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INTEGRATED CAPACITATED LOT SIZING AND SCHEDULING PROBLEMS IN A FLEXIBLE FLOW LINE

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Abstract

The lot sizing and scheduling problem in a Flexible Flow Line (FFL) has extensive real-world applications in many industries. An FFL consists of several production stages in series with parallel machines at each stage. The decisions to be taken are the determination of production quantities (lots), machine assignments and production sequences (schedules) on each machine at each stage in an FFL. Lot sizing and scheduling problems are closely interrelated. Solving them separately and then coordinating their interdependencies is often ineffective. However due to their complexity, there is a lack of mathematical modelling and solution procedures in the literature to combine and jointly solve them.

Up to now most research has been focused on combining lotsizing and scheduling for the single machine configuration, and research on other configurations like FFL is sparse. This thesis presents several mathematical models with practical assumptions and appropriate algorithms, along with experimental test problems, for simultaneously lotsizing and scheduling in FFL. This problem, called the '*General Lot sizing and Scheduling Problem in a Flexible Flow Line*' (GLSP-FFL). The objective is to satisfy varying demand over a finite planning horizon with minimal inventory, backorder and production setup costs. The problem is complex as any product can be processed on any machine, but these have different processing rates and sequence-dependent setup times & costs. As a result, even finding a feasible solution of large problems in reasonable time is impossible. Therefore the heuristic solution procedure named Adaptive Simulated Annealing (ASA), with four well-designed initial solutions, is designed to solve GLSP-FFL.

A further original contribution of this study is to design linear mixed-integer programming (MILP) formulations for this problem, incorporating all necessary features of setup carryovers, setup overlapping, non-triangular setup while allowing multiple lot production per periods, lot splitting and sequencing through ATSP-adaption based on a variety of subtour elimination.

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Chapter 1

Introduction

1.1 Motivation

The increasing intensity of competition in global market leads manufacturing companies to become more efficient. A key success factor in achieving this is having an elaborate production planning system. Due to rapid growth in their size and complexity, the mathematical modelling and optimization of manufacturing systems is an important challenge for Operational Research (OR). This work focuses on two important challenges in managing a flexible flow line, namely the sizing and scheduling of production lots.

The flexible flow line also commonly referred to as hybrid flow shop, is a very prevalent production system and can be found in a vast number of industries, such as automotive, chemical, electronics, steel making, pharmaceutical, food and textile. FFL is a flow line with several parallel machines on some or all production stages and all products follow the same linear path through the system (Quadt, 2004).

Lotsizing and scheduling are closely interrelated and considerably combined in the literature for single machine production system. However, it can be more complicated and challenging to integrate both problems in complex production systems like FFLs. Quadt and Kuhn (2005) explicitly identified a lack of literature not only on combined lot sizing and scheduling but also on stand-alone lot sizing in FFLs. They presented an integrative solution approach for the combined lot-sizing and scheduling problem in FFLs which is limited to necessity of bottleneck stage identification.

Fandel and Stammen-Hegene (2006) formulated the Multi Level General Lot sizing and Scheduling Problem with Multiple Machines (MLGLSP-MM). In the MLGLSP-MM, products are produced on different machines with general production structure in job shop production. However the paper contains only a mathematical model for the MLGLSP-MM without any numerical tests or solution procedure, possibly because the authors themselves recognized that the model's complexity limits optimal solutions to just small instances. Recently, Mohammadi and Jafari (2010) developed an MIP (Mixed Integer Programming) model for flexible flow shop system based on Fandel and Stammen-Hegene (2006) formulations. They assumed the vertical interaction or "inter-level synchronization" between production stages means a production on a production stage can only begin if there is sufficient amount of the product from the previous production stage. The shortage and lot-splitting are not allowed and sequence-dependent setup costs and times are triangles.

For the first time, in this thesis I research new challenges such as lot splitting and shortages, the practiced assumptions in flexible flow shop manufacturing systems (Özdamar and Barbaroso lu, 1999). Moreover sequence-dependent setup times can be "non-triangular" as is the case in many industries such as chemical, pharmaceutical, food and oil as some contamination occurs between certain products. For example, a product p contaminates some other product r, but in order to decontaminate, either an additional cleaning operation must be done as part of a substantial setup time setup st_{pr} that consumes the scarce production time, or a third product q that can absorb the contamination must be produced. Such intermediate "cleansing" or shortcut products can cause non-triangular setup times i.e. product q's ability to absorb p's contamination presents a shortcut opportunity and could result in shorter non-triangular setup times such that $st_{pr} > st_{pq} + st_{qr}$ and product q cleans the machines whilst being processed.

Furthermore the "lead-time synchronization" between production stages is assumed, means a product which is produced at a stage is available for production at the next stage only in the next periods.

Responding to the challenge, the thesis hypothesis is creating the new mathematical models for General Lot sizing and Scheduling in FFL (GLSP-FFL) while considering practical assumptions and solving them which come up with production plans that are more efficient than it would get by solving existing models. GLSP-FFL determines both lot sizes and sequences on parallel machines in multi-stage production system. The problem is complex as any product can be processed on any machine but with different process rates and sequence-dependent setup times & costs. Firstly, we designed three mathematical models (FFL-FS, FFL-CC and

FFL-FM) for GLSP-FFL and presented in 7^{th} international industrial engineering conference 2010 (Mahdieh et al., 2010). Later, it was published in Journal of Industrial and Systems Engineering (Mahdieh et al., 2012). The first model (FFL-FS) is "dynamic" since a decision variable appears as an upper limit index in many constraints, so the model cannot be solved as a MIP, whereas the second and third can. The efficiency of two latter MIP models was assessed and evaluated using numerical tests.

In this thesis, I present a new linear MIP model (FFL-ATSP) through adaption of Asymmetric Travelling Salesman Problem (ATSP) and show through the numerical tests that the new ATSP adaptation for GLSP-FFL has significant improvement of problem's solution in comparison with FFL-CC and FFL-FM.

Fleischmann and Meyr (1997) have showed that the General Lot sizing and Scheduling Problem (GLSP) for single machine with non-zero minimum lot sizes is a very difficult combinatorial problem and even finding a feasible solution is NP-complete. Thus, it can be concluded that the feasibility of our problem, the GLSP with non-zero sequence-dependent setup times/costs in a complex production system, flexible flow line, is also NP-complete. Hence it is necessary to develop an efficient solution procedure for GLSP-FFL. Here an Adaptive Simulated Annealing (ASA) with four well-organized initial solutions is designed to solve GLSP-FFL.

The main restriction of conventional ATSP based models is allowing one lot per product per periods so multiple lots of shortcut products cannot be produced per period when non-triangular setup exits. In a very recent work, Clark and I modelled multiple lots per period via different subtour elimination constraints for single machine and presented in 43rd Annual Symposium of the Brazilian Operational Research Society (Clark and Mahdieh, 2011) and its revision has been submitted to the International Journal of Production Research (Clark et al., 2012). In chapter 4 its extension to parallel machine and FFL system while incorporating all features of setup carry-over and setup-overlapping is modelled.

1.2 Characteristics of the Problem

This thesis breaks new ground by modelling lot sizing and scheduling in a flexible flow line simultaneously instead of separately while incorporating a variety of practical assumptions such as lot-splitting, shortage and non-triangular setup. The objective is to satisfy varying demand over a finite planning horizon with minimal inventory, backorder and production setup costs. The following system characteristics are explicitly noted:

The production line consists of several processing stages in series, separated by finite intermediate buffers, where each stage has one or more parallel identical machines. Multiple products can be produced at stages and production at each stage involves unrelated parallel machines with different production rates. All machines can produce any product. The available capacity of each machine is limited and can vary between periods and stages.

The finite planning horizon is divided into T macro-periods. The independent demand for all products is felt at the final stage at the end of each macro-period. It is known with certainty, but varies dynamically over the planning horizon. Demand for items in other stages is dependent on the production of the next stage. Backlog shortages are permitted for products at the final stage but are upper-bounded by a given percentage of demand in each macro-period. This is the practiced assumption in flow shop manufacturing systems (Özdamar et al., 1999).

The products may be manufactured in lots of varying size on any one of the parallel machines in each stage. The production rate can vary between products and machines, but is constant over the planning horizon. A changeover from one product to another requires a setup time during which the machine is unproductive. Setup times and costs are sequence dependent and can vary between machines. The setup state is conserved when no product is being processed (setup carryover). At the beginning of the planning horizon, each machine is setup for a specified product.

A two-level time structure is assumed. Each macro-period consists of a variable number of micro-periods with variable length. Each machine has its own micro-period segmentation, i.e., the number of micro-period can differ between machines. Micro-periods do not have to be of equal durations on the same machine. At the start of a micro-period, a machine is setup and then produces just one product until the end of the micro-period. Lot-splitting is permitted at any stage, i.e., each product can be simultaneously produced on more than one machine at any given stage. In order to obtain viable schedules, it is assumed that there is the lead time of one period between different production stages (lead-time synchronization). In this case, a product which is produced at a stage is available for production at the next stage only in the next periods.

1.3 Outline of the chapters

The work is divided into six chapters. The reminder of the thesis is as follows.

Chapter 2 provides the review of the literature and recent developments of deterministic dynamic lotsizing problems. The focus of this review is on capacitated lotsizing with sequence-dependent setup which is closely interrelated to scheduling and considerably combined in the literature. However, it can be more complicated and challenging to integrate both problems in complex production systems like FFL. This review discusses a modelling perspective of this challenge on a variety of machine configurations and points out fertile opportunities for future research.

Chapter 3 presents a novel linear MIP model (FFL-ATSP) for the problem of integrating lot sizing, loading, and scheduling in capacitated flexible flow lines with sequence-dependent setups through adaptation of ATSP. In comparison to our former models (FFL-CC and FFL-FM), fewer variables and constraints of FFL-ATSP model makes it more efficient and faster to be solved. Computational tests demonstrate the superiority of FFL-ATSP and its fast speed of solution compared to FFL-CC and FFL-FM.

Chapter 4 is devoted to heuristic solution procedure called an Adaptive Simulated Annealing (ASA) with an effective adaptive temperature control scheme for solving large instances in GLSP-FFL. The adaptive temperature control scheme changes temperature based on the number of consecutive improving moves and maintains it above the minimum level. Four initial solutions and neighbour operators are designed for ASA. The third and fourth novel initial solutions are obtained by solving well-organized model which extracts from the GLSP-FFL and ATSP model respectively. The numerical test compares the efficiency of different initial solutions.

Chapter 5 is presented the new mix integer programming formulations for capacitated lot sizing and scheduling with non-triangular and sequence-dependent setup times and costs incorporating all necessary features of setup carryover and overlapping on different machine configurations. The innovation of the new formulation is the modelling of non-triangular sequence-dependent setups within lot sizing model based on ATSP problem that allows multiple lots per product per period with polynomial number of disconnected subtours prohibition constraints.

To assess how effectively the multiple lot model with setup overlapping takes advantage of shortcut product and setup overlapping feature to reduce backlogs and inventory, three models including one-Lot (1L), Multiple Lots (ML) and Multiple Lots with setup overlapping (MLOV) are compared for three production systems: Single Machine (SM), Parallel Machines (PM) and FFL.

Finally, Chapter 6 summarizes the work and suggests directions for future research.

Chapter 2

Literature review

Generally, production planning in manufacturing determines what product is to be produced on which machine at what time. Production planning problems are typically classified according to hierarchical structure of long-term or strategic, medium-term or tactical and short-term or operational (Bitran and Tirupati, 1993). Long-term planning uses aggregated demand forecasts and makes strategic decisions such as aggregate resource planning to mainly achieve financial targets. Medium-term planning is more detailed and uses partially disaggregated demand to often determine Material Requirement Plan (MRP) and production quantities over planning horizon to optimize both operational and financial criteria while satisfying capacity limitations. Short-term planning uses totally disaggregated or actual demands to make day-to-day decisions on lot sizing, scheduling and loading problems (Heizer and Render, 2004, Karimi et al., 2003). Firstly, Gelders and Van Wassenhove (1981) gave an overview on medium- and short-term production planning and then in (Gelders and Van Wassenhove, 1982) they focused on the issue of integrating various decision level in hierarchical planning. So far considerable amount of research has been done on the various aspects of production planning and inventory management and large amount of models and techniques are already available (Graves et al., 1993, Pochet and Wolsey, 2006, Quadt and Kuhn, 2008, Silver et al., 1998, Thomas et al., 1993, Vollmann et al., 1997).

Lot sizing models can be classified either as medium-term or short-term models according to their level of aggregation and decision horizon (Clark et al., 2011, Jans and Degraeve, 2007b). This study focuses on lot sizing problems with sequence dependent setup times/costs which include more operational and scheduling issues.

2.1 Basic Lotsizing models

Lot sizing aims to determine the optimal timing and level of production. Research on lot sizing started with the economic order quantity (EOQ) model (Harris, 1913). The main assumption for the EOQ models is constant demand for a product over an infinite planning horizon. Since there is no capacity constraint during a single level production process in the EOQ model, the economic lotsizing problem (ELSP) model is developed for multi-product or multi-item considering capacity constraint (Elmaghraby, 1978, Zipkin, 1991). However both EOQ and ELSP are based on a constant demand over an infinite time. The Wagner–Whitin (WW) model (Wagner and Whitin, 1958) is one of the first models for a dynamic demand where a finite planning horizon is subdivided into several discrete periods and demand is given per period and may very over time. The WW problem is single-level, single-item without capacity constraints. The capacitated lotsizing problem (CLSP) can be considered as the extension of the WW problem to capacity constraints and multi-item problem (Bitran and Yanasse, 1982, Haase, 1996, Karimi et al., 2003).

The CLSP is called large bucket problem since several item can be produced per period (Eppen and Martin, 1987). Subdividing the (macro-) periods of CLSP into several (micro-) periods leads to discrete lotsizing and scheduling problem (DLSP) which is called a small bucket problem (Salomon, 1991, Salomon et al., 1991, Salomon et al., 1997, Fleischmann, 1990). The main assumption of the DLSP is all-or-nothing production that means only one item can be produced per period and uses the full capacity. A step towards more realistic situations is the continuous setup lotsizing problem (CSLP) (Bitran and Matsuo, 1986, Karmarkar et al., 1987) and the proportional lot sizing and scheduling problem (PLSP) (Drexl and Haase, 1995, Kimms and Drexl, 1998a, Kimms and Drexl, 1998b) which both do not include the strict all-or-nothing assumption of the DLSP. However, at most one item can be produced per period in the CSLP and two items in the PLSP. Comparing the small bucket lotsizing and scheduling models with the CLSP, the point reveals that through little changes the sequence decisions can be modelled in large bucket lotsizing problem. Thus in contrast to CLSP, lotsizing and scheduling is considered simultaneously (Haase, 1996, Haase and Kimms, 2000). The general lotsizing and scheduling problem (GLSP) is a large bucket problem where due to simultaneously determine lot sizes and sequences, the planning periods divide into a predetermined number of (small bucket) micro-periods with at most one setup (Fleischmann and Meyr, 1997).

Several features or assumptions can be taken into account within the basic lot sizing models such as backlogging, sequence-dependent setup cost or/and time, setup carry over, setup overlapping and different machine configurations like single or parallel machine and single or multi stage. Therefore the numerous extensions of basic models and solving algorithms can be found in the literature. Furthermore there are excellent review papers on these extensions which are worthwhile to discuss here.

2.2 Previous reviews

Firstly Bahl et al. (1987) classified lot sizing models into four categories based on demand type including single- and multi-level, and presence or absence of resource constraints. Wolsey (1995) and Brahimi et al. (2006) focused on single item lot-sizing problem and discussed different extensions of this problem for real-world applications. Potts and Van Wassenhove (1992) reviewed the literature on the integration of lot sizing and scheduling from a scheduling perspective and pointed out the lack of work on combining lot sizing and scheduling. Later Drexl and Kimms (1997) gave an overview on lotsizing and scheduling models. They explained the differences of mathematical formulations for CLSP, DLSP, CSLD, PLSP and GLSP and also reviewed the extension of these models for multi-level structure. They underlined the importance of the extensions on sequence-dependent setup time, backlogging and parallel machines for future research. Karimi et al. (2003) discussed single-level lot sizing problem in both capacitated and uncapacitated cases and classified the literature based on different solution approaches applied for CLSP. They concluded similarly to Drexl and Kimms (1997) that there has been little literature regarding problems such as CLSP with backlogging or with setup time and setup carryover.

Staggemeier and Clark (2001) reviewed meta-heuristics applied to the solution of lot-sizing and scheduling problems. Zhu and Wilhelm (2006) mainly reviewed the optimization and heuristic methods for scheduling problems with sequence-dependent setup times (costs) based on a variety of machine configuration. They also discussed the integration of these models with lot sizing problems and emphasized that most research has been done on single-machine configuration rather than other configurations. In an outstanding review Jans and Degraeve (2007b) gave an extensive overview of modelling deterministic single-level dynamic lotsizing problems and discussed the solution approach in Jans and Degraeve (2007a). They organized the extensions of these models in two directions. The first direction focuses on operational aspects including setups, production characters, inventory, demand side and rolling horizon. The second direction is towards more tactical and strategic aspects such as integrated production-distribution planning or supplier selection. They indicated that with introduction of sequence-dependent setups boundaries between lot sizing and scheduling are fading. They also noted that further integration of lot sizing, sequencing and loading (for example on parallel machine) is a challenging area for future research.

Quadt and Kuhn (2008) present a literature review on CLSP problems that incorporate one of the following extensions in the: back-orders, setup carry-over, sequencing, and parallel machines. Buschkühl et al. (2010) reviewed different modelling and algorithmic solution approaches for the multi-level capacitated lot-sizing problem (MLCLSP) while ignoring the sequencing and scheduling aspects.

2.3 Capacitated Lot Sizing and scheduling with sequence-dependent setups

The classic CLSP does not sequence or schedule products within a period. In addition it does not allow a setup to be carried over from one period to the next, even when the last product in a period is the same as the first product in the next period. Gopalakrishnan et al. (1995) developed a modelling framework for formulating CLSP with setup carry over by introducing additional binary variables. Later Gopalakrishnan (2000) modified the modelling of Gopalakrishnan (2000) for incorporating sequence-independent and product-dependent setup times and costs. Different studies has demonstrated that considering the setup carry-over significantly saves costs by decreasing the number of setups and releasing production capacity (Gopalakrishnan et al., 2001, Gupta and Magnusson, 2005, Porkka et al., 2003, Sox and Gao, 1999). This problem also called the capacitated lot sizing problem with linked lot sizes (Suerie and Stadtler, 2003).

A further step for capacitated lot sizing is to determine a sequence for all products within a time period certainly if setup times or costs are sequence-dependent. One of the first studies regarding sequence-dependent setup cost is DLSPSD (DLSP with sequence-dependent setup cost) by Fleischmann (1994). He reformulated DLSPSD as Travelling Salesman Problem With Time Windows (TSPWTW) formulation to propose a heuristic solution. Salomon (1997) incorporated sequence-dependent setup time into DLSPSD by applying the same TSPWTW approach. The main serious restriction of DLSP as a small bucket is not allowing setup time to be fraction of period capacity.

Hence this study focuses on CLSP as a big bucket problem which is more flexible at integrating of lot sizing and sequencing decisions. The CLSP partitions the planning horizon into a number of lengthy time periods, allowing setup of several products within a same period (bucket). Gupta and Magnusson (2005) classified the CLSP literature according to extensions on sequence dependency of setup costs and times. They extended the framework proposed by Gopalakrishnan (2000) to include sequence-dependent setup times and costs. Hasse (1996) modelled Capacitated Lot sizing problem with Sequence-Dependent setup costs (CLSD) and included setup times (Haase and Kimms, 2000) by assuming of predetermined efficient production sequences and null inventory for a production of an item in a period. The GLSP (Fleischmann and Meyr, 1997) is very close to the CLSD but more flexible since eliminates the restrictions of CLSD. Meyr (2000) included sequence-dependent Setup Times, resulting in the GLSPST and extended to the GLSPPL for parallel machines (Meyr, 2002). The GLSP has been known as the most flexible lotsizing and scheduling formulation in large bucket for representing different environment under slight modifications (Koçlar, 2005, Koçlar and Süral, 2005). Moreover the restriction of holding setup triangular inequality is relaxed in GLSP which allows many time production of a product in a period as long as it does not exceed the number of position (macro-periods) in a period. The non-triangular setup times can happen in many industries such chemical, food, beverage and oil. For example, in the animal feed industry, some products can cause contamination of other families therefore equipment must be cleaned in order to avoid it. Cleaning results in substantial setups that consuming scarce production time. In this case the amount of cleaning can be minimised by producing the intermediate cleansing or shortcut products which can cause non-triangular setup times. In an alternative approach to GLSP, Clark and Clark (2000) designed a mixed integer programming (MIP) model for simultaneous sequencing and lot sizing production lots on a set of parallel machines. They assumed non-triangular sequence-dependent setup times, no setup cost and backlogging possibility.

The problem of sequencing a set of lots with sequence dependent setups is related to the travelling salesman problem (TSP) and the vehicle routing problem (VRP) (Laporte, 1992a, Laporte, 1992b). Almada-lobo et al. (2007) presented two models for CLSP with sequence-dependent setup times and costs using Miller-Tucker-Zemlin subtour prohibition constraint (Desrochers and Laporte, 1991). They incorporated all necessary features of setup carryover which allows a product sets up at the end of one period and the actual production starts in the next period. To model this, triangular inequality for setup times and costs must be hold. Clark et al. (2010) formulated sequencing and lotsizing with non-triangular setup times based on Asymmetric Travelling Salesman Problem (ATSP) at animal feed plant. To solve the model optimal solution methods based on iterative subtour elimination and patching are developed. In the ATSP-based models (Almada-lobo et al., 2007, Clark et al., 2010) at most one lot per product can be produced in periods (no subtour is permitted). Therefore in case of non-triangular setup, the multiple production of shortcut product is not allowed. Menezes et al. (2011) relaxed this restriction and allowed production of multiple lot per period (included connected subtours) by using an iterative model and method based on a potentially exponentially number of subtour elimination constraints (to exclude disconnected subtours). They also modelled the setup cross-over or setup overlapping which is beneficial in tight capacity conditions or whenever setup times are significant. Therefore there is no need to interrupt a setup at the end of a period and resume it at the beginning of the next period due to physical separation between periods. Setup overlapping has been studied by Suerie (2006) for small bucket and by Sung and Maravelias (2008) for big-bucket but with sequence-independent setup times and cost. Clark and Mahdieh (2011) presented the stronger formulation in comparison with Menezes et al. (2011) for modelling the production of multiple lots of a product per periods using a polynomial number of multi-commodity-flow-type constraints (Claus, 1984) to exclude disconnected subtours while allowing ones connected to the main sequence.

2.4 Capacitated lot sizing and scheduling on different machine configurations

Fading boundaries between lotsizing and scheduling poses special challenges for integrating lotsizing, sequencing and loading decisions on a variety of machine configurations. Machine configuration includes single machine, parallel machines, flow shop, flexible flow shop and job shop system. Most research has been focused on combing lotsizing and scheduling for the single machine configuration and research on other configurations is sparse.

Table 2-1: Literature review of capacitated big bucket lot sizing models with respect to back-orders, setup carry-over, sequencing on different machine configuration excluding single-machine. X: covered in reference; (X) partly covered in the reference.

References	Back-orders	Setup times	Setup carry-over	Sequencing	Machine configuration	Over-time	Multi-level
Dillenberger et al. (1993) and (1994)		X	X		Parallel machine		
Gopalakrishnan et al. (1995)		(X)	X		Parallel machine		
Derstroff (1995)		X			Job shop		X
Hindi (1995)					Parallel machine		
Özdamar and Birbil (1998)		X			Partly Parallel machine	X	
Özdamar and Barbarosolu (1999)	X	X			Multi-stage with identical parallel machine	X	X
Kang et al.(1999)			X	X	Parallel machine		
Clark and Clark (2000)	X	X	X	X	Parallel machine		
Belvaux and Wolsey (2000)	X	X			Multiple machine		X
Meyr (2002)		X	X	X	Parallel machine		
Stadtler (2003)		X			Multiple machine	X	X
Quadt (2004)	X	X	X		Flexible flow line		
Fandel and Stammen-Hegene (2006)		X	X	X	Job shop		X
Quadt and Kuhn (2009)	X	X	X		Parallel machine		
Mahdieh et al.(2010)	X	X	X	X	Flexible flow shop		
Mohammadi et al (2010a), Mohammadi (2010b), (2010c) and Mohammadi and Ghomi (2011)		X	X	X	Flow shop		X
Mohammadi (2010) and Mohammadi and Jafari (2010)		X	X	X	Flexible flow shop		X
James and Almada-Lobo (2011)		X	X	X	Parallel machine		

Quadt and Kuhn (2008) in a well-structured paper has given a literature review of capacitated big bucket lot sizing models and solution procedures that extend the standard CLSP formulation by with respect to back-orders, setup carry-over, sequencing or parallel machines. They indicated that only Meyr (2002) and Kang et al. (1999) combined sequencing and lotsizing on parallel machines.

This thesis updates the literature review of capacitated lot sizing not only on parallel machines but also on multi-stage production system given in table 1. Stand alone capacitated lotsizing on parallel machines has been studied by Dillenberger et al. (1993) and (1994), Gopalakrishnan et al. (1995), Hindi (1995), Özdamar and Birbil (1998), Belvaux and Wolsey (2000), Stadtler (2003) and Quadt and Kuhn (2009).

Kang et al.(1999), Clark and Clark (2000), Meyr (2002) and James and Almada-Lobo (2011) integrated lotsizing and scheduling on parallel machines with different extensions as shown in table 2-1.

Moreover some recent work inspired by a specific real-world problem addresses capacitated lotsizing and scheduling on different machine configurations (Almada-Lobo et al., 2010, Almada-Lobo et al., 2008, Almeder and Almada-Lobo, 2011, Ferreira et al., 2012, Ghosh Dastidar and Nagi, 2005).

Flexible Flow Lines (FFL) are flow lines with parallel machines on some or all production stages and occur in many different environments, including automobile manufacture and printed circuit board manufacture (Kurz and Askin, 2003). The survey by Linn and Zhang (1999) reviewed the state of FFL scheduling research and described a variety of different configurations. They noted the lack of research on FFLS with more than two stages and the extensive using of dispatching rules in practice. Their survey did not include any research or mention of lot sizing and scheduling within FFLS. Six years later, Quadt and Kuhn (2005) explicitly identified a lack of literature for lot sizing and scheduling in FFLS and went on to describe a hierarchical 3-phase approach consists of bottleneck planning, schedule roll out and product-to-slot assignment for integrative lot-sizing and scheduling. The second phase consisted of capacitated lot-sizing problem (CLSP) model (Bitran and Yanasse, 1982) generalised to the sequencing of lots of product families lot, the possibility of back-orders and parallel machines. While more general than needed for FFLS, the approach of Quadt and Kuhn (2005) is limited partly due to its aggregation of products into families, but primarily because of the necessity of bottleneck stage identification and stability during the planning run.

Subsequently, in Quadt and Kuhn (2007b), they gathered a wide range of literature on the FFL scheduling problem and built a taxonomy for FFL scheduling procedures (excluding lot-sizing), classifying them by general solution approach. They concluded by noting again that very little research has been published combining both lot sizing and scheduling in FFL, although in the same year Quadt and Kuhn (2007a) did deal with batch scheduling.

Even research on stand-alone lot sizing for FFL is very limited. Derstroff (1995) considered a multi-level job shop problem and extended to include alternative routing on parallel machines where FFL can be interpreted as a special case of such a system. Özdamar and Barbarosolu (1999) considered the lotsizing problem for FFLs called the multi-stage capacitated lot sizing and loading problem (MCLSLP).

Relevant to the sequential stages of FFLs, Fandel and Stammen-Hegene (2006) formulated the Multi Level General Lot sizing and Scheduling Problem with Multiple Machines (MLGLSP-MM), based on the GLSP for single level production and parallel machines. However, the paper contains only a mathematical model which is not a MIP since a variable is used as an index limit and also without any numerical tests or solution procedure, possibly because the authors themselves recognized that the model's complexity limits optimal solutions to just small instances. To recall, a Mixed Integer Programming (MIP) is the optimization of a linear objective function subject to linear constraints in which some or all of the variables are restricted to be integers. In many settings the term refers to Mixed Integer linear programming (MILP).

Recently, Mohammadi et al (2010a) and Mohammadi (2010b) developed an exact MIP formulation for lotsizing and scheduling in pure flowshop based on Fandel and Stammen-Hegene (2006) model. In fact they applied Clark and Clark's (2000) sequencing technique to the Fandel's Model to make it into a MIP. They designed novel heuristics, all based on solving a sequence of smaller Mixed Integer Programs (MIPs). To solve larger instances of the problem, they proposed an algorithmic approach (Mohammadi et al., 2010c) and the genetic algorithm-based heuristic (Mohammadi and Ghomi, 2011). Furthermore Mohammadi extended the model for flexible flowshop system (Mohammadi, 2010) and developed it into a more efficient MIP model in Mohammadi and Jafari (2010). Similar to Fandel and Hegene (2006), they assumed the vertical interaction or "inter-level synchronization" between production stages by defining of shadow product variables. In inter-level synchronization a product cannot be produced earlier in a period than the production of its required component is finished. In other words, a production on a production stage can only begin if there is sufficient amount of the product from the previous production stage. The shortage is not permitted and sequence-dependent setup costs and times are triangles (i.e. it is never faster to change over from one product to another by means of a third product). Furthermore at stages with more than one machine, each product is produced entirely on one machine (lot splitting is not allowed).

Clark, Bijari and I extended MCLSLP to General Lot sizing and Scheduling in FFL (GLSP-FFL) which determines both lot sizes and sequences on parallel machines in multi-stage production system (Mahdieh et al., 2010, Mahdieh et al., 2012). However in contrast of MCLSLP the lot-splitting was allowed due to give more flexibility in the system through lot sequencing. The shortage was permitted and sequence-dependent setup costs and times could be “non-triangle”. Three models were presented (FFL-FS, FFL-CC and FFL-FM) based on Fandel and Stammen-Hegene (2006), Clark and Clark’s (2000) and Fleischmann and Meyr (1997) sequencing formulation technique. It was also assumed the “lead-time synchronization” between production means a product which is produced at a stage is available for production at the next stage only in the next period.

Fred Glover defined a meta-heuristic as a "master strategy that guides and modifies other heuristics to produce solutions beyond those that are normally generated in a quest for local optimality" (Glover and Laguna, 1997). Meta-heuristic algorithms can find a good solution to difficult optimization problems in a reasonable amount of time but do not guarantee that optimal solutions can be reached. So far meta-heuristics like tabu search (TS), simulated annealing (SA) and genetic algorithms (GA) have been widely applied to almost every complex combinatorial problem such as lot sizing problems. Jans and Degraeve (2007a) (2007a) provided a review of the variety of meta-heuristics that have been used to solve lot sizing problems. Tang (2004) discussed simulated annealing techniques and their application in lot sizing problems and presented a binary matrix to represent the decision configuration. He showed that the lot sizing model very well fits into the proposed simulated annealing framework and solution procedure is flexible. In this thesis SA algorithm has been developed for lot sizing and scheduling in flexible flow line in chapter 4.

2.5 Conclusion and final remarks

This literature review focuses on modelling perspective of dynamic deterministic capacitated lot sizing problems (i.e. demands are known with certainty but may vary over time) with sequence dependent setup. The numerous extensions of the basic lotsizing models as Jans and Degraeve (2007b) cited nearly 250 references show that it can be applied to a variety of real-world industrial problems.

According to time structure capacitated lot sizing problems mainly are classified into small bucket (small time window) and big bucket (big time window) (Eppen and Martin, 1987, Gupta and Magnusson, 2005). Introducing sequence dependent setup leads lot sizing models to necessarily incorporate more scheduling aspects. Hence, fading boundaries between lotsizing and scheduling poses special challenges for integrating lotsizing, sequencing and loading decisions on a variety of machine configurations. The big bucket capacitated lotsizing is more flexible at integrating of lot sizing and scheduling decisions. Therefore GLSP (Fleischmann and Meyr, 1997) has been known as the most flexible simultaneous lotsizing and scheduling model in large bucket for representing different environment.

The adaptation of Asymmetric Travelling Salesman Problem (ATSP) is an alternative approach for lotsizing and scheduling with sequence dependent setup (Almada-lobo et al., 2007, Clark et al., 2010). Clark et al. (2010) showed that ATSP approaches were competitive with GLSP ones. Computationally comparing GLSP approach with different ATSP approaches based on a variety of subtour elimination method is another research opportunities to explore.

Several reviews have emphasized that there has been a little literature regarding capacitated lotsizing on a variety of machine configurations (Jans and Degraeve, 2007b, Karimi et al., 2003, Quadt and Kuhn, 2008, Zhu and Wilhelm, 2006). To our knowledge, there are only four papers on parallel machines (Clark and Clark, 2000, James and Almada-Lobo, 2011, Kang et al., 1999, Meyr, 2002), one paper on job shop (Fandel and Stammen-Hegene, 2006), one group of work on flow shop (Mohammadi et al., 2010a, Mohammadi and Ghomi, 2011, Mohammadi et al., 2010c) and including our work, two groups of work on flexible flow line (Mahdieh et al., 2010, Mohammadi, 2010, Mohammadi and Jafari, 2010) to model lotsizing and scheduling simultaneously. Therefore there is a fruitful research area on considering different variants such as lot-splitting, back-orders, non-triangular setup and allowing multiple lot production per periods, sequencing through ATSP-adaption based on a variety of subtour elimination, all features of setup carry-over and setup-overlapping for different machine configurations.

Chapter 3

Lot sizing and scheduling in FFL

Fading boundaries between lotsizing and scheduling poses special challenges for integrating lotsizing, sequencing and loading decisions on a variety of machine configurations. Most research has been focused on combing lotsizing and scheduling for the single machine configuration and research on other configurations is sparse. In 2010, Clark, Bijari and I presented three different mathematical models to consider General Lotsizing and Scheduling Problem in Flexible Flow Line (GLSP-FFL) simultaneously (Mahdieh et al., 2010, Mahdieh et al., 2012). The first model, FFL-FS, cannot be solved as a MIP, whereas the second, FFL-CC, and third, FFL-FM, can. However due to complexity of GLSP-FFL, none of the MIP models could find optimal solution even for small problems and terminated with large value of optimality gap. In this chapter the novel linear MIP model through adaption of Asymmetric Travelling Salesman Problem (ATSP) is presented which makes an enormous reduction in number of variables and constraints and becomes much faster in comparison with the previous ones. Computational experiments are reported.

3.1 Problem definition

Lot sizing and scheduling problems are closely interrelated. Solving them separately and then coordinating their interdependencies is often ineffective and has been broadly researched for single machine production systems (Almada-lobo et al., 2007, Fleischmann and Meyr, 1997, Jans and Degraeve, 2007b). However, it can be difficult and complex to combine both models particularly in production systems with more than one machine in parallel or series. As a result, for complex production systems such as flow shops and flexibles flow line, they are often modelled and solved independently in spite of their interdependencies (Mahdieh et al., 2012, Mohammadi, 2010, Quadt, 2004, Quadt and Kuhn, 2005). Depending on the characteristic features of the problem, there are several interdependencies between lot sizing and scheduling models (Fandel and Stammen-Hegene, 2006).

One of the most important interdependencies which makes the integrating of these two models crucial is the relationship between lot sizing and scheduling when setups are sequence dependent. In a lotsizing model, the optimal lot sizes are determined in order to minimise setup, holding and in some cases backorder costs. In the case of sequence dependent setups, the minimum-cost lot sizes also depend on the schedule on the machine since it influences the machine capacity.

This chapter presents three mathematical models with practical assumptions for simultaneous lotsizing and scheduling in one of the complex production systems called a flexible flow line.

Flexible Flow Line (FFL) is a very prevalent production system which has extensive real-world applications in industry especially automotive, chemical, electronics, steel making, food, paper, pharmaceutical and textile (Linn and Zhang, 1999). A flexible flow line or hybrid flow shop can be considered as an extension of two classical systems, namely the flow shop and the parallel shop. The production line consists of several processing stages in series, separated by finite intermediate buffers, where each stage has one or more parallel identical machines (Pinedo, 1995). The layout of FFL is shown in Figure 3-1.

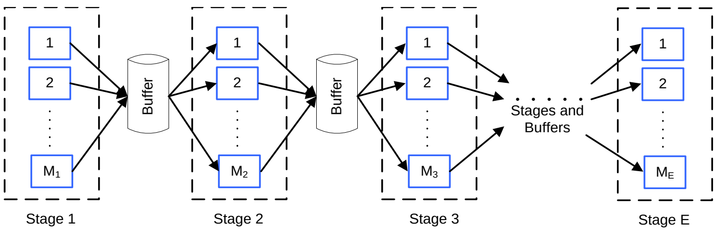

Figure 3-1: Flexible Flow Line

The GLSP-FFL was developed from the single-level GLSP of Fleischmann and Meyr (1997) and the multi-stage capacitated lot sizing and loading problem (MCLSLP) of Özdamar and Barbarosoglu (1999). According to different formulations, three distinct MIP models are introduced for GLSP-FFL. The first and second models, FFL-CC and FFL-FM, are based on Clark and Clark’s (2000) and Fleischmann and Meyr’s (1997) sequencing formulation technique respectively while the third model is formulated through adaptation of ATSP problem. All models are based on the following assumptions: Multiple products can be produced at stages in the flexible flow shop. Production at each stage involves unrelated parallel machines with different production rates. All machines can produce any product. The available capacity of each machine is limited and can vary between periods and stages. The finite planning horizon is divided into T macro-periods. The independent demand for all products is felt at the final stage at the end of each macro-period. It is known with certainty, but varies dynamically over the planning horizon.

The main assumptions of the problem were described in the following:

-Demand for items in other stages is dependent on the production of the next stage.

-Backlog shortages are permitted for products at the final stage and also are upper-bounded by a given proportion of demand (BP) in each macro-period for adding more flexibility to the production system. This is the practiced assumption in flow shop manufacturing system and is consistent with literature (Özdamar and Barbaroso lu, 1999). The backlog policy is appropriate in some situations and it can be other situations where we do not want to impose that. For example some products are more important than others so backlogs are allowed for them (by considering $BP = 1$) and not for the others (by considering $BP = 0$).

-The products may be manufactured in lots of varying size on any one of the parallel machines in each stage.

-The production rate can vary between products and machines, but is constant over the planning horizon.

-A changeover from one product to another requires a setup time during which the machine is unproductive. Setup times and costs are sequence dependent and can vary between machines.

-The setup state is conserved when no product is being processed.

-At the beginning of the planning horizon, each machine is setup for a specified product.

-A two-level time structure is assumed. Each macro-period consists of a variable number of micro-periods with variable length. Each machine has its own micro-period segmentation, i.e., the number of micro-period can differ between machines. Micro-periods do not have to be of equal durations on the same machine.

-At the start of a micro-period, a machine is setup and then produces just one product until the end of the micro-period.

-If setup costs and times are triangular, then it is not economical to produce a product in more than one lot on the same machine in the same micro-period. Thus there will be at most one setup per product per macro-period on each machine and so the number of micro-periods on a machine will be at most the number of products.

-Lot-splitting is permitted at any stage, i.e., each product can be simultaneously produced on more than one machine at any given stage.

-In order to obtain viable schedules, “lead-time synchronization” is assumed means that there is the lead time of one period between different production stages. In this case, a product which is produced at a stage is available for production at the next stage only in the next periods. However in some industries, assuming a lead time of period may be unrealistic and lead to inferior model solutions.

The parameters and indices are:

J Number of total products i, j, k

E Number of different stages e

M_e Number of different machines m_e available for production at stage e (so that the total number of machines over all stages is $M = \sum_e M_e$)

T Number of macro-periods t in the planning horizon

F_{mt} The number of micro-periods f in macro-period t on machine m_e

Note that in the definition of F_{mt} above, to avoid notational clutter such as $F_{m_e t}$, the simple index m is used when strictly speaking the subscripted index m_e should have been used. Similarly, the simple index f will be used when strictly speaking the subscripted index $f_{m_e t}$ should be used. From now on, this convention will be used so that the subscripts e and t are implied wherever the indices m and f are used. Figure 3-2 illustrates the segmentation of macro-periods into micro-periods on a machine m at any stage e. Note how the varying lengths of macro-periods differ between macro-periods.

Macro-period t = 1				…	Macro-period t = T			
$f = 1$	$f = 2$	…	$f = F_{m1}$	…	$f = 1$	$f = 2$	…	$f = F_{mT}$
Micro-periods f				…	Micro-periods f			

Figure 3-2: Micro-period segmentation on a machine differs between macro-periods

The data required are:

d_{it} Demand for product i realised at the end of macro-period t

C_{mt} Available capacity of machine m in macro-period t

st_{ijm} Time needed to setup on machine m from product i to product j

sc_{ijm} Cost needed to setup on machine m from product i to product j

b_{im} Capacity (processing time) on the machine *m* required to produce a unit of product *i*

h_{ite} Cost of holding a unit of product *i* from period *t* to *t+1* at stage *e*

g_{it} Cost of backordering a unit of end-item demand for product *i* from period *t* to *t+1*

BP Maximum permitted proportion of total end-item demand that can be backordered. In case of having different backlog policies for products, it can be setup for each product distinctively (BP_i).

i_{0m} The product setup on machine *m* at the end of period 0, i.e., the starting setup configuration

P_{im} Cost of producing one unit of product *i* on machine *m*

UB_{imt} Upper bound C_{mt}/b_{im} on the quantity of product *i* produced in macro-period *t* on machine *m*

LB_{imt} Lower bound on the quantity of product *i* produced in macro-period *t* on machine *m*

The objective of all three models presented below is to minimise backorders, inventory and setup costs of producing the J products over the T macro-periods in the planning horizon.

3.2 FFL-CC

Clark and Clark (2000) developed a mixed integer programming model for the multi-product lot-sizing problem with sequence-dependent set-up times that allows multiple set-ups per planning period. In the first model, the setup constraints are based on Clark and Clark's (2000) formulation. The decision variables are:

I_{iet} Inventory level of product *i* in stage *e* at the end of macro-period *t.*

B_{iEt} Backordered amount of end-product *i* at the last stage *E* at the end of macro-period *t*.

x_{imf} Production quantity of product *i* on machine *m* in micro-period *f.*

y_{ijmf} Binary variable, = 1 if there is a changeover from product *i* to product *j* on machine *m* at the start of micro-period *f* , = 0 otherwise.

The objective function minimises backorders, inventory and setup costs:

$$\sum_{ijemtf} sc_{ijm}\ y_{ijmf} + \sum_{ite} h_{ite}\ I_{iet} + \sum_{it} g_{it}\ B_{iEt} \tag{3-1}$$

Note how the implied summation limits and indices *e* and *t* avoid notational clutter in the first term in expression (3-1). The full cluttered version would be:

$$\sum_{i=1}^{J}\ \sum_{j=1, i\neq j}^{J}\ \sum_{e=1}^{E}\ \sum_{m_e=1}^{M_e}\ \sum_{t=1}^{T}\ \sum_{f_{m_e t}=1}^{F_{mt}} sc_{ij\,m_e}\, y_{ij\,m_e f_{m_e t}} \qquad \text{(cluttered 3-1)}$$

From now on, expressions will similarly be kept as concise as possible without sacrificing precision. Just occasionally, some clutter will be unavoidable, for example in constraints (3-3) and (3-9) below. If need be, production costs can be included in the objective function by appending the term $\sum_{iemtf} P_{im}\ x_{imf}$. Figure 3-3 shows the flow of production, inventory and backorders over different periods and stages.

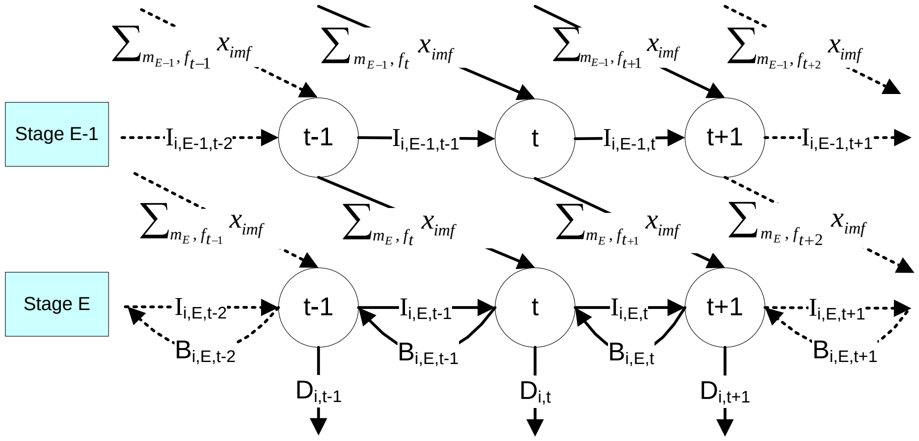

Figure 3-3: Flow diagram of GLSP-FFL

Constraints (3-2) and (3-3) follow from Figure 3-3:

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E, f_t} x_{jmf} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\ j, t \quad (3\text{-}2)$$

Constraint (3-2) expresses the material balance for end items, including backorders. Some clutter is required in order to be clear that the term $\sum_{m_E f} x_{jmf}$ refers only to the final stage E. However, note again how the implied use of the index $f_{m_e t}$ in $\sum_{m_E f} x_{jmf}$ avoids further notational clutter. The context (∀ t) of (3-2) makes it reasonable to assume that the values of *f* apply respectively to just the micro-periods within the specific macro-period *t*. The fully cluttered version would be:

$$\sum_{m_E=1}^{M_E} \sum_{f_{m_E t}=1}^{F_{mt}} x_{j\, m_E f_{m_E t}}$$

Constraint (3-3) expresses the material balance for work in process. Again, some clutter is required in order to be clear that the right-hand side refers to the successor stage e + 1 of the left-hand side's stage *e*:

$$I_{je,t-1} + \sum_{m_e, f_t} x_{jmf} - I_{jet} = \sum_{m_{e+1}, f_{t+1}} x_{jmf} \qquad \forall\, j, t \text{ and } e = 1, \dots, E-1 \quad (3\text{-}3)$$

Constraint (3-4) bounds backorders of end items in any macro-period to be within a specified proportion of demand:

$$B_{itE} \leq BP \cdot d_{it} \qquad \forall\, \mathrm{i}, \mathrm{t} \quad (3\text{-}4)$$

Constraint (3-5) represents the limited capacity:

$$\sum_{ijf} st_{ijm}\; y_{ijmf} + \sum_{if} b_{im}\; x_{imf} \leq C_{mt} \qquad \forall\, e, m, t \quad (3\text{-}5)$$

Constraints (3-6) and (3-7) specify the initial setup configuration in period one. L_{mt} refers to the first Micro-periods of period *t* on machine *m*.

$$y_{ijm\,L_{m1}} = 0 \qquad \forall\, i \neq i_{om}, j, e, m \quad (3\text{-}6)$$

$$\sum_{j} y_{i_{om}\,jm\,L_{m1}} = 1 \qquad \forall\, , e, m \quad (3\text{-}7)$$

Constraints (3-6) to (3-9) ensure that a setup on a machine in each micro-period may only occur between a single pair of different products.

$$\sum_{i} y_{ijmf} = \sum_{k} y_{jkm,f+1} \qquad \forall\; j, e, m, t \text{ and } f = 1, \dots, F_{mt} - 1 \quad (3\text{-}8)$$

$$\sum_{i} y_{ijm\,F_{m,t-1}} = \sum_{k} y_{jkm\,L_{mt}} \qquad \forall\; j, e, m \text{ and } t = 2, \dots, T \quad (3\text{-}9)$$

Constraint (3-10) enforces the appropriate setup before production:

$$x_{jmf} \leq UB_{jmt} \sum_{i} y_{ijmf} \qquad \forall\, j, e, m, t, f \quad (3\text{-}10)$$

Constraint (3-11) enforces minimum lot sizes or specified lower bounds in order to avoid a setup change without subsequent production. If set-up costs or times do not satisfy the triangle inequality ($sc_{ijm} + sc_{jkm} \geq sc_{ikm}\ \ \forall i, j, k, e, m$), then (3-11) prohibits that a setup from *i* to *k* passes through a third product *j* without minimal production of *j*.

$$x_{jmf} \geq LB_{jmt} \sum_{i \neq j} y_{ijmf} \qquad \forall\, e, m, t, j, f \quad (3\text{-}11)$$

However, when some setups are non-triangular, an optimal solution can feature multiple lots of a product on the same machine in the same period. Constraints (3-12) to (3-14) simplify the model by ensuring that a product cannot be produced in more than one lot on a machine in a macro-period:

$$\sum_{i,f,(i \neq j)} y_{ijmf} \leq 1 \qquad \forall\, j, e, m, t \quad (3\text{-}12)$$

$$\sum_{j,f,(i \neq j)} y_{ijmf} \leq 1 \qquad \forall\, i, e, m, t \quad (3\text{-}13)$$

$$\sum_{j,f,(i \neq j)} y_{ijmf} + y_{iim1} + \sum_{k} y_{kimF} \leq 2 \qquad \forall\, i, e, m, t \quad (3\text{-}14)$$

Constraints (3-12) and (3-13) ensure that there is at most one changeover from each product to a different one ($i \neq j$). As shows in figure 3-4 Constraint (3-14) prohibits reproduction of a product which was the last product of the previous period (*t-1*) at the first and end of a current period (*t*).

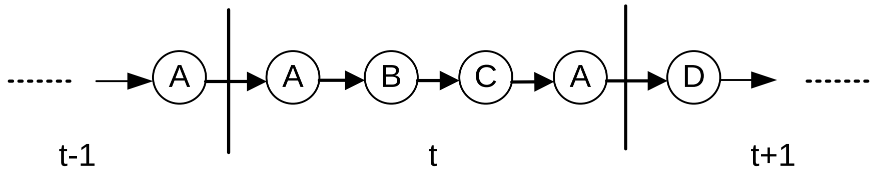

Figure 3-4: Reproduction of the last product of the previous period t-1, at the first and end of a current period t.

Note that F_{mt} is fixed by *J*, the number of products, therefore the number of setups may be less than *J*, but the remaining ones are treated as phantom setups from a product i to itself ($y_{iimf} = 1$) with zero setup time ($st_{iim} = 0$) and no consequent production.

3.3 FFL-FM

Fleischmann and Meyr (1997)'s adaptation of the General Lot sizing and Scheduling Problem (GLSP) to sequence-dependent setup times and parallel machines (Meyr, 2002) can be extended to the FFL. The parameters and continuous decision variables for this new model, denoted FFL-FM, are the same as for the FFL-CC model. However, to be consistent with Meyr's notation, the variable y_{ijmf} is renamed z_{ijmf} , and *y* becomes a new setup-state variable as follows:

y_{imf} = 1 if machine *m* is setup for product *i* in the micro-period *f*, otherwise = 0.

z_{ijmf} = 1 if there is a setup changeover from product *i* to product *j* on machine *m* at the start of micro-period *f*, otherwise = 0.

Note that there is no need to define z_{ijmf} a binary variable in the model since z_{ijmf} as a positive variable will take on the value 0 or 1 in any optimal solution (Fleischmann and Meyr, 1997). As in model FFL-CC, the number F_{mt} of micro-periods within a macro-period is fixed at the number J of products and the objective function also minimises backorders, inventory and setup costs:

$$\sum_{ijemtf} sc_{ijm}\ z_{ijmf} + \sum_{ite} h_{ite}\ I_{iet} + \sum_{it} g_{it}\ B_{iEt} \tag{3-15}$$

Constraints (3-16) - (3-18) are identical to (3-2) - (3-4) of model FFL-CC .

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E,f_t} x_{jmf} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\, j,t \tag{3-16}$$

$$I_{je,t-1} + \sum_{m_e,f_t} x_{jmf} - I_{jet} = \sum_{m_{e+1},f_{t+1}} x_{jmf} \qquad \forall\, j,t\ and\ e = 1,\dots,E-1 \tag{3-17}$$

$$B_{itE} \le BP \cdot d_{it} \qquad \forall\, i,t \tag{3-18}$$

Constraints (3-19) and (3-20) are (3-5) and (3-7) adapted to the new variables y_{jmf} and z_{ijmf} :

$$\sum_{ijf} st_{ijm}\ z_{ijmf} + \sum_{if} b_{im}\ x_{imf} \le C_{mt} \qquad \forall\, e,m,t \tag{3-19}$$

$$\sum_{j} z_{i_{om}\, jm\, 1} = 1 \qquad \forall\, e,m,t = 1 \tag{3-20}$$

Note that this formulation has no strict equivalent of constraint (3-6) which states that the first setup in a macro-period t cannot be from a product which is not i_{om}. However, constraint (3-21) prohibits the value of y_{imf} from indicating that the initial setup-state on a machine is any product which is not i_{om}:

$$y_{im1} \le z_{i_{om}\, im\, 1} \qquad \forall\ i,e,m\ and\ t = 1 \tag{3-21}$$

Constraint (3-22) imposes a minimum initial lot-size except for i_{om}:

$$x_{jm\,1} \ge LB_{jmt} \cdot z_{i_{om}\, jm\, 1} \qquad \forall\ j \ne i_{om}\ , e,m\ and\ t = 1 \tag{3-22}$$

Constraint (3-23) is requires that a product can only be processed on a machine if it is setup for that product:

$$x_{jmf} \le UB_{jmt} \cdot y_{jmf} \qquad \forall\, e,m,j,t,f \tag{3-23}$$

Constraints (3-24) and (3-25) enforce minimum lot sizes, again avoiding intermediate non-zero production setups if set-up costs/times do not satisfy the triangle inequality:

$$x_{jmf} \geq LB_{jmt}\left(y_{jmf} - y_{jm,f-1}\right) \qquad \forall\, e, m, j, t, f = 2, \dots, F_{mt} \quad (3\text{-}24)$$

$$x_{jm1} \geq LB_{jmt}\left(y_{jm1} - y_{jm\,F_{m,t-1}}\right) \qquad \forall\, e, m, j, t = 2, \dots, T \quad (3\text{-}25)$$

Constraint (3-26) ensures that only one setup state is defined in each micro period:

$$\sum_{j} y_{jmf} = 1 \qquad \forall\ e, m, t, f \quad (3\text{-}26)$$

Constraint (3-27) ensures that only one setup changeover occurs in each micro period:

$$\sum_{ij} z_{ijmf} = 1 \qquad \forall\ e, m, t, f \quad (3\text{-}27)$$

Constraints (3-28) - (3-30) are (3-12) - (3-15) adapted to the new variable z_{ijmf} :

$$\sum_{i,f,(i \neq j)} z_{ijmf} \leq 1 \qquad \forall\, j, e, m, t \quad (3\text{-}28)$$

$$\sum_{j,f,(i \neq j)} z_{ijmf} \leq 1 \qquad \forall\, i, e, m, t \quad (3\text{-}29)$$

$$\sum_{j,f,(i \neq j)} z_{ijmf} + z_{iim1} + \sum_{k} z_{kimF} \leq 2 \qquad \forall\, i, e, m, t \quad (3\text{-}30)$$

Constraints (3-31) and (3-32) relate the setup state variables and changeover variables:

$$z_{ijmf} \geq y_{im,f-1} + y_{jmf} - 1 \qquad \forall\ e, m, i, j, t, f = 2, \dots, F_{max} \quad (3\text{-}31)$$

$$z_{ijm1} \geq y_{jm1} + y_{im\,F_{m,t-1}} - 1 \qquad \forall\ e, m, i, j, t = 2, \dots, T \quad (3\text{-}32)$$

3.4 FFL-ATSP

The Asymmetric Travelling Salesman Problem has been very extensively researched and can be adapted to model the problem of sequencing a set of lots with sequence dependent setups between them (Gupta and Magnusson, 2005). For example the CLSP with sequence-dependent setup times is related to the Travelling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP) (Laporte, 1992a, Laporte, 1992b). Here, a novel MIP model is presented for GLSP-FFL via adaptation of ATSP.

The decision variables are:

I_{iet} Inventory level of product *i* in stage *e* at the end of macro-period *t*.

B_{iEt} Backordered amount of end-product *i* at the last stage *E* at the end of macro-period t.

x_{imt} Production quantity of product i on machine m in period t.

y_{ijmt} Binary variable, = 1 if there is a changeover from product i to product j on machine m at the period t, = 0 otherwise.

α_{imt} Equals to 1 if product i is the setup state at the start of period t on machine m, otherwise = 0.

v_{imt} Auxiliary variable to assign product i to machine m at period t.

To avoid notational clutter, the simple index m is used when strictly speaking the subscripted index m_e except in some occasions, some clutter will be unavoidable. Note that the main novelty of ATSP adaptation is the elimination of the micro-period index $f_{m_e t}$ from changeover variables that causes the significant reduction in the number of binary variables in comparison with FFL-CC and FFL-FM. Thus, there is no need to pre-define a fix number of micro-periods in each period and assign products to them.

The objective function minimises backorders, inventory and setup costs:

$$\sum_{ijemt} sc_{ijm}\ y_{ijmt} + \sum_{ite} h_{ite}\ I_{iet} + \sum_{it} g_{it}\ B_{iEt} \tag{3-33}$$

Constraints (3-34) and (3-35) express the material balance including backorders for end items and work in process respectively. Some clutter for m is required in order to be clear that the right-hand side refers to stages in both constraints:

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E} x_{jmt} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\ j,t \tag{3-34}$$

$$I_{je,t-1} + \sum_{m_e} x_{jmt} - I_{jet} = \sum_{m_{e+1}} x_{jm,t+1} \qquad \forall\, j,t\ and\ e = 1,\dots,E-1 \tag{3-35}$$

Constraint (3-36) bounds backorders of end items in any macro-period to be within a specified proportion of demand:

$$B_{itE} \le BP \cdot d_{it} \qquad \forall\, i,t \tag{3-36}$$

Constraint (3-37) represents the limited capacity:

$$\sum_{ij} st_{ijm}\ y_{ijmt} + \sum_{i} b_{im}\ x_{imt} \le C_{mt} \qquad \forall\, e,m,t \tag{3-37}$$

Constraint (3-38) indicates the first setup of each period which ensures that the machine is set up for exactly one product at the beginning of each period. The initial setup configuration at first period is expressed by constraint (3-39). Note that the α_{imt} is not necessary to be integer. Since, in constraint (3-39) α_{imt} equals to one for the first period and from constraint (3-40), it follows that α_{imt} is an integer for *i* in the other periods. Finally constraints (3-38)-(3-40) enforce α_{imt} to be a binary variable.

$$\sum_{i=1}^{J} \alpha_{imt} = 1 \qquad \forall\ e, m, t \quad (3\text{-}38)$$

$$\alpha_{i_{om}mt} = 1 \qquad \forall\ e, m, t = 1 \quad (3\text{-}39)$$

Constraints (3-38)–(3-41) entirely determine the sequence of products on a machine in each period and cause a setup carryover of the machine between periods.

$$\alpha_{imt} + \sum_{j} y_{jimt} = \sum_{j} y_{ijmt} + \alpha_{im(t+1)} \qquad \forall\ e, m, i \text{ and } t = 1 \ldots, T-1 \quad (3\text{-}40)$$

$$\alpha_{imT} + \sum_{j} y_{jimT} \geq \sum_{j} y_{ijmT} \qquad \forall\ e, m, i \quad (3\text{-}40a)$$

To simplify constraints (3-40) and (3-40a) into a single constraint, it is considered that set $t = \{1, .., T+1\}$ for the variable α_{imt}. Thus the constraint (3-40a) is cancelled and the domain of *t* in constraint (3-40) is changed to for all *t* (∀t).

Constraint (3-40) keeps a balanced network flow of the machine set up state and carries to next period. It means that if there is an input setup for product *i* ($\sum_j y_{jimt} = 1$) and no output setup ($\sum_j y_{ijmt} = 0$) in period *t* then this setup is the last one in period *t* and the machine carries the setup configuration of product *i* into the next period ($\alpha_{im,t+1} = 1$). On the other hand, if there is an output setup and no input setup, then the machine is configured for product *i* at the beginning of period *t* ($\alpha_{im,t} = 1$). Moreover if no setup is performed in period t, then setup is carried to the next period.

$$v_{imt} - v_{jmt} + J \times y_{ijmt} \leq J - 1 \qquad \forall\ e, m, i, j, t \quad (3\text{-}41)$$

Constraint (3-41) prohibits product subtours which is based on Miller, Tucker and Zemlin's (MTZ) subtours elimination constraint and it was originally proposed for a Vehicle Routing Problem (VRP) (Miller et al., 1960). It uses the unrestricted v_i variables to define the order in which each city *i* is visited on a tour. Öncan et al (2009) presented a comparative analysis of several asymmetric travelling salesman problem formulations where discussed 24 classifications of ATSP. Desrochers and Laporte (1991) improved MTZ's subtours elimination constraint (3-41) to be constraint (3-41a) by using a lifting technique and extended it to various type of vehicle routing problems. The MTZ's constraint is lifted by adding $(J-2)\times y_{jimt}$ to the left hand side of the constraint:

$$v_{imt} - v_{jmt} + J \times y_{ijmt} + (J-2) \times y_{jimt} \leq J-1 \qquad \forall\, e,m,i,j,t \text{ (3-41a)}$$

Desrochers and Laporte (1991) compared the lifted MTZ formulation with the original formulation for different TSP and VRP problems, though the lifted MTZ formulation has shown low improvement in the case of Asymmetric TSP test problems. Similarly in this thesis, both constraints (MTZ constraint with and without lifting) will be tested for all the problems in the next section.

Constraint (3-42) enforces the appropriate setup before production, either at the beginning or within a period:

$$x_{jmt} \leq UB_{jmt}\left(\sum_i y_{ijmt} + \alpha_{jmt}\right) \qquad \forall\, e,m,j,t \text{ (3-42)}$$

Constraint (3-43) enforces minimum lot sizes to avoid a setup change without subsequent production in case of non-triangle setup. Furthermore it does not enforce a minimum lot size to the product which already setup at the start of a period.

$$x_{jmt} \geq LB_{jmt}\left(\sum_i y_{ijmt} - \alpha_{jmt}\right) \qquad \forall\, e,m,j,t \text{ (3-43)}$$

3.5 Comparison of variables and constraints in FFL-CC, FFL-FM and FFL-ATSP

The main difference between models FFL-CC and FFL-FM is the setup variables. As Clark and Clark (2000) did, the FFL-CC setups are modelled with just one set of binary variables, y_{ijmf}, whereas in the FFL-FM model setups are formulated with one set of binary variables y_{imf} and one set of positive variables z_{ijmf}, similar to Fleischmann and Meyr (1997). However in FFL-ATSP, as a result of ATSP adaptation, there is no need to pre-define a fix number of setups for a machine in each period. Table 3-1 shows the number of variables and constraints in models FFL-CC, FFL-FM and FFL-ATSP. Note that FFL-ATSP has a much smaller number of continuous and total variables than FFL-CC and FFL-FM with one fewer order of magnitude. The order of magnitude refers to the number of powers of J (the number of products). Number of binary variable in FFL-ATSP and FFL-FM is equal whereas FFL-CC has more binary variables with one more order of magnitude.

Table 3-1: Number of variables and constraints in FFL-CC , FFL-FM and FFL-ATSP

Number of:	FFL-CC	FFL-FM	FFL-ATSP
Continuous variables	$J^2TM + JT(E+1) + 1$ $\boldsymbol{O(n^2)}$	$J^3TM + J^2TM + JT(E+1) + 1$ $\boldsymbol{O(n^3)}$	$JT(3M + E + 1) + 1$ $\boldsymbol{O(n^1)}$
Binary variables	J^3TM $\boldsymbol{O(n^3)}$	J^2TM $\boldsymbol{O(n^2)}$	J^2TM $\boldsymbol{O(n^2)}$
Total variables	$J^3TM + J^2TM + JT(E+1) + 1$ $\boldsymbol{O(n^3)}$	$J^3TM + 2J^2TM + JT(E+1) + 1$ $\boldsymbol{O(n^3)}$	$JTM(J+3) + JT(E+1) + 1$ $\boldsymbol{O(n^2)}$
Constraint	$JT(3JM + E + 1) + JM(J-2)$ $+M(T+1) + 1$ $\boldsymbol{O(n^2)}$	$J^2TM(2+J) + JM(1-J)$ $+JT(E+1) + TM(1+2J) + 1$ $\boldsymbol{O(n^2)}$	$JTM(J+4) + M(2T+1)$ $+JT(E+1) + 1$ $\boldsymbol{O(n^2)}$

FFL-ATSP has fewest constraints while the order of magnitude of the number of constraints is the same in all the three models. The computational tests in the next section will provide more insights into the relative efficiencies of the three models.

3.6 Experimental design

The aim of this section is to compare the novel linear MIP model FFL-ATSP with FFL-CC and FFL-FM through computational tests on small and larger problems. Özdamar and Barbarasoglu (1999) designed test problems to solve the CLSP in FFLs. Later Quadt(2004) also used their testing method. This thesis will do the same by varying the attributes of the problem data to test the performance of the models under different conditions. These attributes are conspicuously used in various lotsizing problems in the literature (Buschkühl et al., 2010) and tend to have a significant effect on solution quality.

The experimental design used the following factors:

1. Model formulation
2. Attribute and Dimensionality of Problem
3. Variability of demand
4. Inventory holding cost
5. Tightness of capacity

The problem parameters outside the statistical experimental design are randomly generated as follows: Processing times b_{im} (in hours) are generated from uniform distribution U(1,5) for all products i and machines m. Setup costs sc_{ijm} are generated from U(300,500). Set-up times are related to the total processing time:

$$st_{ijm} = \frac{S \sum_{jtm} b_{jm} \cdot d_{jt}}{T \cdot Maxfac} \tag{3-44}$$

where S is generated from U(0.05,0.10) and $Maxfac = max_e\{M_e\}$ is the maximum number of machines at any stage. In the other words setup times are proportional to the mean production time per machine-period.

The factor levels within the experimental design are randomly generated as follows:

1. Model Formulation: FFL-ATSP, FFL-CC and FFL-FM
2. The attributes and dimensionality of Problem (discussed further below):
    a. Small: $E = 2, M_e = 2, J = 4, T = 6$
    b. Large: $E = 3, M_e = 3, J = 8, T = 6$
3. Demand variability is either low, d_{it} being generated from U(90,110), or high, from U(50,150).
4. Holding and backordering costs assume that successive stages add value, so that work-in-process holding costs will increase as material progresses along the line. To reflect this, a value-added percentage factor VAP is used, whose value is 1.1 (low) or 1.3 (high). Inventory costs are then generated consecutively as follows: The first stage's unit holding cost h_{it1} for product i is generated from U(1,20). For subsequent stages, $h_{ite} = VAP \cdot h_{it,e-1}$ for $e \geq 2$. The backordering cost for product i is $B_{it} = 1.25 \cdot h_{itE}$.
5. Capacity tightness is measured by a factor CAT with value 1.2 (tight) or 1.6 (loose). The mean capacity requirement C per machine at each stage is calculated as:

$$C = \max_e \left\{ \frac{\sum_{jtm} b_{jm} \cdot d_{jt}}{T \cdot Maxfac} \right\} \tag{3-45}$$

which is the maximum, over all stages, of the mean production time per machine-period. The capacity C_{mt} on machine *m* in macro-period *t* is then given by $C_{mt} = CAT \cdot C$.

Özdamar and Barbarosoglu (1999) did not specify the permitted percentage BP of end item demand that can be backordered, but this work considers the value of 1.0 for BP which permits backlogging. Minimum lot size or lower bound on the quantity of product is considered 10 for all products.

Considering the last three experimental attributes above, $2^3 = 8$ combinations were generated for each of the three models and the two sets of test problems (small and large) of very different dimensionality.

3.7 Computational results

To obtain some insight into the dimensionality of the models and problems, the number of continuous and binary variables of the Özdamar and Barbarosoglu (1999) model, and FFL-CC and FFL-FM models for the small and large problems attributes in the former's paper (OzBa 1999) was computed in (Mahdieh et al., 2012) and shown in Table 3-2. The conspicuous features are the huge number of binary variables in FFL-CC and continuous variables in FFL-FM, some 400 and 280 times more respectively than in OzBa (1999) for the big instance as a result of modelling of sequence-dependent setups. Initial computational test indicated that the FFL-CC model is faster and more effective than the FFL-FM (Mahdieh et al., 2012).

Table 3-2: OzBa (1999) attributes and comparing with FFL-CC and FFL-FM

Model	Small problems $E = 3, M_e = 3, J = 5, T = 6$			Large problems $E = 4, M_e = 5, J = 20, T = 6$		
	Continuous Variables	Binary Variables	Total Variables	Continuous Variables	Binary Variables	Total Variables
OzBa 1999	534	270	804	3,600	2,400	6,000
FFL-CC	1,471	6,750	8,221	48,601	960,000	1,008,601
FFL-FM	8,221	1,350	9,571	1,008,601	48,000	1,056,601

In this thesis, the new model for GLSP-FFL problem; FFL-ATSP, causes an enormous reduction in number of binary variables through the adaption of Asymmetric Travelling Salesman Problem constraints, being more efficient in comparison with the previous models.

Table 3-3: Number of constraints and variables in models FFL-ATSP, FFL-CC and FFL-FM

Number of:	Small problem $E = 2, M_e = 2, J = 4, T = 6$			Large problem $E = 3, M_e = 3, J = 8, T = 6$		
	FFL-ATSP	FFL-CC	FFL-FM	FFL-ATSP	FFL-CC	FFL-FM
Constraints	893	1,285	2,545	5,494	11,056	35,167
Continuous variables	361	457	1,993	1,489	3,649	31,297
Binary variables	384	1,536	384	3,456	27,648	3,456
Total variables	745	1,993	2,377	4,945	31,297	34,753

Table 3-3 illustrates the number of constraints and variables of the three models, calculated from the problem attributes of GLSP-FFL. As shown in Table 3-3, FFL-ATSP has considerably fewer binary variables than the FFL-CC and FFL-FM models for both small and large problems. The models are implemented in the optimisation modelling software GAMS build 23.6.5 (Brooke et al., 1988) and solved using the industrial-strength CPLEX 12.0 solver (CPLEX., 2010) on a computer with a 2.1 GHZ CPU and 2 GB of RAM. A one-hour time limit is set for CPLEX to run the models.

Twenty replications were generated for each combination of small problems, so in total 20*8=160 small problems were generated. Each replication was corresponded to different random seeds for generating demand, processing time, holding cost, setup time and cost.

For all (160) small problems, the FFL-ATSP model either with lifting constraint (3-41a) or without lifting constraint (3-41a) found the optimal solution in a mean time of 10 seconds, while the FFL-CC and FFL-FM models not only could not find an optimal solution in one hour but also ended the search with a large optimality gap: 47% for FFL-CC and 83% for FFL-FM on average. Table 3-4 shows an average percentage difference $\%(\frac{Cost-Opt}{Opt})$ between the objective function of FFL-CC and FFL-FM and the optimal cost obtained by FFL-ATSP for each combination. Observe that, similar to the result of previous study (Mahdieh et al., 2012), the FFL-CC model has a much smaller optimality gap than FFL-FM with better best possible solution.

Table 3-4: Average percentage difference between FFL-CC and FFL-FM and the optimal solution overall small problems for each combination

problem attributes for each combination with 20 problems			Average percentage of opt gap $\%(\frac{Cost-Opt}{Opt})$	
Demand	**Holding cost VAP**	**Capacity tightness CAT**	**FFL-CC** model	**FFL-FM** model
Low	1.1	1.2	4.44	14.39
Low	1.1	1.6	5.37	15.54
Low	1.3	1.2	4.50	14.03
Low	1.3	1.6	5.72	16.99
High	1.1	1.2	4.43	17.75
High	1.1	1.6	4.84	14.15
High	1.3	1.2	4.27	13.61
High	1.3	1.6	5.95	16.11
Overall all (160) small problems			**4.94**	**15.32**

Due to the long solution computing times, just one replication of the big problems (8 problems) were generated. The one-hour time limit was removed so that CPLEX terminated when the 2 GB of available RAM was exhausted by the branch-&-cut search. Table 3-5 shows the result of the three models for the big problems indicating the means of the CPU time, percentage of optimality gap and RAM usage.

Observe that both the FFL-CC and FFL-FM models left very large optimality gaps, 81.9% and 99.1% respectively, at termination after about 6 to 7 hours of running time, while FFL-ATSP with original MTZ formulation (i.e. without lifting, FFL-ATSP^{L-}) terminated in less than an hour with 12.5% optimality gap and better solutions for all problems in comparison with FFL-CC and FFL-FM.

Table 3-5: Mean CPLEX results for big problems

Models	Average of CPU time (secs)	Average of Percentage of optimality gap (OPTCR)	Average of RAM usage (MB)
FFL-ATSP^{L-}	3,310	**12.5%**	1886
FFL-CC	21,628	81.9%	1867
FFL-FM	23,919	99.1%	1756

The results of FFL-ATSP model with lifting (ATSP^{L+}) and without lifting (ATSP^{L-}) for big problems are also shown in Table 3-6 and indicate that the MTZ original formulation (ATSP^{L-}) results in an overall lower optimality gap (12.5%) and less CPU time (3,440 seconds) rather than the lifted formulation with 15.4% optimality gap in the average of 4,746 seconds.

Table 3-6: Results of CPLEX optimality gap, Best Possible (BP) solution and CPU time for FFL-ATSP with and without lifting

problem attributes for each combination with one problem			CPLEX opt gap % $\left(\frac{\lvert BI-BP\rvert}{BP}\right)$		Best Possible (BP) solution cost		CPU time (secs)	
Demand	**Holding cost VAP**	**Capacity tightness CAT**	ATSP^{L-}	ATSP^{L+}	ATSP^{L-}	ATSP^{L+}	ATSP^{L-}	ATSP^{L+}
Low	1.1	1.2	12.5	18.8	27,998	29,732	3,216	2,982
Low	1.1	1.6	12.6	16.1	31,651	32,656	3,762	3,822
Low	1.3	1.2	12.7	16.2	31,355	32,655	3,210	3,752
Low	1.3	1.6	12.7	8.2	30,524	29,153	3,200	3,191
High	1.1	1.2	13.7	17.8	28,246	28,961	3,365	4,751
High	1.1	1.6	11.0	20.9	25,761	27,834	3,491	3,609
High	1.3	1.2	13.0	8.2	31,111	29,152	3,618	12,011
High	1.3	1.6	12.2	16.8	28,011	29,732	3,661	3,851
Overall all (8) big problems			**12.5**	15.4	---	---	3,440	4,746

Note that All GAMS solvers use LP based branch-and-bound algorithms for solving MIPs and keep “best integer” (BI) and “best estimate or best possible” (BP) while they run. The best integer is the best solution that satisfies all integer requirements found so far. The best estimate provides a bound for the optimal integer solution. As we don’t have the optimal solution, the quality of a solution can be measured as the distance between best integer and a bound for the optimal solution (best estimate). This value is called the absolute gap (GAMS notation OPTCA). The absolute gap depends on the magnitude of the best estimate and the best integer therefore GAMS defined the relative gap as the optimality gap (in GAMS notation OPTCR) which equals to absolute gap divided by best possible ($\frac{|BI-BP|}{BP}$).

As the numerical test shows, the new ATSP adaptation for GLSP-FFL has significant improvement in both small and large problems solutions. Two statistical tests including a Balanced ANOVA and the non-parametric Friedman test were carried to test significant differences between the means and the medians of the solution values of the three models respectively. Note that both tests indicate highly significant differences (p=0.00) on all the combinations. A Two-Way, also known as a Two-Factor ANOVA Analysis with Replication was carried to test the existence of an interaction between the models and combinations for small problems with 20 replications. Table 3-7 provides the details of the ANOVA results highly indicating no significant interaction between two factors: models and combinations (p=1.00) and also no significant difference between 8 combinations (p=0.788).

Table 3-7: Results of Two-Factor ANOVA with Replication for small problems

Source of Variation	Sum of squares	Degrees of freedom	Mean square	F_0	p-value
Combinations	7	1784465	254924	0.56	0.788
Models	2	115426399	57713199	126.88	**0.000**
Interactions	14	843450	60246	0.13	**1.000**
Error	456	207417627	454863		
Total	479	325471940			

3.8 Final remarks

In this chapter, three mathematical models have been presented for the simultaneously lot sizing and scheduling of flexible flow lines considering practical assumptions in FFL manufacturing systems like lot splitting and shortage. The first model (FFL-CC) is based on Clark and Clark's (2000) sequencing formulation technique while the second model (FFL-FM) is based on Fleischmann and Meyr's (1997). The computational tests indicate that the FFL-CC is more effective and has a much smaller optimality gap than FFL-FM. The third novel MIP model (FFL-ATSP) is based on adaptation of ATSP that shows significant improvement of problems solutions for both problem sizes in very much shorter time than FFL-CC and FFL-FM. For example, for small problems FFL-ATSP found the optimal solution in a mean time of 10 seconds, while the FFL-CC and FFL-FM models not only could not find an optimal solution in one hour but also ended the search with a large optimality gap. Three different ANOVA tests including a Balanced ANOVA and the non-parametric Friedman test and Two-Factor ANOVA Analysis with Replication have been carried to compare the models. The result indicated highly significant differences (p=0.00) between three models and no significant interactions between models and combinations (p=1.00), and different levels of combinations (p=0.788).

GLSP-FFL is an NP-complete problem and even a well designed exact MIP model FFL-ATSP, cannot find any feasible solution in reasonable computing time for some large problems. Hence, it is necessary to develop an efficient solution procedure. The next chapter is devoted to a meta-heuristic algorithm, Adaptive Simulated Annealing, with three initial solutions for solving GLSP-FFL.

Chapter 4

An Adaptive Simulated Annealing for GLSP-FFL

The General Lotsizing and Scheduling Problem in Flexible Flow Line (GLSP-FFL) optimizes the lot sizing and scheduling of multiple products at multiple stages, each stage having multiple machines in parallel. The problem is complex as any product can be processed on any machine but with different process rates and sequence-dependent setup times & costs. Therefore, it is impossible to find even a feasible solution for large problem within a reasonable time period by solving an exact MIP model and it is necessary to develop efficient algorithm for GLSP-FFL. Moreover working with population based algorithm like genetic algorithm is too complex as even finding small size initial population within a reasonable time is impossible. Previous research found that Simulated Annealing (SA) results in better solution for GLSP-FFL than Tabu Search. Moreover SA is relatively easy to code even for complex problems like GLSP-FFL and generally gives a good solution. Hence this chapter is devoted to design an efficient neighbourhood search algorithm, Adaptive Simulated Annealing with four initial solutions. The effectiveness of the proposed simulated annealing and the initial solutions is evaluated by numerical tests.

4.1 Adaptive Simulated Annealing

Simulated Annealing (SA) is a local search method that finds its inspiration in the physical annealing process studied in statistical mechanics (Metropolis et al.,

1953) and was initially proposed by Kirkpatrick and Gelatt(1983) for combinatorial optimization problems. An SA algorithm repeats an iterative neighbour generation procedure and follows search directions that aim to improve the objective function value towards a global optimum.

In a minimisation problem, if the cost value of the neighbour solution $(cost_n)$ is lower than that of the current solution$(cost_c)$, then a move to the neighbour solution is made. However, if the neighbour does not improve the current solution, then there is still a chance of transition by comparing a uniform random number with the transition probability function (4-1). If the probability value is greater than or equal to the random number, then the transition to the worse solution is accepted.

$$P(Transition) = \exp\left(-\frac{\Delta C_i}{T_i}\right) \tag{4-1}$$

where ΔC_i is the cost difference between the neighbour solution and the current solution in iteration i and T_i is the SA temperature. As the algorithm progresses, the temperature decreases according to a function called the cooling schedule. The initial and most frequently used cooling schedule is Geometric $(T_{i+1} = \propto . T_i)$ where the temperature reduces by a constant factor, known as the cooling factor $(0 < \propto < 1)$, after predetermined number of iterations. The advantage of geometric cooling schedule is its simplicity and that it can provide some baseline for comparison with more sophisticated schedules. However regardless of whatever cooling factor is chosen, it seems that the search is less likely to move out of local minimal at a critical stage either as it is not cooling quickly enough or as it is cooling too quickly (Dowsland, 1993, Triki et al., 2005). To overcome this, several theoretical and empirical cooling schedules have been proposed in the literature such as monotonic schedules, adaptive schedules and quadratic cooling schedules (Geng et al., 2011, Gong et al., 2001, Hajek, 1988, Huang et al., 1986, Laarhoven and Aarts, 1987, Otten and Van Ginneken, 1984, Schneider and Puchta, 2010, Thompson and Dowsland, 1996, Van Laarhoven et al., 1992).

In contrast to monotonic schedules, adaptive cooling schedules calculate the next temperature value based on the past search history. Dowsland (1993) considered two functions, one for cooling down $(T/(1 + \beta . T)$ that reduces the temperature when a move is accepted, and another for heating up $(T/(1 - \propto . T)$ that increases the temperature gradually when a move is rejected. Özdamar and Barbarosolu(1999) used only Dowsland's cooling function for multi-stage capacitated lot sizing problem, where β is an adaptive parameter according to the search status. If the last predetermined number of consequent moves result in improving cost values, then $\beta$is decreased by subtracting a step size of less than 1.0 and is increased if they are non-improving. Özdamar and Barbarosolu set the initial value of temperature and β to 1.0 and considered 0.01 as a step size for β.

In this thesis, the Geometric cooling scheme by Kirkpatrick and Gelatt(1983) and adaptive cooling schedule by Dowsland (1993) were originally applied for GLSP-FFL, but through an initial computational test it was observed that they are not effective temperature control schemes. Since there is an exponential term in the transition probability function, using the absolute cost difference (ΔC) drives the transition probability close to zero in problems with big values in the objective function. To alleviate this problem, the relative cost difference ($\Delta C/cost_c$) has been used by some researchers rather than ΔC(Özdamar and Barbaroso lu, 1999). However, the relative cost difference could get very small and drive the transition probability close to zero.

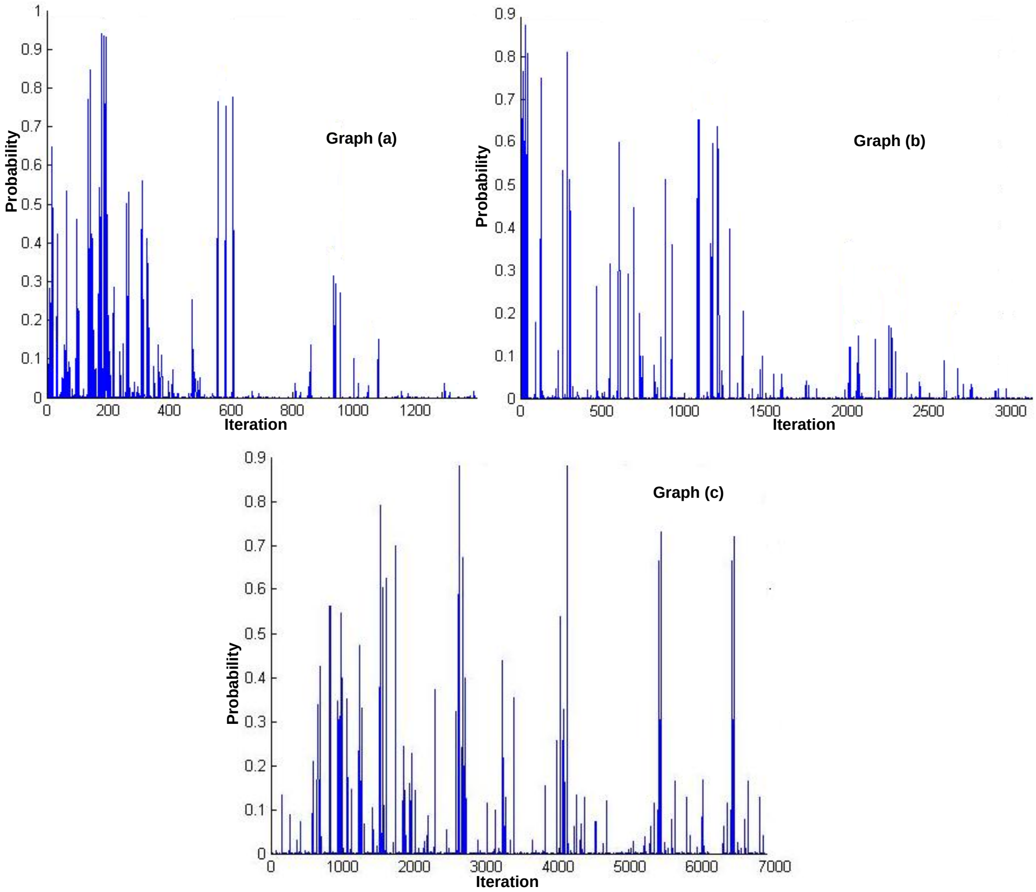

Figure 4-1: Transition probability of; graph (a) Geometric SA, graph (b): Dowsland's SA and graph (c): Azizi and Zolfaghari's SA

Both SAs were run with random seeds for GLSP-FFL problem and, as shown in Figure (4-1) graph (a) Geometric cooling schedule and Dowsland's cooling schedule graph(b), were driven to zero after 1000 and 2000 iterations respectively. Dowsland's adaptive cooling schedule works better but not for iterations more than about 2500, because when the temperature declines as the search continues, β does not have a significant effect on changing a relatively low temperature towards the end of the search. Therefore this approach could not be very useful particularly if local optimal are not near the start point of the search. Azizi and Zolfaghari(2004) designed a new adaptive temperature control that maintains the temperature above minimum level and applied it on job shop scheduling problems. Their temperature is controlled by a single function: $T_i = T_{min} + \lambda \ln(1 + r_i)$ where T_{min} is the minimum value that the temperature can take ($T_0 = T_{min}$), λ is a coefficient that controls the rate of temperature rise, and r_i is the number of consecutive upward moves at iteration i with initial value of zero. If a move results in a higher cost value (upward move) then the counter r_i increases by 1.0 and but remains unchanged if the new solution has the same cost value and if a move results in improving cost then r_i is equal to zero. Both parameters T_{min} and λ were set to 1.0 in Azizi and Zolfaghari's paper.

$$r_i = \begin{cases} r_{i-1} + 1 & if \quad \Delta C_i > 0 \\ r_{i-1} & if \quad \Delta C_i = 0 \\ 0 & if \quad \Delta C_i < 0 \end{cases} \tag{4-2}$$

This dynamic temperature control scheme gives a higher chance of an uphill move once the search starts climbing up regardless of the iteration number i and treats neighbouring solutions at the beginning of the search in the same way as the neighbouring solutions near the end of the search. Thus in this thesis Azizi and Zolfaghari's cooling schedule was applied for to the GLSP-FFL problem and the SA is a form of Adaptive Simulated Annealing (ASA) (Azizi and Zolfaghari, 2004). To obtain the best value for the ASA's parameters (T_{min}, λ), several values have been tested and the consequent behaviour of transition probability function and temperature during the search has been analyzed. By increasing the value of λ, the search spends less time looking for good solutions in its current neighbours and might go to unfavourable regions of solution space. On the other hand, a very small value of λ reduces the effect of the temperature dynamic adjustments and the chance of escaping local optimal. Moreover another advantage of ASA is avoiding the transition probability to be driven to zero because of using either absolute or relative cost differences. As shown in Figure (4-1) graph (c), even after 6,000 iterations, there is still a chance of uphill moves with high probability. Regardless of how big or small the objective function, the transition probability is adjusted by choosing the appropriate value of $\mathrm{T_{min}}$. In this study, the value of $\{1,5,10,15,20,25\}$ tested for λ and T_{min} and the value of 10 for both λ and T_{min}, revealed the best results for 5 samples of small size and 5samples of big sizes GLSP-FFL problem.

Figure (4-2) dramatically shows the effect of ASA's parameters on the search. The green dots indicate the cost of solutions at the iterations and the best found solutions are shown with red dots. As demonstrated in graph (a), better solutions are always accepted (green dots = red dots) and there is no chance of uphill moves and escaping from local optimal while both λ and T_{min} are set to 1. On the other hand, in graph (b), the big value of the parameters ($\lambda = T_{min} = 100$) makes causes too many upward moves thus going in undesirable directions of search from which it is hard to get back on track.

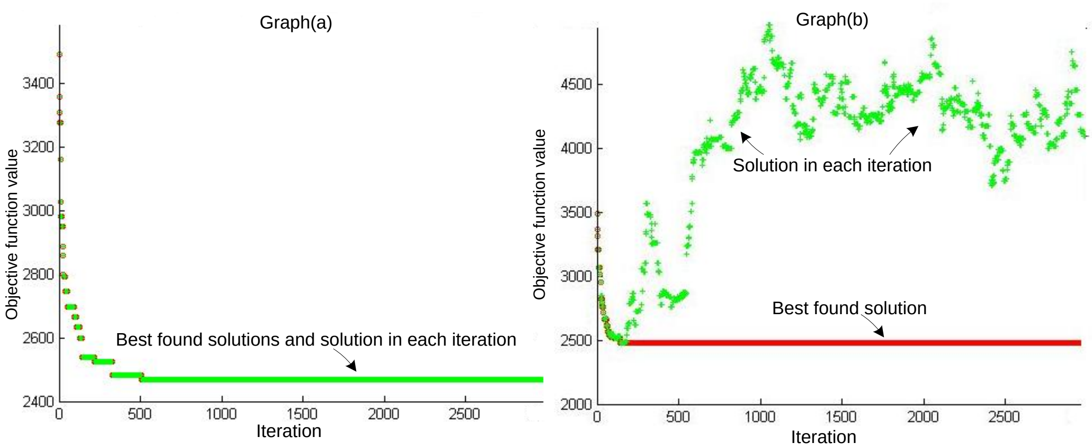

Figure 4-2: Progress of ASA, graph (a): ($\lambda = \mathrm{T_{min}} = 1$) and graph (b): ($\lambda = \mathrm{T_{min}} = 100$)
(+ dot): solution in each iteration; (O dot): best found solution;

4.2 Initial solution

In this thesis, four initial solutions are designed for the ASA algorithm. All the initial solutions are inventory-feasible solutions and satisfy constraints (4-1) and (4-2), while they can be capacity-infeasible solutions and do not satisfy constraint (4-3). The ASA tries to improve the objective function in a given number of moves while capacity infeasibilities are heavily penalized in the objection function. It is important that inventory-feasibility is always preserved when generating neighbours by the neighbour operators. Inventory and capacity constraints are as following:

Inventory constraints:

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E} x_{jmt} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\ j,t \quad (4\text{-}1)$$

$$I_{je,t-1} + \sum_{m_e} x_{jmt} - I_{jet} = \sum_{m_{e+1}} x_{jm,t+1} \qquad \forall\, j,t \text{ and } e = 1,\dots,E-1 \quad (4\text{-}2)$$

Capacity constraint:

$$\sum_{ij} st_{ijm}\ y_{ijmt} + \sum_{i} b_{im}\ x_{imt} \le C_{mt} \qquad \forall\ e,m,t \quad (4\text{-}3)$$

In order to simplify ASA, the one-period-backward shifted demand is considered for intermediate stages ($e < E$), means that $x_{jm,t+1}$ in the right hand of equation (4-2) changes to x_{jmt} . The new inventory diagram is shown in Figure (4-3). The final solution obtained by the ASA is equivalent to the origin GLSP-FFL and it can simply shift forward to be the same as the original one.

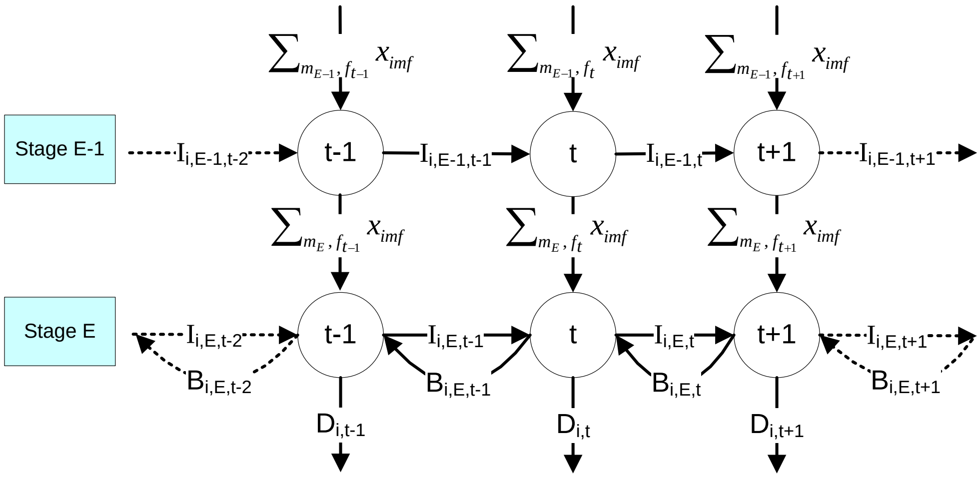

Figure 4-3: New inventory diagram

4.2.1 First initial solution

The first initial solution is generated by randomly assigning all the products to the machines of each stage. The same procedure is repeated for all the periods to generate a sequence. Then the sequence is given to the linear lot sizing model of GLSP-FFL in order to determine the optimal lot size of the given sequence and to test its feasibility. Thus, in the flexible flow line lot sizing model $y_{ij\,m_e f_t^{m_e}}$ is a parameter and the decision variables are:

I_{iet}	Inventory level of product *i* in stage *e* at the end of macro-period *t*.
B_{iEt}	Backordered amount of end-product *i* at the last stage *E* at the end of macro-period *t*.
x_{imf}	Production quantity of product *i* on machine *m* in micro-period *f*.

The data is the same as previous models and constraints are as follows:

$$\sum_{ijemtf} sc_{ijm}\ y_{ijmf} + \sum_{ite} h_{ite}\ I_{iet} + \sum_{it} g_{it}\ B_{iEt} \tag{4-4}$$

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E, f_t} x_{jmf} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\, j,t \tag{4-5}$$

$$I_{je,t-1} + \sum_{m_e, f_t} x_{jmf} - I_{jet} = \sum_{m_{e+1}, f_t} x_{jmf} \qquad \forall\, j,t\ and\ e = 1,\dots,E-1 \tag{4-6}$$

$$B_{itE} \le BP \cdot d_{it} \qquad \forall\, i,t \tag{4-7}$$

$$\sum_{ijf} st_{ijm}\ y_{ijmf} + \sum_{if} b_{im}\ x_{imf} \le C_{mt} \qquad \forall\, e,m,t \tag{4-8}$$

$$x_{jmf} \le UB_{jmt} \sum_{i} y_{ijmf} \qquad \forall\, j,e,m,t,f \tag{4-9}$$

$$x_{jmf} \ge LB_{jmt} \sum_{i \ne j} y_{ijmf} \qquad \forall\, e,m,t,j,f \tag{4-10}$$

The objective function (4-4) minimises backorders and inventory costs. Note that setup cost ($\sum_{ijemtf} sc_{ijm}\ y_{ijmf}$) is a constant value in the objective function. The material balance, bounding backorder and capacity limit are expressed in constraints (4-5) to (4-8). Constraint (4-9) determines the appropriate production lot sizes for a given sequence and the minimum lot sizes are enforced by constraint (4-9).

The CPU time for solving the GLSP-FFL lot sizing model by GAMS is less than one second even for big problems. If the model results in a feasible solution then the sequence with its optimal lot size is considered as a first ASA initial solution otherwise the process is repeated until the feasible solution is achieved. Most of the time, the first sequence has feasible lot sizes, because of considering the production of the all the items in each stage and each period.

4.2.2 Second initial solution

Firstly the quantity of products in each stage and period are given by the external demand of last stage ($X_{ejt} = d_{jt}$). Then the product lot sizes (X_{ejt}) are given to the Loading Heuristic (LHR) for assigning to the machines of each stage.

Because of the assumption of $X_{ejt} = d_{jt}$, the LHR solution is inventory-feasible but may be capacity-infeasible. To check capacity-feasibility and find the optimal lot sizes of LHR solution, the sequence of LHR is given to the linear lot sizing model of GLSP-FFL. If the model results in a feasible solution, then the sequence with its optimal lot size is considered as a second ASA initial solution otherwise the solution of LHR is given to ASA and most often it becomes capacity-feasible in a few number of ASA moves as capacity infeasibilities are heavily penalized in the objection function of ASA. The LHR algorithm is as follow:

In the LHR, the given lot sizes (X_{ejt}) are scheduled on the parallel machines of each stage. Firstly, the products with positive lot size ($X_{ejt} > 0$) are assigned to a product list for each period and stage. Then a capacity factor of machines on that stage and period are computed by equation (4-11) for all the listed products. The capacity factor is the summation of production time, setup time by considering the previous product (i') and used capacity of the machine.

$$capfactor(j, m_e) = usecap(m_e) + st_{i' j m_e} + X_{ejt} \times b_{j m_e} \tag{4-11}$$

After computing capacity factors, the machine and the product (m_e^*, j^*) with minimum capacity factor is determined. The product $j^*$is assigned to the machine m_e^* and then j^* is eliminated from the list and machine used capacity ($usecap(m_e^*)$) is updated. This is repeated until the list becomes empty. The steps of LHR are as follows:

Step 1: Select a pair (e, t) and do the steps 1-1 to 1-5 for this pair.

Step 1-1: Assign all the items with positive lot size ($X_{ejt} > 0$) into the product list.

Step 1-2: Calculate the capacity factor for all the listed products by (4-11).

Step 1-3: Select a pair of product and machine with minimum capacity factor from the list (m_e^*, j^*) and assign product j^* to the machine m_e^*.

Step 1-4: Eliminate product $j^*$from the list and update the machine m_e^* used capacity.

Step 1-5: If the list is empty then go to the next step else go to the step 1-2.

Step 2: If all the pairs (e, t) are selected then stop else go to the step 1 and select another one.

4.2.3 Third initial solution

The third initial solution is generated by solving the compressed and well-organized model which extracts from the GLSP-FFL. The model is Capacitated Lot Sizing Problem for Multi Stage systems (MS-CLSP) with single machine in each stage. The MS-CLSP is the CLSP problem for single flow line which is much smaller than the GLSP-FFL and is optimally solved less than a second even for the big size problem. For example the number of binary variables in the MS-CLSP model for big size problem with $E = 3, M_e = 3, J = 8, T = 6$ is 144 however this number is 3,456 in FFL-ATSP and FFL-FM and 27,648 in FFL-CC.

The MS-CLSP is a MIP model and Variable x_{ejt} indicates a production lot size of product *j* in stage *e* and period *t*. Binary variable for setup of product *j* in stage *e* and period *t* is shown by the y_{ejt} . The model is as follows:

$$Min \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{e=1}^{E} I_{jte} \times h_{jte} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_{jt} \times B_{jtE} + \sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{j=1}^{J} y_{ejt} \times sc_{je} \qquad (4\text{-}12)$$

$$I_{j(t-1)E} - B_{j(t-1)E} + x_{Ejt} - I_{jtE} + B_{jtE} = d_{jt} \qquad \forall\, j,t (4\text{-}13)$$

$$I_{j(t-1)e} + x_{ejt} - I_{jte} = x_{(e+1)jt} \qquad \forall\, j,t,e = 1,\dots,E-1 (4\text{-}14)$$

$$B_{jtE} \le BP \times d_{jt} \qquad \forall\, j,t 4\text{-}15)$$

$$\sum_{j=1}^{J} st_{je} \times y_{ejt} + \sum_{j=1}^{J} b_{je} \times x_{ejt} \le C_{et} \qquad \forall\, j,t (4\text{-}16)$$

$$x_{ejt} \le y_{ejt} \times \left(\frac{C_{et}}{b_{je}}\right) \qquad \forall\, j,t,e (4\text{-}17)$$

The model parameters are obtained from the GLSP-FFS parameters. Therefore the capacity parameter is computed by $C_{et} = \sum_{m_e=1}^{M_e} C_{m_e t}$ for each stage and period and b_{je} is the required time to produce *j* in stage *e* which is given by $b_{je} = max_{m_e}\{b_{jm_e}\}$. The setup time and cost for producing *j* in stage *e* is calculated by $st_{je} = max_i\{st_{ijm'_e}\}$ and $sc_{je} = max_i\{sc_{ijm'_e}\}$ respectively, where m'_e is a machine with maximum production time for producing *j* in stage *e*. The objective function (4-12) minimises backorders, inventory and setup costs. The material balance, bounding backorder and capacity limit are presented in constraints (4-13) to (4-16). Constraint (4-17) enforces the appropriate setup before production. After solving model, lot sizes (x_{ejt}) are given to the LHR ($X_{ejt} = x_{ejt}$) to be scheduled on the parallel machines of each stage. The result of LHR is considered as the third initial solution of ASA.

The first initial solution is feasible but the second and third solutions may be capacity-infeasible solutions. However, because of the MS-CLSP model structure and assigning and scheduling lots by LHR, the third initial solution is most often a feasible solution. Nevertheless in the ASA procedure capacity infeasibilities are penalized by (4-18) in the objective function.

$$PenaltyCost = extracap(m_{et}) \times PenalFactor \qquad (4\text{-}18)$$

The amount of capacity violation of machine in each stage and period is shown by $extracap(m_{et})$ and penalized by the big number PenalFactor.

4.2.4 Fourth initial solution

For fourth initial solution, firstly the sequence is generated by solving the sequencing model of the GLSP-FFL. Similar to ATSP, the sequencing model determines the product sequences on machines of each stage in order to minimise the sequence-dependent setup cost (4-19) without considering capacity and inventory constraints. Then the sequence is given to the linear lot sizing model of GLSP-FFL (explained in first initial solution) to determine its optimal lot size. The sequencing model of GLSP-FFL is MIP and binary variable y_{ijmt} indicates the setup from i to j on machine m, stage e and period t. The model is as follow:

$$\sum_{ijemt} sc_{ijm}\ y_{ijmt} \qquad (4\text{-}19)$$

$$\sum_{i=1}^{J} \alpha_{imt} = 1 \qquad \forall\ e,m,t \quad (4\text{-}20)$$

$$\alpha_{i_{om}mt} = 1 \qquad \forall\, e,m,t = 1 \quad (4\text{-}21)$$

$$\alpha_{imt} + \sum_{j} y_{jimt} = \sum_{j} y_{ijmt} + \alpha_{im(t+1)} \qquad \forall\ e,m,i \text{ and } t = 1 \ldots, T-1 \quad (4\text{-}22)$$

$$v_{imt} - v_{jmt} + J \times y_{ijmt} \leq J - 1 \qquad \forall\, e,m,i,j,t \quad (4\text{-}23)$$

$$\sum_{j,m} y_{jimt} + \sum_{m} \alpha_{imt} \geq 1 \qquad \forall\, e,i,t \quad (4\text{-}24)$$

$$\sum_{j,i} y_{jimt} + \sum_{i} \alpha_{imt} \leq \frac{J}{M_e} + 1 \qquad \forall\, e,m,t \quad (4\text{-}25)$$

Constraints (4-20)-(4-23) similar to constraints (3-38)-(3-41) of FFL-ATSP entirely determine the sequence of products on a machine in each period and cause a setup carryover of the machine between periods. Constraint (4-24) forces the production of all items in each stage and period. So items are distributed on machines in each stage with the minimal setup cost. However the optimal setup cost sequence may not be a feasible sequence particularly capacity feasible solution for the GLSP-FFL as for example one machine may have too many products and another one may not be assigned even one item. To alleviate this problem, for each period, constraint (4-25) balances the number of items on machines of each stage by not allowing production of more than $(\frac{J}{M_e} + 1)$ on each machine. Consider, for example if there are $J = 10$ products and $M_e = 3$ machines in each stage, the number of products on each machine should be less or equal to *4* $(\frac{10}{3} + 1 \cong 4)$ items which results in more likely capacity feasible sequence.

4.3 Neighbour operators

Neighbour generation is a crucial issue in ASA algorithm. In this thesis three neighbour operators are developed for GLSP-FFL in order to efficiently search the solution space by changing lot sizes and sequences while the solution inventory feasibility is preserved. These operators consist of shifting part or whole lot sizes forward to the next period or backward to the previous period or to the same or different machine of stage at the current period. They are called forward, backward and machine-to-machine shifting operators respectively.

For generating a neighbour from the current solution, firstly the triplet (e, j, t) is selected randomly while the lot size of the product j in the period t and the stage e is positive. Then, it is randomly decided whether the selected lot size will be forward $(t^* = t + 1)$ or backward shifted $(t^* = t - 1)$, or transferred to the same or another machine in the same period $(t^* = t)$. The maximum amount of the selected lot size that will be forward shifted $(\mathrm{t}^* > t)$ with considering inventory feasibility preservation is given by (4-26). It is obvious that $B_{e,j,t}$ and BP are zeros for the intermediate stages $(e < E)$.

$$\Delta_{max} = min\{x_{e,j,t}, I_{e,j,t} - B_{e,j,t} + BP \times d_{e,j,t}\} \tag{4-26}$$

In fact the maximum amount that can be shifted is limited both by the lot size, the backorder and inventory limit of period t. On the other hand the maximum amount of the lot size that will be backward shifted $(t^* < t)$ is given by (4-27).

$$\Delta_{max} = min\,\{x_{e,j,t}, I_{e-1,j,t^*}\} \qquad (4\text{-}27)$$

If stage e is the first stage then Δ_{max} equals to $x_{e,j,t}$. Here, the maximum amount of shifting is limited by the lot size and the inventory of the previous stage of period t^*. Finally the amount that will be shifted forward or backward (Δ) has a 50-50 chance of being $\Delta = \Delta_{max}$ or being a random number within the interval zero to Δ_{max} $(\Delta = Random[0, \Delta_{max}])$. For the machine-to-machine shifting $(t^* = t)$, a machine is selected among the machines which produce j. Then the quantity of lot j on the selected machine is assigned to Δ. The lot (Δ)could be transferred to any sequence of the same or different machine in the stage e.

After determining the Δ by each neighbour operator the quantity of product j is updated by $(x_{e,j,t} = x_{e,j,t} - \Delta)$ and $(x_{e,j,t^*} = x_{e,j,t^*} + \Delta)$. The inventory level for forward shifting is updated by $(I_{e,j,t} - B_{e,j,t} = I_{e,j,t} - B_{e,j,t} - \Delta)$ and $(I_{e-1,j,t} = I_{e-1,j,t} + \Delta)$. Furthermore it is updated by $(I_{e,j,t^*} = I_{e,j,t^*} + \Delta)$ and $(I_{e-1,j,t^*} = I_{e-1,j,t^*} - \Delta)$ for backward shifting. However, the inventory level for machine-to-machine shifting does not change.

In forward and backward shifting for subtracting Δ from the quantity of product j in stage e and period t i.e., $(x_{e,j,t} = x_{e,j,t} - \Delta)$, a machine among the machines which produces product j is randomly selected. Then the Δ subtracts from the quantity of product j on the selected machine. Moreover, If Δ is greater than the lot size of product j on the selected machine then for the remained part of Δ another machine is selected randomly for subtraction. This process repeats until Δ gets zero. In machine-to-machine shifting the Δ is subtracted from the origin machine which the Δ is picked from.

Finally, for all the neighbour operators there is the same procedure in order to schedule the Δ on the machines of stage e and period t^* $(x_{e,j,t^*} = x_{e,j,t^*} + \Delta)$. For this purpose a sequence is determined by searching over all sequences on the different machines and selecting the best sequence with minimum incremental cost. For example if there are two machines in stage e as shown in Figure (4-4), then there are totally 7 choices, 4 sequences on machine 1 and 3 sequences on machine 2 for loading the lot j. In another example, suppose that product j is produced in the first sequence of machine 1 rather than product i in Figure (4-4). Then because of zero setup time and cost the best sequence on the machine 1 for loading the lot j is the first sequence and so there are 4 choices (1 sequence on machine 1 and 3 sequences on machine 2) to load the lot j on.

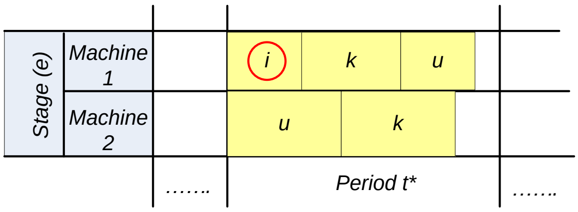

Figure 4-4: An example of searching sequence

The neighbour generating steps are as follows:

Step 1: Select a random triplet (e, j, t) with positive lot size $(X_{ejt} > 0)$.

Step 2: Select a random period t^* among $(t-1, t, t+1)$.

Step 3: If $t^* = t$ then select a machine (in period t^*) among the machines which produces j and assign the quantity of lot j on the selected machine (m^*) to Δ.

Step 4: If $t^* \neq t$ then go to the next step else go to the step 5.

Step 4-1: Calculate Δ_{max} for $t^* > t$ by (4-26) and for $t^* < t$ by (4-27).

Step 4-2: Generate a random number R within zero and one $(R = Random[0,1])$.

Step 4-3: If $R < 0.5$ then $(\Delta_{max} \rightarrow \Delta)$ else $(Random[0, \Delta_{max}] \rightarrow \Delta)$.

Step 5: If $\Delta = 0$ then go the step 1.

Step 6: Update X_{ejt} and X_{ejt^*} by $(x_{e,j,t} = x_{e,j,t} - \Delta)$ and $(x_{e,j,t^*} = x_{e,j,t^*} + \Delta)$.

Step 7: Update the inventory levels (in forward shifting: $(I_{e,j,t} - B_{e,j,t} = I_{e,j,t} - B_{e,j,t} - \Delta)$ and $(I_{e-1,j,t} = I_{e-1,j,t} + \Delta)$ and in backward shifting: $(I_{e,j,t^*} = I_{e,j,t^*} + \Delta)$ and $(I_{e-1,j,t^*} = I_{e-1,j,t^*} - \Delta)$.

Step 8: If $t^* \neq t$ then $\Delta' = \Delta$ go to the next step else go to the step 9.

Step 8-1: Select a machine randomly which produces j in stage e and period t.

Step 8-2: If lot size j on the selected machine is greater than the Δ' then reduce it by Δ' and $\Delta' = 0$ else subtract lot size j from Δ' and then set the lot size j to zero on the selected machine.

Step 8-3: If $\Delta' \neq 0$ then go to the step 8-1.

Step 9: If $t^* = t$ then reduce the lot size j on machine m^* by Δ.

Step 10: Set variable Incrcost to a very big number and select a machine in stage e and period t^{*}.

Step 10-1: If item j is produced on the selected machine then assign its sequence to the lot Δ and $\text{Incrcost}' = 0$ else find the sequence with minimum incremental cost ($\text{Incrcost}'$) for the Δ.

Step 10-2: If $\text{Incrcost}' \leq \text{Incrcost}$ then save the sequence as the best sequence and update Incrcost.

Step 10-3: If all the machines of stage e are considered then go the next step else select another machine and go to the step 10-1.

Step 11: Select the best sequence over all the machines with minimum Incrcost.

4.4 ASA computational test

GLSP-FFL is an NP-hard problem, hence, even a well designed exact MIP model FFL-ATSP, cannot find any feasible solution in reasonable computing time for some large problems. To provide more detailed insight into the complexity of GLSP-FFL, a variety of problem sizes are solved without any time limitation. Figure (4-5) indicates the CPLEX optimality gap for each sizes and details of each problem including number of product, stage, facility and period, percentage of optimality gap and CPU time are shown in Table 4-1.

Table 4-1: Percentage of optimality gap of FFL-ATSP for different problem sizes

Problem Size				Percentage of optimality gap	CPU time
Product	Stage	Facility	period		
6	2	2	6	0%	176
6	2	3	6	0%	194
6	2	4	6	0%	593
6	3	2	6	6.06%	3398
6	3	3	6	14.09%	2516
6	3	4	6	23.71%	2286
6	4	4	6	34.46%	3430
8	2	2	6	1.4%	8622
8	2	3	6	2.15%	3481
8	2	4	6	2.5%	5755
8	3	2	6	13.3%	9222
8	3	3	6	20.5%	3150
8	3	4	6	29.5%	3887
8	4	4	6	No Feasible Solution	3913

Note that for both product sizes (6 and 8), adding a stage or a machine, significantly increases the optimality gap and CPU time. Moreover CPLEX could not find a feasible solution for any problem with the attributes bigger than $E =$

$4, M_e = 4, J = 6, T = 6$ and emphasizes the need of an efficient heuristic solution procedure for large problems more strongly.

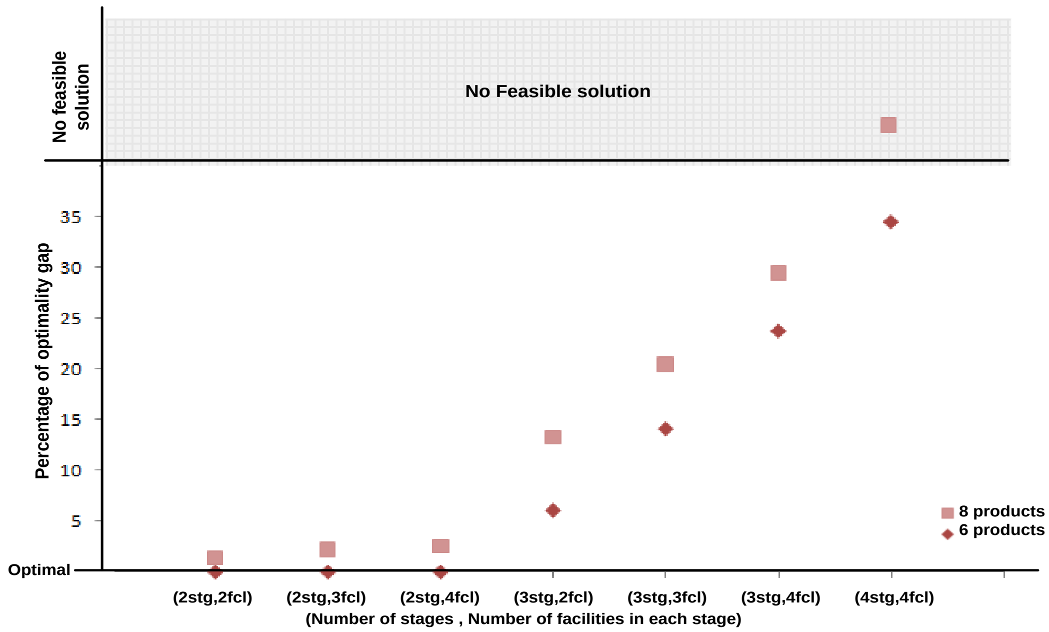

Figure 4-5: Percentage of optimality gap for different problem sizes

Following the results of CPLEX, two sets of test problems; medium size (which is also called small problems) and large problems are considered for testing the ASA algorithm. The attribute and dimensionality of small problems are $E = 2, M_e = 3, J = 6, T = 6$ and for large problems are $E = 4, M_e = 4, J = 8, T = 6$. Since the results of FFL-ATSP model for different combination of all small and big problems in chapter 3 show that there is no significant difference between combinations, I did some tests to analyse the effect of problem generator factors such as capacity tightness, setup cost and holding cost on solution quality. The initial test indicates that the capacity tightness and how big setup costs are in comparison to holding and backorder costs have a significant effect on solution quality. I also noticed that Özdamar and Barbarosolu's (1999) capacity formula (3-45) which was used in chapter 3 is quite loose regardless of the tightness factor (CAT) as it considers the production of all products on each machine (not all machines) in each stage. Therefore to provoke tighter capacity, the Özdamar and Barbarosolu's capacity formula is divided by the maximum number of machines to consider the production of all products on **all** machines per stage. The new capacity formula is as follows:

$$C = \max_{e} \left\{ \frac{\sum_{jtm} b_{jm} \cdot d_{jt}}{T \cdot Maxfac^2} \right\} \tag{4-28}$$

The capacity C_{mt} on machine m in period t is then given by $C_{mt} = CAT \cdot C$ and the capacity tightness is measured by a factor CAT with value 1.0 (tight) or 2.0 (loose).

To test the effect of setup cost magnitude on solution quality, setup costs are generated from $sc_{ijm} = SCM \times st_{ijm}$ with value 1.0 or 5.0 for SCM. In the case of $SCM = 5$, setup costs are much bigger than holding and backorder costs which has a significant effect on solution quality. Therefore there are four scenarios to investigate here: tight and loose capacity, each with small and big values of setup cost. The rest of the problem parameters are generated in the same way as explained in chapter 3 with *VAP = 1.2* (medium) and demand from *U(90,110)*. Five replications were generated for each scenario, so that in total 4*5=20 small problems and 20 big problems were generated. ASA was programmed in MATLAB R7 and run ten times with four initial solutions (in total 40 times) for each problem and the number of moves in each run is limited by 5000 for both problems as there is no particular solution improvement after 2000 moves.

Table 4-2: Computational results (solution's cost) of ASA and FFL-ATSP for small problems. (* indicates best possible solution which is not optimal)

Problem cost (Objective function) scenarios	Number	First initial solution	Second initial solution	Third initial solution	Fourth initial solution	GAMS result Cost (obj)	GAMS result Time (sec)
Tight capacity, $SCM = 1$	No1	2241	2124	2208	1982	1940	3183
	No2	2751	2530	2563	2304	2293	21
	No3	2549	2531	2636	2301	2167	57
	No4	2225	2161	2106	1935	1916	2582
	No5	2467	2431	2385	2184	2100	481
Loose capacity, $SCM = 1$	No1	2844	2904	2909	2468	2329	121
	No2	2717	2702	2667	2317	2291	111
	No3	2320	2345	2259	2151	2061	13650
	No4	2597	2495	2504	2220	2085	363
	No5	2404	2301	2315	2134	2006	102
Tight capacity, $SCM = 5$	No1	11970	11920	10479	10081	9193	6631
	No2*	12108	12610	12746	10856	9374*	3458*
	No3	10650	10700	10112	9465	8439	392
	No4	12560	12385	10824	10786	9384	354
	No5	10185	10435	10677	9180	8103	49
Loose capacity, $SCM = 5$	No1	11040	11090	11487	9885	8907	498
	No2	11707	11270	11442	10150	9133	1376
	No3	12965	12200	11711	11165	9842	427
	No4	14215	13500	12752	12401	10326	1036
	No5	12144	11406	11382	10529	9130	1870

Table 4-2 shows the results of GAMS (FFL-ATSP model) and ASA's best solution cost (objective function) of ten runs for each small problem. Note that the FFL-ATSP model found the optimal solution for all small problems except one problem shown with * as CPLEX exhausted the 2GB of available RAM and left a 1.05% optimality gap. In Table 4-3 a percentage difference $\%(\frac{Cost-Opt}{Opt})$ between the solution of ASA and the optimal cost obtained by FFL-ATSP is shown for small problems.

Table 4-3: Optimality gap of ASA $\%(\frac{Cost-Opt}{Opt})$ for small problems.

Problem optimality gap $\%(\frac{Cost-Opt}{Opt})$ scenarios	Number	First initial solution	Second initial solution	Third initial solution	Fourth initial solution
scenario1: Tight capacity, $SCM = 1$	No1	15.52	9.48	13.81	2.17
	No2	19.97	10.34	11.77	0.48
	No3	17.63	16.80	21.64	6.18
	No4	16.13	12.79	9.92	0.99
	No5	17.48	15.76	13.57	4.00
	Average	**17.3**	**13.0**	**14.1**	**2.8**
scenario2: Loose capacity, $SCM = 1$	No1	22.11	24.69	24.90	5.97
	No2	18.59	17.94	16.41	1.13
	No3	12.57	13.78	9.61	4.37
	No4	24.56	19.66	20.10	6.47
	No5	19.84	14.71	15.40	6.38
	Average	**19.5**	**18.2**	**17.3**	**4.9**
scenario3: Tight capacity, $SCM = 5$	No1	30.21	29.66	13.99	9.66
	No2*	29.16	34.52	35.97	15.81
	No3	26.20	26.79	19.82	12.16
	No4	33.84	31.98	15.34	14.94
	No5	25.69	28.78	31.77	13.29
	Average	**29.0**	**30.3**	**23.4**	**13.2**
scenario4: Loose capacity, $SCM = 5$	No1	23.95	24.51	28.97	10.98
	No2	28.18	23.40	25.29	11.14
	No3	31.73	23.96	18.99	13.44
	No4	37.66	30.74	23.49	20.09
	No5	33.01	24.93	24.67	15.32
	Average	**30.9**	**25.5**	**24.3**	**14.2**
Overall small problems		**24.2**	**21.8**	**19.8**	**8.7**

A Two-Factor ANOVA with Replication is designed to evaluate the effect of different initial solutions of ASA and scenarios on solution quality. As ANOVA Table 4-4 shows, there are highly significant differences (p=0.000) between four initial solutions and scenarios with no significant interaction between them (p= 0.23).

Table 4-4: Results of Two-Factor ANOVA with 5 Replication for small problems.

Source of Variation	Sum of squares	Degrees of freedom	Mean square	F_0	p-value
Scenarios	1972.6	3	657.5	56.93	**0.000**
Initial solutions	2760.2	2	1380.1	119.48	**0.000**
Interactions	97.5	6	16.26	1.41	0.23
Error	554.4	48	11.5		
Total	5384.7	59			

The fourth novel initial solution based on the sequencing model has better solutions than the other initial solutions for all small problems with a mean optimality gap of 8.3%. As shown in Table 4-3 when setup costs are not much bigger than holding and backorder cost ($SCM = 1$), the fourth initial solution found better solutions with 2.8% and 4.9% optimality gap for first and second scenario respectively in comparison to the big values of setup costs ($SCM = 5$) with 13.2% and 14.2% optimality gap for third and fourth scenario respectively. The reason for not getting better results when ($SCM = 5$), is the main assumption of sequencing model which is the production of all items in each stage and period. When setup costs are high it may be more beneficial to produce some of the products in a period and backorder the others to the next period(s) in order to save setup costs.

For big problems, FFL-ATSP could not find any feasible solution with no time-limit, and also the sequencing model of the fourth initial solution could not find an optimal solution in an hour's time-limit whilst it found an optimal solution in less than a minute for small problems. Thus, only ASA with first, second and third initial solution was run for big problems and the third initial solution based on *MS-CLSP* has obtained better solution than the first and second one. Furthermore the second initial solution based on Loading Heuristic algorithm (LHR), results in better solutions than the first random-based initial solution and an ANOVA test indicated highly significant differences (p=0.00) between the three initial solutions. The mean run times of ASA for small and big problems with 5000 iterations are 199.1 and 456.6 seconds respectively.

4.5 Final remarks

In this chapter, a simulated annealing algorithm with an effective adaptive temperature control scheme has been developed. The adaptive temperature control scheme changes temperature based on the number of consecutive improving moves and maintains it above the minimum level. The Adaptive Simulated Annealing (ASA) is based on Azizi and Zolfaghari's cooling schedule (Azizi and Zolfaghari, 2004) and applied for GLSP-FFL. The main advantage of ASA is providing a higher chance of an uphill transition once the search traps in a local minimum regardless of the iteration number by dynamically adjustment of the temperature based on the profile of the search path.

Afterward four initial solutions and three neighbour operators are designed for ASA. The first initial solution is based on generating a random sequence of products on machines of each stage and then running the linear model of GLSP-FFL to find the optimal lot size of the sequence. For the second initial solution, the external demand of each product is considered as the product lot size in each stage and period and the sequences of lots are determined by Loading Heuristic algorithm (LHR). The third novel initial solution is obtained by solving well-organized model which extracts from the GLSP-FFL. The model is Capacitated Lot Sizing Problem for Multi Stage systems (MS-CLSP) with single machine in each stage. MS-CLSP gives the inventory feasible lot sizes which need to be scheduled by loading heuristic algorithm on parallel machines of stages in FFL. The fourth initial solution is generated by solving the sequencing model of the GLSP-FFL to find the sequence and then it is given to the linear lot sizing model of GLSP-FFL to determine its optimal lot size.

The first initial solution is feasible but the other initial solutions may be capacity-infeasible solutions. In the ASA procedure capacity infeasibilities are heavily penalized in the objective function and inventory-feasibility is always preserved when generating neighbours by the neighbour operators. The numerical test shows that for small problems, ASA with the fourth initial solution and for big problems ASA with the third initial solution is able to find much better solutions than other initial solutions.

Chapter 5

Lot sizing and Scheduling with Non-Triangular, Period Overlapping and Carryover Setups

This chapter considers efficient mixed integer programming formulations for capacitated lot sizing and scheduling with non-triangular and sequence-dependent setup times and costs incorporating all necessary features of setup carryover and overlapping on different machine configurations. When setup times and/or costs are non-triangular, it can sometimes be optimal for a shortcut product to be produced in more than one lot in each period. To model this, the ATSP-based formulations are developed which allow multiple lot production within a period and are more efficient than other models as it used polynomial number of disconnected subtours prohibition constraints. Moreover all necessary features of setup carryover and overlapping are modelled including: conserving setup state when no product is being processed over period(s); starting setup in a period and ending in the next period; ending setup at a period and starting production in the next period(s); crossing an imposed minimum lot size over periods. This comprehensive mathematical formulation relaxes all limitation of physical separation between the periods which contrasts the nature of production system. In this chapter firstly the new model is explained for a single machine which then extends to other machine configurations including parallel machines and flexible flow line. Finally computational tests are reported.

5.1 Introduction

As discussed in chapter three, Asymmetric Travelling Salesman Problem (ATSP) is an alternative approach for modelling lot sequencing problem with sequence dependent setups and results in better solutions in much shorter time for flexible flow line systems compared to the other lot sizing and scheduling models on every problem tested.

However the main restriction of conventional ATSP based models is restricting the production of one lot per product per period that it may not be optimal when non-triangular setup exists. Non-triangular setups occur in some industries like food, animal feed, beverage and oil where there are intermediate "cleaning" or "shortcut" products. For example in animal feed industry, some products can contaminate other product and lead to serious effects on animal's health. To avoid this, machines must be cleaned, resulting in substantial setups that consume scarce production times. Alternatively the production of enough amount of intermediate or cleaning product can clean the machines and reduce overall setup times (costs). In this situation the setup to and from cleaning or shortcut product (k) is less costly and time consuming than direct setup between two products (i,j) means that $st_{i,j} \geq st_{i,k} + st_{k,j}$. Therefore shortcut product may need to be produced more than once within a period.

Menezes et al. (2011) modelled the production of multiple lot per period by using an iterative model and method based on a potentially exponentially number of ATSP subtour elimination constraints. In a very recent work, Clark and I presented the more efficient formulation than Menezes et al. (2011) for modelling the production of multiple lots of a product per periods using a polynomial number of constraints based on Claus's formulation (1984) to exclude disconnected subtours while allowing ones connected to the main sequence. The model was presented in 43rd Annual Symposium of the Brazilian Operational Research Society (Clark and Mahdieh, 2011) and its revision has been submitted to the International Journal of Production Research (Clark et al., 2012). This chapter is devoted to the extension of multiple lots model to parallel machine and FFL system incorporating all features of setup carry-over and setup-overlapping.

Setup overlapping has been studied by Suerie (2006) for small bucket and by Sung and Maravelias (2008) for big-bucket but with sequence-independent setup times and cost. Almada-lobo et al. (2007) incorporated setup carryover features for capacitated lot sizing and scheduling problem which allows a product sets up at the end of one period and the actual production starts in the next period. They used Miller-Tucker-Zemlin subtour prohibition constraint (Desrochers and Laporte, 1991) to formulate sequence-dependent setup times and costs while holding triangular inequality. Menezes et al. (2011) modelled the setup cross-over that allows setup starts in a period and ends in the next period.

Here, the first mixed integer linear programming is presented for lot sizing and scheduling with non-triangular sequence-dependent setup times and costs that allows multiple lot production with polynomial number of constraints and incorporates all necessary features of setup carryover and overlapping. The features model the production system more realistically by relaxing all the limitation of physical separation between the periods. Thus a setup can start at the end of a period and end at the beginning of the next period or a setup can end at the end of a period and production starts in the next period. Furthermore an imposed minimum lot size can cross over the periods and setup state is conserved when no product is being processed over period(s). All these features increase the model flexibility and lead to finding better solutions particularly in tight capacity conditions or whenever setup times are significant. The extensions of the model to parallel machines and flexible flow line are presented and discussed via computational tests.

5.2 Modelling multiple lots per product per period for single machine

This section presents the modelling of multiple lots per product per period using a polynomial number of multi-commodity-flow-type constraints (Claus, 1984) to exclude disconnected subtours while allowing ones connected to the main sequence. Figure 5-1 shows an example of main sequence (S) with different type of subtours (A, B, C, D). The Multiple Lot model for Single Machine, denoted ML-SM, allows connected subtours B and C and excludes disconnected A and D.

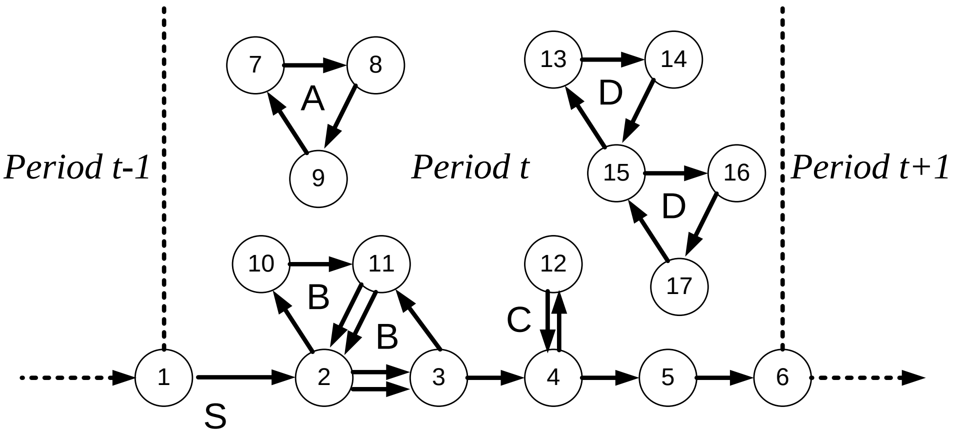

Figure 5-1: A main sequence (S) and different types of subtours (A, B, C, D)

5.2.1 Data and decision variables

The parameters and indices of the ML model are:

J Number of total products *i,j,k*

T Number of periods *t* in the planning horizon

The input data required by the model are:

d_{it} Demand for product *i* realised at the end of period *t*

C_t Available capacity time in each period *t*

st_{ij} Time needed to setup from product *i* to product *j*

sc_{ij} Cost needed to setup from product *i* to product *j*

b_i Time needed to produce a unit of product *i*

h_{it} Cost of holding a unit of product *i* from period *t* to *t*+1

g_{it} Backlog cost per period for product *i* from period *t* to *t*+1

UB_{it} Upper bound C_t/b_i on the quantity of product *i* produced in period *t*

i_0 The product setup at the end of period 0, i.e., the starting setup configuration

ml_j Minimum lot size of product *j*.

The decisions variables by the model are represented by following variables:

I_{it} Inventory level of product *i* at the end of period *t*.

B_{it} Backordered amount of product *i* at the end of period *t*.

x_{it} Production quantity of product *i* in period *t*.

Slk_t Number of unites of slack capacity in period *t*.

x_{it}^F The quantity produced in period *t* of the first (crossover) lot of product *i* in period *t* if it was setup in period *t*-1, otherwise 0.

x_{it}^L The quantity produced in period *t* of the last (crossover) lot of product *i* in period *t* if its production continues into period *t*+1, otherwise 0.

y_{ijt} Number of times that production is to be changed over from product *i* to product *j* in period *t*, Integer non-negative. For example in figure 5-1, $y_{12t} = 1,$ and $y_{23t} = 2$.

z_{it} Number of times that product *i* is in a setup state in period *t*, Integer non-negative. For example in figure 5-1, $z_{1t} = 1,$ and $z_{2t} = 3$.

α_{it} = 1 either because *j*-to-*i* is the last setup in previous periods to *t* or because *j*-to-*i* is the setup operation that overlaps from *t*-1 to *t*.

The objective function minimises backorders, inventory and setup costs:

$$Minimise \sum_{ijt} sc_{ij}\ y_{ijt} + \sum_{it} h_{it}\ I_{it} + \sum_{it} g_{it}\ B_{it} \tag{5-1}$$

5.2.2 Main lot size and setup constraints

Constraint (5-2) balances inventory, backlogs, production and demand over consecutive periods:

$$I_{jt-1} - B_{jt-1} + x_{jt} - I_{jt} + B_{jt} = d_{jt} \qquad \forall\, j,t \quad (5\text{-}2)$$

Constraint (5-3) represents the limited capacity and calculates any slack capacity:

$$\sum_{i} b_i\ x_{it} + \sum_{ij} st_{ij}\ y_{ijt} + slk_t = C_t \qquad \forall\, t \quad (5\text{-}3)$$

Constraint (5-4) enforces the appropriate setup before production:

$$x_{jt} \leq UB_{jt} \times z_{jt} \qquad \forall\, j,t \quad (5\text{-}4)$$

Constraint (5-5) prohibits setup between the same products:

$$y_{jjt} = 0 \qquad \forall\, j,t \quad (5\text{-}5)$$

Constraint (5-6) indicates the first setup of each period which ensures that the machine is set up for exactly one product at the beginning of each period. The initial setup configuration at first period is expressed by constraint (5-7).

$$\sum_i \alpha_{it} = 1 \qquad \forall\ t = 1,..,T+1 \quad (5\text{-}6)$$

$$\alpha_{i_o t} = 1 \qquad \forall\, t = 1 \quad (5\text{-}7)$$

5.2.3 Imposing a minimum lot size

Some cleansing products *k* require a minimum lot size ml_k in order to force the proper cleaning of a previous product *i* contaminants, that is, to avoid a setup from *i* to *j* via zero production of *k* rather than directly. Constraints (5-8) to (5-11) achieve this and also allow a minimum lot size to cross over the periods.

Recall that x_{jt}^F is the quantity produced in period *t* of the first (crossover) lot of product *j* in period *t* if it was setup in period *t*-1, but is otherwise 0, as imposed by Constraints (5-8):

$$x_{jt}^F \leq UB_{jt}\ \alpha_{jt} \qquad \forall\, j,t \quad (5\text{-}8)$$

Similarly x_{jt}^L is the quantity produced in period *t* of the last (crossover) lot of product *j* in period *t* if its production continues into period *t*+1, otherwise 0, as imposed by constraints (5-9).

$$x_{jt}^L \leq UB_{jt}\ \alpha_{j,t+1} \qquad \forall\, j,t \quad (5\text{-}9)$$

Then $x_{jt}^L + x_{j,t+1}^F$ is the size of a crossover lot of a product *j* that has been started in period *t* and completed in period *t*+1. Constraints (5-10) oblige this crossover lot to be of size at least ml_j:

$$x_{jt}^L + x_{j,t+1}^F \geq ml_j\ \alpha_{j,t+1} \qquad \forall\, j,t \quad (5\text{-}10)$$

At last constraint (5-11) imposes minimum lot sizes for both crossover and non-crossover lots using auxiliary variables x_{jt}^L , x_{jt}^F.

$$x_{jt} - x_{jt}^F - x_{jt}^L \geq ml_j\ (z_{jt} - \alpha_{jt} - \alpha_{j,t+1}) \qquad \forall\, j,t \quad (5\text{-}11)$$

These work as follows. When a setup state *j* is neither inherited from the previous period t-1 nor passed on to the next period t+1 then $\alpha_{jt} = \alpha_{j,t+1} = 0$ and so $x_{jt}^L + x_{j,t+1}^F = 0$ by constraints (5-8) and (5-9). In this case, constraints (5-11) oblige the total x_{jt} of the lot sizes to be at least $z_{jt}\,ml_j$ and so it can be split into z_{jt} separate lots, each of which is at least ml_j units in size. However, if a setup state *j* is either inherited from the previous period *t*-1 or passed on to the next period *t*+1 (or both) then $\alpha_{jt} + \alpha_{j,t+1} = 1$ (or 2). In this case constraints (5-11) impose only that the $(z_{jt} - \alpha_{jt} - \alpha_{j,t+1})$ lots of *j* produced entirely within period *t* should be of total size at least $z_{jt}\, ml_j$, again splittable into z_{jt} separate lots, each of which is at least ml_j units in size.

Example 1: Here is an example to show how the new imposing minimum lot constraints can span the lot over the periods with no demands and imposes the minimum lot size (ml_j) for the whole crossover lot.

Consider that there is a demand for product A in period 1, for product B in period 3 and no demand in period 2. Here is investigation of imposing minimum lot size with the existence of shortcut product C. In this case there are two possibilities as below:

In the first possibility setup A to C and C to B can both happen either in period two or, one can happen in period two and the other one in period 1 or 3. So the minimum lot size will be enforced by constraint (5-11). Second possibility consists of happening setup A to C in period 1 and setup C to B in period 3 while there is no setup in period 2 as shown in Figure 5-2.

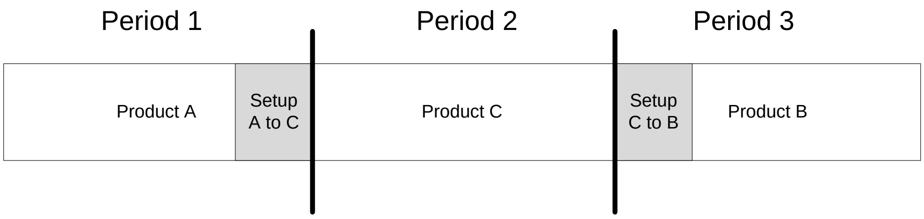

Figure 5-2: Example (1) lot crossover

So according to constraint (5-10):

$$x_{C1}^{L} + x_{C2}^{F} \geq ml_C \qquad \text{(C1)}$$

$$x_{C2}^{L} + x_{C3}^{F} \geq ml_C \qquad \text{(C2)}$$

and according to constraint (5-11):

$$x_{C2} - x_{C2}^{F} - x_{C2}^{L} \geq -ml_C \qquad \text{(C3)}$$

$$x_{C1} - x_{C1}^{L} \geq 0 \qquad \text{(C4)}$$

$$x_{C3} - x_{C3}^{F} \geq 0 \qquad \text{(C5)}$$

In order to impose minimum lot size for C it is needed to justify that the total production of product C (at the end of period 1, in period 2 and at the beginning of period3) is at least ml_C.

$$x_{C1} + x_{C2} + x_{C3} \geq ml_C$$

To justify, first constraints C1 and C2 are add:

$$x_{C1}^{L} + x_{C2}^{F} + x_{C2}^{L} + x_{C3}^{F} \geq 2ml_C \qquad \text{(C6)}$$

Then constraints C3, C4 and C5 are sum up:

$$x_{C1} + x_{C2} + x_{C3} \geq x_{C1}^L + x_{C2}^F + x_{C2}^L + x_{C3}^F - ml_C \qquad \text{(C7)}$$

Finally combining constraints C6 and C7, concludes that the crossover lot of product C ($x_{C1} + x_{C2} + x_{C3}$) is at least ml_C and constraint (5-10) imposes ml_C (not $2ml_C$) for the whole crossover lot. Moreover this conclusion can be extended for more than one period with having no demand.

$$x_{C1} + x_{C2} + x_{C3} \geq x_{C1}^L + x_{C2}^F + x_{C2}^L + x_{C3}^F - ml_C \geq 2ml_C - ml_C \geq ml_C$$

Note that constraints (5-8) to (5-11) is more efficient than the conventional constraint: $x_{jt} \geq ml_j \sum_i y_{ijt}, \forall j, t$(5-27), used in the well known lot sizing and scheduling models (Clark and Clark, 2000, Fleischmann and Meyr, 1997) to impose minimum lot size. Because in conventional constraint, the whole setup and production of minimum lot size should be done in one period so minimum lot size neither can crossover to the next period(s) nor can be produced at a period when the setup ending at the end of previous period(s). All these restrictions are relaxed in new constraints (5-8) to (5-11). Examples 2 and 3 in the section 5.3 show explicitly the difference of two types of constraints for imposing minimum lot size.

5.2.4 Lot sequencing constraints

In this section, the ATSP-related constraints are demonstrated for sequencing product lots. Constraints (5-12) and (5-13) are flow conservation constraints that relate the α_{it} and z_{it} setup state variables to the y_{ijt} changeover variables, to and from a product respectively as shown in Figure 5-3.

$$\alpha_{it} + \sum_j y_{jit} = z_{it} \qquad \forall i, t \quad (5\text{-}12)$$

$$\sum_j y_{ijt} + \alpha_{i,t+1} = z_{it} \qquad \forall i, t \quad (5\text{-}13)$$

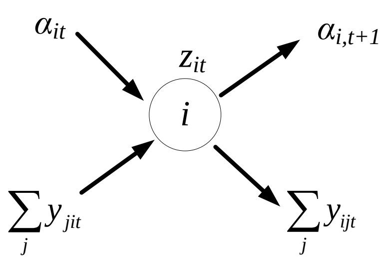

Figure 5-3: Node flow modelled by constraints (5-12) and (5-13)

For example, referring to Figure 5-1, if $i = 1$, then the values in constraints (5-12) and (5-13) are $1 + 0 = 1$ and $1 + 0 = 1$ respectively. If $i = 2$, then the values are $0 + 3 = 3$ and $3 + 0 = 3$ respectively.

The optimal solution to the model specified by expressions (5-1) to (5-13) will consist of single sequence starting with product $i|\{\alpha_{it} = 1\}$ and ending with $k|\{\alpha_{k,t+1} = 1\}$ (possibly with embedded connected subtours), and maybe one or more disconnected subtours, some examples of which are illustrated in Figure 5-1. Subtours connected to the main sequence S are permitted (e.g. subtours B and C), but disconnected subtours must be prohibited (e.g. subtours A and D). The main sequence S and 4 subtours are:

S: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$

A: $7 \rightarrow 8 \rightarrow 9 \rightarrow 7$

B: $2 \rightarrow 10 \rightarrow 11 \rightarrow 2 \rightarrow 3 \rightarrow 11 \rightarrow 2$

C: $4 \rightarrow 12 \rightarrow 4$

D: $13 \rightarrow 14 \rightarrow 15 \rightarrow 16 \rightarrow 17 \rightarrow 15 \rightarrow 13$

Öncan (2009) reviews and analytically compares many ATSP formulations. It highlights the tightness of the multi-commodity-flow (MCF) formulation by Claus constraints (1984) which is the inspiration for the formulation that prohibits disconnected subtours a priori in the proposed model ML-SM. The main idea of this formulation is to ensure that, in any period *t*, there is always a walk from the crossover product ($i|\{\alpha_{it} = 1\}$) to any other product *k* in period *t*'s sequence. To allow a clear explanation, p_t^α will be used in text when referring to crossover product $i|\{\alpha_{it} = 1\}$.

First define additional binary variables a_{ijt}^k which are adopted from Claus (1984) as follows:

a_{ijt}^k =1 if the arc $i \rightarrow j$ is on a walk from crossover product p_t^α to product *k* within period *t*'s sequence of lots, otherwise 0.

For any product *k* produced in period *t*, the variables a_{ijt}^k encode a walk from p_t^α to *k*. It can be called an *k* -walk. The existence of an *k* -walk ensures that product *k* is connected to the main production sequence, maybe within a connected subtour.

Figure 5-4 shows part of *k*-walk from p_t^α crossover product to product *k* passing through the arc $i \rightarrow j$. In this case, $a_{ijt}^k = 1$. Constraints must be formulated to enforce an k-walk for all products k produced in period t. To begin with, the arc $i \rightarrow j$ must be part of a solution in order for a_{ijt}^{k} to have value 1. Thus values of a_{ijt}^{k} must obey constraints (5-14):

$$a_{ijt}^{k} \leq y_{ijt} \qquad \forall\, i,j,k,t \quad (5\text{-}14)$$

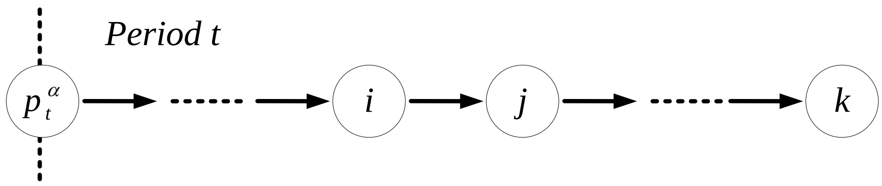

Figure 5-4: An k-walk from p_t^{α} crossover product to product k

Consider once again the infeasible sequence in Figure 5-1. Product $k = 10$ in connected-subtour B is reachable from crossover product $p_t^{\alpha} = 1$ by traversing arcs $1 \rightarrow 2 \rightarrow 10$. This reach-ability is indicated by following non-zero values of a_{ijt}^{10} that constitute an k-walk: $a_{1,2,t}^{10} = a_{2,10,t}^{10} = 1$. In contrast, product $k = 9$ in disconnected-subtour A in Figure 5-1 is not reachable from crossover product $p_t^{\alpha} = 1$. No k-walk exists for $k = 9$. This is indicated by the impossibility of finding values of a_{ijt}^{9} that also obey constraints (5-15) – (5-19) below.

To prohibit disconnected subtours, further binary decision variables z_{it}^{bin} are needed:

z_{it}^{bin} =1 if product *i* is ever in setup state in period *t*, otherwise 0.

Note that $z_{it}^{bin} = 1 \Leftrightarrow z_{it} \geq 1$ and that $z_{it}^{bin} = 0 \Leftrightarrow z_{it} = 0$. This is enforced by following constraints:

$$z_{it} \geq z_{it}^{bin} \qquad \forall\, i,t \quad (5\text{-}15)$$

$$z_{it} \leq ZUB_i z_{it}^{bin} \qquad \forall\, i,t \quad (5\text{-}16)$$

where ZUB_i is a prespecified upper bound (UB) on the value of z_{it} and must be greater than one. ZUB_i is automatically calculated in the computational tests below as the lesser of J (the number of products) and the size of the ordered set $\{(i,j)|st_{ij} \geq st_{ik} + st_{kj}\}$, which can be very large, but is often 1 for non-shortcut products. More detailed analysis of setup times and available production capacities might bring down the value of ZUB_i.

The three sets of constraints (5-17) to (5-19) explained below will now allow connected subtours, and prohibit disconnected ones *a priori*.

Firstly, constraints (5-17) ensure that the k-walk reaches product k (Figure 5-5) and is imposed only when the setup state is configured for k at least once during period t (that is, only when $z_{it}^{bin} = 1$), but not when the setup state is never configured for k during period t, (that is, when $z_{it}^{bin} = 0$):

$$\alpha_{kt} + \sum_{i} a_{ikt}^{k} = z_{kt}^{bin} \qquad \forall\, k, t \quad (5\text{-}17)$$

For example, the k-walk $1 \rightarrow 2 \rightarrow 10$ in Figure 5-1 is forced to reach product $k = 10$ by the following instance of constraint (5-17):

$k = 10 : \alpha_{10,t} + \sum_i a_{i,10,t}^{10} = z_{10,t}^{bin}$ which becomes $0 + 1 = 1$ and enforces that $a_{i,10,t}^{10} = 1$ for a given i.

If a product k is not produced in a period t, then $z_{kt}^{bin} = 0$, and so constraint (5-17) forces $a_{ikt}^{k} = 0\ \forall i$ (constraint (5-14) also forces this via $a_{ikt}^{k} \leq y_{ikt} = 0$).

Secondly, the k-walk in period t specified by the variables $\{a_{ijt}^{k} \mid \forall\, i, j\}$, must start at crossover product p_t^{α} and then traverse further products on its way to product k, as shown in Figure 5-6. If $\alpha_{kt} = 1$ then no k-walk is needed. If $\alpha_{kt} = 0$, then constraint (5-17) means that $\sum_i a_{ikt}^{k} = 1$, i.e., $a_{ikt}^{k} = 1$ for exactly one product i that is the 2nd last product on the k-walk. Constraint (5-18) then forces $a_{jit}^{k} = 1$ for exactly one product j that is the 3rd last product on the k-walk, and so on, going backwards along the k-walk, obliging the a_{ijt}^{k} along the k-walk to have value 1, until it reaches back to the initially-setup product $i = p_t^{\alpha}$ (for which $\alpha_{it} = 1$).

$$\alpha_{it} + \sum_{j} a_{jit}^{k} \geq \sum_{j} a_{ijt}^{k} \qquad \forall\, k, i \neq k, t \quad (5\text{-}18)$$

For example, in Figure 5-1, consider the k-walk $1 \rightarrow 2 \rightarrow 10$ to product $k = 10$. The following two instances of constraint (5-18) obliges the a_{ijt}^{k} along this k-walk to have value 1, reaching back to an initially-setup product $p_t^{\alpha} = 1$ (for which α_{1t} is thus forced to have value 1):

$i = 2$: $\alpha_{2t} + \sum_j a_{j2t}^{10} \geq \sum_j a_{2jt}^{10}$ becomes $0 + \sum_j a_{j2t}^{10} \geq 1$, resulting in $\sum_j a_{j2t}^{10} = 1$.

$i = 1$: $\alpha_{1t} + \sum_j a_{j1t}^{10} \geq \sum_j a_{1jt}^{10}$ becomes $\alpha_{1t} + 0 \geq 1$, resulting in $\alpha_{1t} = 1$.

Thirdly and finally, constraint (5-19) requires that the k-walk from p_t^{α} stops at product k (Figure 5-7) and need go no further:

$a_{kjt}^{k} = 0$ $\forall\, k, j, t$ (5-19)

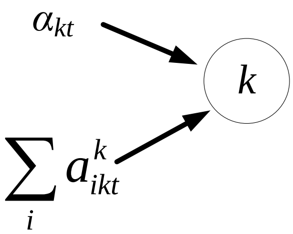

Figure 5-5: The k-walk from p_t^{α} much reach product k (if and only if $z_{kt}^{bin} = 1$)

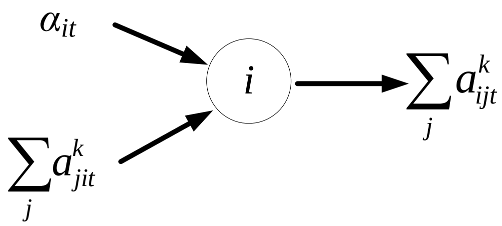

Figure 5-6: The k-walk from p_t^{α} to k only traverse those product i for which $z_{it}^{bin} = 1$ (and if and only if $z_{kt}^{bin} = 1$)

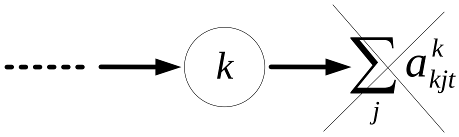

Figure 5-7: The k-walk from p_t^{α} must stop at product k (if and only if $z_{kt}^{bin} = 1$)

For example, the k-walk $1 \rightarrow 2 \rightarrow 10$ in Figure 5-1 stops at product $k = 10$ as enforced by the following instance of constraint (5-19):

$k = 10$: $a_{10jt}^{10} = 0\ \forall\, j, t$

If k is not produced in period t, then constraint (5-19) simply forces $a_{kjt}^{k} = 0$ which has no impact given that constraint (5-17) already obliges $a_{ijt}^{k} = 0$. Thus constraints (5-17, 5-18, 5-19) exclude disconnected subtours. For example, in Figure 5-1, there are no instances of constraints (5-17) - (5-19) that would show that product $k = 9$ in disconnected-subtour A is reachable by an k-walk from crossover product p_t^{α} . This is also true for all the other disconnected products. Thus the setup sequence in Figure 5-1 is infeasible and will be correctly excluded by our formulation.

5.2.5 Concluding ML-SM model formulation

Note that constraints (5-4) are valid but loose: the value of z_{jt} need only be 1, and not ≥ 2. Constraints (5-4) can thus be tightened by replacing z_{jt} by z_{jt}^{bin} .

$x_{jt} \leq UB_{jt} \times z_{jt}^{bin}$ $\forall\, j, t$ (5-20)

Thus model ML-SM is specified by expressions (5-1) to (5-3) and (5-5) to (5-20), and restated completely in the Appendix A. In Expressed as function of the number of products *J* and periods *T*, model ML-SM has $J^2T + 7JT + T$ variables and $J^3T + 2J^2T + 11JT + 2T$ constraints. The ML-SM formulation is thus polynomial-sized. This does not means that the model is solvable in polynomial time - it cannot be, given that the NP-hard ATSP is embedded within it. Rather, the innovation of this model has been (a) the modelling of non-triangular sequence dependent setup within a lot sizing model and (b) the derivation of a polynomial-sized MILP formulation for this problem.

5.3 Modelling period overlapping setup operations for single machine

Multiple Lots model can be generalized to allow setup operations to overlap periods, i.e., to permit a setup to begin in a period and end in the next period. The model is called MLOV-SM relaxes all limitation of physical separation between the periods which contrasts the nature of production system. The MLOV-SM is advantageous when capacity is tight and lot sizing and sequencing decisions need more flexibility to reduce backlogs.

Consider the following additional decision variables:

OLS_{ijt} =1 if the overlapping setup operation *i* to *j* begins in period *t* and finishes in period *t*+1, otherwise 0.

S_t The amount of setup time that overlaps into period *t*+1, having begun at the end of period *t*.

The value of S_t must be zero if there is no overlapping last setup at the end of period *t*:

$$S_t \leq \sum_{ij} st_{ij}\ OLS_{ijt} \qquad \forall\ t \quad (5\text{-}21)$$

Last setup and at most one setup in period *t* can overlap from period *t* to *t*+1:

$$\sum_{j} OLS_{jit} \leq \alpha_{i,t+1} \qquad \forall\ i, t \quad (5\text{-}22)$$

The value of OLS_{ijt} must be zero if *i* to *j* is not a setup initiated in period *t*:

$$OLS_{ijt} \leq y_{ijt} \qquad \forall\ i, j, t \quad (5\text{-}23)$$

The capacity constraint (5-3) now becomes:

$$\sum_{i} b_i\, x_{it} + \sum_{ij} st_{ij}\ y_{ijt} + S_{t-1} - S_t + slk_t = C_t \qquad \forall\, t \quad (5\text{-}24)$$

When the last setup is overlapped; $OLS_{ijt} = 1$, then product j cannot be produced as the last (crossover) lot in period t. Thus constraint (5-4) and (5-9) now become (5-25) and (5-26).

$$x_{jt} \le UB_{jt} \times (z_{jt} - \sum_{i} OLS_{ijt}) \qquad \forall\, j,t \quad (5\text{-}25)$$

$$x_{jt}^{L} \le UB_{jt}\ (\alpha_{j,t+1} - \sum_{i} OLS_{ijt}) \qquad \forall\, j,t \quad (5\text{-}26)$$

Thus model MLOV-SM is specified by expressions (5-1) and (5-2), (5-5) to (5-8), (5-10) to (5-19) and (5-21) to (5-26), and restated completely in the Appendix B.

Example 2 and 3: Here are two examples to show the effectiveness of the new imposing minimum lot constraints (5-8) to (5-11), in comparison with the conventional constraint (5-27) and also the solution's improvement obtained by modelling setup overlapping features. Thus following examples are solved by three models consist of ML-SM, MLOV-SM and Conventional which has the same constraints as ML-SM but imposes minimum lot by constraint (5-27) rather than constraints (5-8) to (5-11).

$$x_{jt} \ge ml_j \sum_{i} y_{ijt} \qquad \forall\, j,t \quad (5\text{-}27)$$

The following data are used for both examples: $C_t = 100$, $ml_j = 10, T = 3, J = 2$, $i_0 = 1, st_{ij} = 20$, $b_j = 1$, $h_{jt} = 15$, $sc_{ij} = 600$, $g_{it} = 1000$; and the demands are shown in the table 5-1. The models are implemented in the optimisation modelling software GAMS build 23.6.5 (Brooke et al., 1988) and solved using the industrial-strength CPLEX 12.0 solver (CPLEX., 2010) on a computer with a 2.1 GHZ CPU and 2 GB of RAM. All models were solved less than a second for both examples.

Table 5-1: Demand data for example 2 and 3.

Demand	***Example(2)***			***Example(3)***		
$\boldsymbol{d_{it}}$	*t*=1	*t*=2	*t*=3	*t*=1	*t*=2	*t*=3
***i*=1**	75	0	90	75	0	90
***i*=2**	0	90	0	0	95	0

The production diagram and the results of Example 2 are shown in Figure 5-8 and Table 5-2 respectively. Note that how modelling of all necessary features of production improves the solution remarkably. As shown in Figure 5-8 Conventional model cannot use the machine's capacity efficiently and there are 5 units idle or slack time in period 1 as the setup and minimum lot production should be done totally in a single period (constraint (5-27)). This restriction is relaxed in the ML-SM model so setup ends in period 1 and minimum lot is produced in period 2 that significantly results in reduction of the number of inventory and backlogs as shown in Table 5-2. However there are still 10 units slack time in period 2 as in the ML-SM model setup cannot overlap i.e., setup begins in period 2 and ends in period 3. In the new lot sizing and scheduling model, MLOV-SM, all the limitations caused by previous models are relaxed and the production system is modelled realistically. Thus the scarce production capacity is used more efficiently.

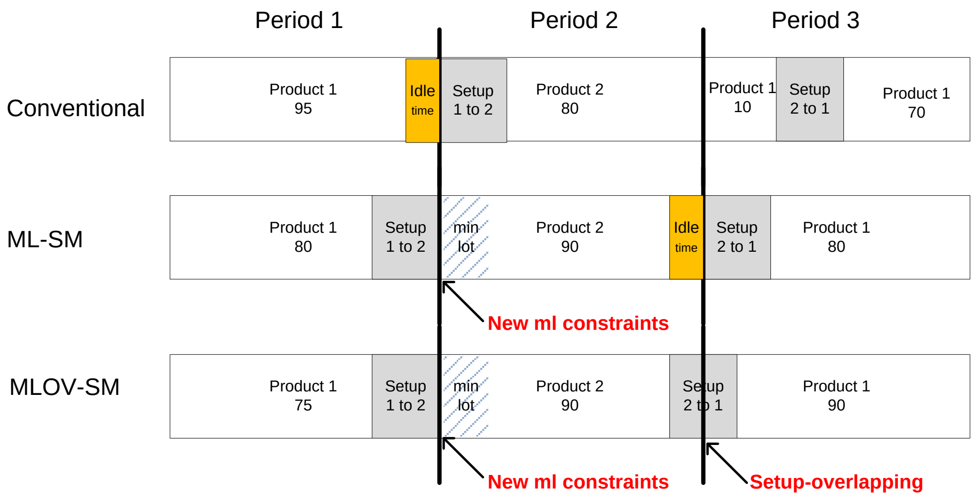

Figure 5-8: Production diagram of Example 2 obtained by Conventional, ML-SM and MLOV-SM models

Table 5-2: Results of Example 2 obtained by Conventional, ML-SM and MLOV-SM models

Example 2	Conventional	ML-SM	MLOV-SM
Slack capacity	5	10	0
Total Inventory	40	10	0
Backlogs	10	5	0
Total cost = Cost of (Backlogs + Inventory +Setup)	11800 (10000+600+1200)	6350 (5000+150+1200)	1200 (0+0+1200)

In example 2 the optimal solution is obtained by the MLOV-SM model with no shortage or inventory. In order to stimulate the tight capacity even more, the demand of product 2 is increased to 95 in example 3. The production diagram and the results of Example 3 are shown in Figure 5-9 and Table 5-3 respectively. Note that the Conventional model found the solution with high total inventories (50) and backlogs (15) while the optimal solution found by MLOV-SM has no backlogs and only 5 inventories. Furthermore as shown in MLOV-SM's production diagram Figure 5-9, the minimum lot crossovers from period 1 to 2. Lot crossover is another feature which is modelled via the new imposing minimum lot size (ml) constraints (5-8) to (5-11) and improves the solutions and gives more flexibility to lot sizing model.

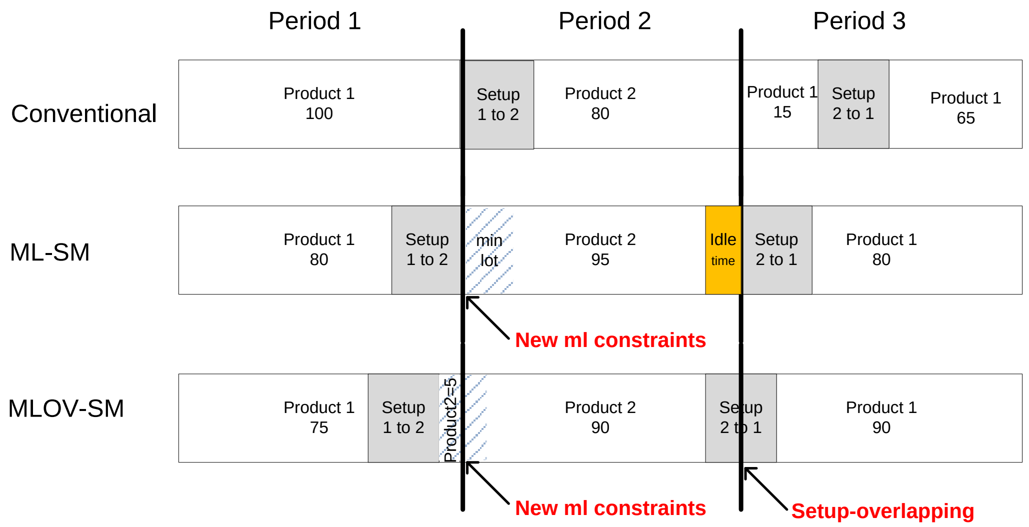

Figure 5-9: Production diagram of Example 3 obtained by Conventional, ML-SM and MLOV-SM models

Table 5-3: Results of Example 3 obtained by Conventional, ML-SM and MLOV-SM models

Example 3	Conventional	ML-SM	MLOV-SM
Slack capacity	0	5	0
Total Inventory	50	10	5
Backlogs	15	5	0
Total cost = Cost of (Backlogs + Inventory +Setup)	16950 (15000+750+1200)	6350 (5000+150+1200)	1275 (0+75+1200)

Examples 2 and 3 showed that how the new comprehensive mathematical formulation, MLOV-SM, relaxes all limitation of physical separation between the periods which contrasts the nature of production system. The MLOV-SM modelled the new features consists of starting setup in a period and ending in the next period; ending setup at a period and starting production in the next period(s); crossing an imposed minimum lot size over periods.

5.4 Extensions to Parallel Machines and Flexible Flow Lines

In this section the multiple lot models are extended to Parallel Machines (PM) and Flexible Flow Lines (FFL). An index *m* is used to model Parallel Machine and *M* is the total number of machines. Data, variables and constraints of Multiple Lot models for single machine are adapted to parallel machines by considering index *m*. Multiple Lot model for Parallel Machines, denoted ML-PM, and Multiple Lot model with Setup-Overlapping for Parallel Machines, denoted MLOV-PM, are the extensions of ML-SM and MLOV-SM respectively.

5.4.1 Parallel Machines

The input data required by the PM models are:

d_{it} Demand for product *i* realised at the end of period *t*

C_{mt} Available capacity time of machine *m* in each period *t*

st_{ijm} Time needed to setup from product *i* to product *j* on machine *m*

sc_{ijm} Cost needed to setup from product *i* to product *j* on machine *m*

b_{im} Time needed to produce a unit of product *i* on machine *m*

h_{it} Cost of holding a unit of product *i* from period *t* to *t*+1

g_{it} Backlog cost per period for product *i* from period *t* to *t*+1

UB_{imt} Upper bound C_{mt}/b_{im} on the quantity of product *i* produced in period *t* on machine *m*

i_{0m} The product setup at the end of period 0 on machine *m*, i.e., the starting setup configuration

The decisions variables by the PM model are represented by following variables:

I_{it} Inventory level of product *i* at the end of period *t.*

B_{it} Backordered amount of product *i* at the end of period *t*.

x_{imt} Production quantity of product *i* in period *t* on machine *m.*

Slk_{mt} Number of unites of slack capacity of machine *m* in period *t.*

x^F_{imt} The quantity produced in period *t* of the first (crossover) lot of product *i* on machine *m* in period *t* if it was setup in period *t*-1, otherwise 0.

x^L_{imt} The quantity produced in period *t* of the last (crossover) lot of product *i* on machine *m* in period *t* if its production continues into period *t*+1, otherwise 0.

y_{ijmt} Number of times that production is to be changed over from product *i* to product *j* on machine *m* in period *t*, Integer non-negative.

z_{imt} Number of times that product *i* is in a setup state on machine *m* in period *t*, Integer non-negative.

α_{imt} = 1 either because *j*-to-*i* is the last setup of machine *m* in previous periods to *t* or because *j*-to-*i* is the setup operation that overlaps from *t*-1 to *t*.

a_{ijmt}^{k} =1 if the arc $i \rightarrow j$ is on a walk from crossover product p_t^{α} to product *k* within period *t*'s sequence of lots on machine *m*, otherwise 0.

z_{imt}^{bin} =1 if product *i* is ever in setup state on machine *m* in period *t*, otherwise 0.

OLS_{imt} =1 if the overlapping setup operation *j*-to-*i* on machine *m* begins in period *t* and finishes in period *t*+1.

S_{mt} The amount of setup time that overlaps into period *t*+1 on machine *m*, having begun at the end of period *t*.

All the ML-PM and MLOV-PM's constraints are similar to ML-SM and MLOV-SM respectively with the new adapted data and variables. The ML-PM and MLOV-PM models are presented completely in Appendix C and D.

5.4.2 Flexible Flow Line

To model different machines at each stage *e* of FFL, an index m_e is used. *E* is the number of different stages e and M_e is the number of different machines m_e available for production at stage *e*. Apart from inventory and backlogs variables, the FFL's data and variables are similar to PM's where index m is replaced by index m_e. The new inventory and backlogs variable of FFL are as follow:

I_{iet} Inventory level of product *i* at stage *e* at the end of period *t*.

B_{iEt} Backordered amount of product *i* at the last stage *E* at the end of period *t*.

Thus the new inventory balance constraints are:

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E} x_{j\,m_e t} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\, j,t \quad (5\text{-}28)$$

$$I_{je,t-1} + \sum_{m_e} x_{j\,m_e t} - I_{jet} = \sum_{m_{e+1}} x_{j\,m_{e+1},t+1} \qquad \forall\, j,t \text{ and } e = 1,\dots,E-1 \quad (5\text{-}29)$$

$$B_{itE} \leq BP \cdot d_{it} \qquad \forall\, i,t \quad (5\text{-}30)$$

Constraints (5-28) and (5-29) express the material balance including backorders for end items and work in process respectively. Constraint (5-30) bounds backorders of end items in any period to be within a specified proportion of demand. This is the practiced assumptions in flexible flow shop manufacturing systems (Özdamar and Barbaroso lu, 1999). Moreover the holding cost will be different at each stage so h_{it} now becomes h_{iet} which shows the cost of holding a unit of product *i* from period *t* to t+1 at stage *e*. Multiple Lot model for Flexible Flow Lines, denoted ML-FFL, and Multiple Lot model with Setup-Overlapping for Flexible Flow Lines, denoted MLOV-FFL, are presented completely in Appendix E and F respectively. Apart from inventory balance constraints, the FFL's constraints is similar to PM's where substituting index m with index m_e.

5.5 At Most One Lot models

Multiple Lot model for any machine configurations is valid irrespective of whether there are non-triangular setups or not. However, when setups are triangular then there exists an optimal solution with zero or one lots per product per period (Clark and Clark, 2000). In this case, the formulation (ML-SM) can then be simplified to a model that assumes *At Most One Lot* per product per period (denoted 1L-SM) by merging z_{jt} and z_{jt}^{bin} to be a binary variable z_{jt} for single machine. Thus constraints (5-15) and (5-16) disappear. Similarly the ML-PM and ML-FFL models can be simplified to the 1L-PM and 1L-FFL models respectively by merging z_{jmt} and z_{jmt}^{bin} to be a binary variable z_{jmt} for PM and merging $z_{j\,m_e t}$ and $z_{j\,m_e t}^{bin}$ to be a binary variable $z_{j\,m_e t}$ for FFL. Thus constraints (15) and (16) of ML-PM (Appendix C) and constraints (17) and (18) of ML-FFL (Appendix E) disappear.

One Lot model is also valid irrespective of whether the setups are triangular or not, but in the latter case, One Lot model's solution could be suboptimal given its limitation of zero or one lots per product per period. In the presence of triangular setups, multiple lots per product per period could occur but this is not required for optimality and so in general it is avoided in models for triangular setups. The computational tests in the next section explore the impact of this limitation.

5.6 Computational tests

Many models in the literature assume that there will be at most one lot per product per period. What are the pros and cons of this assumption? On the one hand, the model will be smaller with fewer variables and constraints, so we might expect faster solution. On the other hand, the solutions with Multiple Lots (ML) per product per period will be excluded, so we will expect worse solutions in some cases. Clark, Rangel and I investigated this supposed trade-off via the computational tests for single machine and showed that often it may not exist (Clark et al., 2012). We demonstrated that the multiple lots features of models enables more efficient production than when the formulation is restricted to single lot per product per period.

The aim of the tests in this thesis is to assess how effectively the ML model took advantage of shortcut products to reduce the total time spent on setups, compared to the equivalent One Lot (1L) model for different machine configurations including SM, PM and FFL. The tests also evaluated the consequences of modelling all necessary features of production in the multiple lot model with setup overlapping, denoted MLOV, on reducing demand backlogs, total inventory and cost in the case of tight production capacity for SM, PM and FFL production system. The models were implemented in the optimisation modelling software GAMS build 23.6.5 (Brooke et al., 1988) and solved using the industrial-strength CPLEX 12.0 solver (CPLEX., 2010) on a computer with a 2.1 GHZ CPU and 2 GB of RAM. The CPLEX optimizer was allowed to run for a maximum of 1 hour of running time, at which point the incumbent solution (i.e., the best found up to then) was used.

To obtain initial insight, the performance of the three models (1L-SM, ML-SM and MLOV-SM) was first compared with 20 problem instances on a single machine. Then via one test problem, the efficiency of the three models for PM and FFL production systems were showed in details.

5.6.1 Results of Single Machine

Consider a production system with a single machine and $J = 10$ products whose lot sizes and sequences were to be scheduled over a horizon of $T = 4$ demand periods. The following data were used: $C_t = 100$, $ml_j = 5$, $i_0 = 1$, $b_j = 0.5$, $h_{jt} = 10$, $g_{jt} = 10000, \forall j, t$ for all instances. In our recent paper (Clark et al., 2012) the setup times were initially set to be $st_{ij} = (j - i) if\ j \geq i$ otherwise $(10 + j - i)$, so the product 2 would normally be setup immediately after product 1. However, product 5 was then made an extreme shortcut with zero setup times: $st_{5j} = st_{i5} = 0$. In this thesis to make setup times more tangible particularly in case of setup overlapping, all setup times were increased by 3 so setup times were changed to $st_{5j} = st_{i5} = 3$ and $st_{ij} = (3 + j - i)\ if\ j \geq i$ otherwise $(13 + j - i)$. Setup costs are proportional to setup times, i.e. $sc_{ij} = 50 \times (j - i)\ if\ j \geq i$, otherwise $50 \times (10 + j - i)$, and for shortcut product are: $sc_{5j} = sc_{i5} = 50$.

The periodic demand forecasts d_{it} varied randomly over product i and period t to provoke non-uniform lot-sizes and avoid lot-for-lot production. To show the effectiveness of the setup overlapping model, the demands in two consecutive periods are set to be non-zero for different products. For example, if there are 10 products then for period t, 5 random products have non-zero demand with the other 5 having demand zero, while in period t+1 those products with zero-demand in period t now have non-zero demand with other 5 having zero demand. When capacity is loose, then there is much flexibility about when setups can occur in an optimal solution, so we expect that period-overlapping setups will not make a difference. However, in tight capacity, there will be little such flexibility, so it is important to use scarce production capacity efficiently via relaxing all restrictions of physical separation between the periods. To simulate tight capacity the overall demand was adjusted so that setup times could take up to 20-25% of capacity. Thus a total of 20 problem instances were generated for single machine and each problem solved by the 1L-SM, ML-SM and MLOV-SM models. Table 5-4 compare the performance of three models on 6 criteria calculated over the planning horizon.

1. Total time spent on setups = $\sum_{ij} st_{ij}\ y_{ijt}$
2. Amount of unused (slack) capacity = $\sum_{t}\ slk_{t}$
3. Inventory = $\sum_{it} I_{it}$
4. Backlogs = $\sum_{it} B_{it}$
5. CPU time
6. Total cost = Backlogs + Inventory + Setup = $\sum_{it} g_{it}\ B_{it} + \sum_{it} h_{it}\ I_{it} + \sum_{ijt} sc_{ij}\ y_{ijt}$

Table 5-4: A mean results of 1L-FFL, ML-FFL and MLOV-FFL for 20 single machine problems

Single Machine	1L-SM	ML-SM	MLOV-SM	P-value
Mean Setup time	77.75	78	77.95	0.902
Mean Slack capacity	5.575	5.975	6.725	0.061
Mean Inventory	**14.45**	**10.5**	**6.1**	**0.000**
Mean Backlogs	**7.55**	**5.8**	**4.1**	**0.000**
Mean CPU time (seconds)	5.4	3.95	4.55	0.136
Mean Total cost = (Backlogs + Inventory +Setup)	76842 (75500+144.5+1197.5)	59260 (58000+105+1155)	42208.5 (4100+61+1147.5)	**0.000**

For each criterion, the difference between the mean values for the three models was statistically tested using a balanced analysis of variance test. The test used the data instance (that is the run) as a random blocking factor. The null hypothesis is that the difference between the models means is zero. The results in Table 5-4 and also the paired T-test's P-values in Table 5-5 show the highly significant decrease in backlogs, inventory and total cost for the model MLOV-SM compared to those for the ML-SM and 1L-SM. The ML-SM is also more efficient than 1L-SM because of using the shortcut product 5 to economise on setups and reduce numbers of backlogs, inventory.

Table 5-5: The paired T-test results between 1L-FFL, ML-FFL and MLOV-FFL for 20 single machine problems

P-values of the paired T-test	1L-SM & ML-SM	ML-SM & MLOV-SM	1L-SM & MLOV-SM
Setup time	0.296	0.467	0.384
Slack capacity	0.175	0.076	0.016
Inventory	**0.005**	**0.049**	**0.003**
Backlogs	**0.000**	**0.000**	**0.000**
CPU time (seconds)	0.020	0.096	0.189
Total cost	**0.000**	**0.000**	**0.000**

5.6.2 Results of Parallel Machines

Consider the production system with 2 machines in parallel. The aim is to satisfy the demand shown in Table 5-6 for 10 products over the 4 planning periods with minimal backorders, inventory and setup costs. The capacity of each machine is $C_{mt} = 50$, thus a total capacity of $\sum_m C_{mt} = 100$ is available for each period. The remained PM data is the same as SM problem: $ml_j = 5$, $i_{0m} = 1$, $b_{jm} = 0.5$, $h_{jt} = 10$, $g_{jt} = 10000, \forall j, t$, also setup times and costs of each machine replicate those for single machine.

Table 5-6: Demand data for PM and FFL.

d_{it}	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$i = 1$	33	0	34	0
$i = 2$	33	0	0	0
$i = 3$	31	0	33	0
$i = 4$	33	0	0	0
$i = 5$	30	0	34	0
$i = 6$	0	33	30	33
$i = 7$	0	33	0	33
$i = 8$	0	24	32	33
$i = 9$	0	33	0	31
$i = 10$	0	31	0	33

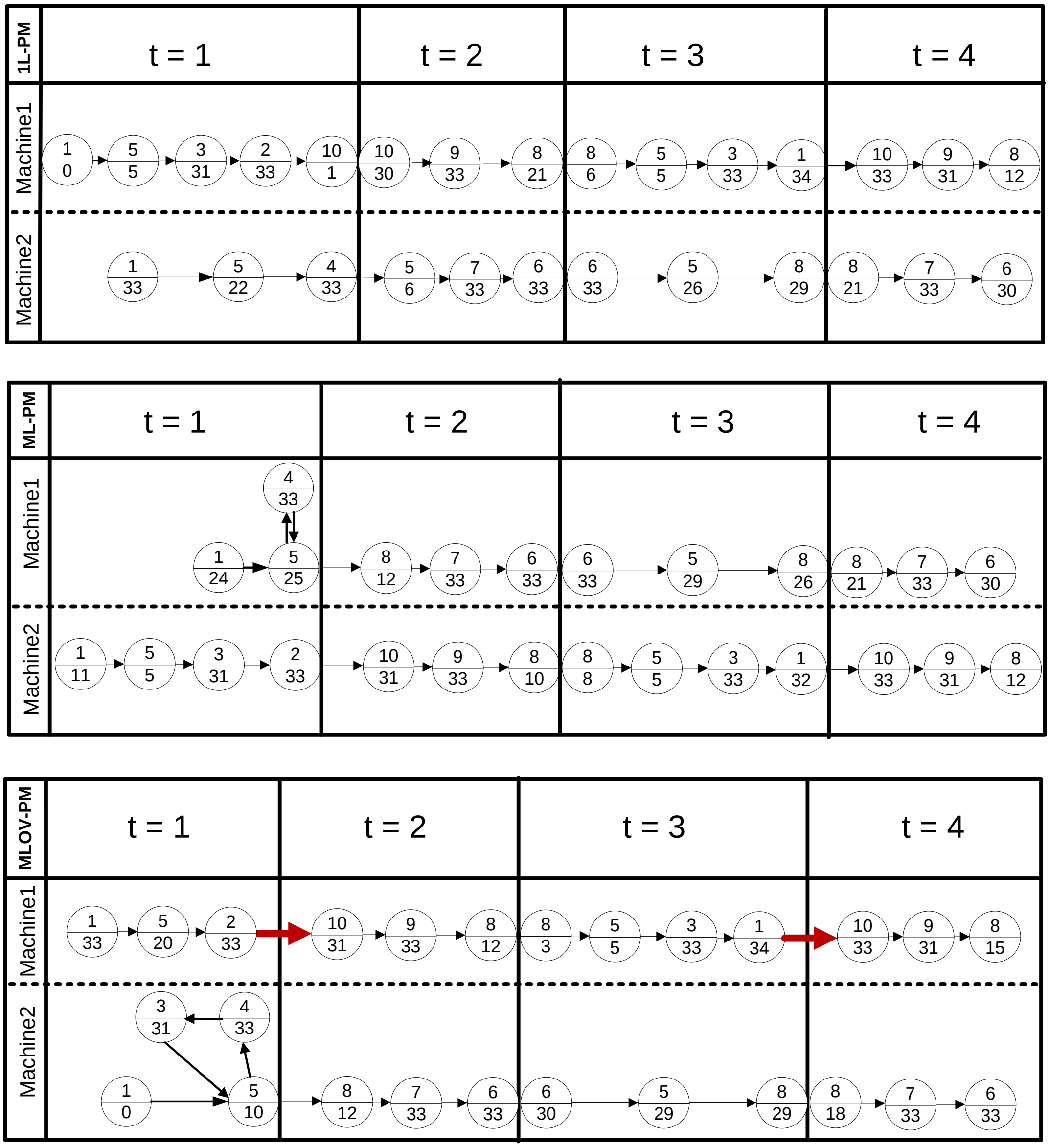

Figure 5-10: The production diagrams of 1L-PM, ML-PM and MLOV-PM

The production diagrams and the results obtained by solving the 1L-PM, ML-PM and MLOV-PM models are shown in Figure 5-10 and Table 5-7 respectively. Note that in Table 5-7, the 1L-PM and ML-PM model found the solution with the same number of inventory 7, and 6 and 2 backlogs respectively while the optimal solution found by MLOV-SM has no backlogs and inventory.

In the production diagram Figure 5-10, each node or circle represents a product at the top and its lot size at the bottom, and each arrow demonstrates a setup and an overlapped setup in bold as below:

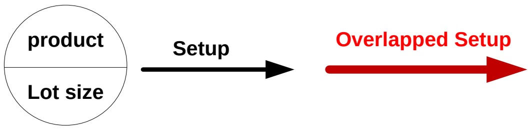

Table 5-7: Results of 1L-PM, ML-PM and MLOV-PM

Parallel machine	1L-PM	ML-PM	MLOV-PM
Setup time	76	80	80
Slack capacity	4	0	0
Inventory	7	7	0
Backlogs	6	2	0
CPU time (seconds)	774	315	451
Total cost = Cost of (Backlogs + Inventory +Setup)	61220 (60000+70+1150)	21270 (20000+70+1200)	1200 (0+0+1200)

Note that how effectively the MLOV-FFL model (Figure 5-10), took advantage of overlapping setup on machine 1 twice to use up machine capacity and reduce inventory, backlogs and slack time. Furthermore both multiple lot models, ML-FFL and MLOV-FFL took advantage of shortcut product 5 and reduce the backlogs, compare to the one lot model 1L-FFL.

5.6.3 Results of Flexible Flow Lines

If the parallel machines production system is duplicated in series then we have a Flexible Flow Lines (FFL) production system with two stages in series and two parallel machines for each stage. In this case, the FFL data for each stage is exactly the same as PM. Holding costs assume that successive stages add value, so that work-in-process holding costs will increase as material progresses along the line. To reflect this, a value-added percentage factor VAP is used, whose value is 1.2. The first stage's unit holding cost h_{it1} for product i is 10 and for the subsequent stages,

$h_{ite} = VAP \cdot h_{it,e-1}$, $e \geq 2$. Thus the second stage's unit holding cost h_{it2} for product i is $h_{it2} = 1.2 \times 10 = 12$.

To analyse the FFL in details, firstly it was solved by the three models 1L-FFL, ML- FFL and MLOV- FFL models considering the demand of first and second period in Table 5-6. The production diagrams and the results of FFL for two periods are shown in Figure 5-11 and Table 5-8 respectively. Then the FFL was solved by three models considering demands of 4 periods, Table 5-6, and the results are presented in Table 5-9.

In order to simplify the FFL production diagram, the one-period-backward shifted demand is considered for intermediate stages ($e < E$), means that $x_{jm_{e+1},t+1}$ in the right hand of equation (5-29) changes to $x_{jm_{e+1}t}$. Thus for first stage, the inventory balance equation would be $I_{j1,t-1} + \sum_{m_1} x_{jm_1t} - I_{j1t} = \sum_{m_2} x_{jm_2t}$, $\forall j, t$.

Note that for two periods demand, the ML-FFL model took advantage of shortcut product in both stages and efficiently used the capacity of all four machines to reduce inventory, backlogs and slack capacity, compared to the ML-FFL. As shown in Table 5-8, the backlogs and inventory fell to 2 and 0 respectively for the ML-FFL model and they both fell to 0 for the MLOV-FFL. Thus the MLOV-FFL, used the total scarce production capacity of 4 machines more efficiently by taking advantage of overlapping setup three times (Figure 5-11) and left no inventory, shortage and slack capacity.

Table 5-8: Results of 1L-FFL, ML-FFL and MLOV-FFL for FFL problem with two periods

Flexible Flow Line	1L-FFL	ML-FFL	MLOV-FFL
Setup time	78	86	86
Slack capacity	9	2	0
Inventory	3	0	0
Backlogs	6	2	0
CPU time (seconds)	623	662	656
Total cost = Cost of (Backlogs + Inventory +Setup)	61236 (60000+36+1200)	21300 (20000+0+1300)	1300 (0+0+1300)

All the three models were solved the FFL problem with two periods in about 10 to 11 minutes. However for the FFL problem with four periods, only the 1L-FFL model could find the optimal solution within the maximum of one hour running time (took exactly one hour) while the MLOV-FFL and MLOV-FFL left large optimality gaps provided by the CPLEX as shown in Table 5-9.

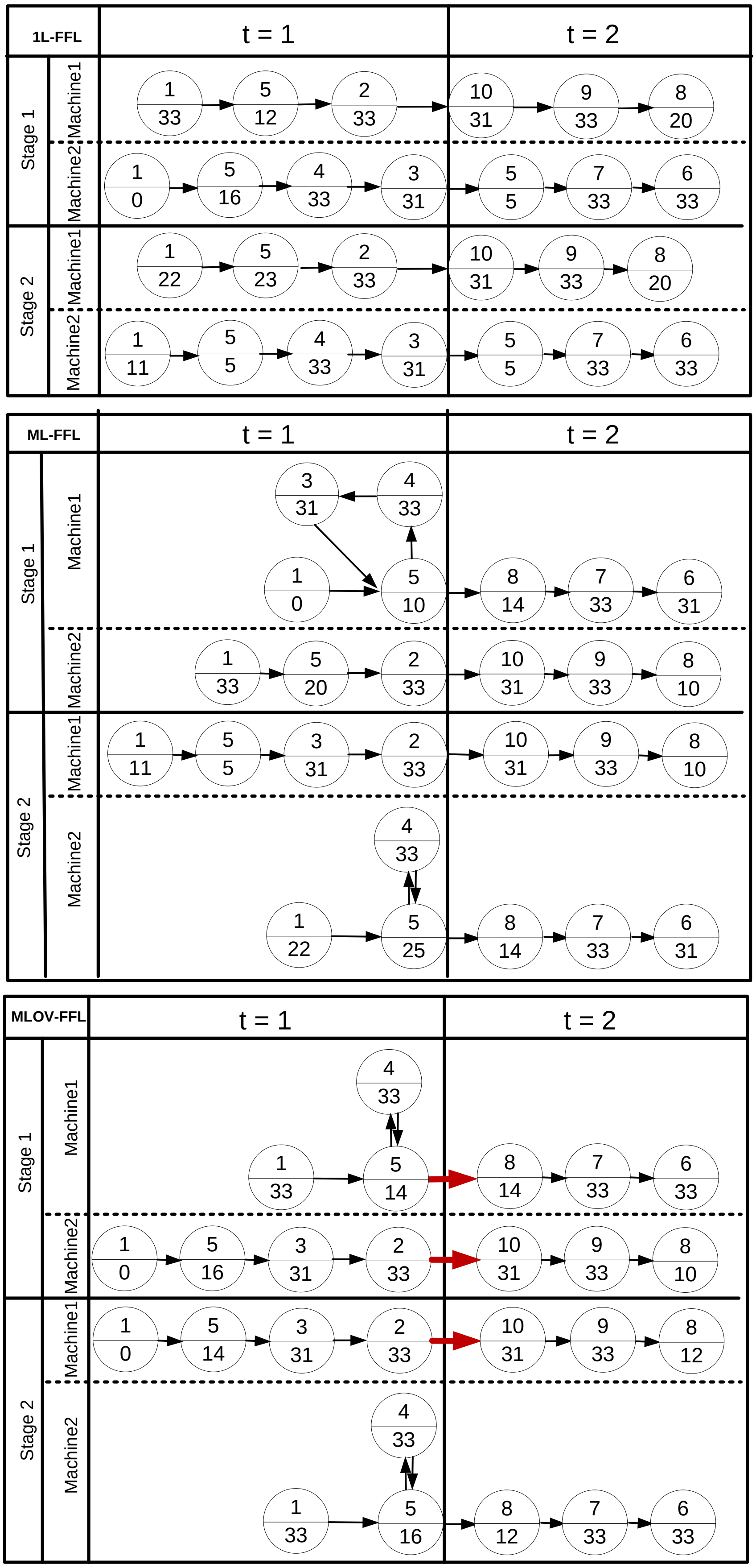

Figure 5-11: The production diagrams of 1L-FFL, ML-FFL and MLOV-FFL with two periods

Table 5-9: Results of 1L-FFL, ML-FFL and MLOV-FFL for FFL problem with four periods

Flexible Flow Line	1L-FFL	ML-FFL	MLOV-FFL
RAM usage	102MB	140MB	1362MB
Optimality Gap%	0%	96.71%	97.5%
Backlogs	6	6	8
Total cost	62408	62384	82410

Due to the importance of number of binary variables in large instances, the MLOV-FFL exhausted the 1362 MB of 2 GB available RAM before terminating the CPLEX branch-&-cut search, leaving large optimality gap because of the extra binary variable of setup overlapping $OLS_{ij m_e t}$ compared with the other models.

5.7 Final remarks

This chapter presented the new mix integer programming formulations for capacitated lot sizing and scheduling with non-triangular and sequence-dependent setup times and costs incorporating all necessary features of setup carryover and overlapping on different machine configurations. These features relax all limitation of physical separation between the periods which contrast the nature of production system and give more flexibility to the lot sizing model. Moreover the innovation of the new formulation has been the modelling of non-triangular sequence-dependent setups within lot sizing model based on ATSP problem that allows multiple lots per product per period with polynomial number of disconnected subtours prohibition constraints.

To assess how effectively the multiple lot model with setup overlapping took advantage of shortcut product and setup overlapping feature to reduce backlogs and inventory, three models 1L, ML and MLOV were compared for three production systems SM, PM and FFL. The one-Lot (1L) model is the ML simplified model that assumes at most one lot per period. The computational results showed that the multiple-lots and setup overlapping features of the model enable more efficient production than when the formulation excludes setup overlapping or is restricted to single lot per product per product.

On single machine (SM) the results of 20 instances showed the highly significant decrease in backlogs, inventory and total cost for the model MLOV-SM compared to those for the ML-SM and 1L-SM. Furthermore the ML-SM is more efficient than 1L-SM because of using the shortcut product 5 to economise on setups and reduce numbers of backlogs, inventory. The results of PM and FFL also confirmed the effectiveness of the new formulation however because of increasing of number of binary variables in large instances, the MLOV exhausted the 2 GB available RAM before terminating the CPLEX branch-&-cut search, leaving large optimality gap because of the extra binary variable of setup overlapping. To sum up, the test results above, although merely probing, and not conclusive, indicate that model MLOV for all machine configurations finally obtains a better solution in small problem sizes. However due to the importance of number of binary variables in large instances, the 1L model is solved in much shorter time compared with ML and MLOV. Thus there is the need for future research to develop efficient solution method for MLOV on different machine configurations particularly PM and FFL. Future work will also computationally compare the different demand data pattern with variables sizes on SM, PM and FFL.

Chapter 6

Conclusion

By increasing pressure of competition towards a globalized economy, many companies focus on high product quality, low costs and quick response in order to satisfy turbulent market demands. To achieve this goal they need the high capability of production planning systems which are too elaborate to be considered in a monolithic way. Thus production planning systems are classified to long-term, medium-term and short-term. Among these three levels of planning, short-term has a crucial role and consists of scheduling and lot sizing problems.

This thesis breaks new ground by modelling lot sizing and scheduling in a Flexible Flow Line (FFL) simultaneously instead of separately. The problem, called the 'General Lot sizing and Scheduling Problem in a Flexible Flow Line' (GLSP-FFL), optimizes the lot sizing and scheduling of multiple products at multiple stages, each stage having multiple machines in parallel. The main novelty of this thesis is to develop linear Mix Integer Programming (MIP) formulations for this problem incorporating a variety of practical assumptions. Here the contributions of the chapters are briefly summarized along with the presentation of future research directions.

6.1 Summary

Firstly, chapter 2 reviews the literature and recent developments of deterministic dynamic lot sizing problems. The focus of the review is on capacitated lotsizing with sequence-dependent setup which is closely interrelated to scheduling and considerably combined in the literature. However, it can be more complicated and challenging to integrate both problems in complex production systems. The review updates the literature regarding the modelling perspective of this challenge on a variety of machine configurations and points out the sparse research on production systems with more than one machine despite of its extensive real-world applications.

The Flexible Flow Line (FFL) is a very prevalent production system and can be found in vast number of industries such as automotive, chemical, electronics, steel making, food and textile. Multiple products can be produced at stages and production at each stage involves unrelated parallel machines with different production rates. All machines can produce any product. The available capacity of each machine is limited and can vary between periods and stages. A changeover from one product to another requires a setup time during which the machine is unproductive. Setup times and costs are sequence dependent and can vary between machines. The finite planning horizon is divided into T periods and the independent demand for all products is felt at the final stage at the end of each period. It is known with certainty, but varies dynamically over the planning horizon. The decisions to be taken are the determination of production quantities (lots), machine assignments and production sequences (schedules) on each machine at each stage in a FFL. The objective is to minimise setup, inventory holding and backorder costs. Moreover in this thesis, the new challenges have been researched such as lot splitting and shortage, the practiced assumptions in flexible flow shop manufacturing systems, and non-triangular setup times and costs. Non-triangular setups occur in certain industries like food, textile and oil where the production of intermediate or cleaning product can clean the machines and reduce overall setup times (costs). In order to obtain viable schedules, "lead-time synchronization" is assumed i.e., there is the lead time of one period between different production stages. In this case, a product which is produced at a stage is available for production at the next stage only in the next period.

Fleischmann and Meyr (1997) integrated the lot sizing and scheduling of several products on a single capacitated machine, calling their model the *General Lot sizing and Scheduling Problem* (GLSP). Afterwards, Meyr (2002) extended this problem for parallel machines and in an alternative approach Clark and Clark (2000) designed a mixed integer programming (MIP) model for simultaneous sequencing and lot sizing production lots on a set of parallel machines in the presence of sequence-dependent setup times. According to different formulations, three distinct models are introduced for GLSP-FFL in chapter 3. The first model FFL-CC is based on Clark and Clark's (2000) sequencing formulation technique when the second model FFL-FM is based on Fleischmann and Meyr's (1997). The computational tests indicate that the FFL-CC is more effective and has a much smaller optimality gap than FFL-FM. The third novel MIP model FFL-ATSP is based on adaptation of ATSP that shows significant improvement of problems solutions for small and big problem sizes in very much shorter time than FFL-CC and FFL-FM. For example, for small problems FFL-ATSP found the optimal solution in a mean time of 10 seconds, while the FFL-CC and FFL-FM models not only could not find an optimal solution in one hour but also ended the search with a large optimality gap. Three different ANOVA tests including a Balanced ANOVA and the non-parametric Friedman test and Two-Factor ANOVA Analysis with Replication have been carried to compare the models. The result indicates highly significant differences between three models and no significant interactions between models and combinations and different levels of combinations.

GLSP-FFL is an NP-hard problem and even a well designed exact MIP model FFL-ATSP, cannot find any feasible solution in reasonable computing time for some large problems. Hence, it is needed to develop an efficient solution procedure. Chapter 4 is devoted to a meta-heuristic algorithm, Adaptive Simulated Annealing (ASA) with an effective adaptive temperature control scheme. The adaptive temperature control scheme changes temperature based on the number of consecutive improving moves and maintains it above the minimum level. The ASA algorithm is based on Azizi and Zolfaghari's cooling schedule (Azizi and Zolfaghari, 2004) that applied for GLSP-FFL. The main advantage of ASA is providing a higher chance of an uphill transition once the search traps in a local minimum regardless of the iteration number by dynamically adjustment of the temperature based on the profile of the search path.

Four initial solutions and three neighbour operators are designed for ASA. The first initial solution is based on generating a random sequence of products on machines of each stage and then running the linear model of GLSP-FFL to find the optimal lot size of the sequence. For the second initial solution, the external demand of each product is considered as the product lot size in each stage and period and the sequences of lots are determined by Loading Heuristic algorithm (LHR). Finally, the third novel initial solution is obtained by solving well-organized model which extracts from the GLSP-FFL. The model is Capacitated Lot Sizing Problem for Multi Stage systems (MS-CLSP) with single machine in each stage. MS-CLSP gives the inventory feasible lot sizes which need to be scheduled by loading heuristic algorithm on parallel machines of stages in FFL. The fourth initial solution is generated by solving the sequencing model of the GLSP-FFL to find the sequence and then it is given to the linear lot sizing model of GLSP-FFL to determine its optimal lot size.

The first initial solution is feasible but the other initial solutions may be capacity-infeasible solutions. In the ASA procedure capacity infeasibilities are heavily penalized in the objective function and inventory-feasibility is always preserved when generating neighbours by the neighbour operators. The numerical test shows that for small problems, ASA with the fourth initial solution and for big problems ASA with the third initial solution is able to find much better solutions than other initial solutions.

Finally, chapter 5 presents the novel mix integer programming formulations for lot sizing and scheduling with non-triangular and sequence-dependent setup times and costs incorporating all necessary features of setup carryover and overlapping on different machine configurations including: conserving setup state when no product is being processed over period(s); starting setup in a period and ending in the next period; ending setup at a period and starting production in the next period(s); crossing an imposed minimum lot size over periods. These features relax all limitation of physical separation between the periods which contrast the nature of production system and give more flexibility to the lot sizing model. Furthermore when setup times and/or costs are non-triangular, it can sometimes be optimal for a shortcut product to be produced in more than one lot in each period. To models this, the ATSP-based formulation is developed that allows multiple lot production within a period and is more efficient than other models as used polynomial number of disconnected subtours prohibition constraints.

Firstly the new model is explained for a Single Machine (SM) which then extends to other machine configurations including Parallel Machines (PM) and Flexible Flow Lines (FFL). To assess how effectively the model takes advantage of shortcut product and setup overlapping feature to reduce backlogs and inventory, three models one-Lot (1L), Multiple Lot (ML) and Multiple Lot with Overlapping setups (MLOV) are compared for three production systems SM, PM and FFL. The 1L model is the ML simplified model that assumes at most one lot per period. The computational results showed that the multiple-lots and setup overlapping features of the model enable more efficient production than when the formulation excludes setup overlapping or is restricted to single lot per product per product.

The results of 20 instances on SM showed the highly significant decrease in backlogs, inventory and total cost for the model MLOV-SM compared to those for the ML-SM and 1L-SM. Furthermore the ML-SM is more efficient than 1L-SM because of using the shortcut product to economise on setups and reduce numbers of backlogs, inventory. The results of PM and FFL also confirmed the effectiveness of the new formulation however due to the importance of number of binary variables in large instances, the MLOV exhausted the 2 GB available RAM before terminating the CPLEX branch-&-cut search, leaving large optimality gap because of the extra binary variable of setup overlapping.

6.2 Future research directions

There are several interdependencies between lot sizing and scheduling models which make the integrating of these two models crucial particularly when setups are sequence dependent. However, it can be difficult and complicated to combine both models for complex production systems such as flow shops and flexible flow lines, thus they are often modelled and solved independently in spite of their interdependencies. The main novelty of this thesis is to model these two problems simultaneously while incorporating a variety of practical assumptions and develop MIP formulations for it. The major limitation of the mathematical modelling of this problem, is that even a well designed exact MIP model (FFL-ATSP) cannot find any feasible solution in reasonable computing time for some large problems. This highlights the importance role of developing efficient solution algorithm(s) for GLSP-FFL.

One interesting area for future research is the integration of Simulated Annealing (SA) and Lagrangean Relaxation (LR) for lot sizing and scheduling in FFL. The hybrid heuristics like LR/SA have already show promise in multi-stage capacitated lot sizing problem (Özdamar and Barbaroso lu, 1999) as the performance of LR can be enhanced by using SA within the Lagrangean heuristics in each iteration.

According to the result of chapter 5, there is the need for future research to develop efficient solution method for MLOV on different machine configurations, possibly via exact methods such as Lagrangian Relaxation coupled with decomposition into single periods where the sub-models can be solved very rapidly, or via heuristic methods such as *Relax-&-Fix* methods of various types or local branching. Future work will also computationally compare the ML model against a functionally equivalent GLSP model and Menezes et al. (2011)'s ATSP-based iterative method which allowed non-triangular setups. Moreover in case of triangular setups, computationally comparing GLSP approach with different ATSP approaches based on a variety of subtour elimination method such as Miller, Tucker and Zemlin's subtours elimination method (Miller et al., 1960) and Claus's multi-commodity-flow (MCF) formulation (1984) is another research opportunities to explore.

Given that in case of existing non-triangular setups the enough production of intermediate or cleaning product can clean the machine less costly, the question arises as to whether the quantity of cleaning product called minimum lot size is sequence dependent. This poses another research challenge about how to model the sequence dependency of minim lot sizes in lot sizing and scheduling problems.

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Appendices

Appendix A: ML-SM model

Minimise $\sum_{ijt} sc_{ij}\ y_{ijt} + \sum_{it} h_{it}\ I_{it} + \sum_{it} g_{it}\ B_{it}$ (1)

$I_{jt-1} - B_{jt-1} + x_{jt} - I_{jt} + B_{jt} = d_{jt}$ $\forall\ j,t$ (2)

$\sum_{i} b_i\ x_{it} + \sum_{ij} st_{ij}\ y_{ijt} + slk_t = C_t$ $\forall\ t$ (3)

$x_{jt} \leq UB_{jt} \times z_{jt}^{bin}$ $\forall\ j,t$ (4)

$y_{jjt} = 0$ $\forall\ j,t$ (5)

$\sum_{i} \alpha_{it} = 1$ $\forall\ t = 1,..,T+1$(6)

$\alpha_{i_o t} = 1$ $\forall\ t = 1$(7)

$x_{jt}^{F} \leq UB_{jt}\ \alpha_{jt}$ $\forall\ j,t$ (8)

$x_{jt}^{L} \leq UB_{jt}\ \alpha_{j,t+1}$ $\forall\ j,t$ (9)

$x_{jt}^{L} + x_{j,t+1}^{F} \geq ml_j\ \alpha_{j,t+1}$ $\forall\ j,t$ (10)

$x_{jt} - x_{jt}^{F} - x_{jt}^{L} \geq ml_j\ (z_{jt} - \alpha_{jt} - \alpha_{j,t+1})$ $\forall\ j,t$ (11)

$\alpha_{it} + \sum_{j} y_{jit} = z_{it}$ $\forall\ i,t$ (12)

$\sum_{j} y_{ijt} + \alpha_{i,t+1} = z_{it}$ $\forall\ i,t$ (13)

$a_{ijt}^{k} \leq y_{ijt}$ $\forall\ i,j,k,t$ (14)

$z_{it} \geq z_{it}^{bin}$ $\forall\ i,t$ (15)

$z_{it} \leq ZUB_i z_{it}^{bin}$ $\forall\ i,t$ (16)

$\alpha_{kt} + \sum_{i} a_{ikt}^{k} = z_{kt}^{bin}$ $\forall\ k,t$ (17)

$\alpha_{it} + \sum_{j} a_{jit}^{k} \geq \sum_{j} a_{ijt}^{k}$ $\forall\ k, i \neq k, t$ (18)

$a_{kjt}^{k} = 0$ $\forall\ k,j,t$ (19)

Appendix B: MLOV-SM model

Minimise $\sum_{ijt} sc_{ij}\ y_{ijt} + \sum_{it} h_{it}\ I_{it} + \sum_{it} g_{it}\ B_{it}$ (1)

$$I_{jt-1} - B_{jt-1} + x_{jt} - I_{jt} + B_{jt} = d_{jt} \qquad \forall\ j,t\ (2)$$

$$\sum_i b_i\ x_{it} + \sum_{ij} st_{ij}\ y_{ijt} + S_{t-1} - S_t + slk_t = C_t \qquad \forall\ t\ (3)$$

$$x_{jt} \leq UB_{jt} \times (z_{jt} - \sum_i OLS_{ijt}) \qquad \forall\ j,t\ (4)$$

$$y_{jjt} = 0 \qquad \forall\ j,t\ (5)$$

$$\sum_i \alpha_{it} = 1 \qquad \forall\ t = 1,..,T+1 (6)$$

$$\alpha_{i_o t} = 1 \qquad \forall\ t = 1 (7)$$

$$x_{jt}^F \leq UB_{jt}\ \alpha_{jt} \qquad \forall\ j,t\ (8)$$

$$x_{jt}^L \leq UB_{jt}\ (\alpha_{j,t+1} - \sum_i OLS_{ijt}) \qquad \forall\ j,t\ (9)$$

$$x_{jt}^L + x_{j,t+1}^F \geq ml_j\ \alpha_{j,t+1} \qquad \forall\ j,t\ (10)$$

$$x_{jt} - x_{jt}^F - x_{jt}^L \geq ml_j\ (z_{jt} - \alpha_{jt} - \alpha_{j,t+1}) \qquad \forall\ j,t\ (11)$$

$$\alpha_{it} + \sum_j y_{jit} = z_{it} \qquad \forall\ i,t\ (12)$$

$$\sum_j y_{ijt} + \alpha_{i,t+1} = z_{it} \qquad \forall\ i,t\ (13)$$

$$a_{ijt}^k \leq y_{ijt} \qquad \forall\ i,j,k,t\ (14)$$

$$z_{it} \geq z_{it}^{bin} \qquad \forall\ i,t\ (15)$$

$$z_{it} \leq ZUB_i z_{it}^{bin} \qquad \forall\ i,t\ (16)$$

$$\alpha_{kt} + \sum_i a_{ikt}^k = z_{kt}^{bin} \qquad \forall\ k,t\ (17)$$

$$\alpha_{it} + \sum_j a_{jit}^k \geq \sum_j a_{ijt}^k \qquad \forall\ k, i \neq k, t\ (18)$$

$$a_{kjt}^k = 0 \qquad \forall\ k,j,t\ (19)$$

$$S_t \leq \sum_{ij} st_{ij}\ OLS_{ijt} \qquad \forall\ t\ (20)$$

$$\sum_j OLS_{ijt} \leq \alpha_{i,t+1} \qquad \forall\ i,t\ (21)$$

$$OLS_{ijt} \leq y_{ijt} \qquad \forall\, i,j,t \;(22)$$

Appendix C: ML-PM model

Minimise $\sum_{ijmt} sc_{ijm}\, y_{ijmt} + \sum_{it} h_{it}\, I_{it} + \sum_{it} g_{it}\, B_{it}$ (1)

$$I_{jt-1} - B_{jt-1} + \sum_{m} x_{jmt} - I_{jt} + B_{jt} = d_{jt} \qquad \forall\, j,t \;(2)$$

$$\sum_{i} b_{im}\, x_{imt} + \sum_{ij} st_{ijm}\, y_{ijmt} + slk_{mt} = C_{mt} \qquad \forall\, m,t \;(3)$$

$$x_{jmt} \leq UB_{jmt} \times z_{jmt}^{bin} \qquad \forall\, j,m,t \;(4)$$

$$y_{jjmt} = 0 \qquad \forall\, j,m,t \;(5)$$

$$\sum_{i} \alpha_{imt} = 1 \qquad \forall\; m, t = 1,..,T+1 (6)$$

$$\alpha_{i_{om}mt} = 1 \qquad \forall\, m, t = 1 (7)$$

$$x_{jmt}^{F} \leq UB_{jmt}\, \alpha_{jmt} \qquad \forall\, j,m,t \;(8)$$

$$x_{jmt}^{L} \leq UB_{jmt}\, \alpha_{jm,t+1} \qquad \forall\, j,m,t \;(9)$$

$$x_{jmt}^{L} + x_{jm,t+1}^{F} \geq ml_j\, \alpha_{mj,t+1} \qquad \forall\, j,m,t \;(10)$$

$$x_{jmt} - x_{jmt}^{F} - x_{jmt}^{L} \geq ml_j\, (z_{jmt} - \alpha_{jmt} - \alpha_{jm,t+1}) \qquad \forall\, j,m,t \;(11)$$

$$\alpha_{imt} + \sum_{j} y_{jimt} = z_{imt} \qquad \forall\, i,m,t \;(12)$$

$$\sum_{j} y_{ijmt} + \alpha_{im,t+1} = z_{imt} \qquad \forall\, i,m,t \;(13)$$

$$a_{ijmt}^{k} \leq y_{ijmt} \qquad \forall\, i,j,k,m,t \;(14)$$

$$z_{imt} \geq z_{imt}^{bin} \qquad \forall\, i,m,t \;(15)$$

$$z_{imt} \leq ZUB_{im}\, z_{imt}^{bin} \qquad \forall\, i,m,t \;(16)$$

$$\alpha_{kmt} + \sum_{i} a_{ikmt}^{k} = z_{kmt}^{bin} \qquad \forall\, k,m,t \;(17)$$

$$\alpha_{imt} + \sum_{j} a_{jimt}^{k} \geq \sum_{j} a_{ijmt}^{k} \qquad \forall\, k, i \neq k, m,t \;(18)$$

$$a_{kjmt}^{k} = 0 \qquad \forall\, k,j,m,t \;(19)$$

Appendix D: MLOV-PM model

Minimise $\sum_{ijmt} sc_{ijm}\ y_{ijmt} + \sum_{it} h_{it}\ I_{it} + \sum_{it} g_{it}\ B_{it}$ (1)

$$I_{jt-1} - B_{jt-1} + \sum_{m} x_{jmt} - I_{jt} + B_{jt} = d_{jt} \qquad \forall\, j,t \quad (2)$$

$$\sum_{i} b_{im}\ x_{imt} + \sum_{ij} st_{ijm}\ y_{ijmt} + S_{m,t-1} - S_{mt} + slk_{mt} = C_{mt} \qquad \forall\, m,t \quad (3)$$

$$x_{jmt} \le UB_{jmt} \times (z_{jmt} - \sum_{i} OLS_{ijmt}) \qquad \forall\, j,m,t \quad (4)$$

$$y_{jjmt} = 0 \qquad \forall\, j,m,t \quad (5)$$

$$\sum_{i} \alpha_{imt} = 1 \qquad \forall\ m,t = 1,..,T+1 (6)$$

$$\alpha_{i_{om}mt} = 1 \qquad \forall\ m,t = 1 (7)$$

$$x^{F}_{jmt} \le UB_{jmt}\ \alpha_{jmt} \qquad \forall\ j,m,t \quad (8)$$

$$x^{L}_{jmt} \le UB_{jt}\ (\alpha_{jm,t+1} - \sum_{i} OLS_{ijmt}) \qquad \forall\ j,m,t \quad (9)$$

$$x^{L}_{jmt} + x^{F}_{jm,t+1} \ge ml_j\ \alpha_{jm,t+1} \qquad \forall\, j,m,t \quad (10)$$

$$x_{jmt} - x^{F}_{jmt} - x^{L}_{jmt} \ge ml_j\ (z_{jmt} - \alpha_{jmt} - \alpha_{jm,t+1}) \qquad \forall\, j,m,t \quad (11)$$

$$\alpha_{imt} + \sum_{j} y_{jimt} = z_{imt} \qquad \forall\, i,m,t \quad (12)$$

$$\sum_{j} y_{ijmt} + \alpha_{im,t+1} = z_{imt} \qquad \forall\, i,m,t \quad (13)$$

$$a^{k}_{ijmt} \le y_{ijmt} \qquad \forall\, i,j,k,m,t \quad (14)$$

$$z_{imt} \ge z^{bin}_{imt} \qquad \forall\, i,m,t \quad (15)$$

$$z_{imt} \le ZUB_{im}\, z^{bin}_{imt} \qquad \forall\, i,m,t \quad (16)$$

$$\alpha_{kmt} + \sum_{i} a^{k}_{ikmt} = z^{bin}_{kmt} \qquad \forall\, k,m,t \quad (17)$$

$$\alpha_{imt} + \sum_{j} a^{k}_{jimt} \ge \sum_{j} a^{k}_{ijmt} \qquad \forall\, k, i \ne k, m,t \quad (18)$$

$$a^{k}_{kjmt} = 0 \qquad \forall\, k,j,m,t \quad (19)$$

$$S_{mt} \le \sum_{ij} st_{ijm}\ OLS_{ijmt} \qquad \forall\ m,t \quad (20)$$

$$\sum_j OLS_{jimt} \leq \alpha_{im,t+1} \qquad \forall\, i,m,t \; (21)$$

$$OLS_{ijmt} \leq y_{ijmt} \qquad \forall\, i,j,m,t \; (22)$$

Appendix E: ML-FFL model

Minimise $\sum_{ijemt} sc_{ij\,m_e}\, y_{ij\,m_e t} + \sum_{it} h_{iet}\, I_{iet} + \sum_{it} g_{it}\, B_{iEt}$ (1)

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E} x_{j\,m_e t} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\, j,t \; (2)$$

$$I_{je,t-1} + \sum_{m_e} x_{j\,m_e t} - I_{jet} = \sum_{m_{e+1}} x_{j\,m_{e+1},t+1} \qquad \forall\, j,t \; and \; e = 1,\dots,E-1 \; (3)$$

$$B_{itE} \leq BP \cdot d_{it} \qquad \forall\, i,t \; (4)$$

$$\sum_i b_{im_e}\, x_{im_e t} + \sum_{ij} st_{ij\,m_e}\, y_{ij\,m_e t} + slk_{m_e t} = C_{m_e t} \qquad \forall\, e,m,t \; (5)$$

$$x_{j\,m_e t} \leq UB_{j\,m_e t} \times z^{bin}_{j\,m_e t} \qquad \forall\, j,e,m,t \; (6)$$

$$y_{jj\,m_e t} = 0 \qquad \forall\, j,e,m,t \; (7)$$

$$\sum_i \alpha_{im_e t} = 1 \qquad \forall\; e,m,t = 1,..,T+1 (8)$$

$$\alpha_{i_{om_e} m_e t} = 1 \qquad \forall\, e,m,t = 1 (9)$$

$$x^F_{j\,m_e t} \leq UB_{j\,m_e t}\, \alpha_{j\,m_e t} \qquad \forall\, j,e,m,t \; (10)$$

$$x^L_{j\,m_e t} \leq UB_{j\,m_e t}\, \alpha_{j\,m_e,t+1} \qquad \forall\, j,e,m,t \; (11)$$

$$x^L_{j\,m_e t} + x^F_{j\,m_e,t+1} \geq ml_j\, \alpha_{m_e j,t+1} \qquad \forall\, j,e,m,t \; (12)$$

$$x_{j\,m_e t} - x^F_{j\,m_e t} - x^L_{j\,m_e t} \geq ml_j\, (z_{j\,m_e t} - \alpha_{j\,m_e t} - \alpha_{j\,m_e,t+1}) \qquad \forall\, j,e,m,t \; (13)$$

$$\alpha_{im_e t} + \sum_j y_{ji\,m_e t} = z_{im_e t} \qquad \forall\, i,e,m,t \; (14)$$

$$\sum_j y_{ij\,m_e t} + \alpha_{im_e,t+1} = z_{im_e t} \qquad \forall\, i,e,m,t \; (15)$$

$$a^k_{ij\,m_e t} \leq y_{ij\,m_e t} \qquad \forall\, i,j,k,e,m,t \; (16)$$

$$z_{im_e t} \geq z^{bin}_{im_e t} \qquad \forall\, i,e,m,t \; (17)$$

$$z_{im_e t} \leq ZUB_{im_e} z^{bin}_{im_e t} \qquad \forall\, i,e,m,t \; (18)$$

$$\alpha_{km_e t} + \sum_i a^k_{ik\,m_e t} = z^{bin}_{km_e t} \qquad \forall\, k,e,m,t \; (19)$$

$$\alpha_{im_et} + \sum_j a^k_{jim_et} \;\geq\; \sum_j a^k_{ijm_et} \qquad \forall\, k, i \neq k, e, m, t \;(20)$$

$$a^k_{kjm_et} = 0 \qquad \forall\, k, j, e, m, t \;(21)$$

Appendix F: MLOV-FFL model

Minimise $\sum_{ijemt} sc_{ijm_e}\, y_{ijm_et} + \sum_{it} h_{iet}\, I_{iet} + \sum_{it} g_{it}\, B_{iEt}$ (1)

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E} x_{jm_et} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\, j, t \;(2)$$

$$I_{je,t-1} + \sum_{m_e} x_{jm_et} - I_{jet} = \sum_{m_{e+1}} x_{jm_{e+1},t+1} \qquad \forall\, j, t \text{ and } e = 1, \ldots, E-1 \;(3)$$

$$B_{itE} \leq BP \cdot d_{it} \qquad \forall\, i, t \;(4)$$

$$\sum_i b_{im_e}\, x_{im_et} + \sum_{ij} st_{ijm_e}\, y_{ijm_et} + S_{m_e,t-1} - S_{m_et} + slk_{m_et} = C_{m_et} \qquad \forall\, e, m, t (5)$$

$$x_{jm_et} \leq UB_{jm_et} \times (z_{jm_et} - \sum_i OLS_{ijm_et}) \qquad \forall\, j, e, m, t \;(6)$$

$$y_{jjm_et} = 0 \qquad \forall\, j, e, m, t \;(7)$$

$$\sum_i \alpha_{im_et} = 1 \qquad \forall\; e, m, t = 1, \ldots, T+1 (8)$$

$$\alpha_{i_{om_e}m_et} = 1 \qquad \forall\, e, m, t = 1 (9)$$

$$x^F_{jm_et} \leq UB_{jm_et}\, \alpha_{jm_et} \qquad \forall\, j, e, m, t \;(10)$$

$$x^L_{jm_et} \leq UB_{jm_et}\, (\alpha_{jm_e,t+1} - \sum_i OLS_{ijm_et}) \qquad \forall\, j, e, m, t \;(11)$$

$$x^L_{jm_et} + x^F_{jm_e,t+1} \geq ml_j\, \alpha_{m_ej,t+1} \qquad \forall\, j, e, m, t \;(12)$$

$$x_{jm_et} - x^F_{jm_et} - x^L_{jm_et} \geq ml_j\, (z_{jm_et} - \alpha_{jm_et} - \alpha_{jm_e,t+1}) \qquad \forall\, j, e, m, t \;(13)$$

$$\alpha_{im_et} + \sum_j y_{jim_et} = z_{im_et} \qquad \forall\, i, e, m, t \;(14)$$

$$\sum_j y_{ijm_et} + \alpha_{im_e,t+1} = z_{im_et} \qquad \forall\, i, e, m, t \;(15)$$

$$a^k_{ijm_et} \leq y_{ijm_et} \qquad \forall\, i, j, k, e, m, t \;(16)$$

$$z_{im_et} \geq z^{bin}_{im_et} \qquad \forall\, i, e, m, t \;(17)$$

$$z_{im_et} \leq ZUB_{im_e} z^{bin}_{im_et} \qquad \forall\, i, e, m, t \;(18)$$

$$\alpha_{km_et} + \sum_i a^k_{ikm_et} = z^{bin}_{km_et} \qquad \forall\, k, e, m, t \quad (19)$$

$$\alpha_{im_et} + \sum_j a^k_{jim_et} \geq \sum_j a^k_{ijm_et} \qquad \forall\, k, i \neq k, e, m, t \quad (20)$$

$$a^k_{kjm_et} = 0 \qquad \forall\, k, j, e, m, t \; (21)$$

$$S_{m_et} \leq \sum_{ij} st_{ijm_e}\, OLS_{ijm_et} \qquad \forall\; e, m, t \; (22)$$

$$\sum_j OLS_{jim_et} \leq \alpha_{im_e,t+1} \qquad \forall\, i, e, m, t \; (23)$$

$$OLS_{ijm_et} \leq y_{ijm_et} \qquad \forall\, i, j, e, m, t \; (24)$$