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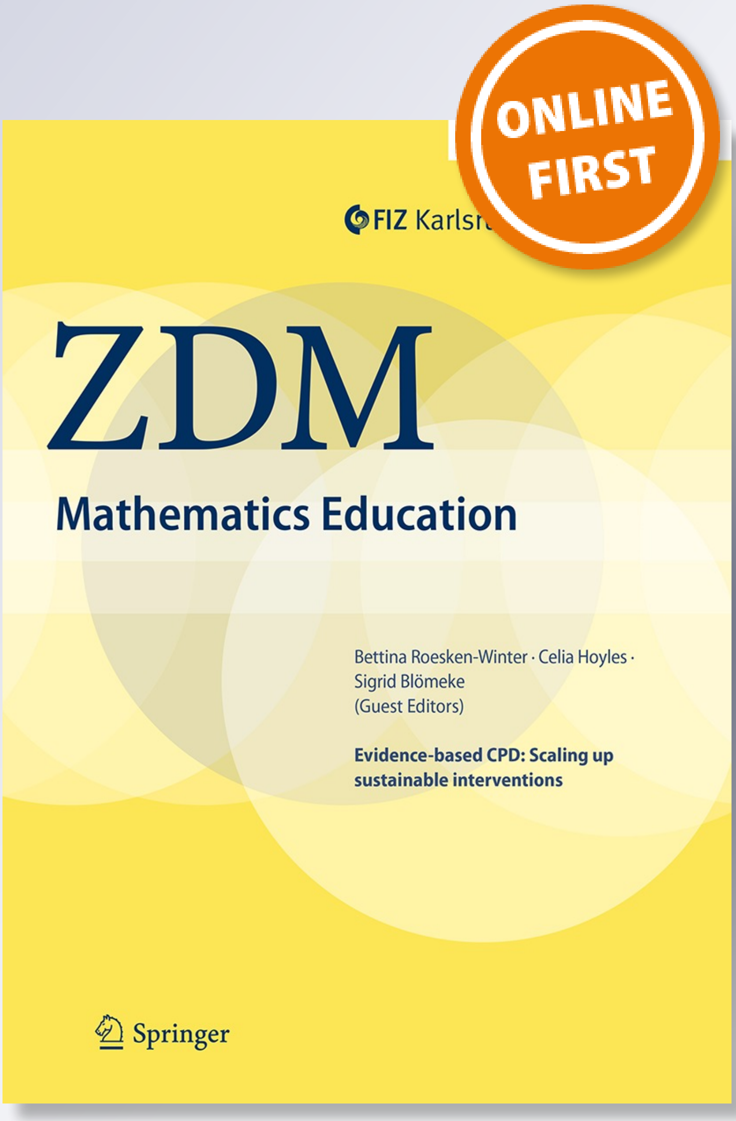

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ORIGINAL ARTICLE

Boundary objects and boundary crossing for numeracy teaching

Hamsa Venkat · Mark Winter

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Abstract In this paper, we share analysis of an episode of a pre-service teacher's handling of a map artefact within his practicum teaching of 'Mathematical Literacy' in South Africa. Mathematical Literacy, as a post-compulsory phase subject in the South African curriculum, shares many of the aims of numeracy as described in the international literature—including approaches based on the inclusion of real-life contexts and a trajectory geared towards work, life and citizenship. Our attention in this paper is focused specifically on artefacts at the boundary of mathematical and contextual activities. We use analysis of the empirical handling of artefacts cast as 'boundary objects' to argue the need for 'boundary crossing' between mathematical and contextual activities as a critical feature of numeracy teaching. We pay particular attention to the differing conventions and extents of applicability of rules associated with boundary artefacts when working with mathematical or contextual perspectives. Our findings suggest the need to consider boundary objects more seriously within numeracy teacher education, with specific attention to the ways in which they are configured on both sides of the boundary in order to deal effectively with explanations and interactions in classrooms aiming to promote numeracy.

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1 Introduction

Internationally, school curricula and policy documents focused on adult skills emphasize 'numeracy' as vital for productive participation in society. In spite of this consensus, understandings about how mathematical learning in formal settings might be structured to support the development of the numeracy competences needed to participate fully and critically in adult life remain relatively piecemeal. Goos' (2007) model of numeracy, comprising four elements, presents a succinct bringing together of a range of prior research on features that have been described as important for numeracy development. These elements are: attention to real-life contexts; the deployment of mathematical knowledge; the use of representational, physical and digital tools; and consideration of students' dispositions towards the use of mathematics.

In 2006, South Africa introduced 'Mathematical Literacy' as a subject in the post-compulsory school curriculum (Grades 10–12: learners aged 15 and above in Grade 10). The Mathematical Literacy curriculum statements, in earlier and current iterations (SA DoE 2003; SA DBE 2011) emphasize within their rhetoric the notion of a subject that aims to prepare students for the mathematical demands of everyday life:

> Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of

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> mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems. (SA DoE 2003, p. 9)

Together with the life-preparation orientation of numeracy, the inclusion of real-life contexts is viewed as a central tenet of Mathematical Literacy teaching within this curriculum (SA DBE, p. 10), mirroring its centrality in Goos' (2007) model of numeracy. There is also emphasis within the curriculum rhetoric on the need for mathematical reasoning around problem situations to be brought into play alongside other contextual considerations:

> Alongside using mathematical knowledge and skills to explore and solve problems related to authentic real-life contexts, learners should also be expected to draw on non-mathematical skills and considerations in making sense of those contexts. […] In other words, mathematical content is simply one of many tools that learners must draw on in order to explore and make sense of appropriate contexts. (SA DBE, p. 11)

Within this statement, there is explicit encouragement to view situations and problems from both mathematical and contextual perspectives in order to make informed decisions. This 'multiplicity of tools' view tends to contrast with the more singular emphasis on mathematical tools in the OECD (2010; 2013) mathematization cycle for Mathematical Literacy working: model formulation, intra-mathematical working, interpretation of mathematical results and validation of results. While Goos et al. (2012) emphasize both mathematical tools and the disposition to use mathematical tools within Goos' (2007) model, they note that the critical orientation within which the key features of numeracy are enveloped goes beyond the use of mathematical tools to being 'aware of appropriate and inappropriate uses of mathematical thinking to analyse situations and draw conclusions' (p. 212). Writing in the critical numeracy literature places particular emphasis on the shift in vantage point needed to critique the limitations of mathematical tools (e.g. see Skovsmose and Yasukawa 2009; Frankenstein 2001; Jablonka 2003). While the South African Mathematical Literacy curriculum has been criticized for falling short of this critical perspective (Christiansen 2006), the curriculum rhetoric does encourage focus on the orientations and limitations of mathematical tools through a shift in vantage to a contextual standpoint.

In these terms, teaching and teacher development for Mathematical Literacy, and therefore for numeracy, require facility and flexibility with viewing situations from mathematical and contextual vantage points. This facility and flexibility lie at the heart of the in-depth analysis of the empirical episode considered in this paper. The episode is selected as a 'telling case' (Mitchell 1984) that suggests, with backing from both literature and theory, that such facility and flexibility cannot be assumed within numeracy teaching, even when the four essential elements of Goos' (2007) model are present. This, in turn, suggests that such flexibility of standpoint is important to take up within any mathematics teacher education that seeks to promote the development of numeracy.

Our focus on problem contexts centres on artefacts within those contexts that can be taken up from mathematical and contextual perspectives. Following the work of Celia Hoyles and Richard Noss, summarized later in this paper, we interpret these artefacts in terms of 'boundary objects' with the boundary constituted by mathematical activity and conventions on one side, and contextual activity and conventions on the other side. We view any traversing between these two activities related to the focal boundary object within numeracy teaching as 'boundary crossing' activity. Our research questions relating to the focal episode presented in this paper are therefore framed in these terms:

- What conventions around the boundary object are communicated within numeracy teaching, and which activities (contextual and/or mathematical) can these conventions be associated with?
- What does analysis of the nature of artefact-related boundary crossing suggest for numeracy teacher education?

The artefact, or boundary object, in focus in this paper was a section of a map of Johannesburg that was used for a coordinate spatial locating task. Seen from the contextual side, the map is an artefact for locating places and charting movement. It can also be used to estimate distances and journey times. Locations on the contextual side are notated using longitude and latitude notation. Seen from the perspective of mathematical activity, the map can be perceived as a small section of a four-quadrant coordinate grid configured around the equator as the 0° latitude line and the prime meridian as the 0° longitude line. Spatial locating coordinate activities in mathematics are generally related to conventions and notations associated with the Cartesian plane involving x- and y-axes intersecting at the origin.

In this paper we use theory relating to boundary objects and boundary crossing to present analysis of an empirical episode that points to the need for numeracy teaching and teacher development to incorporate attention to working with situational artefacts from mathematical and contextual standpoints. Facility within each of these standpoints and flexibility across them appears, to us, to be central to the development of numeracy in ways that can start to build a critical orientation towards mathematics and other tools.

Our argument is developed through a structure that begins with some background on Mathematical Literacy as a school subject in South Africa, and national attempts at developing mathematics teacher education routes in higher education institutions for producing Mathematical Literacy teachers. We go on to provide an overview of literature examining differences in the nature of mathematical working when working within mathematical and contextual orientations, as well as how these different perspectives affect ways of working with artefacts at the boundary. Theoretical perspectives on boundary objects and boundary-crossing activities with some methodological implications are detailed, followed by an outlining of data sources. Findings, analysis and some concluding comments are then presented.

2 Mathematical literacy as a school subject in South Africa

Since 2006 in South Africa, Mathematical Literacy and Mathematics have been structured together as a 'core' strand, with all post-compulsory phase students having to choose one or other of these subjects. The introduction of Mathematical Literacy was aimed at addressing a prior situation in which over 40 % of all post-compulsory phase students dropped mathematics entirely (Perry 2004), and a significant further proportion failed mathematics. The consequences of this situation were flagged in terms of the problems with adult innumeracy that have been widely documented in other countries as well (Paulos 1988; Parsons and Bynner 2007). The introduction of Mathematical Literacy was presented as a pathway for life participation as citizens and workers (SA DoE 2003), with Mathematics offered as the pathway for students who wanted to proceed into mathematically-related disciplines at tertiary level.

A corollary to this introduction was the need to train teachers to teach Mathematical Literacy. This process occurred predominantly through in-service 're-skilling' courses for (non-Mathematics) teachers. The urban university that both of us worked in opted, in 2011, to offer a pre-service route into Mathematical Literacy teaching by packaging a new elective course: 'Concepts and Literacy in Mathematics' (CLM) within its undergraduate Bachelor of Education (BEd) programme. This course, run over years 2–4 of the 4-year BEd programme, sought to attract primary teachers wanting to offer Mathematics as a specialism within their teaching and secondary (non-Mathematics) teachers wanting to offer Mathematical Literacy as a subsidiary teaching subject. This single offering that worked across primary Mathematics and secondary Mathematical Literacy teacher education was justified on the basis that the Mathematical Literacy curriculum stated explicitly that the mathematical content drawn on was based on compulsory phase (up to Grade 9) content, and thus overlapped with the fundamental mathematical knowledge base required for primary mathematics (Grades 1–6) teaching.

Studying pre-service teacher learning among the secondary-level teachers who opted into this Mathematical Literacy training route formed the focus of the second author's in-depth qualitative doctoral study of four of the eight prospective Mathematical Literacy teachers in this group (Winter 2014). All of these students had taken Mathematical Literacy within their own prior schooling backgrounds, achieving over 70 % in the exit National Senior Certificate Mathematical Literacy examination—a level achieved by 6–14 % of Mathematical Literacy candidates between 2008 and 2013 (SAIRR 2014). The broader study sought to explore these students' mathematical understandings and contextual problem-solving competences on course tasks and in classrooms during practicum periods. PISA's mathematization process and cognitive demand elements (OECD 2010; 2013) framed Winter's (ibid.) analysis, given the CLM course aim to address both mathematical and contextual sense-making. As noted already, PISA's mathematization cycle places emphasis on the deployment of mathematical tools within contextual problem-solving processes.

Data analysis in Winter's broader study, based on students' course task responses and observations of their practicum Mathematical Literacy teaching, indicated particular shortcomings relating to 'model formulation' from a contextualized situation, and weaknesses for some students within their intra-mathematical working. In adapting the framework for use in the context of problem-solving within Mathematical Literacy teacher education rather than student problem-solving, Winter (2014) added 'pedagogic link-making' as an additional process element, to encompass pre-service teacher working with intra-mathematical tasks (in the course and in classrooms) with the addition of a contextual situation to support explanation. He found that some teachers were more willing and more skilled in engaging with pedagogic link-making through attaching contextual situations and problems appropriately and specifically to the intra-mathematical working they were engaged with.

While the Mathematical Literacy curriculum points strongly to the need for mathematical working to be driven by situational demands (i.e. by contextual requirements), Winter (2014) noted that some of the pre-service teachers he studied brought predominantly mathematical perspectives to bear on the artefacts that they worked with in Mathematical Literacy, with few, if any, moves to the contextual orientations that are emphasized in the curriculum rhetoric. This lack of dialectic between mathematics and context has been noted earlier in Graven and Venkat's (2007) spectrum of pedagogic agendas seen within South African Mathematical Literacy teaching. However, the focal teaching episode in this paper was drawn from a teacher who showed willingness to engage with artefacts from mathematical and contextual perspectives widely in his teaching. He thus showed the disposition that Goos (2007) has argued is important in order to use mathematics for contextual sense-making and problem-solving. The other teachers in Winter's (2014) sample showed more extensive traces of mathematical orientations to activity, with less frequent moves to situational perspectives.

While the OECD's sequential mathematization process cycle was useful in Winter's (2014) study for pin-pointing elements within problem-solving that pre-service teachers had particular difficulties with, this analysis tended to leave unattended episodes where teaching and classroom interaction suggested possibilities for more bifurcated ways of handling artefacts across contextual and mathematical orientations. The teacher's work with the map artefact, interpreted in this paper as a boundary object, brought this need into particularly sharp relief, and is therefore used to illuminate our argument. As noted already, examples drawn from the critical numeracy literature, in particular, lend support to our claim that fluent and flexible working with more bifurcated mathematical and contextual considerations is more generally useful within numeracy teaching.

Part of the complexity of mathematical/contextual boundary crossing work relates to two key issues. Firstly, mathematical working is involved on both sides of the contextually-driven/mathematically-driven boundary in numeracy. Secondly, the same boundary object can be perceived differently based on whether the perspective taken towards activity is driven by contextual or mathematical goals. We begin with a literature review that details the ways in which mathematical working in numeracy situations frequently differs from mathematical working in mathematical situations. We also note literature relating to the differing perspectives on artefacts related to working with them from mathematical or contextual perspectives.

3 Mathematical working in mathematics/numeracy

Steen (2001, p. 6), discussing the nature of mathematics in numeracy/quantitative literacy, notes that there are differences in the nature and level of mathematical working when contrasted with more traditional mathematical working:

> quantitative literacy involves mathematics acting in the world. Typical numeracy challenges involve real data and uncertain procedures but require primarily elementary mathematics. In contrast, typical school mathematics problems involve simplified numbers and straightforward procedures but require sophisticated abstract concepts.

Steen points here to substantive differences in the problem-solving processes associated with mathematics and quantitative literacy, echoing the bifurcated view that we noted above. For Steen, these differences are driven by differences in the nature of problem contexts—involving 'real data' in quantitative literacy and 'simplified numbers' in mathematics. Hoyles et al. (2010, p. 7), elaborate the view of differences in the nature of mathematics used within adult life-related problem-solving, but locate this difference in goals and activities rather than in the situations themselves:

> most adults use mathematics to make sense of situations in ways that differ quite radically from those of the formal mathematics of school, college and professional training. Rather than striving for consistency and generality, which is stressed by formal mathematics, problem-solving at work is characterised by pragmatic goals to solve particular types of problems, using techniques that are quick and efficient for these problems.

A highly 'situated' mathematical working is pointed at here—the kind of working that has been described in the lineage of studies that developed and fleshed out the theory that all learning and knowledge (with studies of mathematical reasoning in real-world situations providing exemplary evidence) was situated and contextualized, rather than transferable and decontextualized (Lave and Wenger 1991; Scribner 1986). Within this situation-driven orientation, attention to context-sensitive, constraint-aware mathematical working and 'good enough' estimates of quantity or relation or chance are at the fore, rather than the generality, abstraction, accuracy, precision and process emphases that are frequently at the heart of more formal mathematics—what Pimm (2009, p. 159) has noted as 'an almost complete absence of hedging or doubt in its language'. The goals pointed to in the rhetoric of the Mathematical Literacy curriculum statement (previous and current versions), with its emphasis on critical analysis and problem-solving in everyday life, suggest this kind of context-oriented mathematical working:

> The mathematical content of Mathematical Literacy is limited to those elementary mathematical concepts and skills that are relevant to making sense of numerically and statistically based scenarios faced in the everyday lives of individuals (self-managing individuals) and the workplace (contributing workers), and to participating as critical citizens in social and

> political discussions. In general, the focus is not on abstract mathematical concepts. (SA DBE 2011, p. 10)

The Mathematical Literacy curriculum in South Africa thus echoes the position espoused in the numeracy literature of a quite different mathematical working in the service of contextual goals in comparison to the kinds of abstraction and generality-driven mathematical working associated with traditional mathematical goals.

While the nature of mathematical working is at the core of these considerations, artefacts within situations at the interface of different activities and communities are not centrally in focus. Steen's quote above (2001) argues that artefacts tend to be configured differently in mathematical situations in comparison to numeracy situations. In contrast, Venkat (2014) draws attention to 'common' artefacts that emerge and can be viewed in different ways depending on whether the activity that forms the vantage point is mathematical or contextual. Discussing a task based on the Gini coefficient measure of inequality that she used within mathematics teacher education for the discussion and development of notions of numeracy, she provides an example of graphical cumulative frequency representations emerging out of collation of data on income distributions. Two such representations are produced—a straight line, which represents a hypothetical situation of equality, and a curve that is based on collected data. Pre-service teachers are faced with a familiar mathematical object (the $y = x$ line) that has been produced through an unfamiliar process (a cumulative income representation of hypothetical equality). While not the focus of her chapter, this work shows that artefacts presented or produced within numeracy can be associated with different production processes and viewed quite differently from the ways in which they would traditionally be viewed within mathematics.

In similar vein, Hoyles et al. (2004), overviewing their studies of technology integration in mathematics education, describe students' inputs in a LOGO software environment as stated in contextual terms based on:

> the specificity of situations and the contingencies of mathematical expression on tools and technologies and on the communities in which they are used. (p. 312)

Citing Balacheff (1993) in support in their theorization of technology use in mathematics learning, they draw attention to the need for teaching that incorporates:

> the dialogue that must take place between standard ("official") mathematical knowledge, knowledge about the tool, and the "computational transposition" of the mathematical knowledge. (Hoyles et al. 2004, p. 315)

While our attention is not on technological tools but artefacts within situations in numeracy classrooms, a similar interest in the need for teachers to manage the dialogue between 'contingent' expressions linked to the artefact on the situational side and the 'official' expressions relating to it on the mathematical activity side guides the analysis presented in this paper.

Noss (2002), reviewing research on mathematical thinking in workplace situations, points to the need for 'interpretive flexibility' around artefacts, and particularly so in contexts of 'breakdown' when 'oldtimers' in communities are confronted with the need to explain meanings and processes related to artefacts to 'newcomers' in various parts of a workplace. Interpretive flexibility in Noss' (ibid.) argument relates to traversing between the ways that an artefact is used by different subgroups within workplace communities.

Taken together, this research points to a multitude of contexts—numeracy teaching, technology use situations and workplace coordination activities—where flexible movement between the vantage points of mathematical and contextual activities appears to be required. This literature review leads into an introduction to the notions of boundary objects and boundary crossing and the ways in which this theorization led to a grounded data analysis. We focus particularly on Hoyles et al.'s (2010) work on artefacts at the boundaries of mathematics and other discourses.

4 Boundary objects and boundary crossing

Star and Griesemer (1989) developed the notion of a boundary object in their work looking at the need for coordination between interacting communities, with artefacts providing the potential to support coordinating activity:

> Boundary objects are those objects that both inhabit several intersecting worlds and satisfy the informational requirements of each of them … [They are] both plastic enough to adapt to local needs and the constraints of the several parties employing them, yet robust enough to maintain a common identity across sites. They are weakly structured in common use, and become strongly structured in individual site use (Star and Griesemer 1989, p. 393)

The 'common' artefact idea is central to this description, alongside the need for 'coordination' around it. Suchman (1994) developed the concept of boundary crossing to describe workplace moves into unfamiliar activities configured around boundary objects. Engeström et al. (1995, p. 319) went on to describe boundary crossing as involving 'negotiating and combining ingredients from different contexts'.

Akkerman and Bakker, in their review of research focused on boundary objects and boundary-crossing activities, note that both of these concepts, unlike the concept of transfer, are viewed in relation to 'ongoing, two-sided actions and interactions between contexts' (Akkerman and Bakker 2011, p. 136). This view provided a useful lens for thinking about simultaneous awareness of artefacts from mathematical and contextual perspectives.

The studies summarized in Hoyles et al.'s work (2010) on 'techno-mathematical literacies' in workplace settings emphasize focus on artefacts viewed as boundary objects as central to coordination activity. Their ethnographic studies of workplaces focus particularly on communicative breakdowns around the use of technological artefacts, and thus, their theorizations of boundary objects and boundary crossing were particularly salient for our analysis. Within this, the acknowledgement in their design phase of 'technology-enhanced boundary objects' for workplace settings, that 'our mathematical perspective gave only a partial view of how problems could be effectively solved within the workplace' is relevant to the emphasis on mathematics as only one of a range of possible tools that we noted in the South African Mathematical Literacy curriculum.

Fuglestad et al. (2010, p. 296) acknowledge, in their teacher education studies aiming to integrate technology into mathematics teaching, that teachers' histories with boundary objects impact on the ways in which they perceive and take up artefacts:

> we cannot assume that the meanings that we build into the microworld (or any other artifact) are transparent to the teacher. Teachers will construct their own meanings, which will be influenced by their past experiences and beliefs as well as their interactions with these objects.

Akkerman and Bakker (2011) have noted that the concepts of boundary crossing and boundary objects have been developed predominantly within two theoretical traditions: cultural-historical activity theory (Engeström 1987) and situated learning theories focused on communities of practice (Wenger 1998). In this paper, we locate our use of boundary objects and boundary crossing in the activity theory tradition for two reasons: firstly, artefacts are central to our analysis; and secondly the conventions or 'rules' associated with these artefacts from the perspective of contextual and mathematical activities emerged as an important feature in our analysis. Both of these aspects—artefacts and conventions—are strongly represented within the activity theory tradition (Engeström 1993).

Following Hoyles et al. (2010) we consider artefacts drawn from problem situations in classrooms focused on numeracy as boundary objects. How these artefacts are handled within teaching, and the ways in which this handling suggests activities and conventions that work across contextually or mathematically oriented activity systems, forms the basis of the grounded analysis that we present in this paper. As emphasized in the literature section, this handling can include mathematical tools in terms of content and processes that can be deployed with either contextual or mathematical orientations in the foreground. The data excerpts that we draw on to exemplify our argument on the need for both orientations within numeracy teaching are selected to be illustrative of this empirical claim, through the application of the theorizing of numeracy teaching in terms of boundary objects located at the interface of mathematical and contextual activity systems. In methodological terms, we pay specific attention to teacher utterances related to the focal artefact noting the ways in which these discursive fragments connect with goals and conventions relating to mathematical/contextually-driven activity. Informed consents and ethical permissions for the original study and subsequent writing from it were gathered from all study participants and the university.

5 Data sources

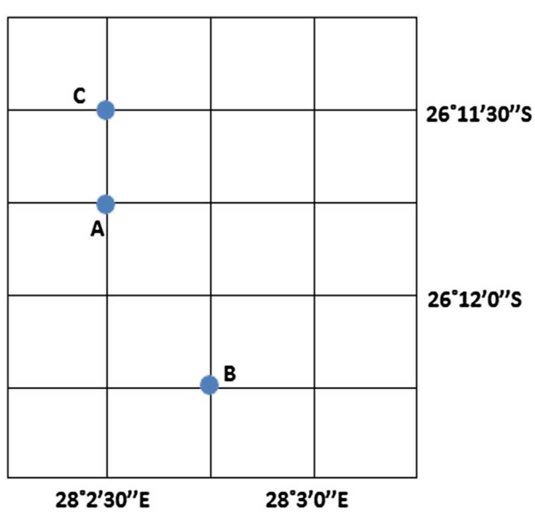
The data excerpts that we draw on were generated from a transcript of a video-taped practicum lesson taught by Mark (pseudonym)—one pre-service Mathematical Literacy teacher within the CLM course. In terms of broader background, Mark was a first language English speaker and a relatively high performing student in terms of the CLM course, the first year BEd compulsory mathematics course that had preceded the CLM course, and his own school background Mathematical Literacy performance. We point this out simply to note that Mark's deployment of mathematics was generally strong across the mathematical activities in the foreground in the first year BEd course and in the contextual activities that were discussed in the CLM course. Across the four lessons observed as part of Winter's (2014) broader study, Mark was largely able to deploy mathematical ideas appropriately, in many cases combined with provision of explanations of his problem-solving procedures that were both mathematically correct and contextually sensitive and appropriate. There were two occasions in which disruptions were observed within Mark's lesson episodes—the first, detailed in this paper, was in the context of the task focused on 'map reading' (Lesson 1) and the second was in the context of a 'personal finance' problem situation (Lesson 2). The disruption in the second case was centred on disconnection between the problem situation and his selections of mathematics deployed to solve the problem within a percentage change focused task. Boundary crossing activity did not occur in this case, and it is therefore of less interest for the focus of this paper (although our sense is that boundary crossing to a contextual vantage point in this episode could well have allowed for critical engagement with the incorrect mathematical procedure that was selected there—see Winter (2014, p. 201) for original analysis). Our focus is therefore on an episode within the map-reading lesson where attempts at what could be interpreted as boundary crossing were seen.

We focus specifically on a teaching episode that utilized a map of Johannesburg as an artefact for a coordinate spatial locating task. Our analysis deals with Mark's discourse and interactions about and with this artefact and its sub-features. His handling suggested awareness of how to produce appropriate answers from the contextual perspective but incorporated explanations that pointed to lack of clarity and distinction between mathematical and contextual conventions associated with this map.

In the next section we present an overview of the instructional task that was utilized during the map-related lesson. We home in on specific episodes within this teaching and provide our interpretations of the teacher's actions and explanations in relation to the notions of boundary objects and boundary crossing.

6 The map-reading problem situation

The focal lesson utilized a map drawn from a geographical South African context that was familiar to the class—the Johannesburg Central area. The figure below shows a simplified diagram of the section of gridlines of the map showing points A, B and C together with the key questions that the teacher used and discussed in the lesson. We include this simplified diagram for the reader's benefit. The problem situation as presented in the lesson was based on the 'real' map artefact presented in the "Appendix", and the task was based on the questions below:



a. Give the degrees, latitude and longitude of the points marked A, B and C.
b. What is the closest road to 26°11′30″S and 28°02′30″E?
c. Using the scale, write down the length and breadth of Joubert Park.

7 Mark's work with the artefact: findings and analysis

Problems occurred in the context of question (a) so we zoom in on the detail of this task in our analysis. Within the lesson preamble, Mark provided explanations of the map-reading context with a focus on definitions of map-related terminology such as scale of a map, and latitude and longitude lines:

Mark:	On Thursday we discussed using scales in terms of the classroom and we discussed different terminologies in terms of scale. What is a scale?
Learner 1:	It is the ratio between the diagram and real life.
Mark:	Yes, it is the ratio between the diagram and real life. So for instance you have 1 cm of the image represents 50,000 cm in real life. This is geography oriented but it is relevant because we need to do calculations to make sure that we understand maps. So when you look at the map, there are two types of lines, or grids, what are those lines?
Learner 2:	Longitudes and latitudes
Mark:	So the two terms are longitudinal lines and latitudinal lines. Now, you have got latitude and longitude, which one is horizontal?
Learner 3:	Latitude is horizontal
Mark:	And the longitude line is the …
Learners:	[Chorus answer] vertical line
Mark:	Those lines on a map formulate grids, they create the grids…

In this short excerpt, we see attention to a range of sub-features of the map artefact. Mark makes reference to a contextually oriented way of looking at scale on the map following an exemplification described in terms of: '1 cm of the image represents 50,000 cm in real life'. This follows a student's offer of 'ratio between the diagram and real life', with this description including a more mathematically oriented language that Mark navigates towards a more contextually located and contingent perspective. Thus, the notion of 'scale' is discussed with emphasis on mathematics being deployed with a contextual orientation: 'we need to do calculations to make sure that we understand maps'.

Here, we see a deft traversing that acknowledges the mathematical orientation offered and exemplifies it from a contextual perspective.

Later in the excerpt we see another deft traversing, this time from the contextual to the mathematical perspective. Here a student offers a contextual language for the lines on the grid, describing them as the 'longitudes and latitudes'. In this instance, Mark uses the more mathematical language of 'horizontal and vertical' to remind students about how to distinguish between them. Thus far, elements of the map as a boundary object are handled smoothly and simultaneously from mathematical and contextual standpoints.

Problems emerged in Mark's subsequent handling of task a), in which coordinates had to be assigned to grid-lines. Mark began his exposition of how to work out the coordinates thus:

Mark: There are 60 s in a minute. Now if you want to find A, then you have to look at the lines [latitude and longitude lines pointed out] that meet at A. Another important point when referencing, you always put north or south first. So if you are referencing something in the northern hemisphere, you start with the 'north' coordinates and then 'east' or 'west' coordinates, separated by a semi-colon. If you do it the other way round you get it wrong. Similarly in the southern hemisphere you start with the 'south' coordinates.

So looking at, ah, before we find point A, there is no coordinate here [points at the longitude line that passes through point B] so we need to find coordinates for that. So looking at the latitude line at the bottom, you got 28 degrees, 2 min 30 s E and 28 degrees, 3 min 0 s E [Teacher refers to the marked longitude values on either side of point B's longitude line here.] What do you think this line [longitude line that passes through point B] will be?

Learner Each two blocks is 30 s so that's the difference between the two given coordinates.

A range of contextually important information is communicated here. There are the 'facts' of '60 s in a minute' and there are conventions associated with recording locations on maps—that the North/South coordinate value precedes the East/West value. However, in dealing with point B, Mark begins by drawing attention to the 'East' value first and thus contradicts the situational convention he has just communicated. Thus, while Mark's communication of the situational rules for grid referencing were appropriate, his subsequent identification of the longitude value for point B first disrupted the contextual rule he had offered.

Within this excerpt, Mark associates the East values shown on the map with the 'base' latitude line on the grid ('looking at the latitude line at the bottom, you got 28°, 2 min 30 s E and 28°, 3 min 0 s E′) rather than with the vertical longitude lines on the grid, although this is subsequently clarified.

Mark's order of working here indicates the conventions associated with mathematically-oriented activity with a coordinate grid: the 'East value' of position B is given before proceeding to its 'South value' (i.e. x-value then y-value). This is in spite of his communication of the contextual convention of giving North/South position prior to East/West position.

Following this, and instead of following through with the 'other' ordinate value for point B, Mark then asks for Point A's latitude value (after acknowledging the 30 s difference between the given longitude lines for point B in the excerpt above) but makes an error in his offer:

Mark: That's right. The degrees aren't changing; the minutes and the seconds are changing. So there is 30 s difference to the next longitude line. So each block represents 15 s. So this line is gonna be 28°, 2 min, 45 s East. Now let's do the one on the vertical (indicating point A's latitude value now). So 26°, 12 min 45 s, is that correct?
Learner: No sir
Mark: No? Why do you disagree with me?
Learner: Because ah it should be 26°, 11 min 45 s.
Mark: Sure? Did anyone see the mistake I made? [Makes correction.] So it is 26°, 11 min 45 s South

Here, neither the contextual nor the mathematical conventions associated with coordinate values are followed through, but some learners, at least, appear to understand and apply understandings related to map-related proportional reasoning skills where 60 s make up a minute. Mark appears to recognize his own error, and offers contextual clarity by adding in the word 'South' to the learner's offer of the correct value.

In spite of the lack of appropriate handling of mathematical or contextual conventions, some learners show awareness of contextual conventions related to seconds and minutes relationships and order of coordinates—evident in their providing the correct answer, above and subsequently:

Mark: Now what are the coordinates for A?
Learner: 26°, 11 min, 45 s South; 28°, 2 min, 30 s East.
Mark: Do you agree with her?
Learners: Yes.

Mark: Now lets look at B. Yes! [Points at a learner.]
Learner: 26°, 12 min, 15 s South; 28°, 2 min, 45 s East.
Mark: Is he right or wrong? [Teacher pauses.] What's the coordinates for point C?
Learner: 26°, 11 min, 30 s South; 28°, 2 min 30 s East.
Mark: Is he right?
Learners: Yes.
Mark: Well done. That's how you reference points on the map.

These interactions suggest that Mark is aware of the conventions associated with situational grid-referencing rules, in spite of a pedagogical ordering that did not work consistently with these conventions, nor with the goal of identifying spatial positions of A, B and C initially in a systematic way in his own teaching. In relation to the convention, Mark's practice indicates awareness of the difference between mathematical and contextual conventions, but without these differences being made explicit or followed through consistently in teaching—and thus potentially problematic for learners unable to internalize the appropriate conventions in their own working. Shortcomings in this lack of explicit communication of differences between mathematical rules and contextual rules became more apparent in a follow-up excerpt focused on a learner question:

Learner: Do the minutes increase when you um.. um.. going down and when you are going up, does it decrease?
Mark: It depends with the map, but yes generally. In this case, yes, it increases as you go down and decreases as you go up.
Learner: It also increases as you go to the right.

In this excerpt, the learner's question suggests his noticing of a phenomenon on the map that surprises him, that 'minutes increase' when movement is downward and decrease when movement is upward. A hypothesis, in the form of a rule, is offered for the teacher to consider and explain. Mark's response in this instance is somewhat ambiguous: 'It depends with the map, but yes generally. In this case, yes, it increases as you go down and decreases as you go up'. There is a reference here to the kind of situational contingency that Hoyles et al. (2004) have noted as important, in that Mark's rule is true for the Southern hemisphere map section that is being used in the lesson. The ways in which magnitudes work in the context of the specific map in question appear to be understood, but there is no communication of what Goldenberg and Mason (2008) refer to as 'range of permissible change', that is, how/when this rule would change if the map section had been drawn from the Northern hemisphere. Instead, the student states a mathematical convention that would be universally true on an *x*–*y* Cartesian plane—that numbers increase 'as you go to the right'—while another convention that contradicts mathematical rules on the Cartesian plane: 'yes, it increases as you go down and decreases as you go up' is also stated.

The student's question points to difficulties with shifts in convention between mathematical rules that Mathematical Literacy learners are likely to have been introduced to earlier in their schooling. Mark's teaching suggests contextually sensitive working in a 'local' sense in that the rules he provides are true in the terrain of the specific map artefact being used, but his response indicates problems in relation to communicating the extent of generalizability of these rules. There is also no follow-up on the extent to which the general statement offered at the end by a student is applicable. In this case too, the conventions associated with the labelling of axes vary depending on whether the artefact is viewed from the perspective of contextual activity around the map as a boundary object, or mathematical activity around the Cartesian grid as a boundary object. In the latter activity, the mathematical convention (incorporating the use of negative numbers) would ensure that numbers increase upwards and to the right. In contrast, in the contextual 'boundary object as map' activity, the hypotheses offered have a more constrained applicability and differing conventions have to be clarified: here, numbers increase both northwards and southwards, and eastwards and westwards, with compass directions replacing negative number values.

8 Concluding comments

Mark made fewer intra-mathematical errors and showed more willingness to work with contextual perspectives in comparison to the other pre-service teachers in Winter's (2014) study, bringing the possibility of working with artefacts from mathematical and contextual perspectives more sharply into view in his teaching than in other cases. The analysis presented above points, in his case, to the need to support him to flexibly move between mathematical and contextual perspectives in his work with the map as a boundary object. While some sub-features of the map as boundary object were traversed with smooth boundary crossing, this was not always the case. Even though Mark was able to both acknowledge his errors and communicate rules that were locally correct in the context of the Johannesburg map as a boundary object, our analysis suggests the need for greater attention within teacher education to the ways in which rules and conventions associated with particular artefacts may overlap and differ depending on whether the artefact is viewed from mathematical or contextual perspectives.

Some more general questions also arise from this analysis. Difficulties for Mark in communicating differences in the rules associated with grid line readings confirm that boundaries can be a 'source of tension and a source of learning' (Lofthouse and Wright 2012, p. 90). Hoyles et al.'s (2010) work has dealt more extensively with the need for communicational clarity across communities/activities, involving personnel located in one or other community. A key difference in the context of numeracy teaching and teacher education—which our data illuminates—is that the numeracy teacher needs familiarity with artefacts at the boundary from the perspectives of both mathematical and contextual activities. Thus, rather than being a member of one or other activity, the numeracy teaching role is centrally configured at the boundary of both activities with the need for extensive comfort with boundary crossing around boundary artefacts.

This, in turn, raises possibilities for extension to Goos' (2007) model when the focus is on numeracy teaching rather than numeracy per se. Our findings point to the importance of 'explication of vantage point' within numeracy teaching—mathematical or contextual—and awareness of the ways in which situational artefacts at the boundary of these perspectives can be viewed differently depending on the vantage point, with the nature of mathematical deployments varying on this basis too. Whether located as a separate subject in the post-compulsory phase as Mathematical Literacy is in South Africa, or integrated within mathematics teaching in the middle years, as is the case in Australia and other countries, students bring awareness of traditional mathematical goals and conventions alongside numeracy goals and conventions. Our analysis indicates that successful numeracy teaching requires the ability and willingness to negotiate these boundary-related ambiguities. The teaching excerpts presented here, in the context of few breakdown incidents in his broader teaching, suggest that Mark was already close to achieving this kind of boundary competence.

Appendix

Map artefact and task drawn from Clarke et al. (2006).

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