Mathematics education for low-achieving students
Effects of different instructional principles on multiplication learning

Rekenwiskunde onderwijs aan zwak presterende leerlingen
Effecten van verschillende instructieprincipes op het leren vermenigvuldigen

(met een samenvatting in het Nederlands)

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Woord vooraf

Hoe leren kinderen eigenlijk rekenen? Het is een lastige vraag, omdat het moeilijk is te achterhalen wat er in het hoofd van een ander omgaat. De voorkant van dit proefschrift illustreert dit. Daar is te zien hoe een meisje van 9 jaar de toetsopgave “Eén konijn kost 15 gulden. Hoeveel kosten drie konijnen samen?” heeft opgelost. Als je drie konijnen tekent, weet je nog steeds het antwoord niet. Maar het is wellicht wel een handige manier om te begrijpen dat het hier om '3 x 15' gaat. Maar hoe dit meisje vervolgens op 45 uit is gekomen, blijft toch onduidelijk. Heeft zij bijvoorbeeld eerst 5 + 5 + 5 = 15 en vervolgens 15 + 30 (3 x 10 =) gedaan? Eén van de lastigste en meest tijdrovende taken van dit onderzoek was om van alle leerlingen te achterhalen op welke manier zij de toetsopgaven gemaakt hadden. Deze en andere resultaten van mijn promotieonderzoek naar de effecten van verschillende instructievormen op de manier waar-op kinderen leren vermenigvuldigen, worden in dit proefschrift gepresenteerd.

Aan dit onderzoek hebben vele leerlingen en leerkrachten van verschillende scholen meegewerkt: de Klimop-school te Almere, de Koningin Wilhelminaschool, de Prins Willem Alexanderschool en de Wingerd te Amersfoort, de Annie M.G. Schmidt school en de Witte School te Arnhem, de Spreekhoorn te Breda, de Klim-op en de Piramide te Capelle a/d IJssel, de Kaardebol en de Prins Willem Alexanderschool te Culemborg, De Ark te Den Haag, het Coolsma Instituut te Driebergen, de Franciscusschool en de Koningin Beatrix school te Ede, de Calvijnschool te Ederveen, de Koningin Julianaschool en de Prins Bernhardschool te Ermelo, Het Spectrum, De Bijenkorf en de Koningin Wilhelmi-naschool te Gouda, de Pollenhof te Hedel, de C. Hummelingschool te Hilversum, de Wissel te Houten, de Hemrik-school te Kamerik, de Morgenster en de Arend te Nunspeet, de Dr. F. Hogewindschool, de Nelson Mandelaschool en de Valentijnschool te Rotterdam, de Anne Frankschool, de Alfonso Corti School, de dr. C. Brouwerschool, de Jan Nieu-wen-huyzenschool, de Koekoek, de Kohnstammschool, de Koppeling, de Nieuwe Regen-tesseschool, de Openbare Tuindorpschool, de Sint Maartenschool en de Taalkring te Utrecht, de Achtbaan te Veenendaal, de Wiekslag te Vianen, de dr. Hordijkschool te Waalwijk, de Driehoek te Wijk bij Duurstede, de Keerkring te Woerden, de Brug en de Openba-re Montessorischool te Zeist en de Sluis te Zwolle. Dankzij hun inzet en medewerking was het mogelijk om dit onder-werp op deze manier te bestuderen. Ik ben hen daarvoor dan ook erg dankbaar.

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Ede, augustus 2002

Contents

Chapter 1
Introduction and theoretical background

1.1 Introduction

This thesis is concerned with mathematics instruction for students having difficulties with the learning of mathematics in general and multiplication in particular. The focus lies on the kind of instruction that these students need to adequately master the mathematics skills required by the elementary school curriculum. One of the most important questions in this light is whether students with difficulties learning mathematics actually benefit from the instruction used for their normal-achieving peers or perhaps require another form of instruction specifically adapted to their needs. In this thesis, the results of a research project devoted to this question are reported. More specifically, the question of whether guided instruction (generally viewed as adequate for use with normal-achieving students) is just as effective as the use of directed instruction (generally advised for low achievers) was investigated.

The research project reported on here is part of a national research program (Van Lieshout, 1997) aimed at optimizing the teaching and learning of arithmetic knowledge. More specifically, the study was initiated to investigate the possibilities and to optimize the use of Realistic Mathematics Education (RME), which is a form of instruction that has introduced a number of changes in the mathematics curriculum in The Netherlands, such as a more important role for the students' own contributions (Streefland, 1990; Treffers, 1991). One of the central questions in this research program is whether mathematics instruction based on students' own contributions (i.e., guided instruction), is always better than clearly structured instruction (i.e., directed instruction). This question is particularly important when it comes to the education of students with limited cognitive capacities. Given that such students often have special needs, a form of instruction other than the form of instruction used with normal-performing students may be called for (Carnine, 1997; Mercer, 1997; Miller & Mercer, 1997). That is, findings showing RME to benefit the student population in general cannot be automatically generalized to special populations.

The research reported in this thesis addresses the effectiveness of changing forms of mathematics instruction for use with a special population of students, namely those students who have difficulties learning mathematics. Recent changes have occurred in the mathematics curriculum, and it is therefore important to know what the effects of these changes are on the learning and performance of all students, including such special populations as students with difficulties learning mathematics. The effects of these changes are investigated in an intervention study concerned with the effects of the use of two different instructional principles for the teaching of multiplication skills.

In the present chapter, the relevant theoretical background will first be briefly reviewed. Thereafter, the specific research questions guiding the research project will be presented and the contributions of the various studies described in the different chapters to answering the research questions briefly outlined.

1.2 Theoretical background

In this section, the theoretical background to the present research project will be briefly reviewed. An overview of the most important theories of learning and instruction will first be provided. This overview may provide insight into the selection of instructional forms for examination in the present research project. Thereafter, the population of students participating in the present project will be described, namely students encountering difficulties with the learning of mathematics. Finally, the current practices with regard to mathematics education in the Netherlands will be described in order to understand how students are expected to learn the various aspects of mathematics and basic multiplication in particular and just how these current practices relate to the learning of students encountering difficulties with learning mathematics.

1.2.1 Learning and instruction

One of the first issues calling for review is the question of just how students learn and what the role of instruction is in this process. Several theories of learning have been put forth over the years in the psychological and educational research literature, and any consideration of instruction cannot ignore such theories because assumptions about how students learn will always influence the manner in which they are taught and, conversely, just how students are taught will always influence the manner in which they learn. For this reason, an overview of the most important learning theories will be presented below.

The different theories of learning distinguish themselves on the basis of the assumptions made about the most important factors in learning. Behavioral theories emphasize the role of the environment and just how events in the environment influence behavior. Cognitivist theories emphasize the role of mental processes in learning and perception. Interactionist perspectives take both the environment and mental processes as important. Interactionists assume that the environment, mental processes, and behavior all interact during the process of learning (Gredler, 2001). The behavioral and cognitive frameworks constitute the major paradigms for studying the phenomenon of human learning (Mercer, 1997).

The behavioral approach

The behavioral approach is based on the premise that the environment greatly influences behavior (Mercer, 1997). According to behaviorists, moreover, the goal of science is to discover the lawful relations between environmental events and behavior (Bredo, 1997). In this approach, learning is also viewed as the shaping of behavioral repertoires. For learning in the classroom, carefully selected stimuli and reinforcers are therefore called for. And because the teacher cannot provide the individual reinforcement needed for all students, computers and other modern technologies are seen as essential.

In the behaviorist approach, the two most important factors in education are student behavior (learner characteristics) and those stimuli that lead to behavioral change (instructional characteristics). Important learner characteristics are individual differences, readiness for learning, and motivation. Behaviorists recognize the existence of several different stages of learning (Mercer, 1997): acquisition, proficiency, maintenance, generalization, and adaptation. Given the behaviorist's emphasis on the environment as a critical factor for learning, considerable emphasis is also placed on the teacher's arrangement of the classroom for optimal learning. And a failure to arrange the classroom in an optimal manner is considered a major cause of learning difficulties. In other words, both learner characteristics and teacher characteristics are considered responsible for the learning process within the be-havioral approach.

One of the essential components of the behavioral approach to learning is direct instruction. The key principle underlying direct instruction is that both the curriculum materials and the teacher presentation of these materials must be very clear and unambiguous (Mercer, 1997). This includes an explicit step-by-step strategy, development of mastery at each step in the learning process, strategy corrections for students errors, gradual fading of teacher-directed activities and increased independent work, use of systematic practice with an adequate range of examples, and cumulative review of newly learned concepts. Task analysis is used to determine the sequence of skills to be taught.

The cognitive approach

The cognitive approach involves the study of the human mind, and developed a model for how people receive, process, and recall information (Bredo, 1997). At present, numerous cognitive theories and considerable research are concerned with the process of human problem solving and strategies for the management of learning. The main theory within the cognitive approach is the information-processing theory, which construes learning as the process of obtaining, coding, and remembering information (Gredler, 2001). Although different perspectives on the exact nature of human memory exist, a multilevel conceptualization is currently the predominant perspective. Based on a computer metaphor, it is assumed that information is processed in sequential stages and that these sequential stages are linked to different memory systems: the various sensory registers, short-term memory, working memory, and long-term memory. The according processes that take place are perception, encoding, storage, and retrieval.

Within the cognitive approach, two important concepts are metacognition and problem solving. Metacognition involves thinking about thinking. The main components are knowledge of cognition and regulation of cognition. Knowledge of cognition involves knowledge and awareness of one's own thinking along with knowledge of when and where to use specific strategies. Regulation of cognition involves the planning, evaluation, and monitoring of cognition. Problem solving is a broad term but generally defined as the handling of new and unfamiliar tasks when the relevant solution methods are not known (Bottge, 2001).

The focus of most cognitive approaches to learning is on instruction consistent with how students actually think during specific learning tasks. What happens to the learner internally during the learning process is considered just as important as what happens to the learner externally (Mercer, 1997). The teacher and teaching materials are only considered important insofar as they help students construct new meanings. In keeping with the main information-processing approach, two general instructional paradigms can be distinguished: reductionism and constructivism (although constructivism is also sometimes described as a third approach to learning in addition to the behavioral and cognitive approaches; Bredo, 1997). Reductionism is based on the premise that new concepts must be analyzed and reduced to simpler, smaller, and more understandable components in order to be learned. An example of reductionism is cognitive behavior modification and the self-instructional program. The idea is that cognitions influence behavior and that behavior can thus be changed by modifying cognitions. The constructivist approach to learning constitutes a more purely cognitive approach.

In the past two decades, increasingly greater emphasis has been placed on the active role of the learner. Constructivism views knowledge as a human construction, and thus subjective as opposed to part of an objective, external reality (Cobb, 1994). Constructivism also underlies an instructional ideology that assumes students to be naturally active learners who are constantly constructing new and highly personalized knowledge with the establishment of links between their prior knowledge and new information (Mercer, Jordan, & Miller, 1996). A distinction can be made between radical constructivism and social constructivism. Radical constructivism is based on Piaget's theory of cognitive development with the exception that 1) knowledge is viewed as a human construction and not, as in Piagetian theory, as an external reality and 2) the focus of radical constructivism is specifically on the learning of school tasks and not the development of logical thinking in general. Radical constructivism construes learning as an internal process occurring in the mind of the individual, and radical constructivists thus call for problem-based learning as opposed to teacher-directed instruction (Gredler, 2001). In keeping with Vygotsky's discussions of knowledge as constructed by societies or cultures, social constructivists construe the classroom as a community that develops its own knowledge (Gravemeijer, 1997).

Constructivists do not agree on the nature of teacher-student interactions. A distinction can be made between endogenous and exogenous constructivism with a continuum of positions occurring in between. The endogenous constructivists think that instruction should be structured to help students discover new knowledge without explicit instruction (Mercer, 1997). Exogenous constructivists think that teachers should engage students by providing explicit instruction via the provision of descriptions, explanations, modeling, and guided practice with feedback. An expanded interpretation of constructivism to include both explicit and implicit instruction is receiving growing acceptance, particularly for the instruction of students with learning difficulties (Mercer et al., 1996).

In this section on learning and instruction, two main approaches were reviewed: beha-viorism and cognitivism. The current mathematics curricula appear to be based on largely cognitivist views. Information processing theory provides a good framework for the understanding of learning. And the majority of the recent mathematics instruction programs and textbooks are based on constructivist principles. However, the application of distinct theories within the educational setting is not always clear. Many psychologists and educators have merged elements from the different theories to create new educational programs. For example, cognitive-based instruction may be supplemented with direct-instruction elements. Or a teacher utilizing a basically constructivist curriculum may consider the use of a reductionist self-instruction approach to be particularly effective for the acquisition of a specific part of the curriculum with a particular group of students. In addition, it is in-creasingly being recognized that no single instructional approach is suited for the education of all students. Given that learning clearly depends on a number of different factors in-cluding the individual learner and his or her needs but also such factors as the time and place of learning, instruction should always be adapted to the particular learner and particular situation. Different approaches could thus be adopted and different elements combined to meet the particular needs of the students involved in a particular learning situation.

1.2.2 Students with difficulties learning mathematics

Given that the present thesis is concerned with students having difficulties with the learning of mathematics, the population will be described more thoroughly at this point. Before the presentation of some general characteristics of the population, a few remarks will first be made about the definition of the group. Students with serious learning problems are often referred to as students with learning disabilities or difficulties (ld) or students with specific learning disabilities (sld) in the case of students with specific spelling, reading, and/or mathematics problems. However, considerable controversy surrounds the definition and diagnosis of ld. One of the controversies concerns the role of intelligence. Disagreement exists about the students who perform very low in reading or mathematics and also have low IQ scores. When ld is defined as a discrepancy between a child's cognitive abilities and his actual performance, such students are not diagnosed as ld but as mild mentally retarded (mmr). Some have therefore argued that it is simply more practical to define ld as performing below a particular level for a particular subject (i.e., students who score below the 25^{th} percentile on a criterion-based math test are diagnosed as having a math learning disability; Siegel, 1999). Exclusion criteria such as serious attention or behavioral disorders should still be included, however (Kaufman & Kaufman, 2001).

The focus of the present thesis is on all students encountering difficulties with the learning of mathematics, which thus encompasses students with learning disabilities but also students with mild mental retardation and students performing below the norm without a specific disability. Although the group is quite varied, the students all have low mathematics performance and thus a need for special instruction in common (Kavale & Forness, 1992). The students in this group will therefore be referred to as *students with difficulties learning math* or *low-performing students* and defined as students performing below the 25^{th} percentile on a criterion-based math test. The terms *learning disability* and *mild mental retardation* will only be used to refer to those students explicitly diagnosed as such by professionals using the official criteria.

General characteristics

The group of students with difficulties learning math is very heterogeneous. Although the students may have difficulties with only mathematics, many of them have other difficulties as well. It is also possible that difficulties with reading, language, and writing may negatively influence the students' math performance (Mercer, 1997). Despite this diversity, it is nevertheless possible to present some general characteristics of students who have difficulties learning math.

First, students who have difficulties learning math often show memory deficits (Rivera, 1997) and particularly problems with the storage of information in long-term memory and the retrieval of such information (Geary, Brown, & Samaranayake, 1991). These same students show greater difficulties than their peers with the automatized mastery of such basic facts as addition up to 20 or the multiplication tables. As a result, they tend to make more mistakes on tests of basic skills and often have to calculate the answers that others know directly (Pellegrino & Goldman, 1987). Given that clear mastery of such basic facts is needed for further math performance, deficits in this area can certainly influence students' later math performance and their mastery of the remainder of the math curriculum.

A second characteristic is that students with difficulties learning math often show inadequate use of strategies to compute answers or solve word problems. This can be explained at least in part by the aforementioned memory deficits, which produce slower development of the relevant strategies than in normal achieving students (Rivera, 1997). Moreover, students with such delays may also fail to apply the strategies they have learned in an adequate manner. Inadequate strategy use may also be caused by metacognitive deficits (Goldman, 1989). Metacognitive knowledge and skills are clearly necessary for the identification, selection, and application of appropriate problem-solving strategies.

A third general characteristic of students with difficulties learning math is deficits in other metacognitive regulation processes such as the organization, monitoring, and evaluation of information (Mercer, 1997). As a result of these deficits, the students often produce mistakes showing the incorrect application of solution. Such students also remain unaware of their mistakes because they do not attempt to evaluate the procedures they apply as good problem solvers have been found to. An additional characteristic is that such students show deficits in the generalization and transfer of information and/or problem-solving strategies. They have difficulties with connecting different tasks and with the application of already acquired knowledge and skills to new or different tasks (Goldman, 1989).

In addition to these general (meta-)cognitive characteristics of students with difficulties learning math, they frequently have other problems such as attention deficits or motivational problems. Various aspects of motivation can be distinguished as particularly important for learning. One aspect is the role of attributions or the explanations that students provide for their successes and failures. Students with an internal locus of control tend to explain the outcomes of particular actions on the basis of their own abilities and effort. In contrast, students with an external locus of control tend to think that factors outside their control (such as luck or task difficulty) determine their results (Mercer, 1997). Students with learning difficulties are more likely than normally achieving students to attribute their successes to external factors. Students with difficulties learning math obviously have a history of academic failure, which may also result in a lack of confidence with regard to their intellectual abilities and doubts about anything that might help them perform better. This situation can lead to marked passivity in the domain of math and possibly other domains of learning. A second aspect of motivation that has been found to be important for learning is the type of goal orientation. A distinction can be made between goals focused on mastery of a task (i.e., a task orientation) and goals focused on performance (i.e., an ego orientation; Nicholls, 1984). Research has shown task-oriented students to generally do more their best than ego-oriented students (Ames & Ames, 1989). A third important aspect of motivation for learning is the self-concept of the individual involved. Just how a student perceives his or her abilities can influence his or her learning. Students with a poor self-concept may lack sufficient self-confidence and resist academic work due to a fear of failure (Mercer & Mercer, 1998).

Instructional needs

Students with difficulties learning math require special attention and instruction adapted to their specific needs. Given the heterogeneity of this group of students, their educational needs are likely to be quite diverse. Nevertheless, many educators argue that most of the students with mathematical difficulties (including students with learning disabilities and mild mental retardation) have more or less the same educational needs as their learning patterns do not differ qualitatively from each other (Kavale & Forness, 1992; Van Lieshout, Jaspers, & Landewé, 1994). It is thus recognized here that students may differ in their educational needs but still have a lot in common. In keeping with the general characteristics described above, a number of general educational needs can thus be identified and seen to reflect those areas in which the students encounter the most difficulties: automaticity, strategy use, and metacognitive skills (Rivera, 1997). Nevertheless, when planning the instruction for a student with special needs, one should always take the specific individual needs of the student into consideration as well.

In addition to the possible heterogeneity of instructional needs highlighted above, there is no complete agreement on the type of instruction that students with difficulties learning math may need. One's vision of learning and instruction basically determines the type of instruction one considers suitable. For instance, a behaviorist will focus on the student's environment and probably attempt to change the behavior of the teacher or the instructional program when a student does not perform adequately. A cognitivist, in contrast, will concentrate on the information-processing capacities of the student and probably attempt to improve the student's strategy use.

Nevertheless, the general characteristics of the students with difficulties learning math point to the type of instruction needed. One of the major problems confronting these students is attaining automaticity. Special attention should therefore be devoted to the automatization of basic facts. Automaticity can be attained by practicing the skill in question. This means that such students will need extra time and possibilities to practice. In addition, such students must learn how to proceed when they do not know an answer directly or, in other words, to apply backup strategies (Lemaire & Siegler, 1995). For instance, when a student does not automatically know the answer to 6 x 4, it is useful for him or her to know that one simply has to add 4 to his or her knowledge that 5 x 4 = 20 or that one can double 3 x 4 = 12 to attain the answer.

A second major problem confronting students with difficulties learning math is that many of them show deficits in the adequate use of strategies. For adequate strategy use, students must have an adequate repertoire of strategies (strategy acquisition) and also know just how and when to apply the various strategies (strategy application). In general, elementary school students with difficulties learning math rely more heavily on counting strategies than normally achieving students (Pellegrino & Goldman, 1987). An adequate repertoire of math strategies can be built in several ways; it should be noted, however, that students with difficulties learning math do not have an exhaustive repertoire of strategies and that teaching them only a limited (but effective) number of strategies may be sufficient (Jones, Wilson, &

Bhojwani, 1997). The acquisition of many different strategies may only lead to confusion. It is often recommended to make the relevant strategies explicit and clear and that the different steps in the strategy must be made as overt and explicit as possible (Carnine, 1997). Such instruction gives students the opportunity to learn the different strategies on a step-by-step basis. However, it is also possible to use more cognitively-oriented instruction, such as self-instruction procedures. Although positive results have been found with self-instruction procedures, many theorists think that increased teacher assistance and special forms of instructional scaffolding are needed for students with difficulties learning math (Carnine, 1997). In scaffolded instruction, the teacher gradually withdraws as the student becomes more competent and confident. Such scaffolding can be combined with the Concrete-Semiconcrete-Abstract (CSA) sequence, and many researchers and teachers believe that this is an excellent manner of instruction for students with problems understanding math concepts, operations, and applications (Mercer & Mercer, 1998). From a constructivist perspective, a repertoire of strategies can be built via exposure to and practice with different problems; the students are not told how to solve the problems and must therefore discover which strategies to use in discussion with other students. In such a manner, students learn from their own experiences. Such instruction is rarely recommended for students with difficulties learning math, however (Kroesbergen & Van Luit, in press).

In order to select and apply different strategies, students must have considerable metacognitive knowledge and skills. Students with difficulties learning math often need extra help with the selection of the appropriate strategies, monitoring the application of a particular strategy, and evaluation of the effectiveness of the chosen strategy. Given that they typically do not do this on their own, explicit instruction is often recommended to teach students effective strategy use (Jones et al., 1997). According to this view, students with learning difficulties can also be expected to generalize knowledge and skills to situations outside the instructional setting less effectively and less frequently than other students when not explicitly instructed. This observation may be one of the reasons for the increased popularity of the constructivist approach to learning as this approach encourages students to participate in instructional activities that enable them to construct their own knowledge of various concepts, skills, and hierarchies of such. There is still too little research along these lines to state what the effects of such an instructional approach may be for students with difficulties (Kroesbergen & Van Luit, in press).

To conclude, students with difficulties learning math need special attention for the acquisition and automatization of basic math facts and the mastery of various cognitive and metacognitive strategies. Direct instruction is one of the most popular methods of instruction employed with these students (Bottge, 2001). And, indeed, research consistently shows carefully constructed explicit instruction to be very effective for students with math learning difficulties (Carnine, 1997). The majority of constructivists also appear to be shifting towards the explicit end of the implicit-explicit instruction continuum for students with difficulties learning math (Mercer & Mercer, 1998). Of course, adequate learning only takes place when the students are sufficiently motivated to learn. Attitudes, beliefs, and motivation thus play an important role in the learning of mathematics (Mercer, 1997), and fostering motivation and a positive attitude towards mathematics is just as important as the other instructional procedures reviewed above.

1.2.3 Learning mathematics

In this section, how mathematics is currently taught and particularly how multiplication is taught will be briefly reviewed. First, the mathematics curriculum will be described along with the place of multiplication instruction within the curriculum. Second, the current teaching methods will be briefly described. And third, a few remarks will be made with regard to low performers and special mathematics instruction.

The mathematics curriculum

The mathematics curriculum starts with the development of number sense in kindergarten and first grade. At this time, a basic understanding of the arithmetic operations is to be established (Correa, Nunes, & Bryant, 1999). Formal math instruction usually begins with addition and subtraction, and then proceeds to multiplication and division before such more advanced skills as fractions, decimals, and percentages are taught. Mathematics has a logical structure, which means that the mastery of lower-level math skills is essential for learning higher-order skills. The concept of learning readiness is thus important for math instruction as well (Mercer, 1997).

When formal math instruction begins, students must master the different operations and basic axioms in order to acquire basic computational and problem-solving skills. The most important axioms are the commutative property of addition, the commutative property of multiplication, the associative property of addition and multiplication, the distributive property of multiplication over addition, and the inverse operations for addition and multiplication (Mercer, 1997). Two different goals can be distinguished within the mathematics curriculum: the automatization of basic skills and the development of sufficient flexibility and adaptability for adequate problem solving (Goldman, 1989). In the present study, the focus lies on the learning of multiplication skills requiring both automaticity and problem solving. As mentioned above, multiplication instruction can start once students have mastered the basic addition and subtraction skills. Normally, multiplication instruction is initiated in second grade.

The basis for multiplication is understanding that it involves repeated addition. When students understand the basic axioms for multiplication, they can move on to other strategies that can then improve their multiplication skills. The ultimate goal of basic multiplication instruction is for the students to have memorized the multiplication facts up to 10 x 10. The instruction typically begins with the simpler tables, such as the tables for 1, 2, 5, and 10. The tables for 5 and 10 are particularly important because they are often used to solve other problems. Thereafter, students must learn to apply the different multiplication strategies to solve unknown multiplication problems.

According to Vygotsky's cultural-historical theory, the learning process develops in several stages from material to mental actions (Bredo, 1997). For this reason, it is generally recommended that early math instruction follow three sequential stages: concrete (e.g., sticks, blocks), semi-concrete (e.g., pictures, number lines), and abstract (e.g., numerals, symbols). This sequence can also be followed to help students acquire an understanding of basic multiplication. For instance, a teacher can demonstrate using concrete materials (e.g., blocks or chair legs) that 3 x 4 is the same as 4 + 4 + 4. In keeping with a constructivist approach to learning, concrete materials can help students discover this math relation on their own. In other words, students should be encouraged to explore, discover, and summarize their own representations of problems. In fact, such self-invented models clearly fit within the semi-concrete stage of instruction. When students understand the basic axioms of multiplication, they can continue with the automatization and discovery of different solution strategies.

Strategy instruction can also be provided in several different ways and thus in keeping with the different theories of learning. However, it is always very difficult to know how people decide what to do (i.e., which strategy to use; Siegler, 1988). Siegler has proposed a model, the distributions of associations model, based on the assumption that people make at least some strategy choices without reference to explicit knowledge. Within this model, individual problems and their answers are represented in an associative network. In order to find the answer to a problem, then, three sequential phases can be followed: retrieval, elaboration of the representation, and application of an algorithm. Retrieval appears to be innate to human beings. When students do not know the answer to a problem directly, they must then rely on elaborations of the representations and the application of solution algorithms that appear to be acquired largely via direct instruction (Siegler, 1988). Nevertheless, it is also possible to offer students problems and have them invent their own solution procedures, as suggested by constructivists.

Current practices

Most of the schools in the Netherlands today use a teaching method based on the principles of realistic mathematics education. This practice is in line with the changes that have occurred in other countries, for instance with the reforms in mathematics in the United States. Realistic mathematics education is based upon constructivist theories (Cobb, 1994; Klein, Beishuizen, & Treffers, 1998). Learning mathematics is conceptualized as familiarizing oneself with the ideas and methods of the mathematical community (Gravemeijer, 1997). The classroom is a community that develops its own mathematics. The children in the classroom jointly acquire knowledge. Students work and learn together; discuss possible solutions with each other; and must explain and justify their answers to each other. Using realistic situations or context problems, students' mathematical reasoning is stimulated. Research suggests that even very young students can invent and acquire their own arithmetic routines for the solution of computational problems that, in some cases, can lead to deep understanding (Singer, Kohn, & Resnick, 1997). However, students allowed to proceed entirely on their own could establish misconceptions with regard to certain math concepts and/or algorithms (Bottge, 2001).

Although instruction based on the principles of realistic mathematics education has shown promising results (Cobb et al., 1991; Gravemeijer et al., 1993; Klein et al., 1998), its beneficial value for students with learning difficulties is highly doubted (Klein et al., 1998; Van Zoelen, Houtveen, & Booij, 1997; Woodward & Baxter, 1997). As described above, children with difficulties learning math appear to need more directed instruction than provided within the framework of realistic mathematics education. Special educators thus tend to employ instructional methods based on cognitive behavior modification principles or direct instruction principles. When students appear to need special instruction, moreover, they are often entered into a special program specifically designed to remedy their problems and instruct those who do not appear to learn sufficiently from the regular teaching program. However, the choice for a special program or instructional method is often very difficult and always depends on the student, the task, and the teacher providing the special instruction. In the next section, such special programs will be considered in greater detail.

Low performers and special instruction

Given the special instructional needs of students with difficulties learning mathematics (see section 1.2.2 above), special programs have indeed been developed to help these students where the regular instruction fails. A first step in the remediation of mathematics problems is diagnosing the problem and mapping the specific needs of the student in question. Special instruction can take a number of forms from a few minutes of extra instruction each lesson to a special program specifically designed to replace regular instruction. The form selected for use depends on the needs of the student, the task to be learned, and the resources of the teacher.

Although a variety of recommendations with regard to the instruction of low performers exist, most of them have in common that the provision of clear structure is recommended (e.g., Archer et al., 1989; Carnine, 1997; Mercer & Mercer, 1998; Ruijssenaars, 1992). Both the form and content of the lessons should be clearly structured. With regard to the form of the lessons, it is recommended that the lessons always be built up using the same pattern including an opening phase with reflection on the previous lesson, a brief presentation of the material to be learned, a practice phase with both guided and individual practice, and a closing phase (e.g., Archer et al., 1989; Veenman, 1993). The instructional principles recommended for use with low performers include the modeling of explicit strategies, cumulative introduction of information, isolation of independent pieces of information, separation of confusing elements and terminology, use of a concrete/semi-concrete/abstract sequence, use of explicit-implicit math instruction, emphasis on relations, explicit generalization instruction, building retention, and instructional completeness (e.g., Carnine, 1989; Mercer & Mercer, 1998; Ruijssenaars, 1992). The main difference between regular instruction and special instruction is that nothing is left to chance in the latter (Ruijssenaars, 1992).

In the final part of this section, the MAthematics Strategy Training for Educational Remediation program (MASTER; Van Luit, Kaskens, & Van der Krol, 1993; see also Van Luit & Naglieri, 1999) will be described. This is an example of a remediation program employing many of the aforementioned instructional principles. The program was also used for the experimental intervention undertaken as part of this research project. The MASTER program was specifically designed to encourage strategy utilization for the solution of multiplication and division problems. The program is largely based on information processing theory although elements from other theories of learning have been incorporated where suited. Behavioral elements underlie the conduct of a task analysis and the use of direct instruction, for example. Cognitive elements underlie the information-processing characterizations of the students' learning difficulties, the use of cognitive behavior modification and self-instruction principles to deal with the problems, and the following of a concrete/ semi-concrete/abstract sequence of instruction.

The goals of the MASTER program are to increase student's:

1. understanding of multiplication as repeated addition;
2. understanding of the number system and the premises of such strategies as reversibility, association, and doubling;
3. memorization of the basic multiplication facts up to 100;
4. understanding of division as repeated subtraction;
5. understanding of the relation between division and multiplication;
6. memorization of the basic division facts up to 100;
7. application of multiplication and division facts in both real and imaginable situations.

The program contains 23 lessons in multiplication and 19 lessons in division and is divided into 6 series of lessons. Each series teaches the steps needed to solve the problem or problems related to specific tasks. A series of lessons starts with an orientation phase in which the child is taught to solve the task with the help of materials. In the next phase, the connection is made to a mental solution in keeping with the concrete/semi-concrete/abstract sequence of instruction. Thereafter, the phases of control, shortening, automatization, and generalization are undertaken. The teacher tries to realize the various phases of instruction with the use of self-instruction. For students who need greater assistance, explicit suggestions or repetition of a particular problem-solving strategy by the teacher may be called for. For example, explicit modeling is included in the program. The intensity of the instruction is clearly adapted to the needs of the student and/or the particular task: a brief explanation of the task to be solved is less intense than explicit modeling of the solution of the task, for example. A small repertoire of multiplication and division strategies is explicitly taught to the students as part of the MASTER program. However, the students are also given the opportunity to develop their own strategies and solutions to various problems. The MASTER program has been found to be very effective for the instruction of both students with learning disabilities and students with mild mental retardation (Van Luit & Naglieri, 1999).

1.3 Research questions

The focus of the present research project is on mathematics instruction for low performing students. The central question concerns the type of instruction required by such students. A distinction can be made between instruction that encourages the students to contribute to their own learning (guided instruction) and instruction that is largely directed by the teacher and thus leaves little or no space for students to contribute to their own learning (directed instruction). This distinction reflects the different instructional theories currently being promoted for mathematics education (constructivist-based instruction) and instruction promoted by special educators (behavioral and/or cognitive instruction; see sections 1.2.1 and 1.2.3 above). In the present research, it is attempted to determine which method of instruction appears to be most suited for use with low-performing students. This was done by designing two different types of intervention for the teaching of multiplication to low-performing students. The two interventions involved adaptation of the multiplication part of the MASTER remediation program described in section 1.2.3 above to the different instructional principles. The effectiveness of the interventions was measured using both multiplication and motivation tests. For multiplication, the areas of automaticity, multiplication ability, and strategy use were investigated as these areas have been found to be clear problem areas for low performers (see section 1.2.2 above). The following motivational aspects were measured: goal orientation, beliefs, attributions, and self-concept. The effects of the interventions were also compared to the effects of regular instruction. In addition, the potential effects of such child characteristics as sex, age, ethnicity, and cognitive abilities, were also studied. Specific attention was devoted to the relation between mathematics knowledge and cognitive abilities.

The following research questions were then addressed.

1. Which instructional principle is more effective: guided or directed instruction?
2. How do students acquire strategies during both interventions (guided and directed)?
3. Does special intervention aimed at strategy use appear to be more effective than regular instruction?
4. What are the effects of different child characteristics (sexe, age, ethnicity, IQ, math level, special needs) on the child’s ability to learn multiplication?
5. What is the relation between mathematics knowledge and cognitive abilities?

The above five questions will be answered in the different chapters of this thesis.

1.4 Outline of this thesis

This thesis reports on the effectiveness of two different mathematics interventions for use with low performing students. Before the effectiveness of the interventions is described, the results of a meta-analysis of the current research literature on mathematics interventions are reported in Chapter 2. Articles examining a particular type of mathematics intervention were sought and just which variables appear to make certain interventions particularly effective was then investigated in the meta-analysis.

In Chapter 3, the results of an initial pilot study are presented. In this study, the effects of guided versus directed instruction were examined (questions 1 and 3). The aim of the pilot study was to examine the suitability of the research design, the adequateness of the intervention programs, and the appropriateness of the test materials. On the base of the findings and experiences attained in the pilot study, the testing materials were adapted and expanded for further use. In Chapters 4 and 5, the results of the main study are reported. In Chapter 4, the effects of guided versus directed instruction on automaticity, ability, and motivation in the domain of multiplication are described (question 1). The effects of such special instruction are also compared to the effects of regular instruction (question 3). In this chapter, the effects of different child variables are also considered (question 4). In Chapter 5, the processes that appear to underlie the learning fostered by the different forms of intervention are considered in greater depth and detail. It is shown how the students in both of the experimental conditions acquired new strategies and just what the effects of the different forms of instruction on the students' strategy use appeared to be (question 2). In Chapter 6, the issue of how students' cognitive abilities relate to their math learning problems is examined (question 5). And finally, the results of the present research project are integrated and further discussed in Chapter 7.

Given that the present thesis is a compilation of research articles, some overlap between the different chapters is unavoidable. More specifically, the research reported in Chapters 3, 4, and 5 of this thesis involves intervention studies following more or less the same design, which means that the method sections in these chapters and the descriptions of the experimental conditions are highly similar. Another consequence of this form is that the sample sizes in the different chapters could lead to confusion. Although the total sample of the main study consisted of 283 students, the studies described in Chapters 4 and 5 report on 265 participants and Chapter 6 reports on a study with 267 participants. Differences in sample sizes (both of the total sample and of the three conditions separately) are due to missing data. In the analyses for Chapter 4, cases with one or more missing values in one of the four multiplication tests have been deleted. Chapter 5 reports on all students except for those who did not fill in their strategy use. The analyses for Chapter 6 have been conducted with the data of only those students from who the CAS data were completely available.

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Chapter 2
Mathematics interventions for children with special educational needs: A meta-analysis[1]

Abstract

This article presents the results of a meta-analysis of 58 studies of mathematics interventions for elementary students with special educational needs. Interventions in three different domains were selected: preparatory mathematics, basic skills, and problem-solving strategies. The majority of the included studies describe interventions in the domain of basic skills. In general, these interventions were also the most effective. Furthermore, a few specific characteristics were found to influence the outcomes of the studies. In addition to the duration of the intervention, the particular method of intervention proved important: Direct instruction and self-instruction were found to be more effective than mediated instruction. Interventions involving the use of CAI and peer tutoring showed smaller effects than interventions not including such.

[1] Kroesbergen, E.H. & Van Luit, J.E.H. (in press). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education.*

2.1 Introduction

Students with difficulties learning mathematics can be found in almost every classroom. About five to ten percent of the students in schools for general elementary education have difficulties with mathematics (Rivera, 1997). Students with difficulties learning math include all students who have more trouble with learning math than their peers, students who perform at a lower level than their peers, and students who need special instruction to perform at an adequate level. The seriousness of these difficulties can vary from (temporary) difficulties in one domain (i.e., a particular area of the math curriculum) to severe learning disabilities affecting several different domains. The difficulties can also manifest themselves at different points in a child's school career: not only in the learning of basic facts or learning to apply previously acquired knowledge but also in the learning of such preliminary mathematics skills as counting, and seriation (Van de Rijt & Van Luit, 1998). The potential causes of these difficulties are numerous and can partly be explained by such child characteristics as intellectual functioning, motivation, problem-solving skills, memory skills, strategy acquisition and application, and vocabulary. Another important cause of such difficulties may be a poor fit between the learning characteristics of individual students and the instruction they receive (Carnine, 1997). In the case of a poor fit, the instruction must be adapted to the students' needs. In other words, all students with mathematics difficulties require special attention (Geary, 1994). These students have special educational needs, need extra help, and typically require some type of specific mathematics intervention, which is the focus of the present meta-analysis.

Given that difficulties can be encountered at different ages and in different mathematical domains, intervention may be called for at different points in a child's school career and in different domains. Studies have shown most math difficulties to have a relatively early onset (i.e., problems emerge between the ages of five and seven years with the learning of basic skills; Schopman & Van Luit, 1996). During kindergarten and first grade, children typically develop number sense, which then grows along the lines of the various Piagetian operations (e.g., number conservation, classification, seriation) and in combination with various counting skills. A basic understanding of the arithmetic operations is es-tablished at this time (Correa, Nunes, & Bryant, 1999). The first category of interventions thus focuses on these preparatory arithmetic skills. For example, Malabonga, Pasnak, Hendricks, Southard, and Lacey (1995) have studied the effects of specific seriation and classification instruction versus regular instruction using academic materials (e.g., counting, shape, and color knowledge) on children's reasoning and math achievement.

The next step is to learn the four basic mathematical operations (i.e., addition, subtraction, multiplication, and division). Knowledge of these operations and a capacity to perform mental arithmetic also play an important role in the development of children's later math skills (Mercer & Miller, 1992; Van Luit & Naglieri, 1999). Most children with math-related learning disorders are unable to master the four basic operations before leaving elementary school and thus need special attention to acquire the skills. A second category of interventions is therefore aimed at the acquisition and automatization of basic math skills. The domain of basic math skills is very large and constitutes an important aspect of elementary math teaching. The studies reported on here, address the learning of such simple addition facts as 5 + 3 (e.g., Beirne-Smith, 1991) but also more complicated operations such as four-digit addition or division facts (e.g., Skinner, Bamberg, Smith, & Powell, 1993).

Mastery of the basic operations, however, is not sufficient: Students must also acquire problem-solving skills (e.g., flexibility and adaptability) in addition to the basic computational skills (i.e., the development of automaticity; Carnine, 1997; Goldman, 1989). Mathematics frequently involves the solution of both verbal and nonverbal problems via the application of previously acquired information (Mercer & Miller, 1992). For mathematical problem solving, that is, children must not only possess the basic mathematical skills but also know how and when to apply this knowledge in new and sometimes unfamiliar situations. The third category of interventions addresses problem-solving skills. In solving mathematics (word) problems, moreover, just which operation to apply or strategy to adopt is not always clear and must therefore be learned. In keeping with these steps, a distinction can be made between interventions that focus on: (1) preparatory arithmetic, (2) automatization of basic facts, or (3) mathematical problem-solving strategies. The distinction between basic skills and problem solving has also been put forth in other research syntheses (e.g., Mastropieri, Bakken, & Scruggs, 1991).

In the present meta-analysis, interventions will be discussed in terms of the domains and skills that they address. Interventions can nevertheless differ from each other with regard to a number of other factors: the use of a computer (computer assisted instruction), the size of the groups used for intervention, the duration and frequency of the intervention, the instructional procedures, the subdomains targeted, etc. The numerous interventions available today clearly reflect the many possibilities for intervention. Different children also need different interventions, but just which interventions and which characteristics of the interventions are most effective for which groups of children is still unclear.

In recent years, several reviews, research syntheses, and meta-analyses have been published on the topic of math intervention. Mastropieri et al. (1991) and Miller, Butler, and Lee (1998) have published reviews of the math interventions available for students with mental retardation and with learning disabilities, respectively. Jitendra and Xin (1997, Xin & Jitendra, 1999) have conducted interesting research on interventions aimed at the word problem solving skills of students with mild disabilities. Swanson and colleagues (Swanson & Carson, 1996; Swanson & Hoskyn, 1998, 1999) published impressive meta-analyses of intervention studies for students with learning disabilities.

In contrast to the preceding reviews, interventions pertaining to three different domains of math-related skills are distinguished in the present analysis because we think that certain interventions may be more effective in one domain than in the other. Another difference between the aforementioned studies and the present study is the method of analysis used. Although some of the former studies are quantitative, they nevertheless only compare (weighted) mean effect sizes to estimate the contributions of different variables and simply assume that other characteristics are equally distributed across the different groups being compared. The method of analysis used in the present study, namely multilevel regression analysis of variance, clearly takes the amount of within and between groups variance into consideration. The method used in the present study also makes it possible to combine baseline and experimental designs within the same analysis while these different types of designs were studied separately in the other reviews. A final difference is that the interventions for all groups of students with difficulties learning math are combined in the present study: interventions for students with mild disabilities, learning disabilities, and mental retardation. Because these children need similar types of instruction (Kavale & Forness, 1992), it seems reasonable to combine them in one analysis.

Research questions

The focus of the present study is on the characteristics of the most effective interventions. An intervention is judged as effective when the students acquire the knowledge and skills being taught and thus appear to adequately apply this information at, for example, posttest. The question, then, is what makes a particular mathematics intervention effective? In order to answer this question, a meta-analysis was undertaken of those studies concerned with mathematics intervention for students with special educational needs in both general and special elementary education. The effects of a number of variables were analyzed.

The following research questions are addressed:

1. Which domain (preparatory skills, basic skills, problem-solving) is mostly investigated, and which domain produces the highest effect-sizes?
2. Is there a trend in outcomes as a function of
   a. Study characteristics, such as year of publication, and design?
   b. Sample characteristics, such as number of subjects, age, and special needs?
   c. Treatment parameters, such as duration, total instruction time, and content?
   d. Treatment components, such as direct instruction, self-instruction, CAI, peer-tutoring?
   e. Treatment components related to recent reforms in the mathematics curricula, such as mediated/guided instruction and realistic mathematics education?
3. Which variables can explain the largest part of the between-studies variance in
   a. The total sample
   b. The three different domains separately

Before presenting the results of our meta-analysis, the method of analysis will first be described: just how the studies were located and selected for closer consideration, how the effect sizes were calculated, and which characteristics of the studies were coded.

2.2 Method

2.2.1 Search procedure

To find studies on the effectiveness of mathematics interventions for students with special educational needs in both general and special elementary education, the following search procedure was followed. The articles examined in the meta-analysis included empirical studies published between 1985 and 2000. A computerized search was conducted of the following databases: Current Contents, ERIC (Educational Resources Information Center), Psychlit, and SSCI (Social Sciences Citation Index). The search descriptors included *math(ematics), arithmetic, addition, subtraction, multiplication, division,* or *number-concepts*; and *intervention*, *instruction, training* or *teaching-method*; and *primary/elementary (education),* or *children*; and *disabilities, difficulties, mild/educable mental retardation, disadvantaged, at-risk, underachieving, low-performing, below-average,* or *lagging in cognitive development*. The adequacy of this set of search terms was checked via a hand search of recent volumes of the most well-known journals in which most of the empirical studies in the field of special education are published, namely *Remedial and Special Education*, *Journal of Learning Disabilities*, *Journal of Special Education*, *Exceptional Children*, *Learning and Instruction*, and *Learning Disabilities: Research & Practice*. In addition to this, the reference lists from other recently published research syntheses, meta-analyses, and reviews were carefully checked (Miller et al., 1998; Swanson & Hoskyn, 1998, 1999; Xin & Jitendra, 1999). Only English-language articles were included. The inclusion of only published journal articles may also have weakened the external validity of the present study due to a tendency to publish only studies with significant positive effects (White, 1994) and should therefore be kept in mind when interpreting the results of the meta-analysis.

2.2.2 Selection criteria

The initial computerized search produced 656 references, including 264 articles that did not have mathematics intervention as the main topic or did not address mathematics intervention in the manner we expected. In addition to this, 172 of the articles were either review articles themselves or not based on empirical analyses. Exclusion of the aforementioned references resulted in 220 publications. The publications to be used in the meta-analysis were next selected by the two authors using the criteria outlined below. While the authors agreed on 95% of the articles, the other 5% were simply discussed to obtain full agreement on the selection of articles. The following criteria were used to select the set of articles for the meta-analysis.

The first criterion for inclusion in the meta-analysis was that the study be concerned with elementary mathematics skills as most math difficulties appear to have their origin in the acquisition and automatization of these skills (Tissink, Hamers, & Van Luit, 1993). The focus of the meta-analysis was therefore on interventions with children in kindergarten and elementary school, and those studies with a mean subject age over 12 were excluded (33 references excluded).

The second criterion was that the study reports on an intervention involving mathematics instruction. An intervention is defined as a specific instruction for a certain period to teach a particular (sub-) domain of the mathematics curriculum. This domain could vary from ‘addition up to ten’ to ‘third grade math’. 71 of the references did not report on an intervention and simply described, for example, the solution procedures used by children without the instruction they received. These studies were therefore excluded. Instruction aimed at other skills, such as planning or metacognition, and interventions based on homework or parent training were also not included (15 references excluded).

The third criterion was that the study reports on subjects with mathematical difficulties, as based on the given description. These difficulties varied from being at risk for difficulties or lagging behind to having mathematical learning disabilities. Studies reporting on subjects with mathematical difficulties as a consequence of severe mental disabilities or weaknesses were also not included (12 references excluded).

A fourth criterion was that only those studies using a between-subjects or within-subjects control condition were included in the meta-analysis. All of the studies therefore had at least an experimental and a control condition or a repeated measures design (8 studies excluded). For statistical reasons, studies with less than three children were also excluded (6 references) and sufficient quantitative information had to be reported to allow calculation of effect sizes according to our methodology (15 references excluded).

Finally, articles describing mathematics or arithmetic instruction without reporting systematic use of instructional strategies were excluded (2 references).

Application of the aforementioned selection procedure produced a total of 58 studies for inclusion in the meta-analysis.

2.2.3 Calculation of effect sizes

For the studies with a between groups design, Cohen’s d was calculated by dividing the difference between the scores for the control and experimental groups at posttest by the pooled standard deviation. For the studies with a repeated measures design, the baseline scores were treated as control scores and thus subtracted from the treatment mean score, with the difference then divided by the pooled standard deviation. Although several authors have raised reservations about this method (see Scruggs & Mastropieri, 1998), mainly because it does not take into account the regression of behavior on time, we think it is still the best method to use when the effects of single subject designs are combined with the effects of group design studies. Because the effect sizes of single subject research are usually higher than those of group design studies, we included the design of the study as a dummy variable in the analyses, to correct for a possible effect of design. Some studies showed very high effect sizes (even up to 12 or 14; Jitendra & Hoff, 1996; Van Luit & Van der Aalsvoort, 1985). In light of the fact that differences between effect sizes above 3 are essentially meaningless (Scruggs & Mastropieri, 1998), we adopted a maximum effect size of 3 for such studies.

For those studies that did not report means and standard deviations, the effect sizes were calculated on the basis of other statistical information (for exact procedures, see Rosenthal, 1994). When the total N was reported but not the Ns for the experimental and control groups, we simply divided the total N by two. When a t-test was conducted but no means and standard deviations or t-values were reported, we used the given significance level to calculate the effect size.

Whereas many of the studies examined the effects of intervention on a variety of tests including tests of mathematics performance, motivation, perceived competence, and transfer, we only used the scores for mathematics performance (and not motivation, general achievement, or attribution) in the meta-analysis. When more than one test or subtest was used to measure mathematics performance, we calculated the effect sizes for all of the tests and then used the mean effect size in the meta-analysis. This is because we would otherwise have several outcome measures for some studies and unequal weightings across studies as a result (Rosenthal & Rubin, 1982; Swanson & Carson, 1996). Several of the studies also used more than one posttest. To calculate the effect sizes, however, we only used the scores for the first posttest. In other words, one effect size was calculated for each study, unless the study made use of more than one experimental condition. In that case and provided the conditions differed significantly from each other, we calculated one effect size for each experimental condition. This procedure was also followed for studies conducted with different groups (e.g., children with learning disabilities [LD] and children with mild mental retardation [MMR]). These procedures resulted in 61 effect sizes for the 58 reported studies.

2.2.4 Coding of the studies

Three categories of coding variables were distinguished. The first category of report identification and methodology included the following variables: (1) year of publication; (2) design (single-subject versus group); and (3) control (whether the control condition also received intervention or not).

The second category was sample description and included the following variables: (4) number of subjects in the experimental and control groups (when necessary, these numbers were estimated on the basis of the data reported); (5) mean age of the subjects (some of the studies did not report the mean age for the subjects and other studies reported only grade level; in those cases, the mean age was estimated on the basis of the available data); and (6) type of special needs (low performing or at risk; learning disabled; mildly or educable mentally retarded; mixed groups or other disorders such as behavior or attention disorders).

The third category was treatment and included the following variables: (7) duration of the intervention; (8) total intervention time; (9) content (preparatory skills such as counting skills or number sense; basic facts such as addition/subtraction and multiplication/division; or problem solving strategies); (10) method (direct instruction, self-instruction, or mediated/assisted performance models; for a description see Goldman, 1989); (11) use of computer assisted instruction (CAI) or not; (12) peer-tutoring or not; and (13) characteristics of RME (Realistic Mathematics Education principles such as “guided reinvention”, phenomenological exploration, the use of self-developed models and meaningful contexts, student contribution, and interactivity; for a more detailed description, see Gravemeijer, 1994).

2.2.5 Statistical analyses

For the meta-analysis, we used a random effects model as described by Raudenbush (1994), by means of the program VKHLM (Bryk, Raudenbush, & Congdon, 1994). This model assumes that study outcomes vary across studies, not only because of random sampling effects, but also because there are real differences between the studies. The advantage of using multilevel regression analysis lies in the flexibility of the method and the ease with which the mean outcomes and variances can be estimated (Hox, 1995; Hox & De Leeuw, 1997). The parameter variance found for the studies is compared to the residual variance in the model including other characteristics of the studies. This results in a measure estimating the amount of variance explained by different study characteristics. In other words, the effects of different explanatory variables are calculated while also taking a number of other characteristics of the studies into consideration, just as in other regression analysis methods.

2.3 Results

Appendix 2.1 presents an overview of the descriptive information for each study along with the authors, subjects, procedural information, and results. A total of 2509 children with special mathematics needs was studied. First, the distribution across the different domains is presented, secondly, the effects of the separate variables are described (models with only one variable included), and finally, the results of the multilevel meta-analyses will be described (models with more variables included).

Question 1: Distribution across the three domains

The studies were distributed across the three domains of elementary mathematics as follows: 13 interventions for preparatory arithmetic, 31 for basic facts, and 17 for problem solving. The domain of basic facts has been investigated mostly ($X^2(2) = 7.897$, $p = .019$). However, no significant differences were found between the effect-sizes of the three different domains.

Table 2.1

Effects of nominal variables

Variable	*N*	*K*	Mean *d*
Design**			
baseline	21	155	2.16
experimental	40	2354	0.62
Control condition**			
no intervention	30	701	1.51
intervention	31	1808	0.51
Special needs*			
low performing	8	192	0.74
learning disabilities	7	293	1.36
mild mental retardation	23	1608	0.80
mixed groups	23	416	0.73
Content			
preparatory	13	664	0.92
basic facts	32	1324	1.14
problem solving	16	521	0.63
Method**			
direct instruction	35	1671	0.91
self instruction	16	372	1.45
mediated/assisted	10	466	0.34
Medium**			
teacher	49	1635	1.05
computer	12	874	0.51
Peer-tutoring			
no	51	1954	0.96
yes	10	555	0.87
Realistic math			
no	47	1709	1.04
yes	14	800	0.70

* $p < .10$, ** $p < .05$

Question 2: Effects of single variables

In Tables 1 and 2, an overview is given of the distribution across the different variables, together with the single effects of these variables. Of the 61 empirical investigations included in the meta-analysis, 21 had single-subject designs and 40 had group designs. The single-subject design studies have significant higher effect-sizes than the group-design studies. Studies in which the control condition received a different intervention (N = 30), show higher effect-sized than studies without a different intervention in the control condition (N = 31). No effect of year of publication was found.

The average number of subjects in the studies was 41.1, with a range of 3 to 136. A negative correlation was found between number of subjects and effect-sizes, which indicates that small studies show higher effects than large studies. The mean age was 8.6 years (*sd* = 1.8), with a range of 5 to 12 years. Interventions with older students had more effect than interventions for younger students. Also, an effect of the special needs of the students was found, because the interventions for students with learning disabilities showed higher effect-sizes than the studies for the other groups. 23 studies described interventions for low performing/at risk children, 23 for children with learning disabilities, 8 for children who are mildly or educable mentally retarded, and 7 for a mixed group or children with several disabilities. The length of the interventions varied from 2 to 140 sessions (M = 10.6, sd = 9.0); the duration of the sessions ranged from 10 minutes to 60 minutes, with a mean of 35 minutes (sd = 13.8). The total instruction time varied from one weak to one year. Both the instruction time as the duration showed a negative correlation with effect-sizes. No effect was found of content (preparatory, basic, or problem-solving skills).

Most of the studies (N = 35) used direct instruction in the intervention. However, self-instruction (N= 16) led to higher effect-sizes. A total of 12 studies used computer assisted instruction (CAI), 10 studies peer tutoring. No effect of peer tutoring was found, but the studies using CAI showed lower effect-sizes than studies in which the teacher instructed the students.

Table 2.2
Effects of continuous variables

Variable	γ
Year of publication	-.020
N	-.007**
Mean age	.106*
Duration	-.018**
Total instruction time	-.001**

* $p < .10$, ** $p < .05$

A last variable concerns reform-based characteristics of interventions. Mediated or assisted instruction asks, contrary to direct-instruction, from the students that they discover and develop their own math skills, while assisted by a teacher. Ten studies used mediated-assisted instruction models, but this did not led to higher results. Fourteen studies used one or more instructional principles from the realistic mathematics education, such as “guided reinvention”, the use of self-developed models and meaningful contexts, or student contribution and interactivity. Although a trend was found in favor of more traditional interventions, no significant differences were found.

Question 3: Multilevel meta-analysis
The aim of the meta-analysis is to discover which of the variables explain at least part of the variance not explained by sampling variance. The results were significantly heterogeneous (estimated parameter reliability for the 58 studies σ^2 = 0.510, p = .000), which justifies the use of a random effects model; because the difference between studies are large, it would be wrong to assume that the students included are selected from one population.

Four variables were found to explain 69% of the between-studies variance. The best explanatory variable was design (γ = 1.4, p = .000). Studies with a single-subject design produced higher effect sizes than those with a group design. A second important explanatory variable was the duration of the intervention (γ = -0.01, p = .032). Interventions that lasted longer, had less effect than shorter interventions. A third explanatory variable was the domain of intervention (preparatory – basic: γ = 0.24, p = .177; preparatory – problem solving: γ = 0.70, p = .004; basic – problem solving: γ = 0.46, p = .018). Interventions in the domain of problem solving were found less effective than in both other domains. A final explanatory variable was the method used (direct instruction – self-instruction: γ = -0.42, p = .051; direct instruction – mediated/assisted instruction: γ = 0.24, p = .199; self-instruction – mediated/assisted instruction: γ = 0.66, p = .009). Self-instruction was found to be more effective than direct instruction or mediated/assisted instruction. These results are also found when only analyzing the studies with group designs. The studies with single subject designs were too few and too homogeneous to analyze separately.

Separate meta-analyses were next conducted for each of the three categories of intervention. Due to the high number of variables and small Ns for the category of “preparatory arithmetic”, the separate meta-analysis for this category was conducted with only a few variables.

Preparatory arithmetic. Because of the low number of studies that used interventions with CAI (1), peer tutoring (0), self-instruction (1), and mediated instruction (1), these variables were not analyzed. None of the studies in this domain had a single-subject design. So, the analyses were conducted with the following variables: year of publication, intervention in control group, number of students, mean age, duration, total time, and realistic math. For the interventions directed at preparatory tasks, the between-studies variance was found to be low ($\sigma^2 = 0.262$, $p = .001$). The variables explaining the most variance were duration of the intervention ($\gamma = -0.08$, $p = .001$), and total instruction time ($\gamma = 0.01$, $p = .001$). Together, these variables explained 99% of the variance.

Basic facts. In the category of interventions aimed at basic skills, 85% of the between-studies variance ($\sigma^2 = 0.943$, $p = .000$) could be explained by the following variables: intervention in control group ($\gamma = 1.49$, $p = .000$), peer tutoring ($\gamma = -0.76$, $p = .011$), mean age ($\gamma = 0.39$, $p = .000$), and method (direct instruction – self-instruction: $\gamma = 0.76$, $p = .043$; direct instruction – mediated/assisted instruction: $\gamma = 1.55$, $p = .001$; self-instruction – mediated/assisted instruction: $\gamma = 0.79$, $p = .070$). The main results are thus that studies in which the control group received an intervention too, produced lower effect sizes than studies with no intervention in the control group, that interventions using peer tutoring are less effective than those not using this method, that the interventions for older students proved more effective, and that direct instruction is more effective than mediated/assisted instruction and self-instruction.

Problem solving. The effect sizes for the studies concerned with problem solving were also found to be heterogeneous ($\sigma^2 = 0.170$, $p = .005$); 99% of the between-studies variance was explained by the following variables: intervention in control group ($\gamma = 0.54$, $p = .052$), year of publication ($\gamma = 0.08$, $p = .061$), type of special needs (low-performing: $\gamma = 0.77$, $p = .068$; learning disabled: $\gamma = 0.32$, $p = .223$; mild mentally retarded: $\gamma = 1.09$, $p = .143$), peer tutoring ($\gamma = -1.62$, $p = .011$), and the use of CAI ($\gamma = -0.78$, $p = .025$). The main results for interventions aimed at problem solving skills are that studies with an intervention in the control group showed lower effect sizes than studies with a no-intervention control group; that peer tutoring and computer assisted instruction are less effective than other intervention methods; and that interventions for children with mild mental retardation are more effective than those for children with learning disabilities, and that interventions for students with diverse and mixed problems are less effective than the interventions for low-performers.

2.4 Conclusions and discussion

With the use of a random effects model in the present study, it was possible to determine which study characteristics appear to be most important for the prediction of effect size. In contrast to more traditional forms of meta-analysis, the different study characteristics can now be compared to each other while also taking any other effects into consideration. The single-variable analyses showed that several variables have a significant influence on the study outcomes. However, when analyzed together, it appears that only four variables were found to explain a significant part of the variance in the effect sizes for all of the studies considered together. For instance, both the duration and the total instruction time proved significant when analyzed separately. However, when analyzed together, it appears that the variable duration explains already a significant part of the variance, and that the total instruction time does not contribute any more to the amount of explained variance. Apparently, there is a strong relationship between total instruction time and duration of the intervention. The same phenomenon is found for the variables design and control condition and for age and domain. Therefore, the analyses from the multi-variables analyses are taken as the basis for our conclusions. It should be noted that the conclusions are only based on published studies, which may weaken the external validity of the findings and therefore call for caution in their interpretation.

The first conclusion is that the majority of the studies describe an intervention in the domain of the basic skills. The interventions in this domain also show the highest effect-sizes. The domain of basic math skills is very large and constitutes an important aspect of elementary math teaching. For this reason, it is not surprising that many of the studies are concerned with this domain. And it appears to be a domain in which interventions are effective. It may be easier to teach basic skills to the students with special needs, than to teach problem solving skills.

The second conclusion is that the most important predictor in the present study was found to be the research design. Similar to the findings of other studies, these analyses show that studies with a single-subject design show more powerful results than those with a group design do. Several possible explanations can be given. One of them is that children in a single-subject design are often administered the same test or parallel versions of the same test and therefore grow accustomed to the test and perform better over time. Furthermore, it should be noted that the effect sizes for single-subject studies are calculated in a slightly different manner than the effect sizes for (quasi-) experimental group studies. An intra-group comparison is made in single-subject studies while inter-group comparisons are made in experimental group studies. The effect sizes for the difference between pretest and posttest in the single-subject studies generally tended to be larger than the effect sizes for the differences between the control and experimental groups in the group studies. In most group designs, however, the control group receives some form of intervention; in single-subject designs, in contrast, the baseline consists of no intervention. This difference may also explain the varying effect sizes observed for the different study designs. The significant effect of intervention in the control group, found in the preparatory and basic skills interventions, supports this explanation. Finally, it should be noted that the training in a single-subject repeated measures study is often criterion-based; training continues until a set criterion is reached (usually 80% correct). Given that the training is only stopped when the results are sufficiently high, the large effect sizes under such circumstances are really not surprising.

A third conclusion concerns the effects of sample characteristics. Overall, no differences were found between studies that reported interventions for students with different special needs. However, in the studies concerned with problem solving, the interventions for students with mild mental retardation were more effective than those for students with learning disabilities were. A possible explanation for this unexpected result could be found in the nature of the interventions. Children with mild mental retardation often receive intensive training across an extended period of time, mainly focused on basic skills. When the intervention involves a new method or domain (in this case: problem solving), the children may then become very motivated. Students with specific learning disabilities may have already a history of failure in the topic concerned, and have motivational problems as a result of such a history.

The fourth conclusion is that the duration of the intervention correlates negatively with effect size, especially for the interventions focused on preparatory tasks. We originally expected this correlation to be positive. A possible explanation is that short interventions focus on a very small and specific domain of knowledge, such as addition up to ten. Prior to intervention, the children score very low; after a short period of intervention, however, they have fully acquired the relevant knowledge and thus score quite high. Longer interventions, in contrast, may focus on a broader domain of knowledge, cost more time, and therefore produce smaller effect sizes than shorter interventions. In addition, the testing of an intervention in a broad domain of knowledge may be complicated by numerous interacting variables.

The fifth conclusion, regarding treatment components of interventions, is that it appears from the present meta-analysis that, in general self-instruction is most effective. However, for the learning of basic skills direct instruction appears to be the most effective. Another conclusion we can draw is that the use of the computer to aid instruction cannot replace the teacher. The interventions with computer assisted instruction produced lower effect sizes than other interventions. This finding corresponds to the findings of other studies (e.g., Hativa, 1994) and suggests that the computer is less effective than a human teacher. Also the interventions making use of peer tutoring were found to be less effective than other interventions. This can be explained by a number of different factors. One important factor is that peers are less capable of knowing the needs of other students than teachers. Young students are also often not accustomed to working together and need experience to develop the necessary skills (Wilkinson, Martino, & Camilli, 1994). The role of the teacher thus appears to be critical to help students and evaluate their progress.

A final conclusion concerns the effects of reform-based interventions. Mediated/ assisted instruction is less effective than direct instruction or self-instruction. No effects were found of the variable realistic mathematics education. In other words, these analyses confirm that the recent changes in the mathematics education does not lead to better performance of the students with special needs.

Implications for practice

From this study, a few interesting conclusions can be drawn with regard to math interventions. When choosing and organizing an intervention, one should keep in mind the following findings. The first concerns the method used to teach students mathematics. Both self-instruction and direct instruction seem to be adequate methods for the students with special needs. For the learning of basic facts, direct instruction appears to be most effective. For the learning of problem-solving skills, self-instruction methods are also quite effective.

A second finding concerns the use of Computer Assisted Instruction (CAI), which can be very helpful when students have to be motivated to practice with certain kinds of problems. With the use of a computer, it is possible to let children practice and automatize math facts and also provide direct feedback (e.g., Koscinski & Gast, 1993). However, the computer cannot remediate the basic difficulties that the children encounter. The results of the present study show, in general, that regular interventions with humans as teachers and not computers are most effective.

We often have children work together in order that they help and teach each other. It appears, however, that children with special needs do not particularly profit from this strategy. Of course, peer tutoring may be helpful and effective at times, but the present study shows that it can not replace or be as effective as instruction by an adult teacher.

Finally, this study gives reasons to think that not all of the changes as proposed by math reformers are as effective as more traditional approaches. However, it always costs time to adjust to changes, and this variable should therefore reexamined thoroughly when more data becomes available.

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RPT higher scores than PC, $p < .05$ $d = 0.52$
Structure + reward highest scores; main effect for reward, not for structure $d = 0.48$
LD students performed lower than low-achieving students, no main effect for treatment; $d = 0.40$
RPT higher math achievement $F(1,37) = 15.38$, $p < .001$ $d = 0.72$
Positive results for peer collaboration ($p < .01$; $d = 0.34$) and for problem solving ($p < .05$; $d = 0.51$), no interaction effect
Mnemonic training most effective, $F(1,21) = 79.74$, $p = .000$; $d = 1.19$

Appendix 2.1

Reference	M age	N	Special needs**	Procedure/ design	Intervention	Sessions/ duration	Results/effect size
Ainsworth, Wood, & O'Malley (1998)	7.3	29	Low-performing	Pretest-intervention-posttest; randomized block design	Computer intervention COPPERS to teach that one (multiplication) problem can have many different correct solutions (exp. condition); use of concrete and everyday knowledge. Control condition: single answer	2 x 35-min sessions in 2 weeks	Both conditions improved significantly; condition with multiple answers improved more than the condition in which single answers were required $F(1,50) = 5{,}912$, $p < .019$ $d = 0.89$
Beirne-Smith (1991)	8.7	30	LD, low-performing	Pretest-intervention-posttest; random assignment	Single-digit addition facts: Counting-on peer tutoring (method A), rote-memorization peer tutoring (method B), and no-treatment control	20* x 30-min sessions in 4 weeks	Peer tutored students (A/B) performed higher than students who received no peer tutoring, $p < .01$ $d = 0.82$
Case, Harris, & Graham (1992)	11.3	4	LD, low-performing	Baseline, phase 1 probes, phase 2 probes, generalization, maintenance	Self-regulated strategy development for addition and subtraction word problems	9 x 35-min sessions in 4 weeks	Students' overall performance on mixed sets of addition and subtraction word problems improved $d = 1.56$
Cassel & Reid (1996)	9.1	4	LD, MMR	Multiple-baseline across subjects	Self-regulated strategy intervention; word problem-solving	8 x 35 min-sessions in 10 weeks	Performance increased to 80% mastery level, maintained 8 weeks; $d = 1.14$
Dunlap & Dunlap (1989)	11	3	LD	Multiple-baseline across subjects	Self-monitoring checklists; subtraction	13 sessions in 3 weeks	All students showed immediate and dramatic gains in intervention period; $d = 2.57$

Appendix 2.1 (continued)

Intervention	Sessions/ duration	Results/effect size
Multiplication facts 0 to 81; concrete-to-representational-to-abstract instructional sequence	10 lessons in 8 weeks	Significant improvement; $d = 2.71$
Contextualized learning environments versus direct instruction. Word problem solving	86 x 30-min lessons	Experimental group showed more improvement than control group; $d = 0.27$
Reciprocal peer tutoring; also effect of parent involvement; math computation	60* x 45-min sessions in 8 months	RPT higher scores than control, $p < .01$ $d = 1.00$
Training in place-value concepts; 15 poor arithmetic children got training; two control groups (good/poor)	5 x 60-min sessions in 5 weeks	Positive effects on place-value understanding, addition, and subtraction, $p < .05$; $d = 0.85$
CAI: word-problem solving ability: focused on the construction of adequate problem representation	10 sessions in 2 weeks	Significantly increased performance from baseline to training phase $d = 1.26$
Schema training versus traditional; word problem solving	20 x 45-min sessions in 4* weeks	Significant main effect for group; schema > traditional, $F(1,31) = 1.8$, $p = .005$; $d = 0.64$

Fantuzzo, Davis, & Ginsburg (1995)	10.6	47	At risk / low-performing	Pretest-intervention-posttest; random assignment	Reciprocal peer tutoring (RPT) versus Practice control (PC); also effect of parent intervention on computation skills	20 x 45-min sessions in 10 weeks
Fantuzzo, King, & Heller (1992)	10.5	68	At risk / low-performing	Pretest-intervention-posttest; random assignment	Effects of offering structure in Reciprocal peer tutoring (RPT); also effect of rewards on computation problems	50* x 45-min sessions in 5 months
Fuchs et al. (1997)	9.4	40	Low-performing with and without LD	Pretest-intervention-posttest	Effects of task-focused goals	44 x 35-min sessions in 26 weeks
Ginsburg-Block & Fantuzzo (1997)	10.5	40	At risk / low-performing	Pretest-intervention-posttest; random assignment	2 conditions: Reciprocal peer tutoring (RPT) and Practice control (PC), working in dyads on computational problems	20 x 45-min sessions in 10 weeks
Ginsburg-Block & Fantuzzo (1998)	9.6	104	Low-performing	Pretest-intervention-posttest; random assignment	NCTM Standards based intervention: problem solving, peer collaboration, both, or nothing (control); computation and word problems	14 x 30-min sessions in 7 weeks
Greene (1999)	10.7	23	LD	Pretest-intervention-posttest; random assignment	Mnemonic instruction versus more traditional instruction on multiplication facts	5 x 20-min sessions in 1 week

3	LD, low-performing	Multiple baseline	Schema-based direct instruction strategy; word-problem solving	5 x 40-min sessions in 2 weeks	Increased percentage of correct solutions to word problems; $d > 3$
16	MMR	Pretest-intervention-post1,2; random assignment	Addition with regrouping; Self-instruction vs external instruction; instruction in small groups (3-5)	7 x 30-min sessions in 2 weeks	Self-instructional training higher results than external instruction; $F(1,12) = 3{,}27$, $p = .05$; $d = 0.67$
3	LD	Multiple baseline across subjects	Generalization of question asking; videotape watching; structuring opportunities for question asking, providing feedback	6 x 50-min sessions	Gradual increase of accuracy scores during treatment; $d = 0.63$
6	LD/ ADHD	Multiple baseline	CAI constant time delay delay in teaching unknown multiplication facts	6 sessions in 2 weeks	Results indicate that this procedure was an effective method of teaching multiplication facts to LD students; $d = 2.18$
58	MMR	Pretest- intervention-posttest; random assignment	CAI (drill & practice) versus paper-pencil; addition/subtraction facts; MMR versus nonhandicapped	13 x 15-min sessions in 1 month	No significant differences between two conditions; $d = -0.44$
6	LD	Multiple baseline	Effects of self-monitoring of attention, accuracy, or productivity	40 x 30-min sessions in 10 weeks	Self-monitoring of accuracy most effective; $d = 1.53$
17	At risk	Pretest-intervention-posttest; random assignment	Exp: seriation, classification training; control: training on academic subjects	30* x 15-min sessions in 3 months	Posttest scores of the experimental children were higher than those of the control children, $t(15) = 1.86$, $p < .05$; $d = 1.61$

Reference	M age	N	Special needs	Procedure/ design
Harris, Miller, & Mercer (1995)	8.3	13	LD	Baseline, pretest, lessons 1-10, posttest, lessons 11-21
Hasselbring & Moore (1996)	8	24	EMR, LD, SED	Pretest-intervention-posttest
Heller & Fantuzzo (1993)	9.9	54	At-risk	Pretest-intervention-posttest; random assignment
Ho & Cheng (1997)	7.0	45	Low-performing	Pretest-intervention-posttest; matched groups
Jaspers & Van Lieshout (1994)	10.3	5	EMR	Multiple baseline design across subjects
Jitendra et al. (1998)	10.4	34	LD, EMR, SED	Pretest-intervention-posttest; randomized block

Appendix 2.1 (continued)

Reference	M age	N	Special needs	Procedure/ design	Intervention	Sessions/ duration	Results/effect size
Marsh & Cooke (1996)	9	3	LD	Multiple baseline across students	Use of concrete manipulatives to teach mathematical problem solving	8 x 20-min sessions	Significant improvement during intervention; $d > 3$
Mattingly & Bott (1990)	11.6	4	LD, EMR, BD	Multiple probe design	Constant time-delay procedure in teaching unknown multiplication facts	6-19 sessions in 2 weeks	Time delay procedure was effective in teaching the targeted multiplication facts; $d > 3$
Mevarech (1985)	7	204	At risk	Intervention - posttests	CAI in individualized or more traditional training	80 x 20-min sessions in 1 year*	Significant main effects for the use of CAI ($p < .001$, $d = 0.73$), not for individualized instruction; $d = -0.20$
Mevarech & Rich (1985)	8	376	At risk	Intervention-posttests	CAI (TOAM): testing and practice; individualized instruction	80 x 20-min sessions in 1 year*	Experimental group scored higher on arithmetic than traditional instruction group; $p < .0001$; $d = 0.54$
Miller & Mercer (1993)	9.4	9	LD, EMR	Multiple baseline across subjects design	Concrete-semiconcrete-abstract teaching sequence for basic math facts and coin sums	13 x 20-min sessions in 3 weeks	CSA was effective for acquisition and short-term retention; $d = 2.28$
Naglieri & Gottling (1995)	11.9	4	LD	Baseline-intervention	Cognitive instruction in addition and multiplication which facilitated planning	7 x 25-min sessions	The intervention did not improve multiplication performance for average planners, but was effective for those with poor planning scores; $d = 0.96$

Jitendra & Hoff (1996)	10.1
Keogh, Whitman, & Maxwell (1988)	10.6
Knapczyk (1989)	10
Koscinski & Gast (1993)	9.5
Lin, Podell, & Tournaki-Rein (1994)	8.4
Maag, Reid, & DiGangi (1993)	9.3
Malabonga, Pasnak, Hendricks, Southard, & Lacey (1995)	5.8

Appendix 2.1 (continued)

Sessions/ duration	Results/effect size
14 sessions in 6 weeks	During treatment, performance increased to average level; $d > 3$
10 x 15-min sessions in 4 weeks	No significant differences in accuracy; CAI > paper and pencil in response time $F(1,48) = 5.22$, $p < .05$ $d = 0.62$
24* x 30-min sessions in 3 months	Experimental group scored higher than control group $F(1,56) = 35.57$, $p < .001$ $d = 0.89$
6 x 40-min sessions in 6 days	Gradual acquisition model significantly enhanced children's performance I: $F(1,71) = 45.46$, $p < .001$; $d = 1.43$II: $F(1,71) = 10.49$, $p < .01$; $d = 0.91$
3 x 30-min sessions in 1 week*	Significant improvement in all conditions, no differences between conditions; $d = 0.32$
14 sessions in 4 weeks	Two of the three students showed increased performance, the third student needed feedback and goal-setting; $d = 2.63$

Naglieri & Gottling (1997)	10.1	12	LD	Baseline-intervention	Cognitive mathematics instruction, planning; Low/high planning (exp-control)	21 x 30-min sessions in 8 weeks	Low-planning students most improvement; $d = 0.71$
Pasnak (1987)	5	22	Low-performing	Pretest-intervention-posttest; random assignment	Classification, seriation, conservation training versus normal math instruction	40* x 15-min sessions in 4 months	Significant positive effect for experimental condition $F(1,20) = 2.72$, $p < .05$; $d = 0.73$
Pasnak, Hansbarger, Dodson, Hart, & Blaha (1996)	5	64	Low-performing	Intervention-posttest; random assignment	Cognitive intervention: classification, seriation, conservation; control children received instruction on verbal and mathematics materials	40 short lessons in 3 months	Experimental instruction produced higher scores than control instruction; $F(1,42) = 6.20$, $p < .02$; $d = 0.53$
Pasnak, Holt, Campbell, & McCutcheon (1991)	5	57	Low-performing	Intervention-posttest; restricted randomization	Piacceleration curriculum: classification, seriation, number conservation; control condition: standard curriculum	40* x 15-min sessions for 3 months	Piacceleration curriculum was significantly superior to the existing curriculum Walk's $\lambda = 8.43$, $p < .01$; $d = 1.09$
Pearce & Norwich (1986)	10.5	8	LD	Baseline, teaching, maintenance; matched group, random assigned	Computer assisted instruction versus direct teaching; simple number estimation skills	18 x 30-min sessions in 6 weeks	Computer group higher increase in scores than direct teaching group; direct teaching group had higher scores in maintenance period; $d = 0.13$
Perry, Pasnak, & Holt (1992)	8.2	23	MMR	Pretest-intervention-posttest; matched pairs	Classification and seriation versus verbal/number concepts	90* x 15-min sessions in 8 months	Significant difference between the two groups in favor of the experimental group, $F(1,16) = 2.71$, $p < .05$; $d = 0.25$

Manipulatives (MAN) and number sentences (NUM) in computer aided word problem solving	12 x 30-min sessions	No effect of MAN, significant effect of NUM, $p < .05$; MAN: $d = 0$; NUM: $d = 0.43$
Demonstration-plus-model; equal addition method of subtracting	30 x 15-min sessions	Instruction was effective for student's computation of subtraction with re-grouping; $d = 2.88$
Cognitive behavioral training in addition and subtraction problems	22 x 20-min sessions	The training favorably improved academic performance; $d > 3$
Specific self-instruction; general self-instruction; didactic instruction; addition problems	4 x 45-min sessions in 2 weeks*	Main effect for instruction group on generalization; specific self-instruction scored highest, $p < .05$. $d = 0.35$
Guiding-structuring; Additional Early Mathematics program	26 x 30-min lessons in 3 months	Experimental group scored higher than control group on posttest; $F(3,02) = 7.19$, $p = .000$; Guiding: $d = 1.04$; Structuring: $d = 1.24$
Self-instructional arithmetic training program	50* x 30-min sessions in 20 weeks	Significant improvement; $t(1,15) = 11.69$, $p = .000$; $d = 2.38$
Structural (SC) vs Realistic (RC); addition and subtraction with renaming	I: 100* x 45-min sessions in 6 months II: 140* x 45-min sessions in 8 months	I: no significant differences between groups; $d = 0.09$; II: SC>RC on trained tasks ($p = .002$); $d = 0.66$

Reference	M age	N	Special needs	Procedure/ design	Intervention
Pigott, Fantuzzo, & Clement (1986)	11	12	Low-performing	Multiple baseline	Reciprocal peer tutoring with group reinforcement; basic arithmetic operations
Podell, Tournaki-Rein, & Lin (1992)	8.2	28	MMR/LD	Pretest-intervention-posttest; random assignment	CAI/paper-and-pencil; automatization of basic skills in addition
Schopman & Van Luit (1996)	6	60	Developmental lag	Pretest-intervention-posttest; matched groups	Preparatory arithmetic strategies, directing and guiding versus control
Schunk, Hanson & Cox (1987)	I:10.6 II:10.9	80 80	Low-performing	Pretest-intervention-posttest	Influence of attributes of peer models: rapid versus gradual acquisition of fraction skills I: also effect of same- or opposite sex model II: also effect of one model versus three
Shiah, Mastropieri, Scruggs, & Mushinski (1995)	10.2	30	LD	Pretest-instruction-posttests 1&2;random assignment	CAI on math word problem solving, two conditions with cognitive strategy-training, with one using animated pictures
Skinner, Bamberg, Smith, & Powell (1993)	11.5	3	BD	Within-subjects, across-problems, multiple-baseline design	Cognitive cover, copy, and compare intervention on division facts

Appendix 2.1 (continued)

Special needs	Procedure/ design	Intervention	Sessions/ duration	Results/effect size
I: MMR II: LD	Pretest-intervention-posttest	MASTER training program: strategy training, teacher assists children in selecting strategies, self-instruction; multiplication facts.	50 x 45-min sessions in 17 weeks	Experimental groups higher posttest scores than comparison groups; I: $F(1,40) = 54.44$, $p < .001$, $d = 2.80$'II: $F(1,40) = 81.82$, $p < .001$, $d = 2.28$
LD/MMR	Pretest-intervention-posttest; matched groups	Early numeracy based on perceptual gestalt theory	50 x 30-min sessions in 6 months	Experimental group scored higher on posttest than comparison group $t(124) = 3.29$, $p = .001$, $d = 0.66$
EMR	Baseline, intervention, follow-up	Self-instruction in solving addition and subtraction problems	30* x 30-min sessions in 2 months	At the end all subjects had reached the goal: solve subtraction problems with renaming; $d > 3$
Low-performing	Pretest-instruction-posttest; matched and randomly assigned	Experimental: classification, number conservation; control: verbal and arithmetic skills	50 x 15-min sessions in 20* weeks	Significant improvement from pretest to posttest; No significant differences between experimental and control groups; $d = 0.06$
LD	Single-subject, alternating treatment design	CAI vs. teacher-directed instruction; multiplication facts	13-31 x 30-min sessions	All students mastered more facts under teacher-directed instruction; $d = 1.95$

Stellingwerf & Van Lieshout (1999)	11.3	100	LD, EMR	Pretest-instruction-posttest; matching, random assignment
Sugai & Smith (1986)	10	7	LD	Multiple-baseline across students
Swanson (1985)	9.5	3	SED, Low-performing	Multiple-baseline design
Thackwray Meyers, Schleser, & Cohen (1985)	8.6	60	Low-performing	Pretest-intervention-posttest; random assignment
Van de Rijt & Van Luit (1998)	5.5	136	Low-performing	Pretest-intervention-post 1,2; matched groups
Van Luit (1987)	11.4	52	LD/EMR/ impulsive	Baseline-intervention-follow-up
Van Luit (1994)	I: 11.7 II: 9.7	I: 16 II: 28	I: EMR II: LD	Pretest-intervention-posttest; matched groups

Wilson & Sindelar (1991)	9	62	LD	Pretest-intervention-post 1, 2 stratified random assignment	Strategy plus sequence instruction vs. strategy only instruction vs. sequence only instruction. Addition and subtraction word problems	14 x 30-min lessons in 1 month*	Strategy plus sequence higher than sequence only; $F(1,57) = 7.49$, $p < .05$; Strategy only higher than sequence only; $F(1,57) = 4.33$, $p < .05$), $d = .21$
Wood, Frank, & Wacker (1998)	10	3	LD	Multiple baseline	Instructional package for multiplication, involving categorizing multiplication facts, mnemonic strategies, steps to be completed for solving facts in each category	3 sessions	Substantial improvement during intervention $d > 3$
Wood, Rosenberg & Carran (1993)	9.5	9	LD	Baseline-instruction-generalization; randomly stratified	Individualized self-instruction vs. observation of self-instruction training vs. control condition	2 sessions	Both experimental groups higher scores on posttest than control group; $d > 3$
Woodward & Baxter (1997)	10	38	Low-performing	Pretest-intervention-posttest	Program aligned with 1989 NCTM- standards versus a more traditional approach in 3rd grade curriculum	Daily sessions in 1 year	For low-achieving students no significant differences; $d = -0.24$

* estimated on the basis of available information
**LD=Learning Disabilities; EMR/MMR=Educable/Mild Mental Retardation; SED=Seriously Emotionally Disturbed; ADHD=Attention Deficit/Hyperactivity Disorder; BD=Behavioral Disorder

Reference	M age	N
Van Luit & Naglieri (1999)	I: 12.7 II: 10.8	42 42
Van Luit & Schopman (2000)	6.2	124
Van Luit & Van der Aalsvoort (1985)	12	4
Waiss & Pasnak (1993)	6	24
Wilson, Majsterek, & Simmons (1996)	10.2	4

Chapter 3
Teaching multiplication to low math performers: Guided versus structured instruction[1]

Abstract

The results of an intervention program for students with difficulties learning mathematics are reported. Two kinds of math intervention, guided versus structured instruction, were compared to regular math instruction. A total of 75 students from regular and special education, aged seven to thirteen, participated. Ability and automaticity multiplication tests were administered before and after the four-month training period. The results show that the students in both of the experimental conditions improved more than the students in the control condition. Some additional differences were found between the two experimental interventions. Guided instruction appeared to be more effective for low performing students than structured instruction and especially for those students in regular education. Special education students appear to benefit most from structured instruction for the automaticity of multiplication problems. A three-month follow-up test showed the acquired knowledge to be well established in both groups.

[1] Kroesbergen, E.H. & Van Luit, J.E.H. (in press). Teaching multiplication to low math performers: Guided versus structured instruction. *Instructional Science.*

3.1 Introduction

In the last decades, mathematics education has been the topic of considerable research and many changes resulting in new teaching approaches. The teaching and learning of mathematics has shifted worldwide towards more realistic education. In the Netherlands, almost every regular elementary school now uses a so-called "realistic mathematics method", which is a method based on the principles of Realistic Mathematics Education (RME). In the US, more than ten years ago, a comparable process was initiated on the basis of the curriculum and evaluation standards of the National Council of Teachers of Mathematics (NCTM, 1989). More than thirty years ago, the foundations for RME were laid by Freudenthal (1968) and his colleagues, who stressed the idea of mathematics as a human activity. Education should give students the "guided" opportunity to "re-invent" mathematics by doing it. The term "realistic" refers to the emphasis that RME places on providing students with problem situations that they can clearly imagine, make real in their minds (Freudenthal, 1991). This can be achieved by offering problems in real-world contexts although such contexts are not a necessary prerequisite. The use of context problems nevertheless plays a significant role in RME and is also promoted by the NCTM standards. They can function as a source for the learning process. By practicing with several different mathematical problems in several different contexts, students learn to use diverse strategies (Van Luit & Naglieri, 1999). During the learning process, not only the interactions between the teacher and the student are important but also those between the students and their peers. Students can learn from each other and each other's strategies. Interaction with peers also promotes conceptual learning (Cobb, 1994).

Another characteristic of RME, which is also promoted by the NCTM standards, is the use of students' own productions and constructions. Students must actively participate in the learning process to become active learners. Whereas the teacher formerly passed on mathematics knowledge in small and basically meaningless parts, students now play an important role in the construction of their own knowledge base. This vision of RME is in line with (socio-) constructivist methods of education (Cobb, 1994; Klein, Beishuizen, & Treffers, 1998). The challenge of teaching from such a constructivist perspective is to create experiences that engage students and encourage them to invent new strategies and learn to understand math. Socio-constructivists consider learning mathematics as a process of becoming familiar with the ideas and methods of the mathematical community (Grave-meijer, 1997). The classroom becomes a community that develops its own mathematics. Children in the classroom acquire new knowledge together. Students work together, learn together, and discuss possible solutions to a problem with each other. They develop problem-solving strategies, which they must then explain and justify to each other. Such learning can be promoted by guided instruction, which forms the focus of this study.

Instruction based on constructivist principles asks students to be proactive participants in the learning process. In practice, the task of the teacher is to structure interaction in such a manner that the students discover new knowledge. This is realized by offering meaningful problems and not teaching strategies. By finding ways to solve problems, the purpose of various strategies becomes apparent to students along with how to apply the necessary strategies. The teacher structures the lessons by asking questions and posing problems. The contribution of the students consists of thinking and discussing possible solutions. Although the teacher can lead the discussion by posing more questions and summarizing what the students say, the students are responsible for bringing forth the strategies to be discussed. By discussing different strategies and solution procedures, students can discover how to solve the relevant problem and thus acquire new knowledge as a result.

Research suggests that instruction based on constructivist principles leads to better results than more direct, traditional mathematics education (Cobb et al., 1991; Gravemeijer et al., 1993; Klein, 1998). And many researchers have observed that learning in such a manner is more motivating, exciting, and challenging (Ginsburg-Block & Fantuzzo, 1998). Students who learn to apply active learning strategies are also expected to acquire more useful and transferable knowledge because, for example, problem solving requires active participation on the part of the learner (Gabrys, Weiner, & Lesgold, 1993).

The question that remains is whether constructivist approaches and RME are also beneficial for *low* performing students (Klein et al., 1998). Van Zoelen, Houtveen, and Booij (1997) conclude that although the average and good students profit from RME, the weaker students appear to benefit much less from this method. Woodward and Baxter (1997) also state that special educators have raised objections to the instructional methods and materials put forth in the NCTM standards because they are too discovery-oriented, and not very sensitive to teaching students with math difficulties. In research, they also show this kind of instruction to benefit most students but those with learning disabilities and low-achievers to a much lesser extent.

Children with learning disabilities and low-performers need special attention to acquire basic mathematics skills (Geary, 1994). Researchers have documented specific mathematical deficiencies. In the domain of computation, for example, students may exhibit deficits and limited proficiency related to fact retrieval, problem conceptualization, speed of processing, and use of effective calculation strategies (Rivera, 1997). These same students also show deficits in the domain of metacognitive skills. Geary, Brown, and Samaranayake (1991), for example, have shown that children with math difficulties frequently use strategies normally used by younger children. More specifically, they usually have difficulties with the use of effective cognitive and metacognitive problem-solving strategies, various memory and retrieval processes, and generalization or transfer. These difficulties express themselves as an inability to acquire and apply computational skills, concepts, reasoning skills, and problem-solving skills. Due to the special needs of children with difficulties learning math, thus, this group of children needs specific instruction (Rivera, 1997).

Instruction should obviously take the particular difficulties of children with special needs into account and especially the lack of spontaneous use of adequate information processing strategies. Van Luit and Naglieri (1999) suggest that teaching step-by-step from concrete to abstract, working with materials to mental representations and providing task-relevant examples can certainly help. Many researchers (e.g., Jones, Wilson, & Bhojwani, 1997; Wood, Frank, & Wacker, 1998) also state that the instruction for children with special needs looks different from regular instruction. Students with math learning difficulties, whether severe or mild, clearly need structured and detailed instruction, explicit task analysis, and explicit instruction for generalization and automatization. This can be realized with direct instruction.

The main characteristic of direct instruction is, in fact, that it is very structured. In practice, direct instruction is teacher-led, because the teacher provides systematic explicit instruction (Jones et al., 1997). New steps in the learning process are taught one at a time, and the teacher decides (guided by the instructional program) when new steps are taught. The lessons are generally built up following the same pattern (e.g., Archer & Isaacson, 1989). In the opening phase, the students' attention is gained, previous lessons are reviewed, and the goals of the lesson are stated. In the main part of the lesson, the teacher demonstrates how a particular task can be solved and then allows the students to work together on the task. When the students appear to have sufficient understanding of the task, they are given new tasks to practice independently. The teacher monitors the students during such practice and provides feedback on completed tasks. Interventions in which students receive direct structured instruction have been frequently found to be very effective (e.g., Harris, Miller, & Mercer, 1995; Jitendra & Hoff, 1996; Van Luit, 1994; Wilson, Majsterek, & Simmons, 1996).

The recommendations mentioned in the literature for teaching students with learning disabilities or low-performing students appear to be in clear opposition to the constructivist principle of guided re-invention. The question is whether teachers can ask low-performing children to actively contribute to lessons by inventing new strategies. Asking for such a contribution actually appears to deny the special status of these children, who clearly have more difficulties with knowledge generalization, connecting new information to old, and the automatization of basic facts.

Recent studies nevertheless suggest that children with learning difficulties can benefit from interventions based on instructions that give students more freedom to develop and use their own strategies (Kroesbergen & Van Luit, in press; Woodward & Baxter, 1997). In other words, RME can perhaps be adjusted in such a manner that it is also very effective for students with math learning difficulties. Mercer, Jordan, and Miller (1994) give a starting point with their description of an "explicit-to-implicit continuum of constructivism". They state that students with learning difficulties clearly need the help of the teacher to become active and self-regulated students. Therefore, a more explicit instruction is needed, but this can nevertheless be designed in accordance with constructivist principles. However, very few studies have examined the possibilities and effectiveness of such interventions (e.g., Woodward & Baxter, 1997), and more research is certainly necessary.

The central question in the present study is therefore whether RME is also appropriate for the instruction of students with learning difficulties and whether or not low performers can benefit from the same type of math instruction as their average performing peers. One characteristic of RME will be examined in particular, namely the contributions of the students themselves. More specifically, it will be examined whether an intervention calling for student's own findings and contributions in connection with guided instruction is more effective than more structured intervention requiring students to apply strategies and procedures modeled by the teacher. In other words, we will attempt to answer the question of whether guided instruction is suitable for teaching children with special learning needs, or if structured instruction is always needed for these children, instead of, or in addition to guided instruction.

3.2 Method

3.2.1 Procedure and design

A quasi-experimental design containing two experimental and one control condition was conducted. Pre-, post-, and follow-up tests were conducted to measure achievement in mathematics ability, automaticity, and transfer. The post-test was administered directly after the training period; the follow-up test was administered three months later, after the summer vacation. The follow-up test was administered to only 58 of the 75 participating students due to a number of students changing schools.

The experimental groups received 30 half-hour multiplication lessons across a period of four months. Research assistants trained and coached by the experimenter conducted the intervention. By means of video observations it was checked if the research assistants implemented the programs as instructed. The lessons were conducted twice weekly in small groups of four to six students each, at the time that the children would normally receive math instruction. On the other three days of the week, the children in the experimental condition followed the regular math curriculum (i.e., the same as their classmates in the control condition), except for the instruction in multiplication. The children in the control condition also followed the regular curriculum and also received a minimum of multiplication instruction twice a week. The regular curriculum included a mixture of different instructional procedures. The instruction that the control children received differed across the different schools but was always somewhere in between guided and structured instruction.

3.2.2 Participants

Seventy-five students from schools for regular and special education participated in the experiment. The students were selected on the basis of low math performance in the opinion of the teacher and as indicated by their scores on general mathematics tests. Only students scoring below the 25th percentile on the national criterion-based tests were selected for inclusion in the study. The students also had to meet the following criteria: they had to be able to count and add to 100 (in order to learn multiplication on the basis of repeated addition) and they had to score below the 50% level on a test of multiplication facts up to ten times ten.

Within each participating school, students were selected according to the above-mentioned criteria. The students were then assigned randomly to an experimental or the control condition. In each school, only one experimental condition was implemented, with one or two groups of five students. In general, the learning difficulties of the students in special education were found to be more severe and complex than the learning problems of those students with special needs in general education.

A total of 75 students from 7 schools were selected: 30 for Guided Instruction (GI), 20 for Structured Instruction (SI), and 25 for the control group (C). As can be seen from Table 3.1, the study included 35 boys and 40 girls, with a mean age of 9.4 years (*sd* = 1.4) and a mean IQ of 88.5 (*sd* = 11.0). Univariate and multivariate analyses showed no differences across the three conditions for age, IQ, or months of multiplication instruction.

However, the children from the regular schools and the children from the special schools were found to differ significantly from each other with regard to age and IQ. As can be seen from Table 3.2, the children in the schools for special education were older (p = .000) and had a lower IQ (p = .000) on average when compared to the children in the regular schools.

Table 3.1
Comparison of groups

Condition	N	Sex		Mean IQ (sd)	Education		Mean	Experience
		male	female		regular	special	age (sd)	in months*
Guided	30	16	14	89.7 (10.1)	20	10	9.5 (1.3)	8.2 (4.9)
Structured	20	8	12	84.1 (11.9)	10	10	9.7 (1.6)	6.5 (6.1)

Control	25	11	14	90.6 (10.8)	18	7	9.1 (1.3)	6.1 (5.9)
Total	75	35	40	88.5 (11.0)	48	27	9.4 (1.4)	7.0 (5.6)

*Experience in months: the time children had received instruction in multiplication before the intervention period

Table 3.2
Comparison of schools

Education	N	Mean IQ (sd)	Mean age (sd)	Mean experience in months (sd)
Regular	48	92.7 (9.7)	8.7 (0.9)	8.0 (5.3)
Special	27	81.0 (9.1)	10.7 (1.3)	5.4 (5.8)

3.2.3 Materials

At pre- and post-test, two multiplication tests were administered: an ability test and an automaticity test (see Appendix 3.1). The ability test contains 20 items with multiplication problems up to 10 x 20. The test consists of ten items in the form of 6 x 7 = ___, and ten in the form of a short story. The children are also asked to write down their solution procedure. The answers are scored as right or wrong, and the solution strategies used by each child are recorded. The ability test includes a total of 10 items beyond 10 x 10. As the students were taught only tasks below 10 x 10, these more difficult items constituted a transfer measure. Transfer refers to the effects of prior learning on later learning in practice (Schopman, 1998).

The automaticity test contains 40 multiplication items of ascending difficulty; 25 of the items are multiplication problems up to 10 x 10; the other 15 items consist of problems with one number below 10 and the other between 10 and 50. The students are asked to solve as many problems as possible within a one-minute period. The answers are then scored as right or wrong.

Instruction program

In this study, two interventions are compared: guided versus structured instruction. To delineate an area within the broad domain of mathematics to be taught within a few months, the study was restricted to the teaching of multiplication problems. For the two experimental interventions, adjustments were made to the MASTER Training Program (Van Luit, Kaskens, & Van der Krol, 1993; see also Van Luit & Naglieri, 1999), which is a remedial program for multiplication and division. In this study, only the multiplication part was used. The program originally consisted of 23 multiplication lessons; in the adjusted versions, two lessons were added. The new program contains three series of lessons: (1) 8 lessons on basic procedures; (2) 11 lessons on multiplication tables; and (3) 6 lessons on 'easy' problems above 10 x 10. In each lesson, a new kind of task is introduced. Each of the series teaches new steps for the problem solving related to specific tasks. The steps between the lessons are small. In order to study the effects of the content of the intervention, it is tried to keep the structure of the two instruction programs as much the same as possible.

Structured instruction. The lessons in the structured condition are always built up in the same pattern. After a repetition of what was done in the preceding lesson, the teacher provides an introduction to the topic of the current lesson. The teacher explains how to solve the task in question, when necessary with the help of special materials like the numberline, blocks or other objects. After the introduction, several tasks are practiced in the group, guided by the teacher, who asks the questions and gives feedback. The students then practice on their own, when necessary with the help of the teacher. All of the tasks are discussed after their completion. During the practice phase, the children familiarize themselves with the kinds of tasks and establish connections to the mental solution of the problems. The goals of the final phase are control, shortening, automatization, and generalization.

The teacher always tells the children how and when to apply a new strategy. The children are then instructed to follow the example of the teacher. A concrete to abstract teaching sequence is followed in teaching new learning steps. The program also promotes the use of the self-instruction method and modeling, by means of a strategy decision sheet, which contains the following questions: "What is the multiplication problem?", "Do I know the answer directly?", "Do I know the answer if I reverse the problem?", "Do I know the answer if I say the multiplication table aloud?", and "Do I know the answer if I do long addition?". Later on, the sheet will be expanded to include the following strategies: splitting by five or ten, using a neighbor problem, and doubling. The strategy decision sheet helps students learn to use different strategies.

To make a clear distinction between both instructions, it was decided that in the structured instruction there would be no room for contributions from the children themselves that are not in line with the procedures the teacher teaches them. It is asked from the children that they contribute to the lessons by answering the teacher's questions and applying the strategies that were taught by the teacher. When a child applies a strategy that has not been taught, the teacher may state that the particular strategy is a possible strategy for solving the problem in question but that they are being taught a different strategy and then request application of the strategy being taught.

Guided instruction. The lessons in the guided instruction condition also start with a review of the previous lesson. What the students do and say in this phase is then taken as the starting point for the current lesson. The teacher states what the topic of the lesson will be (e.g., "we will now practice with the table of 6"). However, the discussion is then centered on the contributions of the children themselves, which means that other topics may sometimes be discussed. Just as in the structured instruction condition, the guided instruction condition contains an introductory phase, a group practice phase, and an individual practice phase. However, in the guided instruction condition, greater attention is devoted to the discussion of possible solution procedures and strategies than in the structured instruction condition.

In the guided instruction condition, much more space is provided for the individual contributions of the students. The main idea is that the teacher presents a problem and the children actively search for a possible solution. The students work together on the solution of the problems, and are given the opportunity to demonstrate their own strategies. The teacher can encourage the discovery of new strategies by offering additional and/or more difficult problems. The teacher supports the learning process by asking questions and promoting discussion between the students. The teacher never demonstrates the use of a particular strategy. As a consequence, when the children do not discover a particular strategy, the strategy will not be discussed within the group. The teacher does, however, structure the discussions during the lessons by helping the students classify various strategies and posing questions about the usefulness of particular strategies, for example.

Control condition. The students in the control condition received instruction as based on the regular curriculum method. Because the students in the control condition are also students with math difficulties, their teachers give generally more attention and extra instruction to them, as compared to their peers. It is important to notice that the control condition is not a 'no intervention' condition, but much more a condition with 'the usual instruction', implemented as optimal as possible. The experimental conditions thus have to compete with a control condition in which the given instruction has already proved to be a good instruction.

Much variance is found in the way the lessons in the control condition were given, because of the variety in teaching methods in the participating schools. The majority of the participating schools used a realistic mathematics teaching method. However, many teachers said that they adapted their instruction to the specific needs of the students by making the instruction less guided and more directed. The teachers also spent more attention to the automatization of the multiplication tables than to strategy use. The regular lessons contain as well instruction as practicing phases. Multiplication is part of the regular program, and in almost every lesson attention is given to multiplication skills. In general, the amount of time spent on multiplication is about one hour a week. The group size in the control condition varied from little groups of three to five students working at the same level to class wide instruction for groups of 25 to 30 students.

The experimental students received the same instruction as their classmates on the days that they did not receive the experimental instruction, except for the multiplication instruction. The teachers were asked to try to give multiplication instruction only at the times that the experimental students received their own instruction, and otherwise the experimental students received worksheets, so that they would not join in the class multiplication instruction.

3.3 Results

In this study, the effects of two mathematics interventions were investigated: guided instruction (GI) and structured instruction (SI). Tables 3.3 and 3.4 show the results of the three conditions for each school type. The effects of instruction in small groups were first compared to the effects of regular instruction (i.e., the two experimental conditions together were compared to the control condition). In addition to this, whether children with math learning difficulties can benefit from a teaching method that requires them to contribute actively to lessons was examined (i.e., the two experimental conditions were compared).

3.3.1 Experimental versus control conditions

First, the difference is studied between both experimental groups together versus the control group. No significant differences were found between the control and experimental groups at pre-test. The results show both the experimental and control groups to improve on both the ability and the automaticity test; however, the amount of improvement differed substantially for the ability test with the experimental groups improving significantly more than the control group, $t(1, 73) = 3.56$, $p = .001$. No significant differences between the experimental and the control group were found for automaticity and transfer. The training thus affected the children's use of strategies the most, which allows them to solve the given problems adequately. The three month follow-up tests showed that the experimental and control groups continued to perform at the same level.

Table 3.3
Means, standarddeviations and effect sizes for ability test (max. score: 20)

condition	school	pretest	posttest	effect size[1]	follow-up
GI	regular	7.4 (4.3)	14.3 (4.1)*	1.67	15.6 (4.4)
	special	4.0 (4.2)	13.8 (5.1)*	2.10	11.6 (3.0)
	both	6.2 (4.5)	14.1 (4.4)*	1.79	14.7 (4.4)
SI	regular	2.8 (2.7)	5.8 (4.3)*	0.83	10.5 (5.2)*
	special	5.0 (4.4)	13.2 (4.9)*	1.75	10.1 (4.3)
	both	3.9 (3.8)	9.5 (5.9)*	1.14	10.3 (4.6)
Control	regular	7.3 (6.5)	9.7 (5.7)*	0.39	13.8 (4.3)*
	special	8.1 (6.8)	13.9 (3.9)*	1.03	13.0 (4.6)
	both	7.5 (6.5)	10.8 (5.5)*	0.55	13.6 (4.3)*

[1] $d = (M_{post}-M_{pre})/\sigma_{pooled}$
* significant improvement compared with previous measurement ($\alpha \leq .05$)

Table 3.4

Means, standarddeviations and effect sizes for automaticity test

condition	school	pretest	posttest	effect size	follow-up
GI	regular	11.8 (2.7)	17.7 (4.4)*	1.61	17.8 (3.9)
	special	8.9 (1.5)	10.7 (1.7)*	1.12	14.2 (2.6)
	both	10.8 (2.7)	15.3 (5.0)*	1.13	17.0 (4.0)
SI	regular	7.6 (2.6)	9.8 (2.9)	0.81	16.1 (6.5)*
	special	4.9 (4.6)	15.2 (4.6)*	2.25	13.7 (4.3)
	both	6.3 (3.9)	12.5 (4.6)*	1.47	14.9 (5.4)
Control	regular	10.6 (5.6)	13.3 (5.7)*	0.69	16.0 (2.7)
	special	7.4 (5.2)	15.6 (2.2)*	1.91	15.6 (1.5)
	both	9.7 (5.6)	14.0 (5.0)*	0.79	15.9 (2.4)

* significant improvement compared with previous measurement ($\alpha \leq .05$)

3.3.2 Structured versus guided instruction

The most important question in this study concerns the potential differences between the two experimental conditions as the purpose of the study was to determine which form of instruction is most effective for low performing children. Before we address this question, some basic differences between the two groups of children in the experimental condition should be mentioned. The children were not randomly assigned to one of the experimental conditions. While the schools themselves were randomly assigned to one of the study conditions, the children within a particular school were assigned to either the experimental condition of the school or the control condition. In other words, differences between schools may contribute to differences between the study conditions. With regard to the pre-test scores, a difference was found on the automaticity test. The children in the SI condition showed lower scores on automaticity than the children in the GI condition, $t(1, 48) = 4.617$, $p = .000$. The ability test showed no differences at pre-test.

Due to the differences between the students in the schools for special education and those in the schools for regular education, a 2 x 2 MANOVA was used to analyze the effects of school type (regular versus special) and condition (guided versus structured). The automaticity pre-test scores were included as a covariate in the analyses. A significant main effect of condition was found. Univariate analyses showed a significant effect of condition on the ability post-test, $F(1, 48) = 11.640$, $p = .001$. The GI group showed the highest scores. No differences were found for the automaticity post-test.

No main effect was found for school type. Interaction effects were found for the ability post-test ($F(1,48) = 9.047$, $p = .004$) and the automaticity post-test ($F(1,47) = 43.469$, $p = .000$). With regard to the ability test, the SI group in regular education showed little improvement while the other groups appeared to clearly benefit from the training. With regard

Table 3.5

Means, standarddeviations and effect sizes for transfer

condition	school	pretest	posttest	effect size	follow-up
GI	regular	2.2 (1.9)	5.7 (3.3)*	1.35	5.9 (2.6)
	special	1.5 (2.5)	6.4 (3.3)*	1.69	3.4 (1.3)*
	both	1.9 (2.0)	5.9 (3.3)*	1.51	5.4 (2.6)
SI	regular	0.5 (1.0)	0.9 (1.1)	0.38	3.4 (2.3)*
	special	1.3 (1.6)	4.9 (3.4)*	1.44	2.3 (2.8)
	both	0.9 (1.3)	2.9 (3.2)*	0.89	2.9 (2.5)
control	regular	2.5 (3.4)	3.1 (2.8)	0.19	5.4 (3.6)*
	special	3.3 (4.1)	5.7 (3.9)*	0.60	5.4 (2.5)
	both	2.7 (3.6)	3.8 (3.3)*	0.32	5.4 (3.3)*

* significant improvement compared with previous measurement ($\alpha \leq .05$)

to the automaticity test, an interesting interaction effect was visible: the children in special education benefited most from the SI condition while the children in regular education benefited most from the GI condition.

After a period of three months, the differences between the conditions appeared to fade somewhat but the acquired knowledge was nevertheless still present for all groups. The SI and control groups in regular education even showed improvement. The strategy use of the children was also studied. However, no significant differences were found. The use of strategies was more or less the same in the different conditions. Although the experimental children were better able to talk about and reflect on their strategies, they did not use alternative or additional strategies when compared to the control children.

A final result concerns the transfer of learned tasks. As can be seen from Table 3.5, the post-test results show significant main effects for condition (p = .003) and school type (p = .002). The students in the GI condition outperformed the students in the other conditions although a significant difference was not found at follow-up. With regard to the effects of school type, it appears that the children in special education improved more on the transfer test than the children in regular education. At follow-up, however, the scores of the children in regular education had improved while those of the children in special education had declined.

In sum, all of the groups improved significantly on the ability and transfer tests although the children receiving guided instruction performed higher than the children receiving structured instruction in both the regular and special schools. The SI group in regular education showed the lowest scores. No main effects of school or instruction were found for the automaticity test although an interaction effect was found to suggest that guided instruction may be better for children in regular education while structured instruction may be better for children in special education.

3.4 Conclusions and discussion

In this study, the effects of guided and structured instruction for the teaching of multiplication skills to low math performers were explored. Seventy-five students participated in the study. Because of this relatively small amount of participants, the results should be interpreted with caution. Further research is necessary before the conclusions could be generalized. In this study, the effects of intervention in small groups versus no intervention at all were first tested. Both of the experimental groups and the control group improved significantly on ability and automaticity. The two experimental groups together showed greater improvement than the control group on the ability test, however. No significant differences were found for improvement in automaticity or transfer. In other words, the two interventions appear to be more effective than regular math instruction. However, this may be partly explained by the fact that the experimental students received the instruction in small groups, which is, in general, more effective than class instruction.

The lack of a difference between the experimental and control groups for automaticity can be partly explained by the fact that the students in the control condition received more instruction in automaticity than the students in the experimental conditions. The accent in the intervention was on insight and strategy use while the teachers in special education in particular tend to emphasize automaticity. It is also worth noting that the experimental students received two times a week special instruction, but the other three days they received the same math instruction as their classmates, except for multiplication instruction. It is plausible that the general math instruction also influences the students' learning of multiplication, which would decrease the effects of a special intervention. Therefore the focus of this study is on the differences between the two experimental conditions, in which the influence of general instruction will be more or less the same for both conditions.

The two experimental groups were found to differ significantly from each other on the ability test with the GI group showing greater improvement than the SI group. This difference was particularly apparent in the schools for regular education. The GI group also outperformed the SI group on the transfer test. The ability test requires students to adequately use and apply various problem-solving strategies. Because the students in the GI condition have learned to actively think and talk about these strategies, it is not surprising that they performed particularly well on this test. The students in the SI condition have learned to apply strategies taught by the teacher and therefore apply these strategies less flexibly in the test. The automaticity test shows a different picture. In regular education, the GI group shows greatest improvement; in special education, the SI group shows greatest improvement. It thus appears that significant differences exist between the needs of children in different types of schools. It seems that guided instruction appears to be particularly well-suited to students in regular education and structured instruction to students in special education. However, this study is limited by the fact that the students were not randomly assigned to one of the experimental conditions, although the schools were. In fact, some small non-significant differences were found between the students of both the conditions. It appeared that the students in the structured condition had a slightly lower IQ, that they had received less multiplication instruction before the training, and showed a lower starting level. Although these differences are not significant, it may have influenced the results.

A remarkable difference between the different types of schools was found for the transfer test. The children in special education showed significantly greater improvement than their peers in regular education during the training period. However, at follow-up measurement, a small decline in the transfer scores for the children in special education was found while the children in regular education showed some slight improvement. It thus appears that the students in special education have learned adequately to apply strategies to new tasks; however, the same skills disappear when intensive training is no longer provided. Conversely, the students in regular education appear to acquire the same skills but also generalize these skills to non-trained problems after completion of intensive training. It should be mentioned here that the follow-up test was administered to only 58 out of the 75 students, and therefore may be less representative for the whole group.

In the period after training, regular math instruction continued, which means that the low-achieving students also received the same instruction as their normally-achieving peers. In keeping with this observation, the follow-up tests also show the differences observed at post-test to have diminished. In other words, the impact of instruction appears to be sufficient to undo the differences produced by intervention.

In conclusion, the results show both forms of math intervention to be more effective than regular math instruction for low performing students. The students in regular education were found to benefit most from guided instruction while the students in special education benefited most from structured instruction. As the learning difficulties of the students in regular education were less severe than the difficulties of the students in special education, however, we should examine the differences between these two groups of low math performers in greater detail.

The children with mild learning difficulties can benefit from the same instruction as their peers: that is, realistic mathematics education appears to be effective provided they are taught at their own level. The children in the present study also received instruction in small groups with the same math level. This means that the teachers were able to monitor the children's understanding of the instruction. However, when these same low performers receive instruction in regular classes, the level of instruction may be too high for them, they may not understand what is happening during instruction, and they may not be able to follow the steps in the learning process, and the extent to which teachers are actually aware of the problems may widely vary. In practice, low performers often receive remedial math instruction, which is usually based on structured instruction. However, the results of the present study show guided instruction to be quite effective for the instruction of such low performing students.

In contrast to the above, the students in special education appeared to benefit most from clearly structured, direct instruction, which is in line with the findings of other studies. However, this finding may also be due to the fact that most students in special education receive only very structured, direct instruction. The duration of the training in this study was four months and perhaps too short for this group of students to get accustomed to a different kind of instruction. Future research involving longer interventions is therefore needed to draw more firm conclusions.

The results of this study show that guided instruction can be quite effective for low performing students. It therefore seems legitimate and very interesting to continue research on realistic math education for such students. In future research, greater attention should be paid to the specific characteristics of the students such as seriousness of the learning difficulties, intelligence, motivation, and strategy use. Greater research should provide greater insight into the effects of the different aspects of realistic math education, and the involvement of a larger number of children, teachers, and schools should make the results more powerful.

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Appendix 3.1
Examples of test items

Automaticity test (40 items)

1 x 4 = ___	6 x 5 = ___
2 x 9 = ___	12 x 4 = ___
6 x 3 = ___	7 x 12 = ___
5 x 9 = ___	2 x 36 = ___

Ability test (20 items)

In a box of chocolates are 4 rows of 7 chocolates each.
How many chocolates does the box contain?

8 x 5 = ___

There are 40 apartments in one building.
How many apartments are in 3 buildings together?

6 x 40 = ___

An accordion has 4 rows of 16 buttons each.
How many buttons does an accordion have?

4 x 15 = ___

Chapter 4
Mathematics instruction for low-achieving students: An intervention study[1]

Abstract

The aim of this study was to compare the effects of two mathematics interventions based on guided versus directed instruction for low-achieving students. A total of 265 students from general and special elementary education schools participated in the study. The students in the experimental groups received a special intervention (guided or directed instruction) aimed at basic multiplication. The effects on the students' automaticity, multiplication ability, and motivation were compared to a control group who followed the regular curriculum. The results show the math performance of the students in the directed instruction condition to improve more than the performance of the students in the GI condition and the performance of the students in both of the experimental conditions to improve more than the performance of the students in the control condition. However, only a few effects on motivational aspects were found. We therefore conclude that the current reforms in the mathematics curriculum requiring students to construct their own knowledge are not very effective for low-achieving students.

[1] Kroesbergen, E.H., Van Luit, J.E.H., & Maas, C.M. (2002). Mathematics instruction for low-achieving students: An intervention study. *Manuscript submitted for publication.*

4.1 Introduction

The mathematics curriculum for elementary school students has changed over the past decades. In the Netherlands, these reforms are mainly based on the didactic principles of Realistic Mathematics Education (RME; Freudenthal, 1991). Also in other countries, the math curriculum has undergone sweeping changes such as the reforms proposed by the National Council of Teachers of Mathematics (NCTM, 1989) in the United States. These changes have led to the creation of curricula in which students are expected to actively contribute to their own mathematics lessons by explaining their mathematical reasoning to each other and constructing their own personal understandings of mathematical concepts. This means that students must listen to both the teacher and their peers, that they be able to explain their mathematical reasoning to others, and that they thus build their own mathematical knowledge (Baxter, Woodward, & Olson, 2001). Research has shown such an approach to be quite promising (Ginsburg-Block & Fantuzzo, 1998; Gravemeijer et al., 1993). Positive effects have been found for both student performance and motivation. Such guided instruction appears to motivate students because they find it more pleasant and more challenging to study in such a manner (Ames & Ames, 1989).

In recent years, the question of whether such reform-based mathematics instruction is as effective for low-achieving students as for normally achieving students has been raised (e.g., Woodward & Baxter, 1997). One of the assumptions underlying the reforms, in fact, is that the new mathematics curricula are effective for all students and therefore for low achievers as well. However, the results of a few recently published studies show this assumption to not always be true. For example, Baxter et al. (2001) studied the responses of low-achieving students to reform-based mathematics instruction and concluded that both the form and content of the mathematics instruction should be adapted to the specific needs of low-achieving students before they can benefit from such reform-based instruction. In an earlier study, Woodward and Baxter (1997) also found an innovative curriculum to clearly benefit average and above average students but students with learning disabilities and low achievers only marginally. In a recent study (Kroesbergen & Van Luit, in press), the effects of instruction requiring active student contributions were compared to the effects of direct instruction, and the instructional method requiring students to construct their own mathematical knowledge was found to be most effective for low-achieving students.

Nevertheless, many researchers are of the opinion that students with learning difficulties need more direct and explicit instruction to learn basic facts and problem-solving skills (Jitendra & Hoff, 1996). Direct teaching is also one of the most popular methods for helping learners acquire greater automaticity (Bottge, 2001; Harris, Miller, & Mercer, 1995). Different studies have shown carefully constructed explicit instruction to be quite effective for the teaching of computational skills (Carnine, 1997). Similarly, Jones, Wilson, and Bhojwani (1997) argue that although the effectiveness of explicit instruction is being questioned in the current mathematics reforms, it is very clear that students with learning disabilities and low math achievers require explicit teaching of the various math concepts, skills, and relationships; in fact, the presentation of multiple approaches and alternative strategies for the computation and solution of problems may only lead to confusion on the part of such students. Furthermore, explicit instruction can increase the motivation of such low-achieving students in addition to facilitating their performance as explicit instruction enables them to handle difficult tasks and thereby motivates them, in many cases, to tackle new tasks (Ames & Ames, 1989).

A discrepancy thus exists between the application of general learning theories, as promoted by the current mathematics reforms, and the application of more specific learning theories for the instruction of low-achieving students. In the present study, the effectiveness of teaching methods based on these different theoretical viewpoints is therefore examined. In order to evaluate the different teaching methods, it is essential to take into consideration how students learn and particularly how low-achieving students learn. Two important goals of the current elementary mathematics curriculum are the automatized mastery of basic operations and the acquisition of adequate problem solving strategies. Given that children with learning difficulties generally have less than adequate memory skills, concomitant storage and retrieval problems, and limited development of the strategies needed for successful problem solving (Rivera, 1997), it is exactly in the two aforementioned areas of mathematics that students with learning difficulties encounter problems. They frequently show motivational deficits as well (Mercer, 1997). In the present study, the focus is on one specific area of the mathematics curriculum, namely the acquisition of basic multiplication facts.

With regard to the acquisition of basic multiplication facts, two important aspects can be distinguished: the development of automaticity (i.e., direct retrieval from long-term memory) and the adequate use of backup strategies. The main instructional goal is the automatized mastery of the basic multiplication facts, which can be reached with practice. Until students have attained full automaticity, however, they must rely on other strategies for the adequate solution of multiplication tasks. For the solution of a given task, the student must typically choose between several strategies (Geary, Brown, & Samaranayake, 1991) and a gradual shift from the use of counting and other backup strategies towards the increased use of direct retrieval with fewer retrieval errors is typically observed (Lemaire & Siegler, 1995). The backup strategies used by students with difficulties learning mathe-matics often resemble the backup strategies used by younger but otherwise normally achieving students. In addition, the long-term memory representations of students with difficulties learning mathematics appear to develop both differently and slower than those of normally achieving students (Bull & Johnston, 1997). As a consequence, low-achieving students have greater difficulties with the retrieval of the relevant information from long-term memory and also make more mistakes than their peers when relying on retrieval strategies.

The central question in the present study is whether low-achieving students benefit more from instruction that requires them to actively contribute to the lessons and construct their own mathematical knowledge under the guidance of a teacher (guided instruction) or explicit instruction that is clearly structured and presented by the teacher (directed instruction). The effects of the two kinds of instruction are investigated on automaticity, multiplication ability, and such motivational variables as goal orientation, self-concept, and beliefs about mathematics.

Our expectation is that students who have to build their own understanding and concepts will gain greater insight into what they have learned and may therefore be more willing to use the strategies that they develop than other students. Furthermore, guided instruction may motivate and challenge students more than directed instruction because the students must actively discover the relevant math facts and solution strategies in the case of guided instruction. Such instruction may also, therefore, result in the more adequate use of backup strategies and thereby higher achievement on multiplication tests. In contrast, direct instruction provides unambiguous information with regard to which strategy to use to solve a multiplication task. When the teacher demonstrates the solution of certain tasks, only correct strategies and solutions are typically presented. Given that low-achieving students often fail to select the most appropriate strategies, direct instruction may lead to improved strategy selection and thus a quicker transition from the use of backup strategies to direct retrieval and thus automatized mastery of the basic multiplication facts. A possible danger of directed instruction, however, is that students may lose their motivation because they are not given opportunities to experiment with their own ideas and solutions (Weinert, Schrader, & Helmke, 1989). Conversely, Ames and Ames (1989) state that directed instruction may encourage and motivate low-achieving students more than guided instruction because the latter may actually discourage them in many cases.

4.2 Method

4.2.1 Procedure and design

In order to compare the effectiveness of the two instructional methods, a group intervention design appeared to be most powerful (Gersten, Baker, & Lloyd, 2000). The group design involved two experimental conditions (guided instruction or GI versus directed instruction or DI) and a control condition (regular curriculum instruction). Pre-, post-, and follow-up tests were conducted to measure achievement with regard to multiplication automaticity and ability. In addition, a motivation questionnaire was administered before and after the training period.

The students in the experimental conditions received a total of 30 multiplication lessons with a duration of 30 minutes each outside the classroom across a period of four to five months. Research assistants, who were intensively trained and coached by the experimenter, conducted the intervention. Logbooks and video observations were used to control for similar implementation of the instructional programs. The lessons were conducted twice weekly in small groups of four to six students each at the time that the students would normally receive math instruction. On the other three days of the week, the students in the experimental condition followed the regular math curriculum with the exception of the multiplication instruction; the teachers were asked to provide regular multiplication instruction at those times when the experimental students were out of the room and otherwise to give the experimental students worksheets to work on during the presentation of the regular multiplication instruction in the class. The students in the control condition followed the regular curriculum on all five days of the week, including the multiplication instruction. Information on the regular curriculum was obtained via a teacher interview and the administration of weekly questionnaires with regard to the amount and content of the mathematics instruction in general and multiplication instruction in particular.

4.2.2 Participants

In this study, 24 elementary schools for general and special education participated. The schools for special education included students with learning and/or behavior disorders and students with mild mental retardation. The students were selected on the basis of low math performance as measured by a national criterion test that divides the population into five categories of percentile scores (A: 75-100%; B: 50-75%; C: 25-50%; D: 10-25%; E: 0-10%). Only students performing at the D or E levels were included in the present study. The students also had to meet the following criteria: they had to be able to count and add to 100 (in order to learn multiplication on the basis of repeated addition) and they had to show no mastery of the multiplication facts up to ten times ten. Two selection tests were conducted to measure the students' performance levels for addition and multiplication.

Within each participating school, the students were selected according to the above-mentioned criteria. The students were then assigned randomly to one of the two experimental conditions or the control condition. In each school, only one experimental condition was implemented with one or two groups of five students. Although all of the students were required to have difficulties learning mathematics, it can be stated that the math learning difficulties of the students in the special education schools were generally accompanied more by additional learning and/or behavior problems than the math learning difficulties of the students in the regular schools. For this reason, whether differences also existed in the intervention outcomes for these two populations of students was also investigated.

Table 4.1
Overview of groups

Condition	N	% Male	Mean IQ (sd)	Education general	special	Mean age (sd)	Mean experience (sd)[a]
Guided	85	62.4	87.4 (12.3)	43	42	9.9 (1.4)	11.2 (6.9)
Directed	83	53.0	88.1 (11.4)	43	40	9.5 (1.3)	8.5 (6.7)
Control	97	57.7	88.8 (12.1)	50	47	9.8 (1.3)	11.5 (7.0)
Total	265	57.7	88.1 (11.9)	136	129	9.7 (1.3)	10.5 (7.0)

[a]*Experience in months: the average time students had received instruction in multiplication before the intervention period*

Table 4.2
Overview of schools

Condition	N	% Male	Mean IQ (sd)	Mean age (sd)	Experience (sd)
General	136	40.4	94.1 (10.6)	8.9 (1.0)	10.8 (7.6)
Special	129	76.0	81.9 (9.9)	10.6 (1.1)	10.2 (6.3)

A total of 283 students were initially selected for the study. Due to missing data and students moving during the school year, a total of 265 students were available for the analyses (see Table 4.1). The study included 153 boys and 112 girls, with a mean age of 9.7 years. Twenty-two percent of the students had an ethnic minority background; these students were equally divided across the three conditions, however. Analyses of variance showed no differences across the three conditions for age or IQ but did reveal a significant difference in the number of months of multiplication instruction, $F(2,262) = 5.876$, $p = .003$. The students in the DI condition had received less instruction on average when compared to the students in the other conditions.

In Table 4.2, an overview of the relevant variables for the students in general education and the students in special education can be found. As expected, the students from the regular schools and the students from the special schools differ significantly from each other with regard to age and IQ. The students in the schools for special education were older ($t(263) = 12.796$, $p = .000$) and had a lower IQ ($t(263) = 9.722$, $p = .000$) when compared to the students in the regular schools. The number of boys and girls was not equally divided across the two types of education ($X^2(1) = 34.245$, $p = .000$); more girls than boys were found to meet the selection criteria in the regular schools while the reverse was true for the special schools. The special education group also contained fewer students from an ethnic minority group than the general education group (8% versus 35%; $X^2(1) = 31.444$, $p = .000$).

4.2.3 Materials

In order to measure the effects of the interventions, four multiplication tests were administered: two automaticity tests and two multiplication ability tests. Automaticity 1 and Ability 1 contain relatively easy tasks while Automaticity 2 and Ability 2 contain more difficult tasks. Automaticity test 1 contains 40 multiplication problems up to 10 x 10. This test is administered orally. The students are asked to solve as many problems as possible within a two-minute period. Automaticity test 2 contains 30 multiplication problems of the type ‘a’ x ‘b’ or ‘b’ x ‘a’ with ‘a’ being a number between 0 and 10, and ‘b’ being a number between 10 and 20. The students are asked to write down the answers to as many problems as possible within 10 minutes. The answers on the tests are then scored as correct or incorrect.

The ability tests contain multiplication problems taken from the test items used to measure the mathematics levels of students in schools for special education in the Netherlands (Kraemer, Van der Schoot, & Engelen, 2000). Ability test 1 is a paper-and-pencil test with 20 items; 16 of the items are multiplication tasks from the tables up to 12 and the other 4 items are: 2 x 110, 3 x 50, 8 x 50, and 4 x 55. The test consists of 12 bare problems and 8 brief story problems. The students are given 20 minutes to complete the test. Ability test 2 is also a paper-and-pencil test containing 20 - mostly context - multiplication problems. This test is more difficult than ability test 1: 5 of the problems are under 10 x 10; 13 are problems with one number under 10 and the other between 10 and 50; and the remaining 2 problems are even more difficult. The answers are scored as correct or incorrect, and the solution strategies used by each student are recorded (for a more detailed description, see Kroesbergen & Van Luit, 2002).

To measure motivational variables, the MMQ 8-11 (Motivation Mathematics Questionnaire; Vermeer & Seegers, 2002) was administered. This questionnaire contains 40 items and consists of 7 scales. Two of the scales measure goal orientation: ego orientation (focus on performing better than one's peers and showing how well you can perform a task) and task orientation (focus on learning new skills and enlarged insight). Two other scales measure beliefs about mathematics: traditional beliefs (in keeping with directed instruction)

Scale	Example
Ego orientation	I like it when I am the only one who knows the answer to a problem
Task orientation	I like it when I can solve a problem that I could not solve before
Traditional beliefs	In math, it is important to remember how to solve a certain problem
Constructivist beliefs	In math, it is important to understand how others solved a problem
External attributions	If I do better than usual, it's because the problems were easier
Effort attributions	If I do better than usual, it's because I tried harder
Self concept	How good are you in mathematics?

Figure 4.1 Examples of motivational scale items.

and constructivist beliefs (in keeping with guided instruction). Two more scales measure attribution style: external attributions (failure and success are ascribed to external factors,) and effort attributions (failure and success are ascribed to the amount of effort exerted). The final scale provides information on the student's self-concept (the picture that the student has of his or her own mathematical abilities). The items are formulated in the form of statements (see Figure 4.1) and the students are asked to indicate the extent to which they agree with a particular statement (1 = 'not at all' and 5 = 'completely').

Instruction program

For purposes of the present study, adjustments were made to the multiplication part of the MASTER Training Program (Van Luit, Kaskens, & Van der Krol, 1993; see also Van Luit & Naglieri, 1999), a remedial program for multiplication and division. A different version of the instructional program was constructed for each of the two experimental conditions. The instructions were changed to create the two different conditions with the worksheets also adjusted to the instructional condition. The instructional program contains three series of lessons: (1) basic procedures; (2) multiplication tables; and (3) "easy" problems over 10 x 10. In each lesson, a new kind of task is introduced. Each of the series teaches new steps for the problem solving related to a specific task. The teacher always keeps the students' existing knowledge in mind and always proceeds at the tempo that appears to fit the students. As already mentioned, the two experimental conditions reflect the use of two different forms of instruction: guided versus directed instruction.

Guided instruction. The lessons in the GI condition always start with a review of the pre-vious lesson. What the students do and say in this phase is then taken as the starting point for the current lesson. The teacher states what the topic of the lesson will be (e.g., "Today we are going to practice with the table of 4"). However, the discussion is then centered on the contributions of the students themselves, which means that topics and strategies other than the ones the teacher had in mind may be discussed. The introductory phase is followed by a group practice phase, and then an individual practice phase. Considerable attention is paid to the discussion of possible solution procedures and strategies.

In the GI condition, considerable space is provided for the individual contributions of the students. The main idea is that the teacher presents a problem and the students actively search for a possible solution. The teacher can encourage the discovery of new strategies by offering additional and/or more difficult problems. The teacher supports the learning process by asking questions and promoting discussion between the students. The teacher never demonstrates the use of a particular strategy. As a consequence, when the students do not discover a particular strategy, the strategy will not be discussed within the group. The teacher does, however, structure the discussions during the lessons by helping the students classify various strategies and posing questions about the usefulness of particular strategies, for example.

Directed instruction. The lessons in the DI condition follow the same pattern. After a repetition of what was done in the preceding lesson, the teacher provides an introduction to the topic of the current lesson (e.g., the table of 4). The teacher explains how to solve the task in question and gives an example of a good solution strategy with the aid, when necessary, of such concrete materials as blocks or the number line. For example, when discussing the table of 4, the teacher mentions the multiplication facts with 4 and discusses how to solve these problems. After such introduction, several tasks from the table are practiced within the group; the students then work on their own. All of the tasks are discussed after their completion. During the practice phase, the students familiarize themselves with the different kinds of tasks and it is assumed that they establish connections to the mental solutions for the problems in such a manner.

The teacher always tells the students how and when to apply a new strategy. The students are then instructed to follow the example of the teacher. The DI program also promotes the use of self-instruction and modeling. There is little room for contributions from the students themselves (i.e., the students must follow the procedures the teacher teaches them). When a student applies a strategy that has not been taught, the teacher may state that the strategy is also a possible strategy for solving the problem in question but that a different strategy is currently being worked with and then request specific application of that strategy.

Control condition

The students in the control condition received instruction based on the regular curriculum used in the school. Given that these students also had math learning difficulties, however, their teachers generally gave them more attention and extra instruction when compared to their peers. It is important to note that the control condition is not a “no intervention” condition but a control condition insofar as the “usual” form of instruction is implemented as optimally as possible.

Considerable variance was found to occur in the manner in which the lessons in the control condition were taught due to the variety of teaching methods used in the partici-pating schools. While the majority of the participating schools utilized a realistic mathematics teaching method, many of the teachers reported adjustment of their instruction to the specific needs of the students by making the instruction more directed and less guided as a consequence. The control condition thus included a mixture of different instructional methods with the specific form of instruction differing across the participating schools but always falling somewhere between guided instruction and directed instruction. The regular lessons contained both instruction and practice phases. Multiplication was part of the regular curriculum, and almost every lesson involved attention to multiplication skills. The average amount of time thus spent on multiplication was approximately one hour a week. The size of the instructional group within the control condition also varied considerably from small groups of three to five students working at the same level to class-based instruction of 25 to 30 students. The size of the instructional groups within the special education schools was generally smaller (3 to 15 students) than the size of the instructional groups within the general education schools (12 to 30 students).

4.2.4 Data analysis

The data analyses were conducted using the MLwiN software (Rasbash et al., 2000). Given the hierarchical structure of the data, multilevel modeling was used to analyze the data. In general, observed differences in the outcome variables can be due to individual characteristics and/or group characteristics. However, the students in the present study often come from the same class and thus have identical scores for such group characteristics as group size or experience of the teacher, which means that the assumption of independence is violated. Multilevel modeling corrects for such dependence. The total amount of variance is thus split into variance at the level of the student and variance at the level of the class, for example. The amount of variance in outcome measures explained by the variables at the different levels is then analyzed (e.g., the individual, class, and school levels).

A first step in multilevel modeling is to analyze the amount of variance at the different levels (with the *empty model* containing no explanatory variables). These analyses called for a two-level model for all of the tests administered (i.e., a model including student- and teacher- or class-level) as the amount of variance was significant for both levels. This means that the children in the same class are more alike than the children in other classes. Analyses based on a model with an additional school level showed no significant variance in any of the outcome variables at that level. Note that the second level consisted of the teacher or class variables and not the experimental intervention groups. This choice was consciously made because the students spent most of their time in their regular classes and only about an hour a week in the intervention group.

Firstly, multivariate multi-level analyses were conducted on the post-test multiplication scores, the follow-up multiplication scores, and the post-test motivation scores. Thereafter, univariate multi-level analyses were undertaken for the different tests. For each test, which of the variables included in this study accounted for the differences observed in the outcome measures was examined to determine the *explanatory model* (the model containing all of the significant variables). The effects of the following variables on multiplication were tested: pre-test, condition (GI, DI, or control), previous multiplication experience (number of months of multiplication instruction already received by student), sexe, ethnicity (native versus foreign), age, general math level, IQ, and type of education (general versus special). For the motivational variables, the effects of improvement in automaticity and ability were added. The variable condition was included in the model as a dummy variable.

4.3 Results

The multiplication outcomes will first be described. In Table 4.3, the means and standard deviations at pre- and post-test for the four multiplication measures are reported for the groups considered together. Paired-samples t-tests show significant improvement on allfour of the multiplication tests. The multivariate analyses showed significant variance in the outcome measures (p = .000), which justifies separate univariate tests. The univariate analyses for the four multiplication tests subsequently showed most of the variance to be located at the student level (respectively 72%, 60%, 58%, and 56%). Which of the variables contributed significantly to the amount of explained variance was considered next. Table 4.4 shows the Beta coefficients for the significant variables and the amount of variance explained by those variables at both student and teacher level. Those variables with no accompanying Beta coefficient were thus not significant. Two nonsignificant (n.s.) effects were nevertheless included in the models because the variable condition was entered as a dummy variable, and it is not possible to include one experimental condition in the model without including the other and because the amount of variance changed significantly when both of the variables DI and GI were excluded.

The variance in the scores on Automaticity test 1 can be partly explained by the variables pre-test, type of education, previous multiplication experience, general math level, and condition. Students in special education perform lower than students in general education. Students who had already received a reasonable amount of previous multiplication instruction improved more than their peers with less previous instruction. Students who performed higher on the general math test did better on the Automaticity test 1 than those students who performed lower. The students in the two experimental conditions performed better than the students in the control condition; however, no significant difference was found between the GI condition and the DI condition.

A significant part of the variance in the Automaticity test 2 is explained by the pre-test score, general math level, and condition. Students with a higher general math level

Table 4.3
Mean, and standard deviations for multiplication tests

Test	Max. score	Pre-test	Post-test	t	p
Automaticity 1	40	17.4 (7.3)	25.8 (7.5)	19.303	.000
Automaticity 2	30	6.7 (5.8)	12.9 (8.2)	15.407	.000
Ability 1	20	10.2 (5.4)	14.8 (3.9)	16.335	.000
Ability 2	20	8.4 (5.2)	11.8 (5.2)	13.610	.000

Table 4.4
Beta coefficients for significant variables on outcome measures of multiplication

Variable	Aut 1[1]	Aut 2	Ab 1	Ab 2
Pre-test score	.54	.49	.33	.55
GI-condition [2]	.16	.19	.15	.08 (n.s.)
DI-condition [2]	.11 (n.s.)	.21	.32	.21
Experience	.16			.13
Sexe			.11	
Ethnicity				
Age				
Math level	.12	.19	.43	.30
IQ				
Education	.33		.17	
% Explained variance				
Teacher level	45	67	62	79
Student level	38	28	43	38

[1] Aut 1, 2: Automaticity tests 1 and 2; Ab 1, 2: Ability tests 1 and 2
[2] Given that the intervention was conducted in small groups and not in the regular classes, corrections were made for the different group and class sizes

improved more than students with a lower general math level. The students in both of the experimental conditions did better than the students in the control condition, but no differences were found between the GI condition and the DI condition.

A considerable part of the variance in the results for the Ability test 1 is explained by the variables in this study. In addition to the pre-test scores for this test, the general math level of the students plays a significant role. Similar to the outcomes for the two automaticity tests, the students with a lower general math level performed worse than students with a higher general math level. The effects of condition are also significant. The students in both of the experimental conditions performed better than the students in the control condition, and the effect of directed instruction was found to be greater than the effect of guided instruction (p = .004). The students in special education did not perform as high as their peers in general education. Remarkable is a significant effect of sex with boys outperforming girls on the Ability test 1.

A large part of the variance in the results for the Ability test 2 is explained by the following variables. Once again, the pre-test scores for this particular test and general math level of the students show the greatest effects. In addition, an effect of months of previous multiplication instruction is found. Type of education shows no significant effect. Condition showed a significant effect, although only the effect of the DI condition is significant. The DI condition was also more effective than the GI condition (p = .011).

To summarize, for all four multiplication tests a significant effect is found for pre-test score, general math level, and condition. The effect of pre-test score means that students with higher pre-test scores also attain higher post-test scores, which is not particularly surprising. The general math level of the students also shows a positive effect, which means that those students with a higher general math level at pre-test improved more than students with a lower general math level. Of special interest are the effects of the two experimental conditions in particular. The automaticity tests show no differences between the two conditions. However, the ability tests clearly show the students in the DI condition to achieve higher than the students in the GI condition at post-test. Two tests, Automaticity 1 and Ability 1, show the students in special education to perform lower than the students in general education. Also an effect of previous multiplication instruction is found for Automaticity 1 and Ability 2.

To investigate whether the acquired knowledge remained after a retention period, follow-up tests were administered three months after post-test. This means that the follow-up test took place after the summer holidays. Thirty-one of the students had changed schools due to removals or entering secondary education. Sixteen of these students were tested in their new schools, but it was not possible to test the other fifteen students. In these analyses we only included the 250 students who participated at all three measurement points. Table 4.5 shows the scores for this group of students. It is remarkable that the students generally performed the same at follow-up as at post-test or did not decrease much. Only Auto-maticity test 2 and Ability test 2 showed a significant decline. Multivariate analyses showed significant multilevel variance (p = .011). However, the univariate tests showed no child variables to cause the differences in improvement in the period of experimental training.

Table 4.5

Means and standard deviations for multiplication tests (N = 250)

Test	Pre-test	Post-test	Follow-up test	*t*	*p*
Automaticity 1	17.2 (7.1)	25.7 (7.5)	25.8 (7.6)	0.410	.682
Automaticity 2	6.7 (5.8)	12.8 (8.2)	12.0 (7.9)	-2.176	.031
Ability 1	10.1 (5.4)	14.7 (3.9)	14.8 (3.9)	0.345	.731
Ability 2	8.2 (5.2)	11.6 (5.2)	11.0 (4.8)	-2.410	.017

Table 4.6
Means and standard deviations for the motivation questionnaire (range 1-5)

Scale	Pre-test		Post-test			
	m	*sd*	*m*	*sd*	*t*	*p*
Ego orientation	3.97	.91	3.84	1.01	2.247	.025
Task orientation	4.09	.68	4.25	.72	-3.207	.002
Constructivist beliefs	3.18	1.14	3.42	1.02	-3.334	.001
Traditional beliefs	4.03	.76	3.82	.83	3.639	.000
Effort attributions	4.28	.67	4.16	.81	2.217	.027
External attributions	3.84	.84	3.61	.96	4.070	.000
Self concept	3.56	.81	3.59	.77	-0.772	.441

As previously stated, not only the effects of the different forms of instruction on the multiplication performance of the students are of interest but also the effects on motivational aspects. In Table 4.6, an overview of the pre- and post-test scores for the motivation questionnaire is presented. It was found that the students, on average, showed significant changes from pre-test to post-test. They became less ego oriented and more task oriented; their beliefs about mathematics also became more in accordance with a constructivist view of learning. Finally, both their external attributions and effort attributions were found to decrease. None of these changes were accompanied by changes in self-concept, however.

The multivariate multilevel analysis proved significant (p = .000), which justifies further univariate analyses. These analyses showed the largest part of the variance in the different measures of motivation to occur at the child level (61, 81, 68, 73, 76, 78, and 78%, respectively). For an overview of the significant variables, the reader is referred to Table 4.7. The results show a differential effect of condition on the scale ego orientation. According to our expectations, the GI condition was less ego oriented than the other two conditions (p = .005). The only other motivation scale influenced by condition was traditional beliefs. The DI group had less traditional beliefs regarding mathematics than the control group. However, the two experimental conditions did not differ from each other on this scale (p > .05). A number of other variables were found to explain part of the variance in the motivation outcomes. Girls had lower scores on self-concept than boys. Also, a negative relation was found between intelligence and the motivation scales of ego orientation, constructivist beliefs, and external attributions. Students from ethnic minorities showed higher scores on the scales of task orientation and external attributions than their native peers. And with regard to the relations between math improvement and motivation, the following observations can be made. Students who improved more on the ability tests showed higher scale scores for task orientation, traditional beliefs, external attributions, and effort attribu-

Table 4.7
Beta coefficients for significant variables on outcome measures of motivation

Variable	Ego	Task	Constr.	Trad.	Effort	External	Self
Sex							-0.22
Age	-0.17		-0.24				
Ethnicity		0.13				0.16	
Education							
DI	0.04			-0.17			
GI	-0.14			-0.10			
IQ	-0.17		-0.16			-0.16	
Pre-test	0.49	0.30	0.32	0.22	0.38	0.42	0.51
Automaticity		-0.14					
Ability		0.12		0.18	0.12	0.18	
% Explained variance							
Teacher level	53	36	42	32	36	84	74
Student level	27	18	12	5	12	16	29

tions when compared to students who improved less on the same tests. However, a negative correlation was found for improvement on the automaticity tests and task orientation. No effect of previous multiplication instruction or type of education was found for any of the motivation scales.

4.4 Conclusions and discussion

In only a few studies to date have the effectiveness of the recent mathematics curriculum changes for low-achieving students been examined (e.g. Baxter et al., 2001; Kroesbergen & Van Luit, in press). One of the central characteristics of these reforms is an emphasis on students' own contributions to their own learning of new knowledge and skills (Freudenthal, 1991). In the present study, the effectiveness of instruction based on student's own contributions (guided instruction) was investigated and compared to the effectiveness of explicit instruction (directed instruction), which is generally recommended for use with low achievers (e.g. Bottge, 2001; Carnine, 1997; Mercer, 1997).

When compared to regular instruction, the GI intervention was found to be very effective. The students in the GI condition improved more on automaticity (both under and above 10 x 10) and multiplication ability; they also became less ego oriented and showed a decline in traditional beliefs regarding mathematics after the intervention. Apparently, low achievers are able to build their own mathematical knowledge and, in contrast to what, for example, Carnine (1997), Jones et al. (1997), and Mercer (1997) have argued, do not necessarily need explicit instruction. Guided instruction was also found to positively influence the students' motivation although the differences were smaller than expected. It is possible that the period of intervention (30 lessons) may have been too short to establish significant differences in motivation.

The DI intervention was also very effective when compared to regular instruction. The students receiving directed instruction showed greater improvement in automaticity and problem-solving ability accompanied by a decline in traditional beliefs regarding mathematics. In the interpretation of these results, however, one should keep in mind that the size of the instructional groups in the control condition was larger than the size of the instructional groups in the experimental conditions. Although the total amount of instruction was the same across the three conditions, the students in the generally smaller groups within the experimental conditions received more intensive training, which probably influenced the results.

Although guided instruction was clearly effective, directed instruction was found to be even more effective and particularly for the students' ability to solve multiplication problems. This finding supports the results of other research (Baxter et al., 2001; Woodward & Baxter, 1997) although the differences from guided instruction were not particularly large in the present study. These results are remarkable because we expected even higher levels of automaticity to be attained in the directed instruction condition and dif-ferences in favor of guided instruction to occur on the ability tests (Kroesbergen & Van Luit, in press). Both instructional methods proved equally effective for the training of automaticity, and directed instruction was found to be more effective than guided instruction for improving the students' problem-solving abilities. These findings confirm the assumption that low-achieving students benefit most from instruction that involves the explicit teaching of a relatively small but adequate repertoire of strategies and just how and when the different strategies can be applied (Jones et al., 1997; Mercer, 1997). Supplying these students with guided opportunities to discover their own strategies and problem solutions, as promoted by the recent mathematics reforms (NCTM, 1991), also appears to be effective but not as effective as direct instruction. One possible explanation for the less than optimal results obtained in the GI condition is that the students in this condition ex-perienced both correct and incorrect solutions, which could produce confusion for low-achieving students in particular (Jones et al., 1997). In the DI condition, only correct solutions were presented and very little confusion could therefore arise.

Although differences were found between the conditions at post-test, no changes were found in the three months following the training. While students' performance is often seen to decline after the period of training has ended (e.g., Case, Harris, & Graham, 1992), the students' performance in the present study barely declined but also did not improve. It should be noted that the six-week summer vacation occurred between post-test and follow-up test, which may explain why the students basically performed at the same level. These findings also suggest that continued domain-specific training should be provided for students with special instructional needs and that teachers should thus not stop once a particular goal has been reached.

Of the seven aspects of motivation measured here, only one aspect was found to vary depending on condition: The DI condition was found to produce higher ego-orientation scores. The focus on the provision of clearly adequate strategies and correct answers in this condition may have fostered the assumption that it is important to do better than the rest by providing the best answers and the best strategies (Nicholls, 1984). A focus on the task itself and what one can learn is presumably better for the learning process. Nevertheless, the motivation questionnaire showed no other differences between the conditions, which means that we cannot conclude that directed instruction is more motivating for low performers than other forms of instruction (Ames & Ames, 1989). The finding of no differences in the students' beliefs regarding mathematics is particularly surprising. In keeping with the instructional principles utilized in the different experimental conditions, we expected those students in the GI condition to develop clearly constructivist beliefs and those students in the DI condition to develop more traditional beliefs. One explanation for the detection of no differences may be that the motivation questionnaire addressed mathematics in general and not multiplication (i.e., the topic of the intervention) in particular. It is possible that the students' beliefs reflect their experiences with the general math curriculum and not the intervention itself. In other words, their general attitudes and beliefs regarding math did not change as a result of the present intervention.

In addition to the instructional interventions themselves, some other variables were found to influence and sometimes unexpectedly not influence the multiplication scores of the students studied here. The students in the regular schools improved more than the students in the special schools. This is not surprising in light of the fact that students in special education tend to have greater problems learning than students in general education. Surprising is that two of the four multiplication tests showed no effects of type of education. This means that the differences between the students in special versus general education are not as great as we think and that the students often have very similar instructional needs (Kavale & Forness, 1992). Along these lines, it is quite remarkable that no effects of intelligence were found while a relatively strong association between intelligence and mathematical ability is normally assumed (Naglieri, 2001). Further research should be conducted to gain a better picture of this relation. Finally, general math level was found to be an important predictor of the intervention outcomes with those students initially showing a higher math level benefiting most from the intervention. This may reflect a generally higher learning ability or the idea that only after students have mastered certain math skills can they more easily acquire other math skills.

To conclude, directed math instruction was found to be more effective than guided math instruction although the guided instruction was still more effective than regular math instruction for the low-achieving students examined in the present study. This means that the current reforms in the mathematics curricula are not based on the most adequate instruction principles for low-achieving students. Low-achieving students have special needs, and their instruction should clearly be adapted to these special needs.

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Chapter 5
Effects of different forms of instruction on acquisition and use of multiplication strategies by children with math learning difficulties[1]

Abstract

This chapter describes the results of an intervention study for students with math learning difficulties. A total of 265 students from elementary schools for regular and special education participated. The students were assigned to one of three conditions: the Guided Instruction (GI) condition, the Directed Instruction (DI) condition, or a control condition. The students in the two experimental conditions received training in multiplication for 4 to 5 months. In the GI condition, the students received an instruction that left considerable room for their own contributions and discoveries. In the DI condition, the teacher clearly and continually steered the instruction. To investigate the effects of the training, two tests were administered before and after the training period: a written and an oral multiplication test. The multiplication performances of the students from all three groups were found to improve equally. They all showed improved strategy use. A few significant differences were nevertheless found between the experimental versus control groups, in favor of the experimental groups, while the differences between the two experimental groups proved small.

[1] Kroesbergen, E.H. & Van Luit, J.E.H. (2002). Effects of different forms of instruction on acquisition and use of multiplication strategies by children with math learning difficulties. In P. Ghesquière & A.J.J.M. Ruijssenaars (Eds). *Children with learning disabilities: A challenge to teaching and instruction.* Submitted for publication.

5.1 Introduction

This chapter focuses on how children with math learning difficulties learn multiplication, and particularly how they learn adequate strategy use. Most students easily learn the multiplication tables up to 10 x 10. However, some students really do not understand the meaning of the symbol “x” and, for example, that 5 + 5 is not the same as 5 x 5. These students have many difficulties remembering all of the multiplication facts. Almost every teacher will recognize a student such as Bob, a 9-year-old boy who participated in this study. Bob is in fourth grade and has had about two years of multiplication instruction. Due to considerable instruction and extra help, he now knows most of the multiplication tables, although he still makes many mistakes. One of the facts that he knows well is 2 x 4 = 8. However, when the teacher asks Bob to lay down ‘2 x 4’ with blocks, he lays down one group of two blocks and one group of four blocks; he then starts to count and, when he arrives at six, thinks he must have made a mistake and starts counting again. Two times four really makes 6? He does not understand what is wrong. Bob is only one example of a student with serious math learning difficulties.

About 25 percent of students have difficulties learning math, and about five to ten percent have serious math difficulties (Rivera, 1997). These difficulties can manifest themselves in different ways and can have different causes. The difficulties can be explained by such student factors as intelligence, motivation, memory skills, metacognitive skills, or vocabulary. Another explanation may lie in the instruction; students with difficulties have special needs and therefore require special instruction (Carnine, 1997). Instruction should be adapted to the specific needs of students (Geary, 1994). In addition, special remedial teaching programs may be needed to teach and support students with learning difficulties. This chapter reports on a study in which two different teaching programs were offered to low performing students. Before describing the design and the results, however, we will first describe the problems encountered by children with math learning difficulties and how multiplication strategies can be taught in particular.

5.1.1 Children with math difficulties

Given that every student is unique and every student with math learning difficulties may encounter other difficulties, it is almost impossible to describe the typical student with math learning difficulties. Nevertheless, many similarities can be found within a group of such students. In fact, researchers have documented a number of specific mathematical deficiencies. For example, in the area of computation, students may exhibit deficits in the skills related to fact retrieval, problem conceptualization, speed of processing, and the use of effective calculation strategies (Rivera, 1997). Cawley and Miller (1989) have reported that the mathematical knowledge of students with serious learning difficulties tends to progress at a rate of approximately one year for every two years of school attendance. Most of the students with such learning difficulties also appear to reach a plateau during sixth or seventh grade and continue to encounter math difficulties throughout high school and into adult life.

Some of the students with math problems simply lag behind in the acquisition of procedural knowledge; others show developmental differences that do not necessarily disappear with age. Research suggests that the math deficiencies of students with learning difficulties emerge in the early years (Mercer & Miller, 1992; Schopman & Van Luit, 1996). An ability to form and remember associations, understand basic relationships, and make simple generalizations appears to be a basic cognitive skill needed to follow initial (formal) math instruction. More complex cognitive skills are needed as students progress to more complicated mathematical skills. In addition to this, the mastery of lower level skills is essential for the acquisition of higher level skills (Mercer & Miller, 1992). Stated concretely: Difficulties learning early numeracy during kindergarten may impede the learning of basic math skills during grade one and later (Schopman & Van Luit, 1996).

During first grade, students start with the learning of basic addition facts and progress to the learning of basic subtraction, multiplication, and division facts. These basic facts are necessary for the acquisition and understanding of higher level mathematical knowledge. Automaticity is also reached with lots of practice, but students with learning difficulties are often found to have many more problems with the development of such automaticity than their normal achieving peers (Schopman & Van Luit, 1996). Research shows, for example, that students with math difficulties must often calculate basic facts while other students simply know the facts directly (Pellegrino & Goldman, 1987). The development of long-term memory representations also proceeds more slowly or differently for children with math difficulties when compared to their peers (Geary, Brown, & Samaranayake, 1991). This leads also to difficulties in fact retrieval. In addition, such students continue to make more mistakes on basic skills than their peers.

Even when the students with learning difficulties master the basic computational skills, the problem-solving strategies they use may still differ from those of their peers. In general, students with learning difficulties have been found to use fewer and less adequate strategies than their normal-achieving peers (Geary et al., 1991; Rivera, 1997). The low math performance of many students can be explained by a lack of adequate strategies for the selection and processing of information and metacognitive deficiencies. Students with math learning difficulties can simply be overwhelmed by the memorization, strategies, vocabulary, and language coding required when the instruction is verbal, the introduction of concepts is rapid, and there is not sufficient time for review and practice. Such difficulties can then express themselves as an inability to acquire and apply computational, reasoning, and problem-solving skills (Rivera, 1997). The gap between the math achievement of students with learning difficulties and that of their classmates will only increase and become particularly apparent in the area of problem solving because the strategies needed for problem solving require not only mathematic competence but also linguistic competence (Carnine, 1997).

Math has a logical structure. Students construct simple relations first and then progress to more complex tasks. As the student progresses with the construction of different math tasks, the content and skills they have discovered are usually transferred as well (Mercer & Miller, 1992). However, students with math difficulties often fail to make the necessary connections between different problems and strategies. They do not automatically apply learned strategies to other situations, and the instruction for such students should therefore be adapted to their specific characteristics and needs.

5.1.2 Teaching multiplication strategies

For adequate problem solving, different types of knowledge are required. A distinction can be made between the more general metacognitive knowledge of students or their awareness of the cognitive processes and strategies that they use to approach, organize, and evaluate the solution to a problem and the more task-specific knowledge and strategies that they apply to solve a problem. Three components of metacognitive knowledge can be distinguished. To start with, students must have declarative knowledge: knowledge of the relevant quantitative concepts, operations, algorithms, and problem-solving strategies (Montague, 1992). Second, students must have procedural knowledge to apply their declarative knowledge: procedural knowledge is knowing how to apply learned skills to achieve certain goals. Third, students must have conditional knowledge, which enables them to select the appropriate strategies and apply these for the solution of specific tasks.

Students with learning difficulties are typically poor problem solvers. They show deficits in all types of knowledge (Montague, 1992). This means that they experience many difficulties with the learning of both general and specific math knowledge, tend to have insufficient information processing skills, do not apply information adequately, and experience troubles with the efficient organization of an approach to the problem-solving task (Case, Harris, & Graham, 1992). Instruction should therefore focus on these deficiencies, both on promoting automaticity and on acquiring adequate strategies. As Goldman (1989) states, the emphasis during instruction should lie on the procedures for solving problems. Students should be explicitly informed about the appropriate strategies for the solution of a given task. Most researchers state that students with learning difficulties need structured, teacher-directed instruction (Jitendra & Hoff, 1996), particularly when they have not had many learning experiences. In addition to this, of course, the instruction should involve activities to promote conceptual understanding.

With regard to strategy instruction, a distinction can be made between direct versus guided instruction (Goldman, 1989). Direct instruction is mostly used to teach task-specific strategies. Students are explicitly taught the steps for the solution of particular types of problems. The instruction is usually scripted and therefore structured step by step to insure mastery of a particular strategy before proceeding to the next type of problem. The focus of direct instruction when it comes to metacognitive knowledge is mainly on declarative knowledge (task-specific strategies), partly on procedural knowledge (how to apply task-specific strategies), and less on conditional knowledge (how to select strategies). Guided instruction, in contrast, is aimed at enabling students themselves to discover what multiplication is all about and just how multiplication problems can be solved. During guided learning, the learning process stands central and not the strategies or solution procedures that the teacher has in mind. In addition to this, the three components of metacognitive knowledge all play an important role in such instruction.

5.1.3 Research questions

The focus of this study is on students' acquisition and use of multiplication strategies. The emphasis is on the use of task-specific strategies for the solution of multiplication problems. For multiplication, a distinction can be made between such basic strategies as counting and repeated addition versus more advanced strategies such as splitting at five and doubling. However, the goal of teaching multiplication tables is always the automatization of multiplication facts. The central question in this study is therefore whether a differential effect of instructional method can be found for the strategies students apply during multiplication. For this purpose, the methods of directed versus guided instruction were compared. The first research question addresses just how the multiplication strategies used by the students develop during the intervention period in which the students received one of both instructional methods. The second question addresses the differential effects of the instructional methods immediately after intervention on both the test scores of the students (question 2a) and the strategies used by the students (question 2b).

5.2 Method

5.2.1 Procedure and design

In order to measure the effects of different instructional methods, a quasi-experimental design containing two experimental conditions and one control condition was utilized. Pre- and post-test scores were calculated before and after instruction.

For a period of four to five months, the students in the experimental groups received twice weekly multiplication lessons with a duration of 30 to 45 minutes. Research assistants, trained and coached by the experimenter, conducted the lessons in small groups of four to six students each. To follow the development of the students, the research assistants kept a logbook for every lesson. In addition, 5 of the 30 lessons were videotaped. Given that the special instruction was undertaken during the time that the students would normally receive math instruction, the total amount of math instruction was comparable across the groups. On the days when the students in the experimental groups followed the regular math instruction in the class, they received the same instruction as their classmates with the exception of the multiplication instruction.

The students in the control condition took part in the regular mathematics curriculum, which included multiplication instruction. Information on their mathematics instruction was obtained via a teacher questionnaire. The instruction that the control children received differed across schools but was always found to be somewhere between directed and guided instruction.

5.2.2 Participants

The participants in this study were students from elementary schools for regular and special education. The students were selected for low math performance on the basis of 1) their scores on a general mathematics test (below the 25th percentile) and 2) the opinion of the teacher. Further selection was conducted on the basis of two tests. The first test contained addition and subtraction problems up to 100. Students who scored below the 60% level on this test were excluded from the study as simply not ready to learn multiplication on the basis of repeated addition. The second selection test contained multiplication facts up to 100. Students who scored above the 50% level were excluded as already having a reasonable multiplication performance level.

Within each participating school, the students meeting the aforementioned criteria were next assigned randomly to an experimental or control condition. Only one experimental condition was implemented per school. A total of 265 students from 24 schools were selected for inclusion in the study: 88 for Guided Instruction (GI), 87 for Directed Instruction (DI), and 90 for the Control condition (C). An overview of the descriptive information for the different groups is presented in Table 5.1. Multivariate analyses showed no significant differences across the three conditions for age or IQ but a significant difference for months

Table 5.1
Comparison of groups

Condition	N	Sex		Mean IQ (sd)	Education		Mean	Experience
		male	female		regular	special	age (sd)	(sd)[1]

Guided	88	55	33	87.3 (12.1)	42	46	9.8 (1.4)	11.0 (6.8)
Directed	87	45	42	88.4 (11.7)	44	43	9.4 (1.3)	8.2 (6.7)
Control	90	53	37	88.7 (12.3)	47	43	9.7 (1.3)	10.8 (6.0)
Total	265	153	112	88.1 (12.0)	133	132	9.7 (1.4)	10.0 (6.6)

[1]Experience: number of months that children had received instruction in multiplication before the intervention period

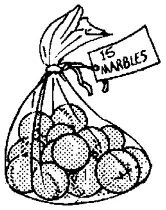
of multiplication instruction received, $F(2,262) = 5.278$, $p = .006$. The students in the DI condition had, on average, received less multiplication instruction when compared to the students in the other conditions.

5.2.3 Materials

In order to gain insight into the solution procedures used by the students, they were given a paper-and-pencil test with a scratch paper to write down the strategy they used for every problem. In addition to this, each child was interviewed and asked to "think aloud" while they solved the given problems. Both the paper-and-pencil test and the interview test were conducted before and after the intervention period.

Paper-and-pencil test

The paper-and-pencil test contained 20 multiplication problems taken from the test items used to measure the mathematics levels of students in schools for special education in the Netherlands (Kraemer, Van der Schoot, & Engelen, 2000). Most of the problems were context problems: multiplication problems embedded in a short story that is recognizable for the students; the reader is referred to Figure 5.1 for an example of such a problem. Only four of the problems were bare math problems with no accompanying text or picture. The difficulty of the problems varied from 3 x 3 and 5 x 5 to 8 x 12 and 5 x 1.25. Only five of the problems were below 10 x 10. Two of the problems were actually division problems that could also be solved using multiplication strategies (e.g., "My aunt wants to paste 50 photos in an album. She wants to paste 5 photos on a page. How many pages will she fill?"). The students were also asked to write the solution procedure they used down on a scratch paper for each problem. The answers were scored as right or wrong, and the solution strategies used by each child recorded.



Fred buys 5 bags.

That makes … marbles

Figure 5.1: Example from the paper-and-pencil test

Interview test

Ten bare problems from the multiplication tables up to twelve were used for the interview. The students were asked to "think aloud" while solving the problems. When only the answer was mentioned and not the solution strategy, the research assistant asked the student how he or she knew the answer and continued to question the student until the strategy was clear. The answers were scored as right or wrong, and the solution strategies used by each child recorded.

Instructional program

In this study, use was made of adapted versions of the multiplication part of the MASTER Training Program (Van Luit, Kaskens, & Van der Krol, 1993; see also Van Luit & Naglieri, 1999). The experimental programs consisted of a series of 25 multiplication lessons: 8 lessons on basic procedures; 11 lessons on multiplication tables; and 6 lessons on "easy" problems above 10 x 10. The emphasis in the lessons was on: (1) automated mastery of the multiplication facts as this knowledge is necessary for further learning and adequate problem solving, and (2) the use of strategies including metacognitive knowledge of how to apply strategies and how to select the most appropriate strategies. The teacher always kept the students' existing knowledge in mind and always proceeded at a tempo that fit the students. As already mentioned, the two experimental conditions reflect the use of two different forms of instruction: guided versus directed instruction.

Guided instruction. The lessons in the GI condition are always started with a review of the previous lesson. What the students do and say in this phase is taken as the starting point for the current lesson. When it appears that the students do not fully understand the tasks discussed in the previous lesson, the focus is again on these tasks. Otherwise, the teacher introduces the next topic (e.g., "Today we are going to practice with the table of 3"). During guided instruction, the discussion is always centered on the contributions of the children themselves, which means that topics and strategies other than those that the teacher has in mind may sometimes be addressed. Just as in the DI condition, the GI condition contains an introductory phase, a group practice phase, and an individual practice phase. However, in the GI condition, greater attention is devoted to the discussion of possible solution procedures and strategies than in the DI condition.

In the GI condition, much more space is provided for the individual contributions of the students. The main idea is that the teacher presents a problem and the children actively search for a possible solution by making a selection from their own repertoire or exploring new strategies. The teacher can encourage the discovery of new strategies by offering additional and/or more difficult problems. The teacher facilitates the learning process by asking questions and promoting the discussion of specific tasks and solutions by the students. The teacher never demonstrates the use of a particular strategy; this means that if none of the children discover a particular strategy, the strategy will not be discussed within the group. The teacher can, however, structure the discussions by asking the students to classify various strategies and posing questions about the usefulness of particular strategies, for example.

Directed instruction. The lessons in the DI condition are also always started with a repetition of what was done in the preceding lesson. When the students show sufficient knowledge of the preceding lesson, the teacher can proceed to the actual lesson. After a short introduction, the teacher explains how to solve the task in question and gives an example of a good solution strategy with the aid of concrete materials when necessary. After the presentation of one or two examples, several tasks are next practiced within the group with an emphasis on the use of strategies. The students then practice on their own, and the various strategies used to solve the tasks are discussed thereafter. During the practice phase, the children familiarize themselves with the kinds of tasks involved and establish connections to the mental solution of the problems. The goals of the final phase are control, shortening, automatization, and generalization.

The explicit teaching of new strategies is intended to help the students expand their strategy repertoires. When a new strategy is taught, it is always the teacher who tells the children how and when to apply the strategy. The children are then instructed to follow the example of the teacher. In the DI condition, there is little room for input from the children themselves (i.e., the children must follow the procedures the teacher teaches them). When a child applies a strategy that has not been taught, the teacher may observe that the particular strategy is certainly a possible strategy for solving the problem in question but that they are being taught a different strategy and then ask the child to apply the strategy being taught. The students in the directed instruction condition learn to work with a strategy decision sheet to help them choose the most appropriate strategy. The strategies taught in this condition are: reversal, splitting at five or ten, neighbor problem, doubling, saying the table, and repeated addition.

5.2.4 Coding of strategies

As already noted, the answers provided by the students on the paper-and-pencil test and the interview test were scored as right or wrong. In addition to this, the strategies the children used to solve the problems were coded. If the strategy (or strategies) used by a student was not apparent or completely unclear, this was coded as "unknown." When a student indicated that he "knew the answer out of his head" (on the scratch paper or during the interview), this was coded as "automatized." The following strategies were coded:

1. automatized;
2. reversal (5 x 6 = 6 x 5);
3. splitting at five or ten – addition (8 x 7 = 5 x 7 + 3 x 7; 13 x 5 = 10 x 5 + 3 x 5);
4. splitting at five or ten – subtraction (9 x 7 = 10 x 7 – 1 x 7);
5. doubling (6 x 4 = 3 x 4 + 3 x 4);
6. neighbor (8 x 9 = 9 x 9 – 1 x 9);

7. reciting aloud or writing down the multiplication table;
8. repeated addition;
9. use of concrete materials or drawing;
10. division;
11. and all possible combinations of such strategies.

For every student, the number of different strategies used was counted (strategy repertoire). For example, when a student used six times repeated addition, two times splitting at five, one time reversal and doubling, and one time reversal and repeated addition, his repertoire score was four as he used four different strategies.

In addition to the number of different strategies used, the relative frequency and efficiency of the students' strategy use was considered. To measure the efficiency of strategy selection, the strategies were assigned to one of the following five categories, based on the steps needed to solve the problem.

1. retrieval;
2. adequate solution requiring one step (e.g., reversing or splitting);
3. semi-adequate solution requiring two steps (e.g., reversal and splitting, splitting and adding: 7 x 4 = 5 x 4 + 4 + 4);
4. not very adequate solution requiring more than two steps (e.g., 4 x 8 = 8 x 4; 2 x 4 = 8; 8 + 8 = 16; 16 + 16 = 32)
5. repeated addition (more than two steps) or counting.

A mean efficacy score was then calculated for each student. For the example mentioned in connection with the number of different strategies used, the student's efficacy score would be 4.2 (6 x 5 + 2 x 2 + 3 + 5 / 10). The lower the score, the higher the efficacy.

5.3 Results

To compare the effects of the different instructional methods on the strategies the students apply for multiplication, two questions were raised. First, how do the strategies used by the students for multiplication develop during the intervention period? Second, do the different methods of instruction produce significant differences in the performance of the students at post-test?

5.3.1 The development of the strategies for multiplication during intervention

Based on the video recordings of lessons 2 (learning that multiplication is the same as repeated addition), 8 (using different strategies), and 14 (the multiplication table of 4), the learning of one group of students receiving GI and one group of students receiving DI will be described. Both of the groups had four students and, at the start of the intervention, both of the groups had already received about 12 months of multiplication instruction. For the GI group, the IQs ranged from 78 to 116; for the DI group, the IQs were more homogeneous and ranged from 92 to 98.

Group 1: Guided instruction (lessons 2, 8, and 14)

Lesson two starts with a repetition of the content of the preceding lesson. In response to the teacher, one of the students mentions that they practiced making "table sums" and demonstrates this by laying down 4 groups of 2 blocks and saying "4 times 2, that makes 8." The teacher responds by saying: "Very good, there are 4 groups of blocks, and 2 blocks in every group…that makes 4 x 2." They continue with this kind of practice, with the teacher asking the students to mention several problems matching the situation. For example, when the teacher asks about four bikes with two wheels each, one can say 2 + 2 + 2 + 2, 4 x 2, or 2 x 4. The teacher asks more questions to prompt the students to provide other solutions. Sometimes the students correct each other's mistakes. In the case of three groups of five blocks lying on the table, for example, one of the students says "5 plus 5 minus 5." One of the other students immediately responds by telling him that this is wrong. When the teacher asks for an explanation, he explains that you have to do 5 + 5 + 5 or you won't have 15 blocks. In this same lesson, one of the students already uses the doubling strategy. The students are working on the following workbook problem: "How many wheels do two cars have?" (see Figure 5.2). Note that the problems and not the answers stand central during the first lessons. The "answer" to this problem is 2 x 4 or 4 + 4. Now the problem is: "How many wheels do four cars have?" One of the students answers "2 x 8, because 8 is 2 x 4, and then double…that makes 16." The teacher then demonstrates, by moving the blocks, what the student has done and thus the doubling strategy. The students also demonstrate in this lesson that they understand that multiplication problems can be reversed.

In the eighth lesson, the teacher asks more specifically about the reversibility of multiplication. By asking how many 5 x 3 makes and how many 3 x 5 makes, the teacher helps the students see that the answer is the same for both problems. The teacher then asks whether this is always true and why. After discussion of the fact that multiplication problems can always be reversed, the discussion moves to the application of such reversibility. The students initially have some difficulties with the idea but decide in the end, when you have to calculate 4 x 9 and you do not know the table of 9 but *do* know the table of 4, that you might as well reverse the problem and find the answer to 9 x 4. From this lesson on, the focus of the lessons is on the use of different strategies. Just which strategies are discussed in the lessons depends on the contributions of the students themselves.

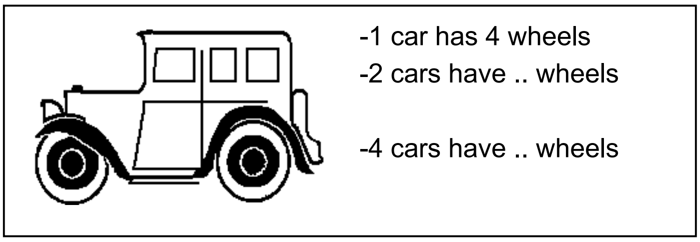


Figure 5.2 Textbook problem for GI condition

The fourteenth lesson starts with the teacher telling the students what the topic is and asking them if they already know one or more problems from the table of four. The discussion then proceeds as follows (s = student, t = teacher):

s1: 1 x 4 = 4
s2: and 0 x 4 = 0
s3: I don't know for sure, but I think that 4 x 4 = 16
s2: yes, that's right
s4: 9 x 4 = 36
s1: 10 x 4 = 40
s2: 7 x 4 = 24
s3: no, that's wrong, that's 28, because 6 x 4 = 24
s1: or 7 x 3 = 21, and 7 makes 28
t: so you can also use the table of three, when you don't know the answer…
s2: but how do you know that you have to add 7?
s3: because it's about the table of 7
t: look, what s1 did, when you don't know 7 x 4, you can do 7 x 3 and 7 x 1, that equals 7x4
s3: yes, because it is in fact the table of 7
s4: or 8 x 4 = 32, minus 4 is 28.

As can be seen, the students already demonstrate different strategies to solve the same problem. The teacher's role can be seen to be small, as the students clearly correct and help each other. The strategies used in this lesson were: neighbor problem (7x4=8x4-1x4), reversal (7x3=3x7), splitting at five (6x4=5x4+1x4) or ten (9x4=10x4-1x4), and doubling (6x4=6x2+6x2). The students were also seen to clearly discuss the use of different strategies with each other.

Group 2: Directed instruction (lessons 2, 8, and 14)

In lesson two, the teacher starts with a repetition of what they did in the preceding lesson: "Do you remember what we did the last time? We made groups with blocks; for example, 3 groups of 3 blocks, like this, and then we said that the matching sum was 3 x 3." The group continues with such tasks. For example, the teacher asks the students to lay down two groups of three blocks and asks which multiplication problem is then represented by this. One of the students answers 2 x 3, and another answers 3 x 2. The teacher then responds that 3 x 2 is not exactly the same as 2 x 3 and that you can also say 3 + 3. The group continues with such tasks until all of the students are able to state both the multiplication and the addition problem in response to a given task. The workbook problem with the wheels of a car is also discussed in this group (see Figure 5.3 and note the difference from Figure 5.2).

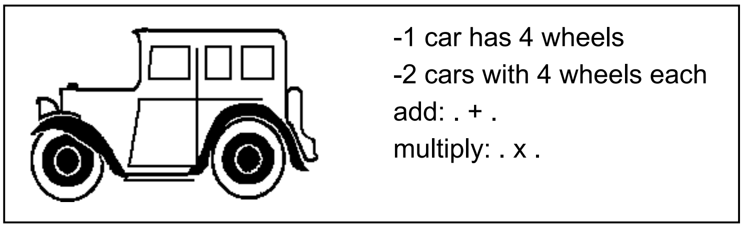


Figure 5.3 Textbook problem for DI condition

After the students have seen that 2 cars have 4 + 4 wheels, the teacher asks the students to depict the problem using blocks. When the students do not know how to do this, the teacher shows them how and explains that “this is one car with four wheels, and this is another car with four wheels, the addition sum is 4 + 4.” She also explains why the multiplication sum is 2 x 4. She then proceeds to the next question: What is the addition sum for four cars? When the first student says 4 + 4, the teacher asks how many more should be added. Another student answers 4 x 4. However, this is not a correct answer because the teacher asked for addition. An answer of 8 + 8 is also not correct. The teacher continues systematically until the correct answer is provided or, if the answer is not provided by one of the students, she provides the answer herself. The only strategy for multiplication used in this lesson is the repeated addition strategy.

In the eighth lesson, the teacher explains the reversal strategy. She demonstrates, using the blocks, that both 3 x 4 = 12 and 4 x 3 = 12. She explains that you can always reverse a multiplication problem. After the students have practiced with such reversibility, the teacher continues on with an explanation of the strategy decision sheet. The sheet presented in this lesson is relatively simple; each time that a new strategy is learned, this strategy will be added to the sheet in the form of a possible new step. The sheet handed out in this lesson contains the following questions: “What is the multiplication problem?”, “Do I know the answer directly?”, “Do I know the answer if I reverse the problem?”, “Do I know the answer if I say the multiplication table aloud?”, and “Do I know the answer if I do long addition?”. Later on, the page will be expanded to include the following strategies: splitting by five or ten, using a neighbor problem, and doubling. The strategy decision sheet helps students learn to use different strategies.

The topic of the fourteenth lesson is the table of four. The teacher lays strips of paper with four presents printed on each strip in the multiplication box (see Figure 5.4). As she lays down a strip, she asks: “How many presents do I have when there are 1 (2, 3, 4,…10) rows?” The students then answer in turn. When they have reached 5 x 4 = 20, she adds “yes, this is an important one.” The student who is supposed to respond 8 x 4 does not know the answer, so the teacher helps by saying: “We just said that 7 x 4 makes 28, so what is 8 x 4?”. The student then counts on until he reaches 32. When the students proceed to work individually in their workbooks, the teacher explains the strategy decision sheet once again. She explains that the students can use this sheet to solve difficult problems and that, by doing this, they can learn to use different strategies.

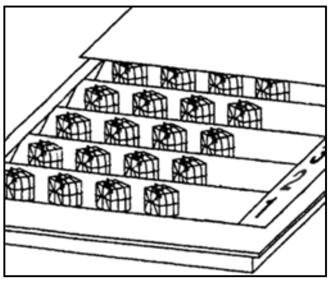


Figure 5.4: Multiplication box

5.3.2 Effects of instruction on test performance and strategy use

In order to compare the different methods of instruction, the test performances of the students in the different conditions were compared. Furthermore, the particular strategies and any changes in the strategies they use were examined. In addition to this, the differences between the two methods of testing (written versus oral) were considered.

In Table 5.2, the results for the pre- versus post-test performance of the students in the three conditions are presented. No significant differences were found between the three conditions, neither at pre-test or post-test. All of the groups improved significantly from pre- to post-test ($p < .05$), but the effect sizes differed with the DI group improving more on the paper and pencil test than the other two groups, $F(2,261) = 7.826$, $p = .001$. Further analyses revealed some differences between the students in regular versus special education. The students in special education performed better on the paper-and-pencil test, which is most likely due to the fact that these students are older and have had more multiplication instruction. When corrected for these differences, the effects of school type almost disappear. Given no further qualitative differences between the students from the two types of schools, the data were collapsed together for the remaining analyses.

Table 5.2
Means and standarddeviations for test performance

Condition	Paper-and-pencil test (max. score 20)					Oral test (max. score 10)				
	pre-test		post-test		d	pre-test		post-test		d
GI	8.28	5.09	11.33	4.97	0.61	5.34	3.14	7.27	2.37	0.70
DI	7.32	5.31	12.11	5.49	0.89	4.24	3.01	6.47	2.85	0.76
C	8.45	4.71	10.96	4.68	0.53	5.07	2.77	6.58	2.56	0.57

Research question 2b concerns the strategy use (quality of problem solving) demonstrated by the students. First, the number of different strategies used by the students (their strategy repertoires) at pre- versus post-test were compared (see Table 5.3). The paper-and-pencil test showed the strategy repertoires of the students from the two experimental conditions but not those from the control condition to increase (GI: $t(1,87) = 3.051$, $p = .003$; DI: $t(1,86) = 5.064$, $p = .000$; C: $t(1,89) = 1.907$, $p = .060$). No differences between the GI and DI conditions were revealed by the written test. In the interview, however, the DI group used a smaller number of different strategies than the other two groups, both at pre-test and at post-test (pre-test: $F(2,262) = 7.610$, $p = .001$; post-test: $F(2,262) = 5.960$, $p = .003$). The differences also remained even after correction for age and months of instruction. However, no differences between the three groups were found for improvement from pre- to post-test, $F(2,264) = 1.811$, $p = .166$.

An overview of how many students in each condition used a particular strategy on at least one occasion (either in combination with another strategy or alone) is presented in Table 5.4. Significant changes in the use of strategies are observed for the experimental groups. At post-test, more students knew at least one of the problems out of their head than at pre-test (pp: $X^2(1) = 22.185$, $p = .000$; o: $X^2(1) = 17.332$, $p = .000$). Also an increase in the use of the splitting strategy on the written test was found ($X^2(1) = 33.667$, $p = .000$). This was mainly due to the increased ability of the students to solve multiplication problems above 10 x 10 by "splitting at ten". Repeated addition was used less at post-test than at pre-test on the interview test (X^2 (1) = 4.719, $p = .030$), presumably because the students have discovered more efficient strategies to use. The written test shows an increase in the students' use of "writing down the table" (X^2 (1) = 4.518, $p = .034$). All these changes in strategy use were only observed for the two experimental groups; the control group did not show any changes in strategy use from pre-test to post-test. Also, some differences are found between the two experimental groups: The interview pre-test scores show relatively fewer students in the DI group to use the strategies "automatized," "splitting," "neighbor," and "repeated addition" ($p < .05$). The same difference is still

Table 5.3
Means and standarddeviations for total of different strategies used (strategy repertoire)

	Paper-and-pencil test				Oral test			
	pre-test		post-test		pre-test		post-test	
GI	2.90	1.55	3.47	1.45	4.03	2.00	4.00	1.76
DI	2.84	1.35	3.60	1.27	3.00	1.98	3.17	1.94
C	3.03	1.39	3.34	1.26	3.94	1.85	3.97	1.69

Table 5.4
Students (%) using the strategies at paper-and-pencil test (pp) and oral interview (o)

Strategy	GI pre-test		GI post-test		DI pre-test		DI post-test		C pre-test		C post-test	
	pp	o	pp	o	pp	o	pp	o	pp	o	pp	o
automatized	54.5	70.5	72.7	87.5	48.3	47.1	79.3	67.8	60.0	66.7	58.9	78.9

reversing	12.5	53.4	15.9	46.6	5.7	55.2	14.9	55.2	4.4	56.7	13.3	52.2
Splitting +	54.5	52.3	81.8	54.5	50.6	39.1	81.6	47.1	58.9	61.1	72.2	65.6
Splitting -	4.5	50.0	8.0	53.4	4.6	27.6	9.2	25.3	3.3	40.0	3.3	24.4
doubling	22.7	28.4	25.0	22.7	19.5	19.5	26.4	25.3	41.1	34.4	32.2	33.3
neighbor	10.2	50.0	13.6	53.4	13.8	28.7	16.1	28.7	12.2	48.9	14.4	52.2
mult. table	18.2	37.5	29.5	25.0	25.3	32.2	32.2	24.1	17.8	32.2	34.4	30.0
rep. add.	37.5	67.0	85.2	54.5	94.3	41.4	90.8	24.1	93.3	52.2	96.7	45.6
drawing	12.5	5.7	5.7	1.1	8.0	6.9	9.2	1.1	10.0	4.4	6.7	3.3
dividing	0.0	0.0	9.1	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

visible at post-test but has not increased. The post-test shows more of the students in the GI group to use division strategies and fewer to use addition strategies when compared to the students in the DI group.

A comparison between the two methods of testing shows that the strategy of "reversal" is more frequently observed on the oral than on the written test, $t(264) = 10.112$, $p = .000$. The same holds for the strategies of "splitting based on subtraction" ($t(264) = 10.662$, $p = .000$) and "neighbor" ($t(264) = 8.332$, $p = .000$). The strategy of "repeated addition" is displayed more often on the written test than on the oral test ($t(264) = 14.390$, $p = .000$). Conversely, "saying the table" is used more often on the oral test at pre-test than on the written test at pre-test ($t(264) = 2.629$, $p = .009$), but the difference disappears at post-test.

Because many different strategies were used, both alone and in combination with other strategies, it seems useful to study the *adequacy* of the students' strategy use. A distinction can be made between the use of retrieval strategies (automatized problems) and the use of back-up strategies (calculation of answers). Within the back-up strategies, a further distinction can be made between the use of adequate strategies (with only one intermediate step), semi-adequate strategies (with two intermediate steps), not very adequate strategies (with more than two intermediate steps), and counting strategies (repeated addition, the use of drawings or other materials). The use of the different categories of strategy adequacy is shown in Table 5.5.

Significant differences in the adequacy of the strategies used by the different groups of students were found at post-test. The students from both of the experimental conditions were found to use adequate strategies more often than the students from the control condition on the written test at post-test, $F(2,262) = 5.131$, $p = .000$. In addition to this, the

Table 5.5
Relative frequency of strategies (in %) for paper-and-pencil test (pp) and oral interview (o)

Strategy	GI pre-test		GI post-test		DI pre-test		DI post-test		C pre-test		C post-test	
	pp	o	pp	o	pp	o	pp	o	pp	o	pp	o
no answer	30.1	24.9	21.1	11.1	39.0	32.2	23.2	16.1	35.3	22.8	24.9	9.8
retrieval	11.6	19.1	17.6	31.1	8.5	10.3	15.9	22.8	7.3	15.2	13.2	25.1
adequate	9.4	23.3	18.4	26.1	7.8	17.4	19.5	25.5	9.1	28.2	13.2	27.6
semi-adequate	1.0	7.7	1.3	6.1	1.2	6.2	1.7	6.2	1.2	7.9	1.0	9.0
little adequate	3.5	2.8	2.1	2.7	3.4	2.8	3.6	1.6	4.8	3.6	3.4	3.8
counting	22.1	19.7	22.2	17.6	21.3	14.5	20.0	11.4	23.5	16.4	21.5	17.7
unknown	22.4	1.3	16.1	0.6	19.0	1.7	16.3	1.4	17.7	0.3	21.5	0.4
total	100	100	100	100	100	100	100	100	100	100	100	100

students in the GI group more frequently used retrieval strategies than those in the DI group on the oral test at post-test, $t(264) = 3.468$, $p = .016$. The students in the GI condition were also better able to write down their solution strategies at post-test than the students in the DI condition (category "unknown", $p = .000$).

From pre-test to post-test, significant changes in the adequacy of the strategies used by the students were found for both the oral and written tests, indicating that the students use more frequently the strategies retrieval and adequate at post-test ($p = .000$). The difference was significant for all of the conditions, and a general decrease in the number of problems with no answer was also observed across conditions ($p = .000$).

The mean efficacy scores (see Table 5.6) also changed significantly from pre-test to post-test for both the written test ($t(264) = 8.720$, $p = .000$) and the oral test ($t(264) = 5.599$, $p = .000$). This shows the students to generally use more adequate strategies at post-test with the exception of the control group on the oral test. In other words, certain differences in the students' use of strategies were found to depend on the type of instruction they received but no differences in the efficacy of the strategies they used.

Table 5.6
Means and standarddeviations for strategy efficacy

	Paper-and-pencil test				Oral test			
	pre-test		post-test		pre-test		post-test	
GI	3.61	1.05	3.03*	1.03	2.82	0.99	2.42*	0.89
DI	3.77	1.02	3.00*	1.00	2.98	1.04	2.46*	1.15
C	3.71	0.94	3.26*	0.91	2.75	0.83	2.56	0.92

* Significant difference ($p < .05$) between pre- and post-test

In sum, the students' strategy use is found to change with the students in the two experimental conditions using a greater number of different strategies at post-test than at pre-test. The adequacy of the strategies used is also found to improve for the two experimental groups but not for the control group.

5.4 Conclusions and discussion

In this study, the process of learning multiplication was investigated. The effectiveness of two experimental programs was compared to that of a control program (i.e., standard math instruction). In the following, the research questions will be examined in light of the present findings. Thereafter, some general conclusions will be presented and discussed.

Question 1: The development of students' multiplication strategies
In the two experimental conditions, the students learned to use different strategies. In the GI condition, the students were encouraged to use those strategies familiar to them or the strategies they discover. Sometimes the students used the strategies of doubling or splitting at ten without actually knowing it or knowing the name of the strategy, and the role of the teacher was then to name the particular strategy or strategies and stimulate the discussion of the use of different strategies. In other words, the students in this condition were already familiar with a number of strategies and encouraged to use the different strategies according to their own preferences. This means that certain strategies were learned in a different sequence than in the DI condition. All of the strategies explicitly addressed in the DI condition were spontaneously addressed in the GI condition, interestingly enough.

In the DI condition, the students also learn to use different strategies but only those dictated by the program and taught by the teacher. The teacher explains every strategy selected to be taught, which means that the students do not discover just how the strategy works on their own. In general, the students in the DI condition learned the strategies taught to them. However, it proved difficult for them to keep to the structure provided by the teacher and not use their own preferred strategies at times. For example, some of the students had considerable troubles with the number line while others did not see the benefit of using the splitting strategy and therefore did not learn much from the exercises concerned with this strategy.

From the lessons we observed, one particular form of instruction did not appear to be better than the other. What we did see, however, is that some of the students react better to guided instruction while others react better to directed instruction. For example, a young girl who tends to have a fear of failure will probably not spontaneously tell the group about a newly invented strategy for fear of being wrong. Such a student will probably benefit most from directed instruction and then during the early stages of instruction in particular. However, a boy with an autistic disorder who rigidly but adequately applied the strategy of "doubling" to every problem did not benefit much from directed instruction and the requirement that other strategies be applied. In other words, taking the characteristics of the individual child into account to select the most appropriate method of instruction appears to be critical.

Question 2a: The effects of different methods of instruction on test scores
As already mentioned, the math test scores for the students in all three of the study conditions were found to improve on both the written and oral tests. The effect sizes nevertheless show the two experimental groups ($d > 0.60$) to improve

more than the control group ($d < 0.60$) on both tests. On the written test, the DI group ($d = 0.89$) improved even more than the GI group ($d = 0.61$). However, these results should be interpreted with caution as significant between-group differences were not found on either the pre-test nor at post-test. Caution is also demanded because the DI students had received less multiplication instruction at the start of the intervention, but still showed the same multiplication level as the students in both other conditions. This could reflect a higher learning capacity or learning speed of the DI students, which could explain the differences found in effect sizes. Moreover, the strategies used by the students to obtain their answers are just as important as the results.

Question 2b: The effects of different methods of instruction on strategy use

With regard to the students' use of strategies, the following conclusions can be drawn. First, the experimental conditions were both found to enlarge the students' strategy repertoires (i.e., the number of different strategies used) while the control condition did not. While the students in the DI group used a smaller number of different strategies than the students in the GI group, no significant differences in the expansion of the students' strategy repertoires were observed. Both of the experimental groups enlarged their strategy repertoires and also showed a change towards a more effective use of strategies: less use of repeated addition and more use of automatized multiplication facts. With regard to the adequacy of the students' repertoire use, all of the students were found to use more adequate strategies at post-test relative to pre-test (i.e., more "retrieval" and "adequate" strategies and fewer "no answer"). Only the two experimental groups showed increased efficacy scores at post-test. In addition to these findings, some small differences were found between the two experimental groups; namely that the students in the GI group used adequate retrieval strategies more often than the students in the DI group and were better able to write their strategies down than the other students. A few of the students in the GI group even applied division strategies to solve some problems spontaneously. In other words, the two experimental groups showed more improved strategy use than the control group, and the GI group improved slightly more than the DI group.

General conclusions

The students who participated in the present study showed clear changes in their strategy use. The experimental (GI and DI) students in particular showed an increased strategy repertoire and a shift towards more effective strategy use. These changes were also found to occur for both the students attending regular education and special education. In other words, even students with learning difficulties are able to expand their strategy repertoires and become good problem solvers when provided with appropriate instruction.

Although the students in both experimental groups learned to use more adequate problem-solving strategies and to discuss the application and use of the different strategies, the exact learning process was found to differ for the two groups. In the GI condition, the students discovered or learned to discover new strategies on their own; they also showed considerable commitment to the discussion of the different solutions. In the DI condition, the students more or less adopted the solution procedures presented to them by the teacher. Given that both of the instructional methods proved effective for students with math learning difficulties, directed instruction is not necessarily needed, as shown in the findings of former research and recommended in other literature (Jones, Wilson, & Bhojwani, 1997; Van Luit, 1994; Woodward & Baxter, 1997). However, the present results are based on mean group scores. It is possible, and also seen in the execution of the programs, that there are individual differences.

Some of the students in the GI condition appeared to be in the right place and managed to discover new strategies and solutions while others clearly needed a more structured example as provided in the DI condition. Some of the students in the GI condition could not handle the "freedom" they were given and appeared to be overwhelmed by the different possible solutions and discussions of strategy use. Conversely, some of the students in the DI condition appeared to feel restricted by the strategies presented by the teacher and showed a preference for selecting their own strategies. In other words, one form of instruction cannot be judged as better than the other; the choice of instructional method, rather, must be based on the individual learning style and individual learning needs of the student in question. It is therefore necessary to diagnose the specific learning capacities of students prior to the initiation of an intervention program. When students need structure or are afraid of making mistakes, it may be best to start with directed instruction. When students find it difficult to use strategies other than the ones they already master, and when they are capable of inventing or acquiring new strategies and also applying the strategies they know to new tasks, guided instruction may be most suitable. It is also, of course, necessary to take a particular student's history of instruction into consideration. When a student is not accustomed to guided instruction, for example, he will need more time to adjust than a student who is already accustomed to such instruction.

A closer look at the results also allows us to draw some more specific conclusions about the students' strategy use. Ten different strategies were distinguished, and some of the strategies were obviously used more frequently than others. The strategies "automatized," "splitting," and "counting" were used most frequently at both pre-test and post-test. Given that all ten strategies were found in the tests but the students used an average of only three or four different strategies, it appears that different students may use different sets of strategies. Indeed, the students in both the GI and DI groups showed clear personal preferences with regard to strategy use. This could be expected for the GI group because the students in this group were clearly encouraged to rely on their own personal preferences. The finding is more surprising for the DI group, which was taught to strictly follow the strategies demonstrated by the teacher. Apparently as these students learn to apply different strategies, they also learn to solve problems in their own way. Contrary to expectations based on former research (e.g., Klein, Beishuizen, & Treffers, 1998), thus, the mathematical problem solving of the students in the DI condition proves to be just as flexible as that of the students in the GI condition.

A final conclusion concerns the differences between the oral and written tests. The oral test was used to measure relatively easy problems while the written test was used to measure both relatively easy and more difficult problems. With regard to the students' strategy repertoires, an increase was found to occur for the written test from pre-test to post-test but not for the oral test. At both pre- and post-test, however, the students demonstrate a greater repertoire of strategies on the oral test than on the written test. The students' mean efficacy is also found to be higher on the oral test at both pre- and post-test than on the written test. Clear differences were present in the strategies used on the oral versus written tests: Whereas "reversal," "splitting," and "neighbor" were used relatively more often on the oral test, "repeated addition" was used relatively more often on the written test. The students' use of the strategy of "saying/writing down the multiplication table" increased over time for the written test and decreased for the oral test. The question that thus arises is just how the differences in strategy use found for the different methods of testing can be explained. One explanation may lie in different degrees of test difficulty. The tests clearly differed in difficulty, and it is therefore possible that the students used different strategies for easier problems (i.e., on the oral test) than for more difficult problems (i.e., on the written test). Another explanation may lie in the mode of testing itself (i.e., oral versus written testing). It was easiest to determine which strategy (or strategies) the students used on the oral test because the test assistant could always question the students. On the written test, we had to depend on what the students wrote down and it was basically impossible to discover what the students did when they did not write their strategy (or strategies) down. Nevertheless, the students were given scratch paper for the written test and this provided clear and relatively reliable insight into the strategy (or strategies) used by the students. On the oral test, students may be predisposed to give the answers that they think the test assistant wants to hear. In other words, the two methods of testing have clear advantages and disadvantages that merit further investigation. In other research (Verschaffel, De Corte, Gielen, & Struyf, 1994), oral report has been shown to provide a good picture of strategy use. Future research should nevertheless be undertaken to provide greater information on the reliabilities of both methods of testing.

In sum, it can be concluded that students who receive guided instruction acquire math problem-solving strategies in a different manner than students who receive directed instruction but that the differences in their actual test results are minimal. In both instructional conditions, the performance of the students with math learning difficulties from both regular and special education was found to increase. Their strategy use was also found to change over time with most of the students showing a larger repertoire of strategies and more adequate strategy use at post-test. Given the few differences between the guided versus directed instruction results, the effectiveness of the two instructional methods can be judged as equal and it is therefore recommended that the method of instruction always be tailored to the specific needs of the student in question.

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Chapter 6
Mathematics learning difficulties and PASS cognitive processes[1]

Abstract

This study examined the relations between mathematics learning difficulties and the Planning, Attention, Simultaneous, Successive (PASS) theory of cognitive processing. The Cognitive Assessment System (CAS) that is used to measure the PASS processes was administered to a group of 267 students with math learning problems attending either regular or special education. The results showed that students with math difficulties performed lower than their peers on all CAS scales and that this group contained many students with cognitive weaknesses in planning or successive processing. In addition, those students with specific difficulties with the acquisition of basic math facts, the automatization of such facts, or word problem solving were found to have distinct PASS profiles. In order to investigate the relations between cognitive abilities and improvement in the mastery of basic math facts and problem solving, 165 of the students with math difficulties were given a special multiplication intervention. It appeared that for the students with attention weaknesses this particular intervention was effective for the learning of basic facts, and for the students with simultaneous weaknesses it was particular effective for the learning of word problem solving.

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[1] Kroesbergen, E.H., Van Luit, J.E.H., & Naglieri, J.A. (2002). Mathematical learning difficulties and PASS cognitive processes. *Manuscript submitted for publication.*

6.1 Introduction

The focus of this study is on the relations between the learning of mathematics by students with math learning difficulties and the cognitive processes included in the Planning, Attention, Simultaneous, and Successive (PASS) theory. It is known that relations exist between certain cognitive processes and math learning, and between PASS processing and effective mathematics instruction (Naglieri & Johnson, 2002). However, these relations are not without controversy, particularly with respect to the abilities of students with learning disabilities (LD). In addition to the many questions regarding the relations between cognition and mathematics (or mathematics learning disabilities), there are also questions regarding the role of intelligence and the role of intelligence testing in the diagnosis of learning disabilities (Kaufman & Kaufman, 2001).

Intelligence tests are mostly used to measure a student's general ability level. In the identification of learning disabilities, IQ tests are also commonly used to compare a student's ability to his or her actual achievement. Unless the discrepancy is beyond some pre-determined value, a learning disability has not been indicated (Mercer, 1997). However, the use of an IQ-achievement discrepancy has been under attack for some time (e.g., Siegel, 1999; Stanovich, 1999). One reason is that the cut-off points for the general intelligence scores used to define learning disabilities are often based, at least in part, on tests that have a clear achievement component (Kaufman & Kaufman, 2001; Naglieri, 1999). Another reason is that intelligence cannot always be measured exactly; there is always some kind of error, which complicates the use of an IQ score in, for example, a LD discrepancy formula. Some authors (e.g., Siegel, 1999) argue that the identification of LD should be based on achievement scores alone and simply encompass those students who consistently score in the 25th percentile, for example, without any consideration of intelligence whatsoever.

Conceptually, intelligence tests are not only used to measure the IQ-achievement discrepancy, they can also be used to map children's cognitive strengths and weaknesses. Although findings generally do not support using IQ tests in this way (Kavale & Forness, 2000; Naglieri, 1999), recent research on cognitive processing has yielded promising results (Naglieri, 1999, 2000). The development of new approaches to intelligence testing, such as the Kaufman Assessment Battery for children (K-ABC; Kaufman & Kaufman, 1983) and the Cognitive Assessment System (CAS; Naglieri & Das, 1997a), is of obvious relevance for both diagnostic (Naglieri, 1999) and instructional purposes (Naglieri & Gottling, 1995, 1999; Naglieri & Johnson, 2000). Because these theory based tests measure ability as a multidimensional perspective, they may provide greater information on specific components and processes than a test designed to measure general intelligence (such as the WISC-III; Wechsler, 1991). And the specific information provided by these tests may be particularly useful during the diagnostic process, the design of instructional programs, and the development of specific interventions.

An example of such a new intelligence test is the Cognitive Assessment System (CAS; Naglieri & Das, 1997a), which is based on a theory of cognitive processing that has redefined intelligence in terms of four basic psychological processes: Planning, Attention, Simultaneous, and Successive (PASS) cognitive processes. The CAS provides information on students' strengths and needs. In addition, CAS scores have been found to be strongly related to achievement ($r = .70$; Naglieri, 2001; Naglieri & Das, 1997b), which is quite remarkable as the test does not contain the verbal and achievement components found in other measures of IQ (i.e., the WISC). These and other reasons have led researchers in the Netherlands (Kroesbergen & Van Luit, 2002; Kroesbergen, Van Luit, Van der Ben, Leuven, & Vermeer, 2000) to study the validity of the CAS when used in that country. This study is an example of one that examines the relations between the CAS and math learning difficulties with particular intent to examine the potential of PASS theory, on which the CAS is based, for the remediation of math learning difficulties.

6.1.1 PASS and math performance

The CAS consists of 12 subtests (3 subtests covering each of the four basic PASS processes). The subtests provide information on a child's cognitive functioning, which includes: (1) Planning processes to provide cognitive control, and utilization of processes and knowledge, intentionality, and self-regulation to achieve a desired goal; (2) Attentional processes to provide focused, selective cognitive activity over time; and two forms of operating on information, namely (3) Simultaneous processes by which the individual integrates separate stimuli into a single whole or group; and (4) Successive processes by which the individual integrates stimuli into a specific serial order that forms a chain-like progression (Naglieri & Das, 1997b).

Naglieri and Das (1997b) have found each of the four sets of PASS processes to correlate with specific types of achievement in math and other academic areas as well. Although all of the PASS processes related to achievement, particular processes such as planning appear to be specifically related to particular aspects of academic performance, such as math calculation (Das, Naglieri, & Kirby, 1994). This specific example is theoretically logical because planning processes are required for making decisions with regard to how to solve a math problem, monitor one's performance, recall and apply certain math facts, and evaluate one's answer (Naglieri & Das, 1997b). Simultaneous processes are particularly relevant for the solution of math problems as these often consist of different interrelated elements that must be integrated into a whole to attain the answer. Attention is important to selectively attend to the components of any academic task and focus on the relevant activities. Successive processes are also important for many academic tasks but in mathematics, probably most important when the children leave the sequence of events and for the memorization of basic math facts. For example, when the child rehearses the math fact 8 + 7 = 15, the child learns the information as a serially arranged string of information that makes successive processing especially important. Successive processing is also important for the reading of words that are not known by sight and may therefore be particularly important for the solution of math word problems.

CAS scores have been found to correlate strongly with achievement scores (Naglieri & Das, 1997b). The overall correlation with the Woodcock-Johnson Revised Tests of Achievement (WJ-R; Woodcock & Johnson, 1989) Skills cluster has been found to be .73. The correlations with mathematics skills have been found to range from .67 to .72, with the highest subscale correlations occurring for Simultaneous processes and Math (.62) and Planning and Math (.57). These correlations are quite high when compared to research with other intelligence tests (e.g., WISC-R, Raven's SPM), where the correlations between intelligence and math performance have been found to range from .30 to .50 (Ruijssenaars, 1992). These findings thus suggest that the CAS may be a good predictor of academic achievement in general and math achievement in particular.

Research has also suggested that a child's PASS profile is related to the effectiveness of particular intervention programs. Naglieri and Gottling (1995, 1999) and Naglieri and Johnson (2000), for example, have shown students to differentially benefit from instruction depending on their PASS cognitive profiles. The implication is that instruction can be made more effective when clearly matched to the cognitive characteristics of students. Along these lines, Naglieri and Johnson (2000) found the math computation of children with a planning weakness to benefit considerably from a cognitive strategy instruction that emphasized planning; those children with no planning weakness but nevertheless receiving the same planning-based instruction also did not show the same level of improvement in math computation as the other children. Similar insights into the relations between the intelligence profiles of students and the effectiveness of particular intervention programs may also aid the planning of remedial education programs and therefore call for further investigation.

6.1.2 Research questions

In the present study, the relations between PASS processes and mathematics achievement were investigated. Two questions were posed. The first question concerns the relations between cognition and math learning difficulties. In light of the current controversy surrounding the diagnosis of learning disabilities, we will use the term "math learning difficulties" to refer to all students performing below the 25th percentile on standardized math tests. Our first question is then: Do students with math learning difficulties exhibit different cognitive profiles than their normally achieving peers? In order to answer this question, a distinction was made between students who have difficulties learning basic math facts and students who have difficulties learning how to solve word problems. If these specific learning difficulties are associated with distinct cognitive profiles, then the CAS may also be of use for diagnostic purposes.

The second question concerns the relations between cognition and improvement in mathematics achievement. It is known that students who perform poorly in mathematics are not very good at the automatization of math facts and not very good at problem solving (Naglieri & Johnson, 2000). In the present study, the changes in the performance of students with different cognitive profiles in response to a mathematics intervention focused on the promotion of both automaticity and adequate problem solving was therefore examined. Stated more generally, we examined whether students' cognitive profiles differentially relate to the effectiveness of a particular math intervention. When such a difference is detected, it can be concluded that the CAS is not only useful as an instrument for the diagnosis of a student's cognitive characteristics, but also as a basis to effectively match the form of instruction or intervention to a student's particular needs.

6.2 Method

6.2.1 Participants

The Dutch version of the CAS was administered to two groups of children. The first group or the reference group consisted of 185 children without specific learning difficulties. The students were randomly selected from the participating elementary schools. The mean age for this group of students was 9.8 years (*sd* = 1.2); 51% were boys and 49% girls. The
second group consisted of 267 children with specific math learning difficulties. The students in this group were selected on the basis of their low performance on a national criterion-based math test (below the 25th percentile). This group consisted of 137 students attending regular elementary schools (*M* age = 8.9, *sd* = 1.3; 44% boys, 56% girls) and 130 students attending special elementary schools for students with learning and/or behavior problems (*M* age = 10.5, *sd* = 0.9; 72% boys, 27% girls).

6.2.2 Procedure

The Dutch version of the CAS was administered by research assistants trained by the first author. The English version was adapted into a Dutch CAS following careful procedures by Kroesbergen and Van Luit (2002). The preliminary reliability and validity analyses produced acceptable results. However, additional research with larger samples is still necessary to better evaluate the Dutch version of the test. In addition, it should be noted that the study reported here is part of a larger research program concerned with the usefulness of the CAS in the Netherlands.

In order to address the second research question, part of the students with math learning difficulties were given special instruction focused on the learning of multiplication (N = 165). Attention was devoted to the automatized mastery of the basic multiplication facts and improvement of the students' use of multiplication strategies. To measure the effects of the intervention, three multiplication tests were administered before and after the intervention period: a test to measure knowledge of basic multiplication facts up to 10 x 10; a speed test with 40 basic multiplication facts to measure the automatized mastery of them; and a word problem solving test consisting of 20 relatively difficult multiplication problems.

6.2.3 Intervention program

For the intervention, the multiplication part of the MAthematics Strategy Training for Educational Remediation (MASTER) was used (Van Luit, Kaskens, & Van der Krol, 1993; see also Van Luit & Naglieri, 1999, and Kroesbergen & Van Luit, in press). This program was designed to encourage strategy utilization with multiplication problems. The program contains three series of lessons: (1) basic procedures; (2) multiplication tables; and (3) "easy" problems above 10 x 10. Each series teaches new steps for the solving of specific tasks. A series starts with an orientation phase in which the child can solve the task with the help of materials. In the next phase, a connection is made to a mental solution. The subsequent control, shortening, automatization, and generalization phases are then completed.

The intervention involved 30 lessons of 30 minutes each presented twice a week to groups of five students. The emphasis in the lessons was on: (1) the use of strategies including metacognitive knowledge of how to select and apply the most appropriate strategies and (2) automated mastery of the multiplication facts as this knowledge is necessary for further learning and adequate problem solving. The discussion of possible solution strategies and procedures by the students was encouraged. The teacher assisted the students in such discussions, promoted reflection on the choices made, and ensured that each student understood the different solutions, and prompted selection of the most efficient strategy. Students were thus taught to flexibly apply different strategies.

6.3 Results

In this study, the Dutch version of the CAS was used although Dutch norms are not as yet available. For this reason, the experimental groups were also compared to a Dutch reference group in addition to the U.S. norms. Before the research questions can be addressed, thus, just how the Dutch reference group performed in relation to the American norms must first be considered.

6.3.1 Performance of Dutch reference group

As already mentioned, the Dutch version of the CAS was administered to a sample of children with no specific disabilities. The scores for these children were then compared to the U.S. based norms. Table 6.1 shows the test scores for the reference group standardized using the American norms. Remarkably, the Full Scale of 101.76 for the Dutch version of the CAS is very similar to the normative mean of 100 for the U.S. standardization sample. One sample t-tests nevertheless show significant deviations from the American norms for three of the four PASS scales and for the full scale. The differences for the Simultaneous processing scale is particularly large with the mean score for the Dutch reference group being more than five points higher than the American norm. It should be noted, however, that this difference represents only one-third of the standard deviation for this scale. On the successive processing scale, the Dutch children do not differ from the American children. It is remarkable that the Planning scores for the Dutch reference group are below the American average while the scores of this group on the other scales are either average or above average.

Table 6.2 shows the mean subtest standard scores and deviations from the normative value of ten. The means for five of the twelve subtests deviate significantly from ten al though only one deviation (Figure Memory) is larger than one. These results are in keeping

Table 6.1
PASS scale scores for Dutch reference group (N=185)

	Mean	SD	Range	Deviation from 100	t	p
Planning	98.27	11.00	63-129	-1.73	2.138	.034
Attention	101.96	11.69	71-138	+1.96	2.278	.024
Simultaneous	105.18	12.95	69-138	+5.18	5.445	.000
Successive	100.90	12.96	65-135	+0.90	0.941	.348
Full scale	101.76	11.54	72-137	+1.76	2.077	.039

Table 6.2
Mean CAS subtest scores and deviations from the norm (= 10)

Scale	Subtests	M (SD)	t	p
Planning	Matching Numbers	9.97 (2.59)	-0.170	.865
	Planned Codes	9.05 (1.89)	-6.829	.000
	Planned Connections	10.18 (2.28)	1.064	.289
Attention	Expressive Attention	10.14 (2.41)	0.795	.428
	Number Detection	10.85 (2.32)	4.977	.000
	Receptive Attention	9.91 (2.39)	-0.492	.623
Simultaneous	Nonverbal Matrices	10.89 (2.69)	4.484	.000
	Verbal-Spatial Relations	9.99 (2.72)	-0.027	.978
	Figure Memory	11.76 (2.76)	8.662	.000
Successive	Word Series	9.78 (2.46)	-1.227	.221
	Sentence Repetition	10.64 (2.66)	3.263	.001
	Sentence Questions	10.17 (2.62)	0.870	.385

with the Dutch students' PASS scores and should clearly be kept in mind when interpreting the results presented below.

6.3.2 PASS cognitive processes and math learning difficulties

In order to examine the relations between the various PASS cognitive processes and math learning difficulties, the CAS was administered to a group of students with such difficulties. Whether the students with math learning difficulties showed different PASS profiles than their normally achieving peers (i.e., the reference group) was then examined. In Table 6.3, the means and standard deviations for the different PASS scales are presented for the reference group and the group with math learning difficulties, with the latter group divided into students enrolled in special versus regular education. The students with math difficulties perform lower than their peers on all of the PASS scales ($p < .01$). Further analyses show similar differences to occur on all 12 of the CAS subtests. In addition, the students in special education showed even lower scores than their peers with math difficulties but in regular education. In accordance with the results for the reference group, the scores of the group with math difficulties on the Simultaneous processing scale were relatively higher than their scores on the other PASS scales.

Given that the group of students with difficulties learning mathematics can be very diverse, different types of math difficulties were next distinguished: (1) students encountering difficulties with the learning of basic multiplication facts; these students score at least one standard deviation below the mean on the basic multiplication test and have

Table 6.3

PASS-scores for the reference group, and the group with math difficulties

Group*	N	Planning	Attention	Successive	Simultaneous	Full scale
Reference	185	98.3 (11.0)	102.0 (11.7)	100.9 (13.0)	105.2 (13.0)	101.8 (11.5)
MD	267	89.0 (12.2)	91.1 (12.6)	87.9 (13.9)	97.9 (12.1)	88.3 (12.0)
- Special	137	85.6 (11.8)	86.6 (12.9)	82.6 (12.7)	95.0 (12.2)	82.9 (11.2)
- Regular	130	92.1 (11.9)	95.2 (10.8)	92.9 (13.0)	100.5 (11.4)	93.3 (10.5)

**MD = math difficulties; Special = special education; Regular = regular education*

Table 6.4

Mean PASS scores (SDs) for students with specific math difficulties (N = 45)

Low scores in	N	Planning	Attention	Successive	Simultaneous	Full scale
Basic skills	15	90.7 (13.0)	93.1 (10.2)	86.7 (16.1)	93.1 (11.3)	87.7 (12.8)
Automaticity	16	84.7 (9.9)	88.3 (8.1)	86.0 (10.5)	98.3 (9.2)	85.3 (8.2)
Word problems	14	96.1 (14.2)	92.6 (11.7)	93.4 (15.0)	99.2 (11.9)	93.1 (13.4)

received at least one year of multiplication instruction; (2) students encountering difficulties with the automatized mastery of basic facts; these students produce average scores on the basic multiplication test but below average scores on the automaticity test; and (3) students encountering difficulties with the solution of math word problems but no difficulties with basic multiplication facts. In the sample of 167 students with math difficulties, 45 students were found to clearly fit into one of the three groups. As can be seen from Table 6.4, no significant differences were found in the performance of the three groups on the four PASS scales or relative to the total group of students with math learning difficulties with the exception of the students encountering difficulties with the solution of math word problems: this group scored significantly better on the Planning scale than the other groups. Within-group analyses showed those students with difficulties learning basic multiplication skills to score particularly low on the Successive processing scale (86.7). Those students with automaticity problems produced particularly low scores on the Planning (84.7), Attention (88.3), and Successive processing (86.0) scales together with relatively high scores on the Simultaneous processing scale (98.3). The group of students with difficulties solving word problems produced relatively lower scores on the Attention (92.6) and Successive pro-cessing (93.4) scales.

In the next set of analyses, whether the group of students with math difficulties contained a greater number of students with cognitive weaknesses than the reference group was examined. A cognitive weakness meant that the child's scale score was significantly lower than the child's mean and less than 85 (1 SD below average). Inspection of Table 6.5 shows more of the students in the group with difficulties learning mathematics relative to the reference groups to have a cognitive weakness in planning ($X^2(2) = 10.333$, $p = .006$) or in successive processing ($X^2(2) = 33.936$, $p = .000$). The group in special education tended to have even more students with a Successive weakness than the group of students with math learning difficulties in regular education ($X^2(1) = 7.119$, $p = .008$), and more of the students in the two groups of students with math learning difficulties considered together were found to have a Successive weakness than in the reference group ($X^2(1) = 9.646$,

Table 6.5

Number of students in different groups with specific PASS weaknesses

Group*	N	Planning weakness	Attention weakness	Successive weakness	Simultaneous weakness

Reference	185	9 (4.9%)	4 (2.2%)	12 (6.5%)	6 (3.2%)
MD	267	38 (14.2%)	17 (6.4%)	65 (24.3%)	8 (3.0%)
- Special	130	18 (13.8%)	10 (7.7%)	41 (31.5%)	4 (3.1%)
- Regular	137	20 (14.4%)	7 (5.1%)	24 (17.5%)	4 (2.9%)

**MD = math difficulties; Special = special education; Regular = regular education*

p = .002). Further analyses showed that the group with a cognitive weaknesses in simultaneous processing scored relatively low on the word problem solving test, $t(152) = 2.081$, $p = .048$.

To summarize, students with different math learning difficulties produce lower PASS scales on average than their normally achieving peers and are also more likely to have a cognitive weakness in planning or successive processing. Students with a simultaneous weakness show particular difficulties with the solution of math word problems. Similar results were found for both groups of students with math learning difficulties although the deviations from the reference group are largest for the special education group.

Table 6.6

Pre-post differences for three math achievement areas and samples with a cognitive weakness in one PASS area.

Test/group*	N	Pre-test (M/SD)		Post-test (M/SD)		Effect size
Basic skills						
MD	89	5.1	2.9	8.1	2.1	1.2
PW	21	5.8	3.5	8.5	2.2	1.0
AW	7	6.0	3.7	9.6	0.8	1.6
SUW	41	5.0	3.1	8.2	2.3	1.1
SIW	7	5.4	3.6	7.9	3.5	0.7
Automaticity						
MD	89	16.2	7.9	26.2	7.9	1.3
PW	21	17.9	8.6	28.3	9.3	1.2
AW	7	15.9	7.9	24.6	3.6	1.5
SUW	41	17.1	6.6	25.6	7.8	1.2
SIW	7	19.3	7.9	25.6	8.5	0.8
Word problems						
MD	89	7.5	5.4	11.4	5.3	0.7
PW	21	9.8	5.0	13.8	4.8	0.8
AW	7	7.7	6.7	11.9	4.7	0.7
SUW	41	7.7	5.6	11.4	5.5	0.7
SIW	7	6.0	4.0	11.6	5.5	1.2

*MD: students with math learning difficulties without specific cognitive weakness; PW, AW, SUW, SIW: groups of students with respectively planning, attention, successive, and simultaneous weaknesses

6.3.3 Mathematics performance and CAS

The second question to be addressed is whether a relation can be detected between improvement in mathematics performance (as a result of special instruction) and students' PASS scores. This was investigated by comparing the effects of the mathematics intervention on children with specific cognitive weakness to the effects on children with no specific cognitive weakness. An overview of the students' scores on the three math achievement tests at pre- and post-test is presented in Table 6.6. As can be seen, all of the groups improved as a result of intervention. However, no significant differences on students' improvement during intervention were found between the samples with a specific cognitive weakness, and no significant differences were found between the students enrolled in special versus regular education. However, students with a simultaneous cognitive weakness showed a tendency towards less improvement in their knowledge of basic multiplication facts and automaticity and most improvement in their word problem solving. Conversely, students with an attention cognitive weakness tended to improve most with regard to their knowledge of basic multiplication facts and automatization of these facts.

6.4 Conclusions and discussion

In this study, two main questions regarding the relations between mathematics learning difficulties and cognition were investigated. The first question is relevant for the diagnostic procedure, while the second question concerned the effects of treatment.

Whether students with math learning difficulties exhibit cognitive profiles that are different from the cognitive profiles of their normally achieving peers was first examined. Students with math learning difficulties were indeed found to show relatively lower scores on the four PASS scales and therefore on the CAS Full Scale as well. The group of students with math learning difficulties performed highest on the Simultaneous processing scale although the reference group also performed higher on this scale than the U.S. norm, which means that this result should be taken as tentative until further testing is undertaken. Additional research and standardization of the CAS with respect to a Dutch norm group is necessary to clearly settle this issue.

More detailed analyses of the present sample revealed a relation between specific math difficulties and specific PASS processes. It appeared that, in accordance with the theory (Naglieri & Das, 1997b), students who encounter difficulties with the learning of basic multiplication facts perform, in general, lower on successive processing. Students who encounter difficulties with the automatization of basic facts show problems with not only successive processing but also planning and attention. The latter processes are particularly important for the automaticity test because a time limit requires the efficient production of correct answers. Finally, those students who encounter difficulties with the solving of math word problems showed relatively weak attention and successive processes and relatively strong planning and simultaneous processes. Although both planning and simultaneous processes are important for the solution of math word problems, these findings suggest that attention and successive processing, which play an important role in reading, also play a key role in this type of math. The present results show that the PASS profiles of students with math learning difficulties differed from those of students with no such difficulties and thus demonstrate the diagnostic value of the CAS, especially in conjunction with other relevant information.

It was also found that more of the students in the group of students with math learning difficulties had a cognitive weakness in planning (14%) or successive processing (24%). This is consistent with the results of the study conducted by Naglieri (2000). Planning is, according to the literature, an important process in mathematics (Naglieri & Das, 1997) along with the simultaneous processes. It was found that those students with simultaneous weaknesses have, on average, greater difficulties with word problem solving although the reverse was not found: Students who encounter difficulties with word problem solving do not produce lower simultaneous processing scores. In solving math word problems, the successive processes also play a critical role, which may explain the lower scores on this scale for the group of students with specific difficulties solving math word problems. Given that a large part of the Dutch math curriculum consists of word problems, it is very understandable that students with a successive weakness may encounter difficulties. However, the present results suggest that the group of students with math learning difficulties is heterogeneous and being comprised of students with a specific planning weakness, with a specific successive processing weakness, with generally low processing scores, and even a few students with attention and/or simultaneous weaknesses. The results also suggest that a child's PASS profile alone is not sufficient to diagnose math learning difficulties, a child's PASS profile can, however, help identify specific cognitive weaknesses and thereby facilitate both diagnosis and treatment. It was the question of treatment that was the second part of this study.

The relations between specific PASS cognitive profiles and the effectiveness of a special math intervention devoted to the learning of basic multiplication facts, the automatization of these facts, and word problem solving skills were carefully examined in the next set of analyses. Previous research showed students with a cognitive planning weakness to benefit from a cognitive intervention with specific attention to planning more than students without a cognitive weakness and more than students with other cognitive weaknesses (Naglieri & Johnson, 2000). Although these results for the entire sample were not confirmed in the present study, the group of students with an attention weakness showed a tendency to improve the most on the (automatized) mastery of basic multiplication facts while the group of students with a simultaneous weakness showed a tendency to improve most on word problem solving. An explanation for the discrepancy in the results of these different studies may lie in the fact that the intervention utilized in the present study was less focused on planning than the interventions used in previous research (e.g., Naglieri & Johnson, 2000). The intervention described here was mainly concerned with the acquisition of the basic math facts and the adequate use of strategies. Although planning is certainly part of strategy use, it was not explicitly taught. Nevertheless, the intervention appeared to be particularly effective for those with a simultaneous cognitive weakness.

To conclude, the results of the present study revealed some important relations between PASS cognitive processes and math learning difficulties. Although the relations were not very strong, the findings nevertheless highlight the importance of particular cognitive processes for the functioning of students within certain areas of the mathematics curriculum. Simultaneous and successive processes appear to be of particular importance for the solution of math word problems, for example, and attention appears to play a role in the automatization of basic facts. Previous research has also shown the CAS to be a valuable diagnostic instrument and also useful for the planning of special instruction or intervention. We therefore encourage further research along these lines. Future research might also address the specific difficulties that students encounter with the mathematics curriculum in connection with the development of special instructional methods based on the PASS cognitive processing theory and specific weaknesses.

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Chapter 7
General discussion

Although mathematics education has recently undergone several changes, the effects on the learning of low-achieving students have rarely been studied. The aim of the present thesis was to contribute to the discussion and research on this topic. In Chapter 1, the students with difficulties learning mathematics were described. It was noted that it is often recommended to give such students explicit, direct instruction based on behaviorist or reductionist principles. However, it was also noted that the current mathematics curriculum call for instruction based on constructivist principles. This discrepancy became the main theme of this thesis and prompted the following question: Do students with difficulties learning mathematics require explicit direct instruction or can they also benefit from guided instruction based on constructivist principles?

In this final chapter, the results from the different chapters in this thesis will be summarized and integrated. In doing this, the five research questions posed in Chapter 1 will be addressed. Some directions for further research in light of the present conclusions will be mentioned and, in the final section, some implications for actual practice will be pointed out in order to make the outcomes of this study relevant for (remedial) teachers working in the field of (special) education.

7.1 Research questions

Which instructional principle is more effective: guided or directed instruction?

In the meta-analysis presented in Chapter 2, directed instruction proved to be more effective than guided instruction for the acquisition of basic mathematical skills. However, only a few studies employed an instructional component comparable to our guided instruction component. In Chapter 3, where the results of the initial pilot study are presented, guided instruction was found to be more effective than directed instruction. This difference was found to characterize the students' level of multiplication ability but not their level of automaticity. An interaction effect between type of school and type of instruction was found for automaticity, with directed instruction proving more effective for special education students and guided instruction more effective for regular education students. In the main study (see Chapters 4 and 5), conversely, no differences in automaticity were found whatsoever. In addition, directed instruction was found to be more effective than guided instruction for the teaching of multiplication skills. In the main study, moreover, no interaction between type of school and type of instruction was found. Finally, the two groups differed from each other on only one of the four aspects of motivation, with the students in the the guided instruction condition being less ego oriented relative to the students in the directed instruction condition after intervention.

The above summary shows the results of the pilot study to not be congruent with the results of the meta-analysis and main study undertaken as part of the present thesis. While the pilot study showed guided instruction to be more effective than direct instruction, the meta-analysis and results of the main study showed direct instruction to be more effective than guided instruction. Given that the number of subjects in the pilot study was relatively small and the sample groups were not completely comparable (see Chapter 3), the pilot results appear to be less valid than the results for the main study. Our conclusion is therefore that directed instruction is generally more effective than guided instruction. These results also show that constructivist-based instruction is not always the most effective method for the teaching of students with difficulties learning mathematics, which confirms the findings of other research (Klein, Beishuizen, & Treffers, 1998; Van Zoelen, Houtveen, & Booij, 1997; Woodward & Baxter, 1997), and is congruent with the theory of special needs on the part of low-achieving students (Carnine, 1997; Jones, Wilson, & Bhojwani, 1997; Mercer & Mercer, 1998). It should nevertheless be mentioned that the differences between the two conditions were small, which implies that guided instruction was also effective but not as effective as directed instruction. It should also be noted at this point that our guided instruction intervention falls on the implicit side of the implicit-explicit continuum of constructivist-based instruction (see Chapter 1) while the relevant literature recommends moving towards the more explicit side of the continuum for students with difficulties learning mathematics (Mercer, Jordan, & Miller, 1996). Further research should examine the effectiveness of more explicit constructivist teaching, which seems quite promising in light of the fact that implicit constructivist teaching has already been found to be quite effective.

How do students acquire strategies during both interventions (guided and directed)?
In both of the experimental conditions, the students have acquired new multiplication strategies adequately, as described in Chapter 5. Students in the guided instruction condition were encouraged to discover their own strategies and apply the strategies that they prefer most. The results show the students in this condition to indeed acquire a variety of strategies, including the strategies taught in the directed instruction condition. Some of the students even used a number of unexpected strategies at post-test, such as division to solve a multiplication problem. The students in the guided instruction condition also showed a gradual shift from pre-test towards the use of more adequate strategies at post-test. The students in the directed instruction condition were taught to use the strategies modeled by the teacher. They, too, acquired the different strategies adequately. However, their strategy repertoires were found to be slightly smaller than the repertoires of the students in the guided instruction condition. Furthermore, the students in the guided instruction condition were better able to write their strategies down than the students in the directed instruction condition.

Apparently students with difficulties learning mathematics are also able to build their own knowledge and invent their own solutions to problems, just as normally achieving students can (Gravemeijer, 1997). When the students with difficulties learning mathematics are found to invent their own solutions, they also appear to be more conscious of the strategies they use despite their sometimes limited metacognitive abilities (Goldman, 1989; Mercer, 1997). This suggests that it is not always necessary to make strategies explicit and clear or to teach only a limited number of strategies, as usually recommended (e.g., Carnine, 1997; Jones et al., 1997). It should be noted, however, that the students only received multiplication instruction. An extension of the instruction to other domains of mathematics implies many more strategies. Further research is therefore needed to investigate whether and when students can handle a larger number of strategies. At this point, it should be noted that the differences between the two groups with regard to strategy use were not accompanied by differences in their multiplication performances. The question then arises as to what exactly the advantages of a varied strategy repertoire may be when students with a smaller repertoire are also able to adequately solve the problems they confront. Further research is therefore called for to gain greater insight into this issue. One possible advantage of the guided discovery of various solution strategies is that students accustomed to discovering their own strategies may be particularly well-equipped to handle other new and unknown problems via the transfer or generalization of strategies, for example.

Does special intervention aimed at strategy use appear to be more effective than regular instruction?
The interventions described in this thesis were focused on the adequate use of strategies for basic multiplication. An understanding of basic multiplication and multiplication strategies was assumed to constitute the basis for the automatized mastery of multiplication facts. The effects of these special interventions were compared to the effects of regular instruction as provided in the control condition. Both the pilot study and the main study showed the experimental groups to improve more than the control group with regard to both the multiplication tests and strategy use. In the pilot study (Chapter 3), the experimental groups improved more on the ability test but performed at the same level as the control group on the automaticity and transfer tests. The main study (Chapter 4) showed the experimental groups to perform better than the control group on all four of the multiplication tests. It was also found that the experimental groups enlarged their strategy repertoires and used more adequate strategies at post-test while no such changes were found for the control group (Chapter 5).
At first sight, the conclusion appears to be that the experimental forms of instruction were more effective than the control instruction. It is therefore recommended to focus on the use of such strategies in the teaching of multiplication. Both guided and directed instruction proved effective although some differences were detected. A few other points should be kept in mind when interpreting these results. First, the experimental instruction was always provided in small groups. The effects of such instruction are usually large when compared to the effects of regular classroom-based instruction (Kroesbergen & Van Luit, in press). The total time spent on multiplication was the same across the three conditions, but the instruction provided in the experimental conditions was more intense because the teacher had to divide his or her attention among fewer students. A possibly greater focus on multiplication in the twice weekly experimental conditions was largely compensated for by the fact that multiplication instruction was provided five times a week in the control condition. Second, the intervention involved a relatively small area of the mathematics curriculum. The consequence of studying only one small area of the mathematics curriculum is that the results cannot easily be generalized to other areas of the curriculum. Further research is therefore necessary to conclude which type of

instruction is most effective across the entire curriculum. Third, exactly what happened in the control condition (i.e., the amount of time spent on multiplication and the content of the multiplication instruction) was not always clear. As a result, the control condition was very heterogeneous and it is difficult to specify the type of instruction to which the experimental conditions were compared. While the majority of the schools used realistic mathematics textbooks, the teachers reported often adapting the instruction to the needs of the students and making it more direct. Finally, the finding that the experimental groups had less traditional beliefs with regard to mathematics than the control group may simply reflect more traditional attitudes on the parts of the teachers involved in the control group and therefore calls for further research.

What are the effects of different child characteristics (sexe, age, ethnicity, IQ, math level, special needs) on the child's ability to learn multiplication?
The effects of the different child characteristics measured in this study were described in Chapter 4. Boys are often found to perform better than girls in the domain of mathematics (e.g., Noteboom, Van der Schoot, Janssen, & Veldhuijzen, 2000; Vermeer, 1997). However, the boys in the present study performed better than the girls on only one of the four multiplication tests with the other tests revealing no differences.

No differential effects of age or ethnicity were found in this study. Note that the variables amount of previous multiplication instruction, student's general math level, and type of education (i.e., special versus regular) are all related to age and were included in the model we tested.

It was very remarkable that no effect of IQ was found while normally correlations of .30 to .60 are found between intelligence and mathematics achievement (see Chapter 6). This association was examined more closely in Chapter 6, and results are discussed in connection with the next research question.

Students with higher general math levels and greater previous multiplication instruction improved more than students with lower general math levels or less previous instruction. Given that all of the students included in the present study were selected on the basis of low multiplication performance, a higher general math level means that such a student had greater difficulties with the learning of multiplication than the learning of other areas of mathematics. It is also therefore not very surprising that such students improved more as a result of the intervention than students with average or lower levels of general math knowledge. That is, multiplication is easier for students with higher levels of addition and even subtraction knowledge. It is striking that the students with the largest amount of previous multiplication instruction benefitted the most from the interventions. In other words, those students who have had more instruction initially perform at the same level as those students who have had less multiplication instruction and seem to have a lower learning capacity in this domain of math but improve the most in the end. It may be that the change of instructional method provided a new learning impulse or that the form of instruction received in the present study was simply more effective than the form of instruction formerly received.

Finally, only a few differences were detected between the students in special versus general education. The results of the main study showed the general education students to improve more than the special education students although this difference was only found to hold for the easier multiplication tests. In addition, no interaction of type of education with condition was found in the main study and no differences in the quality of the strategies used by the students from special versus general education schools were detected. Given these results, we can now conclude that qualitative differences do not exist in the learning of multiplication by students from the two types of schools studied here while quantitative differences do exist with the special education students showing less improvement than the general education students.

What is the relation between mathematics knowledge and cognitive abilities?
The results presented in Chapter 6 show the students who participated in this research study to have IQ scores falling below the mean on average. With regard to the four cognitive processes measured, it appears that in this group of students with difficulties learning mathematics, more students than usual have a specific cognitive weakness, especially in the domains of planning or successive processing. Within this relatively heterogeneous group, moreover, a number of subgroups with specific cognitive profiles can be distinguished. Those students with difficulties learning basic multiplication facts show lower scores for successive processing. Those students with difficulties automatizing basic multiplication facts show lower scores for planning, attention, and successive processes. Those students with problem-

solving difficulties score lower on both attention and successive processes, but relatively high on planning and simultaneous processes. Moreover, relations were found between the students' PASS cognitive profiles and their improvement during the multiplication intervention. The observed relations between particular cognitive profiles and specific learning difficulties have implications for the instruction of the students involved. Further research is nevertheless necessary to provide greater insight into the relations and the type of instruction that appears to be most effective for different students with distinct cognitive profiles.

7.2 Suggestions for further research

The results of this study have prompted a number of new questions with regard to mathematics education for low-achieving students. In this section, some major directions for future research will be pointed out. The areas for further research stem partly from some shortcomings or limitations on the present study and partly from the new questions raised by the results of the present research.

Effectiveness of realistic mathematics education

The aim of the present study was to examine the effectiveness of recent changes in mathematics teaching for (special) elementary education students in the Netherlands. The study is part of a larger research program investigating a number of different areas within the mathematics curriculum. In the present study, the area of multiplication was studied. The central question was whether it is better to provide students with the freedom to actively contribute to their lessons by developing their own solutions or whether students need more structured and direct instruction. Although student contributions to their own learning represent one of the most important characteristics of RME, there are other characteristics in need of further or more explicit research, such as the role of self-developed models and the effects of using realistic contexts. The decision to study only the learning of basic multiplication facts thus constitutes a restriction on the present study. In order to provide clear conclusions regarding the effectiveness of the RME curriculum, all aspects of the teaching method should be studied from kindergarten through the end of elementary school.

Perhaps a more critical limitation on the present study is the fact that the students who participated in the present study were provided small-group instruction outside the class and then compared with a control group of students who received regular instruction in the class. Obviously, the results can be made more reliable when the control group also receives small-group instruction outside the class or when the intervention groups receive instruction on a regular class basis. It is recommended that intact classes be used in future research in order to examine the effects of a total curriculum with experimental instruction provided five (as opposed to two) times a week, which then enables comparsion to a regular control condition.

A final suggestion involves comparison of the results from studies in other countries and international studies. The present study was limited to the Dutch population of students with special instructional needs. However, as the mathematics curriculum has undergone comparible changes in other countries, it may be very informative to compare the different programs and their effects for clear similarities and differences. By doing so, greater information can be gathered on the most and least effective components of the new instructional methods for the learning of low-achieving math students.

Mathematics instruction for low achievers

The students who participated in our study were selected on the basis of low math performance, which resulted in a heterogeneous research group. That is, the research group consisted of both students with "true" mathematics learning disabilities and students with other problems in addition to their mathematics difficulties, such as hyperactivity, reading disabilities, or mild mental retardation. The choice of selection criteria was made in light of the focus of the present study on all kinds of mathematics learning difficulties. It is also very difficult to find a sufficient number of students representing the population in question for a research design involving independent groups. As a consequence, our research sample encompassed many different subgroups of students with specific mathematics difficulties. Further research should therefore address the differences between these subgroups, both with regard to the specific characteristics of the students with different types of mathematics difficulties and their special educational needs. Future research should also consider those variables that can influence the mathematics performance of students with math difficulties in greater detail. For instance, the pilot study showed some clear differences between the students enrolled in a general versus special education program. It is therefore important that we further investigate the differences between these populations and the instructional needs that they have. In this light, a comparison to students with no math learning difficulties may also be very informative.

Given that the main problems in the area of mathematics learning lie in the development of automaticity and problem-solving strategies (both cognitive and metacognitive), research should address the effects of instruction on exactly these skills in the future. In the present study, only a few differences in automaticity were found. That is, for automaticity, almost no differences in the effectiveness of the different types of instruction could be detected. This suggests that further research should more thoroughly examine the development of children’s problem-solving strategies and use of such although these may be more difficult to measure than automaticity. From this perspective, the effects of instruction on the cognitive and metacognitive strategies used by students should be examined in greater depth. Further research should also clarify the effects of various forms of instruction on the transfer and generalization of what is learned. For instance, our pilot study showed that students who have learned to develop their own problem-solving strategies (guided instruction) are better than other students at finding solutions for new and otherwise unlearned problems. Finally, the focus of the present study was predominantly on students’ math performance with a minimum of attention paid to their motivation to learn math. Given that motivation is critical to learning, this topic also merits greater attention in future research.

Methodological issues

In this study, a group design was used to compare students receiving different types of instruction. The disadvantage of such a design is that any individual differences between the students are not visible. As described in Chapter 5, different students may respond differently to the same type of instruction. Additional qualitative studies of the effects of instruction on individual students are therefore needed. Ideally, student characteristics that actually predict the effects of different types of instruction should be identified. Following students across a longer period of time (longitudinal research) can also provide information on the stability of certain characteristics and how certain characteristics appear to develop over time.

Another methodological issue concerns the various methods that can be used to evaluate the effectiveness of a particular type of instruction. In the present study, several methods were used: paper-and-pencil tests either with or without scratch paper, speed tests, oral interviews, and video observations. Different tests were found to produce different results. It is therefore necessary to carefully select one’s measurement instruments and combine various methods whenever possible.

A final suggestion concerns the use of multilevel modeling. It should be recognized that students are influenced by their classes or instructional groups, which means that differences between groups can influence individual outcomes. This was demonstrated in Chapter 4, where anywhere from 28% to 44% of the variance in the study outcomes was found to occur at the group level. Such a group influence may similarly have played a role in the pilot study where differences between the conditions largely resembled the differences between the schools. The small number of schools participating in the pilot study rendered multilevel analyses impossible. For this reason, it is recommended that further research along these lines be conducted with larger samples in the future.

7.3 Implications for practice

In this final section of my thesis, the implications of the present study for teaching and school learning are discussed. The most important question, of course, is which form of instruction is best suited for teaching students with difficulties learning mathematics. Unfortunately, there is no single, explicit answer to this question as every student is unique and has his or her own instructional needs. Before the presentation of some general conclusions, I would therefore like to emphasize the need to view students as individuals. Just which type of instruction is most effective for an individual student depends on child variables (e.g., creativity, adaptibility, motivation, reading skills, metacognitive awareness) and both teacher and class variables (e.g., instructional preferences, time available, textbooks used, group size). For this reason, it is always necessary to prepare special instruction or a remediation program thoroughly. In addition, one should clearly diagnose the level of the student and his or her specific needs with the aid of the teacher and the student him/herself.

Some general conclusions can now be drawn with regard to the effectiveness of the different types of instruction studied here, namely guided versus directed instruction. Although the detected differences were small, directed instruction proved most effective. Many students were nevertheless found to benefit from guided instruction as well. Furthermore, differential effects of instruction on students enrolled in the special versus general education were detected, which can be partly explained by differences between students (students in special education tend to have more severe and/or a greater number of problems than students in general education encountering learning problems), differences in the educational structure (the instruction provided in special education is usually provided in small same-level groups while the instruction provided in general education typically occurs within the larger and often more heterogeneous class group), and differences in the usual instruction (special education programs typically involve more directed instruction than general education programs).

It is recommended on the basis of the present findings that special or remediating programs not be made too different from regular instruction programs, unless the circumstances absolutely call for this. Given that the regular teaching of students is now based on guided instruction in many schools, the provision of special teaching should also be based on guided instruction whenever possible. When the students cannot handle the freedom that guided instruction gives them, when they appear to be overwhelmed by the various possible solutions to a problem or the large amount of verbal input they receive, or when they have troubles developing their own strategies, the instruction should be made more directed. Similarly, when students receive directed instruction as part of the normal course of their education (as often seen in special education), special instructional programs should be based on directed instruction as well. When the students are able to find their own solutions or appear to be restricted by the more or less fixed structure of directed instruction, then one should consider making the instruction more guided. Students who detect and develop their own problem-solving strategies tend to be better at applying the strategies and learning new strategies at a later point in time when compared to students who have troubles detecting or developing their own strategies. Modification of the teaching of such students in the direction of more guided instruction can thus give them increased autonomy, more of a challenge, and greater motivation.

One final conclusion that follows from the present results is that, independent of the particular type of instruction provided, it is important to continue the instruction even after specific targets or learning goals have been attained. Immediate termination of the instruction causes performance to remain the same or even drop over time. In other words, it is necessary to provide students with opportunities to continue working with their newly acquired knowledge and skills and to possibly call upon already established skills in the building of new knowledge and skills.

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Summary

This dissertation contains a number of articles reporting on research on instruction for students who have difficulties learning mathematics. The focus lies on the kind of instruction that these students need to adequately master the mathematics skills required by the elementary school curriculum. Recent developments in mathematics education ask for guided instruction. In the study reported on here, the effectiveness of guided instruction for low-achieving students was investigated by conducting an intervention study in schools for regular and special elementary education. The effects of guided instruction were compared to the effects of directed instruction, which was found to be effective in former research. This study showed that guided instruction is almost equally effective as directed instruction. It is therefore concluded that both directed instruction and guided instruction can be effective for teaching multiplication to students with difficulties learning mathematics.

In Chapter 1, the relevant theoretical background of this study is briefly reviewed. An overview of the most important learning theories is given, including the theories underlying the interventions applied in this study. The population of students participating in this research project is also described, namely the students encountering difficulties with the learning of mathematics in general and multiplication in particular. In general, these students have memory deficits leading to difficulties in the acquisition and remembering of math knowledge. In addition, they often show inadequate use of strategies for solving math tasks, caused by problems with the acquisition and the application of both cognitive and metacognitive strategies. As a consequence they also show deficits in the generalization and transfer of learned knowledge to new and unknown tasks. The instruction should therefore be adapted to the specific instructional needs of these students. In chapter 1, it is also described how students normally receive mathematics education and how instruction could be adapted to their specific needs. Furthermore, a description of the MASTER intervention program, which was used in this study, is given.

Next to the empirical study described in this thesis, a literature study has been undertaken to investigate the effectiveness of different forms of instruction. The results of this study are presented in Chapter 2. Fifty-eight studies that reported on the effects of different interventions for elementary school students with math learning difficulties are included in a meta-analysis. These interventions could be divided into three domains: preparatory math, basic skills, and problem solving. Interventions that focused on the learning of basic math skills proved to be the most effective. The results also show that directed instruction and self-instruction are more effective than guided instruction. Finally, it was found that interventions with computer assisted instruction or peer tutoring were less effective than instruction provided by an adult teacher.

A pilot study has been conducted to investigate the suitability of the intervention program for both experimental conditions. The program was therefore adjusted to suit both a guided instruction version and a directed instruction one. It was also studied if the testing materials were appropriate to measure the students' ability in order to investigate the research questions. A total of 75 low-achieving students from seven elementary schools for general or special education have participated in the pilot study. The results are presented in Chapter 3. The results showed that both interventions were more effective than the regular math curriculum. Furthermore, it was found that guided instruction resulted in higher effects on the students' multiplication ability than directed instruction and especially for the students in regular education. Students attending special education schools seem to profit most from directed instruction, in particular for reaching automatized mastery of multiplication facts up to 10 x 10.

The findings from the pilot study have directed a few changes to the instruments for use in the main study. The main study is described in the Chapters 4 and 5. A total of 283 students from 24 schools for either special or general education have participated. The students have been selected on the basis of their math performance: (1) general low math performance, i.e., below the 25th percentile; (2) sufficient knowledge of addition and subtraction up to 100; and (3) insufficient knowledge of multiplication facts up to 10 x 10. The group of selected students was divided into three conditions: guiding instruction, directed instruction, and regular instruction (control group). The students in the experimental conditions (guided and directed) have received a multiplication intervention by means of the aforementioned intervention program. They received instruction two times a week during a period of four to five months. The control students followed the regular curriculum during the same period.

The effects of both forms of instruction on the students' multiplication ability, automaticity, and motivation are described in Chapter 4. The results show that the students who had received directed instruction improved more than the students who had received guided instruction, although the differences between the two conditions proved to be small. It was also found that both experimental conditions were more effective than the control condition. However, only marginal effects on motivational variables were found.

How the students acquired new strategies and how their strategy use developed under influence of the instruction is described in Chapter 5. The students' strategy use was mapped by asking the students to think aloud, and to write their solution strategies down on scratch papers. It was found that the students' strategy use improved significantly during the intervention period. The experimental students showed greater improvement than their control peers. At the end of the training, the experimental students used more and different, more effective, strategies compared to the pretest. In addition, they were found to better apply the learned strategies. The differences between both experimental conditions were nevertheless small. It was found, however, that the students who had received guided instruction, were better able to report on their solution procedures than the students in the directed instruction condition.

The study described in Chapter 6 was meant to investigate the relation between mathematics learning difficulties and cognition. More specifically, it was studied how math difficulties are related to specific cognitive profiles as measured with the Cognitive Assessment System (CAS). This test is meant to measure four different processes important in information processing: Planning, Attention, Simultaneous, and Successive processes (PASS). The CAS has been administered to the students who participated in the main study. The results showed that students with math learning difficulties perform on average lower on the PASS scales than their peers without specific learning difficulties. Furthermore, it was found that a higher than usual percentage of this group of students have a specific cognitive weakness in one of the PASS processes. The results also showed that students with specific math difficulties, such as difficulties with automaticity or with problem solving, have other cognitive profiles than both students without math difficulties and students with general math difficulties. For instance, students with automatization difficulties showed lower scores on planning and successive processing, while students with difficulties learning word problem solving scored low on successive processing too, but this group also had relatively low attention scores.

The results of the different chapters are integrated and discussed in Chapter 7. The main conclusions are that both instructional forms are suitable to teach students with difficulties learning math to use new strategies, although directed instruction is a little more effective than guided instruction for teaching multiplication. Furthermore, instruction especially focused on strategy use is more effective than the regular instruction. These results have led to a number of suggestions for further research. Future research could be focused on different aspects of realistic mathematics education, or on further analyses of different subtypes of students within the larger group of students with math difficulties, and on their special instructional needs.

This study has shown that a specific intervention can be an effective tool to teach multiplication to students having difficulties learning multiplication. Direct instruction is in general the most effective. However, some students benefit most from guided instruction. It is therefore important to always take into account the individual needs of a student when choosing a specific instruction. Furthermore, the choice for instruction methods should also be made on the basis of the usual instruction and resources of both the teacher and the school.

Samenvatting

Deze dissertatie bevat een aantal artikelen waarin verslag wordt gedaan van onderzoek naar kinderen die problemen ondervinden bij het leren rekenen. De centrale vraag hierbij is welke instructievorm deze kinderen nodig hebben om de basisrekenvaardigheden te leren. Aansluitend bij ontwikkelingen in het huidige rekenwiskunde-onderwijs is nagegaan of instructie die is gebaseerd op de eigen inbreng van leerlingen (banende instructie) ook effectief is voor de zwakkere leerlingen. Om dit te onderzoeken is een interventiestudie uitgevoerd in scholen voor regulier en speciaal basisonderwijs. De effectiviteit van banende instructie is vergeleken met een duidelijk gestructureerde, directe instructie, waarvan uit empirisch onderzoek blijkt dat het een effectieve instructiemethode is voor zwakkere leerlingen. In dit onderzoek is gebleken dat banende instructie niet effectiever is dan directe instructie, hoewel de verschillen klein zijn. De conclusie is dat zowel directe als banende instructie effectief kan zijn voor leerlingen die problemen ervaren bij het leren rekenen.

De achtergronden van dit onderzoek zijn beschreven in hoofdstuk 1. In dit hoofdstuk is een overzicht gegeven van de belangrijkste leertheorieën, inclusief de theorieën waar de genoemde interventies op zijn gebaseerd. Bovendien is een beschrijving gegeven van de leerlingen waar dit onderzoek over gaat: leerlingen die moeilijkheden ondervinden bij het leren rekenen, in het bijzonder bij het leren vermenigvuldigen. Deze leerlingen hebben in het algemeen geheugenproblemen, waardoor zij moeite hebben met het verwerven en memoriseren van rekenkennis. Bovendien vertonen zij vaak een inadequaat gebruik van strategieën bij het oplossen van rekentaken, omdat zij problemen hebben met de verwerving en de toepassing van benodigde cognitieve en metacognitieve strategieën. Dit veroorzaakt ook tekorten in de generalisatie en transfer van geleerde kennis naar nieuwe en onbekende taken. Instructie zou daarom gericht moeten zijn op de specifieke instructiebehoeften van deze groep leerlingen. In hoofdstuk 1 is tevens weergegeven hoe leerlingen normaal gesproken rekenwiskunde-onderwijs ontvangen en hoe instructie aangepast kan worden aan de specifieke behoeften van kinderen met rekenproblemen. Ter illustratie hiervan is een beschrijving opgenomen van het interventieprogramma dat in dit onderzoek is gebruikt, het Speciaal Rekenhulpprogramma Vermenigvuldigen.

Naast de genoemde empirische studie is ook een literatuurstudie uitgevoerd naar de effectiviteit van verschillende instructievormen. In hoofdstuk 2 worden de resultaten gepresenteerd van een meta-analyse waarin 58 studies zijn betrokken die de effecten beschrijven van een bepaalde rekenwiskunde-interventie voor basisschoolleerlingen met rekenproblemen. De hier beschreven interventies kunnen worden onderverdeeld in drie domeinen: voorbereidend rekenen, basisvaardigheden en probleemoplossen. De interventies in het domein van de basisvaardigheden bleken over het algemeen het meest effectief te zijn. De resultaten laten ook zien dat directe instructie en zelf-instructie effectiever zijn dan banende instructie. Tot slot is gevonden dat interventies die gebruik maken van computer ondersteund onderwijs en instructie door leeftijdgenootjes minder effect hebben dan interventies die dit niet doen.

Voorafgaand aan het hoofdonderzoek is een vooronderzoek uitgevoerd. Hiermee is nagegaan of het interventieprogramma geschikt was om beide instructievormen uit te voeren. Het rekenhulpprogramma is hiervoor aangepast aan de twee verschillende instructieprincipes: banende en directe instructie. Bovendien is nagegaan of de gebruikte rekentoetsen geschikte instrumenten waren om de onderzoeksvragen te bestuderen. In totaal hebben 75 zwak presterende leerlingen uit zeven scholen voor regulier en speciaal basisonderwijs aan dit onderzoek deelgenomen. De resultaten van dit vooronderzoek, die in hoofdstuk 3 worden gepresenteerd, hebben tegelijkertijd ook een eerste indruk gegeven van de effecten van beide interventies. Ten eerste is gebleken dat beide interventies effectief zijn in vergelijking tot het reguliere rekenwiskunde-onderwijs. Bovendien wijzen de resultaten van dit onderzoek er op dat banende instructie effectiever is dan directe instructie om zwak presterende kinderen, met name zij die regulier basisonderwijs volgen, te leren vermenigvuldigen. De kinderen in het speciaal basisonderwijs lijken iets meer te profiteren van directe instructie, en dan met name voor het geautomatiseerd leren beheersen van de vermenigvuldigingen tot 10 x 10.

Naar aanleiding van de bevindingen in het vooronderzoek zijn de instrumenten enigszins aangepast voor het hoofdonderzoek. Aan het hoofdonderzoek, dat is beschreven in de hoofdstukken 4 en 5, hebben 283 leerlingen uit 24 scholen voor regulier en speciaal basisonderwijs deelgenomen. Deze leerlingen zijn geselecteerd op basis van hun rekenprestaties: zwak in rekenen in het algemeen (behorend bij de laagste 25%), voldoende kennis van optellen en aftrekken en geen of onvolledige beheersing van het vermenigvuldigen tot 10 x 10. De leerlingen zijn verdeeld over drie condities: banende instructie, directe instructie of controle (reguliere instructie). De leerlingen in de beide experimentele condities (banende en directe instructie) hebben een interventie gekregen op het gebied van vermenigvuldigen, met

behulp van het genoemde interventieprogramma. De instructie is gedurende 4 tot 5 maanden twee keer per week gegeven. De controleleerlingen hebben in dezelfde periode het reguliere instructieprogramma gevolgd.

In hoofdstuk 4 zijn de effecten beschreven van beide interventies op de vermenigvuldigvaardigheid, de automatisering van de tafels van vermenigvuldiging en de motivatie van de leerlingen voor rekenen. De resultaten laten zien dat de leerlingen die directe instructie hebben ontvangen gemiddeld meer vooruit zijn gegaan in vermenigvuldigvaardigheid en automatisering dan de leerlingen die banende instructie hebben ontvangen, hoewel deze verschillen niet groot zijn. Bovendien bleek dat beide experimentele condities effectiever waren dan de controleconditie. Er zijn echter weinig verschillen tussen de condities gevonden wat betreft de motivatie van de leerlingen.

In hoofdstuk 5 wordt beschreven hoe de leerlingen in beide experimentele condities nieuwe strategieën leerden en wat de effecten van de interventies zijn op het strategiegebruik van de leerlingen. Om het strategiegebruik in kaart te brengen, is de leerlingen gevraagd om hardop te denken tijdens het maken van een tiental vermenigvuldigopgaven en hebben zij bij het maken van de toets een kladblaadje gebruikt om op te schrijven op welke manier zij tot een antwoord waren gekomen. Er is gevonden dat alle leerlingen gedurende de interventieperiode hun strategiegebruik verbeterden, hoewel de kinderen in de beide experimentele condities meer vooruitgingen dan de controleleerlingen. Na afloop van de interventie gebruikten de experimentele leerlingen meer en andere strategieën en vertoonden zij ook efficiënter strategiegebruik. Bovendien bleken zij beter in staat om de strategieën adequaat toe te passen. Desondanks waren de verschillen tussen de beide experimentele condities klein, hoewel de leerlingen die banende instructie hadden ontvangen na afloop van de interventie beter in staat waren hun strategieën te verwoorden.

In het onderzoek dat wordt beschreven in hoofdstuk 6 is de relatie tussen rekenproblemen en cognitie nader onderzocht en in het bijzonder de relatie tussen rekenproblemen en specifieke cognitieve profielen zoals gemeten met het Cognitive Assessment System (CAS). Dit instrument geeft informatie over vier processen van cognitieve informatieverwerking: Planning, Aandacht, Simultane en Successieve verwerking (PASS). De CAS is afgenomen bij alle leerlingen die aan het hoofdonderzoek hebben deelgenomen. De resultaten laten zien dat leerlingen met rekenproblemen in het algemeen lager presteren dan hun leeftijdgenoten op alle PASS processen en dat in deze groep relatief veel kinderen met specifieke cognitieve tekorten worden gevonden. Ook blijkt dat leerlingen die specifieke rekenproblemen hebben, zoals problemen met de verwerving of automatisering van basisvaardigheden of juist met probleemoplossen, andere cognitieve profielen vertonen dan leerlingen zonder rekenproblemen of leerlingen met algemene rekenproblemen. Leerlingen met automatiseringsproblemen hebben bijvoorbeeld relatief lagere scores op planning en successieve verwerking, terwijl leerlingen die laag scoorden op de toets met woordproblemen ook relatief lagere scores vertoonden op successieve verwerking, maar daarnaast ook op aandacht.

In hoofdstuk 7 worden de resultaten van de verschillende hoofdstukken geïntegreerd besproken. De belangrijkste conclusies zijn dat beide instructievormen geschikt zijn om leerlingen met rekenproblemen nieuwe strategieën te leren gebruiken, maar dat directe instructie effectiever is dan banende instructie om deze leerlingen te leren vermenigvuldigen, hoewel de verschillen klein zijn. Bovendien blijkt dat instructie waarbij de nadruk ligt op het strategiegebruik betere resultaten oplevert dan reguliere instructie. De gevonden resultaten hebben ook geleid tot een aantal suggesties voor verder onderzoek. Vervolgonderzoek zou gericht kunnen zijn op andere aspecten van het realistisch rekenwiskundeonderwijs, of op een nadere analyse van verschillende subgroepen binnen de groep van leerlingen met rekenproblemen en hun specifieke instructiebehoeften.

Dit onderzoek heeft laten zien dat een specifieke interventie een effectief middel kan zijn om leerlingen die hier moeite mee hebben, te leren vermenigvuldigen. In het algemeen is een directe instructievorm hierbij het meest effectief. Voor een aantal leerlingen is echter een banende instructievorm geschikter. Daarom is het belangrijk om bij de keuze voor een bepaalde instructievorm altijd naar de individuele behoeften van de betreffende leerling te kijken. Bovendien moet rekening gehouden worden met de instructievorm die de leerling normaliter krijgt en de mogelijkheden van de leerkracht en/of de situatie in de klas.

Curriculum Vitae

Evelyn Kroesbergen was born in Ede, the Netherlands, on February 16th, 1975. After finishing secondary school (VWO) in 1993, she studied Educational Sciences with a major in Special Education at Utrecht University. She graduated in September 1998, after which she directly started working on a Ph.D.-project (financed by the Netherlands Organization for Scientific Research NWO) at the Institute for the Study of Education and Human Development (ISED) of the Utrecht University. This dissertation on mathematics education for low-achieving students is the result of that project. In addition to her research activities, she has worked as a teacher. She has given a course in statistics and supervised students writing their master theses. During her work as a Ph.D.-student she has been a co-ordinator of EARLI's JURE (Junior Researchers of the European Association for Research on Learning and Instruction) for two years. She has also represented the Ph.D.-students of her faculty in the BAU, the association of Ph.D.-students of the Utrecht University.