ON THE THEORY OF BETA-RADIOACTIVITY I

THE USE OF LINEAR COMBINATIONS OF INVARIANTS IN THE INTERACTION HAMILTONIAN

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Synopsis

The theory of β-radioactivity, based on the use of a linear combination of all five relativistic invariants, is developed for allowed transitions. The interaction Hamiltonian is taken as $H_\beta = G \Sigma_{k=1}^{5} C_k J_k$, where the C_k are coefficients that determine the combination of the invariants J_k with 1 = scalar, 2 = vector, 3 = tensor, 4 = axial vector and 5 = pseudo-scalar. The formula for the shape of β^+ and β^--spectra is calculated, taking into account the nuclear charge Z (E energy of the electron; p and q are the momenta of the electron and the neutrino respectively):

$$P_{\pm}(E) = (G^2/2\pi^3)pEq^2 F(Z, E) \, [(C_1^2 + C_2^2) \, | \textstyle\int 1 \, |^2 + (C_3^2 + C_4^2) \, | \textstyle\int \sigma \, |^2 +$$
$$+ C_5^2 \, | \textstyle\int \beta\gamma_5 \, |^2 \mp (2\gamma/E) \, (C_1 C_2 \, | \textstyle\int 1 \, |^2 + C_3 C_4 \, | \textstyle\int \sigma \, |^2)]$$

The probability for K-capture becomes for this interaction:

$$P_K = (G^2/4\pi^2) \, (E_0 + E_K)^2 g_{s0}^2 \, [(C_1^2 + C_2^2) \, | \textstyle\int 1 \, |^2 + (C_3^2 + C_4^2) \, | \textstyle\int \sigma \, |^2 +$$
$$+ C_5^2 \, | \textstyle\int \beta\gamma_5 \, |^2 + 2C_1 C_2 \, | \textstyle\int 1 \, |^2 + 2C_3 C_4 \, | \textstyle\int \sigma \, |^2]$$

The angular correlation between electron and neutrino is determined by (neglecting the influence of the nuclear charge):

$$P_{\pm}(E, \mathbf{p}, \mathbf{q}) = \{G^2/(2\pi)^5\}pEq^2 \, \{(C_1^2 + C_2^2) \, | \textstyle\int 1 \, |^2 + (C_3^2 + C_4^2) \, | \textstyle\int \sigma \, |^2 +$$
$$+ C_5^2 \, | \textstyle\int \beta\gamma_5 \, |^2 \mp (2/E) \, (C_1 C_2 \, | \textstyle\int 1 \, |^2 + C_3 C_4 \, | \textstyle\int \sigma \, |^2) -$$
$$- (\mathbf{q} \cdot \mathbf{p}/qE) \, [(C_1^2 - C_2^2) \, | \textstyle\int 1 \, |^2 - \tfrac{1}{8} \, (C_3^2 - C_4^2) \, | \textstyle\int \sigma \, |^2 + C_5^2 \, | \textstyle\int \beta\gamma_5 \, |^2]\}$$

A general principle, viz. complete symmetry for the processes of β^+ and β^--emission, is proposed, which has as a consequence that only two types of combinations can exist: a) combinations of the invariants 1, 4 and 5, b) combinations of the invariants 2 and 3. We then obtain the result that in both cases the $1/E$-term in the β-spectrum drops out.

Some considerations are given on the comparison of theory and experiment. Thus it is discussed that recoil-experiments with ^{6}He are not

yet sufficient to decide if the interaction is given by a "pure" invariant or by a linear combination. They should be completed by recoil-experiments with other nuclei e.g. ^{19}Ne.

§ 1. *Introduction*. The theories of β-radioactivity that have been developed since F e r m i[1]) can all be characterized by saying that they are a logical development according to the perturbation theory of quantum mechanics, if one has assumed a certain form for the interaction Hamiltonian. All theories are based on the neutrino hypothesis and the Dirac equation for leptons and nucleons. F e r m i has chosen a special form for the interaction Hamiltonian. However, the interactions that may be assumed are only restricted by the condition of relativistic invariance, which makes a more general choice possible than the one made by F e r m i.

The present problem in β-radioactivity is to develop the theory for the different possible forms of the interaction Hamiltonian and to see if it is possible to get agreement with the experiments by choosing a special form for this interaction. If this is possible the selected interaction Hamiltonian must be considered a fundamental property concerning nucleons and leptons.

Already in 1935 a form for the interaction, which differed from F e r m i's form by the introduction of the derivative of lepton wave functions, was proposed by K o n o p i n s k i and U h l e n-b e c k[2]) to get a better agreement with the measured form of β-spectra. However, it appeared later that the first measurements contained experimental errors and that this new form for the interaction was certainly not in agreement with later more accurate measurements[3]). Even if no derivatives of wave functions are included in the interaction, the form for the interaction is not completely determined by the condition of relativistic invariance. According to the Dirac theory five independent relativistically invariant expressions can be chosen for the interaction Hamiltonian[4]), which are usually called the scalar, vector, tensor, pseudovector (or axial-vector) and pseudo-scalar interaction (and denoted respectively with S, V, T, A, P). Linear combinations of these five invariants can also be chosen; they are the most general possibility of interaction if no derivatives of wave functions are included[5]). However, practically the whole discussion of experimental results in β-radioactivity has been made under the rather special assumption that the interaction is determined by only one of the

five invariants and not by a linear combination (cf. e.g. [3]). The form of the β-spectra for allowed transitions is the same for the five invariants. Hence the form of forbidden spectra is usually chosen to distinguish between the different interactions. However, no distinct results have as yet been obtained. It must be remarked that the theory for forbidden transitions is not only more complicated and more difficult to analyze but also probably less certain than the theory for allowed transitions for the following reason: if the general form of the theory of β-radioactivity is a reasonably good first approach, but not rigorously correct (which might well be the case) then the theory of allowed transitions might still be a good first approximation while the theory for forbidden transitions may have to be altered seriously (imagine e.g. that the true interaction Hamiltonian has a slight "admixture" of an invariant with derivatives of wave functions). Therefore in this series of articles the phenomena which may lead to a choice for the interaction, will be investigated under the general assumption of a linear combination of the invariants for the interaction and special attention will be focused on allowed transitions.

Even if a choice for the interaction could be made by the study of forbidden transitions e.g. the shape of forbidden β-spectra, a check of the result by means of a comparison of such phenomena as electron-neutrino angular correlation (recoil experiments), observation of polarisation of the β-rays in combination with alignment of the nuclear spins, would be extremely interesting and from a theoretical point of view, more straighforward and convincing if carried out for allowed than for forbidden transitions.

In this article we investigate first the results of the theory of β-radioactivity if a general linear combination of invariants is used. Theoretical considerations are then given on a certain type of symmetry of the interaction Hamiltonian which restricts the number of possible linear combinations of invariants. F i e r z has first given calculations in this direction [5]).

In a following article a new phenomenon in connection with allowed β-transitions will be discussed viz. the polarization of β-particles emitted by nuclei with aligned nuclear spins. Though this has not yet been observed experimentally, this might be possible in the near future. Preliminary notes of these studies have appeared earlier [6]).

§ 2. *Calculation of the transition probabilities, neglecting nuclear charge.* We suppose for the nucleon-lepton interaction Hamiltonian a linear combination of expressions each of the form:

$$H \equiv (\psi^* \Omega_L \varphi)\, Q\, \Omega_N + (\psi^* \Omega_L \varphi)^* Q^+\, \Omega_N^+ \tag{1}$$

with ψ wave function of the electron

    φ wave function of the neutrino

    Ω_L and Ω_N operators from Dirac theory for the leptons and for the nucleons.

$Q = \begin{bmatrix} 00 \\ 10 \end{bmatrix}$ and $Q^+ = \begin{bmatrix} 01 \\ 00 \end{bmatrix}$ transition operators for the nucleons.

The first term of (1) should be used in case of β^--emission (because Q transforms a neutron into a proton) and the second in case of β^+-emission and K-capture (because Q^+ transforms a proton into a neutron). Below we shall only write one term of (1) to simplify the notation.

We shall write $J_1, \ldots, J_5$ for the following invariant expressions, which are possible for the interaction energy and in which the interaction is said to be respectively of the scalar, vector, tensor, pseudovector, pseudoscalar type (These J's are connected with (1) according to: $J = \Psi_f^* H \Psi_i$):

$$
\begin{array}{lll}
\text{S} & J_1 = (\Psi_f^* \beta Q \Psi_i)\,(\psi^* \beta \varphi) & \\
\text{V} & J_2 = (\Psi_f^* Q\, \Psi_i)\,(\psi^* \varphi) & - (\Psi_f^* \boldsymbol{\alpha} Q \Psi_i).(\psi^* \boldsymbol{\alpha} \varphi) \\
\text{T} & J_3 = (\Psi_f^* \beta \boldsymbol{\sigma} Q \Psi_i).(\psi^* \beta \boldsymbol{\sigma}\, \varphi) & + (\Psi_f^* \beta \boldsymbol{\alpha}\, Q \Psi_i).(\psi^* \beta\, \boldsymbol{\alpha}\, \varphi) \\
\text{A} & J_4 = (\Psi_f^* \boldsymbol{\sigma} Q \Psi_i).(\psi^* \boldsymbol{\sigma}\, \varphi) & - (\Psi_f^* \gamma_5 Q\, \Psi_i)\,(\psi^* \gamma_5 \varphi) \\
\text{P} & J_6 = & (\Psi_f^* \beta \gamma_5 Q \Psi_i)\,(\psi^* \beta\, \gamma_5 \varphi)
\end{array} \tag{2}
$$

$$\underbrace{\hphantom{XXXXXXXXXXXXX}}_{\text{great}} \qquad \underbrace{\hphantom{XXXXXXXXXXXXX}}_{\text{small}}$$

The words "great" and "small" point to the fact that the terms called "small" are relativistic terms in the velocities of the nucleons and therefore can be considered of a smaller order of magnitude. Ψ_i and Ψ_f are the wave functions of the initial and final nucleus. The complete expression for the interaction energy becomes:

$$H_\beta = G\, \Sigma_{k=1}^5\, C_k J_k \tag{3}$$

The real constants C_k give the extent to which the invariants are mixed. We must impose a condition on the C_k's in order to determine G and the C_k's completely; we can take e.g. (cf. however § 7):

$$\Sigma_{k=1}^5\, C_k^2 = 1 \tag{4}$$

G is the Fermi constant (or rather: analogous to the Fermi constant in the theories with pure invariants).

The shape of the β-spectrum for β^+ and β^--emission can be calculated according to the following formula (if the influence of the nuclear charge is neglected) for the total transition probability for emission of electrons with energy between E and $E + dE$:

$$P(E)dE = (2\pi)^{-5} \Sigma_e \int d\omega_e \Sigma_\nu \int d\omega_\nu \mid H_\beta \mid^2 pEq^2 \, dE \qquad (5)$$

with: E energy of the electron

E_ν energy of the neutrino

p momentum of the electron

q momentum of the neutrino

E_0 maximum energy of the electrons in the β-spectrum

We have the relation: $q = E_\nu = E_0 - E$

$d\omega_e$ differential for solid angle in which the direction of emission of the electron lies

$d\omega_\nu$ differential for solid angle in which the direction of emission of the neutrino lies

Σ_e sum over the two polarization states of the electron

Σ_ν sum over the two polarization states of the neutrino

$$\mid H_\beta \mid^2 = G^2 \Sigma_{m_f} \mid \Sigma_h \Sigma_k \int d\tau \, [C_k(\Psi_f^* \Omega_k Q_h \Psi_i) \cdot (\psi^* \Omega_k \varphi)_h] \mid^2 \qquad (6)$$

Σ_{m_f} is the sum over the different possible orientations of the final nucleus; each state is characterized by the magnetic quantum number m_f

Q_h is the transition operator for the hth nucleon in the nucleus; the sum Σ_h for all nucleons must be taken

(the index h of $(\psi^* \Omega_k \varphi)_h$ denotes that this quantity must be evaluated at the place of the hth nucleon)

Ω_k operator of Dirac theory in the expression for J_k in (2) (Ω_k may be a vector or tensor in which case the product must be considered as inner product)

$\int d\tau$ denotes integration of the nuclear wave functions

In the formula (5) it is supposed that for ψ and φ plane wave solutions are chosen which are normalised to one particle per unit volume. For several phenomena, however, more detailed knowledge is wanted on the transition probability than given by (5). In this article we calculate the transition probability if certain directions of emission of electron and neutrino are assumed, while we take the

sum for both orientations of the spin for electron and neutrino. The transition probability to a state in which the momenta **p** and **q** of electron and neutrino have directions within $d\omega_e$ and $d\omega_\nu$, respectively, is given by:

$$P(E, \mathbf{p}, \mathbf{q}) \, dE \, d\omega_e \, d\omega_\nu = (2\pi)^{-5} \, \Sigma_e \, \Sigma_\nu \mid H_\beta \mid^2 pEq^2 dE \, d\omega_e \, d\omega_\nu. \quad (7)$$

We use the following form for the Dirac equation in case there is no electromagnetic field (this is the same as in [2]) and [7]); many others use, however, different forms; we use relativistic units: unit of mass: the electron mass m_e; unit of velocity c; unit of action $\hbar$; we write the equation for an arbitrary mass m of the particle; later we put $m = m_e = 1$ for the electron and $m = 0$ for the neutrino):

$$\left.\begin{array}{l} (E + m)\psi_1 + (p_x - ip_y)\psi_4 + p_z\psi_3 = 0 \\ (E + m)\psi_2 + (p_x + ip_y)\psi_3 - p_z\psi_4 = 0 \\ (E - m)\psi_3 + (p_x - ip_y)\psi_2 + p_z\psi_1 = 0 \\ (E - m)\psi_4 + (p_x + ip_y)\psi_1 - p_z\psi_2 = 0 \end{array}\right\} \quad (8)$$

E and p are the operators: $E = -(\hbar/i)\partial/\partial t$ and $p_x = (\hbar/i)\partial/\partial x$ etc.

The plane wave solutions of (8) can be written as:

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = \begin{bmatrix} \hat{\psi}_1 \\ \hat{\psi}_2 \\ \hat{\psi}_3 \\ \hat{\psi}_4 \end{bmatrix} \exp i(\mathbf{p}.\mathbf{x} - Et) \quad (9)$$

It is easily seen that (8) is also valid for the $\hat{\psi}_i$ if E and **p** are considered as numbers; the relation between E and **p** is given by:

$$E^2 - m^2 = p^2 \quad (10)$$

or:

$$E = \pm \sqrt{p^2 + m^2} \quad (11)$$

(For convenience sake below we drop the $\wedge$ in our notation) The most general positive energy solution is given by:

$$\psi = \sqrt{\frac{E + m}{2E}} \begin{bmatrix} -\dfrac{p_z}{E + m} A - \dfrac{(p_x - ip_y)}{E + m} B \\ -\dfrac{(p_x + ip_y)}{E + m} A + \dfrac{p_z}{E + m} B \\ A \\ B \end{bmatrix} \quad (12)$$

This solution is normalized if $|A|^2 + |B|^2 = 1$. It is an immediate consequence of (8) that if:

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}$$

is a positive energy solution of (8), the corresponding negative energy solution is given by (cf. [8]), [9]) and [10]); the negative energy solution, so obtained, is the solution of the Dirac equation for a particle with opposite charge; this makes, however, no difference if no external fields exist as in the case treated here):

$$\begin{bmatrix} \psi_4^* \\ -\psi_3^* \\ -\psi_2^* \\ \psi_1^* \end{bmatrix} \tag{13}$$

Hence, the general negative energy solution can be written in the form:

$$\psi_- = \sqrt{\frac{E+m}{2E}} \begin{bmatrix} B^* \\ -A^* \\ \dfrac{(p_x - ip_y)}{E+m}A^* - \dfrac{p_z}{E+m}B^* \\ -\dfrac{p_z}{E+m}A^* - \dfrac{(p_x + ip_y)}{E+m}B^* \end{bmatrix} \tag{14}$$

(The energy and momentum of the plane wave (14) are given by $-E$ and $-\mathbf{p}$; the energy and momentum of the positon corresponding to the unoccupied negative energy state are therefore given by E and $\mathbf{p}$; we characterize everywhere the negative energy states by the physical quantities for the corresponding positon). Below the matrices D^+ and D^- will be used, defined by their matrix-elements according to:

$$D_{\mu\rho} = \Sigma_{spin}\, \psi_\rho^*\, \psi_\mu \tag{15}$$

(Σ_{spin} denotes that the sum for both spin states must be taken) D^+ and D^- are the matrices defined by (15) for positive and negative energy solutions respectively. Calculation gives:

$$D^+ = \tfrac{1}{2}[1 - (\boldsymbol{\alpha}.\mathbf{p} + \beta m)/E] \tag{16}$$

$$D^- = \tfrac{1}{2}[1 - (\boldsymbol{\alpha}.\mathbf{p} - \beta m)/E] \tag{17}$$

We shall now calculate $\Sigma_e\, \Sigma_\gamma\, |H_\beta|^2$ for allowed transitions and

neglect the effect of the nuclear charge (hence especially for light nuclei). For convenience the signs Σ_{m_j} and Σ_h are no longer written, further the index h is dropped to simplify the notation. The factors $\psi^* \Omega_k \varphi$ can be put before the integral sign, as they vary slowly. The following abbrevation for the nuclear matrix elements is used:

$$\int \Omega_k = \int d\tau \, \Psi_f^* \Omega_k \, Q \, \Psi_i, \text{ if we have } \beta^-\text{-emission} \qquad (18)$$

and $\qquad \int \Omega_k = \int d\tau \Psi_f^* \Omega_k Q^+ \Psi_i, \text{ if we have } \beta^+\text{-emission} \qquad (19)$

We shall further neglect all the "small" terms of (2), which means that relativistic effects for the nucleons are neglected except $\beta\gamma_5$, which will be retained because the coefficient C_5 might be greater than C_1, C_2, C_3 and C_4.

We get in this way for the case of β^--emission:

$$| H_\beta |^2 = G^2 \, | \, \Sigma_{k=1}^5 \, C_k(\psi^* \Omega_k \varphi) . \int \Omega_k \, |^2 \qquad (20)$$

As the Ω_k for T and A are vectors and the product a scalar product of two vectors we shall alter the notation to avoid confusion and put:

$$A^k(k = 1, 2, \ldots, 9) = \beta, 1, \beta\sigma_1, \beta\sigma_2, \beta\sigma_3, \sigma_1, \sigma_2, \sigma_3, \beta\gamma_5 \qquad (21)$$

and $\quad C_k(k = 1, 2, \ldots, 9) = C_1, C_2, C_3, C_3, C_3, C_4, C_4, C_4, C_5 \qquad (22)$

and we can write with ordinary products only:

$$| H_\beta |^2 = G^2 \, | \, \Sigma_{k=1}^9 \, \bar{C}_k(\psi^* A^k \varphi) \int A^k \, |^2 \qquad (23)$$

or: $\quad | H_\beta |^2 = G^2 \Sigma_{k,l=1}^9 \bar{C}_k \bar{C}_l \, (\psi^* A^k \varphi) \, (\varphi^* A^l \psi) \, (\int A^k) \, (\int A^l)^* \qquad (24)$

We use the following reduction to calculate the result and write the Dirac wave functions in their 4 components (Tr = trace):

$$\Sigma_e \Sigma_\nu (\psi^* A^k \varphi) \, (\varphi^* A^l \psi) = \Sigma_e \Sigma_\nu \, (\Sigma_{\rho\sigma} \, \psi_\rho^* \, A_{\rho\sigma}^k \varphi_\sigma) \, (\Sigma_{\lambda\mu} \varphi_\lambda^* A_{\lambda\mu}^l \psi_\mu) =$$
$$= \Sigma_e \Sigma_\nu \Sigma_{\lambda\mu\rho\sigma} A_{\rho\sigma}^k A_{\lambda\mu}^l \, \psi_\rho^* \, \psi_\mu \, \varphi_\lambda^* \, \varphi_\sigma = \text{Tr} \, (A^k D_\nu A^l D_e) \qquad (25)$$

Hence we van write:

$$\Sigma_e \Sigma_\nu \, | \, H_\beta \, |^2 = G^2 \Sigma_{k,l=1}^9 \bar{C}_k \bar{C}_l \, (\int A^k) \, (\int A^l)^* \, \text{Tr} \, (A^k D_\nu A^l D_e) \qquad (26)$$

For the evaluation of the terms: $\text{Tr}(A^k D_\nu A^l D_e)$ we use the multiplication properties of the Dirac matrices (cf. e.g. [7]) and the property that the only matrix for which the trace is $\neq 0$ is I, for which $\text{Tr}(I) = 4$. In this way, we find e.g.:

$$\text{Tr}[\beta D_\nu^- \beta D_e^+] = 1 - \mathbf{q} \cdot \mathbf{p}/E_\nu E \qquad (27)$$

This is a term of the sum for the case of β^--emission, in which case negative energy solutions must be taken for the neutrino and po-

sitive energy solutions for the electron. Because of the non-rela-
tivistic approximation for the nucleons, we can put in the calcula-
tions: $\int \beta = -\int 1$ and $\int \beta \sigma = -\int \sigma$.

Collecting the terms of the sum we get as result for the transition
probability in the case of β^--emission:

$$P_-(E, \mathbf{p}, \mathbf{q}) = (G^2/(2\pi)^5) p E q^2 \{(C_1^2 + C_2^2) \mid \int 1 \mid^2 + (C_3^2 + C_4^2) \mid \int \sigma \mid^2 +$$
$$+ C_5^2 \mid \int \beta \gamma_5 \mid^2 - (\mathbf{q} \cdot \mathbf{p}/qE) [(C_1^2 - C_2^2) \mid \int 1 \mid^2 - \tfrac{1}{3}(C_3^2 - C_4^2) \mid \int \sigma \mid^2 +$$
$$+ C_5^2 \mid \int \beta \gamma_5 \mid^2] + (2/E) [C_1 C_2 \mid \int 1 \mid^2 + C_3 C_4 \mid \int \sigma \mid^2]\} \qquad (28)$$

Analogously we can calculate the result for β^+-emission; we have
only to take negative energy solutions for the electron and positive
energy solutions for the neutrino. Starting with (1) it is found that
for β^+-emission instead of (24) the following formula must be used:

$$\mid H_\beta \mid^2 = G^2 \Sigma_{k,l} \bar{C}_k \bar{C}_l (\psi^* A^k \varphi)(\varphi^* A^l \psi)(\int A^k)^* (\int A^l) \qquad (29)$$

The result of the calculation is:

$$P_+(E, \mathbf{p}, \mathbf{q}) = (G^2/(2\pi)^5) p E q^2 \{(C_1^2 + C_2^2) \mid \int 1 \mid^2 + (C_3^2 + C_4^2) \mid \int \sigma \mid^2 +$$
$$+ C_5^2 \mid \int \beta \gamma_5 \mid^2 - (\mathbf{q} \cdot \mathbf{p}/qE) [(C_1^2 - C_2^2) \mid \int 1 \mid^2 - \tfrac{1}{3}(C_3^2 - C_4^2) \mid \int \sigma \mid^2 +$$
$$+ C_5^2 \mid \int \beta \gamma_5 \mid^2] - (2/E) [C_1 C_2 \mid \int 1 \mid^2 + C_3 C_4 \mid \int \sigma \mid^2]\} \qquad (30)$$

In the calculation of (28) and (30) the average for all directions of
the nuclear spins has been taken.

§ 3. *The shape of β-spectra, neglecting nuclear charge.* We get the
formula for the shape of β-spectra from (28) and (30) by sum-
mation for all directions of emission of electron and neutrino:

$$P_\pm(E) = (G^2/2\pi^3) p E q^2 [(C_1^2 + C_2^2) \mid \int 1 \mid^2 + (C_3^2 + C_4^2) \mid \int \sigma \mid^2 +$$
$$+ C_5^2 \mid \int \beta \gamma_5 \mid^2 \mp (2/E) (C_1 C_2 \mid \int 1 \mid^2 + C_3 C_4 \mid \int \sigma \mid^2)] \qquad (31)$$

It is tacitly understood in this and the following formulae that the
sum for the different states of the final nucleus is taken, hence e.g.

$$\mid \int \sigma \mid^2 = \Sigma_{m_f} [\mid \int \Psi_f^*(m_f) \sigma_x \Psi_i(m_i) d\tau \mid^2 + \mid \int \Psi_f^*(m_f) \sigma_y \Psi_i(m_i) d\tau \mid^2 +$$
$$+ \mid \int \Psi_f^*(m_f) \sigma_z \Psi_i(m_i) d\tau \mid^2] \qquad (32)$$

(this sum is independent of m_i).

The term with $1/E$ can only occur if we have "mixed" invariants.
However, mixing of invariants is possible for which the term never
occurs (viz. combinations of either J_1 or J_2 with either J_3 or J_4).
Further combinations can be given (e.g. of J_1, J_2 and J_3) so that for

several nuclei the term occurs and for others it does not, depending on the magnitude of $|\int 1|^2$ and $|\int \sigma|^2$. This $1/E$-term was first calculated by F i e r z [5]) and later by R o z e n t a l [11]) with the aid of the meson theory. (F i e r z simplifies the state of affairs for convenience by putting all nuclear matrix elements $= 1$; R o-z e n t a l gets a much more complicated formula as he calculates all the terms according to the meson theory of M ø l l e r and R o s e n f e l d).

The influence of the nuclear charge will be discussed in § 5. A discussion of the possibility of a determination of $C_1 \ldots C_5$, using amongst others (31) and the measured shape of β-spectra, will be given in § 7.

§ 4. *The angular correlation of electron and neutrino, neglecting nuclear charge.* The general formula for the angular correlation of the electron and the neutrino is already given by (28) and (30), which we can write together as:

$$P_{\pm}(E, \mathbf{p}, \mathbf{q}) = (G^2/(2\pi)^5)pEq^2 \{(C_1^2+C_2^2) \, |\int 1|^2 + (C_3^2+C_4^2) \, |\int \sigma|^2 + $$
$$+ C_5^2 \, |\int \beta\gamma_5|^2 - (\mathbf{q}\cdot\mathbf{p}/qE) \, [(C_1^2-C_2^2) \, |\int 1|^2 - \tfrac{1}{3}(C_3^2-C_4^2) \, |\int \sigma|^2 + $$
$$+ C_5^2 \, |\int \beta\gamma_5|^2] \mp (2/E) \, [C_1 C_2 \, |\int 1|^2 + C_3 C_4 \, |\int \sigma|^2]\} \qquad (33)$$

If ϑ is the angle between $\mathbf{q}$ and $\mathbf{p}$ and if:

$$\beta = v/c = p/E \qquad (34)$$

we can write:

$$\mathbf{q}\cdot\mathbf{p}/qE = \beta \cos \vartheta \qquad (35)$$

According to (33) the general law for the angular correlation can be given as:

$$1 + (B/E) + A \, \beta \cos \vartheta \qquad (36)$$

in which A and B are constants given by:

$$A = - \frac{(C_1^2 - C_2^2) \, |\int 1|^2 - \tfrac{1}{3}(C_3^2 - C_4^2) \, |\int \sigma|^2 + C_5^2 \, |\int \beta\gamma_5|^2}{(C_1^2 + C_2^2) \, |\int 1|^2 + (C_3^2 + C_4^2) \, |\int \sigma|^2 + C_5^2 \, |\int \beta\gamma_5|^2} \qquad (37)$$

and:

$$B_{\pm} = \mp \frac{2C_1 C_2 \, |\int 1|^2 + 2 C_3 C_4 \, |\int \sigma|^2}{(C_1^2 + C_2^2) \, |\int 1|^2 + (C_3^2 + C_4^2) \, |\int \sigma|^2 + C_5^2 \, |\int \beta\gamma_5|^2} \qquad (38)$$

Hence $B = 0$ if $C_1 C_2 = 0$ and $C_3 C_4 = 0$. For the case of pure invariants it is found that $B = 0$ and $A = -1$ for the scalar and

pseudo-scalar, $A = 1$ for the vector, $A = \frac{1}{3}$ for the tensor and $A = -\frac{1}{3}$ for the pseudo-vector (cf. [5]), [12]), [13]), [14]) and [15]) *).

Experiments on the angular correlation of electron and neutrino are possible by the study of the recoil of the nucleus in β-desintegration. These experiments can give valuable information for the determination of $C_1 \ldots C_4$. The technique has been improved very much in the last few years. Though at present the material on allowed transitions does not yet give clear results, it may be hoped that more accurate data will soon be available. It follows from (37) that $| A | \leqslant 1$. The factor B/E does not vary rapidly and hence the present experimental technique of recoil experiments can probably not give more than an average value $A/(1+B/\bar{E})$ ($\bar{E}$ is a mean value of the electron energy). B can, however, be determined from experiments on the shape of β-spectra.

§ 5. *The shape of β-spectra, taking into account the influence of the nuclear charge; K-capture; the life-time of β-emitters.* The formula (31) gives the shape of the β-spectrum without taking into account the influence of the nuclear charge; this is a reasonable approximation for light nuclei and energies that are not too small. To compare theory and experiments for other cases the effect of the nuclear charge must be calculated. We give the result and main features of the calculation.

Coulomb field solutions for the electron and spherical wave solutions for the neutrino are used. With the same notations as in § 2 we can write for the probability that an electron is emitted with energy between E and $E + dE$ (for allowed transition):

$$P_-(E)dE = 2\pi G^2 \Sigma_{m_f} \Sigma_{\substack{j_\nu l_\nu m_\nu \\ j_e l_e m_e}} | \Sigma_k C_k (\psi^* A^k \varphi) \int A^k |^2 dE \qquad (39)$$

in which ψ and φ are now spherical wave solutions of electron and neutrino normalized to the energy. We must take the sum for all possible quantum numbers $(j_e l_e m_e)$ and $(j_\nu l_\nu m_\nu)$ for these solutions.

If we introduce the matrices:

$$V^\nu_{\mu\rho} = \Sigma_{j_\nu l_\nu m_\nu} \varphi_\rho^{(j_\nu l_\nu m_\nu)*} \varphi_\mu^{(j_\nu l_\nu m_\nu)} \qquad (40)$$

and:

$$V^e_{\mu\rho} = \Sigma_{j_e l_e m_e} \psi_\rho^{(j_e l_e m_e)*} \psi_\mu^{(j_e l_e m_e)} \qquad (41)$$

---

*) It must be mentioned that the results in [5]) and [13]) are contradictory; we find the same results as in [13]); [12]), [14]), and [15]) do not give the results for all invariants. H e b b [14]) and R o s e [15]) take into account the influence of the nuclear charge, but their results disagree.

we can write:

$$P_-(E) = 2\pi G^2 \Sigma_{kl} \, \bar{C}_k \, \bar{C}_l \, \mathrm{Tr}(A^k V^\nu A^l V^e) \, (\smallint A^k) \, (\smallint A^l)^* \qquad (42)$$

The solutions that must be taken for the electron are:

$$\psi \quad \begin{Bmatrix} - \\ Y_{00}f_{-2} \\ - \\ - \end{Bmatrix} \quad \begin{Bmatrix} Y_{00}f_{-2} \\ - \\ - \\ - \end{Bmatrix} \quad \begin{Bmatrix} - \\ - \\ - \\ Y_{00}g_0 \end{Bmatrix} \quad \begin{Bmatrix} - \\ - \\ Y_{00}g_0 \\ - \end{Bmatrix} \qquad (43)$$
$$\quad\; l=1\ m=-\tfrac{1}{2} \quad l=1\ m=\tfrac{1}{2} \quad l=0\ m=-\tfrac{1}{2} \quad l=0\ m=\tfrac{1}{2}$$

Analogously we have for the neutrino (negative energy solutions):

$$\varphi \quad \begin{Bmatrix} - \\ - \\ -Y_{00}Q \\ - \end{Bmatrix} \quad \begin{Bmatrix} - \\ - \\ - \\ Y_{00}Q \end{Bmatrix} \quad \begin{Bmatrix} Y_{00}Q \\ - \\ - \\ - \end{Bmatrix} \quad \begin{Bmatrix} - \\ -Y_{00}Q \\ - \\ - \end{Bmatrix} \qquad (44)$$

from this it is calculated:

$$V^e = \tfrac{1}{2} Y_{00}^2 \, [(g_0^2 + f_{-2}^2) - \beta(g_0^2 - f_{-2}^2)] \qquad (45)$$

$$V^\nu = Y_{00}^2 \, Q^2 \qquad (46)$$

In (43),..., (46) the following notations have been used:

$$Y_{00} = (4\pi)^{-\frac{1}{2}}$$

$\left.\begin{array}{l} g_0 \\ f_{-2} \end{array}\right\}$ functions that give the radial dependence of the electron wave functions, evaluated at the surface of the nucleus (cf. [16]).

$$Q = q/\sqrt{\pi}$$

From the expressions for the wave functions we find:

$$g_0^2 - f_{-2}^2 = (\gamma/E) \, (g_0^2 + f_{-2}^2) \text{ with } \gamma = \sqrt{1 - a^2 Z^2} \text{ and } a = e^2/\hbar c \qquad (47)$$

and we get the result:

$$P_\pm(E) = (G^2/2\pi^3) p E q^2 F(Z, E) \, [(C_1^2 + C_2^2) \, |\smallint 1|^2 + (C_3^2 + C_4^2) \, |\smallint \sigma|^2 +$$
$$+ C_5^2 \, |\smallint \beta\gamma_5|^2 \mp (2\gamma/E) \, (C_1 C_2 \, |\smallint 1|^2 + C_3 C_4 \, |\smallint \sigma|^2)] \qquad (48)$$

with:

$$F(Z, E) =$$
$$= 2(1+\gamma) \, [\Gamma(2\gamma+1)]^{-2} \, (2pR)^{2\gamma-2} \, [\exp(\pi a Z E/p)] \, |\Gamma(\gamma + ia Z E/p)|^2 \qquad (49)$$

R is the radius of the nucleus.

The result (48) ror the shape of β-spectra taking the nuclear charge into account, must be compared with (31), in which the in-

fluence of the nuclear charge is neglected. It is seen that the effect of mixed invariants may be an additional factor $(1 + a/E)$ in comparison with the case of "pure" invariants, in (48) as well as in (31).

The total transition probability $P_\pm$ can be calculated by integrating over the energy; if $\tau_\pm$ is the mean life and $t_\pm$ is the half-life we have:

$$P_\pm = 1/\tau_\pm = (\ln 2)/t_\pm = (G^2/2\pi^3) \int_{E=1}^{E_0} pEq^2 F(Z,E) [(C_1^2 + C_2^2) |\int 1|^2 + (C_3^2 + C_4^2)|\int \sigma|^2 + C_5^2 |\int \beta\gamma_5|^2 \mp (2\gamma/E) (C_1 C_2 |\int 1|^2 + C_3 C_4 |\int \sigma|^2)] dE \quad (50)$$

If we put:

$$f(Z, E_0) = \int_{E=1}^{E_0} pEq^2 F(Z, E) dE \quad (51)$$

$$g(Z, E_0) = \int_{E=1}^{E_0} pq^2 F(Z, E) dE \quad (52)$$

$$M^2 = (C_1^2 + C_2^2) |\int 1|^2 + (C_3^2 + C_4^2) |\int \sigma|^2 + C_5^2 |\int \beta\gamma_5|^2 \quad (53)$$

$$N = 2C_1 C_2 |\int 1|^2 + 2 C_3 C_4 |\int \sigma|^2 \quad (54)$$

this result can be written as follows:

$$P_\pm = (G^2/2\pi^3) [f_\pm(Z, E_0) M^2 \mp \gamma g_\pm(Z, E_0) N] \quad (55)$$

The transition probability for K-capture can be calculated in an analogous way, starting with the formula:

$$P_K = 2\pi G^2 \sum_{m_f} \sum_{\substack{j_\nu l_\nu m_\nu \\ j_e l_e m_e}} | \sum_k C_k (\psi^* A^k \varphi)^* (\int A^k) |^2 \quad (56)$$

in which the wave function ψ of the electron is a wave function of the discrete spectrum; φ a wave function of the neutrino normalized to the energy. The wave functions of electron and neutrino are given by:

$$\psi \begin{bmatrix} - \\ - \\ - \\ Y_{00}g_0 \end{bmatrix} \qquad \begin{bmatrix} \bullet \\ - \\ Y_{00}g_0 \\ - \end{bmatrix} \qquad (57)$$

$$1 = 0 \quad m = -\tfrac{1}{2} \qquad\qquad 1 = 0 \quad m = \tfrac{1}{2}$$

and:

$$\varphi \begin{bmatrix} - \\ - \\ - \\ Y_{00}Q \end{bmatrix} \begin{bmatrix} - \\ - \\ Y_{00}Q \\ - \end{bmatrix} \begin{bmatrix} - \\ Y_{00}Q \\ - \\ - \end{bmatrix} \begin{bmatrix} Y_{00}Q \\ - \\ - \\ - \end{bmatrix} \quad (58)$$

$$1 = 0 \; m = -\tfrac{1}{2} \quad 1 = 0 \; m = \tfrac{1}{2} \quad 1 = 1 \; m = -\tfrac{1}{2} \quad 1 = 1 \; m = \tfrac{1}{2}$$

hence:

$$V^e = \tfrac{1}{2}Y_{00}^2 g_0^2(1 - \beta) \tag{59}$$

$$V^\nu = Y_{00}^2 Q^2 \tag{60}$$

in which $Q = q/\sqrt{\pi}$; the expression for g_0^2 is given by (62). The determination of the traces gives the final result:

$$P_K = (G^2/4\pi^2)\,(E_0 + E_K)^2 g_0^2[(C_1^2 + C_2^2)\,|\textstyle\int 1\,|^2 + (C_3^2 + C_4^2)\,|\int \sigma\,|^2 + \\ + C_5^2\,|\int \beta\gamma_5\,|^2 + 2C_1C_2\,|\int 1\,|^2 + 2C_3C_4\,|\int \sigma\,|^2\,] \tag{61}$$

$E_K \approx \sqrt{1 - \alpha^2 Z^2} \approx 1$ energy of an electron in the K-shell. The value of g_0^2 is given by:

$$g_0^2 = \frac{1 + E_K}{2\Gamma(2\gamma + 1)}\,(2\alpha Z_{eff})^3\,(2\alpha Z_{eff} R)^{2\gamma - 2} \tag{62}$$

Especially important is the ratio P_K/P_+ as this can be determined experimentally; we find:

$$\frac{P_K}{P_+} = \frac{(\pi/2)\,(E_0 + E_K)^2 g_0^2\,(M^2 + N)}{f_+\,(Z, E_0)M^2 - \gamma g_+\,(Z, E_0)N} \tag{63}$$

If $C_1C_2 = 0$, $C_3C_4 = 0$ so that $N = 0$, M^2 disappears from (63) which takes the simple shape:

$$\frac{P_K}{P_+} = \frac{(\pi/2)(E_0 + E_K)^2 g_0^2}{f_+(Z, E_0)} \tag{64}$$

(Remark: A possible deviation of the P_K/P_+ value from (64) as a consequence of mixed invariants was also considered by M e r c i e r [17]); as he uses the not entirely correct result of F i e r z [5]) his result differs from (63)).

§ 6. *Theoretical arguments for certain linear combinations of invariants.* In the present state of the theory, no other well-founded theoretical arguments exist than relativistic invariance to determine the interaction Hamiltonian.

In the meson-theories of β-radioactivity certain linear combinations of invariants are found for H_β, but as these theories are no longer accepted, we will not investigate these combinations.

We will, however, consider the consequences for the interaction Hamiltonian of certain symmetries that can be imposed on it. Of course no strict a priori reasons can be given for the validity of

such symmetries, but it is interesting to investigate if such a symmetry exists by comparison with experiment.

Symmetry principle: To give a precise formulation of this principle, we consider the expressions for an arbitrary J_k for β^- and β^+-emission, according to (1) and (2):

$$J^- = (\psi_+^* (- e)\Omega_L \varphi_-(\nu)) \, (\Psi_f^*(p)\Omega_N \Psi_i(n)) \qquad (65)$$

$$J^+ = (\psi_-^* (- e)\Omega_L \varphi_+(\nu))^* \, (\Psi_f^*(n)\Omega_N \Psi_i(p)) \qquad (66)$$

n and p indicate: neutron and proton wave function (the notation with isotopic spin is not used in (65) and (66)). The indices $+$ and $-$ of ψ and φ indicate that we have to use positive or negative energy solutions; $- e$ indicates that we use the Dirac equation for electrons with a negative charge so that positons must be regarded as "holes".

The expression for the total interaction is given by

$$H_\beta^\mp = G \, \Sigma_{k=1}^5 \, C_k J_k^\mp \qquad (67)$$

We now formulate the *symmetry principle* as follows:

The processes of negaton and positon emission must be symmetrical, apart from Coulomb interactions, in such a way that, if this Coulomb interaction is neglected, the expressions for H_β^- and H_β^+ are equal (possibly with exception of the sign) in any two cases of negaton and positon emission in which the wave functions of the emitted (positive and negative) electrons and neutrino's are physically equivalent and if further: $\Psi_f(p)$ and $\Psi_i(n)$ for negaton emission are respectively the same as $\Psi_f(n)$ and $\Psi_i(p)$ for positon emission. (68)

If we say that the wave functions of a positon and negaton are "physically equivalent" we mean that they represent particles that have e.g. the same momentum if both wave functions are plane waves. However, one particle may be represented as a "hole" with negative energy in such a way that the wave functions are not identical.

We will prove that according to this principle only two kinds of linear combinations are possible: 1) combinations of S, A and P, 2) combinations of V and T. In the first case H_β changes its sign, if we change from H_β^- to H_β^+, in the second case the sign remains the same for this change. The change of sign in the former case does not give a different rôle to negatons and positons: all physical conse-

quences as transition probabilities remain the same, even if the sign changes.

The exact mathematical expression that H_β is symmetrical, respectively antisymmetrical is, for the case of a pure invariant $(H_\beta = J)$:

$$(\psi_-^*(-e)\,\Omega_L\varphi_+(\nu))^* = \varepsilon(\psi_+^*(-e)\,\Omega_L\varphi_-(\nu)) \qquad (69)$$

($\varepsilon = +1$ symmetrical case; $\varepsilon = -1$ antisymmetrical case) if for $\psi_-^*(-e)$ and $\psi_+^*(-e)$ as well as for $\varphi_+(\nu)$ and $\varphi_-(\nu)$ solutions are chosen that are physically equivalent (except for the sign of the charge) which choice is possible if no rôle is played by Coulomb fields.

Remark: Between neutron and proton a slight mass-difference exists. This difference has for result that the wave functions $\Psi_i(n)$ and $\Psi_f(p)$ cannot be rigorously the same as respectively $\Psi_i(p)$ and $\Psi_f(n)$, if (positive and negative) electron and neutrino are emitted with physically equivalent wave functions for both cases. In passing from (68) to (69) this difference between the nucleon wave functions is neglected. (69) is the starting-point for further deductions, which are rigorous.

In order to prove the above-mentioned results we also use a Dirac equation for electrons with positive charge for the positons and hole theory for the negatons; the expressions corresponding to (65) and (66) are in this formulation:

$$\tilde{J}^- = (\psi_-^*(+e)\,\Omega_L\,\varphi_+(\nu))^* \,(\Psi_f^*(p)\,\Omega_N\,\Psi_i(n)) \qquad (70)$$

$$\tilde{J}^+ = (\psi_+^*(+e)\,\Omega_L\,\varphi_-(\nu))\,(\Psi_f^*(n)\,\Omega_N\,\Psi_i(p)) \qquad (71)$$

Between the solutions of the Dirac equation with negative and positive charge a $(1, 1)$-correspondence exists, determined by a matrix C, in such a way that we can write [8] [9] [10]:

$$\begin{aligned}
\psi_+^*(-e) &= C\varphi_-(+e) & \varphi_+^*(\nu) &= C\varphi_-(\nu) \\
\psi_-(-e) &= C^{-1}\psi_+^*(+e) & \varphi_-(\nu) &= C^{-1}\varphi_+^*(\nu)
\end{aligned} \qquad (72)$$

The matrix C has the property:

$$C^* = C^{-1} \qquad (73)$$

In the ordinary representation of the Dirac matrices as is used here (the same as in [7])) C is given by:

$$C = i\beta a_2 = \begin{bmatrix} & & & 1 \\ & & -1 & \\ & -1 & & \\ 1 & & & \end{bmatrix} \tag{74}$$

According to (72), (69) can be written in another way, for we have:

$$(\psi_-^*(-e)\,\Omega_L\varphi_+(\nu))^* = [(C^{*-1}\psi_+(+e))\,\Omega_L(C^*\varphi_-^*(\nu))]^* =$$
$$= (C^{-1}\psi_+^*(+e))\,\Omega_L^*(C\varphi_-(\nu)) = (\psi_+^*(+e)\tilde{C}^{-1})\,\Omega_L^*(C\varphi_-(\nu)) =$$
$$= \psi_+^*(+e)\,(C^+\,\Omega_L^*C)\varphi_-(\nu) \tag{75}$$

hence (69) becomes:

$$\psi_+^*(+e)\,(C^+\Omega_L^*C)\varphi_-(\nu) = \varepsilon\psi_+^*(-e)\Omega_L\varphi_-(\nu) \tag{76}$$

which condition is imposed for every case that $\psi_+(+e)$ and $\psi_+(-e)$ are the same for the electrons as well as the $\varphi_-(\nu)$ for the neutrino wave functions, and (76) is equivalent with:

$$C^+\Omega_L^*C = \varepsilon\Omega_L \qquad (\varepsilon = \pm 1) \tag{77}$$

This is the mathematical expression of (68) for pure invariants, but it is clear that it can be extended immediately to the case of mixed invariants. We shall prove:

$$\left.\begin{array}{l} \text{S, A, P satisfy (77) with } \varepsilon = -1 \\ \text{V, T satisfy (77) with } \varepsilon = +1 \end{array}\right\} \tag{78}$$

hence it is clear that linear combinations of S, A, P respectively V, T also satisfy (77) with $\varepsilon = -1$ respectively $\varepsilon = +1$. Further (77) cannot be satisfied if linear combinations are used in which invariants of both groups, (S, A, P) and (V, T), occur.

The proof of (78) can be given by the use of the expression (74) for C and using the ordinary representation of the Dirac matrices. We will give, however, a proof which does not depend on the representation chosen for the Dirac matrices and it will also be shown that (77) is a Lorentz-invariant condition for the interaction Hamiltonian, which property is of course necessary for a physical condition.

We write the Dirac-equation in a relativistic form with real time coordinate:

$$[\gamma^\mu(p_\mu + (e/c)A_\mu) + imc]\psi = 0 \tag{79}$$

The γ's, generally not Hermitian, satisfy the commutation rule:

$$\tfrac{1}{2}(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) = g^{\mu\nu} \tag{80}$$

with:

$$g^{\mu\nu} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix} \tag{81}$$

We introduce the matrices A, B, C according to (this introduction of the matrices is slightly different from [8]) because of the use of a real time coordinate):

$$\overset{+}{\gamma}{}^\mu = -A\gamma^\mu A^{-1} \tag{82}$$

$$\overset{*}{\gamma}{}^\mu = C\gamma^\mu C^{-1} \tag{83}$$

$$\overset{\sim}{\gamma}{}^\mu = -B\gamma^\mu B^{-1} \tag{84}$$

A and C can be chosen in such a way that:

$$A^+ = A \tag{85}$$

$$C* = C^{-1} \tag{86}$$

Further the following relation between A and C exists:

$$A = -C^+ A * C \tag{87}$$

We put:

$$\psi^+ = \psi * A \tag{88}$$

If we perform a Lorentz-transformation of the coordinate system, the wave function transforms with a matrix S, which defines a representation of the Lorentz-group:

$$\psi' = S\psi \tag{89}$$

A and C than transform according to:

$$A' = S^{+-1}AS^{-1} \tag{90}$$

$$C' = S* CS^{-1} \tag{91}$$

ψ^+ transforms according to:

$$\psi^{+\prime} = \psi^+ S^{-1} \tag{92}$$

Further:

$$\gamma'^\mu = S^{-1}\gamma^\mu S = \sum_\nu a_{\mu\nu}\gamma^\nu \tag{93}$$

The $a_{\mu\nu}$ are real and determine the Lorentz transformation.

The invariants of the Dirac theory can be given in the form:

$$\psi^+ \Gamma \psi = \psi^* A \Gamma \psi \tag{94}$$

in which Γ is a product of matrices γ^μ. (We do not write the indices explicitly in Γ). If we impose the physical condition on the invariants that the $\psi^+ \Gamma \psi$ are real quantities we get the condition:

$$(\psi^+ \Gamma \psi)^* = \psi^+ \Gamma \psi \tag{95}$$

which can be reduced to:

$$A \Gamma A^{-1} = \Gamma^+ \tag{96}$$

By means of (96), (82), (80) it can be deduced if a factor i must be used in the γ-product Γ (in Γ a real factor remains undetermined), and the following list of Γ's for the various invariants is found:

$$\left.\begin{array}{ll} \text{S} & \Gamma = 1 \\ \text{V} & \Gamma = i\gamma^\mu \\ \text{T} & \Gamma = i\gamma^\mu\gamma^\nu \\ \text{A} & \Gamma = \gamma^\lambda\gamma^\mu\gamma^\nu \\ \text{P} & \Gamma = \gamma^\lambda\gamma^\mu\gamma^\nu\gamma^\rho \end{array}\right\} \tag{97}$$

Hence we can write for the different invariants:

$$\Gamma^* = \delta C \, \Gamma C^{-1} \tag{98}$$

in which $\delta = \pm 1$ is a number that can be determined from (80), (83) and (97); it is found:

$$\left.\begin{array}{l} \delta = \quad 1 \text{ for S, A, P} \\ \delta = -1 \text{ for V, T} \end{array}\right\} \tag{99}$$

We will deduce (78) and write (77) in the form that is obtained by putting $\Omega_L = A\Gamma$:

$$C^+ A^* \Gamma^* C = \varepsilon A \Gamma \tag{100}$$

In order to investigate if this condition can be satisfied we reduce it in the following way (by the use of (98)):

$$C^+ A^* C \Gamma C^{-1} C = A\Gamma . \varepsilon\delta \tag{101}$$

$$C^+ A^* C = A . \varepsilon\delta \tag{102}$$

Comparing (102) with (87) we see that (100) is satisfied by every (pure) invariant, and that the value of ε is given by:

$$\varepsilon = - \delta \tag{103}$$

According to (99) and (103), (78) is proved.

We now check the Lorentz-invariance of the condition (100). Some calculation shows that if ψ transforms according to (89), (100) is transformed into:

$$C^+ A^*(S^{-1} \Gamma S)^* C = A(S^{-1} \Gamma S) \qquad (104)$$

Hence it is clear that this is a Lorentz covariant condition, for the expressions for $S^{-1}\Gamma S$ are determined by (93) and as the $a_{\mu\nu}$ are real, $(S^{-1}\Gamma S)^*$ has an expression with the same coefficients as $S^{-1}\Gamma S$.

A different symmetry principle has been given by C r i t c h-f i e l d and W i g n e r [18]) (cf. also [3])). These authors treat the four particles participating in the interaction on an equal footing. This is possible because all the particles are assumed to satisfy the Dirac equation. It is clear that this symmetry is a heavier restriction than our symmetry principle and that it must be expected that the possible linear combinations will also be more severely restricted. The result is indeed that only the linear combination:

$$S - A - P \qquad (105)$$

satisfies this symmetry principle. This combination is antisymmetrical in the four particles; a combination symmetrical in the four particles does not exist.

We think that it is preferable not to treat nucleons and leptons on the same footing and therefore introduced the above-mentioned less restrictive symmetry principle. Even then the same important conclusion as can be drawn from C r i t c h f i e l d and W i g n e r's symmetry principle remains valid viz. the nonexistence of a $1/E$-term in the expression for the shape of the β-spectrum. For it is an immediate consequence of our symmetry principle that $C_1C_2 = 0$ and $C_3C_4 = 0$. Whether this is true must be tested by careful experiments (cf § 7).

§ 7. *The present experimental data and the determination of the Hamiltonian.* If we take the general starting point ((3) from § 2):

$$H_\beta = G \Sigma_{k=1}^5 C_k J_k \qquad (106)$$

for the nucleon-lepton interaction energy, it can be asked how the constants G, C_1, C_2, C_3, C_4, C_5 that define the interaction can be determined from the experiments. These constants are subjected to

a condition e.g. the condition (4) from § 2; however, it is easier to use the following condition (which will be used below):

$$C_1^2 + C_2^2 + 3C_3^2 + 3C_4^2 (+ C_5^2) = 1 \qquad (107)$$

as in this way G is uniquely determined by the half life of the neutron (cf. no 3 below).

For the determination of the constants it is important to remember the selection rules for the nuclear matrix-elements (allowed transitions):

$$\left. \begin{array}{lll} |\int 1 \,|^2 & \Delta J = 0 & \text{no change of parity} \\ |\int \sigma \,|^2 & \Delta J = 0, \pm 1 \ (\text{no } 0 \to 0) & \text{no change of parity} \\ |\int \beta \gamma_5 \,|^2 & \Delta J = 0 & \text{change of parity} \end{array} \right\} \qquad (108)$$

We now give a short survey of the present experimental data on β-radioactivity for allowed transitions and the conclusions that can be drawn from them:

1) The results on the shape of allowed β-spectra give a check on the theory and give the possibility to see if a $1/E$-term is present. The latest measurements with very thin sources still show some deviation from the original Fermi distribution (which has no $1/E$-term) for small energies, but it seems probable that these deviations are due to the remaining source thickness or the electric charge of the source. It seems that no $1/E$-term exists: the deviations that still remain have an entirely different shape. If the measured shape is compared with the Fermi distribution with an extra factor $(1 + a/E)$, it can be concluded for many cases that have been measured accurately that $0 < a < 0,1$ (it is difficult to give a very low upper limit for a, because $1/E$ is a slowly varying function; $E \geqslant 1$). We mention the following nuclei for which accurate experimental determinations have been carried out: ^{3}H[19]), ^{12}B[20]), ^{13}N[21]), ^{15}O[22]), ^{61}Cu[23]), ^{64}Cu[24]).

If $a = 0$ it follows that $C_1 C_2 = 0$ and $C_3 C_4 = 0$, hence only the following combinations of invariants can be realized: S with T, S with A, V with T or V with A.

2) It is practically certain that G a m o w–T e l l e r selection rules are valid i.e. the nuclear spin may change with one or remains the same for allowed transitions. The main argument is that ^{6}He → ^{6}Li is an allowed transition according to life-time and energy. ^{6}He has as even-even nucleus probably spin 0; the spin of ^{6}Li has been measured to be 1; we refer for other arguments to K o n o p i n s k i[3]).

The validity of G a m o w–T e l l e r selection rules means that $C_3^2 + C_4^2$ is not too small in comparison with $C_1^2 + C_2^2$, and that $C_3^2 + C_4^2 > 0$ (though it may be that $C_1^2 + C_2^2 = 0$) cf. (50).

If C_5 should be $\neq 0$ and not too small, transitions had to exist, which must be classified as allowed, according to their lifetime and maximum energy and which show change of parity. As the parity of nuclei cannot be measured but must be deduced for example from the not very certain shell-model, conclusions on C_5 are not yet wholly conclusive. Available evidence makes probable, however, that no allowed transitions with change of parity exist [25] [26]). Hence below we shall put $C_5 = 0$.

3) For a determination of the *Fermi-constant* G it is necessary to determine half-life and maximum energy for a β-transition for which the nuclear matrix element is known. The simplest β-transition is: $n \to p + \beta^- + 780$ keV. In this case no theory of nuclei is needed for the nuclear matrix-elements; we have namely $|\int 1|^2 = 1$ and $\frac{1}{3}|\int \sigma|^2 = 1$, hence $M^2 = 1$ for the neutron independent of the values of $C_1 \ldots, C_4$ and we can calculate G from a determination of the half-life of the neutron, which is, however, very rough at present (between 9 and 18 min [27])). Other β-transitions may also be used to determine G; the nuclear matrix elements must be estimated in this case. $^3H \to {}^3He$ is a suitable transition for this purpose.

It is found that $G \approx 4.10^{-12}$ relativistic units.

4) The angular correlation between electron and neutrino can be determined by recoil experiments. The only recoil experiment to determine A in the angular distribution law:

$$(1 + A\beta \cos \vartheta)$$

for an allowed transition has been made on ^6He by A l l e n et al [28]). However, these fine experiments still had large statistical errors; with a reasonable certainty one can only say that

$$-1 \leqslant A < 0,5$$

Determination of A for various allowed transitions can give the following combinations of the constants C_i (cf. (37)): $(C_1^2 - C_2^2)/(C_1^2 + C_2^2)$ and $(C_3^2 - C_4^2)/(C_3^2 + C_4^2)$ which would be very valuable information on the C_i's. As according to 1) $C_3 C_4 = 0$ we must expect $A = \frac{1}{3}$ or $A = -\frac{1}{3}$ for ^6He, because it results from $\Delta J = 1$ that $|\int 1|^2 = 0$. It is clear that a measurement that gives $A = \frac{1}{3}$ or $-\frac{1}{3}$ for ^6He

does not imply at all that the interaction is given by a pure invariant (cf. (37)). In order to decide this point, A must also be measured for a case with $\Delta J = 0$ e.g. ^{19}Ne. If A for ^{19}Ne and ^{6}He should turn out to be the same, it would follow that the interaction is indeed given by a pure invariant; if they should be different a linear combination must be used and it could even be concluded which linear combination: if e.g. $A = -\frac{1}{3}$ was measured for ^{6}He and $A < -\frac{1}{3}$ for ^{19}Ne, it would follow that the expression for the interaction energy is a combination of S and A, but if $A > -\frac{1}{3}$ for ^{19}Ne, it would follow that we have a combination of V and A.

5) An accurate check on the theory of β-radioactivity is possible by the determination of the ratio P_K/P_+ for simultaneous positon emission and K-capture .Within the limits of experimental accuracy the agreement is very satisfactory [29].

6) An accurate determination of half-lives and maximum energies for K o n o p i n s k i 's group 0 A [3] allows:

a) A test of the theory, namely of the law for the (rough) dependence of halflife on maximum energy. With the present data this check gives a satisfactory result (half-life is approximately inversely proportional to the fifth power of the maximum energy of the β-spectrum).

b) A determination of $(C_3^2 + C_4^2)/(C_1^2 + C_2^2)$ must be possible if measurements are made with great accuracy, and if nuclear matrix elements can be estimated from knowledge on nuclear wave functions (cf. (50)). The present data are rather scarce for this purpose, we hope to come back to this point in the future.

7) Recently L o n g m i r e, W u and T o w n e s [30] have discussed a forbidden β-spectrum (^{38}Cl) on the assumption of different combinations of invariants; it is concluded that several of them can explain the shape of the spectrum, while this is not possible by any single invariant. MM. B o u c h e z et N a t a f have, however, pointed out to us that this case might also be explained with a pure invariant, if an alteration of the ordinary selection rules is taken into account, which might be necessary for light nuclei (cf. [31]).

The *symmetry principle*, we proposed in § 6 has as result: $a = 0$, i.e. no $1/E$-term exists. This is also a consequence of the symmetry principle of C r i t c h f i e l d and W i g n e r. According to 1) of this section the absence of a $1/E$-term is in accordance with the

experimental data. As both symmetry principles give, however, a more severe restriction on the linear combinations than $C_1C_2 = 0$ and $C_3C_4 = 0$ (equivalent to $a = 0$) further experimental data will have to be used to decide if one or the other or perhaps neither of these symmetry principles is true. E.g. our symmetry principle could be tested according to 4) using recoil experiments with ^{6}He and ^{19}Ne.

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