Forecast and Monte Carlo simulation of Zaire River flow

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ABSTRACT

Based on daily Zaire flow data in Kinshasa from 1905 to 1985, a periodic autoregressive model (PAR-model) is constructed which allows Monte Carlo flow generation over long periods (1,000 years for example), and point and interval forecasts with time steps of one month and one week. Model tests on monthly flow distributions, and on maximum and minimum flow volumes of different durations (extreme events) give positive results. Forecasting precision is better than obtained previously. A trial to improve forecast precision by rainfall at a number of stations in the basin (leading indicator model) was a failure; this is attributed to the very slow response of the river. Using results obtained in the past, a technique for obtaining flows at Inga and Boma from Kinshasa flows is suggested. The forecasts can be used in planning the dredging of the river downstream of Boma in order to maintain navigation. It can also be used for real-time control of the hydro-electric power plant at Inga. The simulated series can be used to obtain an optimal operation rule and design of the Inga plant. The models developed can be easily implemented without any specific requirement for computing facilities like processing speed and storage capacity.

KEY WORDS: Periodic Autoregressive Model — Monte Carlo Simulation — Streamflow Forecast — Monthly Flow — Weekly Flow — Zaire River Flow.

RÉSUMÉ

PRÉVISION ET SIMULATION MONTE CARLO DES DÉBITS DU FLEUVE ZAÏRE

En se basant sur les mesures du débit journalier du Zaïre à Kinshasa, un modèle autorégressif périodique (modèle du type Par) est construit, qui permet la simulation Monte Carlo de séries de débit de longue durée (1 000 années par exemple), ainsi que le calcul de prévisions ponctuelles et d'intervalles de prévision, le tout à l'échelle mensuelle ou hebdomadaire. Les tests sur les distributions mensuelles et hebdomadaires ainsi que sur les volumes d'eau maximaux et minimaux de différente durée donnent des résultats positifs. Par rapport aux résultats obtenus dans le passé les prévisions sont meilleures. Un essai d'amélioration de ces prévisions à l'aide de données pluviométriques en plusieurs stations du bassin se solde par un échec ; ceci est attribué à la lenteur de la réponse du fleuve. En utilisant des résultats obtenus dans le passé, les débits à Inga et Boma sont obtenus. Les prévisions sont utiles lors de la planification du dragage en aval de Boma en vue de maintenir la navigation. Elles sont utiles aussi pour le contrôle en temps réel de la centrale hydro-électrique à Inga. La série simulée peut être utilisée pour faire fonctionner cette centrale d'une façon optimale. Les modèles développés peuvent être mis en œuvre facilement sans exigences spéciales concernant la rapidité de calcul ou la capacité de mémoire.

MOTS CLÉS : Modèle Autorégressif Périodique — Simulation Monte Carlo — Prévision du débit — Débit Mensuel — Débit Hebdomadaire — Débit du fleuve Zaïre.

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1. INTRODUCTION

The Zaire river is one of the largest rivers in the world. It is about 4,700 km long and has a drainage area of 3,747,320 km^2. The river cuts through the Crystal Mountains between Kinshasa and Matadi with high velocities, waterfalls and large depth. The river downstreams Boma is a composite system where shoals and islands divide it into numerous channels (Fig. 1). The sedimentation in this reach is remarkable.

Due to the huge area and the flatness of the basin, and also to the different climatic conditions over the two hemispheres, the flow is very smooth with composite seasonal variations. Fig. 2 shows the seasonal variation of monthly mean and monthly standard variation at Kinshasa. By comparing standard deviations with means, it is evident that the variation of the flow is relatively small.

Bultot and Dupriez (1987) extensively investigated the statistical properties of water level and flow in Kinshasa. A time series analysis was carried out. A monthly flow forecasting model was also developed by using a complicated recursive parameter estimation technique, of which the results are mentioned further on.

Van Ganse (1959) investigated the flow from the tributaries between Kinshasa and Inga and discussed how to compute flows at Inga from flow measurements at Kinshasa.

In the present paper the rate curve as computed by Demarée (1987) has been used.

On the basis of previous studies, this paper mainly deals with forecasting and Monte Carlo simulation of the Zaire river flow since these are key factors which affect management of water resources. The forecast is helpful for planning of dredging in the river section downstreams Boma and for real-time operation of the hydro-electric power plant at Inga. The Monte Carlo simulation can offer very long flow series (a thousand years for example) which enable the analysis of water resources systems under a great variety of conditions and to approach the optimal management and operation more closely than would be possible with historical flows only. Actually it may be applied to construct or improve the operation rule of the power plant, and also be useful in planning and design of future hydraulic projects. Although 80 years of observed flow are available, this is not sufficient to compute return periods of 10 or 20 years with a good precision.

The objectives of this study are:

1. to develop a monthly and weekly flow model resulting in point forecasts and interval forecasts and performing Monte Carlo simulation of flows at Kinshasa;
2. to improve the monthly flow forecasting by a leading indicator model using rainfall as the indicator, and
3. to transfer the forecasted and simulated flows at Kinshasa towards Inga and Boma to fulfill the practical requirements, mentioned above.

Among the above mentioned items the only ones which seem to have been studied before are the computation of monthly forecasts and the transfer from Kinshasa to Inga and Boma flows. Neither the weekly time step nor simulation were seemingly considered up to now.

Section 2 defines the periodic auto-regressive (Par) model with periodic AR orders, including model structure and model identification, calibration and test. Sections 3 and 4 treat the Par model application to monthly and weekly flows at Kinshasa. Section 5 discusses transferring flow from Kinshasa to Inga and Boma. In the final section the principal conclusions are drawn.

2. METHODOLOGY OF PERIODIC AUTO-REGRESSIVE (PAR) MODELS

2.1. Justification for using the Par model

When applying time series analysis techniques to hydrological series, seasonality is always one of the main factors to be tackled. There are many models proposed and applied in hydrologic practice. Delleur, Tao and Kavvas (1976) and Stedinger and Taylor (1982a, 1982b) discussed several such models like the Thomas-Fiering model, the Arma model of deseasonalized flow, Fractional Gaussian models and Disaggregation models. Noakes *et al.* (1985) investigated the periodic auto-regressive (Par) model and a number of other models for forecasting monthly river flow and concluded that the Par model including the identification technique by the partial autocorrelation function (PACF) is best. Experience shows however that the PACF technique does not always work, especially for short series of data. Recently, different kinds of conceptual models were proposed such as the shot noise model (Weiss, 1975, 1977) and Non-Gaussian Multicomponent (NGM) model (Vandewiele and Dom, 1989).

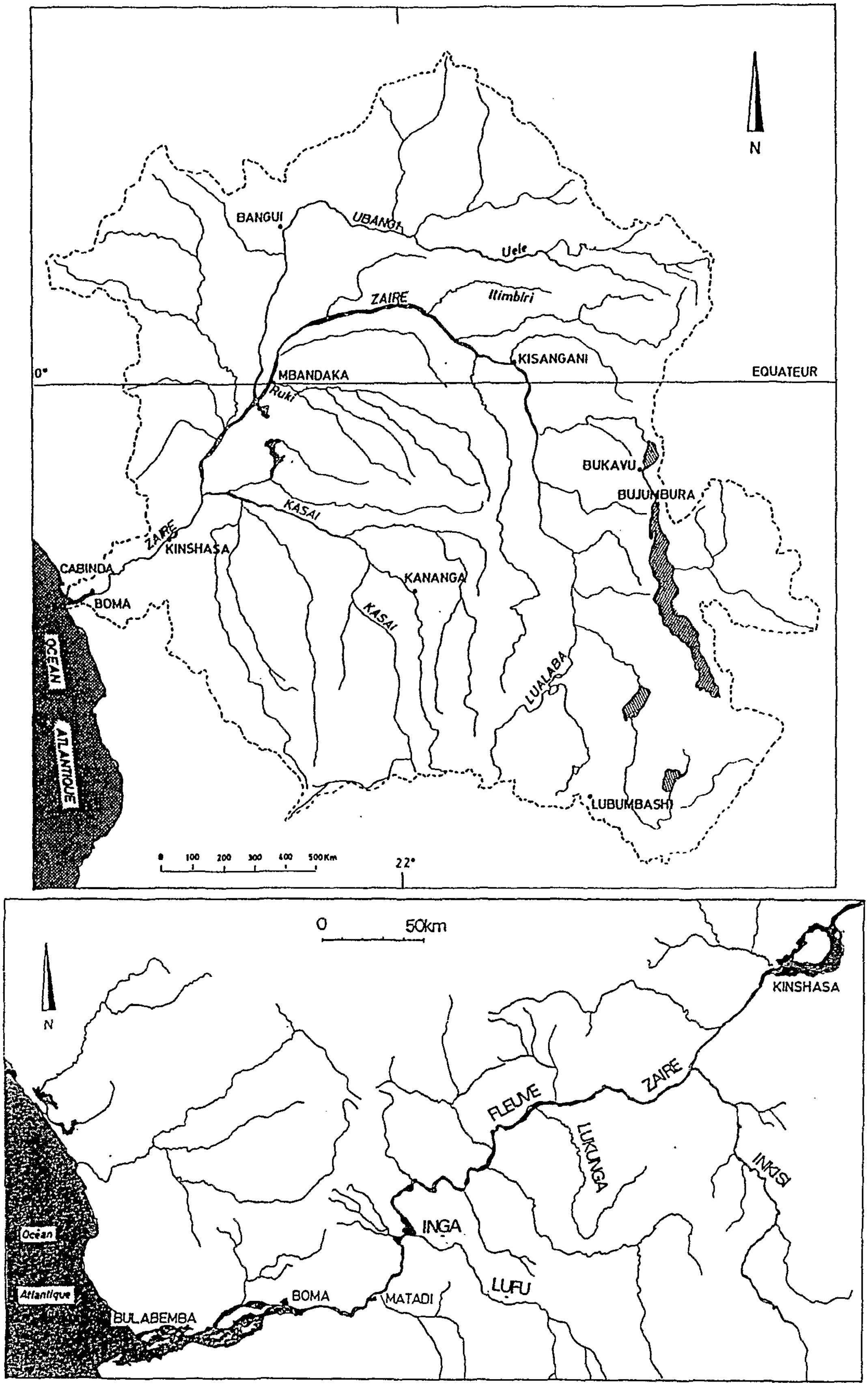

FIG. 1. — The Zaire Basin and its downstream reach (*after* Bultot and Dupriez, 1987).
*Bassin versant du fleuve Zaire et sa partie inférieure (*d'après *Bultot et Dupriez, 1987).*

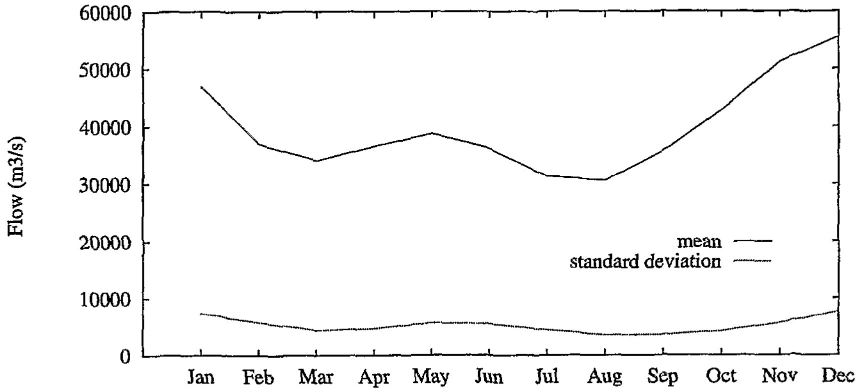

FIG. 2. — Monthly means and standard deviations of the Zaire river flow at Kinshasa (1905-1964).
Moyennes et écarts types mensuels du débit du fleuve Zaire à Kinshasa (1905-1964).

Because of the smooth behavior of the Zaire river flow and the quite long records of flow observations available, the periodic auto-regressive (Par) model is probably sufficient. Other models like the Non-Gaussian Multicomponent model may introduce unnecessary complication in such a special case. Technically, the Par model is well developed in its identification and calibration and easily implemented in practice. A special feature is that the PACF identification technique is not used here. A new identification procedure is introduced.

2.2. MODEL STRUCTURE

Let qij, with i = 1,2,...,Y; j = 1,2,...,P be the flow during season j in year i, where Y is the total number of years of observations and P denotes period (for example 12 for monthly flow) and let $\bar{q}_j$ be periodic means, then

$$\bar{q}_j = \frac{1}{Y} \sum_{i=1}^{Y} q_{ij} \tag{1}$$

By subtracting the periodic mean from q_{ij}, a new series z_{ij} is obtained

$$z_{ij} q_{ij} - \bar{q}_j$$

A Par model is written

$$A_j\,(B)\; z_{ij} = e_{ij} \tag{2}$$

where B stands for the backward shift operator, $A_j(B)$ is a polynomial in B of order w_j for season j, and e_{ij} denotes residual during season j in year i. The residuals are assumed to be uncorrelated and $N\left(0, \sigma_j^2\right)$-distributed. Apparently, the Thomas-Fiering model is a special Par with $w_j = 1$ for all seasons.

The polynomial $A_j(B)$ has the following form

$$A_j\,(B) = 1 + a_{1j}\, B + a_{2j}\, B^2 + ... + a_{w_j j}\, B^{w_j}$$

and Eq. (2) is expressed as

$$z_{ij} = -a_{1j}\, z_{ij-1} - a_{2j} \cdot z_{ij-2} - ... - a_{w_j j}\, z_{ij-w_j} + e_{ij} \tag{3}$$

It is evident that when the second index of the z variable is nonpositive, the complement with respect to P has to be taken, and the first index has to be diminished by a unit. In model identification the orders w_j have to be decided and the parameters a_{ij} are estimated by model calibration.

The application of the Par model possibly requires a preliminary transformation of the flow, for instance the logarithmic transformation or a Box-Cox transformation, in order to obtain normally distributed residuals. Whether or not such a transformation is necessary is judged after model identification and calibration by testing the normality of the residuals by means of statistical techniques like the Kolmogorov-Smirnov test.

2.3. Model identification and parameter estimation

Because the likelihood function is factorized according to the seasons, order and parameters of a season can be separately identified and estimated. This property simplifies very much the model identification and calibration.

For the purpose of easy presentation, Eq. (3) for given j is written in matrix form. Let

$$Z_j^T = \left[z_{1j}, z_{2j}, \ldots, z_{Yj}\right]$$

$$X_j = \begin{bmatrix} z_{1j-1} & z_{1j-2} & \ldots & z_{1j-w_j} \\ z_{2j-1} & z_{2j-2} & \ldots & z_{2j-w_j} \\ \ldots & \ldots & \ldots & \ldots \\ z_{Yj-1} & z_{Yj-2} & \ldots & z_{Yj-w_j} \end{bmatrix}$$

$$A_j^T = \left[-a_{1j}, -a_{2j}, \ldots, -a_{w_j j}\right]$$

$$E_j^T = \left[e_{1j}, e_{2j}, \ldots, e_{Yj}\right]$$

where $[.]^T$ denotes the transpose of a matrix. The sum of squares of residuals is given by

$$Q(A_j) = \left(Z_j^T - A_j^T X_j^T\right)\left(Z_j - X_j A_j\right)$$

The maximum likelihood and least squares estimations of A_j are identical and equal to

$$A_j = \left(X_j^T X_j\right)^{-1} X_j^T Z_j$$

The covariance matrix of A_j is estimated by

$$\mathrm{Cov}A_j = \sigma_j^2\left(X_j^T X_j\right)^{-1}$$

where

$$\sigma_j^2 = \frac{Q(A_j)}{Y - w_j - 1}$$

The half width of the 95% confidence interval (HWCI) of a_{kj} is

$$\mathrm{HWCI}(a_{kj}) = 1.96\sigma_{kj}$$

where σ_{kj} is the square root of the k-th diagonal element of $\mathrm{Cov}(A_j)$.

Identification is performed by feedback: first a sufficient great order w_j is used (3 for the monthly case and 5 for the weekly case) and the parameters and their HWCI are estimated; in a second phase w_j is diminished by one unit if there are non-significant parameters, and again estimations are carried out. This procedure is repeated until all parameters are significantly different from zero. The resulting model is tested according to the methodology explained in the next section. This procedure seems to be a good alternative for the procedures used by Noakes *et al.* (1985).

2.4. Model tests

Model tests are related to the two main applications of the model: Monte Carlo simulation and forecasting.

In order to test the quality of the simulated series, its main statistical properties are compared with those of the observed series. Two aspects are retained in this comparison. First, simulated and observed distributions for each month (or week) are compared by two sample Kolmogorov-Smirnov tests. Second, distributions of maximum and minimum flow volumes of given duration in each year are compared by the same test, because extreme events are important for applications.

Forecasting ability is also tested. Let $\hat{z}_{ij}(h)$ be the forecast of z_{ij+h} with flow information up to and including month (or week) j in year i; then h is the lead time or forecast horizon. According to Eq. (3) one has

$$\hat{z}_{ij}(h) = -a_{1j+h-1}\,\hat{z}_{ij}(h-1) - a_{2j+h-2}\,\hat{z}_{ij}(h-2) - \ldots - a_{w_j j+h-w_j}\,\hat{z}_{ij}(h-w_j) \quad (4)$$

It is understood that for $h - k \leqslant 0$, $\hat{z}_{ij}(h-k)$ is the observation, i.e.

$$\hat{z}_{ij}(h-k) = z_{ij+h-k}$$

By adding the mean $\bar{q}_{j+h}$ of Eq. (1) to $\hat{z}_{ij}(h)$, the point forecast of the flow itself is obtained. The root mean squared error (RMSE) is taken as a quality measure.

The interval forecasts are obtained by Monte Carlo simulations, conditional on the flows observed up to and including month (or week) j in year i. To estimate the conditional standard deviation 300 such simulations were generated each time.

3. MODEL APPLICATION TO MONTHLY FLOW AT KINSHASA

3.1. Parameter estimation

The 81-year observed monthly flow series is divided into two parts, the first 60-year series (1905-1964) is used for model identification and calibration and the second 21-year series (1965-1985) is preserved for Monte

Table I

Orders, parameter values and half widths of their 95 % confidence intervals (HWCI) of monthly flow model. The symbol *j* is the serial number of the months

Ordres, valeurs des paramètres et demi-longueurs de leurs 95 % intervalles de confiance (HWCI) du modèle mensuel. La lettre j *indique le numéro d'ordre des mois*

j	w_j	a_{j1}	a_{j2}	a_{j3}
1	3	1.351 ±0.235	-1.059 ±0.506	0.483 ±0.450
2	3	1.014 ±0.195	-0.629 ±0.298	0.481 ±0.262
3	2	0.837 ±0.171	-0.141 ±0.135	
4	1	0.927 ±0.142		
5	3	1.105 ±0.296	-0.493 ±0.461	0.324 ±0.282
6	1	0.904 ±0.093		
7	3	0.889 ±0.263	-0.411 ±0.325	0.311 ±0.221
8	3	0.754 ±0.223	-0.376 ±0.300	0.299 ±0.234
9	1	0.891 ±0.126		
10	1	1.024 ±0.144		
11	1	1.182 ±0.156		
12	3	1.763 ±0.333	-1.240 ±0.629	0.551 ±0.493

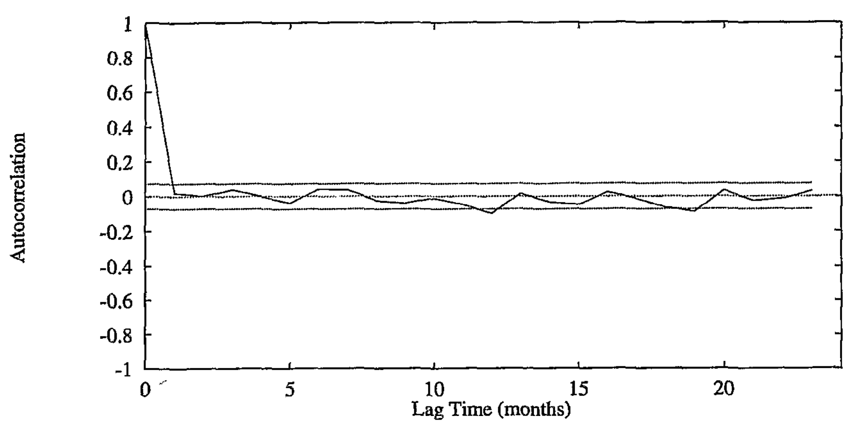

Fig. 3. — Autocorrelation of residuals of monthly Par model of Zaire flow at Kinshasa with 5 % acceptance band.
Autocorrélation des résidus du modèle mensuel du type Par avec bande d'acceptation au niveau de 5 %.

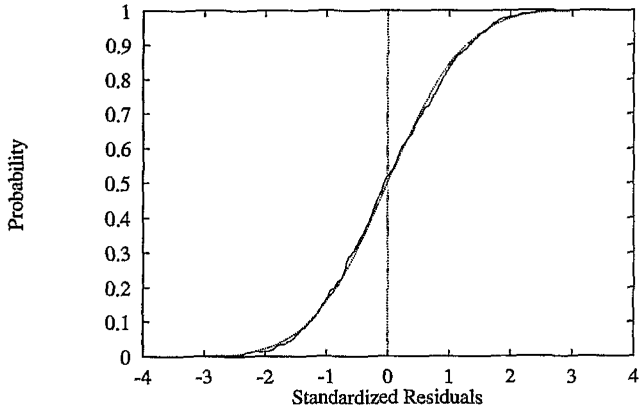

FIG. 4. — The normality test of residuals of monthly Par model of Zaire flow at Kinshasa.
Le test de normalité des résidus du modèle mensuel de type Par.

Carlo simulation and forecasting tests. It should be mentioned that due to the flow in the beginning of the 1960s being abnormally high (Bultot and Dupriez, 1987), the data in those years are included in a relatively long calibration period so as to mitigate the effect of the abnormality.

The orders and parameter values as well as the half widths of their 95% confidence intervals are shown in Table I. It has to be remarked that part of those coefficients are hardly significant. It can be seen that the coefficients in some seasons are greater than unity, for instance, in October and November. However, the overall model is stationary since there is no computational overflow when generating a very long series.

Fig. 3 shows the autocorrelation of the residuals, and Fig. 4 compares distribution of residuals and the theoretical normal distribution. It follows that the residuals are uncorrelated and normally distributed so that a preliminary flow transformation is unnecessary.

3.2. MODEL TEST ON MONTE CARLO SIMULATION

In order to carry out model test by Monte Carlo simulation, a thousand year series is generated. Simulated and observed distributions are compared by a two sample Kolmogorov-Smirnov test. The results are shown in Table II. It is seen that there are no significant differences between the two distributions at 5% significance level.

Maximum and minimum flow volumes of given duration (one and two months durations are considered here) in each year are taken and the resulting simulated and observed distributions are compared by a two sample Kolmogorov-Smirnov test. The results are shown in Table III. Again there are no significant differences at 5% significance level.

The conclusion is that the observed and generated flow series behave in the same way as for all aspects which are important in the applications using generated series.

TABLE II
Two sample Kolmogorov-Smirnov test results comparing simulated and observed flow distributions by month.
The 5 % percentage point is 0.2968
Les résultats des tests de Kolmogorov-Smirnov comparant les distributions du débit observé et simulé pour chaque mois.
La valeur correspondant au niveau de signification de 5 % est de 0,2968

month	K-S statistic	month	K-S statistic
1	0.177	7	0.101
2	0.258	8	0.258
3	0.153	9	0.291
4	0.188	10	0.146
5	0.103	11	0.170
6	0.146	12	0.175

TABLE III
Two sample Kolmogorov-Smirnov test results comparing simulated and observed maximum and minimum flow volume distributions. The 5 % percentage point is 0.2968
Les résultats des tests de Kolmogorov-Smirnov comparant les distributions observé et simulé des volumes d'eau annuels maxima et minima. La valeur correspondant au niveau de signification de 5 % est 0,2968

maximum yearly flow volume	duration (months)	K-S statistic	minimum yearly flow volume	duration (months)	K-S statistic
	1	0.190		1	0.183
	2	0.194		2	0.179

3.3. MODEL TEST ON FORECASTING ABILITY

Figs. 5 and 6 illustrate the point and interval forecasts with horizons of 1 and 2 months, the observed flow and the forecast errors in the period 1965-1974. The root mean squared error (RMSE) is computed for all seasons and horizons, and also for fixed horizons. Moreover the relative RMSE, which is defined as the ratio between the root mean squared error and the overall average of the flow (41,289 m^3/s) is computed as a function of lead time for given horizons and compared with the result of Bultot and Dupriez (1987) and of the Thomas-Fiering model (a special form of Par) in Fig. 7. Fig. 7 shows that the Par model performs better than Bultot and Dupriez's model and the Thomas-Fiering model.

4. MODEL APPLICATION TO WEEKLY FLOW AT KINSHASA

4.1. PARAMETER ESTIMATION

The 60-year weekly flow series (1905-1964) is taken for model identification and calibration and the remaining 21 years (1965-1985) is preserved for Monte Carlo simulation and forecasting tests. The maximum order is taken equal to 5.

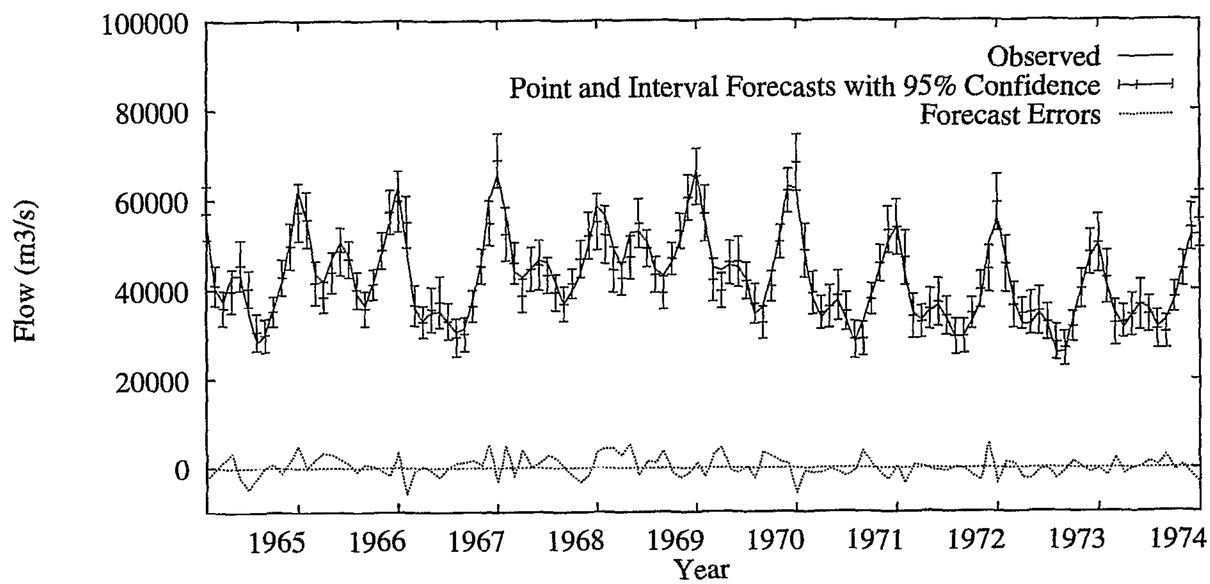

FIG. 5. — Observations, point and 95 % interval forecasts and forecast errors with lead time 1 of monthly Kinshasa flow in the period 1965-1974.
Observations, prédictions ponctuelles, intervalles des prédictions de niveau 95 % et erreurs de prédiction à horizon d'un mois du débit mensuel pendant la période 1965-1974.

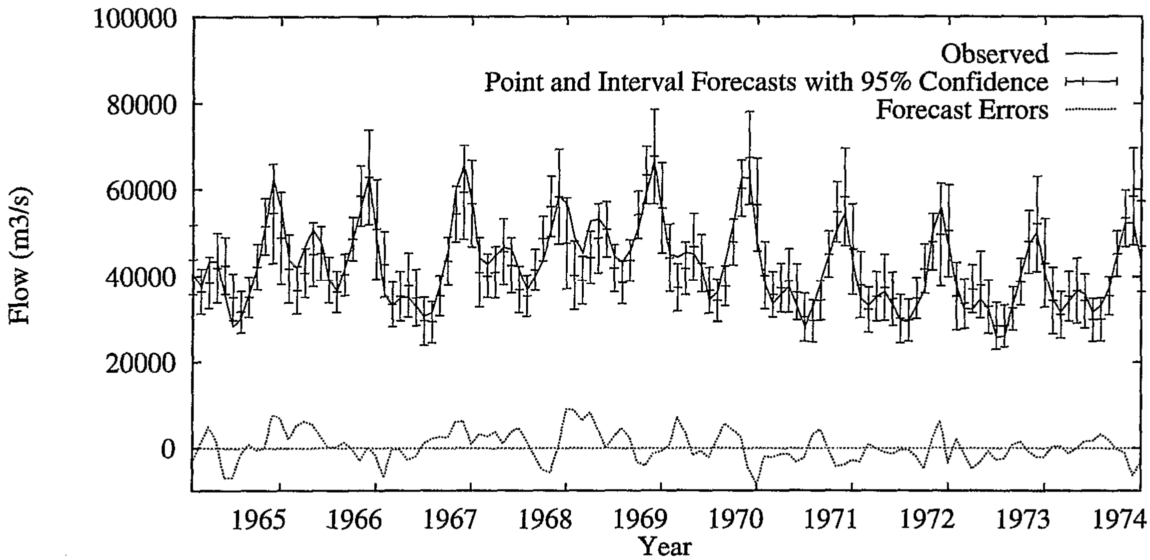

FIG. 6. — Observations, point and 95 % interval forecasts and forecast errors with lead time 2 of monthly Kinshasa flow in the period 1965-1974.
Observations, prédictions ponctuelles, intervalles des prédictions de niveau 95 % et erreurs de prédiction à horizon de deux mois du débit mensuel pendant la période 1965-1974.

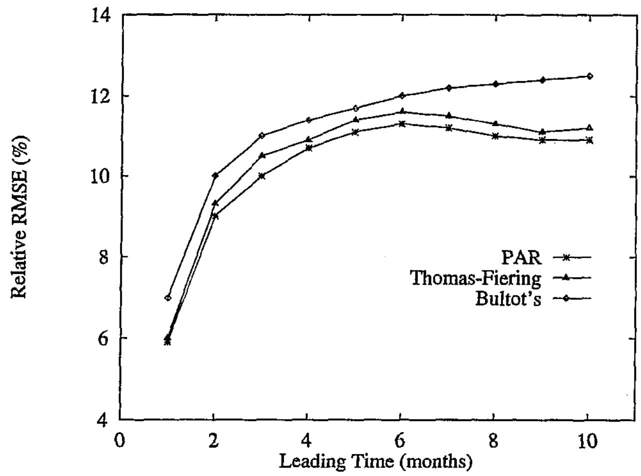

FIG. 7. — The relative root mean squared error of the monthly flow forecast of the Zaire in Kinshasa as a function of lead time.
Racine carrée de l'erreur quadratique moyenne (RMSE) divisée par le débit moyen de la prévision du débit mensuel en fonction de l'horizon de prévision.

The orders, parameter values and the half widths of their 95% confidence intervals are shown in Table IV. It is seen that model orders of most weeks vary from 2 to 3, and only a few weeks need orders of 4 and 5.

Fig. 8 shows the autocorrelation of the residuals, and Fig. 9 compares the distribution of the residuals with the theoretical normal one. Again a preliminary transformation is not necessary.

4.2. MODEL TEST ON MONTE CARLO SIMULATION

Monte Carlo simulation test is performed on a generated series of one thousand years. The distributions for all 52 weeks are tested by the two sample Kolmogorov-Smirnov test. Table V shows the test result. It is seen that there

TABLE IV
Orders, parameter values and half widths of their 95 % confidence intervals of weekly flow model.
The symbol j is the serial number of the weeks.
Ordres, valeurs des paramètres et demi-longueurs de leurs intervalles de confiance à 95 % (HWCI) du modèle hebdomadaire.
La lettre j *indique le numéro d'ordre des semaines.*

j	w_j	a_{j1}	a_{j2}	a_{j3}	a_{j4}	a_{j5}
1	2	1.666 ±0.164	-0.729 ±0.164			
2	3	1.885 ±0.255	-1.213 ±0.455	0.316 ±0.247		
3	3	1.706 ±0.201	-0.946 ±0.356	0.234 ±0.189		
4	2	1.495 ±0.224	-0.540 ±0.223			
5	3	1.790 ±0.221	-1.095 ±0.383	0.259 ±0.226		
6	4	1.817 ±0.247	-1.405 ±0.491	0.814 ±0.457	-0.308 ±0.227	
7	2	1.534 ±0.184	-0.575 ±0.167			
8	4	1.433 ±0.237	-0.726 ±0.443	0.489 ±0.423	-0.265 ±0.212	
9	3	1.621 ±0.191	-0.968 ±0.298	0.264 ±0.154		
10	2	1.424 ±0.205	-0.462 ±0.185			
11	3	1.678 ±0.293	-1.019 ±0.479	0.308 ±0.252		
12	2	1.600 ±0.177	-0.645 ±0.173			
13	3	1.657 ±0.303	-0.976 ±0.528	0.302 ±0.283		
14	2	1.599 ±0.259	-0.614 ±0.258			
15	3	1.727 ±0.196	-1.134 ±0.372	0.401 ±0.232		
16	3	1.889 ±0.246	-1.136 ±0.421	0.234 ±0.229		
17	3	1.862 ±0.300	-1.196 ±0.572	0.387 ±0.336		
18	3	1.893 ±0.212	-1.295 ±0.394	0.366 ±0.236		
19	2	1.554 ±0.189	-0.589 ±0.201			
20	2	1.679 ±0.213	-0.726 ±0.219			
21	3	1.504 ±0.216	-0.810 ±0.404	0.277 ±0.242		
22	3	1.633 ±0.168	-0.944 ±0.252	0.278 ±0.131		
23	2	1.412 ±0.238	-0.421 ±0.229			
24	5	1.986 ±0.218	-1.828 ±0.462	1.398 ±0.546	-0.930 ±0.403	0.393 ±0.170
25	2	1.575 ±0.286	-0.604 ±0.297			
26	4	1.830 ±0.192	-1.422 ±0.415	0.905 ±0.537	-0.344 ±0.302	
27	2	1.412 ±0.187	-0.500 ±0.186			
28	4	1.778 ±0.281	-1.465 ±0.518	0.918 ±0.566	-0.325 ±0.301	
29	2	1.442 ±0.202	-0.528 ±0.181			
30	5	1.727 ±0.173	-1.291 ±0.319	0.677 ±0.380	-0.377 ±0.363	0.214 ±0.179
31	4	1.948 ±0.274	-1.700 ±0.510	0.904 ±0.457	-0.212 ±0.206	
32	3	1.656 ±0.254	-1.155 ±0.418	0.453 ±0.236		
33	3	1.586 ±0.218	-1.023 ±0.368	0.350 ±0.219		
34	3	1.613 ±0.178	-0.975 ±0.293	0.314 ±0.170		
35	3	1.637 ±0.259	-0.990 ±0.407	0.341 ±0.207		
36	5	1.809 ±0.248	-1.473 ±0.500	0.889 ±0.556	-0.504 ±0.433	0.253 ±0.205
37	3	1.480 ±0.198	-0.746 ±0.361	0.259 ±0.220		
38	3	1.586 ±0.267	-0.940 ±0.405	0.328 ±0.210		
39	4	1.537 ±0.229	-0.968 ±0.435	0.652 ±0.423	-0.219 ±0.203	
40	2	1.703 ±0.224	-0.711 ±0.228			
41	2	1.665 ±0.180	-0.662 ±0.188			
42	2	1.477 ±0.187	-0.485 ±0.198			
43	2	1.685 ±0.253	-0.687 ±0.263			
44	2	1.661 ±0.227	-0.659 ±0.238			
45	3	2.056 ±0.244	-1.570 ±0.460	0.546 ±0.279		
46	2	1.571 ±0.151	-0.549 ±0.164			
47	2	1.915 ±0.267	-0.938 ±0.290			
48	3	2.089 ±0.209	-1.510 ±0.456	0.482 ±0.308		
49	4	1.896 ±0.299	-1.361 ±0.672	0.879 ±0.700	-0.453 ±0.389	
50	4	1.927 ±0.273	-0.587 ±0.574	-0.840 ±0.670	0.499 ±0.350	
51	2	1.818 ±0.162	-0.859 ±0.176			
52	2	1.758 ±0.217	-0.839 ±0.228			

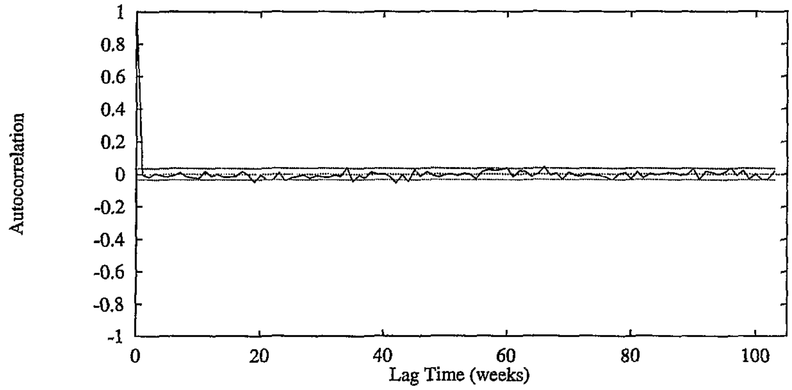

FIG. 8. — Autocorrelation of residuals of weekly Par model of Zaire flow at Kinshasa with 5 % acceptance band.
Autocorrélation des résidus du modèle hebdomadaire du type Par avec bande d'acceptation au niveau de 5 %.

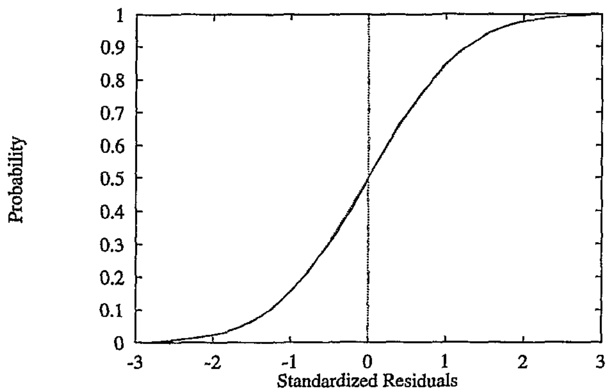

FIG 9. — The normality test of residuals of weekly Par model of Zaire flow at Kinshasa.
Le test de normalité des résidus du modèle hebdomadaire de type Par.

TABLE V
Two sample Kolmogorov-Smirnov test results comparing simulated and observed flow distributions by week.
The 5 % percentage point is 0.2968.
Les résultats des tests de Kolmogrov-Smirnov comparant les distributions du débit observé et simulé pour chaque semaine.
La valeur correspondant au niveau de signification de 5 % est de 0,2968.

week	K-S test statistics	week	K-S test statistics	week	K-S test statistics	week	K-S test statistics
1	0.208	14	0.161	27	0.177	40	0.179
2	0.197	15	0.169	28	0.125	41	0.171
3	0.139	16	0.158	29	0.154	42	0.180
4	0.152	17	0.159	30	0.100	43	0.147
5	0.185	18	0.101	31	0.138	44	0.155
6	0.241	19	0.112	32	0.215	45	0.144
7	0.230	20	0.128	33	0.286	46	0.125
8	0.222	21	0.137	34	0.260	47	0.171
9	0.206	22	0.139	35	0.218	48	0.140
10	0.134	23	0.133	36	0.227	49	0.137
11	0.141	24	0.110	37	0.257	50	0.189
12	0.156	25	0.108	38	0.294	51	0.226
13	0.164	26	0.117	39	0.240	52	0.192

TABLE VI

Two sample Kolmogorov-Smirnov test results comparing simulated and observed maximum and minimum flow volume distributions. The 5 % percentage point is 0.2968

Les *résultats des tests de Kolmogorov-Smirnov comparant les distributions observées et simulées des volumes d'eau annuels maxima et minima. La valeur correspondante au niveau de signification de 5 % est 0,2968.*

maximum	duration (weeks)	K-S statistic	minimum	duration (weeks)	K-S statistic
yearly	1	0.242	yearly	1	0.178
flow	2	0.226	flow	2	0.194
volume	4	0.212	volume	4	0.174
	8	0.185		8	0.188

is no significant difference between the observed and simulated distributions. Furthermore the test on the maximum and minimum flow volumes of fixed durations also shows a good fit between observation and simulation (Table VI). All tests are performed at the 5% significance level.

4.3. MODEL TEST ON FORECASTING ABILITY

The same techniques are used as in section 3.3. As examples, Figs. 10 to 12 show point and interval forecasts, observations and forecasting errors of the years 1965 and 1966. The root mean squared error (RMSE) is computed for all seasons and horizons, and also for fixed horizons and is compared with that of the Thomas-Fiering model, which is illustrated in Fig. 13.

Figs. 11 and 12 show an important autocorrelation of forecasting errors. However it is impossible to take advantage of this property in real time, since the corresponding observations, and consequently the errors, are not known in real time.

5. TRANSFORMING KINSHASA FLOWS INTO INGA AND BOMA FLOWS

The flows at Inga and Boma, which are important for applications, can be obtained by a correction of the simulated and forecasted flows at Kinshasa. The delay of the order of two days can be neglected with a monthly time step. and can be easily taken into account with a weekly time step.

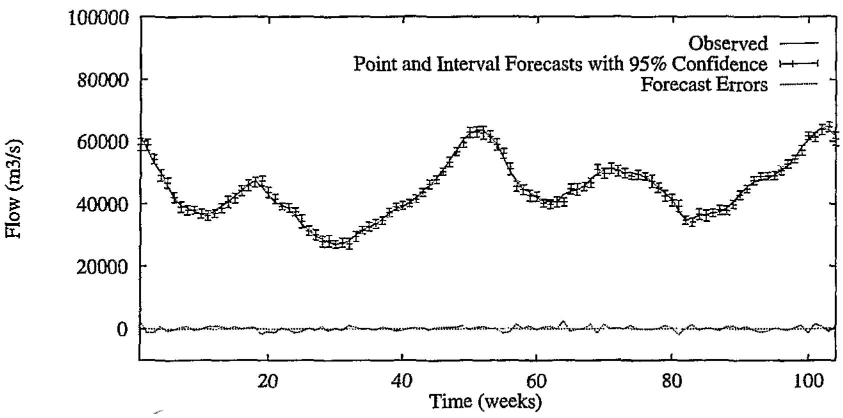

FIG. 10. — Observations, point and 95 % interval forecasts and forecast errors with lead time 1 of weekly Kinshasa flow in the period 1965-1966.

Observations, prédictions ponctuelles, intervalles de prédiction de niveau 95 % et erreurs de prédiction à horizon d'une semaine du débit hebdomadaire pendant la période 1965-1966.

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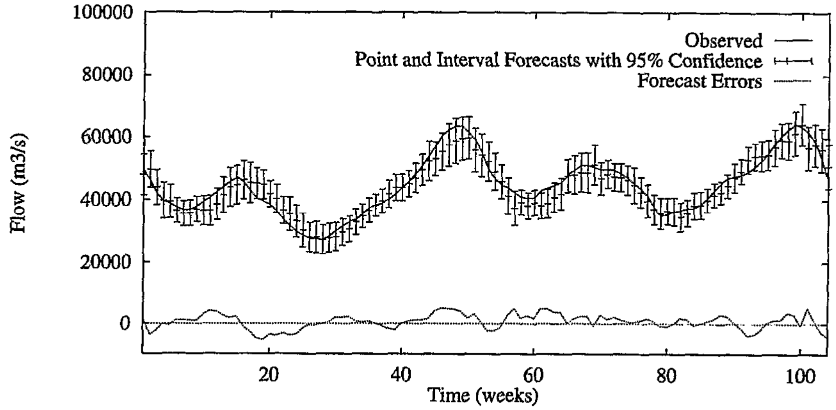

FIG. 11. — Observations, point and 95 % interval forecasts and forecast errors with lead time 4 of weekly Kinshasa flow in the period 1965-1966.

Observations, prédictions ponctuelles, intervalles de prédiction de niveau 95 % et erreurs de prédiction à horizon de quatre semaines du débit hebdomadaire pendant la période 1965-1966.

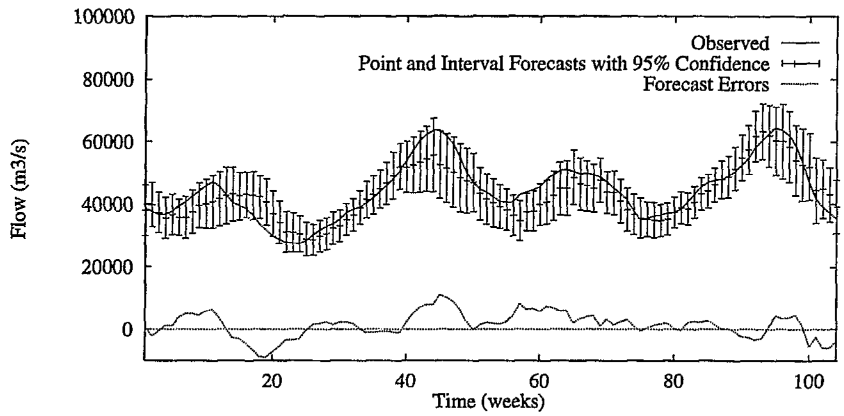

FIG. 12. — Observations, point and 95 % interval forecasts and forecast errors with lead time 8 of weekly Kinshasa flow in the period 1965-1966.

Observations, prédictions ponctuelles, intervalles de prédiction de niveau 95 % et erreurs de prédiction à horizon de huit semaines du débit hebdomadaire pendant la période 1965-1966.

The contribution of the tributaries in the reach Kinshasa-Inga has been studied by Van Ganse (1959). Since this contribution is of the order of only 2% of the Kinshasa flows, it is sufficient to add the mean monthly flows according to Table VII (after Van Ganse, 1959). The ratio of the catchment areas of the tributaries in the reaches Kinshasa-Inga and Kinshasa-Boma is approximately 1.265. Multiplying Van Ganse's corrections at Inga with the latter ratio the corrections at Boma in Table VII are obtained.

TABLE VII
Flow correction values at Inga and Boma (m^3/sec)
Corrections du débit à Inga et Boma par rapport au débit à Kinshasa (m^3/sec)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Inga	792	711	895	1376	1129	561	455	383	341	443	921	1195
Boma	964	900	1132	1741	1428	710	576	485	431	561	1165	1512

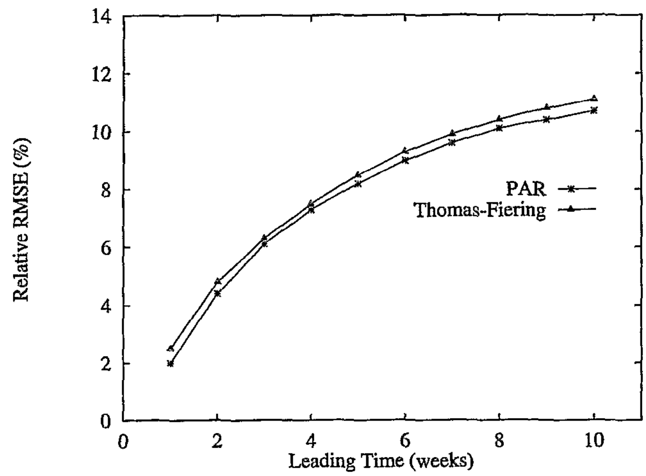

FIG. 13. — The relative root mean squared error of the weekly flow forecast of the Zaire in Kinshasa as a function of lead time.
Racine carrée de l'erreur quadratique moyenne (RMSE) divisée par le débit moyen de la prévision du débit hebdomadaire eu fonction de l'horizon de prévision.

6. CONCLUSIONS

Monthly and weekly flows of the Zaire river at Kinshasa are successfully modeled by a periodic autoregressive model (Par-model) with periodic AR orders. The fact that the likelihood function is a product of factors each corresponding to one season is used in order to estimate separately the parameters belonging to each season. In that way it is easy to find the AR-orders. This procedure is an improvement of the often uncertain method used by Noakes *et al.* (1985).

On the basis of this model a thousand year long flow series has been generated in a Monte Carlo simulation. Observed and simulated flow distributions and distributions of maximum and minimum flow volumes of given durations have been compared and differences have been found statistically non-significant. The simulated flow series can be applied to the planning and management of water resources of the Zaire River, especially to constructing a more rational operating rule of the hydro-electric power plant at Inga as well as to planning and design of future hydraulic projects.

By means of the Par-model point and interval forecasts were computed and relative root mean squared errors (RMSE) turned out to be smaller than with the model of Bultot and Dupriez (1987). The relative RMSE of the monthly model turns out to be less than 6% for one month lead time and about 9% for two months lead time. In the weekly case the relative RMSE is 2% for one week lead time and about 4.5% for two weeks lead time. Point and interval forecasts are helpful in decision-making related to the operation of the water system. The forecast is especially useful in planning of dredging to maintain navigation, and in the real-time operation of the hydro-electric power plant at Inga.

A leading indicator model which takes rainfall in five stations in the basin as leading indicators shows no improvement of the forecast performance.

The result of Van Ganse (1959) is used in order to find flows at Inga and Boma from Kinshasa flows.

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