# Super-Resolution for Unregistered Satellite Images 

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## Summary

Image Super-Resolution refers to a class of image processing techniques used to produce a high-resolution image from a sequence of low-resolution images shifted with respect to each other. In many imaging applications, the imaging sensors used have poor resolution and the use of very high-resolution CCD cameras is often not a viable option. In such cases, we can resort to super-resolution techniques to improve the quality of the images. Even with high precision optics, super-resolution presents an inexpensive solution to achieve the desirable resolution. Medical imaging, surveillance and target recognition are some examples of the possible applications of Super-Resolution.

Super-Resolution is a computationally intensive process and must be robust with respect to different image degradation factors whether they are related to the sensor or to external perturbations (sensor noise, changing conditions, unknown camera characteristics, etc). The algorithm must be driven only by the data provided with no knowledge about the scene or the sensor.

The goal of this thesis is to propose an efficient and less consuming algorithm, in terms of computing cost, to achieve image Super-Resolution suitable for translated satellite images. A new approach for super-resolution is proposed and the various components of this process are analyzed and resolved in this work. The reconstruction in this method is performed using an orthogonal set of Walsh functions. The shift between the input images is estimated using a new measure based upon the sharpness property of edges in the reconstructed image. The registration and the interpolation steps are performed simultaneously in this approach. This point constitutes one of the key points of this method. Furthermore, we propose a new idea to perform edge detection based on the effect of mis-registration in this method. This may improve the performance of the algorithm by targeting only these regions which may contain new information. The edge detection is the consequence of the registration step. We also consider ways to reduce noise in the reconstruction based on the least square fitting of segmentation boundaries. New faster techniques are presented for these different components of Super-Resolution.

The effectiveness of the proposed method is demonstrated for each stage of the SuperResolution process.

Key words: resolution, super-resolution, image registration, geometric corrections, image restoration, Walsh functions, image enhancement, aliasing, Fourier transform, spatial resolution, edge detection and image resolution.

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## Glossary of Terms

CCD Charge-couple Device
FOV Field of view
LEO Low-Earth orbit
OBC Onboard computer
SSTL Surrey Satellite Technology Limited
SR Super-Resolution
HR High resolution
LR Low resolution
DMC Disaster Monitoring Constellation
PSF Point Spread Function
ROI Region of Interest
FT Fourier Transform
CFT Continuous Fourier Transform
DFT Discrete Fourier Transform
FFT Fast Fourier Transform
LSE Least Square Estimation
IFOV Instantaneous Field of View
GCP Ground Control Point
MSE Mean Square Error
SNR Signal to Noise Ratio
PSNR Peak Signal to Noise Ratio
Alsat1 Algerian Satellite 1
GSD Ground Sampling Distance
SAR Synthetic Aperture Radar

## Chapter 1

## 1 Introduction

### 1.1 Introduction

In many applications such as remote sensing, medical imaging and surveillance, images with high resolution are often required. Currently, Charge-Couple-Devices (CCD) are used to capture these high resolution images digitally. However, this technology cannot keep up with the higher demand for more and more resolution. In addition, the presence of shot noise, which affects any imaging system, limits the resolution of these CCDs. This limit arises from the fact that while reducing the area of each CCD increases the resolution, this also reduces the signal strength and therefore less energy is received, while the noise strength remains roughly the same. The current limit on the size of each CCD is about $50 \mu \mathrm{~m}^{2}$. With a smaller CCD area, the Signal-to-Noise ratio (SNR) is too low for images to be useful. Aside from these constraints, the cost is another parameter which needs to be taken into consideration. It is more cost-efficient to launch a cheaper camera with a lower resolution into orbit if high-resolution images can be obtained on the ground using image post-processing techniques.

Recently a great deal of research has been done on Super-Resolution techniques. These techniques increase the resolution of images by combining multiple images to improve the spatial frequency bandwidth [1,2]. There is a rich variety of techniques which seek to reconstruct a high-resolution image from a sequence of low-resolution images captured over the same scene and displaced with respect to each other by a small amount. If the images are captured over some interval of time then we are using temporal resolution to enhance the spatial resolution. Naturally, there is always a large demand for high quality images: Medical imaging, surveillance and identification systems are just some potential applications of Super-Resolution, which require highly detailed images. Furthermore, the application of Super-Resolution for video has also been investigated [3,4,5,6,7], as it has for text recognition [8], astronomical data fusion [9] and applications to mosaics [10].

Super-resolution using Synthetic Aperture Radar (SAR) images have demonstrated important information extraction from applying the techniques to this type of data [11].

Super-Resolution is a process which increases spatial resolution by extracting nonredundant information among the input images. Each low-resolution and aliased image provides a different view of the same scene. Non-redundant information about the scene can be obtained by imaging the scene under different lighting conditions, using different sensors or from different views. We can compare the problem of Super-Resolution to that of data fusion. We require some relative motion from image to image, which will be responsible for providing different information. If the images are shifted by an integer number of pixels, then each image will contain the same information about the scene at different locations, and thus no new information can be extracted. However, if the images have a sub-pixel offset and aliasing is present, then any one image cannot be derived from the rest. New information is therefore contained in each low-resolution image, which we shall exploit to construct the high-resolution image. Examples of different solutions to this problem can be found in $[\mathbf{1 2}, \mathbf{1 3}, \mathbf{1 4}]$.

In the case of satellite imaging, the offset between images is generated by time delayed image capture, exploiting the motion of the satellite in orbit about the Earth (see figure 1.1). In other applications, the camera is fixed and the object is moving within the scene. If the scene motion is known or can be estimated within sub-pixel accuracy, SuperResolution is possible.


Figure 1.1: Example of shifted images capture in satellite imaging

Several people have already proposed a variety of methods to accomplish SuperResolution. Some of the seminal methods will be presented in more detail in this chapter starting from the earliest approach to Super-Resolution proposed by Tsai and Huang in 1984.

Super-Resolution algorithms must take into consideration the computational cost and must be robust with respect to the different sources of image degradation, sensor noise, atmospheric effects, etc. Furthermore, these algorithms must be driven only by the data provided by the sensor as in general we have no information about the noise statistics or the camera.

The goal of this thesis is a complete Super-Resolution algorithm, which can be applied to translated satellite images as well as other types of images from other applications. All the aspects related to this problem are investigated in this thesis. The results obtained demonstrate the performance of our proposed algorithm. A large number of standard images representing different textures and features are used in our experimentations through the whole thesis along with some examples of real images acquired by Alsat1 (Algerian Satellite 1). Most of the standard images are known by image processing community

### 1.2 Problem definition

We describe in this part a general model for Super-Resolution which has been used by several people before and will serve as the basic model for this problem. This model was proposed by [15]. Some assumptions are made here that all the images have consistent lighting conditions, negligible optical distortions and that the image capture is under orthographic projection. More details on this projection and our justification to use can be found in Chapter 4.

The problem of Super-Resolution can be stated as follows:
Given a set of $R$ degraded low-resolution frames $\left\{f_{k}\right\}_{k=1, \ldots R}$, each of $M x N$ pixels, all incorporating a single scene seen from different viewpoints, construct a single high resolution image of dimensions $r M x r N$ for some integer $r>1$.

Figure 1.2 illustrates schematically the problem of Super-Resolution. One of the input images will be chosen as a reference image. The mapping between the remaining images and the reference image may include translation, rotation and magnification. The objective of Super-Resolution is to interpolate the input data at the high-resolution grid after accurately defining the mapping. This is a registration problem. Only translation motion will be considered throughout this thesis. This assumption seems to be very limiting, but in our case it is valid when using the Disaster Monitoring Constellation (DMC) of satellites which employs a pushbroom imaging technology. M. Elad in [16] considered also the same problem by dealing only with pure translation but proposed an improvement to an existing approach for Super-Resolution.


Figure 1.2: A general description for the Super-Resolution process

The relation between low-resolution and high-resolution pixels is shown schematically in figure 1.3.


Figure 1.3: Low-resolution pixels on high-resolution grid

The CCD imaging devices consist generally of arrays of light detectors. The corresponding pixel intensity value depends upon the amount of light entering each detector. The density of the detector array determines the resolution of the image. Figure 1.4 shows a simple model of a CCD camera.


Object


CCD arrays

Figure 1.4: Simple CCD camera model

The optical devices produce a blurred view of an object; the CCD array turns this analogue view into a discrete 2D image. On the other hand, various sources of noise will affect the transformed image. Generally, this noise is additive and can be due to many sources like the quantization errors, sensor errors and others external parameters.


Figure 1.5: Relation between low-resolution and high-resolution pixels

Figure 1.5 above, illustrates the relationship between low-resolution and high-resolution pixels. Each low-resolution image is modelled as a degraded and under-sampled version of the high-resolution image.

Following [15], this relationship can be given by:

$$
\begin{equation*}
f_{k}=D C_{k} E_{k} x+n_{k}, \quad 1 \leq k \leq R \tag{1.1}
\end{equation*}
$$

where D is the under-sampling operator, $C_{k}$ 's are the blurring operators; $E_{k}$ 's are the affine transformations that map the high-resolution grid coordinate system to the lowresolution one. $x$ is the unknown image and $n_{k}$ 's are the additive noise vectors.

The low-resolution images $f_{k}$ and the decimation operator are known. Also, the blurring operator and the camera characteristics may be known. However, the relative motion between the images is not known and has to be estimated from the data provided by the low-resolution images. Finally, with a variety of independent sources of error, the central
limit theorem allows us to assume a Gaussian distribution for the additive noise $n_{k}$ with unknown variances. All these problems will be analyzed in more details in the subsequent chapters with some illustrated examples.

### 1.3 Super-Resolution Restoration

The Super-Resolution solution is a remedy for aliasing in image processing. This problem occurs when the sampling rate is too low and results in distortion in the details present in an image, especially at the edges, which contain the important information in an image. In addition to that, there is a loss of high-frequency component due to the low-resolution point spread function (PSF) and the optical blurring due to the motion of the camera or the focus problems related to the camera. One way to increase the sampling rate is to increase the number of photo-detectors by decreasing therefore their size. But there is a limit to which this can be done beyond where the shot noise degrades the image quality. Also, most of the high-resolution sensors are very expensive. Therefore, we propose to employ Super-Resolution techniques to enhance the images [17,18]. In order to increase the sampling rate, more samples of the images are needed, which is possible by the capture of multiple images of the scene through sub-pixel motion of the camera. Such images are already available in satellite imaging. For the DMC satellites $[\mathbf{1 9}, \mathbf{2 0}, \mathbf{2 1}]$ for example, these satellites will provide more images at a daily period of the same area (see figure 1.6).

With this availability of images, Super-Resolution is largely known as a technique whereby multi-frame motion is used to overcome the limitations of low-resolution cameras.


Figure 1.6: Disaster Monitoring Constellation (SSTL) An example of scenario for multi-temporal images acquisition

### 1.4 Outline of the Thesis

The main goal of this thesis is the development of a complete Super-Resolution algorithm suitable for translated satellite images from a pushbroom imager. The low-resolution images are provided and the output of the algorithm is an image at higher resolution than the input images (doubled in both directions). No knowledge about the scene is assumed in this process and the computational cost must be taken into account for real applications. The translation parameters are also assumed unknown in this process.

Chapter 1 provides a general introduction to the Super-Resolution problem. The definition of the problem is presented with more details concluding with a general model of the problem as well as the principle of this new technique of image enhancement in image processing. In Chapter 2, we present with details the method for Super-Resolution proposed by Tsai and Huang in 1984 for use with Landsat 1 satellite images. This method was implemented and some results are presented in that chapter. The new approach we propose for Super-Resolution using the Walsh functions is presented in Chapter 3 for the 1D case. Chapter 4 handles the problem of registration for satellite images and introduces our approach to register translated images and an example for reconstruction using
synthetic and real data is described in Chapter 5 with detailed discussions about all the problems associated with trying to combine images. Then, in Chapter 6 we develop a new technique to perform image segmentation based upon the new method and Chapter 7 introduces the generalized model to Super-Resolution for 2D cases. Finally, conclusions, remarks and future directions of this research are given in Chapter 8.

### 1.5 Contributions of the Thesis

The main contribution of this thesis is a complete and fast image Super-Resolution algorithm suitable for implementation in satellites using the pushbroom principle. The robustness of this approach is proven according to the results obtained and can be improved in the future exploring more the noise removal and the deblurring steps. For each stage of this algorithm, we have developed a new technique to handle a sub-problem related to the Super-Resolution process.

- In Chapter 2, we present a complete and detailed implementation of the first method for super-resolution proposed by Tsai and Huang along with an example of reconstruction using this frequency domain approach. A new motion estimation method using new measures is proposed in this chapter. Simulation results for the motion estimation and the reconstruction step are presented in more detail.
- In Chapter 3, we describe a fast interpolation method using the Walsh functions. The mathematics behind this method are detailed leading to the derivation of a new mathematical expression for the missing information which exists in an image of resolution $2^{\mathrm{M}+1}$ but absent in a second image with resolution $2^{\mathrm{M}}$. We shall show then, how to retrieve this missing information from another image of resolution $2^{\mathrm{M}}$ shifted from the first image by an arbitrary amount.
- In Chapter 4, we describe the image registration problem. We show how the imaging geometry used affects this operation and particularly in our case for LEO orbits along with the geometric errors present in satellite data. A new measure based on the information obtained by the degradation of the edges during a mis-registration case in the proposed algorithm is proposed. We will use this measure to perform accurate registration
between the input low-resolution images. We present Super-Resolution results from applying a combination of this interpolation algorithm with the registration step simultaneously using some test images.
- In Chapter 6, a new and fast approach for image segmentation derived from the new method for Super-Resolution is proposed. Some examples are presented to segment input images. We shall show how is it possible to use this information to focus the Super-Resolution process in some Regions of Interest (ROI) in the image rather than processing the whole image. We note here that the registration, the image segmentation and the interpolation steps are performed simultaneously in this new algorithm. This constitutes another novelty of this thesis.


### 1.6 The Super-Resolution process

The demand for very high definition images is needed more and more nowadays and sometimes cannot be accommodated affordably by the current CCD technology. On the other side of the problem, and during the process of imaging other factors including the motion between the Earth and the platform, atmospheric perturbations, etc, may degrade the captured images. In such cases, Super-Resolution techniques can be used to extract more information than provided by a single image.

Three sub-problems must be faced here. Let $f_{L R}^{i}, i=1,2, \ldots, R$ be the observed, degraded and shifted from each other images. The first image is generally taken to be a reference. All the shifts $\left(\delta_{x}^{i}, \delta_{y}^{i}\right), i \neq 1$ must be accurately estimated, this is the registration problem. Then, the images must be restored to compensate for blur and noise, the restoration problem. Finally, the resulting data have to be interpolated to the desired high-resolution grid and this is an interpolation problem. The ability to achieve image Super-Resolution is controversial according to authors in [22]. However, the existence of algorithms has demonstrated Super-Resolution in many application areas. Figure 1.7 shows schematically the block diagram of the Super-Resolution process and figure 1.8 illustrates the idea behind this technique using a simple example.


Figure 1.7: The Super-Resolution reconstruction process


Figure 1.8: The Super-Resolution principle: 1D signal

It must be noted here, that throughout this work, the registration and the interpolation subproblems are solved simultaneously using the minimum of input images. This constitutes one of the advantages of this new approach for image reconstruction. The registration and interpolation sub-problems are solved simultaneously as a good reconstruction depends on accurate estimation of the shifts.

### 1.7 Previous work

Image Super-Resolution is a mature problem, which has been analyzed for a long time now. Various methods have been proposed in the recent years, driven by the growing interest in high-resolution imagery. In this section, we will briefly review some of the methods for the generation of super-resolution images. The main differences between the proposed methods lie in the interpolation method used, assumptions made to facilitate the process and the degree of degradations caused by the optics. A detailed review of the state of the art of Super-Resolution restoration techniques can be found in [23].

Tsai and Huang [24] were the first to propose a solution to this problem using a sequence of low-resolution images of a translated scene. They assumed only translation between the input images and solved for the dual problem of registration and restoration. They assumed that the observations are free from degradations and noise. Super-Resolution reconstruction from a sequence of under-sampled and shifted images can take advantage of the available spatio-temporal data provided by the low-resolution images. SuperResolution reconstruction is an example of an ill-posed inverse problem [23]. This is one of the fundamental ideas we shall repeatedly find in the literature. A multiplicity of possible solutions exists given a set of observed images.

In such situations, we can tackle this problem by constraining the solution space using $a$ priori knowledge of the expected solution such as smoothness, positivity, etc.

Mainly, super-Resolution methods may be divided into two categories: frequency domain techniques and spatial domain techniques.

### 1.7.1 Frequency domain methods

First proposed by Tsai and Huang in 1984, this frequency domain method is based on three properties of the Fourier transform [ $\mathbf{2 5}, \mathbf{2 6 , 2 7 , 2 8 , 2 9}$. They assumed only translation between the low-resolution images and solved for the dual problem of registration and interpolation. They assumed that the observed data was noise free and not degraded. They proposed a registration approach based on minimizing the energy at the high frequencies. The three properties used are:

- The shifting property of the Fourier Transform (FT),
- The aliasing relationship between the Continuous Fourier Transform (CFT) and the Discrete Fourier Transform (DFT),
- The fact that the original scene is band-limited.

These listed properties allow the formulation of a system of equations relating the aliased DFT coefficients of the observed images to samples of the CFT of the unknown scene. This system of equations is solved yielding the frequency domain coefficients of the original scene. The resolution of this system needs the knowledge of the relative shift between the frames to subpixel accuracy. The final image is then recovered by applying the inverse DFT.

We can write this in matrix form as follows:

$$
\begin{equation*}
\mathrm{Y}=\Phi \mathrm{F} \tag{1.2}
\end{equation*}
$$

Where Y contains the DFT components of all input low-resolution images, F contains the samples of the CFT of the unknown image, and $\Phi$ is a transfer matrix based upon the motion between low-resolution frames.

Tekalp, Ozkan and Sezan [30] extended the idea proposed by Tsai and Huang by including the effects of a LSI (Linear Shift Invariant) PSF (Point Spread Function) $h(x, y)$ as well as the noise. Again only translation was considered in their case. Using $\mathrm{L}^{2}$ observed images, each $L$ times under-sampled in $x$ and $y$ directions, a system of $L^{2}$ equations with $L^{2}$ unknowns is formulated. This system of equations is solved by the Least Square formulation to cope with the observed noise.

Kaltenbacher and Hardie [31] utilize the frequency domain formulation proposed by Tsai and Huang for restoration of aliased, under-sampled low-resolution images. The difference in this approach occurs only in the estimation of the translational shift required for the whole process. Particular attention was given to the computational cost by the authors.

### 1.7.1.1 Recursive Least Squares Techniques

Kim, Bose and Valenzuela [32] used exactly the same theoretical framework by Tsai and Huang as well as the global translation observation model. However, they extended the formulation to take into consideration the effects of noise and spatial blurring. Recursive Least Squares and Weighted Recursive Least Squares solutions for equation (1.2) in the presence of noise are derived. Sufficient conditions for non-singularity of the matrix $\Phi$ are also derived. Almost the same idea was followed by Tekalp, Ozkan and Sezan who extended Tsai and Huang's method to include blur for translated images which they solve using the Least Squares estimation.

Robustness to errors in observations as well as errors in the matrix $\Phi$ motivated the use of a total least squares (TLS) approach in [33] which was implemented using a recursive algorithm for computational considerations.

This frequency method was further refined by Kim and Su [34] who considered the case of different blurs in each low-resolution image and use Tikhonov regularization to determine the solution of an inconsistent set of linear equations.

Though the advantages of the proposed approach, the restriction to global translation model remains. The translation parameters are assumed known a-priori.

### 1.7.1.2 Multichannel sampling Theorem Based Techniques

An alternative approach based upon the generalized sampling theorem of Papoulis [35] was proposed by Ur and Gross [36]. A non-uniform interpolation of an ensemble of spatially shifted low-resolution images was proposed. A deblurring process follows this step. Although the implementation of the reconstruction is achieved in the spatial domain, the technique is fundamentally performed in the frequency domain. Global translation parameters are assumed known and no noise was considered.

Generally, Super-Resolution reconstruction methods in the frequency domain are very simple to implement, as the principles behind the frequency domain approach are really understandable in terms of basic Fourier transform. Also, they are very attractive in terms of computational cost and the process of Super-Resolution reconstruction is not computationally complex, based upon the de-aliasing techniques given by Tsai and Huang.

However, these methods are limited only to the global translation model and it is difficult to include spatially varying degradation models. Since Super-Resolution is an ill-posed inverse problem, regularization is often required in order to achieve the desired quality of images. Often the most useful a-priori knowledge used to reduce the solution space is via spatial domain constraints. Frequency domain methods intrinsically find it difficult to accommodate such constraints.

In the next section, we shall present the literature on spatial domain methods and conclude with a comparison between these two categories for image Super-Resolution.

### 1.7.2 Spatial domain methods

In this second category of Super-Resolution reconstruction methods, the observation model is formulated and the reconstruction is performed in the spatial domain. The spatial domain observation model can accommodate general motion, optical blur and other degradations. The inclusion of spatial domain a-priori constraints is possible and more flexible. Multiple low-resolution images $Y_{r}, r \in\{1,2 \ldots \mathrm{R}\}$, are used to estimate a highresolution image $f$. The images are written in lexicographic order. $Y_{r}$ and $f$ are related by:

$$
\begin{equation*}
Y_{r}=H_{r} f \tag{1.3}
\end{equation*}
$$

where the matrix $H_{r}$, which is to be estimated, incorporates the motion parameters, degradation effects and sub-sampling. $Y_{r}$ is an $\mathrm{A} \times 1$ vector representing the $r^{\text {th }}(M \times N)$ low-resolution image in lexicographic order. If $k$ is the resolution enhancement factor in each direction, $f$ is an $r^{2} \mathrm{~A} \times 1$ vector representing the $k M \times k N$ high-resolution image in lexicographic order, with $\mathrm{A}=M^{*} N$. This equation can be generalized to:

$$
\begin{equation*}
Y=H f+N \tag{1.4}
\end{equation*}
$$

Where $Y=\left[y_{1}^{T}, y_{2}^{T}, \ldots, y_{R}^{T}\right]$ and $H=\left[H_{1}^{T}, H_{2}^{T}, \ldots, H_{R}^{T}\right]$. N represents the noise matrix. Or, in matrix form we have:

$$
\left[\begin{array}{c}
y_{1} \\
y_{2} \\
\cdot \\
\cdot \\
y_{R}
\end{array}\right]=\left[\begin{array}{c}
H_{1} \\
H_{2} \\
\cdot \\
\cdot \\
H_{R}
\end{array}\right] f+\left[\begin{array}{c}
n_{1} \\
n_{2} \\
\cdot \\
\cdot \\
n_{R}
\end{array}\right]
$$

### 1.7.2.1 Interpolation of non-uniformly spaced samples

The registration of a sequence of low-resolution images results in a single dense composite image of non-uniformly spaced samples. A restored image may be derived from this composite image using reconstruction techniques from non-uniformly spaced samples. Restoration techniques can be applied to compensate for degradations. However, these methods were incapable of reconstructing significantly more frequency samples than those present in the low-resolution images. Degradation models are limited and no apriori constraints were used.

### 1.7.2.2 Iterated Backprojection

Given a high-resolution image estimate $\hat{f}$ and the imaging model H , it is possible to simulate the production of low-resolution images $\hat{Y}$ as $\hat{Y}=H \hat{f}$. Iterated Backprojection methods (IBP) update the estimate of the estimated high-resolution image by backprojecting the error between the $j^{\text {th }}$ iteration of the simulated low-resolution image $\hat{Y}^{(j)}$
$\hat{Y}^{(J)}$ and the observed low-resolution images $Y$ via the backprojection operator HBP.
Algebraically this can be written as:

$$
\begin{align*}
& \hat{f}^{(j+1)}=f^{(j)}+H^{B P}\left(Y-\hat{Y}^{(j)}\right) \\
& \hat{f}^{(j+1)}=f^{(j)}+H^{B P}\left(Y-H \hat{f}^{(j)}\right) \tag{1.5}
\end{align*}
$$

This last equation is iterated until a criterion dependent on $Y$ and $\hat{Y}^{()}$is minimized. Applications of the IBP method may be found in [37]. The shift in their case was estimated using a method described by other authors, followed by the iteration process described above. Since the registration is done separately from the reconstruction, the accuracy of the method depends largely on the accuracy of the estimated shifts. Unfortunately, the solution is often not unique as the Super-Resolution is an ill-posed inverse problem. Also, the inclusion of a-priori constraints is very difficult to achieve in these methods. Noise is handled by simple averaging of all the contributions of lowresolution (LR) pixels.

### 1.7.2.3 Stochastic restoration methods

Stochastic methods have also been applied to the Super-Resolution problem. These techniques treat the SR as a statistical problem and have proven to be efficient since they provide a powerful theoretical framework for the inclusion of a-priori constraints necessary to find a solution of the ill-posed inverse problem. Cheeseman et al. at NASA [38,39] developed the first method using this approach for planetary images. The observed data Y , noise N and the high-resolution image z are assumed stochastic. We consider the stochastic equation:

$$
Y=H z+N
$$

The maximum a-posteriori probability (MAP) approach to estimate $z$ seeks the estimate $\hat{z}_{\text {MAP }}$ for which the a-posteriori probability $\operatorname{Pr}\left\{\mathrm{z}_{\mid} \mathrm{Y}\right\}$ is maximum. This can be understood to find $\hat{z}_{\text {MAP }}$ such as:

$$
\begin{gathered}
\hat{z}_{M A P}=\arg \max [\operatorname{Pr}\{z ; Y\}] \\
\hat{z}_{M A P}=\arg \max [\log \operatorname{Pr}\{Y \mid z\}+\log \operatorname{Pr}\{z\}]
\end{gathered}
$$

Examples of application of these Bayesian methods can be found in $[38,39]$ for the integration of multiple satellite images observed by the Viking orbiter. Some extensions of this method to 3D reconstruction are also presented in the last work.

### 1.7.2.4 Set-theoretic restoration methods

Set-theoretic methods and particularly the method of Projection Onto Convex Sets (POCS), are very popular because of their simplicity and allow convenient inclusion of apriori information. In these methods the space of SR images is intersected with a set of constraint sets representing the desirable SR image characteristics to yield a reduced solution space. POCS refers to an iterative procedure which given any point in the space of SR image, locates a point which satisfies all the convex constraints sets.

Convex sets, which represent constraints on the solution space of $z$, are defined. For example, positivity by $\left\{z: z_{i}>0 \forall i\right\}$, bounded energy by $\{z:\|z\| \leq E\}$ and so on.

For each convex constraint set defined, a projection operator is determined. The projection operator $\mathrm{P}_{\alpha}$ associated with the constraint set $\mathrm{C}_{\alpha}$ projects a point in the space of $z$ onto the closest point on the surface of $\mathrm{C}_{\alpha}$.

It can be shown that repeated application of the iteration $z^{(n+1)}=P_{1} P_{2} \ldots P_{k} z^{(n)}$ will result in convergence to a solution on the surface of the intersection of the k convex constraint sets. These methods have the disadvantages of non-uniqueness of solution, dependence of the solution on the initial guess, slow convergence and high computational cost.

Set theoretic estimation of high-resolution images was first proposed by Stark and Oskoui [40], where a projection onto convex sets (POCS) was used. Their method was extended by Tekalp et al. to include noise. POCS restoration methods have proven to be very efficient with sophisticated observation and degradation models.

### 1.7.2.5 Optimal and Adaptive Filtering methods

Inverse filtering approaches to Super-Resolution have also been proposed. However, these techniques are limited in terms of inclusion of a-priori constraints as compared with POCS and Bayesian methods. Some applications of these techniques can be found in [41].

Elad and Feuer proposed a methodology that combines the three main estimation tools in image restoration, viz., ML estimator, MAP estimator and the POCS. The method assumes explicit knowledge of the blur and the motion constraints.

Recently, Elad et al. [42,43] proposed a different implementation using the $L_{1}$ norm minimization and robust regularization to deal with different sources of noise.

The interdependence of registration, interpolation and restoration has been taken into account by Tom and Katsaggelos $[44,45]$ where the problem is posed as a maxiumumlikelihhod (ML) estimation problem, which is solved by the expectation-maximization (EM) algorithm. The general equation describing the formation of the low-resolution images contains the shifts, blur and the noise parameters. The structure of the matrices involved in the objective function enables efficient computation in the frequency domain. The ML estimation problem then solves simultaneously for the unknown shifts, the noise variances of each image and the high-resolution image. In [46], Komatsu et al. use a nonuniform sampling theorem proposed by Clark et al. [47] to transform non-uniformly
spaced samples acquired by multiple cameras onto a single uniform sampling grid. SuperResolution via image warping is described in [48,49]. Wirawan et al. proposed a blind multichannel high-resolution image restoration algorithm by using multiple finite impulse response (FIR) filters [50]. Improvements to existing reconstruction methods have also been proposed by other authors [51,52,53].

In summary, by reviewing all these methods, it is apparent that the Super-Resolution problem has been analyzed for a long time and has a significant history. Very little work has been done in addressing the three sub-problems; registration, interpolation and restoration, in a complete manner, with the exception of the authors in [44,45]. However, the computation issue still remains a problem for real applications in this method. Accurate degradation modelling typically results in improved restorations.

As a comparison between these two categories of Super-Resolution methods, frequency and spatial methods, Table 1.1 summarizes these differences.

Spatial domain methods are flexible compared to frequency methods though computationally complex. Also, Super-Resolution restoration is critically dependent upon accurate sub-pixel motion estimation.

|  | Frequency domain | Spatial domain |
| :---: | :---: | :---: |
| Observation model | Frequency domain | Spatial domain |
| Motion models | Global translation | Almost unlimited |
| Degradation model | Limited, LSI | LSI or LSV |
| Noise model | Limited | Very flexible |
| SR algorithm | Dealiasing | Dealiasing, a-priori info |
| Computation requirement | Low | High |
| A-priori information | Limited | Almost unlimited |
| Regularization | Limited | Excellent |
| Extensibility | Poor | Excellent |
| Applicability | Limited | Wide |
| Application performance | Good | Good |

Table 1.1: Frequency methods vs. Spatial domain methods (from [22])

## Chapter 2

## 2 The Tsai and Huang method

### 2.1 Introduction

Tsai and Huang [24] were the first to propose a super-resolution algorithm to construct a high resolution image from a sequence of several down-sampled low-resolution images (without blur). Their objective was to enhance the resolution of Landsat images [54]. A Landsat satellite takes pictures over the same area on the ground every 18 days as it orbits around the earth. These pictures were therefore shifted form each other. The problem for them was how to construct a high-resolution image from these low-resolution pictures. This earlier work proposed a frequency domain formulation to solve for the unknown image, based on the shift and aliasing properties of the Fourier transform. The method is detailed below and illustrated with an example of reconstruction using this method.

### 2.2 The Algorithm

Let us denote the continuous scene by $f(x, y)$. With R shifted images of this scene we obtain R functions:

$$
\begin{equation*}
f_{k}(x, y)=f\left(x+\delta_{x k}, y+\delta_{y k}\right) \tag{2.1}
\end{equation*}
$$

where $\delta_{x k}$ and $\delta_{j k}$ are the known shifts of $f(x, y)$ along the x and y coordinates respectively for image $k$, where $k=1,2, \ldots, \mathrm{R}$. The CFT of the original image is $\mathfrak{J}(u, v)$ and those of the translations are $\mathfrak{I}_{\mathrm{k}}(u, v)$.

The shifted images are impulse sampled to yield observed images:

$$
\begin{equation*}
f_{k}(i, j)=f\left(i T_{x}+\delta_{x k}, j T_{y}+\delta_{y k}\right) \tag{2.2}
\end{equation*}
$$

where $i=0,1, \ldots, \mathrm{M}-1, j=0,1, \ldots, \mathrm{~N}-1$ and $k=1,2 \ldots, \mathrm{R}$ ( M and N are the number of pixels in directions x and y respectively), and the sequence of R frames, denoted by $\left\{f_{k}(i, j)\right\}$
corresponds to discrete versions of the shifted images $f_{k}(x, y)$, after uniform sampling with sampling periods $T_{x}$ and $T_{y}$ along the x and y coordinate axes respectively..

The R corresponding 2D DFT's are denoted by $F_{k}(m, n)$. The CFT of the scene and the DFT's of the shifted and sampled images are related via the aliasing relationship by (2.3).

$$
\begin{equation*}
F_{k}(m, n)=\frac{1}{T_{x} T_{y}} \sum_{i=-\infty}^{\infty} \sum_{=-\infty}^{\infty} \mathfrak{I}_{k}\left(\frac{2 \pi m}{M T_{x}}+i f_{x}, \frac{2 \pi n}{N T_{y}}+l f_{y}\right) \tag{2.3}
\end{equation*}
$$

where, $f_{x}=2 \pi / T_{x}$ and $f_{y}=2 \pi / T_{y}$ are the sampling frequencies in x and y directions respectively. From the continuous Fourier transform theory [47, 48, 49], there is a simple way to relate the Fourier transform of the shifted images to non-shifted image:

$$
\begin{equation*}
\mathfrak{I}_{k}(u, v)=e^{j 2 \pi\left(\delta_{x^{\prime}} u+\delta_{y v^{\prime}}\right)} \mathfrak{J}(u, v) \tag{2.4}
\end{equation*}
$$

If we assume that $f(x, y)$ is band-limited then this implies,

$$
\exists \mathrm{L}_{1}, \mathrm{~L}_{2} \text { such as }|\mathfrak{I}(u, v)|=0 \text { for }|u| \geq \mathrm{L}_{1} f_{x} \text { and }|v| \geq \mathrm{L}_{2} f_{y}
$$

Using this assumption and after substituting in the aliasing relation given in (2.3), the DFT $F_{k}(m, n)$ of the $\mathrm{k}^{\text {th }}$ undersampled frame can be expressed as a finite aliased version of the continuous Fourier transform $\mathfrak{I}_{\mathrm{k}}(u, v)$ of the $\mathrm{k}^{\text {th }}$ shifted image $f_{k}(x, y)$ by (2.5).

$$
\begin{equation*}
F_{k}(m, n)=\frac{1}{T_{x} T_{y}} \sum_{i=-L_{1}=-L_{2}}^{L_{1}-1} \sum_{k}^{L_{2}-1} \Im_{k}\left(\frac{2 \pi m}{M T_{x}}+i f_{x}, \frac{2 \pi n}{N T_{y}}+l f_{y}\right) \tag{2.5}
\end{equation*}
$$

for $k=1,2 \ldots, \mathrm{R}$.
Using (2.4) we have:

$$
\begin{equation*}
F_{k}(m, n)=\frac{1}{T_{x} T_{y}} \sum_{i=-L_{l}}^{L_{l=-}-1} \sum_{2}^{L_{2}-1} e^{j 2 \pi\left[\delta_{x+1}\left(\frac{2 \pi n}{M T_{x}}+i f_{x}\right)+\delta_{\mu}\left(\frac{2 \pi n}{N T_{y}}+1 y_{y}\right)\right]} \mathfrak{J}\left(\frac{2 \pi m}{M T_{x}}+i f_{x}, \frac{2 \pi n}{N T_{y}}+l f_{y}\right) \tag{2.6}
\end{equation*}
$$

In (2.6), we can see that $F_{k}(m, n)$ is expressed by the sum of $\left(2 \mathrm{~L}_{1} \times 2 \mathrm{~L}_{2}\right)$ values of the CFT of the unknown high resolution image at a specific point $(u, v)=\left(\frac{2 \pi m}{M T_{x}}+i f_{x}, \frac{2 \pi n}{N T_{y}}+l f_{y}\right)$.

To write (2.6) in matrix form, and by choosing lexicographical ordering: ( $-\mathrm{L}_{1},-\mathrm{L}_{2}$ ), ( -$\left.\mathrm{L}_{1}+1,-\mathrm{L}_{2}\right),\left(-\mathrm{L}_{1}+2,-\mathrm{L}_{2}\right), \ldots,\left(\mathrm{L}_{1}-2, \mathrm{~L}_{2}-1\right),\left(\mathrm{L}_{1}-1, \mathrm{~L}_{2}-1\right)$, we can define a new variable $r=1,2$, $\ldots, 4 \mathrm{~L}_{1} \mathrm{~L}_{2}$ as:

$$
\left(\frac{2 \pi m}{M T_{x}}+i f_{x}, \frac{2 \pi n}{N T_{y}}+l f_{y}\right) \rightarrow r
$$

where $i$ and $l$ can be defined by:

$$
\begin{aligned}
& i=(r-1) \bmod \left(2 \mathrm{~L}_{1}\right)-\mathrm{L}_{1} \\
& l=(r-1) \bmod \left(2 \mathrm{~L}_{2}\right)-\mathrm{L}_{2}
\end{aligned}
$$

Then, the matrix form of (2.6) for $k=1,2, \ldots, \mathrm{R}$ is:

$$
\left[\begin{array}{c}
F_{1}(m, n)  \tag{2.7}\\
F_{2}(m, n) \\
\cdot \\
\cdot \\
F_{p}(m, n)
\end{array}\right]=\frac{1}{T_{x} T_{y}}\left[\begin{array}{ccccc}
\Phi_{1,1} & \Phi_{1,2} & \cdot & \cdot & \Phi_{1,4 L_{1} L_{2}} \\
\Phi_{2,1} & \Phi_{2,2} & \cdot & . & \Phi_{2,4 L_{1} L_{2}} \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\Phi_{p, 1} & \Phi_{p, 2} & \cdot & \cdot & \Phi_{p, 4 L_{1} L_{2}}
\end{array}\right]\left[\begin{array}{c}
\mathfrak{J}_{m n}(1) \\
\mathfrak{I}_{m n}(2) \\
\cdot \\
\cdot \\
\mathfrak{I}_{m n}\left(4 L_{1} L_{2}\right)
\end{array}\right]
$$

where

$$
\mathfrak{S}_{m n}(r)=\mathfrak{J}\left(\frac{2 \pi m}{M T_{x}}+i f_{x}, \frac{2 \pi n}{N T_{y}}+l f_{y}\right)
$$

for $r=1,2 \ldots 4 \mathrm{~L}_{1} \mathrm{~L}_{2}$, and

$$
\Phi_{k, r}=\exp \left[j 2 \pi\left(\delta_{x k}\left(m / M T_{x}+i / T_{x}\right)+\delta_{y k}\left(n / N T_{y}+l / T_{y}\right)\right)\right]
$$

Using this matrix formulation, we can see that, for each $(m, n)^{\text {th }}$ component of the DFT of the $k^{\text {th }}$ low resolution image, and a transformation, which is determined from the motion parameters, we can obtain ( $4 \mathrm{~L}_{1} \mathrm{~L}_{2}$ ) samples of the CFT of the unknown high-resolution image depending on the $(m, n)^{t h}$ component.

For the sake of notational brevity, we can rewrite (2.7) in the following form:

$$
\begin{equation*}
Y=\Phi F \tag{2.8}
\end{equation*}
$$

We can see from (2.8) that Y is a vector (Rx1) with the $k^{\text {th }}$ element being the DFT coefficients $F_{k}(m, n)$ of the observed image $f_{k}(x, y)$. $\Phi$ is a matrix which relates therefore these DFT's of the observed images to samples of the unknown CFT of $f(x, y)$ contained in the vector F with dimensions $\left(\left(4 \mathrm{~L}_{1} \mathrm{~L}_{2}\right) \times 1\right)$. Of course, to calculate matrix $\Phi$, which is of size ( $\mathrm{R} \times \mathrm{L}_{1} \mathrm{~L}_{2}$ ), we need to estimate the shifts between the frames $\delta_{x k}$ and $\delta_{j k}$. This will be explained in the motion estimation section. Therefore, high-resolution image reconstruction requires finding the DFT's of the R-shifted images, determining $\Phi$ (motion estimation), solving the system of equations (2.8) for F and applying the inverse DFT to obtain the high-resolution image. The solution F can be obtained with some simple matrix operations ( $\mathrm{F}=\boldsymbol{\Phi}^{-1} \mathrm{Y}$ ).

We can see that we have taken $4 \mathrm{~L}_{1} \mathrm{~L}_{2}$ values and assigned $(u, v)$ coordinates to them in the frequency domain as samples of the frequency representation of our high resolution image. Therefore, if we generalize for all the rest of points ( $\mathrm{N}-1$ )(M-1), we have ( $\mathrm{N}-1$ )(M1) more matrix equations to solve, and thus $4 \mathrm{~L}_{1} \mathrm{~L}_{2} x(N-1)(M-1)$ more points to add to our frequency representation. In total, we shall have $4 L_{1} L_{2} x M N$ points in our representation whereas we have started just with MN points. So, our resolution has increased by a factor of $4 \mathrm{~L}_{1} \mathrm{~L}_{2}\left(2 \mathrm{~L}_{1} \times 2 \mathrm{~L}_{2}\right.$ in x and y directions respectively). As an example, if we want to increase the resolution by two (2) in each direction and thus by 4 overall, we put $\mathrm{L}_{1}=\mathrm{L}_{2}=1$.

In this section, we describe exactly the execution of the algorithm. Initially, we are interested only in the reconstruction step. We will assume that there is only translation over the entire image. This simplifies the registration as well as the interpolation. The registration part will be discussed afterwards. This means that in our algorithm the motion parameters are supposed known.

The following notation is proposed for this algorithm:

R : the number of low-resolution frames
Deltak: Matrix containing the $x$ and $y$ components of the shift ( $\delta_{\mathrm{xk}}, \delta_{\mathrm{yk}}$ ) of each LR frame.

Y : is a matrix containing the DFT components of the LR frames
F: Fourier transform of the unknown image
Image_HR: high-resolution image


The Tsai and Huang algorithm for image super-resolution

### 2.3 Experimental Results

The method is demonstrated with the help of Sydney image of size $256 \times 256$ taken by TMSAT $^{\circledR}$ in 1998 (figure 2.3a). From the original image, four low-resolution images were obtained by sub-sampling the original image by a factor of two (1/2). Each of these images is of size $128 \times 128$, and displaced by different shifts from each other. The vector shift used was $[(0,0.5) ;(0.5,0) ;(0.5,0.5)]$, where the first frame was chosen as the reference frame (shift null=( 0,0 )). First, the reconstruction step using the Tsai and Huang method was performed. For comparison purposes and the validity of the implementation of the method, it is assumed that the shifts are known exactly for the reconstruction step. Figure 2.1 shows the four low-resolution images used in the reconstruction of size $128 \times 128$. It can be seen from the figure 2.3 d that the Tsai and Huang method is a very effective method of achieving super-resolution. The visual difference between the obtained result using the Super-Resolution algorithm and the other methods for interpolation can be clearly seen at the edges for example or in terms of sharpness of the resulted image compared with the nearest interpolation and bi-cubic interpolation approaches (figure 2.3b, 2.3c). In order to obtain a quantitative evaluation of the results, the Mean Square Error (MSE) between the reconstructed image and the full perfect image was calculated (MSE $=0.0031$ ). This number must be compared with the ones given in Table 2.1, which were obtained when the resolution of each low-resolution frame is increased by some interpolation methods. In this case nearest and Bi -cubic methods were used. These results show clearly that the Tsai and Huang method is a very effective method of achieving Super-Resolution.

In the event that the shifts are not known, registration algorithms which exist in the literature may be used. In our case, many experiments have been carried out in order to analyze the effect of mis-registration on the result of the reconstruction. Figure 2.4 shows an example of reconstruction using the same low-resolution images with slightly wrong shifts namely $(0,0.1),(0.5,0)$ and $(0.5,0.5)$ for the second, third and forth low-resolution image respectively. We can see that the reconstruction contains many sharp lines and we have lost some details of the image. This leads us to propose a measure of the quality of the obtained solution and which can be used to register the input images. This will be developed in the next section.


Figure 2.1: Low resolution images $(\mathbf{1 2 8 x 1 2 8})$ derived from Sydney image used in the reconstruction

| MSE(Original, Frame) | Frame 1 | Frame 2 | Frame 3 | Frame 4 |
| :---: | :---: | :---: | :---: | :---: |
| Nearest neighbour | 0.0186 | 0.0188 | 0.0184 | 0.0185 |
| Bi-cubic | 0.0134 | 0.0122 | 0.0119 | 0.0114 |

Table 2. 1: Mean Square Error calculations


Figure 2.2: TiungSat1 (SSTL) micro-satellite


Figure 2.3: a) Original image (satellite image Sydney, taken by TMSAT, 1998); b) Nearest neighbour interpolation from LR images; c) Bi-cubic interpolation; d) Result of the Super-Resolution (256x256)


Figure 2.4: Example of mis-registration in Tsai and Huang method (SR using wrong shifts)

### 2.4 The new measure of solution quality

Sharpness is defined as a perceived quality of an image. This property is associated with the abruptness of change of tone at edges.

In our case, the sharpness can be one of the important tools to the perceived quality of the reconstructed image using the super-resolution techniques. One easy way to measure the sharpness is to measure the derivative of intensities along $x$ or $y$ directions for the whole image at each pixel.

For us, it was sufficient to do this calculation only along the central horizontal line. Another way to estimate the sharpness is to measure the amount of high frequencies in the image. However, the proposed measure in this case will be the sum of heights of the peaks present in the derivative vector and which are stronger than a certain value which is defined manually.

Figure 2.5 shows an example of an image and its derivative along the central horizontal line and figure 2.6 presents schematically the proposed measure. This measure is expected to be minimum when the shift used in the reconstruction is close to the true shift. This is shown experimentally in figure 2.9.


Derivative of intensities along central horizontal line


Figure 2.5: Example of an image and its derivative of intensities along the central horizontal line

Therefore, we can present schematically the new measure " $M$ " as follows:


Figure 2.6: The new measure of sharpness $M=S u m$ of all heights over "h"

On the other hand, the frequency domain information was investigated as well by analyzing the variation of the power spectrum of the reconstructed images using this method in all possible cases (with and without mis-registration). Experiments have shown that in the case of wrong shifts used, the amount of the energy in the high frequency range is considerable compared with the amount of energy using true shifts.

This has led us to propose another measure $(P)$, which consists of the frequency above which $10 \%$ of the total energy exists (Figure 2.8). This measure is expected to be smaller near the ideal shift that best registers the input images. Figure 2.7 shows an example of two power spectra of an image result using true and wrong shifts in the reconstruction algorithm.


Figure 2.7: Power Spectrum of the result using correct shift (left) and wrong shift (right)


Figure 2.8: The frequency domain measure ( P ) using correct shift (left) and wrong shift (right)

### 2.5 Image registration using the new measures

The reconstruction step using Tsai and Huang method was performed using the four lowresolution images but by varying the shifts for each of them during the reconstruction. The true vector shift is $[(0,0),(0.5,0),(0,0.5),(0.5,0.5)]$. The minimum of the surface generated by calculating the amount of the new measure versus the shifts used in the reconstruction from this test will occurs in the best shift that will register all the input frames.

Figure 2.9 below shows the result of registering the same input images used in our simulation and which have been derived from Sydney image, using both measures. These measures are expected to be minimum near the best shift.


Figure 2.9: Example of registration using the new measures ('Sydney image')
Left: using sharpness measure, Right: using power spectrum measure

The estimated values of shifts for this sequence are shown in Table 2.2:

|  | True shift | Estimated shift <br> (Sharpness) | Estimated shift <br> (Power spectrum) |
| :--- | :---: | :---: | :---: |
| $\left(\delta_{x}^{2}, \delta_{y}^{2}\right)$ | $(0,0.5)$ | $(0,0.7)$ | $(0,0.4)$ |
| $\left(\delta_{x}^{3}, \delta_{y}^{3}\right)$ | $(0.5,0)$ | $(0.7,0)$ | $(0.4,0)$ |
| $\left(\delta_{x}^{4}, \delta_{y}^{4}\right)$ | $(0.5,0.5)$ | $(0.7,0.7)$ | $(0.4,0.4)$ |

Table 2.2: The true and estimated shifts for Sydney image


Figure 2.10: Example of error in shift estimation (Lena image)

It can be seen from these simulations that these measures can be used to estimate the translational shift between images of a sequence before performing the Super-Resolution reconstruction process.

### 2.6 Conclusion

In this chapter, the first method for image Super-Resolution, proposed by Tsai and Huang in 1984, was implemented completely and illustrated with an example of reconstruction using this method. This method provides the advantage of theoretical simplicity and quite high computation complexity. Furthermore, we have proposed two new measures based on the sharpness of the reconstructed image and the energy at the high frequencies range. These measures can be used to estimate the translational shift which best register two input images in Tsai and Huang method. Experimental results demonstrate the use of this approach for both registration and interpolation of low-resolution images to reconstruct better quality images.

The disadvantages of this method can be summarised in high computation cost of the Fourier transform calculations, and also limited ability for inclusion of spatial a-priori knowledge for regularization and therefore less flexibility. The new approach we propose in this thesis uses a spatial domain formulation rather than a frequency domain formulation and will be detailed in the next chapter.

## Chapter 3

## 3 Super-Resolution using the Walsh functions: 1D case

### 3.1 Introduction

The problem we are concerned here is the reconstruction of a high-resolution image from multiple shifted low-resolution images of the same scene. The new approach, which we will describe in this chapter, is based on the representation of a signal by the superposition of members of a set of simple functions, which are easy to generate. Only orthogonal sets of functions can be made to synthesise completely any time function to a required degree of accuracy [55]. Furthermore, the characteristics of an orthogonal set are such that the identification of a particular member of the set contained in a given time function can be made easy using quite simple mathematical operations on the function. It is shown in this work that through very simple projections on these sets of functions, the reconstruction of the high-resolution image is achieved easily by fusing two shifted images, which in some way justify the low computation requirements for this proposed method.

Through this chapter, a new algorithm to achieve image super-resolution suitable for translated satellite images is presented. The reconstruction is performed using a new approach to super-resolution based on the orthogonal set of Walsh functions. We shall derive mathematically an expression for the missing information contained in an image of $2^{\mathrm{M}+1}$ pixels but absent in the lower resolution image of $2^{\mathrm{M}}$ pixels. From this we prove how this missing information can be retrieved from a second low-resolution image, shifted form the first one by an arbitrary amount.

The Walsh functions [55] are an example of these particular functions. They form an ordered set of rectangular waveforms taking only two amplitudes +1 and -1 .

They can be defined by [56]:

$$
\begin{equation*}
W_{2 j+q}(t)=(-1)^{\left[\frac{j}{2}\right]+q}\left\{W_{j}(2 t)+(-1)^{j+q} W_{j}(2 t-1)\right\} \tag{3.1}
\end{equation*}
$$

Where $[j / 2]$ means the largest integer which is smaller or equal to $j / 2 ; q=0$ or 1 and $j=0,1,2 \ldots$.etc.

$$
W_{0}(t)=\left\{\begin{array}{cc}
1 & 0 \leq t<1 \\
0 & \text { elsewhere }
\end{array} \quad W_{1}(t)=\left\{\begin{array}{cc}
1 & 0 \leq t<1 / 2 \\
-1 & 1 / 2 \leq t<1
\end{array}\right.\right.
$$

The first Walsh functions are illustrated in figure 3.1 below. In this work, only the first two Walsh functions $\mathrm{W}_{0}$ and $\mathrm{W}_{1}$ defined above are used.


Figure 3.1: Example of first Walsh functions

We define the original scene by $f(x)$ as an infinitely high-resolution image and by $P_{M}(f)$ the projection of an image $f$ with $2^{\mathrm{M}}$ pixels using the sequences $\mathrm{W}_{0}$. In this case, the intensity of a pixel $n$ in an image of resolution $2^{\mathrm{M}}$ is given by:

$$
\begin{equation*}
f^{(M, n)}=2^{M} \int_{2^{-M}(n)}^{2^{-M}(n+1)} f(x) d x \tag{3.2}
\end{equation*}
$$

The projection of an image using $2^{\mathrm{M}}$ pixel is defined by the set of $f^{(M, n)}$ and can be written as:

$$
\begin{equation*}
P_{M}(f)=\left\{f^{(M, n)} ; \quad 0 \leq n \leq 2^{M}\right\} \tag{3.3}
\end{equation*}
$$

This principle of projection is illustrated in the Figure 3.2 below:


Figure 3.2: The principle of projecting a function.

Figure 3.3 below shows an example of a real signal and its projections with 16 and 32 pixels respectively. In this example $\mathrm{M}=4$ resulting in 16 pixels for $P_{M}(f)$ and $\mathrm{M}=5$ resulting in 32 pixels for $P_{M+l}(f)$. The function is defined by $f(x)=\cos (\pi x)+\sin (10 x)+2$.


Figure 3.3: Top) Example of 1D signal projections $(f(x)=\cos (\pi x)+\sin (10 x)+2)$
Bottom) $\mathrm{P}_{\mathrm{M}}(\mathrm{f})$ and a shifted projection by half pixel

Let us denote by:

$$
y=2^{M} x-n \text {, then if, } 0<y<1 \quad 2^{-M}(n)<x<2^{-M}(n+1)
$$

with

$$
W_{0}(y)=\left\{\begin{array}{lr}
1 & 2^{-M}(n) \leq x<2^{-M}(n+1)  \tag{3.4}\\
0 & \text { elsewhere }
\end{array}\right.
$$

therefore,

$$
\begin{equation*}
f^{(M, n)}=2^{M} \int_{0}^{1} f(x) W_{0}(y) d x \tag{3.5}
\end{equation*}
$$

The new reference we define is based on the sequence $W_{0}$ and is defined by:

$$
\begin{equation*}
\varphi_{M, n}=2^{M / 2} W_{0}\left(2^{M} x-n\right) \tag{3.6}
\end{equation*}
$$

Therefore, equation (3.5) can be written as follows:

$$
\begin{equation*}
f^{(M, n)}=2^{M / 2} \int_{0}^{1} f(x) \varphi_{M, n}(x) d x=2^{M / 2}<f, \varphi_{M, n}> \tag{3.7}
\end{equation*}
$$

with

$$
\begin{equation*}
<f, \varphi_{M, n}>=\int_{0}^{1} f(x) \varphi_{M, n}(x) d x \tag{3.8}
\end{equation*}
$$

Also we have

$$
\begin{equation*}
f(x)=\sum_{n=0}^{2^{M}-1} f^{(M, n)} W_{0}(y)=2^{-M / 2} \sum_{n=0}^{2^{M}-1} f^{(M, n)} \varphi_{M, n}(x) \tag{3.9}
\end{equation*}
$$

this can be rewritten as follows

$$
\begin{equation*}
f(x)=\sum_{n=0}^{2^{M}-1}<f, \varphi_{M, n}>\varphi_{M, n}(x) \tag{3.10}
\end{equation*}
$$

Therefore (3.3) can then be written by

$$
\begin{equation*}
P_{M}(f)=\sum_{n=0}^{2^{\mu}-1}<f, \varphi_{M, n}>\varphi_{M, n}(x) \tag{3.11}
\end{equation*}
$$

In other words, this represents the projection of $f$ over an orthonormal space based on $\mathrm{W}_{0}$ using $2^{\mathrm{M}}$ pixels.

In the other hand we have,

$$
\int_{0}^{1} \varphi_{M, n}(x) \varphi_{M, n^{\prime}}(x) d x=\int_{2^{-M}(n)}^{2^{-M}(n+1)} 2^{M / 2} \cdot 2^{M / 2} \cdot d x=1 \rightarrow \text { only if } n=n^{\prime}
$$

This can prove the orthnormality of the sequences based on $\mathrm{W}_{0}$.

### 3.2 Mathematical background of the new approach

The new approach we propose here is based on the Walsh functions. It is proven in chapter 4 that we can assume orthographic projection in the proposed approach. Given the projection of a scene over an image space with resolution $M$ and the projection of the same scene over another image space with resolution M-1, denoted respectively $P_{M}(f)$ and $P_{M-1}()$ using the sequences of $\mathrm{W}_{0}$, we would like to determine the relation between these two projections using the information contained in the shifted projections of our original signal, and therefore we can say that we have the representation of this scene over two spaces with resolution M and $\mathrm{M}-1$. Thus, this analytical relation between these projections will show us the missing information for an image of resolution $2^{\mathrm{M}-1}$ but present in an image of resolution $2^{\mathrm{M}}$. We shall show then how is it possible to retrieve the missing information with the help of another image of the same lower resolution but shifted from the first one by an arbitrary amount in order to construct an image with higher resolution $2^{\mathrm{M}}$. Schematically, the approach can be illustrated as below in figure 3.4.


Figure 3.4: Diagram of the new approach for Super-Resolution

Here, we can see that we have the projections of the function $f(x)$ on the sequence $\varphi_{M, n}$ which are orthonormal and defined by:

$$
\varphi_{M, n}=\left\{\begin{array}{cr}
2^{M / 2} & 2^{-M}(n) \leq x<2^{-M}(n+1)  \tag{3.12}\\
0 & \text { elsewhere }
\end{array}\right.
$$

We can prove easily that:

$$
\varphi_{M, 2 n}=\left\{\begin{array}{cc}
2^{M / 2} & 2^{-(M-1)}(n) \leq x<2^{-(M-1)}\left(n+\frac{1}{2}\right)  \tag{3.13}\\
0 & \text { elsewhere }
\end{array}\right.
$$

And

$$
\varphi_{M, 2 n+1}=\left\{\begin{array}{lr}
2^{M / 2} & 2^{-(M-1)}\left(n+\frac{1}{2}\right) \leq x<2^{-(M-1)}(n+1)  \tag{3.14}\\
0 & \text { elsewhere }
\end{array}\right.
$$

Therefore,

$$
\begin{equation*}
\varphi_{M-1, n}=\frac{1}{\sqrt{2}}\left(\varphi_{M, 2 n}+\varphi_{M, 2 n+1}\right) \tag{3.15}
\end{equation*}
$$

Or in other ways, this means:

$$
\begin{equation*}
\left.\left.\left.<f, \varphi_{M-1, n}\right\rangle=\frac{1}{\sqrt{2}}\left[<f, \varphi_{M, 2 n}\right\rangle+<f, \varphi_{M, 2 n+1}\right\rangle\right] \tag{3.16}
\end{equation*}
$$

In the other hand we have:

$$
\begin{equation*}
P_{M}(f)=\sum_{n=0}^{2^{N}-1}<f, \varphi_{M, n}>\varphi_{M, n}(x) \tag{3.17}
\end{equation*}
$$

which can be divided for odd and even values for $n$ as follows:

$$
\begin{equation*}
P_{M}(f)=\sum_{n=0}^{2^{M}-1}<f, \varphi_{M, 2 n+1}>\varphi_{M, 2 n+1}(x)+\sum_{n=0}^{2^{M}-1}<f, \varphi_{M, 2 n}>\varphi_{M, 2 n}(x) \tag{3.18}
\end{equation*}
$$

By the same way we have:

$$
\begin{equation*}
P_{M-1}(f)=2^{\frac{M-1}{2}} \sum_{n=0}^{2^{M-1}-1}<f, \varphi_{M-1, n}>\varphi_{M-1, n}(x) \tag{3.19}
\end{equation*}
$$

Using (3.16), and by subtracting (3.18) and (3.19) as well as keeping in mind the orthogonality of the Walsh functions, we end-up with the equation (3.20):

$$
\begin{equation*}
P_{M}(f)-P_{M-1}(f)=\sum_{n=0}^{2^{M-1}-1}<f, W_{1}^{(M-1, n)}>W_{1}^{(M-1, n)}(x) \tag{3.20}
\end{equation*}
$$

with:

$$
\begin{equation*}
\varphi_{M, 2 n}-\varphi_{M, 2 n+1}=W_{1}(x)=W_{0}(2 x)-W_{0}(2 x-1) \tag{3.21}
\end{equation*}
$$

From (3.20) we can see that we have related the projections of the signal with resolution M and $\mathrm{M}-1$. This is interesting and very important as we have represented the signal with two different spaces with $2^{\mathrm{M}}$ and $2^{\mathrm{M}+1}$ pixels. Schematically as shown in figure 3.5 , this means that the missing information from $P_{M-1}(f)$ is only the projection of $f$ on $\mathrm{W}_{1}(\mathrm{x})$ scaled by $2^{\mathrm{M} / 2}$ at each corresponding pixel $n$. in this case, and in order to reconstruct the high-resolution image we need to add and subtract the missing information from each low-resolution pixel and than end-up with two more pixels in the final representation for each pixel.

Missing information


Figure 3.5: The new principle in the reconstruction


Figure 3.6: Relation between Low resolution and high resolution intensities in the new approach

The next problem is how to calculate the projection of $f$ on $\mathrm{W}_{1}$ as $f$, the real scene, is not known in the reality. This calculation can be possible only with the help of information contained in the shifted projections. We define the shift in our case by the following expression: $\Delta=2^{-M}(s+\rho)$, where we refer to the integer part of the shift by " $s$ " and " $\rho$ " to its fractional part (sub-pixel shift). Figure 3.7 shows this definition.


Figure 3.7: Shift definition in the new method

In this case, the shifted version of $\mathrm{W}_{0}$ by shift $\Delta$ can be expressed as below:

$$
W_{0}^{(M, n)}(x-\Delta)=\left\{\begin{array}{cc}
2^{M / 2} & 2^{-M}(n)+\Delta \leq x \leq 2^{-M}(n+1)+\Delta  \tag{3.22}\\
0 & \text { elsewhere }
\end{array}\right.
$$

Then, $P_{M}{ }^{4}(f)$ which represents the shifted projection of $f(x)$ by $\Delta$ will have the following expression:

$$
\begin{equation*}
P_{M}^{\Delta}(f)=\sum_{n=0}^{2^{M}-1}<f, W_{0}^{(M, n)}>W_{0}^{(M, n)}(x-\Delta)=2^{-M / 2} \sum_{n=0}^{2^{M}-1} f_{\Delta}^{(M, n)} W_{0}^{(M, n)}(x-\Delta) \tag{3.23}
\end{equation*}
$$

where

$$
\begin{equation*}
f_{\Delta}^{(M, n)}=2^{M / 2} \int_{0}^{1} f(x) W_{0}^{(M, n)}(x-\Delta) d x \tag{3.24}
\end{equation*}
$$

with $f_{\Delta}^{(M, n)}$ represents the intensity of the pixel $n$ from the shifted projection with shift $\Delta$.

In order to characterise the overlap area between the reference pixel and the shifted pixel, we define a new function as below (see figure 3.8):

$$
h_{\rho}^{(M, n)}(x)=\left\{\begin{array}{cc}
0 ; & 2^{-M}(n) \leq x \leq 2^{-M}(n+\rho)  \tag{3.25}\\
2^{M / 2} ; & 2^{-M}(n+\rho) \leq x \leq 2^{-M}(n+1) \\
0 ; & \text { elsewhere }
\end{array}\right.
$$



Figure 3.8: Definition of the functions $h_{\rho}^{(M, n)}(x)$ and $\bar{h}_{\rho}^{(M, n)}(x)$.

This overlap region can be modelled by a function which can be written with the help of the Walsh functions as below:

$$
\begin{equation*}
h_{\rho}^{(M, n)}(x)=\sum_{k} C_{k} W_{k}^{(M, n)}(x) \tag{3.26}
\end{equation*}
$$

with

$$
\begin{equation*}
C_{k}=\int_{2^{-M}(n)}^{2^{-M}(n+1)} h_{\rho}^{(M, n)}(x) W_{k}^{(M, n)} d x \tag{3.27}
\end{equation*}
$$

By the same way we define the complement of $h_{\rho}^{(M, n)}(x)$ by $\bar{h}_{\rho}^{(M, n)}(x)$. In this case we have:

$$
\begin{equation*}
\bar{h}_{\rho}^{(M, n)}(x)=\sum_{k} D_{k} W_{k}^{(M, n)}(x) \tag{3.28}
\end{equation*}
$$

With

$$
\begin{equation*}
D_{k}=\int_{2^{-M}(n)}^{2^{-M}(n+1)} \bar{h}_{\rho}^{(M, n)}(x) W_{k}^{(M, n)} d x \tag{3.29}
\end{equation*}
$$

It can be proven easily that:

$$
\begin{gathered}
C_{0}+D_{0}=2^{-M / 2} \\
C_{1}+D_{1}=0 \quad \text { and } \quad C_{k}+D_{k}=0 \quad \text { for } \quad k \neq 0
\end{gathered}
$$

Using these two functions, it is clear that we can rewrite the shifted version of $\mathrm{W}_{0}$ as follows:

$$
\begin{equation*}
W_{0}^{(M, n)}(x-\Delta)=h_{\rho}^{(M, n+s)}(x)+\bar{h}_{\rho}^{(M, n+s+1)}(x) \tag{3.30}
\end{equation*}
$$

By substituting (3.30) in (3.24) we get

$$
\begin{equation*}
f_{\Delta}^{(M, n)}=2^{M / 2} \int_{0}^{1} f(x) h_{\rho}^{(M, n+s)}(x) d x+2^{M / 2} \int_{1}^{2} f(x) \bar{h}_{\rho}^{(M, n+s+1)}(x) d x \tag{3.31}
\end{equation*}
$$

Then $h_{\rho}^{(M, n)}(x)$ and $\bar{h}_{\rho}^{(M, n)}(x)$ will be extended using (3.26) and (3.28) with ignoring the terms for $\mathrm{n}>1$.

This can be true only if we assume that $f$ is roughly constant and therefore the product of $f$ with $W_{n}$ for $n>1$ is null as we will add and subtract the same value in each time. Therefore (4.31) can be written as:

$$
\begin{align*}
& f_{\Delta}^{(M, n)}=2^{M / 2} C_{0} \int_{0}^{1} f(x) W_{0}^{(M, n+s)}(x) d x+2^{M / 2} C_{1} \int_{0}^{1} f(x) W_{1}^{(M, n+s)}(x) d x+\ldots  \tag{3.32}\\
& \ldots 2^{M / 2} D_{0} \int_{1}^{2} f(x) W_{0}^{(M, n+s+1)}(x) d x+2^{M / 2} D_{1} \int_{1}^{2} f(x) W_{1}^{(M, n+s+1)}(x) d x
\end{align*}
$$

From (3.32) it can be shown that we have related the intensities of the shifted projection to the intensities of the reference projection as well as the unknown coefficients. The calculation of the relative coefficient for a pixel $n$ requires the coefficient of the pixel $n+1$. Therefore, when solving for $\left\langle f, W_{l}\right\rangle$ from (3.32), one need some assumptions on the values of $f$ outside their scene referred to as boundary conditions. Some people imposed zero boundary condition outside the scene in such situations. We shall show later in our case that assuming a constant evolution of the intensities gives better-reconstructed highresolution image than by assuming zero-intensities beyond the boundaries.

In order to tackle the problem of recurrence, we can start the calculations of $\left\langle f, W_{l}\right\rangle$ from the last pixel towards the first pixel by assuming that these coefficients will vanish beyond the boundary of the image since $f$ is constant and therefore we will be adding and subtracting the same value. In this case equation (3.32) can be simplified as below for the last pixel $n$ :

$$
\begin{align*}
& f_{\Delta}^{(M, n)}=2^{M / 2} C_{0} \int_{0}^{1} f(x) W_{0}^{(M, n+s)}(x) d x+2^{M / 2} C_{1} \int_{0}^{1} f(x) W_{1}^{(M, n+s)}(x) d x+\ldots  \tag{3.33}\\
& \ldots 2^{M / 2} D_{0} \int_{1}^{2} f(x) W_{0}^{(M, n+s+1)}(x) d x
\end{align*}
$$

The reconstruction can therefore be performed pixel by pixel starting from the last pixel and moving towards the first pixel. Equation (3.32) can be written in the general simplified form by:

$$
\begin{equation*}
<f, W_{1}^{(M, n+s)}>=\frac{\left.f_{\Delta}^{(M, n+s)}-C_{0} f^{(M, n+s)}-D_{0} f^{(M, n+s+1)}-2^{M / 2} D_{1}<f, W_{1}^{(M, n+s+1)}\right\rangle}{2^{M / 2} C_{1}} \tag{3.34}
\end{equation*}
$$

With $f_{\Delta}^{(M, n)}, f^{(M, n+s)}$ and $f^{(M, n+s+1)}$ are the intensities of the $(n+s)^{t h}$ shifted pixel, $(n+s)^{t h}$ reference pixel and the $(n+s+1)^{\text {th }}$ reference pixel respectively. For a sub-pixel shift $\rho$, we have $C_{0}=1-\rho ; D_{0}=\rho ; C_{1}=-\rho$ and $D_{1}=\rho$.

The next step following these calculations is to replace these calculated coefficients in (3.20) and construct the unknown high-resolution image pixel by pixel by adding and subtracting the missing information from each corresponding low-resolution pixel.

It can be seen from (3.34) that for the particular case where $s=2^{M}$, we need to guess the information beyond the boundary limit of the image. Some interpolation schemes can be used in this case such us assuming that the function is constant or null. Other sophisticated ideas can be proposed. Some experiments using the same signal used in figure 3.3 are presented in figure 3.9 below for this variety of interpolation methods in order to guess the intensities beyond the boundaries.

As an example of reconstruction using the same function defined up, we present in Figure 3.9, the result of reconstruction from two shifted projections of sixteen (16) pixels to form a new projection of 32 pixels. In this example, we are assuming that we know exactly the information beyond the boundaries, as we know the function itself, and this case, no information will be lost during the reconstruction process.


Figure 3.9: Result of reconstruction from 16 pixels using the exact information beyond the boundary as we know the function


Figure 3.10: Difference between the reconstructed and the original function.

If we assume that $f$ beyond this boundary is constant then this means that we will keep the last graylevel for all the coming pixels. The result of reconstruction is shown in the following figure 3.11, and the difference between the original signal and the reconstructed signal is shown in figure 3.12. It is obvious that we might loose some information due to this boundaries condition assumption.


Figure 3.11: Result of reconstruction assuming $\boldsymbol{f}$ constant beyond the boundary.


Figure 3.12: Difference between the reconstructed signal and the original signal.

Table 3.1 below shows the MSE between the original image and the reconstructed image in both cases where we assume that the function remain constant beyond the boundary and the second case where we assume no information beyond this limit. The results justify our choice by assuming the scene remains constant beyond the boundary. The reconstruction is performed line by line for each image used in these simulations.

| Image | MSE(Original, Result) <br> No information beyond the limit | MSE (Original, Result) <br> Assuming f constant |
| :---: | :---: | :---: |
| Camera man | 0.2712 |  |
| Door | 0.1974 | $8.9307 \mathrm{e}-04$ |
| F16 plane | 0.6400 | $6.4864 \mathrm{e}-04$ |
| House | 0.2720 | 0.0020 |
| Lena | 0.3048 | 0.0022 |
| Sydney | 0.0977 | 0.0017 |
| London | 0.2506 | 0.0140 |

Table 3.1: MSE calculations for boundary condition assumptions


Figure 3.13: Some of the test images used in the experimentation

### 3.3 Experimentation using test images

The results of reconstruction using this new method compared to those obtained by Tsai and Huang methods are compared in Table 3.2. In our case, the reconstruction is performed line by line from only two input images. We evaluate the performance of the different techniques by calculating the Peak Signal to Noise Ratio (PSNR); which is defined as:

$$
P S N R=10 \log \frac{255^{2}}{(X-\hat{X})^{2}}
$$

where, $X$ is the original image and $\hat{X}$ is the reconstructed image. The PSNR results are tabulated in Table 3.2. A set of known test image in image processing are used in this experimentation.

| Image | Tsai \& Huang | Our approach |
| :---: | :---: | :---: |
| F16 plane | 32.7416 | 81.1574 |
| Aerial | 28.7447 | 58.7816 |
| Camera man | 33.2060 | 83.5098 |
| Door | 39.0598 | 83.0204 |
| House | 37.8371 | 79.2653 |
| Lena | 34.6244 | 85.5043 |
| Osaka | 27.6844 | 58.0891 |

Table 3.2: Comparison of PSNR values for different images

We can see that the proposed method performs well for various classes of images. This can be shown by the PSNR improvements. Even though a little amount of artefacts in the reconstructed image, which is due essentially to the boundary condition, the overall quality is good as sharp edges were preserved in the result. The effect of the boundary condition can be overcome by some post-processing techniques, which have been detailed above. More results using real images will be addressed in the chapter 5 with more
analysis and details. Below we illustrate the result obtained for the plane image using Tsai and Huang method from four LR images shifted by half pixel in all directions and the result obtained using our algorithm using only two shifted images horizontally by the same shift, half pixel.


Figure 3.14: left) reconstruction using Tsai and Huang method; right) result of our method

The next chapter will deal with the problem of registration and introduce the new measure we propose to estimate the translation shift between the input images illustrated with some examples of registration.

### 3.4 Conclusion

In this chapter we have presented a new approach to merge a pair of shifted images, based on the orthogonal set of Walsh functions. We have developed all the mathematics behind this approach and presented some examples of reconstruction for one dimensional case. This method is very simple and very flexible for future developments. A good accuracy can be achieved depending on other parameters relative to the Super-Resolution process. The registration problem is one of these problems. The next chapter deals with the registration problem in general for satellite images and particularly for our algorithm.

## Chapter 4

## 4 Geometric Corrections and Image Registration of Satellite Images

### 4.1 Introduction

Image registration is the process by which we seek to align two or more images of the same scene taken at different times by the same sensor or by different sensors. The most crucial task in image registration is to deal with geometric distortions between the images. These distortions are due to different imaging conditions or other internal parameters relative to the sensor used. Typically, image registration is a key step in a lot of image analysis applications in which the resulting information is obtained by combining various data sources. Examples of applications include Multispectral classification, image fusion, image mosaicing, and medical imaging.

In general, image registration is defined by the process of superimposing two images and transforming one of them on top of the other to find the best transform to make them match. A large range of image registration techniques has been well categorized in [57,58]. Our motivation in this chapter is to describe the image registration problem and accurate solutions suitable for Super-Resolution process. We shall also discuss different problems associated with satellite images, which are related to our proposed approach to Super-Resolution.

In order to extract sub-pixel information from the images, each low-resolution image must be accurately registered with respect to a reference frame. Generally, the first frame will be chosen as a reference image. Example of registration with sub-pixel accuracy is proposed by the authors in [59].

In this chapter, we describe some of the common imaging problems encountered in satellite data and which need to be taken into consideration in the Super-Resolution
process. We propose a new measure based upon the sharpness property of edges in the reconstructed image using the new algorithm and which will be used to perform the registration of translated images. We present experimental results verifying our claims and finally, examine the effect of registration errors on the proposed method for SuperResolution to analyze the performance of this new method for Super-Resolution restoration.

Remotely sensed data is usually affected by two types of distortions: radiometric distortions and geometric distortions [60,61,62]. The radiometric distortions result from the effects of the atmosphere and instrumentation errors. These errors can be generally corrected if we know the sensor model used.

The Sources of geometric distortions are:

- Rotation of the Earth during the imaging process
- The finite scan rate of the sensors
- The wide field of view of some sensors
- Curvature of the Earth
- Possible sensor non-idealities
- Variation in satellite attitude and velocity
- Panoramic effects related to the imaging geometry

We shall explain these points in the next section and the effect on the DMC satellites as we may use images provided by these satellites for our simulations.

### 4.2 Sources of geometric distortions

### 4.2.1 Earth rotation Effects

Line scan sensors such as those used in the DMC micro-satellites take a finite time to acquire an image. During the image acquisition time, the Earth rotates from the west to the east so that a line imaged at the end of the image will be further to the west when we start recording the next image. Figure 4.1 illustrates schematically this error. Therefore, it is necessary to offset the bottom of the image to the east by the amount of movement of the ground during image acquisition.


Figure 4.1: Earth rotation effect on imaging geometry
The amount by which the image needs to be skewed to the east at the end of the recorded image depends upon the relative velocities of the satellite and the Earth and also the length of the image recorded. An example is presented for the case of UoSat12 ([19]). In this case, an image requires 360 ms to be recorded. During this time, the surface of the Earth moves to the east by $0.0015^{\circ}$, which is equivalent to 0.1669 km on the ground and in terms of number of pixels, is about 5 pixels. Consequently, the image will contain $0.48 \%$ skew to the east. For Alsat1, the skewing is very small, better than UoSat12 and this is due to the small time needed to record one line ( 5 ms ).

### 4.2.2 Panoramic Effects

Because the field of view of the sensor on the satellites is constant, the effective pixel size on the ground is larger at the extremities than at nadir as illustrated below in figure 4.2. In this case the registration process will be affected strongly because we are comparing two different pixels of different sizes.


Figure 4.2: Panoramic effect

With $\beta$ the Instantaneous Field of View (IFOV) of the satellite and $h$ its altitude. In this case, we can see that the pixel size $P_{\theta}$ at a scan angle $\theta$ can be defined by:

$$
P_{\theta}=h \beta \sec ^{2}(\theta)
$$

But,

$$
P=h \beta
$$

Therefore:

$$
P_{\theta}=P \sec ^{2}(\theta)
$$

For small values of $\theta$, it is clear that the effects of this distortion are negligible. For Alsat1 with $\mathrm{FOV}=26.62^{\circ}$ the distortion in the pixel size along the scan line is 1.06 P and this means that the ground pixel imaged at the extremities of the scan is 1.06 larger than the pixel at nadir and therefore for Alsatl images, this error needs to be corrected before the reconstruction process as we are dealing in the super-resolution with sub-pixels. Actually, this error is not corrected in the released images and one way to avoid this problem is to target in the reconstruction only those pixels located at the center of the image and avoid the extremities. It must be noted also, that we are assuming that the objects observed are acquired under orthographic projection.

### 4.2.3 Earth Curvature

Generally, satellite systems at low altitude and with small absolute swath width on the ground, are not affected strongly by the Earth's curvature. However, wide swath space imaging systems are affected. Figure 4.3 below shows that the effective swath at a given angular field of view is larger than if the curvature of the Earth is ignored.

It can be demonstrated that the real pixel size is given by:

$$
P c=\beta\left[h+r_{e}(1-\cos (\phi))\right] \sec (\theta) \sec (\theta+\phi)
$$



Figure 4.3: Earth curvature effect
For UoSat12 with $\theta=0.0252^{\circ}$ and $\phi=0.1490^{\circ}$, this shows that the effective pixel size is 1.0154 times than that at nadir which means that this error is less significant in LEO orbits but in our case this needs to be corrected for. This error is not corrected in the actual DMC images.

### 4.2.4 Scan Time Skew

Line sensors employing a pushbroom image capture architecture require a finite time to scan across the swath. During this time of scanning the satellite is moving forward causing a skewing in the along track direction. In Alsatl for example, the time to record one line of image is 5 ms . During this time the satellite moves forward at an equivalent ground velocity of $7.51 \mathrm{~km} / \mathrm{s}$. As a result the scan line is advanced by 1.15 pixel size compared with the one before and for this reason we need more images of the same scene in the reconstruction process in order to compensate for the missing ground-pixels.

### 4.2.5 Variation due to Attitude and Orbital Eccentricity

Satellites de not move on precisely circular orbits, and the small eccentricity in the orbit causes a variation in the altitude of the satellite. This leads to a scale change of the pixels as well as the difference in altitude; the eccentricity also causes small changes in the along-track velocity of the satellite, which changes the separation of neighbouring lines of the image. On top of this, the spacecraft has some residual angular velocity which causes the satellite to rotate slightly during image capture. Depending upon the orientation of this
rotation vector, we obtain different types of distortion to the pixel. Schematically this can be illustrated in the figures below (see figure 4.4).


Figure 4.4: Variation in platform altitude, velocity and attitude effects

### 4.2.6 Correction of Geometric Distortions

From what we have presented above, we can see that that the geometric distortions in satellites images are due to many reasons. These distortions can be divided into systematic and non-systematic errors. In most of the cases, it is necessary to correct for these distortions before any manipulation of the images. The geometric correction involves relating the coordinates image of a pixel ( $i, j$ ) to the corresponding spatial coordinates on the Earth's surface. In practice, the Ground Control Points (GCPs) are used to determine the relationship between these two spaces. These points can be understood as points which we can easily identify in the image and for which we know exactly the spatial coordinates on the Earth's surface. There are many ways to estimate these coordinates and the GPS (Global Positioning System) is the most used tool to do that. The precision of the correction depends on the choice of the GCPs. The relationship between the coordinates of a pixel in the image and of the corresponding point on the ground can be expressed analytically. As an example, for a pixel $\left(x_{i}, y_{i}\right)$ in the image and the coordinates of the corresponding point on the surface $\left(x_{s}, y_{s}\right)$, we have:

$$
\begin{aligned}
& x_{s}=f\left(x_{i}, y_{i}\right) \\
& y_{s}=g\left(x_{i}, y_{i}\right)
\end{aligned}
$$

The functions $f$ and $g$ will depend on the type of relationship we assume between both spaces. The simplest model is the general linear model represented by:

$$
\begin{aligned}
& x_{s}=a_{1}+a_{2} x_{i}+a_{3} y_{i} \\
& y_{s}=a_{4}+a_{5} x_{i}+a_{6} y_{i}
\end{aligned}
$$

Although much more complicated models can exist, this model can allow for a shift of origin, rotation, scaling in $x$ and $y$ and some types of skewing. With six unknowns in the system of equations, we need at least three GCPs. The choice of more than three is better to deal with the noise and undetermined cases. The estimation of these parameters is generally carried out using a Minimum Least Squares formulation. Errors that can only be accounted for by the use of GCPs include the Roll, Pitch and Yaw of the satellite or the altitude. Other geometric errors that can be corrected using sensor characteristics and ephemeris data include scan skew, panoramic distortion, platform velocity and perspective geometry.

As a conclusion, the accuracy expected from the Super-Resolution algorithm can not be achieved without the corrections of these geometric errors which we have detailed some of them above. Many techniques are nowadays used and which have achieved acceptable accuracy in remote sensing area. This point needs more investigation in the future work.

### 4.3 Satellite Image Registration

### 4.3.1 Definitions

Image registration is the process of overlaying two images of the same scene but taken at different times and/or by different sensors. In general, this notion can be defined by the process of superimposing two images and transforming one of them to the other to find the best transform to make them match. A large range of image registration techniques for satellite images has been well categorized in the literature such as those proposed by [59]. Due to the diversity of the images manipulated and the various types of distortions it is difficult to propose a unified methodology for image registration. Every method used
should take into consideration the assumed type of geometric distortions, the radiometric deformations and the required accuracy of the registration process.

Nevertheless, most of the registration methods consist of the following common steps [63]:

- Feature detection: distinctive objects are manually or automatically detected. These features can be represented by their point representatives like, centers of gravity, line endings, etc, called control points (CPs).
- Feature matching: at this step, the relationship between the features detected in the sensed image and the reference image is established. Similarity measures are often used for that purpose.
- Transform model estimation: here, the parameters of the mapping functions, aligning the sensed and the reference images are estimated.
- Image resampling and transformation: the sensed image is transformed using the estimated mapping functions on a chosen grid. Interpolation techniques are used then to estimate image values at non-integer coordinates.


### 4.3.2 Registration methods

### 4.3.2.1 Correlation methods

Cross-correlation is the basic statistical approach used in image registration. It gives a measure of the degree of similarity between the sensed image $I$ and the reference image $R$. The two-dimensional normalized cross-correlation measures the similarity for each translation [58].

$$
C(u, v)=\frac{\sum_{x} \sum_{y} R(x, y) I(x-u, y-v)}{\left[\sum_{x} \sum_{y} I^{2}(x-u, y-v)\right]^{1 / 2}}
$$

The cross-correlation will have its peak at the best match at a translation of $(i, j)$. It can be noted here that this measure can be affected strongly in the presence of noise as in the presence of noise, the peak may not be discernible.

### 4.3.2.2 Fourier methods

It is possible to use the properties of the Fourier Transform for image registration. One method to align two images shifted with respect to each other is to use the phase correlation. Given two images $I_{1}$ and $I_{2}$ displaced by $\left(d_{x}, d_{y}\right)$,

$$
I_{2}(x, y)=I_{1}\left(x-d_{x}, y-d_{y}\right)
$$

Their corresponding Fourier Transforms $F_{1}$ and $F_{2}$ will be related by:

$$
F_{2}\left(w_{x}, w_{y}\right)=e^{-j\left(w_{x} d_{x}+w_{y} d_{y}\right)} F_{1}\left(w_{x}, w_{y}\right)
$$

which means that the images have the same Fourier magnitude but a different phase depending to their displacement. From the shift theorem, the phase difference is equivalent to the phase of the cross-power spectrum:

$$
\frac{F_{1}\left(w_{x}, w_{y}\right) F_{2}^{*}\left(w_{x}, w_{y}\right)}{\left|F_{1}\left(w_{x}, w_{y}\right) F_{2}^{*}\left(w_{x}, w_{y}\right)\right|}=e^{\left(w_{x} d_{x}+w_{y} d_{y}\right)}
$$

where * is the complex conjugate. The inverse Fourier Transform of the phase difference is a delta function centered at the displacement. Locality of the peak of the inverse Fourier Transform of the cross-power spectrum phase will give us the shift. Fourier methods can be more effective in the presence of frequency dependent noise but high time consuming.

### 4.3.2.3 Point mapping

The general idea of registration in this approach consists first of computing point features. Secondly, feature points in the reference image are corresponded with feature points in the sensed image. Finally, a spatial mapping, generally a 2D polynomial function, is determined based on Least Squares estimation. The resampling is performed with applying the spatial mapping and an interpolation scheme. More details on the choice of the control points can be found in [58]. Figure 4.5 illustrate the diagram of the registration algorithm using control points.


Figure 4.5: Registration using the GCPs

It must be noted here that the fundamental characteristic of any image registration technique is the type of spatial transformation present. Although many types of distortions can be present in the images, the registration technique must use the class of transformation, which will remove the spatial distortions between images. Examples of these transformations are: affine, projective, perspective and polynomial which we detailed above. One of the problems using this method of registration is that it is difficult to automate the definition of the GCPs.

### 4.4 Image Projection Models

The principal characteristic of any image registration technique is the type of spatial transformation or the mapping model needed to overlay two images. Although in many imaging systems, acquired images are subject to many geometric distortions, the registration technique must use the class of transformation, which will remove only the spatial distortions between the images due to differences in acquisition and not due to differences in scene characteristics. The elementary general transformations used are Affine, Projective, Perspective and Polynomial. In this paragraph, we will define briefly these transformations and their properties.

A transformation $F$ is linear if:

$$
\begin{gathered}
F\left(x_{1}+x_{2}\right)=F\left(x_{1}\right)+F\left(x_{2}\right) \\
F(a x)=a F(x)
\end{gathered}
$$

for some constant $a$.
A transformation is affine if $F(x)-F(0)$ is linear, and linear transformations map straight lines into straight lines. The most popular registration transformation is affine, which is sufficient to match two images of the same scene taken from the same sensor from different positions. This affine transformation is composed of the Cartesian operations of scaling, translation, rotation and shearing. It is a rigid transformation since the whole geometric relationship between points do not change. This is an important advantage for this transformation. It typically has four parameters $t_{x}, t_{y}, s, \theta$ which map a point $\left(x_{1}, y_{l}\right)$ of the input image to a point $\left(x_{2}, y_{2}\right)$ of the second image as follows:

$$
\binom{x_{2}}{y_{2}}=\binom{t_{x}}{t_{y}}+s^{*}\left(\begin{array}{cc}
\cos (\theta) & -\sin (\theta) \\
\sin (\theta) & \cos (\theta)
\end{array}\right)\binom{x_{1}}{y_{1}}
$$

The general 2D affine transformation is:

$$
\binom{x_{2}}{y_{2}}=\binom{a_{13}}{a_{23}}+\left(\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right)\binom{x_{1}}{y_{1}}
$$

with,

$$
A=\left(\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right)
$$

The matrix A can be:

$$
\begin{array}{ll}
{\left[\begin{array}{cc}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{array}\right], \text {,otation }} & {\left[\begin{array}{cc}
S_{x} & 0 \\
0 & S_{y}
\end{array}\right], \text { Scale }} \\
{\left[\begin{array}{ll}
1 & a \\
0 & 1
\end{array}\right], \text { Shearing in } x \text { direction }} & {\left[\begin{array}{ll}
1 & 0 \\
b & 1
\end{array}\right] \text {, Shearing in } y \text { direction }}
\end{array}
$$

Figure 4.10 shows an example of image registration using three transforms: rotation, $x$ axis translation and y-axis translation. In general, image transforms can be categorized based on the geometric transforms for a planar surface element as translation, rotation, scaling, stretching and shearing (see figure 4.6).


Figure 4.6: Elementary image transformations

Translation and rotation transforms are usually caused by the different orientation of the sensor, while scaling transform is the effect of change in altitude of the sensor. On the other hand, the viewing angle causes stretching and shearing. All of these transformations form part of an affine transformation.

The most commonly used models for image projection characteristics are the perspective projection and the orthographic projection.

- Perspective projection

To simplify the explanation of the perspective projection we will make the following assumptions:

1. the center of the projection coincides with the origin of the world
2. the camera axis (optical axis) is aligned with the world's $z$ axis
3. avoid image inversion by assuming that the image plane is in the front of the center of the projection


Figure 4.7: Perspective projection
The perspective transformation accounts for distortion, which occurs when a 3D scene is projected in an optical imaging system. This mapping form 3D to 2D causes an object to
appear smaller the further it is from the camera and more compressed the more inclined it is away from the camera. Assuming that the origins of the world and the image plane coordinate systems are coincident and if the coordinates of the object in the scene are known ( $x, y, z$ ), then the corresponding point in the image ( $X, Y$ ) is given by:

$$
\begin{aligned}
& x=\frac{X f}{Z} \\
& y=\frac{Y f}{Z} \\
& z=f
\end{aligned}
$$

where $f$ is the focal length of the lens and $z$ the distance from the point on the object to the focal point of the imager. As $f$ gets smaller more points are projected into the image plane (wide-angle-camera), and as $f$ gets larger, the field of view becomes smaller (telescopic effect). It can be seen from figure 4.7 that the projection of a point is not unique; any point on the line OP has the same projection. Also, the distances and the angles are not preserved.

- Orthographic projection

The orthographic projection assumes parallel projection from the 3D scene to the image plane as shown in the figure below. Assuming that the origins of the world and the image plane coordinate systems are coincident and $f \longrightarrow \infty$ implies:

$$
\begin{aligned}
& x=X \\
& y=Y
\end{aligned}
$$



Figure 4.8: Orthographic projection

Parallel are projected to parallel lines and sizes do not change with distance from the camera. In the case of remotely sensed images, orthographic projection can be thought of as perspective projection under conditions in which $z$ is very big and roughly constant and $f$ is considered very small relatively. In this case the new coordinates (image plane) are proportional to the input ones (scene plane) so rays joining scene points and image points tend towards being parallel.

- Weak perspective projection

Perspective projection is not a linear transformation. We can approximate perspective projection by scaled orthographic projection if the object lies close to the optical axis and the dimensions of the object are small compared to its average distance $\bar{Z}$ from the camera.


Figure 4.9: Weak perspective projection

The equation of the weak perspective projection is defined by:

$$
\begin{aligned}
& x=\frac{X f}{Z} \approx \frac{X f}{\bar{Z}} \\
& y=\frac{Y f}{Z} \approx \frac{Y f}{\bar{Z}}
\end{aligned}
$$

The scale factor is defined by the term $\frac{f}{\bar{Z}}$.


Figure 4.10: An example of image registration. Here, we have rotation $R, X$ and $Y$ translations
(from [63])

As a conclusion, in Remote Sensing the orthographic projection is usually assumed as a good approximation to the perspective projection. When the objects are relatively far from the camera we can assume that all the lines are parallel to the optical axis and to account for the size differences due to the distance, we introduce a scale factor (weak perspective projection). In our model, we assume orthographic projection and this justifies the fact of using the Walsh functions.

### 4.5 Registration of translated images using the new approach based on the Walsh functions

In order to extract sub-pixel information from the low-resolution images, each frame must be registered with respect to a reference frame to within sub-pixel accuracy. In this part we shall show the effect of registration errors on the super-resolution reconstruction results and introduce a new measure based on these errors, which affect more the edges, and how we are going to use this measure to perform the best estimate for the registration parameters between two input images. We present numerical experimental results verifying our claim.

Equation (3.34) can be simplified in the following way:

$$
\begin{equation*}
X_{n}=Y_{n}+X_{n+1} \tag{4.1}
\end{equation*}
$$

where,

$$
\begin{equation*}
X_{n}=<f, W_{1}^{(M, n)}>2^{M / 2} C_{1} \tag{4.2}
\end{equation*}
$$

and,

$$
Y_{n}=f_{\Delta}^{(M, n)}-C_{0} f^{(M, n)}-D_{0} f^{(M, n+1)}
$$

Where $f^{M, n)}$ is the intensity of pixel $n$ of the reference image and $f_{\Delta}^{(M, n)}$ is the intensity of pixel $n$ of the shifted low-resolution image with shift $\Delta$.

As explained in chapter 3, the reconstruction in this method is based on the accurate estimation of the coefficients $\left\langle f, W_{l}\right\rangle$. However, the estimation of these coefficients is also closely dependant upon the shift parameters used and the effect of noise. It can be seen from equation (4.2) that a small error in the shift estimation can affect the calculation of the unknown $<f, W_{l}>$ s and causes errors to propagate from the last pixel toward the first pixel. This can result in big variations in the $Y_{n}$ 's, therefore resulting in big variations in the vector $X_{n}$ which represents the missing information. By replacing these erroneous coefficients in equation (3.20), it is clear that the reconstructed image will contain more and more banding in each line. This may affect more the edges and results in loss of information.

Figure 4.11 below shows an example of reconstruction from two low-resolution images shifted by half pixel ( 0.5 ) in the x axis, and which were obtained by under-sampling the door image (Figure 4.13). Figure 4.12 shows the result of reconstruction using the same input low-resolution images but assuming a shift of 0.2 . The effect of mis-registration is clearly demonstrated with severe banding and corresponding loss of information.



Figure 4.11: top) Super-Resolution result with true shift, bottom) example of line profile with the missing information

The bottom figure of figure 4.11 shows an example of the image intensities across a line in the low-resolution image and in red the missing information $X_{n}$.


Line 65


Figure 4.12: top) Super-resolution result using wrong shift, bottom) example of line profile with the missing information

This example shows the effect of using an incorrect shift in the estimation of the unknown coefficients $\left\langle f, W_{l}\right\rangle$ which represent the missing information. The original door image is shown in figure 4.13.


Figure 4.13: Original image (door)
We propose to determine the correct shift by using the mean value of the $X_{n}$ as a function of shift. The minimum value for this mean will occur at the correct shift between the images. This is plotted in figure 4.14 for the same line in the door image. We have:

$$
\begin{equation*}
\text { Measure } H=\operatorname{mean}\left(X_{n}\right) \tag{4.3}
\end{equation*}
$$

The mean value of the calculated shifts from each line of the images will provide us the estimated shift between the input images.

### 4.6 Image Registration Experiments

We have tested the accuracy of the method described above for the registration of the low-resolution images. We ran tests on different types of images using different shifts and taking into consideration noise and blur for the second test as well. This is to assess the efficiency of the proposed method in the presence of image degradation. The sequences consist of two simulated images of size $256 \times 128$ obtained by sub-sampling an original image of size $256 \times 256$ in the horizontal axis by a factor of two resulting in half pixel shift between images. The true shift is known. The results of the shift estimation are presented in Table 4.1 for ten very different images. Figure 4.14 shows an example of registration using the "plane F16" image with true shift $=0.5$ pixel.


Figure 4.14: Example of registration using the new measure for F16 image, true shift $=0.5$, estimated shift $=0.5040$

The minimum of the measure occurs exactly at 0.5 which best register the sequence of image.

The second test consisted of degrading the input sequence with a Gaussian white noise with variance of 2 and a blur variance of 0.75 . These parameters are chosen to be low contamination and moderate blur. The true shift was chosen to be different from 0.5 in some experiments. Table 4.2 below presents the results obtained for the registration of the degraded sequence and figure 4.15 shows the result of the reconstruction using true and estimated shift for the plane image.


Figure 4.15: left) result of reconstruction using true shift ( 0.5 ), right) result of super-resolution using estimated shift

| Image | True shift | Estimated shift |
| :---: | :---: | :---: |
| F16 plane | 0.5 | 0.5040 |
| Aerial | 0.5 | 0.4942 |
| Camera man | 0.5 | 0.4987 |
| Door | 0.5 | 0.5016 |
| Sydney | 0.5 | 0.5082 |
| Lena | 0.5 | 0.5002 |
| Blewbury | 0.5 | 0.5016 |
| Baboon | 0.5 | 0.4978 |
| House | 0.5 | 0.4978 |
| Sailboat | 0.5 | 0.5004 |

Table 4.1: Shift estimation test: without degradation

| Image | True shift | Estimated shift |
| :---: | :---: | :---: |
| F16 plane | 0.5 | 0.4898 |
| Aerial | 0.5 | 0.4832 |
| Camera man | 0.5 | 0.4846 |
| Door | 0.5 | 0.4930 |
| Sydney | 0.5 | 0.5161 |
| Lena | 0.5 | 0.5112 |
| Blewbury | 0.5 | 0.4898 |
| Baboon | 0.5 | 0.4791 |
| House | 0.5 | 0.4879 |
| Sailboat | 0.5 | 0.4979 |

Table 4.2: Shift estimation tests: with degradation

Using other shifts, the plane image was used using the same given parameters and the results obtained are presented in Table 4.3 for sub-pixel shifts different from half pixel.

| True shift | 0.15 | 0.25 | 0.75 |
| :---: | :---: | :---: | :---: |
| Estimated shift | 0.1781 | 0.2688 | 0.7135 |

Table 4.3: F16 image: Other examples of shift

Another pair of shifted images was derived from an IKONOS image of Istanbul. The images are shifted by half pixel and were degraded by adding white Gaussian noise with variance 2 and a blur variance of 0.75 . Figure 4.16 shows the plot of the measure versus the shift used in the reconstruction. We can see that the minimum of the plot occurs near the ideal shift which best register the input images.
$\square$



Figure 4.16: Example of registration using second image sequence (Estimated shift=0.5007)

### 4.7 Mean Square Error from mis-registration

In this part, we analyse some qualitative and quantitative effects of the mis-registration on Super-Resolution. We use the plane image shown in figure 4.13 of size $256 \times 256$. We generate two under-sampled and shifted images by half pixel in horizontal direction. $3 \times 3$ pixel Gaussian blur with variance 0.75 was applied to the low-resolution images as well as white Gaussian noise with variance 2 . The down sampling factor was 2 . In this experiment, we examine the effects of mis-registration on Super-Resolution. We simulate registration errors by randomly changing the exact shift. Figure 4.17 shows the MSE as a function of registration parameter. Reconstruction performance degrades as misregistration increases. Naturally, as the error in the shift used increases, the reconstructed image becomes worse with more artefacts along each line.


Figure 4.17: Super-Resolution error in the presence of mis-registration

### 4.8 Conclusion

Image registration is an important aspect required to achieve better super-resolved images. The shift parameters used in this process is very critical and can affect strongly the quality of the reconstructed images. In this chapter, we have shown some of the external factors that may affect the registration of satellite images and shown that these errors have to be corrected in LEO orbits. We introduced also a new measure based on the blurring of the edges during a mis-registration case based on the Walsh functions. For purely translational cases, this measure can be used to estimate the best shift, which registers two input images. We presented some results using degraded low-resolution images. Theses experiments illustrated as expected, that the performance of the reconstruction degrades as registration error increases.

More accuracy is required in the motion estimation by correcting the input images or providing more additional data to become more robust to the mis-registration errors by reducing the effect of noise which can affect the registration process.

## Chapter 5

## 5 Experimental Results Evaluation

### 5.1 Introduction

In chapter3, the new approach for image Super-Resolution has been theoretically presented with all the mathematical details. Extensive preliminary simulations have been carried out on different types of image in order to investigate the feasibility of our proposed method for image Super-Resolution. The projected goal of this research is to improve the resolution improvement for real satellite images. The presented results will provide more proof on that feasibility.

In this chapter, simulations using both real and synthetic image sequences are presented with more details. The practical problems encountered in the reconstruction using real data are discussed.

### 5.2 Examples of reconstruction using test imagery

In this section we illustrate the performance of the method using some test imagery. The first test consists of artificially generated pairs of shifted low-resolution frames. We compare the quality of super-resolution against the original image. The reconstruction is performed line by line from two low-resolution images of size $256 \times 128$, which have been derived from an original image of the F16 plane of size $256 \times 256$. The low-resolution images are obtained by averaging each pair of neighbouring pixels from the original image to form one low-resolution pixel intensity (see figure 5.1 ). We proceed by the same manner for the shifted image but only with one original pixel shift horizontally to obtain a half pixel shift in the low-resolution image. The projection of the line with $2^{(\mathrm{M}+1)}$ refers in these cases to the original intensities and the projections with $2^{\mathrm{M}}$ are the resultant image after this averaging. Figure 5.2 shows these two low-resolution images.

White Gaussian noise has been added to each of the low-resolution frames resulting in a Signal-to-Noise Ratio (SNR) of 30 dB . This is to simulate differences in pairs of images
to be combined for higher resolution, while at the same time providing us ground truth knowledge of the shift and the information content we should ideally see in the higher resolution image. The estimated shift by using the proposed measure earlier is equal to 0.4898 .

For comparison purpose only, an interpolated image from one of the low-resolution images was estimated and figure 5.3 shows this interpolated image together with the result obtained by our Super-Resolution technique. The quality of reconstruction is comparable to that of the original image only with two input low-resolution images. It can be easily seen that the image result is sharper and contain more details. Table 5.1 shows the PSNR calculations for this simulation. The differences in Peak-Signal-to-Noise ratio (PSNRs) between the interpolation schemes and the Super-Resolution approach are significant, and increase with the accuracy of the estimated shifts.


First pixel of the first shifted LR image by $1 / 2$ pixel
Figure 5.1: The Low-resolution images extraction principle.


Figure 5.2: Shifted low-resolution images


Figure 5.3: (upper left): original image of the F16 plane; (upper right): Bi-cubic interpolated image from one LR frame; (lower left): result of SR with estimated shifts; (lower right): result of SR with true shift

| Bi-cubic interpolation | SR (estimated shift) | SR (true shift) |
| :---: | :---: | :---: |
| 59.1659 dB | 74.1276 | 82.3180 dB |

Table 5.1: PSNR calculations

It is clear from these results that the new approach performs considerably better with more accuracy on the shift estimation. We note the increased legibility of the "F16" on the tail of the airplane as well as the words "Air Force". The PSNRs values for all the cases are listed in Table 5.1. The differences in PSNRs between the bi-cubic interpolation and the new method for SR are significant and increase with more accuracy of the estimated shift. It must be noted here that the use of statistical measures such as the MSE and the PSNR is not sufficient to compare the quality of the reconstructed image, as they do not measure the way images are interpreted by the human eye.

In order to test the effect of blurring on the reconstruction, an aerial image of size $256 \times 256$ was used in a second experiment and two shifted and degraded low resolution images were extracted by the same method as in the first experiment. These lowresolution images were then blurred by a $3 \times 3$ Gaussian linear space-invariant filter with variance of 1. Additive noise was added resulting in a blurred SNR of 30 dB for each image. The degraded images are shown in figure 5.4 and the results of the reconstruction are shown in figure 5.5. A zoom on a chosen area in the super-resolved image and in the interpolated image is shown in figure 5.6.


Figure 5.4: Degraded low-resolution images from aerial image


Figure 5.5: (upper left): original image aerial photo; (upper right): Nearest interpolated image from one LR frame; (lower left): result of SR with true shift; (lower right): result of SR with estimated shift


Figure 5.6: example of zoom on an area in the super-resolved image (left) and the interpolated image (right)

Note the additional details afforded by the Super-Resolution method in the presence of noise and blur compared to the interpolated image. Furthermore, no deblurring step was performed here and we expect more sharpness could be achieved by deblurring the images before reconstruction. The PSNR for the bi-cubic image and the SR reconstructed image using the wrong shift are 48.5482 and 48.0607 respectively. In terms of sharpness, we can see from the figure that the super-resolved image is sharper than the interpolated image.

### 5.3 Image noise

One of the common problems in image processing is how to recover an original image from a noisy image [64]. Many algorithms have been proposed in this field and usually a best estimation of the noise variance $\sigma^{2}$ is required to perform a good reconstruction. For the estimation of the noise variance several methods have been proposed [64,65,66]. In most of the cases the accuracy obtained is not satisfactory and depends on the requirements of each application.

There are many sources for noise in remotely sensed images. The first one: photon noise, which is unavoidable and caused by the quantum nature of light. The number of photons that arrive at a detector element is Poisson distributed, according to the literature [67], with expectancy equal to the intensity of the light, and variance equal to the expectancy. This means that this type of noise is not independent of the signal, not additive and not normally distributed. The Readout noise is also another form of the noise in remote sensing and affects CCD cameras. This can be caused by the camera's electronics. This component is additive and normally distributed. It increases as the readout rate increases and this explains why it is not possible to increase the readout frequency. The quantization noise is also another encountered problem in satellite digital imagery. This noise is produced by the quantization of the pixel amplitudes into a finite number of graylevels. One way to reduce these effects is to average the input data (figure 5.7). More data provided will certainly help to reduce the effect of noise in the reconstruction.


Set of 4 noisy images whose PSNR is approximately 45.8786 dB


Noisy-filtered image; PSNR $=46.6267 \mathrm{~dB}$
Figure 5.7: Image averaging to reduce noise

### 5.4 Experimentation using real images

### 5.4.1 Alsat1 imaging model

One of the objectives of this research is to apply our Super-Resolution technique to satellite images in general and SSTL data in particular. The idea was to use Alsat1 images. Alsat1 constitutes the first DMC micro-satellite designed and built by SSTL and was launched on the $28^{\text {th }}$ November 2002 (figure 5.8). The platform of the satellite is an enhanced microsatellite of size $640 \times 640 \times 680 \mathrm{~mm}$, and weighting 100 kg . A low thrust butane propulsion system is carried to enable corrections after the launch, to separate individual spacecraft along the orbit, to perform stationkeeping manoeuvres and finally an end-life manoeuvre to take the spacecraft out of its orbit. More technical data can be found in [19].Our concern here is the imaging payloads. The payload comprises two individual athermalised instruments, each with three linear array imagers with NIR (Near Infrared), R (Red) and G (Green) filters (figure 5.8 right) filters. The instruments are placed side by side with a slight overlap in the field of view, forming a 3-band 20,000 pixel imager. The swath width is about 600 km with 32 m Ground Sampling Distance (GSD). Scenes are defined as $100 x 100 \mathrm{~km}$ size in three bands, and software windowing
functions are used allowing tuning the desired area. Each instrument is cross-connected to two 4Gbit data recorders, and a third functionally reserve 1 Gbit data recorder. Each instrument can be powered separately. This satellite has taken many images, verifying its remote sensing capability.


Figure 5.8: (left): Alsat1; (right) SSTL DMC imager ${ }^{\circ}$
The imager uses a push broom technique during imaging. This imager is a scanner without any mechanical scanning mirror and records one line of an image simultaneously. Figure 5.9 shows this principle. The motion of the satellite is used to move the image line across the surface of the Earth, and the CCD is read out continuously to form one line of the image. Time between two recorded lines is about 5 ms .


Figure 5.9: Push broom imaging principle used in Alsat1

Figure 5.10 below illustrates clearly the imaging geometry in Alsat1 images with all the required parameters for our discussion for the choice of input data in the SuperResolution experimentation.


Figure 5.10: Illustration of the Alsat1 imaging geometry

It can be seen from Figure 5.10, that the actual geometry enables us to observe the same area from two different points only in the overlap region, which is of size 500 pixels representing $5 \%$ of the image width. Therefore, two different images from different points of view cover the same ground area. This overlap region of the two images provides an ideal opportunity to perform Super-Resolution. The overlap region is 500 pixels wide and concerns the full length of the array. An image containing the city of Las Vegas was selected for an experiment to perform Super-Resolution. The size of the images was $16600 \times 10000$ (see Figure 5.11 for an example of the images). The regions contributing to the overlap area in left and right images were extracted. Figure 5.12 shows these images and shows the integer offset which exists between the left and right images and which we need to correct for before the Super-Resolution process. It must be noted here that these are raw images, which have not been corrected for any distortions or deformation.


Figure 5.11: Image from right camera by Alsat1 captured containing the city of Las Vegas. (Near-Infra-Red channel, size of the image: $10000 \times 16600$ )


Figure 5.12: Left and Right image-segments from the overlap area

### 5.4.2 Results of reconstruction using Alsat1 images

First the registration step was performed using the new measure developed in chapter 4, using both images described above of size $1500 \times 497$. The estimated shift was 0.75 in the horizontal axis. Figure 5.13 shows the profile of line 750 in the reference image (left) with a blue colour and profile of the same line in the shifted image (right) with a red colour. From this profile, the first problem that can be noticed is the fluctuations in intensities of neighbouring pixels in the left image. This problem can be corrected for by smoothing the data, and a smoothed version of this data is shown at the bottom of figure 5.13 using a Sobel type smoothing filter. Sobel filter is a simple approximation to the concept of gradient with smoothing. The smoothing in direction $x$ is done with this filter by a convolution with the following mask [112ll $\left.\begin{array}{lll}1 & 1\end{array}\right]$.


Figure 5.13: top) Raw intensities profile of line 750 from the reference and shifted images; bottom) Intensities profile of the same line smoothed with Sobel filter

The difference in intensities between the corresponding pixels in the reference and the shifted images shows some anomalies in this data and this can only be due to the presence of noise in these images. These large differences cause a divergence in equation (4.2) which results in over-estimation of the unknown coefficients $\left\langle f, W_{1}\right\rangle s$ and therefore performing poor reconstruction. To improve the reconstruction, we require more preprocessing of the input images and corrections for geometrical distortions. Basically, this can be understood by normalizing the intensities of the shifted image according to the reference image as these images are shifted by less than one pixel shift. If this balance is not preserved, the vector of $Y_{n}$ 's will deviate from the $\mathrm{x}=0$ axis and cause accumulative errors in the $\left\langle f, W_{l}\right\rangle$ estimation. It must be noted also here, that we are assuming only translation in one direction and have ignored any offset in the orthogonal direction. The transformation between the images may also not be resumed to pure translation, which will also affect the result of the reconstruction. Some of the possible transformations which may exists have been discussed in Chapter 4. More accurate results require more investigation of other types of transformations that exist between two images from the left and the right side of the imager. Unfortunately, there has not been enough time in this thesis to investigate this further.

Figure 5.14 shows an example of the calculated vector $Y_{n}$ for line 750 as well as the estimated vector $\left\langle f, W_{l}\right\rangle$ which represents the missing information. This shows how badly the errors in $\left\langle f, W_{l}\right\rangle$ can affect the reconstruction process. This effect may be reduced by analyzing the information contained in the unshifted images to normalize the intensities of the shifted image by applying a threshold on these intensities. However, this can change the input data and can result in losing information which we are trying to exploit in the reconstruction process.

Vector $Y_{n}$ for line 750


Vector $X_{n}$ for line 750


Figure 5.14: (top) $Y_{n}$ vector for line 750; (bottom) corresponding $\left\langle f, W_{l}>\right.$ 's for the same line: we can notice the divergence of the $<f, W_{l}>\mathbf{s}$ which represents for us the missing information

From these results and due to the impossibility to get some experimental data from Alsat1 which satisfy our requirement in the super-resolution reconstruction process, it was impossible to prove the resolution enhancement using these images.

### 5.4.3 Acquisition of real sequences

A third test sequence consisted of real images captured using one of the SSTL DMC satellite cameras mounted on a optical bench in the University campus. The shift between images could therefore be set manually to half pixel, and so provide real data with ground truth. These images were used along with the known shifts to construct a high-resolution image from the sequence. The enhancement factor sought was two in the horizontal direction.

The camera used in this experiment has a focal length of 150.9 mm . The array is made of CCD pixels of size $7 \mu$. The whole camera system was mounted on a rotatable table monitored by a high precision controller Servo Drives Ultra3000. The whole system is placed on a stable bench. The Ultra3000 is a family of flexible, high performance digital servo drives for a wide range of motion control applications. The resolution of the rotation is about 0.36 arcsec. Figure 5.15 shows the set-up for the image acquisition.


Figure 5.15: Scenario of images acquisition

The focal length of the camera used is 150.9 mm . The width of a CCD pixel is 7 microns. A resolution chart with pixel size 60 mm was placed on a distance of 1.26 km . Figure 5.16 below illustrates the imaging geometry for the SSTL camera.


Figure 5.16: Imaging geometry for the camera used: one ground pixel corresponds to a rotation of 10 arcsec of the table

Before acquiring the images, we were obliged to synchronize the motion of the table with the readout of the pixels in the camera. The velocity of the table was set to 16 counts per second ( 1 count $=1 \mathrm{mDeg}$ ) which is equivalent to $1887.8 \mathrm{arcsec} / \mathrm{sec}$.

Therefore for one pixel shift, we need to rotate the table by:


The pixel clock readout is measured in terms of number of counts, knowing that, according to the camera used we can get 800 ns for 1 count. Thus, we need to setup the number of counts to 6625 to ensure the acquisition of one pixel within 5.3 ms .

### 5.4.3.1 First set acquisition

The images were captured first one by one but starting at different offsets equivalent to half pixel shift for the shifted image ( 5 arcsec angle rotation for the table). Figure 5.17 shows an example of two captured images, of size $1024 \times 768$, shifted by half pixel.

Figure 5.18a shows the profile of the line 645 from the reference image with blue line, the profile of the same lime in the shifted image with red line, and Figure 5.18 b shows an expanded view of part of the line. We can notice on this plot some differences between the corresponding pixels from both images.

The estimated time between two captured images was estimated to be approximately 1 minute. However, the result of reconstruction shows some errors which are due principally to the difference in terms of graylevels between the input images and which causes the divergence of the $\left\langle f, W_{l}\right\rangle$ s. Example of scene changes is presented in figure 5.19. Figure 5.20 shows in blue the low-resolution intensities for line 645 and in red the estimated $<f, W_{l}>$ for this line. The second side of this figure shows how actually the result looks like and on which we can see this severe banding on the reconstructed image. This noise may be due to the CCD pixels response, to the readout process caused by the camera electronics or to the change in atmospheric conditions, This noise component is additive and normally distributed (figure 5.21 ). Other sources of noise may be involved in such situation.


Figure 5.17: Two real captured images shifted by $1 / 2$ pixel

Below are the line profiles (ex. line 645) from the unshifted image and the shifted image. We notice here some non-homogeneities of the data in some locations although the shift between images is only half pixel.


Figure 5.18: a) Intensities profile for unshifted image (blue) and for the shifted image (red) for line 645 ; b) An expansion view of part of the line


Figure 5.19: Example of scene changes between shifted images of the same scene taken at different times, moving objects are an example which cause these differences

The figure above shows clearly the scene changes in less than one minute between two images capture. Moving objects can causes these changes which need to be taken into consideration in the reconstruction process and can result in differences between corresponding pixel in the unshifted and shifted images. Figure 5.20 shows an example of the $\left\langle f, W_{l}\right\rangle$ calculations for one line as an example and the result of the first 100 lines reconstruction. We notice the high banding in the reconstructed image as we move towards the first pixels.


Figure 5.20: left) Low resolution intensities in blue, and the missing information in red Right) Effect of scene change on the reconstruction (reconstruction using $1^{\text {st }} \mathbf{1 0 0}$ lines only)

This change in graylevels values between the input images causes divergence of the vector $Y_{n}$ and therefore causes wrong estimation of the unknown $<f, W_{l}>\mathrm{s}$ which represent the missing information and this result in big outliers in the image result as we will be adding wrong information to the low-resolution image. Figure 5.21 shows the difference between the corresponding pixels in both images (in top figure) and the distribution of this difference where a Gaussian can be assumed (bottom figure)


Figure 5.21: Noise distribution

### 5.4.3.2 Second test acquisition

In order to remedy to this problem, the idea was to derive both shifted images from only one scanned image by reducing the speed of rotation of the table by half way and keeping the remaining parameters as they are. The table will be rotated only by 5 arcsec while the same time for the pixel clock readout $(5.3 \mathrm{~ms})$. In this case, a pixel in the scene will be covered by two image pixels. The unshifted image will be formed then by odd columns of
the image and the shifted image will be formed by the even columns of the same image. In order to ensure that both columns passed over the same electronics ( $\mathrm{A} / \mathrm{D}$ process), as according to the camera used two successive pixels lines pass through two different $\mathrm{A} / \mathrm{D}$ devices, we have halved another time the rotation speed to form two sets of shifted images. By this way, only one image will be used to derive a sequence of shifted image by taking the corresponding odd and even lines. Figure 5.22 shows the image obtained by this idea and figure 5.23 explains the principle schematically.


Figure 5.22: Original image captured after the rotation speed of the table by $1 / 4$


Figure 5.23: Principle of shifted images extraction by reducing rotation speed $1^{\text {st }}$ sequence: pixels $1,3,5,7 \ldots ; 2^{\text {nd }}$ sequence: pixels $2,4,6,8 \ldots$

The effect of reducing the rotation speed of the table is clearly shown in figure 5.22 on the quality of the images which look stretched. Figure 5.24 below shows the variation of the vector $Y_{n}$ for one line as an example and the effect on the estimation of the coefficients $\left\langle f, W_{l}\right\rangle$. It is obvious from these figures that by reducing the acquisition time between the shifted images and which can cause differences in the scale of intensities could reduce these errors and offer better reconstruction. Figure 5.25 shows the result of reconstruction using our Super-Resolution technique compared to the nearest interpolation.


Figure 5.24: left) Low-resolution Intensities profile for the unshifted and shifted line; right) Missing information in red and the low-resolution intensities in blue (line 900)


Figure 5.25: left) Result of the reconstruction; right) interpolated low-resolution image


Figure 5.26: up) Extra sharpness is achieved in the super-resolved image (bottom) compared to the result obtained by nearest interpolation

More sharpness can be seen around the trees and at the resolution chart (Figure 5.26). However more pre-processing is required to achieve better quality result by reducing the banding in the image result.

### 5.5 Reducing image noise

Super-Resolution is a process by which we expect to retrieve some new information and this new information is generally located at the gradient boundaries in the images as the most important features in an image are the edges. In this section, we propose an idea to reduce the effect of noise at the edges. The idea consists on fitting an edge on a proposed model, polynomial in our case, using the Least Squares formulation. The residual between the original and the fitted edge may contain some structure of the image lying in the low frequencies and noise with high frequencies. By smoothing this residual we can expect to
reduce the effect of the outliers present in this residual while keeping some of the structure which may be hidden in this residual. By removing all the residuals completely we may lose some of the data we are currently trying to extract using the SuperResolution. Figure 5.27 illustrates schematically the proposed model for an edge.

We define a residual (noise) $r_{i}$ at a point $i$ as the difference between the observed value of the point $y_{i}$ and the fitted response value $y_{i}$ defined by the polynomial model $Y$ below:

$$
\begin{gathered}
r_{i}=y_{i}-\hat{y}_{i} \\
\text { residual }=\text { data }-f i t
\end{gathered}
$$

and,

$$
Y=f(x)=a_{1} x^{3}+a_{2} x^{2}+a_{3} x+a_{4}
$$

with $a_{1}, a_{2}, a_{3}, a_{4}$ are the coefficients of this model and which must be estimated using the method of least squares. Therefore, this can be interpreted by minimizing the following difference:

$$
S=\sum_{i=1}^{n}\left[y_{i}-\left(a_{1} x_{i}^{3}+a_{2} x_{i}^{2}+a_{3} x_{i}+a_{4}\right)\right]^{2}
$$

Putting the differentiation to zero can then minimize this summation. We have therefore:

$$
\begin{aligned}
& \frac{\partial S}{\partial a_{1}}=-2 \sum_{i=1}^{n} x_{i}^{3}\left[y_{i}-\left(a_{1} x_{i}^{3}+a_{2} x_{i}^{2}+a_{3} x_{i}+a_{4}\right)\right]=0 \\
& \frac{\partial S}{\partial a_{2}}=-2 \sum_{i=1}^{n} x_{i}^{2}\left[y_{i}-\left(a_{1} x_{i}^{3}+a_{2} x_{i}^{2}+a_{3} x_{i}+a_{4}\right)\right]=0 \\
& \frac{\partial S}{\partial a_{3}}=-2 \sum_{i=1}^{n} x_{i}\left[y_{i}-\left(a_{1} x_{i}^{3}+a_{2} x_{i}^{2}+a_{3} x_{i}+a_{4}\right)\right]=0 \\
& \frac{\partial S}{\partial a_{4}}=-2 \sum_{i=1}^{n}\left[y_{i}-\left(a_{1} x_{i}^{3}+a_{2} x_{i}^{2}+a_{3} x_{i}+a_{4}\right)\right]=0
\end{aligned}
$$

After simplifying, we obtain:

$$
\begin{aligned}
& \sum_{i=1}^{n}-x_{i}^{3} y_{i}+a_{1} x_{i}^{6}+a_{2} x_{i}^{5}+a_{3} x_{i}^{4}+a_{4} x_{i}^{3}=0 \\
& \sum_{i=1}^{n}-x_{i}^{2} y_{i}+a_{1} x_{i}^{5}+a_{2} x_{i}^{4}+a_{3} x_{i}^{3}+a_{4} x_{i}^{2}=0 \\
& \sum_{i=1}^{n}-x_{i} y_{i}+a_{1} x_{i}^{4}+a_{2} x_{i}^{3}+a_{3} x_{i}^{2}+a_{4} x_{i}=0 \\
& \sum_{i=1}^{n}-y_{i}+a_{1} x_{i}^{3}+a_{2} x_{i}^{2}+a_{3} x_{i}^{1}+a_{4}=0
\end{aligned}
$$

We can rewrite that in matrix form. We have:

$$
\left[\begin{array}{cccc}
\sum x_{i}^{6} & \sum x_{i}^{5} & \sum x_{i}^{4} & \sum x_{i}^{3} \\
\sum x_{i}^{5} & \sum x_{i}^{4} & \sum x_{i}^{3} & \sum x_{i}^{2} \\
\sum x_{i}^{4} & \sum x_{i}^{3} & \sum x_{i}^{2} & \sum x_{i} \\
\sum x_{i}^{3} & \sum x_{i}^{2} & \sum x_{i} & n+1
\end{array}\right]\left[\begin{array}{l}
a_{1} \\
a_{2} \\
a_{3} \\
a_{4}
\end{array}\right]=\left[\begin{array}{c}
\sum x_{i}^{3} y_{i} \\
\sum x_{i}^{2} y_{i} \\
\sum x_{i} y_{i} \\
\sum y_{i}
\end{array}\right]
$$

The unknown coefficients $a_{1}, a_{2}, a_{3}, a_{4}$ can be estimated from this equation and then we can estimate the best fit for each pixel value. Below we will show an example of an edge fit and the estimation of the residual (noise).


Figure 5.27: Edge model

By this manner, an edge will be reconstructed using this idea and we expect from that the outliers will be smoothed to reduce their effect on the reconstruction. We summarize this new method to clean the edge by the following points (figure 5.28 ):

- Detect an edge
- Fit this edge
- Define a window around this edge and estimate the difference between the original graylevels and the fitted graylevels within this window
- Smooth the difference
- Reconstruct back the edge by putting adding smoothed difference to the fitted values.


Figure 5.28: Edge De-noising

We illustrate that with an example using the same image where one edge is corrected from the line 50 . Here are the different steps followed to reconstruct back the edge illustrated at each step with the obtained result.

- Detect an edge and fit it

- Subtract the original and the fitted and then smooth the difference (noise)

- Reconstructed edge by putting back the smoothed noise to the fitted model



Figure 5.29: left) SR without noise reduction; right) SR with noise reduction

The same experiment to combine the captured images has been carried out another time but this time by reducing the effect of the outliers at the edges using the edge fitting idea presented above. Figure 5.29 shows clearly how much the reduction of these outliers can reduce their effect in the reconstruction process and avoid the divergence of the coefficients $<f, W_{l}>s$.

### 5.6 Conclusion

Our objective in this chapter was to perform Super-Resolution using real images acquired either from real satellites or over real scenes. Super-Resolution using real data has been a real challenge due to the effect of process noise during the image capture. Good quality high-resolution images can be obtained depending up on a good filtering process. This aspect needs more investigation. We have shown the effect of the noise on our method and that this effect can be reduced using some pre-processing techniques in order to reduce the effect of the noise and enhance the edges in the resulting image. There is still work to be done in improving the noise model and the analysis of the noise sensitivity in the proposed algorithm for Super-Resolution.

## Chapter 6

## 6 Segmentation of Images as a Preprocess

### 6.1 Introduction

In the previous chapters we considered the purpose of Super-Resolution to provide globally improved image resolution. For remotely sensed images, however, much of the image does not have small scale information that is accessible from combining a pair of images. Regions such as deserts show little global difference as a result of SuperResolution (see for example figure 5.11). Even for the images taken over Guildford it is hard to find regions in the image which benefit from higher resolution (see figure 5.17).

In the course of this investigation, it has become apparent that to test whether information content is improved, we end up examining small regions of the image where there is rapid variation in intensities due to some texture.

If Super-Resolution were to be incorporated as part of the image capture process on a satellite, then we should only apply Super-Resolution to particular small areas of the image. This makes real time operation of the algorithm feasible provided we can automatically identify regions of interest in the images.

In this chapter, we investigate intensity boundaries in the image using the Walsh functions and by this means segment the images into fairly uniform regions of intensity with boundaries. When the boundaries are known we can gain little extra information from high resolution. Regions where the boundaries become extended imply a texture, may be a city or forestation, for example. These regions are then identified as regions of interest for Super-Resolution.

### 6.2 Edge Modelling

Consider the following model for an intensity edge (ramp) in an image:

$$
f(x)=\left\{\begin{array}{l}
\frac{h}{2}\left(2-e^{-k\left(x-x_{0}\right)}\right), \quad x>x_{0}  \tag{6.1}\\
\frac{h}{2} e^{k\left(x-x_{0}\right)}, \quad x \leq x_{0}
\end{array}\right.
$$

where $h$ represents the height of the edge and $x_{0}$ determine its location. Figure 6.1 shows schematically the representation of this edge. We now consider the effect of our SuperResolution algorithm upon this model of the edge.

We compute first the intensity of pixel $n$ in an image containing this edge, with resolution M. We can compute this directly from equation (6.1) giving:

$$
\begin{equation*}
f^{(M, n)}=2^{M} \int_{2^{-\mu}(n)}^{2^{-\mu}(n+1)} f(x) d x \tag{6.2}
\end{equation*}
$$

In the case when $x>x_{0}$ :

$$
\begin{equation*}
f^{(M, n)}=2^{M} \frac{h}{2}\left[2^{-M+1}+\frac{e^{k\left(x_{0}-2^{-M} n\right)}}{k}\left(e^{-k 2^{-M}}-1\right)\right] \tag{6.3}
\end{equation*}
$$



Figure 6.1: Edge-function

In the same way, the intensity of pixel $n+1$ is given by:

$$
\begin{gather*}
f^{(M, n+1)}=2^{M} \int_{2^{-\mu}(n+2)}^{2^{-\mu}(n+1)}(x) d x  \tag{6.4}\\
f^{(M, n+1)}=2^{M} \frac{h}{2}\left[2^{-M+1}+\frac{e^{k\left(x_{0}-2^{-\mu} n\right)}}{k}\left(e^{-k .22^{-\mu}}-e^{-k 2^{-\mu}}\right)\right] \tag{6.5}
\end{gather*}
$$

The intensity of the shifted pixel $n$ is given by:

$$
\begin{gather*}
f_{\Delta}^{(M, n)}=2^{M} \int_{2^{-M}(n+\rho)}^{2^{-M}(n+1+\rho)} f(x) d x  \tag{6.6}\\
f_{\Delta}^{(M, n)}=2^{M} \frac{h}{2}\left[2^{-M+1}+\frac{e^{k\left(x_{0}-2^{-M} n\right)}}{k}\left(e^{-k\left(2^{-M}+2^{-M} \rho\right)}-e^{-k 2^{-\mu} \rho}\right)\right] \tag{6.7}
\end{gather*}
$$

To compute the missing information we use:

$$
\begin{gather*}
\left\langle f, W_{1}^{(M, n+1)}\right\rangle=2^{M / 2} \int_{2^{-M}(n+1)}^{2^{-M}\left(n+\frac{3}{2}\right)} f(x) d x-2^{M / 2} \int_{2^{-\mu}\left(n+\frac{3}{2}\right)}^{2^{-\mu}(n+2)} f(x) d x  \tag{6.8}\\
\left.<f, W_{1}^{(M, n+1)}\right\rangle=\frac{2^{M / 2} h}{2 k}\left[e^{k\left(x_{0}-2^{-M} n\right)}\left(2 e^{-k \cdot \frac{3}{2} \cdot 2^{-\mu}}-e^{-k \cdot 2 \cdot 2^{-\mu}}-e^{-k 2^{-\mu}}\right)\right] \tag{6.9}
\end{gather*}
$$

We can substitute these parts into the formula for the missing information (equation 3.20):

$$
\begin{equation*}
\left\langle f, W_{1}^{(M, n)}\right\rangle=\frac{\left.f_{\Delta}^{(M, n)}-(1-\rho) f^{(M, n)}-\rho f^{(M, n+1)}-2^{\frac{M}{2}} \rho<f, W_{1}^{(M, n+1)}\right\rangle}{-2^{\frac{M}{2}} \rho} \tag{6.10}
\end{equation*}
$$

$$
\begin{equation*}
\left\langle f, W_{1}^{(M, n)}\right\rangle=\frac{2^{M} \frac{h}{2}\left[\frac{e^{k\left(x_{0}-2^{-\mu} n\right)}}{k}\left(e^{-k\left(2^{-\mu}+2^{-\mu} \rho\right)}-e^{-k 2^{-\mu} \rho}\right)+(3 \rho-1) e^{-k 2^{-\mu}}+1-\rho-2 \rho e^{-k \cdot \frac{3}{2} 2^{-\mu}}\right]}{-2^{M / 2} \rho} \tag{6.11}
\end{equation*}
$$

Referring to (3.20), the high resolution image is given by:

$$
\begin{equation*}
P_{M+1}(f)=P_{M}(f)+\sum_{n=0}^{2^{M}-1}<f, W_{1}^{(M, n)}>W_{1}^{(M, n)}(x) \tag{6.12}
\end{equation*}
$$

this implies for the pixel $n$ in the high resolution image:

$$
\begin{equation*}
P_{M+1}(f)=h+\frac{h}{2} e^{k\left(x_{0}-2^{-\mu} n\right)} \frac{2^{M}}{k}\left[e^{-k 2^{-\mu}}-\frac{1}{\rho} e^{-k\left(2^{-\mu}+2^{-\mu} \rho\right)}-\frac{3 \rho-1}{\rho} e^{-k 2^{-\mu}}-\frac{1}{\rho}+\frac{1}{\rho} e^{-k 2^{-\mu} \rho}+2 e^{-k 2^{-\mu} \frac{3}{2}}\right] \tag{6.13}
\end{equation*}
$$

This equation shows that the location of the edge $\left(x_{0}\right)$ is unaffected by this SuperResolution process, and the slope $k$. Note that the square bracket is independent of which pixel is used ( $n$ ), and only depends on the shift between the pixels $(\rho)$ and the resolution M.

By comparing the expressions between brackets in $P_{M+1}(f)$ and $P_{M}(f)$, it is proven that for $\rho=0$, these projections are the same, and this means that no new information is obtained by the Super-Resolution process. We propose here to analyze the expressions between brackets in the definition of both projections. We can see also that for small values of $\rho$, these terms go to 1 . Figure 6.2 shows the plot of these terms for $\mathrm{M}=13$ versus $k$ which represents the slope of the ramp. The low-resolution image is represented by the blue colour. For $\rho=0.75$, the high-resolution image is represented by black curve and for half pixel shift by red colour. If we assume that we can detect edges in the low-resolution image up to $10 \%$ with a contrast $k_{l}$, we can see from the plot that for the case of SuperResolution we can detect information with a contrast $k_{2}$ bigger than $k_{l}$. And better than that, if we use half pixel shift, more contrast on the reconstructed image can be achieved compared to the low resolution image as $k$ will be bigger and bigger.


Figure 6.2: (left) Plot of the edge amplitude vs. the slope $k$, (right) Percentage of amplitude vs. the slope

### 6.3 Use of the mis-registration effect to perform edge detection

In order to introduce this new idea, we shall present here an example of reconstruction using the new approach using the door image (figure 6.2). Two under-sampled images were derived from the original image by sub-sampling by factor of two and introducing half shift between both images. The first image is chosen to be the reference. The reconstruction was performed using wrong shift. Figure 6.3 shows the profile of the horizontal central line and figure 6.4 shows the effect of mis-registration on the vector $Y_{n}$ for the horizontal central line for example. The variations of $Y_{n}$ are different for the case with shifts compared to the variations using the true shift.


Figure 6.3: Door image


Figure 6.4: Central horizontal line profile


Figure 6.5: Effect of mis-registration on vector $\boldsymbol{Y}_{n}$

Figure 6.4 shows the plot of the vector $Y_{n}$ for the central line when using the true shift $(0.5)$ and a wrong shift $(0.1)$ in the reconstruction. It can be seen from this result that the effect of the mis-registration does not affect strongly the homogeneous regions. We mean by homogeneous regions, the regions where the variation of the intensities is almost the same. This effect is more sensible at the edges where we have big divergence.

Therefore, the idea here was to use this information in order to isolate the edges in an image, as this information will be provided by the variation on the $Y_{n}$ vector.

By defining the slope (gradient) for each pixel using different shifts and defining a threshold by cutting the minor slope variations, we will leave only those regions with high gradient and those are only the edges. Figure 6.5 below shows an example of slope for two pixels; one of them is located at an edge of the image and the other one far from an edge. In this example, the central line was used. The true shift is half pixel. The shift parameter was varied from 0 to 1 sampled by 0.05 and for each pixel the slope was estimated by subtracting the value of $Y_{n}$ for this pixel at the shift extremities ( 0 and 1 ).


Figure 6.6: Slope variation for pixel " 1 " located in a homogeneous area and pixel " 21 " located in an edge

It can be seen from this graph that the slope variation for pixel 21, which is located at an edge, is more important that the variation of pixel 1 which is in a uniform area of the image. The threshold in this case can be adjusted to be more sensible near the edge. Figure 6.7 presents the edge detection of the door image using this approach and for comparison the same result obtained by Canny edge detector in figure 6.8. Our purpose here is not the edge detection as this process can be achieved easily and with better result using Canny edge detector. We have shown that is possible to use this new idea to segment an image using the edge feature and use this information to divide the image on separate segments from which only those textured regions will be chosen to perform the Super-Resolution reconstruction.


Figure 6.7: Door image edge detection using the new idea


Figure 6.8: Canny edge detector

### 6.4 Example of use

In this part, we illustrate with an example using a test image, the use of segmentation in the Super-Resolution reconstruction. Two shifted low-resolution images were extracted from an original image, the Blewbury image shown below in Figure 6.9, by averaging two neighbouring pixels in the original image. The edge detection process was performed on these images. Result is shown in figure 6.11 and the low-resolution frames are shown in figure 6.10. The result shows more concentration of the edges near the woodland which identifies this region of the image as a region suitable to benefit from Super-Resolution. The remaining parts of the image are uniform regions separated by well resolved field boundaries. As a result it is only worth applying our Super-Resolution technique to the region of high edge concentration unlike Fourier techniques that are more global in nature, our technique is ideally suited for application to small localised regions of interest. The result is shown in figure 6.12 . On the left side of figure 6.12 , we see the textured region at low resolution and on the right side the high resolution reconstruction from our technique only in this textured area.


Figure 6.9: Blewbury image


Figure 6.10: Two shifted low resolution images used in the reconstruction


Figure 6.11: Result of segmentation of one of the low-resolution images using our approach


Figure 6.12: left) Nearest interpolation of one of the low resolution images with selection of textured area; right) the same interpolated image with selected textured image enhanced using our approach for Super-Resolution.

### 6.5 Conclusion

We have proposed in this chapter, a very simple and fast algorithm for edge detection based on the reconstruction using the Walsh functions. We have shown that is possible to detect these edges by exploiting the effect of mis-registration on the pixels whether they are located at edges or other smoothed regions. This result can be used to increase the performance of the algorithm by performing Super-Resolution only near these regions which are susceptible to contain new information.

## Chapter 7

## 7 Super-Resolution using the Walsh <br> functions: 2D case

### 7.1 Introduction

In this chapter we present a straightforward model derived for the Two-Dimensional case for the proposed method for Super-Resolution and which has been derived by extension from the 1D model presented in Chapter3. By the same way, we define the intensity of a pixel $(i, j)$ in an image with resolution $2^{(2 \mathrm{M})}$ by the following expression:

$$
\begin{equation*}
f_{(M, i, j)}=2^{(2 M)} \int_{2^{-\mu}(i)}^{2^{-\mu}(i+1)} \int_{2^{-M}(j)}^{2^{-M}(j+1)} f(x, y) d x d y \tag{7.1}
\end{equation*}
$$

Let us denote by $y_{1}=2^{M} x-i$ and $y_{2}=2^{M} y-j$; then if $0<y_{1}<1$ and $0<y_{2}<1$, we have:

$$
\begin{aligned}
& 2^{-M}(i)<x<2^{-M}(i+1) \\
& 2^{-M}(j)<y<2^{-M}(j+1)
\end{aligned}
$$

with

$$
W_{0}\left(y_{1}\right)=\left\{\begin{array}{lc}
1, & 2^{-M}(i) \leq x<2^{-M}(i+1) \\
0, & \text { elsewhere }
\end{array}\right.
$$

and

$$
W_{0}\left(y_{2}\right)=\left\{\begin{array}{lc}
1, & 2^{-M}(j) \leq y<2^{-M}(j+1) \\
0, & \text { elsewhere }
\end{array}\right.
$$

therefore, we can rewrite (7.1) by:

$$
\begin{equation*}
f_{(M, i, j)}=2^{(2 M)} \int_{2^{-\mu}(i)}^{2^{-\mu}(i+1)} \int_{2^{-\mu}(j)}^{2^{-\mu}(j+1)} f(x, y) W_{0}\left(y_{1}\right) W_{0}\left(y_{2}\right) d x d y \tag{7.2}
\end{equation*}
$$

We rewrite (7.2) in the following form:

$$
\begin{equation*}
f_{(M, i, j)}=2^{M} \int_{2^{-\mu}(i)}^{2^{-M}(i+1)} \int_{2^{-M}(j)}^{2^{-M}(j+1)} f(x, y) \varphi_{(M, i)}(x) \varphi_{(M, j)}(y) d x d y \tag{7.3}
\end{equation*}
$$

with

$$
\begin{align*}
& \varphi_{(M, i)}(x)=2^{M / 2} W_{0}\left(2^{M} x-i\right)  \tag{7.4}\\
& \varphi_{(M, j)}(y)=2^{M / 2} W_{0}\left(2^{M} y-j\right)
\end{align*}
$$

Finally we have

$$
\begin{equation*}
f(x, y)=2^{-M} \sum_{i=0}^{2^{M}-1} \sum_{j=0}^{2^{M}-1} f_{(M, i, j)} \varphi_{(M, i)}(x) \varphi_{(M, j)}(y) \tag{7.5}
\end{equation*}
$$

On the other hand, we know that (see proof in Chapter3),

$$
\begin{align*}
& \varphi_{(M-1, i)}(x)=\frac{1}{\sqrt{2}}\left[\varphi_{(M, 2 i)}(x)+\varphi_{(M, 2 i+1)}(x)\right]  \tag{7.6}\\
& \varphi_{(M-1, j)}(y)=\frac{1}{\sqrt{2}}\left[\varphi_{(M, 2 j)}(y)+\varphi_{(M, 2 j+1)}(y)\right]
\end{align*}
$$

therefore, we can prove easily that:

$$
\begin{align*}
& \left\langle f(x, y), \varphi_{(M-1, i)}\right\rangle=\frac{1}{\sqrt{2}}\left[\left\langle f(x, y), \varphi_{(M, 2 i)}\right\rangle+\left\langle f(x, y), \varphi_{(M, 2 i+1)}\right\rangle\right]  \tag{7.7}\\
& \left\langle f(x, y), \varphi_{(N-1, j)}\right\rangle=\frac{1}{\sqrt{2}}\left[\left\langle f(x, y), \varphi_{(N, 2 j)}\right\rangle+\left\langle f(x, y), \varphi_{(N, 2 j+1)}\right\rangle\right]
\end{align*}
$$

we also know that:

$$
\begin{equation*}
f(x, y)=\sum, \sum_{j} f_{(M, i, j)} W_{0}^{(M, i)}(x) W_{0}^{(N, j)}(y) d x d y=2^{-M} \sum, \sum_{j} f_{(M, i, j)} \varphi_{(M, i)}(x) \varphi_{(N, j)}(y) \tag{7.8}
\end{equation*}
$$

or;

$$
\begin{equation*}
f(x, y)=\sum_{i} \sum_{j} \ll f(x, y), \varphi_{(M, i)}(x)>, \varphi_{(M, j)}(y)>\varphi_{(M, i)}(x) \varphi_{(M, j)}(y) \tag{7.9}
\end{equation*}
$$

we know from the definition of the projections that:

$$
\begin{equation*}
P_{M}(f)=2^{M} \sum_{i} \sum_{j} \ll f(x, y), \varphi_{(M, i)}(x)>, \varphi_{(M, j)}(y)>\varphi_{(M, i)}(x) \varphi_{(M, j)}(y) \tag{7.10}
\end{equation*}
$$

By considering both odd and even values for $i$ and $j$, we have:

$$
\begin{align*}
& P_{M}(f)=2^{M} \sum_{i} \sum_{j} \ll f(x, y), \varphi_{(M, 2 i)}(x)>, \varphi_{(M, 2 j)}(y)>\varphi_{(M, 2 i)}(x) \varphi_{(M, 2 j)}(y)  \tag{7.11}\\
&+ \ll f(x, y), \varphi_{(M, 2 i+1)}(x)>, \varphi_{(M, 2 j)}(y)>\varphi_{(M, 2 i+1)}(x) \varphi_{(M, 2 j)}(y) \\
&+ \ll f(x, y), \varphi_{(M, 2 i)}(x)>, \varphi_{(M, 2 j+1)}(y)>\varphi_{(M, 2 i)}(x) \varphi_{(M, 2 j+1)}(y) \\
&+ \ll f(x, y), \varphi_{(M, 2 i+1)}(x)>, \varphi_{(M, 2 j+1)}(y)>\varphi_{(M, 2 i+1)}(x) \varphi_{(M, 2 j+1)}(y)
\end{align*}
$$

Also we have,

$$
\begin{equation*}
P_{M-1}(f)=2^{M-1} \sum_{i} \sum_{j} \ll f(x, y), \varphi_{(M-1, i)}(x)>, \varphi_{(M-1, j)}(y)>\varphi_{(M-1, i)}(x) \varphi_{(M-1, j)}(y) \tag{7.12}
\end{equation*}
$$

By subtracting (7.12) and (7.11) and exploiting the orthogonality of the Walsh functions, we obtain:

$$
\begin{align*}
P_{M}(f)-P_{M-1}(f) & =2^{M} \sum_{i} \sum_{j} \ll f(x, y), \varphi_{(M, 2 i)}(x)>\varphi_{(M, 2 j)}(y)>\varphi_{(M, 2 i)}(x) \varphi_{(M, 2 j)}(y)  \tag{7.13}\\
& +\ll f(x, y), \varphi_{(M, 2 i+1)}(x)>, \varphi_{(M, 2 j)}(y)>\varphi_{(M, 2 i+1)}(x) \varphi_{(M, 2))}(y) \\
& +\ll f(x, y), \varphi_{(M, 2 i)}(x)>, \varphi_{(M, 2 j+1)}(y)>\varphi_{(M, 21)}(x) \varphi_{(M, 2 j+1)}(y) \\
& +\ll f(x, y), \varphi_{(M, 2 i+1)}(x)>, \varphi_{(M, 2 j+1)}(y)>\varphi_{(M, 2 i+1)}(x) \varphi_{(M, 2 j+1)}(y)
\end{align*}
$$

but,

$$
\begin{align*}
& \varphi_{(M, 2 i)}(x)-\varphi_{(M, 2 i+1)}(x)=W_{1}^{(M-1, i)}(x)  \tag{7.14}\\
& \varphi_{(M, 2 j)}(y)-\varphi_{(M, 2 j+1)}(y)=W_{1}^{(M-1, j)}(y)
\end{align*}
$$

By simplifying (7.13) and taking into consideration (7.14), we obtain the final equation which defines the relationship between the projection of the image with $2^{M}$ pixels and the projection of the same image with $2^{\mathrm{M}-1}$ pixels. The difference between these two
projections defines the missing information in the low resolution image and which is needed to reconstruct the high-resolution image:

$$
\begin{equation*}
P_{M}(f)-P_{M-1}(f)=2^{M-1} \sum_{i} \sum_{j} \ll f(x, y), W_{1}^{(M-1, i)}(x)>, W_{1}^{(M-1, j)}(y)>W_{1}^{(M-1, i)}(x) W_{1}^{(M-1, j)}(y) \tag{7.15}
\end{equation*}
$$

Figure 7.1 presents schematically the definitions needed for the Two-Dimensional case.


Figure 7.1: Two-dimensional model representations

By analysing this new model and relating it to the 1 D model, we can assume that the 2 D case can be understood as a combination of two 1D models. Figure 7.2 shows for example two images shifted in $x$ and $y$ directions by a sub-pixel amount $\theta$. If we can imagine an interpolation of the reference image along the vertical axis by $[\theta, 0]$, then the shifted image is only a translated image of this intermediate image along the horizontal axis with shift $[0, \theta]$. Also, if we shift the reference image along the horizontal axis by $[0, \theta]$ we will end up with four images shifted with respect to each other with different shifts. In this case we can perform the reconstruction at each row to reconstruct a new image which will also be fused vertically to form the final image with doubled resolution in both directions. Figure 7.2 illustrates this idea. The simulations in the next section are based upon this principle.


Figure 7.2: 2D reconstruction scenario

### 7.2 Example of 2D images reconstruction

In this section, we present results of reconstruction using various images. Figure 7.3 shows four real low-resolution images which have been derived from an IKONOS highresolution image (size $383 \times 423$ ) over Istanbul. The down sampling factor was two in $x$ and $y$ directions. The vector shift used was $[(0,0),(0.5,0),(0,0.5),(0.5,0.5)]$. The results of interpolating one of the low-resolution images using nearest and bi-cubic algorithms are shown in figure 7.4. The super-resolved image obtained by our algorithm compared with the one obtained using Tsai and Huang method is shown in Figure 7.5. As a visual comparison, some areas are highlighted in the images for comparison purposes only. The structure highlighted in the reference low-resolution image is almost not seen and which becomes more visible in the super-resolved image using our algorithm. The result of the super-resolution contains more detail than any of the other images near the oil tanks for
example, the structure of the buildings and the coast line. Furthermore, the computation time for our algorithm compared to the one of Tsai and Huang method is ten times less and this is an important point required in the real time implementation. Using a personal computer Celeron (R) with CPU frequency $=2.2 \mathrm{GHz}$ and 256 of RAM, the computation time for combining four images each of size $182 \times 200$ pixels using the new algorithm is about 5.0680 seconds. However, we require 63.2800 seconds using Tsai and Huang method to combine the same images.


Figure 7.3: Four low-resolution images of size $182 \times 202$ obtained by subsampling an original IKONOS image by factor of two in both directions and shifted by half pixel


Figure 7.4: (Top) nearest interpolation of one of the low-resolution frames, (Bottom) result of the Bi-cubic interpolation


Figure 7.5: Super-Resolution result using our approach (top),
Result obtained using Tsai\&Huang method (bottom)

In order to prove more the performance of the reconstruction using this new approach, the down sampling factor was increased to $1 / 4$. The images shown in figure 7.7 are the result of the reconstruction using four low-resolution images obtained and shown in figure 7.6.


Figure 7.6: Low-resolution images derived after sub-sampling the original image with $1 / 4$ (size $91 \times 101$ )


Figure 7.7: left) nearest interpolation, middle) Bicubic interpolation, right) super-resolution result

We can see that we can retrieve more structure which has been lost by the down-sampling process. The oil-tanks are visible in the super-resolved image and the coast line is sharper. The structure of the roads is also visible as well as the structure of the buildings.

Another example of reconstruction using another type of images (cosine image) is also presented below in Figure 7.9. The reconstruction is performed from a sequence of four shifted images of size $256 \times 256$ derived from an original image of size $512 \times 512$ with the following vector shift $[(0,0),(0.5,0),(0,0.5),(0.5,0.5)]$ (see figure 7.8).


Figure 7.8: Low resolution images derived from original cosine image

The effect of aliasing has been overcome with the $S R$


Figure 7.9: top) super-resolution result, bottom) Nearest interpolation

### 7.3 Conclusion

In this chapter, the general 2D model for Super-Resolution has been presented and detailed. The results presented so far show the performance of the proposed method and which is very promising in terms of simplicity and efficiency if the all the requirements of the algorithm are met.

## Chapter 8

## 8 Conclusion and Directions for Future

## Research

### 8.1 Contributions

In this last part of the thesis, we summarize the main contributions of this research and propose some directions for future work.

The first objective of our work is the development of an efficient and robust complete algorithm for image Super-Resolution. This algorithm is driven only by the provided data. Each subsystem of the algorithm must take into account the computational cost of processing a large number of images. At the same time, the algorithm must be robust with respect to various sources of degradations that may corrupt the low-resolution images. As described in Chapter 2, the method proposed by Tsai and Huang was described step by step and some results of reconstruction using this method were presented to provide us with a comparison between the frequency domain methods and the spatial domain approaches as the proposed algorithm in this work lies in the latter category. Chapter 3 is entirely devoted to the mathematical development of the new approach based upon the orthogonal set of Walsh functions, while in Chapter 5, we present some results obtained using this approach. Chapter 4 deals with the problem of registering satellite images. We have shown why in our case, LEO satellite imagery, the geometric errors need to be corrected before processing the images with the Super-Resolution algorithm. We proposed a new way to estimate the translational shift between a pair of low-resolution images using a new measure based upon the distortions of the edges resulting from misregistration. Chapter 6 developed a new method for image segmentation which is an intrinsic part of Super-Resolution, as only regions of irregular intensity will benefit from higher resolution. This chapter dealt with the identification of such regions and applying our Super-Resolution algorithm to just those regions. The Two-Dimensional case where the low resolution images are shifted in both axes of the image plane is considered by
extension of the 1D case in chapter 7, and is explained with more details illustrated with some examples of reconstruction using real satellite images.

### 8.2 Image Registration

Super-Resolution from a sequence of shifted low-resolution images is possible only with the presence of motion between these images. The quality achieved in the super-resolved image depends closely upon the accuracy of the determination of this unknown parameter. Our motion estimation idea relies on the proposed approach based on the Walsh functions. A new measure is proposed which is based upon the degradation of the edges resulting from mis-registration. Numerical experiments show the advantages of using this simple measure to estimate the translational shift between two images. The second point of this section focused on the effect of mis-registration on the quality of the reconstructed image. Naturally, the quality of the reconstructed image degrades as the registration errors increase, and with more low-resolution images available, SuperResolution becomes more robust with respect to theses errors. Authors of [69] tried to analyze some of these errors during the reconstruction of high-resolution images.

In the same framework we have proposed a new and simple idea to segment an image by exploiting the behaviour of the pixels during a mis-registration case whether they are located at edges or other regions. This segmentation will help us to target only those regions susceptible to contain more interesting information in an image in the reconstruction process. This process of segmentation and the registration of the input data are performed simultaneously along with the interpolation phase and this constitutes another advantage of this new algorithm, which has never been used before for SuperResolution.

### 8.3 Image Reconstruction

In Chapter 3, we have introduced a completely new algorithm for image SuperResolution. This is a spatial domain method class based on the projection upon the Walsh functions. This contribution is fast and uses very simple mathematics for the reconstruction. The proposed algorithm exploits the projection of an image on Walsh sequences $\mathrm{W}_{0}$ using $2^{\mathrm{M}}$ and $2^{\mathrm{M}+1}$ pixels. We have shown that the difference between these two projections is the projection of the image on $W_{1}$ sequences. Also we have proven
how it is possible to retrieve this missing information from another projection of resolution 2 M but shifted from the reference projection by an arbitrary amount.

The accuracy and efficiency of our algorithm were justified by the results of the numerical experiments for both 1 D and 2D cases which opens the way for many applications. However, the boundary effects on the algorithm need to be analyzed by reducing the resulting banding. One idea to follow is proposed in [70]. The application of Super-Resolution for remotely sensed data was not investigated largely using real data in the Super-Resolution community. Authors in [71,72] talked about that but no real results have been published.

### 8.4 Image Restoration

The noise constitutes one of the important parameters which needs more investigation in many applications of image processing as it can dramatically reduce the quality obtained. We have proposed a simple way to reduce the noise in the input images based upon the least square fitting. Performing more accuracy on the reconstructed result requires more accuracy on removing the noise from the input sequence. One idea we propose according to this model is to analyze the relation between the reference intensities and the shifted intensities and try from this relation to define a model to correct one image according to the second one. This is a radiometric balancing problem. More algorithms for noise removal in image processing field can be also used along with the motion de-blurring part. Authors in [73] have proposed an idea suitable for image sequences and others proposed sub-pixel restoration algorithms [74]. The application of Wiener filtering for noise removal has also been proposed recently in [75]. The concept of digital image restoration has been well detailed by Katsaggelos in [76] along with the relevant problems. Due to time constraint, it was impossible for us to investigate all these aspects for which there is a large literature in the image processing community.

### 8.5 Directions for Future Research

The results achieved by this new algorithm for image Super-Resolution turn out to be very promising and there are many other possibilities to explore in order to improve the quality of the reconstructed images. Probably the most disappointing aspect of this
research was the inability to achieve Super-Resolution for the Alsatl images. This is due to issues such as correction errors, calibration and the optical parameters.

Although the registration framework we adopted in this work was found to be sufficient for our case, a useful extension is suggested to take in consideration other types of motion by accounting also for sensor and motion blur. More accuracy can be achieved when blur degradations are accounted for especially for satellite applications.

It is interesting also to think how to add as much low-resolution images as they become available. This can reduce the effect of the noise on the reconstruction. It is also necessary to explore the pre-processing of the input data. Atmospheric changes, calibration problems are some examples of points that need to be looked for.

This algorithm seems to be suitable for hardware implementation on satellites. The feasibility of how shifted images could be acquired is another point which needs investigating. Some work has already been done concerning the real implementation of Super-Resolution techniques [77].

Nowadays, some people are interested not only by the recovery of the intensity values using the Super-Resolution techniques but more than that; their work includes the recovery of high-resolution depth information in a scene. We call such SR techniques as multiobjective techniques where we are interested in recovering various HR structural or physical properties of a surface. Authors in [78] have introduced the first concepts of this new idea for image enhancement. Super-Resolution using different images from different sensors is also another complicated problem which needs to be looked for. A little work has been done concerning this point [79]. Also, effect of blur is very important on the quality of the reconstructed high-resolution image and which needs to be included in the algorithm. One example is proposed by [80] as well as the performance of geometric corrections as these parameters affect the desired quality [81].

Some results of the proposed new algorithm for satellite Super-Resolution have been presented in some important conferences. Details can be found in [82,83].

## Bibliography

[1] S. Chaudhuri. "Super-Resolution imaging", Kluwer Academic Publishers, 2001.
[2] S. Borman, R. Stevenson. "Image Sequence Processing". Department of electrical Engineering, University of Notre Dame, Notre Dame, USA, October 2002.
[3] H. Stark and P. Oskoui, "High resolution Image Recovery from Image Plane Arrays Using Convex Projections", Journal of Optical Society, vol 6, no 11, pp 1715-1726, November 1989.
[4] P.E. Eren, M.I. Sezan, and A.Tekalp. "Robust, Object-Based High-Resolution Image Reconstruction from Low-Resolution Video". IEEE Transactions on Image Processing, 6(10):1446-1451, 1997.
[5] B. C. Tom and A. K. Katsaggelos. "An iterative algorithm for improving the resolution of video sequences", Proceedings of SPIE Visual Comm., Image Processing, pp. 1430-1438, march 1996.
[6] A. J. Patti, M. Ibrahim Sezan, M. Tekalp. "Superresolution video reconstruction with arbitrary sampling lattices and nonzero aperture time", IEEE Transactions on Image Processing, vol. 6, NO 8, pp. 1064-1076, August 1997.
[7] J. H. Shin, J. H. Jung, J. K. Paik, M. A. Abidi. "Data fusion-based spatio-temporal adaptive interpolation for low-resolution video", http://citeseer.nj.nec.com/shin01data.html.
[8] David Capel, A. Zisserman. "Super-Resolution enhancement of text image sequences", Proceedings of the International Conference on Pattern Recognition, vol. 1, 2000.
[9] H. M. Adorf. "On the combination of undersampled multiframes", Astronomical Data analysis Software and Systems IV, ASP Conference Series, vol. 77, 1995.
[10] A. Zomet, S. Peleg. "Efficient super-resolution and applications to mosaics", IEEE International Conference on Pattern Recognition, vol. 1, 2000.
[11] Frank M. Candocia. "A Unified Superresolution Approach for Optical and Synthetic Aperture Radar Images", PhD thesis, University of Florida, 1998.
[12] L. J. Van Vliet, C. L. Luengo Hendriks. "Improving Spatial Resolution in exchange of temporal resolution in aliased image sequences", Proceedings of the $11^{\text {th }}$ Scandinavian Conference on Image Analysis, Pattern Recognition Society of Denmark, Lyngby, pp. 493-499, 1999.
[13] J. M. Schler, D. A. Scribner, M. R. Kruer. "Alias reduction and resolution enhancement by a temporal accumulation of registered data from focal plane array sensors", SPIE $14^{\text {th }}$ Annual International Symposium on Aerospace/Defence Sensing, Simulation and Controls, Orlando, Florida USA, vol. 4041, 2000.
[14] P. Vandewalle, S. Susstrunk, M. Vetterli. "Superresolution images reconstructed from aliased images", IEEE Visual Communications and Image Processing, 8-11 July 2003, Lugano.
[15] N. Nguyen. "Numerical Algorithms for Image Superresolution", PhD Thesis, Stanford University, July 2000.
[16] M. Elad. "A Fast super-resolution reconstruction algorithm for pure translational motion and common space-invariant blur", IEEE Transactions on Image Processing, vol. 10, no 8, pp. 1187-1193, August 2001.
[17] B. R. Hunt. "Super-Resolution of imagery: Understanding the basis for recovery of spatial frequencies beyond the diffraction limit". Department of Electronics and Computer Engineering, Univeristy of Arizona, Tucson, AZ 85721 (http://www.eleceng.adelaide.edu.au/ieee/idc99/abstracts/hunt2.html).
[18] B. R. Hunt. "Super-Resolution of images: Algorithms, Principles, Performance", International Journal of Imaging Systems and Technology, vol. 6, pp. 297-304, 1995.
[19] http://www.sstl.co.uk
[20] Alex da Silva Curiel, A. Wicks, M. Meerman, L. Boland, M. Sweeting. "Second Generation Disaster-Monitoring Microsatellite Platform", Acta Astronautica, vol. 51, No 1-9, pp. 191-197, 2002.
[21] Wei Sun, J. Paul Stephens and Martin Sweeting. "Micro-Mini-Satellites for Affordable EO constellations: RapidEye and DMC".
[22] S. Baker, T. Kanade. "Limits on Super-Resolution and how to break them", IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 24, no. 9, pp.11671183, September 2002.
[23] S. Borman, R. Stevenson. "Spatial Resolution Enhancement of Low-Resolution Image Sequences: A Comprehensive Review with Directions for future work", Laboratory for Image and Signal Analysis, University of Notre Dame, Notre Dame, July 1998.
[24] R.Y. Tsai and T.S. Huang, "Multiframe image restoration and registration", in advances in Computer Vision and Image Processing, R.Y. Tsai and T.S. Huang, eds, vol.1, pp 317-339. JAI Press Inc, 1984
[25] R. C. Gonzalez and R. E. Woods. "Digital Image Processing", Addison-wesley publishing company, September 1993.
[26] J. F. James. "A Student's Guide to Fourier Transform", Second edition, Cambridge University Press, 2002.
[27] B. Mulgrew, P. Grant, J. Thompson. "Digital Signal Processing: Concepts and Applications", Palgrave edition, 1999.
[28] A. Croft, R. Davison, M. Hargreaves. "Engineering Mathematics", Third edition, Prentice Hall, 2001.
[29] A. K. Jain. "Fundamentals of Digital Image Processing", Prentice Hall, 1989.
[30] A.M. Tekalp, M.K. Ozkan, and M.I. Sezan. "High resolution image reconstruction from lower resolution image sequences and space varying image restoration". In ICASSP, vol III, pages 169-172, san Francisco, 1992.
[31] E. Kaltenbacher, R.C. Hardie. "High-resolution infrared image reconstruction using multiple low resolution aliased frames", in Proceedings of the IEEE National Aerospace Electronics Conference, Dayton, OH, vol 2, pp. 702-709, May 1996.
[32] S.P. Kim, N.K. Bose and H.M. Valenzuela, "Recursive reconstruction of high resolution image from noisy undersampled multiframes", IEEE Transactions on Acoustics, Speech and Signal Processing, vol 38(6) , pp 1013-1027, 1990.
[33] N. K. Bose, H. C. Kim, and H. M. Valenzuela, "Recursive total least squrcs algorithm for image reconstruction from noisy, undersampled mutliframe", Multidimensional Systems and Signal Processing, vol 4, pp 253-268, July 1993.
[34] S.P. Kim and W. Su, "Recursive high-resolution reconstruction of blurred multiframe images", IEEE Transactions on Image Processing. 2:534-539, Oct. 1993.
[35] A. Papoulis. "Generalized sampling theorem", IEEE Transactions on Circuits and Systems, vol. 24, pp 652-654, November 1977.
[36] H. Ur and D. Gross. "Improved resolution from subpixel shifted pictures". CVGIP: Graphical models and Image Processing, 54:181-186, Mar. 1992.
[37] M. Irani and S. Peleg. "Motion analysis for image enhancement: Resolution, occlusion and transparency". Journal of Visual Communications and Image Representation. 4:324-335, Dec 1993.
[38] P.Cheeseman, B.Kanefsky, R.Kruft, J.Stutz, and R.Hanson, "Super resolved Surface Reconstruction From Multiple Images", NASA Technical Report FIA-9412, Dec, 1994.
[39] R. R, Schultz and R. L. Stevenson. "A Bayesian approach to image expansion for improved definition", IEEE Transactions on Image Processing 3, pp. 233-242, may 1994.
[40] H. Stark and P. Oskoui, "High resolution Image Recovery from Image Plane Arrays Using Convex Projections", Journal of Optical Society, vol 6, no 11, pp 1715-1726, November 1989.
[41] M. Elad, A. Feuer. "On Restoration and Super-Resolution for Continuous Image Sequence- Adaptive Filtering Approach". Israel Institute of Technology, July, 1994.
[42] S. Farsiu, D. Robinson, M. Elad, P. Milanfar. "Fast and Robust Super-Resolution", IEEE ational Conference on Image Processing, September 2003.
[43] S. Farsin, D. Robinson, M. Elad, P. Milanfar. "Robust shift and add approach to super-resolution", SPIE Annual Meeting, 3-8 August 2003, San Diego, California, USA, 2003.
[44] B. C. Tom and A. K. Katsaggelos, "Reconstruction of a high-resolution image by simultaneous registration, restoration and interpolation of low-resolution images", in Proc. of International Conference on Image Processing, Washington D.C, pp. 539-542, 1995.
[45] N. A. Woods, N. P. Galatsanos, A. K. Katsaggelos. "EM-Based Simultaneous Registration, Restoration and Interpolation of Super-Resolved Images", IEEE International Conference on Image Processing 2003, Barcelona, September 2003.
[46] T. Komatsu, T. Igarashi, K. Aizawa, and T. Saito. "Very high resolution imaging scheme with multiple different aperture cameras". Signal Processing Image Communication, 5:511-526, December 1993.
[47] J. J. Clark, M. R. Palmer, and P. D. Lawrence. "A transformation method for the reconstruction of functions from non uniformly spaced samples", IEEE Transactions on Acoustics, Speech and Signal Processing, vol. 33, pp. 1151-1165, October 1985.
[48] M.C. Chiang and T. E. Boult. "Efficient super-resolution via image warping", Image and Vision Computing, vol. 18, pp. 761-771, 2000.
[49] M. C. Chaig, T. E. Boult. "Efficient super-resolution via image warping", Image and Vision Computing 18 (2000), pp. 761-771, 2000.
[50] Wirawan, P. Duhamel and, H. Maitre. "Multi-channel high resolution blind image restoration", in Proc. of IEEE ICASSP, Arizona, USA, pp. 3229-3232, 1999.
[51] A. Zomet, A. Rav-acha, S. Peleg. "Robust super-resolution", Proceedings of the International Conference on Computer Vision and Pattern Recongnition, vol 1, pp. 645-650, December 2001.
[52] B. K. Gunturk, Y. Altunbasak, R. M. Mersereau. "Multi-frame information fusion for gray-scale and spatial enhancement of images", International Conference on Image Processing, Barcelona, Spain, September 2003.
[53] D. O. Walsh, P. Nielson. "A new method for super-resolution", http://citeseer.nj.nec.com/walsh93new.html.
[54] http://landsat.gsfc.nasa.gov
[55] K. G. Beauchamp. "Walsh functions and their applications". Academic Press Edition. 1975
[56] M. Petrou, P. Bosdogianni. "Image Processing: The fundamentals". Wiley Edition, 1999, pp 47-59.
[57] L. G. Brown. "A Survey of Image Registration Techniques". Department of Computer Science, Columbia University. ACM computing Surveys, vol 24, n04, January 1992.
[58] Barbara Zitova and Jan Flusser. "Image Registration: A survey", Institute of Information Theory and Automation. Academy of Sciences of the Czech Republic, Pod vodarenskou vezi 4, 18208 Prague 8, Czech Republic.
[59] H. Stone, M. Orchard, Ee-chien chang. "Subpixel registration of images", http://citeseer.nj.nec.com/stone99subpixel.html.
[60] Lillesand Kiefer, "Remote Sensing and Image Interpretation", third edition, John Wiley and Sons Inc, 1994.
[61] W. G. Rees. "Physical Principles of Remote Sensing", Second edition, Cambridge University Press, 2001.
[62] D. L. Hall. "Mathematical techniques in multisensor data fusion", Artech House, 1992.
[63] P. Chalermwat, "High performance automatic image registration for remote sensing", PhD thesis, George Washington University, 1991.
[64] W. B. Davenport, Jr. W. L. Root. "An introduction to the theory of random signals and noise", The IEEE Communications Society, Wiley-Interscience, 1987.
[65] K. Rank, M. Lendl and R. Unbehauen. "Estimation of image noise variance", IEE Proceedings of Image Signal Processing, vol 146, no 2, April 1999.
[66] J. R. Taylor. "An Introduction to Error Analysis", University Science Books, Oxford University Press, 1982.
[67] W. K. Pratt. "Digital Image Processing", Wiley-Interscience publication, 1978.
[68] M. Petrou. "The Differentiating Filter Approach to Edge Detection", Advances in Electronics and Electron Physics, vol. 88, pp. 297-345.
[69] M. K. Ng, N. K. Bose. "Analysis of displacement errors in high-resolution image reconstruction with multisensors", IEEE Transactions on Circuits and Systems, vol. 49, NO 6, pp. 806-813, June 2002.
[70] E. Abreu and S. K. Mitra. "A simple algorithm for restoration of images corrupted by streaks", IEEE International Conference on Circuits and Systems, pp. 730-733, 1996.
[71] M. kohiyama, F. Yamazaki. "Reoslution improvement of frequently observed satellite for urban characterization", $22^{\text {nd }}$ Asian Conference on Remote Sensing, 5-9 November 2001, Singapore.
[72] G. Qin, Z. Geng, Q. Xu. "The application of SR techniques in the Remote Sensing". Institute of Surveying and Mapping, Information Engineering Univeristy, 66 Longhai road, Zhengzhou, China 450052.
[73] B. Bascle, A. Blake, A. Zisserman. "Motion Deblurring and super-resolution from an image sequence", Proceeedings of European Conference on Computer Vision, vol. 2, pp. 573-582, Cambridge, UK, 1996.
[74] Terrana E. Boult. "Local image reconstruction and sub-pixel restoration algorithms", Graphical Models and Image Processing, vol. 55, NO 1, pp. 63-77, January 1993.
[75] F. Jin, P. Fieguth, L. winger, E. Jernigan. "Adaptive wiener filtering of noisy images and image sequences", International Conference on Image Processing, Barcelona, Spain, September 2003.
[76] M. R. Banham, A. K. Katsaggelos. "Digital Image Restoration", IEEE Signal Processing Magazine, March 1997.
[77] C. Hein. "Real-time implementation of Super-resolution imaging algorithm". Lockheed Martin Advanced Technology Laboratory, Canden, NJ 08102, Advanced signal Processing Algorithms, Architectures and Implementations, July 191998.
[78] D. Rajan, S. Chaudhuri. "Multi-Objective Super-Resolution: Concepts and Examples", IEEE Signal Processing Magazine, pp. 49-61, May 2003.
[79] A. Zomet, S. Peleg. "Multisensor super-resolution", $6^{\text {th }}$ IEEE Workshop on Applications of Computer Vision, Orlando, Florida, 2002.
[80] Ming chao chiang, Terrance E. Boult. "Local blur estimation and super-resolution", IEEE Proceedings Conference on Computer Vision and Pattern Recognition, pp. 821-826, Puerto Rico, USA, 1997.
[81] A. Goshtasby. "Registration of images with geometric distortions", IEEE Transactions on Geoscience and Remote Sensing, vol. 2, NO 1, pp. 60-64, January 1988.
[82] N. Omrane, P. Palmer. "Super-Resolution using the Walsh functions: A new algorithm for image reconstruction", IEEE International Conference on Image Processing, Barcelona, Spain, September 2003.
[83] N. Omrane, P. Palmer. "Super-Resolution for translated satellite images using the Walsh fucntions", SPIE Remote Sensing 2003, Image and Signal Processing for Remote Sensing $I X$, vol. 5238, Barcelona, Spain, September 2003.

