Stability of binocular depth perception with moving head and eyes

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Abstract
We systematically analyse the binocular disparity field under various eye, head and stimulus positions and orientations. From the literature we know that certain classes of disparity which involve the entire disparity field (such as those caused by horisontal lateral shift, differential rotation, horisontal scale and horisontal shear between the entire half-images of a stereogram) lead to relatively poor depth perception in the case of limited observation periods. These classes of disparity are found to be similar to the classes of disparities which are brought about by eye and head movements. Our analysis supports the suggestion that binocular depth perception is based primarily (for the first few hundred milliseconds) on classes of disparity that do not change as a result of ego-movement.

Introduction

The separation of the human eyes causes each eye to see a disparate image of the outside world. Generally, it has been accepted that positional disparities are sufficient to generate a three-dimensional (3D) percept (e.g. Wheatstone, 1838; Ogle, 1950; Julesz, 1971). Wheatstone's development of the stereoscope in 1838 was based on this idea. Recently, this knowledge has been used in the field of binocular robots. However, many phenomena relating to disparity and perception of depth are still not understood, including the fact that binocular vision is largely unaffected by eye and head movements (Westheimer & McKee, 1978 concerning lateral eye movements; Steinman, Levinson, Collewijn & van der Steen, 1985 and Patterson & Fox, 1984 concerning head movement).

In binocular robots the quality of 3D analysis is severely reduced by the instability of the cameras (the disparity acquisition system; Eklundh, 1993). By analogy, one would expect the stability of human binocular vision to be reduced by eye and head movements. In the case of a simple object like a chessboard it is immediately clear that the images of the chessboard on our two retinae differ according to whether the chessboard is positioned in front of us or eccentrically. Since the disparity field is composed of the positional differences between the retinal images, the disparity field will depend on the position of the object. On the other hand, if the object is static but the binocular observer makes an eye or head movement, the disparity field before and after the movements will also change. However, this time the disparity of the object and environment change together. In short, during eye and head movements, the images of the entire visual world are continuously changing on both retinae, which means that there are also continuously changing disparities. One would expect these changing disparities to reduce the stability of stereopsis.

In principle, the visual system can utilise the signals that control the eye and neck muscles (efference copies) in order to correct stereopsis for disparities induced by controlled eye and head movements. However, disparities are not only due to controlled eye and head movements, they can also be due to uncontrolled eye and head movements. These uncontrolled movements are caused by noise in the motor system. Experiments have demonstrated large discrepancies between the level of stereoacuity and the relative sloppiness of oculomotor control. Optimal stereoacuity thresholds in the fovea typically attain mean standard deviations of the order of 5 seconds of arc (Berry, 1948; Westheimer & McKee, 1978; McKee, 1983), which is about one-sixth of the diameter of the smallest foveal cones (Westheimer, 1979a). These thresholds for stereoacuity can be obtained even for a 200-ms exposure (Westheimer & McKee, 1978) and are similar in magnitude to the best monocular hyperacuities for motion displacement, vernier tasks and relative width (Westheimer & McKee, 1979; McKee, Welch, Taylor and Bowne, 1990). Given these values binocular vision can be very sensitive. It can be regarded as a hyperacuity mechanism. On the other hand, during natural behaviour, vergence position errors of up to 1-2 deg (Collewijn & Erkelens, 1990), vergence velocity errors of up to 1 deg/s (Steinman & Collewijn, 1980) and errors in cyclovergence of 10 minarc (Enright, 1990; van Rijn, van der Steen & Collewijn, 1994) are easily generated and introduce disparities that are similar in size to the errors. The measured sloppiness of oculomotor control is not due to artefacts in experimental methods (Ferman, Collewijn, Jansen & van den Berg, 1987). Besides oculomotor system instability, there is another factor of uncertainty which affects the interpretation of disparities, namely the exact orientation of the head relative to the body. Head stability is no better than oculomotor stability (Schor, Kearney & Dieringer, 1988).

Although oculomotor and head control is sloppy it is nevertheless possible that a feedback system is at work in binocular depth perception.[1] The noise in the oculomotor system, though producing (cyclo)vergence errors, could be known to the visual system (for instance by means of muscle sensors) and utilised in order to interpret disparities. There is evidence, however, that there is no such feedback system in binocular depth perception. Firstly, fast side-to-side rotations of the head, or pressing against the eye ball, do not influence depth perception (Steinman et al., 1985). Secondly, the results presented in several reports (Foley, 1980; Erkelens & Collewijn, 1985a,b; Regan, Erkelens & Collewijn, 1986; Collett, Schwarz & Sobel, 1991; Cumming, Johnston & Parker, 1991; Logvinenko and Belopolskii (1994); Rogers & Bradshaw, 1995 and Backus, Banks & Crowell, 1996) show that in situations where (conficting) eye muscle information is available changing eye posture does not lead to changing perception of depth in the case of large field stimuli (large displays) or they lead to only weak perception of depth in the case of small field stimuli.

The discrepancies between the sensitivity of stereoscopic vision and the sloppiness of oculomotor control mean that oculomotor stability is at least one order of magnitude less precise than measured stereoacuity (also reported by Nelson, 1977 and Collewijn, van der Steen & van Rijn, 1991). Even if we assume that subjects can obtain very good stereoacuity by using relative depth differences (which are unaffected by noisy eye and head movements; Westheimer, 1979b; Erkelens & Collewijn, 1985a,b) we still do not know why the stereoacuity stimulus as a whole does not tremble in depth as a result of the trembling eye and head movements.

A possible way for the visual system to deal with the effects of sloppy motor control is to utilise all available retinal information. Frisby, Mayhew and co-workers have proposed that gaze (eye posture) parameters theoretically can be calibrated by "shape-from-texture".

[1] Binocular 3D vision involves perception of directions and perception of depth. Regarding binocular perception of directions there is evidence that vestibular and proprioceptive information is used to maintain stability (Howard, 1982; Carpenter, 1988). For instance, fast side-to-side rotations of the head (Steinman et al., 1985), or pressing against the eye-ball, impair the correct coupling between extra-retinal signals and perceived directions and result in impairment of the stability of the visual world in lateral directions.

However, recently they showed that this hypothesis was not confirmed experimentally (Frisby, Buckley, Wishart, Porrill, Gårding & Mayhew, 1995). More importantly, Mayhew and Longuet-Higgins (1981) showed that information about gaze parameters in principle can be calculated from the horizontal and vertical disparities. This gaze parameter information could then be used to interpret disparities. In addition, Gårding, Porrill, Mayhew and Frisby (1995) proposed a decomposition of the disparity interpretation process into disparity correction, which is used to compute three-dimensional structure up to a relief transformation, and disparity normalisation, which is used to resolve the relief ambiguity to obtain metric structure. Discussing the existing literature based on this decomposition into disparity correction and disparity normalisation they showed that in relief tasks depth perception exhibits a large and stable dependence on the structure of the vertical disparity field, whereas metric tasks are hardly affected. Gårding et al. (1995) also reported on the fact that visual tasks that actually require a full metric reconstruction of the three-dimensional visual world are fairly uncommon. The relief transformation preserves many important properties of visual shape, notably the depth order as well as all projective properties such as coplanarity and collinearity. Therefore a disparity processing system that computes a reconstruction of the three-dimensional visual world relying on retinal disparities alone is very attractive even if it does so up to a relief transformation.

Important exceptions to the idea that the metric tasks are hardly affected by vertical disparities are the studies of Rogers and Bradshaw (1993, 1995). Rogers and Bradshaw (1993) showed that subjects can use vertical disparities in order to estimate the perceived peak-to-trough depth of corrugations for large-field stimuli. However, the amount of perceived depth in the full-disparity-cue condition was very much less than would be required for complete depth constancy. In the appendix of their 1995 paper, Rogers and Bradshaw showed that absolute distance from the observer is altered by modifying vertical disparities. (See also the paper of Friedman, Kaye and Richards (1978) who also found that vertical disparity influences metrical perceptual tasks.) Yet, these studies have not been done for limited observation periods.

Despite these findings about a disparity processing system that computes a metrical reconstruction there is no evidence yet that such a system is effective in human vision on a short time-scale. In the next section we report on perceptual studies using simple stereograms which show that several classes of basic stimuli which mimic real world stimuli (containing both realistic horizontal and vertical disparities) do not elicit reliable perception of metric aspects of depth for limited observation periods (up to the order of seconds). On the other hand it has been reported that relief tasks in stereopsis can be effective even to the order of milliseconds (e.g. Kumar & Glaser, 1993; Uttal, Davis & Welke, 1994).

Stereograms and depth perception

Our knowledge about binocular depth perception is obtained to a large extent from experiments with stereograms. In such experiments the subject views (with static head) two half-images of a stereogram, one transformed relative to the other, projected on a screen. In accordance with geometrical rules, local horizontal lateral shift of the half-images relative to each other alters the perceived distance. Horizontal scale between parts of the half-images of a stereogram leads to perceived slant about the vertical axis. Local horizontal shear, on the other hand, leads to perceived slant about the horizontal axis. Figure 1 shows an example of the horizontal scale and shear transformation.

Lateral movements of the entire half-images of a stereogram relative to each other lead to vergence of the eyes (with a gain unequal to one), but are not interpreted as changes in distance (Erkelens & Collewijn, 1985a,b; Regan, et al., 1986). In contrast, differential movements of parts of the half-images give rise to vivid perception of motion in depth. In addition, Regan et al. (1986) showed that under stabilized retinal conditions abrupt changes in the image vergence angle produced no impression of a step change in depth. These authors suggested that the explanation for their results may be that the brain interprets lateral shifts between the entire parts of the stereograms as movements of the eyes and therefore these shifts are best ignored as signals for depth. [As we will see below this explanation is not entirely correct.]

Figure 1 about here.

Another perceptual study (Howard & Zacher, 1991) showed that differential rotation of the entire half-images of a stereogram induces cyclovergence with a gain unequal to one (cyclodisparity) but elicits poor perception of depth. Again, two different cyclodisparities, simultaneously present in the visual field, are required for reliable depth perception. Collewijn, van der Steen and van Rijn (1990) reported that thresholds for perception of depth caused by cyclodisparity increase by a factor of 7 when the visual reference is removed. Cyclodisparities can have considerable magnitudes and can occur frequently during natural behaviour. Howard, Ohmi and Sun (1993) suggested that whole-field cyclodisparities could indicate that the eyes are misaligned and that therefore the perceptual system is inclined to ignore these cyclodisparities when judging slant.

Figure 2 about here.

Finally, slant from horizontal scale and horizontal shear between the entire half-images of a stereogram is relatively poorly perceived (Shipley & Hyson, 1972; Mitchison & Westheimer, 1984, 1990; Stevens & Brookes, 1988; Gillam, Chambers & Russo, 1988). Recently, van Ee and Erkelens (1996a) investigated temporal aspects of slant perception induced by whole-field horizontal scale and horizontal shear. They quantitatively corroborated the earlier finding that when observation periods last up to a few seconds perception of slant caused by whole-field horizontal scale and shear is relatively poor. Using arguments similar to those presented in the previous two paragraphs we now attempt to relate this experimental result to the orientation of the head. Head rotation in a stationary visual world should cause similar disparities as rotation of the entire visual world about the centre of the head when the head is stationary. This idea is explained in figure 2 where two similar drawings are depicted. The drawing in figure 2a represents the geometry of viewing a horizontally scaled stereogram. The drawing in figure 2b represents the geometry of viewing an (initially) frontal plane with rotated head (initially, fixation was at eccentricity α). In the horizontal plane the retinal images of the two situations are the same. Analoguously, one expects disparities caused by forward rotation of the head to correspond to disparities caused by horizontal shear of the half-images of a stereogram. The arguments we use are similar to those used by Erkelens and Collewijn (1985a) and Howard et al. (1993). We suggest that the reason why depth perception of one linear transformation within the stereogram is poor and depth perception of two different linear transformations is vivid is that the disparity field caused by only one linear transformation is ambiguous. In other words, head rotations could induce the same disparity fields as the scaled and sheared stereograms. We argue that the disparity fields caused by horizontal scale and shear are therefore primarily ignored as signals for perception of slant. We also argue that the disparity field caused by two different, simultaneously present, linear transformations cannot be similar to a field caused by ego-movement and, therefore, such a field is an effective stimulus for the slant perception of one plane relative to the other.

Hypothesis

Thus, during (noisy) eye and head movements the disparity field changes continuously. Why do we not perceive a visual world trembling in depth as a result of our trembling disparity acquisition system? One could think of two opposite hypotheses. Either the visual system compensates completely for the disparities induced by these (noisy) eye and head movements or the visual system is blind for these disparities. The findings about (1) using the signals that control the eye and head muscles (efference copies), (2) using a feedback loop based on muscle sensors and (3) using all the available (horizontal and vertical) disparities, suggest that the compensation hypothesis does not provide a sufficient answer to our question, at least not for limited (realistic) observation periods.

Taken together, the above-mentioned suggested explanations for the poor sensitivity of depth perception to several transformations between half-images of a stereogram lead to a generalised hypothesis. We hypothesise that a possible way for the visual system to deal with the effects of sloppy eye and head movements is to use only that part of disparity information which is invariant under eye and head movements. Investigations about the validity of this hypothesis require precise knowledge about what sort of disparity is induced, on the one hand by eye and head movements and on the other hand by transformed stereograms which are known to elicit only poor depth perception. So far, this knowledge has not been supplied by the literature.

The geometry of binocular disparity

Headcentric coordinates and head movement

In order to identify a test point P in three-dimensional space relative to the head we define a right-handed orthogonal coordinate system with the origin above the vertebral column and at the same level as the eyes. The x-axis points from right to left parallel to the interocular axis, the y-axis points vertically upwards, and the z-axis points in the primary direction (straight ahead). After a head rotation or translation the headcentric coordinates (x^H_P, y^H_P, z^H_P) of test point P are (x'^H_P, y'^H_P, z'^H_P). Head translation corresponds to a trivial coordinate modification. For example, a head translation along the y-axis over an arbitrary distance (ad) modifies y^H_P coordinates into $y'^H_P = y^H_P - ad$. Head rotation is not trivial. The coordinates before and after a head rotation are related to each other by an Euler rotation matrix:

$$\begin{pmatrix} x'^H_P \\ y'^H_P \\ z'^H_P \end{pmatrix} = \begin{pmatrix} R^H \end{pmatrix} \begin{pmatrix} x^H_P \\ y^H_P \\ z^H_P \end{pmatrix} \quad ,$$

$$R^H = \begin{pmatrix} \cos\phi^H \cos\psi^H + \sin\phi^H \sin\theta^H \sin\psi^H & \cos\theta^H \sin\psi^H & -\sin\phi^H \cos\psi^H + \cos\phi^H \sin\theta^H \sin\psi^H \\ -\cos\phi^H \sin\psi^H + \sin\phi^H \sin\theta^H \cos\psi^H & \cos\theta^H \cos\psi^H & \sin\phi^H \sin\psi^H + \cos\phi^H \sin\theta^H \cos\psi^H \\ \sin\phi^H \cos\theta^H & -\sin\theta^H & \cos\phi^H \cos\theta^H \end{pmatrix} ,$$

where ϕ^H, θ^H and ψ^H denote angles of head rotation about the vertical, horizontal and primary direction, respectively. The signs of the angles are again defined according to a right-handed coordinate system. The order of rotations is described in a Fick manner, which means that the head is first rotated about the vertical axis, then about the horizontal axis and lastly about the primary direction.[3]

Headcentric coordinates and stereograms

If stereograms are involved, then a separate calculation has to be performed to find the headcentric coordinates for each of the two transformed half-images:

$$\begin{pmatrix} x'^H_P \\ y'^H_P \end{pmatrix}_{\text{left eye image}} = \begin{pmatrix} \cos(0.5\psi^S) + horscale & -\sin(0.5\psi^S) + horshear \\ \sin(0.5\psi^S) & \cos(0.5\psi^S) \end{pmatrix} \begin{pmatrix} x^H_P \\ y^H_P \end{pmatrix}_{\text{left eye image}} + \begin{pmatrix} -0.5 shift \\ 0 \end{pmatrix} ,$$

$$\begin{pmatrix} x'^H_P \\ y'^H_P \end{pmatrix}_{\text{right eye image}} = \begin{pmatrix} \cos(-0.5\psi^S) & -\sin(-0.5\psi^S) \\ \sin(-0.5\psi^S) & \cos(-0.5\psi^S) \end{pmatrix} \begin{pmatrix} x^H_P \\ y^H_P \end{pmatrix}_{\text{right eye image}} + \begin{pmatrix} 0.5 shift \\ 0 \end{pmatrix} ,$$

where ψ^S, $shift$, $horscale$, $horshear$ denote the rotation, lateral shift, horizontal scale and horizontal shear between the entire two half-images of the stereogram, respectively. For simplicity we assume that there is only one transformation at a time between the parts of the stereogram relative to each other.

[3] Fick's coordinate system (Fick, 1854) and Helmholtz's coordinate system (von Helmholtz, 1911) originate from eye movement studies. Rotations do not commute under summation. Decisions should be made about the order in which rotations should be performed. In Fick's system the vertical axis of the eye ball is assumed to be fixed to the skull and the horizontal axis of the eye ball is assumed to rotate gimbal-fashion about the vertical axis. In Helmholtz's system it is the horizontal axis which is assumed to be fixed to the skull (Howard, 1982; page 181). Which system is preferable will depend on the situation. The advantage of Fick's system is that isovergence surfaces are equivalent to isodisparity surfaces. The advantage of Helmholtz's system is that it is based on epi-polar geometry.

Oculocentric coordinates

In addition to defining a coordinate system relative to the head, we also have to define retinal coordinate systems. As before, the x-axis points from right to left, the y-axis points vertically upwards, and the z-axis points in the primary direction. The centre of the oculocentric coordinate system is positioned in the centre of the eye. Initially, we assume that the eye fixates a target at infinity, which means that the visual axis coincides with the primary direction. As shown in figure 3 the coordinates (x^L_P, y^L_P, z^L_P) of a test point P relative to the left eye are parametrised in a Fick manner by its longitude ϕ^L_P and its latitude θ^L_P:

$$\begin{pmatrix} x^L_P \\ y^L_P \\ z^L_P \end{pmatrix} = \begin{pmatrix} x'^H_P - 0.5I \\ y'^H_P \\ z'^H_P - z_l \end{pmatrix} = \begin{pmatrix} \sin\phi^L_P \cos\theta^L_P \\ -\sin\theta^L_P \\ \cos\phi^L_P \cos\theta^L_P \end{pmatrix} ,$$

where z_l is the distance between the centre of the oculocentric coordinate system and the headcentric coordinate system along the z-axis. I denotes the interocular distance. Similar notation is used for the right eye.

Figure 3 about here.

The direction of a new fixation point relative to the left eye is denoted by longitude ϕ^L_F and latitude θ^L_F. The coordinates of a point before and after an eye rotation to the new fixation point are related by an Euler matrix similar to the one given above. After an eye rotation to the fixation point the coordinates of point P relative to the left eye are (x'^L_P, y'^L_P, z'^L_P):

$$\begin{pmatrix} x'^L_P \\ y'^L_P \\ z'^L_P \end{pmatrix} = \begin{pmatrix} \sin\phi'^L_P \cos\theta'^L_P \\ -\sin\theta'^L_P \\ \cos\phi'^L_P \cos\theta'^L_P \end{pmatrix} = \begin{pmatrix} R^L \end{pmatrix} \begin{pmatrix} \sin\phi^L_P \cos\theta^L_P \\ -\sin\theta^L_P \\ \cos\phi^L_P \cos\theta^L_P \end{pmatrix} ,$$

$$R^L = \begin{pmatrix} \cos\phi^L_F \cos\psi^L_F + \sin\phi^L_F \sin\theta^L_F \sin\psi^L_F & \cos\theta^L_F \sin\psi^L_F & -\sin\phi^L_F \cos\psi^L_F + \cos\phi^L_F \sin\theta^L_F \sin\psi^L_F \\ -\cos\phi^L_F \sin\psi^L_F + \sin\phi^L_F \sin\theta^L_F \cos\psi^L_F & \cos\theta^L_F \cos\psi^L_F & \sin\phi^L_F \sin\psi^L_F + \cos\phi^L_F \sin\theta^L_F \cos\psi^L_F \\ \sin\phi^L_F \cos\theta^L_F & -\sin\theta^L_F & \cos\phi^L_F \cos\theta^L_F \end{pmatrix} .$$

From these three equations the longitude ${\phi'}^L_P$ and latitude ${\theta'}^L_P$ in the rotated eye coordinate system can be calculated for arbitrary test points and fixation points. An identical procedure has to be performed for the right eye in order to find ${\phi'}^R_P$ and ${\theta'}^R_P$. Disparity is computed from the differences between retinal coordinates in the two eyes. Horizontal (in fact longitudinal) disparity is defined by subtracting ${\phi'}^R_P$ from ${\phi'}^L_P$. Vertical (latitudinal) disparity is obtained by subtracting ${\theta'}^R_P$ from ${\theta'}^L_P$.

Numerical calculations

The above-mentioned definitions are implemented in a computer program in which we compute disparity fields generated by planar surfaces. The disparity fields are computed for a range of eye, head and object positions, on the one hand, and for several transformations between half-images of a stereogram, on the other hand. Throughout the text and figures disparity is calculated in oculocentric coordinates for a field of 80×80 deg which is centered around the fixation point. This field is provided with a (virtual) lattice of 12×12 evenly distributed directions (the angle between adjacent directions is $80/11 = 7.3$ deg). Disparity is calculated for each of the 144 directions. Results are plotted as a function of longitude (ϕ) and latitude (θ) which are taken relative to the head. In our calculations we use planar surfaces, since planar surfaces have simple computational properties. In addition, the disparity fields of these surfaces have several symmetrical properties, as is shown in the figures throughout this paper, which makes them easier to interpret. However, in principle it is not relevant what the source of the disparity field is. We are not primarily interested in the disparity field per se. We are interested in how a disparity field transforms as a result of eye and head movements. In the calculations we take the centre of head rotation to be 10 cm behind the eyes and the interocular distance to be 6.5 cm. Again, the exact values of these quantities are not relevant for the purpose of our study. We make the assumption that the nodal point and the centre of eye rotation coincide. Cormack and Fox (1985) found almost no effect of nodal point motion for different fixations except under the most extreme conditions.

Disparity and the distance of the object

In figure 4 it is shown how the disparity field depends on viewing distance and direction. In this figure the horizontal and vertical disparity fields are shown for two fronto-parallel (frontal) planes at distances of 250 cm (the white patch, d1, in figure 4) and 50 cm (grey patch, d2). The figure shows that disparity fields of frontal planes are curved when viewed at a finite distance. Objects that are curved along the horopter have zero disparity. For stimuli nearer than the horopter the horizontal disparity is by definition positive. Conversely, for stimuli further away than the horopter, horizontal disparity is negative. Since the horizontal disparity does not depend on latitude (in Fick's description), the horizontal component of the disparity field (figure 4a) does not depend on θ either. Disparity is zero for the fixation point. When fixation is on the plane in the primary direction, points of the frontal plane have negative horizontal disparity because all points are located further away than the horopter.

Figure 4 about here.

The vertical disparity fields of the frontal planes at 250 cm (d1) and 50 cm (d2) in front of the eyes are shown in figure 4b. These fields depend both on ϕ and θ. Each point located outside the plane of fixation and nearer to one eye than to the other eye has vertical disparity. Since fixation is chosen to be in the primary direction, vertical disparity is zero along the directions $\phi = 0$ or $\theta = 0$ and anti-symmetrical with respect to these axes.

Eye and head movements versus stereograms

It will be demonstrated that disparity fields that are brought about by eye and head movements can be adequately simulated by stereograms. In the rest of the paper we compare the disparity field caused by a particular eye or head movement with the disparity field caused by the stereogram that corresponds theoretically to the eye or head movement. The basic stimulus (before the eye or head movement) is always a frontal plane at a distance of 100 cm in front of the eyes.

Cyclovergence versus differential rotation within the stereogram

According to Donders' law (Donders, 1876) and Listing's law (Listing, 1854) the eyes are slightly rotated relative to each other about the line of sight while fixating a tertiary position (for a review see Alpern, 1962). This implies that after a change of fixation cyclodisparity is introduced. The disparity field of the frontal plane caused by pure cyclovergence is depicted in figure 5. The magnitude of cyclovergence is chosen to be 1.26 deg (each eye 0.63 deg). Figure 5a and figure 5b show the horizontal disparity and vertical disparity, respectively, of the plane after such cyclovergence.

Figures 5c and 5d show the horizontal and vertical disparity field of a stereogram with differentially rotated half-images. Each part of the stereogram is rotated over 0.63 deg in opposite directions. The disparity field (both horizontal and vertical) is more curved for negative θ because the points of intersection of the light rays that come from corresponding points of both half-images of the stereogram are nearer for negative θ than for positive θ.

Figure 5 about here.

In the case of cyclovergence the axis of rotation is the visual axis. In the case of differential rotation within a stereogram the axis of rotation of either half-image is perpendicular to the projection screen. Therefore it is not trivial that the disparity fields induced by cyclovergence and differential rotation are equal. The similarity between the numerical results of figures 5a and 5c and the numerical results of figures 5b and 5d implies that the disparity fields caused by cyclovergence and differential rotation of the entire parts of a stereogram are approximately equivalent (figures 5e and 5f).

Head translation in the primary direction versus horizontal shift within the stereogram

Generally, the disparities caused by a head translation towards a frontal plane can be simulated by the following lateral shift between the two half-images of the stereogram:

$$\text{shift} = \frac{T_z \cdot I}{z_0 - T_z} \quad ,$$

Figure 6 about here.

where T_z denotes the translation of the head towards the plane, I the interocular distance and z_0 the distance between the stimulus and the eyes. We can derive this relationship in a straightforward manner as is illustrated in figure 6. From this figure it immediately follows that:

$$\tan\left(\frac{1}{2}\delta\right) = \frac{\frac{1}{2}\mathrm{shift}}{T_z} = \frac{\frac{1}{2}I}{z_0 - T_z} \quad .$$

Figures 7a and 7b show the horizontal and vertical disparity fields of the plane (which was initially positioned at 100 cm) after a head translation of 25 cm towards the plane. Effectively these fields are similar to the fields caused by a plane at 75 cm, which means that both the horizontal and vertical disparity fields are more strongly curved than those of a plane at 100 cm. According to the given relationship between lateral shift and head translation the disparities caused by a head translation of 25 cm towards the frontal plane (at 100 cm in front of the eyes) can be simulated by a lateral shift of 2.2 cm between the half-images of a stereogram (at 100 cm in front of the eyes). Figure 7c shows the horizontal disparity field and figure 7d the vertical disparity field of such a stereogram.

The similarity between figures 7a and 7c as well as between figures 7b and 7d implies that in the disparity domain head translation in the primary direction and lateral shift of the two entire parts of a stereogram are almost equivalent. The lower panels of figure 7 shows the difference in disparity between the upper and middle panels of figure 7.

The results reported so far can be related to the results of Erkelens and Collewijn (1985a,b). They recognized that a change of fixation causes a translation of the retinal images. They also suggested that an offset in the disparity domain corresponds to a lateral shift between the two parts of a stereogram. Figure 7 shows that the latter suggestion is not entirely correct. A lateral shift within a stereogram corresponds to a head translation towards the stimulus, but not to a vergence movement of the eyes.

Figure 7 about here.

Rotation of the head about the vertical axis versus horizontal scale within the stereogram

Consider a frontal plane at 100 cm which is fixated at a longitude of 20 deg. Next, consider a clockwise rotation of the head about the vertical axis over an angle of 20 deg. Figure 8a shows the horizontal disparity of the plane after the rotation. The distance between the plane and the head is shorter for negative than for positive ϕ. As a result the disparity field is more curved for negative ϕ, which is in agreement with the results shown in figure

4a. Figure 8b shows the vertical disparity of the plane under the same viewing conditions. The vertical disparity is anti-symmetrical with respect to the line $\phi = 20$ and with respect to the line $\theta = 0$. The fact that there is a larger difference in distance between the plane and either eye for negative than for positive ϕ results in larger vertical disparities for negative ϕ.

In the introductory section we explained why the disparity fields of horizontal scale can be expected to correspond to the disparity fields caused by head rotation about the vertical axis. Now we will calculate the disparity fields caused by horizontal scale between the two parts of the theoretically corresponding stereogram. In order to know which stereogram corresponds best we use a relationship between slant and the amount of horizontal scale (see van Ee & Erkelens (1996a) for a derivation):

$$\text{slant} = \arctan(\frac{M-1}{M+1} \cdot \frac{2z_0}{I}) \ ,$$

where M is the magnification factor of horizontal scale, I the interocular distance and z_0 the distance from the stimulus.[†] According to this relation, a frontal plane at a distance of 100 cm viewed after a head rotation over 20 deg about the vertical axis corresponds theoretically to a stereogram at a distance[‡] of 106 cm with a horizontal scale of 2.2%.

Figure 8 about here.

Consider a stereogram at a distance of 106 cm in front of the eyes. Figure 8c depicts the horizontal disparity caused by a horizontal scale of 2.2 %. Since the points of intersection of the light rays which come from corresponding points of both half-images of the stereogram are nearer for negative ϕ than for positive ϕ, the disparity field is more curved (as in figure

[†] We adopt Ogle's notation. Ogle (1950), who related slant to horizontal magnification using an aniseikonic lens, found:

$$\text{slant} = \arctan(\frac{M-1}{M} \cdot \frac{z_0}{I}) \ .$$

The relationship between slant and magnification in the case of a stereogram is slightly different from the relationship in the case of aniseikonic lenses.

[‡] From figure 2 it can be inferred that the geometry of a frontal plane at a distance of z_0 viewed after a head rotation over α degree is similar to that of a plane slanted over α degree but at a distance of $z_0 / \cos\alpha$.

8a). Figure 8d shows the vertical disparity that is caused by this transformation. Note that the horizontal scale transformation also has an influence on vertical disparity because the left-hand sides of the half-images of the stereogram are translated in opposite directions, which alters the distance of these parts from the eyes (the same holds for the right-hand sides). As shown in figure 8b, the vertical disparity is anti-symmetrical with respect to the line $\theta = 0$. The lower panels of figure 8 show the difference in horizontal (e) and vertical (f) disparity induced by head rotation about the vertical axis and by the corresponding horizontally scaled stereogram.

Rotation of the head about the horizontal axis versus horizontal shear within the stereogram

Figure 9a shows the horizontal disparity when the initially frontal plane at a distance of 100 cm and fixated at a latitude of 20 deg is viewed after a forward rotation of the head (about the horizontal axis) over an angle of 20 deg. Since the distance between the plane and the head is shorter for negative than for positive θ, the disparity field is more curved for negative θ. Figure 9b shows the vertical disparity for the same viewing conditions. The vertical disparity is anti-symmetrical with respect to the $\phi = 0$ direction but is not anti-symmetrical with respect to the $\theta = 0$ direction. This time vertical disparities for negative θ are larger than those for positive θ.

The relationship between slant and horizontal shear (angle β) is (see van Ee & Erkelens (1996a) for a derivation): [6]

$$\text{slant} = \arctan(\tan\beta \cdot \frac{z_0}{I}) \ .$$

This means that, theoretically, the disparity field of a frontal plane at a distance of 100 cm viewed with the head forwardly rotated over 20 deg corresponds to a horizontally sheared stereogram with magnitude 1.26 deg at a distance of 106 cm. Figures 9c and 9d show the horizontal disparity and vertical disparity induced by the corresponding horizontally sheared stereogram. (Figures 9c and 5c are similar to each other because the horizontal component of disparity induced by a horizontal shear of 1.26 deg is similar to the horizontal disparity caused by differential rotation of 1.26 deg within the stereogram.) The lower panels of figure 9 show that the horizontal (e) and vertical (f) disparity caused by a forward head rotation resembles the horizontal disparity induced by horizontal shear. The equivalence is not very good. A possible reason is that the horizontal shear of a stereogram affects the x-component of the perceived plane, which is not the case with perceived

[6] After completing this paper Prof. Collewijn remarked that Ogle and Ellerbrock (1946) derived a similar equation in order to describe the relationship between the slant of one, in the medial plane positioned, vertical line and the retinal orientation disparity of this line.

frontal planes after a rotation of the head. A slanted plane, induced by horizontal shear of a rectangular stereogram, is perceived as a trapezoid with the small side nearer than the large side.

Figure 9 about here.

Discussion

In this paper we have studied the influence of eye and head movements on the binocular disparity field. We have also investigated what sort of disparity is induced by differential rotation, horizontal lateral shift, horizontal scale and horizontal shear between half-images of a stereogram. We have found that in the disparity domain:
1) cyclovergence resembles differential rotation between the half-images of the stereogram;
2) head translation in the primary direction resembles horizontal lateral shift between the half-images of the stereogram;
3) rotation of the head about the vertical axis (side-to-side rotation) resembles horizontal scale between the half-images of the stereogram;
4) rotation of the head about the horizontal axis (forward rotation) resembles horizontal shear between the half-images of the stereogram.
These numerical results lead to new interpretations of the results of earlier experiments.

Sensitivity of stereopsis

In order to interpret the disparities of the lower panels of figures 5 and 7 to 9 (that is, the differences between eye and head movement-induced disparities and theoretically corresponding stereogram-induced disparities) it is important to realise that sensitivity of human stereopsis varies with eccentricity. The relevant question is: Are the disparities of the lower panels small enough for the visual system not to perceive the difference between a disparity field induced by an eye or head movement and the disparity field induced by the above-mentioned stereograms corresponding to the eye or head movement. Not much is known about the sensitivity of peripheral binocular vision. Several studies showed that stereoacuity strongly degrades outside the foveal area (e.g. Fendick & Westheimer, 1983; Badcock & Schor, 1985; McKee et al., 1990).

Fendick and Westheimer (1983) found that stereoacuity is about 1 arcmin at an eccentricity of 10 deg peripherally. According to Drasdo (1991) the stereoacuity (V) found by Fendick and Westheimer (1983) can be extrapolated with eccentricity (ξ in deg) by a linear function: $V = 0.1 + 0.12\xi$ [in minarc]. In this equation there is no difference between horizontal and vertical eccentricities. The idea of extrapolation is based on similar linear extrapolation functions for several monocular domains (Vernier, Landolt C acuity etc.) but has not been verified for binocular vision.

However, it should be realised that the experimental data concerning stereoacuity have been obtained under ideal and controlled laboratory conditions, usually with experienced subjects. The primary aim of such studies was to investigate the limits of the visual system and not the sensitivity of binocular vision during natural behaviour. Moreover, in view of the fact that in stereoacuity measurements depth judgements are always relative and invariant under whole-field transformations due to eye and head movements, stereoacuity is not an indicator of the precision of all disparity information.

A useful experiment has been done by Schumer and Julesz (1984). They showed by using random-dot patterns that at a pedestral disparity of 0.5 deg, the threshold for detecting a corrugated plane from a flat plane was 33 minarc (their figure 12) almost irrespective of the corrugation frequencies they used. However, as far as we know, the only paper on stereo sensitivity for relatively realistic stimuli is the study by McKee et al. (1990). They showed that in comparison to lateral judgements of distance, stereoscopic judgements are not precise. In addition their study mentioned various examples and provide several references to demonstrate the insensitivity of stereopsis. They argue that in the first place stereopsis is for performing tasks at an arm's distance or for breaking camouflage.

We still have to answer the question of whether the disparities of the lower panels of figures 5 and 7 to 9 are so small that the visual system cannot perceive the difference between a disparity field induced by an eye or head movement and the disparity field induced by the above-mentioned stereograms corresponding to the eye or head movement. Although stereoacuity is too precise to be an appropriate indicator for the sensitivity of stereopsis, we are more or less obliged to use it as an indicator since no other indicators have been investigated in the literature on peripheral vision. We use Drasdo's theoretical linear stereoacuity-threshold function in order to interpret the detectability of the disparity field of the lower panels of figures 5 and 7 to 9. Most (but not all) of these disparity fields fall below Drasdo's thresholds. A tolerance analysis which we conducted revealed that even for planes at a distance of only 40 cm (where the horizontal disparity field is strongly curved, as can be inferred from figure 4) the disparity differences in most situations fall below Drasdo's (1977, 1991) thresholds.

Figure 8e shows the difference between horizontal disparity induced by head rotation about the vertical axis and horizontal disparity induced by a horizontally scaled stereogram. Figure 9e shows the difference between horizontal disparity induced by head rotation about the horizontal axis and horizontal disparity induced by a horizontally sheared stereogram. The latter difference does not fall below stereoacuity thresholds for large eccentricities. The results of figures 8e and 9e can be related to the well-known horizontal/vertical anisotropy[6] in depth perception which has been reported by Rogers and Graham (1983). One might argue that the anisotropy is caused by the fact that the resemblance between horizontal shear and forward rotation is less than the resemblance between horizontal scale and side-to-side rotation. Therefore, one could further argue that horizontal shear is less ambiguous than horizontal scale and is consequently perceived better.

Eye movements and the stability of depth perception

Vergence movements of the eyes lead to a translation of the images over the retinae. A commonly used way to induce translation of a stimulus over the retinae in an artificial way is to generate the images by a haploscope. In a haploscope the displays of the stimuli for the two eyes can be independently rotated about the centre of the eye-ball. However, except in the case of one unique combination of a fixation point and a location of the screens of the haploscope there is in principle a conflict between oculomotor cues and disparity. Cyclovergence leads to a rotation of the images over the retinae. Cyclovergence can be mimicked by differential rotation of the half-images of a stereogram. However, without compensational rotation of the eyes differential rotation cannot mimic a real world stimulus which means that, again, there is a conflict between disparity cues and oculomotor information. Thus, (cyclo)vergence generally cannot be mimicked by a stereogram without introducing this conflict. However, several reports mentioned in the "Introduction" section show that this conflict is not dominant with respect to depth perception. In situations where conflicting eye muscle information is present overall retinal displacements of a stimulus do not lead to changing binocular perception of depth in the case of large stimuli or they lead to only weakly perception of depth in the case of small stimuli.

Head translation in the primary direction and the stability of depth perception

Erkelens and Collewijn (1985a) and Regan et al. (1986) showed that differential lateral translation of the entire dichoptically presented half-images does not elicit perception of depth even when the eyes pursue the lateral motion with a gain unequal to one (Erkelens & Collewijn, 1985b). They found that in the presence of a visual reference (which moved

[6] The report of Rogers and Graham (1983) showed that subjects are more sensitive to horizontal shear than to horizontal scale: perceived slants induced by horizontal shear demonstrate lower detection thresholds and lower latencies than slant induced by horizontal scale.

with a translational velocity different from that of the stimulus) perception of depth was elicited vividly.

In this report we show that disparity induced by a translation of the head towards the stimulus corresponds to a disparity field caused by a lateral shift between the entire half-images of a stereogram. On the basis of this insight we conclude that the experimental results of Erkelens and Collewijn (1985a) and Regan et al. (1986) imply that the class of disparity induced by translations of the head in the primary direction does not elicit depth perception. Of course we realise that cues other than disparity are modified when the head is translated in the primary direction. (When the distance between the head and the stimulus is so large that the stimulus is effectively at infinity, both retinal images are identical and thus disparity has vanished. During the translation towards the object, disparity develops because the retinal projections of the object become different and the retinal images become larger.) However, although changing-size stimulation and changing-disparity stimulation can both produce a sensation of motion in depth (Regan & Beverley, 1979) and eye movements (Erkelens & Regan, 1986), they act largely independently. Both the motion in depth sensation and the eye movements produced by changing-size stimulation can be cancelled by antagonistic changing-disparity stimulation. In our analysis we concentrate on the disparity domain.

Rotation of the head and the stability of depth perception

Steinman & Collewijn (1980) measured eye movements of subjects while they actively rotated their head about a vertical axis. They found that vergence velocity errors of the order of 1 deg/s occurred. They also obtained the impression that vision remained fused, stable and clear. In their 1985-study (Steinman et al., 1985) they examined their impression psychophysically. The study resulted in the conclusion that stereoacuity is not disturbed by large fixation disparities or high vergence velocities. Patterson and Fox (1984) showed that the recognition of a stereoscopically presented Landolt C was not impaired by active head rotations either. Concerning tasks which require stereoscopic slant perception, it still has to be measured how stable these tasks are during head rotations or in situations where neck muscle information and disparity information are decoupled.

Van Ee and Erkelens (1996a) studied the temporal aspects of slant perception with large-field stimuli (no visual reference was present). They found that horizontal scale and horizontal shear between the entire half-images of a stereogram elicit poor perception of depth for observation periods lasting only a few seconds (see also Gillam et al., 1988). Their experimental result implies that the class of disparity which is induced by rotation of the head elicits only poor depth perception.

Jones and Lee (1981) reported on experiments in which human binocular performance and monocular performance were compared in a variety of visuomotor tasks. They found that stereopsis was not important in the performance of visuomotor skills in three dimensions when the subjects were free to move their heads. They concluded that an important benefit of binocular frontal vision with moving head is binocular concordance rather than (changing) binocular disparity. In other words, the benefit of stereopsis may in fact be limited to situations in which the head is stationary (Jones & Lee, 1981).

The relationship between ego-movement-induced disparity and the stability of depth perception

From the previous three subsections we conclude that classes of disparity which can be induced by eye and head movements do not appear to be very relevant for stereopsis, at least if presented in isolation. In table 1 the results are summarised. We suggest that the classes of disparity which can be induced by ego-movement poorly elicit depth perception because they are ambiguous. The classes of disparity which correspond to haploscopic rotation or to differential rotation of the entire half-images of a stereogram are ambiguous because these disparities could also be induced by vergence or cyclovergence, respectively. Disparity caused by a lateral shift between the entire half-images of a stereogram is ambiguous because instead of being caused by the stimulus it could be caused by head translation in the primary direction. The classes of disparity associated with horizontal scale and horizontal shear between the entire half-images of a stereogram are ambiguous because they could also be induced by head rotation.

Table 1 about here.

As explained in the "Introduction" section, tasks based on disparity processing can be distinguished into relief tasks and metrical tasks (Gårding et al., 1995). Insensitivity of stereopsis to disparity fields which result from eye and head movements (for short observation periods) would mean that stereoscopic vision is in principle no able to perform metrical tasks (for short observation periods). On the other hand, relief characteristics are preserved under eye and head movements. From the literature it is known that relief tasks can be done reliably in a couple of milliseconds (e.g. Kumar & Glaser, 1993; Uttal, Davis & Welke, 1994). The result of performing metrical tasks based on stereopsis alone is not veridical (Gogel, 1960; Foley, 1980; Gillam, Flagg & Finlay, 1984; Mitchison & McKee, 1990; Johnston, 1991; van Ee & Erkelens, 1996a). Visual tasks that require a metric reconstruction of the three-dimensional visual world are not very common.

The insensitivity of stereopsis to disparity fields which result from eye and head movements (such as those given in table 1) means that stereopsis is insensitive to global zero and first order modifications between the half-images of a stereogram. Stevens and Brookes (1988) reported that binocular 3D information is best acquired where a stereogram contains curvature features. Their results about curvature features are related to stereograms per se and not to the (retinal) disparity fields caused by these stereograms since even a stereogram without a difference between the half-images gives rise to a curved disparity field (see for instance figure 4). In this study we show why the zero and first order characteristics of the stereogram form a special class for which the visual system is relatively insensitive.

In our view there are other relevant distinctions in addition to the distinction of stereopsis into metrical and relief tasks. One of them is the distinction into short and long observation periods (Gillam et al., 1988; van Ee and Erkelens, 1996a). A second one is the distinction into conditions with and without a visual reference (e.g. Gogel, 1963; Shipley & Hyson, 1972; Gillam et al., 1984, 1988; Regan et al., 1986; Howard & Kaneko, 1994; van Ee and Erkelens, 1995). In the case of long observation periods there is not much of a difference between slant estimation (metrical task) with a visual reference and slant estimation without a visual reference. In both cases there is a large underestimation of slant. However, for short observation periods slant estimation without a visual reference is generally extremely poor (van Ee & Erkelens, 1996a). A third relevant distinction of stereopsis is a distinction into small and large stimuli. A couple of reports (Rogers & Bradshaw, 1993, 1995; Howard & Kaneko, 1994) show that vertical disparities have a smaller influence on depth perception for small stimuli than for large stimuli. Oculomotor cues have a considerable influence for small stimuli but hardly any for large stimuli (Rogers & Bradshaw, 1995; Regan et al., 1986).

Temporal aspects

Taken together, we suggest that depth perception is invariant under eye and head movement-induced disparity. This formulation is probably too general because many authors have found that subjects are able to perceive slant caused by whole-field transformations (in prolonged viewing). Generally, slant estimation induced by whole-field transformations between the two half-images of a stereogram depends on observation time. If subjects are allowed to view the stereogram for more than, say, 10 seconds, then slant estimation is far more veridical than if they view it for, say, one second (Gillam et al., 1988; van Ee & Erkelens, 1996a). That subjects can perceive whole-field slant after prolonged viewing could be caused by the integration of either non-binocular cues or extra disparity signals (for instance vertical disparity). Inspection of the stimulus by actively making eye movements might also contribute to the enhancement of the slant perception over time

(Enright, 1991).

Recently Van Ee and Erkelens (1996b) have suggested that Werner's illusory depth contrast effect[7] (Werner, 1938) may be explained by the idea that stereopsis is relatively insensitive to whole-field horizontal scale and shear. This insensitivity, in turn, results from the fact that these transformations induce disparity fields similar to those induced by head rotations as we have shown in this paper. The fact that slant estimation becomes more veridical over time, makes their explanation consistent with the fact that the illusory slant of a stimulus caused by Werner's depth contrast effect decreases over time (e.g. Kumar & Glaser, 1993).

Robot vision

Three-dimensional imaging has various applications. A possible application is the design of a binocular system (robot) which can produce information about places to which human beings cannot go or do not wish to go. However, in practice during movements of the robot the instability of camera images is such that disparity processing under practical circumstances fails (Eklundh, 1993). The idea that 'ego-movement-induced disparity' is irrelevant for stereopsis may have interesting implications for the future of robotics. Shape perception by means of two cameras could be greatly improved if the types of disparities brought about by the robot's own movements (which are classified in this report) could be filtered out or ignored.

Conclusion

We have calculated the binocular disparity field for a wide range of possible eye, head and stimulus positions. From the literature it is known that certain classes of disparity (such as whole-field horizontal lateral shift, differential rotation, horizontal scale and horizontal shear between the half-images of the stereogram) induce relatively poor perception of depth, at least if presented in isolation. These classes of disparity turn out to be similar to those caused by eye and head movements. Our numerical calculations support the suggestion that binocular 3D vision is based primarily on the classes of disparity that are invariant under ego-movement.

[7] The slant of a surface is not only determined by horizontal scale or shear between its own half-images but also by the scale or shear between the half-images of a visual reference. A positive slant of a visual reference causes a negative slant of the object at hand (and vice versa).

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Figure captions

Fig. 1) An example of a horizontal scale and a horizontal shear transformation between the observed half-images. Horizontal scale between the half-images of the stereogram leads to perceived slant about the vertical axis. Horizontal shear leads to perceived slant about the horizontal axis. M is the magnitude of the horizontal scale transformation expressed as a fraction, β is the magnitude of the horizontal shear transformation, expressed as an angle.

Fig. 2) Figure a) shows the geometry of a horizontally scaled stereogram with unrotated head. Figure b) shows the geometry of an (initially) frontal plane with rotated head. The retinal images in the horizontal plane are identical in both situations. LE and RE denote left and right eye, respectively. Note that the geometry of a frontal plane at a distance of z_0 viewed after a head rotation over α degrees is similar to a slanted plane over α degrees but at a distance of $z_0 / \cos\alpha$.

Fig. 3) In Fick's coordinate system a target is uniquely identified relative to the left eye by its longitude ϕ_P^L and its latitude θ_P^L. In this figure the eye points to a fixation point which is at infinity. The origin of the oculocentric coordinate system is located at the centre of the eye ball. The x-axis of the coordinate system points from right to left, the y-axis points vertically upwards, and the z-axis points in the primary direction. In the direction of the arrow the sign of the angle is defined to be positive.

Fig. 4) Horizontal (a) and vertical disparity (b) of a frontal plane at a distance of 250 cm (d1, white patch) and 50 cm (d2, grey patch) in front of the eyes. ϕ denotes longitude, θ denotes latitude. Both angles are taken relative to the head.

Fig. 5) The upper two panels show horizontal (a) and vertical disparity (b) of a frontal plane at a distance of 100 cm viewed with cyclovergence of 1.26 deg. The second row of panels shows the horizontal (c) and vertical disparity (d) after a differential rotation of 1.26 deg within the corresponding stereogram. The lower two panels show the difference in horizontal (e) and vertical (f) disparity between cyclovergence of the eyes and differential rotation within the stereogram. Note that the dimensions along the disparity axes of the lower panels are different from the other figures.

Fig. 6) A lateral shift between the two half-images of a stereogram leads (within the horizontal plane) to similar disparities as a head translation in the primary direction. T_z denotes the translation of the head towards the plane, I the interocular distance and z_0

the distance between the stimulus and the eyes.

Fig. 7) The upper two panels show horizontal (a) and vertical disparity (b) of the frontal plane (initially at 100 cm) after a head translation of 25 cm in the primary direction. The second row of panels shows the horizontal (c) and vertical disparity (d) after a lateral shift of 2.2 cm within the theoretically corresponding stereogram. The lower two panels show the difference in disparity between head translation and a stereogram-induced lateral shift.

Fig. 8) The upper two panels show horizontal (a) and vertical disparity (b) of the initially frontal plane after a head rotation over 20 deg about the vertical axis. The second row of figures shows the horizontal (c) and vertical disparity (d) after horizontal scaling of 2.2 % within the theoretically corresponding stereogram. The lower two panels show the difference in disparity induced by head rotation about the vertical axis and the stereogram-induced horizontal scale.

Fig. 9) The upper two panels show horizontal (a) and vertical disparity (b) of the initially frontal plane after a head rotation over 20 deg about the horizontal axis. The second row of figures shows the horizontal (c) and vertical disparity (d) after horizontal shearing over 1.26 deg within the corresponding stereogram. The lower two panels show the difference in disparity induced by head rotation about the horizontal axis and stereogram-induced horizontal shear.

Table caption

Table 1) Relevant eye and head movements are presented in the first column. The second column shows the transformations between (the entire) stereogram half-images which give rise to about the same disparity field as the movements listed in column 1. Column 3 gives psychophysical depth perception results relating to experiments where disparity fields caused by the stereograms of column 2 are presented in isolation. Notes: (1): See text of the subsection "Eye movements and the stability of depth perception" and also text of the "Introduction" section. By "haploscopic rotation" we mean a rotation of the displays about the centre of the eye-ball; (2): Howard and Zacher (1991), see also text of the "Introduction" section; (3): Erkelens and Collewijn (1985a); (4): van Ee and Erkelens (1996a).

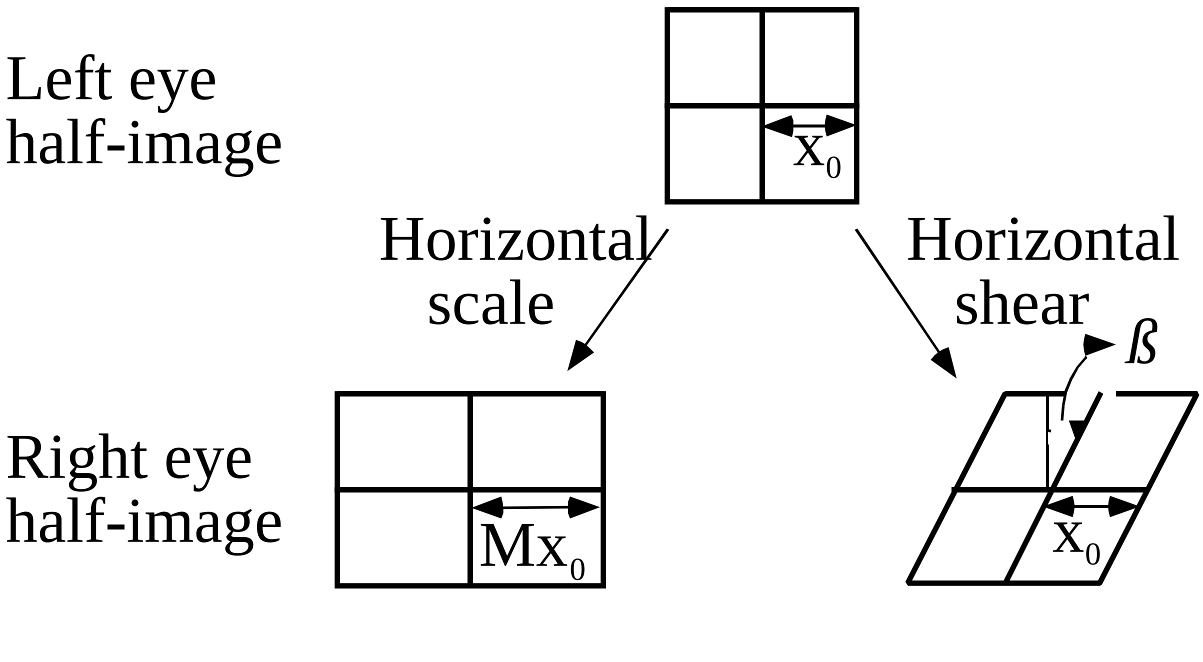

Figure 1, van Ee and Erkelens

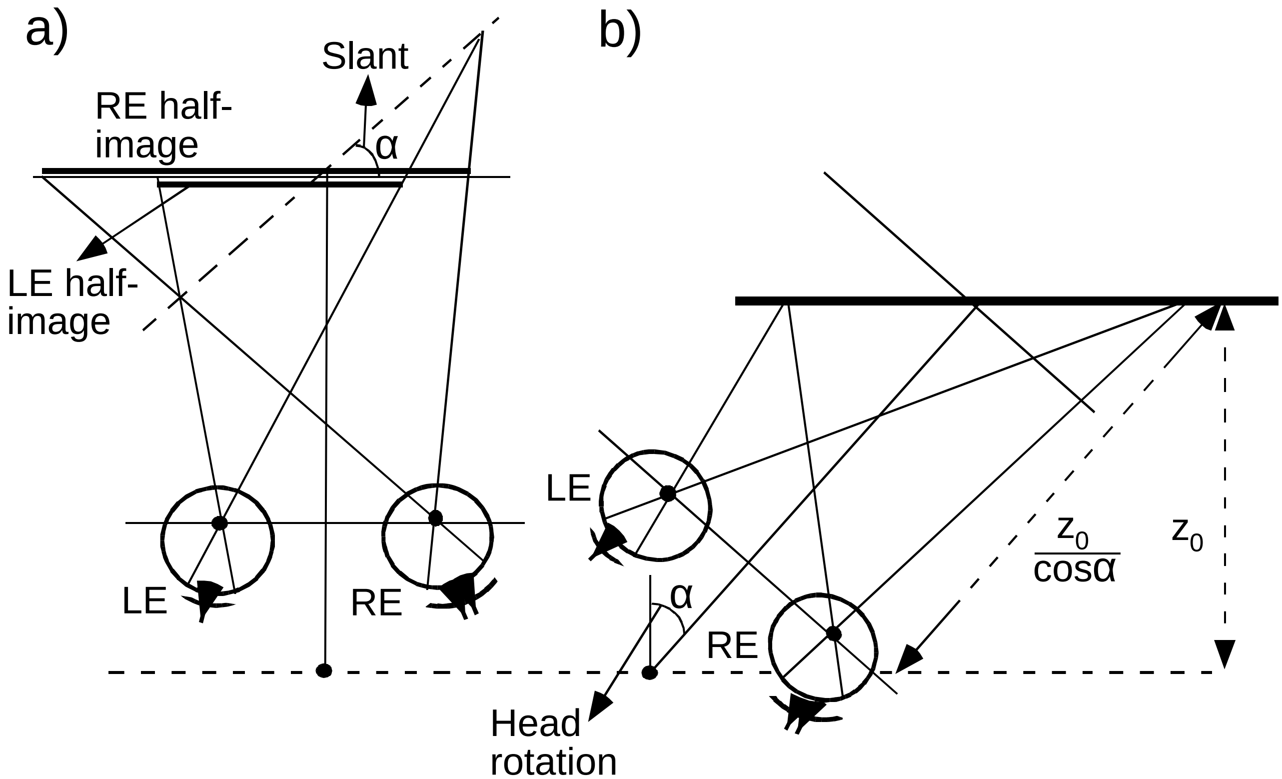

Figure 2, van Ee and Erkelens

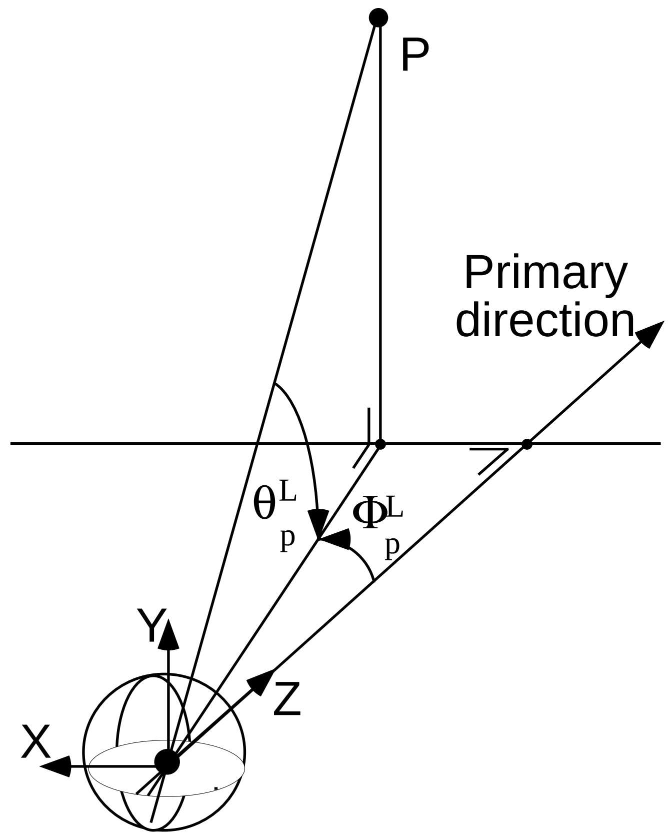

Figure 3, van Ee and Erkelens

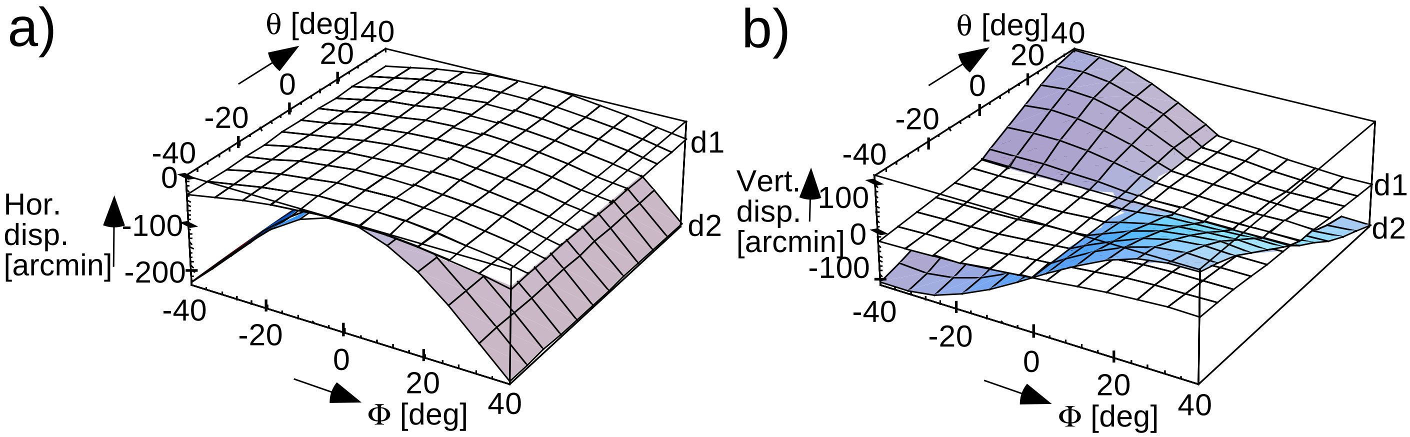

Figure 4, van Ee and Erkelens

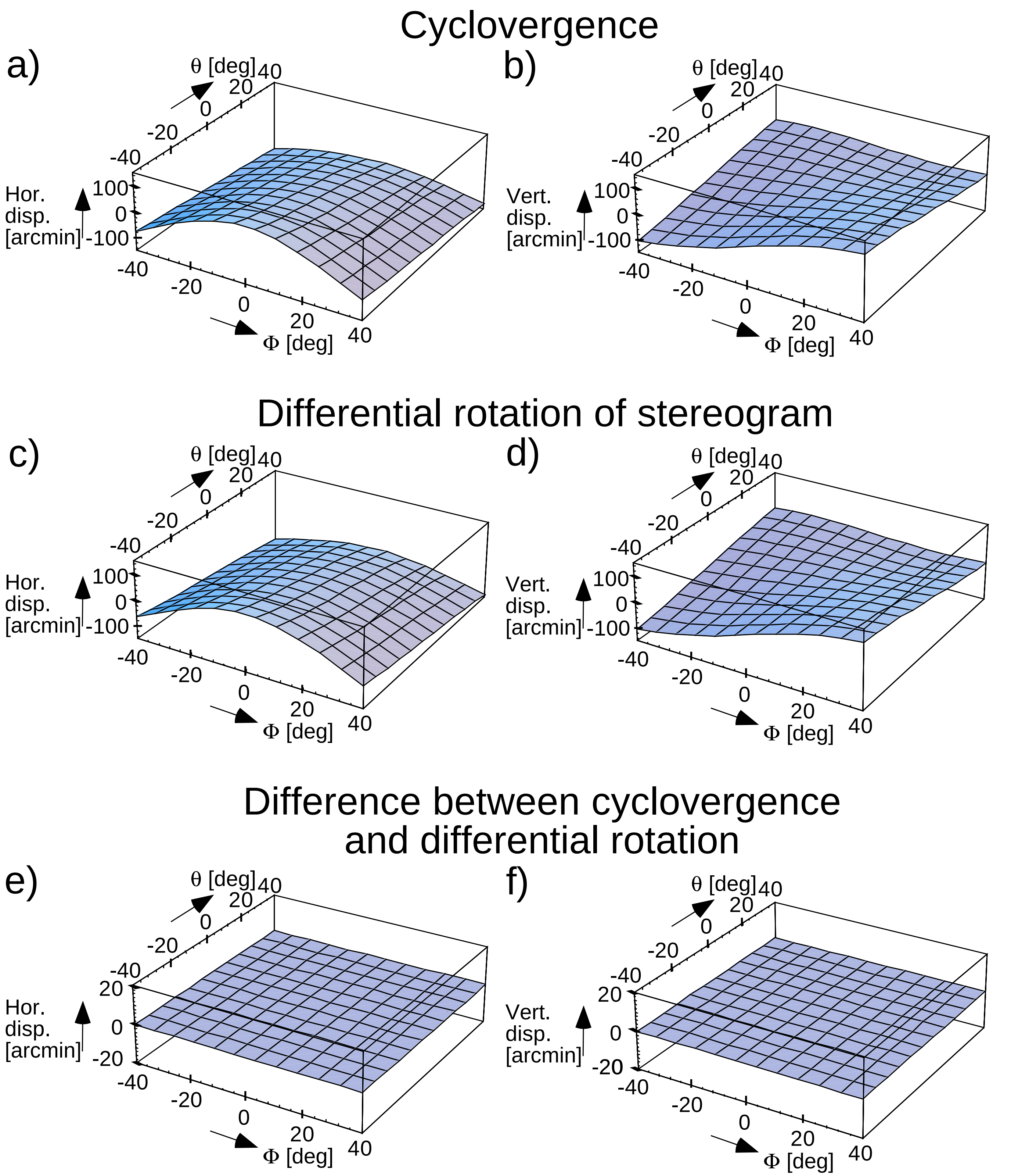

Figure 5, van Ee and erkelens

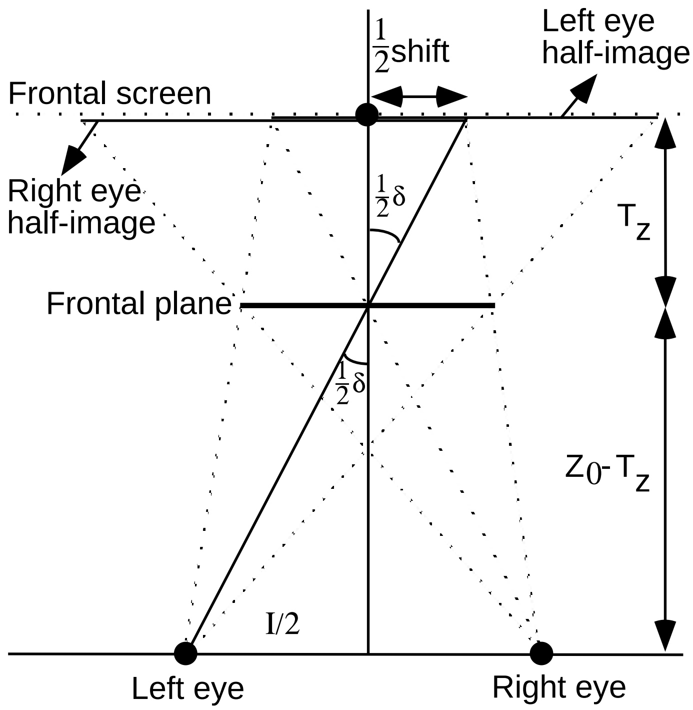

Figure 6, van Ee and Erkelens

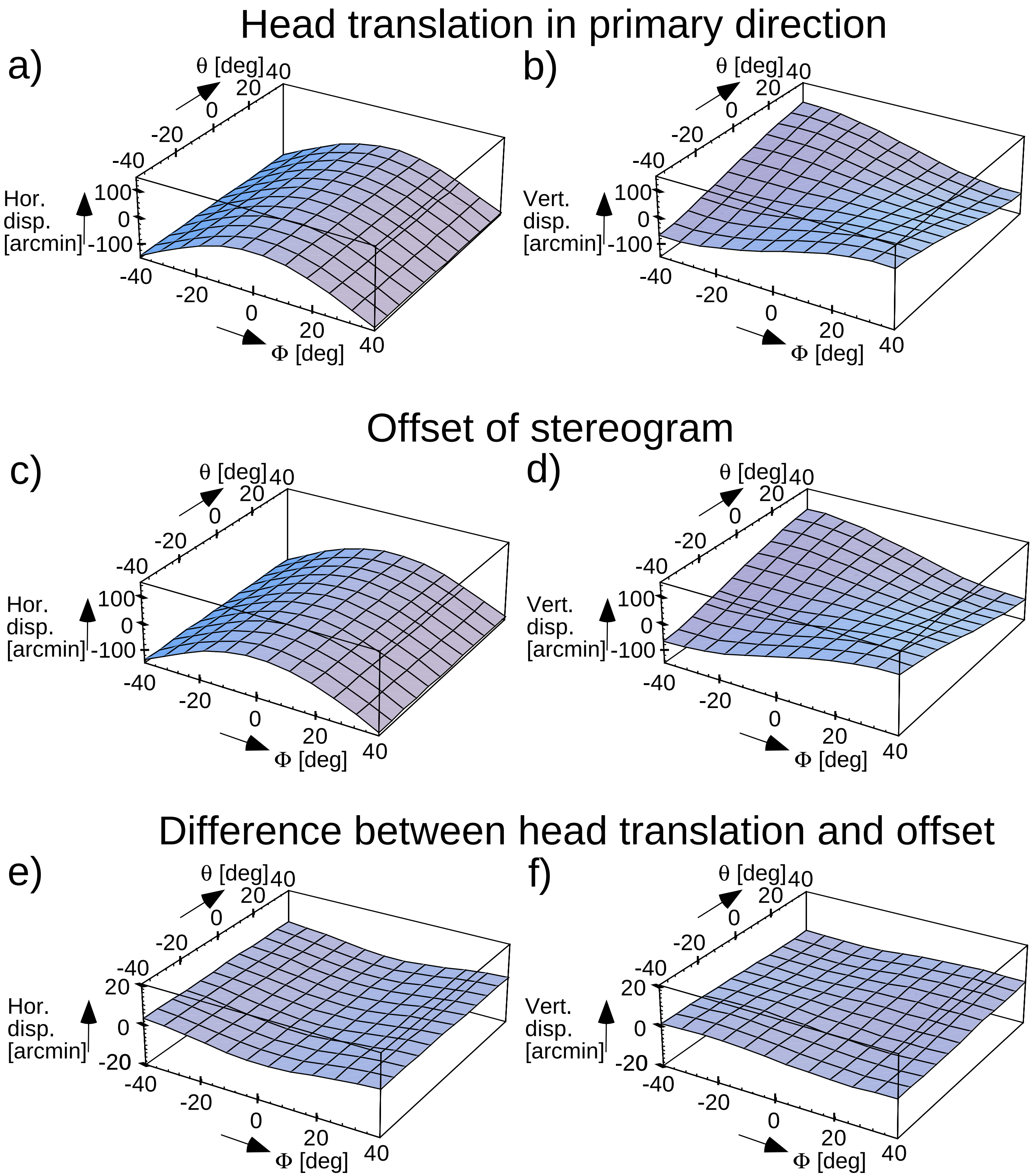

Figure 7, van Ee and Erkelens

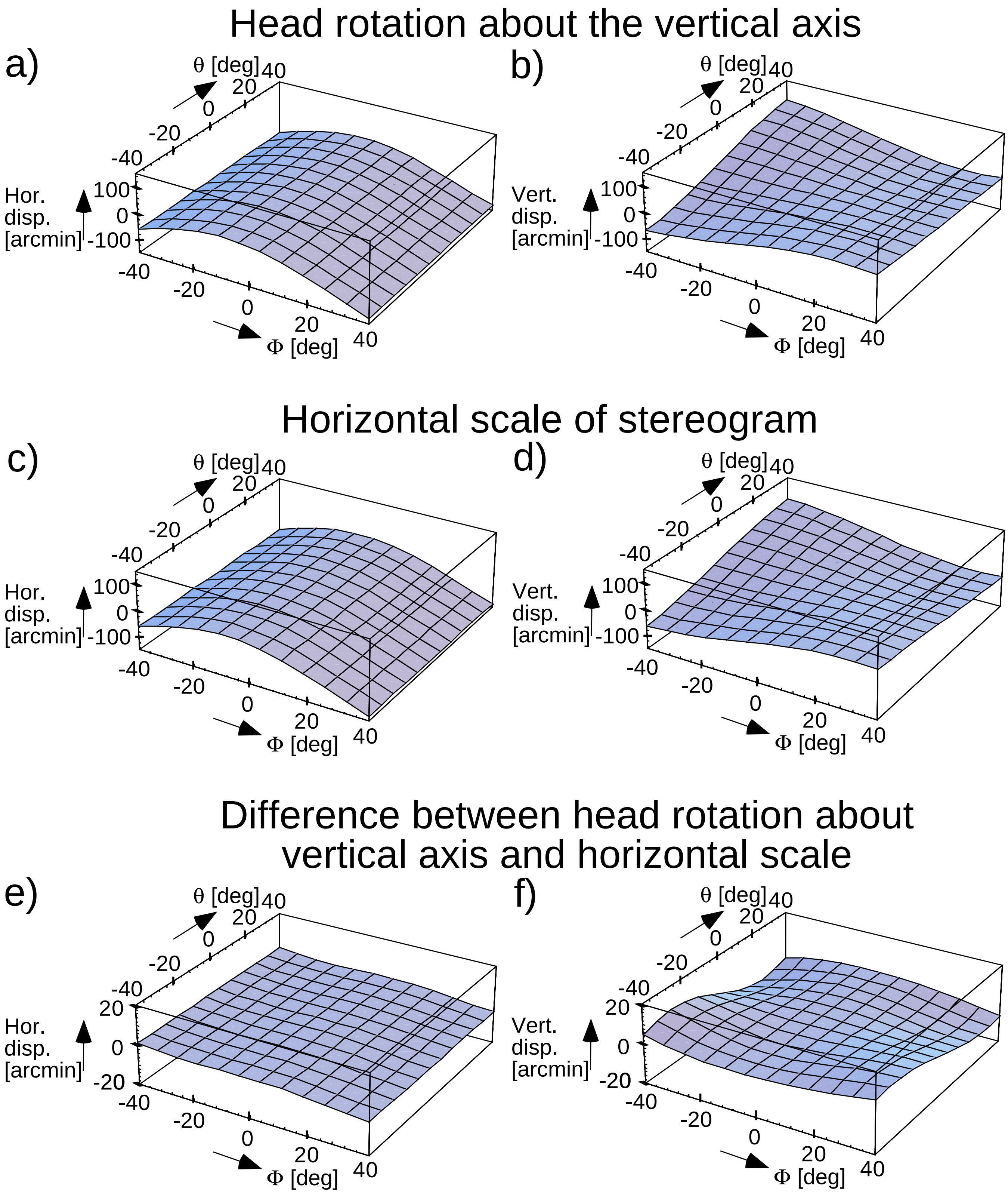

Figure 8, van Ee and Erkelens

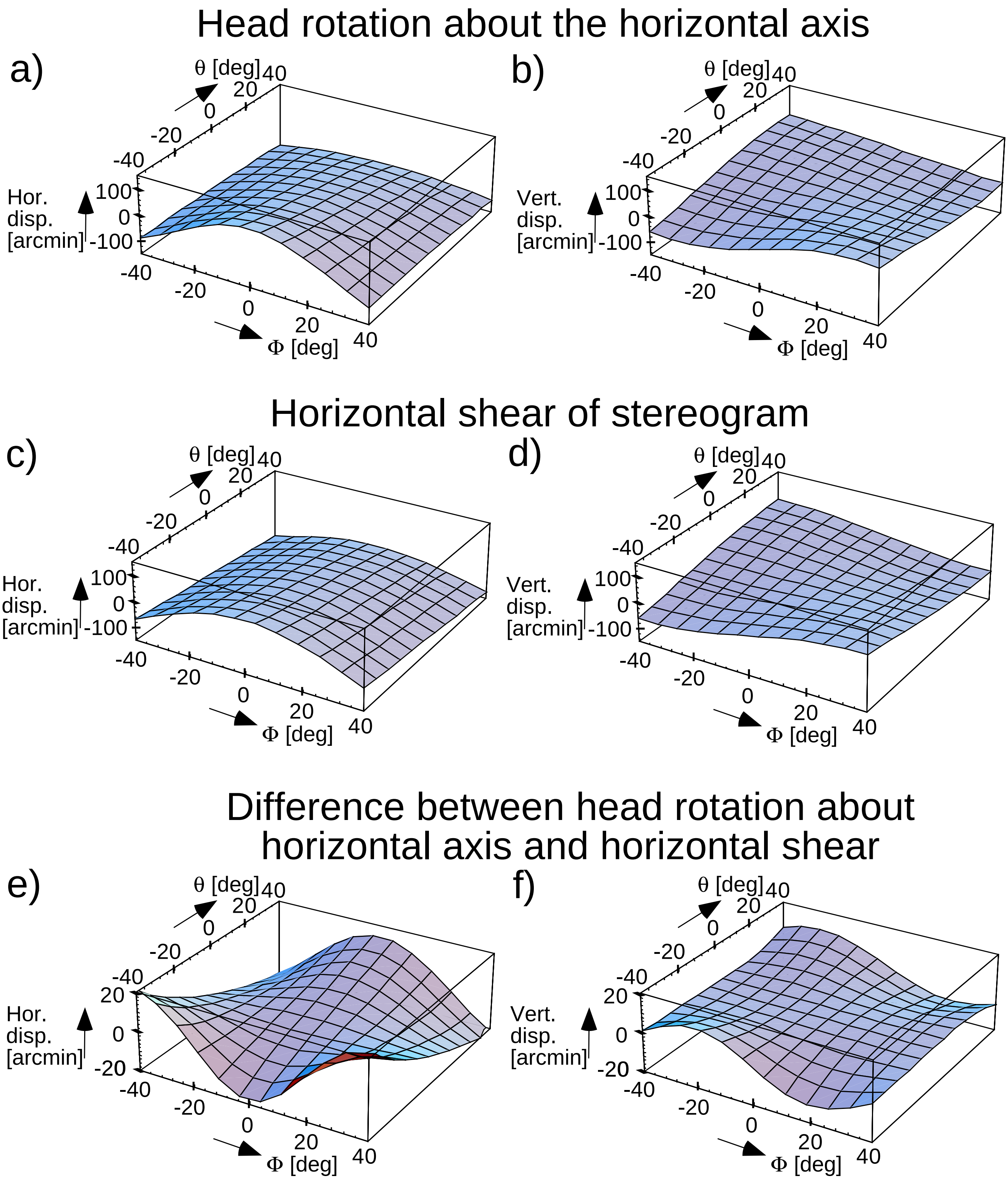

Figure 9, van Ee and Erkelens

Ego-motion	Stereogram	Relation to depth perception
Eye:		
Change of fixation	Haploscopic rotation	Poor (1)
Cyclovergence	Differential rotation	Poor (2)
Head:		
Forward translation	Horizontal translation	No (3)
Sideward rotation	Horizontal scale	Poor (4)
Forward rotation	Horizontal shear	Poor (4)

Table 1, van Ee and Erkelens