Multi-dimensional Numerical Integration on Parallel Architectures

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INTRODUCTION

- **Multi-dimensional** numerical integration is a recurrent need in scientific computing, often required in particle physics, quantum mechanics, and astrophysics applications.
- Current software libraries can struggle with **difficult integrands**, especially in **high dimensions**, leading to prohibitively long execution times and few digits of precision.

MISSION

Develop new algorithms to utilize **highly parallel architectures** such as **GPUs**, to reduce **execution times** and attain greater **precision** than current state-of-the-art adaptive integration methods.

ADAPTIVE INTEGRATION

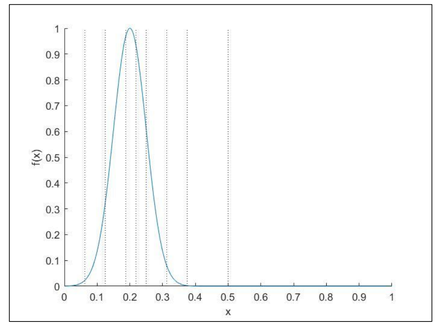
- Divide integration space into sub-regions and locally sample the integrand at different points.
- Complex integrands require more sub-regions.
- Identify where integrand is “easy” and fewer sub-regions will suffice.

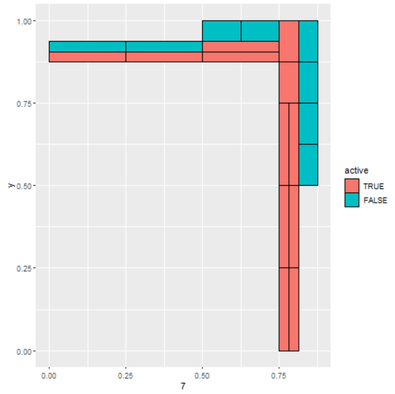
METHODOLOGY

- Utilize fastest existing sequential method for core computations
- Parallelize function evaluations
- Parallelize computation of sub-regions

Main Algorithmic Steps

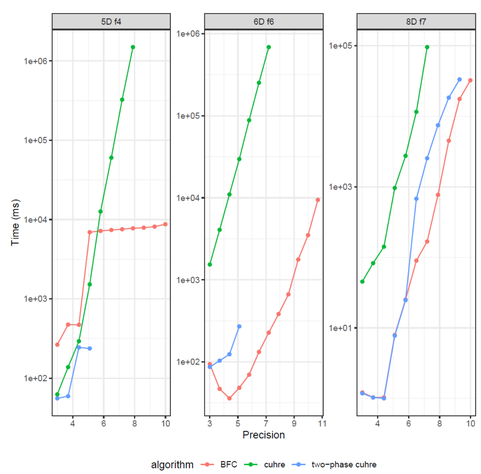
1. Uniform sub-division of integration space
2. Evaluation of all existing sub-regions, returning the integral, error, and coordinate-axis to split for each region.
3. Sum integral and error estimates from evaluated regions to compute cumulative integral and error.
4. Heuristic classification of regions into active or finished (when error is sufficiently small) regions.
5. Accumulate contributions from finished regions and remove them to conserve resources.
6. Terminate when estimated cumulative relative error satisfies the user-requested accuracy.





RESULTS

- Executed on the Wahab cluster’s V100 NVIDIA GPUs.
- Easy integrals are not computed faster due to already low execution times and necessary overhead.
- Orders of magnitude faster on high-dimensional integrands.
- Attainable digits of precision greater than existing methods



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