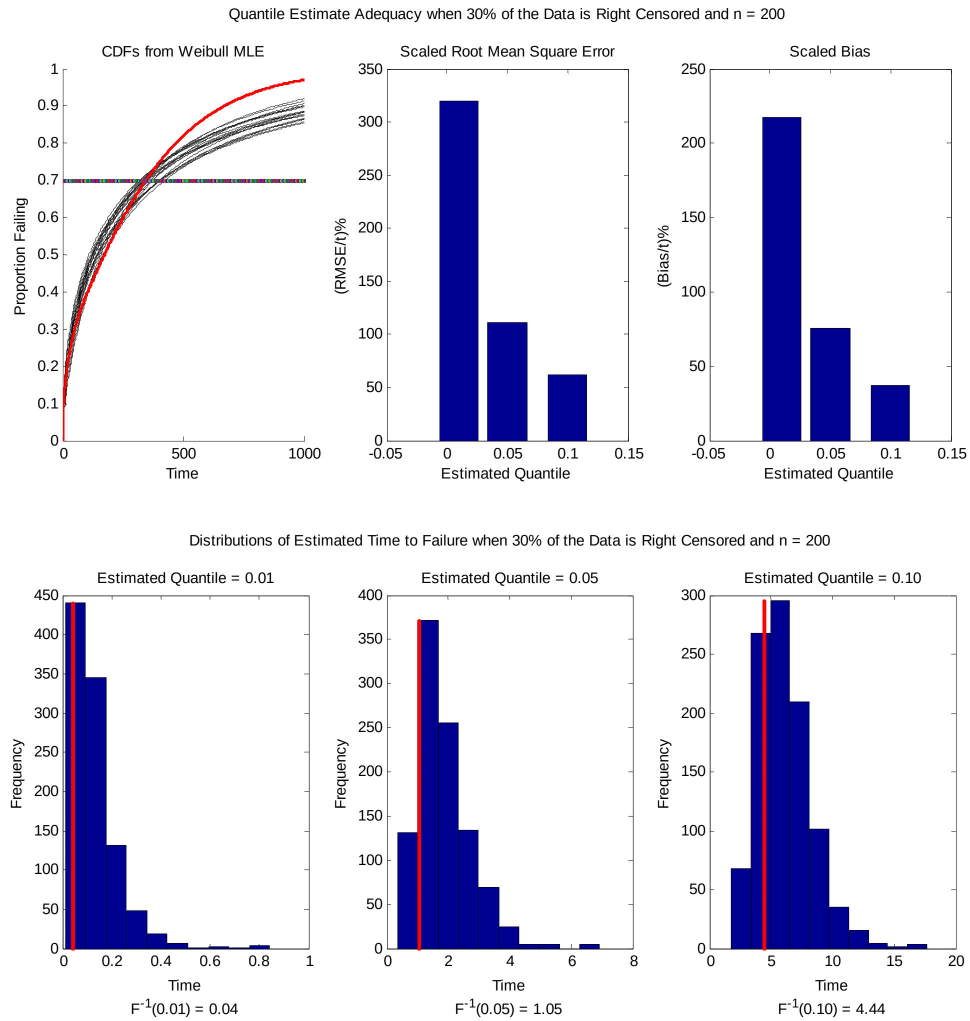
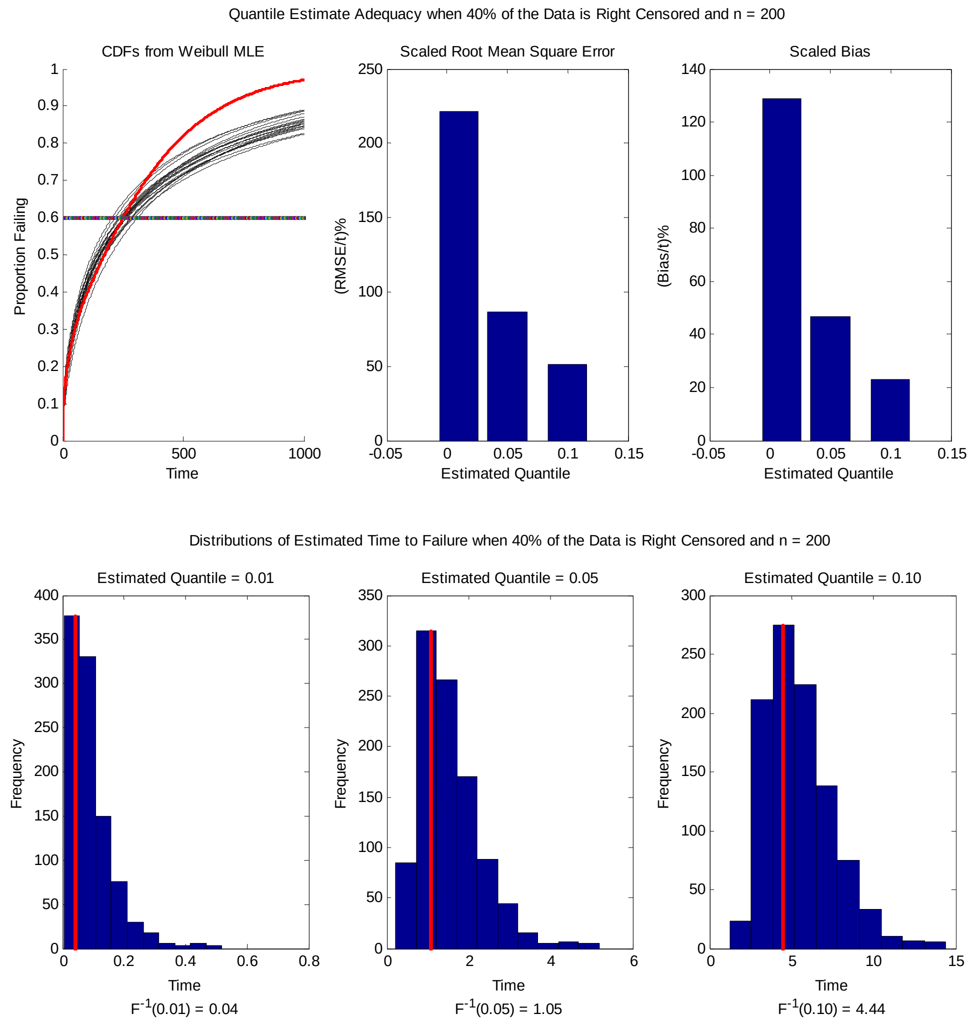
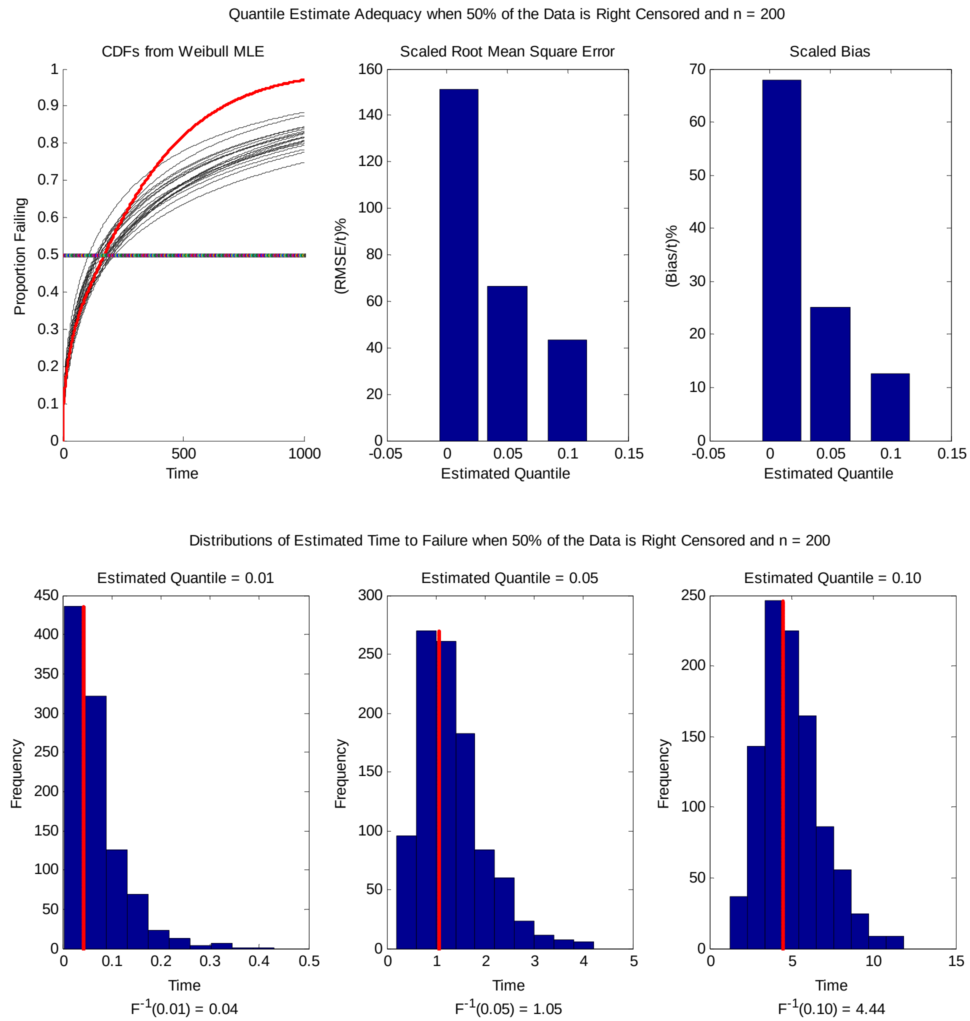

Quantile Estimate Adequacy when 30% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile
Distributions of Estimated Time to Failure when 30% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Estimated Quantile = 0.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.01) = 0.04
F-1(0.05) = 1.05
F-1(0.10) = 4.44

Figure A-7d



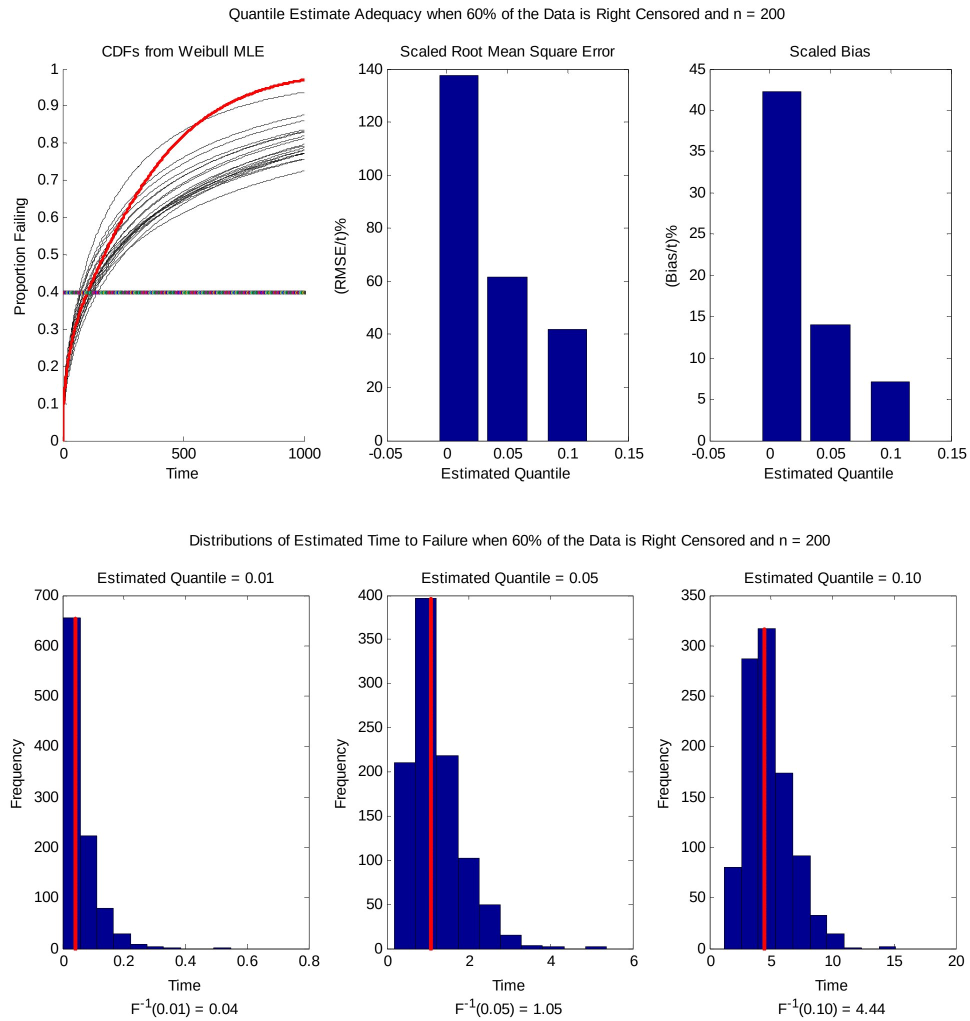
Quantile Estimate Adequacy when 40% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile
Distributions of Estimated Time to Failure when 40% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Estimated Quantile = 0.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.01) = 0.04
F-1(0.05) = 1.05
F-1(0.10) = 4.44

Figure A-7e



Quantile Estimate Adequacy when 50% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile
Distributions of Estimated Time to Failure when 50% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Frequency
Time
F-1(0.01) = 0.04
Estimated Quantile = 0.05
Frequency
Time
F-1(0.05) = 1.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.10) = 4.44

Figure A-7f



Quantile Estimate Adequacy when 60% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile
Distributions of Estimated Time to Failure when 60% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Estimated Quantile = 0.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.01) = 0.04
F-1(0.05) = 1.05
F-1(0.10) = 4.44

Figure A-7g

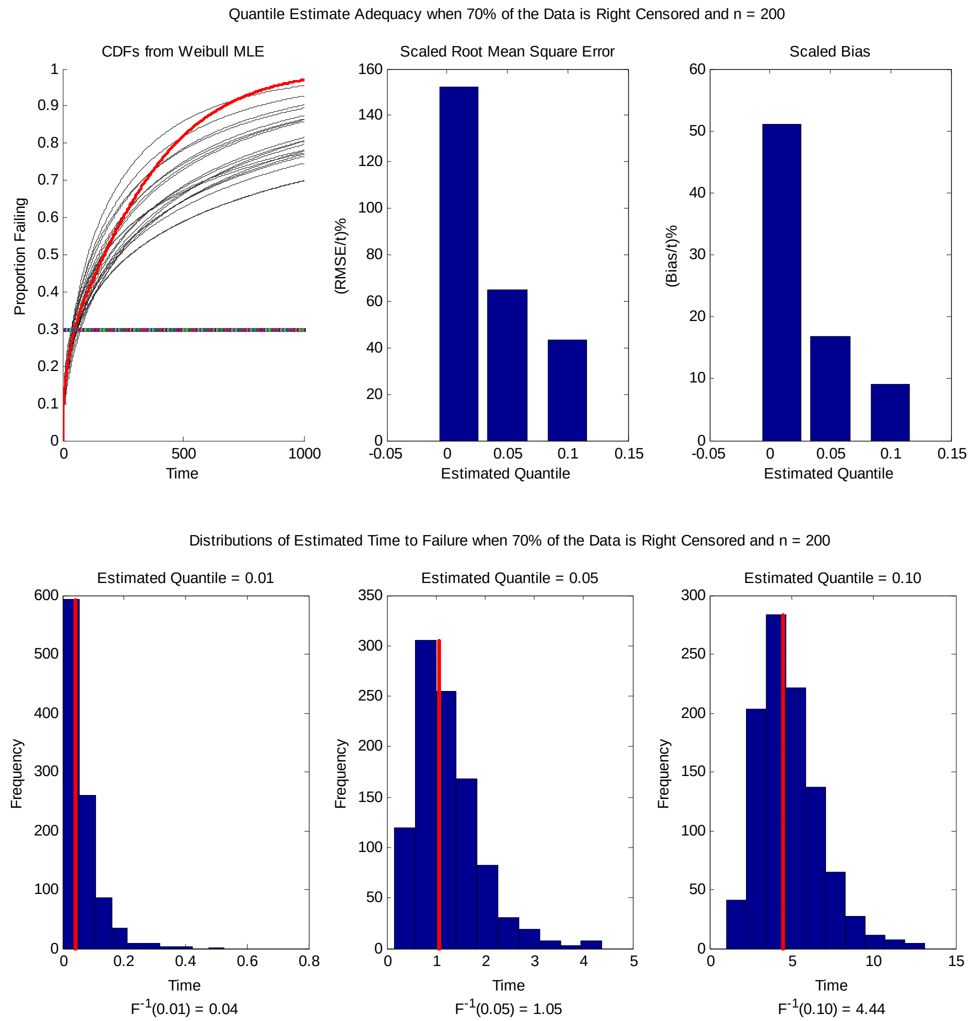

Quantile Estimate Adequacy when 70% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile
Distributions of Estimated Time to Failure when 70% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Frequency
Time
F-1(0.01) = 0.04
Estimated Quantile = 0.05
Frequency
Time
F-1(0.05) = 1.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.10) = 4.44

Figure A-7h

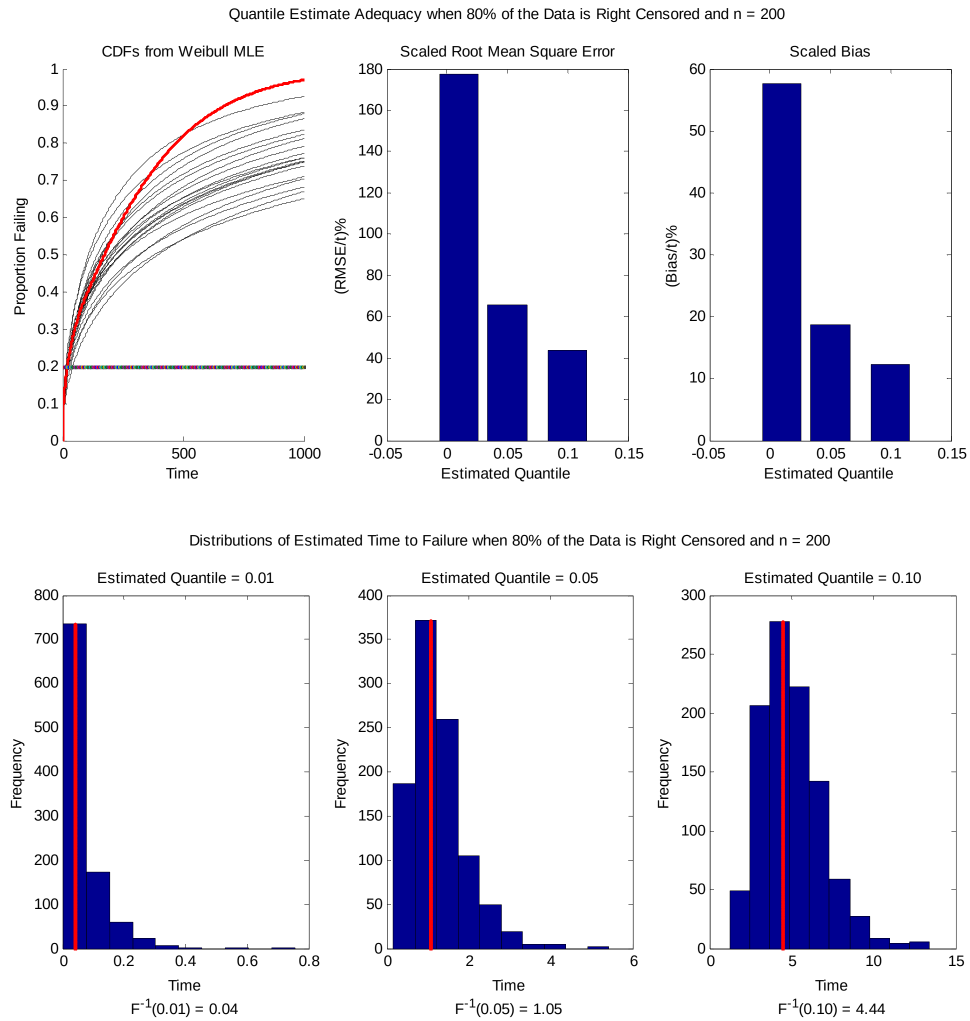
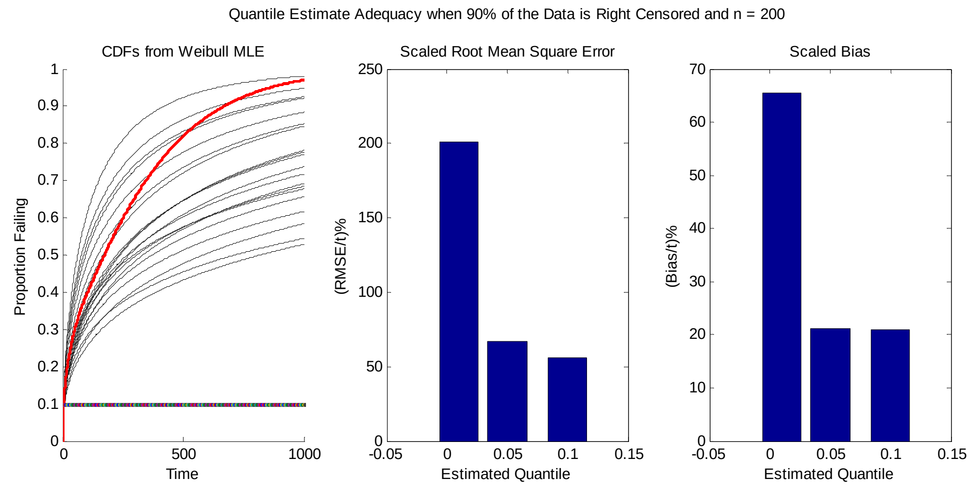

Quantile Estimate Adequacy when 80% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile
Distributions of Estimated Time to Failure when 80% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Frequency
Time
F-1(0.01) = 0.04
Estimated Quantile = 0.05
Frequency
Time
F-1(0.05) = 1.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.10) = 4.44

Figure A-7i


Quantile Estimate Adequacy when 90% of the Data is Right Censored and n = 200
CDFs from Weibull MLE
Proportion Failing
Time
Scaled Root Mean Square Error
(RMSE/t)%
Estimated Quantile
Scaled Bias
(Bias/t)%
Estimated Quantile

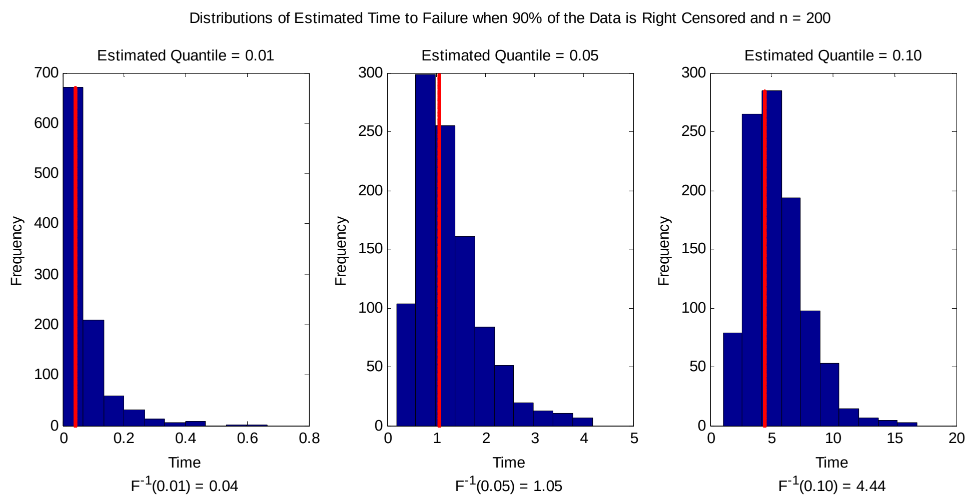

Distributions of Estimated Time to Failure when 90% of the Data is Right Censored and n = 200
Estimated Quantile = 0.01
Frequency
Time
F-1(0.01) = 0.04
Estimated Quantile = 0.05
Frequency
Time
F-1(0.05) = 1.05
Estimated Quantile = 0.10
Frequency
Time
F-1(0.10) = 4.44

Figure A-7j

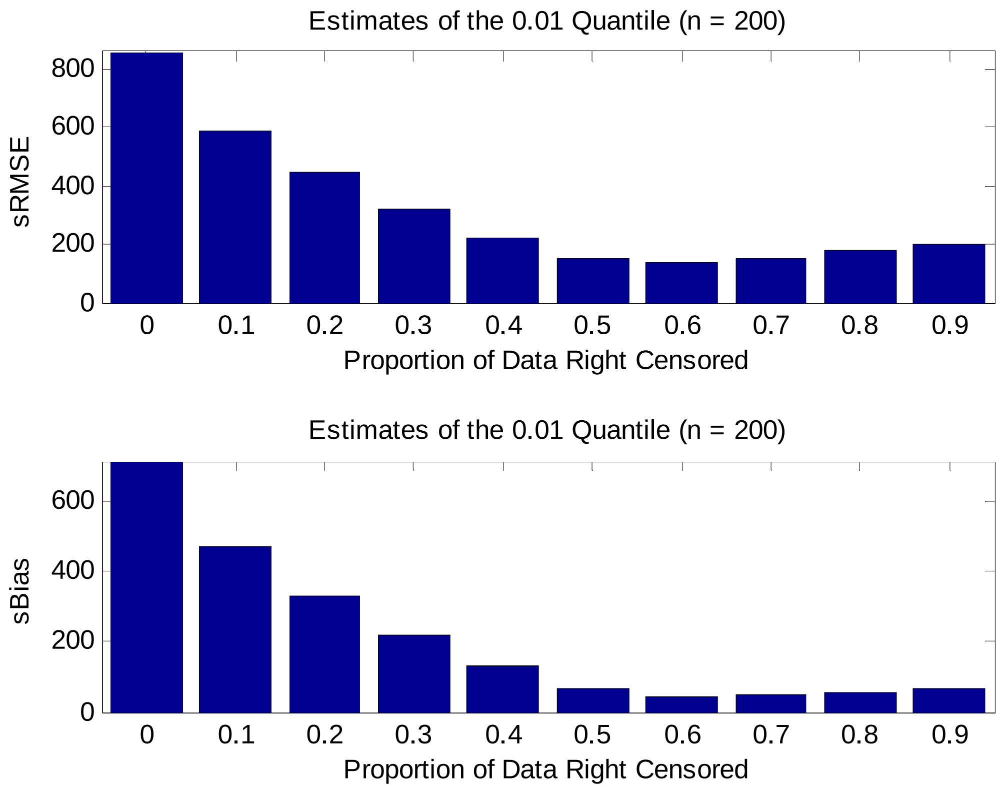

Estimates of the 0.01 Quantile (n = 200)
sRMSE
800
600
400
200
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Proportion of Data Right Censored
Estimates of the 0.01 Quantile (n = 200)
sBias
600
400
200
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Proportion of Data Right Censored

Figure A-7k

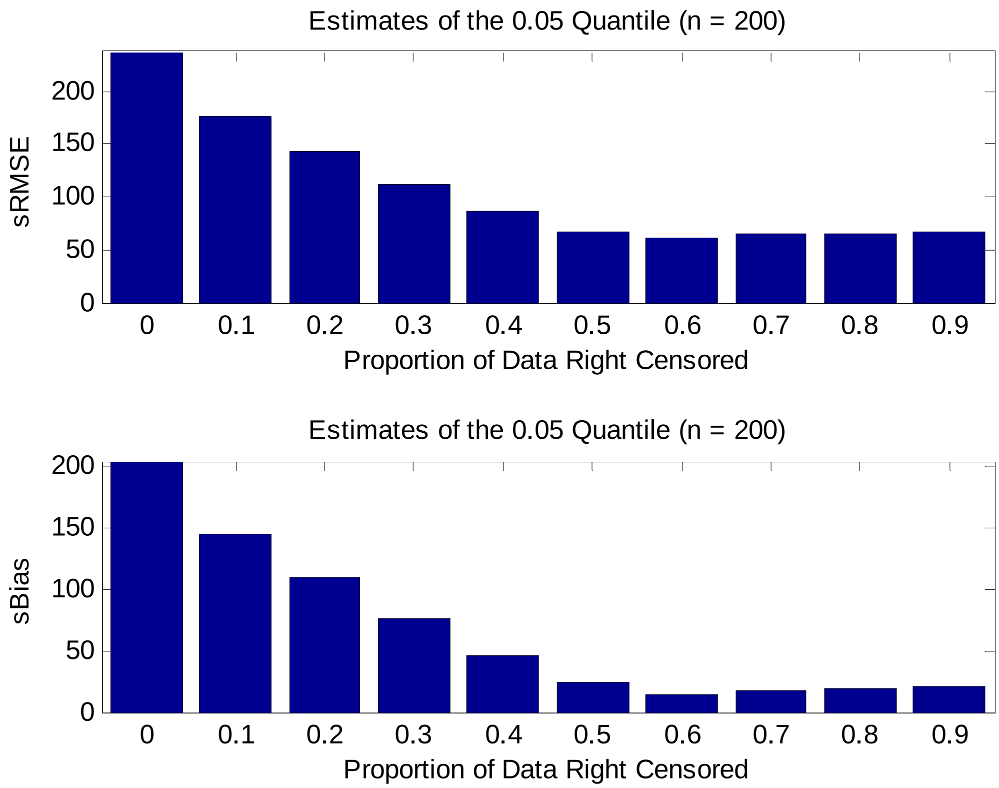

Estimates of the 0.05 Quantile (n = 200)
sRMSE
200
150
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Proportion of Data Right Censored
Estimates of the 0.05 Quantile (n = 200)
sBias
200
150
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Proportion of Data Right Censored

Figure A-7l

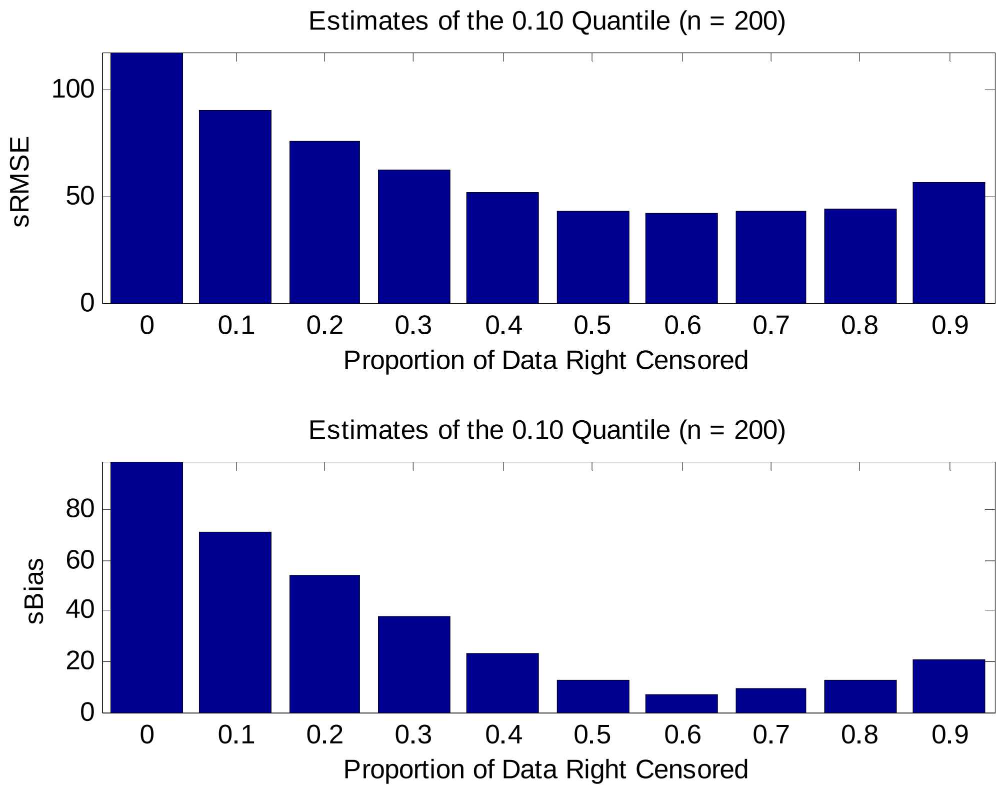

Estimates of the 0.10 Quantile (n = 200)
sRMSE
100
50
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Proportion of Data Right Censored
Estimates of the 0.10 Quantile (n = 200)
sBias
80
60
40
20
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Proportion of Data Right Censored

Figure A-7m

```
%*********************************************************************
% Kevin Crookston
% March 2009
%
% The purpose of this program is to focus on one set of data of size n
and
% incrementally right censor the data, then make Weibull probability
plots
% of each set.
%*********************************************************************

clc
clear all

% The following are parameters that may be changed before running the
% program
_________________________________________________________________

a    = 100;% time at which parameters change
beta1= .5; % 0 < beta1 < 1, decreasing hazard from 0<=t<a
beta2= 1.2;% 1 <= beta2 < 2, constant/slightly increasing hazard for
t>=a
eta1 = 400;% scale parameter for 0<=t<a
eta2 = 400;% scale parameter for t>=a
n    = 200;% sample size

%proportion of data that are right censored (0 <= censd < 1)
censd= [0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9];
% _________________________________________________________________

censdquantile=1-censd;
Fa = 1-exp(-((a/eta1)^beta1)) % Pr(t<a)
const1 = (beta1/eta1)*(a/eta1)^(beta1-1)-(beta2/eta2)*(a/eta2)^(beta2-
1);

%*********************************************************************
% Section 1
% This section generates n values to form a distribution with a bathtub
% hazard rate using the Weibull distribution as a starting point.

options = optimset('Display','off'); %Turns off Display for fsolve
pr=rand(n,1);
for j = 1:n
    if pr(j)<Fa
        wbl(j)=eta1*(-log(1-pr(j)))^(1/beta1);
    else
        const2 = eta2^beta2*((a/eta1)^(beta1)...
                 -(a/eta2)^(beta2)-const1*a+log(1-pr(j)));
        wbl(j) = fsolve(@(t)(t^beta2+eta2^beta2*const1*t+const2)...
                 ,a,options);
    end
end
wbl=sort(wbl);
```

```
% The following vector was generated using the code in Section 1 and is
% provided to reproduce the results in Chapter 5.
%   wbl=[ 0.0039903727136298
%         0.0156068640989011
%         0.0233541881878607
%          0.037391348328135
%         0.0883928873335085
%          0.103457564359056
%          0.168488286280907
%          0.294236714231396
%          0.693388822095304
%          0.902603027896652
%          0.978743437405182
%          0.981698889020608
%           1.44009665264453
%           1.45020743147904
%           1.74474632330818
%           1.80505979702961
%           2.18541181312867
%           2.41657806931289
%            2.5623132333415
%           3.03936041152387
%            3.6654446961043
%           3.80473656231987
%           4.64446925445234
%           5.61777690574127
%           5.87357676996021
%           5.94956688195366
%           7.60520813514189
%           8.18655622178181
%           8.54301429532284
%           9.01958118064765
%           9.06137369986799
%           10.9323211675264
%           12.3768607739793
%            13.595913711246
%           13.7139960833228
%            14.092957311601
%           15.4602021012238
%           16.7521619996055
%             17.03081423702
%           17.7932667495336
%            18.973535127216
%           18.9906468521524
%           20.6671030051526
%           20.9913072183984
%           24.3743197061883
%           29.6932155541685
%           30.1116083468915
%           30.2796686022173
%            30.608463945152
%           30.6262818558629
%           31.8002475615472
%           32.1387768510774
%            34.744676662827
```

```
%            36.0314860864536
%            42.4817405865442
%            45.2693570309753
%             46.701424545085
%            47.6573560577936
%             47.796659476589
%            49.7721371142823
%            50.9286882496906
%            53.1210021764268
%            53.4702908259497
%            55.0914189318471
%            56.4712725539238
%            61.4424134175614
%            65.2530520712491
%            65.9888217478516
%            68.7766581043382
%            74.1068204972085
%            81.0740816526824
%             83.181624043646
%            85.8018912182553
%            87.9068665456484
%            93.7431609491483
%            94.3687955814376
%            109.674222578681
%            111.605724757014
%            115.015313265076
%            120.831500025824
%            131.620428868146
%            145.133948248357
%             146.43884209739
%            147.085179986929
%            147.436986316106
%            147.658971423579
%            149.155201865525
%             150.93044131247
%            154.178442764256
%            156.527451074793
%            162.941222096718
%            164.434830167806
%            167.968533221319
%            178.038553229597
%            181.998440610024
%             194.80622075736
%            201.702182259401
%             209.03099195316
%            210.806522923002
%            214.522891716919
%            223.090644283999
%            228.470984611867
%            230.451846637177
%            235.372613289741
%            236.554847946942
%            239.440060064273
%            240.601896644269
%            250.530116136473
```

```
%            254.465553998878
%            259.798943742332
%            260.684308662467
%              262.3228206664
%            264.318056048119
%            270.245888762459
%            283.198810546011
%            286.719565526528
%            287.799379376865
%            292.800058399115
%            294.665174721823
%            295.083945044709
%            299.300862336036
%            300.395119589006
%            302.906301232311
%            305.021861419089
%            310.224029483174
%            321.762332885458
%            325.868546421857
%            327.734028409453
%            330.024031470434
%            331.452696199791
%            336.333689085617
%            336.664435737215
%            340.027399507907
%            343.696237640881
%            351.540600051611
%            352.424905866367
%            359.505075659292
%            359.632943669973
%            363.910584180208
%            367.459914189383
%            368.074482277033
%            369.623585212897
%            379.426252413984
%            382.284503399289
%            387.530996659472
%            387.754769949858
%            392.417439952775
%            408.937070890796
%            411.581576721946
%            417.983078606343
%            420.034599962274
%             421.38346289986
%            437.799714785453
%            445.728709599269
%            465.940234699165
%            470.409701130986
%            485.569262354863
%            486.505580928325
%            491.767754951698
%            500.848394020581
%            500.925854885773
%              513.1717017822
%            517.735352750196
```

```
%           550.416472047567
%            569.69117123606
%           580.598539685739
%           586.604289223881
%           589.943214580652
%           591.325064221795
%           618.128592966562
%           623.393708284591
%           627.594649988225
%            629.99259154988
%            648.58583545072
%           653.464295912543
%           664.966577194487
%           676.346515586507
%           681.122996237477
%           687.758511567103
%           708.984423197579
%           743.744982886999
%           788.945208572357
%           831.732950613753
%           831.874111037136
%           837.488931847627
%           872.521066665381
%           885.998300403631
%           973.344493129842
%           1002.55489073669
%           1010.45343040958
%           1072.96686947223
%           1111.63474375762
%           1182.10983986554
%           1242.95941881696
%           1244.44325331948
%           1271.89624278634
%           1406.05914944986
%            1440.0611024656
%           1441.78273845109
%           1675.65727986045]';

%*********************************************************************
% Section 2
% This section incrementally induces right censoring at the values
provided
% in the vector censd, and creates Weibull probability plots.
for k=1:length(censd)
    figure (k)
    c=prctile(wbl,censdquantile(k)*100);
    wblc=[wbl(1:floor(n*censdquantile(k))),c];
    censored=[zeros(1,floor(n*censdquantile(k))),1];
    frequ=[ones(1,floor(n*censdquantile(k))),n-length(wblc)+1];

    % negative log likelihood

nll(k)=wbllike(wblfit(wblc,0.05,censored,frequ),wblc,censored,frequ);
    probplot('weibull',wblc,censored,frequ),
        title({'Weibull Probability Plot';...
```

```
        sprintf('when %.0f%% of the Data is Right
Censored',100*censd(k));...
        sprintf('-log likelihood = %.2f',nll(k))});
end

% Section 3
% This section plots the censoring quantile by the negative log
% likelihood
figure (k+1)
plot(censd,nll,'.'),
title('Proportion Right Censored by Negative Log Likelihood'),
xlabel('Proportion Right Censored'),ylabel('Negative Log Likelihood');
```

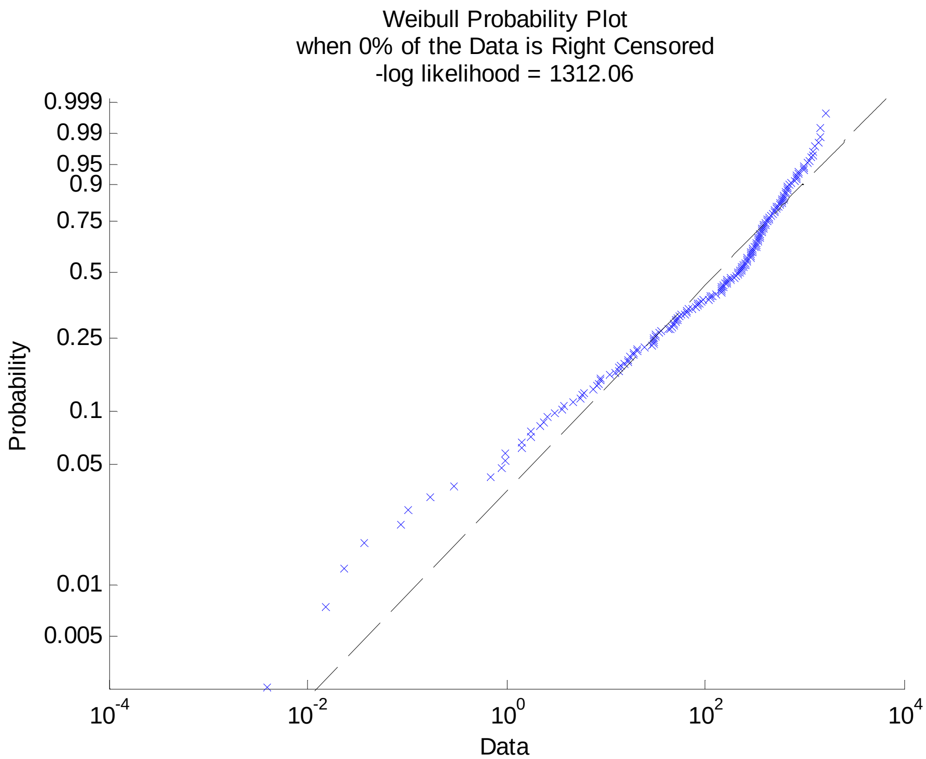


Figure A-8a

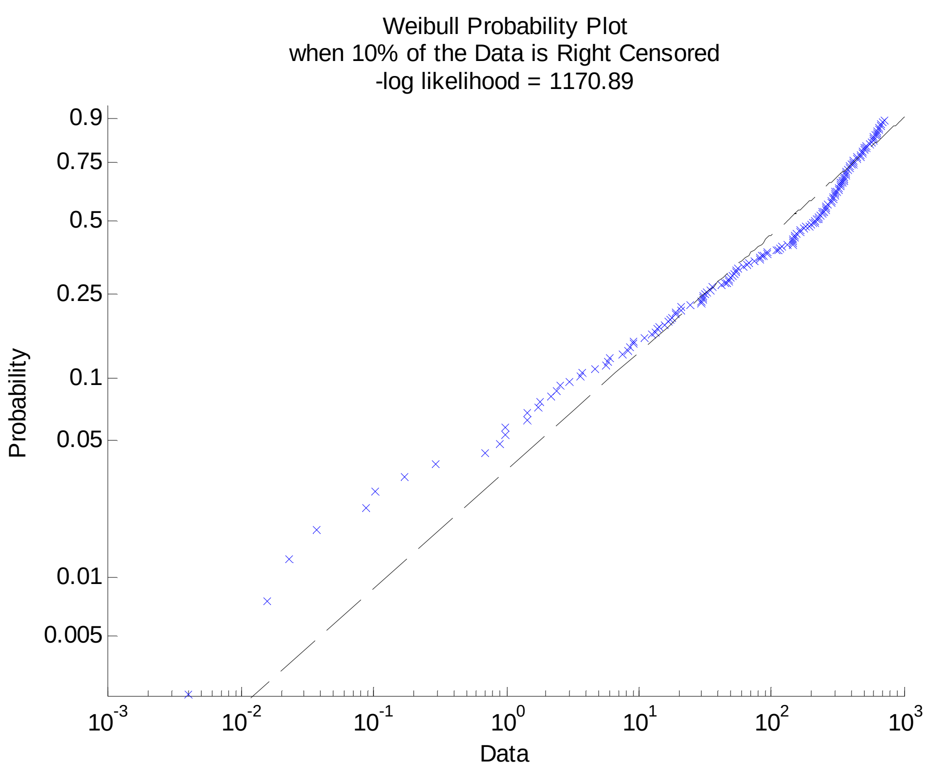


Figure A-8b

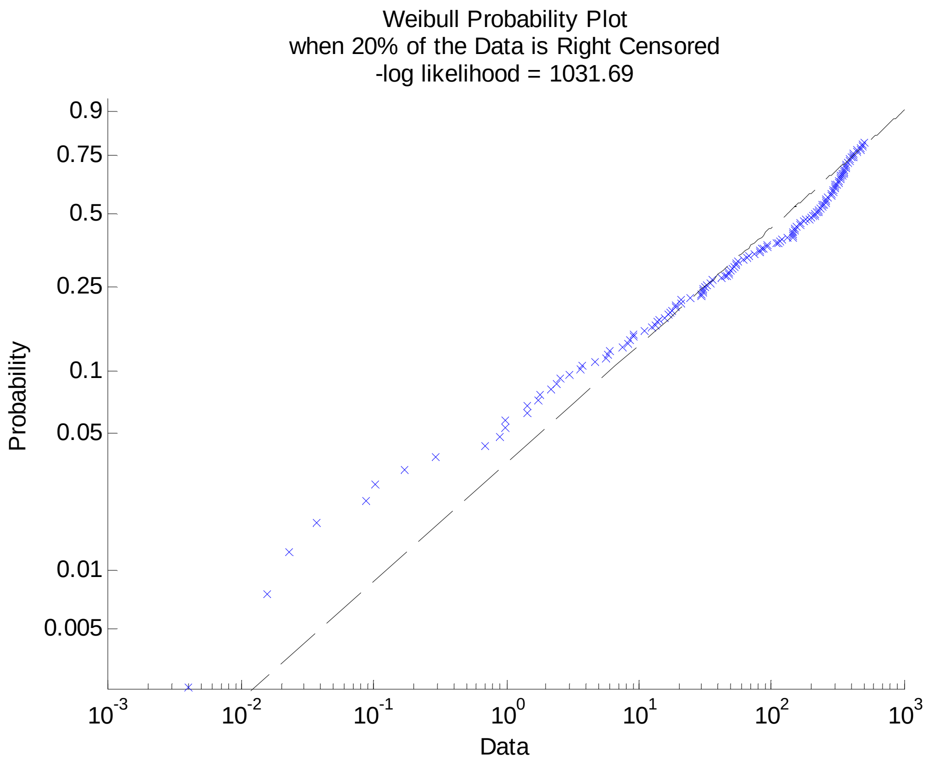


Figure A-8c

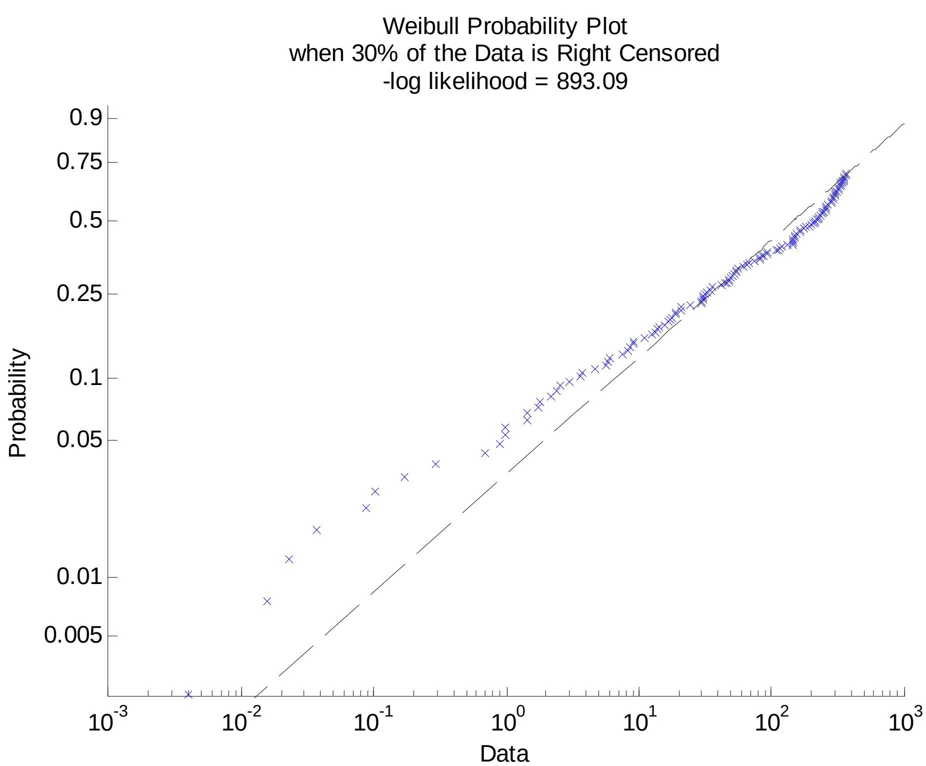


Figure A-8d

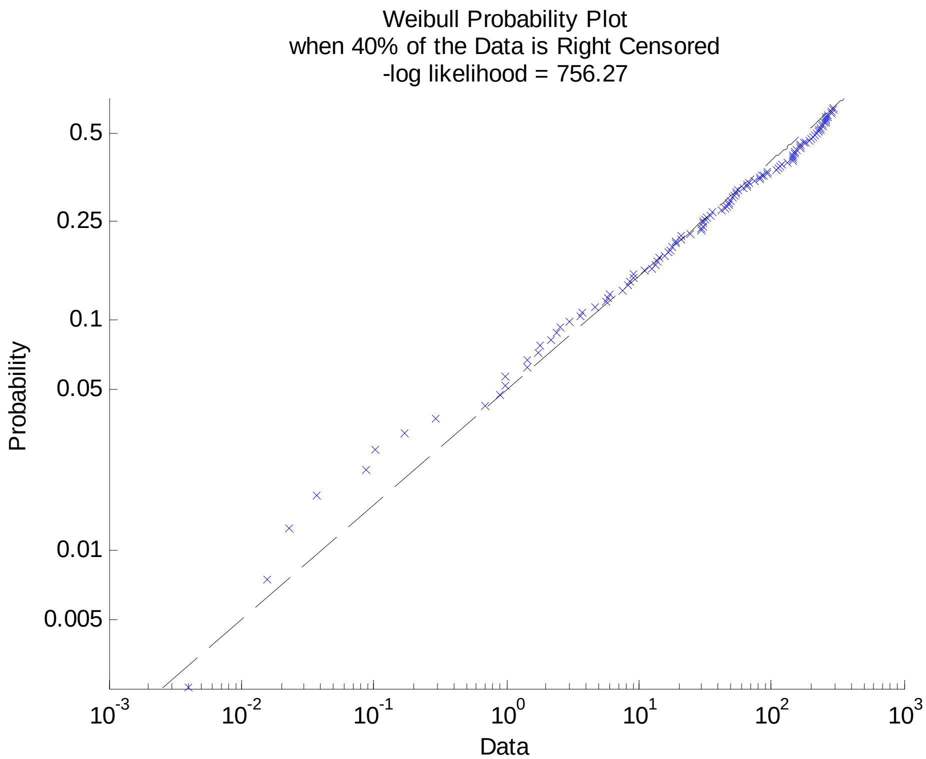


Figure A-8e

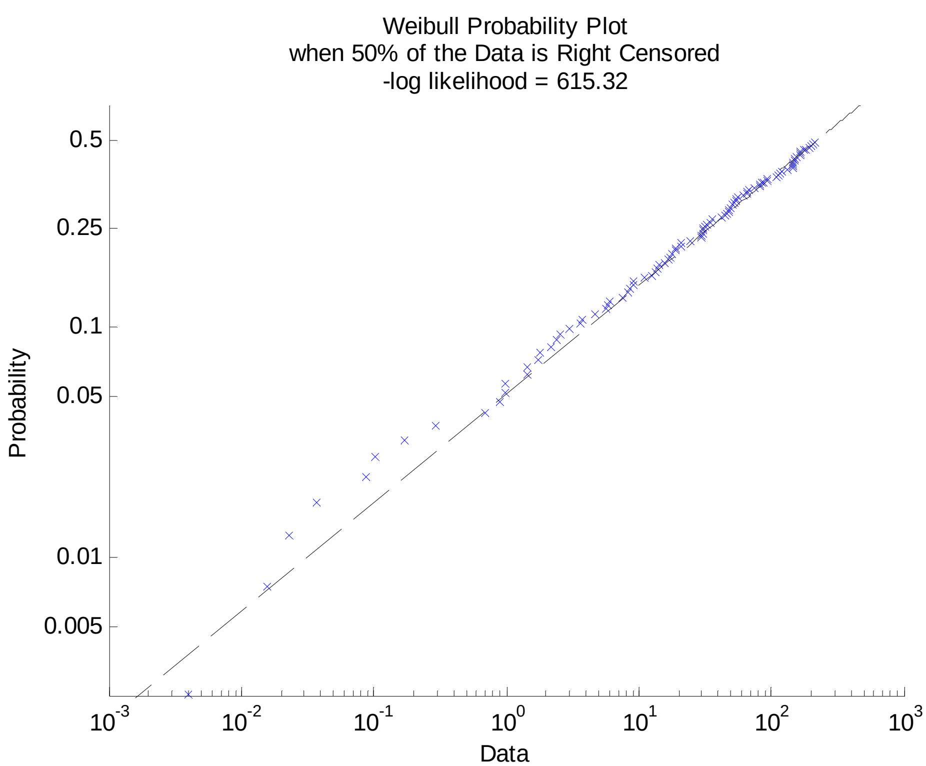


Figure A-8f

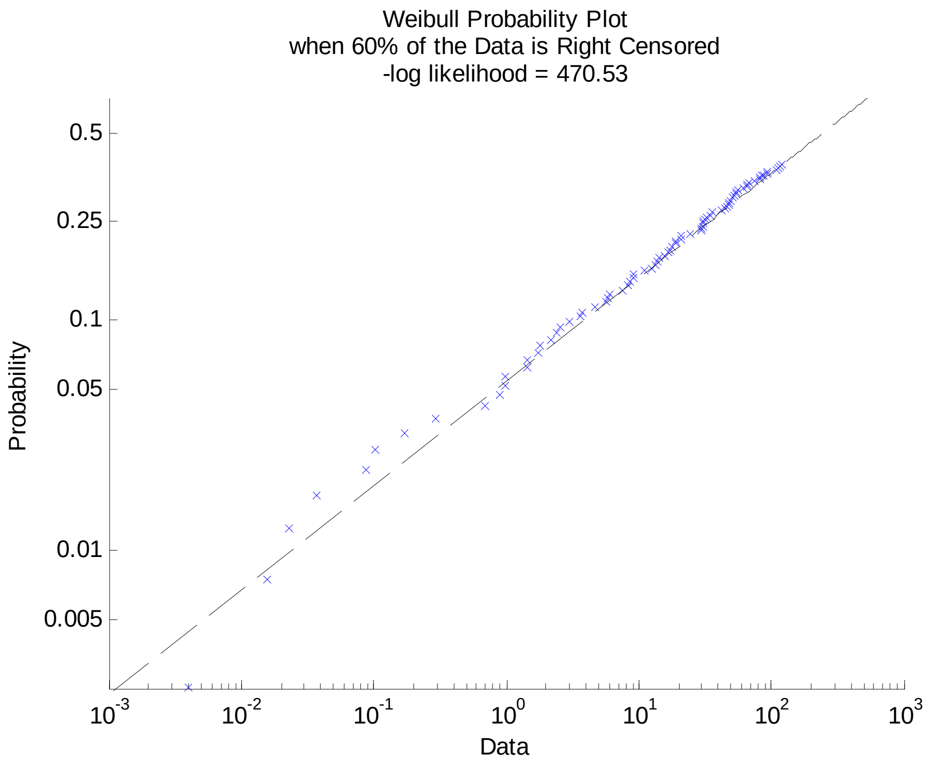


Figure A-8g

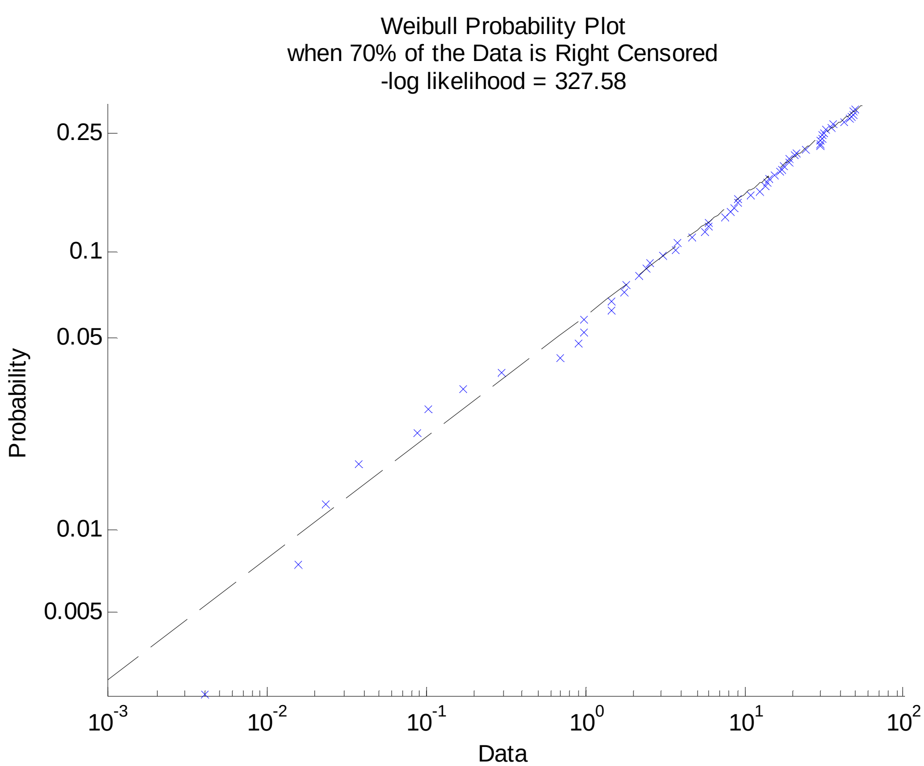


Figure A-8h

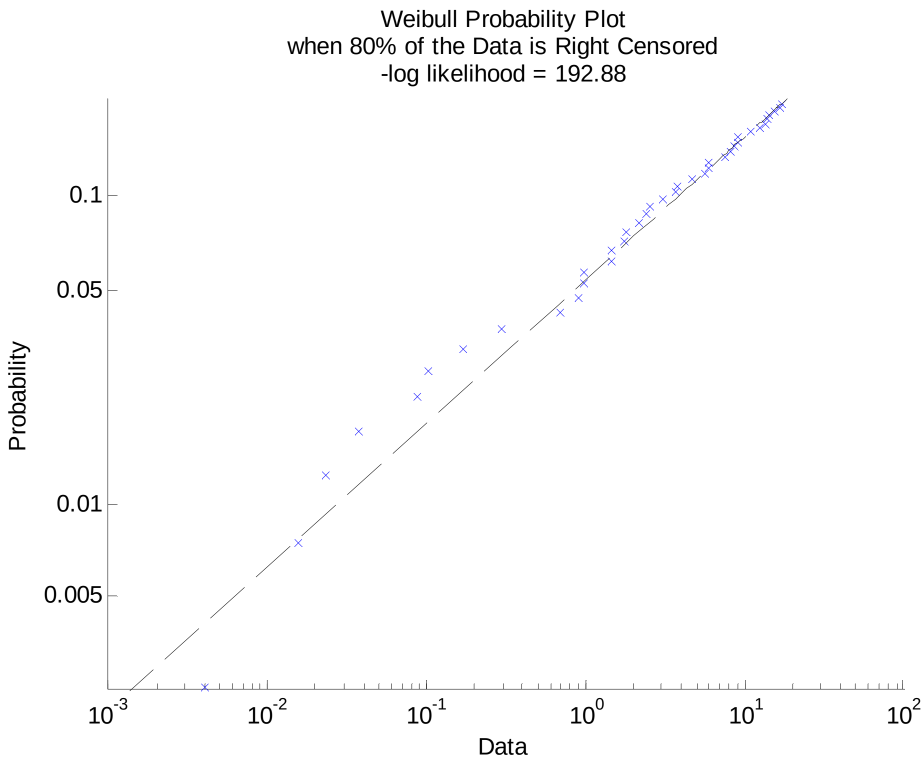


Figure A-8i

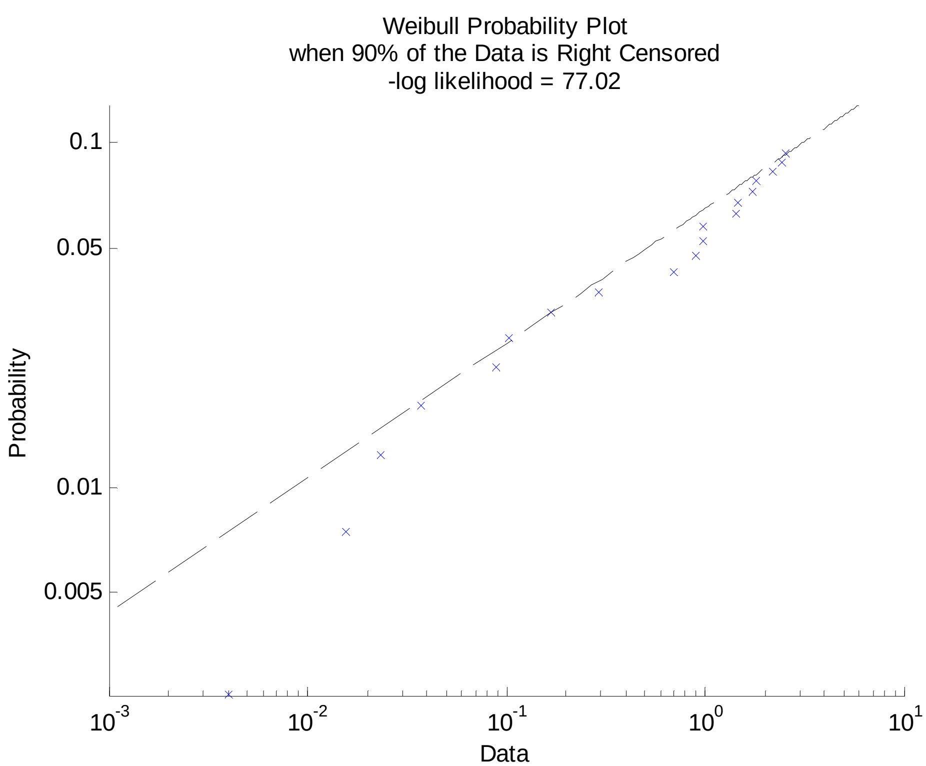


Figure A-8j

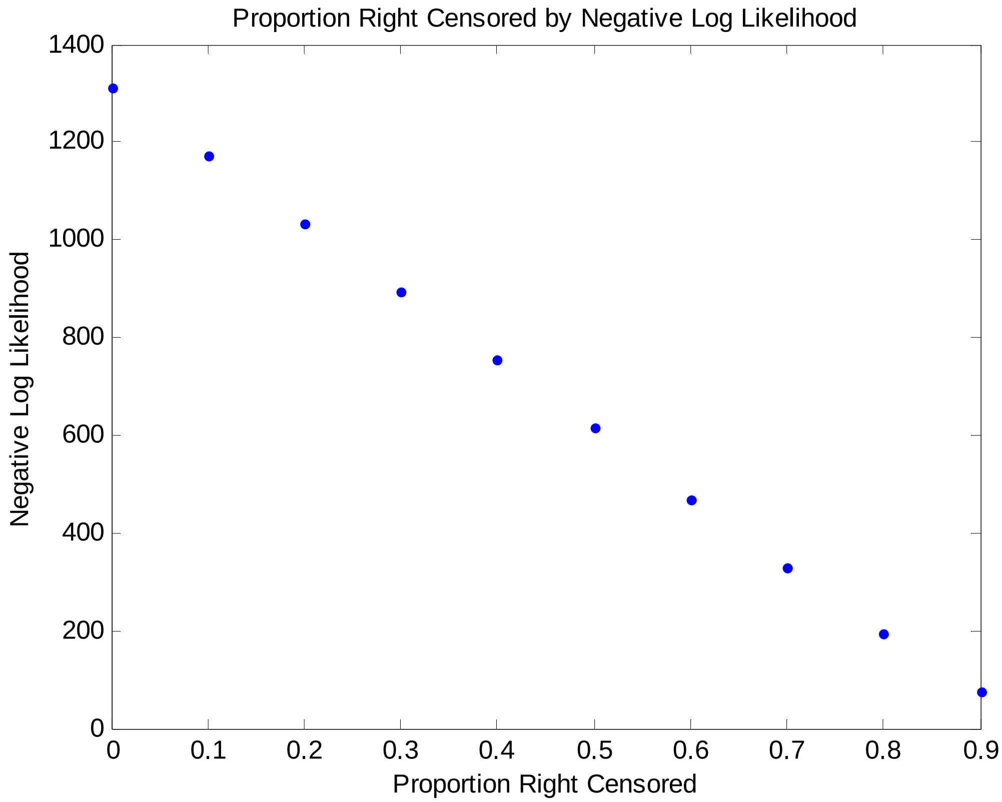

Proportion Right Censored by Negative Log Likelihood
1400
1200
1000
800
600
400
200
0
Negative Log Likelihood
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Proportion Right Censored

Figure A-8k

Vita

Kevin A. Crookston is a Graduate Research Assistant working under the direction of Timothy M. Young in the Forest Products Center at the University of Tennessee, Knoxville. He is working on a Master of Science Degree in Statistics with an emphasis in Industrial Statistics and plans to graduate in May 2009. He earned his Bachelor of Science Degree in Statistics with an emphasis in Quality Science and Manufacturing from Brigham Young University in April 2007, and graduated valedictorian from College of the Canyons with an Associate Degree in Mathematics in 2005.

As an undergraduate, Crookston worked as a teaching assistant for Introductory Statistics, Multiple Linear Regression, and Statistics for Industry. He interned at ITT Aerospace Industries in Valencia, CA and ATK Launch Systems in Promontory, UT. After graduation he plans to obtain a career as a reliability and risk analyst.

During the course of Crookston's education, he has been a member of the American Statistics Association, Mu Sigma Rho, National Golden Key Honor Society, and Society of Manufacturing Engineers. He is an Eagle Scout and continues to work with the Boy Scouts of America as a Cubmaster and a Merit Badge Counselor for Wilderness Survival and Woodcarving.