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The effect of islands on low frequency equatorial motions

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ABSTRACT

We develop a complete linear theory for the effects of islands on low frequency waves of the kind thought to be important in the seasonal and interannual variations of the equatorial ocean circulation. For an island whose meridional extent is small compared to R, the equatorial radius of deformation, the waves pass the island almost undisturbed: the currents incident on the island flow around it to the north and south and continue downstream in the lee of the island. For islands comparable to or greater in extent than R the mass flux incident on the island is largely reflected in a manner similar to the case of a meridionally infinite barrier. An incident equatorial Kelvin wave is reflected as long Rossby waves; symmetric long Rossby waves are reflected as equatorial Kelvin waves while antisymmetric ones stop at the island barrier. In all cases a boundary current composed of short Rossby waves forms at the eastern side of the island and accomplishes the required meridional redistribution of the zonal mass flux.

Calculations carried out for the major mid-ocean low latitude islands (the Gilberts and Galapagos in the Pacific; Sao Tome in the Atlantic; the Maldives in the Indian Ocean) show that the propagation of long waves will not be significantly affected by any of these islands and that small perturbations occur only in their immediate vicinity.

1. Introduction

The equator is a very effective waveguide, with the consequence that the equatorial ocean exhibits a rapid and robust response to seasonal and interannual variations in the wind stress. In the past decade there have been numerous observational and theoretical studies of such low frequency equatorial phenomena (see O'Brien (1979) for an extensive bibliography). Almost without exception, these studies have made heavy use of linear equatorial wave concepts to analyze the ocean's adjustment process (this is true even of the highly nonlinear numerical calculations such as Cane (1979) and Philander and Pacanowski (1980)). These ideas are essential to our understanding of such diverse phenomena as El Niño, the reversal of the Somali Current, upwelling in the Gulf of Guinea and the seasonal transports of heat out of the tropics. All of this work has ignored the presence of islands on or near

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the equator and their possible role as a barrier to wave propagation. Furthermore, it is unclear how measurements of such waves may be distorted by a nearby island. The present paper presents a complete solution to the linear problem and yields the important result that none of the equatorial islands in the world oceans alter the adjustment process significantly.

There have been three previous investigations of similar problems. Rowlands (1982) considered the long time influence of an infinitely thin island on an equatorial Kelvin wave switched on at $t = 0$ and steady thereafter. His analysis relies on the methods of Anderson and Rowlands (1976) and resulted in an integral equation for the solution. He was able to solve this equation analytically only for the asymptotic cases of large and small islands. Yoon (1981) also considered the propagation of waves past equatorial islands in a linear shallow water system. He solved the problem numerically and found that the Kelvin wave is little affected by realistic equatorial islands, but that the $n = 1$ Rossby wave is greatly altered. We agree with the former result but not the latter: as discussed in Section 5, his procedure does not properly isolate the waves of interest. Hendry and Wunsch (1973) looked at measurements at Jarvis Island (0°23′S, 160W) in the Pacific. They sought to explain the influence of the island on the equatorial undercurrent using a nonlinear inviscid steady model that neglected rotation. They obtained some agreement with the observed mass field in the upstream region but pointed out that the dynamics of the wake are obscure.

Our interest is in the effect of islands on the wave motions essential to basin-wide adjustment processes rather than on the details of the perturbations in the vicinity of the islands. Our approach is similar to Rowlands' but we obtain a complete solution with elementary mathematical methods by using the results and techniques of Cane and Sarachik (1976, 1977; henceforth CSI, CSII; also see Cane and Sarachik, 1981; henceforth CSIV). Our approach leads to simple physical interpretations of the results for very large and very small islands. We will solve the linear shallow water equations appropriate to a single baroclinic vertical mode. Throughout we employ the usual American equatorial scaling, namely, the length scale $L = (gH/\beta^2)^{1/4}$ is the equatorial deformation radius and the timescale $T = (gH\beta^2)^{-1/4}$. (H is the equivalent depth of the baroclinic mode, g the acceleration due to gravity and β is the meridional derivative of the Coriolis parameter at the equator.) We choose as our canonical problem the large t asymptotic flow that results when a wave source is switched on at $t = 0$ and remains steady thereafter (i.e., the amplitude has the form $H(t)$, where H is the Heaviside step function). As shown in CSI the results immediately generalize to all low frequency motions.

The plan of the remainder of this paper is as follows: In Section 2 we solve the problem of a Kelvin wave incident on an infinitesimally thin island. Incoming Rossby waves are examined in Section 3, with particular attention to the lowest mode. Results are extended to the case of a finite width island in Section 4, and the influence of friction is considered. Section 5 treats a chain of islands, such as the Maldive Islands in the Indian Ocean. In Section 6 we summarize our results and consider their applicability to the world's oceans.

2. Kelvin wave incident on a thin island

The formal problem that we consider first is as in Rowlands (1982): at $t = 0$ a Kelvin wave of a step function form in time $H(t)$ impinges on an infinitesimally thin island at $x = 0$. The island is oriented north-south, extending from latitude b in the south to a in the north (cf. Fig. 1). We seek the asymptotic response as $t \to \infty$. CSI (see also Anderson and Rowlands, 1976) have shown that for large t, the asymptotic motions are of three kinds:

(i) Equatorial Kelvin waves, propagating energy eastward with u and h proportional to ψ_0, the zeroth order Hermite function: $\psi_0 = \pi^{-1/4}\, e^{-y^2/2}$. The large t response is steady for an $H(t)$ time dependence and is independent of x.

(ii) Long Rossby waves, propagating energy westward. The equatorial Kelvin wave has $v \equiv 0$ and the long Rossby waves have $v \approx 0$; both satisfy the geostrophic relation

$$y\,u + \frac{\partial h}{\partial y} = 0\,. \tag{2.1}$$

Again, the large t asymptotic form is independent of t and x.

(iii) Short Rossby waves (including the mixed Rossby-gravity wave) of the form (see CSII, p. 404):

$$(u^s, v^s, h^s) = \left[\; -\frac{\partial}{\partial y}\;,\; \frac{\partial}{\partial x}\;,\; y \;\right] \{J_o\,(2\sqrt{xt})\,\chi(y)\}\,. \tag{2.2}$$

Note that as $t \to \infty$, $J_o\,(2\sqrt{xt}) \to \delta(x)$; a sum of such modes is an ever-thinning boundary layer trapped to $x = 0$.

Let $u = h = \psi_0(y)H(t)$ for $x \leqslant 0$ be the incoming Kelvin wave. The boundary conditions at the latitude of the island are:

$$u = 0 \quad \text{at} \quad x = 0_+, x = 0_- \quad \text{and} \quad b < y < a \tag{2.3}$$

and the matching conditions are

$$u,h \text{ continuous at } x = 0 \text{ and } y < b \text{ or } y > a\,. \tag{2.4}$$

West of the island, the solution is made up of the incoming Kelvin wave and a set of long Rossby waves (ii) reflected by the island. East of the island it is made up of a transmitted Kelvin wave and a set of short Rossby waves (iii).

Therefore, at $x = 0_-$, west of the island, we may write

$$\begin{aligned} u^w &= u^R + \psi_0(y) \\ h^w &= h^R + \psi_0(y)\,. \end{aligned} \tag{2.5}$$

The components of both the long Rossby waves (u^R, h^R) and of the total solution (u^w, h^w) satisfy the geostrophic relation (2.1).

East of the island at $x = 0_+$ with relations (2.2)

$$u^E = T\,\psi_0(y) - \frac{\partial \chi}{\partial y}$$

$$h^E = T\,\psi_0(y) - y\chi$$

where T is the amplitude of the transmitted Kelvin wave. The matching condition (2.4) for $y < b$ or $y > a$ may be written as

$$u^w = T\,\psi_0(y) - \frac{\partial \chi}{\partial y}$$
$$h^w = T\,\psi_0(y) + y\chi\ . \tag{2.6}$$

Since (u^w, h^w) satisfies (2.1) substituting (2.6) into (2.1) yields:

$$y\left[T\,\psi_0(y) - \frac{\partial \chi}{\partial y}\right] + \frac{\partial}{\partial y}[T\,\psi_0(y) + y\chi] = 0$$

or, since the Kelvin wave is also geostrophic,

$$-y\frac{\partial \chi}{\partial y} + \frac{\partial}{\partial y}[y\chi] = 0\ .$$

Therefore, $\chi = 0$ for $y < b$ or $y > a$. That means there is no short Rossby wave disturbance outside the latitudes of the island, a result noted by Rowlands (1982).

We may immediately conclude that beyond the latitudes of the island (i.e., $y < b$ or $y > a$), the solution west of the island is given by $u^w = h^w = T\,\psi_0(y)$ so that

$$(u^R, h^R) = (T-1)\,\psi_0(y)\ . \tag{2.7}$$

West of the island ($x = 0_-$, $b < y < a$), the relation (2.1) holds along with the boundary condition $u^w = 0$ at $x = 0$. Therefore,

$$h^w = D \quad \text{for} \quad x = 0_- \quad b < y < a$$

with D a constant: the height (or pressure) is constant within the latitudes of the island.

Applying the boundary condition $u = 0$ at $x = 0_+$ with relation (2.2) leads to:

$$\chi(y) = T\int_b^y \psi_0(y)\,dy + C \quad \text{at} \quad x = 0_+, b < y < a \tag{2.8}$$

with C a constant of integration.

At $x = 0$, $h = D$ at $y = a_-$ and $h = T\,\psi_0(a)$ at $y = a_+$, with similar relations at $y = b$. Discontinuities in h are thus possible at $y = a$ and $y = b$. Indeed, if $b \neq -a$ at least one discontinuity is unavoidable. Since west of the island u and h are in geostrophic balance [Eq. (2.1)] for all y, we must allow for the possibility of infinite zonal velocities at the points $y = a$ and $y = b$ to balance the jump in h; that is,

$$u = A\ \delta(y-a) + B\ \delta(y-b) \qquad \text{at} \qquad y = a,b$$

(where we understand the delta functions to be nonzero just off the island at $y = a_+$ and $y = b_-$).

We can now write down the form of the solution at $x = 0$ for all y:

$$\left.\begin{array}{l} u^w = u^E = T\ \psi_0(y) + A\ \delta(y-a) + B\ \delta(y-b) \\ h^w = h^E = T\ \psi_0(y) \end{array}\right\} \begin{array}{l}\text{for} \\ y \geqslant a,\ y \leqslant b\end{array} \tag{2.9}$$

$$\left.\begin{array}{ll} u^w = 0,\ h^w = D \quad \text{at} \quad x = 0_- & \\ u^E = 0,\ h^E = y\left[T\displaystyle\int_b^y \psi_0(y) + C\right] + T\ \psi_0(y) \quad \text{at} \quad x = 0_+ & \end{array}\right\} \begin{array}{l}\text{for} \\ b < y < a\,.\end{array} \tag{2.10}$$

In order to obtain the complete solution we must determine the constants A, B, C, D, and T. We do so by deriving five equations in these five unknowns. First, integrate (2.1) from a_- to a_+ and from b_- to b_+ to obtain:

$$aA + T\psi_0(a) - D = 0 \tag{2.11}$$

$$bB - T\psi_0(b) + D = 0\,. \tag{2.12}$$

It is a given condition of the problem that the solution (u^w,h^w) west of the island has as one component a Kelvin wave of unit amplitude. Using the orthogonality and completeness properties of the eigenfunctions of the shallow water equations (cf. CSI, II or the appendix to CSIV) this condition may be turned into an equation by projecting the Kelvin wave on (u^w,h^w):

$$[(\psi_0,\psi_0),\ (\psi_0,\psi_0)] = [(\psi_0,\psi_0),\ (u^w,h^w)] \tag{2.13}$$

where the inner product is defined by

$$[(u^a,h^a),\ (u^b,h^b)] \equiv \int_{-\infty}^{+\infty} (u_a\, u_b + h_a\, h_b)\ dy\,.$$

Using the expressions (2.9), (2.10) for u^w and h^w (2.13) becomes

$$2 = 2\,T\left\{\int_a^{\infty} \psi_0{}^2\ dy + \int_{-b}^{+\infty} \psi_0{}^2\ dy\right\} + A\ \psi_0(a) + B\ \psi_0(b) + D\int_b^a \psi_0(y)\ dy \tag{2.14}$$

where we have used

$$\int_{-\infty}^{+\infty} \psi_0{}^2(y)\ dy = 1\,.$$

Now integrate the continuity equation over a thin rectangle surrounding the island, i.e., $-\epsilon \leqslant x \leqslant \epsilon$, $b - \epsilon \leqslant y \leqslant a + \epsilon$, $\epsilon << 1$ to obtain

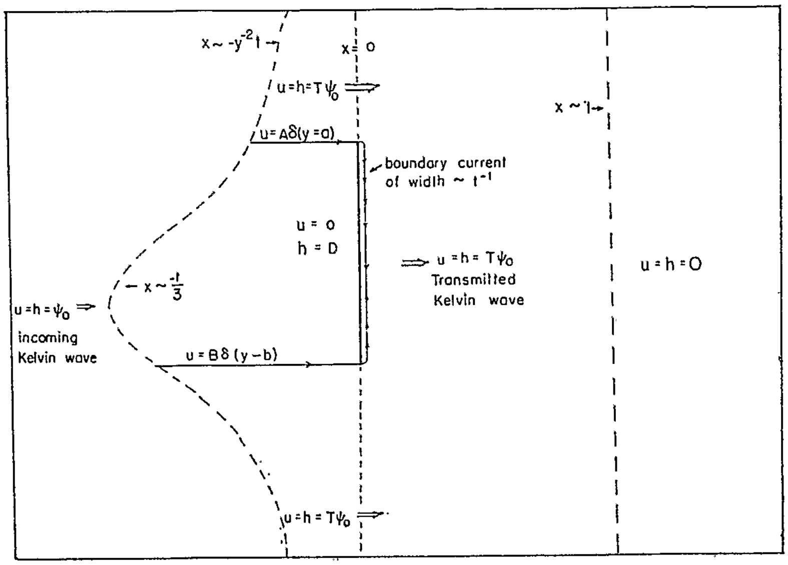

Figure 1. Sketch of the solution for an incoming Kelvin wave. See text.

$$0 = \int_{-\epsilon}^{+\epsilon} dx \int_{b-\epsilon}^{a+\epsilon} dy \frac{dh}{\partial t} + \int_{b-\epsilon}^{a+\epsilon} u^w (x = -\epsilon)\, dy - \int_{b-\epsilon}^{a+\epsilon} u^E (x = +\epsilon)\, dy +$$

$$\int_{-\epsilon}^{+\epsilon} dx\, (v(a+\epsilon) - v\,(b-\epsilon)) .$$

Noting that $v = 0$ for $y > a$ or $y < b$, that $u^w = 0$ for $x < 0$, $b < y < a$, and that $u^s = 0$ for $x > 0$ [cf. (2.2) ff.] so that $u^E = T\,\psi_0(y)$, we obtain, letting $\epsilon \to 0$,

$$A + B = T \int_b^a \psi_0(y)\, dy . \tag{2.15}$$

The constant C may be determined by noting that the boundary current at the southern tip of the island flows into the boundary current along its eastern side. The former has transport B while the latter has a total northward transport of $-C$ at $y = b$. Hence

$$C = -B . \tag{2.16}$$

The same result may be obtained more formally by integrating the continuity equation over the box $0 \leqslant x \leqslant \epsilon$, $b - \epsilon \leqslant y \leqslant a + \epsilon$. Alternately, one may project the Kelvin wave onto the solution (2.9), (2.10) east of the island and make use of (2.15) to obtain (2.16). Note that it follows from (2.2), (2.8), (2.15) and (2.16) that

$$\int_{0_+}^{\infty} v(y = a)\, dx = -A$$

that is, the eastward transport A in the northern boundary current is all carried southward by the boundary current at the eastern side of the island. This is the northside counterpart of (2.16).

The solution is sketched in Figure 1. For $t >> 1$ the asymptotic solution holds out to a longitude $x \sim t$, the farthest eastward the Kelvin wave travels in time t. To the west it will apply behind a front with the form $x \sim -1/3t$ near the equator and $x \sim -y^{-2}t$ for high latitudes (cf. CSII); this front marks the westward extent of the long Rossby waves. Except for the boundary layer at the east side of the island the asymptotic solution is independent of longitude on each side of the island. As remarked above, this eastern side boundary current becomes infinitesimally thin as $t \to \infty$. It is fed by the infinitesimally thin boundary currents at the northern and southern tips of the island.[3] There is no net zonal mass flux associated with the eastern side boundary layer; all of the net mass flux $A + B$ coming around the island is carried further eastward by the Kelvin wave of amplitude T. Equation (2.15) expresses this mass balance. The role of the eastern side boundary layer is to redistribute the mass flux $A + B$ meridionally so that the Kelvin wave may carry it off. This result is the analogue of one pertinent to western boundary currents discussed in CSII (Eq. (19) ff.): all of the mass flux incident on a western boundary is returned in the equatorial Kelvin wave; the boundary current provides the meridional transports needed to make this possible.

Equations (2.11), (2.12), (2.14), (2.15), (2.16) [together with (2.9) and (2.10)] allow us to solve the problem completely:

$$T = 2\,(a-b)\left\{2\,(a-b)\left(1 - \int_b^a \psi_0^2\,dy\right) + 2\,(a\,\psi_0(b) - b\,\psi_0(a)) \int_b^a \psi_0\,dy - \left(\psi_0(a) - \psi_0(b)\right)^2 - a\,b\left[\int_a^b \psi_0\,dy\right]^2\right\}^{-1} \tag{2.17}$$

$$A = -\frac{T}{a-b}\left\{\psi_0(a) - \psi_0(b) + b\int_b^a \psi_0\,dy\right\} \tag{2.18a}$$

$$B = \frac{T}{a-b}\left\{\psi_0(a) - \psi_0(b) + a\int_b^a \psi_0\,dy\right\} \tag{2.18b}$$

and D and C are determined by (2.11) and (2.16). It may be shown (with a great deal of tedious algebra which we shall not reproduce here) that our solution satisfies the integral equation derived by Rowlands. (Rowlands was able to obtain analytic solutions only for certain asymptotic cases.)

We have plotted the parameters as a function of a and b, the latitudes of the island in Figure 2a-d; selected slices through these figures are given in Figures 3 and 4. We note the great sensitivity of T to the position of the island: an island symmetric about the equator is much more effective in blocking a Kelvin wave than an off-equatorial island of the same extent [e.g., $T = .76$ for $a = -b = 1$ while $T =$

3. The reasons for these currents being unrealistically thin are discussed in Section 4.

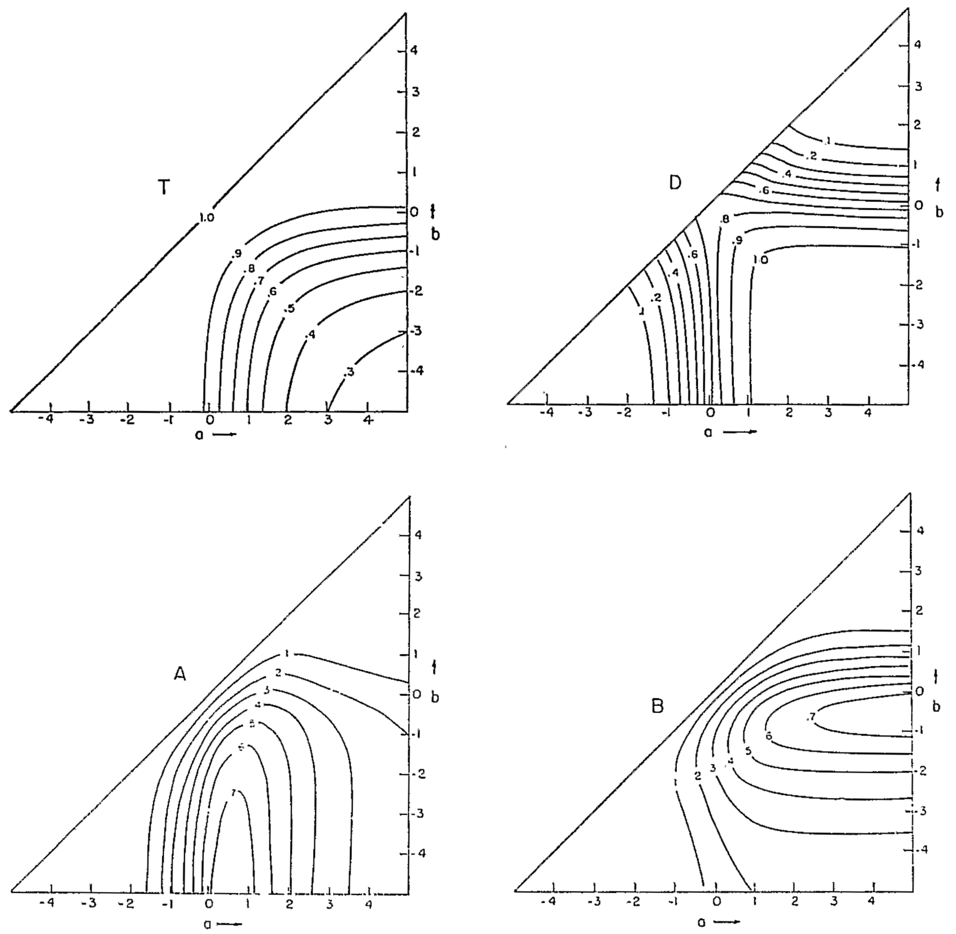

Figure 2. Solution coefficients for the problem of equatorial Kelvin wave incident on an island extending from $y = b$ to $y = a$ $(a > b)$. Cf. Eqs. (2.9), (2.10). (a) T is the amplitude of the transmitted Kelvin wave. (b) D is the height at the west side of the island. (c) A is the amplitude of the zonal current at the northern tip of the island. (d) B is the amplitude of the zonal current at the southern tip of the island.

.96 for $a = 2$, $b = 0$]. This seems an obvious consequence of the equatorially trapped structure of the Kelvin wave.

Asymptotic analysis of the cases of large and small islands will enhance our understanding of the results in Figures 1-4. First, consider islands that block the flow out to latitudes well beyond a radius of deformation of the equator: $a, -b >> 1$. In order to simplify the exposition we will consider only symmetric islands ($b = -a$); it will be clear from the discussion how these results generalize to the asymmetric case. From (2.17), (2.18) and (2.11) one may readily obtain

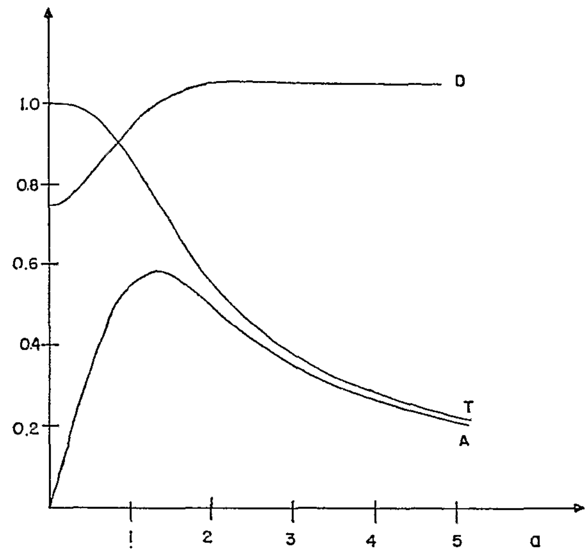

Figure 3. Coefficients as a function of a for a Kelvin wave incident on an island of half width a symmetric about the equator (i.e., $b = -a$). T, D, A are as in Figure 2; note that here $B = A$.

$$T \sim 2\,\pi^{-1/2}\,a^{-1} \tag{2.19a}$$

$$A \sim \sqrt{2}\,\pi^{-1/4}\,a^{-1} \tag{2.19b}$$

$$D = \sqrt{2}\,\pi^{-1/4} + O(a^{-3}) \tag{2.19c}$$

As expected, for $a >> 1$ the incoming Kelvin wave is largely reflected; the transmitted wave has amplitude of $O(1/a)$. To a very good approximation $[O(a^{-3})]$ the height at the western side of the island is the same as that for an infinite barrier.

An interesting physical interpretation may be offered for Eqs. (2.19). If the island is "large enough" then away from the tips of the island the reflected Rossby waves have the same structure as they would if the barrier were infinite; the motions at $y < a$ are unaware that the barrier terminates. In particular, the height sets up to have the same value as it would if the island were infinite [viz. (2.19c)]. The process that brings about this setup has been discussed by Anderson and Rowlands (1976) and CSII. Mass is carried toward the poles in a meridional current that is like a coastal Kelvin wave and is geostrophically balanced:

$$yv \sim h_x\,;$$

therefore the transport in this current is given by

$$\int_{-\infty}^{0} v(y = a)\,dx \sim D/a\,.$$

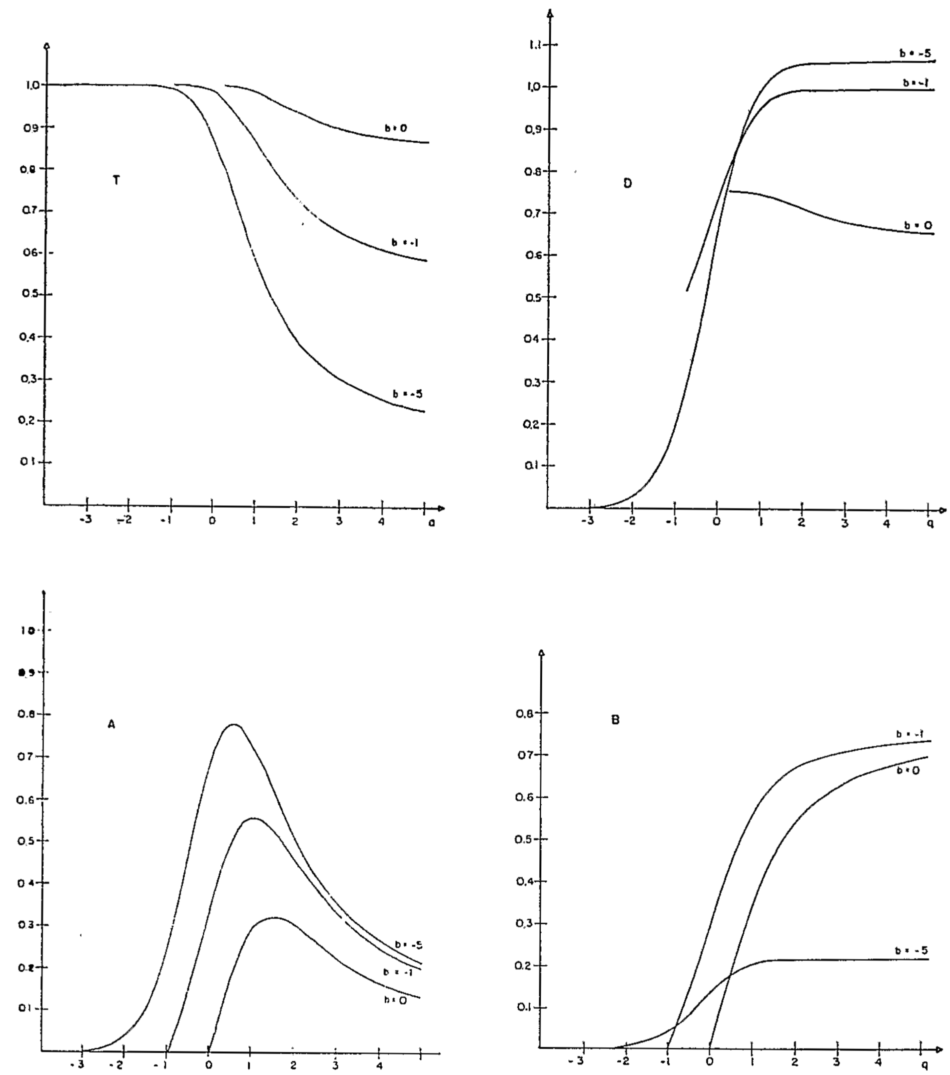

Figure 4. Coefficients for an incoming Kelvin wave as a function of a for $b = -1$, $b = 0$, $b = -5$. T, D, A and B are as in Figure 2.

From (2.19b,c) or (2.11) $D/a \sim A$: the mass transport around the tip of the island is just what the coastal Kelvin wave is able to supply. A is the same as the mass flux that would be flowing northward at latitude a if the barrier were infinite. The amplitude T of the transmitted Kelvin wave is determined by the principle [Eq. (2.15)] that the mass flux in this wave between latitudes $\pm a$ is equal to the rate $2A$ at which mass flows around the island; Eq. (2.19a) then follows. The essence of this analysis is that Eqs. (2.19) follow if a is large enough for the response to the incident Kelvin wave to have the same structure as if the barrier were infinite. From Figure 2 it appears that "large enough" means $a \sim 2$, twice the equatorial radius of deformation.

In the real oceans most of the islands situated close to the equator are small (the Galapagos are a notable exception; cf. Section 5). Therefore it is of interest to consider $a - b = 2\,\epsilon << 1$. In this case $T = 1 + 0(\epsilon)$ and the Kelvin wave is virtually undisturbed by the presence of the island. Also, with $a_* = 1/2(a + b)$

$$A = B = \epsilon\, \psi_0(a_*) + O(\epsilon^2)$$
$$D = \psi_0(a_*) + O(\epsilon^2)\,. \tag{2.20}$$

To leading order, D is the same as the height of the incident Kelvin wave and A and B do what is required for the Kelvin wave to flow around the island.

3. Incident Rossby wave motion

In this section we find the large t asymptotic response to an incoming long Rossby wave of form $H(t)$ impinging on the eastern side of a low latitude island. Again the only possible motions generated at the island are long Rossby waves to the west and Kelvin waves plus boundary-trapped short Rossby waves to the east. Let $\hat{u}$ and $\hat{h}$ be the components of the incoming wave ($\hat{v} \sim 0$). Since it is a long Rossby wave the geostrophic relation,

$$y\,\hat{u} + \frac{\partial \hat{h}}{\partial y} = 0 \tag{3.1}$$

holds and it is orthogonal to the Kelvin wave:

$$\left[(\hat{u},\hat{h}),\,(\psi_0,\psi_0)\right] \equiv \int_{-\infty}^{+\infty} (\hat{u}\,\psi_0 + \hat{h}\,\psi_0)\,dy = 0\,. \tag{3.2}$$

We apply the same methods as in the previous section. From the boundary condition (2.3) and the matching condition (2.4), together with geostrophy (2.1), (3.1), we obtain as before:

$$\chi = 0 \qquad \text{for} \qquad y < b,\, y > a$$

$$h^w = D = \text{constant at } x = 0_-$$

$$\chi = \int_b^y \hat{u} + T_R \int_b^y \psi_0 + C \qquad \text{for} \qquad b < y < a$$

with C a constant and χ a streamfunction for the short Rossby waves defined as before [Eq. (2.2)] and T_R the amplitude of the reflected Kelvin wave. The solution has the form

$$\left.\begin{aligned} u &= \hat{u} + T_R\,\psi_0(y) + A\,\delta(y-a) + B\,\delta(y-b) \\ h &= \hat{h} + T_R\,\psi_0(y) \end{aligned}\right\} \begin{array}{l} \text{at} \\ x=0 \\ y \leqslant b,\, y \geqslant a \end{array} \tag{3.3}$$

$$\left.\begin{array}{ll} u=0,\, h=D & \text{at} \quad x=0_-,\, b<y<a \\ u=0,\, h=\hat{h}+T_R\,\psi_0(y)+y\left[T_R\displaystyle\int_b^y \psi_0\,dy+\int_b^y \hat{u}+C\right] & \begin{array}{l}\text{at}\, x=0_+ \\ b<y<a\end{array} \end{array}\right\}. \tag{3.4}$$

Integrating the geostrophic relation Eq. (2.1) at $x = 0$ between a_- and a_+, and b_- and b_+ leads to:

$$a\,A + \hat{h}(a) + T_R\,\psi_0(a) - D = 0 \tag{3.5}$$

$$b\,B - \hat{h}(b) - T_R\,\psi_0(b) + D = 0\,. \tag{3.6}$$

Projecting the Kelvin wave onto (3.3), (3.4) west of the island, where there is no Kelvin wave, yields:

$$\int_{-\infty}^{+\infty}(u^w + h^w)\,\psi_0\,dy = 0 = 2\,T_R\left\{1 - \int_b^a \psi_0^2\,dy + A\,\psi_0(a)\right\} + B\,\psi_0(b) + \tag{3.7}$$

$$D\int_b^a \psi_0\,dy - \int_b^a (\hat{u}+\hat{h})\,\psi_0\,dy\,.$$

We have used (3.2) in the form

$$\left(\int_a^\infty + \int_{-\infty}^b\right)(\hat{u}+\hat{h})\,\psi_0(y)\,dy = -\int_b^a (\hat{u}+\hat{h})\,\psi_0(y)\,dy\,.$$

For incoming Rossby waves relation (2.15) becomes:

$$A + B = \int_b^a (T_R\,\psi_0(y) + \hat{u})\,dy\,. \tag{3.8}$$

The condition (2.16) that $C = -B$ still holds and may be derived as before either by projecting the Kelvin wave east of the island or integrating the continuity equation between b_- and b_+. Solving (3.5)-(3.8):

$$\begin{aligned} T_R = \frac{T}{2\,(a-b)}\Bigg\{ &(a-b)\int_b^a (\hat{u}+\hat{h})\,\psi_0\,dy + [b\,\psi_0(a) - a\,\psi_0(b)]\int_b^a \hat{u}\,dy \\ &+ (b\hat{h}(a) - a\hat{h}(b))\int_b^a \psi_0\,dy + (\psi_0(a)-\psi_0(b))\,(\hat{h}(a)-\hat{h}(b)) \\ &+ ab\left[\int_b^a \hat{u}\,dy\right]\left[\int_b^a \psi_0\,dy\right]\Bigg\} \end{aligned} \tag{3.9}$$

where T is given by (2.17), and

$$A = -\frac{1}{a-b}\left\{T_R\left(\psi_0(a) - \psi_0(b) + b\int_b^a \psi_0\,dy\right) + \hat{h}(a) - \hat{h}(b) + b\int_b^a \hat{u}\,dy\right\} \tag{3.10}$$

$$B = \frac{1}{a-b}\left\{T_R\left(\psi_0(a) - \psi_0(b) + a\int_b^a \psi_0\, dy\right) + \hat{h}(a) - \hat{h}(b) + a\int_b^a \hat{u}\, dy\right\}. \tag{3.11}$$

We will consider the particular cases where the incoming motions are given by the equatorial Rossby mode with meridional mode number n. As a measure of how well the nth mode is transmitted past the island we also compute a transmission factor γ that is the ratio of the energy in the nth mode west of the island to the incident energy (cf. Yoon, 1981). The comparable measure for the Kelvin wave is T^2, where T is the amplitude of the transmitted Kelvin wave, given by (2.17). Projecting the nth Rossby mode on the solution west of the island we obtain:

$$\gamma = \left[\left(\int_{-\infty}^{b} + \int_{a}^{\infty}\right)(\hat{u}^2 + \hat{h}^2)\, dy + A\,\hat{u}(a) + B\,\hat{u}(b) + T\left\{\left(\int_{-\infty}^{b} + \int_{a}^{\infty}\right)[\psi_0(\hat{u} + \hat{h})\, dy]\right\} + D\int_b^a \hat{h}\, dy\right]\left[\int_{-\infty}^{+\infty}(\hat{u}^2 + \hat{h}^2)\right]^{-1}. \tag{3.12}$$

Figure 5 displays the solution coefficients for an $n = 1$ Rossby mode as a function of the island coordinates a,b while Figure 6 shows a slice along the line $b = -a$ corresponding to symmetric islands. The incoming Rossby mode amplitude has been normalized as in CSIV so that $\int_{-\infty}^{\infty} (\hat{u}^2 + \hat{h}^2)\, dy = 3/8$. Comparison of these figures with the corresponding Kelvin wave figures (Figs. 2 and 3) shows that the $n =$ 1 mode is somewhat more affected by the island but will still pass a small island almost undisturbed. Figure 7 shows the transmission coefficient for the first ten Rossby modes for the case of a symmetric island. For the nth mode the value of γ approaches zero at $a \sim (2n + 1)^{1/2}$, the turning latitude. At that width the island is large enough to block the incident motion and cause it to be reflected as a Kelvin wave. The bumps in the curves are a consequence of the oscillatory nature of the u component.

With an island symmetric about the equator the behavior is qualitatively different for symmetric (e.g., Rossby modes with n odd) and antisymmetric (e.g., even n modes) motions. For symmetric motions a part of the incoming energy is reflected as a Kelvin wave, whereas for antisymmetric motions there is no reflected Kelvin wave: $T_R = 0$. The boundary current at the east side of the island connects westward currents with eastward ones, closing the fluid circuits. There is no height deviation west of the island ($D = 0$) and the currents at the northern and southern tips of the island are in geostrophic balance, $A = -h(a)/a$ and $B = h(b)/b = h(a)/a = -A$. The fluid circuit between the zonal currents A and B is closed by the boundary current east of the island. For example, if A is eastward, then fluid flows eastward from $x < 0$ along the northern tip of the island, turns southward along its eastern side and then flows westward from the island's southern tip.

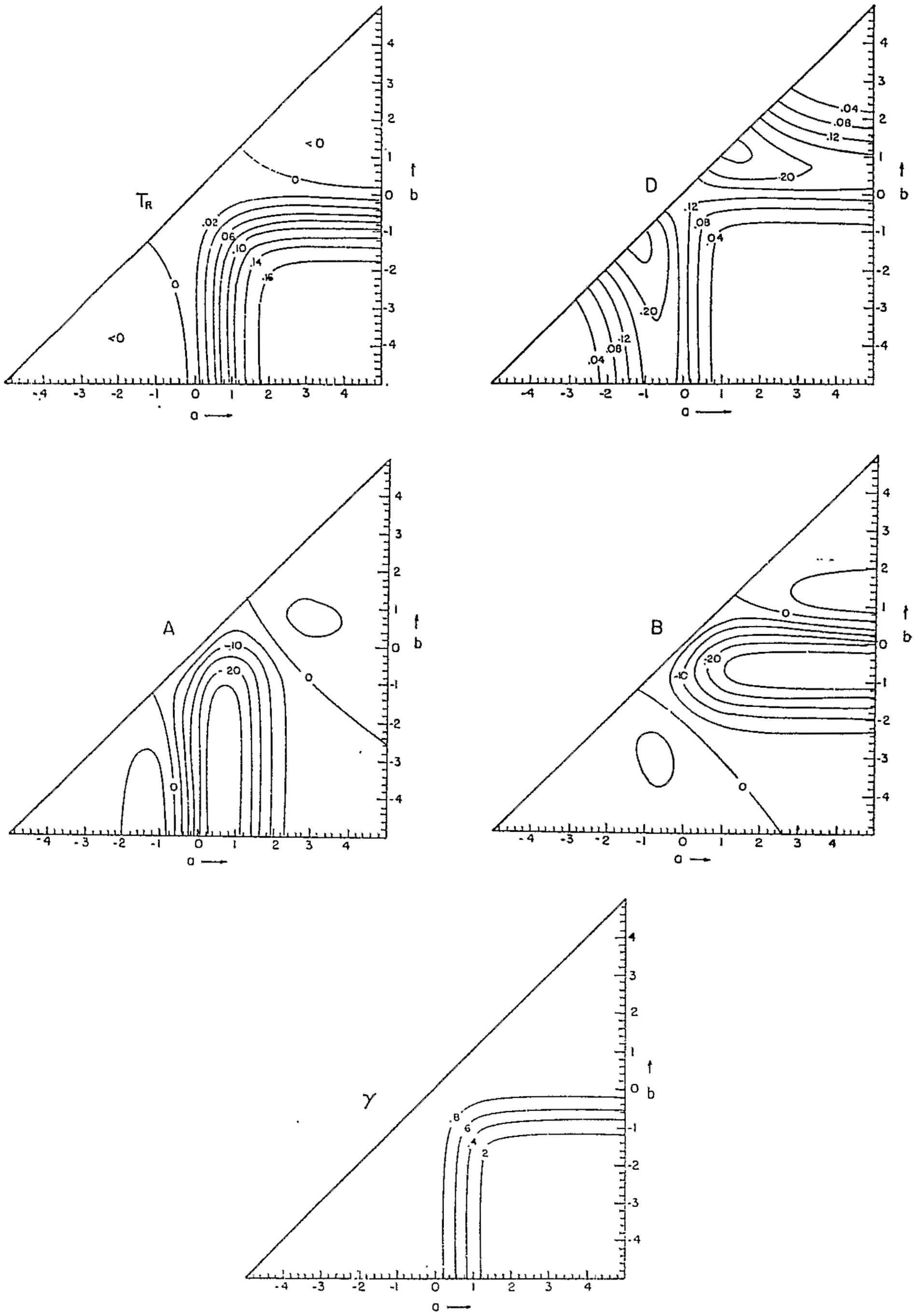

Figure 5. Solution coefficients for the problem of an $n = 1$ Rossby wave incident on an island extending from $y = b$ to $y = a$ $(a > b)$. Cf. Eqs. (3.3), (3.4). (a) T_R is the amplitude of the reflected Kelvin wave. (b) D is the height at the west side of the island. (c) A is the amplitude of the zonal current at the island's northern tip. (d) B is the amplitude of the zonal current at the island's southern tip. (e) γ is the transmission coefficient for the $n = 1$ Rossby wave; viz. (3.12).

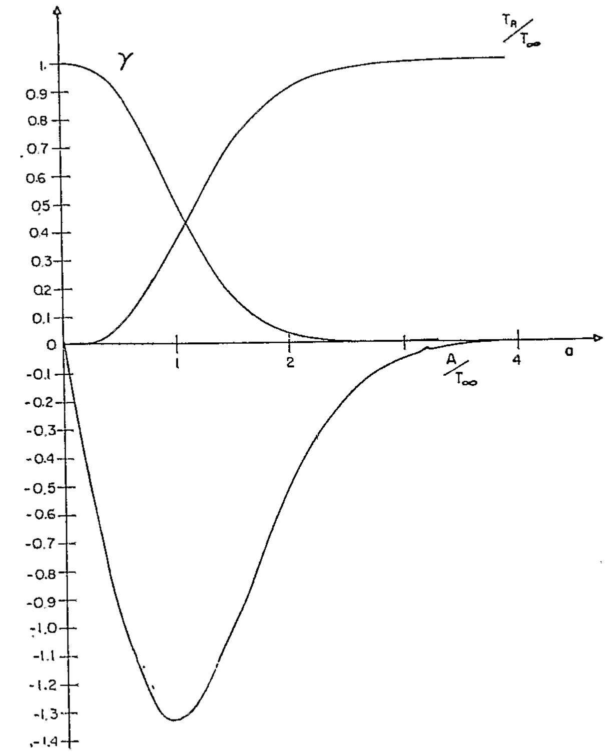

Figure 6. Coefficients as a function of a for an $n = 1$ Rossby wave incident on an island of half width a symmetric about the equator (i.e., $b = -a$). T_R, A, γ are as in Figure 5; note that here $B = A$. T_∞ is the value of T_R as $a \to \infty$.

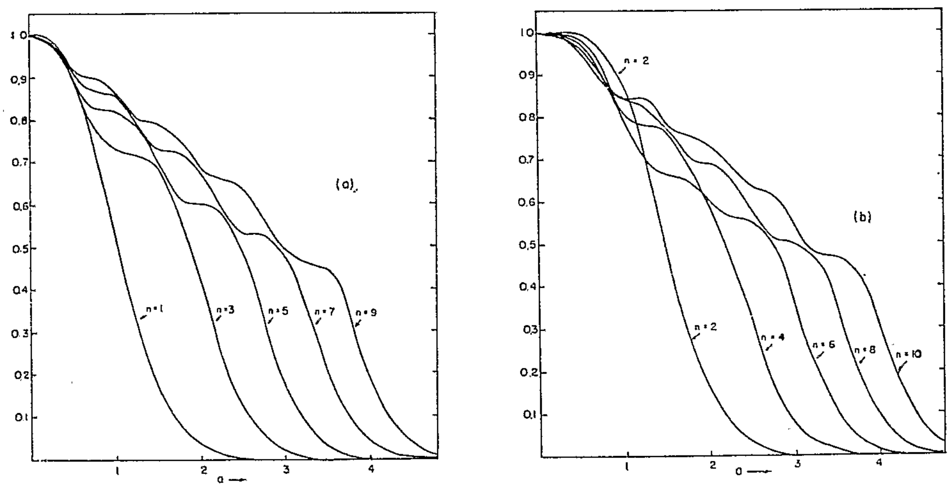

Figure 7. The transmission factor γ for the first 10 Rossby waves in the case of a symmetric island. (a) Symmetric modes, n odd. (b) Antisymmetric modes, n even.

As with the Kelvin wave case, asymptotic analysis for small and large islands is useful. For very small islands, i.e., $a - b = 2\epsilon << 1$, we obtain

$$\begin{aligned} D &= \hat{h}(a_*) + O(\epsilon^2) \\ A &= B = \epsilon\, \hat{u}(a_*) + O(\epsilon^2) \\ T_R &= O(\epsilon^2) \\ \gamma &= 1 - O(\epsilon^2) \end{aligned} \tag{3.13}$$

where $a_* = 1/2(a + b)$. The northern and southern boundary currents carry the mass flux $[\sim 2\epsilon\, u(a_*)]$ around the island to the far side. Otherwise the Rossby wave passes the island unperturbed to $O(\epsilon^2)$: the transmission coefficient is one, the height behind the island is the same as on the incoming side, and the reflected Kelvin wave has zero amplitude.

For the large island case we will again consider the symmetric islands ($b = -a$) explicitly; the results are easily generalized to $b \neq -a$. For $a >> 1$

$$T_R \sim -\frac{1}{\sqrt{2}\,\pi^{1/4}} \int_{-a}^{a} \hat{u}\, dy - \frac{\sqrt{2}}{\pi^{1/4}}\left[\frac{\hat{h}(a)}{a} + \frac{\hat{h}(-a)}{a}\right] + O(a^{-1} e^{-a^2/2}), \tag{3.14a}$$

$$A \sim -\frac{\hat{h}(a)}{a} - T_R \int_{a}^{\infty} \psi_0\, dy + O(a^{-2} e^{-a^2}), \tag{3.14b}$$

$$B \sim \frac{\hat{h}(-a)}{a} - T_R \int_{-\infty}^{-a} \psi_0\, dy + O(a^{-2} e^{-a^2}), \tag{3.14c}$$

$$D \sim O(a^{-2} e^{-a^2/2}), \tag{3.14d}$$

$$\gamma \sim 1 - \left[\int_{-\infty}^{+\infty} (\hat{u}^2 + \hat{h}^2)\, dy\right]^{-1} \left\{\int_{-a}^{+a} (\hat{u}^2 + \hat{h}^2)\, dy + a^{-1} \left[\hat{h}(a)\,\hat{u}(a) + \hat{h}(-a)\,\hat{u}(-a)\right]\right\} + O(a^{-1} e^{-a^2/2}). \tag{3.14e}$$

D is small: the Rossby modes west of the island don't get around behind the island so $h \approx u \approx 0$ there. Note that as $a \to \infty$, $T_R \to 2^{-1/2}\,\pi^{-1/4} \int_{-\infty}^{+\infty} \hat{u}(y)\, dy$; as the barrier becomes infinite all of the incident mass flux is returned in the reflected Kelvin wave, consistent with the results in CSII. Suppose for the moment that $h(a)/a$ and $h(-a)/a$ are negligible. Then (3.14a) says that the amplitude of the reflected Kelvin wave is what is required to return the zonal mass flux incident on the island. If the barrier were infinite the portion of the reflected Kelvin wave north of a would return mass eastward at a rate $T_R \int_{a}^{\infty} \psi_0\, dy$; (3.14b) shows this is now just the rate at which it flows westward along the northern end of of the island. If $h(a)/a$ is not negligible then (3.14b) means that A is the amplitude of the current required to geostrophically balance the height jump from $D \approx 0$ at $y = a_-$ to $h(a)$ at $y = a_+$. Equation (3.14a) shows that the amplitude of the reflected Kelvin wave is reduced by enough so that the mass flux incident on the island can also supply this current and its counterpart south of the island; i.e., from (3.14a,b,c):

$$-T_R \int_{-\infty}^{+\infty} \psi_0 \sim \int_{-a}^{a} \hat{u}\, dy - [A + B]\,.$$

In summary, if $\hat{h}(a)/a$ and $\hat{h}(b)/b$ are small, the reflection generated at the island is like that for an infinite barrier: T_R is determined by the principle that all the incident mass flux is returned in the Kelvin wave and $A(B)$ is equal to the mass flux in the Kelvin wave north (south) of the island. When $\hat{h}(a)/a$ $(\hat{h}(b)/b)$ is not small, this picture is modified: geostrophy demands that $A \sim \hat{h}(a)/a$ $(B \sim \hat{h}(b)/b)$; T_R reduced accordingly, so that some of the incident mass flux is available to supply the zonal boundary currents. From Figure 6 we see that for the $n = 1$ Rossby wave the large a asymptotic regime holds for $a \sim 2$.

4. Finite width islands and the influence of friction

So far we have dealt with an infinitesimally thin island. The solution for a finite width island is essentially the same. To see this consider an island of width L extending from $x = -L$ to $x = 0$; as before its meridional extent is defined by $b < y < a$. We again treat the case of an incoming Kelvin wave with a stepfunction form in time; the Rossby wave case is similar. The boundary conditions $u = 0$ [Eq. (2.3)] apply at the eastern and western ends of the islands and the matching conditions (2.4) hold north and south of it. It is still true that the only possible long time asymptotic motions are long Kelvin and Rossby waves, and short Rossby waves of the form (2.2). The argument given in Section 2 [(2.6) ff.] holds here and establishes that the short Rossby modes can only exist along the eastern side of the island. Hence beyond the latitudes of the island ($y > a$, $y < b$) the solution east of the island must have the form $u^E = h^E = T\,\psi_0$. The matching condition (2.4) then requires the same form directly north and south of the island (i.e., for $-L < x < 0$), so in the region $-L \leqslant x \leqslant 0$, $y \leqslant b$ or $y \geqslant a$ the solution must have the form (2.9). Finally, the matching and boundary conditions at $x = -L$ imply that the solution west of the island has the form (2.9), (2.10). In sum, east and west of the island the finite width solution is identical to the one for the infinitesimally thin island. North and south of the island the solution is given by (2.9): it has the form of a Kelvin wave plus infinitesimally thin currents along the north and south coasts.

Since the infinitesimally thin currents continue to exist for $L \neq 0$, Rowlands' conjecture that they were there in place of coastal Kelvin waves because of the thin island assumption is clearly incorrect. For islands of all thicknesses there in fact is the proper beta-plane version of a coastal Kelvin wave—the equatorial Kelvin wave.

[To see that the two are the same normalize the equatorial wave to be one at $y = a$ and define $\eta = y - a$, the distance from the coast. Then

$$u = h = e^{a^2/2}\, e^{-y^2/2} = e^{a^2/2}\, e^{-(a+\eta)^2/2} = e^{-a\eta}\, e^{-\eta^2/2}\,.$$

Since the scaled radius of deformation at $y = a$ is a^{-1}, for $n << a$ this is the same as the usual f-plane coastal Kelvin wave.]

The presence of this coastal wave does not obviate the need for the thin currents associated with nonzero values of A and B. Recall too that as t becomes infinite the boundary current along the eastern side of the island becomes ever-thinner (cf. Section 2). The unrealistic behavior of these three boundary currents results from the hypothesis of inviscid, steady linear flow. The addition of friction (or time-dependence or nonlinearity) will arrest the thinning of the current along the eastern coast and a steady frictional boundary layer will result (Pedlosky, 1965; CSIV).

To illustrate the effect of friction on the three thin boundary currents we add a small amount of Rayleigh friction and diffusion to the steady shallow water equations:

$$-y\,v + \frac{\partial h}{\partial x} = -r\,u \tag{4.1a}$$

$$+y\,u + \frac{\partial h}{\partial y} = -r\,v \tag{4.1b}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -r\,h \tag{4.1c}$$

where $r << 1$ is the nondimensional Rayleigh friction coefficient. (Taking $r = i\omega$ makes (4.1) the equations for periodic motions of frequency ω.) Equations (4.1) yield a single equation in v:

$$r^3\, v + r\,[\nabla^2 - y^2]\, v + \frac{\partial v}{\partial x} = 0\,.$$

Since $r << 1$ the first term may be neglected [cf. the low frequency approximation]. We expect the flow to have a boundary layer character with a narrow scale, in which case $\nabla^2 v >> y^2 v$ for islands near the equator. This assumption also implies that the flow is approximately nondivergent [cf. (4.1c)] so we may introduce a streamfunction ($v = \psi_x$, $u = -\psi_y$) that satisfies the approximate equation

$$r\,\nabla^2\psi + \psi_x = 0\,. \tag{4.2}$$

The boundary conditions on ψ are

$$\psi \to 0 \quad \text{as} \quad |x| \to \infty \quad \text{or} \quad |y| \to \infty \tag{4.3a}$$

$$\psi = \chi(y) \quad \text{at } x = 0 \quad b < y < a \tag{4.3b}$$

where χ is defined by (2.8); at the northern and southern coasts $0 = v = \psi_x$ so ψ is constant along the coast, i.e.,

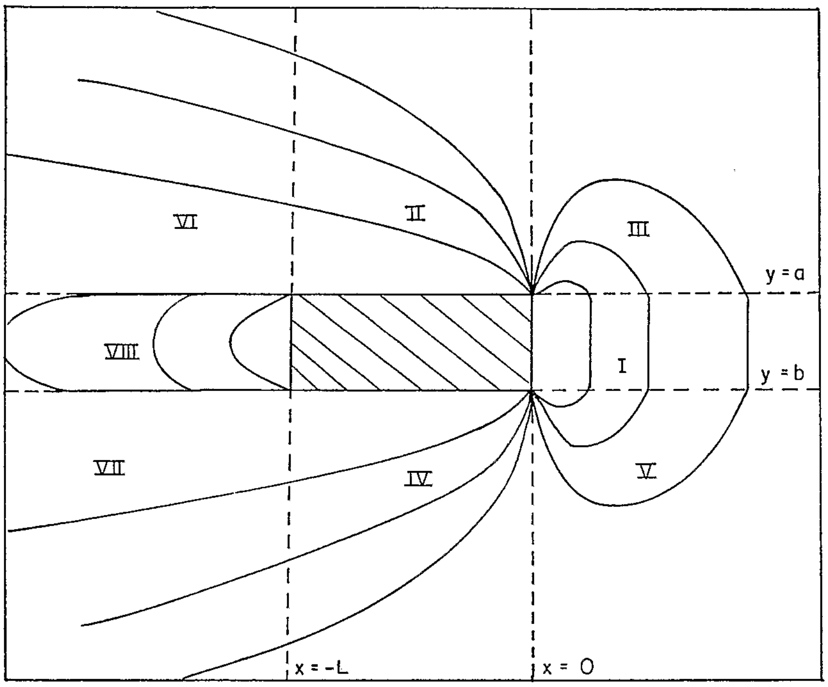

Figure 8. Boundary layer streamfunction pattern for an island similar to Jarvis Island (0° 23′ S, 160W). Dimensionally $a = 2$ km, $L = 5$ km.

$$\psi = \chi(a) \quad \text{at } y = a, -L \leqslant x \leqslant 0 \tag{4.3c}$$

$$\psi = \chi(b) \quad \text{at } y = b, -L \leqslant x \leqslant 0\,. \tag{4.3d}$$

If we now *define* A and B by

$$A \equiv \chi(a), B \equiv \chi(b) \tag{4.3e}$$

it is straightforward to show that if the boundary layer thickness is, say, $0(r\alpha)$ then (2.11) and (2.12) are satisfied and (2.14), (2.15) and (2.16) all hold with an error of $0(r\alpha)$. West of the island (2.9), (2.10) are replaced by

$$\left.\begin{aligned} u^w &\approx T\,\psi_0(y) - \psi_y(x,y) \\ h^w &\approx T\,\psi_0(y) + y\,\psi(x,y) \end{aligned}\right\} \quad x \leqslant -L, y > a \text{ or } y < b$$

$$u^w = -\,\psi_y(x,y);\; h^w = D + y\,\psi(x,y) \qquad x \leqslant -L, b < y < a\,.$$

We find the approximate solution for the boundary layer streamfunction by dividing the area around the island into separate regions (cf. Fig. 8). In region I, ($b \leqslant y \leqslant a, x \geqslant 0$), west of the island, we expect the meridional length scale is $0(1)$ while the zonal one is $0(r)$ so (4.2) simplifies to

$$r\,\psi_{xx}{}^{\mathrm{I}} + \psi_x{}^{\mathrm{I}} = 0$$

and the solution is

$$\psi^{\mathrm{I}} \sim \chi(y)\, e^{-x/r} \tag{4.4}$$

the familiar Stommel western boundary layer. Directly north of the island, in region II ($y \geq a$, $-1 \leq x \leq 0$) we again anticipate a boundary layer structure so $\partial_{yy} >> \partial_{xx}$ and (4.2) may be approximated by

$$r\,\psi_{yy}{}^{\mathrm{II}} + \psi_x{}^{\mathrm{II}} = 0 \tag{4.5}$$

an analogue of the heat conduction equation with $-x$ the timelike variable. This boundary layer will grow to the west, with thickness $y \sim (rx)^{1/2}$. [Westward is the downstream direction since the approximate equation (4.5) is appropriate for the Rossby waves with long zonal wave lengths that propagate westward.] A solution to (4.5) is

$$\psi^{\mathrm{II}} = A\ erfc\left[\frac{y-a}{2\sqrt{r|x|}}\right] \tag{4.6}$$

note that $\psi^{\mathrm{II}} = A\ \delta(y-a)$ at $x = 0$. In the corner region III ($y \geq a$, $x \geq 0$) both zonal and meridional length scales are short and the full Eq. (4.2) applies. The solution matching (4.4) and (4.6) at the boundaries is given by

$$\psi^{\mathrm{III}} \approx A\ e^{-x/r}\ erfc\left[\frac{y-a}{2\sqrt{r\,x}}\right]. \tag{4.7}$$

Equations (4.4), (4.6) and (4.7) solve (4.2) with boundary conditions (4.3); similar solutions apply south and southeast of the island. West and north of the island in region VI ($x < -1$, $y \geq a$) the solution is $\psi^{\mathrm{VI}} = \psi^{\mathrm{II}}$ (and similarly southwest of the island) while directly west of the island ($x < -L$, $b \leq y \leq a$),

$$\psi^{\mathrm{VIII}} = A\ erfc\left[\frac{y-a}{2\sqrt{r}\,|x+L|}\right] + B\ erfc\left[\frac{y-b}{2\sqrt{r}\,|x+L|}\right]. \tag{4.8}$$

The total solution for ψ is plotted in Figure 8 for the case of a small island similar to Jarvis Island at 0°23′ S, 160W in the Pacific (see Hendry and Wunsch, 1973). Note the accumulation of ψ isolines at the eastern corners of the island where the boundary layer is infinitesimally thin. Other physics (e.g., nonlinearity, horizontal viscosity) must be invoked to broaden it there.

Rowlands (1982) applied a linear theory similar to ours to observations of the equatorial undercurrent at Jarvis Island (Hendry and Wunsch, 1973) and the Galapagos (White, 1973). He argues that the undercurrent could be identified with the second baroclinic mode Kelvin wave and the theory interpreted accordingly. Our viscous, finite width island calculation is more realistic than his inviscid thin island theory and does have features in agreement with observation, notably the increased pressure at the upstream side of the island and the wakelike behavior in the lee of the island. The latter is produced by the short Rossby waves in our theory and hence shows the importance of the beta effect even for a small low latitude island. On the other hand, observations (Hendry and Wunsch, 1973) show the wake to be broader than in our calculation and the isolines of height to be more tightly grouped at the upstream side of the island, in contrast to our results (cf. Hendry and Wunsch, 1973, Fig. 6 with our Fig. 8, noting that the latter depicts streamfunction not dynamic topography). Hendry and Wunsch tried a nonlinear theory that neglected rotation to explain their observations. They obtained reasonable agreement on the upstream side but failed to reproduce the wake structure. A successful theory will have to include both nonlinearity and Coriolis effects.

5. Incoming Kelvin wave impinging on a chain of islands

The case of a Kelvin wave impinging on a series of islands oriented north-south (at $x = 0$) can be deduced in a straightforward way from the results of Section 2.

The relation (2.15) still holds for each individual island i; i.e.,

$$A_i + B_i = T \int_{b_i}^{a_i} \psi_0(y)\, dy \tag{5.1}$$

follows from integrating the continuity equation between $x = 0_+$ and $x = \epsilon$, $y = b_i$ and $y = a_i$. A_i, B_i are coefficients for the individual island i and have the same meaning as previously while b_i, a_i are the coordinates of the ith island. Relations (2.11) and (2.12) are valid with index i and (2.14) becomes:

$$2 = 2\,T \left\{ 1 - \sum_i \int_{b_i}^{a_i} \psi_0{}^2\, dy \right\} + \sum_i \Big(A_i\, \psi_0(a_i) + B_i\, \psi_0(b_i) \Big) + \sum_i D_i \int_{b_i}^{a_i} \psi_0\, dy\,. \tag{5.2}$$

We then get the solution

$$T^{-1} = \left\{ 1 - \sum_i \int_{b_i}^{a_i} \psi_0{}^2\, dy \right\} + \sum_i \left(\frac{a_i\, \psi_0(b_i) - b_i\, \psi_0(a_i)}{a_i - b_i} \right) \int_{b_i}^{a_i} \psi_0\, dy - \frac{1}{2} \sum_i \frac{(\psi_0(a_i) - \psi_0(b_i))}{a_i - b_i} - \frac{1}{2} \sum_i \frac{a_i\, b_i}{a_i - b_i} \left(\int_{b_i}^{a_i} \psi_0 \right)^2$$

$$A_i = -\frac{T}{a_i - b_i} \left(\psi_0(a_i) - \psi_0(b_i) + b_i \int_{b_i}^{a_i} \psi_0(y)\, dy \right) \tag{5.3}$$

$$B_i = \frac{T}{a_i - b_i} \left(\psi_0(a_i) - \psi_0(b_i) + a_i \int_{b_i}^{a_i} \psi_0(y)\, dy \right)$$

$$C_i = -B_i$$

$$D_i = T\, \psi_0(a_i) - b_i\, B_i\,.$$

Table 1. Parameter values for the Maldive Islands. The coordinates of the islands are given in nondimensional units. The Equatorial length scale R_1 and R_2 are from Cane and Moore (1981).

First baroclinic mode	Y_S	Y_N	A_i	B_i	D_i
Equatorial length	−2.55	−1.46	0.082	0.042	0.135
scale $R = 349$ km	−1.38	−1.32	0.008	0.007	0.296
	−0.297	−0.263	0.012	0.012	0.712
$T = 0.986$	−0.23	−0.18	0.018	0.018	0.725
	−0.115	−0.06	0.018	0.018	0.738
	0.05	0.307	0.091	0.093	0.735
	0.53	0.663	0.040	0.042	0.621
	0.69	2.282	0.144	0.29	0.384
Second baroclinic mode	−3.20	−1.84	0.044	0.016	0.056
Equatorial length	−1.74	−1.67	0.008	0.006	0.174
scale $R = 227$ km	−0.37	−0.33	0.015	0.015	0.698
	−0.29	−0.22	0.022	0.022	0.719
Transmission coefficient	−0.145	−0.08	0.023	0.023	0.738
$T = 0.989$	+0.07	0.388	0.113	0.116	0.733
	0.67	0.83	0.046	0.048	0.562
	0.87	2.875	0.094	0.26	0.283

These methods can be applied for real chains of islands such as the Gilbert Islands in the Pacific Ocean, or the Maldive Islands in the Indian Ocean to see how a series of islands impedes the propagation of an incident Kelvin wave. The Gilbert Islands are all quite small and we may immediately conclude [viz. (6.3)] that this chain will have a negligible effect on incident equatorial waves. Table 1 shows the results for the Maldive Islands. The Kelvin wave is very slightly perturbed and the transmission coefficient is over 0.98 for the two first baroclinic modes. This is in fact not very surprising because the closer the island is to the Equator, the more effective it is, and in the case of the Maldives the islands close to the Equator are very small ($<$ 0.4 radius of deformation). Therefore we can conclude, as did Yoon (1981), that the Maldive group does not affect the propagation of a Kelvin wave. However, Yoon's numerical calculations gave a smaller transmission coefficient (0.8) than ours. Some of the discrepancy is a consequence of the friction in his model and of a different representation of the islands and smaller radius of deformation, making the Maldives a more substantial barrier. We believe that most of the difference results from the presence of transients in his calculation (cf. his Fig. 5), which make his procedure for calculating γ incapable of isolating the Kelvin wave from the inertia-gravity waves. This problem is more severe for his Rossby wave calculation (where it is compounded by the time dependence chosen for his wind forcing). Unlike Yoon, we find that the $n = 1$ Rossby wave is hardly affected by the Maldives: the transmission coefficient is over .95 for both the first and second baroclinic modes and the amplitude of the reflected Kelvin wave is very small.

The Galapagos [0°15′ N to 1°15′ S, 91°30′ W, taking the 2000 m isobath as a boundary] are the most important islands that might affect the propagation of equatorial waves. In fact, for the first two baroclinic modes the Kelvin wave transmission coefficient T is over .98. [Asymptotic analysis shows that for small islands near the equator $T = 1 - O(\epsilon\, a_*^2, \epsilon^3)$, where ϵ is the half-width of the island and a_* the mean latitude.] For the $n = 1$ Rossby wave γ is over .98 for the first baroclinic mode and over .96 for the second. The pressure field around the Galapagos will differ only slightly from the open ocean values, so measurements near the islands will be representative of the wave (cf. Ripa and Hayes, 1981). In summary, we have the strong result that low frequency equatorial waves will propagate virtually unchanged past any of the world's mid-ocean equatorial islands.

6. Discussion

We have obtained a complete analytic solution for the influence of equatorial islands on steady low frequency waves. If the island is small [$a - b << 1$ radius of deformation] then the waves pass it almost undisturbed, with the mass flux incident on the upstream side flowing around it about equally to the north and south and continuing on downstream in the lee of the island. If the island is large [a, $|b| \gtrsim 2$ radii of deformation] then the principal response is organized as it would be if the island barrier were meridionally infinite. An incident Kelvin wave is largely reflected as long Rossby waves and the height D west of the island is the same as it would be for an infinite barrier. Also, the mass flux A (B) around the island at latitude a (b) is the same as the infinite barrier poleward mass flux at the same latitude. The amplitude T of the Kelvin wave transmitted past the island is then determined by $A + B$, the total amount of mass flowing around the island.

With an incident Rossby wave the height D east of the island is near zero and the strength of the currents flowing around the island are determined by the need to geostrophically balance the jump in height [e.g., from $h(a)$ to $D \approx 0$ at $y = a$]. The mass flux incident on the island is largely reflected in an equatorial Kelvin wave, with some loss to feed the eastward flow at the island's northern and southern tips. If the island is large enough to extend beyond the wave's turning latitudes then virtually all of the mass flux in the wave is incident on the island and most of it is reflected in the Kelvin wave. In this case the mass flowing around the island is equal to the poleward mass flux that would exist at the same latitudes if the island were infinite.

Our solution was first obtained for the case of infinitesimally thin islands and then extended to islands with finite zonal extent. The solution is essentially the same in the two cases. In particular, the infinitesimally thin boundary currents along the islands' western, northern and southern sides continue to exist with a finite width island. The inclusion of additional physics—nonlinearity, time dependence, and/or viscosity—will broaden these boundary currents. We carried through such a calculation with a simple Rayleigh friction formulation.

Much of the previous work on equatorial islands has been concerned with their influences on the equatorial undercurrent. Observations at Jarvis Island [0°23′ S, 160W) in the Pacific (Hendry and Wunsch, 1973) and Sao Tome (0, 9E) in the Atlantic (Hisard *et al.*, 1975) show a thickening of the undercurrent core on the upstream side of the island and a thinning of the core and a meandering current structure on the downstream side. Our theory shows crude agreement with these features but is not consistent with other aspects of the observations. The undercurrent is a highly nonlinear phenomenon and a linear theory cannot be justified; a nonlinear approach like that of Hendry and Wunsch (1973) but also including Coriolis effects is needed for a satisfactory theory. The situation near the Galapagos Islands is even more complex and ambiguous (e.g., White, 1973; Pack and Zaneveld, 1973): the undercurrent has been slowing down for at least 50° of longitude before reaching the Galapagos and is not always apparent east of the islands. It is unlikely that this weakening is due solely to the islands.

Our linear inviscid model is more appropriate to describe the adjustment processes which occur as long equatorial waves are disturbed by low latitude islands than to describe accurately the flow around the islands. These long equatorial waves play an important role in equatorial adjustment and in particular are thought to be crucial to El Nino events along the coast of South America. Therefore, the Galapagos Islands are of special interest. From analysis of sea level fluctuation measurements on the west side of the Galapagos Islands, Ripa and Hayes (1981) have suggested the presence of equatorial waves, especially the first baroclinic mode Kelvin wave. But as they have pointed out, the presence of islands will modify the wave and may induce perturbations in the measurements.

From our theory the transmission coefficient is over 0.98 for the 1st and 2nd baroclinic mode Kelvin waves so the island does not affect the propagation of these waves. This result agrees with Yoon's (1981) numerical calculations. West of the island the sea level signal is slightly enhanced (and constant at the coast) and the thermocline is thickened (assuming it to be described by the second baroclinic mode). East of the island the thickness decreases toward the equator due to the presence of short Rossby waves. The Kelvin wave is not very affected by the Maldive Island chain either, because although they have a greater latitudinal extent the islands close to the equator are small. We also find that the Maldive Islands do not act as a significant barrier for an incoming Rossby wave, in disagreement with Yoon's conclusion. [The discrepancy results from a flaw in his procedure for calculating the Rossby wave transmission coefficient.] For islands the size of those in the Galapagos Archipelago, the Rossby wave is transmitted without major loss of energy and there is only a very weak reflected Kelvin wave. Its propagation will be more severely influenced by the mean currents in the equatorial Pacific (Philander, 1978).

Therefore, we conclude that the islands in the equatorial ocean will not affect the propagation of long equatorial waves significantly, and that the perturbations created by islands will occur only in their vicinity.

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