Teacher Training Syllabus

Mathematics for Adults

by

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INTRODUCTION

Our purpose is to provide an approach to individualizing mathematics learning experiences for adults.

Education is considered communication and the curriculum is the agenda. Determining the agenda is an important part of communicating. Being part of the development will more likely result in effective application. To facilitate application, it is suggested that the instructor and instructor-trainer become involved in the curriculum development activities.

Ralph Tyler's rationale for curriculum development may be summarized as follows:

1. Face the question of objectives or purposes.
2. Incorporate evaluation.
3. Select experiences for the learner.
4. Organize the experiences in a way that they can be useful and brought into the situation so the learner can confront them.
5. Diagnosis of the need to recognize the kinds of experiences appropriate for a learner-group and an individual learner.

Since we are concerned with individualizing learning, there are some components to be incorporated into our mathematics program. These components include the following:

1. Concept/skill directed
2. Pretest (diagnosis)
3. Performance stated tasks in student terms
4. Minimal performance standards
5. Opportunity to recycle or redirect
6. Choice of activities
7. Choice of resources
8. Self-evaluation
9. Post-test (evaluate)
10. Opportunity to question (depth or enrich)

The training of functioning instructors in individualizing basic mathematics is treated here by considering three curriculum activities.

1. The treatment of curriculum content for the learner is considered in what is called: Developing a Mathematics Skills Continuum.

2. Determination of learner current learning level and the evaluation of his success with prescribed learning experiences are developed in the segment entitled: Performing Diagnosis and Evaluation.

3. Curriculum aspects involving operations with the learner are considered in the section: Providing for Instruction and Learning.

As we prepare plans, do so with the idea in mind that they are dynamic--will be modified as we change our purpose and obtain information we did not have when it was conceived.

DEVELOPING A MATHEMATICS SKILL CONTINUUM

Identifying the Scope

Mathematics Program Purpose

The scope of the basic adult mathematics curriculum will be determined by the purpose assigned to it. A statement of purpose will focus attention on the essential. Effective design depends upon what is considered important or intended.

Determining the Content Range

The range within which the curriculum activity is to be performed may be more adequately determined with the guidance of the statement of purpose. Entry and terminal levels, as well as the basic concepts and skills to be included, are the next considerations.

Entry and Terminal Levels - In most adult basic programs, consideration must be given for the student that requires entrance into the program at the "no knowledge" level. The "no knowledge" adult student, of course, is not a reality, but some are not far beyond this in some of the mathematics skills.

Provision for the needs of every student is necessary in an individualized program. This will require that some preparation be made for relatively low level entrance requirements of some students.

If the program purpose does not specify or infer an entry level, it is best to assume a near "no knowledge" entry level requirement. This would be particularly desirable when planning the basic mathematics program for adults.

The terminal level is a variable requirement when considering individual students. For program design, however, the uppermost level required for a student will be the one needed. This again is either specified or inferred by the program purpose.

Frequently, the basic mathematics programs are for GED preparation or its equivalence. In such cases, it would seem appropriate that the mathematics content of the basic high school program be the terminal level for the adult mathematics program.

Basic Concepts and Skills - The three general categories of mathematics skills to be developed are concepts, computation, and applications. Mathematics textbooks, instructional guides, instructional systems, and the like provide many examples of means for developing these general categories.

These instructional systems (or whatever they may be called) are contemporary expressions of the historical development by mathematicians and educators.

It behooves us to review those systems available to us and use them in the development of our program. From a practical point of view, it is advisable that we use what we have and adapt our resources to our requirements in a manner that we are capable of executing.

Designating and Structuring the Units

Following the determination of scope and content range is the task of identifying the units. An identification system should be useful or functional. The functional classification system provides for such things as: arrangement of skills and concepts in order of difficulty; is easily understood by instructors; makes classification of materials simple; allows for modification; and any others desired by the particular planning group.

Area/Level Classification System

It is recommended that some type of area/level classification system be used.

The area/level classification system organizes mathematics skills into areas that describe the content and into levels that identify the difficulty.

Desired is a sequencing of areas according to difficulty with the degree of difficulty increasing as one proceeds from one area to the next. In figure 1 below, Area II, for example, is of greater difficulty than Area I.

LEVELS

AREAS	A	B	C	D	etc.
I	1 2 3 etc.				
II					
III					
etc.					

Figure 1

Levels of difficulty in the classification system provide a sequence of skills within an area. Level A in Area I in Figure 1 contains the lowest level skills for that area. Level B would contain the next higher level skills.

A unit in this system of classification is the skill group in a particular level for one of the areas. An example of a unit in Figure 1 is Area II, Level C. Within the unit (skill group) the skills are arranged in order of difficulty. Selection and ordering of these skills are considered later.

Four examples of area titles that have been used in mathematics programs are provided for guidance in decision making.

Individual Project LOOKING AHEAD
A Curriculum Guide in Adult Basic Education
Federal City College, D.C.

General Mathematics

1. Numerical Systems
2. Basic Operations and their Properties
3. Word Problem Solving Technique
4. Number Patterns
5. Basic Operations with Fractions
6. Decimals--Relationships to Basic Operations
7. Renaming Fractions to Decimals
8. Per Cent of Increase or Decrease (Social Application of Per Cent)
9. Simple and Compound Interest
10. Metric Geometry
11. Indirect Measurement and Square Root
12. Algebra
13. Development of a Selected Vocabulary

A Guide for Conceptual Development of Mathematical Curricula for Undereducated Adults
North Carolina State University

1. The Basis for Quantification
2. Writing Mathematical Statements
3. Addition and Subtraction Concepts
4. Addition and Subtraction Algorithms
5. Multiplication and Division Concepts
6. Multiplication and Division Algorithms
7. Rational Numbers - Fractions
8. Operations with Fractions
9. Rational Numbers - Decimals
10. Operations with Decimals
11. Ratio and Proportion
12. Percentage

Instructional Objectives Exchange
P.O. Box 24095
Los Angeles, California 90024

1. Sets
2. Numbers, Numerals, and Numeration Systems
3. Operations and Their Properties
4. Measurement
5. Per Cents
6. Geometry
7. Probability and Statistics
8. Applications, Problem Solving
9. Relations, Functions, and Graphs
10. Mathematical Sentences--Order, Logic

Behavioral Objectives: A Guide to Individualizing Learning
Westinghouse Learning Press

1. Analysis of Number and System
2. Operations: Numerical and Algebraic
3. Operations: Graphics
4. Geometry
5. Measurement and Probability
6. Sets and Logic
7. Problem Solving

ten components of our numeration system. The following is a list of the concepts and skills needed for number/numeration.

Absolute Value of Number

The absolute value of a number is its value without regard to the sign. It is sometimes called the numerical value.

The absolute value of +5 is the same as the absolute value of -5. Absolute value of a number is indicated by the following symbol:

$$|+5| = |-5|$$

Thinking in terms of the number scale, the absolute value refers to the distance from zero. The two numbers +5 and -5 are the same distance from zero.

Aggregation

Symbols for aggregation are:

Parentheses	( )	(a + b)
Brackets	[ ]	[a + b]
Braces	{ }	{a + b}

Algebraic Expressions

Factors: 3xy has one set of factors, 3x and y.

Prime factors: 3xy has the prime factors 3, x, and y.

Coefficient: Coefficient means two numbers are efficient together in forming the product.

3x is the coefficient of y

y is the coefficient of 3x

3 is the coefficient of xy

x is the coefficient of 3y

xy is the coefficient of 3

3y is the coefficient of x

3, x, and y are the factors of the product 3xy.

A term: A term is any algebraic expression not separated within itself by a plus or minus sign. A term consists only of factors.

$3xy$, $5xyz$, $-2a^2bc$

Similar or like terms: Similar or like terms are exactly alike in their letter or literal parts.

$4xy^2$, $2xy^2$, $-7xy^2$

Unlike or dissimilar terms: Unlike or dissimilar terms are terms that are not exactly alike in their letter or literal parts.

$4xy^2$, $-7x^2y$, $2xy$

Cardinal Number

It tells how many (one, two, three...).

Composite Numbers

An integer that has more than two factors, those integers that are not prime.

Constant

A number whose value is assumed to remain the same in a given problem.

Counting

Three aspects of learning to count:

1. Learning the ordered sequence of number names.
2. Learning to place number names in the correct order, in one-to-one correspondence with the objects to be counted.
3. Learning to assess the significance of step 2 (Where the terminal number of his count is relative to the rest of the sequence).

One-to-one correspondence: The existence of exactly one member of a group to pair with each member in another group.

It involves a reference set comparison.

Comparison: Three conditions that may result from comparison:

equal to	(=)	the reference set
greater than	(>)	
less than	(<)	

Counting is exact:

Example: The number of people in a room.

Measurement is approximate:

Example: The weight of the people in the room.

Counting is infinite: There is always one larger whole positive number than the largest one we consider.

Some counting skills to develop:

Skill 1: Counting as an amount plus 1 (to 10)

Skill 2: Counting by groups of ten: 1 ten
2 tens
3 tens

Skill 3: Counting 1 ten plus 1
1 ten plus 2
1 ten plus 3

Skill 4: Multiple counting:

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30

etc., through 9's

Decimals

Decimals are a special group of common fractions whose denominator is a positive integral power of 10.

.1	one tenth	1/10
.01	one hundredth	1/100
.001	one thousandth	1/1000

Mixed decimal: a number consisting of a whole number and a decimal fraction.

Changing common fractions to decimals:

Even decimal: $\frac{1}{4}$ = .25

Repeating decimal: 1/3 = .3333...

Common fractions and decimal equivalents:

All rational numbers are either terminating or repeating decimals.

Terminating decimal is written .1763
.1763... indicates the decimal is endless.
$.17\overline{63}\ldots$ indicates an endless decimal repeating digits 6,3.

Equations

An equation is a statement of equality between two equal quantities.

Kinds of equations:

1. Identities: An equation that is true for all values of the letter used.

   $3x + 4x = 7x$ or $3x + 4 = 7x$

2. Conditional equations: An equation that is true for only some particular value or values of the letter used.

   $2x + 3 = 7$

Root of an equation: The root or solution of the equation is the value or values for the letter that makes the equation true.

$2x + 3 = 7$
$x = 2$ 2 is a root

Indeterminate equations: An equation in which the numerical value of one of the letters cannot be determined without affecting or being affected by the other letter value, is called an indeterminate equation.

$x + y = 10$

If $x = 4$, then $y = 6$

the pair may be written (4,6)

x and y are called variables

One of the variables is dependent upon the other in regard to value.

If x =	y =
1	9
2	8
3	7
2.5	7.5
-5	15

The variable that is assigned a value is called the independent variable and the other is then the dependent variable.

Systems of Equations:

Two unknowns: $x + y = 11 \qquad x - y = 3$

Only one pair of values satisfies the conditions of both equations. In this case $x = 7$ and $y = 4$ (7, 4) and are the solutions to the system.

Three unknowns:

$$\begin{aligned} 2x + 5y + 3z &= 7 \\ x - 2y + 5z &= 15 \\ 3x - y - z &= 8 \end{aligned}$$

Second Degree Equations: (Quadratic Equation)

An equation containing the second power of an unknown.

General form: $ax^2 + bx + c = 0$

Pure quadratic: When $b = 0$ and there is no x term, the general form becomes $ax^2 + c = 0$

Complete quadratic: contains both the x^2 and x terms.

Third degree equation: (Cubic equation)

Example: $x^3 + 5x^2 + 3x - 4 = 0$

Fourth degree equation: (Quartic equation)

Example: $y^4 - 7y^3 + 3y = 0$

Fifth degree equation:

Example: $x^5 - 6x^3 + 4x + 3 = 0$

Error

Absolute error is one-half of the smallest unit of measure.

Example: 3 1/8 inches

1/2 of 1/8 inch = 1/16 inch (absolute error)

Relative error is the ratio of the absolute error to the measurement.

Example: 3 1/8 inches

(Absolute error) / (Measurement) $\frac{1/16}{3\ 1/8} = 1/50$ (relative error)

Per cent of error is 100 times the relative error.

Example: $1/50 \times 100 = \frac{100}{50} = 2$ (2%)

Exact and Approximate Use of Number

Exactness and approximateness refers to the use of number rather than something inherent in the number.

i.e.: .66 and 2/3

2/3 is approximately .66 if the quantity is .66.

.66 is approximately 2/3 if the quantity is 2/3.

Expanded Notation

46 = 4 tens + 6 ones = 40 + 6

Exponential Notation

$3{,}528 = 3 \times 10^3 + 5 \times 10^2 + 2 \times 10^1 + 8 \times 10^0$

Exponents

The exponent zero:

$x^3 \div x^3 = \frac{x^3}{x^3} = x^{3-3} = x^0$ then $x^0 = 1$

Fractional Exponents

1. $X^2 \cdot X^3 = X^{2+3} = X^5$

2. $N^{\frac{a}{b}} = \sqrt[b]{N^a}$

$(\sqrt{X})(\sqrt{X}) = X$

$(X^{\frac{1}{2}})(X^{\frac{1}{2}}) = X^{\frac{1}{2}+\frac{1}{2}} = X$

$X^{\frac{1}{2}} = \sqrt{X}$

$X^{\frac{1}{3}} = \sqrt[3]{X}$

<u>Fraction</u>

A fraction is a part of a whole number.

$\frac{1}{4}$ 1 (numerator) enumerates; counts
4 (denominator) denominates; names

Proper fraction	1/4	(less than 1)	form of fractions
improper fraction	4/3	(greater than 1)	
mixed number	4 1/2		

Change the form of fractions: Improper to whole or mixed number.

Whole number into improper. Mixed into improper.

Equivalent fractions: Simplifying fractions (reducing fractions) (changing to lower terms)

Example: $\frac{20}{25} = \frac{4}{5}$

$$\frac{20 \div 5}{25 \div 5} = \frac{4}{5}$$

Changing to higher terms:

Example: $\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

<u>Function</u>

Function involves the idea of dependence. When one variable is dependent on another we say the first variable is a function of the second variable. In the example below y is a function of x.

$y = 2x + 3$

Integer

An integer is a whole number.

Literal Numbers (general numbers)

Literal numbers or general numbers are letters of the alphabet that are used to represent numbers.

$x + y = z$ (literal numbers)

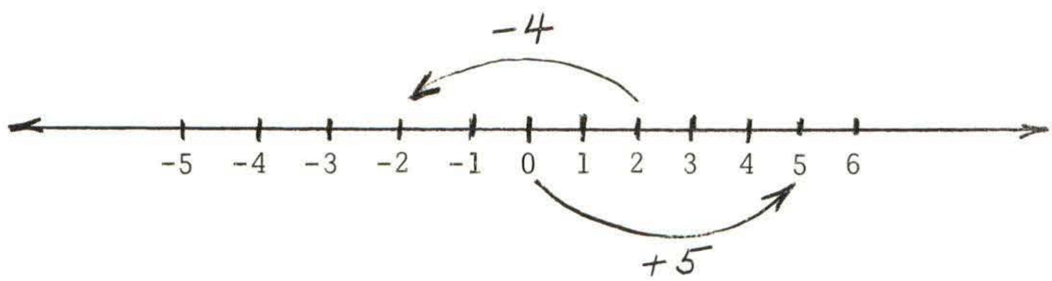
Monomial

A monomial is an algebraic expression consisting of only one term.

$3xy$, $-5mn^2o$ (See algebraic expressions)

Negative Numbers

Positive and negative numbers designate a magnitude and a direction.



When numbers are considered as positive or negative numbers, they are called signed numbers. A number that has no sign before it is considered positive.

When the signs "-" and "+" are used to indicate positive and negative numbers, they are signs of quality. They indicate the kind of number.

Notation

Notation is a system of symbols denoting quantities, operations, etc.

Notation of subgroups. For 5 there are:

2 + 2 + 1,   2 + 3,   4 + 1,   etc.

The number system has the following characteristics:

Positional (in ordered sequence 0 through 9)

Decimal (the tenness)

Place Value (sum of groups)

The symbols used for numbers are called digits and include:

1, 2, 3, 4, 5, 6, 7, 8, 9, and 0

Number

Number is an abstraction. Cardinal number 5 is a property common to all groups of things that have the same number of elements as the number of fingers on the normal human hand.

The word "five" is a name for a number.

5, V, 卌 are symbols to represent a number. They are numerals.

Number Names

Period names:	Billions Millions Thousands Units

Whole numbers: 3,251,628,305 (three billion, two hundred fifty-one million, six hundred twenty-eight thousand, three hundred five)

Whole Numbers										Fractions		
Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths

Decimal point: the separation between the whole number and the fraction.

Decimal fraction: .38 (thirty-eight hundredths)

Mixed numbers: 73.42 (seventy-three <u>and</u> forty-two hundredths)

Numeration

Numeration is a system of number names. It is a process of writing or stating numbers in order of their size. It is a process of numbering.

Ordinal Number

It tells which are (first, second, third...).

Per cent

Per cent means "per hundred." 6 per cent means 6 hundredths 6/100 Per cent is a special kind of decimal with its special symbolism. The symbol is %. 6 per cent is written 6%.

Polynomial

A polynomial is an algebraic expression of more than one term.

$3x + 5mn, \quad 4bc^2 + 6xy - x^3y, \quad 3xy + 6a^2bc - 5xy^2 + 3x$

(See algebraic expressions)

Powers and Roots of Numbers

Powers: 3 x 3 = 9

9 is the square of 3 or the second of 3. Use of the exponent to indicate the power. An exponent is a small number placed at the right and a little above another number, called the base, to show how many times the base is to be used as a factor.

3^2 means 3 x 3 = 9 { (3) is the base; (^{2}) is the exponent }

2^3 means 2 x 2 x 2 = 8 { (2) is the base; (^{3}) is the exponent }

The exponent 3 means that three 2's are to be multiplied together. 8 is the third power of 2.

Roots: The inverse of finding the power is finding the root. The square root of a number is one of the two equal factors of the number. (4 is the square root of 16)

The cube root of a number is one of the three equal factors of the number. (3 is the cube root of 27)

The symbol for a root is the sign $\sqrt{}$ called the radical sign.

$\sqrt{\ }$ $\sqrt[2]{\ }$ square root

$\sqrt[3]{\ }$ cube root

radicand: the number under the radical sign (81)

$\sqrt[2]{81} = 9$

index: the root to be found (2)

A perfect square is a number whose square root can be stated exactly as a fraction or whole number.

A number that cannot be expressed as a whole number or as a common fraction is called an irrational number.

Prime Numbers

An integer that has exactly two factors.

Examples: 2, 3, 5, 7, 11, etc.

Proportion

A statement of equality between two equal ratios is called a proportion.

5 ft. : 15 ft. = 7 ft. : 21 ft.

$$\frac{5 \text{ ft.}}{15 \text{ ft.}} = \frac{7 \text{ ft.}}{21 \text{ ft.}}$$

$$\frac{1}{3} = \frac{1}{3}$$

Ratio

A comparison by division between two like quantities is a ratio between the two quantities.

Compare 12 feet and 9 feet

$$12 \text{ feet} \div 9 \text{ feet} = \frac{4}{3}$$

Note that the answer has no denomination. It is a pure number.

A ratio is indicated by a colon (:).

$$12 \text{ ft.} : 9 \text{ ft.} = \frac{12 \text{ ft.}}{9 \text{ ft.}} = \frac{4}{3}$$

Reciprocal of a Number

A reciprocal is the result of 1 divided by the number.

reciprocal of 6 is $1 \div 6 = \frac{1}{6}$

reciprocal of a is $1 \div a = \frac{1}{a}$

Rounding Off Numbers

Keep only <u>significant</u> digits.

1. If first digit dropped is less than 5, then the last digit kept is left as is.

2. If the first digit dropped is larger than 5, then the last digit kept is increased by 1.

3. If the digit dropped is exactly 5, followed only by zeroes, then: apply a rule of the field you are working.

   In business: round upward.

   In mathematics:
   if last digit kept is even, have it remain.
   if last digit kept is odd, increase it one.

Scientific Notation

$10{,}000{,}000 = 10^7$ ← exponent (superscript) (power)

↑ base

Set Theory

A set is a collection of objects. Individual members of the set are called elements. Braces are used to enclose the elements of a set $\{\ \}$. $b \in A$ expresses "b is an element of set A."

Types of sets:

1. Empty or null set: This is a set containing no elements. It is designated by the Greek letter phi ϕ or by empty braces $\{\ \}$.

2. Overlapping sets: Sets that have some elements in common.

3. Disjoint sets: Sets having no element in common.

4. Subsets: One set is a subset of another if all the elements of the first set are contained within the second set. It is expressed: A is a subset of B.
   Symbolism: $A \subset B$

5. Universal set: The set which has the totality of elements under discussion as its members.
   Symbol: U

The complement of set A is the new set of all things under discussion that are not in A. Symbol: A'

Standard Position

$1.32 \times 10^8 = 132{,}000{,}000$
$1.6 \times 10^{-6} = .0000016$

Trinomial

A trinomial is an algebraic expression of three terms.

$3mn + 5km - 2x^2yc$ (See Algebraic Expressions)

Variable

A number that may change or take on different values in a problem.

In the equation $2x + 7y = 48$
x and y are variables; 2, 7, and 48 are constants.

In the formula $V = \frac{4}{3} \pi r^3$

V and r are variables; 4, 3, and π are constants.

Variation

If variables are interdependent, the one variable changes or varies with changes in the other.

Direct variation: One quantity varies directly as another quantity when the first is equal to a constant times the second.

$y = Kx$

Joint variation: One quantity varies jointly when it depends on the product of two or more other variables. $x = Kyz$

Inverse variation: $y = \frac{K}{x}$

Zero (0)

Zero has three roles:

1. Place holder (makes 23 different from 203).
2. Quantitative (the number of elements in an empty set).
3. An arbitrary point of reference on a directed scale.

Operations

Mathematical operations involve a major emphasis on conceptual bases for a process or algorithm.

An operation should begin with counting. Each terminal objective should be related to a concrete counting operation.

The development of operational skills stresses the "combined" learning of addition and subtraction. Subtraction istreated as the inverse or undoing operation of addition.

Multiplication and division are treated as inverse operations. Multiplication is related to addition as a group addition operation and the repeated subtraction conceptis encountered in division.

Improvement of learner computational facility is improved in a shorter period of time with the combined operations approach. This is possible because the basic facts used in computation may be learned together as inverse operations. Early introduction of the additive identity (o), the "one more counting pattern," and the commutative property of addition reduces the number of basic addition facts to be learned.

Likewise, early introduction of the multiplicative identity (l), the multiplicative property of zero, and the commutative property of multiplication reduces the number of basic multiplication facts to be learned.

The treatment of topics like fractions, decimals, and per cents is made within the Number/Numeration and Operations areas. This provides the learner with a smooth, logical flow from one algorithm to another by building upon the similarities involved.

Signs of Operations:

+	addition
-	subtraction
x	multiplication
÷	division

Addition: (indicated by a + sign)

Addition is combining groups into a single group.

addends: numbers that are added together

sum: result of added numbers

commutative law for addition:

$5 + 3$ or $3 + 5$

$a + b = b + a$

associative law for addition:

$5 + 3 + 7$

$8 + 7 = 15$ or

$5 + 10 = 15$

$(a + b) + c = a + (b + c)$

Addition of Fractions:

fraction with same denominators $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$

fractions with unlike denominators $\frac{a}{b} + \frac{c}{d}$

find the lowest common denominator (LCD)

Addition Number Facts:

Use 0 through 9 number facts. Establish the commutative property of addition before learning the number facts.

Addition Algorithm:

Develop the sequence for whole numbers:

1. a number + 1 concept
2. one digit numbers
3. two digit numbers + one digit (without carrying)
4. two digit numbers (without carrying)
5. two digit number + one digit number (carrying)
6. two digit number + two digit (carrying)
7. column addition
8. multiple digits

163 =	1 hundred + 6 tens + 3 ones
+235 =	2 hundred + 3 tens + 5 ones
398 =	3 hundred + 9 tens + 8 ones

293 =	2 hundred + 9 tens + 3 ones
+164 =	1 hundred + 6 tens + 4 ones
457 =	3 hundred +15 tens + 7 ones

regrouping:

10 tens = 1 hundred

4 hundred + 5 tens + 7 ones

$$\begin{array}{r} 293 \\ +\underline{164} \\ 457 \end{array}$$

Addition and Subtraction of Decimals:

(additon of like place value groups)

$$\begin{array}{r} 25 \\ 7.3 \\ 2.51 \\ 21.6 \\ +\quad .025 \\ \hline 56.435 \end{array}$$

(add the place value columns as shown by arrows)

$$\frac{5}{1000} =$$

$$\frac{1}{100} + \frac{2}{100} =$$

$$\frac{3}{10} + \frac{5}{10} + \frac{6}{10} =$$

$$1 + 5 + 7 + 2 + 1 =$$

$$10 + 20 + 20 =$$

(subtraction of like place value groups)

$$\begin{array}{r} 129.346 \\ -\ 81.183 \\ \hline 48.163 \end{array}$$

Additon of Signed Numbers:

$a + (-b) = (-b) + a = a - b$ if $a > b$

$a + (-b) = (-b) + a = -(b-a)$ if $b > a$

$(-a) + (-b) = -(a + b)$

Inverses:

Two operations are inverses if either one "undoes" the other.

Operation:		Inverse Operation:
add	-	subtract
multiply	-	divide
raising to power	-	extracting roots
combining factors	-	factoring

Subtraction: (inverse operation of addition)

```
   a   ← minuend
 (-)b  ← subtrahend
   c   ← difference or remainder
```

operational symbol

Three Concepts of Subtraction:

1. comparison

   How many more is 9 than 3? (9-3=6)

2. take-away

   How many are left? (7-4=3)

3. missing addend

   How many more are needed?

(Stress No. 3 since you may use what was learned in addition to greater advantage in learning number facts and speeding computation)

Subtraction Facts:

To use the addition number facts to learn subtraction facts is the most economical of time. In reality you will not learn a set of subtraction facts if you treat each subtraction as finding a missing addend.

Example:

```
  6      or     6 - 3 =
 -3
```

Think 3 + what number is 6.

<u>Subtraction Algorithm</u>:

876 = 8 hundreds + 7 tens + 6 ones
- 342 = <u>3 hundreds + 4 tens + 2 ones</u>
5 hundreds + 3 tens + 4 ones

837 = 8 hundreds + 3 tens + 7 ones
- <u>259</u> = <u>2 hundreds + 5 tens + 9 ones</u>

regrouping: (Stress additive decomposition)

8 hundreds + 2 tens + 17 ones
- <u>2 hundreds + 5 tens + 9 ones</u>
8 ones

regrouping:

7 hundreds + 12 tens + 17 ones
- <u>2 hundreds + 5 tens + 9 ones</u>
5 hundreds + 7 tens + 8 ones

7 12
~~8~~~~3~~7
<u>- 259</u>
578

Different types of Subtraction Algorithms:

1. take-away = decomposition 83 = 80 + 3
- <u>46</u> = <u>40 + 6</u>

70 + 13
- <u>40 + 6</u>

2. take-away = equal addition 83 = 80 + 3
- <u>46</u> = <u>40 + 6</u>

80 + 13
- <u>50 + 6</u>

3. additive = decomposition think:
6 + ? = 13
40 + ? = 70

4. additive = equal addition think:
6 + ? = 13
50 + ? = 80

Subtraction of Signed Numbers:

Review the following sequence.

$3 - (-3) = 6$

$3 - (-2) = 5$

$3 - (-1) = 4$

Begin here and work down.

$3 - 0 = 3$ $\qquad$ $a - (-b) = a + b$

$3 - 1 = 2$

$3 - 2 = 1$

$3 - 3 = 0$

$3 - 4 = -1$

$3 - 5 = -2$

Return to $3 - 0 = 3$ and work up.

Look at the order of the answers. Note that subtracting a negative number is the same as adding the number.

Addition and Subtraction of Fractions:

Fractions with same denominators:

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

$\frac{4}{7} - \frac{3}{7} = \frac{1}{7}$

Fractions with different denominators:

$\frac{1}{2} + \frac{1}{3} =$ $\qquad$ $\frac{2}{3} - \frac{1}{5} =$

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$ $\qquad$ $\frac{10}{15} - \frac{3}{15} = \frac{7}{15}$

Finding the lowest common denominator:

Multiple of a number: A multiple of a number is exactly divisible by the number.

Common multiple: A common multiple is a multiple that is exactly divisible by each of the given numbers.

Lowest common multiple (LCM): This is the smallest multiple that is exactly divisible by each of the numbers.

Finding the Lowest Common Multiple (LCM):

1. Write the numbers as products of prime numbers.
2. Write each of the prime factors as many times as it is contained in any one of the given numbers.
3. The LCM is the product of the prime factors listed in step 2.

			prime factors
Example: (step one)	24	=	2.2.2.3
	45	=	3.3.5
	75	=	3.5.5
	210	=	2.3.5.7

(step two) (step three)
2.2.2.3.3.5.5.7 = 12,600

Present fractions and mixed numbers in the following order:

1. With like denominators.
2. With denominators that are factors of largest denominator.
3. With denominators that are not factors of the largest denominator.

Addition and subtraction of Monomials

Combine like terms. Add or subtract the numerical coefficients.

Addition Examples:

2 feet	$2xy$	$6m^2n$	$-9no$
+ 3 feet	$+ 5xy$	$- 3m^2n$	$3no$
5 feet	$7xy$	$3m^2n$	$6no$

Subtraction Examples:

$5xy^2$	$-7a^2c$	$-3cd$
$3xy^2$	$2a^2c$	$-2cd$
$2xy^2$	$-9a^2c$	$- cd$

Addition and subtraction of Polynomials

Combine like terms.

$$\begin{array}{rrrrr} 5x^2 & - 3xy & + 2y^2 & - x & \\ -2x^2 & - 3xy & + 3y^2 & & - 5y \\ \hline 3x^2 & - 6xy & + 5y^2 & - x & - 5y \end{array}$$

Multiplication

1. The addition of equal addends. 2 x 3 = 000 000

2. "Ratio to one" meaning. 2 x 3 = ? What # is 2 times as large as 3? 2:1 = 6:3

Multiplication is indicated by: a.b (a)(b) axb

a multiplicand
x b multiplier
c product

factors of the product

(b x a = c)

Commutative law of multiplication: a x b = b x a

Associative law of multiplication: (a x b) x c = a x (b x c)

Distributive Principle:

4 x(5 + 6) = 4 x 11 = 44
4 x(5 + 6) = (4 x 5) + (4 x 6) = 20 + 24 = 44
a(b + c) = ab + ac

Multiplication number facts:

Use the 0 through 9 number facts.

Establish the commutative property of multiplication before learning the number facts.

Multiplication Algorithm

The multiplication algorithm depends upon the distributive principle and place value.

5 x 7 = sum of five sevens

7 + 7 + 7 + 7 + 7 = 35

```
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
```

8 x 31 = sum of eight thirty-ones
or
8 x (3 tens + 1 one) = (8 x 3 tens) + (8 x 1 one)

[Eight is distributed over thirty (3 tens) and one]

```
 31                       39
x 8                     x 26
240   (8 x 30)             54   (6 x 9)
  8   (8 x 1)             1 80   (6 x 30)
241                       1 80   (20 x 9)
                           600   (20 x 30)
                         1,014
```

```
   39
 x 26
  234   (6 x 9) + (6 x 30) = 54 + 180
  780   (20 x 9) + (20 x 30) = 180 + 600
1,014
```

```
    39                                                  39
  x 26                                                x 26
     4   (6 x 9) + (6 x 30) = 54 + 180                 2 34
  2 30                      = 4 + (50 + 180)            78
    80   (20x9) + (20x30)   = 180 + 600              1,0 1 4
  7 00                      = 80 + (100 + 600)
1,0 14
```

Multiplication of Decimal Fractions and Mixed Decimals

Introduce multiplication and division by powers of ten first.

Example:

$$2 \times 100 = 2 \times 10 \times 10 = 200$$

$$1.3 \times 100 = 1.3 \times 10 \times 10 = 130$$

$$1.3 \div 100 = 1.3 \div 10 \div 10 = .013$$

$$1.3 \div 100 = 1.3 \times \frac{1}{10} \times \frac{1}{10} = .013$$

then consider the following:

$$7 \times .3 =$$

$$7 \times \frac{3}{10} = \frac{21}{10} = 2.1$$

$$3 \times .04 =$$

$$3 \times \frac{4}{100} = \frac{12}{100} = .12$$

$$2.4 \times 5 = 2\frac{4}{10} \times 5 = \frac{24}{10} \times 5 = \frac{120}{10} = 12$$

Multiplication of Signed Numbers

$$2 \times 3 = 3 + 3 = 6$$
$$a \times b = (b + b...) = a \times b$$

$$2 \times (-3) = (-3) + (-3) = -6$$
$$a \times (-b) = (-b - b...) = -(a \times b)$$

$$(-2) \times 3 = -(3 + 3) = -6$$
$$(-a) \times b = -(b + b...) = -(a \times b)$$

$$(-2) \times (-3) = -[(-3) + (-3)] = -[-6] = 6$$
$$(-a) \times (-b) = -[(-b) + (-b)] = -[-(a \times b)] = a \times b$$

Multiplication of fractions

$$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$$

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Types of Computations:

1. A proper fraction times a proper fraction, as $\frac{3}{4} \times \frac{2}{9}$
2. An integer times a proper fraction, as $3 \times \frac{2}{5}$
3. A proper fraction times an integer, as $\frac{2}{3} \times 7$
4. A mixed number or improper fraction times an integer, as $3\frac{2}{3} \times 7$ or $\frac{11}{3} \times 7$.
5. An integer times an improper fraction or mixed number, as $5 \times 1\frac{3}{8}$ or $5 \times \frac{11}{8}$
6. A mixed number or improper fraction times a proper fraction, as $3\frac{1}{3} \times \frac{1}{7}$ or $\frac{10}{3} \times \frac{1}{7}$
7. A proper fraction times a mixed number or improper fraction, as $\frac{1}{7} \times 2\frac{2}{3}$ or $\frac{1}{7} \times \frac{8}{3}$
8. A mixed number or improper fraction times a mixed number or improper fraction, as $3\frac{1}{3} \times 1\frac{2}{5}$ or $\frac{10}{3} \times \frac{7}{5}$

Begin multiplication of fractions with the type two combination (integer times a proper fraction).

Example:

$3 \times \frac{2}{5}$ = three two-fifths

$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$$

Introduce the type one (proper fraction times a proper fraction).

Example: $\frac{1}{4} \times \frac{1}{2}$

By looking at the illustration observe what $\frac{1}{2}$ is when added $\frac{1}{4}$ times.

whole

one half

one fourth of one half. (1/8)

From several experiences with each of the types shown in the above examples the discovery that the product of the numerators over the product of the denominators is the product of the fractions and whole numbers or fractions.

Changing fractions to higher terms:

$$\frac{a}{b} \times \frac{c}{c} = \frac{ac}{bc}$$

<u>Multiplying quantities expressed as powers</u>.

$$2^3 \cdot 2^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$$

$$x^2 \cdot x^5 = (x \cdot x)\ (x \cdot x \cdot x \cdot x \cdot x) = x^7$$

$$x^a \cdot x^b = x^{a+b}$$

<u>Multiplication of monomials</u>.

$$\begin{array}{rl} 2x^2yz & (2)\ x \cdot x \cdot y \cdot z \\ \times\ \underline{3axy^4} & (3)\ a \cdot x \cdot y \cdot y \cdot y \cdot y \end{array}$$

$$(2)(3) \cdot a \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot z \qquad 6ax^3y^5z$$

Steps taken:

1. Determine the sign of the product.
2. Multiply the numerical coefficients of the factors to get the numerical coefficient of the product.
3. Multiply the literal parts by writing all literal factors and adding the exponents of the like literal factors.

Multiplication of a polynomial by a monomial

Multiply each term in the polynomial by the monomial.

$$a(x^2 + 3xy + y^2)$$

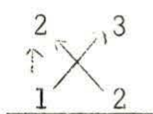
$$ax^2 + 3axy + ay^2$$

Multiplication of a polynomial by a polynomial

Multiply each term of one polynomial by each term of the other and combine any like terms.

$$(a + b)(c + d) = ac + ad + bc + bd$$

$$\begin{array}{r} 23 \\ \underline{12} \end{array}$$



$$\begin{array}{rrr} 3x & - 5y & \\ 2x & + 3y & \\ \hline 6x^2 & - 10xy & \\ & 9xy & - 15y^2 \\ \hline 6x^2 & - xy & - 15y^2 \end{array}$$

<u>Division</u>

Division is to multiplication as subtraction is to addition.

Division is the inverse operation of multiplication.

Division is a process of determining how many times one number is contained in another.

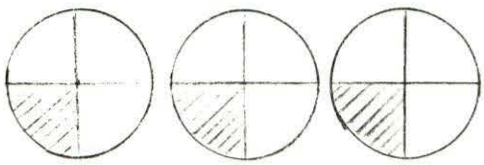
The operation symbol for division is $\div$.

$$\begin{array}{rl} c & \text{(dividend)} \\ \underline{\div\ b} & \text{(divisor)} \\ a & \text{(quotient)} \end{array}$$

$$c \div b = a \qquad\qquad \begin{array}{r} a \\ b\overline{)\,c} \end{array}$$

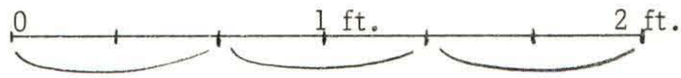
A fraction indicates division. $\frac{c}{b} = c \div b$

Examples: $3 \div 4$



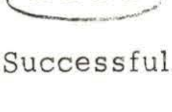
$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$2 \div 3$

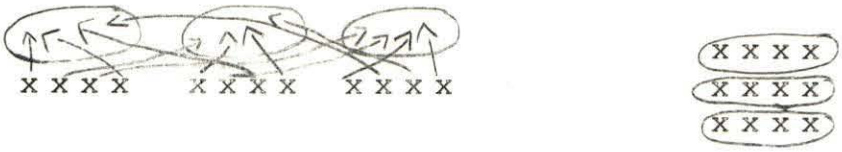


$\frac{2}{3}$

Introduce division with the question: How many fours are in twelve? Pictorially, the array of 12 is divided into groups of 4 as shown below:



Successful experiences with the above should be followed by answering the question: How many items will there be in each of three groups? The total is 12. You have the group of 12 and the three groups in which to distribute them. Put one item at a time in each group in sequence until all items are placed in a group, as shown below:



Division number facts:

Successful experience with the above leads to presenting the division facts. Actually, it is more economical of time to have a student use his multiplication facts to solve his division rather than learn a set of division number facts.

Example: $14 \div 2 =$

Ask how many 2's are in 14? or 2 times what number is 14?

<u>Division algorithm</u>

One digit division: $2\overline{)8}$ 2 times what number is 8? $\begin{array}{r} 4 \\ 2\overline{)8} \end{array}$

(1) $2\overline{)84}$ $84 = 80 + 4$ 2 times what number is 80?
2 times what number is 4?

(2) $\begin{array}{r} 40 \\ 2\overline{)80} \\ \underline{80} \\ 0 \end{array}$ (3) $\begin{array}{r} 2 \\ 2\overline{)4} \\ \underline{4} \\ 0 \end{array}$

$2 \times 40 = 80$ $2 \times 2 = 4$
$80 - 80 = 0$ $4 - 4 = 0$
$40 + 2 = 42$

Combining step (2) and (3):

$$\begin{array}{rll} 2 & (4) & 2 \times \text{what number} = 4 \\ \underline{40} & (1) & 2 \times \text{what number} = 80 \\ 2\overline{)84} & & \\ \underline{80} & (2) & (2 \times 40) \\ 4 & (3) & (84-80) \\ \underline{4} & (5) & (2 \times 2) \\ 0 & (6) & (4 - 4) \qquad (7)\ 40 + 2 = 42 \end{array}$$

```
      2   (5)  3 x ? =   8
     30   (4)  3 x ? =  90
    200   (1)  3 x ? = 600     200 + 30 + 2 plus a remainder
3 /698                                of 2
    600   (2)  (3 x 200)
     98   (3)  (698 - 600)     232  R2
     90   (4)  (3 x 30)
      8   (5)  (98 - 90)
      6   (7)  (3 x 2)
      2   (8)  (8 - 6)
```

```
    4 )
   10 (
  100 ) 114
4/456
  400
   56
   40
   16
   16
    0
```

two digit divisor:

```
      5
    100       105   R26
79/8321
   7900
    421
    395
     26
```

Guess and subtract method:

```
34/74823                                  guess
 - 72000           2,000           34 x 2,000 = 72,000
    2823
   -2720              80           34 x 80 = 2720
     103
   - 102               3           34 x 3 = 102
       1           2,083  R1
```

Division of Signed Numbers

Checking the quotient multiplied by the divisor should equal the dividend applies to the signs as well as the numerical values in division.

$(+) \div (+) = (+)$
$(-) \div (-) = (+)$
$(-) \div (+) = (-)$
$(+) \div (-) = (-)$

Division of Fraction

1. Division of a fraction by a fraction

   $1/2 \div 2/3$

   Change the form to:

$$\frac{\frac{1}{2}}{\frac{2}{3}}$$

If the denominator were 1, the fraction would be in the whole number form $(4/1 = 4)$.

Multiply the numerator and denominator by the reciprocal of the denominator will not change the value of the fraction and make the denominator equal 1.

$$\frac{\frac{1}{2} \cdot \left(\frac{3}{2}\right)}{\frac{2}{3} \left(\frac{3}{2}\right)} = \frac{\frac{3}{4}}{\frac{6}{6}} = \frac{\frac{3}{4}}{1} = \frac{3}{4}$$

Note that the numerator $\frac{1}{2}\left(\frac{3}{2}\right)$ is $\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$ (the solution). Comparing the problem $\frac{1}{2} \div \frac{2}{3} =$ with $\frac{1}{2} \times \frac{3}{2} =$,

one can see that the dividend multiplied by the reciprocal of the divisor is the quotient.

2. Division of a fraction by a whole number:

$$\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

3. Division of a whole number by a fraction:

$$8 \div \frac{2}{3} = \frac{8}{1} \times \frac{3}{2} = \frac{24}{2} = 12$$

Simplifying fractions (Reducing fractions):

$$\frac{ac \div c}{bc \div c} = \frac{a}{b}$$

$$\frac{6}{8} = \frac{(3)(2) \div 2}{(4)(2) \div 2} = \frac{3}{4}$$

Division of Decimals

Cases to consider:

1. A decimal divided by a whole number.

```
   .052
4/.208
```

2. A whole number divided by whole number which does not have the divisor as an exact whole number factor.

```
   4.5
6/27.
  24
   30
   30
    0
```

3. A whole number divided by a decimal. (Change divisor to a whole number and proceed as in case 1.)

```
    204 0.                    2040.
.3/612.0        or        .3/612.0
 ^       ^                  ^     ^
```

4. A decimal divided by a decimal.

$$\begin{array}{r} 20.8 \\ .3\overline{)6.24} \end{array}$$

Per Cent

Percentage problems involve two quantities to be compared and a per cent which compares them ratiowise. One quantity is referred to as the base. The quantity to be compared to the base is the percentage. The per cent, which states the ratio of percentage to base, is called the rate.

Cases of percentage problems:

1. Finding a given per cent of a number,
2. Finding what per cent one number is of another
3. Finding a number when a given per cent of it is known.

The second case will be presented first because it shows more clearly the ratio comparison characteristic.

percentage : base = rate

$$\frac{\text{percentage}}{\text{base}} = \text{rate}$$

Consider the case where six of the eight neighbors have lived there over five years. What per cent is this?

6 out of 8 $\qquad$ 6/8 = rate $\qquad$ 6/8 = 3/4

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{2\,8} \\ 20 \end{array}$$

Per cent (%) is the number of hundredths.

Case one is a form that clearly shows the standard form of multiplication with its two factors and product.

$$\underbrace{\text{rate} \times \text{base}}_{\text{(factors)}} = \underbrace{\text{percentage}}_{\text{(product)}}$$

In our case of the 75% of the 8 neighbors living in the neighborhood over five years, we may obtain the number living over five years by finding the product of rate and base.

$$75\% \times 8 = ?$$
$$.75 \times 8 = 6.00$$

The third case with our problem would be finding the number of families if 75% of them that lived there over five years were 6 families.

$$\frac{\text{percentage}}{\text{rate}} = \text{base}$$

$$\frac{6}{75\%} = \text{base} \qquad \frac{6}{.75} = 8 \qquad .75\overline{)6.00_{\wedge}} = 8. \quad \underline{6\ 00}$$

<u>Dividing quantities expressed in powers</u>

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = x^2$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$x^a \div x^b = x^{a-b}$$

Dividing Monomials

$$\frac{24x^5y^3z}{-4x^2y^3} = \frac{24\ x\ x\ x\ x\ x\ y\ y\ y\ z}{4\ x\ x\ y\ y\ y} = -6x^3z$$

1. Determine the sign of the quotient.

2. Divide the numerical coefficients to get the numerical coefficient of the quotient.

3. Divide the literal parts by writing the literal factors of the dividend and subtracting the exponents of like factors in the divisor.

Division of a Polynomial by a Monomial

Divide each term of the polynomial by the monomial.

$$(12x^3y^4 - 10xy^3 + 8x^2y^2) \div (-2xy) =$$

$$-6x^2y^3 + 5y^2 - 4xy$$

Division of a Polynomial by a Polynomial

1. Arrange the terms of the dividend in descending order of powers. (Write the coefficients including a zero for each missing power.)

2. Arrange the terms of the divisor in descending order of powers.

3. Proceed as in long division.

$$\begin{array}{r|l}
 & 2x^2 - 4x + 3 + \frac{6}{x+2} \\
\hline
x + 2 & 2x^3 + 0 - 5x + 12 \\
 & \underline{2x^3 + 4x^2} \\
 & - 4x^2 - 5x \\
 & \underline{- 4x^2 - 8x} \\
 & 3x + 12 \\
 & \underline{3x + 6} \\
 & 6
\end{array}$$

Operations on Equations

The same operation must be performed to both sides of the equation if the equation is to remain true. (Division by zero is not allowed)

Factoring Algebraic Expressions

Factoring is finding two or more quantities whose product is the original expression.

Types of Algebraic Expressions and their factors:

1. $x^2 + xy = x(x + y)$
2. $x^2 - y^2 = (x + y)(x - y)$
3. $x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$
4. $x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$
5. $x^2 + (a + b)x + ab = (x + a)(x + b)$
6. $acx^2 + (bc + ad)x + bd = (ax + b)(cx + d)$

Solving Quadratic Equations

Solving a Pure Quadratic Equation:

Form $ax^2 + b = 0$

1. Isolate the term containing x.
2. If x^2 has a coefficient other than 1, divide both sides of the equation by the coefficient.
3. Take the square roots of both sides of the equation.

$2x^2 - 8 = 0 \qquad 2x^2 = 8 \qquad x^2 = 4 \qquad x = \sqrt{4} = 2$

Solving a Quadratic Equation by Factoring

1. Factor the equation

2. Set each factor equal to zero.

$$x^2 + 5x - 84 = 0$$

$$(x + 12)(x - 7) = 0$$

$$x + 12 = 0 \qquad x - 7 = 0$$
$$x = -12 \qquad x = 7$$

Solving a Quadratic Equation by Completing a Square

1. See that the coefficient of x^2 is 1.

2. Transpose the constant term to the right side of the equation.

3. Add some quantity to both sides that will make the left side a perfect square. (It will be one-half the coefficient of x.)

4. Write the left side as a square and at the same time combine the terms on the right side.

5. Take the square root of both sides, using both signs (+ and -) on the right side.

6. Solve the resulting equation for x.

$$2x^2 - 12x + 10 = 0$$
$$x^2 - 6x + 5 = 0$$
$$x^2 - 6x + \quad = -5$$
$$x^2 - 6x + 9 = 4$$
$$(x - 3)^2 = 4$$
$$x-3 = \sqrt{4} = \pm 2$$
$$x = 2 + 3 = 5$$
$$x = -2 + 3 = 1$$

Solving a Quadratic Equation by Formula

For the quadratic in the following form: $ax^2 + bx + c = 0$

the formula for solution is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solving Systems of Equations

By addition or subtraction:

(1) $2x + y = 5$	subtracting (2) from (1)
	$x = 4$ and $4 + y = 1$
(2) $x + y = 1$	$y = -3$

By substitution:

(1) $2x + y = 5$	in (2) $y = 1 - x$
(2) $x + y = 1$	substituting $1 - x$ for y in (1)

$$2x + 1 - x = 5$$
$$x + 1 = 5$$
$$x = 4 \quad y = 1 - 4 = -3$$

By comparison:

(1) $2x + y = 5$ making (1) in the form of (2)
(2) $x + y = 1$
(3) $x + y = 5 - x$

Comparing (3) and (2) $x + y = 1$ and $x + y = 5-x$
then $5 - x = 1$ or $x = 4$ and $y = 3$.

Dependent or derived equations:

An equation derived from another equation is dependent upon it and has the same set of values for the variables. An attempt to solve dependent equations results in $0 = 0$.

Example: $3x + y = 9$ $6 + 2y - 18 = 0$

Inconsistent equations:

A pair of inconsistent equations do not have a set of values for the variables. An attempt to solve inconsistent equations results in some impossible condition like $0 = -3$.

Example: $x + y = z$ $x + y = 3$

Independent equations:

Independent equations have a particular pair, or pairs, of values that satisfy the equations. Obviously then, the equations in a system must be independent to secure a solution.

Example: $2x + y - 7 = 0$ $3x - y = 3$

Operations with Sets

$A = \{2, 3, 4, 5, 11, 15, 21\}$

$B = \{1, 2, 4, 6, 8, 12, 16, 25\}$

Intersection of sets:

The intersection of two sets is the set of all elements common to both sets.

Symbol for the operation is $\cap$, sometimes referred to as "cap." $A \cap B = \{2, 4\}$

Union of sets:

The union of two sets is the set consisting of all the individual elements of both sets without repetition of any element.

Symbol for operation is $\cup$, sometimes referred to as "cup."

$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 11, 12, 15, 16, 21, 25\}$

Measurement and Geometry

The measurement and geometry area provides a greater opportunity for a different type of learner participation. Skills and concepts of this particular area are more suited to a laboratory type of experience. A learner may get involved in activities that will often apply the skills and concepts developed in the number/numeration and operation areas. This is usually enjoyed by the adult learner because he may apply what he has learned and readily relate them to the real world.

This area includes common geometric figure recognition and measurement of their physical properties. Such properties as area, volume, and perimeter are included. Included also in the area are systems of measurement like money, time, temperature, weight, capacity, and length. These measurement systems are presented to provide the learner with concepts and skills that will make regular life activities more meaningful.

Major emphasis of this area is on measurement concepts and skills. The geometry portion of this area develops concepts and skills used in identifying shape followed by application to size and other measurement characterisitics.

Nature of Measurement

Properties of measurement to be developed are:

1. All measurements are comparisons. It is a comparison of magnitudes
2. No measurement is exact. Measurement is only approximate. In measurement we are faced with a continuing characteristic of a scale that doesn't handle as nicely as making count of objects.

3. Objects may be compared with each other indirectly by comparing each with a measuring unit. Comparisons are made with some reference unit. Instruments are used to make the reference unit comparison.

4. The desirability of standard reference units.

There are two steps to measuring:

1. A unit of measure is selected.

2. The number of times the unit of measure is contained in what is measured is counted.

Averages

Mean: The sum of the measures divided by the number of measures—average (most representative from the standpoint of size).

Median: The mid-score of the group (most representative from the standpoint of position).

Mode: The measure that appears most frequently in the group (most representative from the standpoint of frequency).

Measurement of Length

Develop the concept of counting the number of units of measure there are in a given distance. Use units that the student is familiar with and then proceed to use those that he has not experienced.

Adults should be able to acquire skill to measure to the nearest eighth inch and nearest millimeter.

Experience in measuring in various English and metric units of length are to include conversion comparisons within the respective systems and between them.

Conversion facts:

12 inches = 1 foot	5,280 feet = 1 mile
3 feet = 1 yard	1,760 yards = 1 mile

Measurement of Area

Square measure:

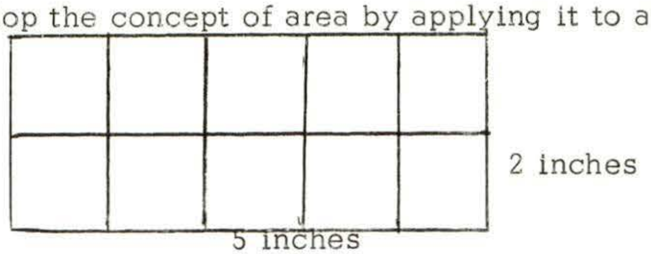
The number of square units contained in section of a plane or surface. Squares of linear measure are used. Examples are square inches and square feet.

Conversion facts:

12 square inches = 1 square foot

9 square feet = 1 square yard

Develop the concept of area by applying it to a rectangle.



Begin by counting the number of square inches. (10)

Follow this by counting the number of square inches in a column (2) and in a row (5).

In this case, the number of rows is identified with the dimension 2 inches (2) and the number in each row with the dimension 5 inches (5).

Thinking: We have an array of two fives.

2 x 5 = 10 squares

The area is 10 square inches.

<u>Measurement of Volume</u>

Cubic measure:

The number of cubic units in a given space.

We use cubes of linear units. Examples are:

cubic inch

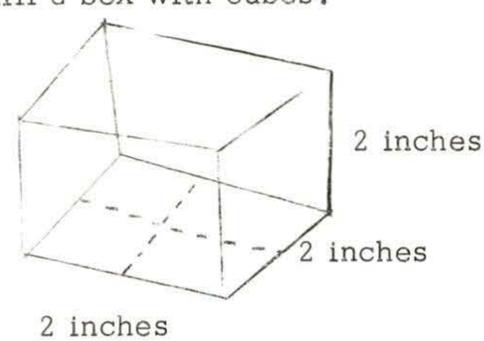
cubic foot

cubic centimeter

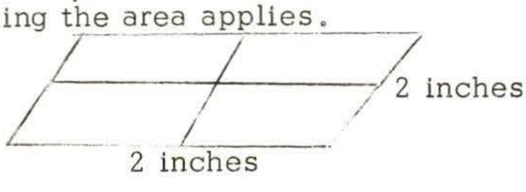
Conversion facts:

1 cubic foot	=	1728 cubic inches
1 cubic inch	=	16.387 cubic centimeters
27 cubic feet	=	1 cubic yard

Develop the concept of volume (space) by applying it to the case where we fill a box with cubes.



Begin by looking at the base or bottom of the box. Our experience with finding the area applies.



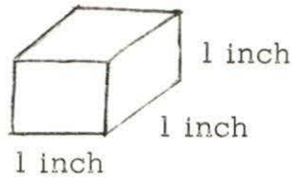
On each square inch of the base can be placed a cube measuring 1 cubic inch.

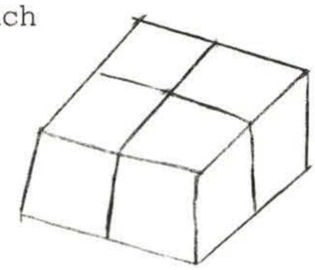
A cubic inch has the following dimensions:

1 inch long

1 inch wide

1 inch high





The number of cubes (cubic inches) on the base of our box is 4. This is the same as the number of square inches in the area of the base.

The height of the box (2 inches) provides the number (2) of layers of cubes.

Thinking: An array of two layers of four cubes each.

2 x 4 = 8 cubes

The volume is 8 cubic inches.

<u>Time</u>

Most adults are able to tell time. Be certain, however, that the learner can measure time in seconds, minutes, and hours. Problems involving work times and sports or job task timing will furnish the clue for what type of time measurement activity to present for the student.

Conversion activities that will be useful in life are best suited for the adult. The common conversion facts include:

60 seconds	=	1 minute
60 minutes	=	1 hour
24 hours	=	1 day
12 months	=	1 year
365 days	=	1 year

Use of the 24-hour time system is helpful when adding and subtracting time.

midnight	2400 hours or 0000 hours
1:30 AM	0130 hours
10:15 AM	1015 hours
Noon	1200 hours
1:00 PM	1300 hours
7:00 PM	1900 hours
10:45 PM	2245 hours

Money

Adults are generally able to recognize and use coin and bill denominations. Check their skill in this unit with activities involving problems where purchase, change required, and counting out of change are necessary.

Usually skills involving the counting back of correct change, without first finding the amount of change, need improvement.

Weight

Common measures of weight:

16 ounces (oz.)	=	1 pound (lb.)
2000 pounds	=	1 ton (T)

Liquid Measure

Common measures

2 cups (cu)	=	1 pint (pt)
2 pints	=	1 quart (qt)
4 quarts	=	1 gallon (gal)
16 fluid ounces	=	1 pint

Dry Measure

The adult generally does not find a great use for these units of measure in his daily life.

The common measures:

2 pints	=	1 quart
8 quarts	=	1 peck
4 pecks	=	1 bushel

Metric System of Measurement

The metric system of measurement is a decimal system.

Prefixes:

deci means 1/10

centi means 1/100

milli means 1/1000

micro means 1/1,000,000

deca means 10

hecto means 100

kilo means 1,000

mega means 1,000,000

Metric linear measurement: the meter is the reference unit.

10 millimeters (mm)	=	1 centimeter (cm)
10 cm	=	1 decimeter (dm)
10 dm	=	1 meter (m)
10 m	=	1 decameter (dkm)
10 dkm	=	1 hectometer (hkm)
10 hkm	=	1 kilometer (km)
100 cm	=	1 m

Conversion facts:

1 m	=	2937 in.
1 in.	=	2.54 cm (approximately)
1 km	=	0.621 mile (or approximately 5/8 mile)

Weight:

The gram is the reference unit.

Conversion facts:

1 kg	=	2.2 lb.
1 gm	=	0.03527 ounce

Temperature:

Reading thermometers to determine temperature improves skills in measurement. By using various temperature scales, practice in use of various unit sizes may be obtained. Experience with thermometers with various markings on the same scale (1°, 2°, and 5°) provides group counting application and scale interpretation skill practice.

The thermometer scale has the number line characteristic that is very good for applying positive and negative number concepts. The application of the signed numbers may be made through change in temperature problems.

Using Centigrade and Fahrenheit temperature scales will set up some measurement conversion activities where formulas may be used. These formulas will relate the mathematical operations skills to measurement concepts and skills.

The formulas are:

$$C^{\circ} = \frac{5}{9} \quad F^{\circ} - 32^{\circ}$$

$$F^{\circ} = \frac{9}{5} \quad C + 32^{\circ}$$

Geometry

Geometry deals with form and size.

The form and size of geometric figures are determined by points, lines, and surfaces.

A point is undefined. It has no geometric size. It is represented by a dot.

Lines:

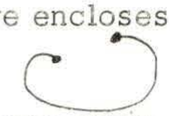
A line has length only.

The symbol for line is:

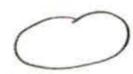
A straight line is generated by a moving point that moves in the same direction.

A curved line is a line, no part of which is straight.

A closed curve encloses a definite amount of area.



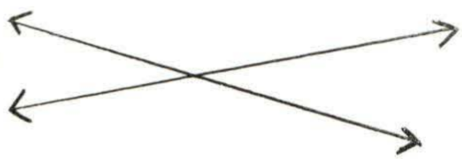
open curve



closed curve

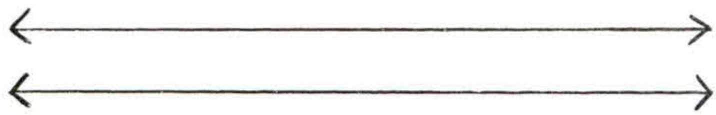
Intersecting lines.

Lines that cross each other in the same plane are intersecting lines.



Parallel lines.

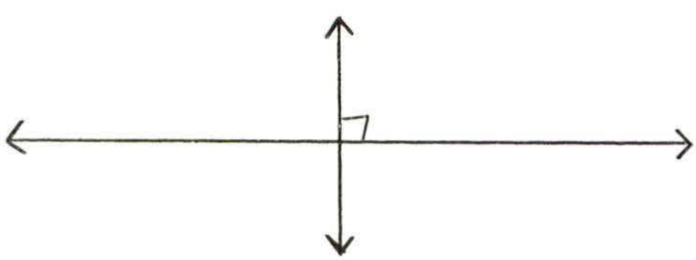
Two lines in the same plane that never intersect.



Perpendicular lines.

Lines that intersect and form equal adjacent angles are perpendicular. The two intersecting lines form a right angle.

Symbol for perpendicular: $\perp$



Line segment.

A line segment is a portion of a line. It has limited length.

The symbol for the line segment is:

The symbol is used with the letters identifying the end points. In the above case line segment AB is written:

$\overline{AB}$ (it is read "line segment AB")

<u>Plane</u>.

Surface has the two dimensions of length and width.

A surface may be:

curved

flat (plane)

A plane figure is one that can be drawn on a flat surface.

<u>Ray</u>.

A ray is a half-line. A ray is understood to have one end point and unlimited extension in one direction.

Symbol: 

<u>Angle</u>.

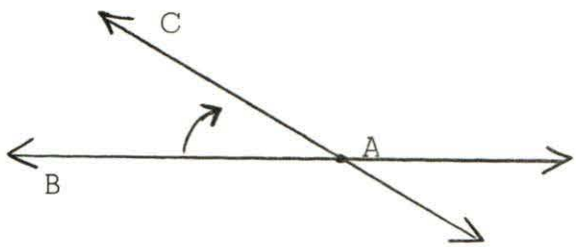
Symbol: $\angle$

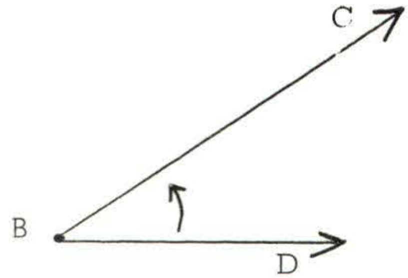
Definition: The amount of opening between two straight lines that meet at a point.

Two other ways of describing an angle.

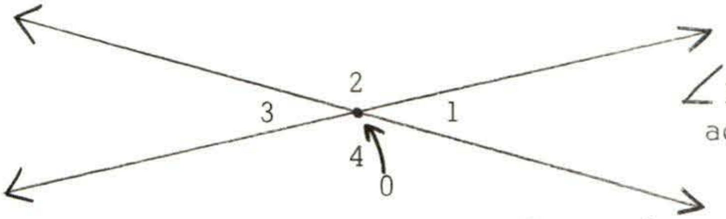
1. An angle is the amount of turning or rotation of a line about a point on the line from one position to a new position.

2. An angle is the amount of rotation of a ray from one fixed position to another fixed position.





Where the two lines or rays intersect is called the vertex of the angle. A and B above and O below are vertices.



∠1 and ∠2 are adjacent angles

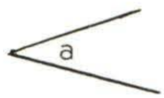
∠1 and ∠3 are vertical angles

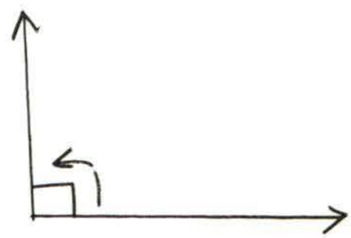
Angles are named several ways:

By the letter at the vertex: ∠A ∠B ∠O

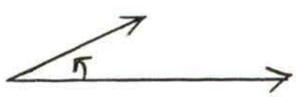
By three letters: ∠BAC ∠CBD

A small letter inside the angle: ∠a

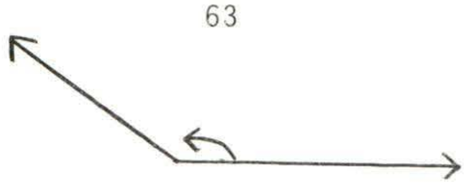




Right angle: sides of the angle are perpendicular.



Acute angle: an angle smaller than a right angle.



Obtuse angle: an angle greater than a right angle but less than a straight angle.

Straight angle: an angle formed by the sum of two right angles. The measure of a straight line.

Measurement of Angles:

One complete rotation is 360° (degrees).

The instrument used to measure angles in degrees is called a protractor.

A reflex angle measures more than 180° and less than 360°.

Angles are complementary if the sum of their measure equals 90°.

Angles are supplementary if the sum of their measure equals 180°.

<u>Polygon</u>.

Definition: A plane closed figure bounded by straight line segments.

Characteristic parts:

sides: line segments that form the boundary of the polygon.

vertices: the points at which the sides of a polygon meet.

diagonal: line segment joining two nonadjacent vertices.

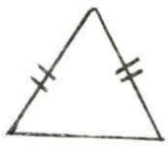
Triangles.

A polygon with three sides.

Symbol: $\triangle$

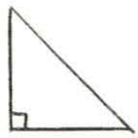
Equilateral Triangle: three sides are equal. All angles are equal.

Isosceles Triangle: has two equal sides.



Scalene Triangle: no two sides equal.

Right Triangle: has one right angle. Side opposite the right angle is call the hypotenuse.



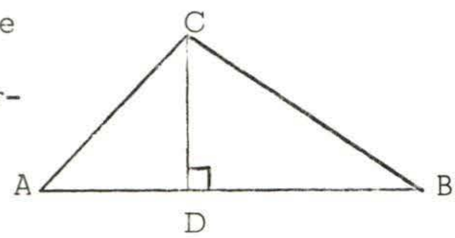
Obtuse Triangle: has one obtuse angle.

Acute Triangle: has all acute angles.

The base of a triangle is a side.

Example: $\overline{AB}$

The altitude of a triangle is the line segment from the vertex perpendicular to the base.



This is $\overline{CD}$ in $\triangle ACB$.

Quadrilaterals.

A polygon with four sides is a quadrilateral.

Trapezoid: a quadrilateral with one pair of parallel sides.

Parallelogram: a quadrilateral with two pairs of parallel sides.

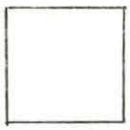
Symbol: ▱

Rectangle: a parallelogram whose angles are right angles.

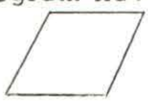
Symbol: ▭

Square: a rectangle with equal sides.

Symbol: □



Rhombus: a parallelogram having equal length sides.



Other polygons:

Pentagon: a polygon with five sides.

Hexagon: a polygon with six sides.

Heptagon: a polygon with seven sides.

Octagon: a polygon with eight sides.

Nonagon: a polygon with nine sides.

Decagon: a polygon with ten sides.

Measurement Geometry:

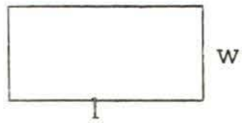
Perimeter: the distance around a polygon.

Area: the number of square units contained in a prescribed surface.

Perimeter and areas of polygons to be determined are:

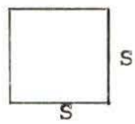
Rectangle: $P = 2l + 2w$

$A = l\ w$



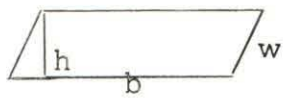
Square: $P = 4s$

$A = s^2$

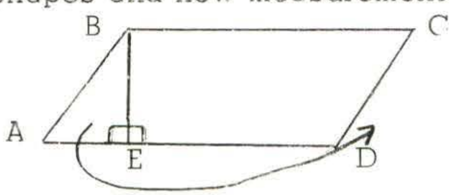


Parallelogram: $P = 2b + 2w$

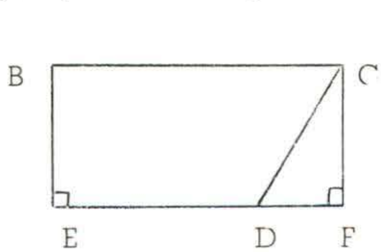
$A = b\ h$



Developing the concepts about how parallelograms area is found will help the student use previously learned skills and concepts in the mastery of others. It will also show the relationships between geometric shapes and how measurement concepts may be applied.



If we draw a parallelogram ABCD, cut off and move triangle ABE to the position suggested by the arrow in the above figure, our new figure will look like the one below.



The parallelogram becomes a special form called a rectangle. Note that the area of rectangle BCFE is the same as parallelogram ABCD.

$\overline{BE}$ is the height of ▱ ABCD and the width of ▭ BCFE.

$\overline{EF}$ is the length of ▭ BCFE and the same length as $\overline{AD}$, the base of the ▱ ABCD.

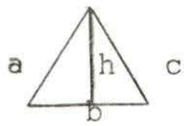
Area = length x width (rectangle).

This is the same as base x height (parallelogram).

Then the area of a parallelogram is: A = b h.

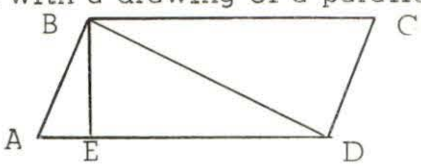
<u>Triangle</u>: P = a + b + c

A = $\frac{1}{2}$ b h



Another valuable draw-and-cut learning activity will show how the area of a triangle may be found.

Begin with a drawing of a parallelogram ABCD.



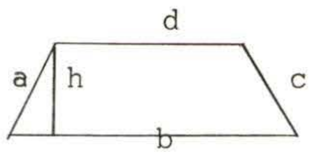
Cut along BD. This divides the parallelogram into two equal sized shapes (check the size) call triangles.

The area of a triangle then is one half the area of a parallelogram with a base and height equal to that of the triangle.

The area of a parallelogram was found to be A = b h so the area of a triangle is A = bh.

<u>Trapezoid</u>: P = a + b + c + d

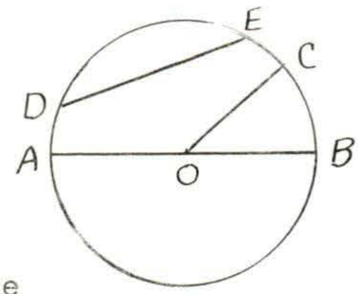
A = $\frac{1}{2}$ h(b + d)



Circle:

A circle is a plane closed curve such that every point on the curve is the same distance from a point within called the center.

Symbol:



Circumference: the length of the curve.

Diameter: a line segment passing through the center with end points on the circumference. $\overline{AB}$ in the figure.

Radius: a line segment joining the center and the circumference. $\overline{OB}$ in the figure.

Chord: a line segment with end points on the circumference. $\overline{DE}$ in the figure.

Semicircle: half a circle.

Semicircumference: half a circumference.

Arc: a part of the circumference.

Symbol: $\frown$

minor arc: $\overset{\frown}{BC}$ in the figure

major arc: $\overset{\frown}{BAC}$ in the figure

Circumference: C

pie: π constant: 3.14 or $\frac{22}{7}$

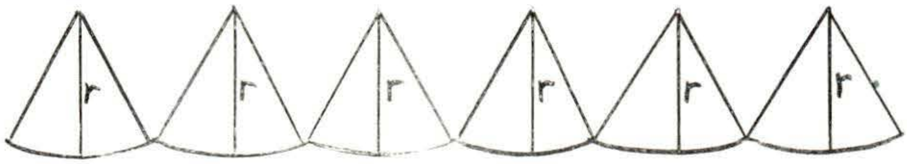
diameter: d

radius: r

$C = \pi d$ or $2 \pi r$

$A = \pi r^2$

If we would cut a circle into small pie-shaped wedges and arrange them as shown below and apply our knowledge about areas of triangles and the circumference of a circle, the formula for the area of a circle does become more meaningful.



In the figure above, we have what appears to be a set of identical triangles. The triangles have a height equal to the radius of the circle from which they were cut. The total length of the bases of all the triangles is the circumference of the circle.

The area of a triangle is: $A = \frac{1}{2}\ b\ h$

Our base is the circumference: $C = 2\pi r$

Substituting $2\pi r$ for the base b we have $A = \frac{1}{2}\ (2\pi r)\ h$.

The height is the radius, so substituting r for h we have

$A = \frac{1}{2}\ (2\pi r)\ r$

$A = \frac{1}{2}\ \underline{(2\pi r)\ r} = \pi r^2$

Polyhedron:

A polyhedron is a geometric solid bounded by planes.

The plane surfaces of a polyhedron are called faces.

Any two faces intersect in a line segment called an edge.

The intersection of three or more edges is called a vertex.

Prism:

A prism is a special kind of polyhedron.

Two of the faces, called bases, are congruent and parallel polygons.

The other faces, called lateral faces, are parallelograms formed by joining the corresponding vertices of the bases with line segments.

A right prism has the edges of its lateral surfaces perpendicular to the bases.

An oblique prism does not have the edges of its lateral surfaces perpendicular to the bases.

Surface Area of Prisms:

The area of the lateral surfaces is found by computing the area of each lateral surface and finding the sum of the lateral areas.

If it is a right prism, the area of the lateral surfaces may be obtained by multiplying the height of the prism by the perimeter of the base.

Total surface area of a prism is found by adding twice the area of the base to the lateral surface area.

Cube:

lateral area: $A = 4e^2$

total area: $A = 6e^2$

Volume of a Prism:

Volume equals the area of the base times the height.

$V = B \times h$

Rectangular prism:

$V = l \times w \times h$

Cube:

Volume equals length of the edge to the third power.

$V = e^3$

Cylinder

Volume: $V = Bh$

$V = \pi r^2 h$

Lateral Surface Area: $LA = 2\pi rh$ or πdh

Pyramids

A pyramid is a polyhedron whose base is a polygon and whose sides, called lateral faces, are triangles having one vertex in common at a point opposite the base. The apex is the vertex in common for the lateral faces.

Regular pyramid: The base is a regular polygon and the apex is directly opposite the center of the base. All the lateral faces are isosceles triangles. The altitude of one of the isosceles triangles is called the slant height of the pyramid. The altitude of a pyramid is the line segment from the apex perpendicular to the base.

Volume: $V = \frac{1}{3} Bh$

Lateral area: $LA = \frac{1}{2} ps$ p = perimeter of base

s = slant height

Cone

A cone is a solid bounded by a plane forming the base and by a lateral curved surface that comes to a point called the apex.

A circular cone is one that has a circle for a base.

A right circular cone has a circular base and the altitude connects the center of the circular base with the apex.

Volume: $V = \frac{1}{3} \pi r^2 h$

Lateral area: $LA = \frac{1}{2} ps$

Since the perimeter is a circle: $C = 2 \pi r$ or πd

the $LA = \frac{1}{2} (2) \pi rs$

$LA = \pi rs$

Sphere

A sphere is a solid bounded by a curved surface such that every point on the surface is the same distance from a point within called the center.

A sphere may be thought of as generated by the rotation of a circle around one of its diameters as an axis.

Great circle: The largest circle that can be cut by a plane intersecting a sphere. The plane would go through the center of the sphere.

Surface area of a sphere: $A = 4 \pi r^2$

Volume of a sphere: $V = \frac{4}{3} \pi r^3$

Congruent Figures:

Congruent figures are those that have exactly the same size and shape.

Line Segments:

Congruent line segments would be those parts of lines that have the same length.

Angles:

Angles that are congruent would be those angles that have the same measure.

Polygons:

Polygons are made up of line segments and angles. Any two polygons that have line and angles the same measure are congruent.

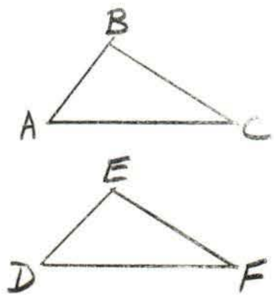
The symbol for "is congruent to" is: $\cong$

Example:

$\triangle ABC \cong \triangle DEF$

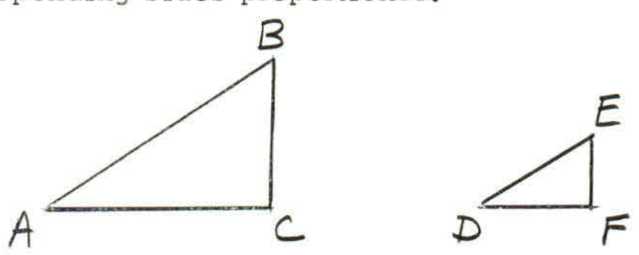
$\overline{AB} \cong \overline{DE}$

$\angle A \cong \angle D$



Similar Figures:

Similar figures have all corresponding angles equal and all corresponding sides proportional.



The symbol for "is similar to" is: $\sim$

In the two triangles on the preceding page the following conditions exist.

$\angle A \cong \angle D$	$\angle A$	corresponds with	$\angle D$
$\angle B \cong \angle E$	$\angle B$	" "	$\angle E$
$\angle C \cong \angle F$	$\angle C$	" "	$\angle F$
	$\overline{AB}$	" "	$\overline{DE}$
	$\overline{BC}$	" "	$\overline{EF}$
	$\overline{AC}$	" "	$\overline{DF}$

$$\frac{\overline{AB}}{\overline{DE}} = \frac{\overline{BC}}{\overline{EF}} = \frac{\overline{AC}}{\overline{DF}} = \frac{2}{1} \begin{cases} \overline{AB} = 2\overline{DE} \\ \overline{BC} = 2\overline{EF} \\ \overline{AC} = 2\overline{DF} \end{cases}$$

Those conditions make the two triangles similar.

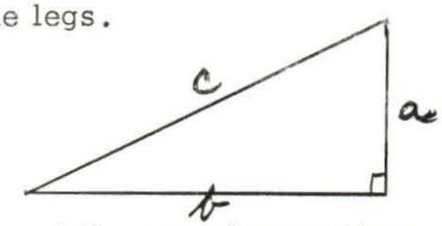
$\Delta ABC \sim \Delta DEF$

<u>Indirect Measurement:</u>

Pythagorean relationship:

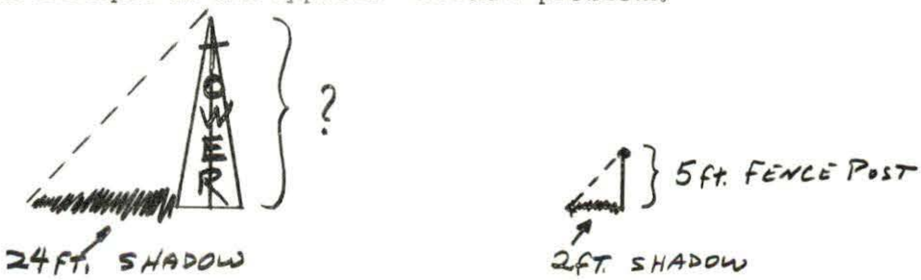
In every right triangle the square of the hypotenuse is equal to the sum of the squares of the legs.

$$c^2 = a^2 + b^2$$



With the pythagorean relationship and the use of proportions as applied with similar figures, many indirect measurements may be performed.

An example is the typical shadow problem:



If the shadow measurements are taken at the same time, the sun rays form similar figures.

In similar figures the corresponding sides are in the same ratio.

$$\frac{5}{2} = \frac{?}{24} = \frac{5}{2}$$

$$? = \frac{5}{2} \times 24 = 60 \text{ feet.}$$

Applications

Applications is the last area of our model classification system.

Social utility is important to adults. The purpose then is to develop this area so it relates the mathematics skills mastered in the other areas to the real world.

It should be realized that an adult learner does want to apply his skills in as realistic a way as possible. Every effort should be made to collect materials and prepare or obtain units that provide the student with options in applying each and every mastered skill and concept. The options, if possible, should be in the subjects of interest to the student at that particular time. Their practical skill application will both reinforce the skills and motivate the student's desire to continue to higher level skills.

Live application problems usually require a combination of mathematics skills to solve them. Activities that require combinations of skills belong in their application area. Some examples of these activities are preparing and using such things as scale drawings, graphs and tables. Any appropriate word problems are always made useful parts of the area.

Consumer topics like taxes, installment buying, banking, borrowing, income, and budgeting are examples of mathematics application. Collections of this type could be an endless undertaking and probably should be in order to upgrade your resources and meet the changing requirements of the students.

Reading is always important. Provide some assistance to the adult student so he may become a better reader of mathematics material, and especially word problems will provide for greater success in his application skills.

When reading in mathematics, be aware of the purpose of reading. A first reading to obtain an over-all picture in broad outline is then followed by a more careful reading for details. A number of subsequent readings may be required:

Student cannot solve the problem so he re-reads in search for what was missed. (What material bears upon the problem at hand?) Scanning to locate the passage wanted and intensive critical reading of the passage.

Search for specific facts. Scan to locate passage and again follow by careful reading of located passage.

Read for review. The purpose is to relate the content to other material, place it in larger perspective, making it a part of a larger whole.

Reading in mathematics usually requires careful attention to every detail.

1. It requires: reflective questioning, critical frame of mind.
2. Being critical toward what is read.
3. Being critical toward one's own comprehension of what he reads.

Not understood statements should be marked or noted. Specific points needing further clarification must be identified. Characteristics of mathematics language which complicate reading are:

1. Conciseness (omission of a word, symbol, punctuation mark may change the entire meaning).

   Examples:

   (1) Think of the difference in meaning of the following statement if a single letter "f" is omitted: Sale, 25% off list price.

   (2) Consider the two questions: Fifteen is 25% of what? and What is 25% of 15?

2. Technical mathematical terms must convey the precise meanings if they are to evoke the desired concept in the mind of the reader. (The correct meanings of the technical mathematical terms are required for adequate reading.)

   The multiple meanings of some technical terms require context association for the precise meaning. (Example: "remainder" used in connection with subtraction and division.)

3. Signs are used.

4. Concepts are developed and have a sequence.

5. Reading of formulas, tables, and graphs.

6. Requires ability to translate a verbal statement to a formula, and vice versa.

Improvement of problem solving ability is a constant requirement for the adult learner. No one method should be considered necessary to obtain an acceptable solution in mathematics. This suggests a variety of methods are to be used. It, however, does not infer that there are a great many methods to be reviewed and mastered. It is suggested that the student be exposed to a variety of methods over a period of time and that he be encouraged to try them. The student should also be encouraged to modify his problem solving procedure as he masters more mathematics concepts and operation skills. Some methods of problem solving that will be helpful to an adult learner are provided next.

Improving Problem Solving Ability

The Analysis Method:

1. Read to determine what is given.
2. Read to determine what is required.
3. Determine from the relationships between the quantities, given and required, what operations are necessary.
4. Estimate the answer.
5. Solve by performing the operations in 3.
6. Check the answer.

Illustration of the analysis method:

Problem:

Mr. James trades cars this year. The new car cost $1250 plus the old car. He averaged 16 miles per gallon of gasoline with his old car and 20 miles per gallon with the new one. Find how much Mr. James saved on gasoline the first year he drove the new car if he drives about 15,000 miles per year and gasoline costs an average of 32 cents per gallon.

Solution:

1. Given: Gasoline mileage with each car. Difference between cost of cars. Miles traveled per year. Cost of gasoline per gallon.

2. Required: Savings in cost of gasoline for the first year.

3. Operations required: We must subtract the annual cost of gasoline for the new car from the annual cost for the old one. The total miles per year divided by the miles per gallon determines the gallons of gasoline used. The number of gallons multiplied by the cost per gallon gives the total cost.

4. Estimate the answer: The old car required about 1,000 gallons and the new one about 750 gallons. At approximately 30 cents per gallon, the saving was about 250 x .30 = $75.00.

5. Solution:

   $\frac{15{,}000}{16} = 938$ gallons used by old car

   938 x .32 = $300.16 cost of gasoline for old car

   $\frac{15{,}000}{20} = 750$ gallons used by new car

   750 x .32 = $240.00 cost of gasoline for new car

   $300.16 - $240.00 = $60.16 saving on gasoline bill

6. Check: Compare the result with the estimate, $60.16 with $75.00. This is not too convincing, but it is not out of reason in view of the approximate nature of the estimate. The problem may be checked by reviewing the computation. The most satisfactory check consists of using a different approach in solving the problem. The old car required 1/16 gallon per mile and the new one 1/20 gallon per mile. Then the new car saved 1/16 - 1/20 gallon per mile. The total gallons saved is 15,000 x (1/16 - 1/20). The total dollars saved is .32 x 15,000 x (1/16 - 1/20) = $60.

Method of analogies:

Relate the problem to a simpler analogous one. This may consist of a substitution of simpler number. Substitution from an unfamiliar to a familiar setting.

Method of dependencies:

Reasoning from the required to known facts, using the Mr. James gasoline problem:

> The amount saved on gasoline depends on the annual cost of gasoline for each car.
>
> The cost for each car depends upon the number of gallons of gasoline used and the cost per gallon.
>
> The number of gallons used depends upon the distance driven and the rate of gasoline consumption.
>
> The distance driven and the rate of consumption are given for each car.

Retrace the steps of reasoning in reverse order, leading to the solution.

Graphic Method:

Use a diagram or graphic picture of the situation presented in the problem as an aid in discovering the relationships that exist between the quantities.

Problem-solving behavior considered good procedures. (Leslie Alfred Dwight)

1. Evaluated solutions by:
   - a. Checking answers with the conditions of the problems
   - b. Checking answers as to reasonableness
   - c. Checking processes chosen

d. Checking computation
e. Re-reading problems after obtaining a solution

2. Used label for each part.

3. Used word cues in relation to the setting of the problem.

4. Interpreted punctuation correctly.

5. Made statements about the problem precise and complete.

6. Gave critical thought to the "required" before computing.

7. After obtaining an impression of the problem as a whole, re-read the problem to note details and to check interpretation.

8. Stated "given" and "required."

9. Planned the solutions and outlined the processes to be used.

10. Determined the relationships of the data in the problem.

11. Used drawings and figures when possible.

12. Noted and discarded irrelevant data.

13. Obtained comprehension of problem before performing computation.

Algebraic Solution of Problems

The solution of problems involved two tasks.

1. Making the equation(s) from the given information.

2. Solution of the equation(s) made.

Russell V. Person's Five Golden Rules for solving problems:

1. Let some letter, such as x, represent one of the unknowns. (This is usually, though not necessarily, the smallest.)

2. Then, try to express the other unknowns by using the same letter.

3. Write a true equation from the information given in the problem. Make your equation say in symbols exactly what the problem says in words.

4. Solve the equation.

5. Check your answer to see whether it satisfies the conditions given in the problem. (Do not check in the equation that you made. Apply them directly to statements and conditions in the problem.)

General Procedure for the Solution of Problems

1. Read the problem carefully once or twice.

2. Be certain that you understand the language and the meaning of the problem.

3. Determine the information which is given, and what is to be found in the problem.

4. Determine which unknown number is to be represented by a letter.

5. Represent as many other numbers as is possible in terms of this letter.

6. Think of any formula that might be of assistance in the problem.

7. Search for a relationship stated in the problem which will help write the equation.

8. Solve the equation carefully.

9. Determine the answer to the question. Be certain to give the answer in terms of the proper unit.

10. Check by testing the answer in the original question.

Scale Drawings

Preparing scale drawings and using them for sources of information to solve problems is a learning activity that involves three essentials to make adequate interpretations. These essentials are:

1. An orientation as to direction.

2. An awareness of shape and its preservation.

3. A knowledge the ratio used.

Graphs

Graphs are found in many publications. Newspapers frequently use them. Adult students are able to apply various mathematics skills and concepts when they prepare to make and obtain data from graphs to solve problems. In life, the adult student will more often be getting information from a graph than he will be preparing one. Substitution of units is an experience involved in the reading and construction of graphs. You will note that two related sets of things are pictured in a graph.

Types of graphs include:

Bar graph: The linear unit used represents either a count or some unit of measure. It shows comparisons.

Pictograph: Each picture represents a specified number of pictured objects. This type of graph is used to show comparisons.

Continuous line graph: The linear unit used represents a given number of units above a base line. This graph is used to show trends and rate of change of a changing quantity.

Circle graph: This graph compares the various parts with the whole.

Table

A table gives essentially the same information as a graph. In the table, the pairs to be pictured are written in some systematic manner. Quite often they are written in two columns.

The adult learner that would prepare a graph would undoubtedly prepare a table of data first.

A student solving problems would use a table very much in the same way he would a graph.

Graphing Equations

Graphing equations by using a rectangular coordinate system is a means to display equations graphically. It will also provide a means to solve equation systems for those students that have progressed to that mathematics level.

Selection of Objectives

Selecting objectives for the mathematics program content that you have selected is an important task.

To make use of the work done by others, it is suggested that the following procedure, or one designed for your needs, be used to obtain an initial set of objectives that fit your mathematics program.

1. Establish criteria for the preferred form of objective statements.

2. Prepare a scale to rate objective statements.

3. Prepare a reviewer's report form.

4. Review mathematics resources like textbooks, curriculum guides, instructional guides, program guides, and program packages and packets. Rate their content objective statements. (Review only the parts that are included in the scope of your program.)

5. Analyze your findings.

6. Select the objective statements from the resources that rated best.

The following list of sources is made as suggestions for review. It would seem desirable that a variety of objective statement forms be reviewed to insure that each of the various requirements will be met in the best possible way.

1. General Mathematics grades 10-12
   Mathematics grades 7-9
   Mathematics grades 4-6
   Mathematics grades K-3

   The Instructional Objectives Exchange
   P.O. Box 24095
   Los Angeles, California 90024

2. Mathematics Skills Continuum

Research for Better Schools, Inc.
Suite 1700
1700 Market Street
Philadelphia, PA 19103

3. Flanagan, Mayer, Shannen
Behavioral Objectives: A Guide to Individualizing Learning

(A 4-volume set: Mathematics, Language Arts, Social Studies, and Science)

Westinghouse Learning Press
Division of Westinghouse Learning Corp.
Palo Alto, California

Ordering of Objectives

It seems that a logical order of mathematical tasks could be determined by considering prerequisite behaviors. Effort to accomplish this should be provided but within limits. An inflexible order is not consistent with individualization.

Objectives are ordered to do several things. These things are:

1. Insure that the learning tasks may be matched to the learner's entry behavior.

2. To provide change and variety among the mathematics topics.

3. To match the level of task complexity which the learner can tolerate.

Objective order is affected by mathematics topic ordering. Two important considerations in topic order are:

1. What important topics can be efficiently taught with the learner's entry behavior.

2. Which of the behaviors is most profitable to future learning.

The ordering of curriculum has important qualities in general, but in reality they are less critical than it seems. If you are individualizing, any error in order will be recognized and adjustments will be made to correct the problem.

Specifying Performance Objectives

After ordering the selected objective statements, it would be appropriate to take the next step. The next step is to state our collection of objectives in performance terms. The exact nature of this statement will unfold after you consider the requirements of your mathematics program.

To explain the performance objective the following is used.

1. General objectives or concepts consider the subject matter area and the student goal.

2. Specific objectives consider the skills that the learner can perform in relation to the concept or general objective.

3. Behavioral objectives are statements of the kinds of behavior accepted as evidence that the learner has achieved the objective, the conditions under which the behavior will occur, and the criteria for acceptable performance.

The third level objective, above, can be recognized as being performance oriented. Its emphasis is on what the student does, how he performs. It is this performance type statement of objective that is recommended for use in your mathematics program.

There are two types of performance objectives to be developed for the program. They are:

1. Terminal Objectives - Terminal objectives are described as major growth points in the order of objectives. These objectives would be selected or developed first. Have them become a part of your developing individualized mathematics program as you complete them.

2. Transitional Objectives - Transitional objectives are described as those objectives that lead the learner to mastery of the terminal objective. It is a sub-objective type of learner performance requirement. The number of transitional objectives you select or develop will depend upon requirements to meet the learning characteristics of the students.

In concluding this section, it should be mentioned that those ready to improve or take any next steps in performance objective development may consider the types of behavior desired in the performance objective. What is referred to are the three major types of educational objectives found in:

1. psychomotor domain
2. affective domain
3. cognitive domain

Describing the types of behavior included in these three domains or categories will make your performance objectives more learner oriented. Consideration of learner performance and needs, of course, is a goal in individualization.

To assist in application of the psychomotor, affective, and cognitive objectives, you are referred to the following resources:

1. Simpson
   The Classification of Educational Objectives, Psychomotor Domain.

   Project Report
   University of Illinois

2. Krathwahl, et al.
   Taxonomy of Educational Objectives; Handbook II, Affective Domain.

   New York: McKay

3. Bloom, et al.
Taxonomy of Educational Objectives; Handbook I, Cognitive Domain.

New York: McKay

4. Johnson and Johnson
Developing Individualized Instructional Material

Palo Alto, California:
Westinghouse Learning Press

PERFORMING DIAGNOSIS AND EVALUATION

Using the definition of individualization as adapting the educational environment to individual differences, we will want to be able to secure information about the individual student.

Securing information about a learner involves diagnosis and evaluation.

Evaluation of student capacities, interests, and stage of development is one of our goals.

Diagnosis is determining the next "thing" to be learned by a student. To accomplish this it is desirable to have a comprehensive system to determine skill objective mastery. It is comprehensive in the sense that tests are provided for all lessons and tests are provided to determine what lessons the student should encounter.

A recommended system for diagnosis and evaluation shall include: entrance diagnosis, pre-unit diagnosis, skill evaluation and pre-skill diagnosis, post-unit evaluation, and exit evaluation.

Entrance Diagnosis

Entrance diagnosis will include those activities used to determine student level of placement in the various areas of the mathematics program.

The type of test used should make a very gross measurement of the terminal objectives, by level in each area. The test need only answer the question: What is the highest level of mastery that the student has obtained in each area?

Details about the specific skills needed to secure mastery of the first nonmastered unit are considered in the next step of student diagnosis.

It is recommended that the entrance test be constructed by using selected items, like those found on the pre-unit tests. The items selected would measure only terminal objectives for the particular level of learning.

Since this is the first test that the student encounters it is wise to make it as adequately brief as possible.

Pre-Unit Diagnosis

The pre-unit diagnosis involves determining student achievement within a single unit of the mathematics program.

A pre-unit test samples all the skills in a unit. It samples in the sense that it measures a skill terminal objective and key transitional objectives. Then when a student shows nonmastery of a skill on the pre-unit test, the items measuring transitional objectives provide information about the specific parts of the skill that a student has mastered and what parts will require learning activities prescribed.

The pre-unit test answers the question: Which skills in the unit has the student mastered?

The nonmastered skills obviously will be the ones prescribed for the student.

Students are dynamic systems. They are dynamic in that they change and are subject to varying degrees of change. Sometimes the changes are sudden and at other times gradual.

Since students change, then diagnosis should reveal this change. Because of this, it is recommended that a pre-unit test be given just before the student is ready to begin work in that unit. This insures the student's current skill requirements.

The student change characteristic also applies to skill diagnosis and will be considered in the suggestions for skill evaluation and pre-skill diagnosis.

Skill Evaluation and Pre-Skill Diagnosis

Skill evaluation is used to determine if or when a student has mastered a particular skill objective.

A skill test measures the terminal behavior called for in the skill objective. Also included on the test are selected test items from key transitional behaviors (sub-behaviors) for the skill.

Key items from the transitional behaviors will serve a diagnostic function when skill nonmastery is shown. Skill test nonmastery requires recycling the student into learning activities for that skill. The transitional behavior items missed on the skill test will provide guidance in determining the appropriate prescription for the student.

A skill test is administered when it is believed that the student has attained mastery of the skill objective. Because of its evaluative and diagnostic function, a skill test should be prescribed without hesitation each time it is believed the student shows mastery of skill terminal behavior.

Consideration of the changes that occur in the student during the skill learning procedure often affects his ability to apply newly formed skills to new situations. This characteristic of learning is particularly applicable to mathematics.

Because of the above characteristic the skill test should have a section that provides for such evaluation. It is particularly important to evaluate what effect student learning may have had on the performance of the next skill, particularly if it was not mastered on the pre-unit test.

A second part of a skill test is a brief test to measure the terminal behavior for that next skill in the mathematics continuum. This evaluation procedure will allow a student to demonstrate any "carry-over" application of developmental skills.

Student current skill mastery combined with previous knowledge often results in the mastery of closely related skills. Efficiency in prescribing for student needs demands a knowledge of this student "carry-over" learning when it does occur.

If the student shows mastery on the second part of the skill test (short pre-test of the next skill), the usual procedure would be the administration of a skill test for the next skill objective to verify his mastery of that objective. It is recommended that two skill tests be constructed for each skill objective.

Post-Unit Evaluation

Post-unit evaluation provides a check of mastery for a unit of mathematic skills.

A post-unit test is administered following the mastery of the last skill in a mathematics unit. This test is another form of the pre-unit test. Because of this characteristic the pre-and post-unit tests may be interchanged.

Exit Evaluation

Exit evaluation may serve two important functions.

1. It may function as a verification of student long range behavioral growth in mathematics.
2. It may be used to provide information to be used in modifying the mathematics program.

In summary, please note that mastery in evaluation is a reasonable criterion. Each item in a test has inherent worth. There is a hierarchy of skills in a skills area. What has been suggested in this section on diagnosis and evaluation is that criterion referenced (refers to content) tests are absolutely adequate in evaluating skill development and diagnosing the needs of students.

PROVIDING FOR INSTRUCTION AND LEARNING

Throughout the syllabus a model system has been formulating or at least there is one that may be composed at this point. This system involves the student activity flow that will use the materials collected and the diagnostic procedure recommended. The student activity flow model may be seen on the following page.

We now want to use the information that we have about the individual student so appropriate educational environments may be prescribed.

An appropriate educational environment is one that provides a student experiences with materials used under conditions or settings that result in efficient mastery of the skill objective.

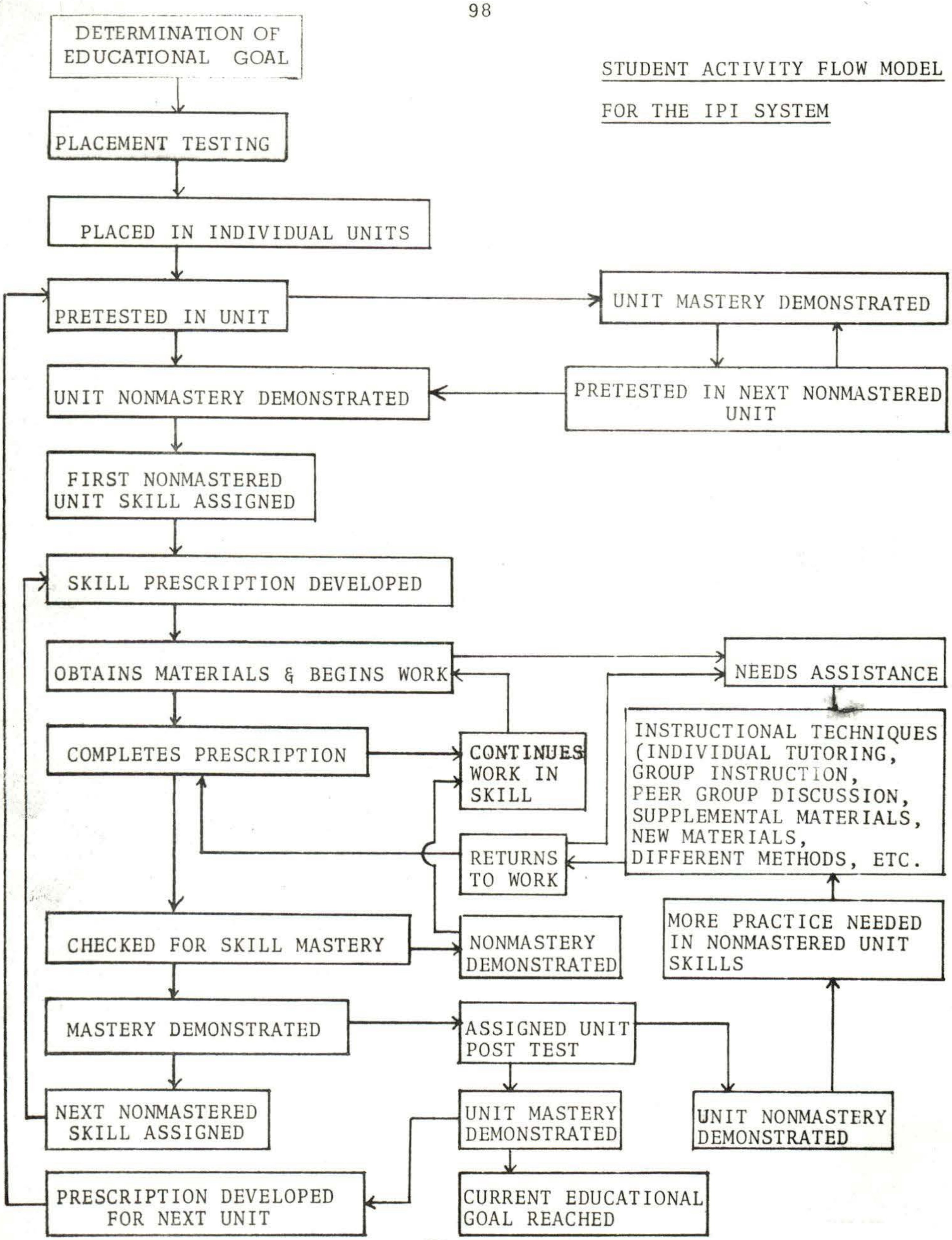
Before discussing settings, experiences, and materials, it would be well to recommend that <u>everything</u> you do for the adult student be a part of the developmental learning process for him. In an individualized program there isn't such a thing as a remedial process. If a student didn't learn something, it wasn't developed properly.

It is further necessary that the instructor of adult students does all he can to:

1. free communication about what the student wants to talk about, know about, and do something about.

2. discover the student's real goal.

STUDENT ACTIVITY FLOW MODEL FOR THE IPI SYSTEM



3. know about the student's world.

4. establish mutual respect with the student

5. establish common obligations with the student as a basis for cooperation.

To meet the needs of the changing characteristics of a student and the differences between students requires individualization of instruction and learning. These conditions require that there be alternative paths to the same objective if we are individualizing.

Settings

Settings or conditions are considered those arrangements of facilities, people and equipment to best present a learning experience.

Although independent learning activity is a predominant part of the individualized mathematics program and a student goal for continual learning, we do have a variety of settings in which the independent activity may be performed.

The following are a few examples of settings for independent activity:

Listening to a number fact tape using a tape record player.

Viewing a single concept film loop about bisecting line segments.

Researching sources at the library or the classroom reference collection to find types of number systems.

Using an abacus or other manipulative device to experience counting and its operation in a place value number system.

Taking a pre-unit test on E level measurement.

A student working word problems from a textbook.

There are settings where interaction with other persons is involved. Examples of these types of settings are given below.

1. An instructor tutors a student about an addition algorithm.

2. A student helps another student construct perpendicular lines.

3. An instructor is demonstrating addition of fractions with unlike denominators to a group of four students.

The settings options are numerous and many settings will be used as the instructor and students become involved under the direction and motivation of the instructor. There are many routes to learning and selection of the one that works best for us each day must be made.

Experiences

Experiences are those activities that promote learning of skills and concepts involved in the mastery of an educational objective. Experiences are the interaction of materials in a setting. These experiences should be selected to meet particular requirements of a student. The selection options for experiences involve modes of representation and modes of presentation.

Modes of representation are:

1. Enactive: these are a concrete, doing, and manipulative type of representation.

2. Iconic: these are a graphic or pictorial type of representation.

3. Symbolic: this type of representation uses symbols like numerals and equations.

Modes of presentation are:

1. Inductive: this is a presentation that develops the concepts and skills involved with the parts and then builds the whole or unites the parts.

2. Deductive: this type of presentation is the reverse of inductive. The result or skill is shown and the method deduced is brought out later.

3. Exemplar: this is a presentation that develops a concept or skill through comparison or presenting examples.

Procedures or experiences should provide for discovery through free and goal directed explorations. This will develop an independence desired for the adult student's continued educational activity.

Materials

We have by now established the need for a variety of materials to use in the many settings and experiences to be called upon to meet a particular student's need to acquire a skill objective. There has been a system for collection and use recommended.

Presented here is a check list of suggestions for selecting materials for adult basic mathematics.

1. Materials have an adult appearance.

2. Covers or identifying markings are appropriate (avoid special labeling other than the content and that is in mature manner).

3. Contents reflect adult tastes.

4. Presents problems of social skills.

5. Presents special information suitable for specific trades or jobs.

6. Entrance test(s) are included in the materials.

7. Entrance test(s) are easily administered.

8. Entrance test(s) quickly place the student into the materials at the appropriate level of difficulty.

9. Materials are self-instructional or programmed.

10. Materials include practice and application experiences.

11. Practice experiences are short.

12. There is a sequentially organized skill building.

13. Vocabulary is appropriate for the student.

14. Includes an instruction manual.

15. Manual includes learning experience plans.

16. Manual includes instructional method suggestions.

17. Manual describes organization of the material.

18. A means of self-evaluation is provided.

19. Self-evaluation is frequent.

20. Group activities included will support the effort of the individual student.

21. Materials have been field tested.

22. Population upon which the material was tested is described.

23. Revisions of the material have been made based upon the findings of field testing.

24. Illustrations are tasteful and accurate.

25. Illustrations augment the instruction-learning.

26. Materials are durable for the type of application intended.

27. Materials are relatively inexpensive.

28. Style of type is pleasing.

29. Layout design is pleasing.