MODELING OF COVID-19 UTILIZING VARIOUS COMPARTMENTAL MODELS TO PREDICT INFECTION RATES THROUGHOUT MICHIGAN

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5 March 2021

Abstract

Compartmental modeling is a method of employing math to create visual representations of a disease interacting with a select population, typically used in epidemiology analyses. This project applies and adapts compartmental modeling equations to data collected on the deaths, infection, and testing of COVID-19 in Michigan. Comparing current data to past predictive models, as well as the visual representations that were developed through the various compartmental modeling methods, allows assessment of the effects of the preventative measures taken by the state, the various rates at which the infection is able to spread, as well as the potential path and spread of the virus in the future.

Introduction

The objective of this study was to observe the way the COVID-19 infection has progressed through Michigan, as well as to compare theoretical infection rates proposed for Michigan in the early onset of COVID-19.

The concept of applying mathematical modeling in epidemiology has existed since the mid-1700s, though modern day compartmental modeling involves the use of math modeling programs such as MATLAB or COMSOL to complete the arduous mathematical tasks. [1] The applications of mathematical modeling in the context of scientific analysis are plentiful, especially in the context of Chemical Engineering.

The modeling method used in this research is compartmental modeling with an SIR model base. The SIR model is made of three equations, the S equation referring to the number of susceptible people in a specific population, I refers to the number of infected, and R is the number of people who have recovered over the specific time frame. [2] Additional equations may be added to the SIR base. As seen in the following equations, an additional variable 'E' has been added to denote the number of exposed individuals.

$$\frac{dS}{dt} = \mu - \beta(t)SI - \mu S \quad \text{[3, eq. (1)]}$$

$$\frac{dE}{dt} = \beta(t)SI - (\mu + \alpha)E \quad \text{[3, eq. (2)]}$$

$$\frac{dI}{dt} = \alpha E - (\mu + \gamma)I \quad \text{[3, eq. (3)]}$$

$$R_0 = \frac{\beta_0 \alpha}{(\mu+\alpha)(\mu+\gamma)} \quad \text{[3, eq. (4)]}$$

The variables in the following equations represent the following parameters

- μ represents birth and death rates
- 1/α is the average incubation period of the disease
- 1/ϒ is the average infectious period
- And β(t) is contact rate (assumed to be constant and equal to β0 as seen in eq. 4)

Methodology

Data from early March to the present was collected on various aspects of COVID-19 as it spread throughout Michigan. The data included (but was not limited to) the number of infectious individuals, tested individuals, deaths, and recovered individuals daily over the time span from March 1st, 2020 to March 1st, 2021.

The initial step in analyzing the data was to estimate the parameters (μ, α, ϒ, and β) by using COMSOL. This was done by simultaneously running collected data of actively infectious over the first ~170 days of virus with the SEIR equations.

number	date	positiveCasesViral	recovered	activeCases
1	3/1/2020	14		14
2	3/2/2020	27		27
3	3/3/2020	50		50
4	3/4/2020	74		74
5	3/5/2020	100		100
6	3/6/2020	140		140
7	3/7/2020	188		188
8	3/8/2020	251		251
9	3/9/2020	372		372
10	3/10/2020	528		528
11	3/11/2020	727		727
12	3/12/2020	996		996
13	3/13/2020	1353		1353
14	3/14/2020	1717		1717
15	3/15/2020	2175		2175

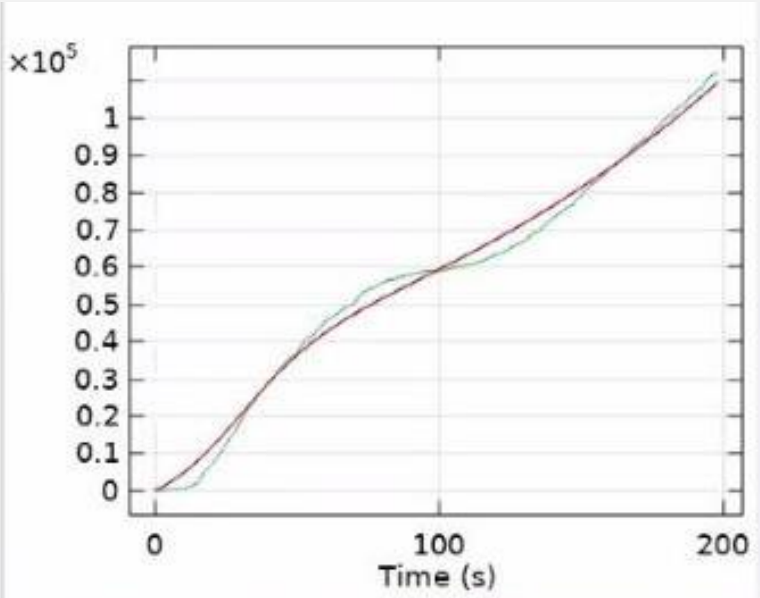

Figure 1 & 2: Raw data (left) and COMSOL graph of raw data in comparison to SEIR graph based off of estimated parameters

The parameters of greatest interest were infection rate (β) and rate of recovery (ϒ), the infection rate was found to be 0.6x10^-8, and the rate of recovery was 0.0173.

These values were then programmed into an SIR model in Matlab and plotted a theoretical graph of the infection rate had no measures been taken to prevent the spread of the disease. [4]

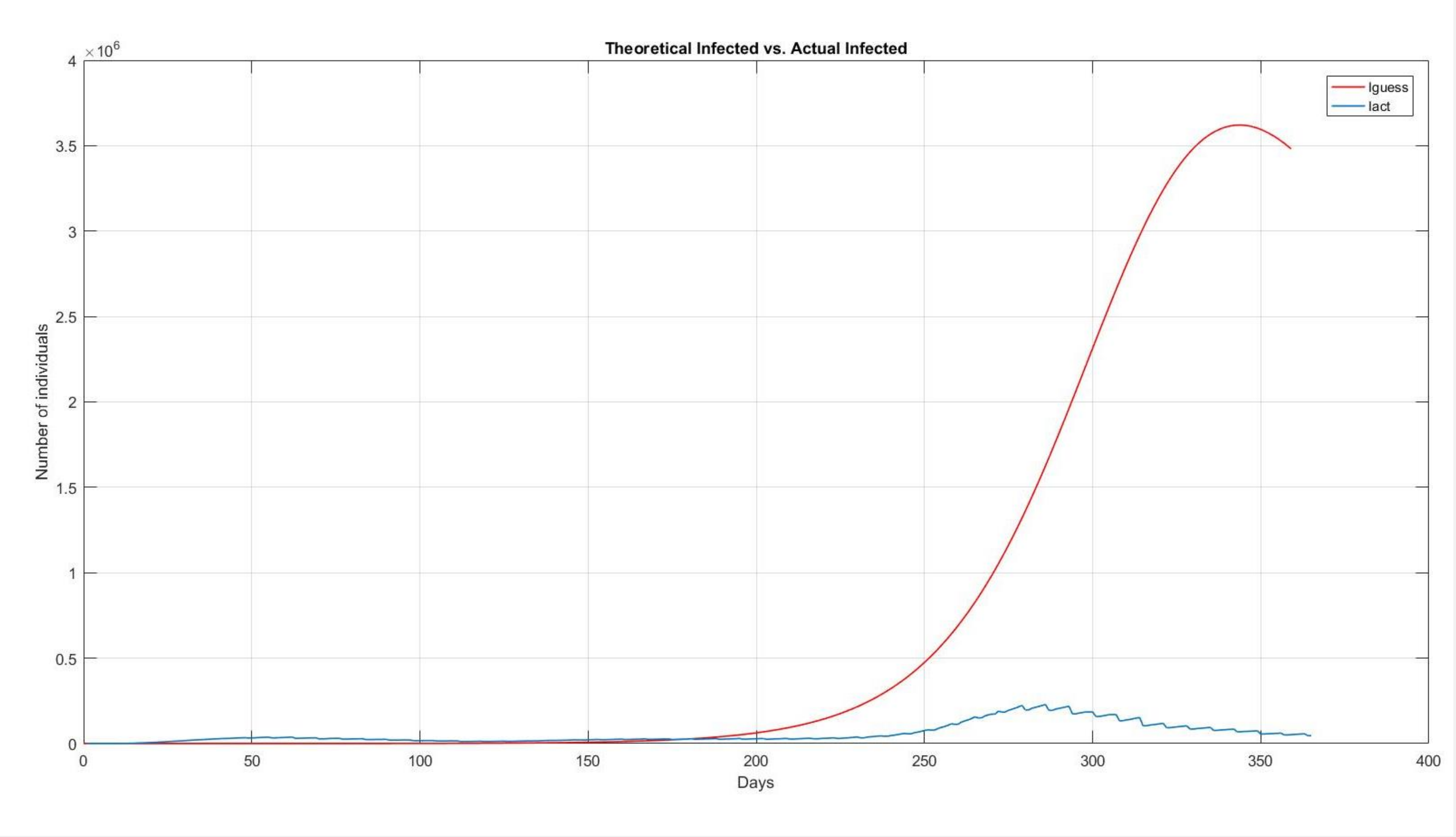

Figure 3: SIR estimation based off predicted COMSOL rates in comparison to active COVID-19 cases in Michigan

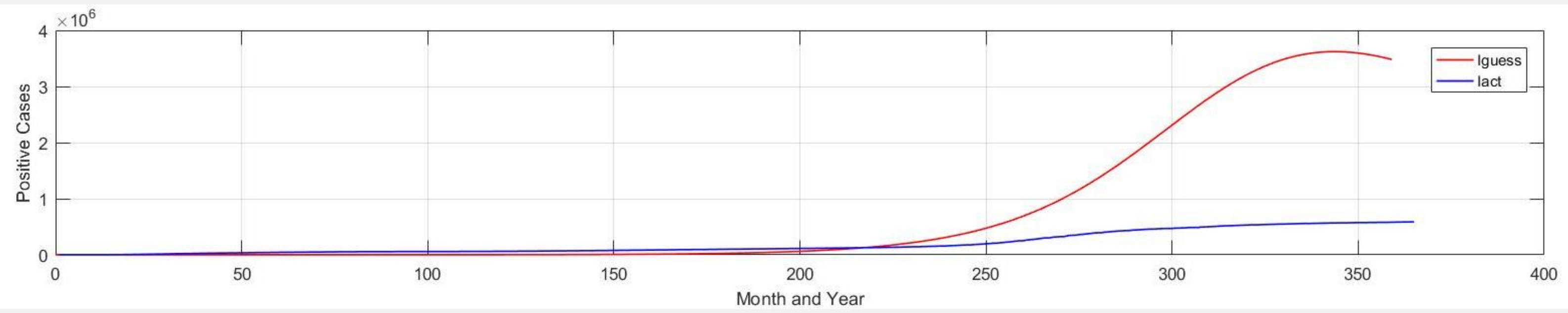

Figure 4: SIR estimation based off predicted COMSOL rates in comparison to all total COVID-19 cases in Michigan

Results

As previously stated, infection rate (β) was found to be 0.6x10^-8, and the rate of recovery (ϒ), was 0.0173, according to this research.

Furthermore, the second graph indicates that the preventative measures taken by the state were valuable to the overall health and wellbeing of the state of Michigan.

Discussion & Limitations

While positive trends are seen in the accuracy of the data gathered, it is difficult to determine the actual value of the data since very little research has been done for specifically the Michigan region, however it is clear to see that quarantine, social distancing, and wearing a mask has considerably mitigated the spread of COVID-19.

There are many limitations to the data. The data shown assumes that the estimated parameters such as contact rate and stay constant throughout the spread of the virus, however this is found to be untrue. Furthermore, the data assumes that a person is unable to be re-infected with COVID-19, which has not been proven true.

In continued iterations of the experiment, additional equations should be added as well as additional parameters. There are developed equations for Quarantine, Social Distancing, etc.

Bibliography

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