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Geometry of Curves and Surfaces in Contemporary Chair Design

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Abstract

In the present work, we focus on some of the current trends used in furniture design, from a dual point of view: differential geometry of curves and surfaces, and the existing perspective deriving from the usual techniques of computer-aided design. The contributions of architects such as Alvar Aalto, Mies van der Rohe, Marcel Breuer, Arne Jacobsen and Charles and Ray Eames to contemporary chair design are related to these techniques. Among them, we point out those which are performed by means of spatial geometric transformations of curves and surfaces, with an emphasis on ruled surfaces.

Introduction

Over history, mathematics have provided a magnificent tool for understanding and representing reality and, at the time, for solving problems considered by the human being. In a first stage, they made it possible to carry out commercial transactions, measures of the ground, etc. Later, the advances in mathematics catalyzed advances in other fields.

As a matter of fact, the foundations of differential calculus, its systematization and the work by researchers such as Newton and Leibniz, made possible enormous technical advances from the seventeenth century on. Their progress, along with those of many others, have promoted wide-ranging and increasing progress in the subject of the geometric study of curves and surfaces. Regarding differential geometry, one may also distinguish the advances achieved in the study of curves and surfaces.

Roughly speaking, the knowledge of the implicit equations defining certain varieties allows verification of whether an object belongs to such variety or not, whereas the use of parametrizations associated with the so-called regular varieties allows construction of the object, at least locally. This process relies on the choice of different values for the parameters describing the variety. The subsequent computational development goes beyond a rudimentary construction, allowing a huge amount of values for the parameters to be checked. As a consequence, the model of the object under study becomes quite satisfactory.

The varieties primarily used in practice are, without doubt, those of dimensions one and two: curves and surfaces. In this respect, mathematics in general, and more particularly differential calculus and geometry of curves and surfaces, have enhanced different advances in multiple disciplines: theoretical development of the famous theory of relativity and electromagnetism, mechanical systems and models, both digital and natural, architecture, econometry, etc. For an approach to such ideas, we refer to the works Goriely et al. (2008), Pottmann et al. (2007) and Marriott and Salmon (2000).

More particularly and from an insight close to our objectives, it is certain that mathematics are intricately linked to spatial skills in 2D and 3D design, as stressed by Chun-Hen Ho and Charles Eastman (2006). The role models in this type of designs are the so-called CAD (computer-aided design) software, allowing the representation of 3D models by using different geometrical procedures and techniques (Lee et al. 2016 and the references therein) or their use in the study of architectural situations (Bhooshan 2017). Chair design has a long tradition, deeply influenced by the improvements in modern engineering. The involvement of qualitative and quantitative criteria by means of the assimilation of genetic interactive algorithms presented by Brintrup et al. (2008) is a clear sample of the huge number of possible scenarios. However, despite these advances having such a crucial influence in the office seating market, which is increasingly driven by tougher health and safety legislation, the domestic chair design still relies on the most intuitive formal working criteria.

The study of an emotional response to chair design was studied through the Kansei technique (Hsu et al. 2017). It was concluded that the images producing the most positive response correspond to the clearest designs: linear designs transmitting their formal structure in a neat way, instead of the chairs with a more organic source of inspiration.

In this paper, we will analyse some of the most common techniques in CAD software such as AutoCAD and Rhinoceros from the point of view of the differential geometry of curves and surfaces, in the actual design of furniture nowadays. More precisely, we focus on the geometrical modelling of 3D structures (Section 1), disposed in different geometric trends in chair design (Section 2). Even though several techniques usually appear in a single piece of furniture, we have tried to propose some illustrative and clear specific examples which apply such techniques in a neat way. Among them, we distinguish those making use of classical geometric curves and/or surfaces and spatial transformations on known curves and/or surfaces, with special attention to ruled surfaces (Section 3).

We decided to make an in-depth study of the geometry of chairs, within furniture design, because chairs show a special complexity due to all the requirements they have to fulfil. A chair should be an independent and particular piece of furniture, light but tough and long lasting, featuring a design that is ergonomic and appealing at the same time, and able to be mass-produced (they often appear in sets of several identical pieces), requiring clear and effective production processes, etc.

1 Geometric Modelling of 3D Structures

In this section, we recall the main ideas in the classical study of regular surfaces from the differential geometry point of view. We refer to (Perdigao Do Carmo 2019) as a classical reference in this respect.

As mentioned in the introduction, 3D digital modelling enables the employment of different designs and structures in practice, tailored to certain needs. These are usually explained from physical or/and aesthetic properties governed through some mathematical formulas.

More precisely, CAD software such as AutoCAD and Rhino are widely-used in the study of 3D architectural structures. In this regard, the geometry described by an inverted catenary curve provides the structure of a catenary arc underpinning itself. Other physical and mathematical motivations solve sensory problems, such as the sonority of a room. In other situations, certain aesthetics are governed by adequate equations, as it happens in the use of algebraic surfaces in architecture and their computational representation (Pottmann et al. 2007).

The use of curves and surfaces by means of the tools provided by differential geometry goes back to the development of geometry, in the eighteenth century. Among the huge number of surfaces which have been subject of study, we focus on curves and surfaces in Euclidean space (i.e., varieties of dimension one and two), which can be locally represented by means of regular parametrizations.

A (nonempty) regular surface is locally represented with a parametrization of the form

$$\begin{aligned} X: U &\rightarrow R^3 \\ (u, v) &\rightarrow (x(u,v), y(u,v), z(u,v)) \end{aligned} \tag{1}$$

where U stands for a (nonempty) open set in which R^2, x, y, z are regular functions defined in U, such that the normal vector to the surface $\frac{\partial X}{\partial u}(u_0, v_0) \times \frac{\partial X}{\partial v}(u_0, v_0)$ does not vanish at any point. The implicit function theorem guarantees that under these hypotheses, at least locally, the surface can be implicitly explained through the set

$$\{(x, y, z) \in R^3 : F(x, y, z) = 0\} \tag{2}$$

for some regular scalar function F, defined in a nonempty set of R^3. Indeed, the two local representations of the surface turn out to be equivalent to one another.

As mentioned above, each representation (1) and (2) becomes more suitable depending on the ultimate goal at the time of handling a variety.

2 Geometric Trends in Chair Design

In the present section, we show some classical mathematical techniques for the construction and adjustment of such surfaces, which are of common use in the most widespread CAD software packages. We also indicate some concrete models applying such techniques in furniture design nowadays. The examples we will refer to are shown in Fig. 1.

Fig. 1. a) Chair of Samuel Gragg (1808); b) chair Nº 14 Michael Thonet (1859); c) Red/Blue chair Gerrit Rietveld (1923); d) chair Nº B33 of Marcel Breuer (1927); e) Barcelona chair of Mies Van Der Rohe (1929); f) Paimio chair model Nº 41 Alvar Aalto (1930); g) Butterfly chair Jorge Ferrari, Juan Kurchan and Antonio Bonet (1938); h) Tripode chair Joseph-André Motte (Fiell and Fiell 2017)

Given the variety of users and functions that must be fulfilled, there are no ideal forms in chair design. The success of a particular chair relies on the quality and variety of connections it establishes with its user. In 1808 Samuel Gragg put forward the design of a chair ahead of its time (Fig. 1a). The curves sculpting the bent wood that this architect and designer set to his chair were neither derived from an anatomical shape nor a dynamical form: rather, they are the results of formal decisions searching for their contemporary references in science and technology. Another outstanding milestone were the chairs designed in bent wood by Michael Thonet in 1859 (Fig. 1b). There were several models which revolutionary formal abstraction was conveyed through three-dimensional curves, pictured as intersections between cylinders drawing quadratic curves (two-cylinder curves). The Nº 14 chair remains one of the most successful industrial designed products of all time (Fiell and Fiell 2017).

Especially interesting was the study of chair design carried out by Erich Dieckmann (Fig. 2). A catalogue of a wide range of possibilities whose curves were intended to be manufactured either in wood or steel, it is an example of a systematic formal research based on the balance of the lines. In the same rigorous line of work, we find the layouts of the architect Marcel Breuer (Fig. 1d). His chairs made of stainless steel tubes are among the most famous of the twentieth century.

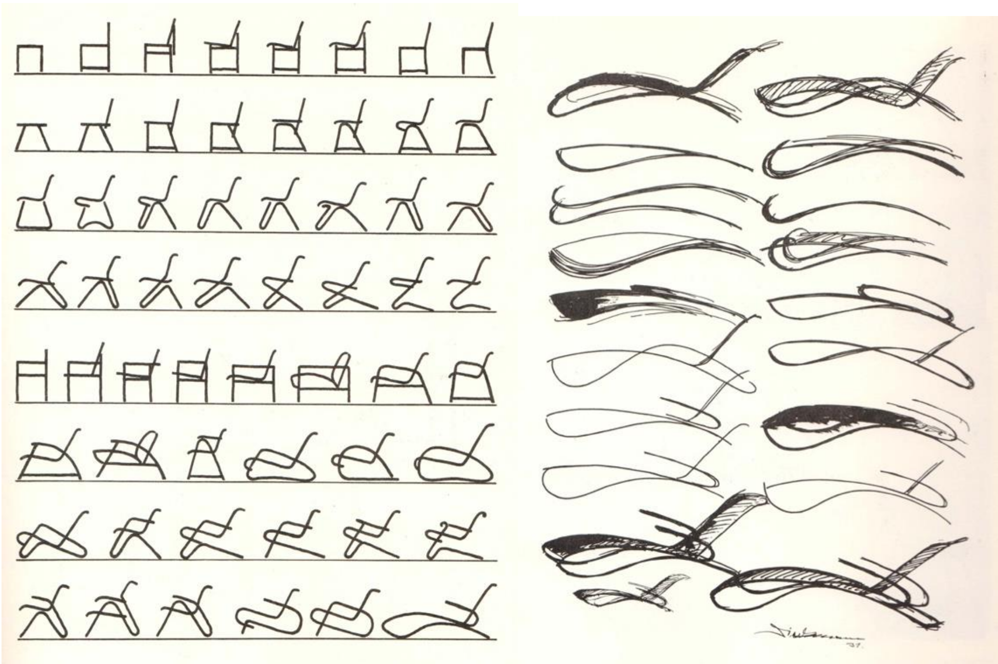

Fig. 2. Drawing in a study made by E. Dieckmann (1931: 41, 79). Images: WikiCommons

2.1 Classical geometric curves and surfaces

Some of the designs of furniture rely recurrently on the shape of classical algebraic curves, giving rise to certain special properties in the element. This is the case of the 'Wooden Chair', produced by Cappellini (https://www.cappellini.it/en/products/sofas-and-armchairs/wooden-chair). This chair is based on a design for a handicraft exhibition in New South Wales, in 1988, by Marc Newson. Subsequently, it was redefined for production by the furniture brand Cappellini, in 1992. This wooden armchair is manufactured in curved natural beech wood. Its design is inspired by the construction of a three-dimensional structure emanating from classical curves close to those in the family of so-called ‘conchoids of de Sluze’, whose implicit equation given by

$$(x-1)(x^2+y^2)=ax^2,$$

for $a<-1$. Such a curve can be parametrized by

$$(x,y)=(\cos(t)(\sec(t)+\mathrm{a}\cos(t)),\sin(t)(\sec(t)+\mathrm{a}\cos(t))).$$

In Fig. 2 we show axonometric, plan and elevation of this model.

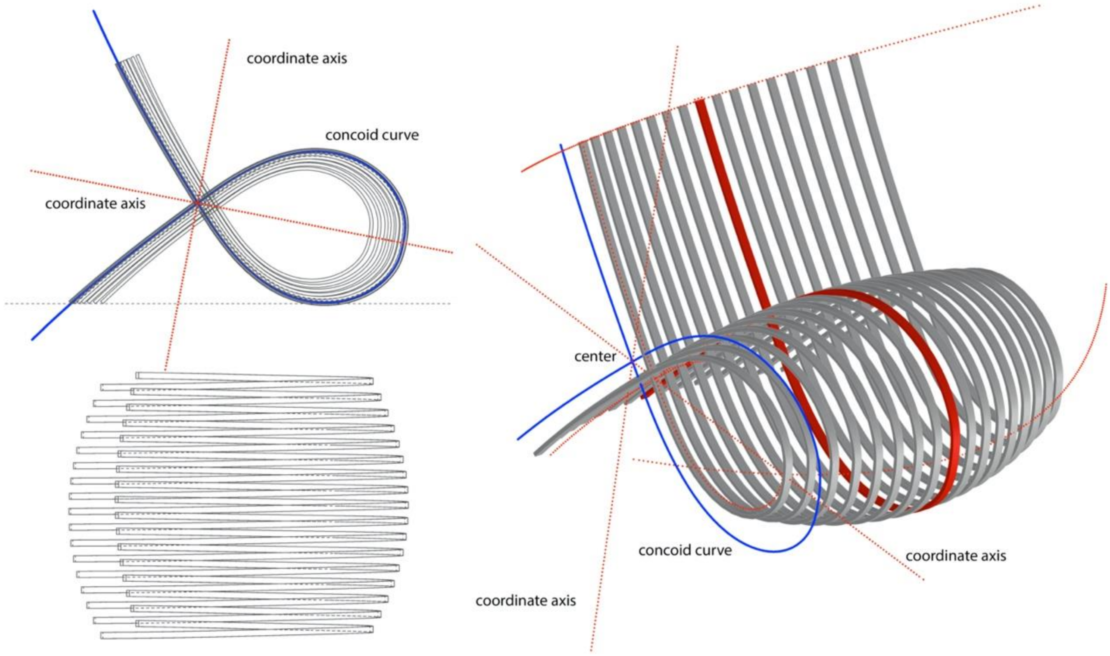

Fig. 1. Plan and elevation views of the geometrical analysis of the Wooden Chair produced by Cappellini

Other models make use of mathematical curves of a classical nature: from conics to other curves featuring certain particular physical or aesthetic properties, such as the catenary curve or the cycloid. In 1929, the architect Mies van der Rohe designed a chair for the Barcelona Pavilion whose structure approximated that of a cycloid (Fig. 1e). In the 1930s, the Finnish architect Alvar Aalto developed several designs of bent laminated wood chairs, among which the chair Paimio chair stands out; this would become part of the Nordic design heritage (Fig. 1f).

It is also usual to find furniture whose design is based on some classical surface. The simplest surfaces, from the mathematical point of view, are planes. Regarding these surfaces, we could highlight examples such as the Red/Blue chair, by Gerrit Rietveld, from 1923 (Fig. 1c). This design is associated to De Stijl Movement, which relies on abstraction to underline the internal structure of its material identity, isolating each element through their colour and position, understanding the whole as a discontinuous group.

Broadly speaking, and increasing the complexity of the variety involved, we could highlight the associated surfaces to zeroes of (nonlinear) polynomials in three independent variables with higher degree. In such a way, quadrics arise (degree two), etc.

The quadrics themselves offer enriching aesthetics and inherent structure, as it may be a double ruled-based surface. Other surfaces do not follow the previous pattern necessarily, and the function F in (2) turns out to have a more general form. This is the case of a bentwood hammock, by an anonymous designer, which we proceed to analyse.

The most common construction of a torus, according to its topological definition, is that of a circle of radius $r > 0$, with center varying among the points of another circle of radius $R > r$, in such a way that the planes containing both circles are always orthogonal and the first one passes through the center of the second circle. More precisely, assume that the fixed circle is located at the plane of the floor. It can be parametrized by $(\mathrm{R}\cos(t), R\sin(t)\,, 0)$, for $t\epsilon R$. The centre of the circle of radius r moves along the points of the first circle in the parameter. For every value of t, the second circle is contained in the plane $\cos(t)\,y - \sin(t)\,x = 0$, and can be parametrized by a second parameter s. The location of any point of the torus is determined in these two respects. Consequently, a parametrization of the torus is

$$(\mathrm{x}, \mathrm{y}, \mathrm{z}) = (\mathrm{R}\cos(\mathrm{t})\,, \mathrm{R}\sin(\mathrm{t})\,, 0) + \big(\mathrm{r}\cos(\mathrm{s})\cos(\mathrm{t})\,, \mathrm{r}\cos(\mathrm{s})\sin(\mathrm{t})\,, \mathrm{r}\sin(\mathrm{s})\big).$$

The implicit equation of the torus can be obtained by solving the previous parametrization in the parameters, and it is given by

$$(R - \sqrt{x^2 + y^2}\,)^2 + z^2 - r^2 = 0.$$

Figure 4 shows different perspectives of this piece.

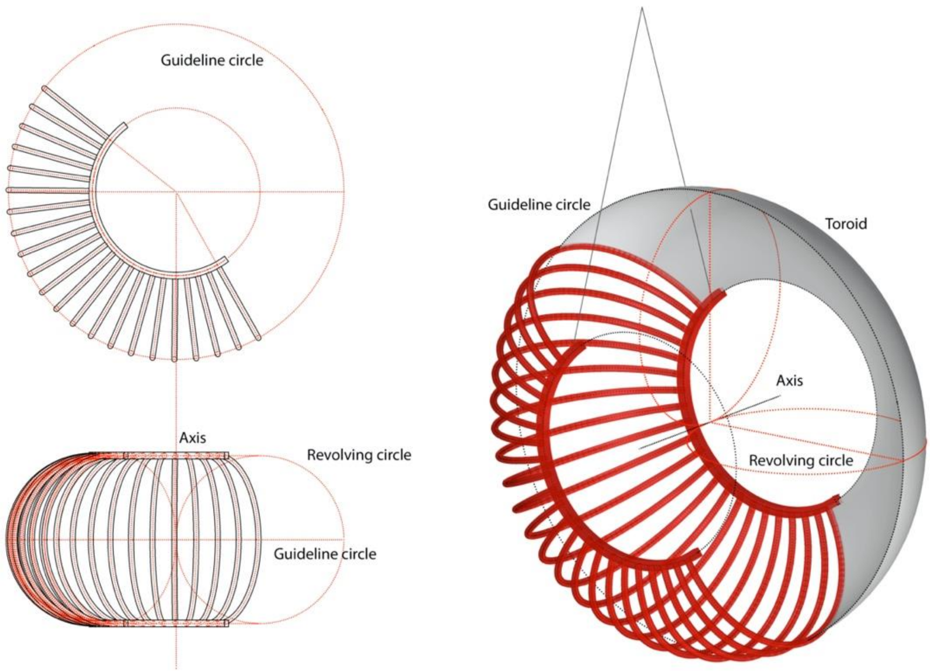

Fig. 2: Plan and elevation views of a toroidal model

We can also point out some other examples of furniture which make use of quadric surfaces, such as the ‘Arc Table’ by Foster + Partners, shown in 2009, in the London design festival (see https://www.fosterandpartners.com/projects/arc-molteni-c-table/). It was inspired by a hyperbolic paraboloid, a recurrent design for chairs and tables that has a solid doubly-ruled structure is the one-sheet hyperboloid. This configuration inspired the ‘Nest Chair’, designed by Markus Johansson in 2011. Figure 5 shows an approximation to this chair.

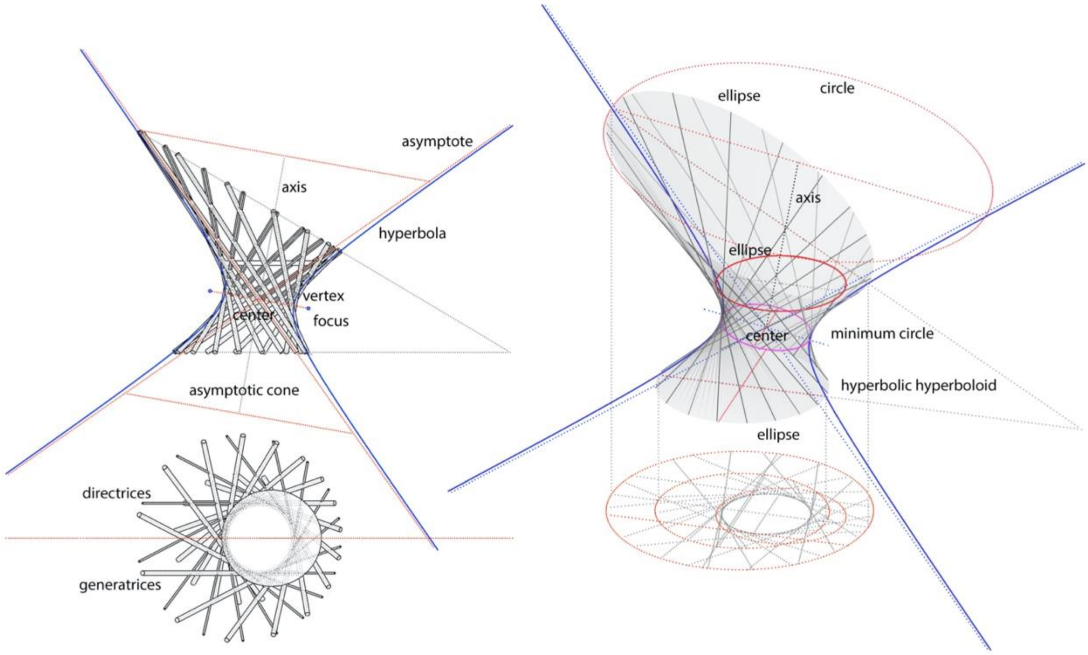

Fig. 5. Plan and elevation views of an approximation to the model Nest Chair (left) and scheme (right)

The geometric construction of a one-sheet hyperboloid leans on the revolution of a line. For the sake of clarity of the geometric construction, we give details for a family of configurations of such surface. Let us consider the line r, joining the point $P = (1,0,-1)$ and $Q = (\cos(s),0,-1)$. The surface of revolution generated by the motion of r around OZ axis is determined by the parametrization

$$x(u,v) = \cos(v)(1 + u\cos(s) - u) - \sin(v)\, u\, sin(s)$$

$$y(u,v) = \sin(v)\,(1 + u\cos(s) - u) + \cos(v)\, u\, sin(\mathrm{s})$$

$$z(u,v) = 2u - 1$$

which describes the hyperboloid.

An important category within the surfaces in chair design are those that can be called anatomic or organic. Some of these models were originally derived from camp stools, used mainly in military field, and had structures formed by folding wooden frames covered by a seat of fabric. The ‘Butterfly Chair’, signed by Ferrari, Kurchan and Bonet in 1938 (Fig. 1g), is an elegant tubular steel frame with a leather sling seat. This chair is a hung shape that seems to be a close approximation to that of an elliptic paraboloid. A similar concept can be found the chairs of Joseph André Motte, made in the late 1940s (Fig. 1h). The ‘Tripod Chair’ profits the versatility of the woven rattan to become a concave surface, in the shape of a cradle or a nest.

In 1940 The Museum of Modern Art in New York organized a competition throughout the United States and the twenty Latin American republics in order to select designers to develop the furniture of the modern American way of life. The ‘Organic Chair’ project presented by Saarinen and Eames received several awards and achieved a great visibility. A manufacturing method never previously applied to furniture was employed to make a light structural shell consisting of layer of plastic glue and wood veneer moulded in three-dimensional forms (Noyes 1941).

In the early 1950s Harry Bertoia created his bent and welded steel rod chairs. The Diamond chair (see Fig. 6) was built out of a mesh of curves similar to catenaries or Gaussian bell curvess, such as those of graph given by

$$(x, y, \exp(-x^2 - y^2)).$$

The outcome is a structure that tries to imprint the movement of the human body on the surface of an orthogonal pattern.

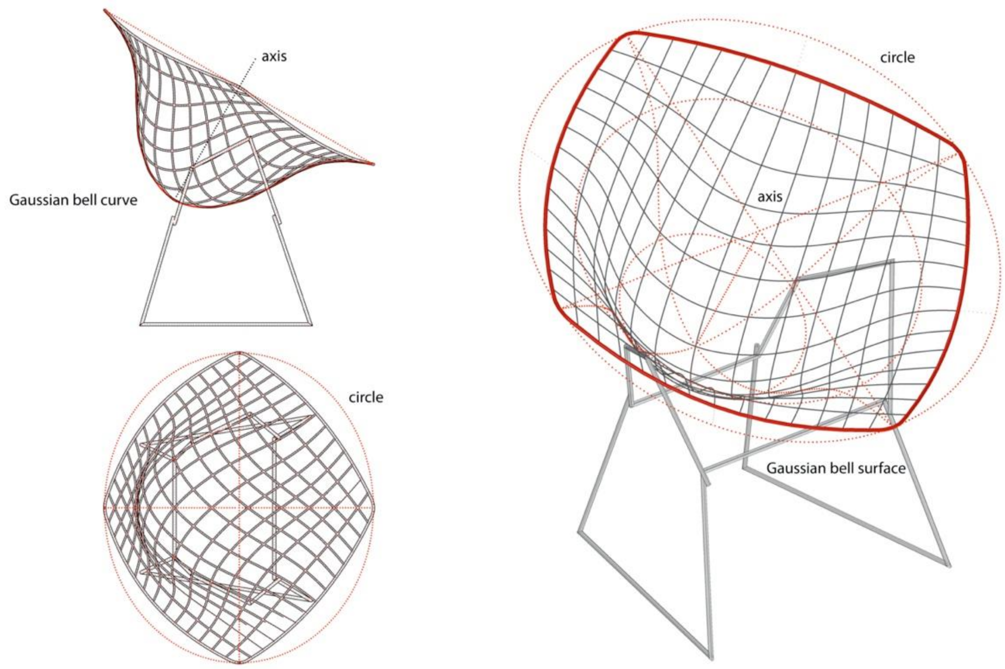

Fig. 6. Plan and elevation views of an approximation to the model Diamond Chair (left) and scheme (right)

2.2 Spatial transformations of curves and surfaces

Among the techniques used in furniture design, one can highlight the use of different spatial transformations of base curves. The specific design detailed in Section 2.1 can also be framed here.

Starting from a curve, the design of furniture is modified through spatial transformations, which can be related transformations such as translations, scale, rotations, reflections or symmetries. This is the case of the ‘Cycle Chair’ created by Thai artist and designer Saran Youkongdee, and presented at the Thailand International Furniture Fair, 2008.

Roughly speaking, the model of this chair could be reduced to the cuts of the elements of a pencil of planes with the elements of a second family of right circular cylinders whose directrices are concentric circles. Indeed, the chair can be approximated by the following parametrizations

$$(r\cos(u), r\sin(u), \mathrm{c}\, r\sin(u)),$$

where $r \in \{\mathrm{r}_1 < \cdots < \mathrm{r}_s\}$ are the radius of the cylinders, $\mathrm{c} \in \{0, \mathrm{c}_1, \ldots, \mathrm{c}_{s-1}\}$ determine the increasing slopes of the ellipses defining the chair; together with the circle at the plan floor. The resource used in the design of this chair can also be considered to some extent as the consequence of the continuous modelling of the offset curves determining the spatial curve.

A model of this design can be found in Fig. 7.

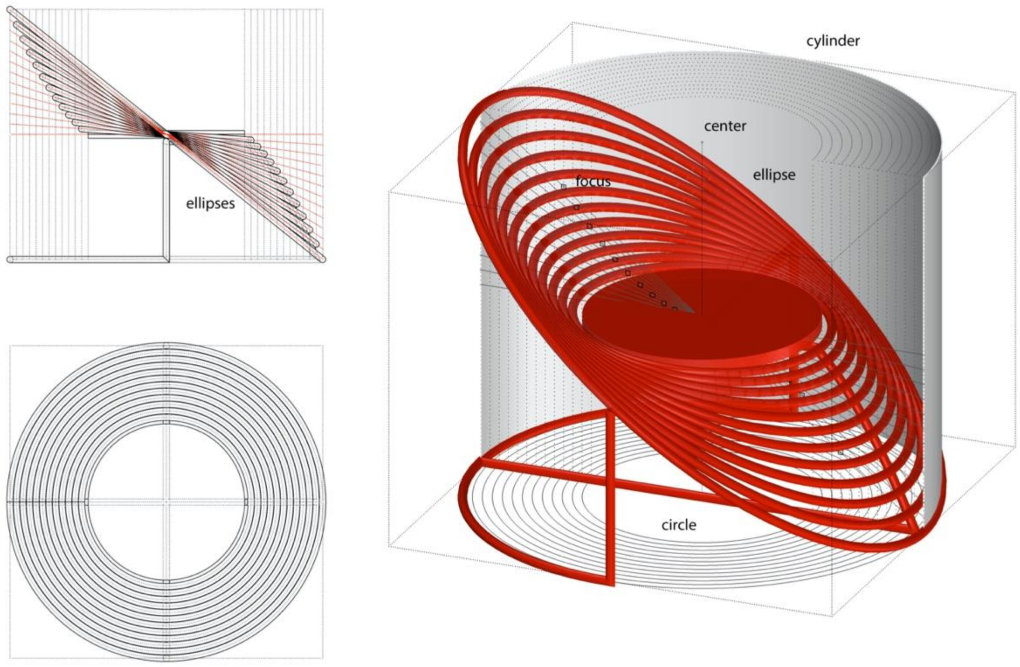

Fig. 7. Plan and elevation views of the model Cycle Chair, by Saran Youkongdee

The use of classical curves has been analysed in Section 2.2. Their arrangement in the space makes it possible to obtain design solutions such as the ‘Hamaca Trinity’. This is a triple hammock designed by Gilbert Tourville, which received an award for innovation in outdoor furniture at the 2013 Las Vegas Hospitality Design Conference and Exhibition. This layout makes an intelligent use of circumferences placed as a stable and balanced structure. Figure 8 displays several views, and also an analysis of the model.

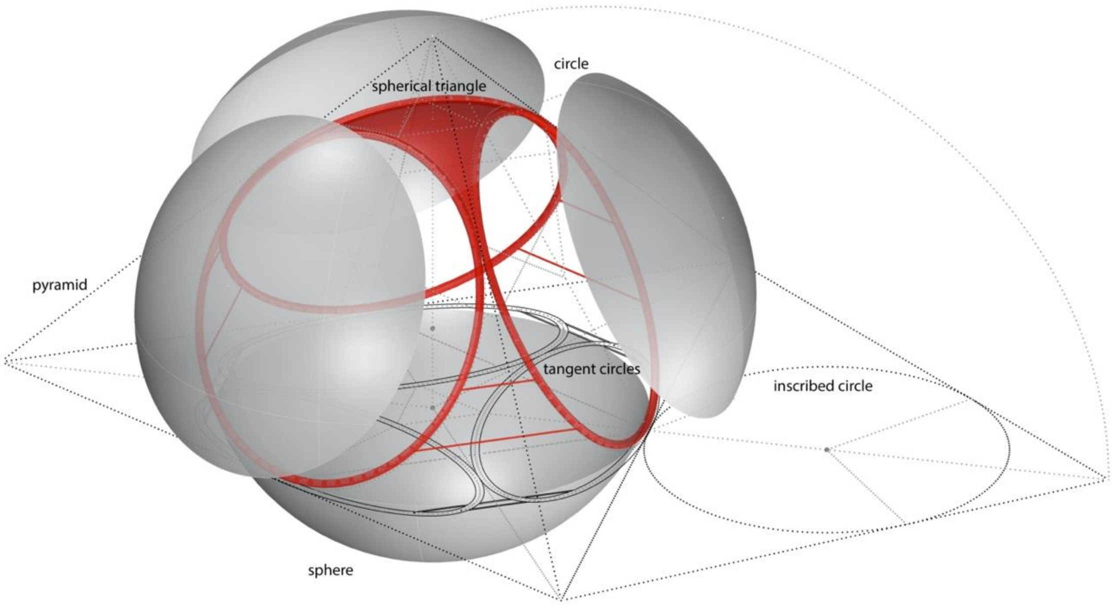

Fig. 8. ~~Plan and elevation views of the model Hamaca Trinity (left) and D~~Ddesign analysis of the model Hamaca Trinity~~(right)~~

A reference also needs to be made to the role of spirals. The special curvature and torsion of these curves make spirals a recurrent tool in artistic works. In the framework of furniture, we refer to the 'Spiral Chair', by Fredrik Mattson, a chair created in 2008 for the Danish firm PP Möber. It consists of an eight-meter-long bentwood spiral (see http://www.fredrikmattson.se/projects/spiral).

Let us consider the line $x = y = 0$ as the axis of revolution of the conical spiral. Then, the parametrization of the Spiral Chair could be approximated in some scale by

$$(au\cos(u)\,, au\sin(u), u)\,,$$

in such a way that the curve is contained in the cone $x^2 + y^2 = (az)^2$, for some a. Figure 9 shows plan and elevation views generated on this model.

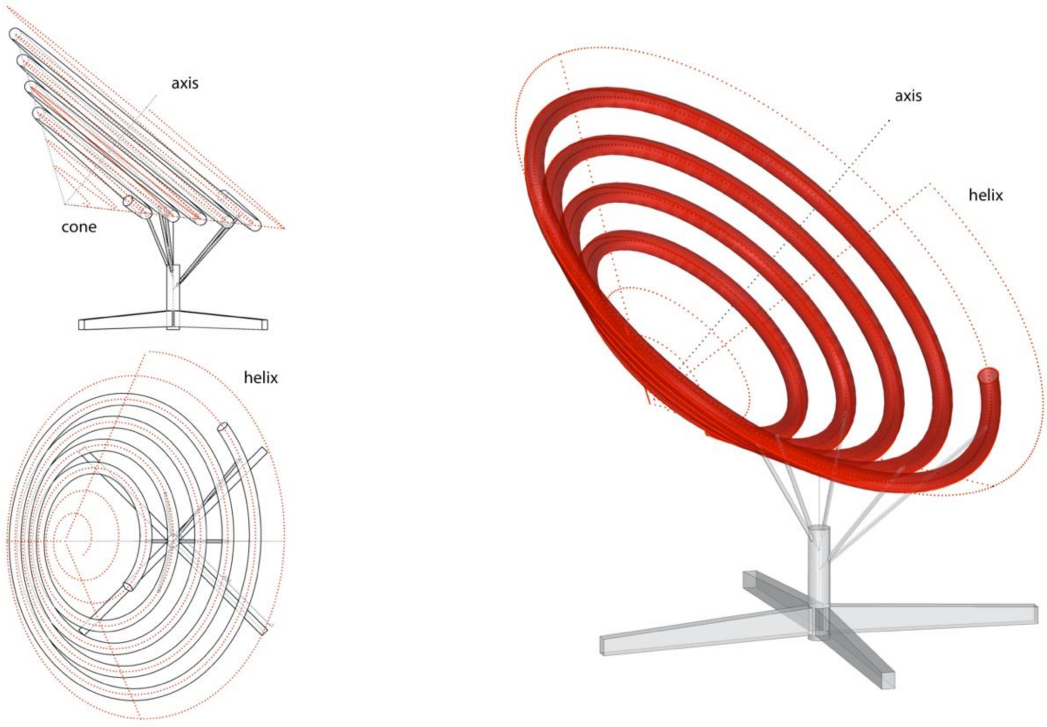

Fig. 9. Plan and elevation views of the model Spiral Chair, by Fredrik Mattson

3 Ruled Surfaces

A natural possible step further from the models defined by transformations on curves towards those described by a regular surface are ruled surfaces. The specificity of such surfaces lies in the fact that they can be built from a family of lines, each of them leaning on a point on a certain curve (base curve or directrix), following directions which depend on such a point. In this way, one constructs surfaces such as cones, cylinders, tangential surfaces, etc. Any parametrization of one such surfaces is given by

$$X(u,v) = \alpha(u) + e(u)v, \quad (u,v)\epsilon\, I \times R, \qquad (3)$$

where $\alpha: I \subseteq \mathrm{R} \rightarrow R^3$ stands for a parametrization of a spatial curve, and where for every $u\epsilon U$, $e(u)\epsilon R^3$ is a non-zero vector. For every value $u\epsilon U$, the previous parametrization (3) is simply the parametrization of a line passing through the point $\alpha(u)$ and direction given by the vector $e(u)$.

Other constructions allow adjustment of the rule-based structure in order to interpolate other varieties. For example, given two spatial curves with associated parametrizations $\alpha, \beta: \mathrm{I} \rightarrow R^3$ the surface which determines

$$X(u,v) = \alpha(u)v + \beta(u)(1 - v), \quad (u,v)\epsilon\, I \times [0,1],$$

is made up of segments joining both curves at their extremes. An example of such a structure is that of the so-called 'Acapulco Chair', designed by the Mexican designer Cecilia León. We refer to Fig. 10 for the plan and elevation views of this model.

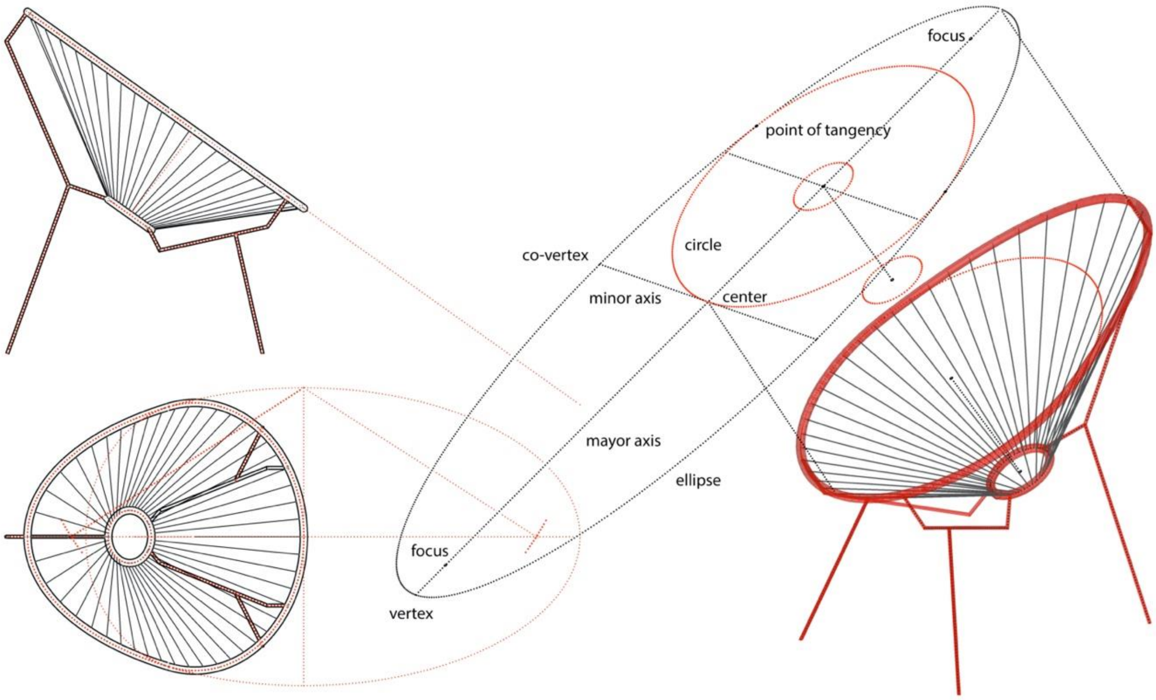

Fig. 10. Plan and elevation views of the Acapulco Chair

Versatility and robustness are two features linked to the physical implementation offered by ruled surfaces, which have been a source of inspiration to a huge number of designs. The 'Parabola Chair' (Fig. 11), designed by Carlo Aiello for the 2013 ICFF (see https://archello.com/product/parabola-chair-2), or the 2011 'Rising Chair', by Robert Van Embricqs (https://www.robertvanembricqs.com/rising-chair) are also really outstanding designs within the ruled surfaces.

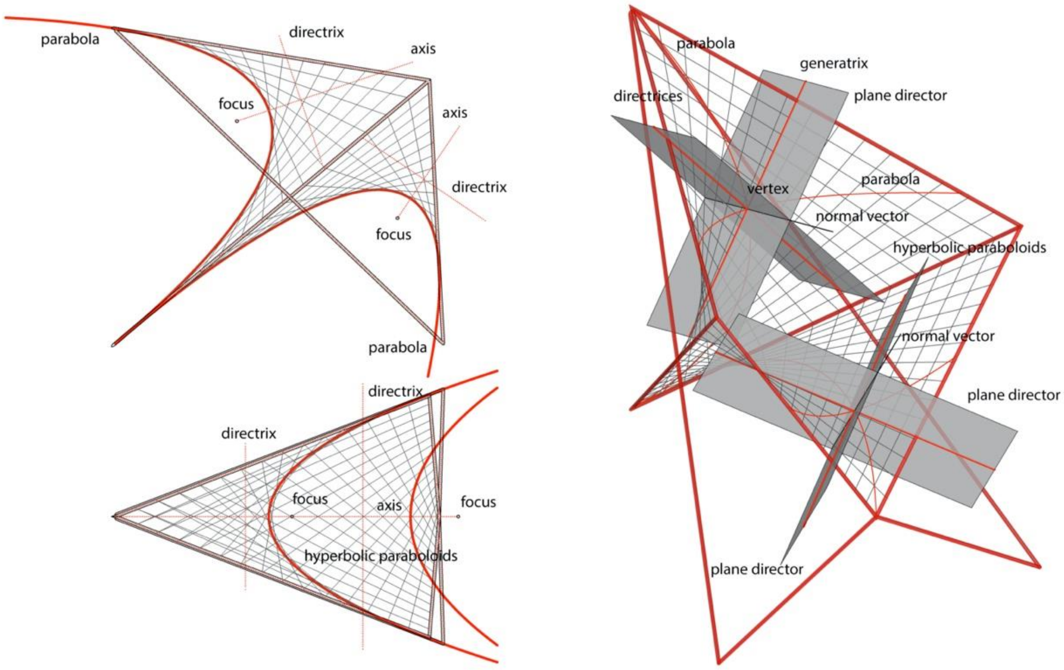

Fig. 11. Plan and elevation views of the Parabola Chair

4 Conclusions

Chair design and architectural design have evolved hand-in-hand during the contemporary age. Traditionally linked to craftsmanship, the arrival of industrialization in the nineteenth century transformed the forms of the most innovative projects for chairs and for architecture. New technologies that allowed production in large quantities and manufacturing processes that required the simplification of shapes and structures were implemented. First bentwood and then plywood prevailed as lighter and more resilient technologies. The same is true of stainless-steel tubular designs, featuring innovative uses of a tenacious material that revolutionized the furniture industry. Many of the most famous chair layouts are due to the work of architects, and the presence of geometry in those designs seems to be part of their success.

In the twentieth century and still today, abstract curves and surfaces are the main modern design resource, both in architecture and furniture. To be sure, there is no single ideal form for a chair. The best design for a chair is not the one that only best fits a particular use or situation, but the one that connects with its users by transmitting a clear formal idea. In that way geometry provides a wide range of possibilities. The use of curves conveys elegance and pragmatism, while extruded, revolved or ruled surfaces give chairs an air of comfort and anatomic sense without even imitating human shapes.

Acknowledgements

All images are by the author unless indicated.

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