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ABSTRACT

Many brown dwarfs have recently been discovered as sub-stellar objects in which deuteron thermonuclear fusion is taking place. Although Jupiter and Saturn emit nearly twice as much heat as they absorb from the Sun, their internal heat-generation mechanisms have been determined to differ from the nuclear fusion that fuels brown dwarfs because they have a mass factor of 0.023–0.077 less than that of brown dwarfs. The possibility for deuteron nuclear fusion in the Earth's core has not been well studied. Here, we compare the conditions for electron degeneracy pressure and temperature for the cores with an Fe–D compound of Earth, Jupiter, and Saturn to the core with deuterium gases of the coldest brown dwarf, WISE 1828+2650, in respect to three-body deuteron nuclear fusion, based on electron capture and internal conversion processes. Our results suggest that deuteron nuclear fusion is possible in the cores of Earth, Jupiter, and Saturn as well the coldest brown dwarf.

I. INTRODUCTION

Brown dwarfs, bridging the gap between giant planets and hydrogen-fusing stars,[1] have recently been discovered as sub-stellar objects that do not have the mass required to sustain the nuclear fusion of ordinary hydrogen. However, they are understood to fuse deuteron as protostars.[2] Thus, they are also referred to as failed stars or super-Jupiters. Recently, temperate Earth-sized planets transiting a nearby ultra-cool brown dwarf (TRAPPIST-1, 2MASSJ23062928-0502285) were discovered.[3] In contrast, Jupiter is the largest planet in the solar system and is known as a failed brown dwarf because its mass M_j is less than 1.3%–7.7% of those of brown dwarfs.[4] Since sunlight, which drives the weather on Earth, is only 4% as strong on Jupiter as on Earth, Jupiter emits nearly twice as much heat as it absorbs from the Sun.[5] It is also known that Saturn and Neptune, as well as Jupiter, radiate about twice as much energy as they receive from the Sun.[6] When mass in a gravitationally bound object shrinks closer to the center of attraction, the gravitational potential energy converts into heat. It can be considered to be due to an exothermic reaction by adiabatic shrinkage.[7] This is known as the "Kelvin–Helmholtz mechanism" for the source of Jupiter's energy,[8] but it has been expanded by Bethe's theory of nuclear fusion.[9] Other ideas, such as internal conversion of incident radiation into mechanical energy[10] and on-going tidal dissipation due to a non-zero planetary obliquity,[11] appear to lack general applicability.[6] The recent discovery of hot Jupiter exoplanets with insufficient eccentricity to be heated internally by tidal dissipation[12] has evoked much discussion as to possible sources of internal heat production.

Following the pioneering work of Kuroda[13] for natural fission reactors, much challenging work, such as planetocentric nuclear reactors by Herndon[14] and nuclear fission at the mantle boundary with the Earth's core by Meijer and van Westrenen,[15] has been carried out to explain thermal energy as the driving force of plate motion. The detection of antineutrinos in KamLand[16] and Borexino[17] was reported as evidence for natural fission. However, judging from a very small amount (12.5 ppm) of decaying product, Pb, in natural rocks and ores, the amount of decaying heat from radioactive materials would be not so large.[18] If radioactive decay has been occurring on other terrestrial planets, such as Mercury, Venus, Mars, and Earth's moon, which are Earth's sister planets with similar composition but smaller size, we would observe plate tectonics. Indeed, plate tectonics do not occur on these planets.[18] On the other hand, Fukuhara[18,19] proposed a model for the origin of thermal energy within the Earth's interior, which is devoid of harmful radioactive waste, in which the generated heat is due to the three-body nuclear fusion of deuteron (D) confined within hexagonal FeDx core–center crystals,

$$^{2}\mathrm{D} + {}^{2}\mathrm{D} + {}^{2}\mathrm{D} \to {}^{4}\mathrm{He} + 2{}^{1}\mathrm{H} + \mathrm{e}^{-} + \bar{\nu}_e + 21.63\ \mathrm{MeV}, \quad (1)$$

where $\overline{\nu_e}$ is an antineutrino. The deuteron-density-dependent fusion rate, R, and heat generation rate were calculated to be 3.5 × 10^{10} fusion/s/m^3 (supplementary material, Sec. 2) and 1.27 × 10^{15} J/m^3 (Ref. 19), respectively. The core volume, where fusion occurred, up through the present is 4.99 × $10^{17}$$m^3$. Because the relative pion exchange force is two and six for D–D and D–D–D simultaneous interactions,[20] we selected the 3D fusion in this study. Thus, the fusion rate of 3D is higher than that of 2D.

Recently, Jackson *et al.*[21] proposed a new model for the formation of the Earth's core, in which it was not extracted from the deep mantle, but derived from large impacts of other early solar system objects (containing FeO-rich silicates) with the proto-Earth. This model is consistent with our hypothesis on nuclear fusion, provided that they were enriched by D_2O. The deuterium concentration in the center core with Fe–D compounds is a necessary condition for the fusion reaction.

In this study, we compare the relations of electron pressure and temperature for the cores with Fe–D compounds of Earth, Jupiter, and Saturn to the core with deuterium gases of the coldest brown dwarf, in respect of deuteron nuclear fusion. However, to the best of our knowledge, the possibility of deuteron nuclear fusion in the cores of Earth, Jupiter, and Saturn, as well as in the coldest brown dwarf, has not been studied.

II. DEGENERATE ELECTRONS AND PION CONDENSATES IN THE CORES OF CELESTIAL BODIES

Various kinds of condensates are expected in the interior of the cores of celestial bodies, including degenerate electrons, neutrons in a superfluid state, protons in a superconducting state, and other exotic states, such as pion and kaon condensates.[22,23] We focus on degenerate electrons and meson condensates in brown dwarfs and the interior of planets with solid cores in the presence of a strongly interacting fermion system and their effects on the dynamics of deuteron thermonuclear fusion.

According to the free electron model by Zel'dovich and Novikov[24] and Al'tshuler and Bakanova,[25] the periodicity of metallic elements disappears under pressures greater than 100 GPa near the dwarf's inner core, which reflects the electronic shell structure of atoms. The outer and degenerate inner electrons behave like free electrons. Thus, we must use the Thomas–Fermi–Dirac model, which is usually thought of as being useful in predicting the properties of matter only at high densities and temperatures, taking into account the exchange energy.[26] In kinetic theory, the electron degeneracy gas pressure is given by the following equation:[27]

$$P = \frac{1}{5}\left(\frac{3}{8\pi}\right)^{\frac{2}{3}} \frac{h^2}{m}\left(\frac{N}{2}\right)^{\frac{5}{3}} \rho^{\frac{5}{3}}, \quad (2)$$

where m, N, and h are the electron mass, Avogadro's number, and Planck's constant, respectively. The relationship is presented as a straight line at pressures above 10^{15} Pa and densities about 10^5kg/m^3. Jensen[27] calculated the isotherms for many metals at pressures below 10^{15} Pa and densities below 10^5 kg/m^3. We calculate the degenerate electron gas pressures of Earth, Jupiter, and Saturn as there are insufficient pressure data available.

In contrast, the meson fields that mediate the interaction between nucleons can condense into a macroscopic state at high densities. This proceeds via a macroscopic excitation of the pion field when the density is high, and hence, the nucleons are interacting strongly, leading to a nonzero expectation value of the pion field in a broken symmetry state, such as the superconducting one. The charged and neutral pion condensed state would occur through a softening of a collective mode. In particular, the neutral condensed state is characterized by a spatially varying finite expectation value of the neutral pion field.[28] The surrounding clouds of pions produced by gluons have an influence on the strong, weak, and electromagnetic interactions under high pressure. The density of the cloud increases with the increase in the density of nucleons accompanied by the strong interaction although the density of the ordinary nucleus is almost constant regardless of elements.

In degenerate electrons and meson condensates in the interiors of solid-core planets with Fe–D compounds and brown dwarfs with deuterons, the degenerate electrons suppress β^--decay from neutron to proton and accelerate K-electron capture from the inner 1s state electrons within the nucleus. The electron capture process, based on the weak interaction, electromagnetically changes a proton to a neutron and simultaneously causes the emission of an electron neutrino ν_e. Indeed, the radioactive decay constant measurement represents a pressure effect on the Be^7 capture rate.[29] Because the deuterium daughter nuclide under high pressure is in an excited state, it emits a gamma ray, γ, by transition from excited to ground states. Since the deuterons in solids are trapped by periodic potential, the pairing partner of deuteron limits to the neighboring deuteron. In this case, a high-energy electron resulting from internal conversion induces the emission of high-speed gamma rays, which may be massive bosons, from the s shell orbit surrounding the deuterium nucleus into the nuclei of the neighboring deuterons. The electron capture and internal conversion processes are simultaneously described as

$$\mathrm{p} + \mathrm{e}^- \to \mathrm{n} + \nu_e \uparrow + \gamma. \quad (3)$$

The neutral pion in Eq. (3) is provided by the high-energy gamma rays via the fundamental process of electromagnetic interactions,[30]

$$\gamma + \gamma = \pi^\circ. \quad (4)$$

From Eqs. (3) and (4), the electrons and protons in two deuterons react with each other to form two neutrons, two electron neutrinos, and one neutral pion,

$$2\mathrm{p} + 2\mathrm{e}^- \to 2\mathrm{n} + 2\nu_e \uparrow + \gamma^\circ. \quad (5)$$

The electron and neutral pion velocities are calculated to be 2.73 × 10^7 m/s and 1.68 × 10^6 m/s, respectively (supplementary material, Sec. 1). The lifetime of the resulting neutral pion is 2.38 × 10^{-17} s (= 4.01 × 10^{-11} m/1.68 × 10^6 m/s), which is slightly less than the lifetime [9.5 ± 1.5 × 10^{-17} s (Ref. 31)] of the neutral pion, provided that the neutral pion is formed in the space [4.01 × 10^{-11} m (Ref. 18)] between the two neighboring shrunken deuterons. Indeed, because the Oppenheimer–Phillips paper indicates that the range of the nucleonic portion of the deuteronic wave-function extends to

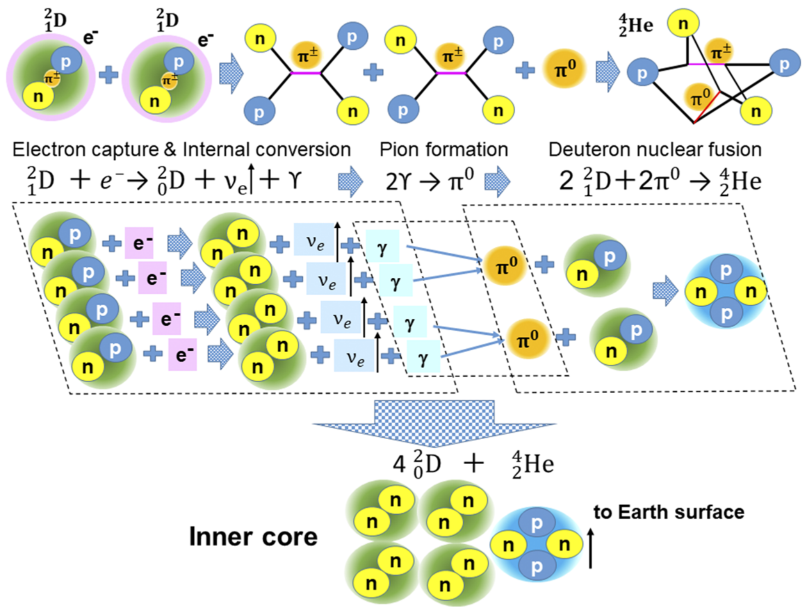

FIG. 1. Schematic presentation of deuteron nuclear fusion based on electron capture and internal conversion processes.

about 5 F (Ref. 32), the lifetime would be less than 2.97×10^{-21} s ($= 5 \times 10^{-15}$ m/1.68×10^{6} m/s). This value is considerably smaller than the lifetime of a neutral pion. Therefore, the formation of the He nucleus from the fusion of two deuterons requires a direct force caused by the exchange of two neutral pions that do not compose the deuteron nucleus because the additional non-exchange part mediated by the neutral pion substantially moderates the n–p force in the He nucleus,[33]

$$ {}^{2}_{1}\mathrm{D} + {}^{2}_{1}\mathrm{D} + 2\pi^{\circ} = {}^{4}_{2}\mathrm{He}. \tag{6}$$

As a result, we can deduce mix products of $4{}^{2}_{0}\mathrm{D}$ and ${}^{4}_{2}\mathrm{He}$ in the center of the core. The neutrinos are released from the Earth's interior to the Universe. These processes are illustrated schematically in Fig. 1.

III. NUCLEAR FUSION IN THE CORES OF CELESTIAL BODIES

A. Earth

For the compressed core of Earth, we calculated the electron degeneracy gas pressure, P, at a high temperature of 5130–6370 K and a high density of 13 780–13 970 kg/m^3 of ε-Fe at 377 ± 8.5 GPa in the Earth's core.[34] From Jensen's calculated isotherm diagrams,[27] we obtain a pressure of 1.0–1.2 TPa. For deuteron thermonuclear fusion reactions that require occurring in celestial bodies, the following conditions at their centers are necessary: a large amount of deuterium and a high-temperature and high-pressure environment to overcome the high Coulomb barrier for fusion reactions. However, the definition for deuteron nuclear fusion needs to be extended from gaseous deuterium objects, such as brown dwarfs, to planets with solid cores containing deuterons, such as Earth, Jupiter, and Saturn.

We next compare the electron pressure effect for the cores of Earth and a brown dwarf. In sharp contrast to the brown dwarf with an inner core of deuterium gas, Earth is a rocky planet with a core of an Fe-based compound. The deuterium atoms of the former are packed by quantum electron degeneracy pressure alone, whereas the deuterium atoms of the latter are served by the squeezing effect of 26 electrons surrounding the Fe nucleus in the tetrahedral sites of the ε-Fe lattice.[34] Figure 2 shows comparative schematics of a D atom squeezed by a tetrahedral Fe atom lattice in the core of Earth and a squeezed D atom in a brown dwarf's core. We obtain a multiplied confinement effect of 0.229 (= 0.517 × 0.443) (supplementary material, Sec. 4) for the D–D distance. Thus, the confinement effect of the latter is more effective than that of the former.

We then consider a physical catalysis effect for the dynamic reactions of deuteron pairs, based on the formation of neutral pions.

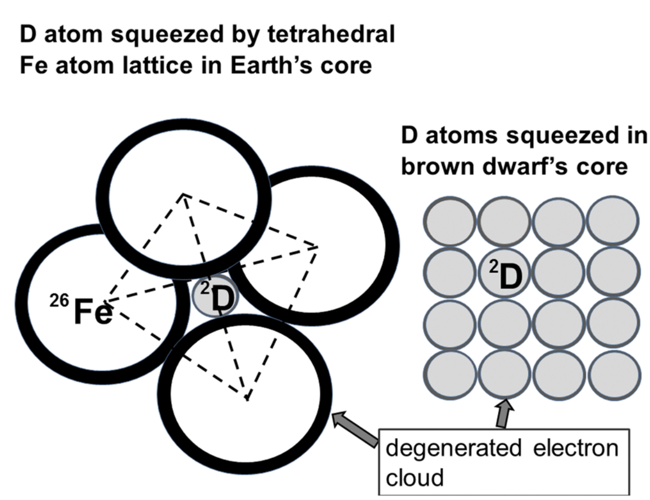

FIG. 2. Schematic of the two squeezing effects by 26 electrons surrounding the Fe in the tetrahedral sites of ε-Fe lattices in the Earth's core and quantum electron degeneracy pressure in a brown dwarf's core.

The effect can be described by the enhancement of the attractive interaction force by a factor of 14 (= 2 × 7) via two neutral pions. Kenny[35] reported that the attraction caused by the exchange of one neutral pion with the spin zero boson is seven times larger than the nucleonic constituents, such as protons and neutrons. He named the mass formula "electropionics." Deuteron fusion results from the reduction in the D–D distance to 0.0284 nm by the multiplied effect of physical catalysis (supplementary material, Sec. 4). Although there is possibility that some momentum transfers to one of the deuterons to cause barrier penetration, we neglect the problem in this study.

B. Jupiter

It is estimated that Jupiter has a central core of solid matter, which is composed of mostly iron and silicate minerals (similar to quartz) and could have a temperature of 50 000K (Ref. 36), which is hotter than the surface of the Sun. The total mass of heavy elements (core + molecular envelope) is between 11 and 45 times that of the Earth's core and 4–14% the total mass of Jupiter.[37] As the core density of Jupiter is estimated to be 2.5×10^4 kg/m^3 (Ref. 38), we can calculate the electron degeneracy gas pressure as 5.2 ± 0.5 TPa from Jensen's isotherm diagram.[27] The amount of deuterium in the core of Jupiter can be derived from the late veneer's bombardment of comets and meteorites with H_2O and D_2O originating from the Kuiper belt or planetesimals between Mars and Saturn, as well as in the primitive dry Earth.[39,40]

C. Saturn

Since Saturn, the second largest planet of the solar system, also radiates 2–3 times [2×10^{17} W (Ref. 41)] as much heat as it receives from the Sun, it must have an internal heat source; however, there is not enough evidence to explain the heat production. Saturn's core is made mostly of rocky and metal elements similar to Jupiter's core.[42] The core's temperature is around 22 000 K (Ref. 43). The mass of the core is larger than Jupiter's core. (For comparison, Jupiter is 317.8 Earth masses and Saturn is 95.2 Earth masses.)[44] Since the core density of Saturn is estimated to be 2.05×10^4 kg/m^3 (Ref. 45), we can calculate the electron degeneracy gas pressure as 3.0 ± 0.5 TPafrom Jensen's isotherm diagram.[27] The amount of deuterium in the core of Saturn could be delivered from comets and meteorites with H_2O and D_2O, as well as in the primitive Jupiter and Earth.[39,40]

Since we cannot get precise temperature, pressure, and density measurements of Neptune's core, we could not calculate the pressure–temperature condition.

D. Brown dwarfs

Brown dwarf interior models provide the following density (ρ_c) condition:[2]

$$10\ \mathrm{g/cm^3} \leqq \rho c \leqq 1000\ \mathrm{g/cm^3}. \tag{7}$$

Recently, Beichman *et al.*[46] discovered the coldest Y brown dwarf, WISEPAJ182831.08 + 265 037.8, with a surface temperature of 300 K. The degeneracy electron gas pressure can be calculated as 370 GPa < P < 31.2 PPa for 1×10^4 kg/m^3 < ρ_c < 1×10^6 kg/m^3 from Jensen's isotherm diagram[27] and Eq. (7) at pressure below and above 10^{15} Pa, respectively.

In contrast, as the mass of the brown dwarf is 0.0029–0.0057M$_\odot$ (solar mass),[46] the peak (Tc) of the mass-dependent core temperature for the brown dwarf can be calculated by the following equation:[47]

$$Tc \sim 2 \times 10^6\ \mathrm{K} \left(\frac{M}{0.05\ M_\odot}\right)^{4/3}. \tag{8}$$

The core temperatures of dwarfs increase with age, reach a peak, and then decrease. From Eq. (8), we obtain a Tc of 4.49×10^4–1.11×10^5 K for the brown dwarf's mass of 0.0029–0.0057M⊙.

IV. PRESSURE–TEMPERATURE RELATIONS FOR DEUTERIUM NUCLEAR FUSION

Here, we note the Lawson criterion, which defines the conditions required for hydrogen nuclear fusion reactors.[48] The "triple product" of plasma (electron) density n_e, confinement time τ, and plasma temperature T is valid for nuclear ignition of homogeneous hydrogen plasma,

$$ne\tau > \frac{12 k_B T}{<\sigma v> Q}, \tag{9}$$

where σ and Q are the cross section and the binding energy of fusion products, respectively, and <> denotes an average over the Maxwellian velocity distribution. According to the Maxwellian molecular speed distribution for gases, the speed distribution of plasma gases with higher density and temperature draws near to Gaussian distribution. Since the confinement time for natural nuclear fusion in the cores of celestial bodies is infinite, we use the modified criterion using the electronic pressure (electron density) and temperature. Thus, it is necessary for the validation of fusion to analyze natural instances of electron degeneracy pressure–temperature conditions. Figure 3 presents the relation of electron pressure and temperature for three instances: possible nuclear fusion in the cores of Earth at 5130–6370 K under 1.0–1.2 TPa, Jupiter at 50 000 K under 5.2 ± 0.5 TPa, and Saturn at 22 000 K under 3.0 ± 0.5 TPa: nuclear fusion in the core of the coldest brown dwarf at 4.49×10^4–1.11×10^5 K under 0.37–31.2 TPa; and hydrogen thermonuclear fusion in the Sun's core at 1.57×10^7 K (Ref. 49) under

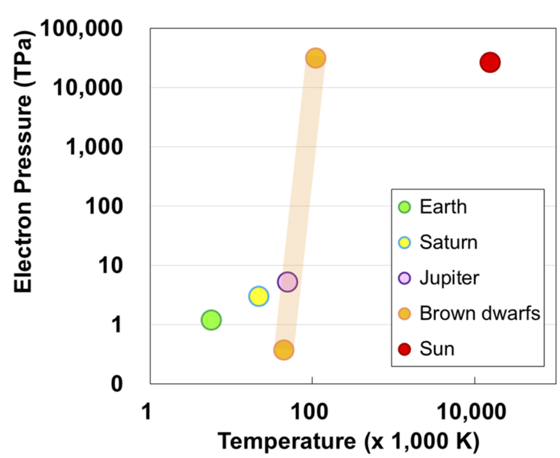

FIG. 3. Electron pressure–temperature conditions in the cores of Earth, Jupiter, Saturn, brown dwarfs, and the Sun.

2.65×10^{11} bar (26.5 PPa).[50] The uncertainty of the data is smaller than the symbol size used in Fig. 3. The pressure–temperature conditions of Earth, Jupiter, and Saturn are near the criterion of the coldest brown dwarf. Jupiter, in particular, could be a brown dwarf, in which deuteron thermonuclear fusion is taking place. It is known that the condition of low temperature and high density lies in low temperature nuclear fusion, such as deuteron fusion in contrast to hydrogen nuclear fusion of high temperature and low density.[51]

Finally, we compare the "double product," ζ, of the pressure (electron density) and temperature for the cores of Earth, Jupiter, and Saturn to those of the coldest brown dwarf with 4.49×10^4 K and 370 GPa. The cores of Earth, Jupiter, and Saturn show ζ = 0.31 [(5130 K/44 900 K) × (1.0 TPa/0.37 TPa)]−0.46 [(6370 K/44 900 K) × (1.2 TPa/0.37 TPa)], 15.65 [(50 000 K/44 900 K) × (5.2 TPa/0.37 TPa)], and 3.97 [(22 000 K/44 900 K) × (3.0 TPa/0.37 TPa)], respectively. Since large ζ plays a decisive role in the deuteron nuclear fusion, Saturn would also be a member of the group of brown dwarfs. Furthermore, there is a possibility that the deuteron nuclear reaction occurs discontinuously in the Earth's core, taking the multiplied effect of D atoms squeezed by tetrahedral Fe atom lattices in the core of Earth into consideration. The details are described in the supplementary material, Sec. 4.

The proposed 3D fusion process requires a solid inner core with Fe–D crystals. The cores of Earth, Jupiter, and Saturn provide a necessary and sufficient condition (temperature over 50 000 K and electron pressure over 0.37 TPa) for deuteron thermonuclear fusion. However, it needs further investigation for 2D and 3D reactions in the cores of celestial bodies because it is not an untestable hypothesis.

V. GEOLOGICAL IMPLICATIONS OF DEUTERON FUSION IN THE INNER CORE OF EARTH

When fusion takes place in a random phase, such as gas or plasma in brown dwarfs, the macroscopic fusion rate is proportional to the square of the deuteron density. However, the microscopic fusion rate in solids is proportional to the deuteron density, because of the drastic decrease of freedom in deuterons in solids.[52] Even if deuteron-nuclear fusion in the cores of Earth, Jupiter, and Saturn does occur, the nuclear reaction does not follow a chain reaction because of the heterogeneous distribution of deuterons squeezed in solid iron and suppression controlled by the deuteron concentration. A collective resonance by three-dimensional electron charge density wave based on the breathing-mode displacement (see the supplementary material, Sec. 4 of Ref. 18) does not occur continuously. Thus, the heat generated by the deuteron thermonuclear fusion would not so much as melt the inner core.

The fusion heat generated after the formation of the inner core transfers to the outer liquid core. A vast sea of electrically conducting molten iron fluid circulates at the outer core, constituting the so-called geodynamo. Evidence from the geologic record shows that the orientation of the dynamo has flipped from north to south and back again hundreds of times during Earth's 4.5-billion-year history. The polarity reversals are explained by the proliferation, growth, and poleward migration of reversed flux patches, which originate at only four broad regions on the core–mantle boundary.[53] Thus, there is a possibility that discontinuous fusion events have an influence on the destruction of the original polarity and generation of the new polarity. Our hypothesis will explain why plate tectonics exist on Earth but not on other terrestrial planets, such as Mercury, Venus, Mars, and moon, provided that the antineutrinos from Jupiter, Saturn, and brown dwarfs are detected.

VI. CONCLUSIONS

We provide a possible model for the origin of thermal energy from interiors of Earth, Saturn, and Jupiter without radioactive wastes, in which heat generation is the result of three-body fusion of deuterons confined within hexagonal FeD core–center crystals. From the viewpoint of deuteron nuclear fusion, we compared the relations of electron degenerate pressure and temperature for the cores of Earth, Jupiter, and Saturn to that of the coldest brown dwarf, using data of published articles. The thermal nuclear fusion would be occurring in the cores of Earth, Saturn, and Jupiter, as well as those of brown dwarfs.

SUPPLEMENTARY MATERIAL

See the supplementary material for degenerate electrons and pion condensates in the Earth's core, the nuclear reaction rate of the deuteron thermonuclear fusion, possible occurrence of excited electrons and neutral pion catalysis, and calculation of the critical temperature and pressure under confinement by high-temperature and physical catalysis effects.

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DATA AVAILABILITY

The data that support the findings of this study are available within the article (and its supplementary material).

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