SOME ASPECTS OF PENDULUMS SYNCHRONIZATION

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In this note we describe some aspects of synchronization problem of a system of pendulums connected with rigid body free in horizontal motion. Results of numerical simulation for the system of four non-identical pendulums are presented. Possible synchronization modes of the system for some sets of data are shown. Amplitudes of motion of the mass centre of the whole system as well as relative motion of pendulums' mass centres are analysed. Evolution of the system motion from transient to unstable synchronization through desynchronization to stable synchronization is shown. Finally the attempt to explain the mechanism of pendulums synchronization was taken.

Synchronization is understood as an adaptation of the system's dynamics due to the interaction between its subsystems, which is achieved by coupling the system's variables.

The synchronisation of two pendulum clocks hanging on the wall for the first time was observed by Huygens [1]. The phenomenon of coupled harmonic oscillators is studied till now by many authors [2, 3, 4]. The synchronisation of horizontally moving beam and n identical mathematical pendulums hanging from the beam has been presented in [5]. Our work deals with the system of n=4 non-identical physical pendulums (Fig. 1).

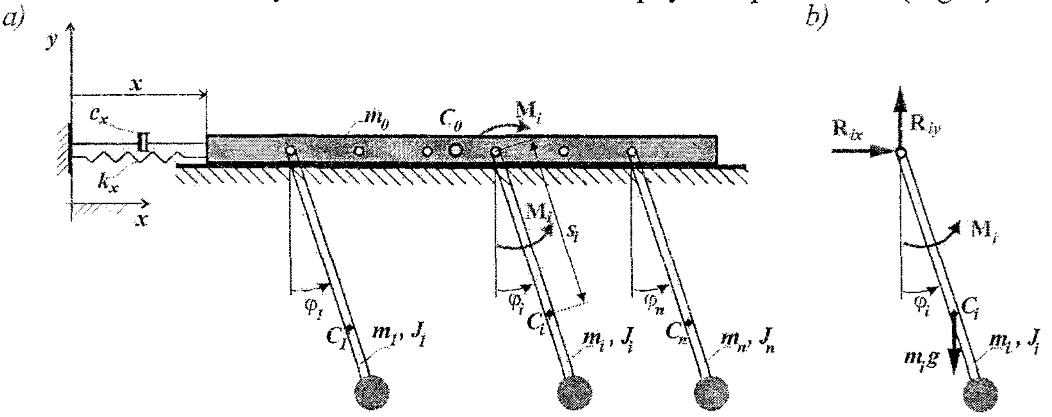


Fig. 1 – Pendulums mounted to the rigid body free to x-motion (a) and the reactions (b)

The equations of motion of the system presented in Fig. 1 are as follows:

$$m\ddot{x} + \sum_{i=1}^{i=n} m_i l_i \left(\ddot{\varphi}_i \cos\varphi_i - \dot{\varphi}_i^2 \sin\varphi_i\right) + c_x \dot{x} + k_x x = 0\,, \tag{1}$$

$$J_i\,\ddot{\varphi}_i + c_{\varphi i}\dot{\varphi}_i + m_i \ddot{x} l_i \cos\varphi_i + m_i g l_i \sin\varphi_i = M_i\,, \qquad i = 1,\dots,n \tag{2}$$

where $m = m_0 + \sum_{i=1}^{i=n} m_i$ denotes the mass of the whole system and M_i describe the energy supplied to the pendulums $\left(M_i = c_{\varphi i}(1 - c_i\varphi_i^2)\,\dot{\varphi}_i^2\right)$.

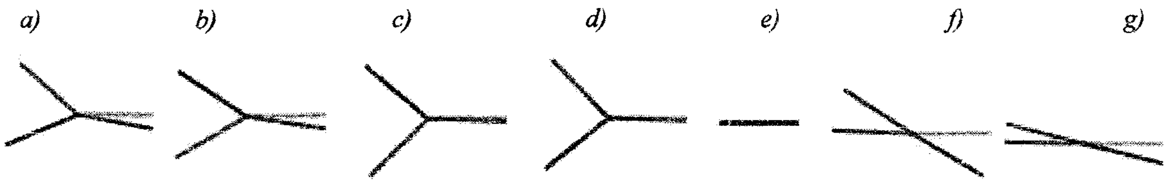
Reaction force (Fig. 1b) in the x-direction acting on pendulum i has the following form

$$R_{xi} = m_i\left(\ddot{x}_i + l_i\ddot{\varphi}_i \cos\varphi_i - l_i\dot{\varphi}_i^2 \sin\varphi_i\right), \qquad i = 1,\dots,n \tag{3}$$

and the total force acting on the body is

$$R_x = -\sum_{i=1}^{i=n} m_i\ddot{x} - \sum_{i=1}^{i=n} m_i l_i \ddot{\varphi}_i \cos\varphi_i + \sum_{i=1}^{i=n} m_i l_i \dot{\varphi}_i^2 \sin\varphi_i\,. \tag{4}$$

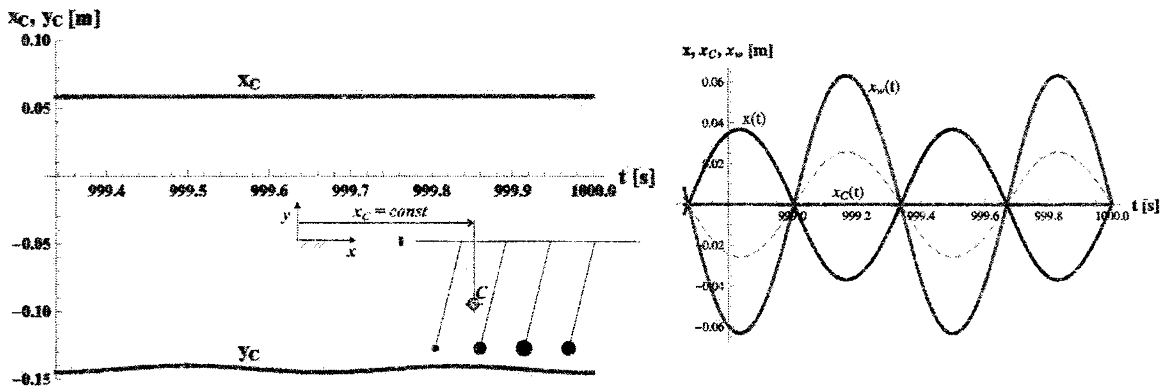
For the system parameters: m_0 = 5 kg, m_n = {1.0, 1.75, 2.25, 2.0}, l_n ={0.248, 0.248, 0.248, 0.248}, c_φ ={-0.01,-0.0306,–0.0506,–0.04}, c ={60., 60., 60., 60.}, k_x= 0.1 N/m, c_x = 0.141 Ns/m, and different initial phase angles of individual pendulums seven modes of system synchronization has been found (Fig. 2).



Final phase differences between pendulums:
a) $\beta_i - \beta_1$ *={123.229, 332.215, 193.016}, b)* $\beta_i - \beta_1$ *={126.056, 213.318, 354.509}, c)* $\beta_i - \beta_1$ *={202.485, 339.768, 135.636}, d)* $\beta_i - \beta_1$ *={358.465, 131.459, 232.27}, e)* $\beta_i - \beta_1$ *={225.663, 39.094, 353.687}, f)* $\beta_i - \beta_1$ *={359.984, 359.973, 359.978}, g)* $\beta_i - \beta_1$ *={358.926, 226.859, 124.352}.*
Fig. 2 – Synchronization modes of n=4 pendulum system.

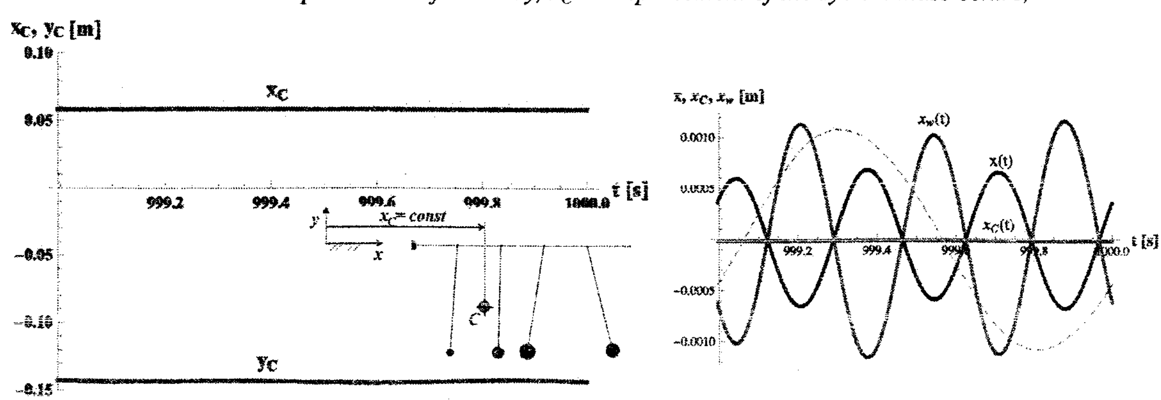
Motion of synchronized system is characterised by the same period and constant relative position of pendulums (constant phase differences). We can classify the types of synchronization shown in Fig. 2 as: multi-phase – cases a) and b), multi-phase with cluster – cases c) and d), in-phase – case e), and two pairs in anti-phase – cases f) and g).

On the basis of the system momentum analysis it can be shown that synchronization is connected with minimization of the system mass centre amplitude. In Figures 3 and 4 it is shown that in the case of in-phase synchronization (Fig. 3) and multi-phase synchronization (Fig. 4) the position of the system mass centre (x_C) is constant, what means that horizontal motion of the system mass centre vanishes.



x – displacement of the body, x_C *– displacement of the system mass centre,*
x_W *– displacement of pendulums mass centre relative to the body.*
Fig. 3 – In-phase synchronization of 4 pendulums

x – displacement of the body, x_C *– displacement of the system mass centre,*

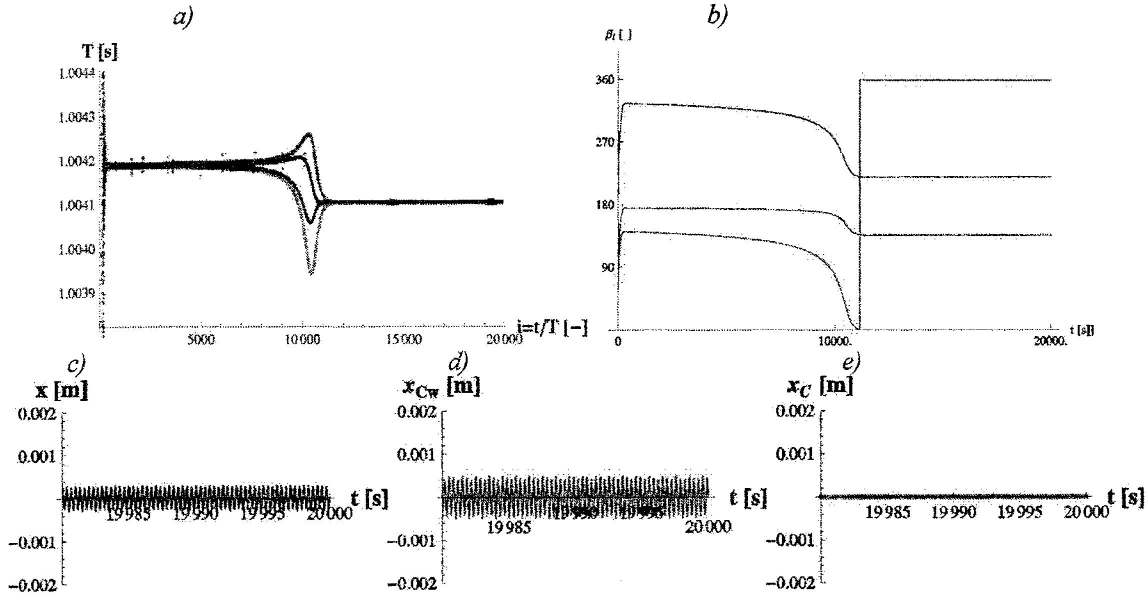


x_W *– displacement of pendulums mass centre relative to the body.*
Fig. 4 – Multi-phase synchronization of 4 pendulums

One can obtain the relation between the position of the system mass centre (x_C), pendulums mass centre relative to the body (x_W) and the body mass centre (x) as follows

$$m\,(\dot{x}_C - \dot{x}) = m_W \dot{x}_W\,. \qquad (5)$$

In Figure 5 the evolution of the system motion from transient (for t < 200) to unstable synchronization (for 200 < t < 6000) and then through desynchronization (for 6000 < t < 11500) to stable synchronization (for t > 11500) is shown.



a) evolution of the pendulums period, b) phase differences between pendulums (β_i-β_1), c) body mas centre amplitudes, d) pendulums mass centre relative to the body, e) system mass centre position.
Fig. 5 – Evolution of the system motion from transient to unstable synchronization and then through desynchronization to stable synchronization

Final result of the motion – time series of pendulums position (φ_i) and the body displacements (x) for stable synchronization of the system (for t >11500) is shown in Fig.6.

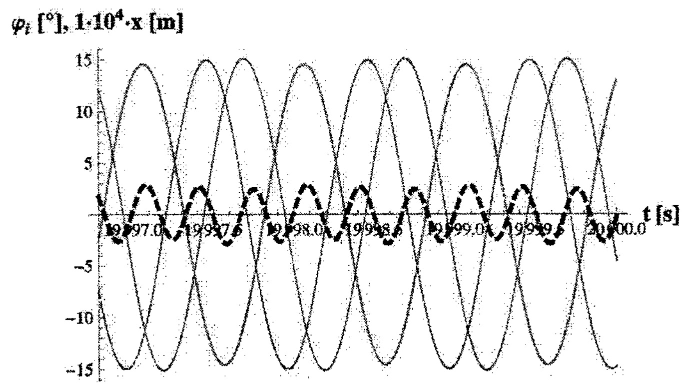


Fig. 6 – Time series of pendulums position (φ_i) and the body displacements (x)

To explain the mechanism of pendulums synchronization we have analysed the reaction forces of individual pendulums acting on the body: $R_{1xi} = -m_i\ddot{x}$, $R_{2xi} = -m_i l_i \ddot{\varphi}_i \cos\varphi_i$, $R_{3xi} = m_i l_i \dot{\varphi}_i^2 \sin\varphi_i$. Changes of the reaction forces of synchronized pendulum system are shown in Fig. 7. It should be pointed out that the components $R_{1x} = -\sum_{i=1}^{n} m_i\ddot{x}$, $R_{2x} = -\sum_{i=1}^{n} m_i l_i \ddot{\varphi}_i \cos\varphi_i$, $R_{3x} = \sum_{i=1}^{n} m_i l_i \dot{\varphi}_i^2 \sin\varphi_i$ have the same period

$T/3$ despite of the fact that periods of individual forces R_{1xi}, R_{2xi}, R_{3xi} are $T/3$, T and T respectively ($T \cong 1{,}0041\ s$).

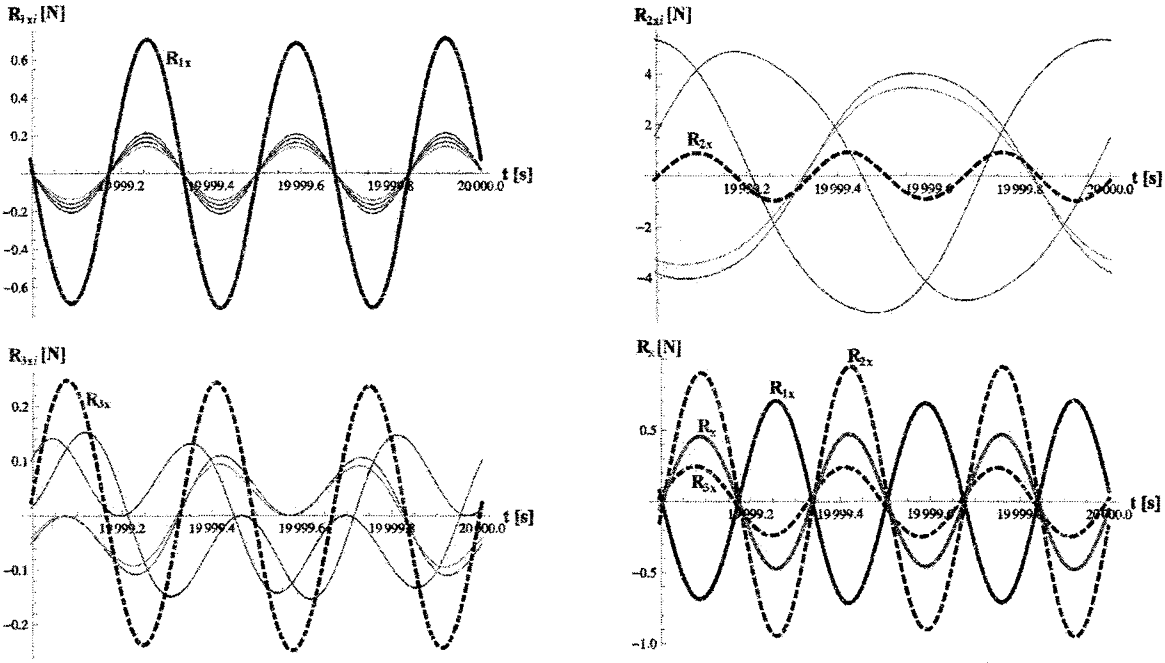


Fig. 7 – Changes of the individual pendulum reaction forces of synchronized system

Concluding, it can be said that deciding factor of the synchronisation mode for a set of data system parameters are initial phase angles β_{0i} of the pendulums. Resultant reaction forces R_{2x} i R_{3x} and the period of these reactions depend on pendulums phase angles β_i. In the case of in-phase synchronization the period of resultant reaction R_x is equal T, whereas the multi-phase synchronization is connected with the period of resultant reaction R_x equal to $T/3$.

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