SPECIMEN MATHEMATICUM

ÆQUATIONUM ALGEBRAÏCARUM CUJUSCUNQUE GRADUS RADICES RATIONALES INVENIENDI METHODUM SISTENS

QUOD

CONS. AMPL. FAC. PHIL. REG. ACAD. ABOËNS.

Publico examini ſubjiciunt

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Philoſophiæ Magiſter nec non Stipend. Brem.

&

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In Audit. Mathematico die XIV Junii MDCCCVI.

h. a. m. f.

ABOÆ, Typis FRENCKELLIANIS.

3.

PROOEMIUM.

Junctæ ſunt omnes in ſcientiarum Mathematicarum Circulo diſciplinæ nexu tam arcto, ut quæ ſupra quamvis ipſius partem effulgeat lux, in omnes alias ſe ſpargat, ſplendorem augens. Neque aliunde arceſſenda eſt cauſſa, curſtatum quem jam tenet, occupare valuerit hæcce ſcientia, & cur tantum jam attigerit perfectionis faſtigium, ut quod efficere valuerit, ipſa miretur ratio humana. Abſolutæ adeo videntur quædam hujus ſcientiæ partes, ut progresſus ulterioris non niſi ſpem reſtare crederes. Quod præcipue de illa valet doctrina, cui Arithmetices univerſalis ſive Algebræ tribuere nomen. In ea imprimis ejus parte, quæ in ſolvendis æquationibus radicibusve eruendis verſatur, ob uſum in tota Matheſi latisſime patentem, cum tali ſollertia, talique ſuccesſu elaboratum eſt, ut in illa amplius quid vix posſit deſiderari. Cum vero materiæ tanti ſane momenti illuſtrandæ, quod vel leviter prodesſet, omni orbandum non fore pretio exiſtimaremus: viſum eſt nobis Specimen Academicum edituris, si methodo, radices æquationum algebraïcarum rationales, ope serierum Arithmeticarum investigandi, quam publicæ nunc subjicimus disquisitioni, aliquam aliis huicce rei magis paribus, ulterius procedendi dederimus ansam, nos non omnem perdidisse operam. — Quare hoc consilium qualitercunque exsecuti, illius a L. B. mitiorem, quo par est officio, expetimus censuram. —

§. I.

Ad radicem æquationis cujusvis hoc modo investigandam, necesse est ut æquatio data in aliam transformetur, cujus singuli termini Summam seriei Arithmeticæ offerant. Quod, quomodo fiat, per æquationem generalem ordinis indeterminati n ostendere conabimur, quæ, licet ob nimiam sui ipsius prolixitatem perfecte resolvi nequeat, ad explicandam tamen methodum nostram maxime est idonea. Hujusmodi æquatione itaque primum considerata, methodi generaliter expositæ ad æquationes determinati ordinis applicationem instituere, animus nobis svadet.

§. II.

Sit æquatio cujuscunque ordinis generalis (A): $A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + \text{------ --} A_n x^n = A$, in qua x radicem ignotam, n numerum terminorum, atque A, A_1, A_2, &c. --- coëfficientes notos designant.

Sub-

Subſtituatur

$$A_1 = \frac{a_1}{1} + \frac{1.a_2}{1.2} + \frac{1.2.a_3}{1.2.3} + \frac{1.2.3\,a_4}{1.2.3.4} + \frac{1.2.3.4.a_5}{1.2.3.4.5} + \text{---}$$

$$A_2 = \frac{a_2}{1.2} + \frac{(2+1)a_3}{1.2.3} + \frac{(3.3+2)a_4}{1.2.3.4} + \frac{(4.11+6.)a_5}{1.2.3.4.5} + \text{---}$$

$$A_3 = \frac{a_3}{1.2.3} + \frac{(3.1+3)a_4}{1.2.3.4} + \frac{(4.6+11)a_5}{1.2.3.4.5} + \text{-----}$$

&c. &c. &c.

Has ſeries quouſque volueris, continuandi nulla erit difficultas, cum ſeriei antecedentis ope, ſeries proxime ſequens, facile determinari poterit. Sit enim C ſemper coëfficiens numericus numeratoris termini antecedentis in ſerie de qua agitur; A_r coëfficiens termini ordinis r^{i} in æquatione (A); $A_{\overline{r+1}}$ coëfficiens termini proxime ſequentis; a_r quantitatum a_1, a_2 --- ordine r^{us}; $a_{\overline{r+1}}$ hanc proxime ſequens; α, β, γ, -- ν, &c. coëfficientes quicunque determinati —

Si fiat

$$A_r = \frac{a_r}{1\ldots r} + \frac{\alpha a_{\overline{r+1}}}{1\ldots\overline{r+1}} + \frac{\beta a_{\overline{r+2}}}{1\ldots\overline{r+3}} + \frac{\gamma a_{\overline{r+3}}}{1\ldots\overline{r+3}} + \text{---}$$

$$\frac{\nu a_{\overline{r+m}}}{1\ldots\overline{r+m}} \text{---}$$ fiet ſeries proxime ſequens $A_{\overline{r+1}} =$

$$\frac{a_{\overline{r+1}}}{1\ldots\overline{r+1}} + \frac{(\overline{r+1}.\,C+\alpha)\,a_{\overline{r+2}}}{1\ldots\ldots\overline{r+2}} + \frac{(\overline{r+2}.\,C+\beta)\,a_{\overline{r+3}}}{1\ldots\ldots\overline{r+3}} +$$

$$\frac{(\overline{r+3}\,C+\gamma)\,a_{\overline{r+4}}}{1\ldots\ldots\overline{r+4}} + \cdots \frac{(\overline{r+m}\,C+\nu)\,a_{\overline{r+m+1}}}{1\ldots\ldots\overline{r+m+1}} + \cdots$$

His in æquatione (A) valoribus ſubſtitutis, ſub hac forma apparebit:

$$\left.\begin{array}{l} a_1 x + \dfrac{1.\,a_2 x}{1.\,2} + \dfrac{1.2.\,a_3 x}{1.\,2\,3} + \dfrac{1.2.3.\,a_4 x}{1.\,2.\,3.\,4.} + \cdots \\ \quad + \dfrac{a_2 x^2}{1.\,2} + \dfrac{3\,a_3 x^2}{1\,2\,3} + \dfrac{11\,a_4 x^2}{1.\,2.\,3.\,4} + \cdots \\ \qquad\qquad + \dfrac{a_3 x^3}{1.\,2.\,3} + \dfrac{6\,a_4 x^3}{1.\,2.\,3\,4} + \cdots \\ \text{\&c.} \qquad\qquad\qquad \text{\&c.} \end{array}\right\} = A$$

Si autem Termini omnes, qui per communes coëfficientes a_1, a_2, a_3, - - - multiplicantur, in unum colligantur, æquatio allata mutabitur in hanc:

$$a_1 x + \frac{a_2}{1.\,2}(x^2+x) + \frac{a_3}{1.\,2.\,3}(x^3+3x^2+2x) + \cdots = A;$$

Quæ autem, terminis ſingulis rite diſpoſitis, redigitur ad ſequentem (B)

$$a_1 x + \frac{a_2 x\,\overline{x+1}}{1.\,2} + \frac{a_3 x\,\overline{x+1}\,\overline{x+2}}{1.\,2.\,3} + \frac{a_4 x\,\overline{x+1}.\overline{x+2}.\overline{x+3}}{1.\,2.\,3.\,4} + \cdots$$

$$\frac{a_r x.\,\overline{x+1}\ldots\ldots\overline{x+r-1}}{1\ldots\ldots r} + \cdots\cdots = A.$$

§. III.

§. III.

Hanc æquationem vel paululum consideraturo facile patebit, primum terminum $a_1 x$ designare summam progressionis Arithmeticæ, in qua terminus primus vocatur a, numerus terminorum x & differentia est $= 0$, sive $a_1 x = a_1 + a_1 + a_1 + a_1 + \text{-------}$

Intelligitur etiam, terminum secundum $\frac{a_2 x . \overline{x+1}}{1.2}$ esse æqualem summæ numerorum naturalium, ductæ in a_2, sive $\frac{a_2 x \overline{x+1}}{1.2} = a_2 + 2a_2 + 3a_2 + 4a_2 + \text{----}$ posito semper numero terminorum $= x$;

Tertius terminus $\frac{a_3 . x . \overline{x+1} . \overline{x+2}}{1.2.3}$. est summa seriei $a_3 + 3a_3 + 6a_3 + 10a_3 + \text{----}$ sive summa numerorum triangularium per a_3 multiplicata; & sic porro.

§. IV.

Transformatione hac, eo, quo jam ostendimus, modo peracta, radix æquationis propositæ non difficulter apparebit. Quoniam enim æquatio (A) æquatur æquationi (B) $= A$, erit etiam

$a_1 +$

$$\left.\begin{array}{l} a_1 + a_1 + a_1 + a_1 + a_1 + --- \\ a_2 + 2a_2 + 3a_2 + 4a_2 + 5a_2 + --- \\ a_3 + 3a_3 + 6a_3 + 10a_3 + 15a_3 + --- \\ a_4 + 4a_4 + 10a_4 + 20a_4 + 35a_4 + --- \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ \&c. \qquad \&c. \qquad \&c. \end{array}\right\} = A.$$

Si omnes primi, ſecundi, &c. termini in unaquaque ſerie unam in ſummam colligantur, nova oritur ſeries, cujus terminus primus erit æqualis Summæ terminorum omnium, qui primi in ſingula ſerie exſtant; ſecundus erit æqualis ſummæ ſecundorum, & ſic porro; ſive (C)

$$(a_1+a_2+a_3+---)+(a_1+2a_2+3a_3+---)+(a_1+3a_2+6a_3+\cdot\cdot)\cdot\cdot\cdot\cdot=A.$$

X autem in unaquaque ſerie numerum terminorum deſignans, eandem etiam in hac retinet ſignificationem; quantitates a_1, a_2, a_3, - - - ſemper determinatæ ſunt; & A quantitatem quovis caſu datam exhibet. Collectis itaque unam in ſummam tot terminis, quot requiruntur ad efficiendum numerum $= A$, numerus terminorum, ad ſummam hanc conficiendam necesſariorum, dabit valorem rationalem integrum & poſitivum ipſius radicis x.

§. V.

Quod ad negativas fractasque radices attinet, facile etiam illæ erui posſunt. Si enim in æquatione qua-

quadam radix x valore gaudeat negativo (id quod primo jam inſpectu apparet), ſubſtituendo $x = - y$ æquatio nova oritur, quæ, eodem ac ſupra jam oſtendimus modo, reſoluta, dabit valores poſitivos quantitatis y, h. e. valores negativos ipſius x. Sive, id quod eodem redit, in æquatione data mutentur ſigna coëfficientium omnium, impares dignitates quantitatis x afficientium, & calculo ut antea inſtituto, radix negativa patebit.

Neque difficilius fractæ eruuntur radices. Obſervandum autem eſt fractas in æquatione radices nullas exiſtere, niſi ſumma poteſtas quantitatis x coëfficiente quodam a gaudeat. Subſtituto itaque $ax = y$, æquatio data mutatur in aliam, radicibus integris gaudentem. Fiat $y = b$, erit $x = \frac{b}{a}$, & æquationis datæ etiam fracta radix determinata.

§. VI.

Methodum noſtram ſic generali modo explicatam, reſtat, ut applicatione ad æquationes determinati ordinis illuſtremus, atque exemplis numericis confirmemus.

Sit æquatio generalis 3:tii ordinis $A_1 x + A_2 x^2 + A_3 x^3 = A$, ſubſtituendo ſecundum

§. 2, $A_1 = a_1 + \frac{a_2}{2} + \frac{a_3}{3}$, $A_2 = \frac{a_2}{2} + \frac{a_3}{2}$, $A_3 = \frac{a_3}{6}$, eruitur $a_3 = 6A_3$, $a_2 = 2A_2 - 6A_3$, $a_1 = A_1 - A_2 + A_3$, & his valoribus in æquatione (C) §. 4 substitutis, obtinetur

$$\left.\begin{matrix} A_1 - A_2 + A_3 \\ +2A_2 - 6A_3 \\ +6A_3 \end{matrix}\right\} + \left.\begin{matrix} A_1 - A_2 + A_3 \\ +4A_2 - 12A_3 \\ +18A_3 \end{matrix}\right\} + \begin{matrix} A_1 - A_2 + A_3 \\ +6A_2 - 18A_3 \\ +36A_3 \end{matrix}$$

$$+\left.\begin{matrix} A_1 - A_2 + A_3 \\ +8A_2 - 24A_3 \\ +60A_3 \end{matrix}\right\} \left.\begin{matrix} A_1 - A_2 + A_3 \\ +10A_2 - 30A_3 \\ +90A_3 \end{matrix}\right\} + \text{----} = A$$

$$= (A_1 + A_2 + A_3) + (A_1 + 3A_2 + 7A_3) + (A_1 + 5A_2 + 19A_3) + (A_1 + 7A_2 + 37A_3) + (A_1 + 9A_2 + 61A_3) + \text{---}$$

Proponatur æquatio 3:tii ordinis completus $x^3 - 6x^2 - x + 30 = 0$, erit $A = -30$, $A_1 = -1$, $A_2 = -6$, $A_3 = +1$, & hanc ob rem $-30 = -6 - 12 - 12 - 6 + 6 + \cdots$; sufficiunt autem & 3 & 5 termini ad efficiendum summam $= -30$, unde colligitur, unam hujus æquationis radicem esse $+3$, alteram $+5$, tertiam autem negativam esse, e conspectu æquationis propositæ intelligitur. Hæc ut inveniatur, mutentur, ut in § 5 monitum est, signa coëfficientium quantitatis x dignitates impares afficientium; unde eruitur $30 = 6 + 24 + \cdots$ patet itaque esse $x = -2$.

Detur æquatio alia 3:tii ordinis $8x^3 + x^2 - 55x + 42 = 0$ erit $A_1 = -55$, $A_2 = +1$, $A_3 = 8$, $A = -42$, atque hinc $-42 = -46 + 4 + \cdots$; unde colligitur $x = 2$;

$x = 2$; Cum autem æquatio hæcce necesſario etiam negativam habebit radicem, fiet $A_1 = -55$, $A_2 = -1$, $A_3 = 8$ & $A = 42$ atque $42 = -48 - 2 + 92 +$ &c. ex quo patet esſe $x = -3$. E forma denique æquationis datæ $8x^3 + x^2 - 55x + 43 = 0$ elucet reliquam radicem esſe fractam; quæ ut pateat ponatur (ſec. § 5) $x = \frac{Z}{8}$ unde æquatio data mutatur in hanc: $Z^3 + Z^2 - 440Z + 2688 = 0$, quæ, eodem ac antea modo calculo inſtituto dabit $-2688 = -438 - 430 - 416 - 396 - 370 - 338 - 300 ----$, unde apparet esſe $Z = 7$, & $x = \frac{Z}{8} = \frac{7}{8}$

Sit adhuc $x^3 - 8x^2 + 21x - 20 = 0$, eodem ac ſupra modo eruetur $20 = 14 + 4 + 0 + 2 + ----$ ſive $x = 4$, & æquatio data per $x - 4$ diviſa, dabit $x^2 - 4x + 5 = 0$ cujus radices $x = 2 \pm \sqrt{-1}$ imaginariæ ambæ ſunt. Patet itaque radices imaginarias inveſtigationi realium non esſe impedimento.

§. VII.

Plura de his afferre & tempus &, quam nobismet propoſuimus ſervandam, brevitatis ratio vetat. Neque opus: facile enim ex his intelligi poteſt, quam arcto inter ſe, doctrina de ſeriebus arithmeticis & illa de æquationibus Algebraicis, cohæreant nexu. Neque in aliud nos ſpectavimus; difficilior enim evadit hæcce methodus in eo caſu, quo major exſtat æquationis propoſitæ radix, quam ut ad uſum communem in quovis caſu, aliis rejectis, poſſit commendari.