TSS, BOD5 AND COD ACCUMULATION AND TRANSPORT OVER URBAN CATCHMENT SURFACES: A MODELLING APPROACH

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Introduction

There is an increasing awareness of pollution problems in urban sewer management. This is the reason why, in France, a national experimental program on urban runoff pollution has been started (3). In the study reported in this paper, the data which have been collected for more than one year from the four experimental catchments (Maurepas and Les Ulis near Paris, Aix-Zup and Aix-Nord in Aix-en-Provence) have been used. We propose here to develop a modeling approach for the production-accumulation mechanisms and for the surface transport of TSS, BOD_5 and COD. For TSS, we will summarize the main results and refer the reader to already published papers (5). Several rainfall-runoff pollution modeling objectives can be theoretically defined. Nevertheless, the nature and accuracy of the available data will set certain model limits. Within the defined framework of the French experimental catchment program, monitoring was carried out at the outfalls of various watersheds. The raw data were based on several samples collected during rainfall events and thus sampling can be viewed as a spatially-varying phenomenon that can be characterized by a pollutant mean concentration value. The modeling approach we have developed is directly derived from the objectives of the national monitoring program to determine estimates of transported pollutant mass over a long duration (about one year) and for a "space scale" which relates to small catchments (less than 50 ha).

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Empirical modeling of the rainfall-runoff process has been shown to possess certain limits (5). Moreover, it appears that the determination of statistical relationsips would be unduly influenced by high magnitude events for which the probability of occurrence is quite low.

For these reasons a conceptual procedure is proposed which is compatible with the representativeness of the sampling program and with the trends arising from the data analysis (5,6).

TSS Accumulation and Transport Modelling Approach

This section summarizes the main results for the modeling approach developed earlier (2,5).

A linear TSS accumulation model, based on the following hypotheses, was selected :

- A constant production rate within a given time interval.
- An initial mass of pollutant (stock) close to the final one on the surface of the catchment when considered over a long time period.
- The transported mass during an event i is less than or equal to the available one, i.e., $Ei \leq Mdi$.

In such cases, the accumulated mass is taken to be proportional to the dry weather period:

$$Ai = Pr \times DTSi \qquad \text{(Eq. 1)}$$

in which :

Ai = accumulated mass (kg) over catchment surface during dry weather period DTSi (in time interval units) which separates rainfall events i and i-1.

Pr = TSS production rate during a time interval (kg/time interval unit).

It is then possible to write (5):

$$Pr = \frac{\sum_{i=1}^{n} Ei}{\sum_{i=1}^{n} DTSi} \qquad \text{(Eq. 2)}$$

Pr, as computed by Equation 2, was checked such that the constraint $Ei \leq Mdi$ was appropriate for each event.

The computed daily production rate for the catchments under consideration are:

- 1.7 kg ha^{-1} d^{-1} in Aix-Zup
- 2.6 kg ha^{-1} d^{-1} in Maurepas
- 3.1 kg ha^{-1} d^{-1} in Les Ulis

(we could not model masses in Aix-Nord because of unresolved problems in volume estimates).

Examination of the chronology of sampled rainfall events can determine successive residual masses over the catchment surfaces within the limits of an undetermined constant. Indeed, we have assumed that a full surface "wash up" has been reached once in order to be able to compute Pr (see Fig. 1, Maurepas example). Given that the previous hypotheses was verified, the proposed linear accumulation model was adopted in preference to others (e.g. asymptotic accumulation, power function accumulation, parabolic accumulation with an upper limited stock, different from the chosen criteria)(5). With regard to TSS transport, the objective of the modeling procedure was to reproduce the transported mass during each event and also the total transported mass within the observed event series. Three control variables were chosen for this step:

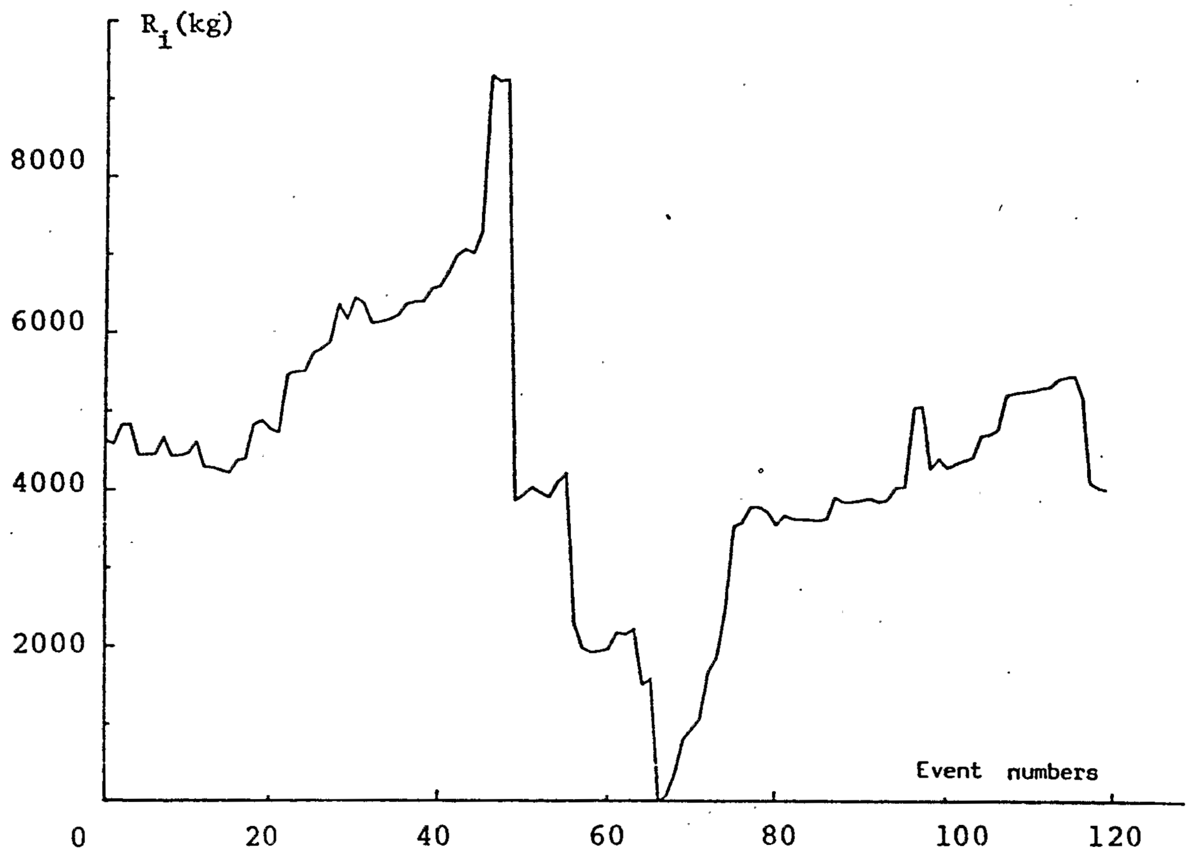

Figure 1 - Residual mass over catchment surface (Ri) after each rainfall event.

Catchment: Maurepas
(Linear accumulation model)

- Available mass, Md (kg)
- Maximum intensity within a five-minute time interval, Imax 5 (mm h^{-1})
- Runoff Volume, VR (m^3)

The chosen model is:

$$E = K.Md^{\alpha}\ Imax5^{\beta}\ VR^{\gamma} \qquad \text{(Eq. 3)}$$

in which:

E = transported mass during any event (kg)
Md, Imax5, VR = defined above
K, α, β, γ, = parameters peculiar to each catchment.

Model parameters were identified using Rosenbrock's method (4), based on the minimization of the sum of the square of the deviations between observed and computed values, subject to the two following constraints:

- The transported mass cannot be negative.
- The transported mass cannot be greater than the available one.

Table 1 summarizes the values of K, α, β and γ.

Table 1 - K, α, β, γ values for each catchment

$E = K\ Md\ \ Imax5\ \ VR$

	AIX-ZUP	LES ULIS	MAUREPAS
K	0.697	0.996	0.412
α	0.324	0.161	0.165
β	0.636	0.818	0.921
γ	0.307	0.421	0.382

In each case the different parameters have comparable values. This is interesting and positive in so far as one one of the aims is to obtain a transport law that could be generalized, at least in terms of its form.

Results expressed as deviations and criterion functions are given in Table 2. The reproduction of the total observed transported mass shows reasonable accuracy as an absolute value. For events with high transported masses, the difference between observed and computed mass varies between $\pm$ 10% and $\pm$ 30%. This is quite good, since it embraces most of the particle masses which are discharged to the receiving waters.

Table 2 - Results for the three exnerimental catchments: Total Suspended Solids - TSS

$$E = K\ Md^{\alpha}\ Imax5^{\beta}\ VR^{\gamma}$$

	AIX-ZUP	LES ULIS	MAUREPAS
Average carried mass (kg)	225.56	417.70	187.25
Average quadratic deviation as a % of the average carried mass	14.1%	10.7%	10.6%
Criterion function	50.981	92.757	20.408
Observed and computed total mass deviation (%)	- 5.8%	+ 5.3%	- 13.0%
Deviation-calibration (%)	- 6.1%	1.4%	12.0%
Deviation-verification (%)	- 5.6%	9.8%	- 33.6%

Rainfall intensity within a five minute time interval is the lowest time boundary used to determine rainfall aggressivity during the French national program. The use of other rain gauges with very low integration times should supply data on rainfall aggressivity which is even more representative. Additional climatic variables such as wind speed, humidity, etc... would further improve the results.

BOD5 and COD Accumulation and Transport Modeling Approach

As a first step, a statistical analysis of the observed mean concentrations and masses was performed (6). The results are similar to those we could get for TSS. Concerning the observed BOD_5 and COD masses, which are the parameters we are interested in, the explanatory variables seem to be Imax5 and VR, and to a lesser degree the dry weather period between two rainfall events (DTS). The transport model will be partially built up from these first results.

Accumulation Modelling Approach

Four pollutant accumulation models were tested including an asymptotic model, a power function, a parabolic model with an upper limited stock, and a linear model.

The asymptotic model:

$$M(t)= M(to)\ e^{-K(t-to)} + \frac{Pr}{K}(1 - e^{-K(t-to)}) \qquad \text{(Eq. 4)}$$

in which:

M(t) = accumulated mass in time t (kg).

K = part of the accumulated particles removed during a time interval.

Pr = pollutant production rate during a time interval (kg/time interval unit).

However, this model was found to produce daily rates much too high, for BOD_5 and COD, except if the asymptote is reached after a two month period (K = 0.025). Such a model is not appropriate for a phenomenon for which the time scale is on the order of a "few days". The power function model can be written:

$$M(t) + a\ t^n \qquad \text{(Eq 5)}$$

in which :

M(t) = accumulated mass in time t (kg).

a = function of the pollutant production rate during a time interval, Pr.

and $n < 1$.

To compute a, we can assume that for tc = x days, we have:

$$\left.\frac{d\ M(t)}{dt}\right|_{t = tc} = 0.05\ Pr \qquad \text{(Eq.6)}$$

It is then possible to compute a and Pr for different values of n. Plausible values of daily production rates of pollutants (BOD_5 and COD) were reached for low n (n = 0.2) and high tc (tc = 60 days) (see Table 3).

Table 3 - BOD_5 and COD Daily Production Rates

($Kg\ ha^{-1}\ d^{-1}$)
n = 0.2 tc = 60 days

	AIX-ZUP	LES ULIS	MAUREPAS
BOD_5	.3	.2	.5
COD	8.6	1.5	3.7

However, as for the previous scheme, these results are not representative of the studied phenomenon. To get such values of Pr, if Equation 6 is confirmed with tc = 10 days, n should be about 10^{-3} or 10^{-4}, which has no meaning.

For the parabolic model with an upper limited stock:

$$M(t) = -\alpha \frac{t^2}{2} + Pr\ t \qquad \text{(Eq. 7)}$$

where:

M(t) = accumulated mass in time t (kg)
α = f (Pr)

It has also been assumed that for tc = x days, Equation 6 is verified. The upper limit of the stock is reached when:

$$\left.\frac{dM(t)}{dt} = 0\right|_{t = t\ lim} \qquad \text{(Eq. 8)}$$

As in the two previous modeling approaches, we have computed Pr for several hypotheses. Results do not agree with the "time-scale" of the phenomenon in so far as they seem to be plausible for t values about 60 days for BOD_5 and for COD, as given by Equation 6.

As a result, the linear accumulation model was chosen. Conceptually, it may not be the most satisfactory model, given that such a model does not consider the degradation phenomenon that should occur with BOD_5 and COD. However, the measuring procedures may not be accurate enough to allow a modeling approach that integrates pollutant degradation.

Given the hypotheses developed for TSS:

- A constant production rate within a time interval.
- An initial stock close to the final one on the surface catchment when considered over a long time observation.
- The transported mass during an event i less or equal to the available mass ($Ei \leq Mdi$).

It is then possible to write Equation 2 for BOD_5 and COD:

$$Pr = \frac{\sum_{i=1}^{n} Ei}{\sum_{i=1}^{n} DTSi}$$

Ei = transported mass during a rainfall event i

DTSi = the dry weather period which separates rainfall events i and i-1.

The computed daily production rates are in Table 4.

Table 4 - BOD_5 and COD Daily Production Rates, Linear Accumulation Model

	Pr ($kg\ ha^{-1}\ d^{-1}$)		
	AIX-ZUP	LES ULIS	MAUREPAS
BOD_5	0.22	0.26	0.16
COD	1.20	1.25	1.10

Results agree with previous hypotheses except for Les Ulis, where the final stock is very different from the initial one, considering BOD_5 as well as COD. This could be due to a measuring procedure failure but this cannot be confirmed. This is the reason why two transport modeling approaches have been tested. The first one considers the available mass as a variable of the model, the second does not (since none of the accumulation models tested is fully satisfying).

Transport Modelling Approach

As mentioned before, two approaches were tested:

- The first one, given by Equation 3, is similar to the TSS model and considers the three same control variables:

  - Available mass, Md (kg)
  - Maximum intensity within a five minute time interval, Imax5 mm/h)
  - Runoff volume, VR (m^3)

- The second one considers a two-level pollution stock on the catchment surface: a first level which is usually requested, and a second one, called "deep level", which is sometimes requested but which cannot be a bounding factor. In this case, two control variables are sufficient, Imax5 and VR. The proposed equation is given by:

$$E = K' \, Imax5^{\beta'} \, VR^{\gamma'} \qquad \text{(Eq. 9)}$$

For BOD_5 and COD, parameter optimization was carried out using Rosenbrock's method (4). In the case of the first approach, two types of constraints were considered for each event:

- The transported mass cannot be negative.
- The transported mass cannot be greater than the available one.

For the second modeling approach, only the first constraint applies.

$$* \; E = K \, Md^{\alpha} \, Imax5^{\beta} \, VR^{\gamma}$$

K, α, β and γ values are given in Table 5. Results expressed as deviations and criterion functions are in Tables 6 and 7 for BOD_5 and COD, respectively.

Table 5 - K, α, β, γ Values for each Catchment

E = K Md Imax5 VR

	AIX-ZUP		LES ULIS		MAUREPAS	
	BOD5	COD	BOD5	COD	BOD5	COD
K	1.255	0.795	0.555	0.820	0.361	0.122
α	-0.069	0.313	0.126	0.175	0.223	0.630
β	0.989	0.780	0.217	0.400	0.500	0.807
γ	0.227	0.226	0.451	0.506	0.291	0.142

Table 6 - BOD_5: Results for the Three Experimental Catchments

$$E = K\ Md^{\alpha}\ Imax5^{\beta}\ VR^{\gamma}$$

	AIX-ZUP	LES ULIS	MAUREPAS
Average carried mass (kg)	37.9	41.8	13.1
Average quadratic deviation as a % of the average carried mass	13.1%	7.9%	13.7%
Criterion function	1.208	0.528	0.145
Observed and computed total mass deviation (%)	10.4%	-6.1%	57.6%
Deviation calibration (%)	5.8%	2.5%	38.5%
Deviation-verification	26.8%	-14.7%	73.4%

Table 7 - COD: Results for the Three Experimental Catchments

$$E = K\ Md^{\alpha}\ Imax5^{\beta}\ VR^{\gamma}$$

	AIX-ZUP	LES ULIS	MAUREPAS
Average carried mass (kg)	213.8	205.2	85.5
Average quadratic deviation as a % of the average carried mass	9.8%	7.6%	12.0%
Criterion function	21.607	11.738	4.699
Observed and computed total mass deviation (%)	-5.2%	6.5%	73.4%
Deviation-calibration (%)	-1.0%	3.0%	26.6%
Deviation-verification (%)	19.8%	11.5%	137.6%

strictly speaking, this model should not have been used with Les Ulis data because one of the assumptions of the accumulation model was not met. The results are not as good as they were with TSS, especially those on the Maurepas catchment. There are large differences between calibration and verification results. Therefore, if the reproduction of the total observed transported mass is quite accurate (excluding Maurepas), we cannot say that the proposed modelling approach is a fully satisfying one.

$$* \ E = K' Imax5^{\beta'} \ VR^{\gamma'}$$

K', β' and γ' values are given in Table 8. Results expressed as deviations and criterion functions are in Tables 9 and 10 for BOD_5 and COD, respectively.

Table 8 - K', β', and γ' Values for each Catchment

$$E = K' \ Imax5^{\beta'} \ VR^{\gamma'}$$

	AIX-ZUP		LES ULIS		MAUREPAS	
	BOD5	COD	BOD5	COD	BOD5	COD
K'	0.994	1.252	1.214	1.875	0.158	0.745
β'	1.085	0.793	0.216	0.398	0.462	0.739
γ'	0.148	0.464	0.459	0.579	0.534	0.487

Deviations between observed and computed masses are similar to those of the previous model. The reproduction of the total observed transported mass is not as good as it was with TSS, since differences between calibration and verification results are not at all negligible. However, results are of the same order as those obtained previously. The investigation shows that for the events with high transported masses, the differ-

Table 9 - BOD_5: Results for the Three Experimental Catchments

$$E = K' \, Imax5^{\beta'} \, VR^{\gamma'}$$

	AIX-ZUP	LES ULIS	MAUREPAS
Average carried mass (kg)	37.9	41.8	13.1
Average quadratic deviation as a % of the average carried mass	12.6%	7.8%	11.5%
Criterion function	1.121	0.520	0.103
Observed and computed total mass deviation (%)	-1.3%	-3.1%	25.1%
Deviation-calibration (%)	-0.25%	2.4%	13.0%
Deviation-verification (%)	-5.4%	-8.7%	35.0%

Table 10 - COD: Results for the Three Experimental Catchments

$$E = K' \, Imax5^{\beta'} \, VR^{\gamma'}$$

	AIX-ZUP	LES ULIS	MAUREPAS
Average carried mass (kg)	213.8	205.2	85.5
Average quadratic deviation as a % of the average carried mass	9.1%	7.4%	10.9%
Criterion function	18.502	11.150	3.948
Observed and computed total mass deviation (%)	-8.2%	10.0%	36.0%
Deviation-calibration (%)	-12.4%	-1.7%	-0.4%
Deviation-verification (%)	6.4%	27.0%	87.5%

ence between observed and computed masses is usually between $\pm$ 10% and $\pm$ 30%, which is interesting in so far as it embraces most of the particle masses discharged to the receiving waters.

For the same level of accuracy, it seems more appropriate to use the second model, because it is not necessary to compute available masses of pollutant. For TSS, we think that the inclusion of climatic variables such as wind speed, humidity, etc... would further improve the results.

Likewise, more rain gauges, having very low integration times, could have shown much higher intensities with shorter time intervals (30 seconds or 1 minute) that would be even more representative of rainfall aggressivity. In urban areas, rainfall aggressivity must be considered as the most important index to explain the transport of pollutants, since once they have been entrained from the ground surface they are almost certain to be flushed by the runoff.

Several other formulations were not adopted (6), including:

$$E = K' \, Md \, (1 - e^{-\beta' \, Imax5}) \; VR^{\gamma'}$$

$$E = K' \, (1 - e^{-\beta' \, Imax5}) \; VR^{\gamma'}$$

$$E = K \, Md^{\alpha'} \left(1 - \frac{1}{Imax5^{\beta'} \; VR^{\gamma'}}\right)$$

$$E = K' \, Imax5^{\beta} + \delta \, DR + \gamma \, VR + \alpha$$

in which:

DR = runoff duration (days).

They were not adopted for several reasons:

- A mean quadratic deviation which varies too much from one catchment to another.

- The criterion function shows high values.
- Differences between calibration and verification are high.

Conclusions

Using data from a national experimental measurement program, the modeling objective was to reproduce the total TSS, BOD_5 and COD loads for selected urban catchments. A two-step approach involving accumulation and transport was first performed. This approach led to good results with TSS. A linear accumulation model was chosen, which depends on a constant daily production rate and on the assumption that over a long time period the total mass produced will be removed. Simulation and rainfall-runoff TSS transport was achieved using a three-variable model (available mass, rainfall intensity within a five minute time interval and runoff volume). The results are good ($\pm$ 5%) in so far as over a long time period the total transported mass can be reproduced by the following model:

$$E = K \ Md^{\alpha} \ \ Imax5^{\beta} \ \ VR^{\gamma}$$

The same two-step approach did not lead to the same level of accuracy for BOD_5 and COD, given that catchments could not satisfy the assumptions in the accumulation models tested.

A one-step approach was then tried to simulateon rainfall-runoff BOD_5 and COD transport. In this case, only two control variables were retained. This model is conceptually different because it assumes that available mass is not a limiting factor. Results are good enough ($\pm$ 10%) so that over a long time period, the total transported masses of BOD_5 and COD can be reproduced using the following relationship:

$$E = K' \ Imax5^{\beta'} \ VR^{\gamma'}$$

The results, however, are not as good as those obtained for TSS.

For the Maurepas catchment, BOD_5 and COD computed balances are always overestimated, irrespective of the approach used. No explanation could be given for the discrepancy. More generally, for each pollutant and for each of the retained approaches, with respect to small events the reproduction of the observed masses is not very good, because the relative influence of one or another of the variables is not well known. For large-scale events, the level of accuracy (from $\pm$ 10% to $\pm$ 30%) seems to be very acceptable.

The modeling of pollutant accumulation and transport phenomena presented here is undoubtedly subject to improvement. Such an improvement, however, would require the acquisition of more and different data such as wind speed, humidity, etc.., or a methodology and a measurement protocol better suited to pollutant sampling. Likewise, other modelling objectives might be be developed. Such formulations would necessarily provide alternative views of the modeled phenomena, since they would depend on shorter time intervals. Such formulations could include pollutogram reconstruction, real-time management, and the prediction of receiving water discharge.

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