

AVERAGE EXCITATION POTENTIALS OF AIR AND ALUMINIUM

by M. BOGAARDT and B. KOUDIJS

Physisch Laboratorium der Rijksuniversiteit te Utrecht, Nederland

Synopsis

By means of a graphical method the average excitation potential I may be derived from experimental data. Average values for I_{air} and I_{Al} have been obtained. It is shown that in representing range/energy relations by means of Bethe's well known formula, I has to be taken as a continuously changing function of the energy of the incident particle.

§ 1. *Introduction.* In theoretical considerations on the subject of range/energy relations the average excitation potential I ranks as an important quantity. Unfortunately its calculation is difficult and the results obtained by a number of authors both theoretically and experimentally, are not very consistent. In addition to a critical reexamination of the previous evaluations of I_{air} and I_{Al} in this article the authors have recalculated these quantities using a new graphical method.

§ 2. *Theoretical considerations.* Many authors have tried to give theoretical expressions for the relation between the range and the energy of charged particles. For computing range /energy relations usually Bethe's theory ¹⁾ ²⁾ is used. Bethe's expression for the mean energy loss is:

$$dE/dx = (4\pi e^4 z^2 n / mv^2) [Z \ln (2mv^2/I) - C_K(\eta)] \quad (1)$$

Here m is the mass of an electron, v is the velocity and ze is the charge of the incident particle, n is the number of atoms per cubic centimeter of the material, Ze is the nuclear charge and I the average excitation potential of the atom. C_K is a correction which deals with the inefficiency in stopping of the K electrons at low energies of the

incident particles; η is the square of the ratio of the velocity of the incident particle to the velocity of a K electron. The quantity

$$B = Z \ln (2mv^2/I) - C_K(\eta) \quad (2)$$

is usually called the stopping number.

The function $C_K(\eta)$ has been derived from theory by Livingston and Bethe²⁾ and represented graphically. A new evaluation of $C_K(\eta)$ made by Walske which differs appreciably from Livingston's and Bethe's results, was recently published by Brown³⁾.

Several authors^{4) 5)} investigated the stopping of charged particles in different materials and deduced values for the stopping power of those materials relative to that of air. According to Livingston and Bethe²⁾ this quantity S is defined by:

$$S = B_{\text{material}}/B_{\text{air}} \quad (3)$$

Now, if a value for I_{air} e.g. $I_{\text{air}} = 80.5 \text{ eV}$ ²⁾, be assumed, it will be possible to find the average excitation potential of the material considered.

In this way Mano⁴⁾, taking $I_{\text{air}} = 87 \text{ eV}$, found that at least in the energy range from 0 to 8.8 MeV for α -particles — corresponding to proton energies from 0 to 2.2 MeV — I could be represented as a linear function of the atomic number Z . His graph of I vs. Z shows that:

$$I = 11.6 Z \quad \text{eV for } Z < 26$$

$$I = 9.1 Z + 65 \quad \text{eV for } Z > 26.$$

Wilson⁵⁾ in a series of measurements determined I for aluminium in the range of proton energies from 0 up to 4 MeV (cfr. section 5). He reports the relation:

$$I = 11.54 Z \quad \text{eV for aluminium } (Z = 13),$$

which is in excellent agreement with Mano's observations.

Recent experiments by Bakker and Segrè⁶⁾, using Wilson's experimental value $I_{\text{Al}} = 150 \text{ eV}$ as a reference value, show that at least for high energies ($E_{\text{proton}} \approx 300 \text{ MeV}$) the following relation is valid:

$$I = 9.2 Z \quad \text{eV for any value of } Z.$$

Apart from these empirical evaluations of average excitation potentials on the basis of Bethe's theory for stopping phenomena,

Bloch⁷⁾ derived a theoretical expression for I on the basis of his stopping formula which, as has been shown by Bethe²⁾, in the energy region where both formulae are valid is essentially the same as Bethe's expression. Bloch's relation between I and Z is:

$$I = 12.85 Z \text{ eV for any value of } Z.$$

§ 3. *Wilson's experiments.* Wilson observed the passage of a beam of protons up to 4 MeV through aluminium foils of thicknesses ranging from 2.68 to 26.9 mg/cm² and determined the transmission current by means of a Faraday cage. He reports the stopping power S of aluminium relative to air to be a constant and therefrom derives an average excitation potential for aluminium. Wilson points out that according to formulae (3), (2) and (1):

$$S = \frac{B_{Al}}{B_{air}} = \frac{Z_{Al} [\ln(2mv^2/I_{Al}) - (C_K/Z)_{Al}]}{Z_{air} [\ln(2mv^2/I_{air}) - (C_K/Z)_{air}]} = \frac{(dE/n dx)_{Al}}{(dE/n dx)_{air}} \quad (4)$$

For equivalent stopping, i.e. stopping over the same energy interval, the right hand side of (4) may be written as:

$$\frac{(n dx)_{air}}{(n dx)_{Al}} = \frac{\text{atomic weight of Al}}{\text{"atomic weight" of air}} \times \frac{(\text{mg/cm}^2)_{air}}{(\text{mg/cm}^2)_{Al}} \quad (5)$$

Taking for the "atomic weight" of air twice its "atomic number", i.e. 14.44, Wilson found S to be 1.19 for the whole energy range investigated. He thus derived the value $I_{Al} = 150$ eV from (4) and (5), using $I_{air} = 80.5$ eV and the function C_K as given by Livingston and Bethe²⁾.

We want to point out that Wilson's procedure is not without objection:

a. The correct "atomic weight" of air is 14.67, i.e. 1.6% higher than the value mentioned above.

b. The values of $C_K(\eta)$ used by Wilson differ appreciably from the values recently calculated by Walske (cfr. section 2).

c. Wilson unfortunately neglected the fact that the function $C_K(\eta)$ has been calculated by Livingston and Bethe as well as by Walske for an effective number of K electrons of 1.81. In the case of aluminium this effective number has a different value. Hönig⁸⁾ calculated the effective numbers of K electrons on the basis of spectroscopic observations. In aluminium 1.54 is found for

this number. Consequently in equation (4) $C_K(\eta_{Al})$ will have to be replaced by $(1.54/1.81) C_K(\eta_{Al})$.

d. Up till now the average excitation potential of air was assumed to be 80.5 eV as has been evaluated by *Bethe* by adapting his theoretical range/energy curve to two experimentally determined values. As will be shown in section 4, the best actual value for I_{air} is 77.5 eV.

e. *Wilson* assumed that the range/energy relation in air could be described by using a constant value for I_{air} . In the next section we will show that in fact *Bethe's* formula can not describe the experimental range/energy curve without taking for I_{air} a continuously changing function of the energy.

§ 4. *The average excitation potential of air.* In their article *Livingston and Bethe*²⁾ state that their range/energy curve

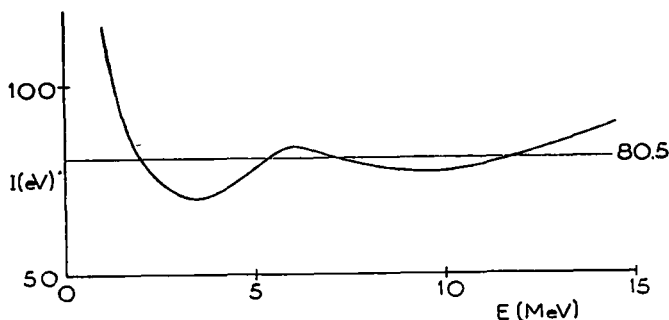


Fig. 1. I -analysis of the former *Bethe* curve for α -particles in air²⁾ (section 4). The inaccuracy of I is about 2.5% for energies up to 8 MeV and then increases to about 7% at 14.5 MeV.

for α -particles in air has been constructed by adapting the theoretical range/energy curve to two experimentally determined values. These two values were determined for the α -particles of polonium and the long range α -particles of *Th C'*. However the curve thus obtained had to be corrected empirically both in the region of the lower energies (1—4 MeV) and in the neighbourhood of $E = 10$ MeV. These empirical corrections may be interpreted as substituting for a constant value of I_{air} a continuously changing function $I_{air}(E)$.

To show the importance of this fact we made an I -analysis of *Bethe's*²⁾ old range/energy relation for air using the old values of

C_K and the following constants as they were available at that time ⁹⁾:

$$\begin{aligned} M_\alpha/m &= 7305 & e &= 4.770 \cdot 10^{-10} \text{ e.s.u.} \\ N &= 6.064 \cdot 10^{23} & 1 \text{ eV} &= 1.591 \cdot 10^{-12} \text{ erg} \end{aligned}$$

From formula (1) one may derive the following expression for I :

$$I = \frac{547.6 E}{\exp [E/1119 \times dE/dx + C_K/7.22]} \text{ eV} \quad (6)$$

E is expressed in MeV and dE/dx in MeV/g/cm².

The result of this I -analysis is plotted in fig. 1. It is evident that in the interval between 5 and 12 MeV the straight line $I_{air} = 80.5$ eV can fairly well represent an average of I_{air} , but that outside this region considerable deviations exist.

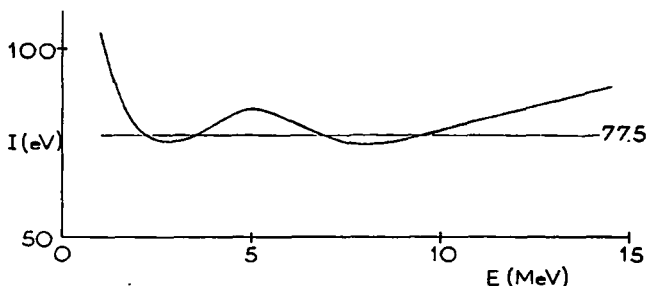


Fig. 2. I -analysis of the new Bethe curve for α -particles in air ¹¹⁾ (section 4). The inaccuracy of I is about 2.5% for energies up to 8 MeV and then increases to about 7% at 14.5 MeV.

The experimental range/energy curve has been changed since then as a result of ionisation measurements by Jesse and Saduiskis ¹⁰⁾. Bethe ¹¹⁾ thereupon recently published a revised range/energy curve in which new purely empirical corrections have been applied. This curve differs appreciably from the old one especially for energies up to 5 MeV for α -particles. We also made an I -analysis of this new curve, using Walske's C_K and the following more recent values of fundamental constants ¹²⁾:

$$\begin{aligned} M_\alpha/m &= 7296 & e &= 4.8024 \cdot 10^{-10} \text{ e.s.u.} \\ N &= 6.024 \cdot 10^{23} \text{ (chemical scale)} & 1 \text{ eV} &= 1.602 \cdot 10^{-12} \text{ erg.} \end{aligned}$$

The expression for I then becomes:

$$I = \frac{548.3 E}{\exp [E/1127 \times dE/dx + C_K/7.23]} \text{ eV} \quad (7)$$

Fig. 2 shows the result of this analysis, which is similar to the curve of fig. 1, but on the average gives somewhat lower values of I_{air} . The region in which $I_{air}(E)$ may be reasonably represented by a straight line is now between 2 and 10 MeV. A careful numerical integration of formula (1) showed that $I_{air} = 77.5$ eV will give the correct difference in range between the polonium and the Th C'-long-range α -particles for which we can take ¹³):

$$\begin{aligned} R(\text{Po } \alpha) &= 3.842 \text{ cm} = 4.710 \text{ mg/cm}^2 & E(\text{Po } \alpha) &= 5.298 \text{ MeV} \\ R(\text{Th C' } \alpha_{long}) &= 11.580 \text{ cm} = 14.197 \text{ mg/cm}^2 & E(\text{Th C' } \alpha_{long}) &= 10.538 \text{ MeV} \end{aligned}$$

§ 5. *The average excitation potential of aluminium.* Wilson's stopping power curve ⁵) is given in units of cm air of 26.85°C and 75.31 cm Hg pressure vs. mg/cm² aluminium. We changed this curve into a range/energy curve in units of mg/cm² aluminium vs. proton energy

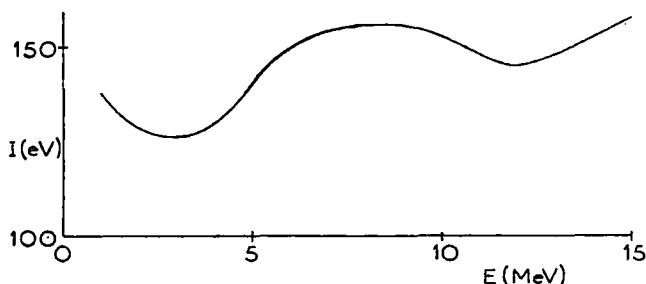


Fig. 3. I -analysis of the aluminium curve ⁵) (section 5). E is the energy of α -particles. The inaccuracy of I is about 3% in the energy region between 2 and 9 MeV and then increases to about 7% at 15 MeV.

in MeV. From this curve the rate of energy loss for protons in aluminium was obtained which we converted into the rate of energy loss for α -particles. For protons the energy loss per g/cm² is just one-quarter of that of an α -particle of the same velocity (i.e. of 3.973 times the proton energy) since the energy loss is proportional to the square of the charge. This is true except in the region of very low energies where the capture and loss of electrons by the particles becomes important. We then made an I -analysis using the modern data available.

Actually we calculated I' , the average excitation potential of the electrons outside the K shell by means of:

$$B = (Z - 1.54) \ln (2mv^2/I') + (1.54/1.81) B_K(\eta) \quad (8)$$

$1.54/1.81 B_K$ is the contribution to the stopping number B due to excitation of the two K electrons.

One can also write:

$$B = Z \ln (2mv^2/I) - (1.54/1.81) C_K(\eta) \quad (9)$$

from which follows:

$$Z \ln I = (Z - 1.54) \ln I' + 11.776 \quad (10)$$

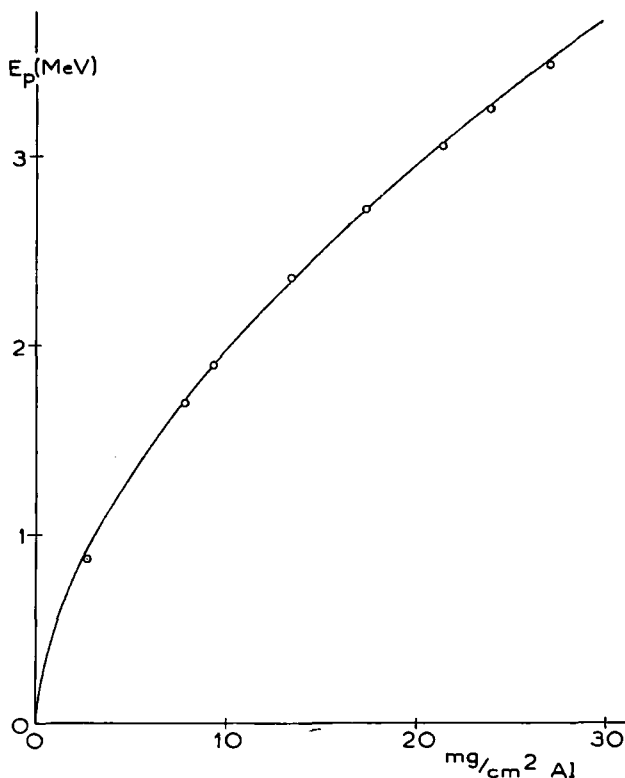


Fig. 4. Theoretical range/energy curve for protons in aluminium. The circles represent Wilson's experimental points ⁵⁾ (section 5).

The value of 11.776 is derived using the theoretical relation between $B_K(\eta)$ and $C_K(\eta)$ ²⁾. Since $C_K(\eta)$ is given by Walske ³⁾ only for values of $\eta > \frac{2}{3}$, while $B_K(\eta)$ is given for values of $\eta < 1.4$ and in our I -analysis we had $0.06 < \eta < 0.93$, we had to calculate I' first instead of turning directly to I .

The values of I_{Al} obtained in this way have been plotted in fig. 3.

The curve is not very dissimilar to the I_{air} -curve (fig. 2). A careful numerical integration shows that $I'_{Al} = 106 \pm 2$ eV represents reasonable average value of I'_{Al} in the region from 1.3 up to about 3. MeV proton energy (fig. 4). This corresponds to $I_{Al} = 151 \pm 3$ eV

TABLE I

Theoretical ranges of protons in aluminium (fig. 4)					
E_p (MeV)	R (mg/cm ²)	E_p (MeV)	R (mg/cm ²)	E_p (MeV)	R (mg/cm ²)
0.25	0.24	1.50	6.34	2.75	17.68
0.50	0.99	1.75	8.22	3.00	20.51
0.75	1.99	2.00	10.30	3.25	23.52
1.00	3.22	2.25	12.57	3.50	26.71
1.25	4.67	2.50	15.03	3.75	30.07

One may note the somewhat surprising consistency of Wilson value $I_{Al} = 150$ eV with this result in the region considered. However it is hardly possible to predict correct values of I_{Al} in the high energy region. Further experimental work will have to be done on this subject to obtain more accurate information.

Acknowledgment. This work is part of the research programme of the „Stichting voor Fundamenteel Onderzoek der Materie” (F.O.M.) made possible by financial support from the „Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek” (Z.W.O.).

Received 5-3-51.

REFERENCES

- 1) Bethe, H., Ann. der Phys. (5) **5** (1930) 325-400.
- 2) Livingston, M. S., and Bethe, H. A., Rev. mod. Phys. **9** (1937) 245-39.
- 3) Brown, L. M., Phys. Rev. **79** (1950) 297-303.
- 4) Mano, G., Ann. de phys. (11) **1** (1934) 407-531.
- 5) Wilson, R. R., Phys. Rev. **60** (1941) 749-753.
- 6) Bakker, C. J., and Segrè, E., private communication.
- 7) Bloch, F., Z. Phys. **81** (1933) 363-376.
- 8) Hönl, H., Z. Phys. **84** (1933) 1-16.
- 9) Birge, R. T., Rev. mod. Phys. **1** (1929) 1-73; Bethe, H. A., and Bacher, R. F., Rev. mod. Phys. **8** (1936) 87.
- 10) Jesse, W. P., and Sadauskis, J., Phys. Rev. **78** (1950) 1-8.
- 11) Bethe, H. A., Rev. mod. Phys. **22** (1950) 213-219.
- 12) Du Mond, J. M. W., and Cohen, E. R., Rev. mod. Phys. **21** (1949) 651-61.
- 13) Holloway, M. G., and Livingston, M. S., Phys. Rev. **54** (1938) 18-37.