BEING ABLE TO DO MATHS BUT YET FEELING KIND OF FREE: USING THE FLAGWAY GAME TO LEARN MATHEMATICS

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BEING ABLE TO DO MATHS BUT YET FEELING KIND OF FREE: USING THE FLAGWAY GAME TO LEARN MATHEMATICS

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Over a two-year period in 2016 and 2017 a team led by Bob Moses worked with teachers in Ireland on a project called The Algebra Project. This paper reports on the implementation of the Flagway Game in two primary schools in Ireland as part of this initiative. Data from teacher interviews and student focus groups are analysed using the theoretical framework of Engeström's activity theory (1987). The findings show that both the physical and mental tools developed by Moses and colleagues do function to develop mathematical thinking and improve enjoyment in learning mathematics. Challenges exist in the form of rules that mitigate against devoting the time needed for this kind of engagement and physical infrastructure to support social learning through physical activity.

INTRODUCTION

This paper reports on an evaluation of the implementation of a teaching and learning curriculum initiative called The Algebra Project in Ireland[1] at primary school level. The Algebra Project (AP) was founded in the USA by Bob Moses in the 1980's and has developed its own methods and curricular materials to improve students' algebraic thinking, giving much attention to the professional development of teachers, community and youth workers (Moses and Cobb, 2001). The initiative reported on here, took place in 2016 and continued into 2017. The aim of the project was to introduce the curriculum and pedagogy of the US AP to Irish teachers and students in order to develop mathematical teaching and learning. It was hoped that exposure to the evidence-based proven methods of the AP curriculum would have a positive impact on mathematics education in Ireland and on students' attitude towards mathematics and their belief that they can do mathematics. This paper reports on the implementation of an AP module called the Flagway Game in two primary schools as part of the project. Data from student focus groups and teacher interviews are presented.

LITERATURE REVIEW

The AP set out to help students in disadvantaged communities in the USA to develop their mathematical understanding so that they could progress to university level education (Moses and Cobb, 2001). The methods employed by the AP are based on experiential learning models such as those of Dewey (1938), Piaget (1952) and Kolb (2015). Moses was also heavily influenced by the work of Quine (1992) who wrote about the development of knowledge and understanding from physical experiences and in particular how theoretical language emerges from ordinary language used to describe experiences. The move from arithmetic to algebraic thinking involves students moving from using numbers as physical quantities to working with abstract variables, while simultaneously generalising operations such as addition and subtraction. A fundamental tenet of the AP is that this move from the physical experience to abstraction can be facilitated by using a five-step programme which begins with a physical event or experience and takes students through to a formal symbolic representation of this

event. AP's curriculum combines inquiry and experiential learning, which involves mathematics emerging from human experience. Mathematics is also made accessible by using real-life situations that embody rich mathematical concepts. Through the process of mathematizing these situations or events, students are encouraged to actively engage in mathematical discourse by using their everyday language for talking about mathematical concepts. This discourse leads to a focus on important mathematical features about the event and to the process of symbolization (Moses and Cobb, 2001, p122). By actively engaging in the mathematics discovery process, students encounter complex mathematical ideas that they learn to work through. This controlled movement from the concrete to the abstract allows students to build their own meanings for algebraic objects as well as helping them see that algebra is not just a collection of mysterious symbols and operations. The five steps are: 1. Physical event; 2. Picture or model of this event; 3. Intuitive (idiomatic) language description of this event; 4. A description of this event in regimented English; 5. Symbolic representation of the event (Moses, Kamii, McAllister Swap, Howard, 1989, p.433).

Dubinsky and Wilson (2013) investigated the effectiveness of the methods of the AP in a study involving low-achieving high-school students, they found that the students made significant gains over the course of the module and developed understanding comparable to that of a university student. The students took part in a seven-week programme which aimed to develop understanding of the concept of function using a module designed by the AP team on the 'Road-Colouring problem' (Budzban & Feinsilver, 2011). The authors concluded that the AP's five-step method of experiential learning allowed the students to engage meaningfully with non-trivial mathematics. Other studies have found that exposing teachers and young people to mathematically rich tasks in a fun and engaging way has the potential to empower them to see the value of mathematics in their lives and also to develop mathematical fluency (Dunphy, Dooley and Shiel, 2014).

THE FLAGWAY GAME

The module used in primary schools as part of this work was the Flagway Game[2]. This game involves skill and speed – both mathematically and physically. It is based on the Mobius function; this function assigns to each positive whole number one of three possible outputs. In the Flagway game these outputs are the colours red, blue and yellow, and so each natural number is either a red, blue or yellow number. To begin with the players are only told the colours of a few numbers (say the numbers 1-6) and are asked to try to guess the colours of other numbers. The students make conjectures based on the information about the colours of numbers they know and these conjectures are tested when new information about the colours of other numbers is given to them. In this way, a spirit of experimentation is encouraged as well as a reliance on reasoning to check conjectures. In practice, figuring out the rules governing how colours are assigned can take some time but simple versions of the game can be played straight away.

A more complicated version of the game has the students taking three cards from the table and running to a circle in the middle of the play area, this circle has a network of coloured paths leading from it (See Figure 1 below). Each path from the centre is made up of three portions, each coloured red, blue or yellow. Players need to arrange their cards in a sequence

and then follow the path dictated by their cards (for example if they have a sequence of 5-6-4 they need to take the red path leading from the centre (for 5), then the blue path (for 6) from that node, and lastly the yellow path (for 4)).

The colours of the numbers are assigned using the Mobius function, and thus depend on the prime factorisation of the number. The three colours correspond to the three possible categories of prime factorisations: the case where the number is divisible by the square of a prime; the case where the number has an odd number of prime factors (with none repeated); the case where the number has an even number of prime factors (with none repeated). Thus, in order to play the Flagway game well (in particular, in order to be quick), it is important to be able to factor numbers and to be able to decide quickly to which category they belong. Pupils are encouraged to represent the factorisations using letters and notice for example that 12 and 18 are both of the form a^2b (and so are both the same colour). Thus, the use of variables is introduced in a natural setting where pupils can appreciate the need for them.

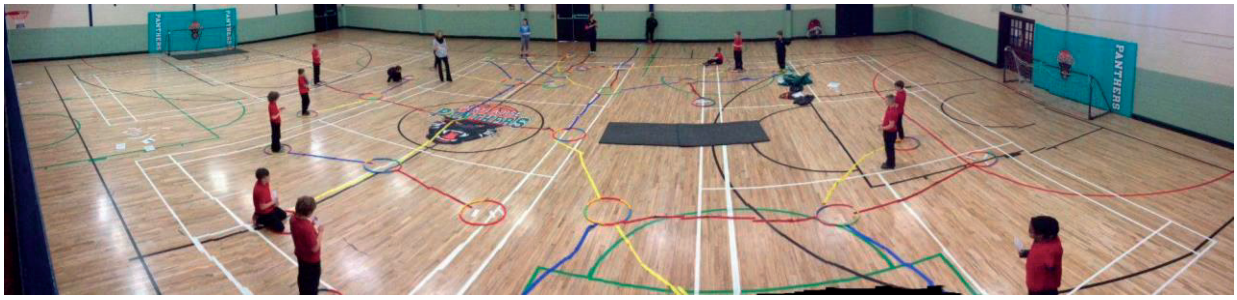

Figure 1: Flagway game in progress

METHODOLOGY

Teachers from nine primary schools around the Kildare region took part in a day-long workshop in February 2016, following an information session with Principals. At this workshop the teachers were introduced to the Flagway game and to the methodology of the Algebra Project. The teachers then introduced these methods into their classrooms over the next few months, under the guidance of the project team. This was followed that summer by a series of intensive workshops facilitated by the US Algebra Project team. Thirteen teachers from six of the original nine primary schools took part in these. At these workshops, the teachers were not told the rules of assigning colours to numbers but had to work together to discover these rules in the same way that pupils would be expected to. The mornings were spent on working through the variants of the Flagway game and discussing how they could be used in the classroom, while the afternoons were spent working with children in a primary school setting along with the AP team.

Four primary teachers from two case study schools who took part in the Flagway module were interviewed at T1 and again a year later at T2 after the participants had used the AP methodologies in their own teaching with 5th and 6th class students. The interviews were analysed to identify any changes to the teaching practices and beliefs of participants as a result of this project. Feedback on the impact of the project from pupils in the case study schools was gathered using focus groups[3]. The researchers also visited schools to observe the methodologies in action [the Flagway game being played].

THEORETICAL FRAMEWORK

The framework for analysis used in this research is Engeström's model of an activity system where contextual artefacts are fundamental in converting external stimuli into internal mental functioning (1987, 2001). There are multiple activity systems at play within any one classroom. This paper takes the student as the unit of analysis and the outcome as developing algebraic thinking and the connections are represented as lines in figure 2 below. In activity theory there are many connections and disconnections that can be observed in any activity system. DeVane and Squire advise that the minimal meaningful unit of analysis is an individual 'engaged in an activity with tools and resources in some social context' (2012, p. 254). In figure 2 we map the AP onto Engeström's (1987) model of an activity system.

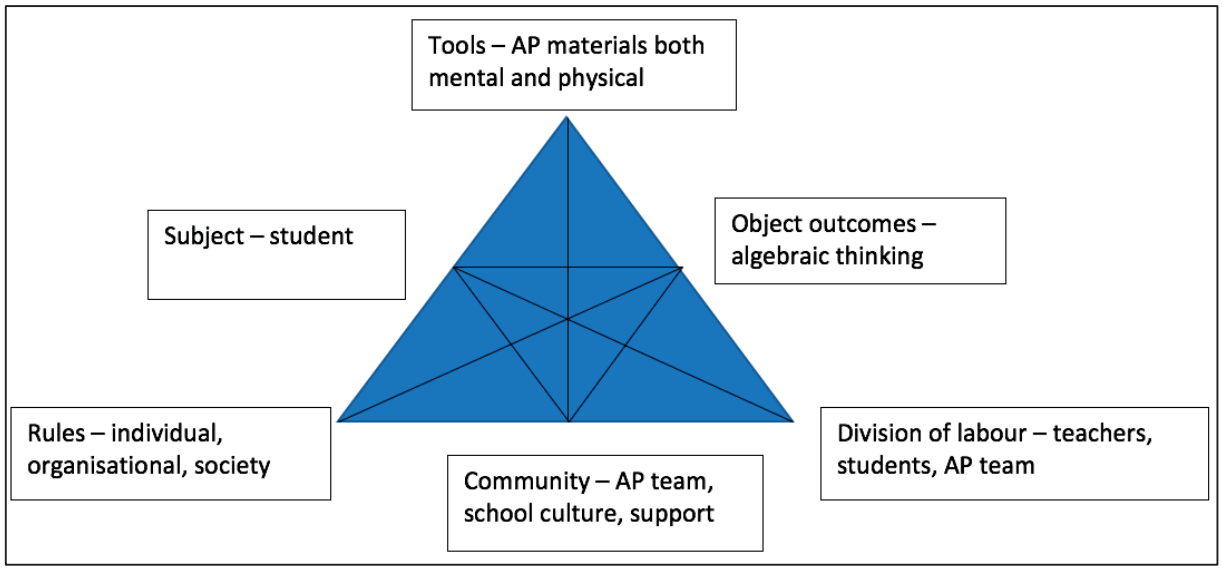

Figure 2: The Algebra Project using Engeström's model of an activity system

RESULTS

We report here on the analysis of the interview and focus group data collected from teachers and pupils who have been involved in the Flagway module. The overall impression from both groups was very positive. For example, one pupil said: *It was not like forced Maths; it was just being able to do Maths but yet feeling kind of free* (T2_CS1_SFG). The themes that emerged concerned the model of professional development used, the implementation of AP methods in schools (benefits and challenges), skills development, opportunities for across school collaboration and fitting Flagway into the Irish curriculum. In this paper we will concentrate on two of these themes.

Implementing the Algebra Project in Schools: Doing Maths and using tools

The teachers spoke about how they implemented the AP methodologies in their schools and what this entails: *It needs an awful lot of drive and it needs an awful lot of commitment to it and it needs an awful lot of organisation but I think it definitely can work* (T2_CS2_T1). Teachers took time to prepare their classes for the mathematics involved. They talked about how they worked ahead of playing the game and the need to work on number sense with

students. They all introduced the Flagway game in the way that it was introduced to them in order not to diminish the learning involved. Stein at al. (2000) advises that teachers can limit the level of cognitive demand of tasks in their attempt to make expectations clear to learners. This can result in the learner following a prescribed set of steps rather than engaging in meaning making. Teachers needed to set up the task in such a way so that the learners got time and space to engage with the problem and construct meaning for themselves. This teacher describes how they introduced the game to their classes: *What the children came up with first of all was they found a way to make the numbers fit which was a pattern which was work that they had come up with which they could explain which was brilliant; we initially did it with the numbers 2 to 10 so then I would test them by saying "OK, where would 11 go, where would 12 fit in your pattern?" and see if they could explain to me and see if it could work and then from that step then I told them that they needed to find what Mobius function was; so then I would give them another number and see if they could work out the pattern, see if they could work out the system between them ….so they wanted to do it because it was a challenge and because I would not give in and I would not tell them* (T2_CS2_T2).

The pupils responded to the challenge. They liked the way that the problem was introduced to them and the freedom that this gave them to explore: *The way our teacher explained it to us was really good, the way he is like a mathematician, he questions us about it, he would not just tell us* (T2_CS2_SFG). This exploration gave opportunities for the pupils to engage in sophisticated mathematical thinking: *even though some of them did not get to it they still learned an awful lot and it was a different way of thinking about Maths where they had to try and come up with their own theorems, it was not just me regurgitating information and them learning it* (T2_CS2_T2).The teachers also spoke about how the AP encouraged independent thinking and problem-solving in their pupils and how it can change pupil's perception of mathematics as being about getting the right answer and nothing else.

The time needed for the learners to work through the process of problem solving and the need for the teacher not to offer clarifying help and to have the confidence to allow the process work was evident in the commentary on implementing the game in class in both case study schools. The Flagway game and the card games and tools created affordances for the activity of the class by structuring the kinds of mathematical knowledge that learners got to use and build. The way the game was played was determined by the individual teachers in interaction with the learners in their classrooms. Their pupils recognised the benefits they were reaping: *Yes, it was a really good experience because it was all to do with prime numbers. It will help us definitely in the future for secondary school* (T2_CS1_SFG). Another said: *We did not really think of it as Maths, we kind of thought of it as a game but in our heads, we were kind of doing the strategies and we were learning but we did not really realise it but we did* (T2_CS1_SFG). The ability to do mental arithmetic was highlighted by teachers and pupils.

Another benefit of the AP methodologies was evident in the fact that both teachers and students found the AP to be inclusive and that the task allowed for all to get involved in the learning. The Flagway game seems to offer opportunities to engage pupils who might otherwise encounter difficulties: *I have a child in my class who would be very very weak and by the end she is smiling and she is running and she is looking involved and it is lovely to see*

everyone getting involved (T2_CS1_T2). At T1 when teachers were asked about a typical maths lesson, they referred to the workbooks and how the learning was very teacher-centred. A typical response was: *It starts out teacher led … and then they would be working a lot with their maths workbooks so it is, they do work individually* (T1_Interview_T2). The descriptions of how the AP methodologies were implemented in the case study schools paint a very different picture; the classes seem to involve inquiry-based learning.

The teachers spoke about two main challenges with the implementation of the AP in schools. One was the physical space needed for the setting up of the game, and the second was the time needed to play the game. CS2 had tried to adapt the game and to do it inside on the whiteboard. CS2_T1 lamented the lack of a recreation space in his school, however, he had improvised and the day the researcher visited the students were very engaged in 'doing mathematics'. *Just trying to do it as much as you can in the small space, clearing the tables and make them walk the game rather than run it, which kind of defeats the active side of the game, and as well as setting it up outside takes time* (T2_CS2_T1). CS1 had a recreation space which meant that they got to play the AP Flagway game more often. They also got to host other schools coming to play the game, however the teachers explained that the Irish weather hampers the availability of space for the game. The pupils expressed very similar opinions; it is difficult to play the game outside unless the weather is good and you need a big hall to play it properly inside. *I prefer to do it outside rather than in the hall because sometimes inside it was really squished because it is smaller. I like doing it outside because we have lots more room to run around* (T2_CS1_SFG).

The time needed to give students the opportunity to problem solve and 'do maths' was a common theme in both teacher and student interviews. One teacher talked about how they had got the setting up time for the game down to a minimum but they also lamented that to really use the game to its potential takes time. *I think there is an awful lot of Maths in it, there is an awful lot of benefit to it but you have to invest in it, we are doing a lot of work, investing time in it, we are able to do it here because we have it in our policy that we do Maths games once a week* (T2_CS2_T1). The division of labour and community are at odds if adequate time for mathematics teaching and learning is not included in the rules of the schools. CS1_T1 talked at T1 about the perceived challenges she anticipated for implementing the AP in her school. At T1 she talked about physical challenges such as the size of the hall, the time table and so on. At T2 she talked about how to integrate the AP more with the curriculum. We would contend that this represents a move in this school to embedding the AP as a part of the curriculum. *The real challenge I think with the Flagway is trying to adapt it now to be most effective within the Irish curriculum; I think that's the biggest challenge*. It is moving from AP being seen as a game to being seen as doing maths.

Development of skills

The development of skills such as working with others, peer teaching, empathy and communication was very obvious from all the student focus groups when they talked about playing the game. They talked about how they taught pupils from the visiting schools, delegated work within teams and encouraged all to get involved. In some of the responses the learning of the mathematics was almost seen as insignificant in comparison to the

development of skills. *I liked it also because it was team work and if you made a mistake nobody was going to give out to you or anything because you just like, it was all to do with your team, one girl might be taking up sets of cards that have not been taken and you might have one girl trying to run around. I liked it because it was pretty fair* (T2_CS1_SFG). Peer tutoring came up in all teacher interviews; the AP was seen as a vehicle for peer tutoring and teachers spoke about how this enhanced the development of skills such as working with others and communication. *If you got the strong kid in with a couple of weak kids some of the games are set up so that it has to be a team answer so the whole team has to stand up and shout the answer, so they've got, it's in their interest to make sure that the weak guy knows what's happening* (T1_CS2_Interview_T1). The ability to build self-efficacy in maths while also doing physical activity was significant for some of the learners in both case study schools. For some, combining the two activities made learning mathematics more enjoyable: *I thought it was good because it gave us an opportunity to get active and to enjoy the Maths, because some people don't enjoy Maths but, in a way, we were kind of doing PE with it and we were still learning* (T2_CS1_SFG). While for others it was the other way around: *I am not that fast as everyone else, I am horrible at running so that is why Flagway, I like the Flagway, it gave me a reason to be good at PE. You need to be fast to win* (T2_CS2_SFG).

DISCUSSION

We have seen the themes that emerged from the interview and focus group data. Overall, pupils and teachers were very positive about the AP and the Flagway game. Both the teachers and the pupils spoke eloquently about the various benefits that they observed. The AP five-step process, with its emphasis on moving from a physical experience to an abstract concept, seems to lead to deeper and more persistent understanding. This echoes the findings of Dubinsky and Wilson (2013). Furthermore, we have seen evidence that the methodologies used can lead to the development of key skills such as group work and communication.

For the pupils in this research the tools, the physical events of working with the colours, numbers and modelling, pushed their problem identification and problem solving skills and encouraged learning. The move from the intuitive language to structured language was evident from what we observed and from teacher and pupil interview data. An outcome not envisaged at the outset was that of the development of the skills of peer tutoring, team work and communication. The tools (the AP pedagogical process) in interaction with the division of labour between the teacher, pupil and curriculum impacted on pupils' development of skills and in doing so enhanced their attitude to maths, as the title says they experienced *being able to do maths and feeling kind of free*. The Flagway game seems to fit very well into the Irish primary school curriculum. However, teachers expressed worry about covering content areas and about preparation for various tests at the end of primary education. This may be indicative of their conceptions of mathematics as a bundle of isolated facts that need to be 'covered', and may indicate a need for a reconceptualization of mathematical teaching and learning.

It is interesting to look at the teacher as a unit of analysis in tandem with the student in two parallel activity systems. What is clear is that the model of professional development used by the AP team where the participants were the pupils in the morning and then implemented their learning in a school setting with the AP team in the afternoon had a big impact on moving the

teachers' thinking and understanding of how the tools support the development of algebraic thinking. This merits further analysis. The dissonance in the activity system was the lack of physical space and time. While this may appear to be easy to solve, it involves a complex and potentially difficult interaction between the rules at organisational level and community to support the investment in this kind of experiential learning and meaning-making.

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[2] Bob Moses developed the Flagway Game in 1995 and patented it in 1996 (Moses, U.S. Pat #5520542 & 5704790).

[3] Data are coded as follows: T1 and T2 refer to pre implementation and post implementation data. CS1 refers to case study school one. SFG is student focus group. For example, T1_CS1_T1 refers to pre implementation interview in case study school one with teacher one.