Reconstruction and Solvability

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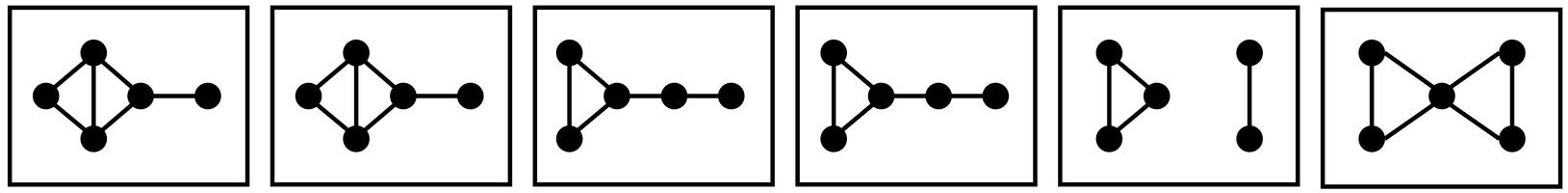
Given a deck of subgraphs of an original graph G where one vertex of G has been removed in each subgraph, can we find G just using this deck?



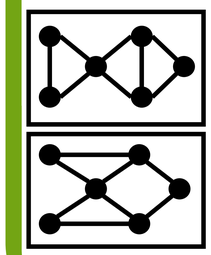
Order (# of Vertices) and Size (# of Edges) of G

The order n is the number of cards. The size m is equal to the sum of sizes m_i of each of the individual subgraphs divided by n-2. This is because each edge is counted in every subgraph except for the two subgraphs where one of their end vertices were removed.

Degree Sequence of G (# of edges coming from each vertex)

The degree sequence is found using m and each m_i. The size of each subgraph is the size of G minus the edges connected to the removed vertex, which is equal to that vertex's degree. The degree sequence will be m-m_i for the m_i correlating to each removed vertex.

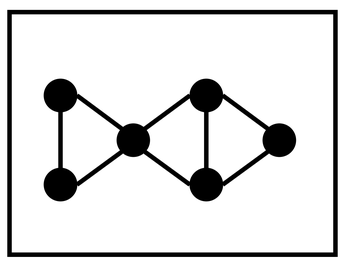
Can we find G from the order, size and degree sequence alone?



No. These two graphs have the same order (6), size (8), and degree sequence (2,2,2,3,3,4), but they are not the same graph. Does the deck give us enough information to find the unique graph G?

Reconstruction Conjecture (1941): Every graph of order 3 or more is reconstructible from its deck of subgraphs

Original Graph G



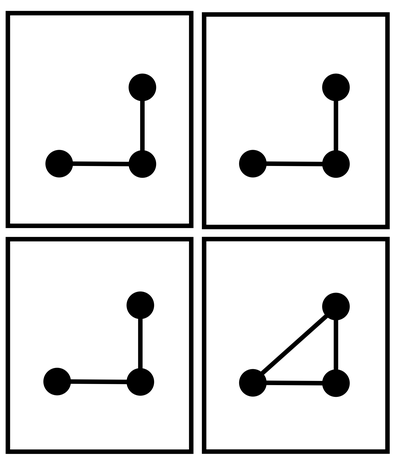
The original graph is constructed using a combination of the knowledge of the order, size, degree sequence, and properties shown in the deck.

Sources

Chartrand, Gary, and Ping Zhang. *A First Course in Graph Theory*. Dover Publications, 2012.

Kratsch, Dieter, and Lane A. Hemaspaandra. "On the Complexity of Graph Reconstruction." Mathematical Systems Theory, vol. 27, no. 3, 1994, pp. 257-273.

A Fake Deck and Identifying It



A fake deck is a deck of cards that is unable to be used in reconstruction because it is either an incomplete or incorrect set of the subsets of G. It is easiest to identify a fake deck if there is a discrepancy in the expected order, size, or degree sequence of G. Looking at the deck to the left, all of the cards point to an order of 4 vertices. We would expect the size to be $\frac{2+2+2+3}{4-2} = \frac{9}{2}$, but size must be an integer. Therefore there is not an integer size and we know that there is no original graph G that this deck of subgraphs was constructed from. Therefore it is a fake deck and cannot be used in reconstruction.