Essays on the Role of Banking Sector in Transmission Mechanism and Business Cycle

by

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Abstract

This thesis tries to explore the relationship between the banking sector and the aggregate economy from three aspects. Chapter one addresses the relationship between financial frictions and the banking sector profit maximization behaviour. Financial frictions stem from the managerial cost associated with intermediation process, factor payments to collateral and labour services. The stake of entrepreneurs is relevant to determine the EFP on two grounds: (i) the ratio of entrepreneurial net wealth to collateral value influences the total managerial cost; (ii) entrepreneurs' holding of collateral help mitigate the EFP from the return on collateral.

Chapter two focuses on the role of supply side disturbance in financial market in shaping the business cycle. Specifically, I generate a DSGE model with a profit maximizing banking sector that predicts a relationship between EFP and corporate balance sheet condition as well as factor price. The shock to the loan productivity and the entrepreneurial net wealth resemble the supply and demand disturbances in financial markets. Constructed shock shows that every post war recession is crashed with the situation that both the two financial shocks are in contraction. The variance decomposition exercise indicates that the loan productivity shock is a very important source of EFP variation, though playing a minor role in determining the rest of the economy.

Chapter three incorporates the bank capital into financial friction analysis. Bank capital is important because it is assumed to be one of the three factors to produce entrepreneurial loans. In consequence, the financial friction is shown to depend on the corporate balance sheet condition, the bank capital ratio, and the liquidity premium. Thus I am able to establish the link between the bank capital channel and the model's transmission mechanism subject to exogenous shocks. The results indicate that an active bank capital channel amplifies the monetary policy shocks (demand) while attenuates the technology shocks (supply) when households' deposit is contracted in nominal term ('Fisher deflation effect'). It is also shown that adverse shocks to bank capital value can lead to sizeable declines in bank lending and economic recession.

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Chapter 1

Collateral, Loan Production and Banking Accelerator

1.1 Introduction

Based on Modigliani-Miller (1958) capital structure irrelevance theorem, mainstream macroeconomic approaches have long ignored the relevance of financial sector to real economic outcomes[1]. Nonetheless, more and more empirical evidences, especially the recent macroeconomic events (2008-2010), have sparked renewed interest in the role of financial factors in the business cycle research. This chapter adds to this research agenda by incorporating a financial intermediary problem into an otherwise standard Dynamic Stochastic General Equilibrium (DSGE) model. The goal is to address the importance of financial factors in business cycle analysis.

It is important to understand two distinct concepts on the effect of the financial frictions on aggregate fluctuations before we move further. The first concept is financial frictions, which is a channel where different shocks are propagated to the rest of the economy and as a result the effects could be amplified or attenuated given its presence. The second concept is financial disturbances, which is a shock directly originated in the financial sector of the economy and has the potential to impact the rest of the variables in the economy. This chapter mainly deals with the first concept while the second concept is left to the following chapter.

This chapter studies the interaction of financial frictions and business cycle in a simple extension of the standard DSGE model. For this purpose, I develop a model that distinguishes households and entrepreneurs in order to motivate lending and borrowing in equilibrium. Financial intermediaries (FIs) exist because they transfer funds from household sector (lenders) to entrepreneurial sector (borrowers). Inspired by a recent paper of Goodfriend and McCallum (2007), I assume that the FIs have access to an explicit loan production function that requires collateral and labour services. There are two advantages for this setup. First, this assumption precludes the possibility that entrepreneurs borrow funds directly from households. Because households do not have access to the loan management technology, it is

[1] Real business cycle (RBC) model and, more recently, the new neoclassical synthesis (NNS) model are both silent about financial relevance. See King and Rebelo (1999) for a detailed exposition of RBC framework and Christiano *et al.* (2005) for NNS.

too costly to conduct direct lending; they delegate monitoring to FIs. Second, this is a model device that can help derive financial frictions easily with first principles. FIs are owned by households and try to maximize their value subject to the loan production function. The financial frictions stem from the cost associated with loan management.

In the baseline model, financial friction manifests itself as the wedge between the interest rate charged on entrepreneurial loans and the interest rate received on households' deposits. The interest differential is used to pay for the cost of loan management: collateral and labour monitoring services. Entrepreneurs need the external funds (loans) to invest in physical capital because they are not able to fully finance with internal funds only. In equilibrium, the borrowing is stopped when the return on capital is equal to the cost of external funds: loan interest rate. Compared to those in a frictionless world, the return on capital is higher and the long-run capital and output level are lower in the model with financial frictions. While in the short-run, dynamic in the baseline model also distinguishes itself from that in frictionless model for endogenous essence of the interest rate wedge; in macroeconomic jargon, the external finance premium (EFP).

The endogeneity of EFP in the baseline model is determined by the following mechanism. In the model, loan is supplied according to the loan production function, which needs labour monitoring and collateral service. Thus for a given level of labour effort, higher collateral value will either boom the loan supply or reduce the cost of loan production; there is a downward pressure on EFP. This conforms well to the economic expansion when asset prices are surging. On the other hand, however, the economic boom also encourages entrepreneurs to invest more. This perhaps drives up the demand for external funds as well, depending on the strength of internal funds relative to total demand of capital. For a given level of labour monitoring, higher loan demand has the potential to push up EFP. The question is which one is stronger. In economic upturns, if loan demand increases more than supply boom in the credit market, the cost of loan production still surges in the economic expansion. So does the EFP. If loan demand increases less or even decreases for the strong upward movement of entrepreneurial net wealth, EFP tends to decline. For the reason above, in the short-run, the mechanism can either accelerate or decelerate the economy subject to other shocks, depending on whether the loan supply or demand boom dominates.

This modelling strategy is motivated along with the recent development of incorporating financial frictions into macroeconomic analysis. Several theoretical claims have been made in this line to rationalize the important role of financial frictions in business cycle fluctuations. One early important attempt in general equilibrium environment is offered by Bernanke and Gertler (1989), where they build on Townsend (1979) costly state verification (CSV) model and show that the presence of asymmetric information and agency costs in credit markets can give the balance sheet conditions of borrowers a role to play in the business cycle through their impact on the cost of external finance. Specifically, the pro-cyclical nature of net worth leads the EFP to fall during economic expansions and to rise during recessions. This can be nontrivial because these agency problems arise in a setting where the Modigliani-Miller theorem does not hold any more. An important insight of Bernanke and Gertler's framework is the theoretical possibility, in qualitative perspective, that financial frictions led by agency cost would enhance the propagation of other underlying exogenous shocks. Carlstrom and Fuerst (1997) afterward demonstrate the quantitative importance of Bernanke and Gertler (1989) mechanism, finding that it could affect an otherwise standard model's transmission mechanism confronting structure shocks (e.g., producing a hump-shaped impulse response for output in an otherwise standard RBC model).

Another framework of credit market imperfections is initiated by Kiyotaki and Moore (1997), in which they assume an environment with ex post renegotiation and the inalienability of human capital. This line of research introduces financial frictions on the quantity side of credit via collateral constraints to borrowers. One implication of this assumption is that borrowing is so tightly constrained by the level of collateral that default never occurs in equilibrium. This framework is recently developed by Iacoviello (2005), who introduces housing as collateral, and by Gerali *et al.* (2010), who embed monopolistically competitive banking sector in the collateral constraints environment.

Among others, one of the most notable settings of financial frictions is derived by Bernanke, Gertler and Gilchrist (1999), BGG hereafter, where they embed a principle-agent problem between entrepreneurs (borrowers) and financial intermediaries (lenders) into a standard NNS framework. In line with the literature focusing on the price of external funds, as in Bernanke and Gertler (1989), BGG incorporates credit market imperfections through the assumption of asymmetric information between entrepreneurs and financial intermediaries and costly state verification of entrepreneurial private return. Entrepreneurial internal funds help mitigate the principle-agent problem. The more of entrepreneurial net worth used in production, the less is the risk of bankruptcy and the cost of state verification. Thus the price of external funds relative to that of internal funds, the EFP, depends inversely on the value of entrepreneurial net worth that is in stake. Procyclical movements in entrepreneurial net wealth tend to cause countercyclical movements in the EFP, enlarging the volatility of entrepreneurs' investment on the one hand, and amplifying the aggregate fluctuations on the other. This mechanism generates the so called "financial accelerator" from the demand side of credit market, and becomes wide-spread in the literature while many studies during 2000s incorporate this type of friction into DSGE models[2].

As is well-known, a key finding of the financial accelerator mechanism from BGG is the derivation of the aggregate reduced form relationship between the dynamic behaviour of EFP and the ratio of entrepreneurial net worth to total value of purchased capital[3]. The foundation of this relationship stems from an optimal contracting problem between the entrepreneurs and financial intermediaries. Since the return on capital is subject to idiosyncratic shock, each entrepreneur has no choice but default when hit by large negative shock. In this case, financial intermediaries can disclose the actual return of default entrepreneurs on the cost of losing fraction as auditing cost. In order to account for the risk of defaulting and monitoring cost ex post, financial intermediaries charge the loan a premium on top of the cost of deposit funds. The lower is the entrepreneurs' own net worth relative to the value of capital (higher leverage), the higher is the probability of insolvency and default, and the higher is the EFP. This leads to a further decline of entrepreneurial net worth the next period as well as another increase of the EFP, shaping the corresponding propagation mechanism.

[2] Christiano *et al.* (2003) incorporate financial frictions in their model to analyse the Great Depression in US. Gertler *et al.* (2007) elaborate the relevance of the financial accelerator in open economy crisis episodes. Meier and Muller (2006), Christensen and Dib (2008) and Queijo (2009) use the friction underlying the financial accelerator to study differences in the transmission of a number of structural shocks.

[3] BGG originally incorporates a standard optimal contracting problem between entrepreneurs and financial intermediaries that maximize entrepreneurial profit subject to the condition that financial intermediaries earn a return equal to risk-free rate with certainty. The external finance premium stems from the monitoring cost. Subsequent studies reduce this financial friction to the elasticity of the external finance premium with respect to the change in the leverage position of entrepreneurs (e.g., Meier and Muller (2006)).

The effect on EFP from the balance sheet condition of borrowers as a way of thinking about credit market imperfections is well developed and accepted in the literature. Despite the widespread usage, there are still some reasons that the financial accelerator framework of BGG is subject to unease. A prominent one among these is the absence of an explicit problem or role for this passive financial intermediary or banking sector, which behaves only as a veil. In this way the financial frictions within the banking sector is ignored completely. Bank's balance sheet condition can be one resolution and motivates some studies seeking to link the financial structure of banks to either the price or the quantity (or both) of external funds so as to emphasize the role of bank capital in the transmission mechanism. See Drumond (2009) for an extensive survey on this on the one hand, and the third chapter of this thesis for a novel exposition on the other. On the other hand, the monitoring cost in BGG is treated as a deadweight loss measured in consumption goods, for which we have no idea how and where this cost is distributed explicitly[4]. This ad hoc assumption can be avoided if we consider the financial frictions generated in the model developed in this chapter.

This chapter introduces an alternative framework that can resemble BGG outcome in the end, at the same time overcome the possible unease discussed above[5]. In the model, EFP is endogenously determined in an environment where banks maximize their market value subject to loan management technology that relies upon collateral evaluation and labour monitoring; addressing the first point. In consequence, financial frictions in this environment stem from the managerial cost associated with intermediation process, which are the factor payments to collateral service and labour monitoring; addressing the third point. As long as the entrepreneurial sector holds fraction of the collateral value, the net wealth, along with borrowing, the return on collateral service distributed to the entrepreneurial sector can mitigate the EFP paid to banks. As shown below, this assumption leads to a similar derivation of the reduced-form relationship between EFP and entrepreneurial leverage position as in BGG. Instead of assuming default risk and asymmetric information, this approach provides a

[4] BGG also point out their model's limitation that is complementary to the issues raised here. I also account for the possible debt-deflation effect, raised by them, by assuming a nominal debt in the economy.

[5] I do not make any attempt to claim that there is any blemish in BGG framework per se. The model exposited in this chapter is considered as a complement to BGG that provides an alternative way to understand financial frictions.

different angle to understand the financial friction and its impact on the real economy[6]. This "loan production/management" framework also provides a fully micro-founded profit maximization problem of banks. If modelled consistently, this framework can be shown as a prominent complement of the BGG financial accelerator mechanism[7].

The main contribution of this chapter can be summarized as follows. First of all, the model constructed with first principle in all sectors resembles the amplification and propagation mechanism a la BGG. This promising result helps us understand the credit market imperfection from a different perspective of managerial cost in intermediation process. Moreover, the effect of Fisher's Debt Deflation hypothesis is also examined, from which we find that financial friction turns to accelerate demand shock while decelerate supply shock in the environment of nominal contract. Thirdly, labour monitoring effort, besides collateral, is essential in loan management, so the dynamic of EFP in current model also depends on factor price of banking labour, which is missing from BGG and many other studies of financial frictions. This additional connection from the supply side of credit market introduces extra mechanism to influence the dynamics of EFP on top of the part that is related to borrower's balance sheet only, and also the transmission of shocks to the whole economy.

The rest of this chapter is structured as follows. Section 1.2 describes the model setup. It starts with the problem of banking sector and then incorporates it into the standard DSGE framework. Section 1.3 defines the model's competitive equilibrium, steady states and log-linear approximation. Section 1.4 calibrates the model and presents the numerical results. Section 1.5 concludes with some final remarks.

1.2 The model

The baseline model shares several features with the original BGG setup and also extends the framework along several dimensions. The ingredients I share with BGG are as follows: (i) I distinguish between households and entrepreneurs to motivate equilibrium lending and

[6] It is fair to claim that since the functional form of loan production contains collateral, motivated by the assumption of possible default by entrepreneurs, it is not completely different from BGG.

[7] Goodfriend and McCallum (2007) use this model mainly in exploring the role of monetary aggregates and financial intermediation for policy decision, but didn't compare it with BGG's framework in a consistent and rigorous fashion.

borrowing behaviour; (ii) Capital producers are introduced to capture the movement of asset price; (iii) Retailers are included as a convenient way to model price stickiness. My extension comprises the following things: (1) Households' wage setting is introduced to better capture the movement of real wage, which is part of my financial accelerator mechanism; (2) Households and financial intermediaries (FIs) sign contract in nominal terms to motivate the debt-deflation effect in the spirit of Fisher (1933); (3) Most importantly, I replace the BGG's original risky debt contract problem between entrepreneurs and FIs with a contract monitored by a loan production/management technology *a la* Goodfriend and McCallum (2007). As shown below, this framework introduces financial frictions from a different perspective. By and large, the baseline model economy includes households, entrepreneurs as wholesale good producers, financial intermediaries/banks, capital producers, retailers as final good producers and a government. Their behaviours are described explicitly one after another.

1.2.1 Financial intermediaries

In the baseline model, the key ingredient which derives financial frictions that influence the transmission mechanism of the model subject to structural shocks is the problem of financial intermediaries (FIs). FIs are owned by households and conduct the main activity of issuing time deposit to collect funds from households in order to finance the lending to entrepreneurs in a perfectly competitive environment. Here I assume the homogeneity of FIs and absence of interbank market for convenience because I only need to track the behaviour of the banking sector in aggregate sense. Also for convenience, reserve requirement is also ignored from the analysis. This can be understood by realising the fact that, as the development of financial markets, financial intermediaries recently and nowadays have easy access to sources of funds that are not subject to reserve requirement. Thus the balance sheet constraint for FIs is simply

$$L_t \leq D_t; \tag{1.1}$$

where L_t stands for loan and D_t for deposit. So long as the interest rate on loan is positive, all available funds will be lent out to firms and balance sheet constraint will hold with equality.

Inspired by Goodfriend and McCallum (2007), the volume of loan supply (equivalent as the demand for deposit funds) is designed to be determined by a model of loan production, or more accurate, loan management, which is involved with collateral assessment and labour monitoring. Effort is more productive in making loans the greater is the collateral value. Collateral is a valuable input in loan production because it enables FIs to enforce the repayment of loans with less monitoring and management effort. FIs expend effort and require collateral but in equilibrium there is no default. This modelling choice is based on the progress along several dimensions that has been made in understanding the implications of credit market imperfections in limited commitment environments where there is no equilibrium default[8], by contrast to the modelling choice of BGG where the authors develop an environment in which default exists in equilibrium because of independent idiosyncratic shocks to entrepreneurial (borrower) sector. The specification of loan production function is in Cobb-Douglas fashion as follows

$$L_{t+1} = F_t (Q_t K_{t+1})^{\gamma} N_t^{F^{1-\gamma}}; \tag{1.2}$$

where L_{t+1} is the amount of loan issued for period $t+1$ determined at the end of period t. Q_t is the capital price in period t, thus $Q_t K_{t+1}$ is the collateral value at the end of time t. N_t^F is the labour effort involved in loan monitoring, and γ denotes for the share of collateral in loan production. F_t represents productivity in loan production to capture the exogenous factor other than collateral value and labour monitoring, following AR(1) process in logarithm:

$$\ln F_t = (1-\rho_f)\ln F + \rho_f \ln F_{t-1} + \varepsilon_{ft}; \tag{1.3}$$

with $\rho_f \in (0,1)$, $\varepsilon_{ft} \sim iid(0, \sigma_f^2)$. It follows the informational and technological advances in the banking sector measured as the total factor productivity in the intermediation process (e.g., Berger 2003). It is noteworthy that Eq. (1.2) distinguishes itself to the original setting in Goodfriend and McCallum (2007) in which both capital and government bond are used as collateral for loan production. The reasons of excluding bond are twofold. First, government bond is not necessary here since the model refrains from the analysis of it; the omission is a

[8] FIs expend effort and require borrowers to post collateral, but in equilibrium there is no default. See Kocherlakota (1996), Hart and Moore (1998), and more recently Jermann and Quadrini (2009).

simplification. Moreover, to resemble BGG's expression of financial friction (shown below), it is more appropriate to exclude bond from the loan production function. Nevertheless, the inclusion of government bond would be straightforward.

In the demand side of the credit market on the other hand, as described in the following section, entrepreneurs obtain the loan to finance the purchase of next period capital in excess of their equity (net worth) NW_{t+1}:

$$L_{t+1} = Q_t K_{t+1} - NW_{t+1}; \tag{1.4}$$

Eq. (1.2) and (1.4) together characterize the equilibrium in credit markets.

We can consider the FIs' problem in the following sense. In marginal perspective, every unit of loan is issued by equalizing the interest rate to cost. The cost consists of two parts: (1) the managerial cost of intermediation for paying the collateral and labour service fees; (2) the interest rate on deposit as the cost of funds. Thus the interest rate on loans would be set to cover the cost of both the two parts; and the credit spread, the difference between interest rates on loan and that on deposit, is just the managerial cost. We can derive this relationship formally in FIs' maximization problem.

The flow of funds of the typical FI at the end of period t is the new arriving deposit funds and gross interest payment on existing loans, less the labour cost for monitoring, cost of collateral service, the new issuing loans and the gross interest payment on existing deposits. The FI chooses the collateral service $Q_t K_{t+1}$, labour monitoring effort N_t^F and newly issued loan L_{t+1} and deposit D_{t+1} to maximize the expected life-time value in favour of the bank owners, households. The profit maximization problem of the bank is given by

$$\max_{\{Q_{t+h}K_{t+h+1}, N^F_{t+h}, L_{t+h+1}, D_{t+1}\}} E_t \sum_{h=0}^{\infty} \beta^h \frac{U_{Ct+h}}{U_{Ct}} \{D_{t+h+1} + R^{ul}_{t+h} L_{t+h} - L_{t+h+1} - R^d_{t+h} D_{t+h} - w_{t+h-1} N^F_{t+h-1} - r^q_{t+h-1} Q_{t+h-1} K_{t+h}\};$$

subject to the FI's balance sheet constraint (1.1) and loan production function (1.2), where $\beta^h U_{Ct+h}/U_{Ct}$ is the household's stochastic discount factor[9]. The first order conditions (F.O.Cs) for this optimization problem are:

$$E_t\{R^{ul}_{t+1} - R^d_{t+1}\} = \frac{r^q_t}{\gamma L_{t+1}/Q_t K_{t+1}}; \tag{1.5}$$

$$E_t\{R^{ul}_{t+1} - R^d_{t+1}\} = \frac{w_t}{(1-\gamma) L_{t+1}/N^F_t}; \tag{1.6}$$

Eq. (1.5) and Eq. (1.6) apply the usual Baumal (1952) conditions, which equalize the credit spread of intermediation to the factor prices of inputs divided by the marginal product of the inputs. As highlighted in Goodfriend and McCallum (2007), this intermediation cost captures the idealized net uncollateralized external finance premium ($UEFP$) in the model, under the condition that entrepreneurs come to the bank for borrowing without posting any collateral. Thus entrepreneurs have to pay full cost of intermediation: labour monitoring plus collateral service. The mathematical expression of UEFP is as follows.

$$UEFP_{t+1} - 1 = \frac{w_t}{(1-\gamma) L_{t+1}/N^F_t}; \tag{1.7}$$

Unanimously, the above is only one extreme case of the situation. In the other extreme, if entrepreneurs possess the full amount of collateral to borrow, they pay the full cost, but get back the total return on collateral services at the same time. Therefore, the net EFP for entrepreneurs is only the labour monitoring cost, which is the fraction $1-\gamma$ of the total cost. This is defined as the fully collateralized external finance premium ($CEFP$) in the model economy, represented as:

$$CEFP_{t+1} - 1 = \frac{w_t}{L_{t+1}/N^F_t}; \tag{1.8}$$

In reality, the actual amount of EFP lies between $UEFP$ and $CEFP$, since entrepreneurs own fraction of the total collateral value in the whole economy, given by $NW_{t+1}/Q_t K_{t+1}$. The

[9] See next section for the households' problem.

service return on the part of collateral owned by entrepreneurs help mitigate the practical EFP, which is determined by the following ratio:

$$EFP_{t+1} - 1 = E_t\{R^l_{t+1} - R^d_{t+1}\} = (UEFP_{t+1} - 1)[1 - \gamma \frac{NW_{t+1}}{Q_t K_{t+1}}]; \tag{1.9}$$

Eq. (1.9) states that the practical EFP is determined by the UEFP discounted by the fraction of collateral service in loan production and the share of collateral owned by entrepreneurial sector. Thus for a given level of UEFP, the higher is the share of collateral owned by entrepreneurial sector, the smaller is EFP. After paying the part of collateral service to entrepreneurs, the rest is collected by FIs, the temporary owner of deposit in the intermediation process, but finally distributed back to households as dividend. The next task is to exposit how UEFP is determined in the intermediation process, corresponding to financial sector productivity, factor price of labour and the entrepreneurial net worth collateral ratio.

For a detailed exposition, let's rewrite the loan production function of Eq. (1.2) by dividing both of the two sides with the collateral value $Q_t K_{t+1}$:

$$\frac{L_{t+1}}{Q_t K_{t+1}} = F_t \left(\frac{N^F_t}{Q_t K_{t+1}} \right)^{1-\gamma}; \tag{1.2'}$$

Eq. (1.2') reveals a very important property of the assumed loan production function: since the ratio of loan to collateral value is a decreasing return to scale (DRS) function to the ratio of labour monitoring to collateral value, the marginal cost (UEFP) of loan production would be an increasing function to the ratio of loan to collateral, and convex if the loan production is collateral intensive ($\gamma > 0.5$). In other words, UEFP is a decreasing function to the ratio of entrepreneurial internal funds (net worth) to total collateral value. To elaborate this property, it is necessary to derive UEFP as a function of the ratio of entrepreneurial net worth to collateral value $NW_{t+1}/Q_t K_{t+1}$ as follows:

$$UEFP_{t+1} - 1 = \frac{w_t}{(1-\gamma)\frac{L_{t+1}}{N^F_t}} = \frac{w_t \frac{N^F_t}{Q_t K_{t+1}}}{(1-\gamma)\frac{L_{t+1}}{Q_t K_{t+1}}} = \frac{1}{1-\gamma} F_t^{-\frac{1}{1-\gamma}} w_t \left(1 - \frac{NW_{t+1}}{Q_t K_{t+1}}\right)^{\frac{\gamma}{1-\gamma}}; \tag{1.10}$$

Combine Eq. (1.9) and Eq. (1.10), after some rearrangements, we get

$$EFP_{t+1} - 1 = \frac{1}{1-\gamma} F_t^{-\frac{1}{1-\gamma}} w_t \left[(1 - \frac{NW_{t+1}}{Q_t K_{t+1}}) \right]^{\frac{\gamma}{1-\gamma}} [1 - \gamma \frac{NW_{t+1}}{Q_t K_{t+1}})] \; ; \tag{1.11}$$

Eq. (1.11) highlights the key relationship between EFP and the ratio of entrepreneurial internal funds to collateral (or purchased capital) value, $NW_{t+1}/Q_t K_{t+1}$, derived from the FI's optimization behaviour. Given that F_t and w_t are exogenous, we can reach the following proposition:

Proposition 1.1: *Assume F_t and w_t are exogenously give in equilibrium, external finance premium is a decreasing and convex function of the ratio of net worth to purchased capital value (collateral value) if* $\gamma > 0.5$.

See Appendix B for the proof of proposition 1.1. This proposition implies a very important inference: The external finance premium is higher the more entrepreneurs rely on the external funds. Figure 1.1 plots the gross EFP against the ratio of net worth to the value of purchased capital with arbitrary calibration ($\gamma = 0.77,\ F = 2.69,\ w = 2.12$):

Fig. 1.1 External finance premium and ratio of net worth to capital value

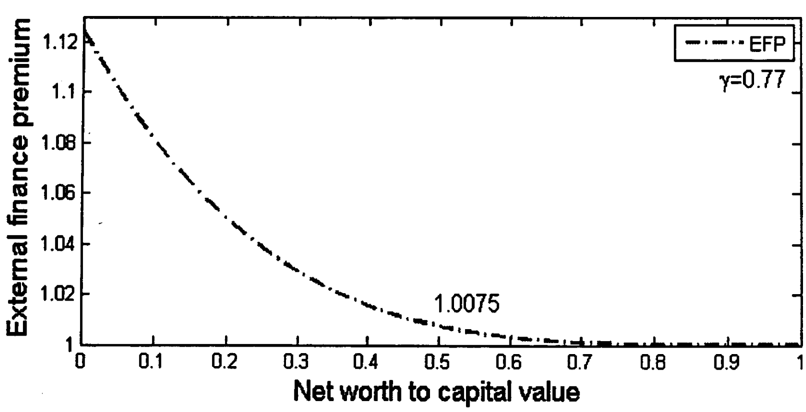

Figure 1.1 shows that the EFP decreases as less external funds is needed with given value of purchased capital (less leverage) in an diminishing rate. This implies that EFP in equilibrium will increase dramatically even after you reduce the internal funds relative to capital value by

only a small amount. This shows the mechanism of the accelerator effect embedded in the banking sector of the baseline model. To see the short-run dynamic relationship, I derive the log-linear form of Eq. (1.11) around the non-stochastic steady state:

$$\begin{aligned} \hat{efp}_{t+1} = & -\frac{EFP-1}{EFP}\left(\frac{\gamma NW/K}{1-\gamma NW/K}+\frac{\gamma}{1-\gamma}\frac{NW/K}{1-NW/K}\right)(\hat{nw}_{t+1}-\hat{k}_{t+1}-\hat{q}_t) \\ & +\frac{EFP-1}{EFP}(\hat{w}_t-\frac{1}{1-\gamma}\hat{f}_t) \end{aligned} \quad ; \quad (1.11L)$$

Eq. (1.11L) elaborates the behind scene accelerator effect from the banking sector in the baseline model. The short run dynamics of EFP depends on the dynamics of the net worth to capital value ratio, real wage for labour monitoring and exogenous loan production technology. Relating to literature, Eq. (1.11L) is highly comparable with the counterpart reduced form equation in BGG framework of the following (see appendix for a detailed derivation):

$$\hat{efp}_{t+1} = -\psi(\hat{nw}_{t+1} - \hat{k}_{t+1} - \hat{q}_t); \quad (1.11L')$$

For Eq. (1.11L'), BGG claims that the elasticity of external finance premium, with respect to the ratio of internal funds to total value of capital, is derived from an optimal contracting problem between entrepreneurs and financial intermediaries. Higher net worth relative to value of purchased capital makes more funds of entrepreneurial sector sink into the project. Thus the incentives are more aligned between entrepreneurs and banks so as to reduce the asymmetric information problem and EFP. The baseline model with loan management also predicts a similar aggregate relationship as in BGG, despite the fact that the corresponding elasticity is shown differently by an expression nesting steady state value of EFP and internal funds to total value of capital ratio, and the parameter value of collateral share in loan production[10]. Besides this, the baseline model also highlights the importance of the real wage to influence the dynamics of EFP before subjecting to the exogenous shock in the banking

[10] The elasticity in BGG equals to ψ only after figuring out the optimal loan contract between entrepreneurs and financial intermediaries; it also depends on micro structure of the contract environment (e.g., average fraction of monitoring cost after the entrepreneurs default). See the appendix or BGG for details.

sector[11]. This promising increment is not revealed in BGG derivation and many other studies of financial frictions following BGG.

1.2.2 Households

Consider a continuum of monopolistically competitive households, indexed by $j \in [0,1]$, who decide between consumption and saving through holding bank deposits, and supply specialized labour services into economy-wide labour market[12]. The bundle of labour services is obtained using the aggregation scheme of Dixit and Stiglitz (1977)

$$N_t = \left[\int_0^1 N_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \ \varepsilon_w > 1; \qquad (1.12)$$

The optimal substitution across labour service leads to the following labour demand equation regarding the j th labour service

$$N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t; \qquad (1.13)$$

where $W_t(j)$ is the nominal wage set by the j th household, W_t is the Dixit-Stiglitz aggregate nominal wage given by $W_t = [\int W_t(j)^{1-\varepsilon_w} dj]^{1/(1-\varepsilon_w)}$, and ε_w gives the constant elasticity of substitution across differentiated labour services.

In each period, the j th household derives utility from consumption of final goods, $C_t(j)$, and leisure, $1 - N_t(j)$ in logarithmic separable form given by

$$U_t = \ln(C_t(j)) + \psi \ln(1 - N_t(j));$$

where ψ measures the households' weight on leisure in the utility function. This specification of preferences is consistent with the existence of the balanced growth path as discussed in King *et al.* (2002). On the other hand, following the assumption of Erceg *et al.* (2000) and

[11] Real wage becomes relevant because it is the factor price in loan management and affects the marginal cost of intermediation activity.
[12] As will be described later, households' labour supply is demanded by entrepreneurs as well as banks.

Christiano *et al.* (2005), the existence of state-contingent securities ensures the homogeneity of households' consumption and asset holdings, left the heterogeneity only with respect to the wage rate they earn and the hours they work. Thus, at the first stage, the intertemporal problem for the j th household is to chooses consumption and deposit holding in order to maximize

$$\max_{\{C_t, D_{t+1}\}} E_t \sum_{h=0}^{\infty} \beta^h \{\ln(C_t) + \psi \ln(1 - N_t(j))\};$$

subject to the intertemporal budget constraint

$$C_t + D_{t+1} = \frac{W_t(j)}{P_t} N_t(j) + R_{t-1}^n \frac{P_{t-1}}{P_t} D_t + \frac{\mathrm{T}_t + \Pi_t + V_t(j)}{P_t}; \tag{1.14}$$

The budget constraint can be understood as follows. Each household enters period t with $P_{t-1}D_t$ units of nominal deposits, that pay a gross nominal interest rate R_{t-1}^n between $t-1$ and t, in a financial intermediary. The consideration of nominal contract between households and FIs is to capture the influential Fisher's Debt-Deflation effect. The FIs real burden will increase/decrease when the economy is subject to deflationary/inflationary pressure. During period t, the j th household also supplies labour to the entrepreneurial firms and banks, for which he receives total factor payment of $W_t(j)N_t(j)$. In addition, he receives/pays a lump-sum transfer/tax from/to the government, T_t, as well as the dividend payments Π_t, and net cash inflows from participating in state-contingent security markets, $V_t(j)$. All these funds are allocated for consumption P_tC_t, and nominal deposit holdings P_tD_{t+1}.

As mentioned above, at the first stage, each household chooses homogenous C_t and D_{t+1} in order to maximize his expected lifetime utility with discount factor $\beta \in (0,1)$, and subject to his budget constraint. The F.O.C for this optimization problem is:

$$\frac{1}{C_t} = \beta E_t \{\frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}} R_t^n\}; \tag{1.15}$$

Eq. (1.15) is the usual intertemporal consumption and saving condition, which states that the marginal cost of giving up a unit of consumption in terms of utility in the current period must be compensated with the expected marginal benefit of holding deposit in the following period.

At the second stage, households decide on nominal wages. Given the fact that households can only set nominal wages in staggered contracts with a constant probability $1-\theta_w$, of renegotiation in each period, the fraction of households who have the opportunity to reset their wages would set it as a mark-up over the marginal rate of substitution of leisure for consumption taking account the probability that he couldn't reset the wage again. The fraction of households who don't have the opportunity to reoptimise must apply the wages that was in effect in the preceding period indexed by the steady state gross rate of wage inflation, ω. This yields the following wage setting problem

$$\max_{W_t^*} E_t \sum_{h=0}^{\infty} [(\beta\theta_w)^h \frac{U_{Ct+h}}{U_{Ct}} \frac{W_t^*(j)\omega^h - MRS_{t+h}P_{t+h}}{P_{t+h}} N_{t+h}(j)];$$

where U_{Ct+h}/U_{Ct} is the households' intertemporal marginal rate of substitution and MRS is the households' intratemporal marginal rate of substitution between consumption and leisure. The F.O.C for this maximization problem is

$$W_t^*(j) = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{h=0}^{\infty} \{(\beta\theta_w)^h U_{Ct+h+1}/U_{Ct+h} N_{t+h}(j) MRS_{t+h}\}}{E_t \sum_{h=0}^{\infty} \{(\beta\theta_w)^h U_{Ct+h+1}/U_{Ct+h} N_{t+h}(j)\omega^h/P_{t+h}\}}; \tag{1.16}$$

Log-linear approximations of the F.O.C imply the following wage inflation curve:

$$\hat{\omega}_t = \beta\hat{\omega}_{t+1} + \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\eta\varepsilon_w)}(\eta\frac{N}{1-N}\hat{n}_t - \hat{\lambda}_t - \hat{w}_t); \tag{1.16L}$$

where ω_t is the gross wage inflation and λ_t is the multiplier of households' budget constraint.

1.2.3 Entrepreneurs

The entrepreneurial sector is modelled largely as in BGG[13]. The only exception is that I do not assume that the entrepreneurial sector is hit by idiosyncratic shock on the return of capital, because, different from BGG, this assumption is not necessary to rationalize the financial frictions so as to generate endogenous EFP. What we rely on is the managerial cost in the FI sector. Aggregate shock in the baseline model is sufficient for the analysis, while extending to include BGG's channel is straightforward if we want to capture financial frictions in both angles. The main purpose of this chapter is to introduce and analyse financial friction within the FI sector rather than to combine it together with that of BGG.

There are a large number of risk neutral entrepreneurs who operate in a fully competitive environment. Each entrepreneur needs to purchase capital in the end of each period for production use in the subsequent period. Capital is combined with labour service to produce output as follows:

$$Y_t = A_t K_t{}^{\alpha} N_t^{G\,1-\alpha}; \tag{1.17}$$

where Y_t is output, N_t^G is hired labour service, K_t is purchased capital and α is capital share in production function. A_t is an exogenous technological factor (TFP), which evolves as

$$\ln A_t = (1-\rho_a)\ln A + \rho_a \ln A_{t-1} + \varepsilon_{at}; \tag{1.18}$$

with $\rho_a \in (0,1)$, $\varepsilon_{at} \sim iid(0,\sigma_a^2)$. At the end of period t, entrepreneurs purchase capital, K_{t+1}, that will be used in period $t+1$, at the real price Q_t (in terms of the consumption good). The cost of capital acquisition $Q_t K_{t+1}$, is financed partly by their end of period net worth NW_{t+1}, and the rest by loan L_{t+1}, borrowing from financial intermediaries[14]. Each entrepreneur is risk-neutral and has finite horizons (discussed below). His demand for capital is determined by comparing the expected marginal return of holding capital with its expected

[13] Also refer to Christiano et al. (2003), (2010), Meier and Muller (2006), Gertler et al. (2007), Christensen and Dib (2008), and Queijo (2009).
[14] For the existence of credit market, entrepreneurs cannot self-finance the purchase of capital.

marginal financing cost. In aggregate terms, the expected gross return of holding one unit of capital from period t to $t+1$, $E_t R_{t+1}^k$, is described as follows[15]:

$$E_t R_{t+1}^k = E_t\left[\frac{X_{t+1}(\alpha Y_{t+1}/K_{t+1}) + Q_{t+1}(1-\delta)}{Q_t}\right]; \qquad (1.19)$$

where δ is the depreciation rate of capital, X_t is the price of intermediate goods relative to final goods, and Q_t, as described above, is the relative price of capital which varies to the capital production technology described below. The term $X_{t+1}\alpha Y_{t+1}/K_{t+1}$ is the marginal product of capital which can be thought of as the income gain of renting capital and the term, $Q_{t+1}(1-\delta)$ is the capital gain of holding it from period t to $t+1$. In equilibrium, this expected marginal return must be equal to the expected marginal cost of external financing, which is the expected interest rate, $E_t R_{t+1}^l$, on loan charged by the financial intermediary. Thus we have

$$E_t R_{t+1}^l = E_t\left[\frac{X_{t+1}(\alpha Y_{t+1}/K_{t+1}) + Q_{t+1}(1-\delta)}{Q_t}\right]; \qquad (1.19')$$

If there is no credit market imperfection in the economy, financial intermediaries would only charge the same interest rate as they pay for the deposit, which is the economy-wide real interest rate[16]. This assumption makes the model collapse to the standard DSGE framework *a la* Christiano *et al.* (2005) and Smets and Wouters (2007). As shown above, the credit market imperfections do exist in the baseline model because of the managerial cost associated with transferring deposit funds into loans. Thus for each unit of loan, the FI will charge a premium on top of the cost of deposit. The detailed exposition of how this premium is endogenously determined has been shown in the section of FI's problem. For now, we should keep in mind that the cost of loan is jointly determined by the cost of deposit and the EFP:

$$E_t R_{t+1}^l = E_t R_{t+1}^k = E_t R_{t+1}^d EFP_{t+1}; \qquad (1.20)$$

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[15] As described in BGG, single entrepreneurial activity is exposed to an idiosyncratic shock which has a mean of one. Thus the return of each entrepreneur may be different from aggregate return.

[16] Here the economy wide real interest rate is equal to the real rate paid on deposit. Since the contract between financial intermediaries and households are in nominal terms, the ex post real return of deposit depends on inflation rate.

On the other aspect, entrepreneurial demand for labour service is determined by equalizing the real wage with marginal product of labour:

$$w_t = X_t(1-\alpha)\frac{Y_t}{N_t^G}; \tag{1.21}$$

Entrepreneurs operate each period to accumulate their net worth. The net increment is through the retained earnings from the operation of production period by period. In order to make sure the necessity of external funds, they are not allowed to accumulate enough funds for fully self-financing. We can achieve this by assuming each of them only has finite expected horizon of life. The probability that each entrepreneur dies in each period is $1-\upsilon$ (i.e., there is a probability υ that he survives to the next period), so his expected lifetime is $1/(1-\upsilon)$. This assumption precludes the probability that entrepreneurs' net worth diverges as well as they accumulate enough wealth to be fully self-financed such that credit markets disappear. They always need to borrow from the FIs to finance desired capital in excess of their net worth. On the other hand, this assumption is also meant to capture the phenomenon of ongoing births and deaths of firms. For the total amount of entrepreneurs to be constant, each period the same amount of new entrepreneurs enter into the economy to replace the departing ones. The newly entered entrepreneurs receive some transferred seed money S_t, as initial funds[17]. The evolution of entrepreneurial net worth can be written as:

$$NW_{t+1} = \upsilon[R_t^k Q_{t-1}K_t - R_t^d EFP_t(Q_{t-1}K_t - NW_t)] + (1-\upsilon)S_t; \tag{1.22}$$

The first term in the square bracket represents the return from holding capital while the second is the repayment term which includes FIs' cost of funds as well as the managerial cost, EFP. Note that R_t^k is the ex post return on capital held in time t, and $R_t^d EFP_t$ is the ex post cost of borrowing. These differ from Eq. (1.20) which describes the ex ante relationship. Eq. (1.22) describes that retained earnings from operation in current period by surviving entrepreneurs and the seed money transferring to new entrepreneurs become next period's entrepreneurial net wealth.

[17] Without this seed money, entering entrepreneurs would have no net worth, and so they would not be able to buy any capital. Also, among the entrepreneurs who survive there are some who are bankrupt and have no net worth. Without a transfer they would not be able to buy capital either.

1.2.4 Capital producers

Entrepreneurial demand for capital is also affected by capital adjustment costs. As emphasized in BGG, it is through the introduction of adjustment costs that volatile asset prices contribute to the fluctuations of entrepreneurial net wealth. Conceptually, it is convenient to consider that there is a perfectly competitive capital producing sector. Each period, capital producers purchase investment goods, I_t, from the entrepreneurs' output and combine it with rented capital, K_t, to produce new capital, $\widetilde{K}_t$, as follows

$$\widetilde{K}_t = \Phi\left(\frac{I_t}{K_t}\right)K_t;$$

with $\Phi(0)=0$, $\Phi'(.)>0$, $\Phi''(.)<0$. This increasing and concave function captures the presence of adjustment costs in the production of capital goods. Capital producers choose the investment expenditure in order to maximize their profit $Q_t\widetilde{K}_t - I_t$, taking the relative price of capital as given. The first-order condition is

$$Q_t = \left[\Phi'(\frac{I_t}{K_t})\right]^{-1}; \qquad (1.23)$$

Here I restrict the capital production function so that the relative price of capital is unity in steady state. Capital producers' decision is linked with the entrepreneurs' capital purchasing decision via the variation in the price of capital.

The aggregate capital stock evolves according to

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t; \qquad (1.24)$$

Note that capital is homogeneous, so there is no difference between newly-produced and old capital. Old capital used by entrepreneurs is rented out for the production of new capital, and then returned at the same price as the newly-produced capital.

1.2.5 Retailers

The retail sector is included to introduce nominal rigidity into this economy. Here I assume that entrepreneurs sell all of their output goods to retailers. Retailers purchase the homogenous wholesale goods from entrepreneurs, differentiate them using a linear technology at no resource cost and sell them as final goods to households, capital producers and the government sector. In this way, the retailers have the monopolistic power to set the prices of these final goods. The reason why retailers are incorporated together with entrepreneurs is to avoid the complication of aggregating individual entrepreneur's demand for capital and his net worth when entrepreneurs themselves are imperfect competitors. Ultimately retailers' monopolistic profits belong to the households who own them, in contrast to entrepreneurs who are independent agents possessing their own wealth. Before exploring the retailers' problem in details, I firstly derive the aggregation of final goods. The final goods Y_t are bundles of differentiated goods $Y_t(j)$, $j \in [0,1]$, provided by the continuum of monopolistically competitive retailers[18]. The aggregation follows the framework of Dixit and Stiglitz (1977) as

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}; \qquad (1.25)$$

where ε_p is the elasticity of substitution between different goods. The optimal allocation of expenditure across differentiated goods implies a downward sloping demand function for goods j:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t; \qquad (1.26)$$

where $P_t(j)$ denotes the price of good $Y_t(j)$, Y_t denotes the aggregate demand, and ε also measures the price elasticity of demand among differentiated goods. P_t denotes the price index of final goods given by

[18] Recall that the assumption of linear technology for retailers ensures that the amount of final goods varies one-for-one with the amount of wholesale goods in the economy.

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon_p} dj\right]^{\frac{1}{1-\varepsilon_p}}; \qquad (1.27)$$

Following Calvo (1983), and a discrete version as in Yun (1996), I assume that each retailer cannot reoptimize its selling price unless it receives a random signal. The probability that each retailer can reoptimize his price in a given period is $1-\theta_p$, independently of other firms and of the time elapsed since the last adjustment. Thus the average length of time a price remains unchanged is $1/(1-\theta_p)$. Retailer j who has the opportunity to reset its price in a given period t choose the price, $P_t^*(j)$, that maximizes its expected discounted profits until the period when they are next able to change their price. On the other hand, the retailer who doesn't have the opportunity to reset price must charge the price that was in effect in the preceding period indexed by the steady state gross rate of inflation, π. Retailer j's optimazation problem is:

$$\max_{P_t^*(j)} E_t \sum_{h=0}^{\infty} [(\beta\theta_p)^h \frac{U_{Ct+h}}{U_{Ct}} \frac{P_t^*(j)\pi^h - X_{t+h}P_{t+h}}{P_{t+h}} Y_{t+h}(j)];$$

Subject to the demand function Eq. (1.26). Note that the stochastic discount factor for expected profits consists of the probability that retailers can change their price and the households' intertemporal marginal rate of substitution. The F.O.C for the optimal problem is

$$P_t^*(j) = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{E_t \sum_{h=0}^{\infty} \{(\beta\theta_p)^h U_{Ct+h+1}/U_{Ct+h} Y_{t+h}(j) X_{t+h}\}}{E_t \sum_{h=0}^{\infty} \{(\beta\theta_p)^h U_{Ct+h+1}/U_{Ct+h} Y_{t+h}(j)\pi^h/P_{t+h}\}}; \qquad (1.28)$$

The aggregate price index is given by

$$P_t^{1-\varepsilon_p} = \theta_p(\pi P_{t-1})^{1-\varepsilon_p} + (1-\theta_p)P_t^{*1-\varepsilon_p};$$

Log-linear approximations of the F.O.C and aggregate price index imply the following New Keynesian Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\hat{\chi}_t; \qquad (1.28L)$$

where $\hat{\chi}_t$ is the log deviation of real marginal cost from steady state.

1.2.6 Government and monetary policy rule

Finally I set the budget constraint for the government and the policy rule of the monetary authority to close the whole model. The aggregate final output goods consist of households' consumption, capital producers' investment expenditure and the government expenditure, G_t. Every period, the market for final goods clears as

$$Y_t = C_t + I_t + G_t; \qquad (1.29)$$

where government expenditure is financed by lump-sum taxes and money creation

$$G_t = \frac{M_t - M_{t-1}}{P_t} + \frac{T_t}{P_t};$$

The monetary authority conducts monetary policy by controlling the gross nominal interest rate R_t^n. Following conventional wisdom, I assume that the central bank adjusts the nominal interest rate according to a simple rule, for which it responses to the lagged inflation rate π_{t-1} and the lagged interest rate R_{t-1}^n. The log-linear version of monetary policy rule is

$$\hat{r}_t^n = \rho_r \hat{r}_{t-1}^n + (1-\rho_r)\kappa_\pi \hat{\pi}_{t-1} + \varepsilon_{rt}; \qquad (1.30)$$

Where ρ_r captures the empirical interest rate smoothing, κ_π is the elasticity of nominal interest rate with respect to deviations of lagged inflation, and ε_{rt} is an exogenous random shock, with zero mean and standard deviation σ_r, to the interest rate, reflecting either failure to track the rule or intentional transitory deviations from the rule (policy shock).

1.3 Equilibrium

In the baseline model, the competitive equilibrium is defined as a set of endogenous variables $\{Y_t, C_t, I_t, K_t, NW_t, L_t, N_t^G, N_t^F, w_t, \omega_t, X_t, \pi_t, R_t^k, R_t^n, R_t^d, Q_t,\}$ which satisfies financial intermediaries' optimal condition of Eq. (1.11), households' optimal conditions of Eq. (1.15) and (1.16), entrepreneurs' efficient conditions of Eq. (1.19), (1.20), and (1.21), capital producers' optimal condition of Eq. (1.23), retailers' optimal condition of Eq. (1.28), monetary policy Eq. (1.30), and resource constraints of Eq. (1.2), (1.4), (1.17), (1.22), (1.24) and (1.29). The exogenous shock processes are given by Eq. (1.3) and (1.18).

1.3.1 Steady state solution

The baseline model's balance growth path is characterized by a deterministic, an exogenous growth mechanism of the key variables. For simplification, the growth rate is assumed to equal zero without any loss of generality for allowing the existence of steady state. The model's non-stochastic steady state system is characterized by above equilibrium conditions in non time varying fashion (see appendix for the summary of the steady state system).

To solve the system, we can follow the method of continuous substitution. Start with the steady state version of Eq. (1.15) and combine it with the Fisher Equation, we have

$$R^d = \frac{R^n}{\pi} = \frac{1}{\beta}; \tag{1.15S}$$

Following Eq. (1.20), we get

$$R^k = R^d \times EFP = \frac{1}{\beta \times EFP}; \tag{1.20S}$$

Meanwhile, we also have the following from Eq. (1.28):

$$\mathrm{X} = \frac{\varepsilon_p - 1}{\varepsilon_p}; \tag{1.28S}$$

Then according to Eq. (1.19), we achieve

$$\frac{Y}{K} = \frac{1/(\beta \times EFP) - 1 + \delta}{\alpha(\varepsilon_p - 1)/\varepsilon_p}; \tag{1.19S}$$

The above attainment predicts

$$\frac{Y}{L}=\frac{Y/K}{L/K}=\frac{Y/K}{1-NW/K};$$

On the other hand, based on Eq. (1.9), we find out UEFP equals to

$$UEFP=1+\frac{EFP-1}{1-\gamma NW/K}; \tag{1.9S}$$

assuming we know the steady state value of EFP and NW/K. Thus combine Eq. (1.6) and (1.21) to eliminate real wage w, we get

$$\frac{N^G}{N^F}=\frac{1-\alpha}{1-\gamma}\frac{Y}{L}\frac{\mathrm{X}}{UEFP-1};$$

where Y/L , X and UEFPare given already. Since $N^G+N^F=N$, we know:

$$N^F=\frac{N}{N^G/N^F+1}; \text{ and } N^G=N-N^F;$$

The next step is to combine Eq. (1.19S) and (1.17) to get

$$\frac{N^G}{K}=\left[\frac{Y}{K}\frac{1}{A}\right]^{\frac{1}{1-\alpha}};$$

where Y/K is given in Eq. (1.19S). Then we can easily calculate the steady state capital stock K, output Y, and the rest variables of the model. Some key steady states are given in table 1.2.

1.3.2 Log-linear approximation

To understand the model's transmission mechanism subject to other shocks, we should obtain the short run properties of the model. One way to capture the model's short run dynamics is to log-linearize all the equilibrium conditions around their non-stochastic steady states. After derivations, the log-linear approximations for the equilibrium system are given by:

$$\hat{l}_{t+1}=\hat{f}_t+\gamma\hat{k}_{t+1}+\gamma\hat{q}_t+(1-\gamma)\hat{n}_t^F; \tag{1.2L}$$

$$\hat{k}_{t+1}=-\hat{q}_t+\frac{NW}{K}n\hat{w}_{t+1}+(1-\frac{NW}{K})l_{t+1}; \tag{1.4L}$$

$$\hat{efp}_{t+1} = -\frac{EFP-1}{EFP}\left(\frac{\gamma NW/K}{1-\gamma NW/K} + \frac{\gamma}{1-\gamma}\frac{NW/K}{1-NW/K}\right)(\hat{nw}_{t+1} - \hat{k}_{t+1} - \hat{q}_t) + \frac{EFP-1}{EFP}(\hat{w}_t - \frac{1}{1-\gamma}\hat{f}_t) \quad ; \quad (1.11L)$$

$$-\hat{c}_t = -E_t\hat{c}_{t+1} + \hat{r}_t^d ; \quad (1.15L)$$

$$\hat{y}_t = \hat{a}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t^G ; \quad (1.16L)$$

$$\hat{r}_t^k = \frac{X\alpha Y/K}{R^k}(\hat{\chi}_t + \hat{y}_t - \hat{k}_t) + \frac{(1-\delta)}{R^k}\hat{q}_t - \hat{q}_{t-1} ; \quad (1.19L)$$

$$E_t\hat{r}_{t+1}^k = \hat{r}_t^d + \hat{efp}_t ; \quad (1.20L)$$

$$\hat{w}_t = \hat{\chi}_t + \hat{y}_t - \hat{n}_t^G ; \quad (1.21L)$$

$$\frac{1}{\upsilon R^k}\hat{nw}_{t+1} = \frac{K}{NW}\hat{r}_t^k - (\frac{K}{NW}-1)\hat{r}_{t-1}^d - (\frac{K}{NW}-1)\hat{efp}_{t-1} + \hat{nw}_t ; \quad (1.22L)$$

$$\hat{i}_t = \frac{1}{\varphi}\hat{q}_t + \hat{k}_t ; \quad (1.23L)$$

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta\hat{i}_t ; \quad (1.24L)$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\hat{\chi}_t ; \quad (1.28L)$$

$$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{i}_t ; \quad (1.29L)$$

$$\hat{r}_t^n = \rho_r\hat{r}_{t-1}^n + (1-\rho_r)\kappa_\pi\hat{\pi}_{t-1} + \varepsilon_{rt} ; \quad (1.30L)$$

Following the convention, all the variables with hat on top denote their percentage deviations from non-stochastic steady state. Using Uhlig's undetermined coefficients procedure yields a state space solution of the form[19]:

$$\hat{s}_{t+1} = \Omega_1\hat{s}_t + \Omega_2\varepsilon_{t+1} ; \quad (1.31)$$

$$\hat{d}_t = \Omega_3\hat{s}_t \quad (1.32)$$

[19] The detailed methodological exposition is given in Uhlig (1999), whereby the Matlab programme is available at his homepage (http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm).

where the state variable vector, $\hat{s}_t$, includes predetermined and exogenous variables; $\hat{d}_t$ is the vector of control variables; and the vector ε_t contains the random innovations. The coefficient matrices, Ω_1 , Ω_2 , and Ω_3 , have elements that depend on the structural parameters of the model. Therefore, the state space solution, (1.31) and (1.32) are used to simulate the model later when showing the results.

1.4 Quantitative experiments

Using the log-linear system above, I am able to conduct some quantitative experiments in this section. The model is calibrated to US economy and I will simulate the baseline model as well as BGG to compare the responses of variables to exogenous shocks in monetary policy as well as goods technology.

1.4.1 Parameterization

To conduct quantitative analysis, first of all, we set parameter values to calibrate the baseline model to the US economy. Since the baseline model generally follows the setup of BGG and the current state of DSGE models, I generally do the calibration based on previous studies. To compromise, I will do sensitivity analysis on some key parameters below to check the robustness of the baseline results. BGG's original parameterization is a good benchmark for the baseline calibration. I will set most of the parameter values based on that. As a complement, I will also refer to more recent studies which estimate those parameters in a full DSGE environment (e.g., Christenson and Dib (2008)). For the parameters in banking sector, I will generally consult the parameterization of Goodfriend and McCallum (2007) as well as other studies that include a banking sector in general equilibrium model.

To summarize, there are four sets of parameters for which we can calibrate the values in sequence. The first set includes all the parameters in the state-of-the-art DSGE models, from which we can borrow those numbers directly. The households' discount factor, β, is set to 0.99 to yield the annual net real risk-free rate 4%. The relative weights of leisure in utility ψ, is picked up to capture the average fraction of hours worked, 31%. The share of capital α is set to equal 0.36 as this value is canonical in RBC literature. For the capital depreciation rate

δ, the value 0.025 is chosen to capture the averages over the post war sample period. The parameters associated with price and wage rigidity are given according to the literature (e.g., Christiano et al. 2005), where the elasticity of demand for goods, ε_p, and labour, ε_w, are 11 and 21 such that the steady state markups are 10% in the goods market and 5% in labour market, and the probability of not reoptimizing for price setters θ_p, and for wage setters θ_w, are both 0.75. The autocorrelation of goods technology shock is following the conventional value of 0.9.

The second set of parameters are introduced in BGG 1999 handbook chapter, which includes the entrepreneurs' surviving rate at the end of each period, υ, the capital price elasticity of investment to capital stock ratio, φ, and the elasticity of external finance premium with respect to the ratio of internal funds to total value of purchased capital, ψ. For υ, I will stick to the value 0.9728 suggested in BGG since this value are also used in many later studies as well. The other two parameters, φ and ψ, are key to BGG's financial accelerator mechanism, since φ measures the level of capital adjustment cost and so the response of investment to shocks and ψ directly captures the degree of financial accelerator effect. Nevertheless, the values of these two parameters are subject to controversies[20]. Originally, BGG set φ equal to 0.25 and ψ equal to 0.05. These define a relatively low level of capital adjustment cost and high degree of financial accelerator effect. After BGG, three studies have estimated these two parameters in full DSGE environment using different estimation methodologies. Christenson and Dib (2008) used classical maximum likelihood method to estimate the value of φ be 0.59 and ψ be 0.042 for US economy in the post Volcker era. Meier and Muller (2006) matched the model impulse response after a monetary shock with that from empirical VAR studies and found an even higher value of φ, 0.65, but insignificant ψ. De Graeve (2008) estimated ψ to be around 0.1 with Bayesian methods. Since he specified investment adjustment cost rather than capital adjustment cost assumed here, there is no counterpart φ over there for reference. Besides above three, Nolan and Thoenissen (2009) calibrated these two values to be 1 and 0.037 for the sample period 1960-2006. To compromise with all above studies, I set the value of φ be 0.5 and ψ be 0.035 in the baseline calibration to get benchmark results for comparison.

[20] BGG calibrate $\psi = 0.05$ based on realistic values for monitoring costs and bankruptcy rates. They also argue that a reasonable value for φ lies between 0 and 0.5.

Table 1.1 Parameters in the model

Parameter	Description	Value
β	*Household's discount factor*	0.99
ψ^x	*Weight on leisure in utility*	2.47
α	*Share of capital in goods production*	0.36
δ	*Capital depreciation rate*	0.025
θ_p	*Retailers' probability of not able to reset price*	0.75
θ_w	*Households' probability of not able to reset wage*	0.75
ε_p	*Goods elasticity of demand*	11
ε_w	*Labour elasticity of demand*	21
υ	*Entrepreneurs' surviving rate*	0.9728
φ	*Curvature of capital adjustment cost function*	0.5
ψ	*EFP elasticity of net worth to collateral value ratio*	0.035
γ	*Share of collateral in loan production*	0.65
ρ_r	*Interest rate smoothing*	0.8
κ_π	*Elasticity of policy rate to inflation deviation*	1.5
ρ_a	*Autocorrelation of goods productivity shock*	0.9

The third set consists of two parameters in the banking sector, which are the share of collateral in loan production/management function γ, and the elasticity of F_t with respect to the state of economy θ. These two parameters are crucial to the banking sector transmission mechanism in the baseline model. The higher are γ and θ, the stronger is the elasticity of external finance premium to the corresponding arguments. To pin down the value of γ, the first reference is Goodfriend and McCallum (2007) where they set γ equal to 0.65, which indicates relatively low elasticity. However, several other studies Benk, Gillman and Kejak (2005), (2008) and (2010), and Leao (2003), which also hold credit production function in their model, choose γ from 0.73 to 0.89. Based on all of these, I set the value of γ equal to 0.65 following Goodfriend and McCallum (2007) for the baseline calibration and alter it later in plausible range of 0.65-0.89. θ is a free parameter only to capture the reduced from elasticity of EFP to the state of the economy, we set it equal to a plausible value of 0.2, which

Table 1.2 Steady state in the model economy

Variable	Definition	Value
I. Steady state values		
A	*Goods sector productivity level*	1
F	*loan productivity level*	2.15
R^d	*Risk-free rate*	1.01
R^k	*Gross return on capital*	1.0176
R^n	*Nominal interest rate*	1.0194
π	*Inflation rate*	1.0092
EFP	*External financing premium*	1.0075
N^G	*Labour service in goods sector*	0.3
N^F	*Labour service in banking sector*	0.005
II. Steady-state ratios		
Y/K	*Output to capital*	0.13
I/Y	*Investment to output*	0.19
C/Y	*Consumption to output*	0.61
G/Y	*Govt expenditure to output*	0.2
L/K	*Leverage ratio*	0.5
N^F/N	*Financial hour to total hour*	1.7%

implies the EFP elasticity to output and nominal interest rate is equivalent with that to real wage in Eq. (1.20L). This value implies 20% of F_t is endogenously determined.

The final set regards those parameters in the monetary policy reaction function. I generally follow Clarida et al. (2000) and set the interest rate smoothing parameter ρ_r equal to 0.8, and the policy response to past inflation κ_π to 1.5.

Finally, some key steady state values are chosen to finish the calibration. The external finance premium is generally unobservable in reality, for which we can only refer to some close indicators to pin down the steady state value. It is set to 1.0075 for baseline value, corresponding to an annual risk spread of 300 basis points, approximating the post war average spread between the corporate bond rate and the three-month treasury bill rate. This is consistent with the estimates in Queijo (2009) and lies within the range reported in De Fiore and Uhlig (2005)[21]. The steady state quarterly gross inflation is set to 1.0092, implying the nominal interest rate of 1.0194. Following BGG, the steady state leverage ratio of entrepreneurs is set to 50%, which means the ratio of net worth to value of purchased capital is 0.5. The steady state consumption, investment and government expenditure share of GDP are given by 0.603, 0.192 and 0.205, respectively to match the historical average. In labour market, the steady state ratio of monitoring hour relative to goods produce hour is 1.7%. Table 1.1 and 1.2 present all parameter and steady state values for the baseline calibration.

1.4.2 Findings

After calibration, this section presents the main findings about the abilities of banking frictions to transmit shocks hitting the economy. I mainly consider the impulse responses to two well known shocks: monetary policy shock (i.e. demand side) and goods technology shock (i.e. supply side). First of all, I show the baseline model's transmission mechanism, stemming from the financial frictions within the banking sector, and compare it to that from BGG. For the sake of consistent comparison, the model, for this moment, is abstracted from nominal contract and wage rigidity, since they are not modelled in BGG. Secondly, I go one step further to show how the performances of banking accelerator is altered by the values of collateral share in loan production function γ. Thirdly, the effect of nominal contract is considered. This is to show how the transmission is altered by the so called Debt-Deflation mechanism. Finally, the role of nominal wage rigidity is highlighted to show the distinct feature of the current model to other models in the literature of financial frictions.

1.4.2.1 Baseline banking frictions versus BGG

Here I examine the responses of the baseline model to two types of shocks: a monetary policy shock and a goods productivity shock. The results are compared with that of the standard no financial friction (NoFF) model to see the amplification and propagation mechanism. The NoFF model is the one with perfect financial intermediation technology so that the banking

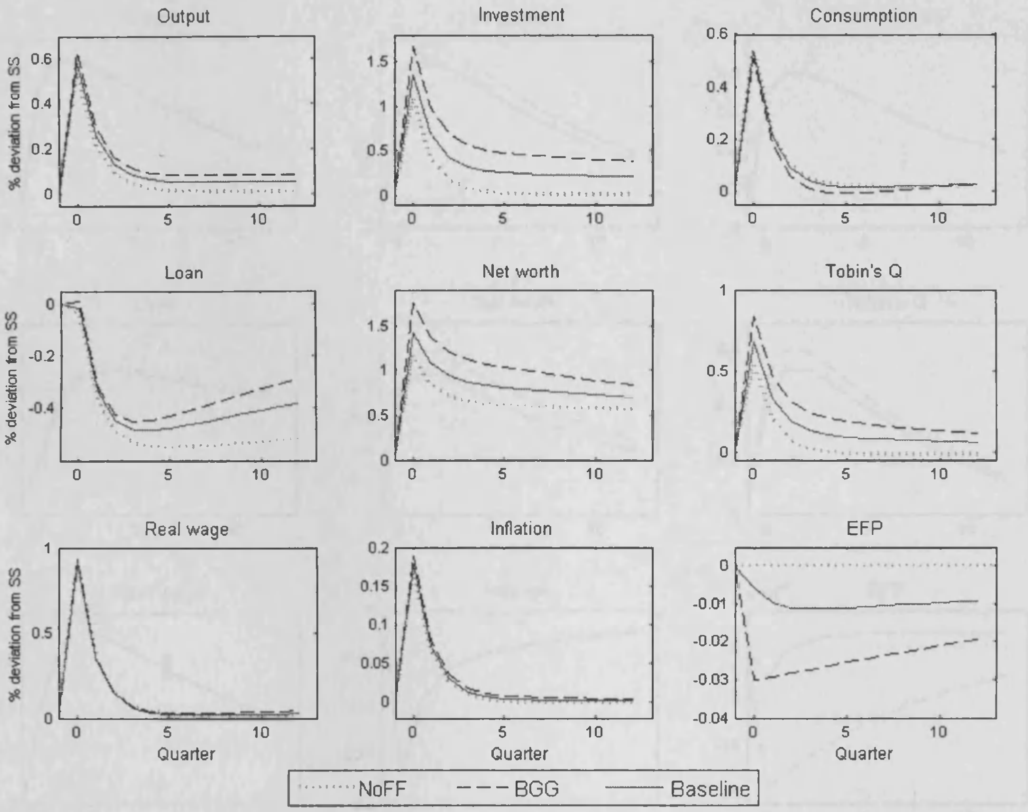
[21] In De Fiore and Uhlig (2005), the reported range for annual risk premiums on bonds and loans in U.S. is between 160 and 340 basis points.

sector is costless. In addition, BGG results are also plotted as a benchmark to see the relative strength of banking accelerator effect with the baseline parameter values. Figure 1.2 plots the responses of output, investment, consumption, loan, net worth, capital price, real wage, inflation and external finance premium to an unanticipated expansionary shock to monetary policy rule. The shock hits the economy at time 0 and the time units here and in all subsequent figures are in quarters. For each single graph the dotted line designated the standard no friction response, generated by fixing the EFP at its steady state level instead of allowing it to respond to changes in the state of the economy. This implies the benchmark simulation are based on model with the same steady state as the BGG model and the baseline banking accelerator model, but in which the additional dynamics associated with the accelerator effect have been turned off. The dashed line stands for the response in BGG financial accelerator framework and the solid line is that from the baseline banking friction model.

The banking accelerator effect is clearly seen in the diagram. In the impact period, an unanticipated drop in interest rate stimulates the demand for capital, which in turn pushes up investment and the price of capital. The rise of asset prices makes the return on capital higher, raising the net worth of entrepreneurs. The speed of increase in the net worth overtakes that of the capital demand, entrepreneurs' leverage ratio decrease, leading to lower marginal cost of bank intermediation and lower EFP despite the real wage that drives up the EFP is also jumping up. Compared with no friction case, amplification mechanism appears. In addition, the decreased EFP further stimulates investment, demand of capital and asset price, causing the economy running into a loop and the increment of net worth is very persistent. Thus we see both the amplification and propagation effect in the baseline banking transmission model. This result resembles that of BGG well qualitatively.

Thus the baseline model can obtain a similar accelerator effect of BGG in the end, but with completely different explanation. BGG attributes the variation of EFP to the cost of state verification (outside the banking sector) assumption while I offer an explanation of changes in EFP from varying marginal cost within the banking sector. However, the acceleration effect in the baseline model is smaller than that of BGG. The initial magnification of output and investment in baseline model only captures half of that in BGG. This reflects the smaller

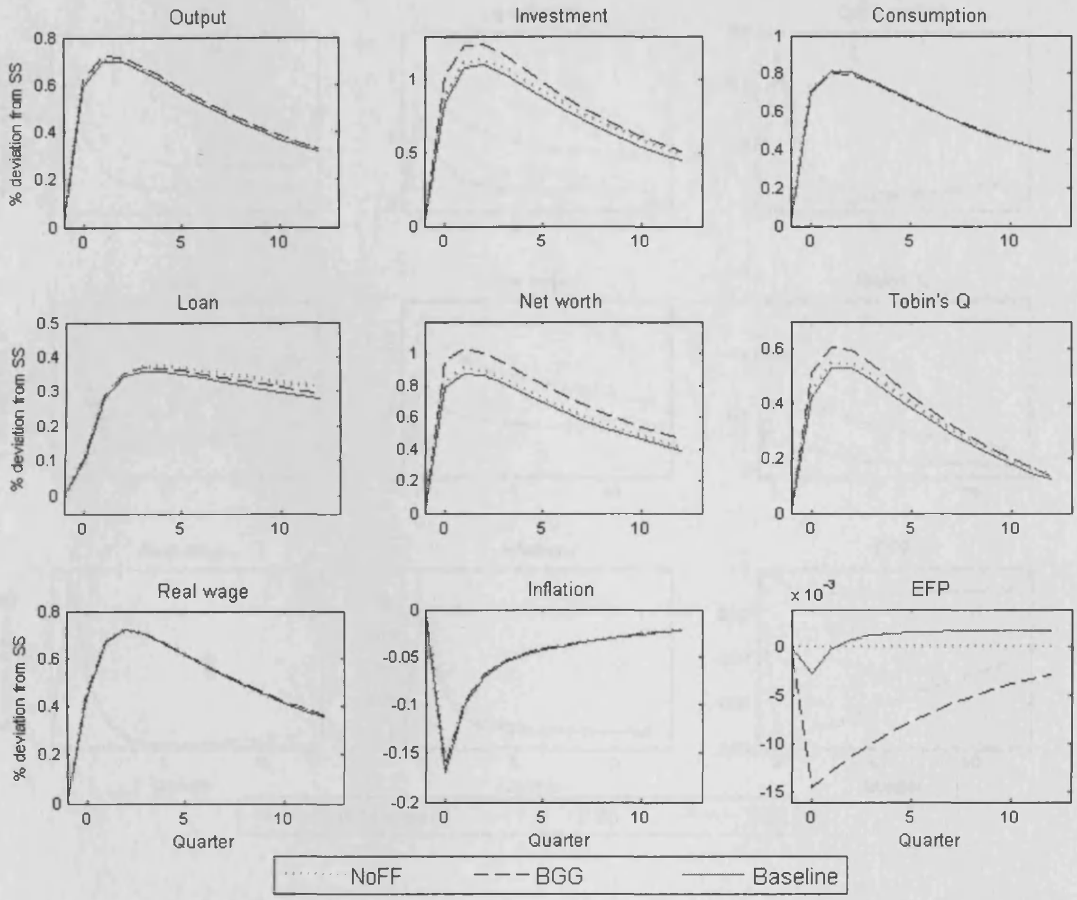
Fig. 1.2 Impulse responses to expansionary monetary policy shock



elasticity of EFP with respect to net worth capital ratio in the baseline model subject to standard calibration (see Eq. (1.11L)).

Figure 1.3 shows the response of the model economy to supply shocks (rise in the goods sector productivity). By contrast to BGG, here the banking frictions generate a slight decelerator effect even not very significant. This can be understood if we focus on the behaviour of EFP. For most of the time, EFP is higher than steady state value. This is because the real wage is procyclical and expands significantly. Remember higher real wage drives up EFP. Thus the net worth increases less than that in frictionless model. In the same way, the behaviours of other variables, like output and investment, is decelerated. On the other hand, the time paths for nominal rate (not shown) and inflation are quite similar in the three models, reflecting that the monetary authority conducts policy concentrating on inflation only.

Fig. 1.3 Impulse responses to positive goods tchnology shock

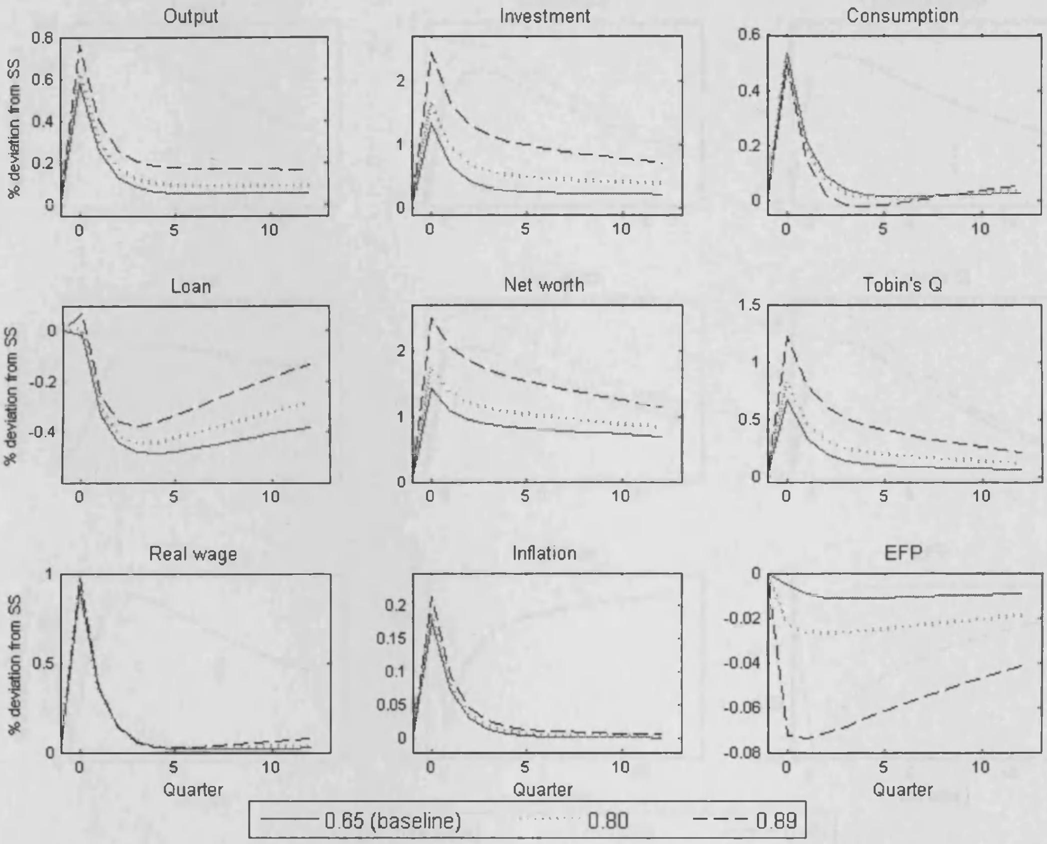


1.4.2.2 Experiments with different collateral share.

After we see the qualitative similarity of the transmission mechanism between the baseline model and BGG, it is natural to further enquire about the quantitative performance of the baseline model. This section shows how the amplification or attenuation mechanism in the baseline model is altered. As shown in Eq. (1.11L), in light of banking sector profit maximization the elasticity of EFP with respect to net worth capital ratio is determined by three factors: Steady state level of external finance premium, steady state leverage ratio and the share of collateral in loan production γ. Since the former two are fixed in many studies since BGG, I will keep it steady and vary the latter[22].

[22] At least one exception is Christiano et al. (2010) where they fixed the leverage ratio to 0.23 in their model and argued that the data between 1998Q1 and 2003Q4 shows that this ratio is in the range of 0.18-0.44.

Fig. 1.4 Impulse responses to expansionary monetary policy shock with different γ.



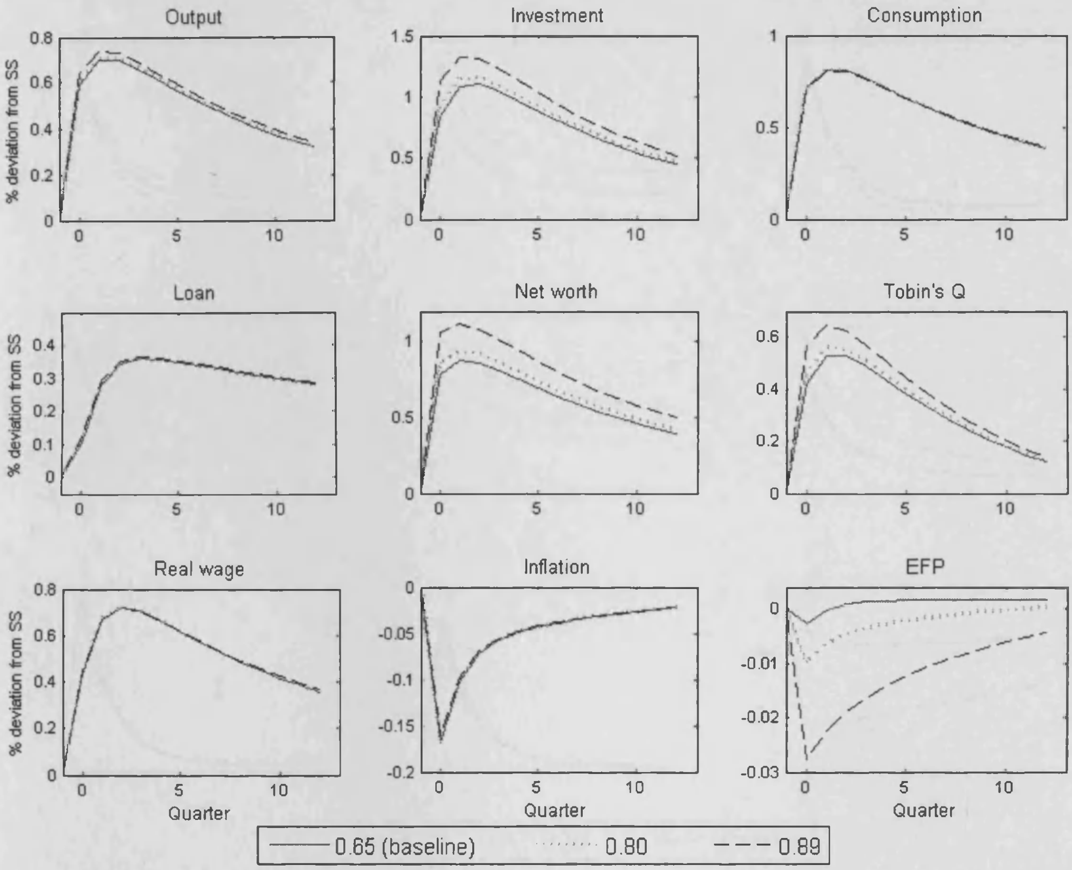
Following the discussion in the parameterization section, the value of γ is subject to discrepancies from 0.65 to 0.89 in several different studies. The banking accelerator model only captures about half of BGG with the baseline value of 0.65, for which we can alter the value of γ to see different degree of elasticities. To remind us, the elasticity is give by

$$Elasticity = \frac{EFP-1}{EFP}\left(\frac{\gamma\, NW/K}{1-\gamma\, NW/K}+\frac{\gamma}{1-\gamma}\frac{NW/K}{1-NW/K}\right)$$

Proposition 1.2: *The elasticity of external finance premium with respect to net worth collateral ratio is an increasing and convex function of the share of collateral in loan production,* γ.

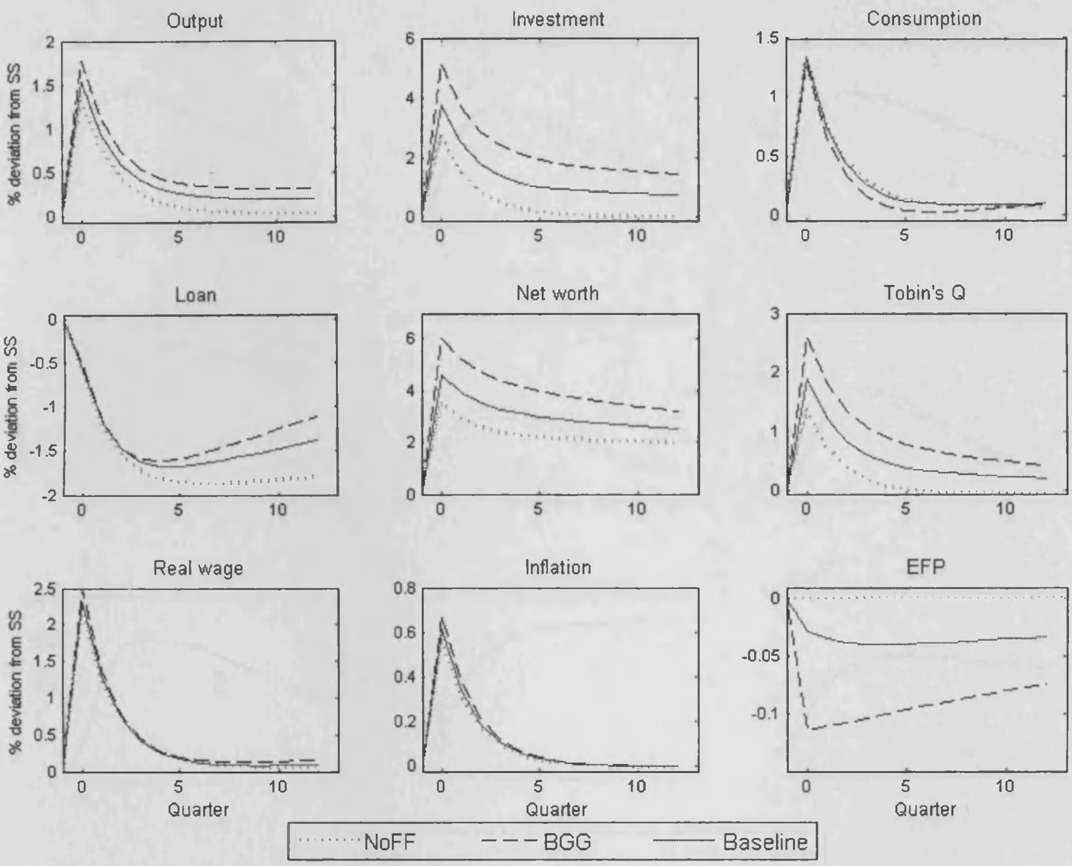
The mathematical proof of proposition 1.2 is shown in Appendix A, while the quantitative exemplification is shown in figure 1.4 and 1.5. Figure 1.4 displays the impulse response for

Fig. 1.5 Impulse responses to positive goods technology shock with different γ.



an expansionary monetary policy. In each graph, the solid line represents the response of the economy with baseline value of 0.65 (lower bound). The dotted and dashed lines stand for the banking accelerator model with γ equals to median value of 0.8 (approximately match BGG), and upper bound 0.89. The increased elasticity of EFP with higher value of γ is clearly seen in all the graphs. With low value of γ, 0.65, the impact period amplification for output and investment is weaker than that of BGG while the magnification catch up and go exceeding BGG as we increase γ to the upper bound 0.89. The increase of the amplification is non-linear since the difference of the initial jump between 0.89 and 0.8 is much larger than that between 0.8 and 0.65, though the value of γ increases by more in the latter. Thus the diagram confirms that the magnification effect is an increasing and convex function of γ. The value of γ that captures the elasticity similar to BGG is 0.8. Therefore, in order to make the banking accelerator effect catching the BGG one, we need a relatively high value of γ. My personal perception is that the value of 0.8 lies within the plausible range and is acceptable.

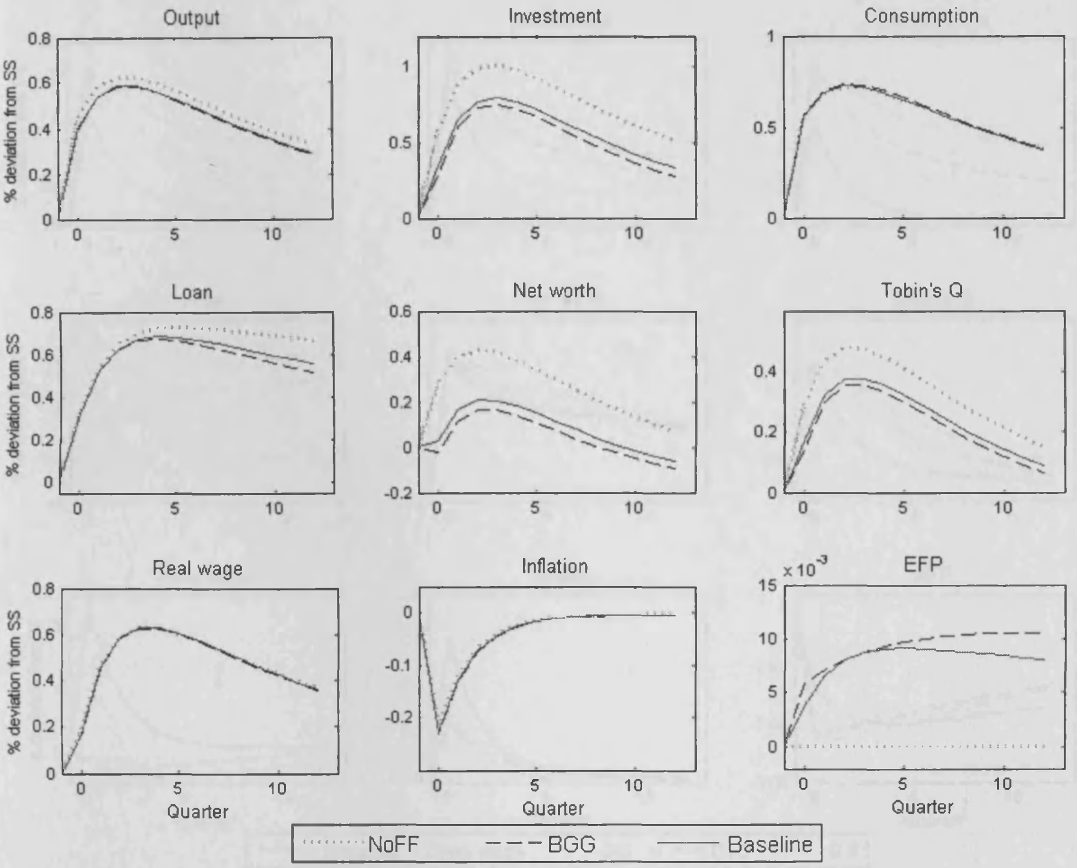
Fig. 1.6 Impulse responses to expansionary monetary policy shock with nominal contract.



Moreover, other aspects of banking accelerator mechanism assumed in this model can also do compensation, which is shown in the next subsection. Despite of the large discrepancies in the amplification effect, the differences in propagation mechanism are relatively small, reflecting the three models with different values of γ follow quite similar loop of multiplicative effect.

Figure 1.5 plots the responses of the economy to a positive goods technology shock. The diagram also confirms the legitimacy of proposition 3 graphically. Compared with the baseline model, models with larger collateral share in loan production function can overturn the decelerator effect and generate more and more accelerator effect. The upper bound value of γ, 0.89, leads to the largest accelerator effect, along with the highest contribution of net worth to capital purchasing and the deepest drop of EFP. The overall picture indicates that

Fig. 1.7 Impulse responses to positive productivity shock with nominal contract.

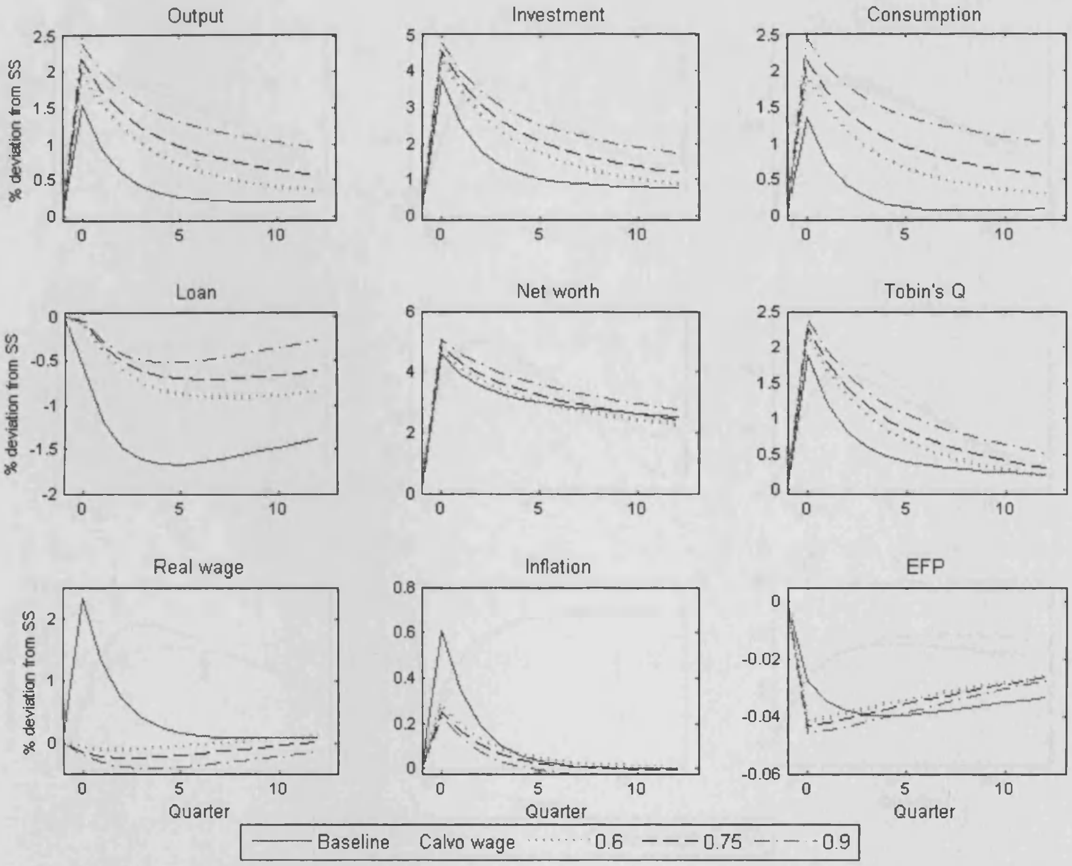


higher value of γ strengthens non-linearly the acceleration effect when both shocks hit the economy.

1.4.2.3 Effect of nominal contract

The baseline model of banking friction is shown to have strong enough power to amplify and propagate the responses of the economy to shocks by altering the transmission mechanism of the no friction model. In this sub-section, we also find out that normally the acceleration effect is stronger when the economy is hit by demand shocks (e.g., monetary shock) than when the economy is subject to supply shocks (e.g., productivity shock). This phenomenon stems from the well known Fisher's Debt-deflation effect: the real burden of borrowers is higher when the economy is experiencing deflation because the nominal debt is now exacerbated in real terms with declining price level. Since supply shock always drives output and price in different direction, the acceleration effect is attenuated in this scenario. In the

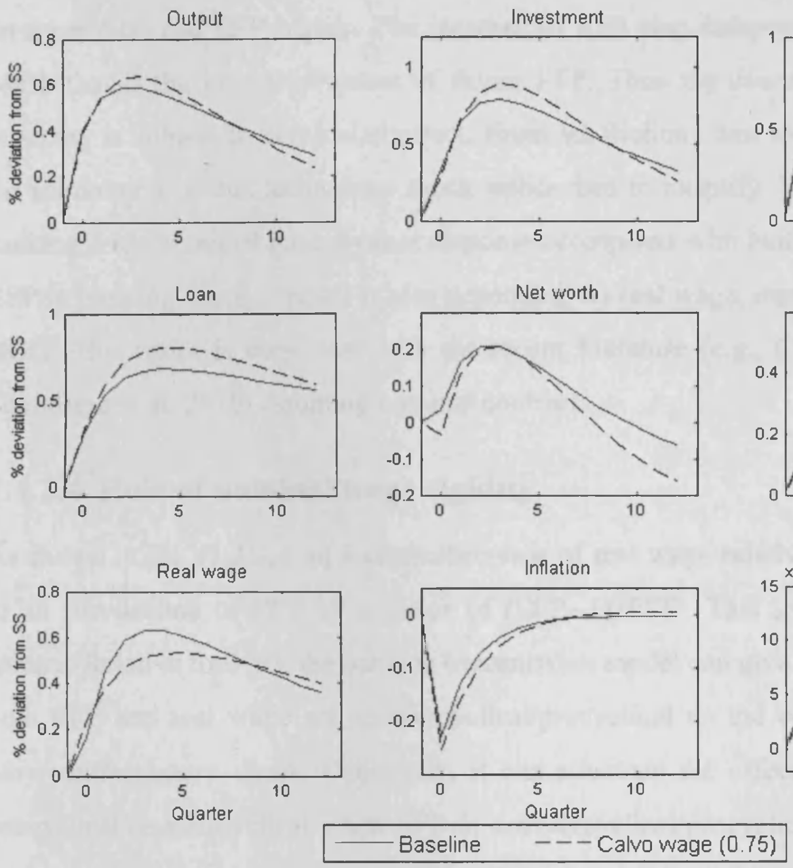
Fig. 1.8 Impulse responses to expansionary monetary shock with Calvo wage



baseline setting, the contract between FIs and households is in real term so that the Debt-deflation effect is not passing through. It is straightforward to conjecture this effect would be magnified when the contract is in nominal term. To see the effect of nominal contract is the object in this sub-section.

Figure 1.6 displays the impulse responses of the model economy with nominal contract between households and FIs to expansionary monetary shock. Since the contract is tied up in nominal term, the initial real cost of deposit is totally determined by inflation, which has upward pressure subject to expansionary monetary shock. The immediate rise of inflation dampens the real cost of deposit fully and is in favour of entrepreneurial net worth accumulation because the FIs will adjust the loan rate according to the real cost of deposit. On the other hand, the increase of net worth reduces the loan needs and decreases the marginal cost in the loan intermediation and so EFP. As shown in figure 1.6, because

Fig. 1.9 Impulse responses to positive technology shock with Calvo wage



inflation has upward pressure subject to demand shock, net worth increases significantly and makes the entrepreneurs less dependent on external funds (loan decreases), the EFP declines dramatically compared with real contract case (figure 1.2). Also, the EFP in banking friction model has a trend-reverting dynamics because of the initial increase of real wage, which causes the EFP not able to reach the lowest value at the beginning.

The impulse responses of the model economy with nominal contract to positive technology shock are shown in figure 1.7. The key difference is that when the economy is hit by a supply shock, inflation is decreasing and driving up the real cost of deposit. Although economic expansion turns to increase the return on capital and entrepreneurial net worth, the increase in real cost of deposit offsets the positive effect partially or even fully so that the overall response of net worth is uncertain. On the other hand, since the response of net worth is not strong, the dependence on external funds is not reduced but increased, making firm's leverage ratio and EFP higher. The increase of EFP also dampens the accumulation of net worth that is the key determinant of future EFP. Thus the overall effect is that when the economy is subject to nominal contract, financial frictions turn to attenuate the response of the economy to goods technology shock rather than to magnify. We can see both BGG and banking friction model have weaker responses compared with NoFF model. The dynamic of EFP in banking friction model is also depending on real wage, distinguishing itself to that in BGG. This result is consistent with the recent literature (e.g., Christenson and Dib 2008, Christiano et al. 2010) assuming nominal contract.

1.4.2.4 Role of nominal wage rigidity

As shown in Eq. (1.21L), an increase/decrease of real wage relative to the steady state leads to an rise/decline of EFP by a factor of $(\mathrm{EFP}-1)/\mathrm{EFP}$. This inspires the following conjecture: Relative to BGG, the banking transmission model can give extra acceleration effect if both EFP and real wage are countercyclical/procyclical on the condition of an expansionnary/contractionary shock. Oppositely, it can attenuate the effect of BGG if real wage is procyclical/countercyclical when EFP is countercyclical/procyclical on the condition of an expansionary/contractionary shock. For instance, assume a shock hits the economy, drives output up and EFP down, displaying amplification effect in BGG. Meanwhile, real wage is countercyclical, thus push down EFP even more, which accelerate the BGG amplification effect even further. Nevertheless, this is not a common situation in reality since the shock stimulating the economy always drives up the real wage simultaneously. Hence, the inclusion of factor price in (1.11L) often dampens the acceleration effect. However, one exception is the model with nominal wage rigidity. In this case, the expansionary monetary shock stimulates the whole economy with the acceleration effect presenting. This stems from the countercyclical movement of real wage. The dynamic of real wage is determined by comparing the movements of wage inflation with that of price inflation. Real wage is positively related to wage inflation, but negatively related to price inflation, both one for one. With wage rigidity, wage inflation can be more sluggish than price inflation. This mechanism is able to make real wage countercyclical subject to monetary shock.

As shown in figure 1.8, real wage is procyclical when the model is without nominal wage rigidity, but countercyclical when nominal wage rigidity presents, making the initial response of EFP stronger. More rigidity there is, stronger the initial response of EFP. Thus relatively large inertia in nominal wage setting is the key for this effect to be empirically relevant. The extra acceleration effect does appear as explained above. Subject to expansionary monetary shock, the decline of real wage causes EFP to drop more for models with wage setting. We can see from the diagram that the EFP's initial response is nearly doubled. However, the dynamics of EFP in the model with wage rigidity is less persistent. This stems from the dynamics of loan which controls the firm's leverage and EFP. As the effect is stronger the more rigid is the nominal wage, it is reasonable to imagine that in the model with wage rigidity only, we can see much more differences in the models' dynamics.

Figure 1.9 plots the impulse responses of the model subject to technology shock. As expected, the response of real wage in the model with wage rigidity is smaller than that in the model without. Interestingly, the response of EFP in the wage rigidity model is still stronger than that in the baseline model without wage rigidity. This stems from the dynamics of other variables that also determine the behaviour of EFP. For instance, the loan is higher in the wage model, making the leverage higher and so EFP higher. Another notable feature is that although output and investment are more amplified in the model with wage rigidity, they are less persistent. The reason can be the behaviour of EFP that is more persistently higher in the model with wage rigidity, causing the output and investment come back quicker.

1.5 Conclusion

In a well-known paper, BGG document the credit market imperfections by referring to the information asymmetry between entrepreneurs and financial intermediaries ex ante and the costly state verification once entrepreneurs default to the contract ex post. Based on this, they are able to derive that the financial frictions are relevant to the entrepreneurial balance sheet condition. However, their framework is subject at least two limitations. One is the absence of an explicit problem or role for this passive financial intermediary or banking sector, which behaves only as a veil. Moreover, the monitoring cost in BGG is treated as a deadweight loss measured in consumption goods, for which we have no idea how and where this cost is distributed explicitly.

This chapter provides an alternative modelling strategy to rationalize the financial frictions on the one hand, and addresses the aforementioned limitations in BGG on the other. By contrast to BGG, the model in this chapter ascribes the financial friction to the internal cost along the loan management in the banking sector. Banks require collateral evaluation and labour monitoring when producing loans. Thus for each unit of loan, the cost includes the factor prices paid to collateral and labour services on top of the return on deposit. Since banks tend to maximize their market value, all the cost is transferred to entrepreneurs. In this setting, the stake of entrepreneurs in the production is relevant to determine the EFP because of the grounds from two aspects: (i) the ratio of entrepreneurial net wealth to collateral (economy-wide capital) value influences the total managerial cost in the banking sector; (ii) entrepreneurs' holding of collateral help mitigate the EFP since part of the service return on collateral is distributed to the entrepreneurial sector. According to these, I can derive an observationally equivalent relationship between EFP and the stake of entrepreneurs in the production as in BGG. This provides a rationale for us to understand the financial frictions from a different angle. Besides the leverage ratio, the EFP is shown to be determined also by factor price of loan management, the variation of capital price and the aggregate economic conditions, for which I believe the model in this chapter derives a mechanism that contains broader ingredients in determining the behaviour of EFP.

Despite the consideration of the financial frictions from supply side of the credit market, however, this model is still silent about the banks' balance sheet condition in determining the EFP on the one hand and the transmission mechanism of the aggregate economy on the other. In light of the recent global financial crisis, it should be necessary to investigate the relevance of banks' balance sheet condition to financial frictions and economic contractions and put this into future research agenda.

Chapter 2

Financial Sector Shocks, External Finance Premium and Business Cycle

2.1 Introduction

The very recent financial crisis and possibly ongoing economic recession demonstrated that the financial sector should be an important factor which can influence the economic activity. As stated in Gertler and Kiyotaki (2009), we no longer need to appeal either to the Great Depression or to the experiences of many emerging market economies to motivate interest on the role of financial factors in aggregate fluctuations since the worst financial crisis and economic downturn of the post war era is currently undergoing.

The importance of financial factor in shaping business cycle has been studied extensively in the literature. One of the most notable contributions in dynamic stochastic general equilibrium (DSGE) context is by Bernanke, Gertler and Gilchrist (1999), BGG hereafter. They develop the so-called financial accelerator mechanism and demonstrate that the existence of an optimal financial contracting, in an environment of information asymmetry between lenders and entrepreneurs, can magnify and propagate the responses of the economy to some main underlying shocks (e.g., monetary and total factor productivity (TFP)). Thus the financial markets may unavoidably increase the volatility of the economy through the endogenous variation of financial frictions.

Nevertheless, it is worth noting that the financial sector in BGG solely plays a role of transmitting shocks originating from other sectors. Thus the framework only captures one branch of the financial factors which should also include the fact that financial structure of the economy can also be an independent source of volatilities as suggested by recent economic events. For this reason, the importance of financial sector as an original source of aggregate fluctuations is still under investigation.

To fill this gap, this chapter tries to explore the quantitative role of financial sector disturbances in shaping the US business cycle. Built on the model developed in the last chapter, I am able to introduce two financial sector shocks into the model from two different sources. One is the shock to the loan management technology. We treat it as the supply side disturbance because it lies within the banking sector. The other shock stems from the demand side of the financial sector, characterizing as the shock to the entrepreneurial net worth. It is very important to introduce the two shocks together so that we have a complete picture in mind how the disturbances originating from the financial sector affect the aggregate economy.

Before moving to the results of this chapter, it is useful to have a brief review of related studies in order to keep the literature on track. One notable contribution recently is by Nolan and Thoenissen (2009), henceforth NT. They extract the entrepreneurial net wealth shock along with the TFP and monetary shocks for US economy from a DSGE model with financial accelerator mechanism a la BGG and name it as a shock to the efficiency of the financial sector. They try to distil the contribution to US business cycle of financial shock on top of the financial friction mechanism. They conclude that their extracted financial shock process is found to (i) be very tightly linked with the onset of recessions, more so than TFP or monetary shocks; (ii) remains contractionary after recessions have ended; (iii) account for a large part of the variance of GDP; (iv) be strongly negatively correlated with the external finance premium (EFP).

Despite these promising findings, the financial shock constructed in NT shouldn't be considered as a complete description of the disturbance in the financial sector because they only considered the demand side. It is important to recognize that both BGG and NT only considered the demand side of the financial markets. The financial friction developed in BGG is built upon the balance sheet of entrepreneurial sector. Entrepreneurial net worth is crucial to determine the cost of external funds which can influence the demand of external funds by entrepreneurs. Similarly, if the net worth is subject to stochastic disturbance, the shock only affects the entrepreneurial balance sheet and the demand side of the financial market. Regard the recent financial crisis, it seems more appropriate to also think about the effects from the supply side of the financial market, the financial intermediaries/banking sector.

Up to date, several studies have been considering the banking sector in determining the financial frictions on the one hand, and the disturbance in financial intermediaries as a source of business fluctuations on the other. Markovic (2006) introduces the bank capital channel in the monetary transmission mechanism on top of the corporate balance sheet channel as in BGG and highlights three sub-channels in the banking sector (supply side): default risk channel, adjustment cost channel and capital loss channel. He concludes that all the three channels in the banking sector reinforce the aggregate credit channel in the monetary transmission mechanism and increase the effects significantly in the event of large shocks to the value of bank capital. Zhang (2009) considers the bank's balance sheet effect from a different angle where banking sector is assumed to share the risk with entrepreneurial sector. When the economy is subject to large adverse shock, both the entrepreneurial and banking sector balance sheets are exposed to the risk and the deterioration of the two balance sheets reinforce each other and drive the economy down further. Aguiar and Drumond (2009) also emphasize the relevance of bank capital channel in determining the aggregate fluctuations, but from the Basel regulatory perspective.

Most of the studies (e.g., above studies) focusing on the bank capital channel in the model's transmission mechanism also stress the significance of disturbance to bank capital per se, but to a limited extend. They only deal with the impulse response conditional on the bank capital shock, but generally ignore the explicit time series process of the shock and its influence on the whole business cycle. One notable exception is Hirakata et al. (2010) who estimate a DSGE model with banking sector using Bayesian methodology and extract the shocks to the bank's net worth. Based on variance decomposition, shocks to the banking sector are found to be a main source of the spread variations and play a significant role for investment volatility.

All these studies convey an important signal that supply side friction and disturbance in the financial sector are also relevant to aggregate fluctuations; thus should be dictated for more attention and exploration. This chapter works along this line and focuses on the role of supply side disturbance in financial market in shaping the business cycle. Specifically, the model, built on the one in the last chapter, generally follows the setup of BGG and NT except that I replace the optimal contracting problem between lender and borrower with explicit profit maximization in banking sector subject to a loan production function. The profit maximization in banking sector can predict a relationship between EFP and corporate balance sheet condition as well as factor price in the banking sector. In this way, both the demand side (entrepreneurial sector) and the supply side (banking sector) contribute to the financial frictions. The shock to the technology in loan production manifests itself as the disturbance in banking sector. With the shock to entrepreneurial net wealth, we can see the perturbation in both of the supply and demand side in financial markets. By and large, this strategy extends the work of NT by allowing another shock in the banking sector on top of the net worth shock to affect the financial sector and the whole economy on the one hand, and distinguish itself to the work of Hirakata et al. (2010) who designate the disturbance in banking sector to bank net wealth on the other[23].

The main contributions of this chapter can be summarized as follows. First of all, use the shock construction procedure discussed in details below, I can extract the four shocks in which TFP, monetary and net worth shocks are close to their counterparts in NT on the one hand, and TFP as well as monetary shocks are observationally similar to the ones constructed with traditional estimation procedure on the other. This can be treated as a robust check that the inclusion of another shock wouldn't alter the processes of shocks originally generated in NT despite the fact that my model setup is slightly different from theirs.

Second, subject to the interesting, but also a little bit confusing result in NT that the net worth shock remains contractionary after recessions have ended, it is promising to find that, after add in the loan productivity shock, all the post war recessions happen only when both of the entrepreneurial net worth and the loan productivity shocks are in contraction, implying either one of them is not strong enough to cause an economy-wide recession.

Third, both of the extracted loan productivity and net worth shock are negatively correlated with proxies of EFP, despite the fact that the correlation between net worth shock and EFP is higher in absolute value. We can conjecture from it, that the loan productivity shock is also significant in shaping the financial business cycle even if it is not the dominant one.

Fourth, consistent with our prediction, the variance decomposition indicates that loan productivity shock is an important source of EFP variation, though the dominant driving force is still net worth shock. This somehow matches the result in Hirakata *et al.* (2010) in which they predict a quantitatively similar feature of bank capital shock in determining EFP. Even though we assume the shock in the banking sector with different essence, Hirakata *et al.* (2010) and I reach similar result in this dimension. Finally, net worth shock is still a dominant factor along several other dimensions of the economy after we include the loan productivity

[23] Current model is different from that in Hirakata *et al.* (2010) in other respective as well. In Hirakata *et al.* (2010), more shocks are used since they apply the Bayesian estimation where I follow the shock construction procedure developed in Benk *et al.* (2005), (2008).

shock, more important than TFP and monetary shock in determining output, investment, loan, hours and federal funds rate, while the loan productivity shock plays a minor role.

The rest of this chapter is structured as follows. Section 2.2 derives the model used in this chapter. Section 2.3 calibrates the model to quarterly data of US economy. The construction of all shock processes and numerical simulation are carried out in section 2.4. Section 2.5 concludes with some final remarks.

2.2 The model

The designated model I develop here is a standard DSGE New Keynesian model largely follows BGG's original setup and NT's extension. One noticeable exception is that I develop the financial frictions with a fully micro-founded loan production/management function and a financial intermediaries' profit maximization problem instead of BGG's original optimal contracting problem. The other one is that I introduce one more shock from the supply side of financial market on top of the net wealth shock from the demand side as in NT.

Besides the banking sector, the model economy is inhabited by households, three types of producers: entrepreneurs, capital producers, and retailers, and a government who conducts monetary policy. Households own differentiated labour service and have the power to set the nominal wage in the labour market as in Erceg *et al.* (2000). Entrepreneurs produce intermediate goods and borrow from banks that convert household deposits into business financing for the purchase of capital. The presence of collateral evaluation and labour monitoring costs create the financial friction, which causes loan interest rate higher than deposit interest rate. This makes the entrepreneurial demand for capital depends on their financial position and the supply of external funds depends on the state of the economy. The interaction between the demand and supply equilibrates the credit market. Capital producers purchase investment goods and build new capital to sell to the entrepreneurs. This captures the up and down movement of asset prices. Retailers present because it is more convenient to introduce nominal stickiness this way to keep track of the development in conventional dynamic New Keynesian framework. They set nominal prices in a staggered fashion a la Calvo (1983) and Yun (1996).

2.2.1 Households

The economy is populated by a continuum of monopolistically competitive households, indexed by $j \in [0,1]$, who consume, work and save. Each of them supplies differentiated labour service to the entrepreneurial and banking sector, which regard each of their labour service as an imperfect substitute for that of others. In this setup, entrepreneurs and banks demand bundles of labour services, which is obtained using the aggregation scheme as in Dixit and Stiglitz (1977)

$$N_t = \left[\int_0^1 N_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \quad \varepsilon_w > 1;$$

The optimal substitution across labour service leads to the following labour demand equation regarding the j th labour service

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t;$$

where $W_t(j)$ is the nominal wage set by the j th household, W_t is the Dixit-Stiglitz aggregate nominal wage given by $W_t = [\int W_t(j)^{1-\varepsilon_w} dj]^{1/(1-\varepsilon_w)}$, and ε_w gives the constant elasticity of substitution across labour service.

To motivate the demand for money, I follow Sidrauski-Brock and include money in the utility function of households[24]. Thus the j th household derives the expected life time utility from consumption (with external habit) of final goods, $C_t(j)$, real balance holding, $M_t(j)/P_t$, and leisure, $1 - N_t(j)$; with discount factor, $\beta \in (0,1)$, this is given by

$$U = E_t \sum_{h=0}^{\infty} \beta^h \left[\frac{(C_{t+h}(j) - \xi \overline{C}_{t-1+h})^{1-\eta^c}}{1-\eta^c} + \psi^m \frac{(M_{t+h}(j)/P_{t+h})^{1-\eta^m}}{1-\eta^m} + \psi^x \frac{(1 - N_{t+h}(j))^{1-\eta^x}}{1-\eta^x} \right];$$

[24] This setup follows Nolan and Thoenissen (2008).

where $\overline{C}_t$ is aggregate consumption, η^c, η^m and η^x measure the intertemporal elasticity of substitution for consumption, real balance and leisure. ψ^m and ψ^x represent the weight on real balance and leisure in the utility function.

The j th household enters period t with $P_{t-1}D_t(j)$ units of nominal deposits in a financial intermediary, and nominal money balances, $M_{t-1}(j)$. While deposits pay a gross nominal interest rate, R^n_{t-1}, between $t-1$ and t, money balances bear no interest. During period t, the j th household supplies labour to the entrepreneur firms and banks, for which he receives total factor payment of $W_t(j)N_t(j)$. In addition, he receives a lump-sum transfer from the monetary authority, $\mathrm{T}_t(j)$, as well as the dividend payments, $\Pi^F_t(j)$, from banks and, $\Pi^R_t(j)$, from retailers, as he owns both of them. All these funds are allocated for consumption, $P_tC_t(j)$, money holdings, $M_t(j)$, and nominal deposit holdings, $P_tD_{t+1}(j)$. Thus the household's intertemporal budget constraint, in real terms, is

$$C_t(j)+\frac{M_t(j)}{P_t}+D_{t+1}(j)=\frac{W_t(j)}{P_t}N_t(j)+\frac{M_{t-1}(j)}{P_{t-1}}\frac{P_{t-1}}{P_t}+R^n_{t-1}\frac{P_{t-1}}{P_t}D_t(j)$$
$$+\frac{\mathrm{T}_t(j)}{P_t}+\frac{\Pi^F_t(j)}{P_t}+\frac{\Pi^R_t(j)}{P_t};$$

The j th household chooses $C_t(j)$, $M_t(j)/P_t$, and $D_{t+1}(j)$ in order to maximize his expected lifetime utility subject to his budget constraint and labour demand constraint. The first order conditions (F.O.Cs) for this optimization problem are[25]:

$$U_{Ct}=R^n_t\beta E_t\{U_{Ct+1}\frac{P_t}{P_{t+1}}\}; \qquad (2.1)$$

$$\frac{U_{Mt}}{U_{Ct}}=\frac{R^n_t-1}{R^n_t}; \qquad (2.2)$$

Eq. (2.1) is the usual intertemporal condition, which states that the marginal cost of fore-going a unit of consumption in the current period must be compensated with the marginal benefit in the following period. Eq. (2.2) is the money demand equation.

[25] The omission of households' index in the F.O.Cs stems from the assumption following Erceg et al. (2000) and Christiano et al. (2005) that the implicit existence of state-contingent securities ensures households' consumption and asset holding are homogenous.

Given that the households set nominal wages in staggered contracts with a constant probability, $1-\theta_w$, of renegotiation in each period, the fraction of households who have the opportunity to reset their wages will set it as a mark-up over the marginal rate of substitution of leisure for consumption (MRS) taking account the probability that he cannot reset the wage again. The fraction of households who don't have the opportunity to reoptimise must apply the wages that was in effect in the preceding period indexed by the steady state gross rate of wage inflation ω. This yields the following maximization problem:

$$MaxE_t\sum_{h=0}^{\infty}[(\beta\theta_w)^h \frac{\lambda_{t+h}}{\lambda_t}\frac{W_t^*(j)\omega^h - MRS_{t+h}P_{t+h}}{P_{t+h}}N_{t+h}(j)];$$

The F.O.C for the maximization problem is

$$W_t^*(j) = \frac{\varepsilon_w}{\varepsilon_w - 1}\frac{E_t\sum_{h=0}^{\infty}\{(\beta\theta_w)^h \lambda_{t+h+1}/\lambda_{t+h}N_{t+h}(j)MRS_{t+h}\}}{E_t\sum_{h=0}^{\infty}\{(\beta\theta_w)^h \lambda_{t+h+1}/\lambda_{t+h}N_{t+h}(j)\omega^h/P_{t+h}\}}; \tag{2.3}$$

Log-linear approximations of the F.O.C imply the following wage inflation curve:

$$\hat{\omega}_t = \beta\hat{\omega}_{t+1} + \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\eta_n\varepsilon_w)}(\eta_n\frac{N}{1-N}\hat{n}_t - \hat{\lambda}_t - \hat{w}_t) \tag{2.3L}$$

where ω_t is the gross wage inflation and λ_t is the multiplier of households' budget constraint.

2.2.2 Entrepreneurs

The entrepreneurial sector largely follows the original BGG setup[26]. In each period, entrepreneurs combine hired labour and purchased capital to produce intermediate goods in a constant return to scale (CRS) technology. This aggregate production function is given by

$$Y_t = A_t K_t^{\alpha} N_t^{G1-\alpha}; \tag{2.4}$$

[26] Other similar expositions are Meier and Muller (2006), Gertler *et al.* (2007) and Christensen and Dib (2008).

where Y_t is produced intermediate goods, N_t^G is hired labour service, K_t is capital purchased in last period and α is capital share in production function. A_t is an exogenous technology measure capturing total factor productivity in goods sector. It follows

$$\ln A_t = (1-\rho_a)\ln A + \rho_a \ln A_{t-1} + \varepsilon_{at}; \tag{2.5}$$

with $\rho_a \in (0,1), \varepsilon_{at} \sim iid(0,\sigma_a^2)$. Consider an entrepreneur's decision making at the end of period t as an example. At that moment, the entrepreneur needs to purchase capital, K_{t+1}, that will be used in period $t+1$, at the price $P_t Q_t$ (Q_t is the relative price of capital goods in terms of the consumption goods). Thus the real cost of capital acquisition is $Q_t K_{t+1}$. The entrepreneur can only afford part of the expenditure, equalling to his net worth NW_{t+1}, and rely on external funds for the rest. This requires a model of explicit credit market and lender, which is the financial intermediary/bank described in details later. The capital demand of entrepreneurs is determined by the equality of expected marginal external financing cost with expected marginal return of holding capital.

$$E_t R_{t+1}^l = E_t R_{t+1}^k = E_t\left[\frac{X_{t+1}(\alpha Y_{t+1}/K_{t+1}) + Q_{t+1}(1-\delta)}{Q_t}\right]; \tag{2.6}$$

where δ is the depreciation rate of capital, X_t is the price of intermediate goods relative to final goods, and Q_t, as described above, is the relative price of capital which varies because of the adjustment cost. Thus the expected return on capital consists of two aspects: the income gain of $X_t \alpha Y_{t+1}/K_{t+1}$ and the capital gain of $Q_{t+1}(1-\delta)$. This return must be equal to the gross loan rate charged by financial intermediary/bank to ensure the optimal holding of capital by entrepreneurs.

Given the existence of credit market imperfections, the gross loan rate $E_t R_{t+1}^l$ will be equal to the multiplication of gross external finance premium EFP_{t+1} and gross deposit rate $E_t R_{t+1}^d$. The determination of EFP_{t+1} is shown in bank's optimal loan production/management in the next sub-section. As described previously, since the bank promises to pay households a non-state contingent nominal rate of R_t^n, the real rate depends on the ex post inflation rate. Thus we would also see a debt deflation effect, a la Fisher (1933), in the credit markets. The key equation to show financial frictions in this model can be written as

$$E_t R^l_{t+1} = E_t R^k_{t+1} = EFP_{t+1}\left(\frac{R^n_t}{E_t(\pi_{t+1})}\right); \tag{2.7}$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate. On the other aspect, entrepreneurial demand for labour service is determined by equalizing the real wage with marginal product of labour:

$$w_t = X_t(1-\alpha)\frac{Y_t}{N^G_t}; \tag{2.8}$$

Let's leave the detailed exposition of financial frictions to the bank's problem discussed below. To finish the entrepreneur's problem, it is necessary to analyse the transition of their net worth. The existence of credit market implies that entrepreneurs are not allowed to fully self finance. In other words, they cannot accumulate their net worth forever. We can achieve this by assuming the exit and entry of entrepreneurs out and into the entrepreneurial sector. The probability that an entrepreneur will survive until the next period is υ (i.e. there is a probability $1-\upsilon$ that he dies in between periods), so entrepreneurs only have finite expected horizon $1/(1-\upsilon)$ for operation. This assumption is vital, as it ensures that entrepreneurs never accumulate enough net wealth to finance new capital expenditure entirely and have to go to the credit market for external funds. The size of the entrepreneurial sector is constant, with new arrivals replacing departed entrepreneurs. The newly entered entrepreneurs receive some transferred seed money, S_t, for operation[27]. We can derive the evolution of entrepreneurs' net worth as follows:

$$NW_{t+1} = x_t\upsilon[R^k_t Q_{t-1}K_t - \frac{R^n_{t-1}}{E_{t-1}\pi_t}EFP_t(Q_{t-1}K_t - NW_t)] + (1-\upsilon)S_t; \tag{2.9}$$

where the first term in the square bracket represents the *ex post* return of holding capital in t and the second is the cost of borrowing, which is the real interest rate implied by the loan contract signed in $t-1$. As borrowers sign a debt contract that specifies a nominal interest rate, the loan repayment in real terms depends on the ex post real interest rate. Thus an increase (decrease) in inflation will reduce (increase) the real cost of debt repayment and

[27] Without this seed money, entering entrepreneurs would have no net worth, and so they would not be able to buy any capital. Also, among the entrepreneurs who survive there are some who are bankrupt and have no net worth. Without a transfer they would not be able to buy capital either.

push up (down) the entrepreneurial net worth. The stochastic nature of net worth evolution is introduced by a random disturbance term x_t, which follows the process

$$\ln x_t = \rho_x \ln x_{t-1} + \varepsilon_{xt}; \qquad (2.10)$$

where $\rho_x \in (0,1), \varepsilon_{xt} \sim iid(0, \sigma_x^2)$. This random term shifts entrepreneurial net worth up and down independently of movements in fundamentals. Christiano *et al.* (2010) interpret this shift factor as a reduced form way to capture what Alan Greenspan has called 'irrational exuberance', or simply asset price bubbles. NT follows Gilchrist and Leahy (2002) to treat this as a shock to the efficiency of contractual relations between borrowers and lenders so as to influence the degree of asymmetric information and costly state verification problem. I interpret this disturbance as a credit demand shock as it perturbs the financial condition of entrepreneurs and their demand for external finance. As shown below, this is justified by looking at the impulse response that x_t drives aggregate level of loan and EFP into the same direction, a distinguished characteristic of demand shock[28].

2.2.3 Banks

The function of external finance channel in the model economy is determined by financial intermediaries/banks. They issue deposits to collect funds from households and then convert those funds into lending as corporate loans to entrepreneurs. To simplify the analysis, I omit any regulation of reserve or the existence of inter-bank markets[29]. The latter justifies the existence of a representative bank in the model economy. The absence of reserve requirement and positive loan rate imply that the bank will lend out whatever is deposited: $L_t = D_t$. Based on the assumption in Goodfriend and McCallum (2007), the volume of loan supply (equivalent as the demand for deposit funds) is designed to be determined by a model of loan production, or more accurate, loan management, which is involved with collateral assessment and labour monitoring. This setup is motivated to capture the supply side of the credit market since in BGG the financial intermediaries exist passively to satisfy the demand of external

[28] Note this explanation is not contradicted with either Christiano et al. (2010) or NT. Specifically, a positive shock to entrepreneurs' net worth (asset bubble) can be thought of as a negative shock to credit demand since more investment can be financed internally; it can also be treated as a shock to the contractual efficiency that pushes down the EFP.

[29] This can be partly justified that the reserve requirement is mostly for demand deposit, not time deposit considered here. Moreover, the bank in the model is in broader sense to capture the economy-wide credit.

funds by entrepreneurs. In what follows, the loan management is assumed to be conducted by combining the collateral for evaluation and labour effort for monitoring. The specification is in Cobb-Douglas fashion as follows:

$$L_{t+1} = F_t (Q_t K_{t+1})^{\gamma} N_t^{F\,1-\gamma}; \qquad (2.11)$$

where L_{t+1} is the amount of loan lending in period $t+1$ determined at the end of period t. Q_t is the price of capital at the end of period t, thus $Q_t K_{t+1}$ is the value of collateral at the beginning of time $t+1$. N_t^F is the labour effort involved in loan monitoring, and γ denotes for the share of collateral in loan production. F_t is an exogenous technology measure capturing total factor productivity in banking sector (loan supply shock), following

$$\ln F_t = (1-\rho_f)\ln F + \rho_f \ln F_{t-1} + \varepsilon_{ft}; \qquad (2.12)$$

with $\rho_f \in (0,1)$, $\varepsilon_{ft} \sim iid(0, \sigma_f^2)$. It is noteworthy that Eq. (10) distinguishes itself to the original setting in Goodfriend and McCallum (2007) that only economy-wide capital is used as collateral for loan production. The reasons are twofold. First, government bond is not necessary here since the model refrains from the analysis of it; the omission is a simplification. Moreover, to resemble BGG's expression of financial friction (shown below), it is more appropriate to exclude bond from the loan production function.

On the other hand, as described in section 2.2.2, entrepreneurs obtain the loan to finance the purchase of next period capital in excess of their net wealth NW_{t+1}:

$$L_{t+1} = Q_t K_{t+1} - NW_{t+1}; \qquad (2.13)$$

Eq. (2.11) and Eq. (2.13) together characterize the equilibrium in credit markets.

The flow of funds of the typical FI at the end of period t is the new arriving deposit funds and gross interest payment on existing loans, less the labour cost for monitoring, cost of collateral service, the new issuing loans and the gross interest payment on existing deposits. The FI chooses the collateral service $Q_t K_{t+1}$, labour monitoring effort N_t^F and newly issued loan L_{t+1} and deposit D_{t+1} to maximize the expected life-time value in favour of the bank owners, households. The profit maximization problem of the bank is given by

$$\max_{\{Q_{t+h}K_{t+h+1},N^F_{t+h},L_{t+h+1},D_{t+1}\}} E_t\sum_{h=0}^{\infty}\beta^h\frac{U_{Ct+h}}{U_{Ct}}\{D_{t+h+1}+R^{ul}_{t+h}L_{t+h}-L_{t+h+1}-R^d_{t+h}D_{t+h}- w_{t+h-1}N^F_{t+h-1}-r^q_{t+h-1}Q_{t+h-1}K_{t+h}\};$$

subject to the bank balance sheet constraint $L_t = D_t$ and loan production function Eq. (2.11), where $\beta^h U_{Ct+h}/U_{Ct}$ is the household's stochastic discount factor. The F.O.Cs for this optimization problem are:

$$E_t\{R^{ul}_{t+1}-R^d_{t+1}\}=\frac{r^q_t}{\gamma L_{t+1}/Q_tK_{t+1}}; \tag{2.14}$$

$$E_t\{R^{ul}_{t+1}-R^d_{t+1}\}=\frac{w_t}{(1-\gamma)L_{t+1}/N^F_t}; \tag{2.15}$$

Eq. (2.14) and Eq. (2.15) apply the usual Baumal (1952) conditions, which equalize the marginal cost of intermediation to the factor prices of inputs divided by the marginal product of the inputs. As highlighted in Goodfriend and McCallum (2007), this marginal cost captures the idealized net uncollateralized external finance premium (UEFP) in the model, under the condition that entrepreneurs come to borrow without any collateral. Thus entrepreneurs have to pay full cost of intermediation: labour monitoring plus collateral service. In the other extreme, if entrepreneurs possess the full amount of collateral to borrow, they pay the full cost at the same time get back the return of collateral services. Therefore, the net EFP for entrepreneurs is only the labour monitoring cost, which is the fraction $1-\gamma$ of the total cost. We call this the fully collateralized external finance premium (CEFP) in the model economy, represented as:

$$CEFP_{t+1}-1=\frac{w_t}{L_{t+1}/N^F_t};$$

In reality, the actual amount of EFP lies between UEFP and CEFP, since entrepreneurs own fraction of the total collateral value in the whole economy, given by NW_{t+1}/Q_tK_{t+1}. The exact EFP is determined by this ratio:

$$EFP_{t+1}-1=(UEFP_{t+1}-1)[1-\gamma\frac{NW_{t+1}}{Q_tK_{t+1}}]; \tag{2.16}$$

Combine Eq. (2.16) and Eq. (2.15), after some rearrangements, to get

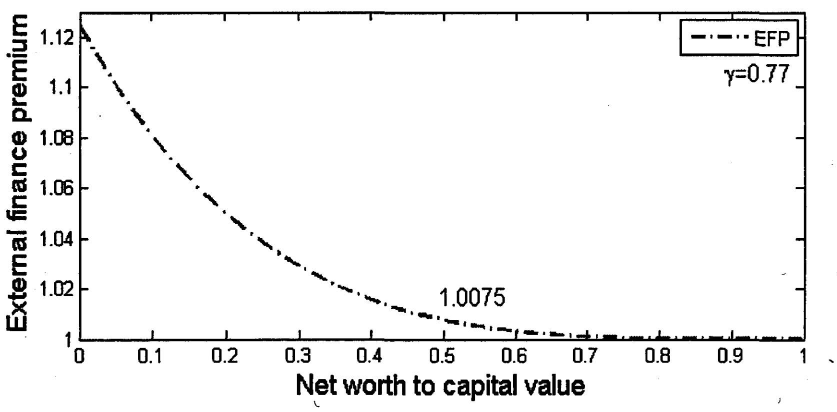
$$EFP_{t+1} - 1 = \frac{1}{1-\gamma} F_t^{-\frac{1}{1-\gamma}} w_t \left[(1 - \frac{NW_{t+1}}{Q_t K_{t+1}}) \right]^{\frac{\gamma}{1-\gamma}} [1 - \gamma \frac{NW_{t+1}}{Q_t K_{t+1}})] ; \qquad (2.17)$$

Eq. (2.17) highlights the key relationship between *EFP* and the ratio of internal funds to purchased capital value, $NW_{t+1}/Q_t K_{t+1}$, from the bank's optimization behaviour. Given that F_t and w_t are exogenous, and capital price is at steady state value of unity, we can derive the following proposition[30]:

Proposition 2.1: *In equilibrium, assume F_t and w_t are exogenously given, and capital price is in steady state value. External finance premium is a decreasing and convex function of the ratio of net worth to purchased capital value.*

This proposition implies a very important inference comparable to BGG: The external finance premium is higher the more entrepreneurs rely on external funds. Figure 2.1 plots the gross EFP against the ratio of net worth to the value of purchased capital with arbitrary calibration ($\gamma = 0.77$, $F = 2.69$, $w = 2.12$):

Fig. 2.1 External finance premium and ratio of net worth to capital value



[30] This proposition has already been shown in the last chapter. The only reason to put it here again is to make this chapter self-contained.

Figure 2.1 shows that the EFP decreases as less external funds is needed with given value of purchased capital (less leverage) in an diminishing rate. This implies that EFP in steady state will increase dramatically even after you reduce the internal funds relative to capital value by only a small amount. This shows the mechanism of the accelerator effect embedded in the banking sector of the baseline model. To see the dynamic relationship, I derive the log-linear form of Eq. (2.17) around the non-stochastic steady state:

$$\begin{aligned} \hat{efp}_t &= -\frac{EFP-1}{EFP}\left(\frac{\gamma\, NW/K}{1-\gamma\, NW/K}+\frac{\gamma}{1-\gamma}\frac{NW/K}{1-NW/K}\right)(\hat{nw}_{t+1}-\hat{k}_{t+1}-\hat{q}_t) \\ &\quad \frac{EFP-1}{EFP}(\hat{w}_t-\frac{1}{1-\gamma}\hat{f}_t) \end{aligned} \quad ; \quad (2.17L)$$

Eq. (2.17L) elaborates the behind the scene accelerator effect from the banking sector in the baseline model. The short run dynamics of EFP depends on the dynamics of the net worth to capital value ratio, real wage for labour monitoring and exogenous loan production technology. Thus Eq. (2.17L) is highly comparable with the counterpart reduced form equation in BGG framework of the form:

$$\hat{efp}_{t+1} = -\psi(\hat{nw}_{t+1}-\hat{k}_{t+1}-\hat{q}_t);$$

BGG claims that the elasticity of external finance premium with respect to the ratio of internal funds to total value of capital is derived from an optimal contracting problem between entrepreneurs and financial intermediaries. Higher net worth relative to value of purchased capital makes more funds of entrepreneurial sector sink into the project. Thus the incentives are more aligned between entrepreneurs and banks so as to reduce the asymmetric information problem and EFP. The baseline model with loan management also predicts a similar aggregate relationship as in BGG, despite the fact that the corresponding elasticity is shown differently by an expression nesting steady state value of EFP and internal funds to total value of capital ratio, and the parameter value of collateral share in loan production[31]. Besides this, the baseline model also highlights the importance of the real wage to influence

[31] The elasticity in BGG equals to ψ only after figuring out the optimal loan contract between entrepreneurs and financial intermediaries; it also depends on micro structure of the contract environment (e.g., average fraction of monitoring cost after the entrepreneurs default). See the appendix of BGG for details.

the dynamics of EFP before subjecting to the exogenous shock in the banking sector[32]. All these promising increments are not considered in BGG and many other studies of financial accelerator.

2.2.4 Capital producers

Capital producers are included to rationalize the fluctuations in the real capital price Q_t, since the volatile asset prices contribute to the fluctuations of entrepreneurial wealth. Consider there are perfectly competitive capital producers in the economy to control the supply of capital. They combine the purchased capital and investment funds to produce new capital, $\widetilde{K}_t$, according to

$$\widetilde{K}_t = \Phi\left(\frac{I_t}{K_t}\right)K_t;$$

with $\Phi(0)=0$, $\Phi'(.)>0$, $\Phi''(.)<0$. This increasing and concave function captures the presence of adjustment costs in the production of capital goods. Capital producers choose the investment expenditure in order to maximize their profit, $Q_t\widetilde{K}_t - I_t$, taking the relative price of capital as given. The first-order condition is

$$Q_t = \left[\Phi'(\frac{I_t}{K_t})\right]^{-1}; \tag{2.18}$$

Here I restrict the capital production function so that the relative price of capital is unity in steady state. Capital producers' decision is linked with the entrepreneurs' capital-purchasing decision via the variation in the price of capital.

The aggregate capital stock evolves according to

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t \tag{2.19}$$

[32] Real wage becomes relevant because it is the factor price in loan management and affects the marginal cost of intermediation activity.

Note that capital is homogeneous, so there is no difference between newly-produced and old capital. Old capital used by entrepreneurs is rented out for the production of new capital, and then returned at the same price as the newly-produced capital.

2.2.5 Retailers

The retail sector is applied to introduce nominal rigidity into this economy. Here I assume that entrepreneurs sell all of their output goods to retailers. Retailers purchase the homogenous wholesale goods from entrepreneurs, differentiate them using a linear technology at no resource cost and sell as final goods to households, capital producers and the government sector. In this way, the retailers have the monopolistic power to set the prices of these final goods. The reason why retailers are incorporated together with entrepreneurs is to avoid the complication of aggregating individual entrepreneur's demand for capital and his net worth when entrepreneurs themselves are imperfect competitors. Ultimately retailers' monopolistic profits belong to the households who own them, in contrast to entrepreneurs who are independent agents possessing their own wealth. Before exploring the retailers' problem in details, I firstly derive the aggregation of final goods. The final goods Y_t are bundles of differentiated goods $Y_t(j)$, $j \in [0,1]$, provided by the continuum of monopolistically competitive retailers[33]. The aggregation follows the framework of Dixit and Stiglitz (1977) as

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}};$$

where ε_p is the elasticity of substitution between different goods. The optimal allocation of expenditure across differentiated goods implies a downward sloping demand function for goods j:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t;$$

[33] Recall that the assumption of linear technology for retailers ensures that the amount of final goods varies one-for-one with the amount of wholesale goods in the economy.

where $P_t(j)$ denotes the price of good $Y_t(j)$, Y_t denotes the aggregate demand, and ε also measures the price elasticity of demand among differentiated goods. P_t denotes the price index of final goods given by

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon_p}\,dj\right]^{\frac{1}{1-\varepsilon_p}};$$

Following Calvo (1983), and a discrete version as in Yun (1996), I assume that each retailer cannot reoptimize its selling price unless it receives a random signal. The probability that each retailer can reoptimize his price in a given period is $1-\theta_p$, independently of other firms and of the time elapsed since the last adjustment. Thus the average length of time a price remains unchanged is $1/(1-\theta_p)$. Retailer j who has the opportunity to reset its price in a given period t choose the price, $P_t^*(j)$, that maximizes its expected discounted profits until the period when they are next able to change their price. On the other hand, the retailer who doesn't have the opportunity to reset price must charge the price that was in effect in the preceding period indexed by the steady state gross rate of inflation, π. Retailer j's optimazation problem is:

$$Max E_t \sum_{h=0}^{\infty}[(\beta\theta_p)^h \frac{U_{Ct+h}}{U_{Ct}} \frac{P_t^*(j)\pi^h - X_{t+h}P_{t+h}}{P_{t+h}} Y_{t+h}(j)];$$

subject to the demand function of $Y_t(j)$. Note that the stochastic discount factor for expected profits consists of the probability that retailers can change their price and the households' intertemporal marginal rate of substitution. The F.O.C for the optimal problem is

$$P_t^*(j) = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{E_t \sum_{h=0}^{\infty}\{(\beta\theta_p)^h U_{Ct+h+1}/U_{Ct+h} Y_{t+h}(j) X_{t+h}\}}{E_t \sum_{h=0}^{\infty}\{(\beta\theta_p)^h U_{Ct+h+1}/U_{Ct+h}\, Y_{t+h}(j)\pi^h/P_{t+h}\}}; \qquad (2.20)$$

The aggregate price index is given by

$$P_t^{1-\varepsilon_p} = \theta(\pi P_{t-1})^{1-\varepsilon_p} + (1-\theta)P_t^{*\,1-\varepsilon_p};$$

Log-linear approximations of the F.O.C and aggregate price index imply the following New Keynesian Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\hat{\chi}_t; \tag{2.20L}$$

where $\hat{\chi}_t$ is the log deviation of real marginal cost from steady state.

2.2.6 Government and monetary policy

Finally I set the budget constraint for the government and the policy rule of the monetary authority to close the whole model. The aggregate final output goods consist of households' consumption, capital producers' investment expenditure and the government expenditure, G_t. Every period, the market for final goods clears as

$$Y_t = C_t + I_t + G_t; \tag{2.21}$$

where government expenditure is financed by lump-sum taxes and money creation

$$G_t = \frac{M_t - M_{t-1}}{P_t} + \mathrm{T}_t;$$

For monetary policy, I assume the monetary authority exogenously sets the gross growth rate of money, μ_t, such that the supply of real money balance evolves according to[34]

$$m_t = \mu_t m_{t-1}\frac{P_{t-1}}{P_t}; \tag{2.22}$$

The money growth rate is assumed to follow a stochastic AR(1) process as

$$\ln \mu_t = \rho_m \ln \mu_{t-1} + \varepsilon_{mt}; \tag{2.23}$$

where $\rho_m \in (0,1)$, $\varepsilon_{mt} \sim iid(0, \sigma_m^2)$.

[34] The choice of money supply rule instead of interest rate rule is because of the large sample span from 1964 to 2009, following NT.

2.2.7 Equilibrium

In the baseline model economy, the equilibrium is defined as a set of endogenous variables $\{Y_t, C_t, I_t, N_t^G, N_t^F, K_t, NW_t, L_t, m_t, Q_t, w_t, R_t^k, R_t^n, \pi_t, EFP_t, X_t\}$ that satisfies households' decision rules (2.1) and (2.2), wage inflation curve (2.3), entrepreneurs' optimal conditions (2.6), (2.7) and (2.8), banks' decision rule (2.17), capital producers' optimal condition (2.18), New Keynesian Phillips curve derived from retailers' problem, (2.20), resource constraints (2.4), (2.9), (2.11), (2.13), (2.19), (2.21), and the money growth rule (2.22). Thus the log-linear version of the system around the non-stochastic steady state can be derived as[35]:

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + \hat{r}_t^n - \hat{\pi}_{t+1}; \tag{2.1L}$$

$$\eta_m \hat{m}_t = \frac{1}{1-R^n}\hat{r}_t^n - \hat{\lambda}_t; \tag{2.2L}$$

$$\hat{\omega}_t = \beta\hat{\omega}_{t+1} + \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\eta_n\varepsilon_w)}[\eta_n(\frac{N^G}{1-N}\hat{n}_t^G + \frac{N^F}{1-N}\hat{n}_t^F) - \hat{\lambda}_t - \hat{w}_t]; \tag{2.3L}$$

$$\hat{y}_t = \hat{a}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t^G; \tag{2.4L}$$

$$\hat{r}_{t+1}^k = \frac{\mathrm{X}\alpha Y/K}{R^k}(\hat{\chi}_{t+1} + \hat{y}_{t+1} - \hat{k}_{t+1}) + \frac{(1-\delta)}{R^k}\hat{q}_{t+1} - \hat{q}_t; \tag{2.6L}$$

$$\hat{r}_{t+1}^k = \hat{r}_t^n - \hat{\pi}_{t+1} + \hat{efp}_t; \tag{2.7L}$$

$$\hat{w}_t = \hat{y}_t + \hat{\chi}_t - \hat{n}_t^G; \tag{2.8L}$$

$$\frac{1}{\upsilon R^k}(\hat{nw}_{t+1} - \hat{x}_t) = \frac{K}{NW}\hat{r}_t^k - (\frac{K}{NW} - 1)(\hat{r}_t^n - \pi_t) - (\frac{K}{NW} - 1)\hat{efp}_t + \hat{nw}_t; \tag{2.9L}$$

$$\hat{l}_{t+1} = \hat{f}_t + \gamma\hat{k}_{t+1} + \gamma\hat{q}_t + (1-\gamma)\hat{n}_t^F; \tag{2.11L}$$

$$\hat{k}_{t+1} = -\hat{q}_t + \frac{NW}{K}\hat{nw}_{t+1} + (1 - \frac{NW}{K})\hat{l}_{t+1}; \tag{2.13L}$$

$$\begin{aligned}\hat{efp}_t = &-\frac{EFP-1}{EFP}\left(\frac{\gamma NW/K}{1-\gamma NW/K} + \frac{\gamma}{1-\gamma}\frac{NW/K}{1-NW/K}\right)(\hat{nw}_{t+1} - \hat{k}_{t+1} - \hat{q}_t) \\ &+ \frac{EFP-1}{EFP}(\hat{w}_t - \frac{1}{1-\gamma}\hat{f}_t)\end{aligned}; \tag{2.17L}$$

$$\hat{q}_t = \varphi(\hat{i}_t - \hat{k}_t); \tag{2.18L}$$

[35] For steady state solution, please refer to chapter one.

$$\hat{k}_{t+1} = \delta\hat{i}_t + (1-\delta)\hat{k}_t; \tag{2.19L}$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta}\hat{\chi}_t; \tag{2.20L}$$

$$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{i}_t; \tag{2.21L}$$

$$\hat{\mu}_t = \hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t; \tag{2.22L}$$

Given the log-linear version of the stochastic processes

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{at}; \tag{2.5L}$$

$$\hat{x}_t = \rho_x\hat{x}_{t-1} + \varepsilon_{xt}; \tag{2.10L}$$

$$\hat{f}_t = \rho_f\hat{f}_{t-1} + \varepsilon_{ft}; \tag{2.12L}$$

$$\hat{\mu}_t = \rho_m\hat{\mu}_{t-1} + \varepsilon_{mt}; \tag{2.23L}$$

Eq. (2.1L) to (2.13L) and (2.17L) to (2.23L) are the log-linear version corresponding to Eq. (2.1) to (2.13) and (2.17) to (2.23). Following the convention, all the variables with hat on top denote percentage deviations from non-stochastic steady state, where I omit the conditional expectations operator on the assumption of 'Certainty Equivalence'. Using Uhlig's undetermined coefficients procedure yields a state space solution of the form[36]:

$$\hat{s}_{t+1} = \Omega_1\hat{s}_t + \Omega_2\varepsilon_{t+1}; \tag{2.24}$$

$$\hat{d}_t = \Omega_3\hat{s}_t; \tag{2.25}$$

where the state variable vector, $\hat{s}_t$, includes predetermined and exogenous variables; $\hat{d}_t$ is the vector of control variables; and the vector ε_t contains the random innovations. The coefficient matrices, Ω_1 , Ω_2 , and Ω_3 , have elements that depend on the structural parameters of the model. Therefore, the state space solution, (2.24) and (2.25) is used later to construct underlying shocks and simulate the model.

[36] The detailed methodological exposition is given in Uhlig (1999), whereby the Matlab programme is available at his homepage (http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm).

parameter values are central to my shock extraction process, I try to keep them as clo possible to standard choice in the literature generally, and to NT specifically. There a total 25 parameters, including those characterizing the shock processes. The discount ra is set equal to 0.99 to match the average annual steady state real interest rate of 4% elasticity of substitution for consumption η^c, real balances η^m and leisure η^xa re a equal to conventional value 1.5, implying a nearly logarithmic utility function. persistence parameter ξ is 0.6. The weight on real balances ψ^m equals to 0.0019 to matc average M1 velocity of consumption. To reconcile the average working time of around the weight on leisure ψ^x is set to 2.47. The share of capital in goods production functi and the capital depreciation rate δ are fairly standard in real business cycle (RBC) liter to which we set value of 0.36 and 0.025. For entrepreneurs' surviving rate in the end of period υ, I will use the value of 0.978, implying entrepreneurial average life of 45 qua The next two parameters, φ and ψ, are key to BGG's financial accelerator mechanism, φ measures the level of capital adjustment cost and so the response of investment to sl and ψ directly captures the degree of financial accelerator effect. Despite the dispute these two parameter values, I follow NT to set them to be 1 and 0.037 respectively[37]. F share of collateral in loan production function γ, I refer to all relevant studies consid loan production (e.g., Goodfriend and McCallum (2007), Benk, Gillman and Kejak (2 (2008) and (2010), and Leao (2003)). Since their chosen values for γ lie between 0.65 (bound) and 0.89 (upper bound), any value between the two bounds are plausible. I pick value of 0.803 to make the elasticity of EFP with respect to the net worth to capital ra the banking model match that set in NT. The parameters associated with price and rigidity also follow that in NT, where the elasticity of demand for goods ε_p, and l

[37] Originally, BGG set φ equal to 0.25 and ψ equal to 0.05. These define a relatively low level of adjustment cost and high degree of financial accelerator effect. Christenson and Dib (2008) used max likelihood method to estimate the value of φ be 0.59 and ψ be 0.042 for US economy in the post Volck Meier and Muller (2006) found an even higher value of φ, 0.65, but insignificant ψ.

Table 2.1 Parameters in the model

Parameter	Description	Value
β	*Household's discount factor*	0.99
η^c, η^m, η^x	*Intertemporal elasticity of substitution*	1.5
ψ^m	*Weight on real balances in utility*	0.0019
ψ^x	*Weight on leisure in utility*	2.47
ξ	*Habit persistence*	0.6
α	*Share of capital in goods production*	0.36
δ	*Capital depreciation rate*	0.025
θ_p	*Retailers' probability of not able to reset price*	0.5
θ_w	*Households' probability of not able to reset wage*	0.75
ε_p	*Goods elasticity of demand*	11
ε_w	*Labour elasticity of demand*	4
υ	*Entrepreneurs' surviving rate*	0.978
φ	*Curvature of capital adjustment cost function*	1
ψ	*EFP elasticity of net worth to collateral value ratio*	0.037
γ	*Share of collateral in loan production*	0.803
ρ_a	*Autocorrelation of goods productivity shock*	0.95
ρ_m	*Autocorrelation of money growth rate*	0.65
ρ_x	*Autocorrelation of loan demand shock*	0.9
ρ_f	*Autocorrelation of loan supply shock*	0.9
$\varepsilon_a, \varepsilon_m, \varepsilon_x, \varepsilon_f$	*Standard deviations of the four shocks*	0.0075

ε_w, are 11 and 4 such that the steady state markups are 10% in the goods market and 33% in labour market, and the probability of not reoptimizing for price setters θ_p, is 0.5 while that for wage setters θ_w, is 0.75.

Now we have calibrated values for 17 out of the 25 parameters, the last 8 are the parameters capturing the process of the underlying 4 shocks. Since the shock processes are constructed in the next section, I give initial values for the 8 parameters in advance so that the model can be solved numerically. Some key steady state values in the model are also highlighted as follows.

Table 2.2 Steady state in the model economy

Variable	Definition	Value
I. Steady state values		
A	*Goods sector productivity level*	1
F	*loan productivity level*	2.15
R^d	*Risk-free rate*	1.01
R^k	*Gross return on capital*	1.0176
R^n	*Nominal interest rate*	1.0194
π	*Inflation rate*	1.0092
EFP	*External financing premium*	1.0075
N^G	*Labour service in goods sector*	0.3
N^F	*Labour service in banking sector*	0.005
II. Steady-state ratios		
Y/K	*Output to capital*	0.13
I/Y	*Investment to output*	0.19
C/Y	*Consumption to output*	0.61
G/Y	*Govt expenditure to output*	0.2
L/K	*Leverage ratio*	0.489
N^F/N	*Financial hour to total hour*	1.7%

The external finance premium is generally unobservable in reality, hence we can only refer to some close indicators to pin down the steady state value. It is set to 1.0075 for baseline,corresponding to an annual risk spread of 300 basis points, approximating the post war average spread between the corporate bond rate and the three-month treasury bill rate. This is consistent with the estimates in Queijo (2009) and lies within the range reported in De Fiore and Uhlig (2005)[38]. The steady state quarterly gross inflation is set to 1.0092, implying the nominal interest rate of 1.0194. Following NT, the steady state leverage ratio of

[38] In De Fiore and Uhlig (2005), the reported range for annual risk premiums on bonds and loans in U.S. is between 160 and 340 basis points.

entrepreneurs is set to 48.9%, which means the ratio of net worth to value of purchased capital is 0.511. The steady state consumption, investment and government expenditure share of GDP are given by 0.603, 0.192 and 0.205, respectively to match the historical average. In labour market, the steady state ratio of monitoring hour relative to goods produce hour is 1.7%. All the parameters and their calibrated values are described in table 2.1 while steady states are summarized in table 2.2.

2.4 Empirical Results

Based on the calibration discussed above, I carry on the evaluation of the empirical performance of the model. First of all, the four underlying shocks are constructed using the method proposed in Benk *et al.* (2005), (2008) and (2010) and NT. To check the robustness, the DSGE extracted TFP and monetary shock processes are compared to their counterparts derived from traditional estimation. Moreover, the two financial shocks are plotted against the post war recessions indicated by NBER on the one hand, and against the proxies of EFP on the other. The empirical performance of the model with financial shocks, both or either one, is evaluated by calculating second moments, historical decomposition and variance decomposition.

2.4.1 Construction of shocks

The assumed processes of the underlying four shocks are not appropriate for simulation until they are specified to be consistent with the baseline model. Two main reasons lie behind this. First of all, while goods sector productivity shock and monetary shock have non-controversy origins and can be easily backed up by conventional approach[39], there are no well agreed counterparts for financial shocks, especially when we are considering the shocks from both supply and demand sides. Assuming different financial structures in the model economy might imply different shock processes. For instance, in Benk et al. (2008) exchange credit model, the autocorrelation for credit shock is 0.93, and the standard deviation of innovation is

[39] Goods sector productivity shock can be estimated from constructed Solow Residuals. Monetary shock can be estimated by using data on money supply.

0.019. While in Atta-Mensah and Dib (2008) of credit creation model, the two corresponding parameters are 0.78 and 0.047 respectively. Christiano et al. (2010) report 0.53 and 0.025 for the financial wealth shock in their model. Based on these, I have to estimate the financial shock processes independently to capture the model consistent ones. Moreover, as argued in Ingram et al. (1994) and studies following up, any model that is in accord with the several time series that make up US macroeconomic data must feature multiple shocks that are correlated at all leads and lags. At least we cannot avoid the possibility of the correlations between the innovations driving the shock process. Therefore constructing consistent shocks nested in the model is not only desirable, but also necessary.

To construct the four underlying shocks, the procedure in NT is adopted[40], which is briefly described as follows. As assumed in section 2.2, the four shocks follow AR(1) processes. By giving initial values for the autocorrelation parameters, we can solve the model and recover the Markov decision rules numerically, which are written in state-space form as shown in Eq. (2.24) and (2.25). The model's endogenous control variables are stacked in vector $\hat{d}_t$, and the endogenous and exogenous state variables are contained in vector $\hat{s}_t$. The sequence of the variables in $\hat{s}_t$is ordered in such a way that the endogenous predetermined state variables appear first and the exogenous states follow up. Eq. (2.25) can now be written more explicitly as

$$\begin{aligned}\hat{d}_t &= \Omega_{31}\hat{s}_{1t} + \Omega_{32}\hat{s}_{2t} \\ &= \Omega_{31}[\hat{k}_t, n\hat{w}_t, \hat{q}_{t-1}, \hat{m}_{t-1}, \hat{w}_{t-1}, \hat{c}_{t-1}, \hat{r}^n_{t-1}]' + \Omega_{32}[\hat{a}_t, \hat{\mu}_t, \hat{x}_t, \hat{f}_t]'\end{aligned}; \tag{2.25'}$$

By solving the model, we recover the two coefficients matrices, Ω_{31} and Ω_{32}. In this case, we can estimate the processes of the four shocks if we assign values to $\hat{d}_t$ and $\hat{s}_{1t}$ from the data. This is straightforward from the ordinary least squares estimators for $[\hat{a}_t, \hat{\mu}_t, \hat{x}_t, \hat{f}_t]'$ via the following transformation:

$$[\hat{a}_t, \hat{\mu}_t, \hat{x}_t, \hat{f}_t]' = (\Omega_{32}'\Omega_{32})^{-1}\Omega_{32}'[\hat{d}_t - \Omega_{31}\hat{s}_{1t}]; \tag{2.26}$$

[40] Benk *et al.* (2005), (2008) and (2010) apply the same procedure extensively in a series of papers. A similar application can also be found in Chari *et al.* (2007) where they are trying to realize all the underlying wedges.

The identification of the four underlying shocks requires the data for at least four variables contained in $\hat{d}_t$. More than four variables simply give an over identification estimation for the shocks. The choice of the preferred combination of variables is discussed below.

Given the estimated series of the four shocks, what should be focused on next is to estimate each autocorrelation coefficient of the four processes. To account for the possible correlations between disturbances (heteroskedasticity), I apply the following seemingly unrelated regressions estimator (SURE)[41]:

$$\begin{bmatrix} \hat{a}_t \\ \hat{\mu}_t \\ \hat{x}_t \\ \hat{f}_t \end{bmatrix} = \begin{bmatrix} \rho_a & 0 & 0 & 0 \\ 0 & \rho_m & 0 & 0 \\ 0 & 0 & \rho_x & 0 \\ 0 & 0 & 0 & \rho_f \end{bmatrix} \begin{bmatrix} \hat{a}_{t-1} \\ \hat{\mu}_{t-1} \\ \hat{x}_{t-1} \\ \hat{f}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{at} \\ \varepsilon_{mt} \\ \varepsilon_{xt} \\ \varepsilon_{ft} \end{bmatrix}; \qquad (2.27)$$

After obtaining the estimates of the first order autocorrelation coefficients for $\hat{a}_t$, $\hat{\mu}_t$, $\hat{x}_t$, and $\hat{f}_t$, I substitute them back to the solution algorithm to get a new matrix Ω_{32}, then estimate the shock process again and proceed in an iterative fashion. Successive versions of Ω_{32} are calculated until ρ_a, ρ_m, ρ_x and ρ_f converge. Then the ultimate estimated autocorrelations and variance-covariance matrix (VCM) are used in the solution algorithm to simulate the model.

As noted earlier, it should be borne in mind that the choice of the preferred combination of variables contained in $\hat{d}_t$ is crucial to generate robust time series of the underlying shocks. Both Benk *et al.* (2005) and NT argue that different combinations of variables in $\hat{d}_t$ yield different shock processes so that it is not easy to identify how to pick up the correct bunch of variables. To solve this potential problem, NT proposed a rule of thumb criteria which states that the sensible combination should produce estimated processes for productivity shock and monetary shock that are highly correlated with their conventionally constructed counterparts from single equation estimation[42]. Based on this, they generate a time series for the shock to entrepreneurial net worth (loan demand shock) on the condition that the constructed TFP and

[41] The reason why the off-diagonal elements in the autocorrelation matrix are zero will be discussed below.
[42] Productivity shock is easily constructed via the detrended Solow residuals, given the data on per capita GDP, capital stock and labour. Monetary shock is more straightforward to recover by using the data on M1.

monetary shock have high correlation with their conventional counterparts: 0.76 and 0.94. I follow the same strategy here to estimate both the loan demand and supply shocks. During the estimation, I tried different combinations of variables in $\hat{d}_t$ and distinguish the most plausible one that gives TFP and monetary shock highly correlated with their counterparts[43]. In particular, I picked up six variables that are suitable from $\hat{d}_t$: $[\hat{y}_t, \hat{n}_t^G, \hat{m}_t, \hat{n}_t^F, \hat{w}_t, \hat{\omega}_t]'$. Note that all of them are logged and linearly detrended, refer to Appendix B for data description. To rationalize the choice, $\hat{y}_t$, $\hat{n}_t^G$ and $\hat{m}_t$ are chosen to make plausible TFP and monetary shock while $\hat{n}_t^F$ is used to capture the dynamics of credit supply shock, $\hat{f}_t$. $\hat{w}_t$ and $\hat{\omega}_t$ are also included in the estimation so that the credit demand shock is recovered close to that in NT[44].

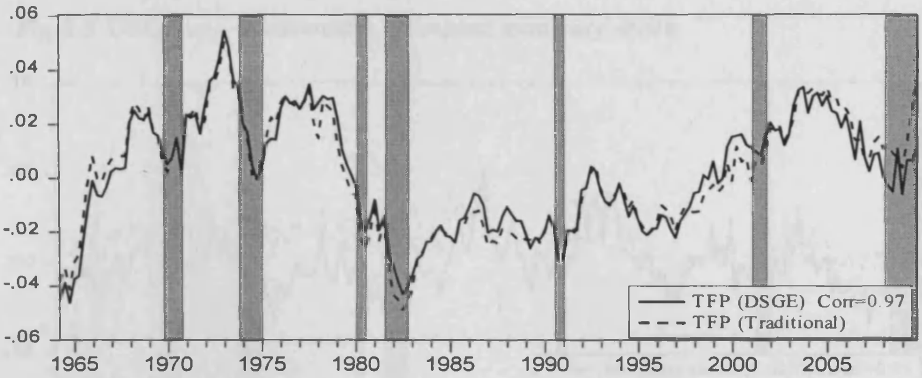
Figure 2.2 plots the DSGE constructed and traditionally estimated TFP processes covering the sample period between 1964Q2 and 2009Q4. It is clear to see that the DSGE derived shock (solid line) mimics the traditionally estimated shock (dashed line) very well, with a very high correlation coefficient of 0.97 between them. On the other hand, the comparison of monetary shock between the two derivations (shown in figure 2.3) is less satisfied with the corresponding correlation of 0.76. The main discrepancy stems from the first half of the sample period, where the DSGE derived shock always underpredicts that from traditional estimation. This feature also presents in NT's estimation of monetary shock, despite the fact that they have a considerably higher correlation of 0.94 between the two. Since there are no extant conventional counterparts of financial shock, it is currently impossible to assess the robustness of the estimation for financial shocks as we did for the previous two shocks. This also rationalizes the use of SURE as a plausible way to get autocorrelation coefficients and VCM discussed above. Nevertheless, the high correlation between the previous two and their corresponding counterparts justifies the validity of the two financial shocks from state-space derivation, plotted together in figure 2.4. The estimation of entrepreneurial net worth shock (left axis) is closely linked with that in NT[45], implying the shock construction process is not

[43] Only the most plausible shock processes are plotted here while those from other combination are available upon request.

[44] Actually, $\hat{m}_t$ and $\hat{w}_t$ are endogenous state variables and belong to $\hat{s}_{1t+1}$. Picking them up is justified since their values are also determined in the Markov decision rulus and can be treated equally as variables in $\hat{d}_t$.

[45] Refer to figure 3 in Nolan and Thoenisson (2009).

Fig. 2.2 DSGE and traditionally estimated total factor productivity



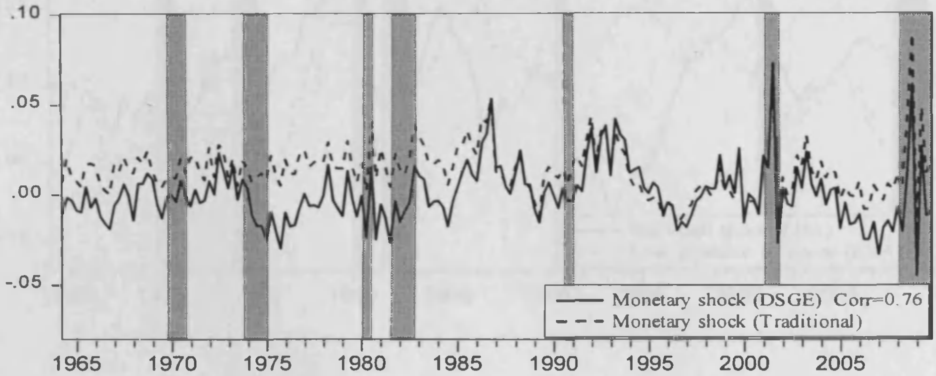
sensitive to the number of shocks. This shows a one step further validation of the loan productivity shock (right axis) generated here.

The time series properties of the four shocks can be summarized as follows. First of all, the first order autocorrelation coefficients for the four shocks are $\rho_a = 0.9433$, $\rho_m = 0.4189$, $\rho_x = 0.9796$, and $\rho_f = 0.8216$. The two financial shocks (demand and supply sides) are both more persistent than the growth rate of M1, but straddle the TFP shock. The net worth shock is more persistent than TFP while the loan productivity shock is less persistent. This ordering is consistent with that described in NT for the three shocks (excluding loan productvity) on the one hand and here on the other for the two shocks (TFP and money) derived from conventional estimation[46]. Turning to the VCM of the disturbances, both of the two VCM from DSGE construction and from traditional estimation are shown below:

$$VCM^{DSGE} = 10^{-4} \times \begin{bmatrix} 0.4123 & 0.1823 & 0.1718 & -0.9288 \\ 0.1823 & 2.4582 & -1.0387 & 1.8035 \\ 0.1718 & -1.0387 & 1.0586 & -0.0648 \\ -0.9288 & 1.8035 & -0.0648 & 23.1235 \end{bmatrix}; \quad VCM^{Tra} = 10^{-4} \times \begin{bmatrix} 0.3965 & 0.1145 \\ 0.1145 & 2.3861 \end{bmatrix};$$

[46] The conventional derived shocks show that TFP is more persistent than the growth rate of M1, with $\rho_a = 0.9556$, $\rho_m = 0.6097$.

Fig. 2.3 DSGE and traditionally estimated monetary shock

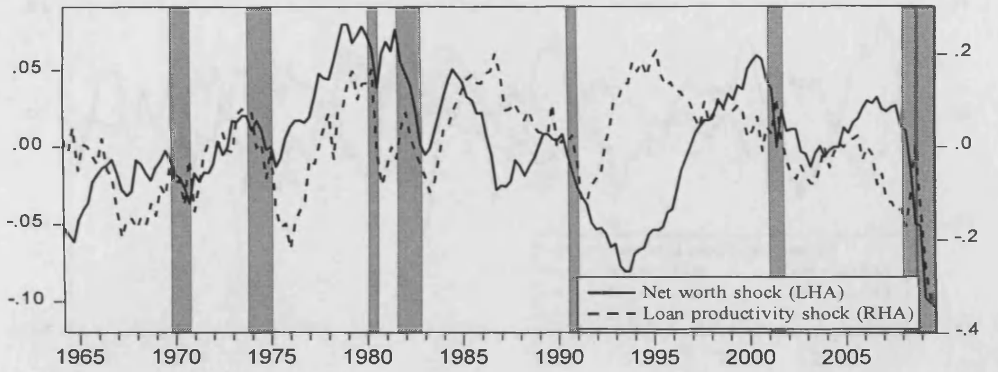


The comparison between DSGE constructed and traditionally estimated VCM shows that both of the DSGE TFP and money growth are slightly more volatile than their traditional counterparts. Moreover, the two are positively correlated in the two cases while the correlation between the DSGE derived ones is a little higher. The positive correlation between TFP and money growth is indicative of an historical accommodation of supply-side shocks by the Fed. Now focus on the VCM of DSGE constructed disturbances per se. The loan productivity (supply shock) is negatively correlated with TFP, but positively correlated with money growth, implying the loan supply side is more accommodative to monetary condition[47]. On the other hand, the net worth shock is negatively correlated with money growth (consistent with NT), but positively correlated with TFP (by contrast with NT). It seems more favourable to positive correlation between TFP and asset bubble since asset prices always burst during recessions. The negative correlation to money growth implies that the Fed goes against asset price bubbles[48]. It is noteworthy that the correlation between loan productivity and net worth shock is slightly negative (-0.0648×10^{-4}), for which it shows the identification of supply side from demand side shock in credit markets. The volatilities of the

[47] Benk et al. (2008) constructed an exchange credit shock process that also possesses positive correlation to money growth and negative correlation to TFP as the loan productivity extracted here. This shows a way of consistency despite that their shock is to consumption credit and my shock is for investment.

[48] There is a literature focusing on whether the central bank should respond to asset price when conducting monetary policy. See Bernanke and Gertler (1999).

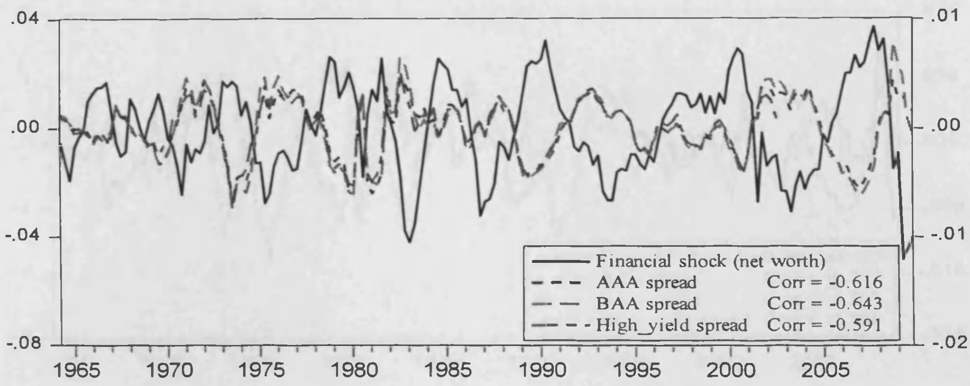
Fig. 2.4 DSGE estimated net worth and loan productivity shocks



four shocks are ordered consistently to relevant studies in the literature. The net worth shock is more volatile than TFP but less volatile than money growth; in line with NT. The loan productivity is the most volatile one; also appears in Benk *et al.* (2005). This can be possibly understood by the fact that shock in a specific sector is much more volatile than TFP which is an aggregate shock that results in the smoothing of all the idiosyncratic shocks from different sectors.

It would be interesting to relate the DSGE extracted shocks with the post war NBER business cycle reference dates, which track recessions starting at the peak of a business cycle and ending at the trough. Figures 2.2 to 2.4 also highlight the 7 main recession episodes in the sample between 1964Q2 and 2009Q4 (including the most recent recession triggered by the sub-prime mortgage crisis in 2007): 1969Q4-1970Q4, 1973Q4-1975Q1, 1980Q1-1980Q3, 1981Q3-1982Q4, 1990Q3-1991Q1, 2001Q1-2001Q4 and 2008Q4-2009Q4 (end of sample). Figure 2.4 shows an apparent picture that every recession happened when both of the entrepreneurial net worth and loan production are in contraction. This is a fairly striking result, implying only one of the two financial shocks, either demand or supply side, is not strong enough to cause an economy-wide recession. For instance, the non recession era such as (1964-1965), (1985-1987) and (1992-1994) witness a contraction of entrepreneurial net worth while the loan productivity is in expansion. The boom in the supply of credit offsets the contraction in entrepreneurial sector and avoids economy-wide recessions. The reverse is

Fig. 2.5 DSGE extracted financial shock(net worth) and EFP proxies

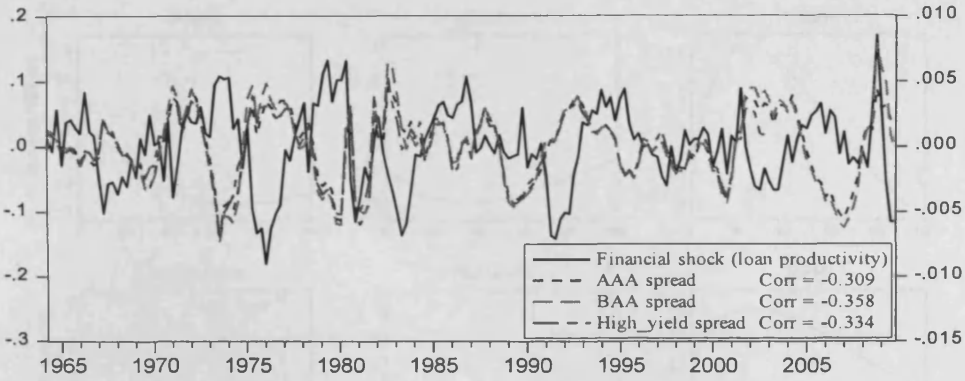


also true as shown in the recession episodes of 1973Q4-1975Q1 and 1981Q3-1982Q4 that the economic down-turn ceases earlier because the entrepreneurial sector recovers sooner than the credit supply. It is noteworthy that the recent recession could have recovered earlier since the loan supply started to expand during 2008 before the breakdown of Lehman Brothers, which reverse the credit expansion into a deeper contraction, as shown in the red line of figure 2.4. This implication is also apparent in Hirakata *et al.* (2010)[49]. On the other hand, the ink between the peaks and troughs of the business cycle and the realized TFP and money growth shocks is less obvious than the financial shocks, of which is also present in NT.

To assess the validity of the DSGE constructed financial shock, NT also plot it against the proxy of external finance premium, the spread between AAA rated corporate bonds and the 3-month Treasury bill rate, and find a strong negative correlation between the two. Here the same assessment procedure is followed, where both the loan productivity and net worth shock are plotted against the proxies of EFP. The spread of AAA rated corporate bonds as well as that of BAA and high yield bonds are used as the proxies[50]. Figure 2.5 and 2.6 plot the HP-filtered net worth and loan productivity shock respectively against the three HP-filtered proxies.

[49] Refer to the figure 4 of their constructed shock processes with Bayesian approach.
[50] Gertler and Lown (1999) argue that the high-yield bond spread emerges as a particularly useful indicator of the external finance premium and financial conditions more generally in the last two or three decades.

Fig. 2.6 DSGE extracted financial shock(loan productivity) and EFP proxies

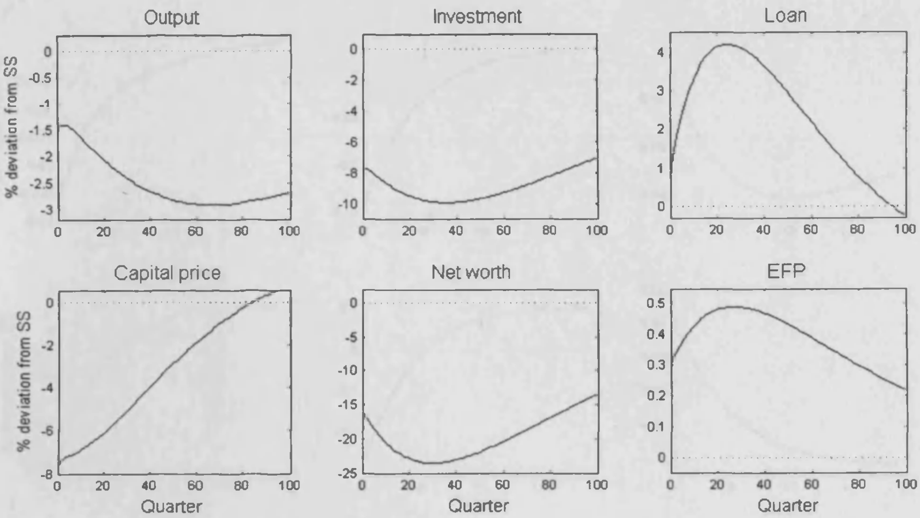


The two figures indicate that both of the two shocks are significantly negatively correlated with the three proxies of EFP. With stronger correlation, it seems that larger fraction of the cyclical EFP is accommodating the demand side of credit market. Nevertheless, the supply side effect shouldn't be ignored completely. It is also found that the loan productivity leads those spreads for two or three quarters since the correlation between contemporaneous spreads and lagged loan productivity is higher in absolute value (not shown here), though the increment is fairly small (to about -0.37). This feature is not appearing for net worth shock. The corresponding correlation between TFP and spreads, or money growth and spreads is fairly weak: (-0.03 to -0.13) and (0.14 to 0.21).

2.4.2 Impulse responses to financial shocks

This section briefly examines the impulse responses of the model economy to the two financial shocks. Figure 2.7 and 2.8 plot the responses of six variables that are attractive: output, investment, loan, capital price (Tobin's q), net worth and external finance premium. It is apparent that the two negative shocks both drive down the economy as expected. The effect of the net worth shock is very strong and persistent; the economy goes to downturn for a very long period, as shown in NT. The responses to loan supply contraction is relatively weak, but still significant. The effect is also less persistent as the economy reverts back to steady state quickly. One interesting thing to notice is that the responses of loan volume are in opposite

Fig. 2.7 Impulse responses to financial shock (net worth)



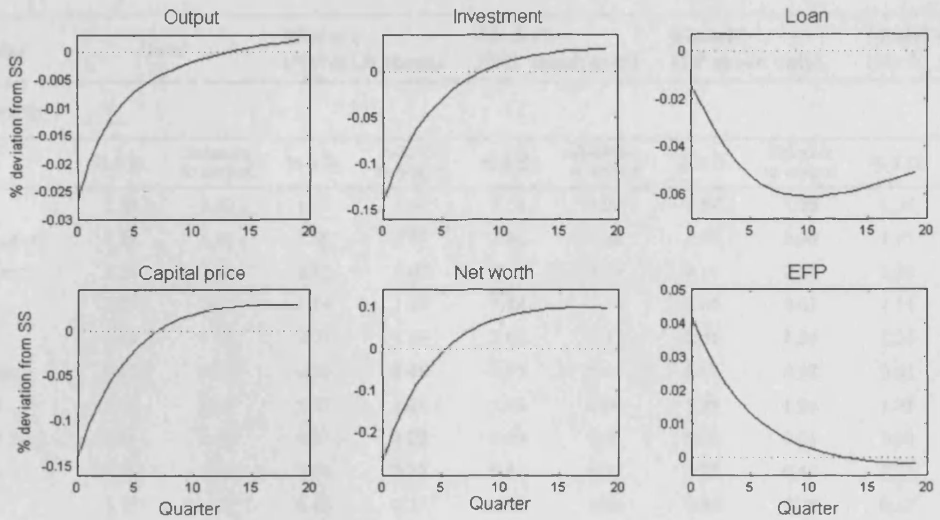
direction subject to the two shocks which are in the same direction. This rationalizes the earlier claim that the entrepreneurial net worth shock behaves more like a credit demand shock, while the loan productivity resembles a supply shock. Subject to a negative shock to net worth, the wealth of entrepreneurs contracts hugely because of the persistent effect; drops much deeper than that of capital demand. This makes the demand for external funds larger than before, pushing up the EFP. Thus a negative shock to net worth behaves like a positive shock to loan demand, driving up loan (quantity) and EFP (price) simultaneously. On the other hand, a negative shock to loan productivity resembles a credit supply contraction, accompanied with declining loan volume (quantity) and increasing EFP (price).

2.4.3 Second moments

Comparing the second moments of the model simulated series with the moments of the empirical series from the data is the traditional way to evaluate the performance of the business cycle models. Here I follow this strategy to assess how the business cycle performance of the model is altered after we add in the two financial shocks individually on the one hand and together on the other[51]. Table 2.3 summarizes the second moments of the

[51] The statistics for the model without financial frictions are not compared since Nolan and Thoenissen (2008) have done that extensively.

Fig. 2.8 Impulse responses to financial shock (loan productivity)



key variables from data (1964:1-2009:4) and compares them with the data generated by four models (estimates from 100 repeated stochastic simulation) that are identical except that: Model 1 has both financial shocks; Model 2 wipes out the supply sided financial shock; Model 3 gets rid of the net worth shock; Model 4 has no financial shock[52].

For the volatility part, as shown in panel A, the models with financial shocks, either one or both, come closer to data for output, investment, loan, hours, M1, inflation and EFP. Among the models with financial shocks, Model 3 comes closer to data for consumption, investment, real wage while Model 2 better matches for loan, hours and M1. For EFP, Model 3 performs as well as Model 2. The latter reconciles with the result highlighted in Nolan and Thoenissen (2008). There are two variables that all the models fail to predict the moments completely. One is the net worth for which all the models overpredict by 4 to 8 times; the other one is the nominal interest rate where all types predict less than 10% to the empirical counterpart.

For the correlation with output, as shown in panel B, all the models (Model 1 to Model 4) correctly predicts the sign of the correlation with GDP for consumption, investment, hours, M1, inflation, EFP and net worth. For nominal interest rate, Model 1 and 2 can predict the correct sign while Model 3 and 4 fail. None of the four models predict the sign for loan and

[52] The shocks in Model 2, Model 3 and Model 4 are constructed separately by the same methodology described above.

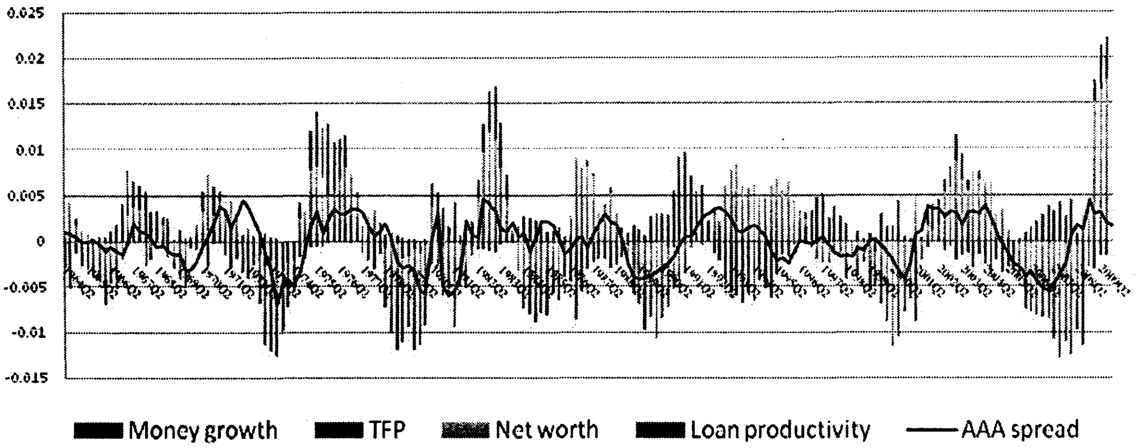
Table 2.3 Second Moments (Data 1964Q2 to 2009Q4)

Variables	Data		Model 1 (NW&LP shock)		Model 2 (NW shock only)		Model 3 (LP shock only)		Model 4 (No F_ shocks)	
A. Volatility										
	% S.D.	Relative to output	% S.D.	Relative to output	% S.D.	Relative to output	% S.D.	Relative to output	% S.D.	Relative to output
Output	1.58	1.00	1.60	1.00	1.61	1.00	1.57	1.00	1.54	1.00
Consumption	1.29	0.82	1.56	0.97	1.60	0.99	1.41	0.90	1.41	0.92
Investment	5.26	3.33	8.12	5.07	8.19	5.09	4.14	2.64	3.88	2.52
Loan	2.29	1.45	1.74	1.09	1.84	1.14	1.05	0.67	1.14	0.74
Hours	1.88	1.19	2.07	1.29	2.03	1.27	2.41	1.54	2.34	1.52
Real wage	0.97	0.61	0.76	0.48	0.75	0.47	0.91	0.58	0.91	0.59
Real M1	3.21	2.03	2.57	1.61	2.63	1.64	1.98	1.26	1.98	1.29
Nominal rate	0.41	0.26	0.03	0.02	0.03	0.02	0.02	0.01	0.02	0.01
Inflation	0.29	0.18	0.53	0.33	0.52	0.32	0.72	0.46	0.72	0.47
EFP	0.35	0.22	0.43	0.27	0.33	0.20	0.33	0.20	0.16	0.10
Net worth	2.25	1.42	17.04	10.65	17.23	10.70	8.59	5.47	8.10	5.26
B. Contemporaneous correlation with output (S.E.)										
Consumption	0.87		0.34 (0.14)		0.34 (0.12)		0.96 (0.01)		0.97 (0.01)	
Investment	0.90		0.82 (0.05)		0.81 (0.05)		0.95 (0.01)		0.96 (0.01)	
Loan	0.26		-0.24 (0.16)		-0.24 (0.13)		-0.17 (0.15)		-0.17 (0.14)	
Hours	0.86		0.86 (0.03)		0.86 (0.03)		0.86 (0.04)		0.85 (0.04)	
Real wage	0.14		-0.05 (0.15)		-0.03 (0.13)		-0.28 (0.15)		-0.25 (0.16)	
Real M1	0.14		0.19 (0.14)		0.19 (0.13)		0.95 (0.01)		0.96 (0.01)	
Nominal rate	0.38		0.19 (0.15)		0.34 (0.12)		-0.10 (0.09)		-0.12 (0.09)	
Inflation	0.15		0.40 (0.09)		0.37 (0.09)		0.69 (0.05)		0.67 (0.05)	
EFP	-0.65		-0.72 (0.07)		-0.77 (0.06)		-0.64 (0.08)		-0.90 (0.03)	
Net worth	0.58		0.81 (0.06)		0.80 (0.06)		0.94 (0.02)		0.94 (0.02)	

real wage correctly despite that both variables have nearly acyclical behaviour. On the quantitative perspective, Model 1 and 2 underpredict the correlation with output of consumption, investment while Model 3 and 4 overpredict. All the four versions of the model overpredict the correlation for M1, inflation and net worth. The only variable that all the four models capture perfectly simultaneously is hours. EFP, the most important variable in this study, is captured better by models with financial frictions and Model 3 performs best.

Comparing across models with and without financial shocks reveals mixed results in the performance assessment. The contribution of financial shocks is not straightforward to identify. This complements some recent findings that financial accelerator plays limited role

Fig. 2.9 Historical decomposition of EFP



in the model's transmission mechanism[53]. The main contribution of financial shocks in terms of matching the data's second moments over the sample period is the ability to match the second moments of the EFP. For both the volatility and correlation with output, Model 3 with loan productivity shock on top of TFP and monetary shock performs the best. Although Model 1 and 2 are not far away from Model 3, the importance of supply side financial shock to determine the dynamics of EFP is revealed clearly.

2.4.4 Historical decomposition of EFP

To further confirm the conjecture from previous section that the loan productivity shock plays an important role in determining the dynamics of EFP, this part decomposes the variation of cyclical EFP into the four underlying shocks. Figure 2.9 displays the time path of AAA spread (EFP proxy) and the contribution of each structural shock. The solid black line is the data of cyclical AAA spread from 1964Q2 to 2009Q4. Red and blue bars refer to the contribution of monetary and TFP shock respectively while green and purple bars represent the contribution of the two financial shocks; net worth and loan productivity. The effect of TFP to the cyclical AAA spread is minor. The contribution of monetary shock seems larger, but still moderate. This can explain why the second moments (both the volatility and correlation with output) generated by model excluding financial shocks are far away from their empirical counterparts. The figure clearly shows that the seven notable economic

[53] See Meier and Muller (2006), Christensen and Dib (2008) who use strict econometric testing.

Table 2.4 Variance decomposition

Variables	Percentage owing to:			
	TFP	Money growth	Net worth	Loan productivity
Output	8.72%	34.78%	51.07%	5.42%
Consumption	19.84%	46.86%	20.95%	11.00%
Investment	1.96%	21.85%	71.16%	5.02%
Loan	18.86%	17.97%	56.89%	6.33%
Hours	13.69%	32.99%	48.40%	4.93%
Nominal rate	12.63%	32.98%	52.07%	2.31%
Inflation	27.01%	34.14%	32.90%	5.94%
EFP	3.63%	16.68%	53.92%	25.76%
Net worth	2.16%	21.85%	71.16%	5.02%

recessions within the sample period, 1969Q4-1970Q4, 1973Q4-1975Q1, 1980Q1-1980Q3, 1981Q3-1982Q4, 1990Q3-1991Q1, 2001Q1-2001Q4 and 2008Q4-2009Q4, corres-pond well with the episodes when the AAA spreads are around cyclical peaks. Meanwhile, the cyclical peaks are contributed mostly by the two financial shocks. Each recession episode is companied by the situation when both of the two financial shocks predict a high EFP. The recent financial crisis is a notable case such that the EFPs driven by net worth as well as loan productivity are both unprecedentedly high. On the other hand, if the two shocks predict different sign for EFP, the AAA spread is only moderate and the economy is not in recession (e.g., mid 1980s and mid 1990s). After combining figure 2.4 and 2.9, it is fair to claim that both of the two shocks from financial sector are significant and important to determine the cyclical behaviour of EFP.

2.4.5 Variance decompositions

In this section, I measure the contribution of each of the four shock processes to EFP as well as other key macroeconomic time series. Here I follow the procedure applied in Hirakata et al. (2010). Table 2.4 reports the variance decompositions for output, consumption, investment, loan, hours, federal funds rate, inflation, EFP and net worth[54]. It is apparent that the net worth shock accounts for the largest fraction of variation for output, investment, loan, hours, federal

[54] In calculating the variance decompositions, I first calculate the volatility of the endogenous variable conditional on each of the shocks, and then sum these volatilities to calculate the share of each shock.

funds rate, EFP and net worth itself. These results are in line with that in NT, predicting the net worth shock is the dominant force for the variation of above variables. Not surprisingly to see the fluctuations of inflation is mainly determined by the monetary shock, which is the genuine natural cause of inflation. For EFP, we can still find the significant influence from the loan productivity shock, despite the main dominant force is still net wealth shock. About one third of the total effect on EFP from financial sector comes from the supply side of the credit market.

2.5 Conclusion

In this chapter, I make one step further based on the previous chapter to quantitatively assess the role played by the shocks originated from the banking sector in the U.S. business cycle. Specifically, I build a model that generally follows the setup of BGG and NT except for the banking sector, where I replace the optimal contracting problem of BGG with an explicit profit maximization problem in the banking sector subject to a loan production function. In consequence, both the demand side (entrepreneurial sector) and supply side (banking sector) of the credit market contribute to the financial frictions. In the model context, four exogenous shocks, including two conventional structural shocks (TFP and monetary) and the two financial sector shocks (entrepreneurial net wealth and loan productivity), are constructed by combining the data and the model's linear state-space solution. The shock to the technology in loan production resembles the disturbance originating in the banking sector. Together with the shock to entrepreneurial net wealth, we can analyse the contribution of the disturbance from both the supply and demand side of the financial market.

The main results can be summarized as follows. First of all, the DSGE extracted TFP and monetary shock are observationally similar to their counterparts constructed with traditional estimation while entrepreneurial net worth shock is close to the one constructed in NT. Secondly, every post war recession is crashed with the situation that both the entrepreneurial net worth and the loan productivity shock are in contraction; either one of them is not strong enough to cause an economy-wide recession. Thirdly, both of the extracted loan productivity and net worth shock are negatively correlated with proxies of EFP. Fourth, the variance decomposition indicates that loan productivity shock is an important source of EFP variation, though the dominant driving force is still net worth shock. Finally, net worth shock is still a dominant factor even after we include the loan productivity shock.

This study rationalizes one interesting point raised in NT that the entrepreneurial net worth shock remains contractionary after recessions have ended. This is because the shock from the other side of the credit market, the loan productivity, starts the expansion. Another notable point from the analysis is subject to the recent economic recession. The terrible economic downturn could stop earlier since the loan supply is found to expand during 2008. However, the breakdown of Lehman Brothers reverses the credit expansion into a deeper contraction. This point is still searching for the empirical support.

For future research, it is always attractive to incorporate the banks' balance sheet condition into the analysis. The importance of bank capital, either for the transmission mechanism or as an independent source of aggregate fluctuations, is still under exploration.

Chapter 3

Banking Capital in Loan Management and Bank's Balance Sheet Channel of Transmission Mechanism

3.1 Introduction

In the empirical literature, vast evidences suggest that the credit market imperfections play an important role in the transmission mechanism of the economy to both nominal and real structure shocks. These evidences have motivated extensive attempts to model the financial frictions and relevant transmission mechanism in theoretical frameworks, especially in state-of-the-art dynamic stochastic general equilibrium (DSGE) environments. Up to date, models including credit market imperfections can be categorised as three distinct types: (i) the corporate balance sheet channel, focusing on the demand side of the credit market (i.e., borrowers' financial position and its effect on the external finance premium (EFP) that borrowers face); (ii) the bank lending channel, arising from the supply side of the credit market (e.g., banks finance loans in part with liabilities that carry reserve requirements); and more recently, (iii) the bank capital channel, also emphasizing the supply side such that monetary policy affects bank lending through its impact on bank capital (e.g., banks' balance sheet)[55].

The earlier model devices related to bank lending channel and the borrowers' balance sheet channel were dictated by the empirical findings of the relevance of these two channels in monetary transmission mechanism (e.g., Bernanke and Blinder (1988), Kashyap and Stein (1994) among others, for bank lending channel; Hubbard (1995), Gertler and Gilchrist (1994) among others, for corporate balance sheet channel[56]). In DSGE setting, the literature has so far focused mainly on the demand side of the credit market, modelling the financial accelerator working via corporates' balance sheets. As reviewed in previous chapters, key notable contributions include Bernanke *et al.* (1999), BGG hereafter, Kiyotaki and Moore (1997), and Carlstrom and Fuerst (1997) for closed economy and Gertler *et al.* (2007) in open economy context. These frameworks mainly emphasize the importance of entrepreneurial stake in the corresponding economic activity because of the asymmetric information and cost of state verification (CSV), but are silent about banks' balance sheet in determining the cost of external finance and the impact on the propagation of the business cycle.

[55] As both the bank lending channel and the bank capital channel build on the hypothesis that monetary policy works, in part, by affecting banks' supply of loans, they are sometimes treated as part of a broader bank lending channel, notwithstanding being based on different transmission mechanisms.

[56] Also see Bernanke and Gertler (1995) for a review.

The reason why the exposition of the effect from banks' balance sheet status in shaping credit market frictions and aggregate fluctuations is not in research agenda until very recently comes from the lack of early evidences in favour of that mechanism. Empirical studies during the 1990s mostly failed to find support for the bank balance sheet channel. In part this was because the methodology was unsuitable insofar as it focused on aggregate data, which can be misleading[57].

More recently, a new empirical approach based on microdata studies on individual loan agreements in the US (Hubbard *et al.* (2002)) finds that bank capital is important for banks' decisions on the loan interest rate in financial distress. Series of papers in Angeloni *et al.* (2003), also find empirical relevance of banks' balance sheets in the monetary transmission mechanism in most euro-area countries. Besides these, other relevant empirical studies try to distil whether bank capital affects banks' supply of loans and real activity. Kishan and Opiela (2000, 2006), Van den Heuvel (2002), Gambacorta and Mistrulli (2004), for instance, show that the real effects of monetary policy are generally stronger when banks are small and low-capitalized. Hubbard *et al.* (2002), in turn, find that, even after controlling for information costs and borrower risk, the capital position of individual banks affects the interest rate at which their clients borrow. Moreover, recent financial turmoil unquestionably has underlined the importance of analyzing the link between banks' balance sheets and economic activity.

Based on these, since the middle of 1990s till now, quite a few papers have been searching for the rationale for the bank capital channel of transmission mechanism. The first thing to sort out is the motivation of bank capital holdings. The majority of the theoretical bank capital channel literature focuses exclusively on bank capital requirements imposed by banking regulation. Blum and Hellwig (1995), who have pioneered this approach, argue that a rigid link between bank capital and bank lending imposed by regulation may amplify the macroeconomic fluctuations. Given that banks cannot issue new capital and that firms do not fully replace bank loans by other sources of finance, if the aggregate banking sector confronts negative return shock, all the banks become undercapitalized and have to decrease lending subject to the regulatory capital adequacy requirement. An alternative explanation for the

[57] Aggregate number for credit can be misleading since funds do not flow freely from banks with excess capital to banks with capital shortages. Moreover, using aggregated data does not adequately control for loan demand, thus failing to isolate the loan supply effects.

decrease in bank loan supply during bad times, under regulatory capital requirements, rests on banks' exposure to the interest rate risk. Given that some long-term bank assets involve fixed interest rates whereas the returns of many short-term bank liabilities are closely linked to market interest rates, an increase in the interest rate after a contractionary monetary policy and, consequently, an increase in the bank's cost of funding leads to a decrease in the bank's profits, under the maturity mismatch on the bank's balance sheet, weakening the bank's future capital position and thus increasing the likelihood that its lending will be constrained by an inadequate level of capital. See this kind of analysis in Van den Heuvel (2002) who concludes that the lending of constrained banks overreacts to the monetary policy shock, compared to unconstrained banks.

The assumption of no equity issuance is the key to let regulatory bank capital requirements take effect in above studies. Some other expositions also consider regulatory capital requirements, but assume that banks may issue equity. However, equity issuance involves costs, as in Bolton and Freixas (2006) for instance, who introduce a cost of outside capital for banks by assuming information dilution costs in the issuance of bank equity: outside equity investors, having less information about the profitability of bank loans, tend to misprice the equity issues of the most profitable banks. In such a context, binding capital requirements may magnify the effects of a contractionary monetary policy, because this policy may cause a decrease in bank capital, as bank loans become insufficiently lucrative when information dilution costs in bank equity issuance are taken into account.

On the other hand, some studies also consider the market force of bank capital requirements instead of regulation. As defined by Berger *et al.* (1995), market capital requirements are associated with capital ratio that maximizes the value of the bank in the absence of regulatory capital requirements. Holmstrom and Tirole (1997) treat the role of bank capital in a different way, featuring two sources of moral hazard problem both between entrepreneurs and banks as well as between banks and depositors (households): entrepreneurs can choose different projects and have an incentive to undertake the riskier projects in order to enjoy private benefits, thus be required to invest their own net worth in the project by banks so that they wouldn't go after those private benefits; meanwhile, banks may not dutifully monitor entrepreneurs because it is costly and publicly unobservable, in the case of which households also require banks put their own net worth (bank capital) in the lending funds. As delegated monitors for depositors, banks must then be well capitalized to convince depositors that they have enough stake in the entrepreneurs' projects.

This double moral hazard framework is further developed by Chen (2001), Aikman and Paustian (2006) and Meh and Moran (2010) in dynamic general equilibrium environment to explore how bank capital transmits structural shock to the whole economy and, consequently, makes the response of the economy more amplified and persistent. One common feature within them can be summarized as follows. When negative shock hits the economy, both the banks and entrepreneurs' net wealth deteriorates at the same time market requires both of them finance a larger share of investment projects on their own. Because banks and entrepreneurs' net worth are largely predetermined, bank lending must decrease to satisfy the market requirements, thereby leading to a decrease in investment. This, in turn, affects negatively banks and entrepreneurs' earnings and also their net worth in the future, leading to the propagation of the initial shock over time.

Another interesting approach is proposed by Sunirand (2003), who also supports the bank capital amplification hypothesis. He applies the CSV framework of Townsend (1979) to two financial contracts, both between entrepreneurs and banks and between banks and depositors. Banks act as delegated monitors on entrepreneurs' investment projects and depositors perform the role of 'monitoring the monitor'. The two-sided CSV leads to a wedge between the internal and external cost of funds that motivates endogenous roles for both firm's and bank's net worth condition in the model. He concludes that embedding the information asymmetry between households and banks into the financial accelerator model strengthen the amplification and propagation mechanism.

From the brief review above (summarized in table 3.1), we can summarize that the bank capital channel thesis overall predicts that the introduction of bank capital requirements, either for regulatory or market reasons, tends to amplify the effects of monetary and other exogenous shocks. However, in most of those models in favour of market determined bank capital requirements, the main motivation is to deter the moral hazard behaviour from borrowers because of the information asymmetries between them and lenders.

Table 3.1 Summary of theoretical models with bank capital channel

Studies	Capital requirements	Effect of shocks on the economy
Chen (2001)	Market	Amplified and more persistent
Sunirand (2003)	Market	Amplified with the introduction of a double CSV approach
Aikman and Paustian (2006)	Market	Amplified and more persistent with the introduction of an asymmetric information problem between depositors and banks and between banks and firms
Meh and Moran (2007)	Market	More persistent
Blum and Hellwig (1995)	Regulatory	Amplified with the introduction of binding CR
Van den Heuvel (2002)	Regulatory	Amplified the standard interest rate channel of MP
Bolton and Freixas (2006)	Regulatory	Potential amplifying effect of MP: tightening in MP causes decreases of incentives to raise bank capital causes further decline of lending
Markovic (2006)	Regulatory	Amplification of output response to a contractionary MP
Aguiar and Drumond (2009)	Regulatory	Amplifies monetary shocks through a liquidity premium effect on the EFP faced by firms

This chapter considers the market force of bank capital requirements from a different perspective. This is motivated by the importance of factors within the banking sector to affect the banking capital channel, but lack of attention is paid on it. I specify a typical functional form of loan production in the banking sector and assume bank capital is necessary because it is one of the factors that help the banking sector produce or manage loans. In such setting, the bank capital or the bank's balance sheet condition can also play a role in determining the transmission mechanism of the economy confronting exogenous shocks. For instance, after a negative shock hitting the economy, the entrepreneurial sector's net wealth deteriorates quickly and more external financing is needed. Higher amount of loans must be accompanied with more bank capital according to the loan production function. Since issuing bank capital is more costly than issuing deposit, banks can only increase the loan rates to cover the higher cost of operation. Facing higher external finance premium, more investment and production are cut and the net worth of entrepreneurial sector deteriorates even further, propagating the decline of economic activities from the initial shock.

Absent from the explicit modelling of information asymmetry or moral hazard problem, the role of banking capital in loan production is sufficient to generate the relevant transmission mechanism. In this framework, not only the demand side (e.g., corporate balance sheet condition), but also the supply side of credit market condition (e.g., bank capital) is under the spotlight. They interact with each other and build up an even stronger transmission mechanism. It is noteworthy to bear in mind that in the previous two chapters financial frictions which stem from the cost of loan management conducted in financial intermediaries also shed light on the supply side of credit market, but are silent about bank capital. This chapter fills this gap and contribute a little bit along the literature.

The detailed exhibition of the financial frictions from the model is shown in the following sections while the brief summary is provided here. After the derivation, the financial friction, expressed in the external finance premium, is shown to depend on three things: the ratio of entrepreneurial net worth to total capital value (corporate balance sheet condition); the ratio of bank capital to lending funds (bank's balance sheet condition); and the difference between the return on bank capital and the return on deposit. The former one follows the claim from the previous chapters that holding of collateral help mitigate the entrepreneurs' payment for the cost associated with loan management. The latter two stem from the role of bank capital in loan production, and, consequently, in the transmission mechanism. The difference between bank capital return and deposit return is defined as liquidity premium. This is rationalized by the assumption that deposit can fulfil the liquidity needs from households, thus bank capital must offer a higher pecuniary return to attract the holding of it. Given that the bank's liability composition is constant, the higher is the liquidity premium, the larger is the EFP. Similarly, given that the liquidity premium is constant, EFP is larger when higher proportion of loans is produced or financed with bank capital.

Virtually similar strategy of modelling bank capital channel is in Markovic (2006) and Aguiar and Drumond (2009). Markovic (2006) develops a model that can account for three bank capital channels: (i) the adjustment cost channel, which builds on the allocation cost necessary to reduce the asymmetric information problem; (ii) the default risk channel, which arises from the possibility of banks defaulting on their capital; and (iii) the capital loss channel based on the assumption that, during a recession, banks' shareholders anticipate a future fall in the value of bank capital. All channels trigger an increase in the required return on bank capital by shareholders, and thus an increase in the cost of bank capital. Current model shares some common features with that in Markovic (2006). For instance, I also assume holding bank capital is subject to adjustment cost such that the adjustment cost channel and capital loss channel are embedded implicitly in current model. However, Markovic's model motivates the bank holding of capital from regulatory basis while the current model from market force. Moreover, the current model excludes the possibility of bank default and attributes to the liquidity premium for the return differential between bank capital and deposit, which is the common strategy I share with Aguiar and Drumond (2009). But they concentrate more on how Basel Accord, the capital regulation, can contribute to a procyclical banking sector.

The main findings of this chapter can be summarized as follows. An active bank capital channel in the model amplifies and propagates the monetary policy shocks (demand shock) while attenuates and dampens the technology shocks (supply shock) when households' deposit is contracted in nominal term. This result stems from the so called 'Fisher deflation effect' channel initiated by Fisher (1933). The Fisher and bank capital effect mechanisms reinforce each other in the case of shocks that move the price level and output in the same direction (demand shock), and they tend to cancel each other in the wake of shocks which move the price level and output in opposite directions (supply shock). On the other hand, it is shown that adverse shocks that cause persistent declines in bank capital value, can lead to sizeable declines in bank lending and economic recession.

The rest of this chapter is structured as follows. Section 3.2 derives the construction of bank capital channel in transmission mechanism. Section 3.3 completes the whole model with other agents and defines the general equilibrium. Section 3.4 calibrates the model to quarterly data of US economy. Section 3.5 shows the results and relevant discussions. Section 3.6 concludes with some final remarks.

3.2 Construction of bank capital channel

In this model environment, the function of external finance channel is determined by banks. They issue deposits and bank equity shares to collect funds from households and convert them to lending as the corporate loans to entrepreneurs. Bank capital is necessary because it is assumed to be one factor to produce entrepreneurial loans[58]. Different from the previous chapters, deposit fund instead of money is assumed to generate utility for households because of the liquidity it provides. To simplify the analysis, I omit any regulation of reserve or the existence of inter-bank markets. The latter justifies the existence of a representative bank in the model economy. The absence of reserve requirement and positive loan rate imply that the bank will lend out both the deposit and equity funds in order to maximize the market value. Thus the bank's assets and liabilities are summarized by the following table of the balance sheet:

Table 3.2 Balance sheet for banks

Assets	Liabilities
Loans: L_t	Deposits: D_t
	Equity: $Q_t^z Z_t$

and the balance sheet constraint is given by

$$L_t = D_t + Q_t^z Z_t; \tag{3.1}$$

Following the assumption made in Goodfriend and McCallum (2007), GM hereafter, the volume of loan supply is designed to be determined by a model of loan production, or more accurate, loan management. In the original setting from GM, loan management is involved with collateral assessment and labour monitoring, while here I add in one more factor, the bank equity. The bank equity can be treated as a securer against bank assets. I model it in the loan production function to follow the spirits of regulatory bank capital requirements (e.g., Basel Accord) indirectly. This setup is motivated to capture the supply side of the credit market in determining the financial frictions through the banks' balance sheet channel. In BGG loan equals to deposit; there is no bank capital involved. In their model banks' balance sheet is not relevant in determining the credit market imperfections. To resolve this limitation, bank capital is modelled to be necessary in loan management in current chapter. Specifically, the loan management is assumed to be conducted by combining the labour effort, collateral and bank equity. The functional form is in Cobb-Douglas fashion as follows:

[58] Bank capital and bank equity are used interchangeably.

$$L_{t+1} = F_t N_t^{F\gamma_1} (Q_t^k K_{t+1})^{\gamma_2} (Q_t^z Z_{t+1})^{1-\gamma_1-\gamma_2}; \tag{3.2}$$

where L_{t+1} is the amount of loan lending in period $t+1$ determined at the end of period t. Q_t^k and Q_t^z are the price of physical capital and bank capital respectively at the end of period t, thus $Q_t^k K_{t+1}$ is the value of collateral at the beginning of time $t+1$. N_t^F and $Q_t^z Z_{t+1}$ are the bank labour and equity involved in loan production. γ_1, γ_2 and $1-\gamma_1-\gamma_2$ denote for the shares of the three factors. F_t is an exogenous technology measure capturing total factor productivity in banking sector (loan supply shock), following

$$\ln F_t = (1-\rho_f)\ln F + \rho_f \ln F_{t-1} + \varepsilon_{ft}; \tag{3.3}$$

with $\rho_f \in (0,1)$, $\varepsilon_{ft} \sim iid(0, \sigma_f^2)$. Eq. (3.2) is the cornerstone to generate the financial friction in this framework. Since managing loans is costly, involving labour, collateral and bank capital, there is a spread between the price of loan and the price of bank's source of funds to cover the managerial cost. It is also noteworthy that by contrast to GM, Eq. (3.2) only uses economy-wide capital, not government bond, as collateral for loan production. The reasons are twofold. First, government bond is not necessary here since the model refrains from the analysis of it; the omission is a simplification. Moreover, to make the expression of financial friction partly observationally equivalent to that in BGG (shown below), it is appropriate to exclude bond from the loan production function.

The instantaneous profit of the bank at the end of period t can be expressed as the new arriving deposit and equity funds and gross interest payment on existing loans, less the labour cost for monitoring, cost of collateral service, the new issuing loans and the gross payment to existing deposits and equities. The bank chooses the collateral $Q_t K_{t+1}$, labour service N_t^F, newly issued loan L_{t+1}, deposit D_{t+1}, and equity $Q_t^z Z_{t+1}$ to maximize the expected life-time profit in favour of the bank owners, households. The profit maximization problem of the bank is given by

$$\begin{aligned} \underset{N^F_{t+h}, Q_{t+h}K_{t+h+1}, L_{t+1}, D_{t+1}, Q^z_t Z_{t+1}}{Max} E_t \sum_{h=0}^{\infty} \beta^h \frac{U_{Ct+h}}{U_{Ct}} \{ & D_{t+h+1} + Q^z_{t+h} Z_{t+h+1} + R^{ul}_{t+h} L_{t+h} - L_{t+h+1} - R^d_{t+h} D_{t+h} - \\ & R^z_t Q^z_{t+h-1} Z_{t+h} - w_{t+h-1} N^F_{t+h-1} - r^c_{t+h-1} Q^k_{t+k-1} K_{t+h} \} \end{aligned};$$

subject to the bank balance sheet constraint Eq. (3.1) and loan production function Eq. (3.2), where $\beta^h U_{Ct+h}/U_{Ct}$ is the household's stochastic discount factor. One notable thing here is that in the maximization problem R_t^{ul} denotes for the interest rate charged on uncollateralized loans because the price on loans should cover all the collateral costs in loan management. Since entrepreneurs own fraction of the economy-wide collateral, part of the return on collateral goes towards entrepreneurial sector, helping mitigate the cost of external funds. The details are shown below. The first order conditions (F.O.Cs) for this optimization problem are:

$$E_t\{R_{t+1}^{ul} - R_{t+1}^{d}\} = \frac{w_t}{\gamma_1 L_{t+1}/N_t^F}; \tag{3.4}$$

$$E_t\{R_{t+1}^{ul} - R_{t+1}^{d}\} = \frac{r_t^c}{\gamma_2 L_{t+1}/(Q_t^k K_{t+1})}; \tag{3.5}$$

$$E_t\{R_{t+1}^{ul} - R_{t+1}^{d}\} = \frac{E_t\{R_{t+1}^{z} - R_{t+1}^{d}\}}{(1-\gamma_1-\gamma_2)L_{t+1}/(Q_t^z Z_{t+1})}; \tag{3.6}$$

Eq. (3.4) and (3.5) apply the usual Baumal (1952) conditions, which equalize the marginal cost of intermediation to the factor prices of inputs divided by the marginal product of the inputs. Eq. (3.6) equalizes the ratio of bank equity spread to loan spread, the uncollateralized external finance premium (UEFP), to the marginal product of equity in loan management, implying the key relationship between EFP and the rest of the economy derived below. As shown in previous chapters, this marginal cost of intermediation is assumed to be equal to the net UEFP, not the practical external finance premium. To cover the marginal cost caused by labour, collateral and bank capital, the bank would charge each unit of loan the amount of the marginal cost on top of that it pays to households. Thus the total marginal cost of intermediation is identical to the UEFP. However, in reality, entrepreneurial sector owns part of the economy-wide collateral which can help mitigate the practical EFP charged on the loan funds. Thus the actual amount of EFP is smaller than UEFP because of the fraction of the total collateral value owned by entrepreneurs, given by the ratio $NW_{t+1}/Q_t^k K_{t+1}$. Thus the exact EFP is related to UEFP according to the following:

$$EFP_t - 1 = (UEFP_t - 1)[1 - \gamma_2 \frac{NW_{t+1}}{Q_t^k K_{t+1}}]; \tag{3.7}$$

$\gamma_2 NW_{t+1}/Q_t^k K_{t+1} * (UEFP_t - 1)$ measures the fraction of collateral return on loan management service distributed to the entrepreneurial sector. Combine Eq. (3.6) and (3.7) can achieve:

$$EFP_t - 1 = \frac{1}{1-\gamma_1-\gamma_2} E_t\{R_{t+1}^z - R_{t+1}^d\} \frac{Q_t^z Z_{t+1}}{L_{t+1}} [1 - \gamma_2 \frac{NW_{t+1}}{Q_t K_{t+1}}]; \qquad (3.8)$$

Eq. (3.8) highlights the determination of the financial friction stemming from the interaction of both corporate balance sheet and banks' balance sheet channel that the practical EFP depends on three distinct factors: the ratio of entrepreneurial net worth to total capital value; the ratio of bank capital to lending funds (assets); and the difference between the return on bank capital and the return on deposit. The former is related to corporate balance sheet effect while the latter two determine the relevance of banks' balance sheet. We can see clearly that EFP is positively related to entrepreneurial leverage ratio (one minus the ratio of net worth to capital value), bank equity to lending ratio and the liquidity premium (difference between bank capital return and deposit return). To see the near steady-state dynamics, take the log-linear form of Eq. (3.8):

$$\begin{aligned} \hat{efp}_t = {} & \frac{EFP-1}{EFP}\frac{R^z}{R^z - R^d}\hat{r}_{t+1}^z - \frac{EFP-1}{EFP}\frac{R^d}{R^z - R^d}\hat{r}_{t+1}^d + \frac{EFP-1}{EFP}(\hat{q}_t^z + \hat{z}_t - \hat{l}_t) \\ & - \frac{EFP-1}{EFP}\frac{\gamma_2 NW/K}{1-\gamma_2 NW/K}(\hat{nw}_t - \hat{q}_t^k - \hat{k}_t) \end{aligned} ; \qquad (3.8L)$$

It is noteworthy that by contrast to the previous chapters, the corporate balance sheet here doesn't seem to be relevant to determine the marginal cost of intermediation per se. This stems from the derivation that corporate balance sheet is substituted out by liquidity premium and bank capital ratio. The entrepreneurial net wealth is effective only because it influences the fraction that UEFP can be mitigated when the cost is passed through to entrepreneurs[59].

Finally, I should define a functional form for the price of bank capital as I assume there is adjustment costs of varying bank capital holdings. Markovic (2006) argues that this definition is necessary to model his capital loss channel. From empirical evidence, the price of bank

[59] Despite that the corporate balance sheet channel is relatively weak in current model, it wouldn't be problematic any more if we add it to the BGG financial accelerator mechanism, which is fairly strong. The combination of BGG corporate balance sheet channel and bank capital channel emphasized here is subject to future research.

shares moves broadly in line with the price of other non-financial firms' shares, which in this model should be reflected in the value of entrepreneurial net wealth. Thus I follow the functional form assumed in Markovic (2006) that in the log-linear terms, the expected price of bank capital depends on the weighted average of current price of bank capital and expected price of firm shares. This is given by

$$E_t\hat{q}^z_{t+1} = \rho_{Qz}\hat{q}^z_t + (1-\rho_{Qz})E_t n\hat{w}_{t+1}; \tag{3.9L}$$

3.3 General equilibrium

Besides the banking sector, the economy also comprises other five types of agents:
(i) Households, who set nominal wage, supply labour to wholesale goods sector and banking sector in monopolistic competitive labour markets, consume and allocate their savings to bank deposits and bank capital; (ii) Entrepreneurs, who need external finance to buy capital, which is used in combination with hired labour to produce wholesale output; (iii) Capital producers, who combine old capital and investment funds to produce new capital and sell to entrepreneurial sector; (iv) Retailers, added in order to incorporate inertia in price setting; (v) Government, who conducts both monetary and fiscal policy.

3.3.1 Households

The economy comprises a continuum of infinitely lived monopolistically competitive risk-averse households, indexed by $j \in [0,1]$, of length unity. To motivate the nominal wage inertia, each household supplies differentiated labour service to entrepreneurial and banking sector, which regard each of their labour service as an imperfect substitute for that of others. In this setup, entrepreneurs and banks demand bundles of labour services, which is obtained using the aggregation scheme as in Dixit and Stiglitz (1977)

$$N_t = \left[\int_0^1 N_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \quad \varepsilon_w > 1;$$

The optimal substitution across labour service leads to the following labour demand equation regarding the j th labour service

$$N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t;$$

where $W_t(j)$ is the nominal wage set by the j th household, W_t is the Dixit-Stiglitz aggregate nominal wage given by $W_t = [\int W_t(j)^{1-\varepsilon_w} dj]^{1/(1-\varepsilon_w)}$, and ε_w gives the constant elasticity of substitution across labour service.

On top of the above, each household also consumes and invests the savings in assets which include deposits, that pay a nominal riskless rate of return between $t-1$ and t of R_{t-1}^n, and risky shares of ownership of banks in the economy, that pay real return R_t^z. Varying bank equity holding is subject to adjustment cost in order to capture the short run variation of bank capital price. The j th household chooses consumption (with external habit) and the asset portfolio to maximize the expected lifetime utility (appropriately discounted) subject to an intertemporal budget constraint (in real terms) that reflects interperiod allocation possibilities. Specifically, the household's problem is given by

$$\max_{C_t, D_{t+1}, EQ_{t+1}} E_t \sum_{h=0}^{\infty} \beta^h [\frac{(C_{t+h}(j) - \xi \overline{C}_{t-1+h})^{1-\eta^c}}{1-\eta^c} + \psi^d \frac{(D_{t+h+1}(j))^{1-\eta^d}}{1-\eta^d} + \psi^x \frac{(1-N_{t+h}(j))^{1-\eta^x}}{1-\eta^x}];$$

subject to

$$C_t(j) + D_{t+1}(j) + Q_t^z Z_{t+1}(j) + \frac{\chi}{2} \cdot \frac{[Q_t^z (Z_{t+1}(j) - Z_t(j))]^2}{Q_t^z Z_t(j)} =$$

$$w_t(j) N_t(j) + \frac{R_{t-1}^n}{\pi_t} D_t(j) + R_t^z Q_t^z Z_t(j) + \mathrm{T}_t(j) + \Pi_t(j);$$

where $C_t(j)$ denotes the j th household's real consumption, $\overline{C}_t$ the aggregate consumption, η^c, η^d and η^x measure the intertemporal elasticity of substitution for consumption, deposit and leisure. ψ^d and ψ^x represent the weight on deposit and leisure in the utility function. $D_{t+1}(j)$ stands for the deposits (in real terms) held from t and $t+1$, $Z_{t+1}(j)$ the bank equity held from t and $t+1$, $N_t(j)$ the hours worked, $w_t(j)$ the real wage, $\mathrm{T}_t(j)$ the lump sum taxes,

$\Pi_t(j)$ the dividends. $\beta \in (0,1)$ is the subjective discount factor. Real deposits are included in the instantaneous utility function to indicate the existence of liquidity services from wealth held in the form of that asset. Compared to bank capital, deposits have an advantage for households in terms of liquidity. The F.O.Cs for households' optimization problem are[60]:

$$U_{Ct} = \beta E_t\{U_{Ct+1}\frac{R^n_{t+1}}{\pi_{t+1}}\} + U_{Dt+1}; \tag{3.10}$$

$$U_{Ct}Q^z_t[1+\chi(\frac{Z_{t+1}}{Z_t}-1)] = \beta E_t\{U_{Ct+1}Q^z_{t+1}[R^z_{t+1}+\frac{\chi}{2}(\left(\frac{Z_{t+2}}{Z_{t+1}}\right)^2-1)]\}; \tag{3.11}$$

Eq. (3.10) describes the equilibrium trade-off between consumption and holding deposit in terms of utility, which states that the utility loss of giving up a unit of consumption in the current period must be compensated with the utility gain of holding deposit, including the liquidity yield as well as the discounted pecuniary yield in the following period. Eq. (3.11) shows the intertemporal condition of holding bank equity after taking the adjustment cost into account.

Finally, given that the households set nominal wages in staggered contracts with a constant probability, $1-\theta_w$, of renegotiation in each period, the fraction of households who have the opportunity to reset their wages will set it as a mark-up over the marginal rate of substitution of leisure for consumption taking account the probability that he cannot reset the wage again. The fraction of households who don't have the opportunity to reoptimise must apply the wages that was in effect in the preceding period indexed by the steady state gross rate of wage inflation, ω. This yields the following maximization problem

$$\underset{i}{Max}E_t\sum_{h=0}^{\infty}[(\beta\theta_w)^h \frac{\lambda_{t+h}}{\lambda_t}\frac{W^*_t(j)\omega^h - MRS_{t+h}P_{t+h}}{P_{t+h}}N_{t+h}(j)];$$

The F.O.C for the maximization problem is

[60] The omission of households' index in the F.O.Cs stems from the assumption following Erceg *et al.* (2000) and Christiano *et al.* (2005) that the implicit existence of state-contingent securities ensures households' consumption and asset holding are homogenous.

$$W_t^*(j) = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{h=0}^{\infty} \{(\beta\theta_w)^h \lambda_{t+h+1}/\lambda_{t+h} N_{t+h}(j) MRS_{t+h}\}}{E_t \sum_{h=0}^{\infty} \{(\beta\theta_w)^h \lambda_{t+h+1}/\lambda_{t+h} N_{t+h}(j) \omega^h / P_{t+h}\}}; \qquad (3.12)$$

3.3.2 Entrepreneurs

In each period, entrepreneurs combine hired labour and purchased capital to produce intermediate goods in a constant return to scale (CRS) technology. This aggregate production function is given by

$$Y_t = A_t K_t^{\alpha} N_t^{G\,1-\alpha} \qquad (3.13)$$

where Y_t is produced intermediate goods, N_t^G is hired labour service, K_t is capital purchased in last period and α is capital share in production function. A_t is an exogenous technology measure capturing total factor productivity in goods sector. It follows

$$\ln A_t = (1 - \rho_a) \ln A + \rho_a \ln A_{t-1} + \varepsilon_{at}; \qquad (3.14)$$

with $\rho_a \in (0,1), \varepsilon_{at} \sim iid(0, \sigma_a^2)$. At the end of period t, the entrepreneur needs to purchase capital, K_{t+1}, that will be used in period $t+1$, at the price Q_t^k. The entrepreneur can only afford part of the expenditure $Q_t^k K_{t+1}$, equalling to his net worth NW_{t+1}, and rely on external funds from banks for the rest. The capital demand of entrepreneurial sector is determined by the equality of expected marginal external financing cost with expected marginal return of holding capital.

$$E_t R_{t+1}^l = E_t R_{t+1}^k = E_t \left[\frac{X_{t+1}(\alpha Y_{t+1}/K_{t+1}) + Q_{t+1}^k (1-\delta)}{Q_t^k} \right]; \qquad (3.15)$$

where δ is the depreciation rate of capital, X_t is the price of intermediate goods relative to final goods. The expected return on capital consists of two aspects: the income gain of $X_t \alpha Y_{t+1}/K_{t+1}$ and the capital gain of $Q_{t+1}^k(1-\delta)$. This return must be equal to the real gross loan rate to ensure the optimal holding of capital by entrepreneurs.

Given the existence of credit market imperfections, the gross loan rate $E_t R^l_{t+1}$ will be equal to the multiplication of gross external finance premium EFP_{t+1} and the real cost of deposit funds as described above. The determination of EFP_{t+1} has been shown in bank's optimal loan management in the previous section. The equation that shows financial frictions in this model can be written as

$$E_t R^l_{t+1} = E_t R^k_{t+1} = EFP_{t+1} E_t R^d_{t+1}; \qquad (3.16)$$

where R^d_t equals to the real opportunity cost of internal funds after taking into account the inflation effect. Entrepreneurial demand for labour is determined by equalizing the real wage with marginal product of labour:

$$w_t = X_t (1-\alpha) \frac{Y_t}{N^G_t}; \qquad (3.17)$$

The last part of entrepreneurial problem is the transition of the net worth. The existence of credit market dictates that entrepreneurs are not allowed to fully self finance. In other words, they cannot accumulate their net worth forever. We can achieve this by assuming the exit and entry of entrepreneurs out and into the entrepreneurial sector. The probability that an entrepreneur will survive until the next period is υ (i.e. there is a probability $1-\upsilon$ that he dies in between periods), so entrepreneurs only have finite expected horizon $1/(1-\upsilon)$ for operation. This assumption is vital, as it ensures that entrepreneurs never accumulate enough net wealth to finance new capital expenditure entirely and have to go to the credit market for external funds. The size of the entrepreneurial sector is constant, with new arrivals replacing departed entrepreneurs. The newly entered entrepreneurs receive some transferred seed money, S_t, for operation[61]. We can derive the evolution of entrepreneurs' net worth as follows:

$$NW_{t+1} = \upsilon[R^k_t Q_{t-1} K_t - R^d_t EFP_t (Q_{t-1} K_t - NW_t)] + (1-\upsilon) S_t; \qquad (3.18)$$

where the first term in the square bracket represents the *ex post* return of holding capital in t and the second is the cost of borrowing.

[61] Without this seed money, entering entrepreneurs would have no net worth, and so they would not be able to buy any capital. Also, among the entrepreneurs who survive there are some who are bankrupt and have no net worth. Without a transfer they would not be able to buy capital either.

3.3.3 Capital producers

Capital producers are included to rationalize the fluctuations in the real capital price Q_t^k, since the volatile asset prices contribute to the fluctuations of entrepreneurial wealth. Consider there are perfectly competitive capital producers in the economy to control the supply of capital. They combine the purchased capital and investment funds to produce new capital, $\widetilde{K}_t$, according to

$$\widetilde{K}_t = \Phi\left(\frac{I_t}{K_t}\right)K_t;$$

with $\Phi(0)=0$, $\Phi'(.)>0$, $\Phi''(.)<0$. This increasing and concave function captures the presence of adjustment costs in the production of capital goods. Capital producers choose the investment expenditure in order to maximize their profit, $Q_t^k\widetilde{K}_t - I_t$, taking the relative price of capital as given. The first-order condition is

$$Q_t^k = \left[\Phi'(\frac{I_t}{K_t})\right]^{-1} \tag{3.19}$$

Here I restrict the capital production function so that the relative price of capital is unity in steady state. Capital producers' decision is linked with the entrepreneurs' capital-purchasing decision via the variation in the price of capital.

The aggregate capital stock evolves according to

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t \tag{3.20}$$

Note that capital is homogeneous, so there is no difference between newly-produced and old capital. Old capital used by entrepreneurs is rented out for the production of new capital, and then returned at the same price as the newly-produced capital.

3.3.4 Retailers

The retail sector is applied to introduce nominal rigidity into this economy. Here I assume that entrepreneurs sell all of their output goods to retailers. Retailers purchase the homogenous wholesale goods from entrepreneurs, differentiate them using a linear technology at no resource cost and sell as final goods to households, capital producers and the government sector. In this way, the retailers have the monopolistic power to set the prices of these final goods. The reason why retailers are incorporated together with entrepreneurs is to avoid the complication of aggregating individual entrepreneur's demand for capital and his net worth when entrepreneurs themselves are imperfect competitors. Ultimately retailers' monopolistic profits belong to the households who own them, in contrast to entrepreneurs who are independent agents possessing their own wealth. Before exploring the retailers' problem in details, I firstly derive the aggregation of final goods. The final goods Y_t are bundles of differentiated goods $Y_t(j)$, $j \in [0,1]$, provided by the continuum of monopolistically competitive retailers[62]. The aggregation follows the framework of Dixit and Stiglitz (1977) as

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} ;$$

where ε_p is the elasticity of substitution between different goods. The optimal allocation of expenditure across differentiated goods implies a downward sloping demand function for goods j:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t ;$$

where $P_t(j)$ denotes the price of good $Y_t(j)$, Y_t denotes the aggregate demand, and ε also measures the price elasticity of demand among differentiated goods. P_t denotes the price index of final goods given by

[62] Recall that the assumption of linear technology for retailers ensures that the amount of final goods varies one-for-one with the amount of wholesale goods in the economy.

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon_p} dj\right]^{\frac{1}{1-\varepsilon_p}};$$

Following Calvo (1983), and a discrete version as in Yun (1996), I assume that each retailer cannot reoptimize its selling price unless it receives a random signal. The probability that each retailer can reoptimize his price in a given period is $1-\theta_p$, independently of other firms and of the time elapsed since the last adjustment. Thus the average length of time a price remains unchanged is $1/(1-\theta_p)$. Retailer j who has the opportunity to reset its price in a given period t choose the price, $P_t^*(j)$, that maximizes its expected discounted profits until the period when they are next able to change their price. On the other hand, the retailer who doesn't have the opportunity to reset price must charge the price that was in effect in the preceding period indexed by the steady state gross rate of inflation, π. Retailer j's optimazation problem is:

$$MaxE_t\sum_{h=0}^{\infty}[(\beta\theta_p)^h \frac{U_{Ct+h}}{U_{Ct}} \frac{P_t^*(j)\pi^h - X_{t+h}P_{t+h}}{P_{t+h}} Y_{t+h}(j)];$$

subject to the demand function of $Y_t(j)$. Note that the stochastic discount factor for expected profits consists of the probability that retailers can change their price and the households' intertemporal marginal rate of substitution. The F.O.C for the optimal problem is

$$P_t^*(j) = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{E_t\sum_{h=0}^{\infty}\{(\beta\theta_p)^h U_{Ct+h+1}/U_{Ct+h}Y_{t+h}(j)X_{t+h}\}}{E_t\sum_{h=0}^{\infty}\{(\beta\theta_p)^h U_{Ct+h+1}/U_{Ct+h} Y_{t+h}(j)\pi^h/P_{t+h}\}}; \tag{3.21}$$

The aggregate price index is given by

$$P_t^{1-\varepsilon_p} = \theta(\pi P_{t-1})^{1-\varepsilon_p} + (1-\theta)P_t^{*1-\varepsilon_p};$$

Log-linear approximations of the F.O.C and aggregate price index imply the following New Keynesian Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\hat{\chi}_t; \qquad (3.21L)$$

where $\hat{\chi}_t$ is the log deviation of real marginal cost from steady state.

3.3.5 Government and monetary policy

Finally I set the budget constraint for the government and the policy rule of the monetary authority to close the whole model. The aggregate final output goods consist of households' consumption, capital producers' investment expenditure and the government expenditure, G_t. Every period, the market for final goods clears as

$$Y_t = C_t + I_t + G_t; \qquad (3.22)$$

where government expenditure is financed by lump-sum taxes

$$G_t = \mathrm{T}_t;$$

The monetary authority conducts monetary policy by controlling the gross nominal interest rate R_t^n. Following Taylor (1993) and related studies thereafter, I assume that the central bank adjusts the nominal interest rate in response to deviations of inflation π_t and output Y_t, from their steady state values, with certain degree of interest rate smoothing. The log-linear version of monetary policy rule is

$$\hat{r}_t^n = \rho_r \hat{r}_{t-1}^n + (1-\rho_r)[\kappa_\pi \hat{\pi}_t + \kappa_y \hat{y}_t] + \varepsilon_{rt} \qquad (3.23)$$

Where ρ_r captures the empirical interest rate smoothing, κ_π and κ_y are the elasticity of nominal interest rate with respect to deviations of inflation π_t and output Y_t, and ε_{rt} is an exogenous random shock, with zero mean and standard deviation σ_r, to the interest rate, reflecting either failure to track the rule or intentional transitory deviations from the rule (policy shock).

3.3.6 Equilibrium

In this extended model that involves bank capital channel, the equilibrium is defined as a set of endogenous variables $\{Y_t, C_t, I_t, K_t, D_t, Z_t, NW_t, N_t^G, N_t^F, w_t, \omega_t, \pi_t, R_t^k, R_t^z, R_t^n, Q_t^k, Q_t^z\}$ which satisfies the EFP determination rule (3.8), the functional form of bank capital price (3.9), households' decision rules (3.10) and (3.11), wage inflation curve (3.12), entrepreneurs' optimal conditions (3.15), (3.16) and (3.17), capital producers' decision rule (3.19), price inflation curve (3.21), monetary policy rule (3.23), and resource constraints (3.1), (3.2), (3.13), (3.18), (3.20) and (3.22).

Steady state solution

Before moving to the calibration of the model, it is useful to look at the steady states of the model economy. The model's non-stochastic steady state system is characterized by above equilibrium conditions in non time varying fashion.

To solve the system, we can follow the method of continuous substitution. Start with the steady state version of Eq. (3.11), we have

$$R^z = \frac{1}{\beta}; \qquad (3.11S)$$

Since the difference between R^z and R^d stems from the liquidity premium $L.P.$, we have

$$R^d = \frac{R^z}{L.P.};$$

where the value of $L.P.$ is given in the calibration section. Following Eq. (1.20) and Fisher Equation, we have

$$R^k = R^d \times EFP; \qquad (3.16S)$$

$$R^n = R^d \times \pi;$$

Meanwhile, we also have the following from Eq. (3.21):

$$X = \frac{\varepsilon_p - 1}{\varepsilon_p}; \qquad (3.21S)$$

Then according to Eq. (1.19), we achieve

$$\frac{Y}{K}=\frac{R^{k}-1+\delta}{\alpha X}; \qquad (3.15S)$$

The above attainment predicts

$$\frac{Y}{L}=\frac{Y/K}{L/K}=\frac{Y/K}{1-NW/K};$$

On the other hand, based on Eq. (3.7), we find out UEFP equals to

$$UEFP=1+\frac{EFP-1}{1-\gamma_2\,NW/K}; \qquad (3.7S)$$

assuming we know the steady state value of EFP and NW/K. Thus combine Eq. (3.4) and (3.17) to eliminate real wage w, we get

$$\frac{N^G}{N^F}=\frac{1-\alpha}{\gamma_1}\frac{Y}{L}\frac{X}{UEFP-1};$$

where Y/L , X and UEFPare given already. Since $N^G+N^F=N$, we know:

$$N^F=\frac{N}{N^G/N^F+1}; \text{ and } N^G=N-N^F;$$

The next step is to combine Eq. (3.15S) and (3.13) to get

$$\frac{N^G}{K}=\left[\frac{Y}{K}\frac{1}{A}\right]^{\frac{1}{1-\alpha}};$$

where Y/K is given in Eq. (3.15S). Then we can easily calculate the steady state capital stock K, output Y, and the rest variables of the model. Some key steady states are shown in table 3.4.

Log-linear approximation

Taking a log-linear approximation of the equilibrium system around non-stochastic steady state gives:

$$\hat{l}_t=\frac{Z}{L}(\hat{z}_t+\hat{q}_t^z)+(1-\frac{Z}{L})\hat{d}_t; \qquad (3.1L)$$

$$\hat{l}_{t+1}=\hat{f}_t+\gamma_2\hat{k}_{t+1}+\gamma_2\hat{q}_t^k+\gamma_1\hat{n}_t^F+(1-\gamma_1-\gamma_2)(\hat{z}_{t+1}+\hat{q}_t^z); \qquad (3.2L)$$

$$\begin{aligned}\widehat{efp}_t = \frac{EFP-1}{EFP}\frac{R^z}{R^z-R^d}\hat{r}^z_{t+1} - \frac{EFP-1}{EFP}\frac{R^d}{R^z-R^d}\hat{r}^d_{t+1} + \frac{EFP-1}{EFP}(\hat{q}^z_t + \hat{z}_t - \hat{l}_t) \\ -\frac{EFP-1}{EFP}\frac{\gamma_2 NW/K}{1-\gamma_2 NW/K}(n\hat{w}_t - \hat{q}^k_t - \hat{k}_t)\end{aligned}; \quad (3.8L)$$

$$\hat{q}^z_{t+1} = \rho_{Qz}\hat{q}^z_t + (1-\rho_{Qz})n\hat{w}_{t+1}; \quad (3.9L)$$

$$E_t\hat{r}^z_{t+1} + E_t(\hat{q}^z_{t+1} - \hat{q}^z_t) + E_t(\hat{\lambda}_{t+1} - \hat{\lambda}_t) = \phi(\hat{z}_t - \hat{z}_{t-1}) - \frac{\phi}{R^z}E_t(\hat{z}_{t+1} - \hat{z}_t); \quad (3.10L)$$

$$\frac{1}{\beta R^d}\hat{\lambda}_t + \frac{\psi^d D^{-\eta^d}\eta^d}{\beta\lambda R^d}\hat{d}_t = \hat{\lambda}_{t+1} + \hat{r}^n_t - \hat{\pi}_{t+1}; \quad (3.11L)$$

$$\hat{\omega}_t = \beta\hat{\omega}_{t+1} + \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\eta_n\varepsilon_w)}[\eta_n(\frac{N^G}{1-N}\hat{n}^G_t + \frac{N^F}{1-N}\hat{n}^F_t) - \hat{\lambda}_t - \hat{w}_t]; \quad (3.12L)$$

$$\hat{y}_t = \hat{a}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}^G_t; \quad (3.13L)$$

$$\hat{r}^k_t = \frac{X\alpha Y/K}{R^k}(\hat{x}_t + \hat{y}_t - \hat{k}_{t-1}) + \frac{(1-\delta)}{R^k}\hat{q}^k_t - \hat{q}^k_{t-1}; \quad (3.15L)$$

$$\hat{r}^k_{t+1} = \widehat{efp}_t + \hat{r}^n_t - \hat{\pi}_{t+1}; \quad (3.16L)$$

$$\hat{w}_t = \hat{x}_t + \hat{y}_t - \hat{n}^G_t; \quad (3.17L)$$

$$\frac{1}{\upsilon R^k}n\hat{w}_t = \frac{QK}{NW}\hat{r}^k_t - (\frac{QK}{NW} - 1)\{\hat{r}^n_{t-1} - \pi_t + \widehat{efp}_{t-1}\} + n\hat{w}_{t-1}; \quad (3.18L)$$

$$\hat{q}_t = \varphi(\hat{i}_t - \hat{k}_{t-1}); \quad (3.19L)$$

$$\hat{k}_t = \delta\hat{i}_t + (1-\delta)\hat{k}_{t-1}; \quad (3.20L)$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\hat{x}_t; \quad (3.21L)$$

$$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{i}_t; \quad (3.22L)$$

$$\hat{r}^n_t = \rho_r\hat{r}^n_{t-1} + (1-\rho_r)[\kappa_\pi\hat{\pi}_t + \kappa_y\hat{y}_t] + \varepsilon_{rt}; \quad (3.23L)$$

Eq. (3.1L) to (3.2L), Eq. (3.8L) to (3.13L) and Eq. (3.15L) to (3.23) are the log-linear version corresponding to Eq. (3.1) to (3.2), Eq. (3.8) to (3.13) and Eq. (3.15) to (3.23). Following the convention, all the variables with hat on top denote percentage deviations from non-stochastic steady state, where I omit the conditional expectations operator on the assumption of 'Certainty Equivalence'. Using Uhlig's undetermined coefficients procedure yields a state space solution of the form[63]:

$$\hat{s}_{t+1} = \Omega_1 \hat{s}_t + \Omega_2 \varepsilon_{t+1}; \tag{3.24}$$

$$\hat{d}_t = \Omega_3 \hat{s}_t \tag{3.25}$$

where the state variable vector, $\hat{s}_t$, includes predetermined and exogenous variables; $\hat{d}_t$ is the vector of control variables; and the vector ε_t contains the random innovations. The coefficient matrices, Ω_1 , Ω_2 , and Ω_3 , have elements that depend on the structural parameters of the model. Therefore, the state space solution, (3.24) and (3.25), is used to simulate the model later when showing the results.

3.4 Calibration

Before using the log-linear system above to simulate the model, it is necessary to set values to all the structural parameters. In what follows, I set parameter values to calibrate the model to quarterly data of the post war US economy. Since most of the structural parameters are standard and identical to that in previous chapters, I set them equal to the originally calibrated values. More attention is paid to newly introduced parameters and steady state values that are relevant to the banks' balance sheet channel.

The first two steady states for which we should set values are the ratio of bank capital to assets (loans) and the liquidity premium in the banking sector; these two values are relevant to bank capital channel. One should bear in mind that the minimum Basel capital requirement is 8%, but the actual bank capital ratio is higher than that for most banks. They hold some buffer of equity above the regulatory minimum, so as to lower the risk of an adverse shock leading to capital inadequacy in the future. Meh and Moran (2010) calibrate the bank capital ratio to be 14% based on the 2002 average risk-weighted capital-asset ratio of US banks. On the other hand, Van den Heuvel (2008) gets a lower value of 10% from the data spanning from 1993 to 2004. Thus I pick a plausible value of 12% that is the average from these two

[63] The detailed methodological exposition is given in Uhlig (1999), whereby the Matlab programme is available at his homepage (http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm).

studies. The liquidity premium should be calibrated with some caution. Since the model abstracts from any default risk of holding bank equity, the difference totally stems from the liquidity disadvantage of bank equity. For this reason, Aguiar and Drumond (2009) equalize the return on bank equity to the return to firm equity and Van den Heuvel (2008) uses the data on subordinated bank debt to measure the liquidity premium. I follow the estimation of Van den Heuvel (2008) and set the liquidity premium to be 3.16% annually in steady state. These two steady state values, together with the EFP and entrepreneurial leverage, help fix the share of bank equity in loan production function from Eq. (3.8). The parameter governing the persistence of bank capital price is set to 0.78, in line with the estimation in Markovic (2006)[64].

The rest structural parameters and steady states are almost identical to those in previous chapters. The discount rate β is set to a lower value of 0.985 than before. This is to match the average annual steady state bank equity return of 5.92%. Combine this with the liquidity premium can reveal the annual return on deposit of 2.76%. The elasticity of substitution for consumption η^c, deposit η^d and leisure η^x are all set to 1.5, implying a nearly logarithmic utility function. Habit persistence parameter ξ is 0.6. To reconcile the average working time of around 30%, the weight on leisure ψ^xi s set to 2.47. The share of capital in goods production function α and the capital depreciation rate δ are set to 0.36 and 0.025. For entrepreneurs' surviving rate in the end of each period, υ, I use the value of 0.9728, implying entrepreneurial average life of 36 quarters. The parameter φ that measures the level of physical capital adjustment cost is set to 0.5 while the one for bank capital adjustment cost is 0.3. The share of collateral and labour in loan production function are 0.65 and 0.24 respectively, implying the share of bank equity to be 0.11. This value is fairly close to the corresponding one 0.096 in Gillman and Kejak (2010), who also put bank capital in credit production function. The parameters associated with price and wage rigidity also follow the literature, where the elasticity of demand for goods ε_p, and labour ε_w, are 11 and 21 such that the steady state markups are 10% in the goods market and 5% in labour market, and the probability of not reoptimizing for price setters θ_p, is 0.5 while that for wage setters θ_w, is

[64] Markovic (2006) originally estimates this for UK, but mentions that the result in US is very close.

Table 3.3 Parameters in the model

Parameter	Description	Value
β	*Household's discount factor*	0.985
η^c, η^d, η^x	*Intertemporal elasticity of substitution*	1.5
ψ^d	*Weight on deposit in utility*	0.0019
ψ^x	*Weight on leisure in utility*	2.47
ξ	*Habit persistence*	0.6
α	*Share of capital in goods production*	0.36
δ	*Capital depreciation rate*	0.025
θ_p	*Retailers' probability of not able to reset price*	0.5
θ_w	*Households' probability of not able to reset wage*	0.75
ε_p	*Goods elasticity of demand*	11
ε_w	*Labour elasticity of demand*	21
υ	*Entrepreneurs' surviving rate*	0.9728
φ	*Curvature of physical capital adjustment cost function*	0.5
χ_z	*Curvature of bank capital adjustment cost function*	0.3
γ	*Share of collateral in loan production*	0.65
ρ_a	*Autocorrelation of goods productivity shock*	0.95
ρ_r	*Interest rate smoothing*	0.9
κ_π	*Elasticity of policy rate to inflation deviation*	1.5
κ_y	*Elasticity of policy rate to output deviation*	0.1
ρ_{Qz}	*Persistence of bank capital price*	0.78

0.75. For the parameters regarding the monetary policy reaction function, I generally follow Clarida et al. (2000) and set the interest rate smoothing parameter ρ_r equal to 0.9, and the policy response to inflation and output deviation, κ_π and κ_y, to 1.5 and 0.1, respectively.

The key steady state values in the model are also highlighted as follows. The external finance premium is set to 1.0075 for baseline, corresponding to an annual risk spread of 300 basis points, approximating the post war average spread between the corporate bond rate and the three-month treasury bill rate. This is consistent with the estimates in Queijo (2009) and lies

Table 3.4 Steady state in the model economy

Variable	Definition	Value
I. Steady state values		
A	*Goods sector productivity level*	1
R^d	*Deposit rate*	1.0069
R^z	*Bank equity return*	1.0152
R^k	*Gross return on capital*	1.0148
R^n	*Nominal interest rate*	1.0162
π	*Inflation rate*	1.0092
EFP	*External financing premium*	1.0075
N^G	*Labour service in goods sector*	0.3
N^F	*Labour service in banking sector*	0.005
II. Steady-state ratios		
Y/K	*Output to capital*	0.13
I/Y	*Investment to output*	0.19
C/Y	*Consumption to output*	0.61
G/Y	*Govt expenditure to output*	0.2
L/K	*Leverage ratio*	0.5
Z/L	*Bank capital ratio*	0.12
N^F/N	*Financial hour to total hour*	1.6%

within the range reported in De Fiore and Uhlig (2005)[65]. The steady state quarterly gross inflation is set to 1.0092, implying the nominal interest rate of 1.0162. The steady state entrepreneurial leverage ratio is set to 50%, which means the ratio of net worth to value of purchased capital is 0.5. The steady state consumption, investment and government expenditure share of GDP are given by 0.603, 0.192 and 0.205, respectively to match the historical average. In labour market, the steady state ratio of monitoring hour relative to goods produce hour is 1.6%. All the parameters and their calibrated values are described in table 3.3 while steady states are summarized in table 3.4.

[65] In De Fiore and Uhlig (2005), the reported range for annual risk premiums on bonds and loans in U.S. is between 160 and 340 basis points.

3.5 Results

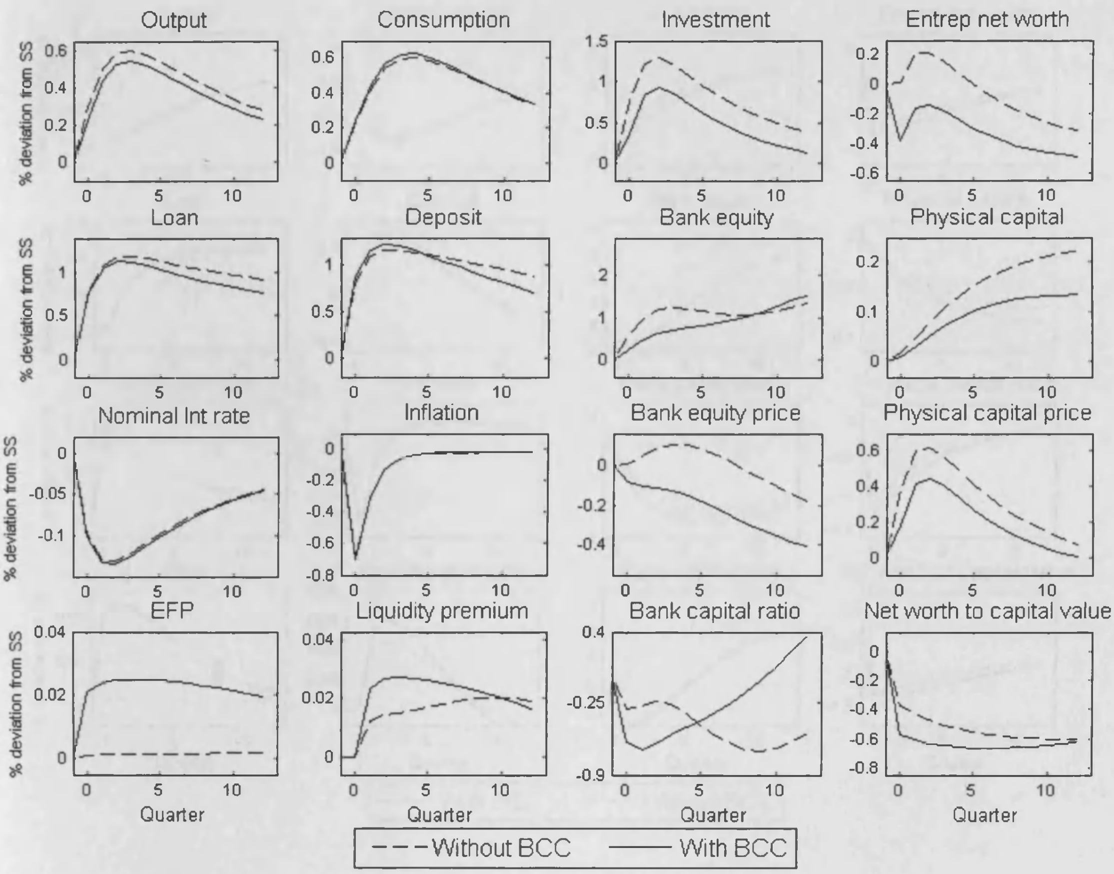
This section presents the main findings about the link between the bank capital channel and the model's transmission mechanism subject to exogenous shocks. I generally consider the impulse responses of the economy to two conventional shocks prevailing in the literature: goods technology shock (supply side) and monetary policy shock (demand side). The latter is corresponding to the monetary transmission mechanism which attracts lots of attention from the policy makers. I also consider the effects of independent shocks from the banking sector itself, defined as the large and persistent fall in bank equity value. This is to resemble the economy-wide write-off of non-performing loans in general, and the recent global financial crisis in particular, which can be simulated as a large deterioration of the banking sector balance sheet. It would be interesting to see the response of the whole economy to such shock.

3.5.1 Technology shocks

Figure 3.1 presents the effects of a one-standard deviation positive technology shock on two versions of the model economy. The first version corresponds to an active bank capital channel and is labelled the *With BCC* economy. Its responses to the technology shock are shown in solid lines. The second version of the model economy is similar to the first one except that the bank capital channel is turned off. Since the bank capital channel is active through the liquidity premium effect and the bank capital ratio variation, we can shut the channel down by cut off the connection between EFP and the above two. This economy is labelled the Without BCC and its responses are displayed in dashed lines.

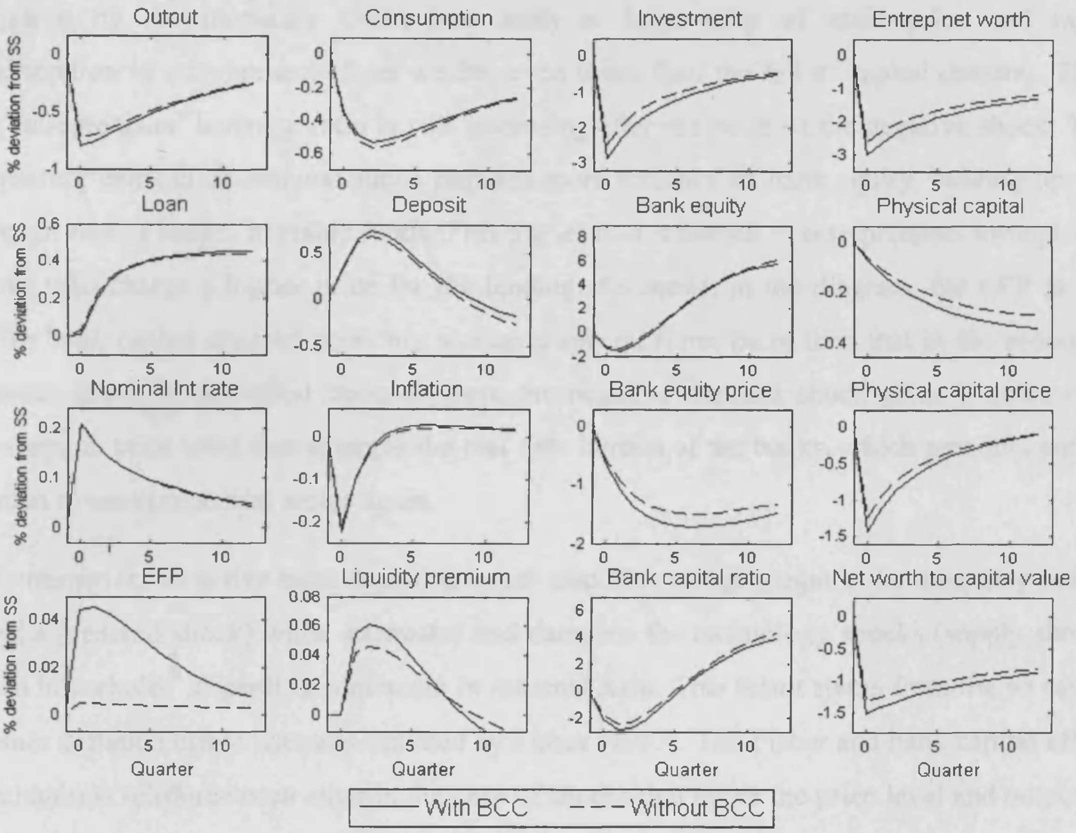
In the impact period, the rise in the technology drives up the marginal product of capital and stimulates the demand for capital, which in turn pushes up investment and the price of capital. We clearly see the jump of output, consumption and investment from the diagram. However, here the banking friction generates an attenuation effect, dampening the technology shock. This comes from the debt deflation effect, where the decline of inflation associated with supply shock raises the real debt burden, depressing the net worth for entrepreneurial sector. The decline in net worth accompanied by the rise of loan, which is stimulated by the positive

Fig. 3.1 Impulse responses to technology shocks



supply shock, pushes up the leverage ratio and the need to raise bank capital. This generates higher cost of bank intermediation (liquidity premium) and requires higher EFP, which attenuates the effect of initial shock and dampens the following propagation as well. This result is consistent with the recent literature (e.g., Christenson and Dib 2008, Christiano *et al.* 2010) assuming nominal contract. Thus we see a reverse order of response in figure 3.1. The no friction version (not reported) would give the highest initial impact which is also in effect for longest period. The model economy with bank capital channel attenuates the most, more than the version without banks' balance sheet effect. Finally, the positive supply shock leads to deflation. In reaction, monetary authorities follow a loose policy after the onset of the shock, acting as an additional force to stimulate the economy. However, this doesn't overturn the attenuation effect.

Fig. 3.2 Impulse responses to monetary policy shocks



3.5.2 Monetary policy shocks

The monetary transmission mechanism is usually analysed by exploring the economy's impulse responses to a policy innovation. In order to assess the importance of channels through which actions of policymakers affect the economy, I simulate the impulse responses of the economy to a temporary increase in the policy rate of 1% per annum. Figure 3.2 presents the responses to a one-standard deviation negative monetary policy shock, for the bank capital channel economy (solid lines) and for the no bank capital channel economy (dashed lines).

Bank capital channel reveals itself clearly in the diagram. Compared with the impulse responses of the economy without bank capital channel, the decline of output, investment, capital, asset price are all more significant in the economy with banks' balance sheet effect.

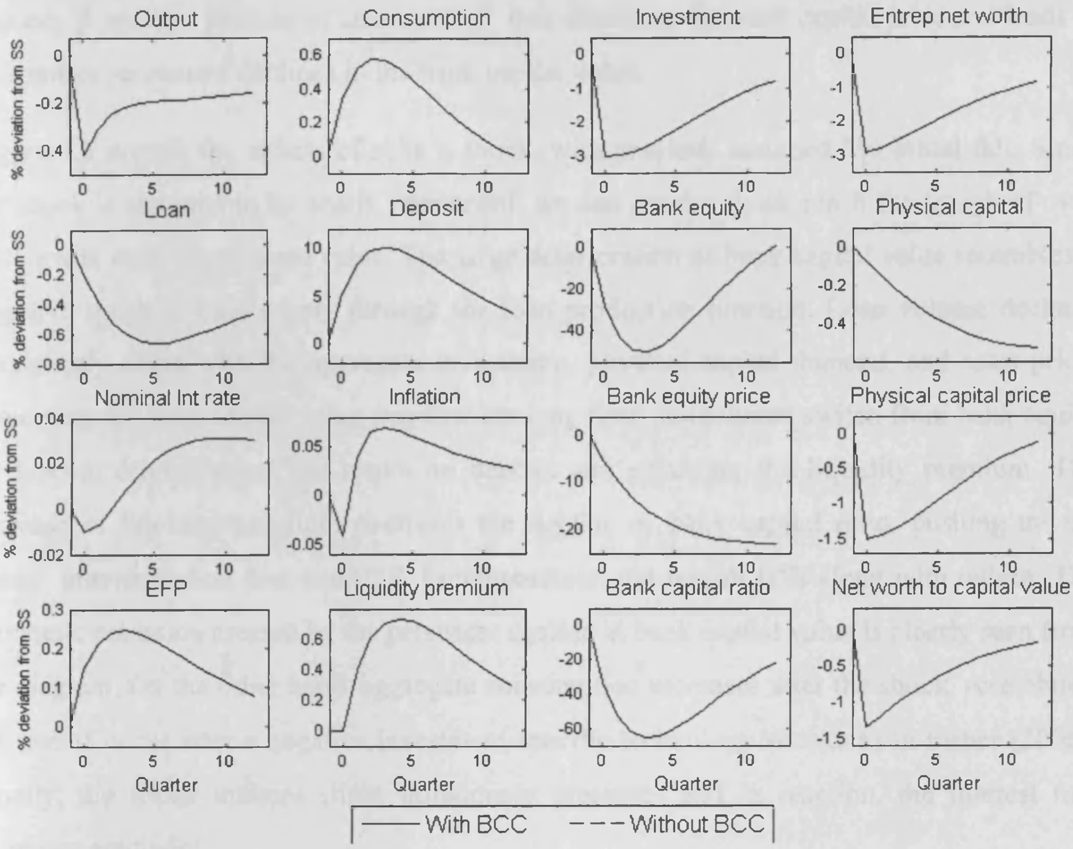
The accelerator effect of the bank capital channel works as follows. The economic recession triggered by the monetary contraction leads to large drop of asset price and rapid deterioration of entrepreneurial net wealth, even faster than the fall of capital demand. Thus the entrepreneurs' leverage ratio is still increasing after the onset of the negative shock. The increasing demand of external funds requires more issuance of bank equity, bidding up the average cost of banks' loanable funds. This higher cost is passed to entrepreneurs through the banks who charge a higher price for the lending. As shown in the diagram, the EFP in the active bank capital channel economy increases several times more than that in the economy without active bank capital channel. Here the negative demand shock gives a downward pressure on price level that enlarges the real debt burden of the banks, which pass this higher burden to entrepreneurial sector again.

To summarize, an active bank capital channel amplifies and propagates the monetary policy shocks (demand shock) while attenuates and dampens the technology shocks (supply shock) when households' deposit is contracted in nominal term. This result stems from the so called 'Fisher deflation effect' channel initiated by Fisher (1933). The Fisher and bank capital effect mechanisms reinforce each other in the case of shocks that move the price level and output in the same direction (demand shock), and they tend to cancel each other in the wake of shocks which move the price level and output in opposite directions (supply shock). As shown in Christiano *et al.* (2010), the Fisher deflation effect is an additional source of nominal rigidity in the economy.

3.5.3 An experiment of Credit Crunch

Past evidences show a record of occasional but large direct shocks to banks' balance sheets. Such shocks can deliver an immediate impact to the value of bank capital, and thus the price of bank shares. A notable example is the recent upheavals in financial markets worldwide, characterized by growing loan loss provisions, large asset writedowns and dramatic reduction in profits of financial institutions. The recognition of the banking sector's inability to recover the principal from non-performing loans implies that the banking sector was not as productive as balance sheets had previously indicated. This would likely trigger a permanent

Fig. 3.3 Impulse responses to a 'Credit Crunch' experiment



fall in the value of bank capital via a fall in the price of bank shares. I now consider the effects of financial shocks interpreted as a 'Credit Crunch' that lead to deep and permanent declines in bank capital value. The economy's impulse responses are simulated for an initial fall in the price of bank equity of 5%, the magnitude of which is to resemble the recent financial distress episodes.[66] We can capture the effects of 'Credit Crunch' experiment by assuming that the bank capital price is subject to episodes of exogenous permanent decline. In this context, the bank capital price evolution equation defined in (3.9L) becomes

$$E_t\hat{q}^z_{t+1} = \rho_{Qz}\hat{q}^z_t + (1-\rho_{Qz})E_t n\hat{w}_{t+1} - E_t d\hat{q}^z_{t+1};$$

[66] This value is also used in Markovic (2006) and Meh and Moran (2010).

Where $d\hat{q}_t^z$ represents the deterioration of bank capital value and follows an exogenous AR(1) process. A positive permanent shock to $d\hat{q}_t^z$ thus decreases the bank capital price and leads to exogenous permanent declines in the bank capital value.

Figure 3.3 depicts the effects of such a shock, with previous assumed 5% initial fall. Since the shock is assumed to be nearly permanent, we can see the shock reach the trough of over 20% lower than steady state value. The large deterioration of bank capital value resembles a negative shock to loan supply through the loan production function. Loan volume declines persistently along with the aggregate investment, physical capital demand, and asset price. Expecting the bank capital value stay low for long time, households switch from bank equity to deposit, driving down the return on deposit and enlarging the liquidity premium. The increase of liquidity premium overturns the decline of bank capital ratio, pushing up the banks' intermediation cost and EFP. Entrepreneurial net wealth falls along with output. The economic recession created by the persistent decline in bank capital value is clearly seen from the diagram. On the other hand, aggregate consumption increases after the shock, resembling the results occur after a negative investment-specific technology shocks as in Fisher (2006). Finally, the shock induces slight inflationary pressures and in reaction, the interest rate increases gradually.

3.6 Conclusion

In this chapter, I provide a framework to incorporate the bank capital into financial friction analysis. Bank capital is important because it is assumed to be one of the three factors to produce entrepreneurial loans. In consequence, the financial friction is shown to depend on the corporate balance sheet condition, the bank capital ratio, and the liquidity premium. The former one follows the claim from the previous chapters that holding of collateral help mitigate the entrepreneurs' payment for the cost associated with loan management. The latter two stem from the role of bank capital in loan production as well as in the transmission mechanism. Given that the banks liability composition is constant, the higher is the liquidity premium, the larger is the EFP. Similarly, given that the liquidity premium is constant, EFP is larger when higher proportion of loans is produced or financed with bank capital.

Based on this framework, we are able to establish the link between the bank capital chan-nel and the model's transmission mechanism subject to exogenous shocks. To this end, the main findings can be summarized as follows. An active bank capital channel amplifies and propagates the monetary policy shocks (demand shock) while attenuates and dampens the technology shocks (supply shock) when households' deposit is contracted in nominal term. This result is ascribed to the 'Fisher deflation effect', which states that the real debt burden is related to the economy-wide inflation. The Fisher and the bank capital effect mechanisms reinforce each other in the case of shocks that move the price level and output in the same direction (demand shock), and they tend to cancel each other in the wake of shocks which move the price level and output in opposite directions (supply shock). On the other hand, it is shown that adverse financial shocks originating from the banking sector, which cause persistent declines of bank capital value, can lead to sizeable declines in bank lending and economic recession.

The current model is a primitive attempt and only considers the market discipline for bank capital requirements. Thus in future research, adding explicit regulatory requirements into this framework would enrich the analysis and possibly affect the business cycle properties of the capital adequacy ratio. It should be interesting to see how the interaction between regulatory discipline and market force of bank capital requirements influence the financial friction and the economic transmission mechanism as a whole.

Appendices

Appendix A: Appendix to chapter 1

A.1 Proof of proposition 1.1

Given the equation of external finance premium expressed in steady state

$$EFP-1=\frac{1}{1-\gamma}F^{\frac{1}{1-\gamma}}w\left[(1-\frac{NW}{QK})\right]^{\frac{\gamma}{1-\gamma}}[1-\gamma\frac{NW}{QK})];\qquad (A1)$$

The first and second derivatives of EFP with respect to net worth capital ratio are

$$\frac{\partial EFP}{\partial(NW/QK)}=-\frac{1}{1-\gamma}F^{\frac{1}{1-\gamma}}w\gamma\left(1-\frac{NW}{QK}\right)^{\frac{2\gamma-1}{1-\gamma}}[\frac{1-\gamma NW/QK}{1-\gamma}+1-\frac{NW}{QK}];\qquad (A2)$$

$$\frac{\partial^2 EFP}{\partial(NW/QK)^2}$$

$$=\frac{1}{1-\gamma}F^{\frac{1}{1-\gamma}}w\frac{\gamma}{1-\gamma}\left(1-\frac{NW}{QK}\right)^{\frac{3\gamma-2}{1-\gamma}}[(2\gamma-1)(\frac{1-\gamma NW/QK}{1-\gamma}+1-\frac{NW}{QK})+1-\frac{NW}{QK}];\qquad (A3)$$

It is easy to verify that the first order derivative comes out with negative sign while the second order derivative is positive, so prove the proposition.

A.2 Proof of proposition 1.2

Given the elasticity of EFP with respect to net worth capital ratio expressed as

$$Elasticity=\frac{EFP-1}{EFP}\left(\frac{\gamma\, NW/K}{1-\gamma\, NW/K}+\frac{\gamma}{1-\gamma}\frac{NW/K}{1-NW/K}\right);\qquad (A4)$$

We fix the value of EFP and NW/K to their conventional steady state values. The first order partial derivative of elasticity with respect to γ is given by

$$\frac{\partial Elasticity}{\partial\gamma}=\frac{EFP}{EFP-1}\frac{NW}{K}[\frac{1}{(1-\gamma\, NW/K)^2}+\frac{1}{(1-NW/K)(1-\gamma)^2}]>0;\qquad (A5)$$

And the second order derivative is

$$\frac{\partial^2 Elasticity}{\partial \gamma^2} = 2\frac{EFP}{EFP-1}\frac{NW}{K}[\frac{NW}{K}\frac{1}{(1-\gamma\, NW/K)^3}+\frac{1}{(1-NW/K)(1-\gamma)^3}] > 0; \quad \text{(A6)}$$

Both of the first and second order partial derivatives of the elasticity with respect to γ are positive, ensuring the upward sloping and convex function.

A.3 BGG's financial contract

In this section of the appendix, I describe the derivation of the standard debt contract problem between entrepreneurs and financial intermediaries (FIs) in a parsimonious fashion, where asymmetric information and costly state verification are the key elements. The derivation is divided into two steps: (1) the optimal contract in the non-stochastic steady-state, where aggregate risk is absent; (2) the log-linear form of the key financial friction equation around the steady-state.

Let NW denote the steady-state level of entrepreneurial net worth, Q the price of capital (equal to one in steady state) and K the steady-state level of the capital stock. The entrepreneur borrows $QK-NW$ to purchase K units of capital in the production. Also let R^k denote the steady-state gross rate of return on capital investment and ω an idiosyncratic shock on the capital return for each individual entrepreneur. Thus the return on a specific project is ωR^k. Following BGG and others, ω is assumed to follow lognormal distribution i.e. $\ln(\omega) \sim N(-0.5\sigma_\omega^2, \sigma_\omega^2)$. The realized payoff on the entrepreneur's capital is $\omega R^k QK$. Note that ω is unknown to both the entrepreneur and the FI prior to the investment decision. Even after the realization of the idiosyncratic shock, the FI can only observe ω by paying a proportionate monitoring cost, $\mu\omega R^k QK$. FIs are assumed to be break-even in equilibrium and are able to perfectly diversify idiosyncratic credit risk. Accordingly, their opportunity cost is the interest rate on deposit, $R^d = 1/\beta$.

The optimal contract specifies a cutoff value $\overline{\omega}$ such that if $\omega \geq \overline{\omega}$, the entrepreneur pays the lender a fixed amount $\overline{\omega} R^k QK$ and keeps the remaining equity $(\omega - \overline{\omega})R^k QK$. Alternatively, if $\omega < \overline{\omega}$, the entrepreneur receives nothing, while the FI monitors the entrepreneur and receives $(1-\mu)\omega R^k QK$ in residual claims net of monitoring costs. In equilibrium, the FI earns an expected return equal to the deposit return implying

$$\left(\overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)d\omega + \int_0^{\overline{\omega}} \omega f(\omega)d\omega - \mu\int_0^{\overline{\omega}} \omega f(\omega)d\omega\right)R^k QK = R^d(QK - N); \quad \text{(A7)}$$

The optimal contract maximizes the payoff to the entrepreneur subject to Eq. (A7). Given constant returns to scale, the cutoff value $\overline{\omega}$ determines the division of expected gross payoff, $R^k QK$, between the entrepreneur and the FI. Let $\Gamma(\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega)d\omega + \overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)d\omega$ denote the gross share of the payoff going to the FI, while $\mu G(\overline{\omega}) = \mu\int_0^{\overline{\omega}} \omega f(\omega)d\omega$ denotes the expected share pertaining to monitoring costs. The payoff share going to the entrepreneur is thus given by $1-\Gamma(\overline{\omega})$. Defining $k = QK/N$ and $s = R^k/R^d$, we can set up the Lagrangian as:

$$L = (1-\Gamma(\overline{\omega}))sk + \lambda[(\Gamma(\overline{\omega}) - \mu G(\overline{\omega}))sk - (k-1)];$$

The following optimality conditions are obtained:

$$\overline{\omega}: \Gamma'(\overline{\omega}) - \lambda[\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})] = 0;$$

$$k: [(1-\Gamma(\overline{\omega})) + \lambda(\Gamma(\overline{\omega}) - \mu G(\overline{\omega}))]s - \lambda = 0;$$

$$\lambda: [\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]sk - k + 1 = 0;$$

Rearranging gives

$$s(\overline{\omega}) = \frac{\lambda}{1-\Gamma(\overline{\omega}) + \lambda(\Gamma(\overline{\omega}) - \mu G(\overline{\omega}))}; \quad \text{(A8)}$$

and

$$k(\overline{\omega}) = \frac{1-\Gamma(\overline{\omega}) + \lambda(\Gamma(\overline{\omega}) - \mu G(\overline{\omega}))}{1-\Gamma(\overline{\omega})}; \quad \text{(A9)}$$

where the Lagrange multiplier λ is now also defined as a function of $\overline{\omega}$, by virtue of the first optimality condition noted above: $\lambda(\overline{\omega}) = \Gamma'(\overline{\omega})/(\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega}))$. BGG has shown that both $s'(\overline{\omega}) > 0$ and $k'(\overline{\omega}) > 0$. This ensures the existence of a relationship

$$k = \Psi(s), \text{ with } \Psi'(s) > 0; \quad \text{(A10)}$$

that links the EFP, s, to the ratio between capital and entrepreneurial net worth, k. This relationship is the key feature of the financial accelerator.

To determine $\bar{\omega}$, I proceed as follows. The steady-state net worth, as shown in Eq. (1.22) is given by

$$NW = \upsilon(1-\Gamma(\bar{\omega}))R^k QK + (1-\upsilon)S;$$

Divide the above equation by $R^d QK$ and rearrange, we get

$$\beta / k(\bar{\omega}) = \upsilon(1-\Gamma(\bar{\omega}))s(\bar{\omega}) + (1-\upsilon)S/K; \tag{A11}$$

Thus $\bar{\omega}$ can be determined for given values of $\{\beta,\mu,\upsilon,\sigma_{\omega}^2,S/K\}$[67].

All derivations above pertain to the non-stochastic steady-state of the model. BGG also establish that, with the addition of aggregate uncertainty, a positive relationship between the external finance premium and the capital to net worth ratio continues to hold. Specifically, following Eq. (A10), this relationship can be written as

$$\frac{Q_t K_{t+1}}{NW_{t+1}} = \Psi\left(\frac{E_t R_{t+1}^k}{E_t R_{t+1}^d}\right); \tag{A12}$$

Eq. (A12) provides a link between the entrepreneur's demand for physical capital relative to his current net worth and the wedge between the expected return to capital and the deposit rate. The log-linear form of Eq. (A12) shows

$$K/N(\hat{q}_t + \hat{k}_{t+1} - n\hat{w}_{t+1}) = \Psi'(R^k/R^d)R^k/R^d\, E_t(\hat{r}_{t+1}^k - \hat{r}_{t+1}^d); \tag{A13}$$

or

$$EFP_{t+1} - 1 = E_t(\hat{r}_{t+1}^k - \hat{r}_{t+1}^d) = -\psi(n\hat{w}_{t+1} - \hat{q}_t - \hat{k}_{t+1}); \tag{A14}$$

with $\psi = \dfrac{\Psi(R^k/R^d)}{\Psi'(R^k/R^d)R^k/R^d}$.

Eq. (A14) corresponds to Eq. (1.11L') in the main text.

[67] S/K resembles the wage earned by entrepreneurs assumed in BGG and can be fixed in steady-state accordingly, see the appendix in Meier and Muller (2006).

A.4 Steady state system

$$L = FK^{\gamma} N^{F^{1-\gamma}};\tag{S1}$$

$$L = K - NW;\tag{S2}$$

$$EFP - 1 = \frac{1}{1-\gamma} F^{-\frac{1}{1-\gamma}} w\left[(1-\frac{NW}{K})\right]^{\frac{\gamma}{1-\gamma}} [1-\gamma\frac{NW}{K})];\tag{S3}$$

$$1 = \beta\frac{R^n}{\pi};\tag{S4}$$

$$\frac{\psi C}{1-N} = \frac{\varepsilon_w}{\varepsilon_w - 1} w;\tag{S5}$$

$$Y = AK^{\alpha} N^{G^{1-\alpha}};\tag{S6}$$

$$R^k = \mathrm{X}(\alpha Y/K) + 1 - \delta;\tag{S7}$$

$$R^k = R^d EFP;\tag{S8}$$

$$w = \mathrm{X}(1-\alpha)\frac{Y}{N^G};\tag{S9}$$

$$NW = \upsilon R^k NW + (1-\upsilon)S;\tag{S10}$$

$$\delta K = I;\tag{S11}$$

$$\mathrm{X} = \frac{\varepsilon_p - 1}{\varepsilon_p};\tag{S12}$$

$$Y = C + I + G;\tag{S13}$$

$$R^n = R^d \pi;\tag{S14}$$

Appendix B: Appendix to chapter 2

B.1 Data description

Data are expressed in per-capita terms using population over 16 (expressed in billions) in quarterly base.

GDP, Y_t: Gross Domestic Product, in billions of dollars, deflated by the Implicit Price Deflator of GDP. Source: Bureau of Economic Analysis.

Consumption, C_t: Personal Consumption Expenditures (non-durables plus services), in billions of dollars, deflated by the Implicit Price Deflator of GDP. Source: Bureau of Economic Analysis.

Investment, I_t: Private Fixed Investment, in billions of dollars, deflated by the Implicit Price Deflator of GDP. Source: Bureau of Economic Analysis.

Inflation, π_t: first difference of the log of the Implicit Price Deflator of GDP. Source: Bureau of Economic Analysis.

Real wage, w_t: Real Average Hourly Compensation for nonfarm business sector. Source: Bureau of Economic Analysis.

Wage inflation, ω_t: first difference of the log of the Average Hourly Compensation for nonfarm business sector. Source: Bureau of Economic Analysis.

Capital, K_t: quarterly series is constructed using annual capital stock data and quarterly data on investment expenditure. Source: Bureau of Economic Analysis.

Tobin's q, Q_t: constructed using Eq. (2.18L) and the data on investment and capital.

Net worth, NW_t: nonfarm nonfinancial corporate business net worth (market value) taken from the flow of funds account.

Money, M_t/P_t: Real per capita M1.

Nominal interest rate, R_t^n: 3-month average of the daily effective federal funds rate. Source: Federal Reserve System.

Hours worked, N_t^G: nonfarm business sector index, hours of all persons. Source: Bureau of Labor Statistics.

Banking hours, N_t^F: product of two series: average weekly hours of production workers and production workers, thousands in the financial sector. Source: Bureau of Labor Statistics.

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