

## Perceived Visual Direction near an Occluder

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### Abstract

When an opaque object occludes a more distant object, the two eyes often see different parts of the distant object. Hering's laws of visual direction make an interesting prediction for this situation: the part seen by both eyes should be seen in a different direction than the part seen by one eye. We examined whether this prediction holds by asking observers to align a vertical monocular line segment with a nearby vertical binocular segment. We found it necessary to correct the alignment data for vergence errors, which were measured in a control experiment, and for monocular spatial distortions, which were also measured in a control experiment. Settings were reasonably consistent with Hering's laws when the monocular and binocular targets were separated by 30 arcmin or more. Observers aligned the targets as if they were viewing them from one eye only when they were separated by 2 arcmin; this behavior is consistent with an observation reported by Erkelens and colleagues. The same behavior was observed when the segments were horizontal and when no visible occluder was present. Perceived visual direction when the two eyes see different parts of a distant target is assigned in a fashion that minimizes, but does not eliminate, distortions of the shape of the occluded object.

### 1 Introduction

A primary visual function is to recover the location, orientation, and shape of objects in the environment from the light reaching the eyes. An important source of information for such recovery is the spatial differences, or retinal disparities, between the two eyes' images. Here we examine some interesting properties of binocular space perception near occluders.

Figure 1A depicts a binocularly fixated surface. Vectors  $\mathbf{B}$ ,  $\mathbf{M}$ , and  $\mathbf{F}$  represent the positions of points B, M, and F relative to the observer. Recovery of the positions of those points requires an estimation of the lengths and directions of  $\mathbf{B}$ ,  $\mathbf{M}$ , and  $\mathbf{F}$ . Most research in binocular vision has concerned the estimation of the lengths, but the directions must be estimated as well in order to recover the positions of the surface points.

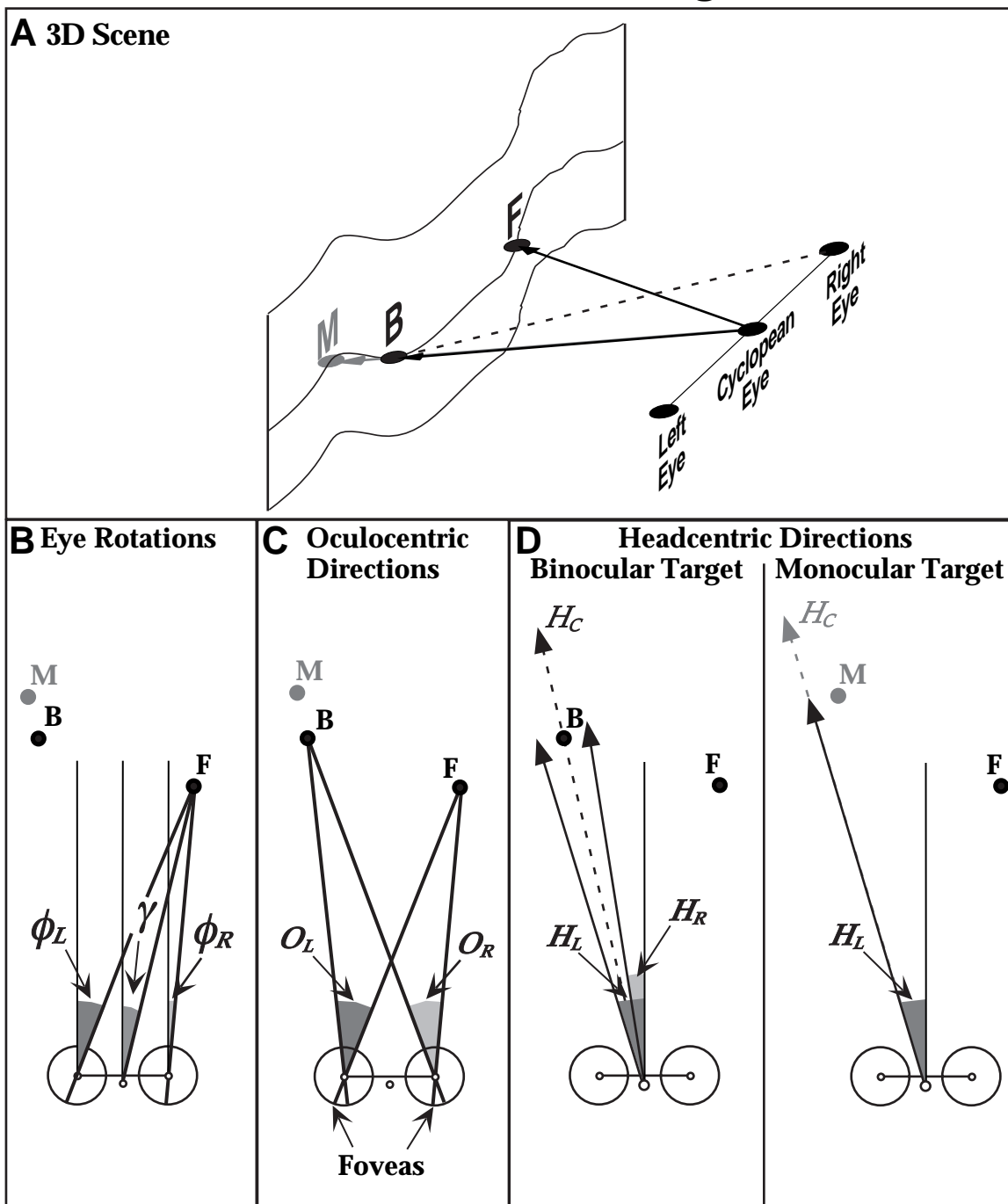
Theoretically, the estimation of visual directions is fairly straightforward for many viewing situations because the oculocentric direction of incoming light is given by the position of its retinal image relative to the fovea. Hering (1879) described a method by which the visual system could estimate the directions of monocular and binocular targets, and this method is now called Hering's laws of visual direction<sup>1</sup>.

In describing Hering's laws, we first consider only points in the visual plane (the plane containing the fixation point and the centers of the two eyes). The points are visible to both eyes as is the case for points F and B in Figure 1A. Hering's laws can be summarized by three calculations (Banks, 1995). First, one calculates the eyes' version as schematized in Figure 1B. The eyes' rotations,  $\phi_L$  and  $\phi_R$ , are defined as the angles through which the eyes are rotated about vertical axes (that is, axes through the eyes' centers of rotation and perpendicular to the visual plane) with respect to planes through the centers of rotation and parallel to the head's median plane. The eyes' version,  $\gamma$  is the average of the rotations. Second, oculocentric directions are calculated as shown in Figure 1C. The oculocentric directions,  $O_L$  and  $O_R$ , are the angles between the visual axes and the visual lines to the object. Third, one calculates headcentric

<sup>1</sup> Hering's laws for perceived visual direction should not be confused with his Law of Equal Innervation, which describes eye movements.

directions as schematized in Figure 1D<sup>2</sup>. These directions,  $H_L$  and  $H_R$ , are the oculocentric directions plus the version:  $H_L = O_L + \gamma$  and  $H_R = O_R + \gamma$ . According to Hering, the origin for perceived visual directions is the cyclopean eye (denoted by C) which lies on the Vieth-Müller Circle and in the head's median plane.

### Binocular Directions & Hering's Laws



<sup>2</sup> The computation of visual direction relative to the head involves measurement of image positions in the retinae and of the positions of the eyes with respect to the head. The order of operations can vary with the same outcome. Equation (1) implies that image positions are converted to head-centric directions which are then averaged, but, theoretically, image positions could be first averaged and then converted to head-centric directions (Ono, 1991) and the predicted directions would be identical with regard to the present study.

**Figure 1.** Binocular visual direction and Hering's laws. A) A surface viewed binocularly. Three surface points are highlighted. F is the fixation point. B is visible to both eyes. M is visible to the left eye only (suggested by interruption of dashed line). The visual system estimates the directions to the three points. Those estimated directions are schematized by vectors **F**, **B**, and **M** from the cyclopean eye to points F, B, and M, respectively. The problem under investigation here is how the direction vectors are determined from the two eyes' inputs. B) Top view of points F, B, and M and rotations of the eyes. The view is a cross-section through the visual plane. The head's median plane is indicated by the line from the cyclopean eye. Planes parallel to the median plane and through the centers of the eyes are indicated by lines through the eyes. The left eye's rotation  $\phi_L$  is the angle between its median line and its visual axis (fixation axis); the right eye's rotation  $\phi_R$  is the corresponding angle in that eye. The eyes' version  $\gamma$  is the average of the two rotations:  $(\phi_L + \phi_R)/2$ . C) Top view showing the oculocentric directions of B in the two eyes. The oculocentric directions of B are the angles between the visual axes and the visual lines to B. In the left and right eyes, they are respectively  $O_L$  and  $O_R$ . D) Top view showing the estimated headcentric directions of both B and M. The direction estimated from the left eye's image is the oculocentric direction plus the version:  $H_L = O_L + \gamma$ . The direction estimated from the right eye is  $H_R = O_R + \gamma$ . When the two eyes' images are fused (as in the left panel of D), the final headcentric direction is the average of the directions reported by the two eyes:  $H_C = (H_L + H_R)/2$ . The right panel of D is a top view showing the estimated headcentric direction of M, which is seen by the left eye only. The estimated direction is the oculocentric direction reported by the left eye plus the version:  $H_C = \gamma + O_L$ . Note that the predicted visual directions of B and M are different; M should be seen to the left of B.

When  $H_L$  and  $H_R$  differ significantly, Hering's laws predict that two directions will be seen: the percept will be diplopic. When  $H_L$  and  $H_R$  are slightly different, the visual system may be able to fuse the two directions into a single perceived direction. In this case, the perceived headcentric direction is assumed to be the average of the calculated headcentric directions:

$$H_C = (H_L + H_R)/2$$

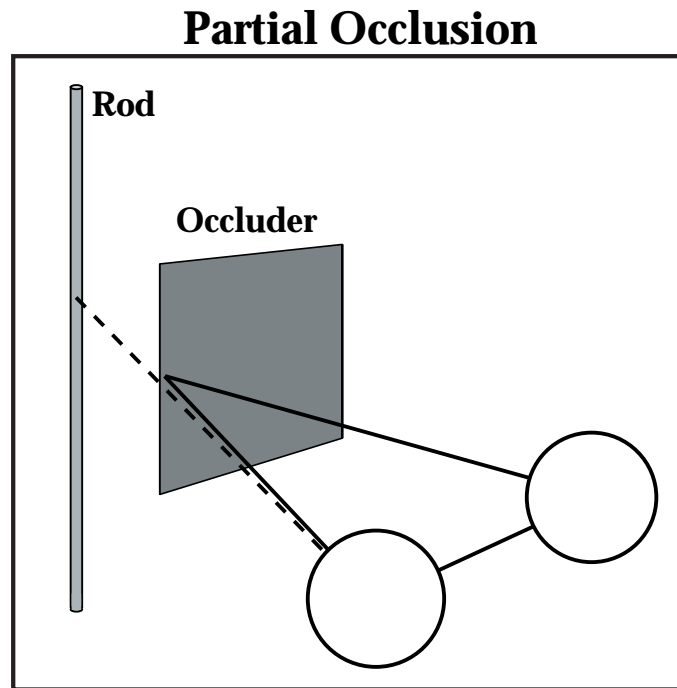
which means that:

$$H_C = \gamma + (O_L + O_R)/2 \tag{1}$$

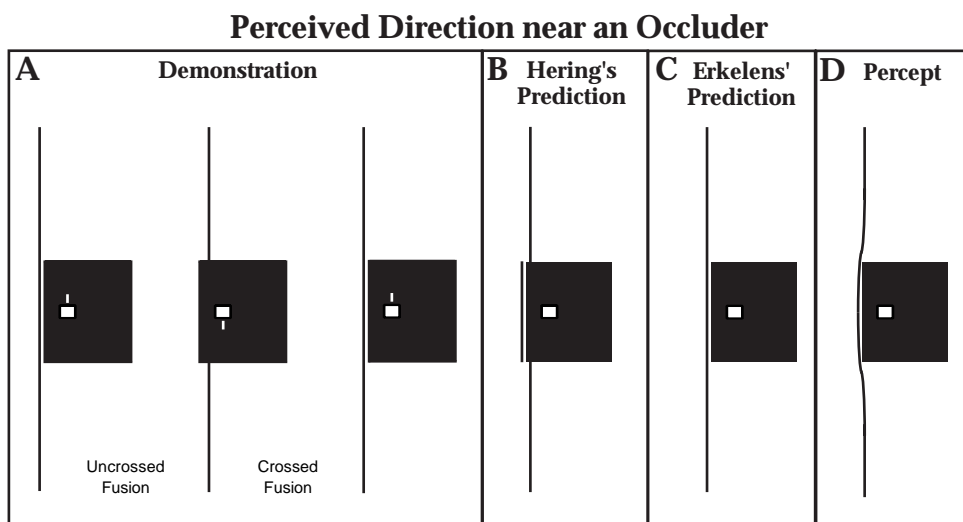
According to Hering's laws, the headcentric direction of a point that is visible to one eye (such as point M in Figure 1) is given by  $H_C = H_V$  where the subscript V refers to the viewing eye. Thus, for monocularly visible points:

$$H_C = \gamma + O_V \tag{2}$$

If the occluding surface in Figure 1 were made transparent so that the right eye could also see point M, then, according to Hering's laws, M's perceived direction would change. Specifically, whenever  $O_L$  and  $O_R$  are similar, but not equal, Equations (1) and (2) yield different visual directions. This fact leads to an interesting prediction for situations in which part of an object is visible to both eyes while another part is visible to one eye only. This situation is demonstrated in Figure 2: A rod is positioned to the left and behind an occluder such that the tops and bottoms of the rod are visible to both eyes, but the middle is visible to the left eye only. Hering's laws predict different perceived directions for the top/bottom and middle of the rod. Specifically, the direction of the middle section which is only visible to the left eye is given by Equation (2) (where  $O_V = O_L$ ) whereas the direction of the top and bottom which is visible to both eyes (but fused) is given by Equation (1). Consequently, for  $O_L \neq O_R$ , the predicted directions of the monocularly and binocularly visible parts of the rod differ. For the partial occlusion situation in Figure 2, the predicted direction of the monocular target is leftward relative to the binocular target.



**Figure 2.** The partial occlusion situation. A rod is placed to the left and behind an occluder such that the rod's top and bottom are visible to both eyes, but the middle is visible to the left eye only. Hering's laws predict different perceived directions for the top/bottom and middle of the rod.



**Figure 3.** Demonstration and predictions of perceived direction near an occluder. A) Demonstration of the situation in Figure 2. Free fuse the white square by uncrossing (left and middle figure) or crossing (middle and right figure) your eyes. Fixate the white square carefully and use the small vertical nonius lines to assess the accuracy of your vergence (they should appear vertically aligned). Notice the perceived shape of the rod. B) Hering's laws, strictly interpreted, predict that the middle section of the line will appear displaced leftward from the tops and bottoms. C) Erkelens' prediction, again strictly interpreted, is that the line will appear straight. D) Most people perceive a curved line, which is not consistent with Hering's nor Erkelens' predictions.

Erkelens & van de Grind (1994) and Erkelens, Muijs, & van Ee (1996) studied perceived directions for partial occlusion situations like the one depicted in Figure 2. They reported that the perceived relative directions of monocular and binocular targets are consistent with the directions estimated by the eye that can see both targets. This phenomenon has been called capture of perceived direction (Erkelens & van Ee, 1997). An implication of this observation is that the assigned directions of neighboring monocular and binocular targets are both given by Equation (2) above. Presumably, the presence of monocular and binocular stimuli in the same vicinity causes the invocation of this means of direction assignment.

Figure 3A is a demonstration that allows one to observe what happens in the partial occlusion situation; it is a stereographic rendition of a vertical rod placed behind a vertical occluding edge. The entire rod is visible to the left eye, but only the tops and bottoms are visible to the right eye. Fixation is in the plane of the occluder, so the binocular portions of the line have non-zero disparity. If the disparity is not large, the binocular portions can be fused. According to Equations (1) and (2), Hering's laws predict that the middle of the line will appear to be horizontally displaced relative to the tops and bottoms; this prediction is portrayed in Figure 3B. We will refer to this as the *Hering prediction* in the remainder of the paper. According to Erkelens and colleagues (e.g. Erkelens & van de Grind, 1994), directions of the top, middle, and bottom of the line are assigned according to Equation (2), so the line will appear straight. This prediction is portrayed in Figure 3C; we will refer to it as the *Erkelens prediction*.

If you examine Figure 3A by free-fusing, you can see that the percept differs from both the Hering and Erkelens predictions<sup>3</sup>: The middle of the line does indeed appear to be displaced leftward from the tops and bottoms (as predicted by Hering), but the displacement occurs smoothly rather than discontinuously. Figure 3D illustrates the percept reported by a number of observers.

In the present study, we measured the relative perceived directions of monocular and binocular line segments in order to better understand the direction assignment near occluding edges. Our main interest was to determine how the separation (change in vector direction in Figure 1) between monocular and binocular targets affects the assignment of direction.

## 2 General Methods

### 2.1 Observers

Four observers with normal stereopsis participated. Three of them—JMH, JVE and SLB—were unaware of the experimental hypotheses. BTB and JVE had considerable experience in stereoscopic experiments; JMH and SLB did not. The refractive errors of all observers were corrected by contact lenses or glasses.

### 2.2 Apparatus

Observers viewed the stimuli on a computer monitor at a distance of 570 cm. The room was otherwise dark. The stimuli were generated in MATLAB. Refresh rate was 67 Hz. Pixel size was 12 x 12 arcsec. The monitor was viewed through a 3 x 2 deg aperture. The head was stabilized with a chin and forehead rest.

Images were presented to the two eyes using the standard red-green anaglyphic technique. The intensities of the red and green stimuli were adjusted to appear equally bright when viewed through the red and green filters placed before the eyes. Dichoptic images were presented in most of the experiments; in those cases, the left and right eyes' half-images were green and red, respectively. There was no visible crosstalk between the half-images. One experiment involved monocular viewing; in that case, the filters were placed on the computer monitor. Apart from the images, the computer screen was black.

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<sup>3</sup> To our knowledge, Hering did not actually discuss direction assignment for nearby monocular and binocular targets. Additionally, Erkelens and colleagues did not specify how close monocular and binocular targets need to be in order for directions to be assigned from one eye. Thus, the work presented here is intended to determine how these two means of direction assignment depend on the proximity of monocular and binocular stimuli.

### 3 Experiment 1: Perceived Alignment & Gap Size

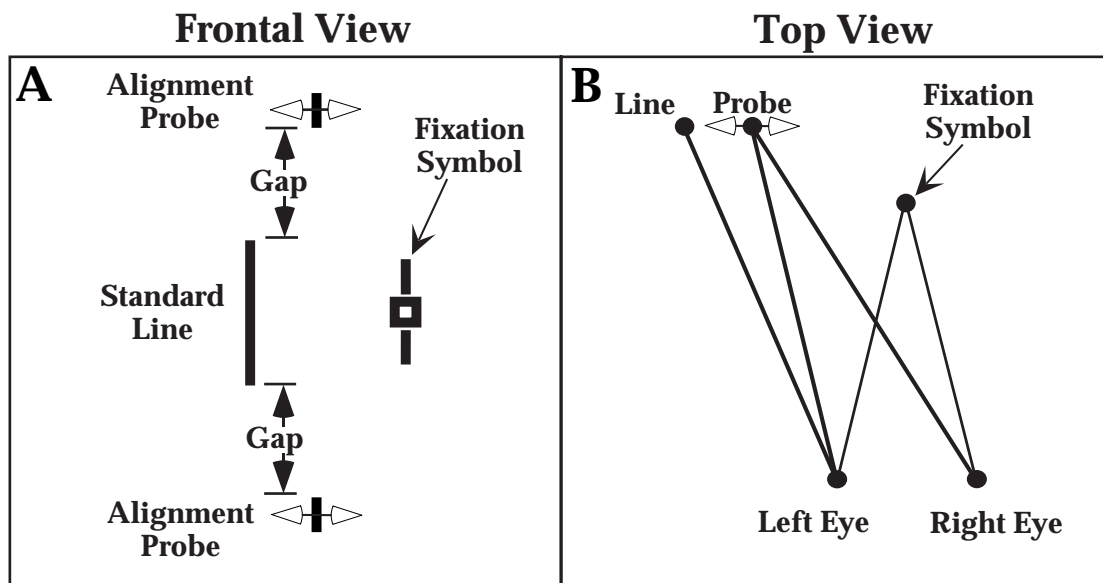
#### 3.1 Method

The stimulus consisted of a fixation symbol, a monocular standard line, and a binocular alignment probe (Figure 4). The fixation symbol consisted of a binocular central square and monocular vertical line segments. Observers were told to fixate the central square and to use the monocular nonius lines to assess the accuracy of their vergence. The central square was 5.8 x 5.8 arcmin and the nonius lines were 8.7 arcmin high. Disparity was introduced into the fixation symbol by shifting the red and green half-images in opposite directions. There was no occluder visible, though stimuli were consistent with a black occluding surface.

The standard line was 39 arcmin to the left or right of the fixation symbol. It was generally visible to the left eye when placed to the left and to the right eye when placed to the right. The line was 1.6 arcmin wide and 39 arcmin high.

The alignment probe was presented above and below the standard line and was visible to both eyes. The two parts of the alignment probe were always co-linear. Its width was 1.6 arcmin; the height of each part was 6.9 arcmin. The probe's disparity relative to the display screen was always 0. The vertical separation between the ends of the standard line and alignment probes was 2, 15, or 30 arcmin; we refer to this separation as the *gap*.

## The Stimulus



**Figure 4.** Schematic of the stimulus in Experiment 1. A) Frontal view of the stimulus. The fixation symbol consisted of a binocular central square and vertical nonius lines that were used to monitor vergence accuracy. The standard line was placed to the left or right of the fixation symbol; when on the left (right), it was visible only to the left (right) eye. The binocular alignment probe was presented above and below the standard line and was visible to both eyes; the two parts were always co-linear. Observers adjusted the probe's horizontal position until it appeared aligned with the standard line. The vertical separation—the gap—between the ends of the standard line and alignment probes was an independent variable in the experiment. B) Top view of the stimulus. The disparity between the fixation symbol and the alignment probe was another independent variable. The distance to the standard line was ill-defined because it was monocular.

Observers judged the perceived azimuth of the alignment probe relative to the standard line. Before each stimulus presentation, they fixated the fixation symbol and assessed their vergence accuracy with the nonius symbols. They initiated a stimulus presentation with a button press. The fixation symbol was then extinguished and the standard line and alignment probe appeared immediately for 100 ms. The

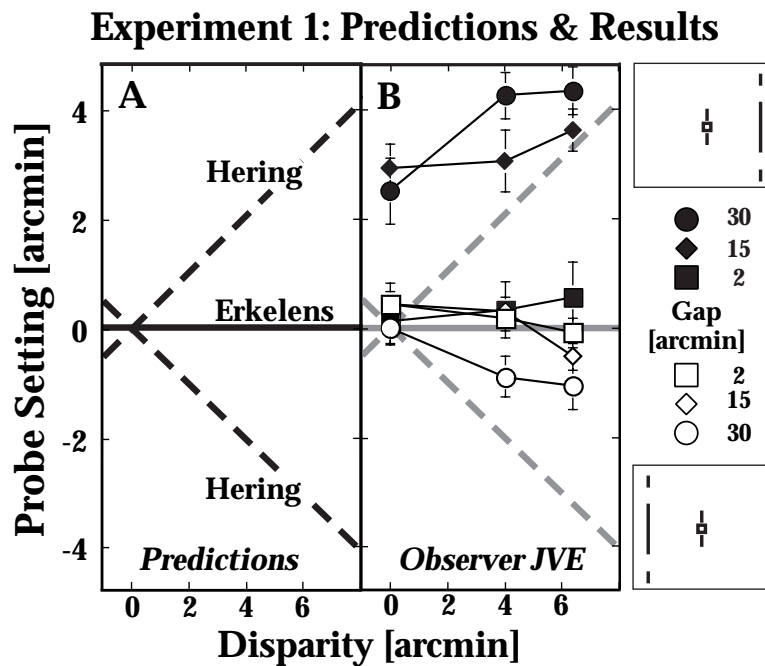
fixation symbol reappeared after the line and probe were extinguished. After each stimulus presentation, observers indicated with a button press whether the probe had appeared to the left or right of the standard line. Initial position of the probe was randomized. No feedback was given.

The horizontal position of the alignment probe was varied according to a 1-down/1-up staircase in order to find the point of subjective alignment. Step size was initially 4 pixels, but was reduced to 2 pixels after the second reversal and to 1 pixel (12 arcsec) after the fourth reversal. The staircase was terminated after 14 reversals and the position of subjective alignment was estimated from the average of probe positions for the last 10 reversals.

The experiment was conducted by running 18 alignment staircases simultaneously: There were two standard line positions (left and right), three gaps (2, 15, and 30 arcmin) and three disparities (0, crossed 4, and crossed 6.4 arcmin). Stimuli from these 18 conditions were randomly interleaved, so the observer could not bias fixation toward the standard line while performing the task.<sup>4</sup>

### 3.2 Predictions

The Hering and Erkelens models clearly make different predictions for the perceived directions of neighboring monocular and binocular targets. The left panel of Figure 5 shows the predicted alignment-probe settings for the two models as a function of the horizontal disparity of fixation relative to the probe. The abscissa is the fixation point's disparity relative to the alignment probe. The ordinate is the horizontal position of the alignment probe relative to the standard line for the eye that saw both.



**Figure 5.** Predictions and results for Experiment 1. A) Hering's and Erkelens' predictions. The azimuth of the predicted probe setting relative to the standard line is plotted as a function of its disparity relative to the fixation point; azimuths are calculated for the eye that sees both the alignment probe and the standard line. Erkelens' prediction is that the probe should be set to the same azimuth as the standard line; thus, the prediction is 0 for all disparities. Hering's prediction is that the probe's azimuth relative to the standard line should be half its disparity. The predictions are the dashed diagonal lines; the upper and lower lines are the prediction when the standard line is to the left and right of fixation, respectively. B) The probe settings of observer JVE are plotted in the same format. The gray lines represent the predictions. The filled symbols represent the settings when the standard line was to the left of fixation and the unfilled symbols represent the settings when the standard line was to the right. The squares, diamonds, and circles represent data for gap sizes of 2, 15, and 30 arcmin, respectively. Error bars represent  $\pm 1$  standard deviation.

<sup>4</sup> This was an important aspect of the procedure. In pilot testing, we found that knowing where the standard line would appear caused observers to make eye movements in the direction of the line while making their judgments.

According to the Hering prediction, the standard line (monocular) and alignment probe (binocular) will appear displaced relative to one another by half the signed disparity when they are physically aligned in the eye that sees both. For example, consider the situation in Figure 3 where the rod has uncrossed disparity; the whole rod is seen by the left eye and so the top, middle, and bottom are physically aligned in that eye. According to Hering, the top and bottom of the rod (equivalent to the binocular alignment probe in the experiment) should appear offset rightward relative to the middle of the rod (equivalent to the monocular standard line). To align the two parts of the rod perceptually, the observer would have to move the top and bottom leftward by half the disparity. Therefore, the Hering predictions in the format of Figure 5 are two lines, one with positive slope when the standard line was to the left of fixation (and seen by the left eye) and another with negative slope when the line was to the right (and seen by the right eye).

The Erkelens prediction is straightforward. The perceived directions of the monocular standard line and binocular alignment probe are assigned by the eye that sees both, so when they are physically aligned in that eye, they will also appear to be aligned. Thus, the observer does not have to adjust the probe's position from the physically aligned position. The Erkelens prediction in the format of Figure 5, therefore, is a line at 0.

The demonstration in Figure 3 shows that perceived direction seems to change smoothly from the monocular to the binocular part of the line. Thus, we suspected that assigned direction depends strongly on the vertical separation between monocularly and binocularly visible targets. Neither Hering nor Erkelens examined the influence of the separation between monocular and binocular targets on their perceived directions, so for simplicity, we assumed that they predict no effect of gap size.

### 3.3 Results

The right panel of Figure 5 shows the alignment settings of observer JVE. Her average probe settings are plotted as a function of disparity and gap size. Although the data are somewhat noisy, it appears that her settings were most consistent with the Hering predictions when the gap was large and with the Erkelens prediction when the gap was small. The data from all four observers are shown in Figure 9 below, but we first describe the manner in which alignment data were corrected for vergence errors and monocular spatial distortions.

## 4 Experiment 2: Fixation Disparity

One can see from Equations (1) and (2) that inaccurate vergence affects the perceived direction predicted by Hering's laws. Specifically, the angular change in direction when the eyes are correctly verged compared with when they are incorrectly verged is:

$$\Delta H_C = \frac{\mu_F - \mu_V}{2}$$

where  $\mu_F$  and  $\mu_V$  refer to the vergence of the fixation symbol and the actual vergence of the eyes, respectively. When the eyes are properly fixated, the two vergences are equal, and there is no change in visual direction. One can also see from Equations (1) and (2) that the *Hering* and *Erkelens predictions* are identical if the eyes' vergence is the same as that of the alignment probe. In this case, the oculocentric directions,  $O_L$  and  $O_R$ , are equal, so Equation (1) reduces to Equation (2). Clearly, for the issues being examined here, it is crucial to know where the eyes were fixating when the alignment experiment was performed. For this reason, we developed a procedure to measure vergence errors at the time the alignment judgments were made.

### 4.1 Method

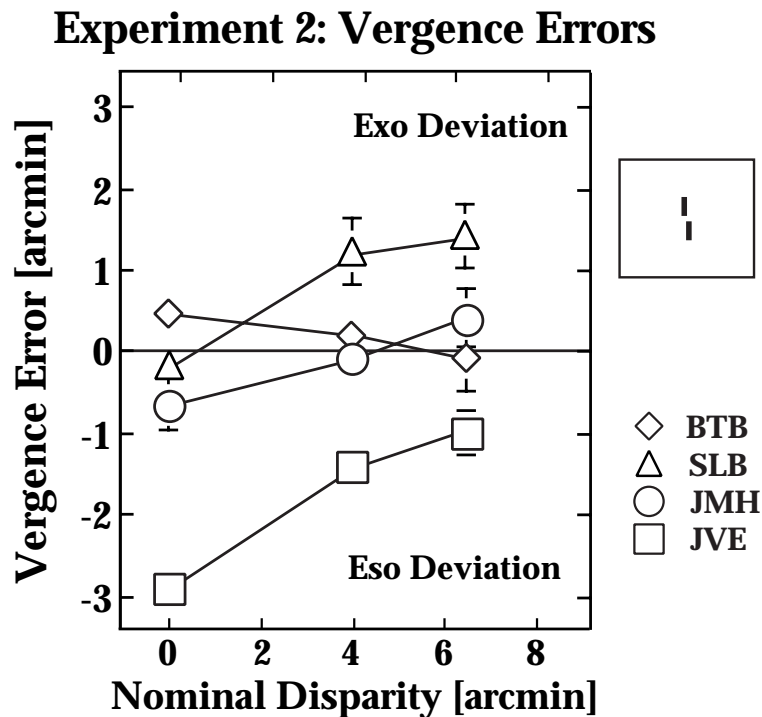
When the observer initiated a stimulus presentation in Experiment 1, there was a probability of 0.14 that a fixation-disparity probe would appear rather than the standard line and alignment probe. The fixation-disparity probe was a 100 ms, dichoptic vernier target that was presented in the position of the fixation symbol. The top and bottom line segments of the vernier target were presented to the right and left eyes, respectively. The vertical separation between the ends of the segments was 2.2 arcmin because vernier acuity is quite precise for this separation (Westheimer & McKee, 1977). Observers indicated whether the upper segment appeared to the left or right of the lower segment. As with a standard nonius fixation marker, the apparent left and right positions of the segments reflected the directions of the lines of sight. The horizontal positions of the upper and lower segments were varied with a staircase



procedure in order to find the position of subjective alignment. The parameters of the staircase were identical to those described above for Experiment 1. Three staircases were conducted with the fixation-disparity probe, one for each of the disparities presented in the primary experiment. These data were used to correct the primary alignment data for effects due to vergence errors.

4.2 Results & Discussion

The results from the fixation-disparity measurements are displayed in Figure 6. Vergence errors are plotted as a function of the disparity of the fixation symbol; positive errors represent cases in which the observers actually fixated behind the fixation symbol. The vergence errors of observers SLB, JMH, and JVE varied as a function of the disparity of the fixation symbol; they tended to verge in front of the symbol when its disparity was small and behind the symbol when its disparity was large. In addition, JVE generally verged in front of the fixation symbol and SLB behind it. The vergence errors in BTB were small.



**Figure 6.** Experiment 2: Vergence errors during the experiment. The difference between the vergence of the fixation symbol and the actual vergence of the eyes is plotted as a function of the disparity of the fixation symbol relative to the alignment probe. The vergence of the eyes was assessed using a flashed fixation-disparity probe (see text for details). If vergence were precise, the data would lie on the horizontal line at 0. The data points represent the estimated vergence errors for the four observers. Exo-deviations (i.e., the eyes verged behind the fixation symbol) are indicated by points above the horizontal line at 0 and eso-deviations (i.e., the eyes verged in front of the symbol) are indicated by points below. Error bars represent  $\pm 1$  standard deviation. These data were used to correct the alignment data (see Figure 8) for vergence errors.

The vergence error measurements were used to correct the alignment data from Experiment 1. The correction procedure is explained in the description of Figure 9 below.

Erkelens & van de Grind (1994) reported alignment behavior consistent with the hypothesis that perceived direction is determined by the eye that sees both the monocular and binocular targets. Recall that Hering's prediction is the same as the Erkelens prediction if the eyes' vergence is the same as the vergence of the alignment probe. For this reason, it is important to know whether observers make vergence errors in the direction of the alignment probe. Our data show that they do make such errors even when they are explicitly instructed to fixate accurately. Thus, the findings of Erkelens & van de Grind (1994) can be questioned on the grounds that vergence errors in the direction of the alignment

probe may have occurred. We will return to this issue once we have corrected our alignment data for vergence errors and monocular spatial distortions.

### 5 Experiment 3: Monocular Spatial Distortions

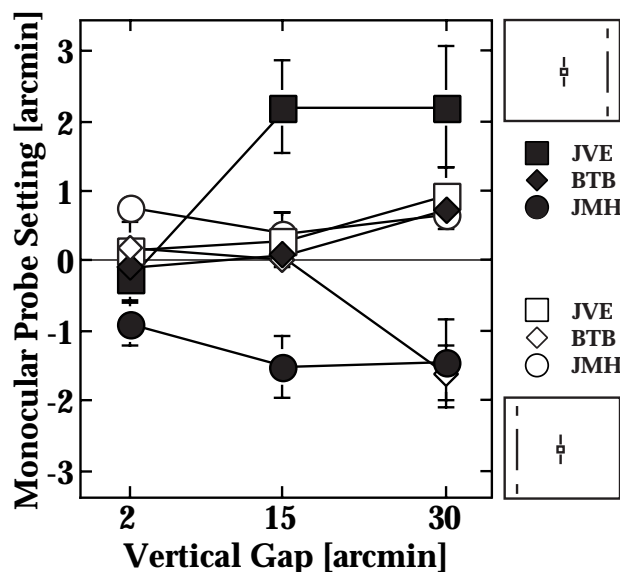
Our model of the task assumes that two small targets will appear vertically aligned when they are imaged at the same horizontal position in the retina. However, this assumption is questionable because the perception of alignment, even in one eye, is subject to error (Bedell & Flom, 1981). We thus attempted to measure monocular spatial distortions that could have affected alignment judgments in Experiment 1. We constructed the monocular experiment such that errors due to perceptual biases and optical aberrations (including chromatic aberration) could be measured and then used to correct the alignment data from Experiment 1.

#### 5.1 Methods

The stimuli were identical to those presented in Experiment 1 except the line and probe were viewed monocularly. Specifically, the standard line and alignment probe had the same dimensions and chromatic content and appeared in the same configuration as before, but the observers viewed both with one eye. If the line and probe appeared to the left of fixation, they were viewed with the left eye; if they appeared to the right, they were viewed with the right eye. The fixation symbol was presented binocularly.

Three of the four original observers (BTB, JMH, and JVE) participated. They indicated after each stimulus presentation whether the monocular alignment probe had appeared left or right of the monocular standard line. Six alignment measurements (three gap sizes and two standard line positions) were conducted; the disparity of the fixation symbol was not varied.

### Experiment 3: Monocular Alignment



**Figure 7.** Experiment 3: Monocular alignment errors. The difference between the azimuth of the monocular standard line and the alignment probe, presented to the same eye, is plotted as a function of the vertical separation between the ends of the line and the probe. Error bars represent  $\pm 1$  standard deviation.

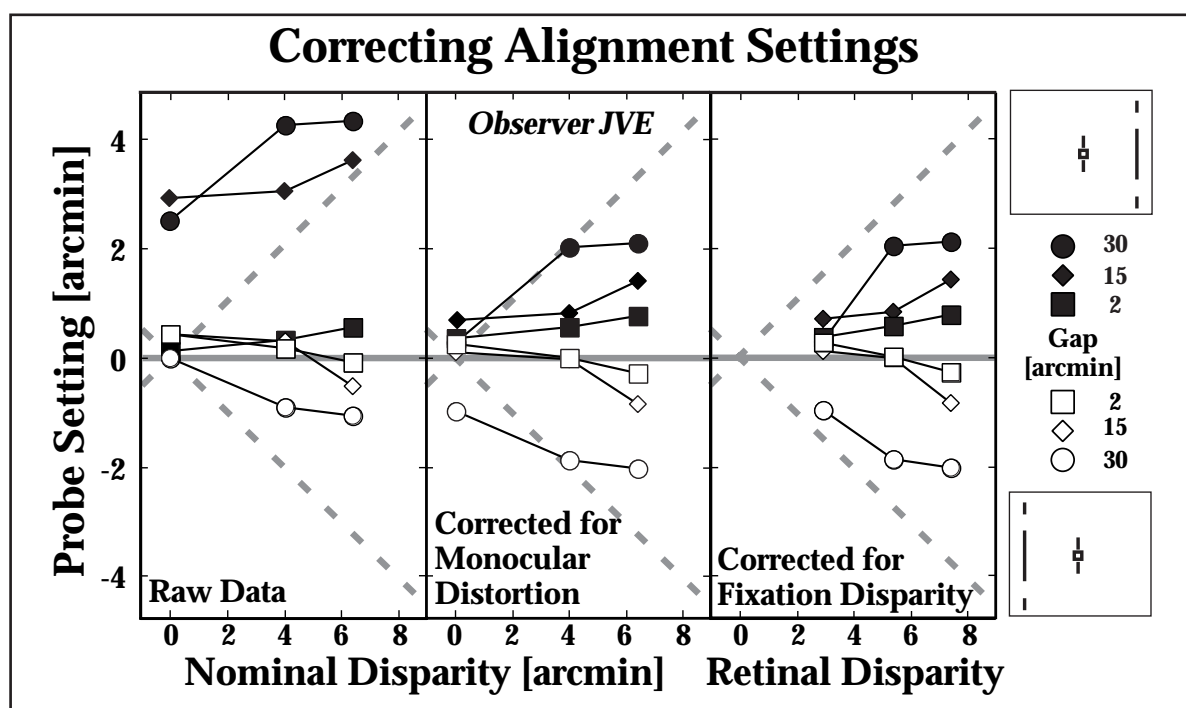
5.2 Results & Discussion

The results are shown in Figure 7 which plots the horizontal position of the alignment probe setting relative to the horizontal position of the standard line. The settings deviated significantly from 0. Observer JVE set the probe to positions 2 arcmin from true alignment when the gap size was 15 arcmin or greater. Observer JMH set it nearly 1 arcmin from true alignment when the gap was only 2 arcmin.

These data reveal that perceived co-linearity is in fact subject to error even in one eye. Many previous experiments on binocular visual direction (e.g., Erkelens & van de Grind, 1994; Shimono, Ono, Saida & Mapp, 1998) have used alignment procedures and, therefore, their data are subject to the monocular errors manifest here.

6 Correction for Vergence and Monocular Alignment Errors

We measured the vergence of each observer for each condition of Experiment 1 and we measured monocular spatial distortions affecting perceived alignment. We now use those error data to correct the alignment data in Figure 5 and thereby obtain a clearer picture of the binocular processes involved in the assignment of perceived direction.

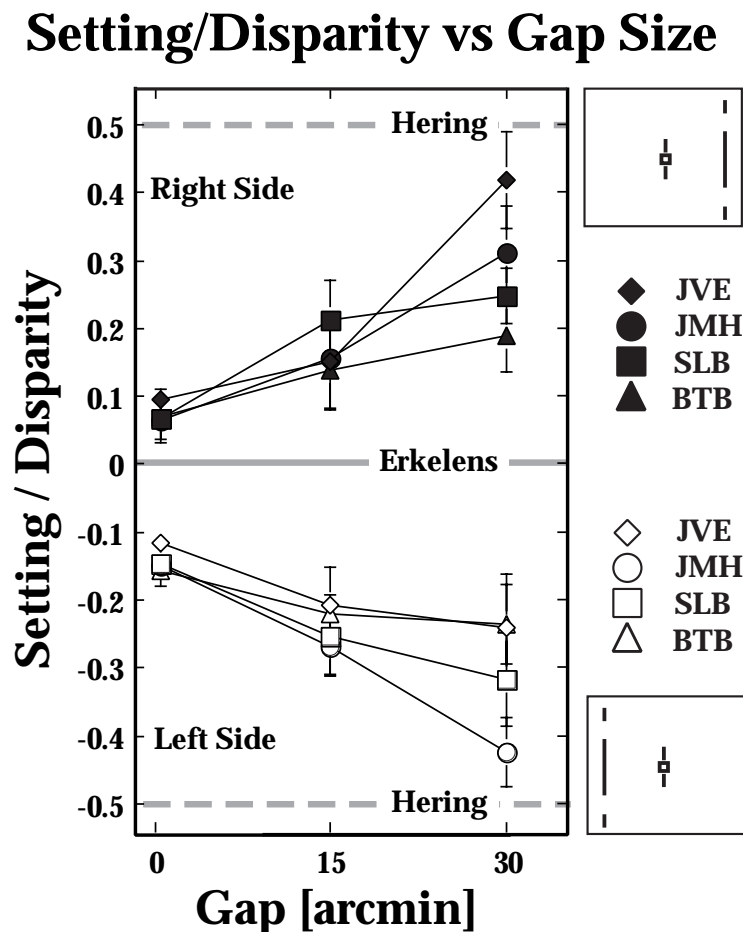


**Figure 8.** Alignment settings and corrections for vergence errors and monocular distortion. The left panel shows the data from observer JVE; the data are the same as in the right panel of Figure 5. The azimuth of the alignment probe setting relative to the standard line is plotted as a function of the disparity of the fixation symbols relative to the alignment probe. The central panel shows the same data once corrected for the monocular distortions measured in Experiment 3 (see Figure 7). These errors are corrected by shifting the data at each gap size vertically in order to undo the azimuthal distortion observed in Experiment 3. The right panel shows the data from the central panel once corrected for the vergence errors measured in Experiment 2 (see Figure 6). These errors are corrected by plotting the retinal disparity on the abscissa rather than the nominal disparity.

We have reproduced the alignment settings of observer JVE in the left panel of Figure 8 (these are the same data as in Figure 5). As before, the filled and unfilled symbols represent settings when the standard line appeared to the left and right of the fixation symbol, respectively. The middle panel shows the same data once it has been corrected for monocular distortions measured in Experiment 3. For example, JVE had an error of 2.2 arcmin in monocular alignment when the vertical gap was 15 arcmin, so all of her data at this gap size were shifted vertically by 2.2 arcmin. By comparing the left and middle panels, you can see that the major effect of this correction is a vertical shift of the data at the larger gap sizes. The right panel shows the same data after it has also been corrected for vergence errors measured

in Experiment 2. To implement this correction, we plotted the data in units of the retinal disparity of the alignment probe; this has the effect of shifting points along the horizontal axis from the middle to the right panel.

The corrected data for JVE (and for the other observers, not shown in the figure) are reasonably consistent with the Hering prediction when the gap size is 30 arcmin and with the Erkelens prediction when it is 2 arcmin. The change from Hering to Erkelens behavior is consistent with what one observes in the demonstration shown earlier (Figure 3): The line appears bowed smoothly away from the occluder because Hering's laws determine the perceived location of the line's center, while Erkelens' prediction determines the perceived location of the line near the transition from monocular to binocular.



**Figure 9.** Summary of corrected alignment data from Experiment 1. Regression lines were fit to data in the format of the right panel of Figure 8 for each observer at each gap size. The slopes of those lines represent the change in azimuthal setting as a function of retinal disparity: setting/disparity. Those slopes are plotted here as a function of the vertical gap between the alignment probe and standard line. Different symbols represent the data from different observers. The filled and unfilled symbols represent data when the standard line was to the left and right of the fixation symbol, respectively. Those stimulus conditions are also schematized by the square icons at the top left and bottom right of the figure. Error bars represent ±1 standard deviation.

We can summarize the alignment data as a function of gap size by fitting regression lines to the corrected data. For each combination of gap size and side on which the standard line appeared, we performed a linear regression on the corrected alignment data. The slopes of the best-fitting lines are plotted in Figure 9 for all observers and conditions of Experiment 1. These slopes represent the change in alignment settings as a function of the retinal disparity of the alignment probe. The Hering prediction is

a slope of  $\pm 0.5$  and the Erkelens prediction is a slope of 0. Notice that each observer's data exhibited a smooth transition from slopes near 0 when the gap size was 2 arcmin to slopes near  $\pm 0.5$  when the gap was 30 arcmin. None of the data is completely consistent with Hering or Erkelens, but the data are most consistent with Erkelens at small gaps and with Hering at large gaps.

## 7 Experiment 4: Influence of an Occluder

In everyday vision, there are a number of situations in which a monocular target can appear near a binocular target. One situation is depicted in Figure 2 where an occluding edge blocks one eye's view of a target or a part of a target; this situation is interesting because it can occur in the central visual field where localization is quite accurate. We found in Experiment 1 that neither the Hering nor Erkelens prediction accounts for the perceived direction of a binocular target in the vicinity of a monocular target. We asked next whether the presence of a visible occluder is important to the assignment of direction. This question is of theoretical interest because researchers have claimed that perception near an occluder is influenced by the ecological plausibility of the alternative spatial configurations (see, for example, Shimojo & Nakayama, 1990).

### 7.1 Methods

The stimulus and procedure were identical to those in Experiment 1 with the following exceptions. A solid 80 x 40 arcmin rectangle was added to the display in the position depicted in Figure 2. The occluder had the same disparity relative to the display screen as the fixation symbol and this disparity was constant at 6.4 arcmin. Vertical separation (gap) between the standard line and alignment probe was constant at 2 arcmin. Alignment settings were conducted with four interleaved staircases (with and without occluder, stimuli to the left and right of fixation). In addition, the fixation-disparity probe (see Experiment 2) was presented randomly with a probability of 0.20. Observers BTB, JMH, and JVE participated.

### 7.2 Results & Discussion

The average alignment settings with and without the occluder were: 0.60 and 0.70 for observer BTB, 0.44 and 0.34 for JMH, and 0.60 and 0.74 for JVE. Clearly, the presence of the occluder had no consistent effect on the perceived direction of the alignment probe relative to the standard line. We conclude, therefore, that the assignment of directions of neighboring monocular and binocular targets does not depend on the presence of a visible occluder, but rather depends simply on the separation between the targets and the retinal disparity of the binocular target.

## 8 Experiment 5: Horizontal Lines

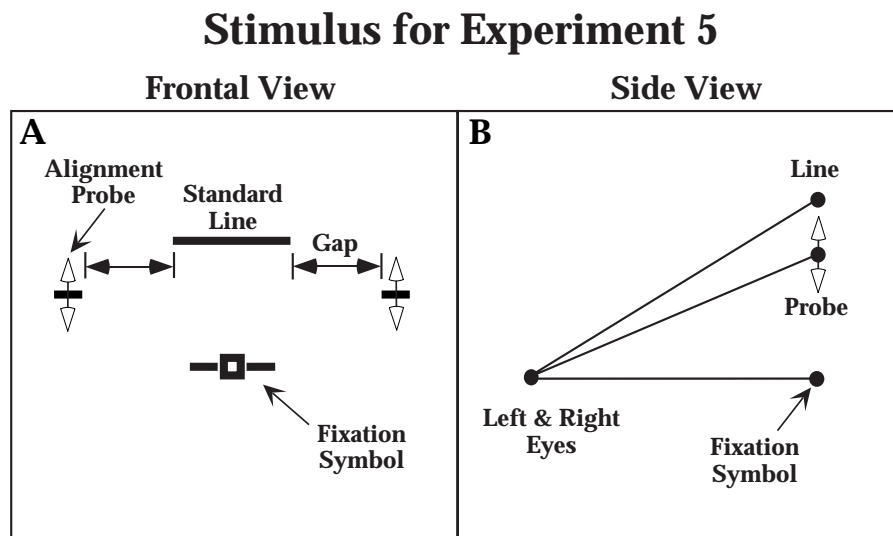
The likelihood of a partial occlusion in natural vision is very dependent on the orientations of the partly occluded object and the occluding edge. For simplicity, let us restrict our discussion to elongated, linear objects (so they have a clear orientation) and linear occluding edges. Furthermore, we consider only cases in which the orientations of the object and occluding edge are the same.

The geometry of occlusion is best described in head-centric terms because it is the locations of the object and the occluding edge relative to the eyes' positions in space that determine whether an occlusion occurs or not; the positions of the eyes in the head are irrelevant (if we ignore the fact that the optical center is not coincident with rotation center in a given eye). We will, therefore, define orientation in head-centric coordinates. Partial occlusion can occur when the occluding edge is vertical; we have seen such a situation in Figure 2. However, partial occlusion can never occur when the occluding edge is horizontal. A horizontal line can be occluded by a horizontal edge in both eyes or neither eye; there can be no partial occlusion.

Given that partial occlusions cannot occur in natural vision with horizontal objects and edges, we wondered how visual direction is assigned when we create such partial occlusions artificially. If the assignment of direction near occluders depends on experience, the visual system might behave differently in the artificial situation. If, on the other hand, the assignment of direction near occluders is a general rule that simply concerns what to do with monocular and binocular targets in the same vicinity, then the visual system might exhibit similar behavior to what we observed in Experiment 1.

### 8.1 Methods

The stimulus was identical to the one used in Experiment 1 with the following exceptions. First, and most importantly, the orientations of the standard line, alignment probe, and fixation symbols were rotated 90 deg such that they were horizontal (Figure 10). As before, the standard line was 39 arcmin from the fixation symbol. Second, the nominal disparity of the fixation symbol was 0 or 4 arcmin relative to the display screen and it was a vertical disparity. As before, the probe's disparity relative to the computer monitor was 0. Third, the gap between the ends of the standard line and alignment probe were horizontal; as before, the gap magnitudes were 2, 15, or 30 arcmin. Fourth, observers had difficulty making vertical vergence eye movements accurately to the fixation symbol when it had a non-zero disparity, so we added two strips of random-dot texture (not shown in Figure 9) with the same vertical disparity as the fixation symbol. The texture strips were 2.6 x 0.25 deg in width and height, respectively, and were positioned 0.6 deg above and below the fixation symbol.



**Figure 10.** Schematic of the stimulus in Experiment 5. A) Frontal view. The fixation symbol consisted of a binocular central square and horizontal nonius lines that were used to monitor the accuracy of vertical vergence. The monocular standard line was placed above or below the fixation symbol. The binocular alignment probe was presented left and right of the standard line; the two parts were co-linear. Observers adjusted the probe's vertical position until it appeared aligned with the standard line. The horizontal separation—the gap—between the ends of the standard line and alignment probes was an independent variable. B) Side view. The disparity between the fixation symbol and the alignment probe was another independent variable. The distance to the standard line was ill-defined because it was monocular.

The procedure was identical to the one in Experiment 1 except observers now indicated whether the alignment probe appeared above or below the standard line. The fixation-disparity probes were presented again so that we could assess the accuracy of vertical vergence; the probes were horizontal, so observers indicated whether the left segment appeared above or below the right one. Observers JMH and BTB participated.

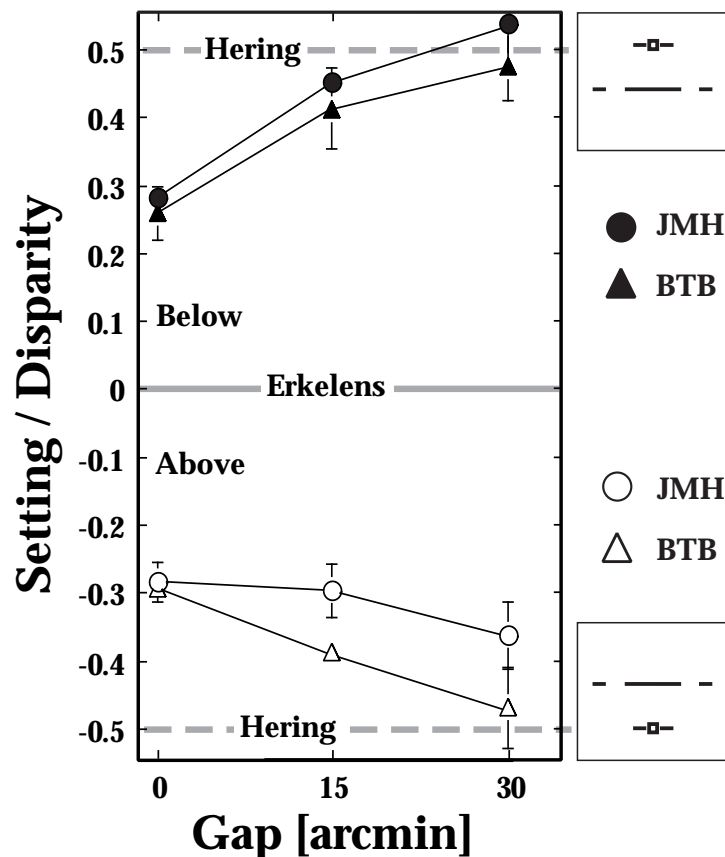
### 8.2 Results

We fit the data from this experiment with regression lines after the data were corrected for vertical vergence errors and then plotted in a format similar to Figure 8. The slopes of those best-fitting regression lines are plotted in Figure 11 as a function of the horizontal gap between the standard line and alignment probe. (Monocular spatial distortions do not affect these slopes, and we did not collect monocular data for this experiment.)

The data show both similarities and dissimilarities when compared with the data for vertical lines (Figure 9). They are similar in that alignment settings become increasingly consistent with Hering's predictions as gap size is increased. They are dissimilar in that the data are uniformly more consistent with Hering's predictions: at small gap sizes, the settings were not as close to Erkelens' predictions as we observed in Experiment 1 and, at large gaps, they were closer to Hering's predictions than we observed.

Thus, there is some evidence that the assignment of perceived direction differs for horizontally as opposed to vertically oriented objects. One cannot determine from our data whether the difference is due to differential experience with horizontal and vertical partial occlusions or is another example of how vertical and horizontal disparities are processed by fundamentally different mechanisms.

### Experiment 5: Results



**Figure 11.** Summary of corrected alignment data from Experiment 5. Regression lines were fit to data in the format of the right panel of Figure 8 for each observer at each gap size. The slopes of those lines represent the change in the elevation setting as a function of retinal disparity: setting/disparity. Those slopes are plotted here as a function of the horizontal gap between the alignment probe and standard line. Different symbols represent data from two different observers. Unfilled and filled symbols represent data when the standard line was above and below the fixation symbol, respectively. Those stimulus conditions are also schematized by the square icons at the top and bottom right of the figure. Error bars represent  $\pm 1$  standard deviation.

## 9 General Discussion

Most research in binocular vision has concerned the estimation of the distance to points in the visual scene. As we pointed out in the discussion of Figure 1, the directions to such points must also be estimated in order to recover the 3-d positions of the surface points. Direction estimation has not received nearly as much theoretical and experimental attention as the distance estimation and perhaps the primary reason is the nearly universal acceptance of Hering's laws of visual direction as a description of the visual system's method for direction estimation.

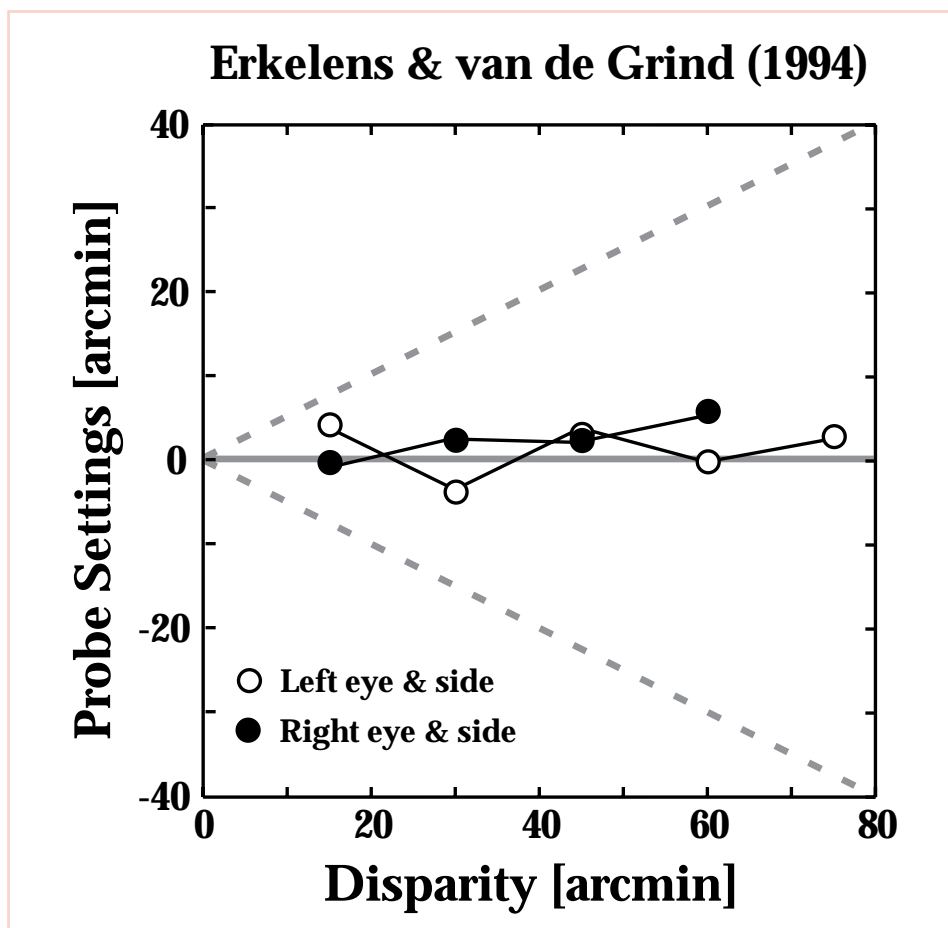
Hering's laws make an interesting prediction when part of an object is occluded to one eye while another part is not occluded at all (Figures 1-3): the binocular part, if it is fused, should be seen in a

different direction than the monocular part. We found that this prediction is not completely confirmed. Specifically, the abruptness of the change in perceived direction predicted from Hering's laws is not observed; rather the change is smooth and apparently reflects a gradual transition from the use of one eye to determine the directions of monocular and binocular targets that are within a few arcmin of one another to the use of Hering's laws for targets that are farther apart. We found that this smooth transition occurs whether a visible occluder is present or not and whether the monocular and binocular targets are vertically oriented or not.

### 9.1 Comparison with Erkelens & van de Grind (1994)

Among a variety of conditions they examined, Erkelens and van de Grind (1994) presented a monocular test line and binocular probe and asked observers to adjust the probe until it appeared in the same direction. This is basically the same as our main experiment. Erkelens and van de Grind's data were not consistent with Hering's prediction and this led them to the hypothesis that the perceived direction is determined solely by the eye seeing both the test line and probe (which we have called the Erkelens' prediction).

Erkelens and van de Grind's stimulus differed in some ways from ours: in their and our experiments respectively, the test line's retinal eccentricities were  $\sim 180$  and  $39$  arcmin, the relative disparities between the probe and fixation point were  $15\text{--}75$  and  $0\text{--}8$  arcmin, and the gaps between the line and probe were  $165$  and  $2\text{--}30$  arcmin. Although our methods differed, Erkelens and van de Grind's results are still not compatible with ours; in particular, they did not observe behavior consistent with Hering's predictions with large gaps and we did. It is worthwhile, therefore, to compare the data explicitly.



**Figure 12.** Probe settings of observer CE in Erkelens & van de Grind (1994) as a function of the probe's disparity relative to the fixation point. The dashed gray lines represent Hering's predictions and the solid gray lines Erkelens' predictions.



There were two conditions in Erkelens and van de Grind (1994) that were similar to conditions presented here. Those data are presented in their Figure 4 and are labelled “Lb+” and “Rb+”.<sup>5</sup> The data were presented in a different format than the one used here, so we have replotted them in Figure 12. The probe settings of observer CE are plotted as a function of the probe’s disparity relative to the fixation point. The dashed gray lines represent Hering’s predictions and the solid gray lines Erkelens’ predictions. The data are clearly much more consistent with Erkelens’ predictions than with Hering’s. In Figures 8 and 9 of the current paper, it is clear that we observed Hering-like behavior when the gap size was as small as 30 arcmin, a factor of 6 smaller than the gap size used by Erkelens and van de Grind. Thus, these data are indeed inconsistent with our observations.

We hypothesize that the evident discrepancy can be attributed to a difference in the stimuli and a difference in the experimental procedure. First, the gap between the monocular test line and binocular probe in the Erkelens and van de Grind experiment contained texture visible to both eyes whereas in our experiment the gap was blank. Perhaps the texture acted as an alignment aid and thereby effectively reduced the gap between the monocular test line and nearest binocular target. In other words, the effective gap in Erkelens and van de Grind (1994) may have been much smaller than 2.75 deg. Second, as we pointed out earlier, the Hering and Erkelens predictions are the same when the eyes’ vergence is such that the retinal disparity of the binocular probe is zero. Erkelens and van de Grind instructed their observers to direct their eyes toward the fixation symbol, but they did not measure the accuracy of fixation nor did they provide nonius elements to enable the observers to assess accuracy themselves. Indeed, one suspects that their observers’ vergence did drift toward the binocular probe because Erkelens and van de Grind presented disparities as high as 75 arcmin at an eccentricity of ~3 deg and the fusion limit is normally less than 20 arcmin at that eccentricity (e.g. Mitchell, 1966). In any event, the discrepancy between our data and those of Erkelens and van de Grind (1994) underline the need to assess vergence accuracy in experiments on binocular visual direction (Banks, van Ee & Backus, 1997) when a comparison between the visual directions of both monocular and binocular objects is involved.

### 9.2 Use of Nonius lines to assess vergence accuracy

Nonius lines are widely used in basic research and clinical tests to determine the eyes’ vergence. The assumption underlying nonius usage is that equation (2) above holds. Specifically, a vergence change affects only the oculocentric positions,  $O_L$  and  $O_R$  (because version is not affected by vergence). If fixation is accurate,  $O_L = O_R$ , and the perceived directions of nonius lines are both equal to the eyes’ version.

Recently, there have been reports that the nonius-line technique is not always a valid measure of vergence accuracy; these can be thought of as violations of equation (2). Shimono, Ono, Saida, and Mapp (1998) claim that nonius lines are only valid measures of vergence accuracy if the lines are not positioned close to binocular texture at a different disparity than the fixation point. Their data bear some resemblance to ours because we also reported conditions in which the perceived direction of a monocular target relative to a binocular target is not predicted by equation (2) unless the monocular and binocular targets are separated by 30 arcmin.

### 9.3 Measurement error and perceived visual direction

Hering’s laws of visual direction have been the standard account of the visual system’s method for estimating directions to binocular or monocular targets (Ono & Mapp, 1995; van de Grind, Erkelens & Laan, 1995). It is interesting to note that discussions of Hering’s laws have involved only descriptions of geometric quantities such as the angles between the visual axes and the visual lines to the target. For example, equations (1) and (2), which represent our instantiation of Hering’s laws, involve only physical angles and do not incorporate errors the visual system might make in measuring them. In Experiment 3 we found evidence for measurement errors that do in fact affect alignment judgments and it seems likely that other such errors exist. Perhaps future investigations of direction perception should seek to uncover other errors and then incorporate them into visual direction models.

<sup>5</sup> “Lb+” stands for test line on left side and visible to the left eye, probe binocularly visible, and disparity of fixation point crossed. “Rb+” stands for the same condition except the test line was on the right side and was visible to the right eye only.

#### 9.4 Concluding remarks

Hering's laws make an interesting prediction when part of an object is visible to both eyes while another is visible to one eye only (Figures 1–3): the binocular part, if it is fused, should be seen in a different direction than the monocular part. Thus, an objectively straight contour would appear to have a break and displacement. This distortion of perceived shape could be avoided if the visual system employed the method suggested by Erkelens and colleagues (Erkelens & van de Grind, 1994; Erkelens, et al., 1996; Erkelens & van Ee, 1997): assignment of the perceived directions of the binocular and monocular parts by one eye only. Because the projection of the contour in that eye is continuous, the shape distortion can thereby be avoided. We found that neither Hering nor Erkelens rules predict perceived direction for the partial occlusion situation. When monocular and binocular targets are within 2 arcmin of one another, direction follows Erkelens' prediction reasonably well. When the targets are separated by 30 arcmin or more, perceived direction is more consistent with Hering's prediction. By making a smooth transition from one form of direction assignment to another, the visual system minimizes, but does not eliminate, the distortion of perceived shape that Hering's laws predict.

#### Acknowledgments

This work was supported by research grants from AFOSR (93NL366) and NSF (DBS-9309820). RVE was supported by Human Frontier of Science (RG-34/96) and the Foundation for Life Sciences (SLW Talent Stipendium, NWO #810-404-006/1) of the Netherlands Organization for Scientific Research. BTB was partially supported by NIH (T32 EY0704-18). MSB was partially supported by the Max-Planck Institute for Biological Cybernetics. We thank Casper Erkelens for help with pilot experiments, Payam Saisan for technical assistance, and Jamie Hillis, Suzanne Baker, and Jacquelin van Ee for doing many alignment settings.

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