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The Structure and Behaviour of the Continuous Double Auction

by

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ABSTRACT

FACULTY OF ENGINEERING, SCIENCE AND MATHEMATICS
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The last decade has seen a shift in emphasis from centralised to decentralised systems to meet the demanding coordination requirements of today's complex computer systems. In such systems, the aim is to achieve effective decentralised control through autonomous software agents that perform local decision-making based on incomplete and imperfect information. Specifically, when the various agents interact, the system behaves as a computational ecology with no single agent coordinating their actions. In this thesis, we focus on one specific type of computational ecology, the Continuous Double Auction (CDA), and investigate market-oriented approaches to decentralised control. In particular, the CDA is a fixed-duration auction mechanism where multiple buyers and sellers compete to buy and sell goods, respectively, in the market, and where transactions can occur at any time whenever an offer to buy and an offer to sell match. Now, in such a market mechanism, the decentralised control is achieved through the decentralised allocation of resources, which, in turn, is an emergent behaviour of buyers and sellers trading in the market. The CDA was chosen, among the plenitude of auction formats available, because it allows efficient resource allocation without the need of a centralised auctioneer.

Against this background, we look at both the structure and the behaviour of the CDA in our attempt to build an efficient and robust mechanism for decentralised control. We seek to do this for both stable environments, in which the market demand and supply do not change and dynamic ones in which there are sporadic changes (known as market shocks). While the structure of the CDA defines the agents' interactions in the market, the behaviour of the CDA is determined by what emerges when the buyers and sellers compete to maximise their individual profits.

In more detail, on the structural aspect, we first look at how the market protocol of the CDA can be modified to meet desirable properties for the system (such as high

market efficiency, fairness of profit distribution among agents and market stability). Second, we use this modified protocol to efficiently solve a complex decentralised task allocation problem with limited-capacity suppliers that have start-up production costs and consumers with inelastic demand. Furthermore, we demonstrate that the structure of this CDA variant is very efficient (an average of 80% and upto 90%) by evaluating the mechanism with very simple agent behaviours. In so doing, we emphasise the effect of the structure, rather than the behaviour, on efficiency.

In the behavioural aspect, we first developed a multi-layered framework for designing strategies that autonomous agents can use for trading in various types of market mechanisms. We then use this framework to design a novel Adaptive-Aggressiveness (AA) strategy for the CDA. Specifically, our bidding strategy has both a short and a long-term learning mechanism to adapt its behaviour to changing market conditions and it is designed to be robust in both static and dynamic environments. Furthermore, we also developed a novel framework that uses a two-population evolutionary game theoretic approach to analyse the strategic interactions of buyers and sellers in the CDA. Finally, we develop effective methodologies for evaluating strategies for the CDA in both homogeneous and heterogeneous populations, within static and dynamic environments. We then evaluate the AA bidding strategy against the state of the art using these methodologies. By so doing, we show that, within homogeneous populations, the AA strategy outperformed the benchmarks, in terms of market efficiency, by up to 3.6% in the static case and 2.8% in the dynamic case. Within heterogeneous populations, based on our evolutionary game theoretic framework, we identify that there is a probability above 85% that the AA strategy will eventually be adopted by buyers and sellers in the market (for being more efficient) and, therefore, AA is also better than the benchmarks in heterogeneous populations as well.

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Nomenclature

Chapter 2

I	set of buyers
J	set of sellers
$\overline{I}$	set of intra-marginal buyers
$\overline{J}$	set of intra-marginal sellers
p^*	competitive equilibrium price
q^*	competitive equilibrium quantity
$\tilde{p}$	transaction price
k	weight in k-pricing rule
α	Smith's coefficient of convergence or price volatility
o_{bid}	outstanding bid
o_{ask}	outstanding ask
Δ	minimum bid or ask increment
p_{max}	maximum bid or ask allowed in the market
ℓ_i	limit price of buyer i
c_j	limit (or cost) price of seller j

Chapter 3

C_j	cost function of supplier j
f_j	start-up cost of supplier j
x_j	quantity of production allocated to supplier j
u_j	marginal cost of supplier j
cap_j	production capacity of supplier j
b	bid price
a	ask price
q_b	bid quantity
q_a	ask quantity
$bid(q_b, b, i)$	bid of buyer i

$ask(q_a, a, j)$	ask of seller j
$C[n, D]$	minimum total production cost given n suppliers and a total demand of D
q_i	demand of buyer i

Chapter 4

$\mathcal{M}$	the market
$\mathcal{I}$	the set of trading agents
$\mathcal{A}$	the set of actions of trading agents
g	the good
$\mathcal{B}$	the set of buyers
$\mathcal{S}$	the set of sellers
$state_{\mathcal{M}}(t)$	state of Market $\mathcal{M}$ at time t
ext_i	external information i
$H(.)$	history
$T(.)$	state transfer function
$price(t)$	market price of good at time t
$bid(t)$	bid at time t
$ask(t)$	ask at time t
$n_i(t)$	number of items to be bought or sold by trading agent i
v_i	set of limit prices of trading agent i
$budget_i(t)$	budget available to agent i
$comp_i(t)$	computational resources currently available to agent i
$T_{CDA}(.)$	state transfer function of CDA

Chapter 5

$\hat{p^*}$	estimate of competitive equilibrium price
r	degree of aggressiveness
τ	target price
θ	property of aggressiveness model
β_1	learning rate of short-term learning mechanism
β_2	learning rate of long-term learning mechanism
η	rate of increase (or decrease) of bid or ask price

Chapter 6

A_s	number of sellers
A_b	number of buyers

S_s	number of strategies available to sellers
S_b	number of strategies available to buyers
p	buyer's mixed strategy or buyers' population mix
q	seller's mixed strategy or sellers' population mix
$\dot{p}$	buyer's dynamics
$\dot{q}$	seller's dynamics
(p_{nash}, q_{nash})	mixed-Nash equilibrium

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to my grandparents

Chapter 1

Introduction

Over the last decade, there has been a shift in emphasis from *centralised* to *decentralised* computer systems, to meet the increasingly demanding requirements of complex systems and the need to be more flexible and robust in dynamic environments. Now, in many of these cases, an agent-oriented approach, with its emphasis on autonomous actions and flexible interactions (Jennings, 2001), is an appropriate computational model. Specifically, in such systems, the constituent agents are typically capable of local decision-making based on incomplete and imperfect knowledge about the system, and, when placed together, the overall system behaves as a *computational ecology* in which the agents interact and strategically compete for resources. However, because there is no centralised system-wide control, it is a major challenge to coordinate behaviour in the system. Against this background, this thesis is concerned with developing techniques for the decentralised control of such complex computer systems.

In many computational ecologies, decentralised control can be achieved by the allocation of (scarce) resources in a decentralised manner. Moreover, decentralised resource allocation is a subject that has long been studied in economics (Mas-Collel et al., 1995). Given this, this work is specifically concerned with using economic metaphors and tools to achieve decentralised control. Broadly speaking, such work can be non-price-based (employing a mechanism that does not involve price or payment for resources) or price-based (using price as an economic motivator). In the former, resource allocation can be based on techniques involving game-theoretic models (with selfish agents that seek to maximise their individual return) (Yemini, 1981) or techniques based on decentralised algorithms (with non-selfish agents that cooperate and have the individual aim of maximising the social welfare) (Kurose and Simha, 1989). The latter is a *market-oriented approach* and is the one we focus on in this thesis. We adopt such an approach because of its ability to facilitate resource allocation based on very little information (i.e. it just works based on price), its flexibility through its distributed nature and its reliance on local decision-making and its ability to be robust in dynamic environments (by adapting effectively to changes).

In more detail, markets are price mechanisms[1] that allow *selfish* and *profit-motivated*[2] agents to buy and/or sell resources. In so doing, the interaction of these self-interested, profit-motivated agents in a free market[3] can result in a close to optimal allocation of resources (Smith, 1962). Thus, efficient resource allocation in markets is an *emergent behaviour* from the interaction of these self-interested agents. Now, market mechanisms can exist in a multitude of forms including fixed-price markets in which a center or central agent arbitrarily sets a fixed price, dynamic-price markets where a center arbitrarily changes the price of goods, and auctions where the price is dynamically set in a decentralised manner. Each of these has its own particular properties and characteristics. However, probably the most popular is the *auction*, here defined as a mechanism that establishes prices based on participants' offers to buy or sell resources (Wurman, 2001). Auctions can be categorised as being single-sided (Krishna, 2002) (such as the first-price open-cry auction commonly known as the English Auction or the online auction eBay (`http://www.ebay.com`) in which there is a single buyer (or seller) and multiple sellers (or buyers), or double-sided (Friedman and Rust, 1992) (such as the clearing-house double auction or the Stock Exchange) in which there are multiple buyers and multiple sellers. Here, our focus is on the double-sided variety as it perfectly addresses our aim to develop decentralised resource allocation solutions in systems with multiple consumers and suppliers. Within this context, probably the most prominent mechanism is the Continuous Double Auction (CDA) (Friedman and Rust, 1992). In this, multiple buyers and sellers compete for resources that are allocated whenever buyers and sellers reach an agreement to trade. Given our focus and objectives, the CDA is particularly interesting in that it allows resource allocation among multiple consumers and suppliers and it is decentralised in nature (no center computes the allocation) and yet very efficient in terms of solving the resource allocation problem (Davis and Holt, 1993). Thus, we focus on the CDA as our exemplar economically-inspired mechanism for decentralised control.

[1] A mechanism defines how traders interact in the market, and how their actions lead to an allocation of resources. In a price mechanism, price is used as an indicator of the resource's value.

[2] The objective of a profit-motivated agent is profit, whether it cooperates or not. Thus, a selfish profit-motivated agent is after profit, but never cooperates to meet its objective. In a market mechanism, we consider such agents, with buyer and seller agents competing for profit.

[3] A free market economy is one in which the allocation for resources is determined only by the demand and supply, through the traders' interactions (Mas-Collel et al., 1995). This contrasts with a centralised system where a single agent (that is aware of all traders' preferences) computes the allocation.

Within this context, there are two aspects to the CDA that need to be considered. The first is the *structural* one, that defines the framework within which traders operate. This covers issues such as what market information should be revealed to which agents, the format of the offers to buy and to sell, when a transaction occurs and the price of that transaction. The second is the *behavioural* one that is concerned with the strategic interactions of the traders that determine the behaviour of the CDA. This covers issues such as the strategies that buyers and sellers should adopt and how efficient these strategies are with respect to how efficient the market is. Given this, in Section 1.1, we first investigate the structure and the behaviour of the CDA in more detail. Then, in Section 1.2, we discuss the research aims of this work, and list our contributions to advance the state of the art in Section 1.3. Finally, Section 1.4 gives the structure of this thesis.

1.1 The Continuous Double Auction

Market trading is governed by a *market mechanism*. Such mechanisms are designed to define the exchange process between buyers and sellers, by specifying the set of messages that can be exchanged (e.g. the traders' actions such as submitting a bid or an ask, or agreeing to a transaction) and by specifying the resource allocation process given the received messages (e.g. when transactions occur given the exchanged messages and at what price these transactions occur). In the CDA, there is usually a fixed-duration trading period (typically referred to as a *trading day*), and buyers and sellers can submit bids and asks, respectively, at any time during the trading day and the market clears continuously. Specifically, the market clears whenever there is a match between open bids and asks (i.e. a transaction is possible). In a single-unit, single-attribute CDA, the market clears (with a single trade) whenever the outstanding bid is at least as high as the outstanding ask. All messages submitted by traders are usually public and announced to all the participants in the market.

CDAs are important and popular because they are highly efficient market institutions:

> 'Markets organised under double-auction trading rules appear to generate competitive outcomes more quickly and reliably than markets organised under any alternative set of trading rules.' (Davis and Holt, 1993)

Given this, the CDA is currently one of the most common forms of electronic marketplaces. It has emerged as the dominant financial institution for trading securities and financial instruments and the major foreign exchanges (FX) or stock exchanges (like the NASDAQ and the New York Stock Exchange - NYSE) use variants of it. Indeed, today, the total value of trades on the NYSE stands at around 12.4 trillion dollars[4] of yearly transactions while foreign exchanges are worth in excess of 1.9 trillion dollars[5] of daily transactions. Thus, while decentralised resource allocation is the main motivation for our work on the CDA, its possible application in CDA-based financial markets is an important additional facet. Specifically, preliminary evidence already exists that software agents can outperform their human counterparts in such settings (see Subsection 2.3.1) and we believe that future marketplaces will increasingly involve ever larger numbers of such agents.

Now, whether we consider decentralised control or electronic trading scenarios, there are two aspects that characterise the CDA; namely the structural and the behavioural (as discussed above). Each of these will now be dealt with in turn.

First, we consider the structural perspective. This is determined by the market protocol which is a set of interaction rules and a set of clearing and pricing rules. The interaction rules define how participants interact through a set of actions. There are usually many interaction rules in a mechanism, ranging from specifying whether a trader can be a buyer and seller to specifying that a bid or an ask that can be submitted in the market must have a particular format. The clearing and pricing rules determine when and at what price a transaction occurs. The clearing rule only determines when a transaction occurs, while the pricing rule only determines the price of that transaction.

When taken together, it is these rules that allow the CDA to be an efficient market mechanism. Specifically, in a CDA populated by selfish profit-motivated agents, there is an equilibration of transaction prices towards an *equilibrium price* whereby the demand is equal to the supply and the allocation of resources is optimal. This equilibration occurs because if the demand is greater than the supply, the price of the resources rises which, in turn, reduces demand (because a segment of the population can no longer afford it) and increases supply (since more suppliers are willing to trade at the higher price). Similarly, when supply exceeds demand, prices fall, which reduces supply and increases demand. Thus, according to the micro-economics of markets, the price approaches a *market equilibrium*, where the demand equals the supply. How that equilibration is brought about during the trading day is known as the *dynamics* of the mechanism. The importance of how this dynamics aggregates privately held information (about preferences) to drive the market towards a solution for the resource allocation problem was described by Hayek:

[4] `http://www.nyse.com/pdfs/movolume0505.pdf`
[5] `http://www.bis.org/publ/rpfx05t.pdf`

> ‘The problem *(of how information that is held privately is accurately co-ordinated through the trading process to reach an equilibrium)* is in no way solved if we can show that all facts *(complete market information)*, if they were known to a single mind, would uniquely determine the solution; instead we must show how a solution is produced by the interactions of people each of whom possesses only partial knowledge. To assume all the knowledge to be given to a single mind in the same manner in which we assume it to be given to us as the explaining economists, is to assume the problem away and to disregard everything that is important and significant in the real world’. (Hayek, 1945)

It is to this dynamics of the CDA that its high efficiency is attributed, and it is this high efficiency as a decentralised resource allocation solution that motivates our work on the CDA mechanism. Now, though the CDA is already very efficient, when we consider the volume of trade in CDA-based financial markets with trillions of dollars worth of transactions, an improvement in efficiency, even of the order of 0.1%, is highly desirable and worthwhile. Thus, the emphasis of our work on the structure of the CDA is on how the market rules can be modified to improve certain desirable properties of the CDA such as price volatility, or fairness of profit distribution among buyers and sellers, as well as efficiency. In particular, to analyse how changing the market protocol really influences these properties of the mechanism, researchers have considered markets populated by very basic strategies, such that the properties of the mechanism can be attributed to the structure, rather than the behaviour, of the market. To date, however, most of this work has been about solving a standard resource allocation problem that is defined by a fixed demand and supply. Thus, research on the structure of the CDA has tended to overlook more complex problems and, specifically, when the demand and supply is complex and a market equilibrium does not exist. Thus, we believe it is important to investigate whether the CDA can still be used to solve the more complex allocation problems in a decentralised manner. If we can show this, then, we will augment the space of resource allocation problems for which the CDA can be considered as a viable solution mechanism.

Having considered the structure of the CDA, we now turn to its behaviour. As stated previously, this behaviour is what emerges from the interactions of the buyers and the sellers in the market and, it depends on the strategies of all the agents in the market. In this context, the agents strategise within the given market mechanism to determine what actions they should take, at what time. However, because the CDA is a complex game that is not amenable to a game-theoretic analysis (Gode and Sunder, 1992)[6], there is no known *dominant strategy* which produces the highest profit in the auction, regardless of what strategies it is playing against. Thus, over the past decade, there has been considerable research endeavour in developing trading strategies that define how agents should behave based on a variety of heuristic approaches (see Section 2.3 for more details). Moreover, these strategies have generally been targeted at static environments in which the market demand and supply does not change. However, we believe there is still significant scope for better strategies and, in particular, for strategies that perform well in both static environments and dynamic environments in which the market demand and supply changes sporadically. Given this, one of our lines of research considers the design of strategies for the CDA and its variants (to complement our work on the structure of the CDA). Moreover, because we are looking at the CDA mechanism for decentralised control, we require the interactions of the participating agents to result in a system that displays certain desirable properties such as efficiency and robustness in a wide variety of situations. To this end, we need to analyse how the strategies adopted by agents bring about these properties in both *homogeneous* populations (where all agents adopt the same strategy) and *heterogeneous* populations (where agents adopt different strategies). Furthermore, because we require our decentralised resource allocation to be robust in dynamic environments and efficiently adapt to changes in the demand and the supply, we need to analyse the behaviour of the CDA in both static and dynamic environments. Unfortunately, the current techniques for performing such analyses and for predicting system behaviour have a number of shortcomings (detailed in Subsection 2.3.5) and, so, work is needed to devise appropriate means of doing this. Work on this aspect therefore also represents an important research strand of this thesis.

1.2 Research Aims

The motivation for our research on the CDA is its widespread application and its general effectiveness as a decentralised solution to resource allocation. Now, given such motivations, our research needs to contribute to both the structural and the behavioural aspects. More specifically, we will now discuss the research aims of this thesis that deal with a number of issues in these two areas:

[6] As a consequence of the large space of actions and the continuous nature of the CDA game, and the multiple players that participate, the problem is too complex for a game theoretic analysis to find a dominant strategy.

1) ***to improve certain desirable properties of the CDA***. This refers to properties such as market efficiency, price volatility and fairness of profit distribution among buyers and sellers that are attributable to the CDA's structure. The reasons for wanting to do this are two-fold: (i) so that the CDA is more widely adopted as a market-based mechanism for resource allocation and (ii) to incentivise more buyers and sellers to join a market governed by such a mechanism.

2) ***to modify the structure of the CDA so that it can solve complex allocation problems.*** While the standard version of the CDA solves a relatively simple resource allocation problem, we want to observe how efficient a CDA can be in more complex situations. The motivation here is to demonstrate the feasibility of applying the CDA to a wider range of problems than has been considered.

3) ***to design a more efficient strategy for the CDA.*** As there is no known dominant strategy for the CDA, a multitude of heuristic-based strategies have been developed. However, these are typically developed for static environments. Thus, we believe that a more efficient strategy can be designed and, furthermore, that equal emphasis should be placed on designing such a strategy for both static and dynamic environments. The motivation here is to improve the behaviour of the CDA as a decentralised allocation system in both types of environments.

4) ***to develop methodologies for evaluating strategies in the CDA.*** Such methodologies should be able to analyse the CDA in homogeneous and heterogeneous populations, within static and dynamic environments, and for different (symmetric and asymmetric) demand and supply. In particular, we need to be able to evaluate properties, including the efficiency of the strategies, and the price volatility and the fairness of profit distribution among buyers and sellers. Moreover, for heterogeneous populations, because strategies can be adopted by buyers and sellers in different proportions in the market, we need to be able to analyse the evolution of these proportions and identify those which are most likely to be adopted. The motivation here is to obtain a better understanding of the complex interplay of the agents' behaviours and to make better predictions about what will happen in various circumstances.

This thesis addresses each of these four aims within the over-arching objective of developing a decentralised system for resource allocation based on the CDA market mechanism and demonstrating its efficiency in a wide variety of circumstances.

1.3 Research Contributions

Given the research aims outlined above, we now highlight the following specific contributions to the state of the art made by the research contained in this thesis:

1) ***We develop an efficient CDA-based mechanism for a complex decentralised resource allocation problem***. To address research *aim (2)*, we develop a CDA variant that solves a more complex allocation problem than the standard CDA. Specifically, this is the first work that modifies the CDA protocol to cope with multiple suppliers with limited production capacities and a cost structure composed of a fixed overhead cost and a constant marginal cost, and multiple buyers that have an inelastic demand[7]. We then go on to empirically evaluate the structure of our CDA variant by using a zero-intelligence behaviour[8], to emphasise the effect of the structure, rather than behaviour, on the efficiency of the mechanism. Despite such simple behaviour, we showed that our mechanism is efficient (which is an average of 80% and up to 90%). Furthermore, to address *aim (1)* for desirable properties, we demonstrate how our modified protocol allows an equal distribution of profits among buyers and sellers.

2) ***We develop a multi-layered framework for designing strategies***. This work addresses the issues in *aim (3)* where we observe that a multitude of strategies have been developed for the CDA and, because there is no known dominant one, we can always expect the design of new strategies. Because there is no current framework for designing strategies for the CDA, we develop such a framework in order to provide a blueprint that will assist the strategy designer with the different aspects involved in this process. Specifically, it will help the designer to identify the issues such as gathering information, processing information and using that processed information to strategise in the market. At present, bidding strategies are typically designed in an ad hoc and intuitive manner with little regard for discerning best practice or attaining reuseability in the design process, and our framework puts the development on a more systematic footing.

[7] A trader with an inelastic demand does not vary its demand according to price and has a positive utility for their requirement, and a utility of 0 for anything else. This contrasts with the standard assumption of elastic demand in which price is more responsive to changes in demand and the trader varies its demand according to price.

[8] A zero-intelligence behaviour is given by a strategy that ignores all market information and makes a random (and uninformed) decision on the offer to buy or sell to submit in the market (see Subsection 2.3.4.2).

3) ***We develop a bidding strategy for the CDA that is more effective than any previous strategy.*** This work addresses *aim (3)* for a more efficient strategy. Our Adaptive-Aggressiveness (AA) strategy is based on *aggressiveness* in the market. Specifically, it has both short and a long-term learning that allows such agents to adapt their bidding behaviour to be efficient in a wide variety of environments. For the short-term learning, the agent updates the aggressiveness of its bidding behaviour (more aggressive means that it will trade off profit to improve its chance of transacting, less aggressive that it targets more profitable transactions and is willing to trade off its chance of transacting to achieve them) based on market information observed after every bid or ask appears in the market. The long-term learning then determines how this aggressiveness factor influences an agent's choice of which bids or asks to submit in the market, and is based on market information observed after every transaction (successfully matched bid and ask). The principal motivation for the short-term learning is to update the agent's aggressiveness to immediately respond to short-term market fluctuations, while for the long-term learning it is to adapt to long-term changes in market conditions and to enable the agent to perform efficiently in dynamic environments. Our strategy addresses the issue that strategies have previously not been explicitly designed for dynamic environments and, in our approach, we identify the market shocks in the market and explicitly adapt the agent's behaviour in response to them.

4) ***We develop a framework for analysing strategic interactions in the CDA.*** This work addresses *aim (4)* of evaluating strategies in heterogeneous populations. In particular, the standard model is predicated upon the assumption that buyers and sellers have to adopt the same strategies. This is a serious simplification and shortcoming and, so, to address it, we develop a new model that separately analyses the strategic behaviour of buyers and sellers in the market. Specifically, we adopt a two-population evolutionary game theoretic approach, where we consider buyers and sellers as separate populations in the market. In so doing, our model offers new insights into how the choices of strategies affect the buyers' and sellers' economic efficiency and, so, we can better evaluate strategies in heterogeneous populations.

5) ***We develop methodologies for evaluating CDA strategies in both homogeneous and heterogeneous populations***. This work fulfils *aim (4)*. In our methodology to evaluate strategies in homogeneous populations, we look at the daily market efficiency and price volatility for different (symmetric and asymmetric) market demand and supply, rather than simply the overall efficiency for a symmetric demand and supply as is commonly done in the literature. This is an advance because our methodology provides better insights into how the efficiency of a strategy changes

as the strategies learn over the different trading days and because it identifies the drastic decrease in efficiency after a market shock. In terms of our methodology to evaluate strategies in heterogeneous populations, we use our novel two-population evolutionary game theoretic framework to analyse the buyers' and sellers' strategic behaviours in the CDA. Specifically, we use our methodologies to benchmark AA against the state of the art CDA strategies in static and dynamic environments under different market settings and, indeed, our evaluation provides insights that were not possible with previous methodologies. In so doing, we also empirically demonstrate the superiority of our AA strategy. In the homogeneous scenario, it outperformed the state of the art by up to 3.6% in the static case and 2.8% in the dynamic case. In the heterogeneous scenario, we identified that there is a probability above 85% that the AA strategy will eventually be adopted by buyers and sellers in all the settings we investigate.

The following papers have been published in support of these contributions:

- *R. K. Dash, P. Vytelingum, A. Rogers, E. David and N. R. Jennings* Market-based task allocation mechanisms for limited capacity suppliers. *IEEE Trans on Systems, Man and Cybernetics (Part A), 2007.* This deals with *contribution 1*.
- *P. Vytelingum, R. K. Dash, M. He, and N. R. Jennings.* A framework for designing strategies for trading agents. *Proc. IJCAI Workshop on Trading Agent Design and Analysis, pages 7-13, 2005.* This deals with *contribution 2*.
- *P. Vytelingum, R. K. Dash, M. He, A. Sykulski and N. R. Jennings.* Trading strategies for markets: A design framework and its applications. *Lecture Notes in Artificial Intelligence, pages 171-186, 2006*. This deals with *contribution 2*.
- *P. Vytelingum, D. Cliff and N. R. Jennings.* Strategic Bidding in Continuous Double Auctions. *Submitted to the Artificial Intelligence Journal, 2006 (paper selected at ECAI 2004 for fast track revision).* This deals with *contributions 3, 4* and *5*.
- *P. Vytelingum, R. K. Dash, E. David, and N. R. Jennings.* A risk-based bidding strategy for continuous double auctions. *Proc. 16th European Conference on Artificial Intelligence, pages 79-83, 2004.* This deals with *contribution 3*.
- *P. Vytelingum, D. Cliff, and N. R. Jennings.* Evolutionary stability of behavioural types in the continuous double auction. *Proc. AAMAS Joint Workshop on Trading Agent Design and Analysis and Agent Mediated Electronic Commerce VIII, pages 153-166, 2006.* This deals with *contribution 4*.

1.4 Thesis Structure

This thesis contributes to the two main avenues of research on the CDA, namely on its structure and its behaviour. In this section, we outline the structure of this thesis and its focus on these two areas.

We begin with a literature review of the CDA. In Chapter 2, we provide an overview of the micro-economics of markets, and on the structural and behavioural aspects of the CDA. First, we look at the work on the structure of the CDA. Specifically, we review existing research on modifying the market protocol to improve market efficiency or to reduce the price volatility in the CDA. Second, we review the work on the behaviour of the CDA. Here, we begin by looking at the software agent-human interaction in the CDA. Thereon, we focus on bidding strategies for software agents and, in particular, we look at frameworks for designing strategies and at how to categorise strategies. We then detail the common CDA strategies and, finally, we describe the standard one-population evolutionary game theoretic approach to analyse the behaviour of the CDA.

Next, we look at the structural and the behavioural perspective of this thesis.

Part I. The Structural Perspective

We describe, in Chapter 3, our novel CDA-based mechanism for decentralised task allocation, where sellers have limited capacity and have a start-up production cost and buyers have inelastic demand. We first describe the resource allocation problem which is to minimise the total production cost of suppliers. We then detail a centralised mechanism to solve the problem and the market protocol of our novel decentralised mechanism, including the pricing and clearing rules. Given the optimal centralised solution, we are then able to calculate the efficiency of our decentralised mechanism as the ratio of the total production cost of the decentralised case to that of the centralised case. Then, we describe in detail the structure of our novel mechanism (i.e. its market protocol).

Given the centralised and decentralised solution, we then evaluate the latter mechanism. To this end, we develop a zero-intelligence behaviour for our mechanism, such that structure, rather than behaviour, affects the efficiency of the system. Given this simple behaviour and the structure of our mechanism, we evaluate our CDA variant for different numbers of market participants.

Part II. The Behavioural Perspective

In Chapter 4, we describe our *IKB* framework that provides systematic guidelines for designing strategies for markets. We apply our model to analyse different strategies in the CDA and, in particular, describe its application in designing a strategy that we entered into the International Trading Agent Competition.

In Chapter 5, we describe our *AA* bidding strategy, designed using our IKB framework. We describe, in detail, the different components of the strategy, including the short-term and the long-term learning mechanisms.

In Chapter 6, we describe our two-population evolutionary game theoretic model to analyse the behaviour of both buyers and sellers in the market. We then describe the application of our model in analysing the CDA given a particular scenario, and compare this analysis to that with the standard one-population EGT model to identify the insights that are overlooked by the latter model.

In Chapter 7, we first develop our methodologies to evaluate the main state of the art strategies in homogeneous and heterogeneous populations. Using these methodologies, we benchmark AA against the state of the art within static and dynamic settings and with symmetric and asymmetric demand and supply. We demonstrate that AA outperforms the current state of the art in all these situations.

Finally, in Chapter 8, we summarise the contributions of this work and conclude, highlighting how this thesis introduces new areas for future work.

Chapter 2

Literature Review

In this chapter, we begin by introducing the classical micro-economic theory of demand and supply of markets, which is generally used to explain the high efficiency of the allocation in the CDA (Section 2.1). Next, we introduce the CDA mechanism, and describe the structure that is typically considered in the literature and that we will be using in our work to be consistent with this body of work. We then review the main related work on the structural and behavioural aspects of the CDA (sections 2.2 and 2.3 respectively). Finally, in Section 2.4, we summarise the reviewed work and, given our research aims, we analyse to what extent the related work on the structure and behaviour of the CDA satisfies these aims and identify the issues that require subsequent research.

2.1 Background on the Microeconomics of Markets

In a market, it is generally accepted that the higher the price of the commodity (good or service), the lower the demand and, conversely, the lower the price, the lower the supply. This demand and supply characteristic can be represented by a demand and supply curve, which is a function of the demand and the supply with respect to price (see Figure 2.1). The demand and supply curves meet at the *competitive market equilibrium*:

Definition 2.1. The competitive market equilibrium is when demand meets supply in a free market populated by profit-motivated and selfish agents. According to the classical micro-economic theory, the transaction prices in the CDA are expected to converge towards that competitive equilibrium price p^*. As p^* can only be calculated if the demand and supply are available, which is not the case here because of the decentralised nature of the CDA and the fact that no single agent knows all the agents' preferences,

p^* cannot be known a priori. The equilibrium is referred to as competitive because it is the competition among buyers and sellers that drive transaction prices to p^*.

At the competitive market equilibrium price, p^* the social welfare of the system (i.e. the benefit of the system as a whole) is maximised (i.e. the profit of all traders is maximised). The market equilibrium quantity, q^*, also known as the clearing quantity, is the optimal number of trades to maximise the profit of all traders. Now, in Figure 2.1, because the demand and supply curves intersect over a range of quantities and we are dealing with discrete quantities, we have a *volume tunnel*, where the equilibrium quantity can be $q^* - 1$ or q^*. However, since we assume goods are desirable, an allocation takes place even though there is no profit involved and, thus, the equilibrium quantity is at q^*. In cases when the demand and supply curves do not intersect over a single price (see Figure 2.2), we have a *price tunnel* between p_s^* and p_b^* and p^* lies somewhere within this range.

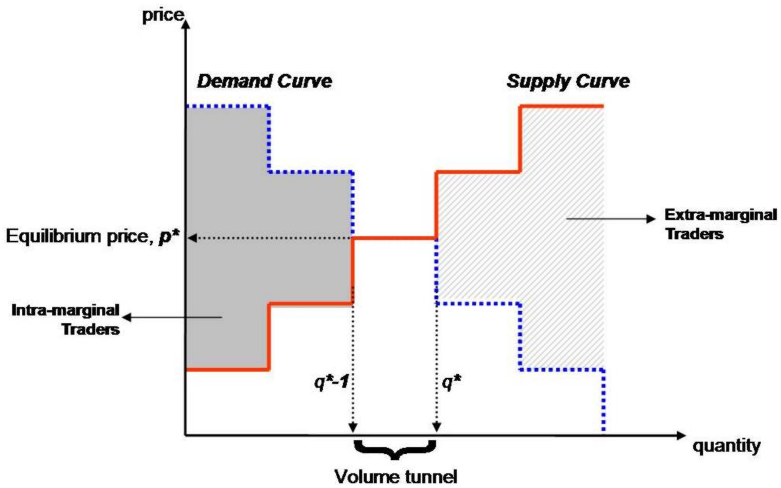


FIGURE 2.1: Demand and supply curve.

Now, in a centralised system, we have an agent (also known as the auctioneer) that has *complete* and *perfect* information (Mas-Collel et al., 1995) of the system. That is, it knows all the agents' limit prices (the maximum and the minimum a buyer and a seller is willing to offer respectively). Knowing this, the auctioneer can maximise social welfare by determining which agents will trade and at what price. Agents on the left of the equilibrium point (buyers with limit prices $\geq p^*$ and sellers with limit (cost) prices $\leq p^*$) are known as *intra-marginal traders* and will be trading at price p^* (with intra-marginal buyers willing to trade above p_s^* and intra-marginal sellers below p_b^*). Agents with limit prices equal to the equilibrium price will be trading at zero-profit (trading

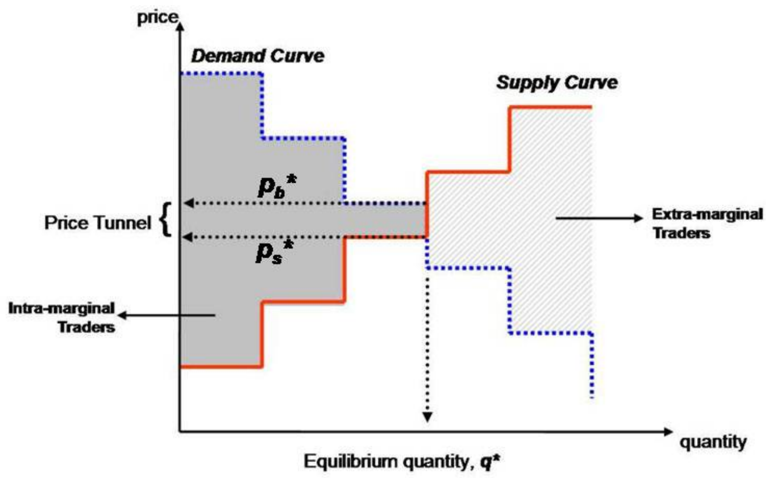


FIGURE 2.2: Demand and supply curve. p^* lies between p_s^* and p_b^*.

at the margin), as we assume that goods are desirable. The other participants are the *extra-marginal sellers* whose cost prices are too high and *extra-marginal buyers* whose limit prices are too low to trade in the market.

Definition 2.2. The limit price is the maximum bid a buyer is currently willing to offer, and the minimum ask a seller is willing to offer.

Definition 2.3. $\boldsymbol{\ell_i}$, is the limit price of buyer i.

Definition 2.4. $\boldsymbol{c_j}$, is the limit price of seller j.

The centralised approach to the resource allocation problem (in such situations), between a set I of buyers and a set J of sellers, is an optimisation problem. Specifically, it is a maximisation of the objective function which is the profit from all traders. The solution is given in Equation 2.1 where c_j is the cost price of seller $j \in J$, $\ell_i \in I$ is the limit price of buyer i as the equilibrium price, p^*, and the set $\overline{I}$ of intra-marginal buyers and the set $\overline{J}$ of intra-marginal sellers that transact. Here, p^* can lie over a range of values when we have a price tunnel (see Figure 2.2). Given the solution to the resource allocation problem, the *market efficiency* is said to be maximised.

$$
\begin{aligned}
p^* &= \arg\max_p \left\{ \sum_{i \in I} \max(0, p - \ell_i) + \sum_{j \in J} \max(0, c_j - p) \right\} \\
\overline{I} &= \{x \subset I |\ \ell_i \geq p^* \ \forall i \in x\} \\
\overline{J} &= \{y \subset J |\ c_j \leq p^* \ \forall j \in y\}
\end{aligned} \tag{2.1}
$$

Definition 2.5. The market efficiency is the ratio of all agents' surpluses in the market to the maximum possible surplus that would be obtained in an allocation where the profits of all buyers and sellers are maximised.

In a centralised market mechanism, market efficiency is optimal. While the objective of a decentralised system is also to maximise social welfare, an efficient allocation cannot be computed since there is no single agent that has the complete and perfect information that is needed to compute such an allocation. Rather, in the case of a CDA, it is the emergent behaviour of the market mechanism that computes the allocation and this is therefore non-optimal. Now, in many practical situations, it is often better to trade-off some of this efficiency for the desirable properties of a decentralised mechanism (as outlined at the beginning of Chapter 1). Given this, in this thesis we consider the CDA as such a decentralised market mechanism.

In his seminal work, Vernon Smith demonstrates that markets governed by the CDA mechanism and populated by selfish and profit-motivated (human) traders, can achieve a close to optimal market efficiency (Smith, 1962). Moreover, there is an equilibration of transaction prices to the competitive market equilibrium price, p^*, that is predicted by the micro-economics of free markets. Thus, despite the selfish nature of participants and the decentralised nature of information, it is shown that a close to optimal market efficiency can be reached in a CDA institution. He also demonstrated that if there was a *market shock* (a sudden change in demand and supply at the beginning of a trading day), transaction prices would converge to the new competitive equilibrium price (if it changes). An example of such a market shock is given in Figure 2.3 where p^* increases from 2.0 to 3.0.

Definition 2.6. A market shock[1] is a sudden change in the agents' preferences (their limit prices) and, hence, in the market demand and supply. Note that changing the demand and supply does not necessarily mean a change in the competitive market equilibrium as the new demand and supply curves could still meet at the same market equilibrium.

Smith also introduced a *coefficient of convergence*, α, given the history of n transaction prices, $\tilde{p}_i,\ i \in \{1..n\}$ (see Equation 2.2). Here, α is proportional to the standard deviation of transaction prices around the theoretical equilibrium price and it can be evaluated

[1] There are different types of dynamic changes in real markets that are not referred to as market shocks, for example rallies (the sustained upward movement of the competitive equilibrium price), sell-offs (the sustained downward movement of the competitive equilibrium price), movements or trends (less sustained upward or downward shifts respectively). However, because it is not a focal aspect of this work, we generalise the meaning of market shocks to cover all of these phenomena for simplicity.

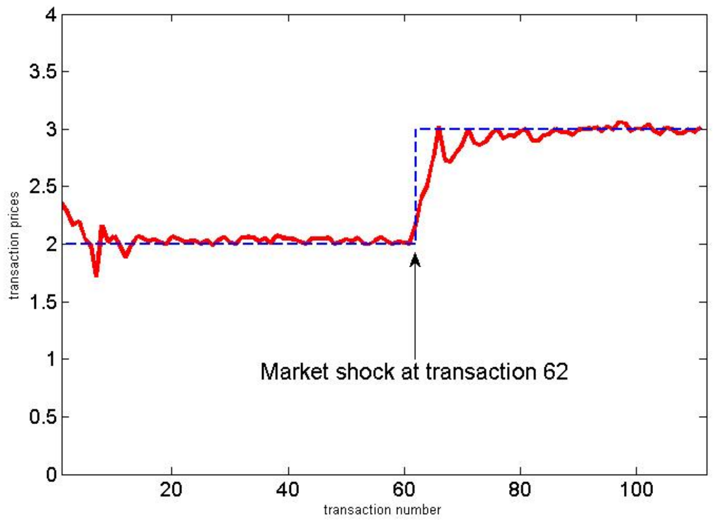


FIGURE 2.3: Example of a market shock. The red full line represents the transaction prices, while the dashed and black line represents the competitive market equilibrium price. p^* increases from 2.0 to 3.0 as from transaction 62.

over different periods in the history of transaction prices to estimate their trend of convergence. This coefficient of convergence can also be considered as the price volatility in the market:

$$\alpha = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\tilde{p}_i - p^*)^2}}{p^*} \tag{2.2}$$

Having looked at the micro-economics of demand and supply of markets and how it can be used to predict the efficient outcome in markets governed by the CDA institution, we now describe the CDA in more detail (looking at the structural element in the next section and the behavioural element in the subsequent one).

2.2 The Structural Aspect of the CDA

A market mechanism is defined by the *market protocol* that determines the nature of bids and asks allowed in the market, the information published to the buyers and sellers in the market, the clearing rule that indicates when a transaction occurs and the pricing rule that indicates the price at which a transaction occurs. The CDA is one such mechanism. However, there exist many variants, based on different market protocols. The most popular example is that used in financial institutions like the NYSE. This variant of the CDA includes things like the fact that a commission is charged for placing a bid or an ask, and that some traders have different levels of privilege with better access to other traders' messages than is available to unprivileged traders (usually to improve the overall efficiency of the system). Dash *et al.* (2007), on the other hand, describe a variant of the CDA for market-based control applications, where the clearing rules ensure that partial clearing[2] of multi-unit bids is not allowed (see Section 3.3).

However, these examples of the CDA are highly domain specific and are, therefore, difficult to generalise from. Thus, most research in this area (e.g. (Gode and Sunder, 1993; Cliff and Bruten, 1997; Tesauro and Das, 2001)) has generally been structured around the market protocol initially proposed in (Smith, 1962). In this, multiple buyers and sellers are allowed to submit shouts (bids and asks) in a market for homogeneous, single-attribute goods, and the market clears (with a single trade) whenever a bid and an ask match (hence, the continuous nature of the CDA), and clears at a price between (and including) the matched bid and ask. The *k-pricing* rule is usually adopted where the market clears at a weighted average of the bid and the ask as given in the following equation:

$$\tilde{p} = k.p_b + (1 - k)p_a \tag{2.3}$$

where $\tilde{p}$ is the transaction price, p_b is the matched bid and p_a is the matched asked. The parameter k is typically set to 0.5 in the CDA.

Furthermore, the protocol includes the NYSE *spread-improvement* and the *no order queuing* rules. The former requires that a submitted bid or ask improves on the outstanding bid (the highest unmatched bid) or the outstanding ask (the lowest unmatched ask) respectively, while the latter specifies that offers are single-unit, are not queued in the system, and are simply erased when a better offer is submitted. The CDA lasts several trading days, with a trading day itself lasting several trading rounds which is the period during which bids and asks are submitted (with the bid-ask spread decreasing) until the market clears. To more formally analyse the CDA, we now explore some of these basic notions in more detail:

Definition 2.7. A trading day is the period (with a deadline) during which traders are allowed to submit bids and asks (resulting in transactions whenever these match), at the end of which the market closes. In Smith's model of the CDA, at the beginning of a

[2] In a classical multi-unit CDA, partial clearing of multi-unit bids (to buy multiple units of goods) and multi-unit asks (to sell multiple units of goods) are allowed (Friedman and Rust, 1992).

trading day, traders are endowed with a set of goods to buy or sell (that determine the market demand and supply).

Definition 2.8. A bid is an offer to buy submitted in the market. The bid is published as a quote that is viewable by the other participants.

Definition 2.9. An ask is an offer to sell submitted in the market. The ask is published as a quote that is viewable by the other participants.

Definition 2.10. A shout is a generic term for a bid or an ask.

Definition 2.11. An outstanding bid or an outstanding best bid, o_{bid}, is the current maximum (uncleared) bid submitted in the market.

Definition 2.12. An outstanding ask or an outstanding best ask, o_{ask}, is the current minimum (uncleared) ask submitted in the market.

Definition 2.13. The bid-ask spread is the difference between o_{bid} and o_{ask}.

Definition 2.14. The minimum increment, Δ is the minimum bid or ask increment in the market.

Definition 2.15. The maximum price, p_{max} is the maximum bid or ask allowed in the market (to prevent unreasonably high asks and speed up the trading process).

Definition 2.16. A trading round is the period during which bids and asks are submitted until there is a match and a transaction occurs. There are typically several trading rounds in a trading day. At the beginning of the trading round, $o_{bid} = 0$ and $o_{ask} = p_{max}$.

Given the standard structure of the CDA, we now review the work on modifying that structure to improve on its desirable properties, specifically by changing the pricing and shout improvement rules.

2.2.1 Designing Pricing Rules

We now review the work by Phelps *et al.* (2003) where they consider optimising the pricing rule for the CDA. Here, the objective is to find the pricing rule that maximises some objective fitness, V, that combines the market efficiency and the buyers' and the sellers' efficiencies (also referred to as buyer and seller market power respectively):

$$V = \frac{\textit{market efficiency}}{2} + \frac{\textit{buyers' efficiency} + \textit{sellers' efficiency}}{4} \tag{2.4}$$

They propose two approaches to designing the pricing rule, which we now describe in detail.

First, the authors propose to optimise the k parameter in the CDA pricing rule (see Equation 2.3). This is a simple enough operation that can be achieved by using brute force and calculating the fitness, V, over the space $k \in [0, 1]$. Having done this, their simulation results showed that the best value for k is shown to be 0.5 (see Figure 2.4), validating the use of $k = 0.5$ in the traditional CDA.

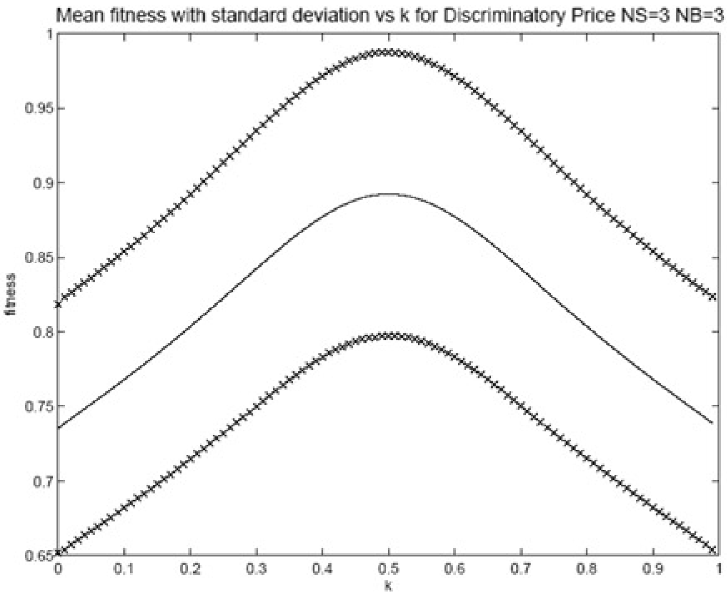


FIGURE 2.4: Fitness V (with standard deviation) plotted against k for a market of 12 traders (taken from (Phelps et al., 2003)).

Second and, perhaps more interestingly, the authors do not limit their approach to the typical pricing rule given in Equation 2.3, but rather consider the space of all arithmetic combinations of p_b and p_a. The pricing rule was then allowed to evolve over such a space using genetic programming. The evolved (and automatically acquired) pricing rule was an extremely complex function (see (Phelps et al., 2003) for more details), which, however, was shown to be approximately equal to $0.5p_b + 0.5p_a$, apart from a small variation when the ask is small or when the difference between the bid and ask is marginal.

In summary, this line of work can be seen to address our first aim of improving the properties of the CDA and, specifically, it shows how the structure of the CDA can be evolved (by evolving its protocol) for this purpose. Furthermore, it indicates that the $0.5p_b + 0.5p_a$ pricing rule is the most efficient and, thus, we shall adopt such a pricing rule in our work on the standard CDA model.

2.2.2 Improving Pricing and Shout Improvement Rules

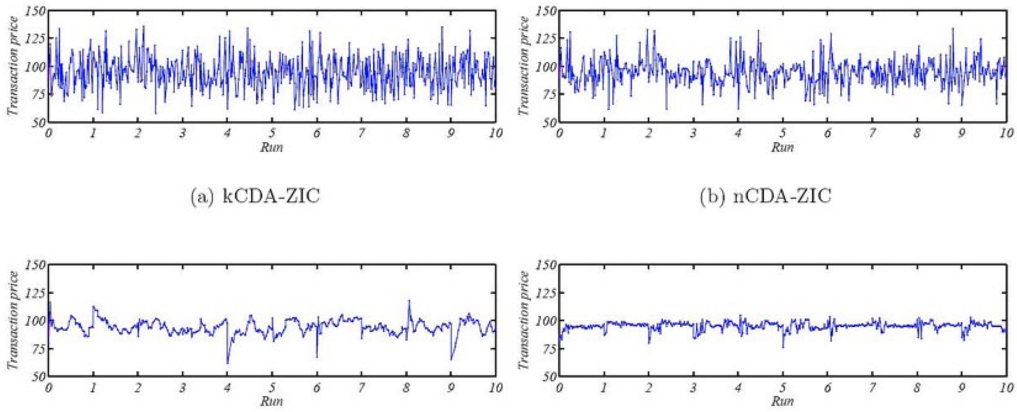
Having seen an automated approach to designing pricing rules, we now consider another notable work on the structural aspect of the CDA where the objective is to reduce price fluctuations. In particular, (Parsons et al., 2006) investigate how modifying the traditional k-pricing rule and the NYSE shout improvement rule can meet their objective. The motivation for such work is that with reduced fluctuations, participants would then be guaranteed transaction prices close to the competitive equilibrium price, and such guarantees for fair transaction prices would incentivise more participants to join the market.

First, the authors look at the *pricing policy* in the CDA. In particular, they modify the traditional policy, referred to as KPricingPolicy (given in Equation 2.3) to the NPricingPolicy commonly used in the Clearing-House double auction (where the market only clears at the end of the auction) (Friedman and Rust, 1992). The NPricingPolicy keeps a sliding window of size N of matching pairs of bids and asks used to set the transaction prices. It is important to understand that though this policy might reduce the price fluctuation in the market, it would not change the market efficiency, but would simply redistribute the profits among the participants and thus change the efficiency of the buyers and sellers. Equation 2.5 describes the policy exactly:

$$p_T = \frac{1}{2N} \sum_{i=T-N+1}^{T} (p_{a_i} + p_{b_i}) \;\; s.t. \; p_{a_T} \leq p_T \leq p_{b_T} \tag{2.5}$$

where p_{b_i} and p_{a_i} are the accepted bid and ask corresponding to the ith transaction respectively, T is the latest transaction and p_T is the price at which the transaction is set using the KPricingPolicy. Here, $\tilde{p}$ is bounded between p_{b_T}, the maximum a buyer is willing to offer, and p_{a_T} the minimum a seller is willing to offer. Note that the NPricingPolicy becomes the KPricingPolicy when $n = 1$, and the auction is then a traditional CDA. On the other hand, when N is equal to the total number of transactions, the auction is then a traditional Clearing-House Double Auction. Thus, N ranges over a continuous space of double auctions, with the Continuous Double Auction and the Clearing-House Double Auction at either end.

The authors then replace the KPricingPolicy by the NPricingPolicy, with $n = 4$, and compare the price fluctuations (by considering Smith's α, see Equation 2.2) of the CDA given a simple and non-intelligent behaviour, with ZI-C agents (see Subsection 2.3.4.2) and a more complex and intelligent behaviour, with GD agents (see Subsection 2.3.4.4). The results, given in figures 2.5 and 2.6, showed that the price fluctuations were reduced in both markets, with the better improvement in the ZI-C markets (as indicated by the relatively larger decrease in α). This is because ZI-C agents randomly submit bids and asks, and the spread of matching bids and asks is typically quite high. On the other hand, the more intelligent GD agents submit bids and asks that tend to converge towards the competitive equilibrium price, such that the spread is then greatly reduced, and the new pricing policy is then only marginally more effective.



(a) kCDA-ZIC (b) nCDA-ZIC (c) nCDAEE-ZIC (d) kCDA-GD

FIGURE 2.5: Transaction prices, plotted as 10 different runs, for the KPricingPolicy (in (a) and (b)) and for the NPricingPolicy (in (c) and (d)) (taken from (Parsons et al., 2006)).

Auction Type	α		E_a		Auction Type	α		E_a	
	mean	stdev	mean	stdev		mean	stdev	mean	stdev
kCDA-ZIC	15.613	1.352	95.732	1.537	kCDA-GD	4.198	1.140	99.224	0.548
nCDA-ZIC	**12.180**	1.532	**95.732**	1.537	nCDA-GD	**4.000**	1.1221	**99.224**	0.548
kCDAEE-ZIC	**9.360**	1.039	93.296	2.061	kCDAEE-GD	5.374	4.909	84.612	26.165
nCDAEE-ZIC	***6.028***	1.353	93.788	1.938	nCDAEE-GD	5.271	5.245	83.460	26.783
nCDAEEd5-ZIC	**6.221**	1.300	**96.072**	1.657	nCDAEEd5-GD	4.242	2.779	95.996	13.126
nCDAEEd10-ZIC	**6.879**	1.117	***96.868***	1.187	nCDAEEd10-GD	**4.054**	1.262	98.768	1.092
nCDAEEd15-ZIC	**8.000**	1.017	**96.864**	1.179	nCDAEEd15-GD	**4.016**	1.256	99.132	0.684
nCDAEEd20-ZIC	**9.220**	1.002	**96.744**	1.144	nCDAEEd20-GD	***3.994***	1.217	***99.224***	0.548

FIGURE 2.6: Metrics for KCDAs, nCDAs and nCDAEEs measured over 10 trading days. Bold face indicates the corresponding market outperforms or equals its kCDA counterpart. Bold italic points out the best results in the ZI-C and GD markets (taken from (Parsons et al., 2006)).

Second, the authors then go on to look at *shout improvement* in the market. They replace the traditional NYSE rule (detailed in Section 2.2) with a novel EE (or Estimated Equilibrium) shout improvement rule. In particular, the latter rule considers an estimate of the competitive equilibrium price, denoted by $\hat{p^*}$, by maintaining a sliding window of the m latest transactions:

$$\hat{p^*} = \frac{1}{m} \sum_{i=T-m+1}^{T} p_{t,i} \tag{2.6}$$

where $p_{t,i}$ is the transaction price as calculated in Equation 2.5.

Thus, rather than improving on bids and asks as in the NYSE rule, the buyers have to bid above $(\hat{p^*} - \delta)$, and the sellers below $(\hat{p^*} + \delta)$. Here, δ is some parameter to relax the range of bids and asks allowed, and was introduced because the transaction prices typically deviate greatly from the competitive equilibrium price at the beginning of the auction. The efficiency and price fluctuations with the new shout improvement rule (denoted by CDAEE$\delta*$) are given in Figure 2.6. First, the authors consider $\delta = 0$. Clearly, α is then considerably smaller for nCDAEE-ZI-C (i.e. the CDA with NPricingPolicy and EE with $\delta = 0$ and populated by ZI-C agents), though the new shout improvement rule decreases efficiency in that case. However, with GD agents, the performance is even worse, with price fluctuations actually increasing and the market efficiency decreasing. The authors conjectured that the GD agent adapts its bids or asks even though it is on the wrong side of the estimate and got rejected, and they have insufficient time to adapt sufficiently to be efficient. This led the authors to consider relaxing the shout improvement rules and consider different values for δ, and the results using such rules are given in Figure 2.6. As can be seen, the markets with $\delta \geq 5$ were indeed more efficient with either ZI-C or GD agents, than with $\delta = 0$, though when they increased δ, the price fluctuations decreased with GD agents, but increased with ZI-C agents.

The work we review here is relevant to our first research aim of reducing price fluctuations by modifying the protocol of the CDA. In particular, Parsons *et al.* change the pricing and shout improvement rules, and empirically show that price fluctuations can be reduced to incentivise agents to adopt the mechanism. However, one shortcoming of the work is that it only demonstrated improvement with non-intelligent agent behaviours, and none with more intelligent agent behaviours. With the ZI-C strategy usually providing the lower bound on market efficiency and price fluctuations because of its zero-intelligence nature (see Subsection 2.3.4.2), the new rules do indeed improve these lower bounds. However, they are not so effective with the higher bounds set by the state of the art intelligent strategies such as GD. With these strategies generally adopted for their higher efficiencies, the insights of this work are still not of great significance, though their future work on analysing the market with other complex and intelligent strategies within a dynamic setting (with market shocks) is indeed promising. Thus, we intend to still use the standard KPricingPolicy, as well as the NYSE shout-improvement rule as we will be working with intelligent strategies of the calibre of GD.

When considering both this work and the previous work on designing pricing rules, we see some endeavour towards our first research aim, though none towards our second of solving more complex resource allocation problems than the standard CDA. Next, we look at the behavioural aspect of the CDA. Before doing so, however, we observe that the literature on the behaviour is considerable greater than on the structure. We believe this is so because the CDA is a well established mechanism whose structure is generally assumed to be efficient (and used as is) and research on the behaviour is driven by the belief that breakthroughs will come through the design of more efficient behaviours for the CDA. However, we believe the impact of the CDA as a decentralised allocation solution will depend on the research on both the structure and the behaviour, which is why this thesis looks at both.

2.3 The Behavioural Aspect of the CDA

In this section, we look at the behaviour of the CDA determined by the strategic interaction of trading agents in the market. Before considering how traders strategise in the CDA, we give some background on their setup within a market. As discussed, most extant work has used a market setup, based on Smith's seminal work in experimental economics with human traders (see Section 2.1). In this case, agents are assigned fixed roles; that is they are either buyers or sellers. There are also typically several trading days and at the beginning of each, participants are given a list of *limit prices*, for each unit to be sold or bought. The allocation of limit prices is held constant over the days, but sometimes the allocations are modified after several days to evaluate the responsiveness of the mechanism to market shocks. Finally, it is assumed that agents are selfish traders and that no collusion occurs in the market. In our work, we adopt this setup so as to conform to the prior work.

Given this background, in this section, we begin by reviewing the seminal study of how human traders and software agents compete within a CDA institution (see Subsection 2.3.1). We then focus on software agents and review the literature on the design of bidding strategies (see Subsection 2.3.2), before looking at some of the most common such strategies available to software agents (see Subsection 2.3.4). Finally, we review the existing work on frameworks and methodologies for evaluating strategy effectiveness and predicting system performance (see Subsection 2.3.5).

2.3.1 Agent-Human Strategic Interaction

Software agents that are endowed with trading strategies (on behalf of profit-motivated humans), are playing an increasingly pivotal role in electronic marketplaces, with human traders competing against autonomous software traders. Thus, the interaction between software agents and human economic agents in a diverse range of market mechanisms is becoming of increasing interest. Specifically, the successful demonstration of the superiority of artificial traders over their human counterparts in these mechanisms is fundamental if humans are to entrust their money, preferences and economic decision-making to software agents.

To this end, Das *et al*. (2001) designed a set of experiments that allowed human traders to interact with software bidding agents in the CDA. In their experiments, in which there was the simultaneous participation of humans and software agents, it was shown that software agents can, on average, consistently outperform their human counterparts. In particular, they noted significant interaction between agents and humans with roughly 30% of all transactions falling into this category. Moreover, the average efficiency of human traders was typically below the very high efficiency noted in Smith's human-human experiments (Smith, 1962), while the efficiency of software traders was higher than 100% (that is the surplus achieved was higher than the surplus that they were expected to achieve in an optimal allocation). The results showed that software agents were consistently exploiting human traders and strongly suggest that agents are usually more performant as a group, in contrast to a single agent in a population of human traders. In prior simulations of the CDA with all human traders, the efficiency of human traders tended to improve during the trading period and this was typically attributed to learning and human intelligence. However in agent-human simulations of the CDA, human traders were consistently outperformed over the length of the trading period.

While this is but just one study, we believe it is very important. In particular, empirically demonstrating that software agents economically outperform human traders motivates the use of these agents in marketplaces, as well as the need for sophisticated strategies that these agents can adopt.

With the rationale for software bidding agents having been motivated, we go on to review the work on *bidding strategies* for software agents trading in the CDA.

2.3.2 Methodologies for Designing Bidding Strategies

First, we consider methodologies for designing bidding strategies, partly addressing our research aim to design strategies. We believe this line of work is important because such a framework would be useful in providing a systematic approach to designing strategies for the CDA mechanism and in analysing the design of the different types of bidding strategies.

One notable work (Vetsikas and Selman, 2003), proposes a methodology for deciding the strategy of bidding agents participating in simultaneous auctions. Their methodology decomposes the problem into sub-problems that are solved by *partial* or *intermediate* strategies and then they advocate the use of rigorous experimentation to evaluate those strategies to determine the best overall one across all the different auctions. However, their methodology is very much tailored to simultaneous auctions in general and the Trading Agent Competition (TAC) in particular. Thus, it cannot readily be generalised to other auction formats or other market mechanisms.

There are a number of other approaches, including (Chavez and Maes, 1996; Gimnez-Funes et al., 1998; Fasli et al., 2002) that look at the strategic behaviour of agents. However, they avoid issues related to how information and knowledge are used by trading agents in their strategies and focus instead mainly on their strategic behaviour. In so doing, they ignore the fundamental steps in the process that leads to a decision in the market.

Thus, in general, we can conclude that there is no systematic framework for designing bidding strategies for the CDA (our principal motivation here). Given its importance, we believe this is a serious shortcoming and, so, we seek to develop just such a methodology in Chapter 4.

2.3.3 Categorising Bidding Strategies

Before we look at the details of different examples of bidding strategies for the CDA, we provide a categorisation of strategies to help identify their main aspects. In this context, it is possible to categorise the behaviour of strategies by distinguishing whether they use a history of market information or not, and whether they consider external information or not:

1. **No History**. Strategies that do not use a history of market information are usually reactive (make a decision based on the immediate changes in market conditions)

and make myopic decisions based only on current market conditions. Such strategies also usually exploit the more complex bargaining behaviour of competing strategies (use them for their own selfish objectives) and thus require less computational resources to strategise. One such example is the *eSnipe* strategy[3] which is frequently used on eBay to submit an offer to buy near the end of the auction.

2. **History**. Strategies that use a history of market information are usually more complex and efficient, being able to monitor changes in the market and exploiting those changes. We further subdivide this into:

   (a) *Non-predictive*: These strategies are typically belief-based (use some belief of the market) and form a decision based on some belief of *the current market conditions*. Given its belief over a set of actions, the agent then determines the best action over the short or long term.

   (b) *Predictive*: These strategies make some form of prediction about the market state in order to adapt to it. Now, because future market conditions that the trading agent adapts to cannot be known *a priori*, this type of strategy typically makes its prediction using the history of market information. By tracking such changes and adapting its behaviour accordingly, the agent aims to remain competitive in changing market conditions.

3. **No External Information**. In this case, the strategy has access only to information pertaining to the market, and it does not consider any signals that are external (e.g. the falling market price of a good affecting the client's preferences for another type of good in another auction or the floods affecting the supply of a commodity). However, the agent can choose whether or not to use the internal information (e.g. the eSnipe strategy uses the internal market information, while the ZI Strategy, described in Subsection 2.3.4.2, does not make use of any market information in the CDA).

4. **External Information**. It is possible that signals external to the market can influence the preferences of the participants. Such signals are independent of the market and cause the clients' preferences in the market to change (e.g. unforeseen weather conditions affecting the production of wheat and thus the market for wheat indirectly). Thus, external information can be a valuable source of information that the agent can use to strategise in the market.

Given our categorisation for bidding strategies, we now review the most common ones for the CDA in this light.

[3] `www.esnipe.com`

2.3.4 Examples of Bidding Strategies

Because there is no known dominant strategy for the CDA, a wide variety of approaches have been developed over the years for trading agents that attempt to maximise their profits. We now describe the most common such strategies. In terms of the nomenclature of sub-section 2.3.3, none of these strategies employ external information (i.e. they have no access to external information). This is because we are looking at a controlled environment, which is more practical for analysing the mechanism.

2.3.4.1 Kaplan

The first strategy that we consider is Kaplan, also known as a *sniping* strategy. It is a simple reactive and non-predictive strategy that outperformed all other strategies in the Double Auction Tournament held at the Santa Fe Institute in 1990 (Friedman and Rust, 1992). The Kaplan strategy does not adapt to market activity or infer the market equilibrium, but rather exploits the bidding behaviour of other strategies and forms an offer whenever one of the following conditions is satisfied:

1. The best ask is less than the minimum transaction price in the previous period.
2. The best ask is less than the maximum transaction price in the previous period, and the ratio of the bid-ask spread and the best ask is less than a spread factor, while the expected profit is more than a minimum profit factor.
3. The fraction of time remaining in the period is less than a time factor (hence the term sniping strategy).

Kaplan won the tournament because it *snipes* at any profitable deal (based on some simple heuristics) and, thus, it lets the other strategies do all the negotiating. However, a system populated by agents using Kaplan does not perform efficiently because there is no longer this competition among buyers and sellers that will drive the market to an efficient outcome, but rather agents that are all waiting for a profitable transaction. On its own, the Kaplan strategy does not constitute a good basis for decentralised control in a complex system.

2.3.4.2 Zero-Intelligence

One of the most prominent strategies that have been developed over the years is Gode and Sunder's *Zero-Intelligence* (ZI) trading strategy (Gode and Sunder, 1993). The ZI agent is not motivated to seek trading profits and ignores all market conditions when forming a bid or ask by selecting a bid or an ask drawn from a uniform distribution over a given range (hence the term zero-intelligence). In terms of our categorisation, ZI is non-history-based and non-reactive as it does not consider the market condition in its decision-making process.

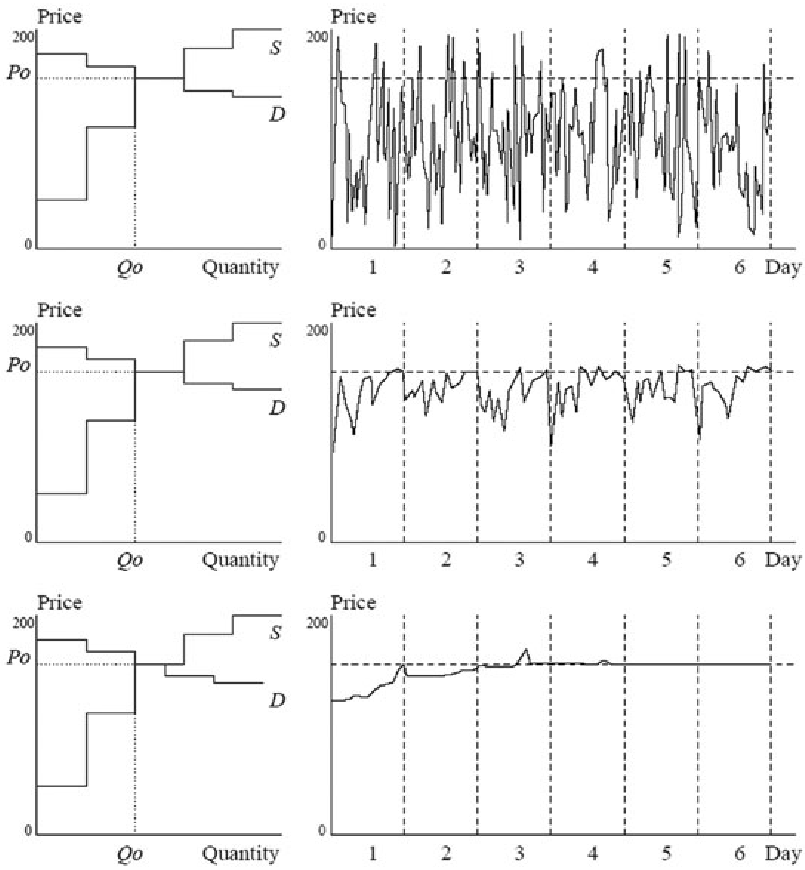


FIGURE 2.7: Result from one of Gode and Sunder's experiments (taken from (Gode and Sunder, 1993)). Results from ZI-U (top), from ZI-C (middle) and from human traders (bottom). Plots on the left-hand side show the demand and supply used in the experiments and those on the right-hand side are the history of transaction prices for the different populations.

Specifically, Gode and Sunder considered the performance of both constrained and unconstrained ZI traders, which they respectively term ZI-C and ZI-U traders. The former are subject to budget constraints and are not allowed to trade at loss. The latter, however, are allowed to enter loss-making transactions (i.e. they are allowed to submit a bid (ask) that is higher (lower) than their limit (cost) prices). In more detail, the ZI-C buyer draws a bid from a uniform distribution between the minimum allowed bid and its limit price. The ZI-C seller forms an ask from a value drawn from a uniform distribution between its cost price and p_{max}, beyond which we assume no transaction can take place. For the ZI-U traders, the shout price is drawn from a uniform distribution between 0 and p_{max}.

The results of the simulations of markets with ZI-C and ZI-U traders are shown in Figure 2.7. The qualitative differences between the price histories of the ZI-U, ZI-C and human traders, can be clearly seen in the figure. Specifically, the ZI-U markets exhibited little systematic pattern and no tendency to converge toward any specific level. In human markets, transaction prices converged to stable values close to the theoretical equilibrium price, as a result of subjecting intelligent and profit-oriented bargaining behaviour to market discipline. Gode and Sunder noted a slow convergence to the equilibrium price during each day of trading in the ZI-C markets (see figure 2.7). They explained this convergence by considering the narrowing of the feasible range of transaction prices, assuming that the traders with the highest valuation (i.e the buyers with the highest limit prices and the sellers with the lowest cost prices) trade first. Thus, intra-marginal ZI-C traders on the left of the demand and supply curve have a higher probability of submitting a shout that results in a transaction. With each good traded, the demand and supply curve shifts to the left until only extra-marginal traders remain. As the demand and supply curve shifts, the range of feasible transaction prices narrows with their mean converging towards the competitive equilibrium price.

The simulations also showed that the CDA institution can sustain high market efficiency even if agents are not profit-motivated and do not exhibit strategic behaviour. In this context, Gode and Sunder argued that the difference in performance between the ZI-U and the ZI-C traders, and between the ZI-C and human traders, could indicate the different extent to which overall market efficiency is dependent on the market structure or behaviour. Thus, if ZI-C traders are considered to have zero rationality, the difference in market performance would be attributable to the systematic rationality of human traders. In this case, the difference between ZI-U and ZI-C traders is attributable to market structure, where the trader is not allowed to submit a loss-making offer. ZI-C traders have the same imposed budget constraints as human traders, but have none of the intelligence or ability to learn from experience. Thus, the performance difference between the ZI-C and human traders could indicate how intelligent bidding behaviour affects market efficiency.

According to Gode and Sunder, given that the efficiency of ZI-C agents is close to that of human traders, market efficiency appears to be almost entirely a result of market structure. Their results therefore question previous assumptions that the efficiency of human markets is a consequence of human intelligence (Smith, 1962). Furthermore, human traders had the lowest profit dispersion[4], while the profit dispersion of ZI-C traders was much closer to that of the humans than the ZI-U traders. From this, they noted that these results suggest that individual aspects of market performance might be sensitive to human intelligence, in contrast to market efficiency.

2.3.4.3 Zero-Intelligence Plus

The *Zero-Intelligence Plus* (or ZIP) strategy was first designed by Cliff and Bruten (1997) to show that more than zero-intelligence is required to achieve efficiency close to that of markets with human traders (see Figure 2.7). ZIP has subsequently been used in a number of works as a benchmark for strategy evaluation (e.g. (Das et al., 2001; Tesauro and Das, 2001; Walsh et al., 2002)).

Specifically, while the ZI strategy ignores the state of the market and past experience, the ZIP strategy uses a history of market information and is of the predictive class, adapting the agent's profit margin to the future market conditions. That is, the agent increases or lowers its profit margin to remain competitive in the market. In this context, the profit margin determines the difference between the agent's limit price and the shout price.

At the beginning of the trading day, the agent has an arbitrarily low profit margin which it increases or decreases, depending on the different market events (submitted offers by buyers and sellers and any successful transactions) during the trading period. The ZIP buyer increases its profit margin whenever events in the market indicate that it could acquire a unit at a lower price than its current shout price, given by its profit margin. For a ZIP seller, if its last shout resulted in a transaction and its shout price was less than the transaction price, this indicates that it could transact at a lower price which would necessarily increase its profit margin.

Conversely, ZIP buyers and sellers reduce their profit margin when this margin is too high to remain competitive. In this case, the buyer would have market power (to influence the trend of ask prices) if the seller were to decrease its profit margin whenever an unsuccessful bid is submitted (since a series of unsuccessful bids would unnecessarily decrease the seller's margin). Similarly, the buyer should not lower its profit margin after each unsuccessful ask submitted in the market. Thus, a buyer lowers its profit margin only after a submitted bid is rejected, while a seller lowers its profit margin only after an unsuccessful ask. It is also necessary to consider the trader's strategy after a

[4] *Profit dispersion* is the root-mean-square error between actual profit and the profit given an optimal allocation.

successful shout. If the last bid was successful, the unsuccessful seller lowers its profit margin so as not to be undercut by the competing sellers. The unsuccessful buyer lowers its profit margin after an ask was accepted by a competing buyer. We summarise the bidding behaviour of the ZIP trader in Figure 2.8, where $p_i^b(t)$ and $p_j^s(t)$ are the most profitable offer to buy or sell of ZIP buyer i and seller j respectively, at any time during the trading period and $s(t)$ denotes the price of the most recent shout. In particular, we have 6 different rules that specify when to increase or decrease the profit margin.

Adaptive Rules for the ZIP Seller:

if (last shout was accepted at price $s(t)$)
  1. any seller j for which $p_j^s(t) \leq s(t)$ should raise its profit margin
  2. if (last shout was a bid)
    1. any active seller j for which $p_j^s(t) \geq s(t)$ should lower its margin
else
  2. if (last shout was an offer)
    1. any active seller j for which $p_j^s(t) \geq s(t)$ should lower its margin

Adaptive Rules for the ZIP Buyer:

if (last shout was accepted at price $s(t)$)
  1. any buyer i for which $p_i^b(t) \geq s(t)$ should raise its profit margin
  2. if (last shout was an offer)
    1. any active buyer i for which $p_i^b(t) \leq s(t)$ should lower its margin
else
  2. if (last shout was a bid)
    1. any active buyer i for which $p_i^b(t) \leq s(t)$ should lower its margin

FIGURE 2.8: The ZIP Trading Strategy.

In this case, the profit margin is modified using a simple adaptive mechanism based on the Widrow-Hoff algorithm (Widrow and Hoff, 1960). This is a continuous-space learning mechanism that back-projects the error between the current value and some desired value onto that current value. At any given time t, the ZIP trader i calculates the shout-price, $p_i(t)$, given its limit price, ℓ_i, and trader's profit margin, $\mu_i(t)$, according to the following equation:

$$p_i(t) = \ell_i(1 + \mu_i(t)) \tag{2.7}$$

The ZIP seller's margin is raised by increasing $\mu_i(t)$ and is lowered by decreasing $\mu_i(t)$, $\mu_i(t) \in [0, \infty)$. Conversely, the ZIP buyer raises and lowers its profit margin by decreasing and increasing $\mu_i(t)$ respectively. The initial profit margin, $\mu_i(0)$, is drawn from a uniform distribution over the range $[0.1, 0.5]$ at the beginning of the simulation. The aim of dynamically modifying $\mu_i(t)$ is for the trader's shout price to remain competitive against that of other participants. The learning mechanism of the profit margin is given in the following equations:

$$\begin{aligned} \mu_i(t+1) &= (p_i(t) + \Gamma_i(t+1))/\ell_i - 1 \\ \Gamma_i(t+1) &= \gamma_i \Gamma_i(t) + (1-\gamma_i)\Delta_i(t), \\ & \qquad where\ \Gamma_i(0) = 0\ \forall i \end{aligned} \tag{2.8}$$

$$\begin{aligned} \Delta_i(t) &= \beta_i(\tau_i(t) - p_i(t)) \\ \tau_i(t) &= R_i(t)s(t) + A_i(t) \end{aligned} \tag{2.9}$$

where the learning coefficient, $\beta_i \in [0.1, 0.5]$, determines the rate of convergence of the trader's shout price toward the target price $\tau_i(t)$. Here, R_i is a randomly generated coefficient that sets the target price *relative* to the submitted shout price $q(t)$, with $R_i \in (1, 1.05]$ to increase $\tau_i(t)$ (when increasing the profit margin) and $R_i \in [0.95, 1)$ to lower $\tau_i(t)$ (when reducing the profit margin). $A_i(t)$ is an absolute perturbation, so the target price differs by at least a few units (of the minimum increment on the outstanding bid or ask), from even relatively small shout prices. $A_i(t)$ is drawn from a uniform distribution over $[0, 0.05]$ for an absolute increase and over $[-0.05, 0]$ for an absolute decrease. If we were to set $\tau_i(t)$ to $q(t)$, the trader would never be able to submit an offer.

Furthermore, Cliff and Bruten (1997) improved their learning mechanism to minimise the effect of high-frequency changes in bids or asks, by considering the momentum (trend) of shout prices. The momentum-based updates are given in Equation 2.8. Specifically, the *momentum coefficient*, $\gamma_i \in [0, 1]$, determines the weight of previous shout prices on the change in the profit margin. When γ_i is equal to 0, the learning mechanism is myopic and ignores past quotes, while a high γ_i gives more weight to the trend of shout prices. In their simulations, γ_i is uniformly distributed over the range $[0.2, 0.6]$. Thus, the ZIP strategy has a set of 8 different parameters (β_i, γ_i, R_i and A_i to increase, R_i and A_i to decrease the margin, $\mu_i(0)$ for the buyer and the seller) that determine how to increase or decrease the profit margin which is specified by the 6 different rules.

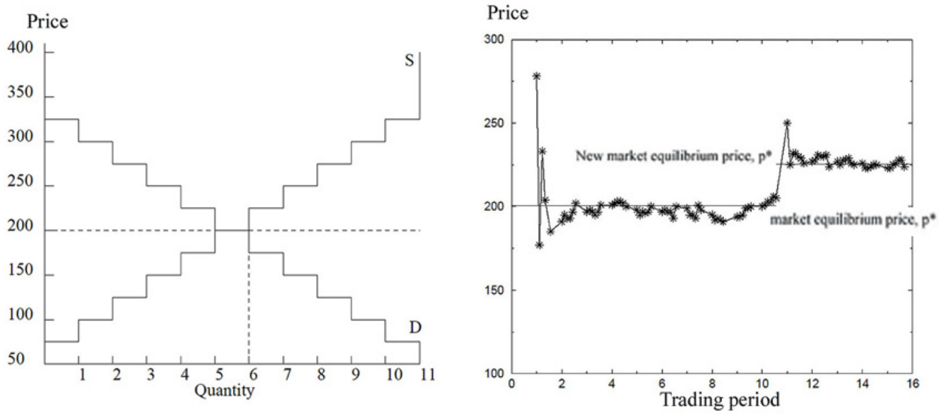


FIGURE 2.9: The right panel shows the results from simulations with ZIP traders (taken from (Cliff and Bruten, 1997)). The left panel illustrates the demand and supply used for the first 11 trading periods. Thereon, the competitive equilibrium price is increased to 225.

The results of simulations of the ZIP strategy in the CDA are presented in Figure 2.9. As can be seen, the transaction prices converge towards the competitive equilibrium price after a few trading days, and remain at that level with low variance. The ZIP strategy was shown to readily achieve results that were closer to human traders than ZI-C traders, even though the initial parameters of the ZIP trader were not optimised for the demand and supply of the market. Cliff and Bruten (1997) also showed that the profit dispersion of ZIP traders was significantly lower than that of ZI-C traders. Furthermore, a sudden change in endowment of limit and cost prices to buyers and sellers respectively (i.e. a market shock) at the beginning of period 12, is considered. In that case, the competitive equilibrium price increases from 200 to 225. The result in Figure 2.9 shows that the ZIP strategy rapidly adjusts to the new equilibrium price given by the new demand and supply.

The ZIP strategy was subsequently extended to *ZIP60*. Because more computation capability meant that the set of 8 parameters employed in ZIP did not need to be the same for all the 6 different learning rules (that specify when to increase or decrease the profit margin), Cliff (2005) proposed an extension of the number of parameters to 60. Thus, ZIP was extended to ZIP60 (Cliff, 2005). In more detail, first, the 8 parameters to update the margin were extended to 10 (see (Cliff, 2005) for further details), and rather than using the same set of 10 parameters for each of the updating rules, he now had a set of 10 parameters for each rule. Thus, 8 parameters are extended to 60 such that the strategy can be better tailored to the market environment. The 60 parameters are chosen as the solution to a Genetic Algorithm (GA) search to find the point in a 60-dimensional space that improves the convergence of transaction prices and effectively minimises α (see Equation 2.2).

The extensive results given in (Cliff, 2005), clearly show that the ZIP60 strategy performs better than ZIP, with up to 10% improvement in certain cases. Furthermore, the author did a principal component analysis of the parameters to identify any correlation between the data and, indeed, he validated his approach by ensuring that the parameters were not simply a combination of the original set of 8 parameters of the ZIP. Furthermore, the author observed that some parameters contributed marginally to the effectiveness of the strategy, such that a ZIP50 would in theory suffice. Such reduction of the parameter space implies faster discovery of the best parameters.

However, one shortcoming with the method is that the strategies are evolved for a specific set of demand and supply. Given that in almost all cases this is unknown *a priori*, some doubts have to be cast on the effectiveness of such an approach. Having said this, this work does provide useful insights on how strategies can be evolved to satisfy a particular market. The approach could be extended to consider a space of market demand and supply (assuming that the ZIP60 agent is only aware that it is participating in a particular market randomly selected from a known space), rather than specific ones.

2.3.4.4 Gjerstad-Dickhaut

We now look at a different class of strategies, namely the history-based and non-predictive GD family. The GD strategy, developed by (Gjerstad and Dickhaut, 1998), is based on a belief function that an agent builds to indicate whether a particular shout is likely to be accepted in the market. It was later extended to GDX (Tesauro and Bredin, 2002) which also considers the time left before the auction closes.

In the GD strategy, buyers form beliefs that a bid will be accepted and similarly sellers form beliefs that an ask will be accepted in the market. The traders form their beliefs on the basis of the history of observed market data and, particularly, on the frequencies of submitted bids and asks and of accepted bids and asks resulting in a transaction. Given this information, the bidding strategy is to submit the shout that maximises the trader's own expected surplus, which is the product of its belief function and its risk-neutral[5] utility function. The GD strategy also implicitly considers the notion of recency, by

[5]The risk-neutral agent uses a linear utility function and submits the price that maximises its expected profit.

limiting the trader's memory length, L, to a few transactions (L was set to 5 in Gjerstad and Dickhaut's simulations). The most recent history of shouts and transactions is considered.

In Gjerstad and Dickhaut's model, the seller's belief function, $\widehat{p}(a)$, is based on the following assumptions. If an ask $a' < a$ has been rejected, then an ask, a, will also be rejected. Similarly, if an ask $a' > a$ has been accepted, then an ask submitted at a will also be accepted. Furthermore, if a bid $b' > a$ is made, then an ask $a' = b'$ would have been taken, since they assume that this ask a' would be acceptable to the buyer who bid b'. Similar assumptions are made about the buyer's belief function, $\widehat{q}(b)$. We now define the bid and ask frequencies $\forall d \in D$, where D is the set of all permissible shout prices in the market, used in the belief function.

Definition 2.17. Bid Frequencies: $\forall d \in D$, B(d) is the total number of bid offers made at price *d*, TB(d) is the frequency of accepted bids at *d*, and RB(d) the frequency of rejected bids at *d*.

Definition 2.18. Ask Frequencies: $\forall d \in D$, A(d) is the total number of ask offers made at price *d*, TA(d) is the frequency of accepted asks at *d*, and RA(d) the frequency of rejected asks at *d*.

Definition 2.19. The *Seller's Belief Function* for each potential ask price, a, is defined as:

$$\widehat{p}(a) = \frac{\sum_{d \geq a} TA(d) + \sum_{d \geq a} B(d)}{\sum_{d \geq a} TA(d) + \sum_{d \geq a} B(d) + \sum_{d \leq a} RA(d)} \tag{2.10}$$

Definition 2.20. The *Buyer's Belief Function* for each potential bid price, b, is defined as:

$$\widehat{q}(b) = \frac{\sum_{d \leq b} TB(d) + \sum_{d \leq a} A(d)}{\sum_{d \leq a} TB(d) + \sum_{d \leq a} A(d) + \sum_{d \geq a} RB(d)} \tag{2.11}$$

The seller's belief function is modified to satisfy the *NYSE spread reduction rule* (see Section 2.2). Thus, for any ask that is higher than the current outstanding ask, the belief function is set to 0 (i.e. that ask cannot be accepted). Similarly for the buyer, the belief that any bid submitted that is below the outstanding bid is accepted is 0 (i.e. that bid cannot be accepted).

Furthermore, because the belief function is defined over the set of all bids and asks within the trader's memory, the belief is extended to the space of all potential bids or asks allowed in the market, constrained by the outstanding bid and ask and the step-size of the belief function. Then, *cubic spline interpolation* is used on each successive pair of data items to calculate the belief of points in between the pair of data. In particular, a cubic function[6], $p(a) = \alpha_3 a^3 + \alpha_2 a^2 + \alpha_1 a + \alpha_0$, that ensures each two successive pair of points is constructed with the following properties:

1. $p(a_k) = \widehat{p}(a_k)$
2. $p(a_{k+1}) = \widehat{p}(a_{k+1})$
3. $p'(a_k) = 0$
4. $p'(a_{k+1}) = 0$

The coefficients, $\alpha_i \; \forall i = \{0..3\}$, that satisfy the above properties, are then given by the solution to the following equation:

$$\begin{bmatrix} a_k^3 & a_k^2 & a_k & 1 \\ a_{k+1}^3 & a_{k+1}^2 & a_{k+1} & 1 \\ 3a_k^2 & 2a_k & 1 & 0 \\ 3a_{k+1}^2 & 2a_{k+1} & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha_3 \\ \alpha_2 \\ \alpha_1 \\ \alpha_0 \end{bmatrix} = \begin{bmatrix} \widehat{p}(a_k) \\ \widehat{p}(a_{k+1}) \\ 0 \\ 0 \end{bmatrix} \tag{2.12}$$

Then, the buyer's belief function, $q(b)$, is constructed similarly using the pairs $(b_k, \widehat{q}(b_k))$ and $(b_{k+1}, \widehat{q}(b_{k+1}))$. Having defined the belief functions, Gjerstad and Dickhaut proved that the beliefs are monotonically non-increasing (see (Gjerstad and Dickhaut, 1998) for further details). Thus, the belief of an ask, $a > a'$ being accepted has to be lower than the belief of a' being accepted, and, similarly, the belief of a bid, $b < b'$, has to be lower than that of b. On this basis, we can clearly see that monotonicity of beliefs is an essential property that the belief function of the GD strategy must satisfy.

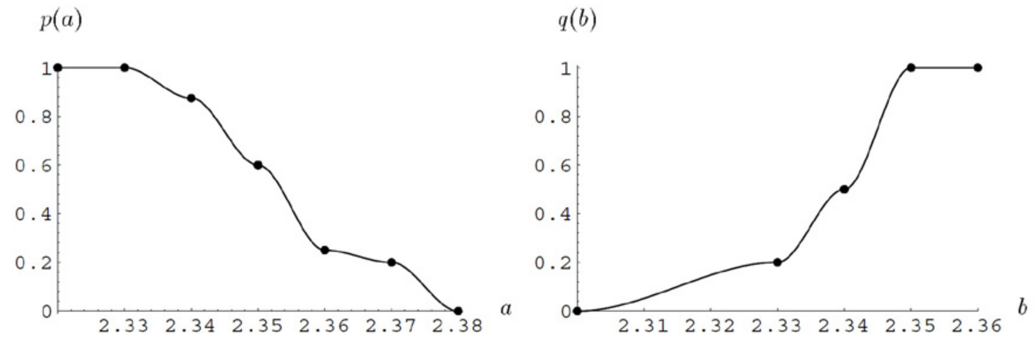


FIGURE 2.10: A typical belief function of a buyer (function $q(b)$ on the right) and a seller (function $p(a)$ on the left).

[6]We noted that the Gjerstad and Dickhaut use a particular type of interpolation where the gradient of the function at the known values is 0. While smoother functions could have been used, experiments demonstrated no further improvement using such functions.

Given the belief function, the GD strategy forms an offer to buy or sell that maximises the trader's expected surplus (which is defined as the product of its belief function and its utility function, $\pi(a)$). Because Gjerstad and Dickhaut consider risk-neutral traders, the utility function is linear and equals the profit of the traders; that is, the difference between the seller's ask price and its cost price, and the difference between the buyer's bid price and its limit price. When the trader's maximum expected surplus is negative, there is no incentive to submit a bid or an ask and the trader abstains from bidding. The trader's utility function and its surplus maximisation is formulated as follows:

For a buyer i,

$$\pi(b) = \begin{cases} \ell_i - b & \text{if } b < \ell_i \\ 0 & \text{if } b \geq \ell_i \end{cases}$$

For a seller j,

$$\pi(a) = \begin{cases} a - c_j & \text{if } a > c_j \\ 0 & \text{if } a \leq c_j \end{cases}$$

$$b^* = \arg \max_{b \in (o_{ask}, o_{bid})} [\pi(b).q(b)] \tag{2.13}$$

$$a^* = \arg \max_{a \in (o_{ask}, o_{bid})} [\pi(a).p(a)] \tag{2.14}$$

The performance of the GD strategy was empirically evaluated through a set of laboratory experiments. The results of the simulations showed that the efficiency of markets with GD traders was close to optimal with rapid convergence of transaction prices to the competitive equilibrium price with an equilibrium quantity of trade (a volume tunnel between 5 and 7) during each trading day (see Figure 2.11). By shifting the demand and supply after five trading days, it was also shown that GD traders responded to the changing market dynamics, and transaction prices adjusted to the new competitive equilibrium. Furthermore, GD was evaluated in heterogeneous populations, against ZI, ZIP and Kaplan. It was shown that the GD (with a slight modification) was the most efficient, closely followed by ZIP. Specifically, GD extracted 1.7% more profit than ZIP in a heterogeneous population of the two strategies. While GD was the state of the art in the non-predictive class, ZIP was the state of the art in the predictive class.

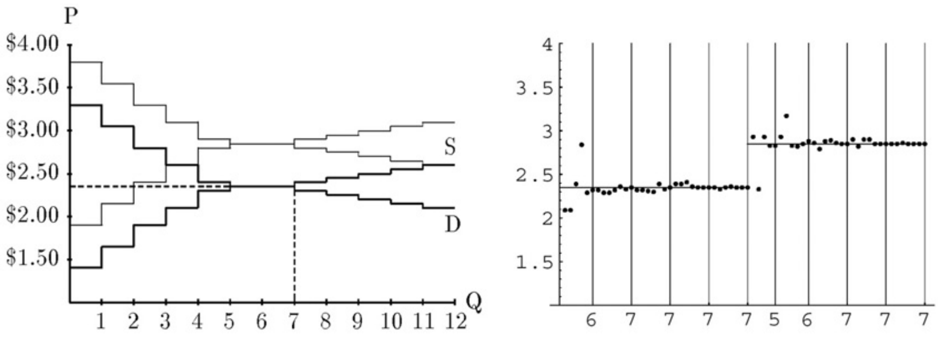


FIGURE 2.11: Left panel illustrates demand and supply of the market. Note the change in demand and supply (after 5 trading days). D denotes the demand curve, S the supply curve, P price and Q quantity. Results from market simulations with GD traders are shown in the right panel. The x-axis is divided into the different trading days, with the x-axis values corresponding to transacted quantities for each period (taken from (Gjerstad and Dickhaut, 1998)).

Now, although GD was successful, Tesauro and Bredin believed they could improve upon it by factoring time into its decision making process as the expected number of bidding opportunities before the auction closes. This means that the GDX agent has the opportunity to trade later on during the trading day and can thus wait for more profitable transactions than GD which assumes it has a single bidding opportunity whenever it is submitting a bid or an ask.

In particular, GDX calculates the belief in a similar manner as GD, but uses dynamic programming (coupled with the expected profit maximisation process) to decide on the best price and when to submit a bid or an ask. The following equation and the algorithm given in Figure 2.12 describe exactly the price formation process in GDX:

$$p^*(T) = \arg \max_{p \in (o_{ask}, o_{bid})} \quad (f(p,T)\,[s_M(p) + \gamma V(M-1, N-1)] + (1 - f(p,T))\gamma V(M, N-1)) \tag{2.15}$$

where $V(m,n)$ is the expected profit of the mth allocation a trader has to bid or ask for at the nth bidding opportunity, and is computed by the algorithm given in Figure 2.12, with $V(m,0) = 0\,\forall m$ and $V(0,n) = 0\,\forall n$. Here, $\gamma \in [0,1]$ represents a discount factor associated with bids or asks and $f(b,T)$ is the belief function, at time T, of the buyer or the seller, and $s_M(p)$ is the profit for an offer p for the Mth unit a trader has to buy or sell. The optimal bid or ask to submit at time T is then $p^*(T)$.

Algorithm Expected value computation

for $n = 1$ to N **do**
  for $m = 1$ TO M **do**
    $V(m,n) = \max_p(\; f(p,t_n)[s(p) + \gamma V(m-1,n-1)] + (1 - f(p,t_n))\gamma V(m,n-1))$
  end for
end for

FIGURE 2.12: Calculating $V(m,n)$ using a dynamic programming algorithm (taken from (Tesauro and Bredin, 2002)).

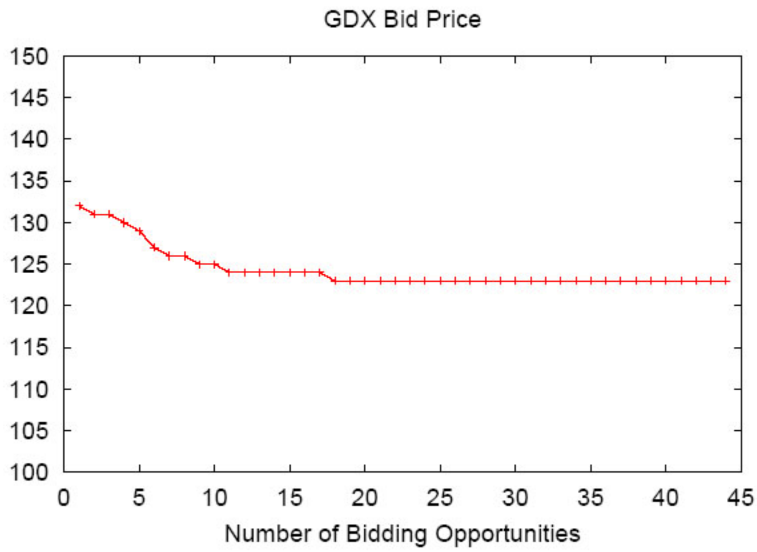


FIGURE 2.13: An example of a GDX bid price based on the number of bidding opportunities (taken from (Tesauro and Bredin, 2002)).

Whenever the GDX agent is triggered to submit a bid or an ask in the market, it estimates the number of bidding opportunities, N, before the auction closes, and calculates the optimal bid or ask given the agent's belief function that a bid or an ask will be accepted in the market. It assumes that the belief function is invariant of time such that GDX and GD use the same belief function. Figure 2.13 gives an example of the different bids calculated given the number of bidding opportunities N. Note that the bid price for a single bidding opportunity is equal to a GD bid price. The strategy is then benchmarked against GD and ZIP in a balanced heterogeneous population (where the two strategies are represented equally), and agents are endowed with either a single unit or a set of units (10 in this case) to buy or sell.

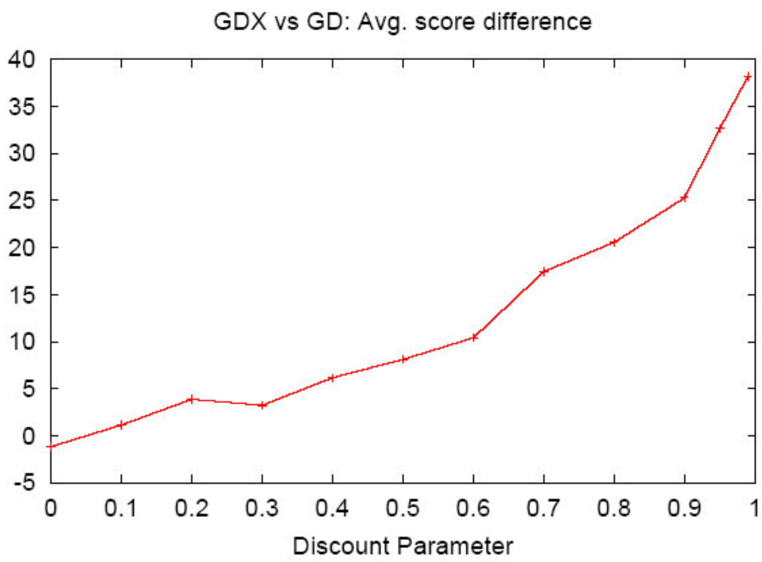


FIGURE 2.14: The average profit difference between GD and GDX in a balanced heterogeneous population, over 1000 runs. Each agent has a single unit to trade and the theoretical population surplus is 1500.0 (taken from (Tesauro and Bredin, 2002)).

First, the results of GDX against GD are given in Figures 2.14 and 2.15, where the difference in average profit between GDX and GD is compared over the space of feasible discount factors, γ, for single unit and multi units respectively. In the former case, GDX extracts more profit than its GD counterpart, with a maximum difference of about 2.5% when $\gamma = 0.99$. In the latter case, with multi-unit allocations, GDX does not always extract more profit, but generally performs better than GD, with a maximum difference in average profit of about 0.5% when $\gamma = 0.9$. We also note a drop in performance of GDX when γ approaches 1 (see Figure 2.15), and the authors suggest that this is due to the poor accuracy of the forecast of their belief function far into the future. Generally, the GDX strategy clearly showed improvement over GD, though in some cases, this improvement was not significant.

Next, the authors benchmarked GDX (and GD) against ZIP in a multi-unit allocation setting, with GDX adopting the best discount factor of 0.9 identified in the results from previous experiments (see Figure 2.15). The results are summarised in Table 2.1. As can be seen, both GD and GDX performed better than ZIP, though the average profit difference was higher for GDX. Thus, the benefits of the time-dependent and dynamic programming approach are indeed validated with the increase of average profit difference from 3.3% to 3.9%.

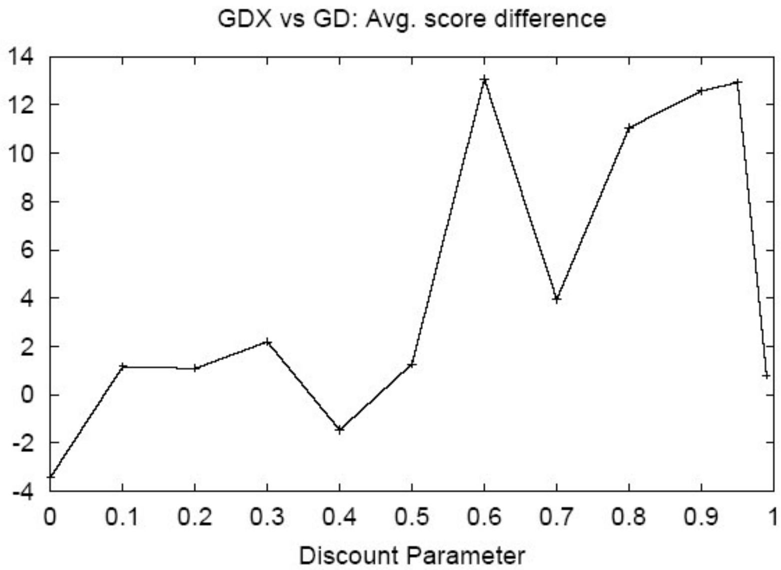


FIGURE 2.15: The average difference between GD and GDX in a balanced heterogeneous population, over 1000 runs. Each agent has 10 units to trade and the theoretical population surplus is 1500.0 (taken from (Tesauro and Bredin, 2002)).

Groups	**Profit Difference (and % of total theoretical surplus)**
GDX vs ZIP	+102.8 (3.9%)
GD vs ZIP	+87.1 (3.3%)

TABLE 2.1: The average surplus difference when GD and GDX ($\gamma = 0.9$) compete against ZIP in a balanced heterogeneous population. The theoretical population surplus is 2612.0.

2.3.4.5 Fuzzy Logic

The FL (fuzzy logic based) bidding strategy (He et al., 2003) is non-predictive and uses a history of transaction prices. It employs heuristic fuzzy rules and fuzzy reasoning mechanisms (Zadeh, 1965) in order to determine the best bid or ask offer given the current state of the market. We begin by introducing some of the notations used.

Definition 2.21. The *reference price*, P_R, is the median of the ordered history of transaction prices.

Definition 2.22. The valid bids set, D_b, is the set of the valid bids that a buyer can submit:

$$D_b = \left\{ b \middle| b_0 < b \leq \min(a_0, l_i^b) \right\}$$

where b is the price at which a buyer submits a bid and l_i^b is the limit price of buyer i.

Definition 2.23. The valid asks set, D_s, is the set of the valid asks that a seller can submit:

$$D_s = \{a | \max(b_0, l_j^s) \leq a < a_0\}$$

where a is the price at which a seller submits an ask and l_j^s is the cost price of seller j.

Definition 2.24. A *triangular fuzzy number*, z, is used to represent a real number, and is represented as follows:

$$z = (m, \theta, \chi)$$

where *m* is the center, and θ and χ are the left and right spreads respectively (see figure 2.16). The triangular number is described in detail in (He et al., 2003).

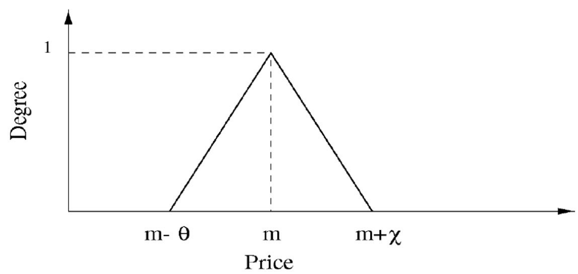


FIGURE 2.16: Triangular fuzzy number $z = (m, \theta, \chi)$ (taken from (He et al., 2003)).

Now, when an FL agent submits an offer, it considers the outstanding bid, the outstanding ask, its limit price or cost price and the reference price P_R (see Definition 2.21). While the authors do not relate it explicitly to the competitive market equilibrium (discussed in Section 2.1), they regard it as a *suitable* transaction price. Now, when $P_R \leq b_0 < a_0$ or $b_0 < a_0 \leq P_R$, a set of heuristic rules is used to decide whether or not to submit an offer, and, if so, at what price. These rules are summarised in Figure 2.17 for both the FL buyer and the FL seller. For example, in rule *SR1*, when b_0 is *much bigger than* P_R, the agent accepts the outstanding bid, b_0. Otherwise, the agent submits an ask given by the triangular fuzzy number $(a_0 - \beta_{s,1}, \theta, \chi)$. The fuzzy sets in Figure 2.18, represent the different relations, such as 'b_0 *is much_bigger than* P_R', used in the fuzzy heuristic rules. Furthermore, the β parameters used in the fuzzy rules, θ and χ, are specified at the beginning of the trading day. Parameters P_1, P_2, P_3 and P_4

- When $P_R \leq b_o < a_o$, the heuristic rule is:

  (SR_1) IF b_o is $much\_bigger$ than P_R
  THEN accept b_o
  ELSE *ask* is $(a_o - \beta_{s,1}, \theta, \chi)$.

- When $b_o < a_o \leq P_R$, the heuristic rule is:

  (SR_2) IF a_o is $much\_smaller$ than P_R
  THEN no new ask
  ELSE ask is $(a_o - \beta_{s,2}, \theta, \chi)$.

- When $b_o < a_o \leq P_R$, the heuristic rule is:

  (BR_1) IF a_o is $much\_smaller$ than P_R
  THEN accept a_o
  ELSE bid is $(b_o + \beta_{b,1}, \theta, \chi)$.

- When $P_R \leq b_o < a_o$, the heuristic rule is:

  (BR_2) IF b_o is $much\_bigger$ than P_R
  THEN no new bid
  ELSE bid is $(b_o + \beta_{b,2}, \theta, \chi)$.

FIGURE 2.17: Fuzzy heuristic rules for the FL buyer (*BR1* and *BR2*) and the FL seller (*SR1* and *SR2*).

(see Figure 2.18) are associated with the different rules and β parameters, and are also decided prior to the CDA game.

When $b_0 \leq P_R < a_0$, however, the bidding process is more complex and is handled by a fuzzy reasoning mechanism. In this case, the rules for the FL buyer are presented in Figure 2.19 and those of the FL seller in Figure 2.20. Here, the distance between a_0 or b_0 and the reference price is expressed using the fuzzy linguistic terms *far_from*, *medium_to* and *close_to* defined in figure 2.21. The λ parameters, θ and χ, are also initialised at the beginning of the trading day.

The fuzzy heuristic rules and the fuzzy reasoning mechanism output a triangular fuzzy number that represents a bid ($z_b(b)$) or an ask ($z_s(a)$). Based on that fuzzy number, $z_s(a) = (m_s, \theta_s, \chi_s)$, the seller submits an ask given by the following formulae:

$$\begin{aligned} DS_s = & \quad \{a \big| a \in D_s \cap \{a \big| z_s(a) \geq \pi_s\}\} \\ \textit{ask} = & \begin{cases} b_0 & \text{if } b_0 \in DS_s \\ \arg\max_{a \in DS_s} \{z_s(a)\} & \text{otherwise} \end{cases} \end{aligned} \tag{2.16}$$

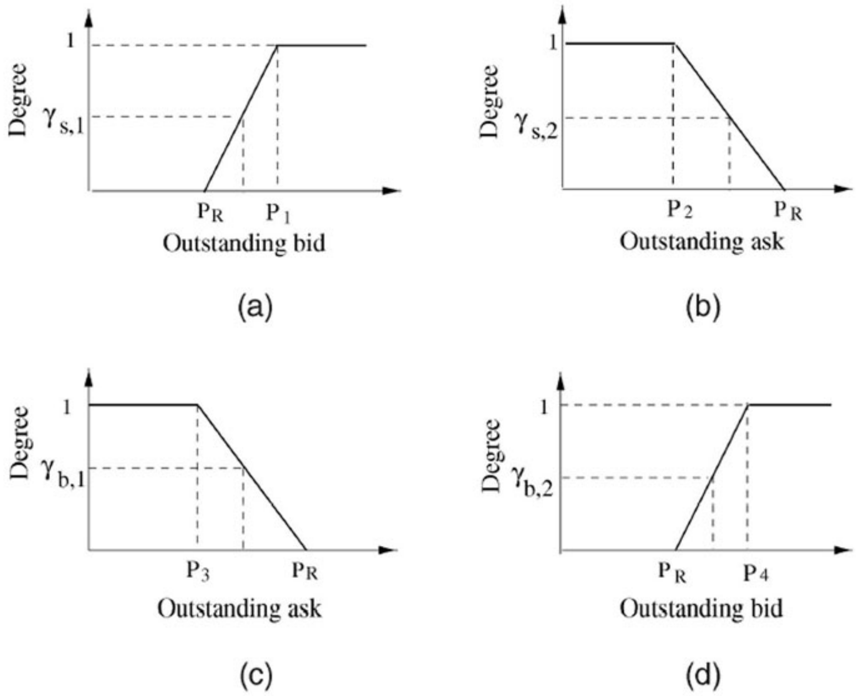


FIGURE 2.18: Fuzzy sets in heuristic rules (taken from (He et al., 2003)). (a) Outstanding bid is much_bigger than P_R (SR1). (b) Outstanding ask is much_smaller than P_R (SR2). (c) Outstanding ask is much_smaller than P_R (BR1). (d) Outstanding bid is much_bigger than P_R (BR2) (taken from (He et al., 2003)).

IF	(a_o is *far_from* or *medium_to* P_R) and (b_o is *far_from* P_R)
THEN	bid is $(b_o + \lambda_{b,1}, \theta, \chi)$.
IF	(a_o is *far_from* or *medium_to* P_R) and (b_o is *medium_to* P_R)
THEN	bid is $(b_o + \lambda_{b,2}, \theta, \chi)$.
IF	(a_o is *far_from* or *medium_to* P_R) and (b_o is *close_to* P_R)
THEN	bid is $(b_o + \lambda_{b,3}, \theta, \chi)$.
IF	a_o is *close_to* P_R
THEN	bid is $(P_R - \lambda_{b,4}, \theta, \chi)$.

FIGURE 2.19: Fuzzy Rule Base for FL buyers.

IF $(b_o$ is *far_from* or *medium_to* $P_R)$ and $(a_o$ is *far_from* $P_R)$
THEN ask is $(a_o - \lambda_{s,1}, \theta, \chi)$.

IF $(b_o$ is *far_from* or *medium_to* $P_R)$ and $(a_o$ is *medium_to* $P_R)$
THEN ask is $(a_o - \lambda_{s,2}, \theta, \chi)$.

IF $(b_o$ is *far_from* or *medium_to* $P_R)$ and $(a_o$ is *close_to* $P_R)$
THEN ask is $(a_o - \lambda_{s,3}, \theta, \chi)$.

IF b_o is *close_to* P_R
THEN ask is $(P_R + \lambda_{s,4}, \theta, \chi)$.

FIGURE 2.20: Fuzzy Rule Base for FL sellers.

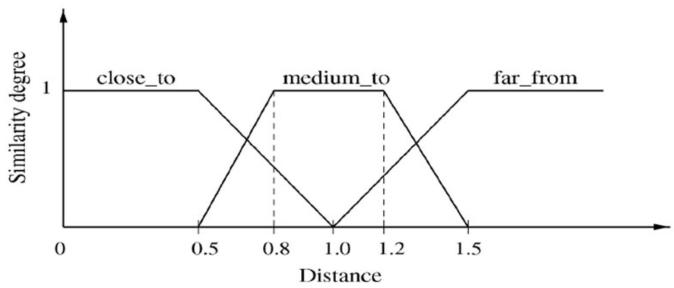


FIGURE 2.21: Fuzzy sets used in fuzzy reasoning (taken from (He et al., 2003)).

Using the output fuzzy number, $z_b(b) = (m_b, \theta_b, \chi_b)$, the buyer submits a bid given by the following formulae:

$$\begin{aligned} DS_b &= \{b | b \in D_b \cap \{b | z_b(b) \geq \pi_b\}\} \\ bid &= \begin{cases} a_0 & \text{if } a_0 \in DS_b \\ \arg\max_{b \in DS_b} \{z_b(b)\} & \text{otherwise} \end{cases} \end{aligned} \tag{2.17}$$

Furthermore, the authors extended the FL strategy and described how a different set of parameters for the fuzzy sets can be chosen for the FL agent to adopt different risk attitudes (see Figure 2.22 where the agent has different utility functions for different attitudes), and how a set of simple learning rules can be used to change the FL agent's risk attitude to be more profitable in the market. The learning is based on how frequently the agent is trading (with 'how frequently' defined by the fuzzy sets given in Figure 2.23) and a set of rules given in Figure 2.24.

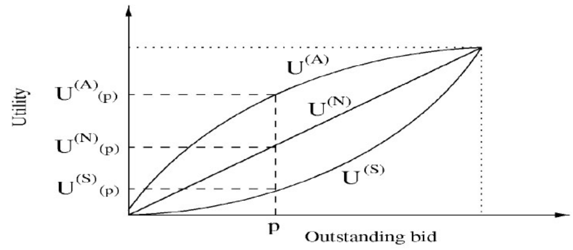


FIGURE 2.22: Utility functions of FL agents with different risk attitudes: risk-neutral (N), risk-seeking (S) and risk-averse (A) (taken from (He et al., 2003)).

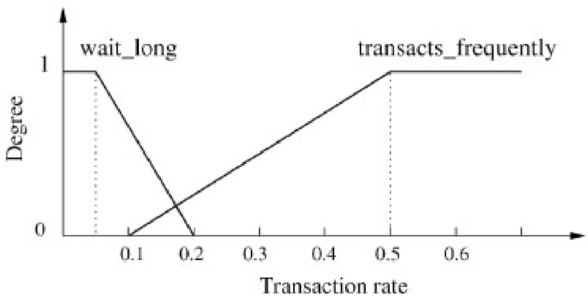


FIGURE 2.23: Two fuzzy sets for transaction rate. This rate is calculated by the number of transactions made by an agent divided by the total transaction numbers in the market after the latest change of the FL agents attitude towards risk (taken from (He et al., 2003)).

IF	agent i waits_long to transact
THEN	$A^{(i)}_{attitude} = A^{(i)}_{attitude} - r\delta$
IF	agent i transacts_frequently
THEN	$A^{(i)}_{attitude} = A^{(i)}_{attitude} + r\delta$

FIGURE 2.24: Learning rule of the FL agent. $A^{(i)}_{attitude}$ denotes the attitude of agent i, r is the learning rate, and δ is the minimum change in the risk attitude.

The FL strategy was benchmarked against some of the most common strategies in the CDA, namely the ZI and the GD which we have seen in this section, and the CP strategy

(Preist and Tol, 1998), a variant of the ZIP strategy in heterogeneous populations with 5 buyers and 5 sellers each adopting one of the 5 strategies we investigate. The results are shown in Figure 2.25. As can be seen, the FL and GD buyers and sellers obtained higher profits than with the other strategies. The authors attribute the performance of FL to two factors. First, the FL agent considers the outstanding bid and ask, as well as a reference, which they believe is a very important factor in bidding. Second, the FL strategy can dynamically vary the rate of increase (decrease) in bid (ask) offers according to the prevailing market conditions. Thus, the FL strategy can jump from a very low bid to a transaction price, as opposed to the benchmark strategies which only increase (decrease) their bids (asks) gradually.

Furthermore, we observe that some of the results are questionable as CP, based on ZIP, is sometimes outperformed by ZI, which was shown otherwise in (Tesauro and Das, 2001). We believe this can be accounted for by their non-conventional methodology with 5 buyers with 5 strategies and 5 sellers with 5 strategies. We also observe that the performance of FL is only marginally better than GD. However, the authors also point out that, in a homogeneous population of FL agents, the efficiency is around 85% which is considerably lower than that of GD and GDX (which is around 99%). Because FL does best when it can exploit the bidding behaviour of other strategies, its behaviour can be compared to that of Kaplan which also does not do well in a homogeneous population. For these reasons, FL cannot be adopted in a decentralised system for resource allocation.

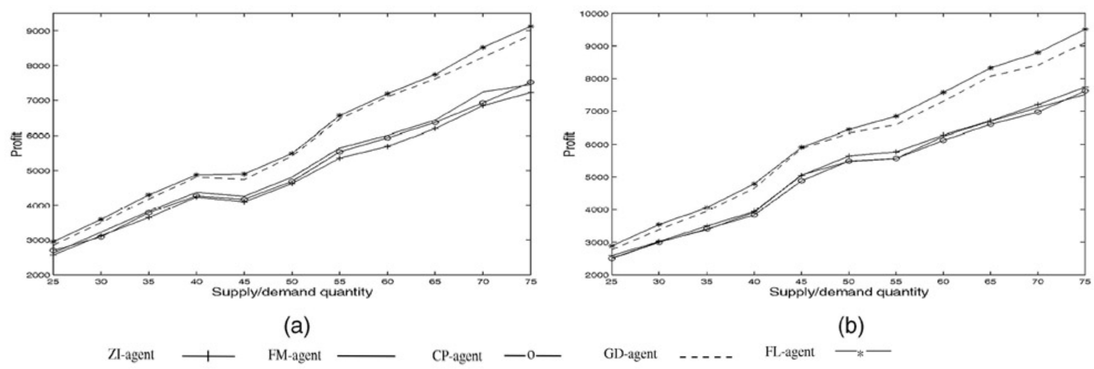


FIGURE 2.25: Performance of agents with different strategies (taken from (He et al., 2003)). The horizontal axis represents the demand and supply of the market and the vertical axis represents the total profit of the corresponding agent using a specific strategy in one session. Panel (a) shown buyers' total profits and panel (b), sellers' total profits.

2.3.5 Evaluating Strategies

Given the avalanche of strategies for the CDA and our research aim to evaluate these strategies, we now review the methodologies that exist for doing this. In particular, we divide the review into separate categories for homogeneous and heterogeneous environments because we wish to observe how a strategy performs on its own and when opposed to other strategies. For the homogeneous case, the emergent behaviour of the market is more interesting from a system designer's perspective. For the heterogeneous case, the emergent behaviour of the market is of interest, though, from an agent's perspective, the efficiency of the different strategies in the market would be insightful. We now look at each case in turn.

2.3.5.1 Homogeneous Populations

Related work on evaluating strategies for the CDA has typically looked at the average efficiency of a strategy over several trading days, in static markets with symmetric demand and supply (e.g. (Gjerstad and Dickhaut, 1998; Tesauro and Das, 2001)). However, we believe this has a number of shortcomings.

First, analysing the daily efficiency of the strategy provides more insight into how effective a strategy is in learning from market interactions. This view is partly supported by (Cliff and Bruten, 1997), though they focus on the daily price volatility. Specifically, we believe that as the agent learns to be more competitive in a static market and the transaction prices converge towards the competitive equilibrium price, we expect its efficiency to improve. Such an analysis would allow us to observe just that. Thus, it is important to measure efficiency on a daily basis because it gives us insights into how the behaviour of the market is changing and, in particular, is improving. Moreover, such observations would not be possible if we just focused on average efficiency because we end up with a scalar value that does not say anything about the trend of the daily efficiencies. Second, we believe that daily price volatility should also be looked at. To date, however, only Cliff and Bruten (1997) consider such a metric. Because the competitive market equilibrium is usually central in a strategy, the price volatility, calculated as Smith's parameter (see Section 2.1), is important because it gives insights into how the agents adjust their behaviours such that the transaction prices converge to that equilibrium. The rate of convergence usually determines how fast the market reaches a high efficiency and, thus, would be useful in analysing the effectiveness of a strategy in a homogeneous population. Third, only Cliff and Bruten (1997) have looked at dynamic environments with different market demand and supply. However, they only describe how transaction prices change, and not how daily efficiency and price volatility change in such environments. This is important because, as discussed in Section 1.1, we are considering decentralised resource allocation in both static and dynamic environments. Thus, it would be interesting to analyse how daily efficiency and price volatility change in both types of environments. Furthermore, because demand and supply cannot be known *a priori*, we must ensure that the strategies are evaluated in markets with different types of representative demand and supply, and not simply the standard cases (with symmetric demand and supply) to ensure the significance of our analysis.

Given these shortcomings, we need to use an analytical method that considers both market efficiency and price volatility, on a daily basis, to highlight this learning in both static and dynamic markets with different market demand and supply.

2.3.5.2 Heterogeneous Populations

When we consider methodologies for evaluating strategies in heterogeneous populations, we come across two principal approaches. The first one (adopted in (Tesauro and Das, 2001; Tesauro and Bredin, 2002; Vytelingum et al., 2004)) consists of comparing the efficiency of strategies in balanced populations (where strategies are adopted in equal proportions). However, this approach fails to consider unbalanced populations where strategies are present in different proportions. The second one proposed by Walsh *et al*. (2002) and adopted in (Phelps et al., 2004; Vytelingum et al., 2006) does allow unbalanced populations. This approach is important because a strategy might perform better or worse based on the number of buyers and sellers that adopt it, an insight which would allow us to better evaluate a strategy and, thus, we consider this approach in this thesis.

In particular, Walsh *et al*. propose an evolutionary game-theoretic (EGT) approach based on computing the mixed-Nash equilibrium of heuristic strategies and the dynamics analysis of equilibrium convergence (Weibull, 1995). Now, because an EGT analysis is infeasible for all but the simplest games (such as the Prisoner's Dilemma (Weibull, 1995)), Walsh *et al*. describe how complex games that involve repeated interactions with more elaborate actions and payoffs, can be made amenable to such analysis. Specifically, their model considers the high-level, heuristic strategies of the trading agents as simple actions, and the payoff to these strategies is the average profit extracted in the market (by so doing, they essentially abstract a complex iterated game to a simple normal-form one). To illustrate their approach, they apply it to two different games, namely the Automated Dynamic Pricing (ADP) game and the CDA game. In the former, they analyse how sellers endowed with a set of heuristic strategies interact in the market, and what strategies these sellers are most likely to adopt. In the latter, they consider the strategic interaction of agents that use the same strategy as a buyer and a seller. Their methodology has now been widely adopted and, in particular, (Phelps et al., 2004) used it to compare two different auction mechanisms (the continuous and the call double auction mechanism) with similar strategies were available for both.

Given this background, we first describe Walsh *et al.*'s EGT model in more detail and, specifically, how they compute the heuristic payoff table that details the expected payoff to each agent (as a function of the S strategies that agents are allowed to play and the combination of the A agents playing those strategies). We then describe how this table can be used to compute the mixed-Nash equilibrium of the game and the well-documented *replicator dynamics* model (Weibull, 1995) (which is a standard way of representing the population distribution changes), to analyse the CDA. Finally, we describe how they apply their model to the CDA.

Computing the Heuristic Payoff Table: With the heuristic payoff table, the expected payoff of a player playing a strategy, j, given the strategies adopted by the other $(A-1)$ players is required. Now, because of the non-deterministic and complex nature of the CDA game, some simplifications are required:

1. The payoff of a strategy is the average payoff of an agent playing that strategy in the CDA game, given the different strategies all the A agents are playing in that game.

2. All agents have the same set of strategies to play, and have the same payoff when playing the same strategy. Thus, as described in (Walsh et al., 2002), we can restrict our analysis to symmetric games (Weibull, 1995), and significantly reduce the complexity of the problem. Rather than having a table of size S^A, we reduce it to $\binom{A+S-1}{A}$ entries.

Given these, a heuristic payoff table[7] can be built by considering the exhaustive set of strategies the A agents can play, and the number of agents playing each strategy (rather than considering which strategy each of the A agents is playing). Now, because payoff in the CDA game is non-deterministic, a significant number of independent simulations

[7] A table entry for a 20-player game with 3 strategies would be $(|S_1|, |S_2|, |S_3|, U_1, U_2, U_3)$ where S_j is the set of agents playing strategy j, $|S_j|$ is the number of agents playing strategy j, and U_j is the average payoff of an agent playing strategy j. Note $\sum_{j=1}^{3} |S_j| = 20$ and there are 231 entries.

are required for each table entry to ensure that these are representative values. Thus, for each entry, a statistically significant number of CDA games is required with A agents, each assigned a strategy and a type (buyer or seller) to play, ensuring there is an equal number of buyers and sellers, with a probability of 0.5 that there will be an additional buyer or seller if A is odd.

Given the heuristic payoff table, we now look at the EGT analysis as is done with a normal-form game.

Computing the Equilibrium: Here, we describe how Walsh *et al*. compute the mixed-Nash equilibrium of the CDA. An agent i chooses the strategy it plays according to its *mixed-strategy*, $\hat{p}_i = (\hat{p}_{i,1}, ..., \hat{p}_{i,S})$ and $\sum_{j=1}^{S} \hat{p}_{i,j} = 1$, where $\hat{p}_{i,j}$ represents the probability that agent i plays strategy j. At the equilibrium, $\hat{p}_i^*$, an agent i cannot receive a higher payoff by unilaterally deviating to another mixed-strategy, assuming that the other agents do not change their strategies (Weibull, 1995). Now, because they assume a symmetric game, all agents have the same mixed-strategy and the same mixed-Nash equilibrium. Furthermore, because a very large population is being considered, they validate that p is equal to the mixed-strategy. Given this, they denote both the population distribution and the mixed-strategy as $p = (p_1, ..., p_S)$, and the mixed-Nash equilibrium as p_{nash} hereafter.

In the EGT analysis, they denote the expected payoff of an agent playing a strategy j, given the mixed-strategy p, as $u(e^j, p)$. To compute $u(e^j, p)$, they consider the results from a large number of CDA games with an agent playing strategy j and $(A-1)$ agents selected from the population, with a mixed-strategy p. For each game and every strategy, they average[8] the individual payoffs (obtained from the heuristic-payoff table) of agents using strategy j. The mixed-Nash equilibrium is then formulated as the argument to the minimisation problem given in Equations 2.18 and 2.19. Specifically, p_{nash} is a mixed-Nash equilibrium if and only if it is a global minimum of $v(p)$ (Walsh et al., 2002; McKelvey and McLennan, 1996), and they validate that p is a global minimum if $v(p) = 0$.

$$v(p) = \sum_{j=1}^{S} \left(\max\left[u(e^j, p) - u(p, p), 0\right]\right)^2 \quad (2.18)$$

where $u(p, p) = \sum_{j=1}^{S} u(e^j, p)p_j$ is the average payoff of an agent in a population with distribution p.

[8] In effect, here, they are not running the CDA game for every simulation, but only selecting the appropriate payoff from our heuristic payoff table each time.

$$p_{nash} = \arg\min_{\overline{p}\in\Delta} [v(p)] \tag{2.19}$$

Solving such a non-linear minimisation problem is non-trivial and computationally demanding. Thus, they use the Amoeba non-linear optimiser (see (Walsh et al., 2002) for more detail) to find the zero-points of the function v. Because the algorithm used is a non-linear *local* minimiser, they restarted the algorithm repeatedly at random points within the unit-simplex until they had found 30 previously-discovered equilibria in a row.

Now, while the mixed-Nash equilibrium gives a theoretical and static perspective of the simplified CDA game, the dynamics of the game and how the equilibria are reached often provide more insight. Given this, they turn to the replicator dynamics which have been shown to be a good model for agent learning (with agents learning using reinforcement learning to reach the equilibrium (Tuyls and Nowe, 2005)).

Computing the Replicator Dynamics: The replicator dynamics, $\dot{p} = (\dot{p}_1, ..., \dot{p}_S)$, describe how the population distribution p changes (where $p = (p_1, p_2, ..., p_S), p \in \Delta$ is an element of a unit-simplex Δ, and $\sum_{j=1}^{S} p_j = 1$). This approach assumes that an agent deviates to another strategy that appears to be receiving a higher payoff. Specifically, $\dot{p}$ is a vector given as follows:

$$\dot{p}_j = \left[u(e^j, p) - u(p, p)\right] p_j \tag{2.20}$$

To observe the dynamics of the game, we calculate *trajectories* (i.e. how the mixed strategies change). In more detail, we start with any mixed strategy p, and calculate the dynamics $\dot{p}$. The replicator dynamics show the strategy trajectories and how they converge to an equilibrium, though they do not necessarily settle at a fixed point (Weibull, 1995). In this context, an equilibrium to which trajectories converge, and settle, is known as an *attractor*, while a *saddle point* is an unstable equilibrium at which trajectories do not settle. The region within which all trajectories converge to a particular equilibrium is known as the *basin of attraction* of that equilibrium. The basin is a very useful measure of the adoption of the attractor equilibrium and its area determines how likely the population is to converge to that equilibrium. More formally:

Definition 2.25. A trajectory is the change in mixed strategy, starting from a particular mixed strategy, and following the replicator dynamics.

Definition 2.26. An attractor is a mixed-Nash equilibrium towards which the replicator dynamics (trajectories) converge.

Definition 2.27. A saddle point is a mixed-Nash equilibrium from which replicator dynamics (trajectories) diverge[9].

Definition 2.28. A basin of attraction of a mixed-Nash equilibrium is the space of mixed strategy from which trajectories will converge to that equilibrium.

Here, they compute $\dot{p}$ by starting at different population distributions p inside the unit-simplex and following the trajectory given by Equation 2.20.

Having outlined the basic concepts, we now show how they can be used to analyse the dynamic behaviour of strategies in a CDA. Specifically, we describe Walsh *et al.*'s EGT analysis of a 20-agent CDA with three different strategies, namely Kaplan, ZIP and GD (see Subsection 2.3.4).

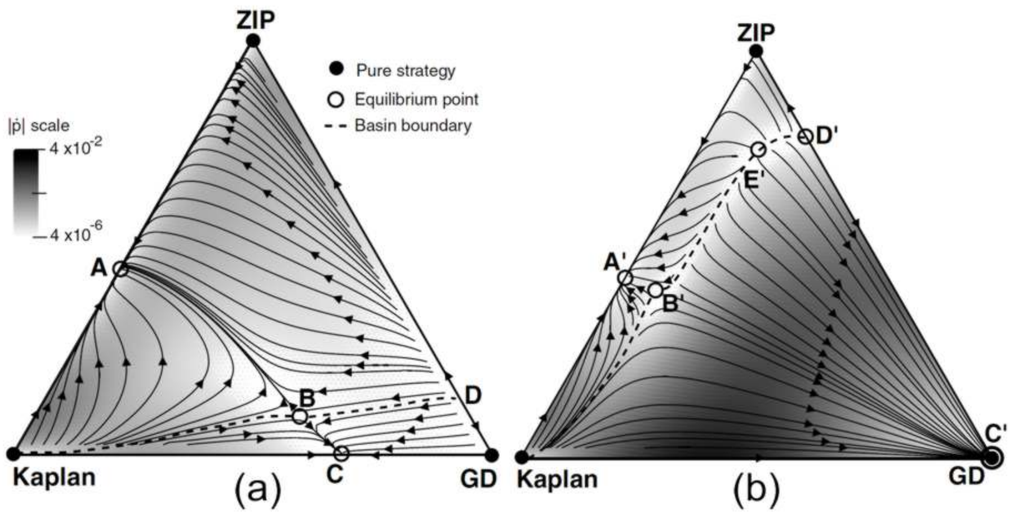


FIGURE 2.26: (a) The replicator dynamics of a 20-agent CDA. Here, there are two attractors: A and C and a saddle point: B. (b) Replicator dynamics if the payoffs were perturbed such that 5% of the payoffs of Kaplan and ZIP were transferred to GD (taken from (Walsh et al., 2002)).

The replicator dynamics of the analysis are given in Figure 2.26(a). The different vertices correspond to one of the pure strategies, while the gray shading denotes the magnitude of the dynamics $|\dot{p}|$. Specifically, we have two attractors A and C, towards which all the trajectories converge, and a saddle point B from which trajectories diverge[10]. When we have a majority of ZIP agents, we can see that it is more profitable to deviate to Kaplan, suggesting that Kaplan does well when in the minority. However, as too

[9]The term *deflector* would be more intuitive. However, we use saddle point to be consistent with the literature.

[10]Note that D is not a saddle point. In the three-strategy game, at D, there are incentives to deviate to Kaplan. Because at a saddle point, there are no incentive to deviate to any strategy, D is one.

many agents adopt Kaplan, there are fewer agents for Kaplan to exploit (see Subsection 2.3.4.1 for more detail). Then, when Kaplan is in the majority, agents are economically motivated to deviate to ZIP. This results in a balance of the population at the mixed-Nash equilibrium A. We observe a similar behaviour with Kaplan and GD, with a mixed-Nash equilibrium B. Furthermore, we generally observe that agents tend to deviate from GD to Kaplan and ZIP when GD is in the majority. As the behaviour of the agents evolves, it usually settles at either A or C. However, there is a considerably larger probability[11] that A will be adopted.

Walsh *et al*. also describe how to predict the replicator dynamics if a strategy were to be improved. Specifically, they showed how the dynamics change when 5% of the profits of Kaplan and ZIP were distributed to GD. These dynamics are given in Figure 2.26(b). In this case, there are a new set of equilibria, with two attractors A′ and C′, and three saddle points B′, D′ and E′. The area of the basin of attraction for C′ is then considerably larger than that of A′, implying that there is now a much larger probability that C' will be adopted. Furthermore, we observe that the equilibrium C′ is now the pure strategy GD such that it nearly always benefits to deviate to GD in a population with Kaplan and GD agents. Thus, if GD could be sufficiently improved, it would then be nearly dominant and always be adopted. Such a method to predict the market dynamics by artificially inflating payoffs could be useful to motivate the improvement of particular strategies. However, it fails to provide useful insights into the design issues with the strategies that need to be addressed.

The EGT approach to evaluating strategies in heterogeneous populations is indeed more insightful than simply comparing the efficiencies of strategies in balanced populations. However, a key assumption of this approach is that an agent will adopt the same heuristic strategy even when it has to perform different roles (such as being a buyer and a seller). In games like the ADP, where agents have a single role (as a competing seller), such an assumption does not constrain the analysis, and their methodology is appropriate. However, in double-sided games, like the CDA, such an assumption is both unrealistic and unnecessarily restrictive. In practice, buyers and sellers usually have different bidding behaviours whose efficiency depends on a number of factors including what strategies other buyers and sellers adopt, and the demand and supply of the market then determines the complex interactions of these strategies which in turn, determines their overall effectiveness. To maximise its profit, we believe an agent should be allowed to select whatever is the best strategy for it when acting as a buyer and whatever is the best for it as a seller. The present constraint of compromising on both and

[11] This probability that an attractor will be reached is equal to the area of the basin of attraction of that attractor.

having to select the same strategy for both roles can only have a negative effect on the agent's economic efficiency. We believe that such an assumption should not be made because this approach would then miss some important phenomena. Thus, an approach to analyse how the buyer *and* seller strategies separately evolve in the market is needed.

2.4 Summary

In this chapter, we began with some background on the micro-economic theory of demand and supply, and the competitive market equilibrium in a free market, and described the structural and behavioural aspects of the CDA. In more detail, in the former aspect, we looked at the somewhat limited work that has analysed the structure of the CDA, and how the market protocols can be modified to improve its effectiveness. In the latter aspect, we first reviewed the work on software agent and human interactions in the market, followed by a comprehensive review of bidding strategies, as well as frameworks for designing and analysing bidding strategies for software agents.

Given the research aims discussed in Section 1.2, we now discuss the extent to which the state of the art addresses these aims. The purpose of such a discussion is to identify the issues which this thesis needs to address to meet its aims:

(a) **The Structure of the CDA**:

- We observe that the work on the structure of the CDA has primarily been concerned with improving the effectiveness (e.g. improving the efficiency or reducing price fluctuations) of the mechanism by modifying the market protocols, including the pricing and clearing rules. Broadly speaking, this satisfies our research *aim (a)*. However, all these lines of work are based on the simple resource allocation problem described in Section 2.1. Thus, to fulfil research *aim (b)*, we need to investigate if the CDA would still be efficient if it were to be modified to solve a more complex resource allocation problem. In particular, we wish to consider markets where sellers have limited capacity and a production function with a start-up cost and buyers have an inelastic demand. Given a CDA variant for such a market, we then need to determine how it can be further modified to achieve certain desirable properties in this new decentralised system and, thus, we would partly be addressing our research *aim (a)*.

(b) **The Behaviour of the CDA**:

- Our research *aim (c)* is about designing new strategies. However, there is presently no framework for designing strategies for market mechanisms. We believe that such a framework is desirable because it would provide an engineering approach to designing bidding strategies by providing guidelines for the strategy designer. In so doing, we would partly fulfil research *aim (c)*. Specifically, we can see from the existing work that an agent's bidding strategy is about collecting (internal and external) market information, processing such information along with the agent's private information (given limited computational resources), and using such processed information (knowledge) in a rational and profit-motivated behaviour to bid efficiently in the market. Our framework should thus reflect all those different aspects of a strategy.

- We believe that existing bidding strategies can be improved upon and, in particular, we believe this to be especially true of the predictive class of strategies. The state of the art in that particular class is the ZIP family of strategies where the profit margin is updated in a linear manner over a feasible range. However, we believe a more flexible approach is possible by having the profit margin change non-linearly over that range (as it might be more efficient to vary the rate of change of the profit margin). Furthermore, we believe that the notion of a reference price (as per the FL strategy − see Subsection 2.3.4.5) is important (because it provides a suitable reference to what a profitable price is). But, we believe that a prediction of the competitive equilibrium price would be a better choice as transaction prices converge towards that price (than the median of the transaction price as in FL). Thus, to tackle research *aim (c)* to design more efficient strategies, we will develop a novel bidding strategy for the CDA based on these intuitions.

- In Subsection 2.3.5.2, we highlighted the shortcomings of Walsh *et al.*'s standard EGT approach for analysing complex interactions in the CDA because of its restrictive assumption that buyers and sellers adopt the same behaviour. Specifically, the evolution of the buyers' and sellers' behaviours should not be the same, since the payoffs for deviating to another strategy are different for the different roles. Thus, we believe that a model that separately analyses the evolution of the buyer and seller strategies is more appropriate and, indeed, more insightful. Given that such an analysis can be used to evaluate strategies in heterogeneous populations, we will develop such a model, and partly fulfil our research *aim (d)* on designing methodologies for evaluating strategies.

- Finally, we observed that the methodologies to analyse the efficiency of strategies in the CDA differ considerably. The evaluation of strategies in homogeneous populations has typically been limited to static environments with symmetric demand and supply. Cliff's work, on the other hand, looked at dynamic settings with market shocks, and different demand and supply, but was limited to the ZIP strategy only. For heterogeneous populations, while some work simply considered a balanced population, other work used the Walsh *et al.*'s EGT model which allows unbalanced situations. Furthermore, evaluating strategies in heterogeneous populations has been limited to a static setting, as well as a symmetric demand and supply. Thus, to meet our research *aim (d)* and address the issues with existing methodologies, we will develop new approaches for evaluating strategies in both homogeneous and heterogeneous populations, that consider the best aspects of the different methodologies. Specifically, because EGT is the best approach for the heterogeneous case, we will use it as our point of departure. Given these methodologies, we will then benchmark the novel strategy we will design against the state of the art strategies from the predictive and non-predictive classes, namely ZIP[12] and GDX, in different market environments.

[12]Note that we do not use ZIP60 as it is tailored to particular markets. Because, we are looking at static and dynamic systems where the problem (represented by the demand and supply in our market-based approach) is unknown *a priori*, we do not have a ZIP60 strategy that is optimized for a range of demand and supply, and we do not believe that an educated guess of the 60 parameters can result in ZIP60 being more efficient than the ZIP. Furthermore, Cliff proposes the set of 8 parameters of the ZIP, evolved for a set of different environments (Cliff, 2001) and, thus, we will use the ZIP with these 8 parameters as a benchmark.

Part I

THE STRUCTURAL PERSPECTIVE

The structure of the CDA defines how the buyers and sellers interact in the market. In particular, it is specified by the market protocol, a set of rules that define who is allowed to participate in the market, what types of bids or asks can be submitted, and when a transaction occurs, and at what price. To this end, in this part of the thesis, we address our research aims to develop a new CDA-based mechanism, with certain desirable properties, to solve a more complex resource allocation problem than that usually looked at in the standard CDA. As stated in Section 1.1, we do this to demonstrate the broader space of application of the CDA as a decentralised resource allocation solution.

In more detail, in Chapter 3, we first describe the resource allocation problem, which in this case is minimising the total production cost of sellers. Next, we describe a centralised mechanism that gives an optimal solution to our problem, and then go on to give the design of the protocol of our decentralised mechanism to satisfy the constraints of the new problem. Furthermore, we describe how we modify this protocol to allow a fair distribution of profits among buyers and sellers. Note that the purpose of the centralised mechanism is to calculate the efficiency of our mechanism as the ratio of total production cost in our decentralised mechanism to the total production cost in a centralised and optimal (globally minimum) solution. Next, we evaluate the efficiency of our decentralised mechanism and, to this end, we develop a simple zero-intelligence strategy. The purpose of such a simple behaviour is to attribute the efficiency of the system to the structure, rather than the behaviour, of the mechanism (as per Gode and Sunder's work). We compare the efficiency of our mechanism with such a behaviour for different market sizes and, in so doing, we empirically demonstrate that our mechanism is very efficient in terms of minimising total production cost.

The overall purpose of this work is to demonstrate that the CDA can be an efficient solution to non-conventional and complex resource allocation problems. Furthermore, because we evaluate our system using a zero-intelligence behaviour, we are then be are able to provide a lower bound on the efficiency of the system. This enables a system designer to choose such a solution if the efficiency trade-off for the desirable properties the decentralised solution offers is sufficiently low.

Chapter 3

Designing a CDA Mechanism for Limited-Capacity Suppliers

In this chapter, we present our work on the structural aspect of the CDA. In particular, we consider how to modify the protocol of the CDA and show that we can develop a variant to solve a more complex resource allocation problem than in the standard CDA. Furthermore, we evaluate the efficiency of the mechanism. Now, in order to calculate the efficiency of this protocol, we have to compare the total production cost of our decentralised mechanism to that of a optimal solution. Thus, we develop a centralised mechanism that solves the problem optimally using dynamic programming. We then develop a zero-intelligence behaviour to find the efficiency of our system that is attributable to the structure, rather than the behaviour of the mechanism by adopting the approach of Gode and Sunder (see Section 2.3.4).

Our problem involves suppliers with a particular form of *cost structure* (consisting of a fixed overhead cost and a constant marginal cost) and *finite production capacities* (which are both privately known to them), and consumers with inelastic demand. We pick this scenario because these traits are typical of many real world applications such as electricity markets and job-shop scheduling. For example, a power plant will typically have a fixed start-up cost and a constant marginal cost of running the plant up to its maximum capacity, and the classic job shop scheduling problem consists of running periods composed of an initial machine set-up time (overhead cost) plus a cost per unit time (the marginal cost) and a finite capacity which these machines can run up to.

Now, most work on CDAs assumes a cost structure that consists of an increasing marginal cost for each unit supplied and no startup cost. This choice of cost structure is quite natural in macro-economic models and it results both in a competitive market equilibrium price for the commodity and in efficient allocations (see Section 2.1). Unfortunately, the particular cost structure of our domain implies that no such equilibrium exists. This is because the average unit cost of producing lower quantities is greater than that when producing larger quantities as a result of the start-up cost (this is akin to models where there are economies of scale in which the start-up cost is shared over a greater product run (Mas-Collel et al., 1995)). The presence of a capacity constraint further complicates matters since, in general, a single seller will not be able to fully satisfy the total demand. Furthermore, since we are developing a protocol for task allocation, we consider buyers with inelastic demand (i.e. buyers do not vary their demand according to price) which, in turn, means that the CDA is focused on finding the cheapest set of seller(s) given an exact demand from the buyers[1]. Given these points, we need to modify the standard CDA mechanism by designing suitable clearing rules and constraining the type of offers allowed in the market in order to deal with the aforementioned issues.

We now look in detail at our allocation problem in Section 3.1, and present a centralised and a decentralised mechanism to solve it in sections 3.2 and 3.3 respectively. Finally, we evaluate the decentralised mechanism in Section 3.4.

3.1 The Market Allocation Problem

Here, we discuss in more detail the problem structure that we consider in the remainder of this chapter. The system which we wish to control consists of n suppliers of a resource and a number of consumers with total demand D. Each supplier, j, is characterised by a maximum capacity that it can provide, cap_j, and a cost function C_j. The cost function is defined as a combination of a fixed price, f_j, payable for any amount of production and a separate per unit price u_j:

$$C_j = \begin{cases} 0 & \text{if } x_j = 0, \\ f_j + x_j u_j & \text{if } 0 < x_j \leq cap_j \end{cases} \tag{3.1}$$

where x_j is the quantity of production allocated to seller j. Thus, an allocation vector $\mathbf{x} \in \mathcal{X}$ is one in which each agent j is asked to supply a quantity x_j. We assume that both the demand and the details of the cost function are private information of

[1]Inelastic demand also ensures a fair comparison with the centralised case. This is because allowing for elastic demand will result in an allocation which satisfies a demand defined by the demand and supply curves rather than a prior demand that has been made by the buyers (which would occur with inelastic demand). It also allows us to characterise the cost of decentralising the market-based mechanism in terms of its efficiency loss.

the producers (also referred to as suppliers or sellers) since they represent distinct self-interested stakeholders. Given this, the overall aim of the system is to satisfy the total demand by allocating production between the different producers. Here, we assume that the resource is bought and sold in small indivisible units (as is common in most billing systems) and thus $x_j \in \mathbb{N}$.

As the designer of the whole system, we are interested in ensuring that the overall allocation, $\mathbf{x}^*$, of the resource under consideration is optimum in the sense that it minimises the total cost of production. In this case, it is an optimisation problem where we minimise the sum of the individual production costs, whilst satisfying the total demand, $\sum_j x_j = D$, and the capacity constraints of each individual producer:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \sum_j (\alpha_i f_j + u_j x_j) \tag{3.2}$$

such that $0 \leq x_j \leq c_j$ and where:

$$\alpha_i = \begin{cases} 0 \text{ if } x_i = 0 \\ 1 \text{ otherwise.} \end{cases}$$

3.2 The Centralised Allocation Mechanism

Our centralised mechanism builds upon the standard VCG mechanism since this mechanism has a number of desirable economic properties with respect to task allocation (see (Mas-Collel et al., 1995) for more details). Specifically, it is *efficient*, incentivises the agents to reveal their costs truthfully to the auctioneer in *dominant strategy* (i.e. an agent finds no better option than to reveal its costs truthfully) and guarantees a non-negative utility to the participating agents.

The standard VCG mechanism for task allocation represents the producers as agents participating in a reverse auction to satisfy the demand of the auctioneer. The agents submit their respective private information about their costs, known as their types, θ_i, in sealed bids to the auctioneer. After this stage, the auctioneer finds the efficient allocation and then calculates the transfers (i.e. the amount of money that is to be paid to each agent). It is this transfer scheme that results in the agents having truthful reporting as a dominant strategy.

However, there are two key differences between our setting and that of a standard VCG mechanism. First, each agent's type has three dimensions that characterise its cost function instead of the usual one. Specifically, these dimensions are the fixed price or setup cost, f_i, the unit cost, u_i, and the capacity, cap_i. Second, the capacity of the agent does not directly impact on the cost of supplying an allocated quantity of a resource, but rather puts a limit on the amount that it can supply. This differs from the standard setting of a VCG where an agent's type directly impacts on its cost. Thus, an agent overstating its capacity does not change its payment in the traditional VCG mechanism (as shown in (Dash et al., 2007)), but does change the efficient set of suppliers calculated by the centre.

To deal with these differences, the standard VCG needs to be extended in three ways. The first change is to have agents report the attributes that define their cost functions rather than a single cost price. The second change is to have a separate allocation and payment phase (as opposed to the traditional VCG mechanism where this is amalgamated into a single phase) since it is the very reports of the agents (i.e that of their capacities) which define the space of *feasible* allocations. The third change is the introduction of a penalty scheme that incentivises the agents to report truthfully on their capacities.

The following describes the centralised mechanism:

1. First the seller agents, S_i, provide reports of their types $\widehat{\theta_i} = (\widehat{f_i}, \widehat{u_i}, \widehat{cap_i})$ (where $\widehat{\ }$ denotes reported) to the centre.
2. The centre, having gathered total demand from the buyer agents, solves equation 3.2 and assigns production to the agents according to the optimal allocation vector $\widehat{\mathbf{x}}^*$ with reported types.

The payment of the center to the agents is conditioned such that the agents report their types truthfully and $\widehat{\theta_i} = \theta_i$ (see (Dash et al., 2007) for more details). The centre can calculate the task allocation to the agents exactly using dynamic programming. Specifically, we wish to calculate $C[n, D]$ – the minimum total cost to satisfy a demand of D with access to n producers. This can be solved using the recursive expressions:

$$C[0,d] = \begin{cases} 0 & \text{if } d = 0 \\ \infty & \text{if } d > 0 \end{cases}$$

$$C[i,d] = \min_x \begin{cases} C[i-1,d] \\ C[i-1,d-x] + f_i + xu_i \end{cases}$$

such that $0 < x \leq cap_i$. As the production allocated to each producer is in indivisible units, we can calculate $C[n, D]$ by evaluating all nD possible values. This results in an algorithm which operates in pseudo-polynomial time.

In particular, a simple algorithm for this solution is presented in figure 3.1. Here we calculate all the values of the array, $C[n, D]$, starting from the known case $C[0, 0] = 0$ and using the recursive expressions above to calculate subsequent values. Moreover, the same approach can be used to calculate the resulting task allocation to the agents.

```
Calculate initial row of matrix C
C[0,0] ← 0
for d = 1 to D do C[0,d] ← ∞
Loop through the total number of producers
for i = 1 to n do
    Loop through the total demand
    for d = 0 to D do
        C[i,d] ← C[i-1,d]
        Loop through the total capacity of producer i
        for x = 1 to min{d,cap_i} do
            Compare the previous result to the current
            result and select the minimum of the two
            C[i,d] ← min{C[i,d],C[i-1,d-x]+f_i+xu_i}
Return the final result
return C[n,D]
```

FIGURE 3.1: Pseudo-code representing the dynamic programming solution to find the optimum centralised solution in pseudo-polynomial time.

3.3 The Decentralised CDA-Based Allocation Mechanism

In this section, we develop a decentralised allocation mechanism based on the CDA. Our allocation problem involves multiple suppliers and multiple buyers, and the matching of the two is determined by the sellers and buyers who successfully transact with one another. The most common CDA format assumes buyers and sellers have an increasing marginal cost and no startup cost and the offers in the trade are via price alone (see Section 2.1). However, in our case, the total production cost depends on both the startup cost and the number of units to be sold (given the marginal cost). In fact, since the startup cost is distributed over the sale quantity, the cost price is not fixed for different numbers of units sold. As a result, the supplier cannot firmly decide on an asking price (based on the production cost per unit or *cost price*) that would allow it to be profitable and to participate in the task allocation (by transacting with potential buyers). This is because the sale quantity cannot be known *a priori*. To overcome this, we assume that it is possible for the supplier to make a prediction about the amount of units it expects to sell (since exact demand can only be estimated)[2]. In traditional cost settings (increasing marginal cost and no fixed cost), a supplier can start making bids for a low quantity and slowly ramp up its price so as to ensure it does not make a loss. However, in our scenario, low quantities correspond to higher unit prices. Thus the supplier is faced with the problem that reducing its price may not guarantee that it transacts and in certain cases may lead to a loss (if a buyer specifies a demand such that the ask price becomes lower than the cost price). We therefore allow sellers to communicate the amount they wish to sell to the market via a multi-dimensional bid consisting of both quantity and price. We also specify in our clearing rules that a transaction only occurs when a buyer's bid can be fully satisfied (i.e. no partial clearing is allowed for the buyer because of its inelastic demand).

Given this background, a key objective for the decentralised mechanism is to be *individually rational* (see Section 2.1). In this case, this means ensuring the suppliers can be profitable in the market so that they are incentivised to enter it in the first place. Furthermore, while the mechanism has to be individually rational, our global objective is to achieve the most efficient outcome (task allocation) that we can. Now, this is equivalent to finding the allocation that minimises the total production cost. In a typical CDA mechanism, the optimal allocation occurs when the total profit of all buyers and all sellers is maximised (see Section 2.1) and this occurs when the combined cost of sellers is minimised, as the sellers with the lowest cost would be successful.

However, given our additional constraints of limited capacity and a startup cost, the seller's strategic behaviour would be more complex than that of the buyer, since, as we mentioned before, it additionally has to strategise over the quantity it is expected to sell.

[2]In fact, in CDA scenarios demand cannot be known even after the bids have been submitted (Cliff and Bruten, 1997). This is why sellers try to predict the demand in order to be more profitable (He et al., 2003).

In this context, we cannot achieve full efficiency because no agent has complete information about every other agent in the market and the sellers do not have the increasing marginal costs which would guarantee an equilibrium price for trade (Mas-Collel et al., 1995).

Given this, our aim is to design a protocol that achieves a level of efficiency that is reasonably close to the optimal solution given by our centralised mechanism. The protocol we propose is a variant of the multi-unit CDA. Specifically, buyers and sellers can submit offers to buy and sell multiple units of the resource, respectively, and those orders are queued in an order book which is cleared continuously (with additional constraints as a result of buyers' inelastic demand). The protocol proceeds as follows:

- Buyer i submits an offer, $bid(q_b, b, i)$, to buy exactly q_b ($q_b \geq 1$) units of the good at the unit price b. The utility of buyer i for a quantity other than q_b is 0.
- Conversely, supplier j submits an offer, $ask(q_a, a, j)$, to sell a maximum of q_a ($q_a \geq 1$) units at unit price a.
- These bids and asks are queued in an *orderbook*, which is a publicly observable board listing all the bids and asks submitted to the market (see table 3.1). The bids in the order book are sorted in decreasing order of price and the asks are in increasing order (higher bids and lower asks are more likely to result in transactions).
- The *clearing rule* in the market is as follows. Whenever a new bid or ask is submitted, an attempt is made at clearing the order book. The order book is cleared *whenever* a transaction can occur (that is, when the lowest asking price is lower than the highest bidding price *and* any bidding offer can be cleared completely and the bidding quantity for each offer is completely satisfied by the supply to be cleared). The transaction price is set at the bidding price which we experimentally find to result in the total market profits being equally divided between the buyers and the sellers[3].

To further illustrate this process, we present a graphical representation of the clearing rule in Figure 3.2. As can be seen, the offers queued in the order book are used to build demand and supply curves. All bids with a unit price lower than the lowest unit ask price

[3]We chose this option because we require the desirable property of fairness of profit distribution among buyers and sellers (see research aim 1). We use the approach described in Subsection 2.2.1 where the k parameter of the k-pricing rule is optimised for a desirable property of the mechanism which, in our case, is the equal distribution of profits.

Order Book	
Bids *(quantity, price, buyer)*	*Asks* *(quantity, price, seller)*
(30, 2.95, 2)	(60, 2.20, 3)
(40, 2.75, 5)	(25, 2.60, 1)
(30, 2.70, 1)	(40, 3.22, 2)
(24, 2.16, 3)	(100, 3.50, 5)
	(25, 3.69, 7)
	...

TABLE 3.1: Multi-unit CDA Order Book before clearing.

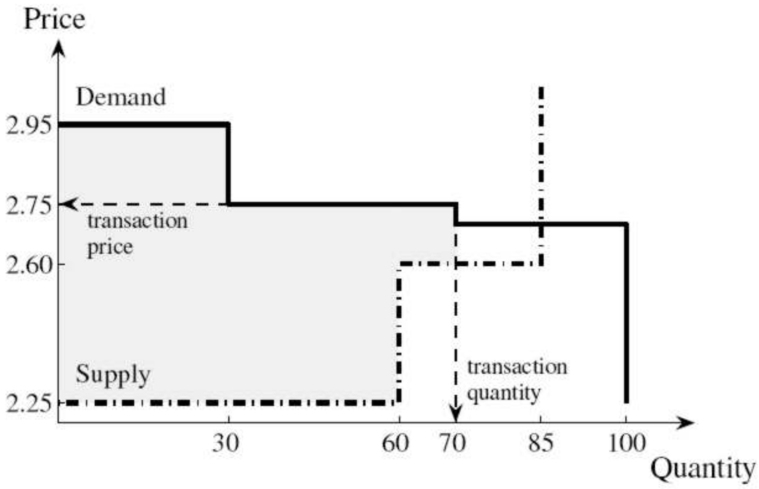
and, similarly, all asks with a unit price higher than the highest unit bid price, cannot result in any transaction and are not represented in Figure 3.2. The transaction price and quantity are clearly shown in Figure 3.2 (2.75 and 70 respectively), as the point where the demand curve crosses the supply curve under the additional constraint that bid offers are not divisible. At this transaction price, the total profit of all participants involved in the transactions is maximised (it is equivalent to the solution of an optimisation problem where we maximise the total profit given the set of all the transactions possible in the order book), with all constraints specified by our protocols satisfied. The order book in Table 3.1 can thus be cleared as shown in Figure 3.2 resulting in the new order book given in Table 3.2.

Order Book	
Bids *(quantity,price,buyer)*	*Asks* *(quantity,price,seller)*
(30, 2.70, 1)	(15, 2.60, 1)
(24, 2.16, 3)	(40, 3.22, 2)
	(100, 3.50, 5)
	(25, 3.69, 7)
	...

TABLE 3.2: Multi-unit CDA Order Book after clearing.

3.4 Evaluating the Decentralised Mechanism

Here, we describe how we evaluate our decentralised mechanism. Specifically, we consider its efficiency, calculated as the ratio of the total production cost of the centralised mechanism to that of the decentralised mechanism. Furthermore, because we wish to



(a)

Order Book	
Bids (quantity, price, buyer)	*Asks (quantity, price, seller)*
(30, 2.95, 2)	(60, 2.20, 3)
(40, 2.75, 5)	(25, 2.60, 1)
(30, 2.70, 1)	(40, 3.22, 2)
(24, 2.16, 3)	(100, 3.50, 5)
	(25, 3.69, 7)
	...

(b)

FIGURE 3.2: Panel A shows the demand and supply (curves) of the order book, with the shaded region representing allocations. Panel B points out the *clearable bids and asks* in the order book. Those bids and asks are considered to generate the demand and supply curves in panel A.

evaluate the structure of the mechanism, we adopt a zero-intelligence behaviour and, to this end, we develop a zero-intelligence strategy which we describe next.

3.4.1 The ZI2 Bidding Strategy

One of the principal concerns in developing a market mechanism is to ensure that it is efficient even when the participants adopt a simple strategic behaviour. The underlying intuition here is that by considering such behaviour, we are able to establish a lower bound on the efficiency of the mechanism and we can consider the extent to which the structure of the market mechanism itself affects the efficiency of the market (as discussed in Subsection 2.3.4.2). The ZI strategy (as discussed in Subsection 2.3.4.2) is widely used for this purpose since it is not motivated by trading profit and effectively ignores the state of the market and past experience when forming a bid or an ask. Since in our mechanism, the asks consist of price and quantity, we extend the ZI strategy to the *ZI2 strategy* which randomises over both price and quantity.

We now focus on the behaviour of the strategy. As discussed earlier, any sophisticated strategy, on the sell side, would make some form of prediction on the number of units it is likely to sell as part of its price formation process (because information about the actual demand is not available and there is uncertainty as to whether the agent is more competitive than the other participating suppliers). Instead, our ZI2 supplier j, randomises over the expected transaction quantity to form a limit price c_j which is used as in the original ZI strategy. Thus the ZI2 strategy is[4]:

For buyer i,

$$\begin{aligned} b_i &\sim \mathcal{U}(0, \ell_i) \\ \textit{offer} &= bid(q_i, b_i, i) \end{aligned} \tag{3.3}$$

For seller j,

$$\begin{aligned} \hat{q}_j &\sim \mathcal{U}^Q(0, cap_j) \\ c_j &= (f_j + \hat{q}_j u_j)/\hat{q}_j \\ a_j &\sim \mathcal{U}(c_j, p_{max}) \\ \textit{offer} &= ask(cap_j, a_j, j) \end{aligned} \tag{3.4}$$

Buyers are endowed with high limit prices at the beginning of the auction (because they have inelastic demand), while sellers are endowed with their cost functions and capacities (collectively referred to as the production function). Buyer i submits offers to buy the quantity, q_i, it requires at a unit price drawn from a uniform distribution ranging from 0 to its limit price ℓ_i (see Equation 3.3). Conversely, seller j submits an ask between its limit price and p_{max} as per Equation 3.4, where cap_j is its total production capacity, f_j is its startup cost and u_j is its marginal cost.

[4] $X \sim \mathcal{U}(A, B)$ describes a *discrete* uniform distribution between A and B, with steps of 0.01, the typical minimum currency. $X \sim \mathcal{U}^Q(A, B)$ describes a *discrete* uniform distribution between A and B, with steps of 1.

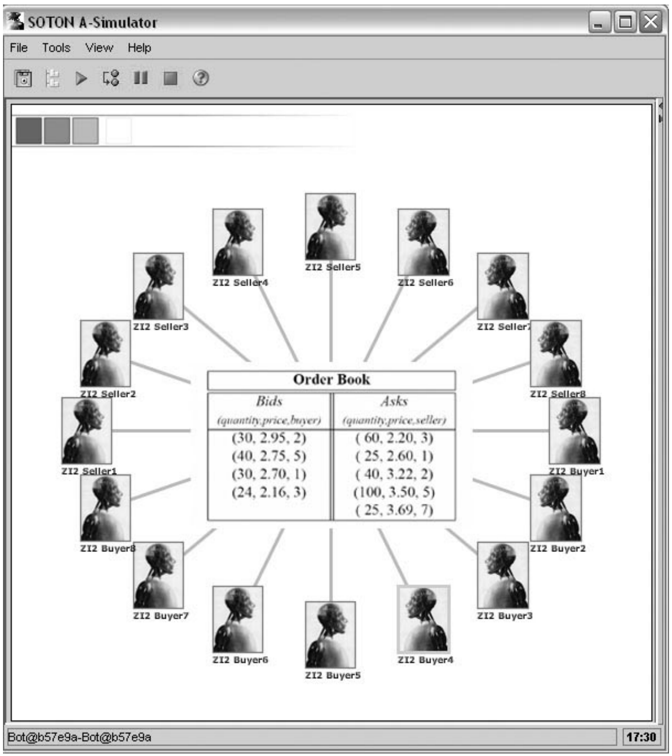


FIGURE 3.3: The multi-unit CDA simulator.

Sellers

	S_1	S_2	S_3
Capacity	100	150	175
Fixed Price	100	200	120
Unit Price	1.5	1	2

TABLE 3.3: A set of three producers bidding to satisfy a demand of 200 units.

3.4.2 The Experimental Setup

In order to perform empirical evaluations, we developed an implementation of this distributed mechanism (see Figure 3.3) based on the protocol and strategies described here. As the experimental setup, we ran the simulations over 2000 rounds[5] for different markets and, here, we analyse in details a small market with 3 buyers and 3 sellers (Market A) and a larger market with 15 buyers and 15 sellers (Market B). We consider both the small and the large markets so as to demonstrate the scaleability of our mechanism.

In each market, each seller was given a production function (supply for Market A is given in Table 3.3), while each buyer was required to procure an exact quantity of

[5]The results were validated using a student t-test with two samples of 2000 runs, assuming equal variance with means $\mu_1 = 0.7198$ and $\mu_2 = 0.7218$ and p-value $p = 0.3660$. This means that the difference between the means is not significant.

units with a relatively *high* limit price. We ran different simulations for each market, with different total demands ranging from 1 to the maximum production quantity. The total demand, D, was equally distributed among the buyers. Thus, the total demand in Market A was varied from 1 to 425 (the maximum supply quantity of Market A), while in Market B with 15 buyers and 15 sellers, the total demand ranged from 1 to 2400.

3.4.3 The Empirical Study

In order to empirically evaluate the efficiency of the mechanism, in terms of minimising the total cost of production, we measure this property and compare it to the optimal solution found in the centralised mechanism. Given each total demand, the mean efficiency of the market (averaged over 2000 independent rounds) is shown in Figure 3.5, where the optimal production cost is normalised to 1, while the total production cost of the centralised mechanism and the decentralised mechanism is shown in Figure 3.4. As can be seen, the mechanism is efficient with an average efficiency of 83% (and a minimum efficiency of 53% when demand is relatively low) for Market B and an average efficiency of 86% (and a minimum efficiency of 67%) for Market A. The minimum efficiency case occurs when the demand is split amongst many excess suppliers (with respect to the optimal allocation). This increases the overall cost of supply as a result of the fixed cost of the extraneous suppliers. However, in the typical CDA, the worst-case analysis considers the average efficiency of ZI agents (Gode and Sunder, 1993). This is because although it is theoretically possible for an allocation of very low efficiency to occur, in almost every run (higher than 99% of the runs), the CDA implemented with agents employing the ZI strategy has a high efficiency. Thus, it is the zero-intelligence nature of the strategy which provides a lower bound on measuring efficiency and, we expect the average efficiency with a more intelligent strategy to be higher (Cliff and Bruten, 1997; Vytelingum et al., 2004). We thererefore adopt this approach in discussing the inherent efficiency of our CDA mechanism.

In experiments with each market, we observe an increasing trend whereby the market efficiency increases as total demand approaches the maximum capacity of the sellers. It can also be seen that there is a high variance when the total demand is relatively low. Considering specifically the set of experiments with Market A (3 buyers and 3 sellers), the intuitions behind these observations are as follows. The variance of the market efficiency is generally higher when the total demand is low. This is because the optimal allocation for a total demand of 100 is completely covered by seller 1 (with a marginal cost of 1.5 and a startup cost of 100). However, our market mechanism does not ensure that only seller 1 will trade and, thus, sellers 2 and 3 may also be part of this allocation for the total demand of 100. The high variance is principally an artefact of the additional startup costs if more than one seller were to trade. As the total demand increases past 175, the optimal allocation is covered by at least two sellers. The variance past the demand of 175 is the result of sellers supplying different numbers of units at different marginal costs, with at most one additional startup cost. When the total demand is very high, close to the total capacity, all the sellers participate in the allocation, and the small variance is solely due to the sellers providing different numbers of units (a difference which is relatively low compared to the total startup cost). The observations in the set of experiments with Market B can also be explained by the same reasoning, with the higher variance occurring when demand that can be covered by a single seller is distributed among multiple sellers.

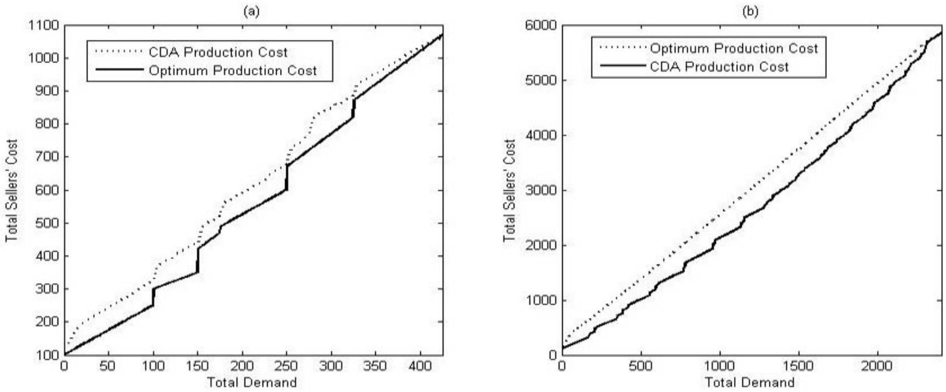


FIGURE 3.4: Optimal and CDA production cost for 3 buyers and 3 sellers in (a) and 15 buyers and 15 sellers in (b).

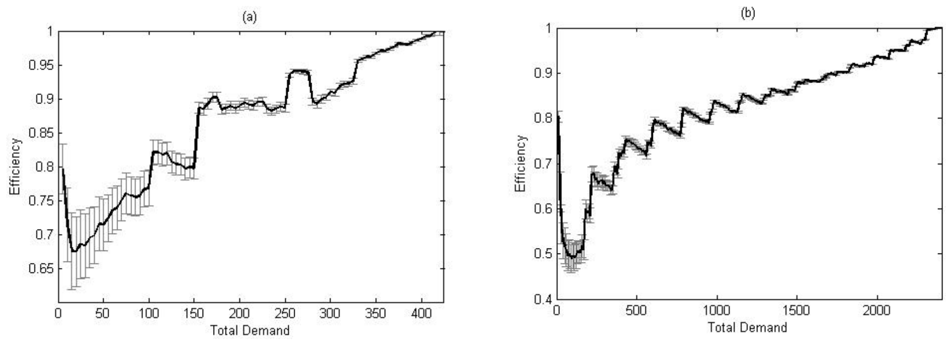


FIGURE 3.5: Average market efficiency for 3 buyers and 3 sellers in (a) and 15 buyers and 15 sellers in (b).

Furthermore, we can explain the increasing trend of the market efficiency seen in Figure 3.4. Considering Market A, a demand of up to 175 can be provided by only 1 seller. The jumps in Figure 3.4 correspond to the optimal allocation changing between a combination of one to three sellers. For example, jumps at 100 and 150 correspond to the optimal allocation starting with seller 1, changing to seller 2 and finally to seller 3. The increase in efficiency as total demand increases is the result of the number of sellers involved in the optimal allocation, changing from a single seller (up to a total demand of 175) to three sellers (past a total demand of 325 which is the highest demand any two sellers can cover). However, in our market, any number of sellers can trade at any time. As total demand increases, the loss in efficiency that arises from the extra startup costs (compared to the optimal allocation) decreases, thus explaining the generally increasing trend. In the simulations with Market B, a similar trend can be observed, with a lower efficiency when demand is lower than the minimum sellers' capacity (210). As in Market A, there are more inefficient allocations that can arise when demand is low (and can be satisfied by a single seller), which would decrease the average efficiency much more than it would, given a smaller number of inefficient allocations. Here, we use the same intuition as in Market A to explain the jumps, which are larger in number given the larger number of participants.

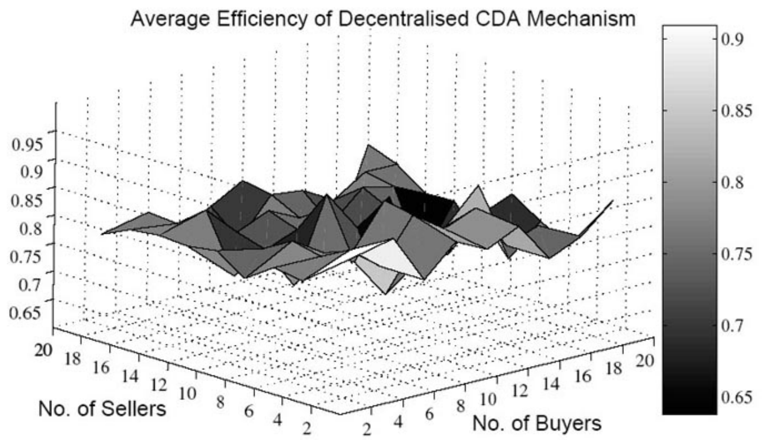


FIGURE 3.6: Performance of the decentralised mechanism in markets with different numbers of buyers and sellers.

Given the results in figures 3.4 and 3.5, we observe that our distributed mechanism is appreciably efficient[6] in both a small and a relatively larger[7] market, with a high average efficiency in either case. In Figure 3.6, we plot the average efficiency over the space of different market sizes varying over 2 to 20 buyers and sellers. We observe the same

[6]What represents a satisfactory efficiency is subjective and depends on the system designer's requirements. For example, in a critical application, an efficiency below 90% would be considered as poor and be unacceptable while in another application, an efficiency of 60% would still be acceptable given the desirable properties of a decentralised system.

[7]In the future, we intend to investigate the efficiency in considerably larger markets of the order of hundreds of agents and observe how the efficiency scales with the market size. We hypothesise that the efficiency curve would approximate a linear function ranging from 0.7 to 1 with the average (lower bound) efficiency at 0.85.

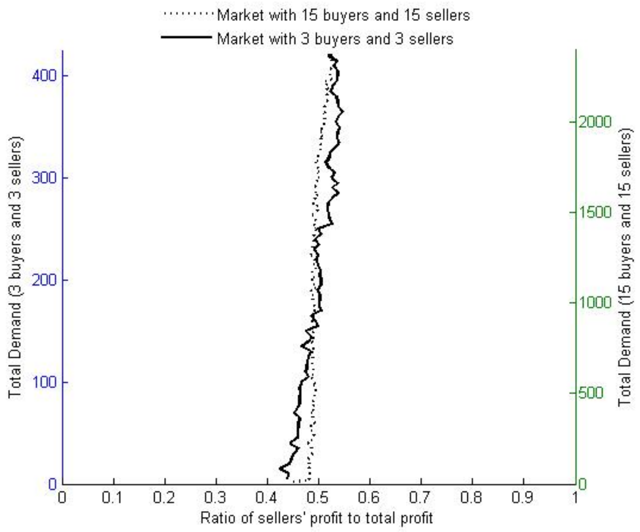


FIGURE 3.7: The profit distribution among buyers and sellers in markets A and B.

high efficiency as in markets A and B, around 80%. Finally, the simulation results in Figure 3.7 show that the sellers and buyers equally share the market profits (the ratio of sellers' profits to total market profit is approximately equal to 0.5) in either market. The fact that our agents can be profitable and thus are incentivised to enter the market means that our distributed mechanism is individually rational.

3.5 Summary

In this chapter, we first presented our work on designing a CDA variant to solve a more complex resource allocation problem than in the standard CDA. Specifically, we showed how the market protocol of the CDA can be modified to satisfy a scenario where suppliers have a production function with a startup cost and limited capacity and, consumers have inelastic demand.

Then, we evaluated our novel decentralised mechanism. For this purpose, we developed a centralised mechanism that computes the optimal solution to the resource allocation problem, in terms of the total minimum production cost of suppliers. The efficiency of the system is then calculated as the ratio of the total production in the centralised mechanism to that in the decentralised one. We then developed the ZI2 bidding strategy to obtain a simple behaviour in the mechanism. Such a simple behaviour allows us to evaluate the efficiency of the mechanism from a structural perspective. Because we expect the efficiency of the mechanism to be much higher given a more sophisticated behaviour, we can thus consider the efficiency with the ZI2 strategy as a lower bound in our decentralised solution to such a complex resource allocation problem.

Though this work on designing a CDA variant focuses on a single complex resource allocation problem, the bigger picture is that the CDA has been shown to be amenable to these more complex problems, even in scenarios where a competitive market equilibrium does not exist. Thus, this work satisfies our aim to modify the CDA structure to solve complex problems and further motivates the more widespread use of the CDA mechanism as a decentralised solution.

Part II

THE BEHAVIOURAL PERSPECTIVE

The behaviour of the CDA is emergent and depends on the strategies adopted by every buyer and seller in the market. In this part, we address our research aims to design and analyse the behaviour of the CDA.

To tackle the design aspect of this part of the research, we first develop a novel multi-layered framework for devising strategies for the CDA (in Chapter 4). We then use this framework to design a bidding strategy for the CDA in Chapter 5. The objective of the new strategy is to address our research aim for more efficient strategies for the CDA. In particular, we design our strategy to be robust in both static and dynamic environments, with a short-term and a long-term learning mechanism to adapt the agent's behaviour to the changing market conditions.

To tackle the analysis aspect of this part of the research, we first develop a novel two-population evolutionary game theoretic framework to analyse the strategic interactions of buyers and sellers in the CDA, which we report in Chapter 6. Then, in Chapter 7, we describe novel methodologies for evaluating CDA strategies in both homogeneous and heterogeneous populations. Specifically, for the latter, we adopt our novel evolutionary game theoretic framework to analyse the evolution of buyers' and sellers' behaviour in the market and use such an analysis to evaluate the performance of strategies in heterogeneous populations.

Chapter 4

A Framework for Designing Bidding Strategies

In this chapter, we present our work on a framework for designing strategies for trading agents in electronic market mechanisms, work which addresses our research aim to design bidding strategies for the CDA. Though more of a black art than a serious engineering endeavour at present, we believe that the design of successful strategies can nevertheless be viewed as adhering to a fundamental and systematic structure. Thus, we look at a general framework for designing bidding strategies; a framework that is simple enough to be applicable in a broad range of market mechanisms, but modular enough to be used in the design of complex bidding behaviour. We believe such a model is important for the strategy designer because it provides a principled approach towards the systematic engineering of such strategies which, in turn, can foster more reliable and robust strategies. Here, we do not solely look at the CDA context, because modifying the structural aspect of the typical CDA mechanism may necessitate changing the whole framework. Thus, we prefer to develop a general framework for market mechanisms that we can then apply to the CDA and its variants.

As there is no systematic software engineering framework currently available for designing strategies for trading agents (as discussed in Section 1.1), this work advances the state of the art by providing the first steps towards such a model. Specifically, our framework is based upon three main principles:

1. An agent requires information about itself and its environment in order to make informed decisions.

2. An agent rarely has full information or sufficient computational resources to manage all the extracted information.

3. Given its limited computational resources and information, an agent needs to employ heuristics in order to formulate a successful strategy.

In more detail, in order to operate in such situations, we advocate a multi-layered design framework. We believe this is appropriate because most strategies can be viewed as breaking down the task of bidding into a clear set of well defined sub-tasks (such as gathering relevant information, processing that information and using that processed information in a meaningful manner). This decomposition can be viewed as a series of (semi-) distinct steps that are handled by different layers. Furthermore, our aim is to ensure the model is sufficiently abstract to be used as the agent model in more general agent-oriented software engineering frameworks, such as Gaia (Zambonelli et al., 2003) and Agent UML (Bauer et al., 2001) and such that it overlays well with existing multi-layered architectures for multi-agent systems (Ashri and Luck, 2001). Given this, our framework consists of three layers: the *Information*, *Knowledge* and *Behavioural* layers (hence we term our framework the *IKB* model hereafter) and each layer corresponds to one of the principles we address above. In more detail, the information layer records raw data from the market environment. This is then processed by the knowledge layer in order to provide the intelligent data which is used by the behavioural layer to condition the agent's strategy. To illustrate the use of our framework, we chose two example marketplaces that are popular for trading agents. Firstly, we consider marketplaces that use the CDA and we place a number of the standard CDA strategies discussed in Subsection 2.3.4 within it. Secondly, we consider a more complex scenario, the Travel Game of the International Trading Agent Competition (TAC), where an agent has to strategise in multiple simultaneous auctions of different formats (Wellman et al., 2002; Fasli et al., 2002). In both cases, we employed our IKB model successfully.

4.1 The IKB Model

In this section, we detail the main components that the designer of a trading agent strategy should pay attention to. In so doing, we develop a framework for designing strategies in trading markets. In our model, we have a market $\mathcal{M}$ regulated by its protocol that is predefined. The collection of variables representing the dynamics of the system at time t is represented by the state variable $state_{\mathcal{M}}(t_k)$. Within this market, there is a set of trading agents, $\mathcal{I}$, that approach the market through a set of actions which are determined by their strategies. In order to formulate its best strategy, an agent *ideally* needs to know which state it is currently in (agent state), the market state and the actions it can take.

Definition 4.1. Agent's State. An agent i's state, $state_i(t)$, at time t is a collection of variables describing its resources (computational and economic) and privately known preferences.

Definition 4.2. Market State. The market state, $state_{\mathcal{M}}(t)$, at time t is a collection of variables describing all the (public and private) attributes of the market.

Definition 4.3. Strategy. A strategy, $\mathcal{S}_i$, for agent $i \in \mathcal{I}$, defines a mapping Γ_i from the history of the agent state $H(state_i)$ and of the market states $H(p_{\mathcal{M}})$, and current agent state $state_i(t)$ and the market state $state_{\mathcal{M}}(t)$ to a set of atomic actions $\mathcal{A}_i = \{a^i_1, a^i_2, \ldots, a^i_k, \ldots\}, a^i_k \in \mathcal{A}_i$ where $\mathcal{A}_i$ is the set of all possible actions for agent i at time t.

The actions chosen by strategy $\mathcal{S}_i$ then affect the external environment such that it causes a change in the market state. In fact, this strategy could interplay with strategies selected by other agents, $\mathcal{I} \setminus i$, as well as some external input(s), ext_n, so as to lead the market to the new state:

$$state_{\mathcal{M}}(t+1) = \quad T(state_{\mathcal{M}}(t), H(state_{\mathcal{M}}), \mathcal{A}_1, \mathcal{A}_2, \ldots, ext_1, ext_2, \ldots) \tag{4.1}$$

where $T(.)$ is the state transfer function. From Definition 4.3, it is clear that in order for an agent to know which strategy is best, it should know the complete description and history of the states (all market information), a complete description of all actions available to it, its preferences over the states, a model of its opponents' states, behaviours and preferences, and the state transfer function.

In practice, however, an agent will typically not have all this information (for a number of reasons, such as limited sensory capabilities, privacy of opponent information and limited knowledge of relevant external signals). Furthermore, an agent's limited computational resources imply that it might not be able to keep a complete history of all past interactions. Given this, there is a need for designing feasible strategies that use limited computational and sensory resources. To this end, we advocate the following design principle in which an agent manages its limited capabilities through its Information Layer (IL), its Knowledge Layer (KL) and its Behavioural Layer (BL) (as shown in figure 4.1).

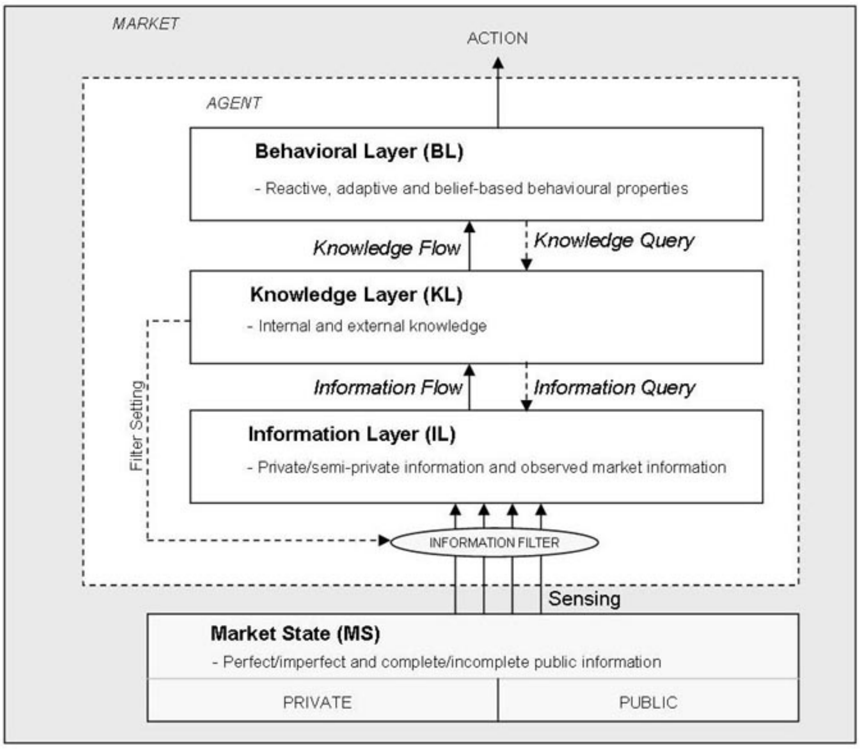


FIGURE 4.1: Structure of the IKB Model.

In more detail, the Market State (MS) contains public information (i.e. information *available* to all agents in the market) and private/semi-private information (i.e. information *available* to one/some agents). We now provide a description of each of the layers that pertain to the agent:

- **Information Layer.** The IL contains data which the agent has extracted from the MS and private information about its own state. This extraction is a filtering process (which we represent as the Information Filter in Figure 4.1) whose objectives are defined by the KL (e.g. filtering out only transaction prices).

- **Knowledge Layer.** The KL represents the gathered *knowledge* that is aggregated from the data in IL (e.g. bids submitted in the market). The BL queries the KL to obtain the knowledge it requires.

- **Behavioral Layer.** The BL determines the agent's strategic behaviour by deciding on how to use the information available to it in order to interact with the market through a set of actions (e.g. submitting a bid). It queries the KL for the relevant knowledge it requires (e.g the belief that a bid will be accepted in the market).

We next describe each of these layers in further detail, whilst explaining the process through which an agent uses a plethora of raw data to select appropriate actions.

4.1.1 The Information Layer

This subsection deals with how an agent gathers information which is then passed on to the KL. The KL will select the data being stored in the IL by modifying the information filter (see Figure 4.1) appropriately. This filter will screen the data from the MS with some noise (due to environmental noise or the agent's sensory limitations). As a result, the IL of an agent will contain a noisy, restricted view of all information which it can observe. Furthermore, the IL will also contain information about the agent's state, $state_i(t)$, as well as its action set $\mathcal{A}_i$.

We distinguish between information and knowledge in the following way:

Definition 4.4. Information. Information is raw data that can be sensed by an agent.

Definition 4.5. Knowledge. Knowledge is the processed data that is computed by an agent from the information it has gathered.

Now, information is typically categorised as follows (Mas-Collel et al., 1995):

- Complete/Incomplete: An agent has complete information if it is aware of the complete structure of the market (that is, its action sets and the result of each action). Otherwise, it has incomplete information.
- Perfect/Imperfect: An agent has perfect information if it is certain of its state, the history of the market's and the agent's states ($H(state_{\mathcal{M}})$ and $H(state_i)$) that have led it into this state. Otherwise, it has imperfect information.

As argued earlier, an agent's sensory and computational limitations imply that it will rarely have perfect and complete information. For example, an agent might not be aware of its complete action set (i.e. an agent might believe that its action set at time t is $\mathcal{A}'_i \subset \mathcal{A}_i$) or it may be unsure of which state it is in (i.e. it expresses an uncertainty over $p_i(t_k)$). Thus, the agent will need to have certain heuristics in order to guide its search for information. This information can be gathered from public, semi-private and private sources. Public information is observable by all agents ($i \in \mathcal{I}$) in the market and includes things such as the market price in a stock exchange, the minimum increment in an eBay auction and the number of lots of flowers on sale in a Dutch flower auction. Semi-private information is that which is available to a subset of the agents ($i \in \mathcal{J} \subset \mathcal{I}$) and includes things such as the amount that a supplier might require from an agent and the code to signalling actions by a bidder ring in an auction (Krishna, 2002). Private information is only observable by a single agent and includes items such as its budget or the goods it is interested in. Thus, given the required information that the KL has requested, the agent will devote its limited resources to obtaining it. Then, having gathered the required information from the market, the agent proceeds to use this information to infer knowledge in the KL.

4.1.2 The Knowledge Layer

The KL connects the information and the behavioural layers (see Subsection 4.1.3). It infers knowledge from the information sensed by the agent and passes it to the BL which acts upon it. In order to do so, the KL is first requested by the BL as to which knowledge to acquire. This knowledge could be, for example, the current Sharpe ratio[1] of a stock or a prediction of the market price based on a particular prediction model. Based on this and the current knowledge of the agent's state, the KL will decide upon the information it requires and set the information filter accordingly. The KL will then use the input from the IL so as to to infer the appropriate knowledge which it will output to the BL.

Mirroring the IL, the KL can be segmented into knowledge about the agent's and the market's state. The former is what the agent knows about itself. This includes knowledge pertaining to its subgoals (such as its risk attitude or the deadline by which a good is to be delivered) and knowledge about its state $state_i(t_k)$. The latter is what the agent knows about the market and would include items such as the degree of competitiveness in the market, the opponents' states and any available market indicators.

4.1.3 The Behavioral Layer

The BL represents the decision-making component of the strategy. In this context, such strategies are targeted towards finding the most profitable action in the market. However, as outlined earlier, more often than not, there is no known optimal action, as many markets are simply too complex and the set of actions too large to determine such an optimal action analytically. Then, as there is no best strategy, a heuristic approach is taken.

[1] The Sharpe ratio is a measure of a stock's excess return relative to its total variability (Sharpe, 1966).

Thus, the BL instructs the KL as to what knowledge it needs to gather from the market which, as described in subsection 4.1.2, is computed from the market information. With the relevant knowledge of the market and its goals, the agent i forms a decision based on its strategy $\mathcal{S}_i$ and interacts with the market through actions $\mathcal{A}_i$. The goal of an agent's strategy is typically profit-maximisation, with the more sophisticated strategies considering both short-term and long-term risk. The formulation of the strategy usually depends on such goals and the market protocols.

Given this insight, we categorise the different behavioural properties of the strategy into different levels and, specifically, we build upon the categorisation in Subsection 2.3.3. In more detail, we distinguish those strategies in terms of the type of information (in Equation 4.1) which is used (i.e. whether they use a history of market information or not and whether they consider external information or not):

1. **No History** (ignores $H(state_{\mathcal{M}})$ from Equation 4.1). Such reactive strategies make myopic decisions based only on the current market conditions $p_{\mathcal{M}}(t)$. The myopic nature of these strategies imply a lower workload on the KL since they require less information to sense and process. Reactive strategies also usually exploit the more complex bargaining behaviour of competing strategies and thus require less computational resources to strategise. One example of such a strategy is the *eSnipe* strategy which is frequently used on eBay to submit an offer to buy near the end of the auction.

2. **History** (considers $H(state_{\mathcal{M}})$ in Equation 4.1). We further subdivide those strategies that use a history of market information as being predictive or not (i.e. whether they predict $\{state_{\mathcal{M}}(t+1), state_{\mathcal{M}}(t+2), \ldots\}$ or not). The non-predictive strategies typically use $H(state_{\mathcal{M}})$ to estimate $state_{\mathcal{M}}(t)$:

   (a) *Non-predictive*: The non-predictive strategy is typically belief-based and forms a decision based on some belief of *the current market conditions*. The agent's belief is computed from the history of market information in the KL, and usually represents the belief that a particular action will benefit the agent in the market (for example an offer to buy that is accepted). Given its belief over a set of actions, the agent then determines the best action over the short or long term.

   (b) *Predictive*: A strategy makes a prediction about the market state in order to adapt to it. Now, because future market conditions (that the trading agent adapts to) cannot be known *a priori*, the adaptive strategy typically makes some prediction using the history of market information. The KL is required to keep track of how the market (knowledge) is changing to predict the future market, while the BL uses this knowledge about the market dynamics to improve its response in the market. Being adaptive is particularly important in situations where the environment is subject to significant changes. By tracking such changes and adapting its behaviour accordingly, the agent aims to remain competitive in changing market conditions.

3. **No External Information** (ignores $ext_1, \ldots, ext_n$ in Equation 4.1). In this case, the strategy does not consider any signals external to the market (e.g. the falling market price of a good affecting the client's preferences for another type of good in an auction). However, the agent can choose whether or not to use the (internal) information (e.g. the eSnipe strategy uses the internal market information, while the ZI Strategy in the CDA does not make use of any market information).

4. **External Information** (considers $ext_1, \ldots, ext_n$ in Equation 4.1). It is possible that signals external to the market can influence the preferences of the participants, such as an event independent of the market causing the clients' preferences in the market to change (e.g. unforseen weather conditions affecting the production of wheat and thus the market for wheat indirectly). Thus, external information can be a valuable source of information that the agent can use to strategise in the market.

Having presented our IKB model for designing trading strategies, we now discuss how our model can be applied to the design of strategies specifically for the CDA mechanism.

4.2 Applying the Model to the CDA

As discussed in Chapter 1, because there is no known dominant strategy in the CDA, several researchers have worked on competing alternatives (see Subsection 2.3.4). In this section, we describe how these strategies can be broken down by our IKB model and, thus, how our framework could be used to build these strategies by reverse engineering. First, we give a formalised definition of the asynchronous CDA specifically for the IKB. The market state of the CDA at time t is $state_{\mathcal{M}}(t) =< g, \mathcal{B}, \mathcal{S}, price(t), bid(t), ask(t) >$ where:

1. g is the good being auctioned off.

2. $\mathcal{B} = b_1, \ldots, b_{n_b}$ is the finite set of identifiers of bidders in the market, where n_b is the number of current bidders.

3. $\mathcal{S} = s_1, \ldots, s_{n_s}$ is the finite set of identifiers of sellers in the market, where n_s is the number of current sellers.

4. $price(t)$ denotes the current market price of good g in the market. This corresponds to the most recent transaction price.

5. $bid(t)$ denotes the outstanding bid at time t.

6. $ask(t)$ denotes the outstanding ask at time t.

The agent state at time t, is $state_i(t) =< id_i, n_i(t), \boldsymbol{v}_i = (v_{i1}, \ldots, v_{n(t)}), \ell_i, budget_i(t), comp_i(t) >$ where:

1. id_i defines the identity of the agent as either a buyer or a seller agent.

2. $n_i(t)$ defines the number of items an agent is currently interested in either buying or selling.

3. $\boldsymbol{v_i} = \{v_{1,i}, \ldots, v_{n_i(t),i}\}$ is the set of limit prices ordered from highest to lowest in the case of a bidder and vice versa in the case of a seller.

4. ℓ_i is the current limit price.

5. $budget_i(t)$ is the budget available to agent i.

6. $comp_i(t)$ is the computational resources (memory and processing power) currently available to agent i.

The action set of the agent depends on its identity (id_i). If it is a buyer, it has $\mathcal{A}_i = \{bid_i, silent\}$ where $bid_i \in [0, p_{max}]$ and $silent$ is no bid. Correspondingly, if it is a seller its action set is $\mathcal{A}_i = \{ask_i, silent\}$ where $ask_i \in [0, p_{max}]$. It should be noted that in the CDA, the elements of $\mathcal{A}_i$ will only be singletons (i.e. an agent can only take a single action at a time). The state transfer function T_{CDA} in the CDA is the rules for acceptance and rejection of bids and asks, as well as the clearing rules (see below). The standard CDA is not influenced by external signals (i.e. the transfer function T_{CDA} has no $ext_1, \ldots, ext_n$ arguments[2]) and the market changes each time an agent submits a bid or an ask and thus simultaneous bidding does not occur. Thus $state_{\mathcal{M}}(t+1) = T_{CDA}(state_{\mathcal{M}}(t), H(p_{\mathcal{M}}), \mathcal{A}_i)$ whereby $T(.)$ is defined by the following rules:

[2] Thus, a CDA strategy does not consider external information.

	Kaplan	*ZI-C*	*ZIP*	*GD*
Information Layer	Limit price and outstanding bid/ask	Limit price	Limit price and transaction price and Current bid/ask and current profit margin	Limit price and history of bid/ask and transaction price
Knowledge Layer	Measures for heuristics	None	Competitive profit margin, success of trade	Belief that bid/ask will be accepted
Behavioral Layer	No history, non-predictive	Random	History, predictive	History, non-predictive

TABLE 4.1: Analysis of four CDA strategies under the IKB model.

- if $\mathcal{A}_i = bid_i$, then
  - if $bid_i < bid(t)$ then bid_i is rejected and $state_{\mathcal{M}}(t+1) = state_{\mathcal{M}}(t)$.
  - if $bid(t) < bid_i < ask(t)$ then $bid(t+1) = bid_i$ and all other market variables remain unchanged.
  - if $ask(t) < bid_i$, then $price(t+1) = cr(ask(t) + bid_i)$ (where $cr(.)$ is a clearing rule stating the transaction price at which the clearing should occur)[3], $bid(t+1) = 0$ and $ask(t+1) = p_{max}$.
- if $\mathcal{A}_i = ask_i$, it follows the same procedure as above.
- if $t + 1 = deadline$, then the auction ends. $deadline$ is the preset time when the market closes.

Furthermore, an agent's state will also change, conditional on whether its bid or ask is accepted in the market. If an agent's bid bid_i results in a transaction, $n_i(t+1) = n_i(t) - 1$, $budget_i(t+1) = budget_i(t) - price(t+1)$ and $\boldsymbol{v}_i = \{v_{2,i}, \ldots, v_{n_i(t),i}\}$ and $\ell_i = v_{2,i}$. If an agent's bid is unsuccessful, then the MS relays this private information to the agent. The agent's visibility is restricted to only bids and asks being submitted in the market (with the agent that submitted a bid or an ask, not disclosed) and successful transactions. This information is publicly available in the MS. Based on the information that describes the market conditions, the agent strategises to submit a competitive offer to buy or sell. Given this background, we now analyse a selection of the most popular strategies for the CDA, from the perspective of the IKB model. We provide a summary of the analysis in Table 4.1.

- **The Kaplan Strategy** (see Subsection 2.3.4.1): This is a non-predictive strategy that makes a decision based only on simple heuristics (see Subsection 2.3.4.1),

[3]This varies according to the CDA; examples include the midway value or the earlier of the bid or the ask (see discussion in Section 2.2).

and ignores the history of market information. Thus, the IL collects the current outstanding bid and ask ($bid(t)$ and $ask(t)$ respectively) from the MS. Thereafter, using this information from the IL, the KL calculates the measures that are used in the heuristic rules of Kaplan's BL. These rules determine what action, $\mathcal{A}_i = \{bid_i|ask_i, silent\}$, the agent i submits in the market.

- **The Zero-Intelligence (ZI) Strategy** (see Subsection 2.3.4.2): The ZI has a random behaviour: it is non-predictive and does not use the history of market information. It effectively ignores the market state (MS) and considers only its limit price, ℓ_i (its private information state in the IL) when submitting a bid or an ask in the market. The KL does not compute any knowledge and simply forwards ℓ_i from the IL to the BL.

- **The Zero-Intelligence Plus (ZIP) Strategy** (see Subsection 2.3.4.3): This is a predictive strategy that uses the history of market information to predict the future market condition and adapt to it. It updates the profit margin of agent i (as per Equation 2.8) to remain competitive based on a set of learning rules given the changing market conditions (see Figure 2.8). The IL collects $bid(t)$, $ask(t)$ and $price(t)$ (as instructed by the KL). The IL forwards this data, as well as the agent's profit margin (private information in its IL), to the KL. That knowledge is then used in the BL to predict the future market and adapt its profit margin, μ_i, to it. The BL then submits $\mathcal{A}_i = \{bid_i|ask_i, silent\}$, where bid_i or $ask_i = (1 + \mu)\ell_i$.

- **The GD Strategy** (see Subsection 2.3.4.4): This is a non-predictive strategy that uses a history of market information. The BL decides on an action, $\{bid_i|ask_i, silent\}$, by solving a risk-neutral utility maximisation problem (see equations 2.13 and 2.14) involving the limit price ℓ_i and the buyer's belief $q(b)$ or the seller's belief $p(a)$ that a bid or an ask at a particular value will be successful in the market, respectively. Thus, the BL instructs the KL that it requires such knowledge. The KL then defines the Information Filter (see Figure 4.1), so that relevant information, namely the history of bids, asks and transaction prices ($H(bid)$, $H(ask)$ and $H(price)$ respectively) are filtered to the IL. That information, along with the agent's limit price is passed to the KL. The KL can then compute the belief and passes it, along with the limit price, to the BL.

Having discussed how the IKB model can be applied to existing strategies for the CDA, we consider in the next section how we can use our framework to engineer a new trading strategy given a particular market mechanism.

4.3 Designing a Trading Strategy for TAC

Here, we describe how we employed our IKB framework to design a novel strategy for the TAC[4]. This competition involves a number of software agents competing against each other in a number of interdependent auctions (based on different protocols) to purchase travel packages over a period of 5 days (for the TACtown destination) for different customers. In more detail, in a TAC Travel Game (each lasting 9 minutes), there are 8 agents required to purchase packages for up to 8 customers (given their preferences) and that compete in 3 types of auctions which we describe next.

1. *Flight auctions*. There is a single supplier for in-flight and out-flight tickets over different days, with unlimited supply, and ticket prices updating every 10 seconds. Transactions occur whenever the bid is equal to or greater than the current asking price of the flight supplier.
2. *Hotel auctions*. There are two hotels at TACtown, namely Shoreline Shanties (SS) and Tampa Towers (TT), with TT being the nicer hotel and each hotel having 16 rooms available over 4 different days. Thus, there are 8 different hotel auctions (given the 2 hotels and rooms being available for 4 different days). Hotel rooms are traded in 16th-price multi-unit English auctions, whereby the sixteen highest bidders are allocated a room for a particular day in a particular hotel, and at the end of every minute except the last, a hotel auction randomly closes, and the 16th and 17th highest price of each hotel auction that is still open is published.
3. *Entertainment auctions*. There are three types of entertainment in TACtown, namely a museum, an amusement park and a crocodile park, and 12 different entertainment auctions (for the three type of entertainment tickets for each of the four days). At the beginning of the game, each agent is randomly allocated 12 entertainment tickets tradeable in the different multi-unit CDAs which clear continuously and close at the end of the game.

Given this background on the TAC environment, our objective is to design a trading strategy for an autonomous software agent participating in such a game. We develop the strategy by using the IKB framework, adopting the multi-layered approach (see Figure 4.2). We now describe the strategy within the different layers prescribed by the IKB.

[4]Our IKB framework is employed in designing our agent, *WhiteDolphin*, which was ranked 3rd in the final of the TAC Travel Game 2006, held at the TADA/AMEC Workshop, Hakodate, Japan (see `http://www.sics.se/tac/page.php?id=60`).

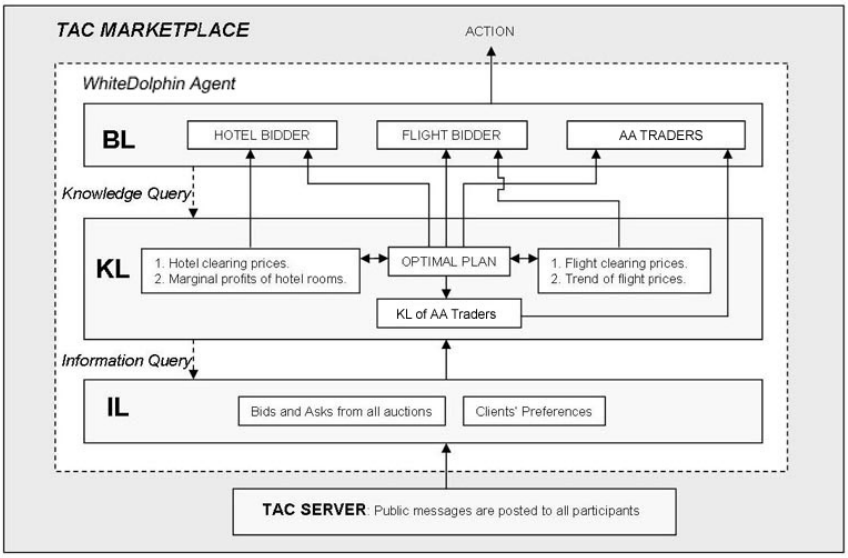


FIGURE 4.2: Structure of the WhiteDolphin Strategy for the TAC Travel Game.

4.3.1 The Behavioural Layer

The issues associated with the bidding behaviour can be summarised as follows:

1. What item to bid for?
2. How much to bid for?
3. When to bid?

We address the first issue by considering the *optimal plan* (see Definition 4.6). Thus, the agent always bids for the set of items (flight tickets, hotel rooms and entertainment tickets) required for the optimal plan, querying the optimal plan from the KL every 10 seconds. As a hotel auction closes every 60 seconds, the set of items available to the agent is further constrained and the optimal plan has to be recalculated. We address the other issues by considering the different auction formats.

Definition 4.6. Optimal Plan. The optimal plan is the set of travel packages, for 8 different clients, that would yield the maximum profit, given the clients' preferences (that determine the utility of the package) and the cost of the packages.

Definition 4.7. Marginal Profit[5]. The marginal profit of a hotel room (in a particular hotel on a particular day) is the decrease in the agent's total profit if it fails to acquire

[5]The marginal profit described here is similar in essence to the marginal value used in (Cheng et al., 2003).

that room. Thus, the marginal profit of a hotel room that is not required in the optimal plan is 0.

First, we consider the 8 flight auctions. Given the manner in which the flight prices update, it is possible to predict the trend of the price update. Such a trend is queried from the KL. If the trend suggests a decrease in price, the BL then queries the predicted lowest ask price of the flight auction, and a bid is placed in that auction when that minimum is reached, if such flight tickets are required in the optimal plan. Conversely, if an increasing trend is predicted in a flight auction, we face a trade-off between acquiring all the tickets in such an auction immediately at the current lowest price, and waiting in case the agent does not manage to acquire the *scarce* hotel rooms required in the optimal plan, which could make the flight tickets redundant (since they are no longer required in the optimal plan and represent a loss). We implement the trade-off by spreading our bids in a flight auction over the remaining length of the TAC game. For example, if 4 tickets are required from a particular flight auction with an increasing trend, we could buy a single ticket every minute over the next 4 minutes, rather than buying all 4 immediately.

Next, we have the 8 hotel auctions, with a random one clearing (and closing) every minute. Thus, every minute, as the optimal plan changes, we update our bid in those auctions that are yet to close. Now, there is uncertainty in being able to acquire all the items required in the optimal plan, particularly at the beginning of the game. Because a bid in a hotel auction can only be replaced by a higher bid, and because the optimal plan typically changes during the game, it does not pay to bid too high for an item at the beginning of the game. This is because that item might no longer be required in the optimal plan as the game progresses, thus might be acquired and not used. Given this, our agent does not bid for a hotel room at its marginal profit (see Definition 4.7), but rather bids low at the beginning of the game (at a fraction of its marginal profit) and gradually increases its bid for a room towards its marginal profit as the game progresses, bidding its marginal cost after the 7th minute before the last hotel auction closes.

Finally, we have the twelve entertainment auctions. Here, we use a preliminary version of our AA strategy (see Chapter 5 for more details). In particular, we have 12 AA traders that bid for the items required in the optimal plan. The agent further instructs the AA trader to buy *cheap* in auctions that do not influence the optimal plan, and sell *high* for all the items that it holds, if the agent can thus be more profitable, rather than using these items in its optimal plan.

We now consider the knowledge required for the bidding behaviour.

4.3.2 The Knowledge Layer

Here, we principally require the optimal plan which is given as the solution to an optimisation problem[6]. Specifically, the agent searches for the plan that maximises its profit, which is the total utility of the packages less their estimated cost. The utility of a package is determined by a client's preferences, which is queried from the IL. Furthermore, the optimisation problem is constrained by different requirements of a feasible package, for example a client needs to stay in the same hotel for the duration of his/her stay or the client is required to stay in a hotel during the length of his/her stay (Wellman et al., 2002), with additional contraints imposed as hotel auctions close. We also consider the additional knowledge of the predicted clearing price of the hotel auctions and of the flight auctions (based on the trend of flight prices in those auctions) to estimate the cost of a plan.

Now, for the hotel auctions, we calculate the marginal profit of hotel rooms required in the plan, to form the bidding price in the active hotel auctions. This is carried out by considering the next best package if a particular hotel room in the optimal plan cannot be acquired. The drop in profit then represents the marginal profit of that hotel room. Next, for the flight auctions, the KL estimates the trend of the flight prices, by considering its history. Such knowledge is used in the BL to decide when to bid for flight tickets, and in this layer, to calculate the minimum asking prices when a decreasing trend is identified. Finally, for the entertainment auctions, the agent has the same KL as the AA traders (see Section 5.2 for more details and for the design of the AA strategy using the IKB framework).

4.3.3 The Information Layer

In this layer, the agent extracts all the information needed for the knowledge it requires. Indeed, it tracks information relevant to the TAC Travel Game, such as the running time of the game and which auctions have closed, as well as the clients' preferences that do not change during the game. When it considers the individual auctions, the agent has to record the history of published information (bids and asks where available). In the flight auctions, the history of flight prices is required to estimate the trend, which represents vital knowledge. In the hotel auctions, the history of the publicly announced 16th highest price can be recorded up to when the auction closes. Such information

[6] We use ILOG CPLEX 9.0 to solve the optimisation problem, with a solution typically found within a few milliseconds.

can be used to estimate the clearing price of the hotel auctions in future TAC games. Finally, for the entertainment auctions, the agent has the same IL as the AA traders.

4.4 Summary

The objective of this line of work is to provide a systematic framework for designing strategies for market mechanisms. In particular, we required a framework that was sufficiently general to be used for the CDA mechanism, as well as variants thereof. To this end, we developed a framework that can be broken down into three principal components: the behavioural layer, the knowledge layer and the information layer. In so doing, we believe this work is an important preliminary step towards guiding the strategy designer by identifying the key models and concepts that are relevant to this task. We demonstrated the application of our approach by showing that CDA strategies can be analysed using our IKB framework and by showing how it can be used to assist in the design of a strategy for the TAC game.

The work addresses our research aim of designing strategies for the CDA. With this framework, we assist the strategy designer by providing a systematic approach to the design of such strategies. Furthermore, our framework is sufficiently flexible to assist the design of strategies for CDA variants as well, which this thesis also investigates. In the next chapter, we demonstrate its use at the preliminary stage of the design of our novel bidding strategy for the CDA.

Chapter 5

An Adaptive-Aggressiveness Bidding Strategy

In this chapter, we design our novel bidding strategy to fulfil our research aim for more efficient strategies for the CDA.

To date, considerable research endeavour has been invested into devising strategies for agents that participate in CDAs. However, there is no known dominant strategy (see Chapter 1). Thus, many strategies have been developed as heuristic-based, decision-making algorithms that attempt to best exploit the observable market information available to the agents in order to maximise their profits (see Subsection 2.3.4). Indeed, several of these strategies have been shown to outperform human traders in laboratory experiments (as discussed in Subsection 2.3.1). However, we believe that more efficient strategies can still be developed and, in this work, we will go on to develop just such a strategy.

In particular, the extant CDA strategies have typically been developed assuming that the market is *static*, meaning there is no change in demand and supply at the beginning of each trading day. However, in this work, one of the motivations for the use of decentralised systems is their being dynamic and, furthermore, real markets such as NASDAQ and the NYSE are typically very dynamic, with reasonably frequent market shocks. Thus, we believe the efficiency of strategies in dynamic environments is central to their application in practice. Now, although some of the designers have made initial attempts to show their strategies will still do well in dynamic markets (see discussion in Subsection 2.3.4), these strategies were not developed explicitly for such environments. We believe this is a mistake because there are some fundamental differences between static and dynamic environments. Specifically, these are primarily to do with the sporadically changing competitive market equilibrium. Given this, the efficiency of a strategy depends on how effective it is at adapting its bidding behaviour to the new market structure and, thus, to the new competitive equilibrium price. From this, our intuition is that different behaviours are needed when the market is relatively stable and when it is changing. In particular, in the static case, the agent can be effective by assuming that the competitive equilibrium does not change significantly, whereas in the dynamic case, it can make no such assumption and must learn, assuming that this competitive equilibrium may change. Furthermore, for maximum generality, we want to ensure that our strategy performs well in *homogeneous* populations in which all the agents use it (as would typically be the case in market-based control applications) and in *heterogeneous* populations in which agents can adopt a range of alternate strategies (as would be the case in financial institutions). Given this, we simply assume that an agent is selfish and tries to maximise its individual return and that it is unaware of whether it is trading in a homogeneous or a heterogeneous environment.

In this chapter, we first discuss, in Section 5.1, how we consider bidding aggressiveness in our strategy. Thereon, in Section 5.2, we describe our strategy and Section 5.3 summarises the work.

5.1 Bidding Aggressiveness

The key intuition behind our strategy is that we can profitably vary how aggressively an agent bids in the CDA. We focus on bidding aggressiveness[1], in particular, as we believe it is the key determinant of success in the market. It is so central because it describes how the agent manages the trade-off between profit and probability of transaction. Here, an *aggressive* trading agent is one that tries to increase its chance of transacting by placing bids, that are not necessarily highly profitable. In contrast, its *passive* counterpart tries to transact at more profitable prices, but has to trade off its chance of actually transacting. When the agent is not able to transact, it could choose to become more aggressive, such that it increases its chances of being able to transact and, conversely, when it can transact, it could choose to become more passive in order to try to increase its profits. In other words, the agent could react to the market information by being more or less aggressive based on how it is performing in the market:

[1]In some work, the trader's risk attitude has been used to describe broadly the same behaviour (Byde, 2003). However, we believe that such a property is intrinsic to the trader, and thus, is not an appropriate term to describe our changing behaviour in this case. Indeed, in the financial world, the behaviour of our strategy is typically referred to as aggressiveness rather than risk and this was our main motivation for the terminology we adopted.

Definition 5.1. Aggressiveness is defined as the inclination to interact more actively in the market. The *aggressive* trader submits better offers than what it believes the competitive equilibrium price to be, to improve its chance of transacting, and trades off profit for that purpose. The *passive* trader is less inclined to transact and more inclined to win a profitable transaction and thus submits offers that are worse than what it believes the competitive equilibrium price to be. The *neutral* trader submits offers at what it believes is the competitive equilibrium price, which is the expected transaction price.

Specifically, the agent can adopt behaviours that have different levels of aggressiveness, $r \in [-1, 1]$, ranging from aggressive ($r < 0$), through neutral at $r = 0$, to passive ($r > 0$), coupled with a learning mechanism to decide upon this level. Specifically, an agent that adopts a *passive* strategy waits for more profitable transactions than at its estimate of p^* (hereafter the estimate is denoted by $\hat{p}^*$) and is willing to trade-off its chance of transacting for higher expected profit. In contrast, an *aggressive* strategy trades-off profit to improve its probability of transacting in the market. The *neutral* agent attempts to transact at $\hat{p}^*$ which is the expected transaction price.

Given this, we employ a short-term learning mechanism to fine-tune the agent's aggressiveness whenever it submits a bid or an ask, or a transaction occurs (if the bid and the ask match) in the market. The actual way in which the degree of aggressiveness translates to a bid or an ask to submit in the market can be fixed or can be linear, in which case the aggressiveness would be similar to the agent's profit margin. However, we believe that this mapping should be updated depending on the prevailing market conditions. Thus, we employ a long-term learning strategy that adapts this mapping, in a non-linear fashion, to the changing conditions and, in particular, to the volatility of transaction prices. We refer to this learning as long-term because the occurrence of bids and asks is a fraction of the number of transactions that occur and because the benefit of learning this mapping is only really observable over a number of trading days. The purpose of the long-term learning is especially evident in dynamic markets where market conditions can change drastically and a different mapping should then clearly be adopted. Hereafter, we refer to our strategy as the *Adaptive-Aggressiveness* (or AA) strategy.

Having given our intuitions, we next describe our AA strategy in more detail. Chapter 7 then evaluates its effectiveness against a number of the state of the art alternatives.

5.2 The Bidding Strategy

As market conditions keep changing, different levels of aggressiveness are likely to be best at different times and so the agent needs a means for updating r. Thus, an AA agent has two principal decision-making components: (i) a bidding one that specifies what bid or ask should be submitted based on its current degree of aggressiveness, and (ii) a learning mechanism to update its behaviour according to the prevailing market conditions.

Given the desired behaviour of our strategy and the knowledge it requires (including the competitive equilibrium estimate), we use our ***IKB framework*** (see Chapter 4) to formulate the structure of our strategy. In more detail, the IL is instructed (by the KL) to record $bid(t)$ and $ask(t)$ to be used to update the degree of aggressiveness of the agent and a history of transaction prices, $H(price)$. The KL then uses $H(price)$ to estimate the market equilibrium price and to learn the best aggressiveness given the market condition. Such information, along with that about the market condition and the agent's limit price, $v_{n_i(t),i}$, is obtained from the IL. It is then relayed through the KL, in a set of bidding rules in the BL (based on knowledge of the market conditions). The latter then decides what offer, $\{bid_i|ask_i, silent\}$, the agent i submits.

Given how market information translates to a bid or an ask to submit in an AA agent, we now give a model that describes how the AA strategy works, along with an indication of how the different components that represent how knowledge (such as the equilibrium estimate) are updated based on observed information from the market (see Figure 5.1). The learning and bidding of the strategy are represented by two distinct components, (i) the bidding and (ii) the adaptive component within our IKB model.

The first component determines which bids or asks to submit given a set of bidding rules (see Subsection 5.2.4). These rules specify how to react to the current market conditions given the target price τ which represents the agent's most competitive[2] price in the market. Now, the *aggressiveness model*, as described in detail in Subsection 5.2.2, gives a mapping function to τ of the agent's current degree of aggressiveness, its limit price, $\hat{p}^*$ (which is provided by the *equilibrium estimator* described in Subsection 5.2.1), and an intrinsic parameter θ. In particular, θ determines the shape of that mapping function (see figures 5.2 and 5.3 for more details).

[2] A bid (or ask) is competitive if it is the agent's most profitable bid (or ask) that it believes would be accepted in the market. Note that a bid is always less than or equal to, and the ask always more than or equal to, its limit price. This is somewhat similar to the price given the ZIP agent's profit margin, and the price that maximised the GDX agent's expected profit.

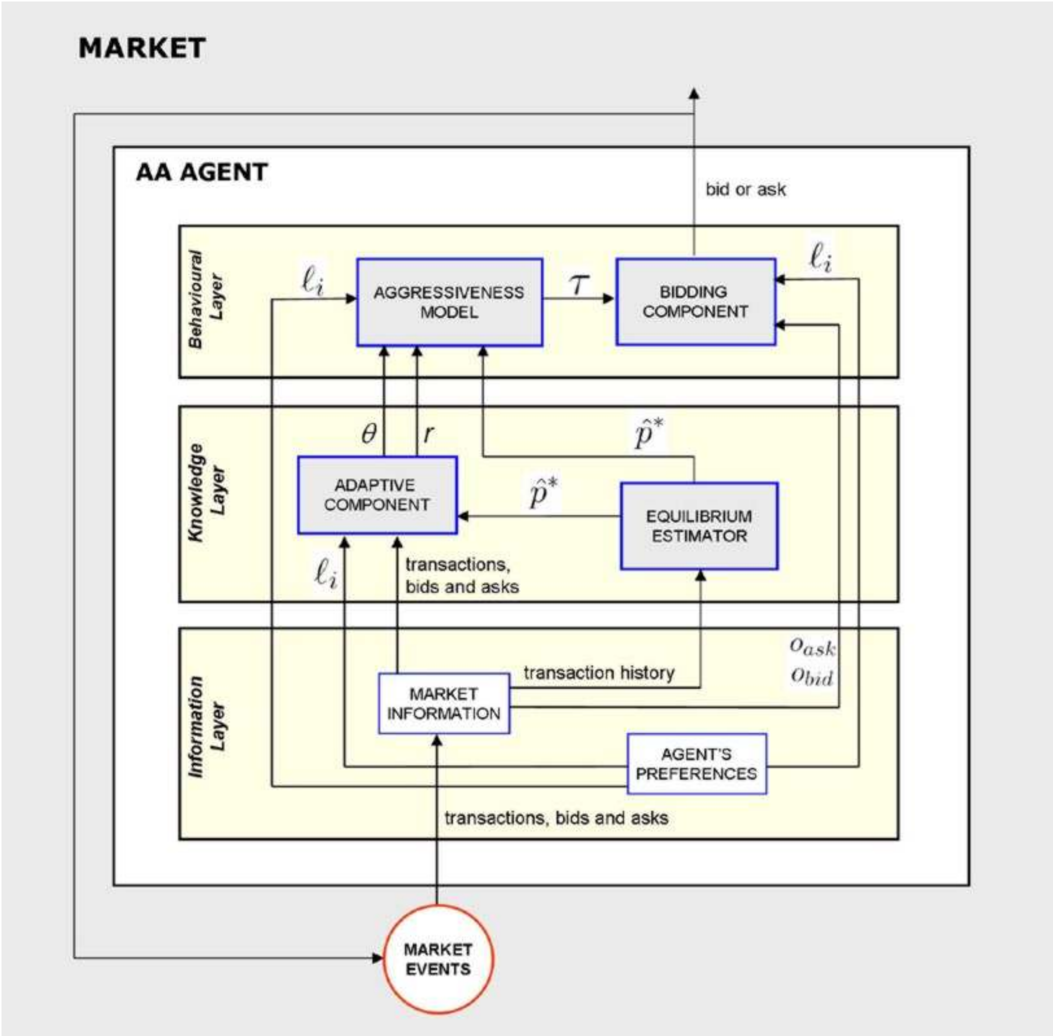


FIGURE 5.1: The AA bidding strategy.

The second component represents the adaptive part of the strategy where the agent updates its bidding behaviour, when triggered by a market event (when a transaction occurs or a new bid or ask is submitted). This update causes the agent to adopt a more passive behaviour if it believes it can transact at a higher profit or a more aggressive one if it believes it is targeting too high a profit to transact. In particular, we have short-term and long-term learning mechanisms that update the agent's bidding behaviour. The former updates the degree of aggressiveness, r, whenever a bid or ask is submitted and is described in more detail in Subsection 5.2.3.1. The latter updates θ in the aggressiveness model after every transaction and is described in more detail in Subsection 5.2.3.2.

In the following subsections, each component of our strategy is now described in turn.

5.2.1 The Equilibrium Estimator

Because p^* cannot be known *a priori*, we use the *moving average* method for calculating its estimate, $\hat{p}^*$, based on the history of transactions (see Equation 5.1). We make this choice because the moving average is an objective analytical tool that gives the average value over a time frame spanning from the last transaction. Moreover, it is sensitive to price changes over a short time frame, but over a longer time span, is less sensitive and filters out the high-frequency components of the signal within the frame. Moving average thus allows us to emphasise the direction of a trend and smooth out large price fluctuations and, thus, we believe this is a reasonable choice. Based on our assumption that the transaction prices converge to the competitive equilibrium price, we introduce the notion of *recency* in the moving average by giving more weight to the more recent transaction prices. Specifically, Equation 5.1 describes how we calculate $\hat{p}^*$ given a set of the N most recent transaction prices:

$$\hat{p}^* = \frac{\sum_{i=T-N+1}^{T} w_i p_i}{\sum_{i=T-N+1}^{T} w_i} \; where \; w_T = 1 \; and \; w_{i-1} = \lambda w_i \tag{5.1}$$

where $(w_{T-N+1}, \ldots, w_T)$ is the weight given to the N most recent transaction prices, $(p_{T-N+1}, \ldots, p_T)$, and T is the latest transaction. Based on simulations, we set λ to a value of 0.9 and N to roughly the number of daily transactions to emphasise any converging pattern in the history (see Figure 5.6 for an example).

5.2.2 The Aggressiveness Model

The role of the aggressiveness model is to generate the current target price, τ, given the agent's current degree of aggressiveness r. In this context, an agent can be of two types; namely, intra-marginal and extra-marginal (see Section 2.1). Recall that a buyer (seller) is intra-marginal if its limit price is higher (lower) than the competitive equilibrium price. In contrast, the extra-marginal buyer's (seller's) limit price is lower (higher) than the competitive equilibrium price. Now, in a centralised mechanism with an efficient allocation (market efficiency is 1), only intra-marginal agents transact, while extra-marginal ones do not (recall the discussion in Section 2.1). However, in a decentralised mechanism, while intra-marginal agents are expected to transact, and extra-marginal counterparts are not, the latter do sometimes succeed in transacting. This is because transaction prices are never exactly at p^*, and, thus, extra-marginal buyers can exploit asks below p^* and extra-marginal sellers bids above p^*. In such cases, when the extra-marginal traders do transact, the allocation is no longer efficient and the market efficiency dips below 1.

Our aggressiveness model differs for these two type of traders, fundamentally because of their limit prices with respect to p^*. Given this, we consider them each in turn.

First, we consider the intra-marginal trader. In its aggressiveness model, a target price equal to $\hat{p}^*$ implies that the trader is neutral. When an intra-marginal agent adopts a passive behaviour, it considers a target price that is below (for the buyer) or an ask that is above (for the seller) p^*, in order to obtain a higher (than expected at $\hat{p}^*$) profit margin. Conversely, an aggressive attitude implies that the intra-marginal trader targets bids above (asks below) the competitive equilibrium price, which improves the probability of its bids (asks) being accepted (but decreases its profit margin).

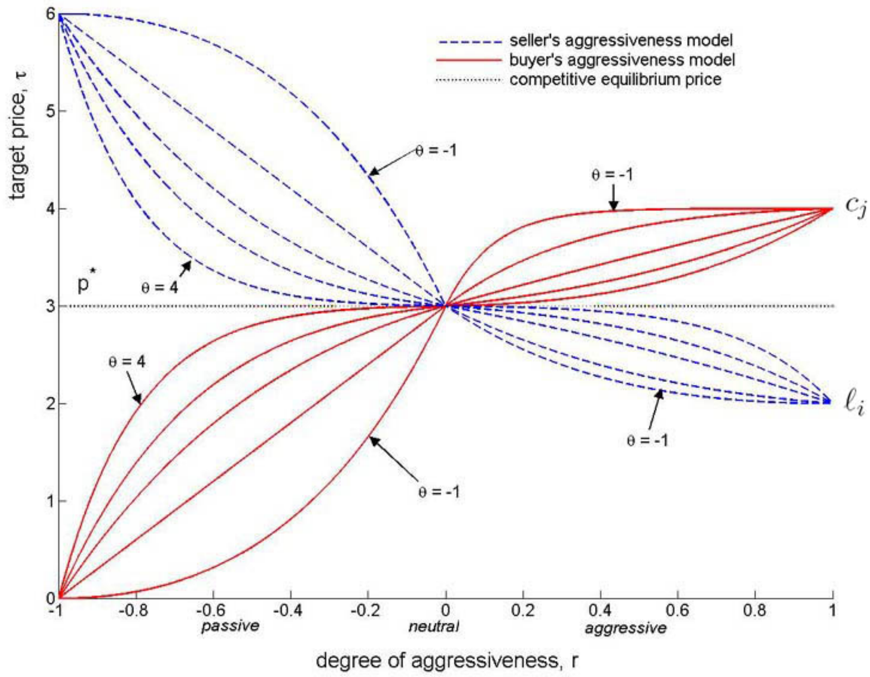


FIGURE 5.2: Aggressiveness for the intra-marginal trader for different θ. Solid lines represent the buyer's function, and the dashed lines, the seller's function.

For the intra-marginal aggressiveness model, we identified the following constraints that it should satisfy over the different degrees of aggressiveness. In particular, when the buyer is completely aggressive ($r = -1$), it targets a bid at its limit price and when it is completely passive ($r = 1$), it targets a bid at 0 (for maximum profit but no chance of actually transacting). The neutral buyer ($r = 0$) targets a bid at $\hat{p}^*$. Therefore, the aggressiveness function is defined at these three specific aggressiveness levels. Similar intuitions apply for the seller's aggressiveness function. However, when ($r = -1$), the seller submits the maximum ask, p_{max} (see Definition 2.15), allowed in the market. Given these constraints, there is an infinite solution space for such a function and so we choose a parameterised function (see Figure 5.2) within the solution space with θ determining the behaviour of the function (i.e. its rate of change with respect to the degree of aggressiveness r).

Specifically, equations 5.2 and 5.3 detail the intra-marginal buyer's and seller's aggressiveness model (and its relationship between r and τ). We adopt these particular functions because they are continuous (and thus, we do not have sudden jumps of τ as r changes) and θ allows the agent to explicitly specify the properties of the function. When θ is high, the magnitude of the gradient tends to 0 at $r = 0$ and increases as θ tends to -1. Conversely, when θ is low, the magnitude of the gradient is high at $r = 0$ and thus allows faster update of the target price as r changes. A slow update is required when the transaction prices are converging to $\hat{p}^*$, while a fast update is required at the beginning of the auction or after a market shock, when market conditions are changing considerably. Indeed, experimental results described in (Vytelingum et al., 2004) suggest that the effectiveness of our bidding strategy depends on the value θ. In particular, we observed from market simulations that a high θ is more beneficial when the prices are converging towards $\hat{p}^*$ and it is not profitable to deviate too much from $\hat{p}^*$. When faced with a high price volatility (with all agents still exploring the market), an agent is then better off with a low θ to also explore the market by allowing a faster update of its degree of aggressiveness. In Subsection 5.2.3.2, we describe how updating θ, and thus the aggressiveness model, after every transaction can be beneficial in the long term.

For an intra-marginal buyer i,

$$\tau = \begin{cases} \hat{p}^*(1 - re^{\theta(r-1)}) & \text{if} \quad r \in (-1, 0) \\ (\ell_i - \hat{p}^*)\ (1 - (r+1)\, e^{r\underline{\theta}}) + \hat{p}^* & \text{if} \quad r \in (0, 1) \end{cases}$$

$$where\ \underline{\theta} = \frac{\hat{p}^* e^{-\theta}}{\ell_i - \hat{p}^*} - 1 \tag{5.2}$$

For an intra-marginal seller j,

$$\tau = \begin{cases} \hat{p}^* + (p_{max} - \hat{p}^*)\, re^{(r-1)\theta} & \text{if} \quad r \in (-1, 0) \\ \hat{p}^* + (\hat{p}^* - c_j)\ re^{(r+1)\underline{\theta}} & \text{if} \quad r \in (0, 1) \end{cases}$$

$$where\ \underline{\theta} = log \left[\frac{p_{max} - \hat{p}^*}{\hat{p}^* - c_j} \right] - \theta \tag{5.3}$$

Now, for the above equations, the marginal trader is a limiting case, where $\ell_i = \hat{p}^*$ and $c_j = \hat{p}^*$. However, these equations are not valid in the extra-marginal case where the seller cannot ask below $\hat{p}^*$ and the buyer cannot bid above $\hat{p}^*$. In such situations, the extra-marginal buyer and seller modify their aggressiveness functions to that of Figure 5.3. This reflects the fact that the extra-marginal trader cannot be aggressive and its degree of aggressiveness, r, is clipped at 0 such that it will submit its limit price to maximise its chance of transacting. For this case, Equations 5.4 and 5.5 describe that aggressiveness function precisely:

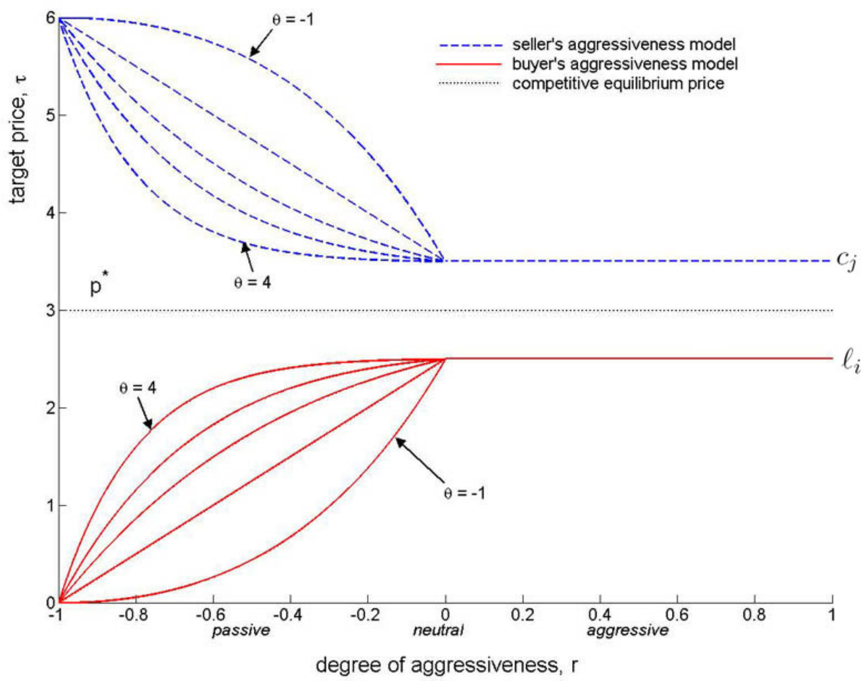


FIGURE 5.3: Aggressiveness for the extra-marginal traders for different θ. Solid lines represent the buyer's function, and the dashed lines, the seller's function.

For an extra-marginal buyer i,

$$\tau = \begin{cases} \ell_i(1 - re^{\theta(r-1)}) & \text{if} \quad r \in (-1, 0) \\ \ell_i & \text{if} \quad r \in (0, 1) \end{cases} \tag{5.4}$$

For an extra-marginal seller j,

$$\tau = \begin{cases} c_j + (p_{max} - c_j)re^{(r-1)\theta} & \text{if} \quad r \in (-1, 0) \\ c_j & \text{if} \quad r \in (0, 1) \end{cases} \tag{5.5}$$

We next look at the adaptive component of the AA strategy, where the agent learns its degree of aggressiveness and its aggressiveness model.

5.2.3 The Adaptive Component

The adaptive component consists of the short-term and long-term learning mechanisms that update r and θ respectively. In the following subsections, we describe each of these in more detail.

5.2.3.1 Short-Term Learning

In the short-term mechanism, the agent uses a set of learning rules (summarised in Figure 5.4) to update its aggressiveness, every time a bid or an ask is submitted or a transaction occurs in the market. It performs this in order to better fit the prevailing market conditions. Specifically, a simple continuous learning algorithm, the Widrow-Hoff algorithm (initially adopted in the ZIP strategy), is used to increase or decrease the aggressiveness, $r(t)$, at time step t (see Equation 5.6).

In more detail, the aim here is to adapt the agent's aggressiveness to the current *desired aggressiveness*, $\delta(t)$, which represents the degree of aggressiveness that would allow the buyer to bid the minimum of its limit price and a price slightly higher than the outstanding bid or the seller to ask the maximum of its limit price and a price slightly lower than the outstanding ask. Here, $\delta(t)$ is a factor of r_{shout}, the degree of aggressiveness that would form a price equal to the bid b, if the agent is a buyer and the last event was a bid, or to the ask, a, if the agent is a seller and the last event was an ask, or to the transaction p_T, whether the agent is a buyer or a seller and if the last event was a transaction (see bidding rules in Figure 5.4). When the agent is decreasing its degree of aggressiveness (to be more profitable), it sets $\delta(t)$ to slightly lower than r_{shout} ($\lambda = -0.05$) so that the target price is higher than the outstanding bid or lower than the outstanding ask. When it is increasing its degree of aggressiveness (to improve its chance of transacting), it sets $\delta(t)$ to slightly higher than r_{shout} ($\lambda = 0.05$). The algorithm then enacts a continuous-space learning process that backprojects a fraction of the error between the desired degree of aggressiveness, $\delta(t)$, and the degree of aggressiveness, $r(t)$, onto the same degree of aggressiveness, $r(t)$. As $r(t)$ updates, it gradually follows the changing $\delta(t)$ at a rate dependent on the learning parameter β_1. A reasonable value of β_1 is chosen, but λ and β_1 are not fundamental to the results we report here. Specifically,

$$\begin{aligned} r(t+1) &= r(t) + \beta_1(\delta(t) - r(t)) \\ \delta(t) &= (1+\lambda)r_{shout},\ \lambda = \{-0.05, 0.05\} \end{aligned} \tag{5.6}$$

where $\beta_1 \in (0,1)$ is the learning rate of the algorithm which influences the rate of change of $r(t)$ and, hence, of the target price τ.

Learning Rules for Buyer i:
if (transaction occurs at price p_T)
  if ($\tau \geq p_T$) *buyer must be less aggressive*
  else *buyer must be more aggressive*
else if (bid, b, submitted)
  if ($\tau \leq b$) *buyer must be more aggressive*

Learning Rules for Seller j:
if (transaction occurs at price p_T)
  if ($\tau \leq p_T$) *seller must be less aggressive*
  else *seller must be more aggressive*
else if (ask, a, submitted)
  if ($\tau \geq a$) *seller must be more aggressive*

FIGURE 5.4: Short-Term Learning Rules.

The learning rules employed here are broadly similar to those of the ZIP strategy. We employ its learning mechanism because it has been shown to effectively exploit market information. However, rather than updating a profit margin, we employ the mechanism to update the agent's degree of aggressiveness. We also simplify the adaptive mechanism by not considering a momentum-based update, since the manner in which the aggressiveness is updated with respect to the competitive equilibrium price minimises any high-frequency change in the bid or ask prices. In more detail, when the buyer's target price is greater than the transaction price, this implies that the buyer can transact and so it should try to be more profitable in the next round by being less aggressive. If its target price is less than the transaction price, this suggests that the buyer cannot transact at its target price, and thus should increase it by being more aggressive. Similar intuitions apply for the seller's learning rules. An example of how the degree of aggressiveness changes in a specific scenario is given in Figure 5.6.

5.2.3.2 Long-Term Learning

As described in Subsection 5.2.2, θ influences the bidding behaviour. Given this, we now describe how we can learn such a parameter on a long-term basis, after every transaction, to improve the efficiency of AA. The underlying intuition here is that different values of θ are best within different market conditions and, in particular, the best values of θ depend on the price volatility. Given this, we update θ (after every transaction) through a learning process based on the price volatility, which we measure as an approximation of Smith's α-parameter (see Section 2.1), given that the agent only has an estimate of the competitive equilibrium price. Equation 5.7 describes the learning mechanism:

$$\begin{aligned} \theta(t+1) &= \theta(t) + \beta_2(\theta^*(\alpha) - \theta_t) \\ \alpha &= \frac{\sqrt{\frac{1}{N}\sum_{i=T-N+1}^{T}(p_i - \hat{p}^*)^2}}{\hat{p}^*} \end{aligned} \tag{5.7}$$

where $\beta_2 \in (0, 1)$ is the learning rate of the algorithm, that determines how θ adapts. In particular, $\theta^*(\alpha)$ is a function (see Figure 5.5) that determines the desired θ parameter given the current price volatility[3], α, calculated over a window of the N latest prices. p_i is the price of transaction i, and T is the most recent transaction. $\theta^*(\alpha)$ is given by Equation 5.8 and is shown in Figure 5.5. Based on simulation results for different environments, we chose this particular function as it approximates[4] well the optimal θ parameter that maximises performance given the price volatility:

$$\begin{aligned} \theta^*(\alpha) = \; & (\theta_{max} - \theta_{min})(1 - (\alpha - \alpha_{min})/(\alpha_{max} - \alpha_{min})) \\ & e^{2((\alpha-\alpha_{min})/(\alpha_{max}-\alpha_{min})-1)} + \theta_{min} \end{aligned} \tag{5.8}$$

where $[\theta_{min}, \theta_{max}]$ is the range over which we update θ, α_{max} is the maximum α that occurs in the market, and α_{min} is the minimum α.

Given the mechanism, we now consider an example of how θ changes in a specific scenario in Figure 5.6. In particular, θ updates after every transaction, as specified by

[3]The price volatility is calculated in the same manner as Smith's coefficient of convergence α.

[4]Our function is only an approximation since it is averaged over the optimal θ for a number of different market environments. The exact environment and thus, the exact optimal θ, are unknown *a priori*.

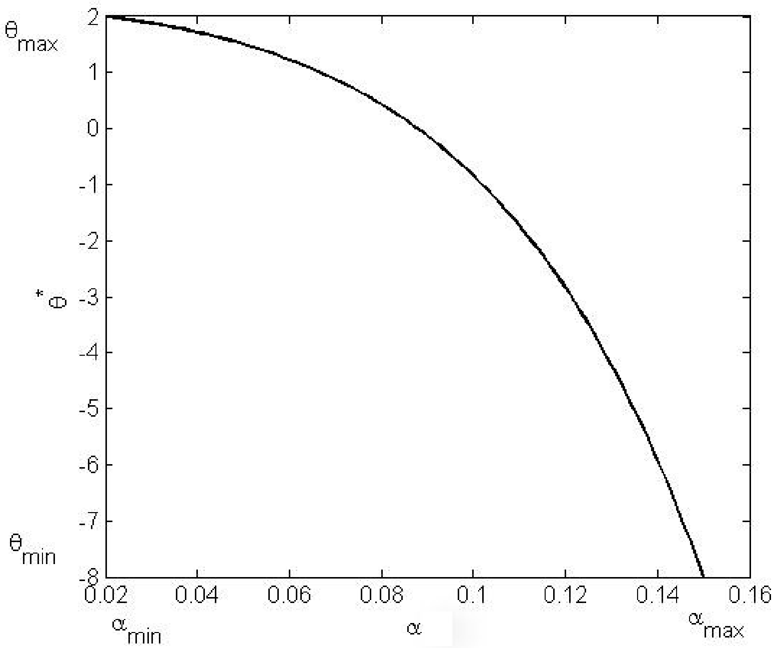


FIGURE 5.5: Function $\theta^*(\alpha)$ gives the desired θ^*.

Equation 5.7, fixed at -4 for the first few transaction prices (until a reasonable estimate of the competitive equilibrium price is obtained) and then updated to settle at θ_{max} (around 2) as the transaction prices converge to the competitive equilibrium price (at 2.65). When a market shock is identified by the sudden increase in α (at transaction 54), θ gradually decreases to θ_{min} (around -8) to give a faster update of the target price τ (see Subsection 5.2.2). As the agents' behaviours gradually adapt to the new market demand and supply and the transaction prices converge towards the new competitive equilibrium price (at 3.82), θ gradually increases back to a high value (around 2) that is more suitable for a low price volatility.

Having looked at the aggressiveness model (that outputs τ given r and θ), and the adaptive component (that updates r and θ), we now need to describe the bidding component where the agent forms a bid or an ask to submit in the market, based on the current market conditions, its limit price and τ.

5.2.4 The Bidding Component

In the bidding component, the agent employs a set of bidding rules to decide whether or not to submit a bid or an ask, and at what price if it decides to do so. If the buyer's (seller's) limit price is lower (higher) than the current o_{bid} (o_{ask}), it cannot submit any bid (ask), and waits for the beginning of the next round. On the other hand, if the agent

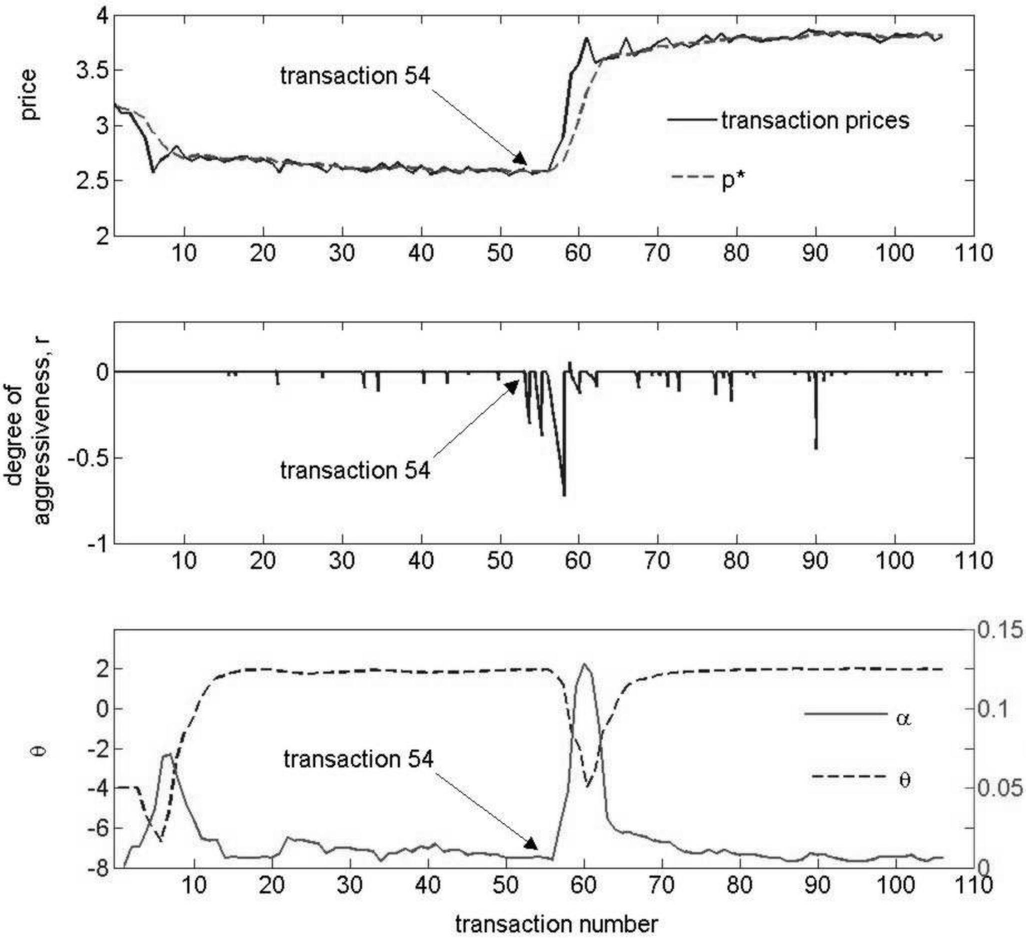


FIGURE 5.6: The history of transaction prices and $\hat{p^*}$ (top plot), the short-term learning of r (middle plot) and the long-term learning of θ (bottom plot). Note that we have a market shock after transaction 54, with θ updated to match the change in price volatility.

can submit a bid or ask in the market, it considers its set of bidding rules to form a price. In this, we identify two cases when an agent bids: during the first trading round, where it cannot estimate the competitive equilibrium price, and the subsequent rounds where it can. In particular, Figure 5.7 gives the bidding rules, and equations 5.9 and 5.10 describe the price formation process in detail in the two cases:

$$bid_i = \begin{cases} o_{bid} + (\min\{\ell_i, o_{ask}\} - o_{bid})/\eta & \text{if first round} \\ o_{bid} + (\tau - o_{bid})/\eta & \text{otherwise} \end{cases} \tag{5.9}$$

Bidding Rules for Buyer i:
if ($\ell_i \leq o_{bid}$) *submit no bid*
else
  if (first trading round) *submit bid given by* Equation 5.9
  else
    if ($o_{ask} \leq \tau$) *accept* o_{ask}
    else *submit bid given by* Equation 5.9

Bidding Rules for Seller j:
if ($c_j \geq o_{ask}$) *submit no ask*
else
  if (first trading round) *submit ask given by* Equation 5.10
  else
    if ($o_{bid} \geq \tau$) *accept* o_{bid}
    else *submit ask given by* Equation 5.10

FIGURE 5.7: Bidding Rules.

$$ask_j = \begin{cases} o_{ask} - (o_{ask} - \max\{c_j, o_{bid}\})/\eta & \text{if first round} \\ o_{ask} - (o_{ask} - \tau)/\eta & \text{otherwise} \end{cases} \quad (5.10)$$

where $\eta \in [1, \infty)$ is a constant that determines the rate of increase (decrease) of the bids (asks).

At the beginning of the first trading round, the agent has *no information* other than its limit price. Now, because if the buyer submits too high a bid, it can transact at a not very profitable price (with respect to p^*), it starts with low bids that progressively approach the minimum of its limit price, ℓ_i, and the outstanding ask, o_{ask}, (see Equation 5.10) to explore the market. Similarly, the seller, j, submits an ask towards the maximum of its cost price, c_j, and the outstanding bid, o_{bid}, (see Equation 5.9). Thus, the agent effectively reduces the bid-ask spread with an exponentially decreasing trend (since the bid increase should be decreasing to reflect the decreasing bid-ask spread) determined by η and its limit price. Here, a low η implies a faster rate of convergence of bids or asks until they are matched at a transaction price and, conversely, a high η implies a more conservative bidding approach and a slower convergence. With the latter, while being more profitable if it transacts, the agent risks missing out on a transaction if other agents adopt a more conservative strategy (similar to that of an AA agent with a lower η). However, with a lower η, the agent might be too hasty, and adopting a more conservative approach might be more profitable. In our simulations, we choose a value of 3 for η, which was observed to be a good compromise over a multitude of environments. Furthermore, the buyer can only submit a bid if its limit price is higher than o_{bid}, or otherwise, it remains idle until the beginning of the next trading round. We use similar intuitions to design the behaviour of the seller.

After the first trading round, the agent has an initial estimate of the competitive equilibrium price, which it subsequently updates after each transaction. Initially, we set the agent's aggressiveness factor, r, to 0 (meaning it adopts a neutral attitude) because of the lack of market information. Based on the target price, τ, and the set of bidding rules that dictate how the agent should react to the current market conditions, the trader then forms a bid or ask to submit in the market. In more detail, if the target price is higher than the outstanding ask at any time during the bidding process, the buyer accepts the outstanding ask (which is a better offer than it was targeting). Otherwise, it submits a bid, given by Equation 5.9, that approaches the (changing) target price in a similar manner as in the first trading round. We use similar intuitions to design the seller's bidding rules. Here, if the target price is lower than the outstanding bid, the seller accepts the outstanding bid. Otherwise, it submits an ask given by Equation 5.10. Furthermore, as in the first trading round, η affects the bidding process in a similar manner and is set to 3 throughout the trading day.

5.3 Summary

In this chapter, we described a novel predictive and history-based bidding strategy that software agents can use to bid in Continuous Double Auctions. The adaptive-aggressiveness strategy is principally based on a short-term and a long-term learning of the agent's bidding behaviour. For the short-term learning, the motivation was to immediately respond to fluctuations in the market conditions, and the agent updates the *aggressiveness* of its bidding behaviour based on market information observed after every bid or ask appears in the market. The motivation for the long-term learning mechanism, on the other hand, was to respond to more systematic changes in the market conditions and, in particular, to market shocks. To achieve this, our strategy updates an aggressiveness model that determines how the agent's degree of aggressiveness influences its choice of bids or asks to submit in the market, based on market information observed after every successful transaction.

This work addresses our research aim to develop more efficient strategies since, as we will empirically demonstrate in Chapter 7, our AA strategy outperforms the state of the art in a wide range of different market environments. Before we do this, however, in the next chapter, we describe a framework for analysing the strategic interactions of such strategies in the CDA.

Chapter 6

A Framework for Analysing Strategic Behaviours

As discussed in Section 1.1, because of the complexity of the CDA and the dominance of heuristic strategies, it is very difficult to determine *a priori* which strategies will be effective in which situations. This is a serious concern for agent designers because they have no principled way of selecting which strategy to adopt; this is important because the various strategies can perform very differently in different market settings. Moreover, it is also a concern for market designers because they want to deploy a mechanism that is efficient and stable, but this depends on the strategies that the various participants adopt. Thus, we require a method of determining which strategies are more likely to be adopted in the market (see research aim 4). Walsh *et al*. propose such a method with an EGT model that analyses the evolution of an agent's strategy in the market. However, as discussed in Subsection 2.3.5.2, their model is constrained by a major assumption that buyers and sellers adopt the same behaviour in the market.

To address this shortcoming, we propose a two-population EGT model to analyse the complex interaction of buyers *and* sellers in the market. Specifically, we consider a game with two distinct populations, each endowed with a separate set of heuristic strategies. The two populations correspond to the two different types of trading agents, the buyers and the sellers, and they are endowed with distinct sets of strategies. Thus, our model makes no assumption that an agent must have the same strategy for buying and selling, although if this is the best thing to do, then, our model will converge to it. In developing this new model, we advance the state of the art as follows. First, we provide a novel analytical model that dissects the buyer and seller trading roles in the market and allows us to analyse the market efficiency and stability from a buyer's and a seller's perspective. Second, to illustrate the effectiveness of our model, we analyse the CDA with the state of the art strategies and discuss how such an analysis of the evolution of buyers' and sellers' strategies can indeed be used to evaluate strategies in heterogeneous populations. Finally, to demonstrate the benefits of our model over Walsh *et al.*'s, we compare a one-population EGT analysis against a two-population one and identify strategic interactions between buyers and sellers that cannot be observed with the former.

6.1 A Two-Population EGT Model

In a two-population game, a player i from population P selects its strategy from a set of S_P *strategies*, where a strategy is a policy that determines the agent's *action(s)* in the game. The payoff to each player is then a deterministic mapping of the strategies of the players from the two populations and is usually read off a corresponding payoff table. For generality, we assume that each player has a mixed strategy, $x^i = (x^i_1, \ldots, x^i_j, \ldots, x^i_{S_P})$ (where x_j is the probability that it plays pure strategy j) that it plays in the game. Then, as rational behaviour dictates that each player will change its mixed strategy for a higher payoff, we have an evolution of behaviours as all the players in the game concurrently change their mixed strategy. EGT, then, allows us to analyse such an evolution. While EGT has commonly been used to analyse simple games (see Subsection 2.3.5.2), here, however, we are interested in more complex two-population market games with A_b buyers and A_s sellers, endowed with a set of S_b and a set of S_s heuristic strategies respectively.

We address this problem by abstracting the complex market game to a simple normal-form game, as per Walsh *et al.*'s model and, we do so as follows. Playing the complex buyer and seller strategies in the game can be considered as high-level actions similar to the *actions* in a normal-form game. The payoff to a buyer or a seller for such actions is then the total profit that they have extracted at the end of the game (which can last over several trading days). In such cases, the payoffs to buyers and sellers are usually referred to as being *heuristic*, because they are the result of the complex, non-deterministic interaction of trading agents in the game. Thus, our analysis begins with the computation of the heuristic payoff table and, thereafter, we analyse our market games, in terms of equilibria and dynamics, using a two-population EGT analytical approach, as the novel step of our model.

6.1.1 Computing the Heuristic Payoff Table

First, we describe how to calculate the entries of the heuristic payoff table with the expected return to each agent as a function of the strategies played by all agents. Now, for a two-population, normal-formal game with S_b buyer strategies, S_s seller strategies, A_b buyers and A_s sellers, we require $\left(S_b^{A_b} \times S_s^{A_s}\right)$ entries in the table. However, because the table size increases exponentially with the number of strategies, some simplifications are necessary to make the analysis tractable. In particular, Walsh *et al.* restrict their analysis to symmetric mixed-Nash equilibrium when they assume that each agent from the single population has the same mixed strategy, and hence, expects the same payoff when playing the same pure strategy. In our case, when considering two populations, the size of the payoff table then reduces considerably to $\binom{A_b+S_b-1}{A_b} \times \binom{A_s+S_s-1}{A_s}$. For example, for a market game of 10 buyers and 10 sellers, each endowed with 2 different strategies, the size of the payoff table reduces from 1.05×10^6 to 121 (from asymmetric to symmetric payoffs in each population).

For the exhaustive set of strategy profiles[1], we calculate the different payoffs for the buyer and the seller to the different strategies in the market (see Appendix A for the complete heuristic payoff table of this particular scenario). Because of the non-deterministic nature of the table, we require a statistically significant number of independent runs for each profile[2]. Given the heuristic payoff table (which represents the most computation-intensive part of our analytical approach), we can now proceed to the equilibrium computation and the dynamics analysis of the market game. In the former, we describe the ideal static properties of the population proportions using the different strategies in the system (i.e. the mixed-Nash equilibria of the game). In the latter, we detail how to calculate the dynamics $\dot{p} = (\dot{p}_1, \ldots, \dot{p}_{S_b})$ and $\dot{q} = (\dot{q}_1, \ldots, \dot{q}_{S_s})$ of the game, which describe how the buyer and seller population distributions, $p = (p_1, \ldots, p_{S_b})$ and $q = (q_1, \ldots, q_{S_s})$, change[3]. Here, because we are considering very large populations, we can validate that the population distributions, p and q, are equal to the mixed strategies of the buyers and sellers. Hereafter, we will refer to these terms as the mixed strategies of the two populations.

[1] A strategy profile $[\rho^b, \rho^s]$ defines the number of buyers $\rho^b = (\rho_1^b, \ldots, \rho_{S_b}^b)$ and sellers $\rho^s = (\rho_1^s, \ldots, \rho_{S_s}^s)$ using the different buyer and seller strategies respectively. An example of the 121 strategy profiles for the above 10 buyers and sellers game would be $[(1, 9), (2, 8)]$, where we have 1 buyer using buyer strategy 1, 9 buyers using buyer strategy 2, 2 sellers using seller strategy 1, and 8 using seller strategy 2.

[2] Non-parametric tests (Hollander and Wolfe, 1973) on the runs for the different profiles showed that 2500 runs was sufficient for statistical significance at the 95%-confidence-interval.

[3] The change is subject to the constraints that $\sum_{h=1}^{S_b} p_h = 1$ and $\sum_{k=1}^{S_s} q_k = 1$.

6.1.2 The Equilibrium Analysis

Having described how to calculate the mixed-Nash equilibrium given the heuristic payoff table, it is now possible to formulate our solution as the global minimum of a real-valued function, $v(p, q)$ (given in Equation 6.1) on a polytope, whose fixed points approximate the mixed-Nash equilibria, (p_{nash}, q_{nash}) (McKelvey and McLennan, 1996):

$$\begin{aligned} v(p, q) &= \textstyle\sum_{h=1}^{S_b}(\max\left[u_b(e^h, p, q) - u_b(p, p, q), 0\right] p_h)^2 + \\ &\textstyle\sum_{k=1}^{S_s}(\max\left[u_s(e^k, q, p) - u_s(q, q, p), 0\right] q_k)^2 \end{aligned} \tag{6.1}$$

where $u_b(e^h, p, q)$ represents the expected payoff to a buyer adopting pure strategy h when the other buyers adopt a mixed strategy p and the sellers a mixed strategy q. $u_b(p, p, q) = \sum_{i=1}^{S_b} u_b(e^i, p, q)p_i$ is the average payoff to a buyer in the market. Similarly, $u_s(e^k, q, p)$ is the expected payoff to a seller adopting pure strategy k when all buyers adopt mixed strategy p and the rest of the sellers adopt mixed strategy q. $u_s(q, q, p) = \sum_{j=1}^{S_s} u_s(e^j, q, p)q_j$ is the average payoff to a seller in the market. Now, when calculating the expected payoff of a buyer using a pure strategy h, we consider a significant number of games where one buyer adopts pure strategy h, $(N_b - 1)$ buyers adopt mixed strategy p and N_s sellers adopts mixed strategy q. The individual profit of all agents using the pure buyer strategy h, is then averaged over all the games as the required payoff. A similar procedure applies when calculating a seller's expected payoff.

Now, solving such a non-linear minimisation problem is non-trivial and, indeed, can be computationally demanding. Thus, we use a non-linear minimisation algorithm based on the Nelder-Mead method (Nelder and Mead, 1965) and provided by the Matlab optimization toolbox to solve the problem of finding the zero-points that minimise $v(p, q)$. Furthermore, because the toolbox can only find local minima, we repeatedly restart the algorithm at random points (p and q) a number of times until no new minima is found for 30 runs (as per (Walsh et al., 2002)).

Having looked at the static perspective of our analysis of the market game, we now consider the dynamic perspective.

6.1.3 The Dynamics Analysis

To analyse the dynamics of the market, we adopt the method used by Walsh *et al.* and, specifically, we use replicator dynamics. As discussed in Subsection 2.3.5.2, the replicator dynamics is appropriate because it has been shown to be a good approximation to an agent learning model, such as reinforcement learning (Tuyls and Nowe, 2005), which we would typically find in such markets. The following equations describe how we calculate the dynamics, $\dot{p_h}$ for pure buyer strategy h and $\dot{q_k}$ for pure seller strategy k:

$$\dot{p_h} = \left[u_b(e^h, p, q) - u_b(p, p, q)\right] p_h \quad (6.2)$$

$$\dot{q_k} = \left[u_s(e^k, q, p) - u_s(q, q, p)\right] q_k \quad (6.3)$$

As in the one-population EGT analysis (see Subsection 2.3.5.2), to observe the dynamics of the game, we look at the trajectories (see Definition 2.25) and identify all the attractors and saddle points, as well as the basins of attraction. Specifically, we calculate a trajectory by starting with any pair of mixed-strategies (p, q), and calculate the dynamics, $\dot{p}$ and $\dot{q}$, given by equations 6.2 and 6.3 respectively, as we progress along that trajectory which converges to an attractor or diverges from a saddle point. Furthermore, as discussed in Subsection 2.3.5.2, by considering the area of the basin of attraction of each attractor, we can calculate the probability that an attractor will be eventually adopted assuming that there is a uniform probability distribution over the starting points.

Given this framework to analyse buyers' and sellers' behaviours, we next analyse the evolving behaviour of the CDA.

6.2 Applying the Model to the CDA

Having detailed our analytical model, we now apply it to the CDA, the subject of this thesis. While our analysis is feasible for any number of buyers and sellers and any number of strategies, a visual representation of the trajectories of the replicator dynamics and the equilibria is only possible when considering at most a set of two strategies in either population (because we can effectively plot the replicator dynamics in a two-dimensional space) and so this is the case to which we limit ourselves here.

In this section, we first demonstrate how we analyse the evolution of buyer and seller strategies in the CDA using our EGT model for a particular scenario. Second, we compare a two-population EGT analysis of the CDA against the comparable one-population one so that we can clearly see the benefits of our approach.

6.2.1 The Analysis

Here, we analyse the evolution of buyer and seller behaviours assuming they can choose between two strategies. First, we generate our heuristic payoff table and we then go on to perform the actual EGT analysis for a particular dynamic scenario.

As we are considering a set of only 2 strategies in either population, we can plot the replicator dynamics, $(\dot{p}_1, \dot{q}_1)$, as vectors at different (p_1, q_1) in a two-dimensional figure. In such cases, the horizontal axis represents the buyer population proportion, p_1, and the vertical axis, the seller population proportion, q_1. Then, the different vertices correspond to different pure buyer and seller strategies. An example of an EGT analysis is given in figures 6.1 and 6.2. The former plot gives the replicator dynamics of the analysis, with the vertices corresponding to different pure strategies, and its shading denotes the magnitude of the dynamics, $(|\dot{p}_1| + |\dot{q}_1|)$, given the mixed strategies of the buyers and sellers. As the magnitude of the dynamics decreases (and the shading is darker), there is less and less incentive to deviate to another strategy, until the magnitude is 0 at a mixed-Nash equilibrium and, then, it does not pay off to deviate to another buyer or seller strategy. Finally, the latter plot gives the magnitude of the buyer's and seller's dynamics, with a mixed-Nash equilibrium occurring when the magnitude of both dynamics is 0. We consider these magnitudes to compare the buyer's and seller's payoff difference when deviating to the more efficient strategy.

More specifically, here we analyse the CDA in a dynamic market where buyers and sellers can choose between the GDX and AA strategies (which we empirically demonstrate to be the two most efficient CDA strategies in Chapter 7). Figure 6.1 is the dynamics, and Figure 6.2 the magnitude of the dynamics in such a scenario. While we are not concerned with the specificities of the scenario we are considering, the purpose of this exercise is to show what the EGT plot reveals.

In this scenario, we have three attractors A (at pure strategy AA), B (at pure strategy GDX) and C, and two saddle points D and E. Specifically, the area of the basin of attraction of A is 0.78, that of B is 0.12 and that of C is 0.10. Based on this, we infer that there is 78% chance that AA will eventually be adopted by all buyers and sellers in the market, a chance of 12% that GDX will eventually be adopted by all buyers and sellers and, finally, a chance of 10% that all buyers will adopt GDX and all sellers will adopt AA. Furthermore, if we aggregate these results, there is 12% chance that GDX will eventually be adopted by all buyer and sellers, and 10% chance that, eventually, all buyers will adopt GDX and all sellers will adopt AA. We can further infer that there is 88% chance the sellers will adopt AA, 12% chance they will adopt GDX, and there is 78% chance the buyers will adopt AA and 22% chance they will adopt GDX.

Now, when we consider the trajectories when GDX buyers are in the minority (right-hand part of the dynamics plot), we observe that the sellers nearly always deviate to AA. As GDX becomes more popular among buyers (with trajectories flowing towards left, i.e. GDX), sellers now deviate to either GDX or AA (shown by the trajectories flowing either to the top at AA or to the bottom at GDX), depending in which basin of attraction they are. When we consider the magnitude of the buyer's and the seller's dynamics (see Figure 6.2), we observe that the latter is larger than the former specifically when AA buyers are in the majority (when p_1 is close to 1). This implies that there is, then, a fast convergence of the seller's strategy to AA, suggesting that AA sellers are most profitable when competing against AA buyers and the strategies quickly evolve to AA. Furthermore, we observe that there is marginally less economic incentives to deviate to another strategy when GDX is in the majority (shown by the low magnitude of the buyer's and seller's dynamics). Thus, it takes longer for the strategy to evolve to either the mixed-Nash equilibrium B or C.

6.2.2 Comparison with the One-Population EGT Model

We have previously argued that our two-population model offers better insights into the behaviour of the CDA (see Subsection 2.3.5.2). Now, in order to see this directly, we compare the two in a given scenario. The EGT plot using the two-population model is the scenario we considered above and the corresponding one-population model is given in Figure 6.3. As we are considering only two strategies here, the dynamics is given by a one-dimensional plot, where GDX is the pure strategy at 0 and AA is the pure strategy at 1.

From Figure 6.3, we observe two attractors A′ at 1 and B′ at 0, and a saddle point C′ at 0.30. By considering the space of trajectories that converge to either attractors, we can infer that there is 30% chance that all agents (buyers and sellers) will eventually adopt the pure strategy GDX, while there is a chance of 70% that all agents will eventually adopt the pure strategy AA. We also observe that AA is more efficient than GDX (hence the trajectory towards AA) when in the majority (shown by the higher magnitude when

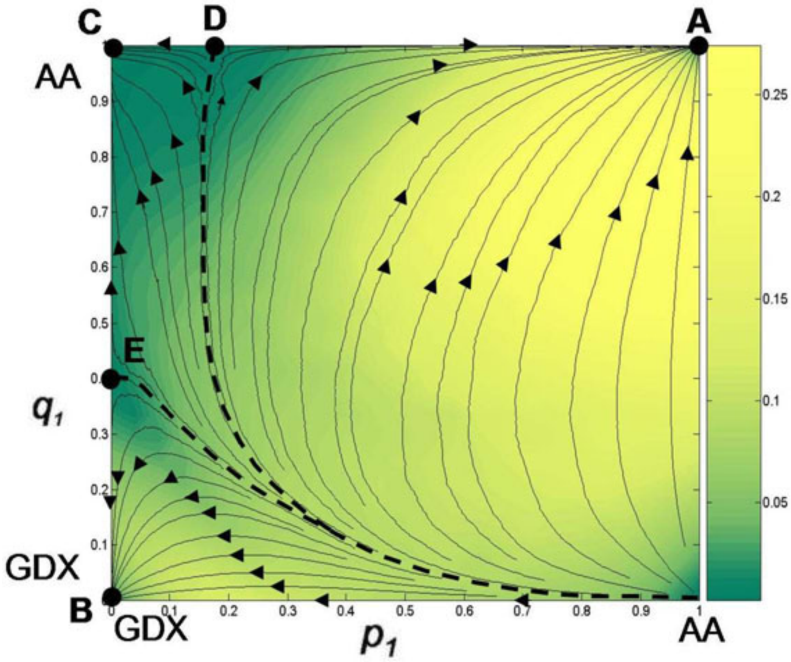


FIGURE 6.1: The replicator dynamics for a dynamic market with AA and GDX buyers and sellers. Here, we have three attractors: A at (1,1), B at (0,0) and C at (0,1) and two saddle points: D at (0.19,1) and E at (0,0.40). The dotted line denotes the boundary between the basins of attraction.

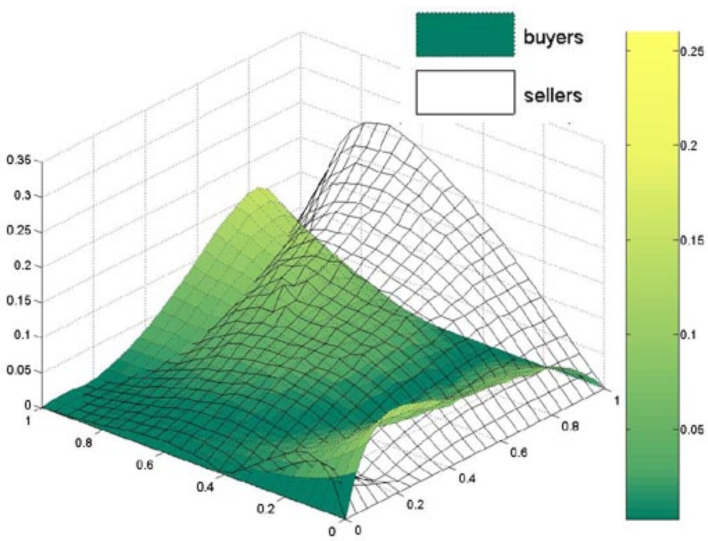


FIGURE 6.2: The magnitude of the buyer's and seller's dynamics.

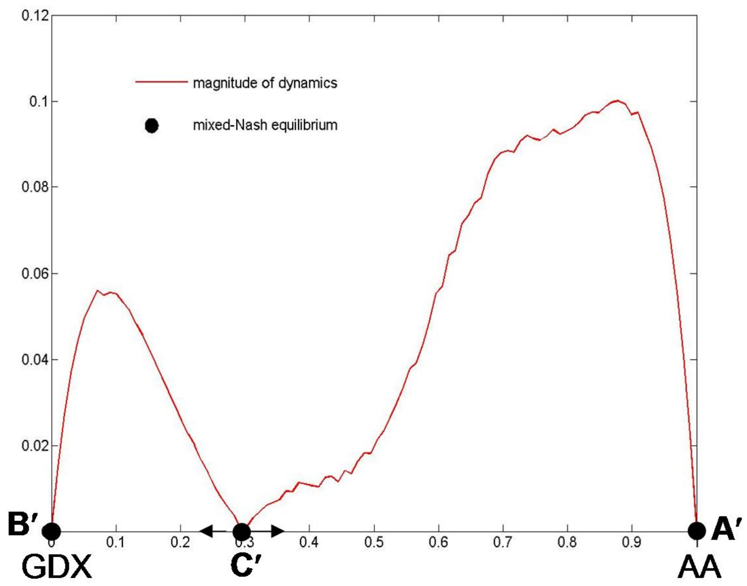


FIGURE 6.3: A one-population EGT analysis. We have two attractors: A′ at 1 and B′ at 0 and a saddle point: C′ at 0.30.

close to 1, i.e. AA) while GDX is more efficient than AA when in the majority (shown by the higher magnitude when close to 0, i.e. GDX). At the saddle point C′, the strategies are equally efficient and, thus, it does not payoff for an agent to deviate to another strategy.

Now, we observe certain similarities with the dynamics of the two-population model. Specifically, we also have two attractors at the pure strategies, AA and GDX, with a probability of 0.78 and 0.12 that they will be adopted respectively. While the one-population model incorrectly predicts the probabilities of the outcomes, it cannot predict that if buyers and sellers can adopt different strategies, there is then a probability of 0.10 that the sellers will adopt AA while buyers will adopt GDX (at the attractor C). It cannot detect this because it assumes that buyers and sellers adopt the same behaviour. Furthermore, the two-population model shows that buyers and sellers do not adopt the same strategies and, thus, this assumption should not be made as it oversimplifies the problem such that the analysis is not significant.

6.3 Summary

This chapter advances the state of the art by developing a novel analytical model of marketplaces with multiple buyers and sellers. Our model removes a key restriction of the previous state of the art, namely that agents adopt the same strategy as a buyer and as a seller. In so doing, our approach provides a better market analysis in terms of how buyers' and sellers' strategic behaviours evolve in the market. We first gave a detailed analysis of such an evolution in the CDA given a particular scenario with AA and GDX strategies. We then gave an analysis of the same scenario using Walsh *et al*.'s one-population model to compare the dynamics. From this, we observed that the latter failed to identify how buyers and sellers can adopt different strategies given that they are more profitable in doing so. While the one-population model does give an approximation, though poor, of how the market behaves, its assumption means that it ignores all cases when it is economically more beneficial to select different strategies as a buyer and a seller. As shown in our two-population model, buyers and sellers do indeed adopt different strategies when they are more profitable in doing so, and such is the case in practice when no such assumption holds. The consequence of this is that the analysis of the one-population model is not significant in practice, a limitation which our new two-population model addresses.

Having outlined the methodology, we will now deploy it, in the next chapter, to evaluate the effectiveness of the AA strategy (and in so doing, we will complete our research aim 3).

Chapter 7

Analysing the Effectiveness of the AA Strategy

In this chapter, we first describe the methodologies for evaluating strategies in homogeneous and heterogeneous populations to meet our last research aim. Then, we use these methodologies to benchmark our novel AA strategy against the state of the art ZIP and GDX strategies. By so doing, we empirically demonstrate that AA is the most efficient bidding strategy for the CDA and, thus, satisfying our third research aim.

7.1 The Experimental Setup

There are three parts to the experimental setup for benchmarking a strategy for the CDA: (i) the market setup, (ii) the agent setup and (iii) the methodology to evaluate the strategy or strategies adopted in the market. First, we describe the market and agent setup, invariant of the strategies adopted and, thus, of the type of population. We then describe how we look at markets with different symmetric and asymmetric demand and supply, and with market shocks. Second, we describe the methodologies for both homogeneous and heterogeneous populations, giving the metrics we use in either case to analyse the performance of a strategy in the market. Furthermore, we also discuss how we ensure the statistical significance of our results.

First, we describe the market setup, that is how we simulate the market and its CDA mechanism. We consider a discrete-time simulator of such a CDA model, and at each time step, an agent is randomly triggered to submit a bid or an ask (between 0 and p_{max}) in the market. In line with previous work, we impose a deadline on the duration of a trading day with the auction closing after 1000 time steps. At the beginning of a trading day, buyers and sellers are endowed with a set of limit prices that correspond to goods to buy and sell respectively. For controlled experiments, we specify the limit prices to induce a desired demand and supply for the market. In particular, limit prices are drawn from uniform distributions U_b and U_s for buyers and sellers respectively. We choose a uniform distribution to obtain an expected linearly decreasing demand curve and an expected linearly increasing supply curve which are commonly used in market models. For the purposes of this thesis, we consider the following different uniform distributions to model representative (symmetric[1] and asymmetric) markets similar to those considered in previous studies (e.g. (Cliff, 2005; Cliff and Bruten, 1997; Tesauro and Das, 2001)):

- Market 1 (M1): $U_b = \mathcal{U}(1.5, 4.5)$[2] and $U_s = \mathcal{U}(1.5, 4.5)$. This is a symmetric market that has an expected equilibrium at 3.0.
- Market 2 (M2): $U_b = \mathcal{U}(1.5, 4.5)$ and $U_s = \mathcal{U}(2.8, 3.2)$. This is an asymmetric market with a flat supply curve. The equilibrium is expected at 3.0.
- Market 3 (M3): $U_b = \mathcal{U}(2.8, 3.2)$ and $U_s = \mathcal{U}(1.5, 4.5)$. This is an asymmetric market with a flat demand curve. The equilibrium is expected at 3.0.
- Market 4 (M4): $U_b = \mathcal{U}(2.5, 5.5)$ and $U_s = \mathcal{U}(2.5, 5.5)$. This is a symmetric market that has an expected equilibrium at 4.0.

By considering limiting cases with a flat demand or supply, we want to see how such extreme asymmetry in Markets M2 and M3, will affect the efficiency of buyer and seller strategies in the CDA. The purpose of Market M4 is to observe how the strategies perform when the competitive equilibrium changes during a market shock.

In particular, we look at different experiments with markets M1, M2, and M3, and market shocks MS14, MS21, MS31 and MS23[3]. In the static environment, we only look at these three markets as we would have the same behaviour in M4 as we would in M1 since there is only an upward shift in the demand and supply and the absolute differences between the agents' preferences remain the same. In the dynamic environment, on the other hand, we are mostly interested in how the strategies adapt from their best behaviour in one market to their best behaviour in the new market. Now, if we have

[1] In a symmetric market, the ratio of gradient of the demand curve and that of the supply curve is -1. M1 is an example of a symmetric market (see Figure 7.1).

[2] $\mathcal{U}(u, v)$ is a uniform distribution between u and v.

[3] MSxy refers to a scenario with a market shock from the demand and supply in Mx to that in My.

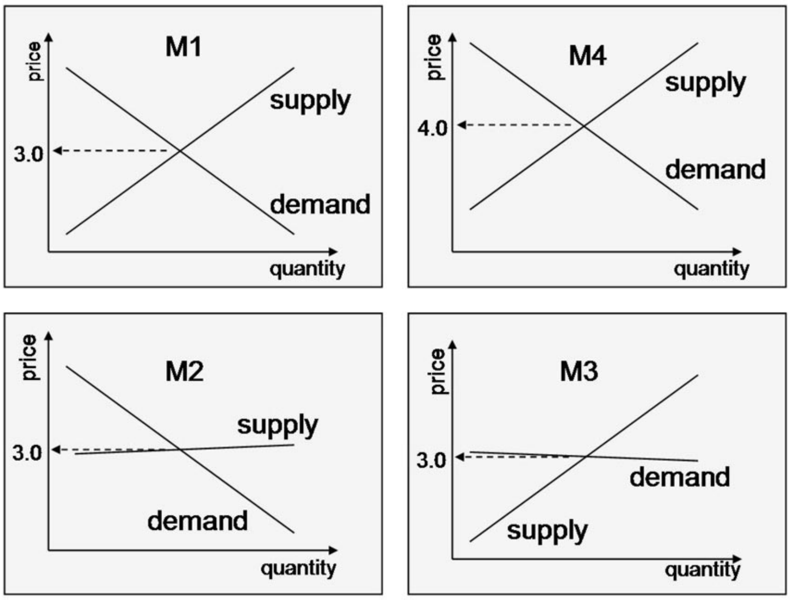


FIGURE 7.1: Demand and supply of Markets M1, M2, M3 and M4.

more than one market shock, as in scenario MS214, we would observe how the strategy adapts from M2 to M1 and finally to M4. Now, we would observe the same behaviour in MS21 and MS14 as the agent's behaviour on Day 20 of MS21 would be the same as on Day 10 in MS14. However, the observations from MS21 and MS12 would be different. Indeed, in the former, we would observe how the agent adapts from the flat supply of M2 to the normal supply of M1, while in the latter, we would observe how it adapts from a normal supply to a flat supply. Because the dynamics is broadly similar in many cases, we only analyse in detail a subset of the single market shocks, and generally look at how the agent adapts from an extreme to a normal demand or supply, or to a change in the competitive equilibrium price. A brief analysis of all of the remaining cases is given in Appendix B.

The market is populated by a set of 10 buyers and 10 sellers in both the homogeneous and heterogeneous scenario. Each agent is endowed with a limit price corresponding to a unit of good to buy or sell. For the static scenario, the CDA lasts 10 days. For the dynamic scenario, the CDA lasts 20 days with the market demand and supply kept constant during the first 10 days and changed thereafter, effectively inducing a market shock on Day 11 (see Figure 7.2 for an example). For the homogeneous scenario, we can avoid redundancy in our experiments when evaluating the strategies within a static environment for markets M1, M2 and M3, by looking at the performance of the strategies before the market shocks for the dynamic cases. Given the market setup, we consider a statistically significant number of runs, namely 2500 runs, of the CDA, each lasting 10 or 20 trading days.

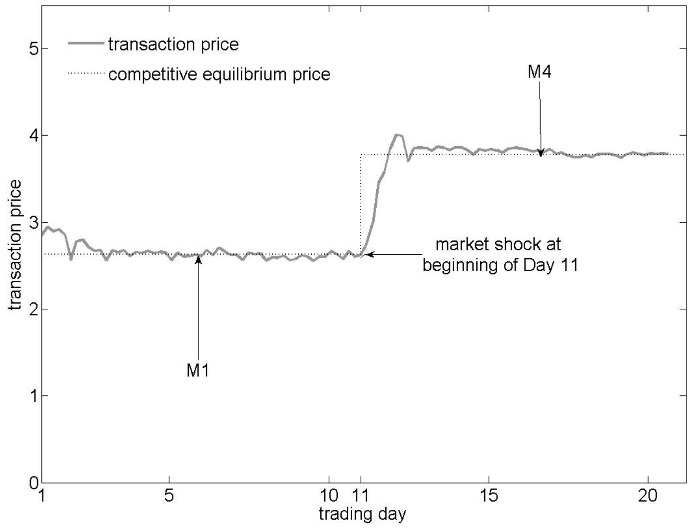


FIGURE 7.2: Example of a transaction history with scenario MS14.

We validate our results at the 95%-confidence-interval by running the non-parametric Wilcoxon rank sum test (Hollander and Wolfe, 1973) on the daily efficiency of the strategies and on the difference between the actual and the expected payoff in heterogeneous populations. We chose such a test because we cannot ensure the normality of our data set and because we want to ensure statistical significance of our dynamic analysis particularly around mixed-Nash equilibria where that difference is significantly smaller. Furthermore, we provide error bars at the 95%-confidence interval in the daily efficiency of strategies within homogeneous populations (as shown in Figure 7.3).

We now look at the agent setup. Each buyer and seller is endowed with a limit price corresponding to a unit of good to buy or sell to induce the market demand and supply in M1, M2, M3 or M4. For the setup of the AA agents, based on simulations, we set the size of window of transactions over which we calculate $\hat{p^*}$, N to 5, the parameter η in the bidding layer (see equations 5.9 and 5.10) to 3.0, λ to 0.05 when increasing the degree of aggressiveness and to -0.05 when decreasing the degree of aggressiveness. The learning rate β_1 and β_2 are drawn from a uniform distribution $\mathcal{U}(0.1, 0.5)$ (similar to the unoptimised parameters of the original ZIP strategy). For GDX agents, γ is set to 0.9 based on Walsh *et al.*'s simulations and, finally, the ZIP agents are initialised with the set of parameters evolved in (Cliff, 2001) (as discussed in 2.4).

Given the experimental setup, we now proceed to evaluating the strategies in homogeneous and heterogeneous populations, in turn.

7.2 Homogeneous Populations

First, we evaluate the performance of the ZIP, GDX and AA strategies in homogeneous populations given the methodology we describe in Subsection 2.3.5.1. In Figures 7.3 to 7.6, we look at the performance of the AA strategy and the benchmarks GDX and ZIP in the different markets highlighted earlier, and Table 7.1 (see Page 127) details the efficiency of the buyers and the sellers, and of the overall efficiency of the strategy in these markets. Note that apart from the symmetric Market M1, buyers and sellers do not expect the same profit due to the asymmetric nature of the demand and supply, and, thus, the efficiency of the strategy is not the average of the buyers' and sellers' efficiency. By dissecting the efficiency of the buyers and the sellers, we can observe whether the buyers or the sellers are performing better given the particular demand and supply. We now consider the static and dynamic scenarios in turn.

7.2.1 The Static Scenario

We first analyse the efficiency of the strategies within a static environment, with markets M1, M2 and M3. In M1 (see Day 1 to 10 in Figure 7.3), we can see that our strategy outperforms both benchmarks on every trading day, with an average efficiency of 0.997. We also note that with AA agents, the transaction prices converge faster (with a lower α) and, on average, remain closer to p^* than with GDX or ZIP agents (AA has the smallest α on Day 10). On the first day, we observe that AA has the highest efficiency because the AA agents assume that there is no information on the first round and adopt a conservative approach (submit bid and ask with a slowing increasing trend) and they have a faster update of their target price. ZIP makes no such assumption and starts with a random profit margin, while GDX suffers from the lack of information (bids, asks and transaction prices). After a few days, the efficiency of all three strategies converges to some value, which is highest with AA agents, and lowest with ZIP. This validates our market setup of 10 days for each market, since we can observe that even if we would consider a larger number of days, the efficiency of the subsequent days would not change. Furthermore, it also validates our analytical method to look at daily efficiency, since we can observe that the efficiency is different on different days for different strategies. Moreover, the daily efficiency converges to different maxima for each strategy, suggesting that the AA strategy is best at learning to be more efficient in the market. With the traditional analytical method (as detailed in Subsection 2.3.5.1), we would only calculate the average efficiency over all the trading days and would not

	AA			**GDX**			**ZIP**		
Scenario	buyer	seller	all	buyer	seller	all	buyer	seller	all
M1	0.969	1.025	0.997	0.981	0.998	0.990	1.010	0.960	0.982
M2	1.212	0.459	0.992	1.145	0.708	0.981	1.143	0.660	0.971
M3	0.389	1.247	0.996	0.595	1.161	0.981	0.896	1.069	0.960
MS14	1.044	0.981	0.992	1.022	0.989	0.988	1.033	0.980	0.979
MS21	1.088	0.754	0.993	1.054	0.876	0.987	1.085	0.817	0.974
MS31	0.667	1.159	0.997	0.778	1.103	0.988	0.932	1.049	0.968
MS23	0.968	1.250	0.994	0.894	1.228	0.980	1.015	0.863	0.968

TABLE 7.1: Efficiency of strategies in homogeneous environments (over all trading days).

observe the fact that efficiency is capped after a few trading days to a maximum, while strategies like AA and GDX learn to be efficient at a much faster rate than ZIP.

In markets M2 and M3 (see days 1 to 10 in Figures 7.4 and 7.5), we also observe that AA is the most efficient (99.7% in M1, 99.1% in M2 and 99.6% in M3). In particular, it does much better than the other strategies in asymmetric markets than it does in the symmetric Market M1 (around 2.1% better in asymmetric cases compared to 1.1% better in the symmetric case – see Table 7.1). This is because the competitive equilibrium price does not change significantly[4] and, thus, the target price remains close to p^* on Day 11. However, in these asymmetric markets, the α-parameter of AA is the highest, while being lowest in the symmetric market. We explain this difference by separately considering the buyers' and the sellers' efficiencies (see Table 7.1). In Market M2 (with a flat supply curve), the fact that the buyers' efficiency is higher than the sellers' means that the transaction prices are, on average, less than p^*, with the buyers having more profitable transactions. This, in turn, indicates that the buyers are more successful at driving the market price (i.e. forcing transaction prices to be lower and be more profitable from their perspective) when the supply is flat. We make similar observations with Market M3 which has a flat demand curve. While α is still highest for AA, the AA sellers' efficiency is higher than the AA buyers', indicating that sellers are driving the market price to be higher than p^*, and are being more profitable from their perspective. As with M1, the daily efficiency converges with all three strategies, with AA still having the highest efficiency on Day 10.

[4]The competitive equilibrium price still changes as we are dealing with uniformly distributed limit prices, and the non-deterministic demand and supply is expected to be as in M1 to M4.

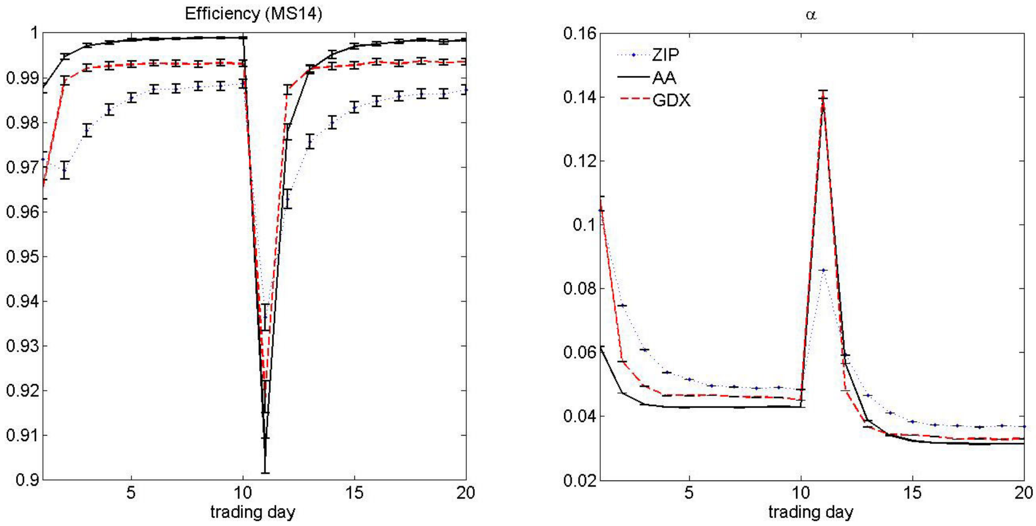


FIGURE 7.3: Scenario MS14. The market efficiency of AA is 0.992, of ZIP, 0.979 and of GDX, 0.988. If we consider the static scenario for Market M1, the market efficiency of AA is 0.997, of ZIP 0.982, and of GDX 0.990.

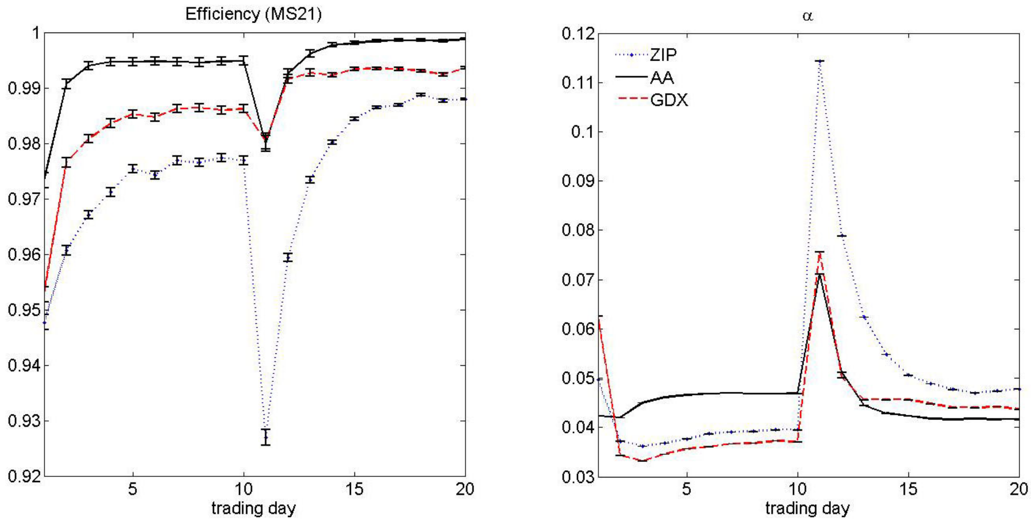


FIGURE 7.4: Scenario MS21. The market efficiency of AA is 0.993, of ZIP 0.974, and of GDX 0.987. If we consider the static scenario for Market M2, the market efficiency of AA is 0.992, of ZIP 0.971, and of GDX, 0.981.

7.2.2 The Dynamic Scenario

We now analyse the daily efficiency of strategies when faced with market shocks. At the beginning of Day 11, the strategies are all tailored to perform best in Day 10. Now, with a market shock, the conditions to which those strategies have adapted are different, forcing those agents to relearn the best strategic behaviour in the market. Essentially, a

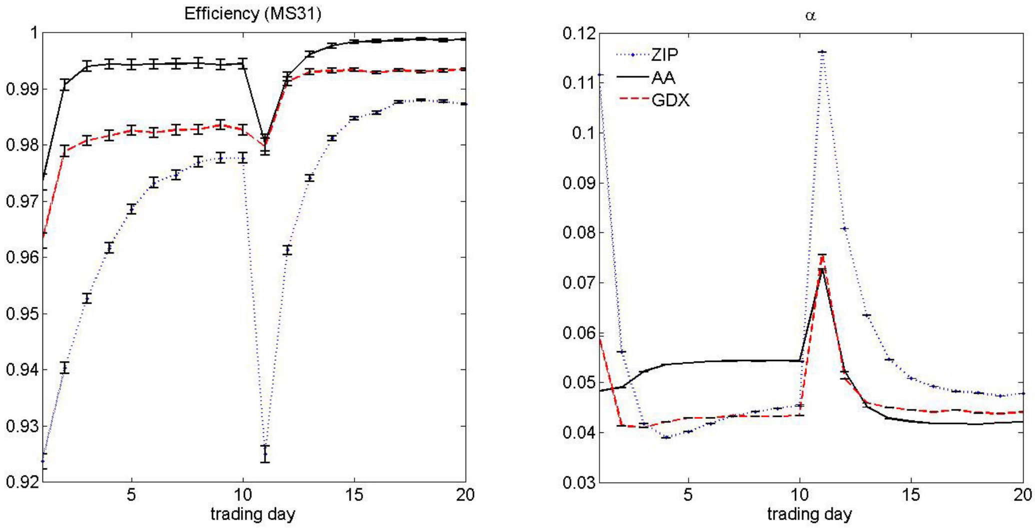


FIGURE 7.5: Scenario MS31. The market efficiency of AA is 0.996, of ZIP 0.968, and of GDX 0.987. If we consider the static scenario for Market M3, the market efficiency of AA is 0.996, of ZIP 0.960, and of GDX 0.981.

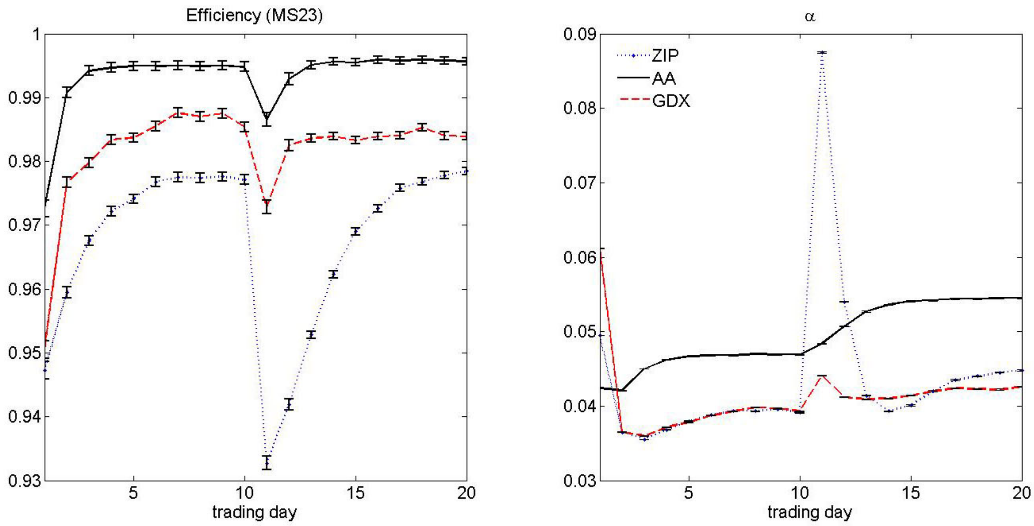


FIGURE 7.6: Scenario MS23. The market efficiency of AA is 0.994, of ZIP 0.968, and of GDX 0.980. As in MS21, when we consider the static scenario in the first 10 days, the market efficiency of AA is 0.992, of ZIP 0.971, and of GDX, 0.981.

robust strategy should be able to rapidly adapt to the new market conditions, since the longer it takes to do this, the more inefficient it is.

In scenario MS14 (see Figure 7.3), the market demand and supply structure remains the same, with an increase in the competitive equilibrium price p^*. In this case, on average, AA still outperforms the benchmarks with an efficiency of 0.992. Because there is a significant shift of p^*, we observe a significant decrease in the efficiency of the AA strategy, as p^* has to be re-estimated gradually (as transaction prices diverge from the old equilibrium and converge to the new one). However, with the higher α, and thus a lower θ, the AA target price changes at a faster rate than it would with a fixed θ (see Subsection 5.2.2), forcing transaction prices to converge at a faster rate to the new p^*. Here, we also observe that the efficiency of the benchmarks, GDX and ZIP on Day 11, is only slightly better than AA, though the latter's efficiency improves after a few trading days to be better than the benchmarks. This can be explained by the fact that $\hat{p}^*$ is a fundamental parameter of the AA strategy, such that a significant change in p^* affects its performance. Furthermore, AA and GDX have the highest α because p^* is central to the AA's aggressiveness model, and because GDX's belief function approximates a step function at p^*. On the other hand, ZIP does not consider p^* explicitly when it forms a bid or an ask. In fact, it only considers its latest profit margin on Day 10 when starting to bid (with a new limit price given the market shock) on Day 11.

In scenario MS21 (see Figure 7.4) where p^* does not change significantly, we initially have a flat supply followed by a symmetric demand and supply. Again, AA performs best with the highest average efficiency and it is the most efficient strategy with the fastest adaptivity to the new market conditions (with the lowest α). Indeed, GDX and AA have the lowest α, which is considerably smaller than in scenario MS14 where p^* changes significantly. ZIP suffers the most from a market shock, with a significant drop in efficiency and slow adaptability. This is because ZIP reuses the same profit margin at the beginning of the following day, and given the significant change in preferences (limit prices) after a market shock, its profit margin is no longer tailored for the new market, and the decrease in efficiency then depends on how different the preferences in the two consecutive markets are. Thus, the decrease is considerable as we are looking at an extreme change for the sellers' preferences. We observe similar behaviour for the three strategies in scenario MS31, with AA outperforming the other strategies.

Furthermore, AA outperformed the benchmarks with the best margin in scenario MS23 (see Figure 7.6), where the market goes from a flat supply to a flat demand. ZIP suffers considerably here as the profit margin, which had been tailored to Market M2, is now used in Market M3 at the beginning of Day 11. With the supply curve now ranging from 1.5 to 4.5 (rather than between 2.8 and 3.2), the same set of sellers' profit margins gives a wider range of asks that are no longer profitable in the market. On the other hand, GDX and AA do not suffer such a drastic change in α as ZIP does. Indeed, we observe that the magnitude of the peak in α on Day 11 for GDX and AA in MS14 is about twice that in MS21 and MS31 where we change either the demand or the supply curve, while there is no peak when we change both the demand and supply curves in MS23. We explain this by considering the limit prices of buyers and sellers. Indeed, in MS14, even though the market demand and supply remain the same, the buyers' and sellers' individual preferences change drastically, with the extreme case where extra-marginal traders become intra-marginal and intra-marginal traders become extra-marginal. In MS21 with a flat supply (MS31 with a flat demand) the change in sellers' (buyers') preferences is not as significant as buyers' (sellers'). Since market behaviour is affected by both buyers' and sellers' behaviours, the change in preferences is then reflected in the change of market efficiency and α. Thus, in MS23 with no extreme changes in demand and supply observed in MS14, in demand in MS21 and in supply in MS31, the drop in efficiency for GDX and AA is even smaller, with no peak in α on Day 11.

7.3 Heterogeneous Populations

Next, we benchmark AA against ZIP and GDX in heterogeneous populations given the methodology we presented in Chapter 6. We consider the static and dynamic scenarios in turn.

7.3.1 The Static Scenario

First, we evaluate the strategies in a static scenario with no market shock, and in turn consider populations with AA against ZIP, and AA against GDX. In Market M1 with AA and ZIP agents, we have a single mixed-Nash equilibrium at AA (see Figure 7.7), implying that all buyers and sellers adopt the *dominant* AA strategy. We also observe that the dynamics have comparable magnitudes and that the magnitude of buyer's dynamics is higher when AA sellers are in the majority, and that the seller's dynamics are higher when AA buyers are in the majority (with higher magnitude here implying faster convergence to AA). Thus, here, AA agents are most efficient when they are in the majority. Similarly, with AA and GDX agents in M1, we have a single *attractor* (mixed-Nash equilibrium towards which trajectories converge) at A and *saddle points* (mixed-Nash equilibrium that trajectories diverge away from) as can be seen in Figure 7.8, with the majority of buyers and sellers eventually adopting the AA strategy (and only 4% of buyers and 21% of sellers adopting GDX). Here, the magnitude of convergence to A is highest when there is a majority of GDX buyers and sellers, implying that AA buyers and sellers are most efficient when they are in the minority. We also observe that buyers and sellers do not necessarily select the same buyer and seller strategy respectively (e.g. when AA buyers and ZIP sellers are in majority, buyers tend to deviate to ZIP, and sellers to AA), which would not have been identified with the traditional one-population model.

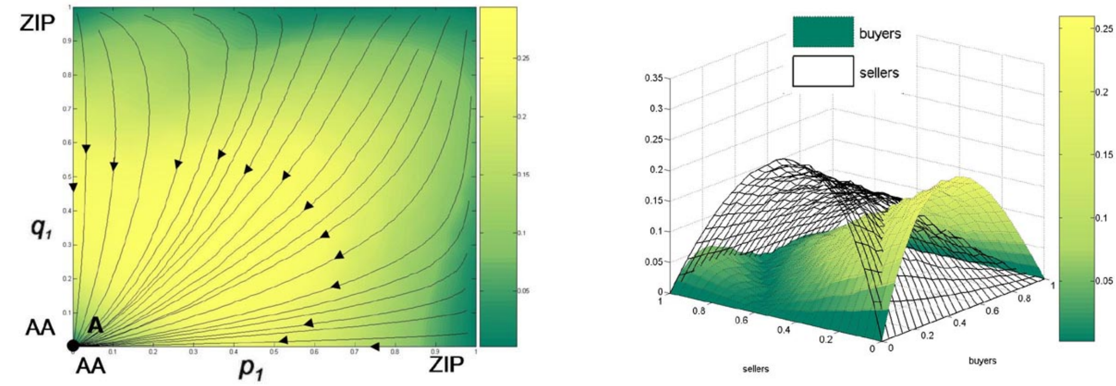


FIGURE 7.7: Scenario M1 with AA and ZIP agents. Here, we have a single dominant strategy at (0,0). The magnitudes of the buyer's and seller's dynamics are of comparable magnitude.

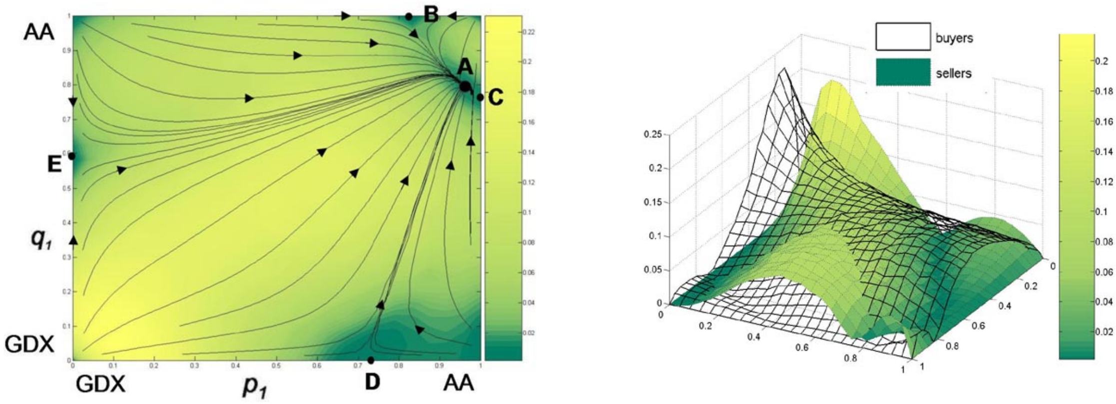


FIGURE 7.8: Scenario M1 with AA and GDX agents. The replicators converge towards the single mixed-Nash equilibrium A at (0.96,0.79). Thus, buyers and sellers are more likely to adopt the AA strategy, with a relatively small proportion adopting the GDX strategy. The magnitudes of the buyer's and seller's dynamics are comparable.

Next, we look at Market M2 with a flat supply. With a population of AA and ZIP (see Figure 7.9), we have a single dominant strategy, A, with all buyers and sellers adopting AA. The obvious observation here is that the magnitude of the seller's dynamics is considerably smaller than that of the buyer's. This suggests that there is more economic incentive for buyers to adopt AA than for sellers to do so. This happens because of the market's flat supply, meaning the sellers have considerably lower expected profits than buyers, and, thus, gain less in profit when deviating to another seller strategy (in contrast

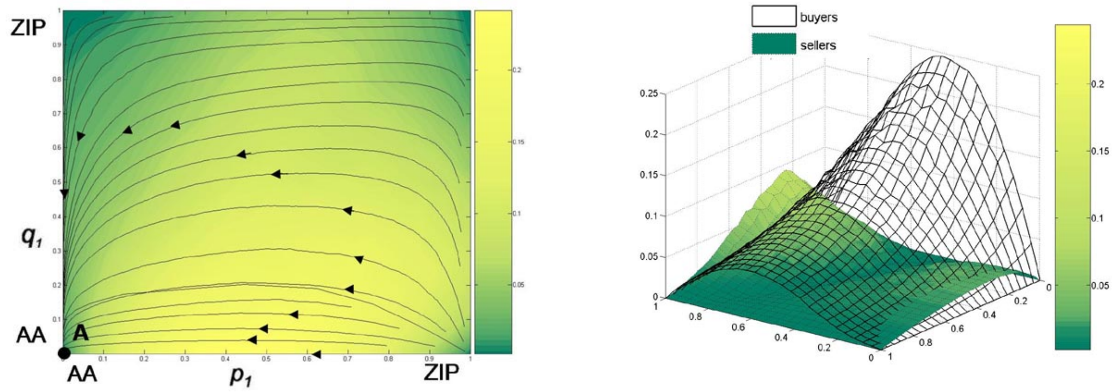


FIGURE 7.9: Scenario M2 with AA and ZIP agents. Here, we have a dominant strategy at (0,0). All buyers and sellers eventually adopt the AA strategy. The magnitude of the seller's dynamics is considerable smaller than that of the buyer's.

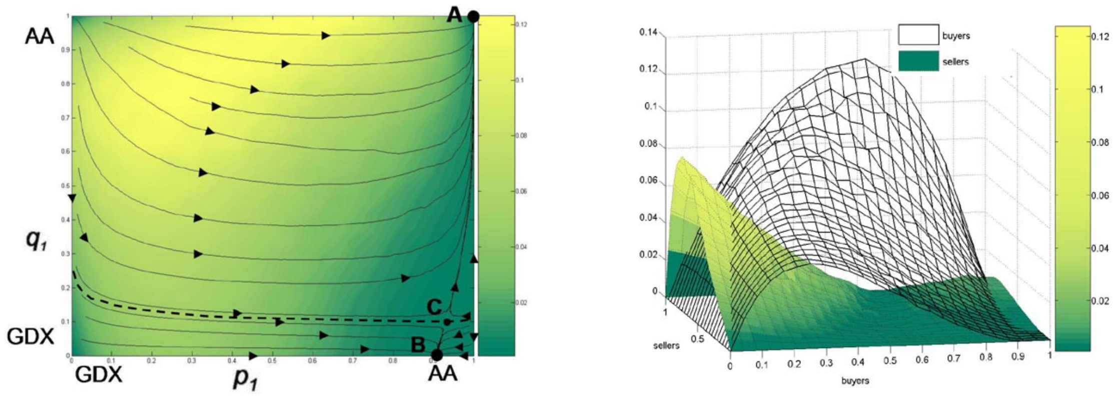


FIGURE 7.10: Scenario M2 with AA and GDX agents. Here, we have two attractors: A at (1,1) and B at (0.92,0), and a saddle point C at (0.93,0.10). The area of the basin of attraction for A is 0.884, and for B is 0.116. The magnitude of the seller's dynamics is higher when GDX buyers are in majority, and considerably lower as AA buyers are represented more.

with the buyer case). Furthermore, we observe that when the majority of buyers adopt ZIP, the sellers tend to adopt ZIP, and when the majority of buyers adopt AA, the sellers tend to adopt AA.

Now, with a population of AA and GDX in M2 (see Figure 7.10), we have two equilibria, A at $(0.92, 0)$ and B at $(1, 1)$. Because the basin of attraction of A is considerably larger than that of B, there is an equally larger probability (0.884 compared to 0.116) that the mixed-Nash strategy A will be adopted (and all agents will eventually select AA). Thus, there is still a small probability of 0.116 that 8.0% of buyers will adopt GDX and all sellers will adopt GDX, such that AA is not dominant. When we consider the magnitude of the dynamics, we observe that the sellers' magnitude is considerably smaller than the buyers', and we explain this with the same intuition as with AA and ZIP in M2. Furthermore, when GDX buyers are in majority, the sellers are more inclined to adopt GDX, and when AA buyers are in majority, sellers tend to adopt AA, though if GDX sellers are in the majority, then sellers are likely to adopt GDX.

Now, because of the reflective nature of M2 and M3, we only report on our analysis of the strategic performance in M2. However, we observe reflective behaviours in M3 (see Appendix B), with the magnitude of the buyers' dynamics being considerably smaller than the sellers' in this case.

7.3.2 The Dynamic Scenario

We now turn to the performance of the strategies in dynamic environments with market shocks. In particular, we look at scenarios MS14 and MS21, and provide some further results for MS31 and MS23 in the appendix (which further validate our claim that AA is better than both ZIP and GDX).

In scenario MS14 with AA and ZIP strategies (see Figure 7.11), we have two attractors at A and B, and a saddle point at C. The basin attraction of A is considerably larger than that of B, with the higher probability of 0.978 that all the buyers and sellers will eventually adopt the AA strategy. As in M1, the magnitude of the buyer's and seller's dynamics is highest when AA agents are in the majority, which again suggests that AA is most efficient when it is in the majority. Furthermore, as in M1, we observe that the magnitude of the buyer's and the seller's dynamics are comparable, and this is because we are still dealing with symmetric markets where buyers and sellers expect similar payoffs. However, unlike in M1, AA is no longer dominant, and there is now a small probability of 0.022 that ZIP will be eventually adopted in the market. Similarly, with AA against GDX in MS14 (see Figure 7.12), we have two attractors at A and B, and a saddle point at C, and the basin of attraction is much larger for attractor A. As with AA against ZIP, the market shock causes AA to no longer be dominant, and there is now a small probability of 0.065 that GDX will be eventually adopted in the market.

In scenario MS21 (see figures 7.13 and 7.14), where the supply changes, we observe a similar set of attractors as in MS14 (but with a probability of 0.961 that AA will be adopted against ZIP, and a probability of 0.869 that it will be adopted against GDX). However, the dynamics of how these equilibria are reached differ, with sellers having a slight tendency to adopt more ZIP or GDX than in MS14 when AA buyers are in the minority. In that case, the magnitude of the seller's replicator dynamics is higher

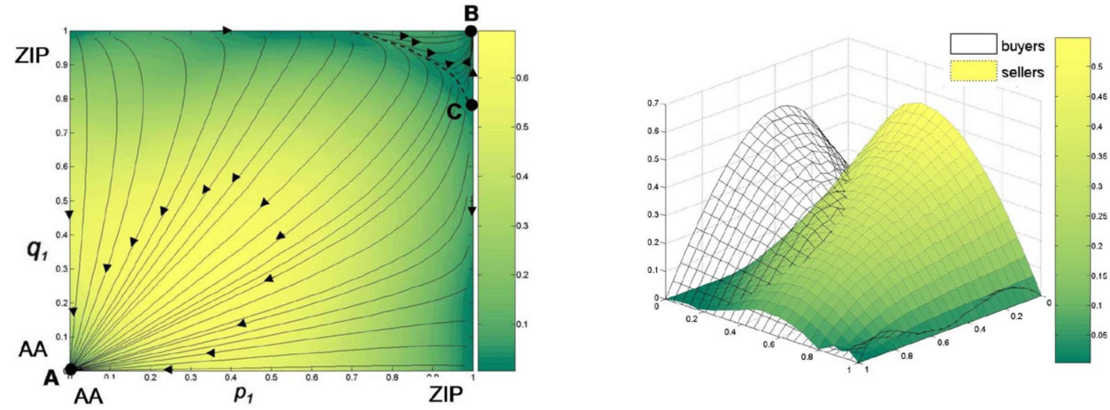


FIGURE 7.11: Scenario MS14 with AA and ZIP agents. Here, we have two attractors: A at (0,0), B at (1,1) and a saddle point, C at (1,0.78). The area of the basin of attraction of A is 0.978, and that of B is 0.022.

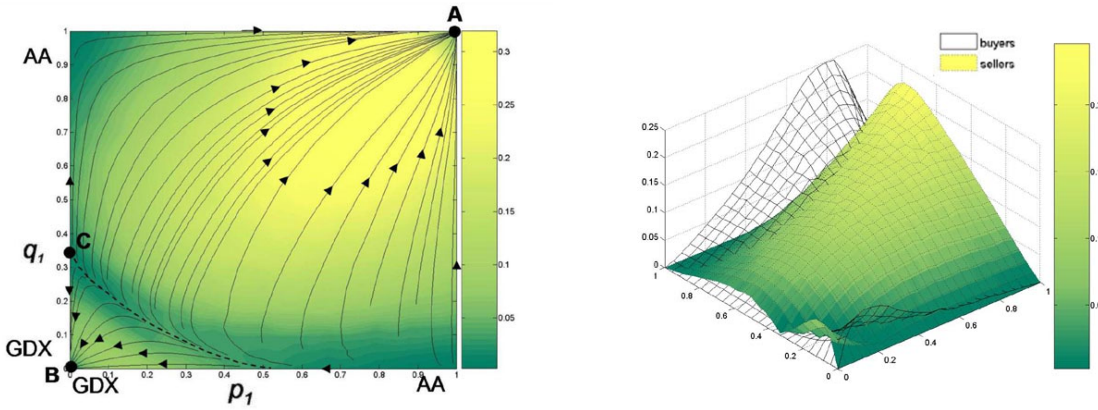


FIGURE 7.12: Scenario MS14 with AA and GDX agents. Here, we have two attractors: A at (1,1), B at (0,0) and a saddle point, C at (0.0.34). The area of the basin of attraction for A is 0.935 and that of B is 0.065.

than that of the buyer's (because of the asymmetric demand and supply, and sellers expect higher profits than buyers) and thus influence more the dynamics of the CDA. As the AA buyer strategy becomes increasingly popular, the buyer's dynamics have increasingly more weight and increasingly influence the dynamics of the market. In some cases (in the basin of attraction of equilibrium B), the change in dynamics is not sufficiently in favour of AA buyers, and the GDX and ZIP buyers then take over.

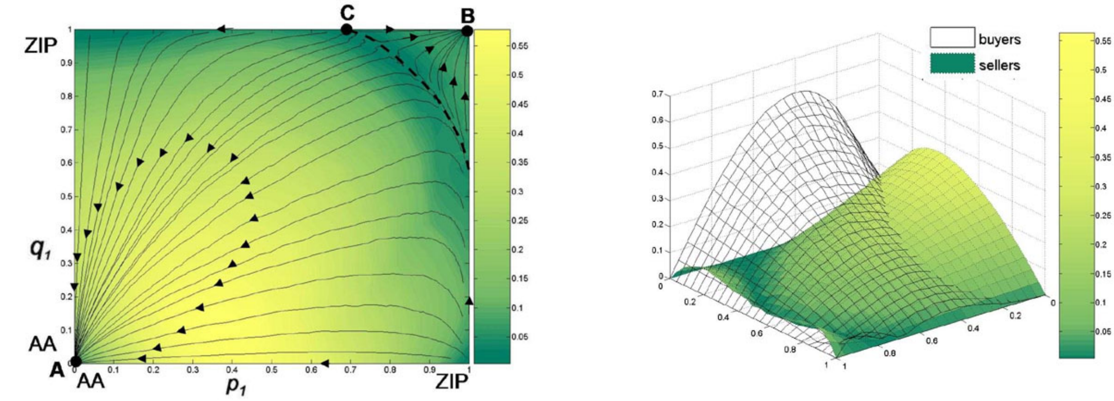


FIGURE 7.13: Scenario MS21 with AA and ZIP agents. Here, we have two attractors: A at (0,0), B at (1,1) and a saddle point, C at (0.68,1). The area of the basin of attraction of A is 0.961, and that of B is 0.039.

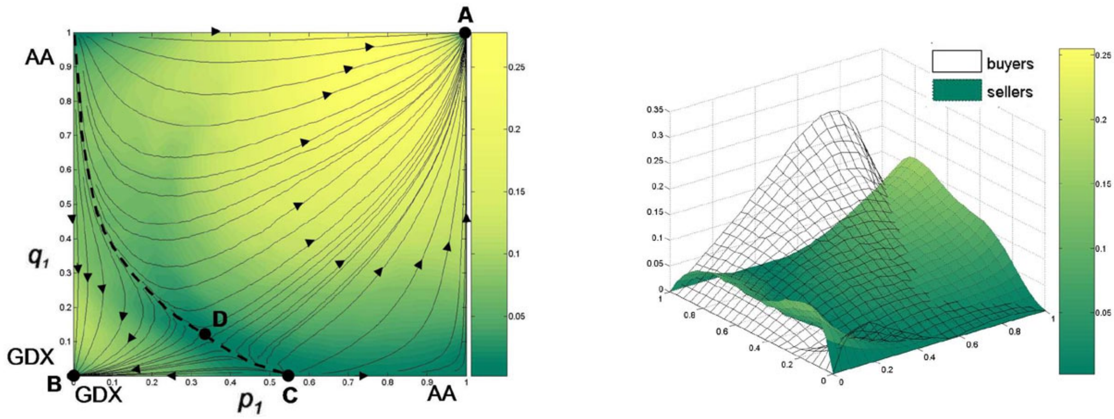


FIGURE 7.14: Scenario MS21 with AA and GDX agents. Here, we have two attractors: A at (1,1) and B (0,0), and two saddle points: C at (0.55,0) and D at (0.33,0.11). The area of the basin of attraction of A is 0.869, and that of B is 0.136.

7.4 Summary

In this chapter, we used our methodologies to benchmark[5] our AA strategy against ZIP and GDX in different environments. In so doing, we empirically demonstrated how AA is the most efficient strategy and outperforms the state of the art in all the cases we consider. Specifically, within homogeneous populations, the AA strategy outperformed the benchmarks, in terms of market efficiency, by up to 3.6% in the static case and 2.8% in the dynamic case. Within heterogeneous populations, based on our evolutionary

[5]Note that we did not benchmark our strategy against FL or Kaplan because we considered strategies that would be efficient in both homogeneous and heterogeneous populations. In the future, it would be interesting to observe whether AA can be exploited by such sniping strategies.

game theoretic analysis, there was a probability above 85% that the AA strategy will eventually be adopted by buyers and sellers in the market. Thus, we addresssed two of our research aims. First, we showed that our new methodologies enabled us to gain insights into behaviour that would not have been observed using the previous methods. Examples of such phenomena include that a strategy might be better at selling than at buying when it is in the majority and that at a mixed-Nash equilibrium, a buyer and a seller adopt different strategies. This addresses our fourth research aim and, more specifically, the scenario with AA and GDX in M2 where the buyer and seller agents adopt different strategies at the mixed-Nash equilbrium. Second, we empirically show that our novel strategy outperforms the state of the art, meeting our third research aim for more efficient strategies.

Chapter 8

Conclusions

This thesis has looked at the Continuous Double Auction, a market mechanism where multiple buyers and sellers compete and where transactions occur whenever a bid and an ask match, on a continuous basis until the market closes. The CDA is today one of the most popular market mechanisms, with applications ranging from market-based control, through decentralised resource allocation, to financial markets. With such valuable applications, there are strong motivations for a better understanding of and improvements in the CDA mechanism. To this end, this thesis has looked at both the structural and the behavioural aspects of the CDA and has made research contributions to both. These contributions are re-capped and matched against our original research aims in Section 8.1. Thereafter, we outline directions for future research in this area in Section 8.2.

8.1 Research Achievements

We began with our work on the structure of the CDA (see Chapter 3). First, we designed a decentralised mechanism based on the CDA, to solve a particular task allocation problem with sellers having a cost structure and buyers having inelastic demand. This addressed our second research aim of showing how more complex resource allocation problems than those encountered in the standard CDA can be addressed. Furthermore, we modified the protocol of the new mechanism to ensure a fair distribution of profits among buyers and sellers, and this was an example of how we can modify the structure of a CDA variant to bring about desirable properties, as motivated by our first research aim. Second, we demonstrated that the structure of our new mechanism was very efficient (on average 80%, but reaching up to 90% in some cases). To do so, we first developed a centralised mechanism with an optimal solution of the resource allocation problem. The purpose of this centralised solution was to evaluate the decentralised mechanism with respect to an optimal solution and analyse the trade-off in efficiency when decentralising the resource allocation task in the system. Furthermore, we developed a zero-intelligence (ZI2) strategy. The purpose of such a simple behaviour was to ensure that the efficiency of the mechanism was attributable to the structure rather than the behaviour. Such a decentralised solution would generally be adopted, instead of the centralised and optimal solution, when the desirable properties of a decentralised mechanism are required and if the trade-off (of an average of 15%), in terms of efficiency, is below a threshold set by the requirements of the system.

Next, we considered the behaviour of the CDA. As stated previously, this is emergent and depends on the strategic interactions of buyer and sellers in the market and, thus, on the strategies they adopt. Given this, the behavioural aspect of research on the CDA is mainly concerned with bidding strategies and, in particular, with their design and evaluation. We first considered the design of strategies to meet our third research aim. We began by developing a multi-layered framework (IKB) for designing strategies for agents operating in market mechanisms, to assist strategy designers through the systematic design of strategies for the CDA and its variants (see Chapter 4). Indeed, we successfully used this framework to engineer the design of our entry to the 2006 Trading Agent Competition (where it came third) and our adaptive-aggressiveness (AA) bidding strategy for the CDA (see chapter 5). The AA strategy is based on both short-term and long-term learning that allows the agent to adapt its bidding behaviour to be efficient in a wide variety of environments. The principal motivation for the short-term learning is to update the agent's aggressiveness to immediately respond to short-term market fluctuations, while for the long-term learning it is adapt to long-term changes in market conditions and to enable the agent to perform efficiently in dynamic environments in which the market demand and supply changes suddenly.

We then went on to consider the means of evaluating bidding strategies in both homogeneous and heterogeneous populations in line with our fourth research aim. Because an EGT approach is best for the latter, we first addressed the issues with the existing state of the art and developed a novel EGT framework to analyse the evolution of both buyers' and sellers' behaviours in the market. This work is reported in Chapter 6. Such a framework allows a strategy designer to evaluate the efficiency of his strategy while, for the system designer, it gives insights into how the behaviour of the system is changing and what it is most likely to evolve into.

Finally, in Chapter 7, we employ the newly developed methodologies to benchmark AA against the state of the art CDA strategies in different static and dynamic environments, in both homogeneous and heterogeneous populations. In so doing, we empirically demonstrate how AA is the most efficient strategy and outperforms the state of the art in all the cases we consider. This thus satisfies our research aim for a more efficient strategy for the CDA. Specifically, within homogeneous populations, the AA strategy outperformed the benchmarks, in terms of market efficiency, by up to 3.6% in the static case and 2.8% in the dynamic case. Within heterogeneous populations, based on our evolutionary game theoretic analysis, there was a probability above 85% that the AA strategy will eventually be adopted by buyers and sellers in the market. Now, a more efficient strategy implies more efficient resource allocation systems for homogeneous populations, as well as more economic benefits in heterogeneous populations. Furthermore, such benefits would incentivise human traders to purchase such a strategy that their agents could use.

When taken together, these contributions are an important step towards improving the structure and behaviour of the CDA, and to demonstrating that the CDA can be a valuable tool for solving decentralised control problems. In making these advances, we have successfully addressed the research aims set out at the beginning of this thesis. However, this work has also opened up new research avenues that require future work and, in the next section, we outline these avenues and highlight potential points of departure.

8.2 Future Work

Research on sophisticated bargaining behaviour for autonomous software agents participating in complex marketplaces, such as the CDA, is only now starting to gather pace. Moreover, because the CDA is such an important tool for decentralised resource allocation and because there is no optimal analytical solution, further research is still needed on both the structural and behavioural aspects. Thus, while this thesis addresses some of the key issues concerning this mechanism, there are still other areas that require subsequent work. We now highlight some of the most important of these:

- In this thesis, we developed a model to analyse the evolution of buyers' and sellers' behaviour in the market and demonstrated its effectiveness. However, we believe that this model can be further extended to also analyse the evolution of the structure of the CDA, if that structure were allowed to change to improve some property of the mechanism (e.g. market efficiency, price volatility or fairness of profit distribution). Thus, in our model, we would have two populations corresponding to buyers and sellers, and a third population of mechanisms with different protocols. The aim of this extension would be to observe the co-evolution of the structure and behaviour of the CDA, and identify how the CDA would evolve under certain circumstances.

- Because the CDA can essentially be considered as an approach to decentralised control, we intend to develop other CDA variants to solve other complex decentralised resource allocation problems such as with non-linear production functions or where consumers can form coalitions and bid as a group. Furthermore, we intend to modify the standard CDA model set by Smith and rigorously observed for the past few decades. In particular, we believe that the CDA should be modelled to more closely reflect real financial markets and, in particular, the stock market. One intrinsic difference between Smith's model of the CDA and the stock market is that in the former, a market shock occurs over trading days, while in the latter, a market shock occurs within trading hours during a trading day. Thus, we intend to develop a CDA model where the market is perturbed by sporadic shocks over trading hours and analyse the implication of such changes on properties of the CDA such as market efficiency, price volatility or convergence towards the competitive market equilibrium.

- Given our new model of the CDA, we intend to change (the behaviour of) our AA strategy for the new structure of the CDA and, thereon, *upgrade* our AA strategy for real financial markets such as the stock markets, where considerably more information has to be factored in (e.g. short-term and long-term history of transaction prices, volume of transactions and market trends).

With the growing popularity of the CDA and its applications, we envisage considerably more research on this mechanism. Given this, we believe this thesis highlights the different areas of research and applications of the CDA and advances the state of the art in both its structure and behaviour. Furthermore, the CDA is shown to be not just another auction mechanism, but rather one that can be built upon and modified to solve complex decentralised problems, and one that has emerged as the dominant mechanism in trillion-dollar financial institutions.

Appendix A

Examplar Strategy Profile for EGT Analysis

A strategy profile $[\rho^b, \rho^s]$ defines the number of buyers $\rho^b = (\rho^b_1, \ldots, \rho^b_{S_b})$ and sellers $\rho^s = (\rho^s_1, \ldots, \rho^s_{S_s})$ using the different buyer and seller strategies respectively. Here, we give an example of a heuristic payoff table with such profiles and, specifically, we consider the scenario used in our EGT analysis of Chapter 6. In this case, we have a market with 10 buyers and 10 sellers each having a set of two different strategies, namely AA and GDX in a dynamic scenario MS31 (see Section 7.1 for more details). Thus, we have a payoff table with 121 strategy profiles. Here, AA is strategy 1 and GDX strategy 2 and for each strategy profile, we give the payoff of buyers and sellers using the different strategies, as well as the efficiency of each strategy. Note that the first and last entries are homogeneous cases for profile (0,10,0,10) with all GDX agents and (10,0,10,0) with all AA agents respectively.

expt	buyer strategy 1 (B_1)	buyer strategy 2 (B_2)	seller strategy 1 (S_1)	seller strategy 2 (S_2)	strategy 1 efficiency	total	payoff B_1	payoff S_1	strategy 2 efficiency	total	payoff B_2	payoff S_2
0	0	10	0	10	0	0	0	0	0.9879	112.06	40.59	71.47
1	0	10	1	9	0.6302	5.62	0	5.62	0.9819	104.72	39.27	65.45
2	0	10	2	8	0.9032	13.5	0	13.5	0.9725	96.95	38.08	58.87
3	0	10	3	7	1.0348	21.65	0	21.65	0.9589	89.27	36.8	52.47
4	0	10	4	6	1.1097	30.24	0	30.24	0.9398	81.16	35.63	45.53
5	0	10	5	5	1.1505	38.54	0	38.54	0.9153	73.14	34.31	38.82
6	0	10	6	4	1.1767	47.07	0	47.07	0.8824	64.7	33.01	31.68
7	0	10	7	3	1.2005	55.97	0	55.97	0.8375	55.81	31.59	24.22
8	0	10	8	2	1.2251	65.05	0	65.05	0.7797	46.72	30.17	16.54
9	0	10	9	1	1.2467	74.5	0	74.5	0.7021	37.19	28.79	8.4
10	0	10	10	0	1.2649	84.07	0	84.07	0.5989	27.43	27.43	0
11	1	9	0	10	0.3676	2.31	2.31	0	0.9914	108.04	37.68	70.35
12	1	9	1	9	0.734	9.43	3.02	6.4	0.987	100.87	36.45	64.41

13	1	9	2	8	0.9085	17.48	3.34	14.14	0.9794	93.28	35.14	58.14
14	1	9	3	7	1.0071	25.52	3.45	22.06	0.9679	85.79	34.02	51.77
15	1	9	4	6	1.0653	33.82	3.44	30.38	0.9506	77.86	32.74	45.11
16	1	9	5	5	1.0998	41.84	3.33	38.51	0.927	69.97	31.42	38.54
17	1	9	6	4	1.1279	50.26	3.22	47.03	0.8948	61.66	30.16	31.49
18	1	9	7	3	1.1524	58.94	3.08	55.86	0.8501	52.92	28.84	24.07
19	1	9	8	2	1.1782	67.9	2.94	64.96	0.7905	43.92	27.44	16.47
20	1	9	9	1	1.2021	77.26	2.78	74.48	0.7079	34.44	26.05	8.39
21	1	9	10	0	1.2227	86.78	2.64	84.14	0.5948	24.7	24.7	0
22	2	8	0	10	0.6447	6.55	6.55	0	0.993	103.58	34.57	69.01
23	2	8	1	9	0.8178	13.88	7.12	6.75	0.9909	96.66	33.42	63.23
24	2	8	2	8	0.9224	21.74	7.32	14.42	0.9859	89.31	32.16	57.14
25	2	8	3	7	0.9873	29.51	7.37	22.13	0.9767	82.03	31.05	50.97
26	2	8	4	6	1.0302	37.51	7.22	30.28	0.9619	74.31	29.84	44.46
27	2	8	5	5	1.0603	45.33	6.98	38.35	0.9408	66.63	28.53	38.1
28	2	8	6	4	1.087	53.54	6.71	46.82	0.9089	58.45	27.27	31.17
29	2	8	7	3	1.1126	62.11	6.41	55.69	0.8633	49.8	25.88	23.91
30	2	8	8	2	1.1376	70.9	6.1	64.79	0.8033	40.97	24.58	16.39
31	2	8	9	1	1.1611	80.14	5.77	74.37	0.715	31.55	23.2	8.34
32	2	8	10	0	1.1835	89.59	5.46	84.13	0.5908	21.9	21.9	0
33	3	7	0	10	0.7961	11.36	11.36	0	0.9926	99.05	31.46	67.59
34	3	7	1	9	0.8773	18.46	11.59	6.86	0.9927	92.34	30.46	61.87
35	3	7	2	8	0.9316	26.02	11.61	14.4	0.9902	85.22	29.35	55.86
36	3	7	3	7	0.9768	33.54	11.52	22.01	0.9842	78.22	28.31	49.9
37	3	7	4	6	1.009	41.28	11.19	30.08	0.972	70.71	27.05	43.66
38	3	7	5	5	1.0306	48.77	10.77	37.99	0.9539	63.29	25.86	37.43
39	3	7	6	4	1.055	56.79	10.29	46.5	0.925	55.34	24.58	30.76
40	3	7	7	3	1.0783	65.13	9.79	55.34	0.8796	46.86	23.22	23.64
41	3	7	8	2	1.1034	73.79	9.26	64.53	0.8182	38.13	21.88	16.24
42	3	7	9	1	1.1255	82.86	8.73	74.12	0.7257	28.86	20.55	8.3
43	3	7	10	0	1.1491	92.24	8.21	84.03	0.5867	19.25	19.25	0
44	4	6	0	10	0.8928	16.62	16.62	0	0.9877	93.88	28.15	65.72
45	4	6	1	9	0.9239	23.49	16.6	6.89	0.9905	87.45	27.12	60.33
46	4	6	2	8	0.949	30.73	16.43	14.29	0.9916	80.65	26.13	54.52
47	4	6	3	7	0.9732	37.86	16.13	21.73	0.9879	73.85	25.07	48.78
48	4	6	4	6	0.9945	45.33	15.69	29.63	0.9792	66.66	24.03	42.62
49	4	6	5	5	1.0102	52.57	15.08	37.48	0.9648	59.5	22.88	36.62
50	4	6	6	4	1.0304	60.36	14.4	45.96	0.9391	51.81	21.63	30.17
51	4	6	7	3	1.0503	68.47	13.64	54.82	0.8961	43.6	20.33	23.26
52	4	6	8	2	1.0728	76.93	12.84	64.08	0.8332	35.05	19.02	16.03
53	4	6	9	1	1.0947	85.83	12.07	73.75	0.735	25.96	17.76	8.2
54	4	6	10	0	1.1183	95.09	11.32	83.77	0.5781	16.45	16.45	0
55	5	5	0	10	0.9606	21.9	21.9	0	0.9785	88.47	24.7	63.76
56	5	5	1	9	0.9586	28.49	21.73	6.75	0.984	82.33	23.82	58.51
57	5	5	2	8	0.9658	35.45	21.42	14.02	0.9878	75.79	22.89	52.89
58	5	5	3	7	0.9748	42.28	21	21.28	0.9875	69.29	22.02	47.27
59	5	5	4	6	0.9873	49.46	20.36	29.1	0.9832	62.44	20.94	41.49
60	5	5	5	5	0.9975	56.43	19.6	36.82	0.9733	55.6	19.9	35.69
61	5	5	6	4	1.0116	63.9	18.69	45.2	0.951	48.18	18.75	29.43
62	5	5	7	3	1.0283	71.77	17.68	54.08	0.9123	40.3	17.54	22.75
63	5	5	8	2	1.0482	80	16.64	63.35	0.8487	31.94	16.26	15.67
64	5	5	9	1	1.0685	88.72	15.57	73.14	0.7452	23.06	15.02	8.04

65	5	5	10	0	1.0906	97.79	14.48	83.3	0.5653	13.72	13.72	0
66	6	4	0	10	1.0256	27.93	27.93	0	0.9607	82.26	21.02	61.24
67	6	4	1	9	0.9981	34.12	27.56	6.55	0.9692	76.45	20.29	56.15
68	6	4	2	8	0.9866	40.68	27.09	13.59	0.9764	70.24	19.54	50.7
69	6	4	3	7	0.9848	47.21	26.54	20.66	0.9801	64.08	18.7	45.38
70	6	4	4	6	0.9867	53.99	25.73	28.26	0.9807	57.6	17.72	39.88
71	6	4	5	5	0.9909	60.68	24.75	35.92	0.9772	51.17	16.76	34.4
72	6	4	6	4	1.002	50.36	23.54	26.81	0.9199	30.04	14.85	15.19
73	6	4	7	3	1.0127	75.45	22.37	53.07	0.9265	36.6	14.57	22.03
74	6	4	8	2	1.0276	83.33	20.96	62.36	0.8613	28.51	13.3	15.2
75	6	4	9	1	1.0454	91.78	19.47	72.3	0.75	19.87	12.06	7.8
76	6	4	10	0	1.0653	100.59	18.03	82.56	0.5423	10.83	10.83	0
77	7	3	0	10	1.0907	34.61	34.61	0	0.9304	75.2	17	58.19
78	7	3	1	9	1.0466	40.45	34.18	6.26	0.941	69.71	16.45	53.26
79	7	3	2	8	1.0186	46.56	33.53	13.02	0.9519	63.93	15.79	48.13
80	7	3	3	7	1.0025	52.58	32.72	19.86	0.9607	58.25	15.07	43.18
81	7	3	4	6	0.995	58.99	31.76	27.22	0.9668	52.22	14.27	37.94
82	7	3	5	5	0.9917	65.29	30.62	34.67	0.9696	46.22	13.44	32.77
83	7	3	6	4	0.9951	72.13	29.37	42.76	0.9605	39.64	12.55	27.08
84	7	3	7	3	1.0017	79.33	27.73	51.6	0.9301	32.5	11.47	21.03
85	7	3	8	2	1.0125	86.86	25.93	60.92	0.8691	24.84	10.3	14.54
86	7	3	9	1	1.0262	94.95	23.93	71.01	0.745	16.56	9.1	7.45
87	7	3	10	0	1.0433	103.48	21.89	81.58	0.487	7.79	7.79	0
88	8	2	0	10	1.1614	42.16	42.16	0	0.8834	67.13	12.33	54.79
89	8	2	1	9	1.1011	47.55	41.63	5.92	0.8953	62	11.98	50.02
90	8	2	2	8	1.0585	53.19	40.85	12.34	0.9091	56.66	11.51	45.14
91	8	2	3	7	1.0298	58.75	39.97	18.77	0.9203	51.36	11.02	40.34
92	8	2	4	6	1.0109	64.65	38.84	25.8	0.9318	45.84	10.39	35.44
93	8	2	5	5	0.9993	70.47	37.5	32.96	0.9421	40.4	9.75	30.65
94	8	2	6	4	0.9945	76.78	35.91	40.87	0.9424	34.45	9.02	25.42
95	8	2	7	3	0.9956	83.55	34	49.55	0.9203	27.86	8.15	19.7
96	8	2	8	2	1.0011	90.65	31.69	58.95	0.8601	20.74	7.14	13.59
97	8	2	9	1	1.0107	98.36	29.12	69.24	0.7162	12.88	5.98	6.89
98	8	2	10	0	1.0238	106.48	26.43	80.04	0.3847	4.67	4.67	0
99	9	1	0	10	1.2423	50.87	50.87	0	0.8153	57.96	6.58	51.37
100	9	1	1	9	1.1689	55.87	50.33	5.54	0.8248	53.09	6.42	46.67
101	9	1	2	8	1.1144	61.12	49.58	11.53	0.8367	48.06	6.22	41.84
102	9	1	3	7	1.0723	66.08	48.5	17.57	0.851	43.33	5.94	37.39
103	9	1	4	6	1.0412	71.38	47.27	24.11	0.8647	38.31	5.65	32.65
104	9	1	5	5	1.0201	76.68	45.88	30.79	0.8793	33.41	5.33	28.08
105	9	1	6	4	1.0053	82.32	44.01	38.3	0.8865	28.1	4.9	23.19
106	9	1	7	3	0.9976	88.43	41.69	46.74	0.8713	22.22	4.33	17.88
107	9	1	8	2	0.9962	94.97	39	55.96	0.8142	15.82	3.64	12.17
108	9	1	9	1	1	102.11	35.71	66.39	0.6449	8.76	2.73	6.02
109	9	1	10	0	1.0077	109.69	32	77.68	0.2135	1.64	1.64	0
110	10	0	0	10	1.3326	60.41	60.41	0	0.7217	47.97	0	47.97
111	10	0	1	9	1.2506	65.22	60.06	5.16	0.7245	43.27	0	43.27
112	10	0	2	8	1.1827	70.04	59.31	10.72	0.7311	38.57	0	38.57
113	10	0	3	7	1.1321	74.7	58.47	16.22	0.7341	34.01	0	34.01
114	10	0	4	6	1.0884	79.4	57.18	22.21	0.7432	29.53	0	29.53
115	10	0	5	5	1.0566	84.06	55.83	28.22	0.7502	25.14	0	25.14
116	10	0	6	4	1.03	88.9	53.77	35.12	0.754	20.48	0	20.48
117	10	0	7	3	1.0117	94.19	51.26	42.93	0.7246	15.37	0	15.37
118	10	0	8	2	0.9993	99.76	48.05	51.7	0.6634	10.14	0	10.14
119	10	0	9	1	0.9937	105.99	43.91	62.08	0.4802	4.72	0	4.72
120	10	0	10	0	0.9947	112.83	39.02	73.81	0	0	0	0

Appendix B

Evaluating Strategies within Heterogeneous Populations

In Chapter 7, we benchmarked the AA strategy against the state of the art ZIP and GDX strategies for different scenarios. Here, we provide an analysis of AA against ZIP and GDX, but also of ZIP against GDX, in the remaining cases that have not been considered in the main thesis. For each case, we give the different attractors and saddle points and the probability that each of these attractors will be adopted.

We observe that AA always outperforms ZIP and GDX in line with our observations in the main thesis, while GDX always outperforms ZIP.

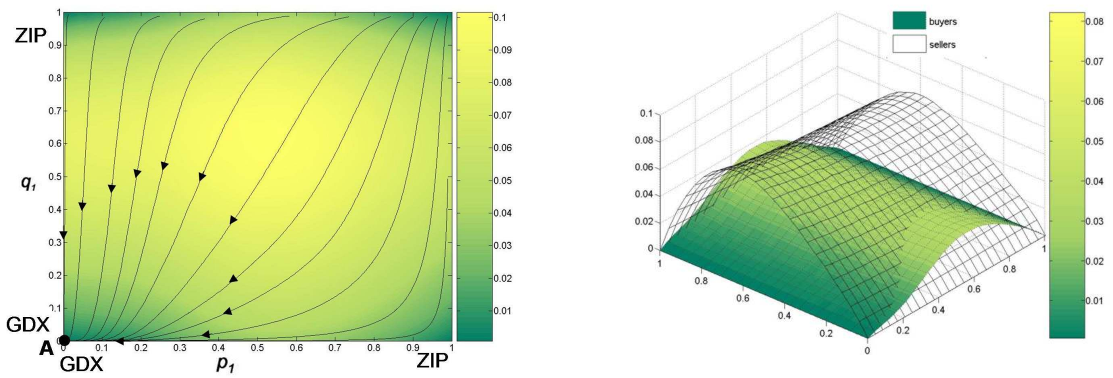


FIGURE B.1: Scenario M1 with ZIP and GDX agents given a symmetric demand and supply. The replicators converge towards the single mixed-Nash equilibrium A at (0,0). The magnitudes of the buyer's and seller's dynamics are comparable.

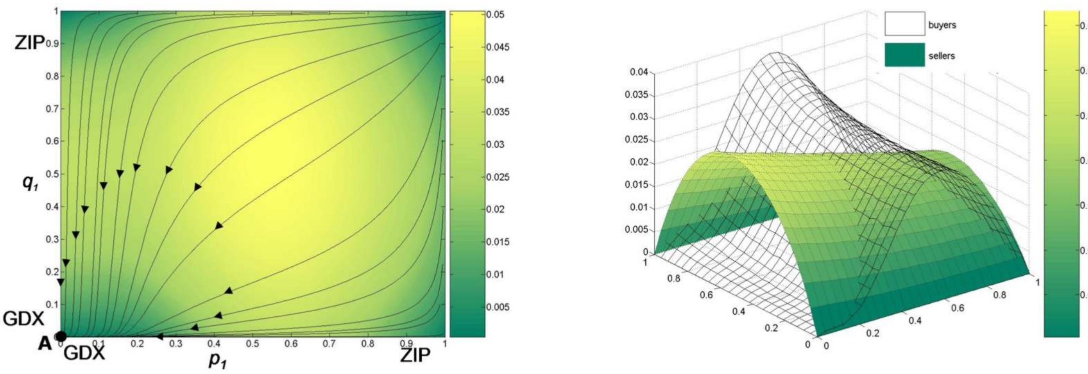


FIGURE B.2: Scenario M2 with ZIP and GDX agents given a flat supply. Here, we have a single attractor: A at (0,0). All buyers and sellers eventually adopt the GDX strategy. The magnitude of the seller's dynamics is smaller than that of the buyer's, though the different when compared to M1 (where the magnitude of buyer's dynamics is smaller than that of seller's) is fairly significant.

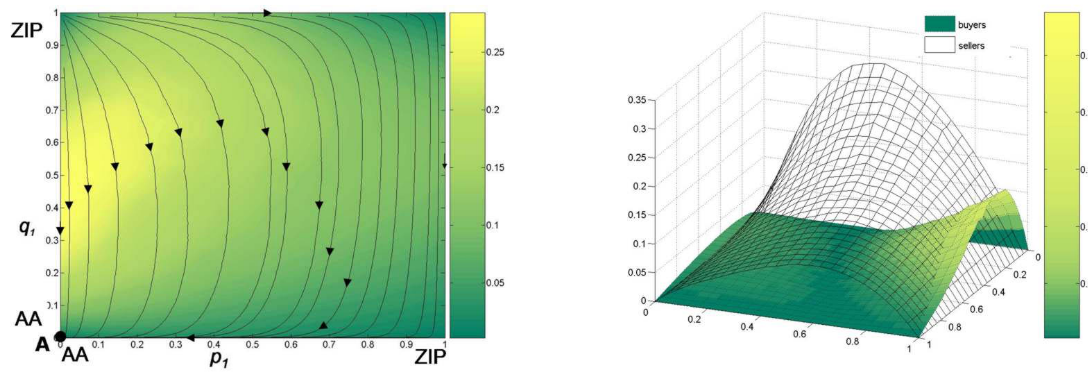


FIGURE B.3: Scenario M3 with AA and ZIP agents. Here, we have one attractor: A at (0,0). AA is a dominant strategy that will eventually be adopted in the market.

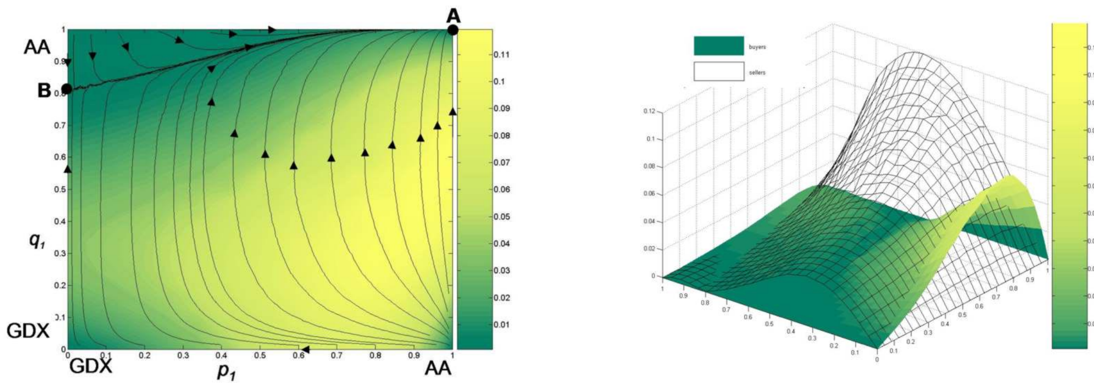


FIGURE B.4: Scenario M3 with AA and GDX agents. Here, we have one attractor: A at (1,1) and one saddle point: B at (0,0.81). AA is a dominant strategy that will eventually be adopted in the market.

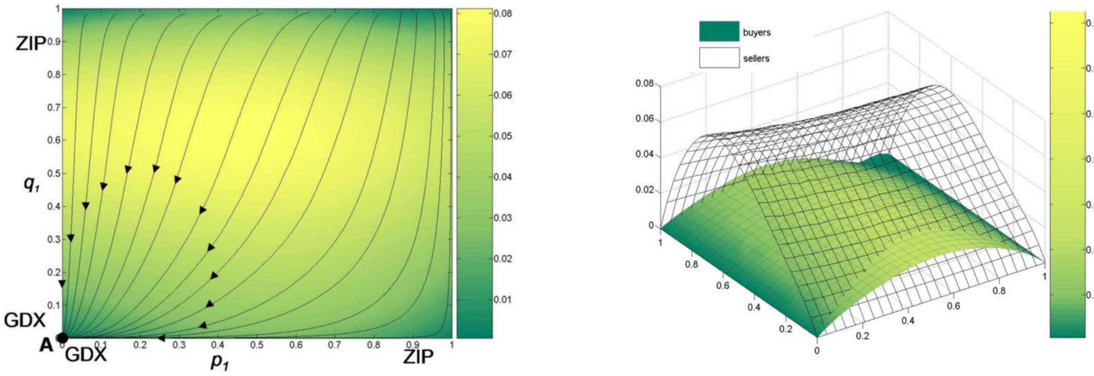


FIGURE B.5: Scenario M3 with ZIP and GDX agents given a flat demand. Here, we have a dominant strategy A at (0,0). The magnitude of the seller's dynamics is considerable larger than that of the buyer's because of the flat demand.

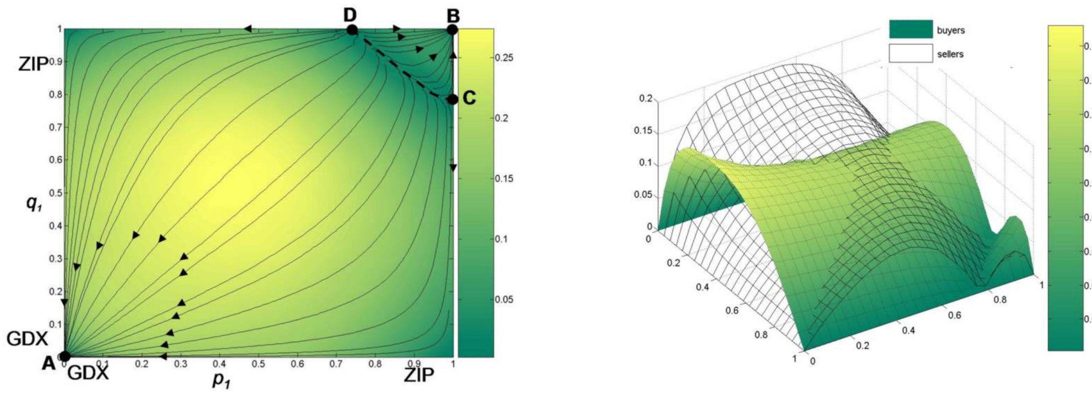


FIGURE B.6: Scenario MS14 with ZIP and GDX agents. Here, we have two attractors: A at (0,0), B at (1,1) and two saddle points: C at (1,0.78) and D at (0.71,1). The area of the basin of attraction for A is 0.967 and that of B is 0.033.

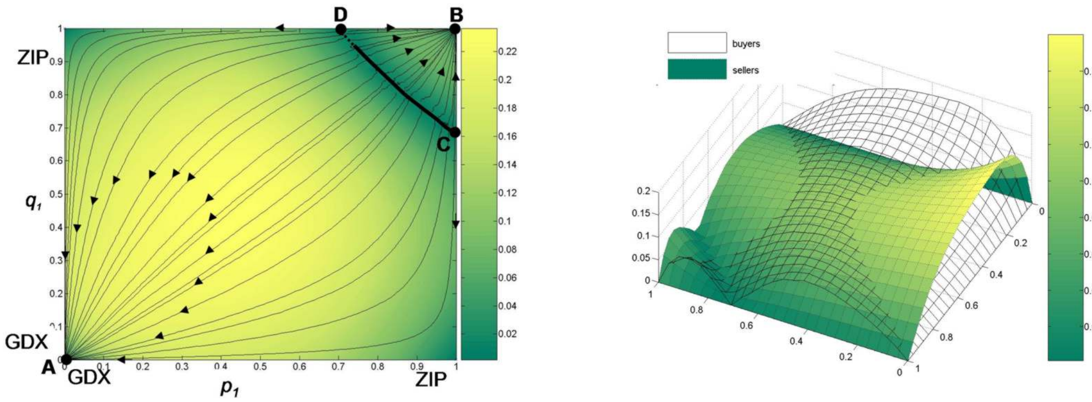


FIGURE B.7: Scenario MS21 with ZIP and GDX agents. Here, we have two attractors: A at (0,0) and B (1,1), and saddle points: C at (1,0.69), D at (0.70,1) and E which is a continuous *line* of equilibria represented by the dark line. The dotted line is the boundary between the basins of attraction of A and B. The area of the basin of A is 0.948, and that of B is 0.052.

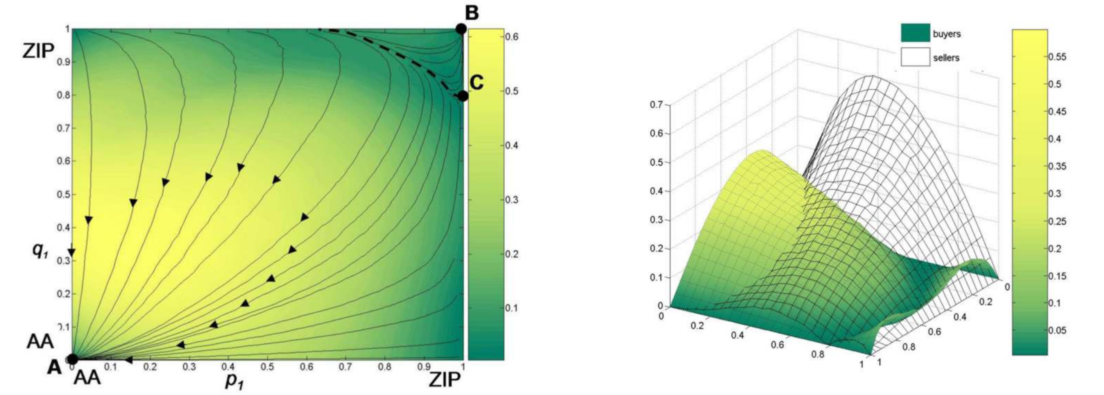


FIGURE B.8: Scenario MS31 with AA and ZIP agents. Here, we have two attractors: A at (0,0) and B at (1,1) and a saddle point: C at (1,0.80). The probability that mixed-Nash equilibrium A will be adopted is 0.952 and that B will be adopted is 0.048.

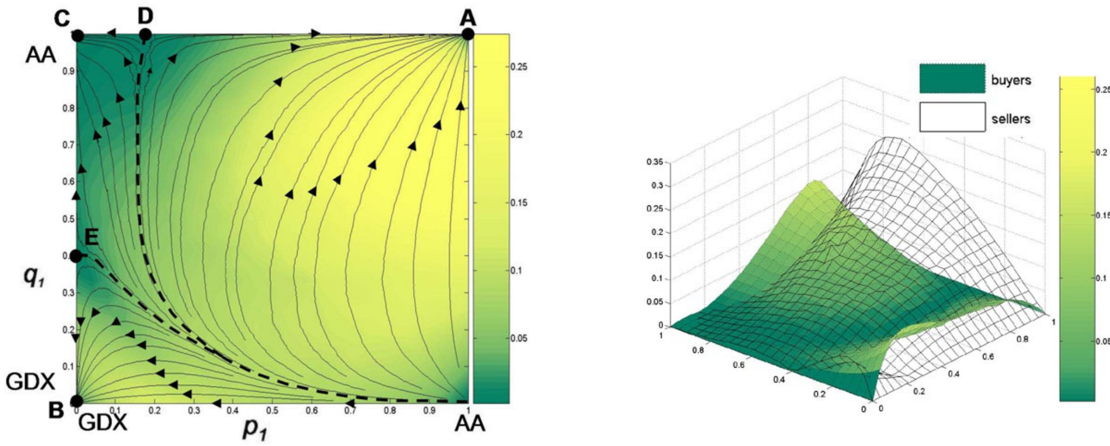


FIGURE B.9: Scenario MS31 with AA and GDX agents. Here, we have three attractors: A at (1,1), B at (0,0) and C at (0,1) and two saddle points: D at (0.19,1) and E at (0,0.40). The probability that mixed-Nash equilibrium A will be adopted is 0.776, that B will be adopted is 0.125 and that C will be adopted is 0.099.

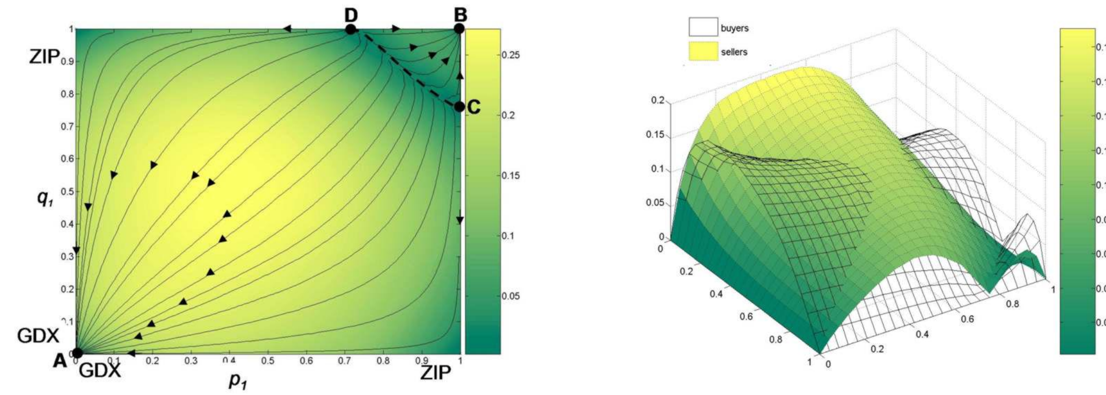


FIGURE B.10: Scenario MS31 with ZIP and GDX agents. Here, we have three attractors: A at (0,0) and B at (1,1) and two saddle points: C at (1,0.76) and D at (0.71,1). The probability that mixed-Nash equilibrium A will be adopted is 0.962 and that B will be adopted is 0.038.

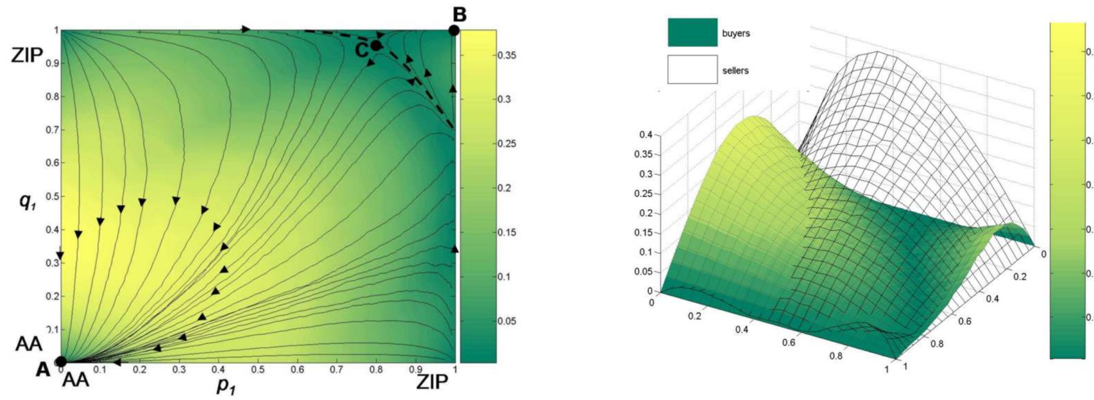


FIGURE B.11: Scenario MS23 with AA and ZIP agents. Here, we have two attractors: A at (0,0) and B at (1,1) and a saddle point: C at (0.80, 0.95). The probability that mixed-Nash equilibrium A will be adopted is 0.965 and that B will be adopted is 0.035.

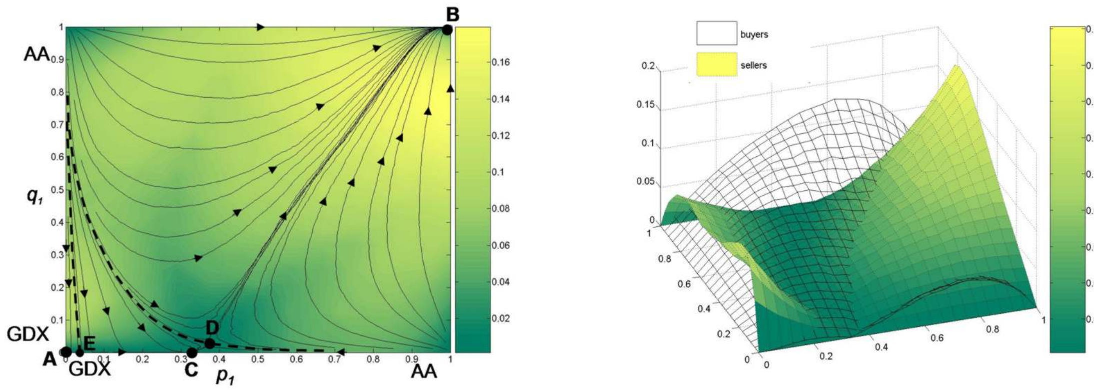


FIGURE B.12: Scenario MS23 with AA and GDX agents. Here, we have three attractors: A at (0,0), B at (1,1) and C at (0.32.0) and two saddle points: D at (0.37,0.04) and E at (0.03,0). The probability that mixed-Nash equilibrium A will be adopted is 0.010, that B will be adopted is 0.902 and that C will be adopted is 0.088.

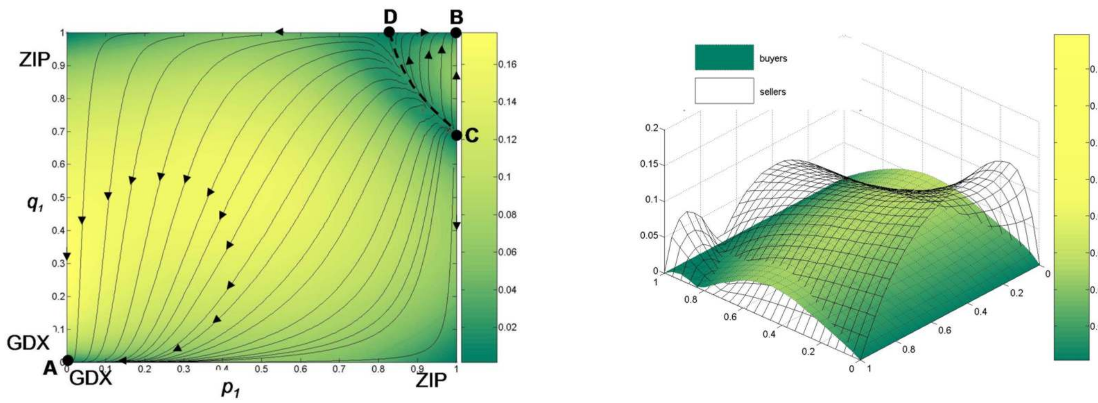


FIGURE B.13: Scenario MS23 with ZIP and GDX agents. Here, we have two attractors: A at (0,0) and B at (1,1) and two saddle points: C at (1,0.69) and D at (0.83,1). The probability that mixed-Nash equilibrium A will be adopted is 0.966 and that B will be adopted is 0.034.

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