HEAT TRANSFER AT ELECTROCHEMICAL TREATMENT OF ORGANIC DISPERSED HYDRAULIC SYSTEMS

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Abstract

The proposed model combines electric and temperature fields, when organic dispersed hydraulic systems are subjected to electric-thermochemical treatment. This model has been applied for developing the technologies and equipment for protein production from potato juice, milk whey and for electrolytic activation of yeast productivity. The given model relations are required to establish optimal treatment parameters of media, to impose restriction on technological and design parameters, to define medium flow diagrams in the reactor, electrode and orifice material, ultimate electric field strength and current values, etc.

Keywords: organic dispersed hydraulic systems, electric treatment, electric field strength, temperature, Joule heating

Agricultural products of vegetable and animal origin (vegetable and fruit juices, milk whey, solutions for growing microorganisms, moist and moistened forage, etc. can be conditionally referred to as organic dispersed hydraulic systems (ODHS). To improve the efficiency of using nutritive potential and increasing extraction or formation of useful materials, and to disinfect ODHSs the latter are treated in different ways – mechanical, thermal, complex [1].

The electric field applied through the medium heats it. Increasing the temperature can have a positive effect on the treatment process in some cases and a negative effect in others. A temperature value depends on electric field density, medium specific conduction, on design of a reactor, where a medium is treated, and on flow diagram. In all cases, the temperature increase means the growth of the energy intensity of the process. This is undesirable. In addition, the treatment quality is affected by a uniform temperature distribution over the medium volume. A higher temperature near the surface of current-conducting electrodes and the separating membrane may worsen the treatment quality or in general, this may hinder the electric field applied to the medium.

The present work considers a flat electrode system, whose the interelectrode space are separated by a membrane into cathode and anode regions with the length L and the widths δ_1 and δ_2, respectively. The objective of this work was to develop a mathematical model able to describe electric and temperature fields in liquid moving in the anode and cathode regions. Temperature, more precisely its uniformity between electrodes, serves as a criterion in defining the character and direction of moving liquid, permissible electrical conduction of membrane material, and electric field strength.

As ODHSs are usually subjected to applied electric fields of relatively small strength (E < 1 kV/cm) and at rather a long interelectrode distance (l_e ~ 1 cm), we can consider the temperature variation in ODHS using the Ohmic conduction model to a good enough approximation [2]. Also, based on a small rate of diffusive and convective transfer of a charge in comparison to a migration velocity of ions in ODHS due to an applied electric field, the equation for the total current in ODHS is of the form

$$\nabla(\sigma E) = \nabla(\sigma \nabla \Phi) = 0 \tag{1}$$

We determine the ODHS temperature from the convective heat conduction equation with the consideration of Joule heating:

$$\rho C_p \left(\frac{\partial T}{\partial \tau} + \vec{u} \cdot \nabla T \right) = \nabla(\lambda \nabla T) + \sigma E^2 \tag{2}$$

At the initial boundary condition $T\big|_{\tau=0} = T_0$ and the boundary conditions $\lambda \nabla T\big|_{\Gamma} = \alpha(T_{env} - T\big|_{\Gamma})$, where λ, ρ, C_P is the thermal conductivity, the density, and the specific heat capacity of a dispersed medium respectively, $\vec{u}$ is the velocity vector, α is the heat transfer coefficient at the boundary Γ, T_0 is the initial temperature, T_{env} is the environment temperature, $T\big|_{\Gamma}$ is the medium temperature at the boundary Γ, σ is the specific electrical conductivity of a medium, E is the electric field strength, Φ is the electric potential.

The flow of the medium can be co-current and counter-current. The electric field is applied from one electrode to another. The temperature in the medium and in the membrane varies from the initial value (room temperature) to some final maximum value assigned by the technological process. At that, the membrane temperature must not exceed the maximum permissible one of the treated material. For example, under coagulation of whey proteins, it is not more than 45 ^{0}C. Such a temperature range shows the heat and electrophysical parameters of the medium and the membrane are weakly temperature-dependent. They can be taken as constant.

Thus, the temperature is the objective function. It is necessary to determine a temperature field of ODHS in the presence of internal heat sources depending on the thermophysical parameters of the moving liquid and the membrane.

For the steady flow of the liquid in the interelectrode space with relative sizes $\delta_1 / L >> 1$ and $\delta_2 / L >> 1$, we can assess the medium state along the channel to a boundary layer approximation for both temperature and electric fields:

$$\frac{\partial}{\partial y}(\sigma(y) E_y) = -\frac{\partial}{\partial y}\left[\sigma(y) \frac{\partial \Phi}{\partial y} \right] = 0, \tag{3}$$

$$\text{for the liquid : } \rho C_p u \frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left(\lambda(y)\frac{\partial T}{\partial y}\right) + \sigma E^2 \qquad (4)$$

$$\text{for the membrane: } 0 = \frac{\partial}{\partial y}\left(\lambda_m \frac{\partial T}{\partial y}\right) + \sigma_m E^2 \qquad (5)$$

At the channel entrance, the boundary conditions are $T = T$ and at the channel exit, the "soft" boundary condition is $\partial T / \partial x|_{x=L} = 0$. In the case of counter-flow, we should take into consideration that for the anode and cathode regions, these boundaries are opposite to each other. At the channel (electrode) walls, the Newton conditions are valid for the heat flux

$$\lambda_1 \frac{\partial T}{\partial y}\bigg|_{y=\delta_m+\delta_1} = \alpha_1 (T_{env} - T|_{y=\delta_m+\delta_1}), \qquad \lambda_2 \frac{\partial T}{\partial y}\bigg|_{y=-(\delta_m+\delta_2)} = -\alpha_2 (T_{env} - T|_{y=-(\delta_m+\delta_2)}). \quad (6)$$

Here δ_m is a membrane half width. We determine the coefficients of heat transfer α_i [$Wm^{-2}K^{-1}$] from a heated fluid to an ambient medium in the cathode and anode regions using the Nusselt number as $\alpha_i = Nu_i \lambda_i/\delta_i$ [3, 4]. In the considered-type channel, for the laminar flow regime we use $Nu_i = 7{,}54$ and for the turbulent flow regime – $Nu_i = 0.0155 \mathrm{Pr}_i^{0,5} \mathrm{Re}_i^{0,83}$ valid for fluids with the Prandtl number $1 < \mathrm{Pr}_i < 25$ [4].

With the boundary conditions on the electrodes for the potential ($\Phi|_{y=\delta_m+\delta_1} = U_0$, $\Phi|_{y=-(\delta_m+\delta_2)} = -U_0$) and the electric field continuity condition at the medium–membrane boundary ($y = \delta_m$ and $-\delta_m$) included, we can show that the solution of equation (3) is the dependence:

$$E(y) = \frac{2U_0}{\sigma(y)\left[\frac{\delta_1}{\sigma_1} + \frac{2\delta_m}{\sigma_m} + \frac{\delta_2}{\sigma_2}\right]} \qquad (7)$$

Here, the electrical conductivity σ is the function of y, because it varies depending on the electrical conductivity of the medium moving above and under the membrane, as well as of the electrical conductivity of the membrane (σ_1, σ_2, σ_m respectively). Substituting formula (7) into the formula for heat source term in temperature equation (4) yields:

$$Q(y) = \sigma E^2 = \frac{(2U_0)^2}{\sigma(y)\left[\frac{\delta_1}{\sigma_1} + \frac{2\delta_m}{\sigma_m} + \frac{\delta_2}{\sigma_2}\right]^2} \qquad (8)$$

The regime of medium flow in the interelectrode space can be either laminar or turbulent, and determined by the Reynolds number $\mathrm{Re}_i = u_i D_{Hi} / \nu_i$ ($i = 1, 2$), where $D_{Hi} = 2\delta_i$ is the hydraulic channel diameter. The analysis [4] shows that the critical Reynolds number at a transition from the laminar to the turbulent regime for a plane-parallel channel also has its traditional value of ~ 2300.

To calculate the dynamics of laminar flow, it is possible to use the theory of laminar flow of incompressible liquid in the flat channel. First, consider the cathode region. Following the approach described elsewhere in [3], for laminated flows it is possible to show that the velocity distribution in this region is

$$V_1(y) = V_1^{\max}\left(1 - \left[\frac{2(|y| - \delta_m) - \delta_1}{\delta_1}\right]^2\right), \qquad V_1^{\max} = \frac{3}{2}u_1 .$$

Here, it is taken into account that for considered-type channels, the pressure drop $\Delta p_1 = \xi_1 \frac{L}{D_{H1}} \frac{\rho_1 u_1^2}{2}$ is assigned by the bulk velocity u_1 and the friction resistance coefficient – by the Reynolds number: $\xi_1 = 96/\mathrm{Re}_1$ [3, 4]. Similarly, for the anode region of the channel we have

$$V_2(y) = V_2^{\max}\left(1 - \left[\frac{2(|y| - \delta_m) - \delta_2}{\delta_2}\right]^2\right), \qquad V_2^{\max} = \frac{3}{2}u_2 .$$

For the dynamics of turbulent flow, bearing in mind that the Reynolds number for the problems of interest is not too high (over the range 5000–30000), the power-law velocity profile ("one seventh" law) is used to assign the velocity profile [3, 4]:

$$\frac{V_1(y)}{V_1^{\max}} = \left(\frac{2(|y| - \delta_m)}{\delta_1}\right)^{\frac{1}{7}}, \qquad V_1^{\max} = u_1/0.817 \text{ at } \delta_m \le y \le \delta_m + \delta_1/2 ,$$

$$\frac{V_2(y)}{V_2^{\max}} = \left(\frac{2(|y| - \delta_m)}{\delta_2}\right)^{\frac{1}{7}}, \qquad V_2^{\max} = u_2/0.817 \text{ at } -(\delta_m + \frac{\delta_2}{2}) \le y \le -\delta_m .$$

The last profiles are symmetric relative to the coordinates $y = (\delta_m + \delta_1)/2$ and $y = -(\delta_m + \delta_2/2)$. Thus, the heat transfer problem remains the only unsolved one.

Using the integro-interpolation method [5] for finding a temperature averaged over the thickness of each of the channels and the membrane, equations (4) and (5) arrive at the following system of equations:

for the medium above the membrane:

$$\rho_1 C_{p1} u_1 \delta_1 n_1 \frac{\partial T_1}{\partial x} = K_1(T_m - T_1) + K_{1env}(T_{env} - T_1) + \sigma_1 E_1^2 \delta_1 , \qquad (9)$$

for the medium under the membrane:

$$\rho_2 C_{p2} u_2 \delta_2 n_2 \frac{\partial T_2}{\partial x} = K_2(T_m - T_2) + K_{2env}(T_{env} - T_2) + \sigma_2 E_2^2 \delta_2 , \qquad (10)$$

for the membrane: $0 = K_1(T_1 - T_m) + K_2(T_2 - T_m) + \sigma_m E_m^2 2\delta_m$. (11)

Here, we determine the bulk velocity in channels as $u_{1,2} = Q_{1,2}/S_{1,2}$, where $Q_{1,2}$ is the volumetric flowrate of the medium in the channel, $S_{1,2}$ is the flow area of the channel, $n_{1,2}$ are the subscripts of the medium direction ($n_{1,2} = 1$ – along the *x*-axis, $n_{1,2} = -1$ – opposite to the *x*-axis), subscripts 1, 2, *m* denote the parameter in the considered region.

The heat transfer coefficients are $K_{1,2} = (1/\alpha_{1,2} + 2\delta_m/\lambda_m)^{-1}$, $K_{1env} = (1/\alpha_1 + 1/\alpha_{env} + \delta_{1e}/\lambda_{1e})^{-1}$, $K_{1env} = (1/\alpha_2 + 1/\alpha_{env} + \delta_{2e}/\lambda_{2e})^{-1}$. Here $\delta_{1,2e}$ and $\lambda_{1,2e}$ are the thickness and thermal conductivity of electrode material, α_{env} is the coefficient of heat transfer from electrode surface to room (we take $\alpha_{env} = 8$ W/m^{2}K).

The determined coefficients $K_{1,2}$ and equation (11) yield that the membrane temperature is equal to:

$$T_m = \frac{K_1}{K_1 + K_2} T_1 + \frac{K_2}{K_1 + K_2} T_2 + \frac{\sigma_m E_m^2 2\delta_m}{K_1 + K_2}. \quad (12)$$

Thus, for the medium temperature variation to be found, the system of the ordinary differential equations must be solved

$$\rho_1 C_{p1} u_1 \delta_1 n_1 \frac{\partial T_1}{\partial x} = \frac{K_2 K_1}{K_1 + K_2}\left(T_2 - T_1 + \frac{\sigma_m E_m^2 2\delta_m}{K_2}\right) + K_{1env}(T_{env} - T_1) + \sigma_1 E_1^2 \delta_1, \quad (13)$$

$$\rho_2 C_{p2} u_2 \delta_2 n_2 \frac{\partial T_2}{\partial x} = \frac{K_1 K_2}{K_1 + K_2}\left(T_1 - T_2 + \frac{\sigma_m E_m^2 2\delta_m}{K_1}\right) + K_{2env}(T_{env} - T_2) + \sigma_2 E_2^2 \delta_2. \quad (14)$$

We can obtain a general solution to the system of equations (13)–(14) by finding the characteristic equation roots and by using the variation method of arbitrary constant [6].

The present work contains the solution for a particular ODHS when protein is produced from milk whey [7]. Table 1 contains the set of characteristics needed for calculation. In our case, according to the parameters, $\Pr_i = \mu_i C_{pi} / \lambda_i \approx 5$ and the liquid flow is a laminar.

Table 1

Thermophysical, Dynamic and Geometric Characteristics

Parameter name	Designation	Unit	Value
Voltage	U	V	36
Distance from cathode to membrane	δ_1	m	0.08
Distance from anode to membrane	δ_2	m	0.07
Membrane thickness	$2\delta_m$	m	0.00015
Channel length	L	m	0.5
Membrane heat capacity	C_{pm}	J kg^{-1}K^{-1}	1500
Membrane material density	ρ_m	kg/m^3	1300
Initial membrane temperature	T_m^0	K	293
Initial medium temperature in cathode region	T_1^0	K	293
Initial medium temperature in anode region	T_2^0	K	293
Medium density in cathode region	ρ_1	kg/m^3	995.7
Medium density in anode region	ρ_2	kg/m^3	995.7
Medium heat capacity in cathode region	C_{p1}	J kg^{-1}K^{-1}	4174
Medium heat capacity in anode region	C_{p2}	J kg^{-1}K^{-1}	4174
Medium thermal conductivity in cathode region	λ_1	W m^{-1} K^{-1}	0.618
Medium thermal conductivity in anode region	λ_2	W m^{-1} K^{-1}	0.618
Membrane thermal conductivity	λ_m	W m^{-1} K^{-1}	0.17
Membrane electrical conductivity	σ_m	Ohm^{-1}m^{-1}	0.0210
Medium electrical conductivity in cathode region	σ_1	Ohm^{-1}m^{-1}	0.48
Medium electrical conductivity in anode re-	σ_2	Ohm^{-1}m^{-1}	0.48

gion			
Medium bulk velocity in cathode region	u_1	m·s^{-1}	0.01
Medium bulk velocity in anode region (volumetric flowrates above and under membrane are the same)	u_2	m·s^{-1}	$u_1 \frac{\delta_1}{\delta_2}$
Coefficient of heat transfer from electrode surface to room	α_{env}	W/m^{2}K	8
Material thickness of electrode 1	δ_{1e}	m	0.001
Material thickness of electrode 2	δ_{2e}	m	0.001
Material thermal conductivity of electrode 1	λ_{1e}	W/m K	236
Material thermal conductivity of electrode 2	λ_{2e}	W/m K	236

For the assigned treatment parameters (Table 1) in the co-current flow of the medium in channels ($n_1 = n_2 = 1$), the analytical equations (13)–(14) are described by the functions

$$T_1 = -1141{,}524\exp(-0{,}00213x) - 12{,}799\exp(-0{,}01123x) + 1447{,}322\,, \quad (15)$$

$$T_2 = -1137{,}392\exp(-0{,}00213x) + 12{,}845\exp(-0{,}01123x) + 1417{,}547\,. \quad (16)$$

In the case of the counter-flow of the medium ($n_1 = -n_2 = 1$), the solution is presented by the following functions

at the channel length L = 1 m:

$$T_1 = -310{,}824\exp(0{,}00491x) - 843{,}498\exp(-0{,}00488x) + 1447{,}322\,, \quad (17)$$

$$T_2 = -790{,}678\exp(0{,}00491x) - 331{,}5896\exp(-0{,}00488x) + 1417{,}547 \quad (18)$$

at the channel length L = 0.5 m:

$$T_1 = -205{,}207\exp(0{,}00491x) - 842{,}969\exp(-0{,}00488x) + 1447{,}322\,, \quad (19)$$

$$T_2 = -792{,}025\exp(0{,}00491x) - 331{,}381\exp(-0{,}00488x) + 1417{,}547\,. \quad (20)$$

The temperature variation in the membrane is

$$T_m = 0{,}5(T_1 + T_2) + 5{,}98935\,. \quad (21)$$

Figure 1 demonstrates the temperature variation according to the above analytical solutions (15)-(21). There is a fairly clear trend that in the case of co-current flow, both the medium and the membrane undergo heating along the length of channels (Fig. 1, *a*). At the same time, in the case of counter-flow, the treated medium is heated, not substantially varying the membrane temperature along the channel length (Fig. 1, *b*, *c*). It should also be mentioned that, when the channel length is the same, the medium temperature drop between the channel entrance and exit in co-current and counter-current flow of the medium (Fig. 1, *a, c)* is identical.

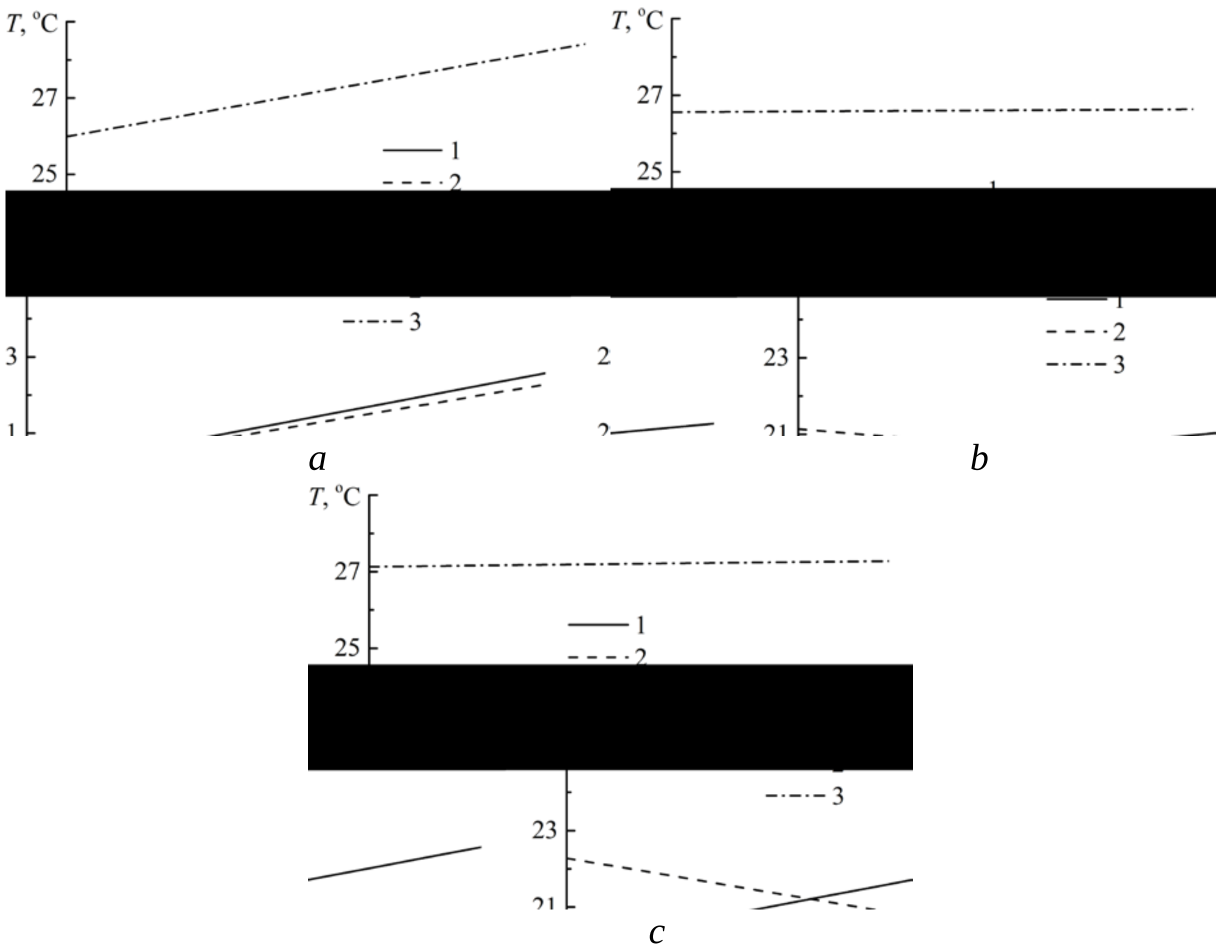

Figure 1 – Temperature variation of the treated medium along the channels and the membrane (1 – temperature T_1; 2 – T_2; 3 – T_m): *a* – co-current flow of the medium, *b* – co-current flow of the medium along the channel at L = 0.5 m, *c* – counter-current flow of the medium at L = 1 m.

The proposed model for a relationship between electrochemical processes and electric and temperature fields, when organic dispersed hydraulic systems are subjected to electric-thermochemical treatment, has been utilized to develop technologies and equipment for protein production from potato juice, milk whey, for electrolytic activation of forage yeast productivity, etc. The above-mentioned relations are needed to set optimal treatment parameters of media, to impose resistance restrictions on technological and design parameters, to define medium flow diagrams in the reactor, electrode and orifice material, limiting values of electric field strength and density, process duration, etc.

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