COSMOLOGY AND THE HIGGS MECHANISM

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ABSTRACT

It is noted that spontaneous symmetry treaking implies a finite cosmological term in the Einstein equation for gravity. The present theories of weak and e.m. interactions disagree violently with the experimental limit on such a term unless an ad-hoc counter curvature is introduced.

1. Spontaneous symmetry breaking is an essential ingredient in gauge theories of elementary particle interactions. Up to now, the breaking is achieved through the introduction of scala'r particles, that subsequently develop a non-vanishing and finite vacuum expectation value. Many physicists feel nowadays the necessity for these Higgs bosons as a cumbersome and not very appealing burden; in this note we wish to point out another feature of this mechanism. The effect discussed here occurs already in the tree approximation, and in this sense it is a classical effect, quite distinct from zero point energy shifts resulting from quantum corrections. Here we want to state explicitly that there is no logical difficulty regarding the now circulating models. However, more ambitious theories of nature, that also include gravitation as part of the symmetry scheme may experience great difficulties.

Briefly stated, the following happens. In case of a spontaneously broken symmetry the vacuum becomes a state in which a certain finite amount of energy-momentum density is smeared over all space. This may be sensed by gravitation, and we may expect a curvature of space-time due to this. If we assume that, before symmetry breaking, space time is approximately euclidean then after symmetry breaking to the amount needed to generate the appropriate masses for the vector mesons of weak interactions, a curvature of finite but outrageous proportions result. The reason that no logical difficulty arises is that one can assume that space-time was outrageously "counter curved" before symmetry breaking occurred. And by accident both effects compensate so precisely as to give the (in this context) very euclidean universe as observed in nature. Actually, in the context of an exploding universe, very entertaining speculations can be made, but we will refrain from that.

2. As a simple model we consider the Lagrangian of gravitation and a scalar complex isospinor field K[2] in interaction with themselves and each other;

$$\mathcal{L} = \sqrt{g}\ \{-\frac{1}{\kappa^2} R - g^{\mu\nu}\ \partial_\mu K^+\ \partial_\nu K - \mu K^+ K - \frac{1}{2}\lambda\ (K^+K)^2\}\ .$$

The K-field is like the Higgs field in the Weinberg model. The gravitational field $h_{\mu\nu}$ enters through the tensor $g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu}$. Further g denotes the determinant of $g_{\mu\nu}$, and $g^{\mu\nu}$ is the inverse of $g_{\mu\nu}$. To first order in κ, $g^{\mu\nu} = \delta_{\mu\nu} - \kappa h_{\mu\nu}$. The constant κ is related to Newton's constant G as expressed by the equation $\kappa^2 = 16\pi G$. If units are such that $\hbar = c = 1$ we find $\kappa = 5.8 \times 10^{-22}\ MeV^{-1}$ from $G = (6.6T3 \pm 0.003) \times 10^{-8}\ cm^3/g\ sec^2$. Tofirst order in κ the equation determining the vacuum expectation value of the field K is as usual:

$$\langle K \rangle_0 = \begin{pmatrix} F \\ 0 \end{pmatrix}, \qquad F = \sqrt{-\frac{\mu}{\lambda}}\ .$$

If we substitute

$$K = \frac{1}{\sqrt{2}} \quad (Z + \sqrt{2}\, F + i\psi^a \tau^a) \begin{pmatrix} 1 \\ 0 \end{pmatrix} ,$$

we obtain the Lagrangian

$$\mathcal{L} = \sqrt{g} \; \{ - \frac{1}{\kappa^2} R + C + \mathcal{L}(Z,\psi) \} ,$$

where $\mathcal{L}(Z,\psi)$ contains terms involving the fields Z and ψ. The constant C is determined by the parameters μ and λ. Expressing C in terms of $M^2 = -f^2 \mu/2\lambda$ and $m^2 = 4 M^2 \lambda/f^2$ we find

$$C = \frac{1}{2} \frac{\mu^2}{\lambda} = M^2 m^2/2f^2 \quad .$$

The constant C has the dimension of a $(\text{mass})^4$, and within the context of weak interaction theories its magnitude, give or take a few orders of magnitude may be expected to be of the order $(50\,\text{GeV})^4$ for the purposes of weak interactions. In fact, in those cases one has M = vector boson mass, m = Higgs particle mass and f = weak interaction coupling constant.

However, there are strong experimental limits on the magnitude of C. If the equations of motion corresponding to the above Lagrangian are written down we find:

$$R^{\mu}_{\nu} - \frac{1}{2} R \; \delta^{\mu}_{\nu} + \frac{1}{2} \kappa^2 \; C \; \delta^{\mu}_{\nu} = \kappa^2 \; T^{\mu}_{\nu} \; ,$$

where T^{μ}_{ν} contains the ψ and Z fields.This is the Einstein equation with a cosmological term. Cosmological considerations yield the limit

$$\kappa^2 \, C < 10^{-57} \text{ cm}^{-2} .$$

This results in:

$$C < 0.23 \times 10^{-35} \text{ MeV} = (1.23 \times 10^{-9} \text{ MeV})^4 ,$$

which is clearly in violent disagreement with the number quoted above ,

The discrepancy can be removed by introducing a term $- C \sqrt{g}$ in the Lagrangian. This ad hoc removal of a finite effect to suit experiment is in our opinion not very satisfactory.

3» To provide for a little perspective we want to exhibit more explicitly the way such symmetry breaking acts to generate a vector meson mass. In the absence of symmetry breaking, the equation of motion for a vector meson field W_μ is:

$$\partial_\mu F_{\mu\nu} \equiv \partial_\mu (\partial_\mu W_\nu - \partial_\nu W_\mu) = j_\nu .$$

The spontaneous symmetry breaking gives rise, in j_ν, to a term of the form $M^2 W_\nu$, and the equation becomes the equation of motion for a massive vector field:

$$\partial_\mu F_{\mu\nu} - M^2 W_\nu = j'_\nu \cdot$$

This is quite analogous to what happens to the Einstein equations of gravity. However, in the vector-meson case the gauge symmetry of the theory forbids the introduction of an ad hoc counter mass term. But in itself, if this mass generation mechanism is taken seriously it is quite illogical not to take it serious with respect to gravitation.

4. Some further consequences of spontaneous symmetry breaking occur if the improved energy-momentum tensor is employed. This amounts to the introduction of a term $1/6 \sqrt{g}\, R\, K^+ K$ in the Lagrangian. After spontaneous breakdown a term $\sqrt{g}\, R$ and a term $\sqrt{g}\, R\, Z$ appear. They are however too small to be of any direct consequence.

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References

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