SECONDARY USER PRICING STRATEGIES IN A COGNITIVE RADIO ENVIRONMENT

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Declaration

I declare that this research report is my own, unaided work, except where otherwise acknowledged. It is submitted for the degree of Master of Science in Engineering at the University of the Witwatersrand, Johannesburg, South Africa. It has not previously been submitted for any degree or examination at any other university.

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Abstract

There has been a growing demand for spectrum availability due to inefficient management of the radio frequency spectrum and underutilization of all spectrum bands. Spectrum has been managed with the same approach for over the last decade and only recently due to the phenomenal growth in mobile and broadband communications has attention been given to it. Intelligent communication systems such as cognitive radio have been identified in assisting the need for the limited resource, wireless spectrum. If spectrum trading becomes commercially successful, it can provide great economic and social benefits for the service provider, primary and secondary users. In order to maintain viability of spectrum trading, a pricing strategy is necessary for secondary users, it is also imperative to find a game theory model that minimally impacts the primary users in terms of their service, however it should aid in decreasing the cost to the primary users. Game theory along with economic theory is used to analyse the relationships/cooperation between the users and service provider. This work contributes to the field of dynamic spectrum access and aims to compare pricing strategies of secondary users in terms of the revenue earned by the primary service providers as well as investigate the impact of regulations on said pricing strategies.

The pricing strategies modelled and simulated in MATLAB include the market-equilibrium pricing strategy and the competitive pricing strategy. These two strategies are chosen as they are the most relevant in South Africa. The two pricing strategies are compared in terms of advantages and disadvantages as well the revenue earned by each of the primary services. The framework for testing is provided along with the test cases. The influence of telecommunication regulations and policy on the frameworks and results are discussed in detail as well as the impact of the telecommunication regulation and policy in South Africa.

Acknowledgement

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List of Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{N}$	Set of natural numbers
$\mathcal{G}$	Game
$\mathcal{U}$	Utility function
$\forall$	For all
$\mathcal{S}$	Supply function
$\mathcal{D}$	Demand function
$\mathcal{D}^{-1}$	Inverse demand function
p_i	price offered by primary service provider i
C	Production cost per unit of commodity
C_f	Fixed cost
$\mathcal{P}$	Profit function
$\mathcal{B}$	Best response function
k	Spectral efficiency
γ	Signal-to-noise ratio
BER^{tar}	Target bit-error-rate
$\bar{b}$	Vector of shared spectrum sizes
N	Total number of primary service providers
v	Spectrum substitutability
$\mathcal{R}$	Revenue gained by primary service provider
c	Constant weight
M_i	Number of on-going primary users serviced by primary service i
W	Total spectrum size
B_i^{req}	Bandwidth requirement per user i
$\mathcal{C}_n$	Cost discount given to primary users
$\mathcal{F}_i$	Spectrum owned by primary service provider i
$\lambda_{j,}\mu_k$ and σ_l	Lagrange multipliers

List of Abbreviations

AIP	Administrative Incentive Pricing
BER	Bit Error Rate
COGEU	Cognitive radio systems for efficient sharing of TVWS in EUropean context
CSIR	Council for Scientific and Industrial Research
DoC	Department of Communications
DSA	Dynamic Spectrum Access
DTT	Digital Terrestrial Television
ECS	Electronic Communications Service
ECNS	Electronic Communications Network Service
FCC	Federal Communications Commission
GHz	Gigahertz
IBA	Independent Broadcasting Authority
ICASA	Independent Communications Authority of South Africa
IMT	International Mobile Telecommunications
ITU	International Telecommunication Union
kHz	Kilohertz
KKT	Karush-Kuhn-Tucker
MATLAB	Matrix Laboratory
MHz	Megahertz
NE	Nash Equilibrium
PDA	Personal Digital Assistant
QoS	Quality of Service
SABC	South African Broadcasting Corporation
SABRE	South African Band Re-Planning Exercise
SATFA	South African Table of Frequency Allocation
SATRA	South African Telecommunications Regulatory Authority
SDR	Software Defined Radio
SIR	Signal to Interference Ratio
SNR	Signal to Noise Ratio
TDMA	Time division multiple access
TENET	Tertiary Education and Research Network of South Africa
TVWS	Television White Spaces

UHF	Ultra High Frequency
VHF	Very High Frequency
vNM	Von Neumann–Morgenstern
WAPA	Wireless Access Providers' Association of South Africa
WRC	World Radiocommunication Conference

Chapter 1

1 Background to Spectrum Trading and Pricing in South Africa

1.1 Introduction

The radio frequency spectrum is natural resource critical for delivering electronic communication services and for building a knowledge-driven economy and society. With the growing demands for spectrum in the country and the poor assignment of spectrum bands, it is regarded as a scarce resource. "*In the last decade, while bandwidth efficiency for high quality multimedia transmission over wireless networks has been dramatically improved in terms of speed, reliability and power level, the demand for wireless spectrum use has been growing as rapidly with the consequence that the wireless spectrum is becoming a scarce resource*."[1] In order to improve the utilization of the radio frequency spectrum, intelligent wireless communication systems such as cognitive radio technologies can be utilized.

Studies have shown that while some frequency bands are heavily used there are many bands that are unoccupied most of the time, dependent on geographical areas and population. These potential unoccupied bands can be optimised by use by unlicensed users. Cognitive radios are able to improve the capability of a wireless receiver by allowing it to operate in multiple frequency bands using multiple transmission bands.

In the past a number of research projects focused on the use of Cognitive Radio, as it was an emerging technology, currently there has been a drive to focus on the various pricing strategies for spectrum sharing between primary and secondary users. The sharing can be achieved by deployment of two specific themes, namely "*Sensing and detection of Primary Radio signals in a Cognitive Radio Environment using modulation identification technology*" and "*Distributed Transmit Power control strategies for Cognitive Radio Networks: Challenges, Requirements and Options*". Prof Nel and Dr Zhu [1] have performed extensive work on pricing strategies pertaining to opportunistic spectrum access.

To design efficient and effective dynamic spectrum access techniques for a cognitive radio network, technical aspects (such as power control and channel allocation) as well as economic aspects (such as pricing) need to be considered. Through spectrum trading, the licensed users (or primary service provider) are able to sell a portion of the unused spectrum to the unlicensed users (or secondary service provider) for a price. Pricing for both the licensed users selling spectrum and the unlicensed users buying the spectrum is important. Therefore, an optimal and stable solution for spectrum trading in terms of price and allocated spectrum is required to maximise the revenue of the seller and utility of the buyer while still satisfying both the seller and buyer and their solutions.

Game theory and Economic theory are used to analyse the relationships and interactions between the players (primary and secondary users). Game theory is "*an important tool in studying, modelling and analysing the cognitive interaction process*" [2] and provides a stepping-stone in analysing various pricing strategies for spectrum sharing. Concepts such as utility theory, market-equilibrium, oligopoly market and auction theory define the incentive for licensed users to yield the right of spectrum access to the unlicensed users.

As radio spectrum is a major component of the infrastructure that enables the information society, an important issue in spectrum trading is policy, regulation and spectrum management. Spectrum regulations and policies define rules of cooperation between primary and secondary users and spectrum management has been practised around the world since the 1920's. The use of spectrum is regulated by the Independent Communications Authority of South Africa (ICASA) and the management and development of the spectrum plays an important role in developing a knowledge-driven economy and society. The regulator directly impacts the pricing model used in the country by the regulations and policy it enforces.

In South Africa, the spectrum management arrangements are a shared responsibility between the policy maker and the regulatory authority, i.e. the Department of Communication (DoC) coordinates spectrum for government services, while ICASA regulates all other spectrum requirements. Currently spectrum trading is seen as illegal and is being investigated by ICASA and the DoC.

There is increasing interest in this technology from researchers in both academia and industry, and engineers in the wireless industry, as well as from spectrum policy makers.

1.2 Problem Definition

Through spectrum trading, the licensed users (or primary service provider) are able to sell a portion of the unused spectrum to the unlicensed users (or secondary service provider). In spectrum trading, pricing for both the licensed users selling spectrum and the unlicensed users buying the spectrum is imperative, hence an optimal and stable solution for spectrum trading in terms of price and allocated spectrum is required to maximise the revenue of the seller and utility of the buyer while still satisfying both the seller and buyer and their solutions. This optimal and stable solution is dependent on the regulations and policies enforced in the country.

A challenge for spectrum trading today would be for regulators and researchers alike in identifying an appropriate band to promote spectrum trading or to facilitate the entry of new market participants.

1.3 Motivation

With the exponential growth in wireless services and technologies, the demand for radio spectrum is steadily increasing. With the current spectrum management policy in which spectrum bands are assigned statically, there is the issue of crowded spectrum as well as underutilization of spectrum at various times and bands. This raises concerns of regulatory authorities about the optimal pricing mechanisms for spectrum usage.

This work is motivated by consideration of dynamic spectrum access from an economic perspective and can benefit the regulator in providing them with a perspective that considers both the regulatory issues as well as engineering concerns. The development of the pricing strategy is closely related with resource allocation.

1.4 Scope and Objectives

The key objective of this research is to use scientific tools to evaluate and compare the two pricing strategies. In achieving this, the scope and objectives covers:

a. Carry out a literature survey on the application of game theory, economic theory and pricing strategies in cognitive radio networks as well as the telecommunication regulations and policy in South Africa pertaining to spectrum trading;
b. Make theoretical enquiry into the two strategies under review and establish deductions based on computer simulations; and

c. Provide an analysis of the spectrum trading regulatory approaches together with deductions based on the computer simulations; and
d. Lay a theoretical foundation for further research.

1.5 Research Question

It has been proved that spectrum can be shared amongst users successfully. Is spectrum trading viable in South Africa? If so, which spectrum band is it most suited for? How does the regulations and policy impact spectrum trading and the selected band?

Would the payoffs received from secondary user spectrum trading be worth charging for or is it acceptable for it to be offered for free?

1.6 Organisation of this Research Report

Chapter 1 is a brief introduction into the subject of spectrum sharing for cognitive radio networks and discusses the motivation and problem definition behind the research as well as the objectives of this research.

Chapter 2 introduces the cognitive radio environment, its components, cognitive radio applications and dynamic spectrum access. The cognition cycle is briefly explained along with its aspects, namely spectrum sensing, spectrum analysis and spectrum decision. An introduction is given to game theory and economic theories in dynamic spectrum access, which presents a method to model the relationship and interactions among primary and secondary users competing for spectrum and pricing strategies to assign the price.

Chapter 3 extends the literature survey in Chapter Two to a review of specific application scenarios considered in some research papers. The literature review is from the perspective of contextual assessment of spectrum sharing, game theory and economic theory models applied in cognitive networks. The concluding section discusses telecommunication regulations with respect to spectrum trading and spectrum licensing, its usage and the importance of spectrum sharing in a South African context.

Chapter 4 presents a framework for the two options of pricing strategies in cognitive radio networks which are under consideration, in sufficient theoretical detail. The approach is to define a simple scenario in which to apply the two strategies separately, given the same conditions and assumptions. This chapter also discusses the algorithms of the pricing strategies in a logical manner.

Chapter 5 presents the research methodology for both the framework depicted in Chapter 4 and the analysis of spectrum trading regulations and policy followed by the simulation results and approaches to spectrum trading regulations.

Chapter 6 presents the research findings, recommendations, future work and conclusion.

1.7 Summary

This chapter is an introduction to the subject of spectrum sharing for cognitive radio networks and discusses the motivation and problem definition behind the research. A summary on the organisation of the research report is presented. The following chapter provides background theory on spectrum trading concepts such as cognitive radio, TV white spaces, game theory, economic theory and spectrum trading and pricing.

Chapter 2

2 Background Theory to Spectrum Trading Concepts

2.1 Introduction

This chapter presents the concept the three fundamental operational processes of the cognition cycle and dynamic spectrum sharing, namely spectrum sensing, spectrum analysis and spectrum decision and provides definitions on a Software-Defined radio and the cognitive cycle. Applications of cognitive radio are discussed with specific reference to spectrum sharing.

The following sections deal with the introduction to game theory and economic theory, and the three pricing strategies, namely market-equilibrium pricing, competitive pricing and cooperative pricing, as solutions to the challenges of pricing strategies in cognitive networks. Spectrum pricing and spectrum trading is discussed in detail.

2.2 Cognitive Radio Environment

"*Cognitive radio is an emerging technique to improve the utilization of radio frequency spectrum in wireless networks"* and is able to advance the adaptability and flexibility of wireless communication systems as mentioned in [3]

The concept of "*cognitive radio*" was first proposed by Mitola in 1999 which he defined as: "*the point in which wireless personal digital assistants (PDAs) and the related networks are sufficiently computationally intelligent about radio resources and related computer-to-computer communications to: (a) detect user communications needs as a function of use context, and (b) to provide radio resources and wireless services most appropriate to those needs*". Recently the Federal Communications Commission (FCC) suggested that any radio with adaptive spectrum awareness is to be referred to as a cognitive radio. [21], [22]

The motivation behind cognitive radios is the scarcity of the frequency spectrum which by exploiting the existence of spectrum holes, as they are aware of its surrounding environment, the efficiency of the frequency spectrum can be improved as discussed in [4] A spectrum hole is defined as the frequency band which is allocated to licenced users and in some locations and in some times not utilised by the licensed users and could therefore be accessed by unlicensed users.

Cognitive radios are able to perform dynamic spectrum management (also known as opportunistic spectrum access), as they are able to detect available channels at a specific time or location in the wireless spectrum and accordingly change its transmission or reception parameters to accommodate more wireless communications in a spectrum band. Dynamic spectrum sharing allows for efficient and fair spectrum allocation among primary users and secondary users.

Secondary users are unlicensed wireless users that are equipped with cognitive radio devices that sense the spectrum they want to use and detect the presence of primary users, also known as legacy spectrum holders. Based on the information detected by the secondary users, they can now dynamically access the licensed bands based on negotiated or opportunistic basis with minimal interference on the primary users. Secondary users make use of a concept known as dynamic spectrum access or opportunistic spectrum access. Dynamic spectrum access exploits the fact that large portions of the spectrum are underutilized as bandwidth demands vary along space and time dimensions, also known as "*white spectrum*".[1] The process of selling spectrum underutilised by primary users to secondary users is known as spectrum trading.

The major factor that leads to underutilised spectrum is the inefficient use of the radio spectrum by the current spectrum licensing scheme. In the current spectrum licensing scheme, the radio spectrum allocated to licensed users cannot be used by unlicensed users while the spectrum allocated is not in use. This static and inflexible allocation of spectrum forces legacy wireless systems to be able to operate only on a dedicated spectrum band, unable to adapt the transmission band according to the changing environment.[27]

An important characteristic of cognitive radios is its cognitive intelligence that allows the secondary user to perform intelligent decisions on spectrum usage and communication parameters based on the sensed spectrum dynamics as well as the primary users decisions. The major functionalities of a cognitive radio include spectrum sensing, spectrum management/spectrum sharing and spectrum mobility and this is assisted by a Software Defined Radio (SDR), the key component to implementing cognitive radios.

Users access a wireless system through the air interface that is a common resource and they transmit information using battery energy. Since the air interface is a shared medium, each user's transmission is a source of interference for others. The signal-to interference ratio (SIR) is a measure of the quality of signal reception for the wireless user. Typically, a user intends to achieve a high quality of reception (high SIR) while at the same time expending a small amount of energy.

2.2.1 Software-Defined Radio (SDR)

The wireless innovation forum defines a software defined radio as "*a radio in which some or all of the physical layer functions are software defined*" [28]. This means that channel modulation and waveforms are defined in software, allowing the SDR hardware to support any waveform at any carrier frequency and any bandwidth. Waveforms are generated as sampled digital signals, and converted from digital to analogue, similarly the receiver utilises a wideband analogue to digital converter to capture all the channels of the software node.

A SDR allows for a reconfigurable wireless communication system in which the transmission parameters (operating frequency band, modulation mode, and protocol) can be controlled dynamically. The main functions of SDR include multiband operation (support of wireless data transmissions over different frequency spectrum used by different wireless access systems), multistandard support, multiservice support and multichannel support (operate (transmit and receive) over multiple frequency bands simultaneously). [27]

2.2.2 Cognitive Cycle

The cognition cycle in cognitive radio architecture includes a temporal organization and flow of inferences and control states, Mitola described the states in this cycle as Observe-Orient-Plan-Decide-Act-Learn as shown in Figure 2-1 courtesy of Hossain et al.[21]. This cycle describes the major functions of cognitive radio which are required to adapt the transmission parameters according to the changing environment.

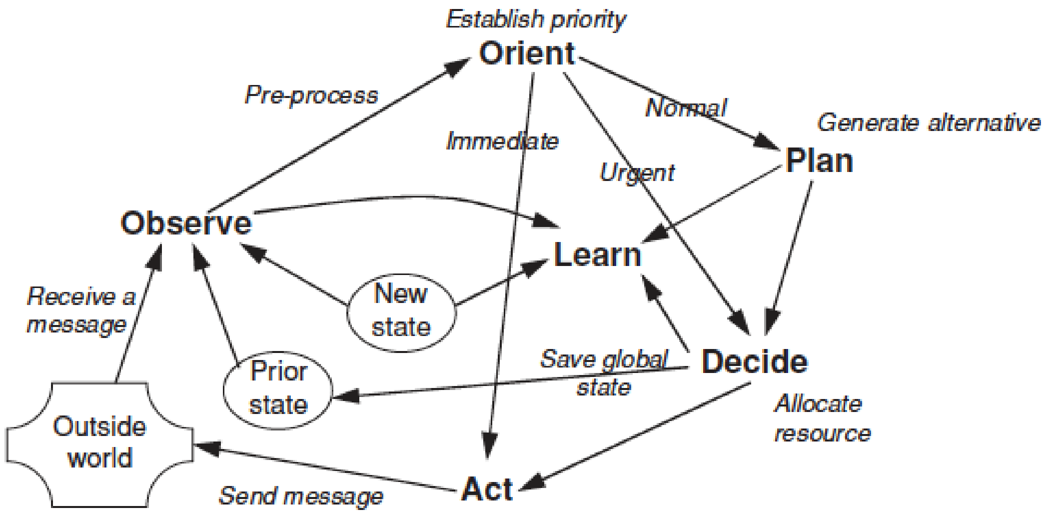


Figure 2-1: Cognitive cycle [27]

The cognitive cycle can be broadly categorised into three dynamic sharing processes, spectrum sensing, spectrum analysis and spectrum decision as shown in Figure 2-2 as adapted from [29].

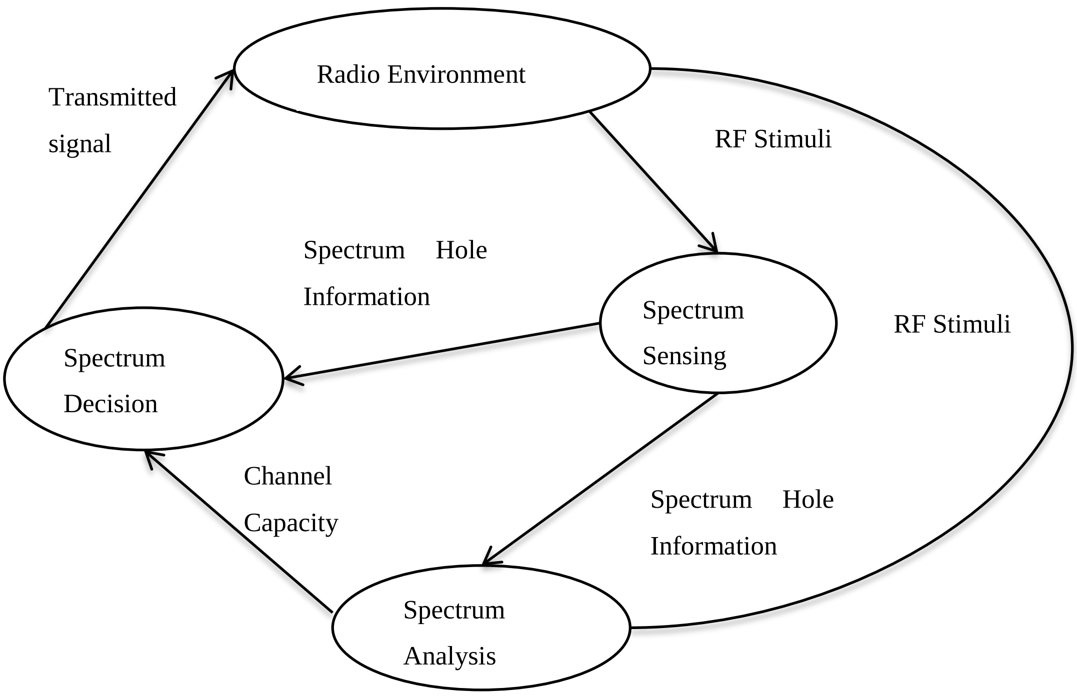


Figure 2-2: Basic cognitive cycle for dynamic sharing [29]

2.2.2.1 Spectrum Sensing

The goal of spectrum sensing is to determine the status of the spectrum by monitoring the target spectrum band to detect spectrum holes (band, location and time) as well as the activity of the licensed users. A cognitive radio transceiver is also able to determine the method of accessing the spectrum hole without interfering with the transmission of a licensed user.

Spectrum sensing can either be centralised or distributed. Centralised spectrum sensing reduces the complexity of the user terminals as a sensing controller senses the target frequency and relays it to the other nodes in the system. In the distributed spectrum sensing, the unlicensed users perform the spectrum sensing independently, if the results are used by individual cognitive radios it is known as non-cooperative sensing, whereas in cooperative sensing, the results are shared with other users. There are three major types of spectrum sensing, non-cooperative sensing, cooperative sensing and interference based sensing as discussed in [27]

Sensed spectra can be classified into the following as highlighted in [30]:

- Black spectrum holes: spectra is fully occupied and should be avoided when their emitters are on;
- Gray spectrum holes: spectra is partially occupied and can act as candidates for prospective service operators;
- White spectrum holes: spectra are free and are candidates for prospective service operators.

2.2.2.2 Spectrum Analysis

The goal of spectrum analysis is to schedule and plan spectrum access by the unlicensed users from the information obtained from spectrum sensing and performs channel state estimation to determine estimation of the spectrum hole to derive a model for increased efficiency. Spectrum analysis characterises the different spectrum bands in terms of operating frequency, bandwidth, interference, primary user activity and channel capacity. [27]

2.2.2.3 Spectrum Decision

Once the information from spectrum sensing is analysed to gain knowledge of the spectrum holes, a decision to access the spectrum is made by optimising the system performance given the desired objectives and constraints as per [27]. The cognitive radio determines the channel capacity, spectrum hole information, data rate and bandwidth of the transmission to determine the appropriate spectrum band for transmission of the signal.

2.2.3 Cognitive Radio Applications

As cognitive radio plays a significant role in making the best use of scarce spectrum, [27] lists where cognitive radio can be applied to a variety of wireless communications scenarios:

- Next generation wireless networks
- Coexistence of different wireless technologies
- eHealth services
- Intelligent transportation system
- Emergency networks
- Military networks
- TV bands for smart grid
- Public safety
- Broadband cellular

2.2.4 Dynamic Spectrum Access (DSA)

The idea behind DSA is inspired by spectrum occupancy measurements that indicated a number of instances where assigned spectrum is only used for brief periods of time [1]. DSA was introduced to maximise flexibility of spectrum use, account all dimensions of spectrum use and promote efficient use of the spectrum, as there was a need for spectrum reform. Dynamic spectrum access is defined by [27] as "*a mechanism to adjust the spectrum resource usage in a near-real-time manner in response to the changing environment and objective (e.g. available channel and type of applications), changes of radio state (e.g. transmission mode, battery status, and location), and changes in environment and external constraints (e.g. radio propagation, operational policy)*".

Dynamic access strategies can be broadly categorized into three models, Dynamic Exclusive Use Model, Open Sharing Model and Hierarchical Access Model as in [23]. Taxonomy of the models is shown below in Figure 2-3 as adapted from [23]. The differences in these models are:

- Dynamic Exclusive Use Model: the spectrum bands are licenced to services for exclusive use under a certain rule. In this model the licenser (government) allocates the spectrum to the licensee. Two approaches have been proposed under this model, viz. spectrum property rights and dynamic spectrum allocation. Spectrum property rights allow licensees to freely sell and trade spectrum and choose their technology which can be seen as long term exclusive use, whereas in the dynamic spectrum allocation spectrum allocation is performed at finer scale and the allocations varies at a much faster scale than the current policy and the spectrum is allocated to secondary users for a relatively short period. In the dynamic exclusive use model, there are three sub models for the secondary market, non-real time secondary market, real time secondary market for homogenous multi-operator sharing and heterogeneous multi-operator sharing. [23][27]

- Open Sharing Model (also referred to as spectrum commons): all cognitive radio users have the same right to access the radio spectrum as a basis for managing a spectral region. The three variants of this model are, uncontrolled, managed and private-commons sub models. The simplest form is the uncontrolled sub model and this could result in tragedy of the commons, where a cognitive radio user suffers from interference.
- Hierarchical Access Model (also referred to as shared use): the radio spectrum can simultaneously be shared between primary users and secondary users. Secondary users can opportunistically access the spectrum if it is not occupied or fully utilised by primary users with limited interference on the active primary users. There are two approaches that allow secondary users to exploit the unused spectrum band without causing interference to active primary users, namely spectrum underlay and spectrum overlay. In the spectrum underlay the transmission power of the secondary user is constrained to operate below the inference temperature limit of primary users, whereas in spectrum overlay no constraints are imposed and secondary users can identify and exploit spectrum holes defined in frequency, time and space. [27]

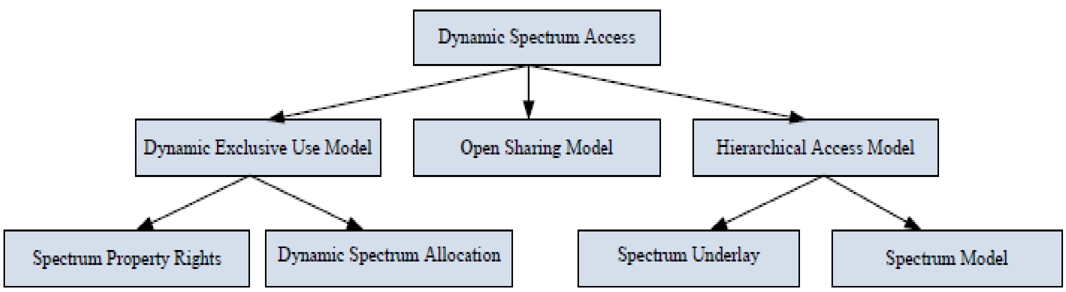


Figure 2-3: Taxonomy of dynamic spectrum access [23]

In a dynamic spectrum model, efficient usage of the spectrum can be achieved by allowing secondary users to utilise the licenced band while the band is not being utilised by the primary users provided that interference to the primary user is kept insignificant.

When the licenced band is not utilised, it is known as spectrum hole, i.e. a band free for utilization. The spectrum usage can thus be modelled as an ON-OFF process, where an OFF indication indicates the spectrum is unoccupied (spectrum hole). Due to each state of the system being independent of the past and future states, [21] suggests that the spectrum dynamics can be modelled as a Semi-Markov process.

There are two methods that spectrum can be shared, horizontally and vertically. Horizontal sharing refers to a sharing scheme between radio systems with equal regulatory priority, without causing interference and vertical sharing refers to a sharing scheme between licensed primary users and unlicensed secondary users. The operations of cognitive radio fall within vertical spectrum sharing.

The two major phases in dynamic spectrum are spectrum exploration (sensing and analysis) and spectrum exploitation (decide and handoff).

From an economic viewpoint, spectrum trading can generate more revenue for the spectrum owner and also enhance the satisfaction of cognitive radio users.

2.3 Television Whitespaces Band

As discussed in [32], *"According to the ITU report "Digital Dividend: Insights for spectrum decisions", TV white spaces (TVWS) are "portions of spectrum left unused by broadcasting, also referred to as interleaved spectrum"".* The spectrum lying in the 470 to 862 MHz, more commonly known as the TV spectrum, has been known to have desirable properties in the market, due to its nature to travel further and penetrate buildings more easily than higher frequencies. TVWS are referred to as the currently unoccupied portions of spectrum in the terrestrial television frequency bands in the VHF and UHF TV spectrum (analogue or digital). "*These TV spectrum gaps, with advantageous propagation properties inherent to UHF spectrum (excellent outdoor and indoor coverage and non-line-of-sight propagation properties) have been identified in some administrations as an alternative for providing commercial wireless services other* *than broadcasting*" is highlighted in [32],. Both properties well exploited by TV broadcasters may now very well be exploited by other operators, with the digital dividend clearing up TV usage from the spectrum band.

The digital dividend refers to the spectrum that will be released in the process of digital TV migration. Currently the broadcasting systems transmit in analogue which inefficiently uses the spectrum, when the broadcasters switch from analogue platforms to digital only platforms; part of the spectrum band that has been used for broadcasting will be freed up as digital broadcasting requires a smaller spectrum band. The bands freed up are high demand spectrum resources due to its desirable properties. Digital technologies are more robust and can accommodate more services on the same spectrum channel hence utilising spectrum more efficiently. Proposed utilization of the released spectrum, [33], includes use for broadband wireless access services such as mobile services as it is cheaper than fixed broadband to provide last mile connectivity.

The available TVWS spectrum available can vary according to geographical features, level of interference potential to the TV broadcasting service, TV coverage objectives and television channels utilization. The availability of the TVWS spectrum can be classified according to, [32]

- Frequency: idle channels of a TV band plan due to frequency separation
- Height: the availability of TVWS in a given area in terms of the height of the TVWS transmission site and its antennae height in relation to the surrounding TV broadcasting coverage reception.
- Space: geographical areas outside the current TV coverage and therefore no broadcasting signal are present.

The classifications are not limited to frequency, space and height and can also be classified by the time domain for example.

Some of the wireless technologies being explored in TVWS, [33], are low-power, machine-to-machine (M2M) communication devices, low-power wireless broadband applications, military applications, private and public mobile electronic communications and broadcasting, capitalizing on the longer coverage (propagation and capacity bandwidth) ranges achievable with VHF/UHF band.

2.4 Introduction to Game theory

Game theory is a mathematical framework of models that formally study conflict and cooperation and allow for analysis and understanding of strategic scenarios. According to [5] it is *"the study of the ways in which strategic interactions among economic agents produce outcomes with respect to the preferences (or utilities) of those agents, where the outcomes in question might have been intended by none of the agents*". Game theoretic concepts apply to several decision makers (known as agents) who are all concerned with their own benefit (known as payoff).

Game theory has been applied to many facets of various study fields, one of which is cognitive radio and the pricing thereof of spectrum. Below a brief history of game theory is given along with concepts general to game theory.

2.4.1 History of Game Theory

In 1838, Antoine Cournot illustrated one of the earliest examples of a game-theoretic analysis of a duopoly. The foundation of the field was later laid in 1944 in the publication of *Theory of Games and Economic Behaviour* by J Von Neumann and O Morgensten [25]. This book provided much of the basic terminology and problem setup such as the two-person zero-sum game.

The concept of equilibrium was defined in 1950 by John Nash as well as named after him as the Nash equilibrium. Nash demonstrate that finite games always have an equilibrium point at which all players choose actions which are best for them given their opponents' choices of action or strategies. Nash equilibrium provides a general solution to mutually consistent strategies of players. The concept of non-cooperative game theory has been a focal point of analysis since then.

During the 1950's and 1960's, game theory broadened theoretically and was applied to problems of war, politics, psychology and sociology. Since the 1970's it has been said to have driven a revolution in economic theory. In 1974, the concept of correlated equilibrium was introduced by R Aumann. In 1994, special attention was awarded to game theory with the receiving of Nobel prizes in economics to J Nash, J Harsanyi and R Selten.

In the late 1990's, the concept of the auction was developed as a high-profile application of game theory. Many auctions were designed with the goal of allocating electromagnetic spectrum to the mobile telecommunications industry more efficiently.

2.4.2 Definition of a Game

The game in game theory is a formal model of an interactive situation and usually involves several players, however one-player games do exist and they are termed a decision problem. A game will lay out the players, their preferences, their information, the strategic actions available to them and how these influence the outcome.

A game consists of three fundamental components: a set of ***players*** (participants in the game), a set of ***strategies*** (plans by each player that describe what action will be taken in any situation), and a set of ***payoffs*** for the given set of actions (rewards for each player for all combinations of strategies).

The payoff (also referred to as utility) represents the motivations of the players, [27], "*a utility function for a given player assigns a number for every possible outcome of the game with the property that a higher (or lower) number implies that the outcome is more preferred*"

A central assumption to many branches of game theory is that players are rational. A player is said to be rational, [26], if they always chose an action which gives the outcome they most prefer (maximising their utilities), given what they expect their opponents to do. Therefore the goal of game theory is to predict how the game will be played by rational players or to give advice on how best to play the game against opponents that are rational.

A famous and simple example of a game is "the prisoner's dilemma"; which is a game in strategic form between two players (also known as the criminals). The two players are arrested for the same crime; however there is not enough evidence to convict either player. Both players are interviewed separately with two available strategies, '*cooperate'* or '*defect',* if a player defects, they testify to convict the other player with the payoff of a reduced sentence or to go free and the convicted player is imprisoned. If both players cooperate, the payoff is high for both players (as neither can be convicted) and if both defect, they both get a reduced sentence (which can be described as a null result). If one player cooperates and one defects, one player will have a higher payoff than the other. Each player's preference is based on the jail time they individually serve. The dilemma each player faces is the choice between the two strategies, where good decisions cannot be made with without information. The player that goes first will always choose to defect due to the rationality of the player; however this leads to an inefficient outcome as the best payoff for each player is cooperation.

Generally game theory can be described formally as either cooperative or non-cooperative.

2.4.2.1 Cooperative Game

Cooperative games also known as coalitional games provide a high-level description specify only what payoffs each potential group can obtain by cooperation of its members. Cooperation of the members entails agreement of the members to the adopted strategy and payoff function that accrues to the group. Cooperative games do not detail the process by which the coalition forms. Coopertive game theory focuses solely on the outcome of the game and investigates games with respect to the relative amounts of power held by various players, or how a successful coalition should divide its proceeds. In a cooperative game, there is no competition between the players in the group and they act as a single entity to maximise the total group utility.

2.4.2.2 Non-Cooperative Game

Non-cooperative games on the other hand are concerned with the analysis of the strategic choices or of the strategic interactions between selfish players competing in a game for their own interest to maximise their own profit. In this branch of game theory the non-cooperation explicitly models the process of the players making choices out of their own interest, cooperation can arise in non-cooperative game models when players find it in their best interest. A non-cooperative game theoretical framework is often used to obtain an equilibrium solution that optimizes the payoffs of all players. A non-cooperative game may be the only choice for the individual to play if the information is strictly limited to local information, however this game may have a very low-efficiency outcome. One of the most widely used solutions for a non-cooperative game is the Nash equilibrium and will be discussed in detail later on.

*"A **non-cooperative game** is one in which players are unable to make enforceable contracts outside of those specifically modelled in the game. Hence, it is not defined as*

games in which players do not cooperate, but as games in which any cooperation must be self-enforcing" as defined in [27].

*"A **static game** is one in which all players make decisions (or select a strategy) simultaneously, without knowledge of the strategies that are being chosen by other players"*, [27]. The game is said to be simultaneous as the players have no information about the decisions of the other players.

A basic type of game studied in non-cooperative game theory is ***strategic-form*** also known as normal form. In the strategic form, a game lists each player's strategies, and the outcomes that result from each possible combination of choices. The outcomes that result are represented by a separate payoff for each player, which is a number (also known as utility) that measures how much the player likes the outcome. In the normal form, a simultaneous game is represented by a matrix; where for two players, one is represented by the rows and the other by the columns. Each row or column represents the payoff for each player for every combination of strategies.

A strategic-form game consists of three objects:

1. Players. A set of agents who play the game, $N = \{1, \ldots, n\}$, with typical element $i \in \mathbb{N}$
2. Strategies. For each $i \in \mathbb{N}$ there is nonempty set of strategies S_i with typical elements $s_i \in S_i$
3. Payoffs. A payoff function $u_i : S \mapsto \mathbb{R}$ assigned to each player i, where S = $\times_{i \in N} S_i$

s ∈ S = $\times_{i \in N} S_i$ is known as a strategy profile

A strategic-form game is represented by:

$$\boldsymbol{\mathcal{G}} = \langle \boldsymbol{N}, \{\boldsymbol{S_i}\}_{\boldsymbol{i} \in \boldsymbol{N}}, \{\boldsymbol{u_i}\}_{\boldsymbol{i} \in \boldsymbol{N}} \rangle \tag{1}$$

A more detailed game than the strategic form is an ***extensive form*** also known as a game tree. Extensive form describes completely how the game is played over time and includes the order in which players take actions, the information that players have at the time they must take those actions, the times at which any uncertainty in the situation is resolved and the rules of the game. In the extensive form, a sequential game is represented graphically and provides information about the players, payoffs, strategies and the order of moves. The graphical representation contains nodes (or vertices) to represent points at which players can take actions and edges to represent the actions that may be taken at that node. The nodes are connected by edges. An initial node will represent the first decision that can be made and terminal nodes represent an end of the game. The terminal node is therefore labelled with the payoff earned by each player.

An extensive-form game with perfect information consists of:

1. Players. A set of players with typical element $i \in \mathbb{N}$
2. Histories. A set of histories with typical member $h \in H. h$ is a sequence of actions by individual players. $\emptyset \in H$, but there is no $(h, a) \in H$ where a is an action for some player, than h is "terminal". Set of terminal histories is denoted as $Z \subset H$
3. Player function. A function $P: H \backslash Z \longmapsto \mathbb{N}$, assigning a player to each non-terminal history
4. Payoffs. vNM payoffs for each $i \in \mathbb{N}$ are defined over terminal histories $u_i : Z \mapsto \mathbb{R}$ (A vNM payoff is when there exists a real-valued function defined by possible outcomes such that every preference of the player is characterized by maximizing the expected value of the function)

An extensive-form game is represented by:

$$\mathcal{G} = \langle \boldsymbol{N}, \boldsymbol{H}, \boldsymbol{P}, \{\boldsymbol{u_i}\}_{\boldsymbol{i} \in \boldsymbol{N}} \rangle \tag{2}$$

When there are multiple strategies available to the players of the game, the player will act rationally and chose the outcome they prefer, [27]. For each user there is a strategy space where some of the strategies are superior to the other due to the player's interests. A strategy is known as ***dominant strategy*** if regardless of what the other players do, the strategy earns the player the higher payoff than any other, [27]. If one strategy is dominant, all other strategies are dominated. A strategy is said to be a ***dominated strategy*** if regardless of what the other players do, the strategy earns the player a lower payoff than another strategy. A rational player will never chose to play a dominated strategy and will always play the dominant strategy in equilibrium regardless of what the other players do, [27].

When all information of past strategies and corresponding payoffs are known to each player, it is called a game with perfect information; otherwise it is regarded as a game with imperfect information.

Nash Equilibrium (NE) also known as strategic equilibrium is defined as, [27], the solution at which any player in the game cannot achieve a better solution by deviating unilaterally; given the actions of the other players (i.e. every player will select a payoff maximising strategy given the strategies of every other player). Nash Equilibrium is used to recommend a strategy to the players when there is no dominated strategy, since the players are assumed to be rational, it is reasonable for each player to expect his opponents to follow the recommendation as well. Players are said to be in equilibrium if a change in strategies by any one of the players will lead that player to earn less than if they remained with their current strategy.

A Nash Equilibrium strategy profile $s^* \in S$ such that for each $i \in N$,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \qquad \forall s_i \in S_i \tag{3}$$

At s^*, no i regrets playing s_i^*; given all the other players' actions, i could not have done better.

In some games, it is possible to have more than one NE, if this is the case, a theory of strategic interaction should guide the players towards the most reasonable equilibrium upon which they should focus. Among all these equlibria, the optimal one needs to be selected, one of the criteria to determine this is by Pareto optimality. The ***Pareto Optimality,*** [27], is a measure of efficiency and the outcome of game is said to be Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player better off. Usually a Pareto optimal outcome cannot be improved upon without resulting in a lower payoff to at least one player. NE is not Pareto optimal; hence the player's payoffs can all be increased, [27].

A game in strategic form does not always a have a NE in which each player deterministically chooses their strategies, players may instead randomly choose from among pure strategies with certain probabilities. A pure strategy describes a strategy of a player concerned in the game that takes it from one state to the other and is a specific move or action that a player will follow in every possible attainable situation in a game. Randomizing ones choice to choose among pure strategies is known as a mixed strategy.

A mixed strategy is defined as an active randomization, with given probabilities, which determine the player's decision. The probability distribution is based on how frequently each move is about to play. A player will play a mixed strategy when they are indifferent to several pure strategies as well as if the opponent benefits from knowing the next move (mixed strategies will allow the opponent to keep guessing). In a special case, a mixed strategy can be the deterministic choice of one of the given pure strategies [26]. From these definitions it is given that every finite strategic game has a mixed strategy NE (as illustrated by Nash in 1951).

Nash equilibria of non-cooperative static games often have low efficiency and this can be overcome with use of pricing and referee-based approaches.

2.4.3 Game Theory and Cognitive Radio Networks

In the cognitive radio environment game theory provides a mathematical tool to model strategic interactions among players (secondary and primary users) using formalized incentive structures as well as deriving well-defined equilibriums criteria to study the optimality of game outcomes under various scenarios, [1]. It aims to model and analyse the cognitive interaction process, designing efficient, self-enforcing, distributed and scalable sharing schemes due to the fact that the surrounding radio environment is constantly experiencing changes. Unreliable wireless channels, traffic variations, and user mobility and dynamic topology bring about these changes in the environment.

In the cognitive radio environment, game theory aims to study the intelligent behaviours and interactions of selfish network users. Users are said to be selfish as they all compete for spectrum resources and may have no incentive to cooperate with other users, e.g. with primary users and secondary users competing for the spectrum resources, if primary users are selfish, the secondary users may never/rarely contract spectrum resources.

"*The importance of studying cognitive radio networks in a game theoretic framework is multifold.*" The network users' behaviors and actions can be analyzed by modeling dynamic spectrum sharing among the users as a formalized game structure, where the theoretical achievements in game theory can be fully utilized. Game theory also allows for various optimality criteria for the spectrum-sharing problem as it provides well-defined equilibrium criteria to measure game optimality under various game setting since it is difficult to analyze and solve multi-objective optimization problems (such as optimization of spectrum usage). An important branch of game theory is non-cooperative game theory, [2], as it enables derivation of efficient distributed approaches for dynamic spectrum sharing using only local information, these approaches are highly desirable when centralized control is not available or flexible self-organized approaches are necessary.

The spectrum sharing schemes can be classified into four categories, non-cooperative, cooperative, stochastic and economic and auction games, [2]. *"With economic and auction games the spectrum resources are traded like exchangeable goods in a spectrum market. Stochastic spectrum sharing games allow users to adapt their strategies accordingly with the changing environment."* [2]

For the cognitive cycle, a cognitive radio observes the environment and sets the intelligent move, which can be mapped into a game. The cognitive radio users are the players in the game, the payoff is the utility, the observations are the arguments of the utility function, and the outside world can be interpreted as the outcome space of the game.

2.5 Economic Theories in Dynamic Spectrum Access

2.5.1 Utility Theory

Utility is defined as the usefulness, the ability of something to satisfy needs or wants. In the cognitive radio environment, the concept of utility can be used to quantify the satisfaction of a cognitive radio entity, [27]. The fundamental concept of utility is preference, which is used to indicate the consumer's preference among the different options. Utilities types include ordinal utility to indicate preference, cardinal utility to indicate the percentage of preference and marginal utility to indicate rate of increase in utility. Marginal utility is useful for utility maximization.

A rational user will always try to maximise their utility by making the best decision, however this is also based on the prices of the commodities and the level of the consumer income. Consumer income will dictate the amount of money that the consumer can spend per unit of time.

Utility maximisation is complex due to the limited amount of information available on the utility of each commodity. To reach the highest utility due to consumption over time, the consumer can learn and adapt the decision over time, known as bounded rationality.

A utility function to quantify user satisfaction can be defined by,

$$\boldsymbol{\mathcal{U}} = \mathbf{f}(\mathbf{x}), \forall\, \mathbf{f}\colon \mathbb{R}_{+}^{\mathbf{N}} \rightarrow \mathbb{R} \tag{4}$$

where $x = [x_1, \dots, x_n, \dots, x_N]$ denotes a vector containing different quantities of each of the commodities, n. The vector,x, is known as the consumption plan of the consumer.

The marginal utility can be determined by the first derivative of the utility function.

The concept of utility in a cognitive radio environment can be used to provide a layer of abstraction for QoS or formulation of a radio resource model and represents the payoff of each entity in a game formulation. Together with game theory and classical optimization, utility functions can be used for radio resource allocation in wireless networks.

One of the major factors that affect customers satisfaction is price, the concept of net utility can be used to indicate the satisfaction on both the price and performance.

2.5.2 Market Equilibrium

Market equilibrium is a condition where a market price is established through competition such that the amount of goods or services sought by buyers (demand function) is equal to the amount of goods or services produced by sellers (supply function). The supply function is represented by $\mathcal{S}(p)$ and is a function of price, p. The demand function is represented by $\mathcal{D}(p)$ and is also a function of price, p. The amount of supply from the seller is an increasing function of price, whereas the demand for the goods or services is a decreasing function of price.

The concept of a market-equilibrium price can be used to satisfy both sellers and buyers and the market-equilibrium price p^* is given by,

$$\boldsymbol{\mathcal{S}}(\boldsymbol{p}^*) = \boldsymbol{\mathcal{D}}(\boldsymbol{p}^*) \tag{5}$$

The market-equilibrium price is the best possible strategy for both the sellers and buyers and it would be undesirable for either to deviate from this price without an incentive. The market-equilibrium price also ensures that there is no excess supply in the market and all spectrum demand is satisfied, [27].

2.5.3 Oligopoly Market

An oligopoly market is a market whereby a small number of firms (known as oligopolists) dominate a particular market, [27]. These firms compete with each other independently to sell a commodity and maximise their profit. The commodity supplied by the firms can either be homogenous or differentiated, where for a homogenous commodity, all firms produce the same commodity and for a differentiated commodity, each firm produces different commodity for which each of them is substitutable. As each commodity may not be perfectly substitutable (could be due to the difference in quality of propagation and/or interference conditions), the competition among the firms is affected by the level of substitutability. These firms also control the amount of supply as well the price of the goods/service and the amount of supply or the offered price not only affects its profit but also the profit of the other firms. [27]

With few sellers, each oligopolist is likely to be aware of the actions of the others. The decisions of one firm therefore influence and are influenced by the decisions of other firms. Strategic planning by oligopolists needs to take into account the likely responses of the other market participants.

"In a cognitive radio network, a firm could be analogous to a primary user or a primary service provider. The primary service providers compete with each other to sell the spectrum opportunities to the secondary users or secondary service providers. The objective of a primary service provider is to maximize the profit of selling the spectrum opportunities" (i.e. rational behaviour). [27]

To analyse and predict the behaviour of the firms in an oligopoly market, the theory of non-cooperative games can be utilised (Section 2.4.2.2). The players in this game are the firms, the strategy is the supplied quantity or the offered price and the payoff is given by the profit of the firm. The three classical game theoretic models used to model the oligopoly market are the Cournot, Bertrand and Stackelberg models. Each model has different market structures and the strategies used in the competition differ.

To illustrate the behaviour of these three models, a market with two firms (a duopoly) is considered for simplicity sake. In a duopoly the total number of players is two. A linear demand function is assumed,

$$\boldsymbol{\mathcal{D}(p) = A - Bp} \tag{6}$$

and the inverse demand function is expressed as,[27]

$$\boldsymbol{\mathcal{D}^{-1}(p) = p(d) = A' - B'd \quad , where\ A' = \frac{1}{A}\ and\ B' = \frac{1}{B}} \tag{7}$$

2.5.3.1 Cournot Model

In the Cournot model, the firms compete in terms of the amount of supply to the market and choose their quantities independently and simultaneously. In Cournot competition there is more than one firm and there is no product differentiation (homogenous product produced). The customers react to the price that they are willing to pay for the supplied quantity. The price is determined by the inverse demand function (7).

As the firms compete for the amount of supply to the market, the supplied quantity of one firm will affect the market price of the commodity, and therefore the profit of the other firms. The firms can be said to be economically rational and act strategically, seeking to maximize their profit given their competitors' decisions never colluding with one another.

Assume a production cost of C per unit of commodity and fixed cost of C_f then the profit can be expressed as, [27]

$$\boldsymbol{\mathcal{P}_i(s_i, s_j) = p(s_i + s_j)s_i - Cs_i - C_f} \tag{8}$$

$$\boldsymbol{= \left(A' - B'(s_i + s_j)\right)s_i - Cs_i - C_f}$$

where s_i and s_j are the supplied quantity for the duopoly market from firms i and j, respectively. As the objective of each firm is to maximise its profit, the best response of each firm can be obtained by finding the optimal amount of supplied quantity which maximises the profit given the amount of the supply from the other firms. The optimal amount supplied can be determined at the point where the derivative of the profit is equal to zero, [27]

$$\boldsymbol{\frac{d\mathcal{P}_i(s_i, s_j)}{ds_i} = 0 = A' - B'(2s_i + s_j) - C} \tag{9}$$

$$\therefore\ \boldsymbol{s_i^* = \mathcal{B}_i(s_j) = \frac{A' - B's_j - C}{2B'}} \tag{10}$$

Similarly the best response of firm *j* can be derived. It can be seen that as the supplied quantity for firm 1 increases, their profit increases to a point until there is an abundance of supply, at this point the profit starts decreasing. The point at which the profit starts decreasing is known as the best response for firm i as it gives the highest profit to firm i.

The solution to a Cournot game is Nash equilibrium, which gives the optimal amount of supply which maximises the profits of the firms. In the Cournot model, the Nash equilibrium can be expressed by, [27]

$$\boldsymbol{\mathcal{B}_i}(\boldsymbol{s_j^*}) = \boldsymbol{\mathcal{B}_j}(\boldsymbol{s_i^*}) \tag{11}$$

Graphically the Nash equilibrium is the point where the best responses of the two firms intersect each other and can be obtained mathematically from,

$$(\boldsymbol{s_i^*}, \boldsymbol{s_j^*}) = \left(\frac{\boldsymbol{A' - C}}{\boldsymbol{3B'}}, \frac{\boldsymbol{A' - C}}{\boldsymbol{3B'}}\right) \tag{12}$$

At Nash equilibrium, none of the firms can have a better profit without adjustment in the supplied quantity from the other firms.

2.5.3.2 Bertrand Model

In a Bertrand model, all firms make their decision simultaneously in terms of price and choose their prices independently; customers choose their quantities (demand) based on the price and the production capacity is unlimited, [27]. Both firms have the same constant unit cost of production, so that marginal and average costs are the same and equal to the competitive price. The firms competing choose their price so that their profit is maximised given the prices chosen by the other firms. The solution of a Bertrand game is dependent on the substitutability of the commodity, in the case of a homogenous commodity, the customer can choose to buy the commodity from any of the firms and will always choose the firm with a lower price, whereas for a differentiated commodity, the demand functions for the commodity from the different firms are different and dependant on the prices of all firms whose commodities are substitutable, [27]. In the case of a homogenous commodity, there is a unique Nash equilibrium where all the prices charged by the firms are identical and the price is identical to the production cost. This is due to the fact that the firm with the cheapest price will supply the entire market; if another firm decreases its price to lower than firm 1 then firm 2 will now supply the entire market and the other firms will have zero profit, therefore they too need to decrease their price. The equilibrium of a homogenous Bertrand game can be seen to be trivial, [27].

Therefore for a Bertrand game, a differentiated commodity can be described by the following demand functions: [27]

$$s_i(p_i, p_j) = A - Bp_i + Dp_j \tag{13}$$

$$s_j(p_i, p_j) = \tilde{A} - \tilde{B}p_i + \tilde{D}p_j \tag{14}$$

where A, B, D, $\tilde{A}$, $\tilde{B}$, $\tilde{D}$are constants of the demand functions (D and $\tilde{D}$ indicate the level of substitutability). The demand functions in (13) and (14) indicate if the price from firm i increases, the demand for the commodity from firm i will decrease and if the price from firm j decreases, the demand for the commodity from firm i also decreases. The profit of firm i is represented by, [27]

$$\mathcal{P}_i(p_i, p_j) = p_i s_i - Cs_i - C_f \tag{15}$$

$$= (A - Bp_i + Dp_j)(p_i - C) - C_f$$

The profit of firm j can be derived similarly to that of i. The best response of firm i can be derived by differentiating the profit with respect to the price of firm i, [27]

$$\frac{d\mathcal{P}_i(p_i, p_j)}{dp_i} = 0 = B(C - p_i) + A - Bp_i + Dp_j \tag{16}$$

$$\therefore\ p_i^* = \mathcal{B}_i(p_j) = \frac{BC + A + Dp_j}{2B} \tag{17}$$

Similarly, the best response of firm j can be derived. Graphically the Nash equilibrium is located at the points where the best responses of the firms intersect with each other, mathematically the Nash equilibrium can be found to be, [27]

$$(p_i^*, p_j^*) = \frac{2B\tilde{B}C + 2A\tilde{B} + \tilde{B}CD + \tilde{A}D}{4B\tilde{B} - D\tilde{D}}, \frac{2B\tilde{B}C + 2\tilde{A}B + BC\tilde{D} + A\tilde{D}}{4B\tilde{B} - D\tilde{D}} \tag{18}$$

The substitutability of the commodity affects the slope of the best response functions as well as the location of the Nash equilibrium.

2.5.3.3 Stackelberg Model

In a Stackelberg model, there is a firm (referred to as a leader) who is able to make decisions (commit to the chosen strategy) on the supplied quantity or price before other firms (i.e. followers). These followers then make their decisions by taking into account the decision of the leader (the decision taken by the follower is the optimal strategy based on the observed strategy chosen by the leader). These firms compete with each other in terms of supplied quantity or price and move sequentially.

The solution of a Stackelberg game can be seen to be the set of strategies where the profit of the leader is maximised and the followers choose their best response; this is known as the ***Stackelberg equilibrium***. The Stackelberg equilibrium can be obtained by backward induction, where the best response of the follower is obtained first and then given the followers best response, the leader optimises its strategy to achieve the highest profit.

The profit attained by follower j can be computed from, [27]

$$\boldsymbol{\mathcal{P}_j(s_i, s_j) = \left(A' - B'(s_i + s_j)\right) s_j - C s_j - C_f} \tag{19}$$

and the best response of this follower is given by, [27]

$$\boldsymbol{s_j^*(s_i) = \frac{A' - B' s_i - C}{2B'}} \tag{20}$$

As the objective of the leader is maximise its profit, the profit of leader i is given by, [27]

$$\boldsymbol{\mathcal{P}_i(s_i, s_j) = \left(A' - B'\left(s_i + s_j^*(s_i)\right)\right) s_i - C s_i - C_f} \tag{21}$$

$$\boldsymbol{= s_i \frac{A' - B' s_i - C}{2} - C_f} \tag{22}$$

By differentiating the profit function (22) with respect to the strategy of the leader (s_i), the subgame perfect Nash equilibrium can be determined, [27]

$$\boldsymbol{\frac{d\mathcal{P}_i(s_i, s_j)}{ds_i} = 0 = A' - \frac{B' s_i}{2} - B'\left(s_i + \frac{A' - B' s_i - C}{2B'}\right) - C} \tag{23}$$

$$\boldsymbol{\therefore\ s_i^* = \mathcal{B}_i(s_j) = \frac{A - C}{2B'}} \tag{24}$$

Based on the optimal strategy of the leader, the optimal strategy for the follower is,

$$\boldsymbol{s_j^* = \mathcal{B}_j(s_i) = \frac{A' - C}{4B'}} \tag{25}$$

The subgame perfect Nash equilibrium is the optimal strategy for the leader if the leader can make a decision before the follower. [27]

Graphically the Nash equilibrium can be represented by the point where the best responses of both firms meet and this is not the same point as the Stackelberg equilibrium. At the Stackelberg equilibrium, the leader offers a larger supplied quantity than that of the follower, consequently the profit of the leader is higher (also known as first-move advantage). Mathematically, the Stackelberg equilibrium is represented by,

$$(\boldsymbol{s_i^*}, \boldsymbol{s_j^*}) = \left(\frac{\boldsymbol{A' - C}}{\boldsymbol{2B'}}, \frac{\boldsymbol{A' - C}}{\boldsymbol{4B'}}\right) \tag{26}$$

2.5.4 Auction Theory

"*An auction is a process used to obtain the price of a commodity with an undetermined value"*, [27]. Typically there are three categories of an auction, supply auction, demand auction and a double auction. In the supply auction, multiple sellers offer their commodities to a buyer whereas in a demand auction multiple buyers bid for a commodity being sold by a seller, [27]. The double auction has multiple buyers bidding to buy commodities from multiple sellers.

The components of an auction market include:

- A seller: Market entity selling a commodity who offers the price and amount of the commodity to be traded by the auction
- The buyer: Market entity buying the commodity from the seller and submits a bid in terms of price and bidding quantity through the auction
- The trading/clearing price: The price of each commodity to be traded in the auction market

In a cognitive radio environment, auction models can be used to sell the allocated band to the highest bidder for a defined period. The different types of single-side auction models include the English auction, the Dutch auction, Sealed High-bid auction and the Vickrey auction. A single-side auction occurs when there is one auctioneer. Double auction based pricing may be used when there are multiple sellers (licensed users) and multiple buyers (unlicensed users).

In the English auction, buyers would bid the highest price they are willing to pay and the item would be sold to the highest bidder. The bid is said to be dynamic as the bidders are aware of the other bidder's bids and can change their bids accordingly. The bid would start at the seller's reserve price which is the lowest the item can be sold for.

In a Dutch auction, the bidder who bids the quickest wins the auction. The item is bid at the highest price and the price is decremented at certain intervals, once a bidder indicates the buying signal, the item is sold. The Dutch auction is also known as a one-shot auction. An extension of the Dutch auction is the Anglo-Dutch auction whereby an ascending auction is run instead and all but two bidders have dropped out. The final bids of the two bidders are presented in sealed offers and the higher bidder wins the auction (provided the bids are higher than the current asking bid).

The Sealed High-Bid auction is also known as the First-Price Sealed-Bid auction, and the auction is awarded to the highest bidder. The auction takes place in one round and all bids are submitted sealed.

Similar to the English auction is the Vickrey auction or the second price auction, however the winner of the auction with the highest bid pays the second highest bids price.

Shortfalls of auctions are winners-curse, where there is a tendency for the winning bid to exceed the value of the item purchased). A comparison of the different types of bids is given in [24].

Spectrum auction can be jointly designed with a resource allocation framework.

2.6 Economics of Dynamic Spectrum Access: Spectrum Trading

2.6.1 Spectrum Trading

The process of selling/leasing spectrum underutilised by primary users to secondary users is known as spectrum trading. The seller would first be the Government for spectrum allocation as the Government owns the spectrum and thereafter or post-allocation the operators would become the sellers, reselling their portion of the underutilised spectrum. The market where the Government is allocating spectrum to primary users is known as the primary market, due to the regulatory requirements from the Government, the process of spectrum allocation is lengthy and inflexible. As the secondary market is not controlled by the government it can be seen as an attractive tool to promote efficient use of the radio spectrum. The Government would aim to maximise their revenue and minimise budget defects, whereas the operators aim to maximise their profit and wealth in the long run. [24] [27]

From economics, trading is defined as *"a process of exchanging a commodity or service in a market"*, [27]. This process can be performed through direct exchange of a commodity or service or through a medium of exchange (generally money). This concept of trading can be applied to spectrum leasing in the secondary market and encompasses different dimensions of spectrum resources (i.e. frequency band, time slot). An important issue in spectrum trading is the pricing for both the licensed users selling spectrum and the unlicensed users buying the spectrum.

As the seller increases the price to achieve a higher revenue, the utility of the buyer decreases due to a higher cost, similarly the QoS performance exhibits the same behaviour when spectrum allocation to unlicensed users are varied. Therefore, an optimal and stable solution for spectrum trading in terms of price and allocated spectrum is required to maximise the revenue of the seller and utility of the buyer while still satisfying both the seller and buyer and their solutions.

When designing spectrum trading models to obtain an optimal and stable solution, different techniques can be applied, namely [27],

- Microeconomic approach: in the microeconomics approach spectrum trading is modelled with two major entities, a spectrum seller and a spectrum buyer. The solution of this approach is based on market-equilibrium where the demand is equal to the supply and the profit of the seller and satisfaction of the buyer are maximised.
- Classical optimization approach: in the classical optimization approach, spectrum trading is formulated as an optimization problem where there is a single objective under a set of constraints. The objective can change subject to the desired outcome, e.g. maximise profit of spectrum owner or maximise the utility of the cognitive radio user. The solution of this approach is "system-wise" optimal for the entire system as it only relies on a single objective function
- Non-cooperative game approach: in the non-cooperative game approach, several entities are involved and they all have different, possibly conflicting, interests. The solution of such a model must satisfy all the entities involved.

- Bargaining game approach: in the bargaining game approach, the system entities can negotiate and bargain with each other to obtain a fair and efficient solution. This approach can be used when cognitive radio users can cooperate and each entity can influence the action of other entities during spectrum trading.
- Auction approach: in the auction approach, the buyers submit their bids for spectrum and profit is maximised by allocating the spectrum to the bidder with the highest price.

From an economic viewpoint, spectrum trading can be considered as a part of spectrum management (along with interference control, spectrum sharing, spectrum regulation, spectrum allocation and transmission adaptation) and is the process between spectrum exploration and exploitation.

The two major structures of spectrum trading are, [27]:

- Single seller (monopoly): There is only a single seller in this market and the seller can maximise its revenue given the demand from the buyers. The buyers only have the option to cooperate or compete with each other to buy spectrum from the seller and the choices made by the buyers affect the revenue directly. The seller has the option to adapt its parameters according to the behaviour of the market.
- Multiple sellers (oligopoly): With multiple sellers in the market, the buyer can choose the offer to maximise their satisfaction in terms of performance and price, hence the revenue of a seller will be less than that in a monopoly.

Alternatively, another option is the commons-use spectrum model where there is no permanent seller and all users have the right to access the spectrum. The issue with this model is that if a particular user requires more spectrum than another user, the other user will need to be compensated.

When developing a spectrum trading model, the following aspects need to be considered, [27]

- Mode: mode refers to two things, namely the change of spectrum ownership and the change of use due to spectrum trading. After spectrum trading is done, the spectrum ownership is transferred from the seller to the buyer, the buyer could also use the spectrum for a wireless service hence a change of use is possible.

- Extent: extent defines the degree of a spectrum owner's rights and obligations that are transferred to the buyer, either shared or transferred completely. In the case of a complete transfer, all rights and obligations of spectrum access are completely transferred to the buyer, whereas in a shared transfer, both spectrum owner and buyer share the rights and obligations.
- Duration: duration determines the length of time the buyer can access the traded spectrum for and the different scales of duration can be defined as short-term lease, long-term lease, sale-and-buy-back and permanent. Buyers are allowed to access the spectrum until the licence expires.

Spectrum trading models can be classified according to infrastructure, configuration, activation and flexibility based on different criteria. The different classifications refer to, [27]

- Infrastructure: the infrastructure used for spectrum trading can either be shared or non-shared (dedicated infrastructure). In the case of the shared infrastructure, there are multiple unlicensed users that share the same equipment, whereas in the dedicated infrastructure, each unlicensed user uses its own equipment to utilise the spectrum.
- Configuration: the configuration for spectrum trading can either be centralised or distributed. For the case of centralised spectrum trading configuration, a spectrum broker is used to control the spectrum trading and the transmission parameters, whereas in distributed spectrum trading configuration each of the unlicensed users negotiates independently with a licensed user for spectrum trading. In the case of distributed configuration, the licensed users can either cooperate or compete to buy the spectrum from a licensed user.
- Activation: activation of spectrum trading can be initiated by three types of users, spectrum owner (licensed user), cognitive radio user (unlicensed user) and jointly by the spectrum owner and cognitive radio user. The activation of spectrum trading can either be periodic or sporadic, where periodic spectrum trading entails spectrum being traded for a fixed period of time and sporadic spectrum trading entails spectrum trading being initiated at any point in time.
- Flexibility: flexibility of spectrum trading can either be multiprotocol, restricted protocol or single protocol. In the case of multiprotocol, there is no restriction on the protocol to be used by an unlicensed user as opposed to restricted or single protocol where the licensed user determines a specific protocol or set of protocols

than an unlicensed user can use. This allows specification on the type of wireless service utilised.

Another important issue in spectrum trading is information management which details the information necessary to be exchanged between buyers and sellers. Information flows required for spectrum trading include, [27]

- Request and acknowledgement messages of spectrum trading from the licensed and unlicensed users: this includes the details of spectrum demand and spectrum supply as well as pricing information.
- Spectrum access parameters: this includes the set of parameters for the unlicensed user to access the spectrum and is known as public information to licensed users.
- Spectrum occupancy information: this includes information for an unlicensed user to identify the spectrum opportunities and initiate spectrum trading as well the spectrum supply which will be sold to an unlicensed user.
- Report on spectrum access: this information is used by the licensed users and contains information on spectrum utilization and interference levels caused to the licensed users.

2.6.2 Spectrum Pricing

Price is defined as the rate at which anything can be exchanged for anything else, the scarcer and more useful a commodity is, the higher the economic price of the commodity. Spectrum is seen as a very useful and scarce resource, hence the price is high. From an economic pricing perspective, the pricing transaction may be considered from three aspects: those of the buyer, the seller and the wider industry or economy as a whole,[24].

The buyer would most likely be telecom operators in the telecom industry and these buyers would act as to maximise their utility under certain constraints. The seller on the other hand would first be the Government for spectrum allocation as the Government owns the spectrum and thereafter or post-allocation the operators would become the sellers, reselling their portion of the spectrum. The Government would aim to maximise their revenue and minimise budget defects, whereas the operators aim to maximise their profit and wealth in the long run, [24]. Each seller scenario would pose different price determination strategies and has to be analysed under different forms of competition. The third aspect for consideration is the industry or economy as a whole pricing which provides great influence on the buyer and seller. The two approaches for pricing in the industry are general equilibrium analysis and macroeconomics, [24].

In a cognitive radio network, the problem of pricing is different to that in a traditional wireless network due to spectrum sharing and the adaptability of the licensed and unlicensed users. A licensed user can charge a price to an unlicensed user for spectrum access and this price can be dynamically adjusted according to the availability of spectrum opportunity. Spectrum opportunity is a function of traffic load in the licensed network and the demand from the unlicensed users. This demand is dependent on the current number of ongoing sessions and applications used by the unlicensed users.

To avoid network congestion which degrades system performance, the number of users sharing the limited spectrum can be limited with use of an admission control mechanism together with the pricing scheme *"To support a secure pricing scheme, an authentication mechanism is required to verify the users to access the spectrum. An authorization mechanism is used to grant access to the users. An accounting mechanism is used to record the usage statistics and calculate the price to be charged to the users"*, [27].

2.7 Summary

This chapter builds on the concepts presented in Chapter 1 and presents summaries on various subjects that relate to the theme of spectrum trading, the technology that makes it viable and the methods of modelling a spectrum trading market. The introductory sections provide the concepts of the fundamental operational processes of the cognition cycle and dynamic spectrum sharing and provide definitions and an overview on a Software-Defined radio and the cognitive cycle. Applications of cognitive radio are discussed with specific reference to spectrum sharing. Sections 2.4 and 2.5 discuss a detailed introduction into game theory and economic theory, and the three pricing strategies, namely market-equilibrium pricing, competitive pricing and cooperative pricing, as solutions to the challenges of pricing strategies in cognitive networks. Under the economic theories in dynamic spectrum access, topics such as utility theory, market-equilibrium, oligopoly market and auction theory are covered. Spectrum pricing and spectrum trading is then discussed in detail.

The subsequent chapter provides a discussion on comparative approaches to spectrum trading models and regulation and is derived from literature and the South African context.

Chapter 3

3 Comparative Approaches to Spectrum Trading Models and Regulation

3.1 Introduction

In this chapter, an overview is given of the existing work in the fields of spectrum sharing, game theory models applied to spectrum sharing, economic theories applied to spectrum sharing as well as the regulations imposed on spectrum trading.

The concluding subsections discuss telecommunication regulations, the regulatory environment and the issue of spectrum licensing, its usage and the importance of spectrum sharing.

This section is derived from literature and the South African context.

3.2 Spectrum Sharing

The scarcity of wireless spectrum, inefficient allocations of frequency and developments in network technologies has prompted a number of studies in this field towards spectrum management and trading/sharing. The spectrum sharing process can be divided into five major steps [6], spectrum sensing, spectrum allocation, spectrum access, transmitter-receiver handshake and spectrum mobility. Akyildiz et al. discusses the existing work performed that is aimed at providing solutions for each of these processes and provides a survey of dynamic spectrum access, cognitive radio wireless networks and next generation wireless networks with cognitive radios (including architecture frameworks) in [6]. Akyildiz et al. findings show extensive development of next generation networks by exploitation of the existing wireless spectrum opportunistically and highlights the necessity to ensure efficient spectrum-aware communication; further research is needed along the lines discussed in their survey. This paper shows insight into the capabilities of cognitive radio techniques, the communication protocols for efficient communication, spectrum management functionalities such as spectrum sensing, spectrum analysis and spectrum decision as well as spectrum mobility; which provides an overview and background of topics pertaining to spectrum trading.

An important aspect of spectrum trading is the viability of spectrum trading within a country; an article in Techcentral, [7], discusses spectrum trading in SA and mentions that the National Planning Commissions National Development Plan proposes that the country should allow companies to trade in scarce radio frequency spectrum and suggests mechanisms such as "*spectrum auctions and reverse bids for underserviced areas*" for radio frequency spectrum allocations. They also state that spectrum "*should be fully tradeable once allocated*". The article goes further into discussing the opinions of industry players, where they all agree it would be a good move. Steve Song, an industry player highlights "*he is not aware of many global examples where spectrum trading has had a big impact*" however he feels "*there's a lot of theoretical potential in the idea*". Steve Song and Henk Kleynhans both believe opening TV white spaces (TVWS) will have a bigger impact, as cognitive radio is the future of wireless telecommunications.

Secondary spectrum trading in TVWS is discussed in [8]; the authors propose the use of a spectrum broker to manage the TVWS secondary spectrum market with two modes of trading operation, auction mode and merchant mode. The results from this study show *"the secondary spectrum market in TVWS has the potential to support wireless communication services of multiple players, including mobile communication operators with continuously increasing spectrum demands"* [8]. A discussion is also provided around successfully applied spectrum trading mechanisms in a real-world test scenario in Munich, Germany which showed the efficiency of the proposed market mechanisms (auction design).

In Europe, an approach known as COGEU (Cognitive radio systems for efficient sharing of TVWS in EUropean context) is being investigated and developed to exploit TVWS. *"COGEU proposes a bicameral (national) geolocation database separating bands for common usage and for secondary spectrum trading. The commons bands are for access without the need for guaranteed quality of service (QoS), while secondary trading bands are for access with guaranteed QoS"*. Mwangoka, Marques and Rodriguez present the utilization of TVWS from the perspective of the COGEU project in [9]. The paper discusses challenges of exploiting TVWS in Europe and the COGEU framework before presenting the COGEU broker model for spectrum trading. The results from [9] show that "*the COGEU project envisions exploiting the TVWS as tradable and flexible spectrum to expand the range of spectrum available over which key services can be provided with QoS guarantees if necessary*."

Although there have been many methods proposed for improved spectrum assignment there are still many issues related to their implementation, such as interference in a multi-provider environment and determining the elements and architectures for feasible implementations of spectrum trading markets. An analysis of these issues is discussed in [10] with relation to the types of trading interactions in a spectrum trading market and the kinds of architectures that can be used to implement them. An issue highlighted in this paper is the difficulty of the regulatory agencies in managing spectrum due to the new technologies and new uses of the spectrum, which is possibly quite relevant to South Africa. The authors discuss the benefits of spectrum trading, [10]; highlighting that "*spectrum trading can improve the efficiency of the initial distributions of spectrum by allowing the licensees to be those who value its use the most and by making use of the technology that provides the best economic gains*." Spectrum trading would allow for competition and stimulation of technological innovation.

In [11], the conditions for viability of spectrum trading markets are discussed by considering scenarios with different market structures, number of trading participant's amount of tradable spectrum. The authors find their models indicate that spectrum markets can be viable in a service if sufficient numbers of market participants exist and the amount of tradable spectrum is balanced to the demand. "*A challenge for regulators and researchers alike will be identifying an appropriate band to promote spectrum trading or to facilitate the entry of new market participants",* [11].

In [12], the authors point out that scholars from as early as 1959 agree that spectrum trading can improve spectrum efficiency and focus on two aspects: "*demonstrating the necessity of the introduction of spectrum trading and discussing how to promote it*". This paper analyses the incentives of spectrum trading from both a microeconomic and macroeconomic viewpoint and then focuses on the relationship between initial spectrum assignment and secondary spectrum trading.

3.3 Game Theory Models Applied to Spectrum Trading/Sharing

"*Game formulations can be used for multiplayer optimization to achieve individual optimal solutions for resource allocation*" [13]. Game theoretic models for resource allocation have mainly focused on admission control, throughput optimization, power control and channel allocation. This section discusses some of the related work.

Zhang and Yu published a survey on spectrum sharing in cognitive radio using game theory [15]. In this paper, they discuss basic elements of modelling by game theory before introducing several dynamic spectrum-sharing algorithms.

Alptekin and Bener [17] discuss the pricing and transmission power control processes in terms of a one-shot non-cooperative game model and aim to determine the optimum price values for unit spectrum bands that maximise the primary service provider's profits while protecting the social welfare of their network. They also discuss the impact of transmission power and flexibility on the offered spectrum size on the profit maximisation as well as unit prices. The results show power limitations directly influence the supply decisions of the primary service providers, spectrum trading with a centralised controller increases their profit and degrading the received quality of the primary users may be profitable for the primary service provider is some cases. Saraydar et al. presented a power control solution based on game theoretical framework for wireless data as discussed in [17].

In [42], Zhu et al investigates a duopoly pricing model which could be used for communication service competitions reselling IP-based service over Wireless Mesh Networks. A two-stage non-cooperative game is used to model the two access point providers, where stage 1 is the where the access point providers set their prices to maximise their profits respectively and stage 2 is when given the price and QoS combinations from both access point providers, the end-users decide whether or not to make use of the services and from which provider. The results show the end-users expected compensated utility (a function of the prices and the QoS offered by the access point providers). It is a key factor of the two access points to determine their prices such that Nash Equilibrium can be reached, [42].

Wang et al propose a novel auction-based model in [43] to characterise and analyse inherent features (such as competition among secondary users and uncertainty about the wireless environment for secondary users) in a dynamic spectrum sharing environment with one primary user and multiple secondary users sharing the same frequency spectrum. The results show the best response of each secondary user is a non-linear function of the other user's strategy, and the Nash Equilibrium varies with different channel qualities, therefore a secondary user with a better channel quality prefers to bid a larger spectrum size, [43].

3.4 Joint Game and Economic Models Applied to Spectrum Trading/Sharing

With the use of different game models, a number of proposals have been made regarding pricing models and/or perspectives. Nel and Zhu in [1] have discussed the expansion of the work performed in [16] to model the use of the opportunistic spectrum access allowing secondary users to share the spectrum resources with primary users on an opportunistic basis using a three player Stackelberg game model. "*In this game model, the service provider as the leader aims to enhance its revenue while improving the utilization of its channel by allowing opportunistic spectrum access without violating the primary users non-zero tolerated interference probability*" [1]. Nel and Zhu aim to demonstrate that the service provider could earn more revenue with less secondary users when the channel is under-utilized by exploiting the secondary users' willingness to pay. The simulation results from [1] show that under certain assumptions, dynamic spectrum access with secondary sharing can greatly improve the revenue for the service provider when the channel is under-utilised or over utilised; however by compensating the primary users due to interference caused by secondary users could result in a loss of revenue. The paper further shows that by exploiting the secondary user's willingness to pay, the service provider could earn more from less secondary users when the channel utilization of the primary users is lower as the secondary users who can be allocated more bandwidth with better channel condition are more willing to pay.

Niyato and Hossain present three different pricing and market models: market-equilibrium, cooperative and competitive as discussed in [31]. These models are used to compare the prices offered by the service providers at equilibrium as well as the profit attained at equilibrium. The model in [31] assumes a primary service provider services a number of primary users as well as secondary users. The service providers can set the offered price accordingly with one of the three pricing models. The market-equilibrium and cooperative pricing models are based on optimization problems whereas the competitive pricing model is based on a non-cooperative Bertrand game assuming the players are selfish and compete against one another for price, [31] . The authors simulate static and dynamic models of the three pricing strategies and determine the cooperative pricing model returns the highest profit with the lowest stability and the market equilibrium has the lowest profit with the highest stability.

The Bertrand model used for competitive pricing is expanded in [14] and [40]. In [14] Niyato and Hossain discussed the problem of spectrum sharing among primary users and multiple secondary users and used a non-cooperative Bertrand game to model a spectrum overlay-based cognitive radio wireless system with one primary user and a number of secondary users. A static and dynamic game is simulated and compared and the inefficiency of Nash Equilibrium is explored. The major observations in this paper are that the spectrum sharing solution in case of the dynamic strategy adaptation depends on the given system parameters as well as the algorithmic parameters (e.g. learning rate) and the Nash Equilibrium does not necessarily maximise the total profit of the secondary users, however it does provide a fair solution, [40]. To expand on the concepts examined in [14], the same authors further investigate the inefficiency of the Nash Equilibrium as well as collusion and show collusion returns a higher profit than Nash Equilibrium by ensuring the primary services are aware of punishment due to deviation by properly weighting the profit in the future.

A variation of the model is discussed in [44], where a Cournot game model is used to model the competition between secondary users for spectrum offered by the primary user. A static and dynamic game is modelled and simulated and shows the results that as the secondary user can achieve a higher transmission rate from adaptive modulation, higher profits can be achieved and hence secondary user prefers to have a larger spectrum size. A static model is used when all the secondary users are able to observe the strategies and payoffs of other secondary users, in the dynamic model, the secondary user adapts its spectrum sharing strategy by observing the marginal profit which is a function of spectrum price offered by the primary user.

Alptekin and Bener [17] have discussed in previous works a proposal for short-term sub-lease of spectrum bands to different service providers and the optimum prices determined with use of a non-cooperative game.

Chen, Zhang, Kuo proposed an adaptive cooperative spectrum-sharing model based on fairness and total profit in cognitive radio networks and demonstrated the effectiveness of the cooperative game in [4]. Salameh, Krunz and Younis performed a study on cooperative adaptive spectrum sharing to determine how the nodes in cognitive radio network cooperate to access the medium to maximise the cognitive radio network in [18].

3.5 Regulations of Spectrum Trading/Sharing

As radio spectrum is a major component of the infrastructure that fortifies the information society, an important issue in spectrum trading is policy and regulation, [33]. Spectrum regulations and policies define rules of cooperation between primary and secondary users. Due to the significance of the frequency spectrum to the economy, the use of spectrum is regulated by the Independent Communications Authority of South Africa (ICASA). The management and development of the spectrum plays an important role in developing a knowledge-driven economy and society, [33] [34].

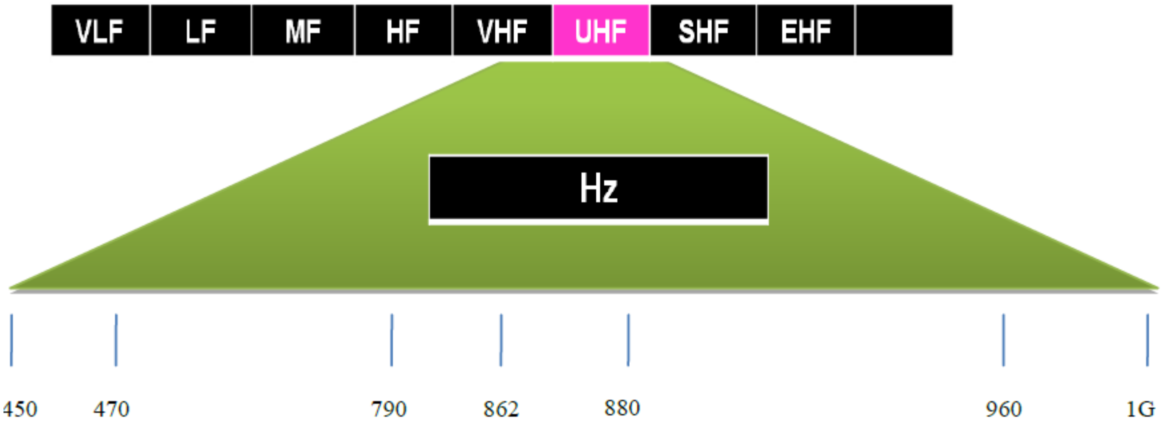
"*The use of radio spectrum has become an integral part of society's infrastructure*" [34]. For decades, viewers have benefited from the reception of clear TV signals, travellers have relied upon assured communications and radio-location for aircraft, and all citizens have benefited from radio connectivity for the public safety services. Recently, the phenomenal growth in personal mobile communications has turned wireless access via mobile phones from a luxury to a necessity for many people, [33]. This growing demand for mobile and broadband communications is fuelling the demand for radio frequency spectrum licensing. As this demand increases, spectrum needs to be managed to avoid excessive interference between the different users, [33] [34].

This section provides an introduction to spectrum management, a discussion on spectrum management reform in South Africa as well as spectrum policy and regulation in South Africa. The concluding subsection delves into on-going discussions in South Africa.

3.5.1 Spectrum Management

The concept of the radio spectrum regulation is known as spectrum management and has been practised around the world since the 1920's, [33]. Spectrum management involves technical and regulatory mechanisms that are designed to achieve the optimal use of the radio frequency spectrum with the key purpose of maximising the value society gains from the radio spectrum. Earlier techniques of spectrum management may have been effective when utilising radio communication systems however due to the technological progress and innovative applications to utilise radio spectrum, the spectrum management process has become rather out-dated as it has not kept up with the major changes in technology, [33] [35].

The two major components of spectrum management are the planning of spectrum (allocation of spectrum) and the licensing of spectrum users (assignment of frequency bands), [33]. The planning of spectrum bands is generally based on a clearly defined sharing criterion, whereas spectrum assignment follows from the planning component and is the detailed identification and coordination of the specific spectrum bands to individual users with specific technical conditions to avoid interference. These spectrum band plans are captured on regulations and are capable of enforcement, [33].



Allocation	Service	Utilisation
450-470 MHz	Fixed/Mobile	Public Access Mobile
470-790 MHz	Broadcasting	Digital TV
790-862 MHz	Broadcasting/Mobile	Digital TV/IMT
862-880 MHz	Mobile	Cellular/IMT
880 – 960 MHz	Mobile	Cellular
960 – 1 GHz	Aeronautical	Aircraft communications

Figure 3-1: Radio Frequency Spectrum Allocations [33]

Figure 3-1: Radio Frequency Spectrum Allocations [33] courtesy of Peter Zimri [33] shows the radio frequency spectrum allocations in South Africa as well as the channel arrangements, which is a detailed version of transmit and receive frequencies. Figure 3-2: Available Radio Spectrum [33] courtesy of Peter Zimri [33] shows the available radio frequency spectrum, where the “sweet spot” of the radio frequency spectrum is approximately from 380MHz to 1000MHz, which lies in the Ultra High Frequency (UHF) band.

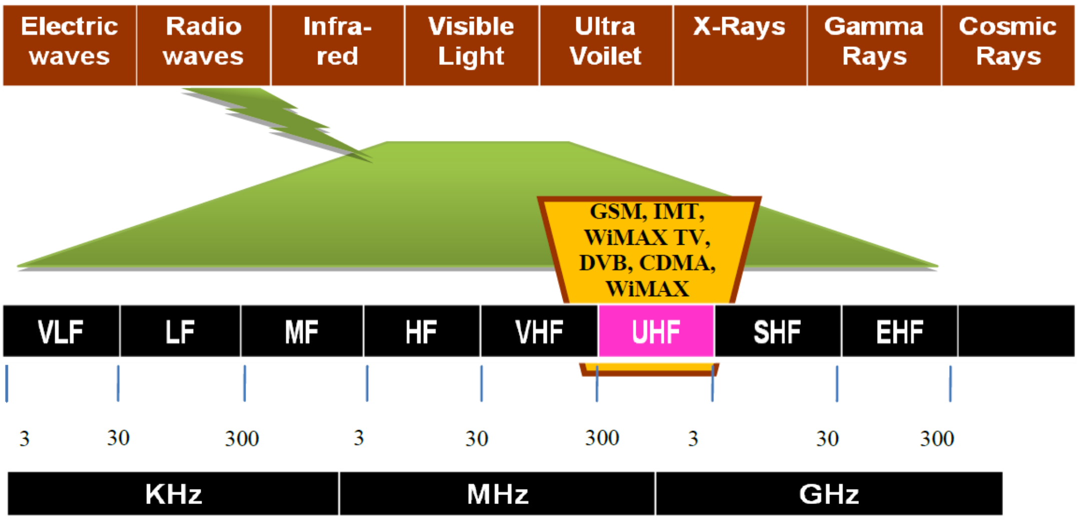


Figure 3-2: Available Radio Spectrum [33]

The allocation of frequencies, as defined by International Telecommunication Union (ITU), can be assigned in three ways, restricted frequency bands, open frequency bands and licenced frequency bands, [30]. Restricted frequency bands are exclusively reserved for radio astronomy; open frequency bands are allocated by the government and are free for use by anyone as long as they operate within transmit power limits and licensed frequency bands are allocated commercially at a cost and only licensed users may transmit within the allocated spectrum range, [30].

In South Africa, the spectrum management arrangements are a shared responsibility between the policy maker and the regulatory authority, i.e. the Department of Communication (DoC) coordinates spectrum for government services, while ICASA regulates all other spectrum requirements. The national government agencies, ICASA in the case of South Africa, are typically responsible for allocation (type of services and technology deployed in the band) and assignment (entities granted licences to use the radio frequency bands) of the radio frequency spectrum as well as the role of administration and the harmonization of spectrum across borders, [33].

Currently, the spectrum management policy in South Africa is for ICASA to assign and allocate spectrum bands statically (known as “command-and-control”), which results in spectral under-utilisation, [30]. Furthermore, traditional regulatory policies of most countries like South Africa conform to the vertical layered model (sharing scheme between primary licensed users and secondary unlicensed users, where secondary users opportunistically exploit the licensed spectrum when the primary users are not active) compared to the horizontal layered mode (sharing scheme between radio systems with equal regulatory priority, without causing interference and the operations of such radios can either be in the licensed or unlicensed frequency bands), [30].

To ensure that the interference between the different users remain manageable while allowing as many efficient users as possible, the spectrum regulator has the role of providing each user with the right to transmit on a particular frequency over a particular area, which is typically in the form of a license. "*The right to access the spectrum (or license) is generally defined by frequency, space, transmit power, spectrum owner (i.e. licensee), type of use, and the duration of license*" [27]. Usually, a license is allocated to one licensee, and the use of spectrum (assignment) by this licensee must conform to the specification in the license (e.g. maximum transmit power, location of base station, type of service assigned to this spectrum band). In the current spectrum licensing scheme, the license cannot change the type of use or transfer the right to other licensee, [27]. This limits the use of the frequency spectrum and results in low utilization of the frequency spectrum. With a low utilization of the frequency spectrum, the economic value derived from the spectrum is not maximised.

The limitations in spectrum access due to the static spectrum licensing scheme can be summarized as follows: [27]

- Fixed type of spectrum usage: The type of spectrum use cannot be changed; causing portions of the TV band could remain largely unused in many locations due to cable TV systems.
- Licensed for a large region: When a spectrum is licensed, it is usually allocated to a particular user or wireless service provider in a large region (e.g. an entire city or state), however, the wireless service provider may use the spectrum only in areas with a good number of subscribers, to gain the highest return on investment. Consequently, the allocated frequency spectrum remains unused in other areas, and other users or service providers are prohibited from accessing this spectrum.
- Large chunk of licensed spectrum: A wireless service provider is generally licensed with a large chunk of radio spectrum, for a service provider, it may not be possible to obtain license for a small spectrum band to use in a certain area for a short period of time to meet a temporary peak traffic load.

- Prohibit spectrum access by unlicensed users: Only a licensed user can access the corresponding radio spectrum and unlicensed users are prohibited from accessing the spectrum even though it is unoccupied by the licensed users.

As the demand grows for spectrum, the regulator receives frequent requests for new spectrum as well as the allowance of existing users to change application, this as well as the current method of allocation and assignment is causing it to become increasingly difficult for the spectrum regulator to manage [36].

3.5.2 Spectrum Management Reform in South Africa

"*Traditional spectrum management practice is predicated on the spectrum being a limited resource that must be apportioned among uses and users by government administration*" [37]. Spectrum management has not been able to keep up with the major changes in technology, business practice and economic policy and hence has led to growing technical and economic inefficiencies as well as obstacles to growing innovation. These inefficiencies have provided a basis for spectrum management reform. Spectrum management reform offers low- and middle-income countries important new opportunities as well as challenges.

In South Africa, spectrum management reform could be seen to have commenced in 1995 when the DoC undertook a detailed spectrum investigation. As discussed in [33] the recommendations from the investigation resulted in the first national spectrum allocation plan, the South African Band Re-Planning Exercise (SABRE-1), which covered spectrum allocations in the frequency range 20MHz to 3400MHz. SABRE-1 was followed up with SABRE-2 and resulted in the development of the South African Table of Frequency Allocations which included spectrum band allocations up to the 80GHz.

The National Table of Frequency Allocations covering the frequency band from 9 kHz to 3000 GHz was finalised by ICASA and the DoC in July 2010 following the publication of the radio frequency policy discussing the framework for management and planning of the spectrum in South Africa, [33]. ICASA then finalised the radio frequency spectrum fees regulations providing a bases for Administrative Incentive Pricing (AIP) in March 2011, [33].

Table 1: Key events towards Spectrum Management Reform [33]

Period	Documented Event	Spectrum Matter
October, 1993	Independent Broadcasting Authority Act 153 of 1993	Independent Institution Managing Broadcasting Spectrum
August, 1995	Notice by P & T on SABRE	Invitation of the Development of the National Spectrum Allocation Plan
November, 1996	Telecommunications Act (Act 103 of 1996)	Spectrum Mandate awarded to the Authority
April, 1997	Final South African Band Replanning Exercise (SABRE)	Publication of Band Plan and Migration Strategy
May, 1997	Amendment to SABRE	Inclusion of 3400 - 3600 MHz
April, 1999	Broadcasting Act	Establishment of Frequency Spectrum Directorate MOC
September, 1999	Feasibility Study into Common Public Safety System	Licensing and award of spectrum available for a common Public safety network
May, 2000	ICASA Act	Reform of the Regulators, IBA and SATRA
August, 2001	SABRE 2	Covering Spectrum 3 to 70 GHz
November, 2001	Telecommunications Act Amendment	Award of 1800 and 3G spectrum to the 5 Major Operators
July, 2004	Final SATFA	Revision of frequency band Plan to consolidate SABRE 1 and 2 spectrum from 20 MHz to 70 GHz.
December, 2005	Broadcasting Frequency Plan 2004	Publication includes Spectrum for DTT
April, 2006	Electronic Communications Act (Act 36 of 2005)	Spectrum Mandate split between the DoC and ICASA
May, 2005	Ministerial Task Team	Develop Digital Migration Report
June, 2006	Regional Radiocommunication Conference	GE-06 plan for Digital Terrestrial Broadcasting
December, 2006	Policy Directions	Finalisation of the band plan till after 2007
September, 2007	World Radiocommunication Conference 2007 (WRC-07)	ITU Spectrum Allocation for Mobile (IMT)
September, 2008	Broadcasting Digital Migration Policy	Transition Period to migrate from Analogue to Digital technologies
2010	ICASA DTT Regulations	Allocation of Spectrum Channels/Multiplexers to incumbents
July, 2010	South African Table of Frequency Allocations	A revised band plan was published taking into account the Ministerial Policy directions
April, 2010	Radio frequency spectrum Policy	Seek to outline policy spectrum usage and processes
June, 2010	ICASA amendment bill	Take away the frequency planning function from the Authority
August, 2010	Radio frequency spectrum fees regulations for ECS/ECNS Licensees	Ensure effective and efficient usage of spectrum through the administrative incentive pricing (AIP)
September, 2010	Review of radio frequency spectrum regulations	Consolidate all spectrum regulations to allow envisaged market based approach and trading and leasing of spectrum

Table 1 above provides a brief overview of the significant spectrum events in South Africa as highlighted by Peter Zimri in [33], as obtained from the policy maker, regulators and spectrum interest groups.

In 2007 at the World Radiocommunications Conference, there was an ITU allocation of a 790-862 MHz for mobile services on a primary basis. This led to what is known as the "digital dividend" [33]. In South Africa, the matter of how the regulatory framework and policy cater for the dividend and how it will be managed is pending with the Department of Communications, which delays the implementation of the digital migration. It is further delayed by finalization of the national frequency plan [33].

It is important to note that in September 2010, ICASA embarked on a public process to review the existing radio regulations established under the Post Office Act and Radio Act of 1952, these regulations aimed to introduce a market-based spectrum management approach as opposed to a command-and-control mechanism [33]. However, regulations were withdrawn in the final radio spectrum regulations. June 2010 saw the introduction of the ICASA Amendment Bill in Parliament, which pursued removing the spectrum planning functions from ICASA; however it has been withdrawn due to controversial issues within the Bill.

"*Despite these reforms, the problems of the delays from allocation of radio frequency spectrum bands to specific electronic communication services to the assignment or licensing of radio frequency channels to respective licensees have not been resolved*" [33]. This is due to the introduction of new electronic communication technologies, inconsistent approaches employed between the policy maker and the authority and the lack of intelligence exhibited in allocating and assigning spectrum.

3.5.3 Spectrum Management Policy and Regulatory Approaches

There are three primary spectrum management regulatory models which are deployed globally, command-and-control (administrative), market-based or spectrum property rights and spectrum commons. These regulatory models are driven by the Government, market and technology respectively.

In the command-and-control approach, also known as administrative, a centralised planning or command and control decision making system exists. In this approach the state dictates what technology and applications are allocated for a range of radio frequency spectrum. To initially award spectrum licences in this approach, a beauty contest is held (“*A beauty contest is a licensing process whereby a regulatory authority decides which firm’s financial, technical, and general services offerings are sound”* [33]*).* The spectrum management controlling body decides the duration of the spectrum usage which may include rollout obligations. In South Africa, this approach is still widely deployed and ICASA has endeavoured to attach rollout obligations for access to the 800 MHz and 2.6 GHz spectrum bands in their latest spectrum migration proposal [33].

The Market-based or spectrum property rights approach is based on the introduction of property rights and can be characterised by three elements [33], i) well defined exclusive rights of the to the use of the spectrum, ii) a market-type primary assignment mechanism for the initial allocation of spectrum rights and iii) a secondary market in which these rights can be sold. The main argument for the market-based approach is that it would dramatically increase the economic efficiency of spectrum use, however a consequence of putting all spectrum on the market would be, that so much spectrum might be freed that the price could drop close to zero,[37].

In South Africa, the recent debates tend to focus on the same ideas as Melody [33], such that the spectrum management regime fosters social and economic objectives. At the turn of the twentieth century, Melody reconsidered the spectrum debate and conveyed the adverse outcome of a third generation spectrum auction, the solution to this was to eradicate monopoly rents associated with scarce public resources (spectrum) by permitting innovative new entrants into the telecommunications market. ICASA has envisaged the design of the auction process for the licensing of the 2.6 GHz, 3.5 GHz and 800 MHz that allows 450 holders of electronic communications network licences to compete fairly for the spectrum [33].

Another major driver in the spectrum debate is technological innovation, radio technologies now coming to market or under development allow for more efficient use and easier sharing of the spectrum and may render spectrum scarcity obsolete. This type of approach is known as the spectrum commons approach, [33]. In the spectrum commons model, radio frequency spectrum is allocated on a non-exclusive rights basis and the licensees and users can use this allocated spectrum unrestrained. This spectrum can be referred to as licence exempt frequency bands. Due to the uses of these frequency bands, there are several rules that the users have to adhere too, such as restricted power levels to avoid interference to other services. Typical services supported in commons bands, [33], include remote control car locking mechanisms, microwave ovens, Bluetooth, etc. (all short-range devices). In South Africa, users are allowed to operate short-range devices in certain bands under specified power constraints and equipment type approval limitations; type approval is the certification of electronic communications equipment against an official standard. A general open access model is currently being proposed by ICASA and the DoC to implement bands for qualifying users and therefore manage the spectrum [33].

Replacement of spectrum management regimes and policy processes cannot be changed overnight as governments must consider spectrum requirements for its country's safety and security and for scientific purposes [33].

3.5.3.1 Primary Spectrum Assignment Models

The initial assignment of radio frequency channels is known as the primary assignment and will always be a function of government, irrespective of whether a spectrum management regulatory model is used. There are four mechanisms of primary assignment, namely [33] [39]:

- First-come, first-served: The first-come, first served model is based on the principle that the right to use the spectrum is assigned to whichever user is first to apply and is characterised by when the demand is less than the supply, it is economically efficient if there is no scarcity, else incumbents dominate the airwaves. It has the benefit that its administrative method has a low transaction cost and it is a very simple process, on the other hand, the downsides are that it is largely subjective and the more efficient operators (with adequate information sources and resources) will have an advantage over a smaller operator.
- Comparative Review (also known as 'Beauty contest'): The beauty contest is the most common method of assigning spectrum and the applicant is required to provide detailed information that is then evaluated on the basis of a set of criteria. Deserving applicants will be scored or weighted on issues such as rapid rollout, viability of the network and its ability to manage competitive issues, which works for developed nations where processes are more transparent and well structured. This assignment model can be seen as subjective judgements and is not economically efficient.

- Lottery: In the lottery spectrum assignment model, spectrum is assigned to applicants at random. This method is intrinsically non-discriminatory and eliminates any competitive distortion. Although this model is quick and transparent, there are many disadvantages, such as there is a strong possibility that it could lead to ineffective award of spectrum and hence inefficient assignment of spectrum resources.
- Auction: An auction is defined as "a market-transaction, conducted on the basis of explicit rules that allocates resources and determines a price by comparing the bids submitted by market participants". An auction is the method of initially assigning the spectrum channels by a regulatory authority and regarded as a market-based approach to achieve maximum economic benefits from the resource. The types of auctions are discussed in section 2.5.4.

In South Africa it is anticipated that the regulator will seek to use beauty contests or auctions or a hybrid methodology of the two, such assignments will only occur after an ITA has been issued by ICASA [38].

3.5.4 Spectrum Policy and Regulation in South Africa

There are a scarce number of sources in the public domain that document spectrum management in South Africa [33], many of the publications focus on regulation development and engineering.

Pre 1994, Telkom and the South African Broadcasting Corporation (SABC) performed the spectrum management function in the telecommunications and broadcasting sectors respectively, spectrum management activities were then governed under the Radio ACT no3 of 1952, [33]. With the declaration of the Independent Broadcasting Authority (IBA) Act No 153 of 1993, the country saw the establishment of the first independent spectrum management function for broadcasting services; thereafter the South African Telecommunications Regulatory Authority (SATRA) was established with the Telecommunications Act No 103 of 1996, [33].

Spectrum management responsibility in South Africa is split between the DoC and ICASA, the policy maker and regulator respectively. With the intensified demand due to commencement of the digital terrestrial broadcasting migration process, controversies surfaced in 2006 causing the DoC to internationally agree on electronic communications standards and a spectrum plan for digital broadcasting. Thereafter ICASA embarked on a similar process to develop the digital frequency plan [33].

Due to conflicts over responsibilities [33], the licensing of the high demand spectrum bands (800 MHz, 1800 MHz, 2.6 GHz and 3.5GHz) were impacted and hence delayed the roll-out of critical wireless broadband technologies. These delays impact the rolling out of new electronic communication networks and the provision of new services, which directly impacts the economy of the country due to limited access to the wireless broadband and the internet [33].

Despite the increased interest in spectrum management and the impact of wireless communications on universal access and services, there has been little research performed on the impact of the DoC and ICASA on the allocation and assignment of spectrum resources. It can be deducted that a split spectrum management scheme with a control-and-command mechanism creates inefficiencies from allocation to the award of spectrum assignments to licensees in South Africa [33].

Spectrum trading in South Africa is seen to be an illegal process that the regulatory framework and policy do not cater for. There is much discussion around this topic in South Africa and ICASA and the DoC have decided to further investigate if and how it should be implemented in South Africa as of April 2014. Spectrum bands are issued using administrative incentive pricing (AIP), where the bands are issued on a first-come first served basis which is coupled with a fee. The fee is based on value of the spectrum for a user with another service, additional costs if the service has to make use of other means and additional costs if the licensee uses less spectrum.

3.5.5 Ongoing Discussions in South Africa

The debate of unused spectrum in South Africa has been going on for many years as discussed in [19] by R Muller. Dominic Cull of Ellipsis Regulatory solutions agrees to the fact that spectrum trading should be allowed however with regulations to prevent profiteering and the spectrum broker model. The article [19] concludes that the market is not immediately excited about spectrum trading.

There is also a debate on whether radio frequency spectrum is a natural resource and that it is not scarce. M Mueller [45] debates that it not a natural resource, nor is it scarce, it is interference that gives rise to scarcity. This sentiment is further shared with others, who believe the spectrum scarcity is due to lack of knowledge when assigning and allocating spectrum resources [33].

With regards to the use of TVWS for secondary users, there is a discussion on what secondary user market the TVWS can meet [41]. In 2013, a group of partners implemented a TVWS trial network covering ten schools in the Western Cape over a six month period during 2013. The trial partners include TENET, CSIR Meraka, e-Schools Network, WAPA and Google, with Comsol Wireless Solutions, Carlson Wireless Technologies and Neul as the vendor partners. The trial aimed at demonstrating TVWS can be used to deliver affordable broadband and Internet services without interfering with TV reception and increase awareness of the potential for TVWS technology in South Africa and across the continent. The trial [41] proved that TVWS can be used to access the Internet over long distances without causing interference; however the trial did not show how the cognitive radio scans the environment and avoid interference. It also omitted spectrum database and where the intelligence comes into the network.

If the suggested market is the rural sector, the cost of erecting base station and electricity consumption needs to be calculated to determine if the benefits outweigh the costs and how these costs can be recovered. Also the benefits of using white spaces over 3G/4G or satellite technologies need to be identified.

3.6 Socio-economic Impact of Spectrum Sharing

"*Spectrum trading can provide significant economic and social benefits only if they become widely available and they are utilized*" [17]. The benefits currently derived from Television Whitespaces (TVWS) that was made possible through Dynamic Spectrum Access (DSA) has enhanced socio-economic development of the end-user through the provision of high-speed Internet access to its citizens [20]. The TVWS has also enhanced radio spectrum availability without any national or regional re-structuring of the current international radio spectrum allocation policy. Likewise, the flexibility involved in DSA permits a dynamic spectrum market where licensed owners can lease out their unused radio spectrum to generate revenue, not only provides more income for the licensed owners, but also enhances radio spectrum availability and its utilization. Furthermore, the lower entry costs provided by DSA has contributed to both product and business model lifecycles by enhancing production of more communication equipment and services as well as promoting more job opportunities. This increase in worldwide production as well as the provision of more job opportunities has positively impacted a number of nations' GDP and worldwide economic growth in general.

3.7 Summary

This chapter reviewed existing research work related to spectrum sharing, game theory models applied to spectrum sharing and economic models applied to spectrum sharing. A brief overview is provided on the socio-economic impact of spectrum sharing.

The concluding subsections discuss telecommunication regulations, the regulatory environment and the issue of spectrum licensing, its usage and the importance of spectrum sharing as well as on-going discussions in South Africa.

The following chapter presents a framework a theoretical framework of three pricing models, competitive pricing model, cooperative pricing model and market-equilibrium pricing model.

Chapter 4

4 Theoretical Framework of Pricing Models

4.1 Introduction

This chapter presents an in-depth review of three different pricing models for spectrum price and each model encompasses different degrees of competition and cooperation among the primary service providers. The pricing models considered are market equilibrium, competitive pricing and cooperative pricing. A simple scenario is defined where each pricing model algorithm can be separately applied, given the same conditions and assumptions.

The system considered is adapted from [27] where secondary users can opportunistically exploit the wireless spectrum licensed to primary users. It can be assumed that the secondary users can intelligently make decisions on the approach to adopt in accessing the spectrum. Figure 4-1 adapted from [27] illustrates the basic system design of spectrum sharing between the primary and secondary users. There are *N* licensed service providers that are said to service the licensed primary service users. Each service provider, i, serves M_i primary users. The service providers can then sell underutilized portions of its spectrum to secondary users at a price (spectrum price). The spectrum price (per unit spectrum) is denoted, p_i.

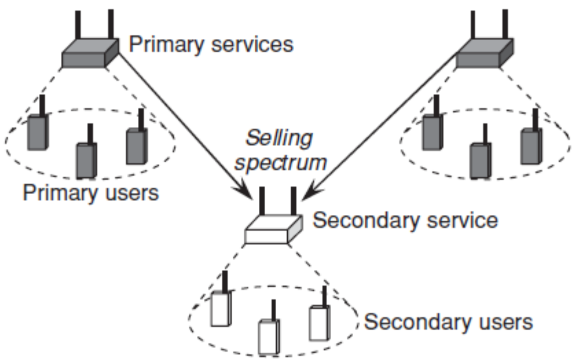


Figure 4-1: Illustration of spectrum trading [27]

It is assumed that the spectrum sharing between the primary and secondary users is performed on a time-division multiple access (TDMA) based wireless-access scheme, such that the primary users sell time slots in the licensed spectrum to the secondary users.

The spectrum demand of the secondary users is dependent on the transmission rate achieved on the allocated frequency spectrum and the price charged by the licensed service providers. As both the primary and secondary users use adaptive modulation, the transmission rate can be dynamically adjusted by the channel quality [27]. The spectral efficiency (k) of a Gaussian channel is given by,

$$\boldsymbol{k = \log_2(1 + K\gamma)}, \qquad \boldsymbol{where\ K = \frac{1.5}{\ln(0.2/BER^{tar})}} \tag{27}$$

where γ is the signal-to-noise ratio (SNR) and BER^{tar} is the target bit-error-rate (BER). $k_i^{(s)}$ is used to represent the spectral efficiency of wireless transmission by secondary users, whereas $k_i^{(p)}$represents the spectral efficiency of wireless transmission by primary users, both for primary service i.

The profit of a primary service provider depends not only on the cost of sharing the spectrum with the secondary service providers (e.g. due to performance degradation of primary users), but also on the strategy chosen by other primary service providers. The price, p_i, can be set by three different pricing strategies, namely market-equilibrium, competitive and cooperative pricing models and this is shown in section 4.4, 4.5 and 4.6 respectively. These different pricing strategies result in different behaviors of the firms in achieving the best and stable decisions.

For the purpose of this research report, a duopoly is considered, therefore there are only two primary services each servicing a number of primary users and a group of secondary users. The application of these pricing models extend beyond two-user games, however the assumptions, derivations and submissions here can be generalized to multi-user scenarios.

4.2 Utility and Spectrum Demand of the Secondary Users

To quantify the satisfaction of spectrum access by secondary users, a utility function is given by Equation (28), where $\mathcal{U}(\bar{b})$ indicates the payoff for each value of bandwidth in the vector, $\bar{b}$. [31]

$$\boldsymbol{\mathcal{U}(\bar{b}) = \sum_{i=1}^{N} b_i k_i^{(s)} - \frac{1}{2}\left(\sum_{i=1}^{N} b_i^2 + 2v\sum_{i\neq j} b_i b_j\right) - \sum_{i=1}^{N} p_i b_i} \tag{28}$$

where b is a vector of shared spectrum sizes from all the primary services, p_i is the price offered by primary service provider i and N is the total number of primary service providers. The quality of the spectrum is taken into account by the spectral efficiency,

$k_i^{(s)}$ and v denotes the spectrum substitutability. Spectrum substitutability indicates the ability of a secondary user to be able to switch among the frequency spectra depending on the price offered by the primary service providers. The spectrum substitutability factor is defined as, when $v = 0.0$, a secondary cannot switch among the frequency spectra, whereas when $v = 1.0$, a secondary can openly switch among the frequency spectra.

To obtain the demand function for spectrum from the primary service provider i, the utility function is to be differentiated with respect to b_i (spectrum sizes for primary service provider, i) to determine a partial solution. Therefore, the demand ($\mathcal{D}_i(\bar{p})$) for primary service provider, i, is represented by,

$$\boldsymbol{\mathcal{D}_i(\bar{p}) = \frac{d\mathcal{U}(b)}{db_i} = 0 = \frac{\left(k_i^{(s)} - p_i\right)(v(N-2)+1) - v\sum_{i\neq j}(k_i^{(s)} - p_j)}{(1-v)(v(N-1)+1)}} \quad (29)$$

where p is a vector of prices offered by all the primary service providers in the market. The demand function can be simplified to $\mathcal{D}_i(p) = \mathcal{D}_1(p_{-1}) - \mathcal{D}_2 p_i$, where p_{-1} denotes the vector of prices of all primary services except service i. $\mathcal{D}_1(p_{-1})$ and $\mathcal{D}_2 p_i$ are both constants for given p_j for $i \neq j$ and are given by, [27]

$$\boldsymbol{\mathcal{D}_1(p_{-i}) = \frac{k_i^{(s)}(v(N-2)+1) - v\sum_{i\neq j}(k_i^{(s)} - p_j)}{(1-v)(v(N-1)+1)}} \quad (30)$$

$$\boldsymbol{\mathcal{D}_2 = \frac{(v(N-2)+1)}{(1-v)(v(N-1)+1)}} \quad (31)$$

4.3 Revenue and Cost functions for a Primary Service Provider

If spectrum trading is allowed on the network, a primary service provider has two sources of revenue, the primary service users and the secondary users. Although spectrum trading can generate higher revenue to the primary service provider by selling spectrum to secondary users, it comes at the cost of degraded QoS performance on the primary users. This is due to interference caused by secondary users sharing the radio spectrum with primary users. The degraded QoS to the primary users causes a loss to the primary service provider in the form of a cost discount.

For the purpose of this research report, it is assumed that the primary users are charged a flat rate for a guaranteed amount of bandwidth, if this required bandwidth cannot be provided, a discount is offered to the primary users.

The revenue gained from primary users served by primary service i ($\mathcal{R}_i^l$) is given by,

$$\boldsymbol{\mathcal{R}_i^l = c_1 M_i} \tag{32}$$

where c_1 denotes a constant weight for the revenue and the revenue gained from sharing spectrum with secondary users ($\mathcal{R}_i^s$) is given by,

$$\boldsymbol{\mathcal{R}_i^s = p_i b_i} \tag{33}$$

where b_i represents the spectrum size shared with the secondary users and p_i the corresponding price per unit of spectrum.

The cost discount ($\mathcal{C}_n(b_n)$) given to primary users is given by,

$$\boldsymbol{\mathcal{C}_n(b_n) = c_2 M_i \left(B_i^{req} - k_i^{(p)} \frac{W_i - b_i}{M_i}\right)^2} \tag{34}$$

where c_2 denotes a constant weight for the cost functions at the primary service. The bandwidth requirement per user is denoted by B_i^{req}, the spectrum size is denoted by W_i and the number of on-going primary users serviced by primary service i is denoted by M_i.

It can be seen that given the revenue from the primary users is a linear function of the number of on-going users, whereas the revenue from the secondary users is a linear function of the shared spectrum size, given the spectrum price. "*The cost is proportional to the square of the difference between the bandwidth requirement and allocated bandwidth to a primary user*" [31].

4.4 Market-Equilibrium Pricing Strategy

In the market-equilibrium strategy, it is assumed that the primary service provider is not aware of any others and hence there is no competition or cooperation and the spectrum price is set naively based on spectrum demand from the secondary users (demand function) [27]. The price is set based on the willingness of the primary service provider to sell spectrum and this is determined by the supply function. The supply function indicates the size of radio spectrum shared by a primary user with the secondary user, whereas the demand function indicates the size of radio spectrum required by secondary users [27].

Given the price per unit of spectrum, p_i the spectrum supply function will indicate the size of the spectrum to be sold by the primary service provider and this spectrum supply function can be derived based on a profit maximisation problem. "*The solution of this* *optimization formulation is the optimal spectrum size, b_i to be shared with the secondary users"* [27].

Based on equations (32), (33) and (34), the profit ($\mathcal{P}_i$) of primary service provider i for owned spectrum $\mathcal{F}_i$, can be expressed by,

$$\boldsymbol{\mathcal{P}_i = c_1 M_i + p_i b_i - c_2 M_i \left(B_i^{req} - k_i^{(p)} \frac{W_i - b_i}{M_i}\right)^2} \tag{35}$$

The optimal spectrum size can be determined by differentiating the profit function ($\mathcal{P}_i$) with respect to b_i,

$$\boldsymbol{\frac{d\mathcal{P}_i}{db_i} = 0 = -p_i + 2c_2 M_i \left(B_i^{req} - k_i^{(p)} \frac{W_i - b_i}{M_i}\right) \frac{k_i^{(p)}}{M_i}} \tag{36}$$

Therefore the optimal value of b_i^* gives the supply function ($\mathcal{S}_i$) as,

$$\boldsymbol{b_i^* = \mathcal{S}_i(p_i^*) = W_i - \frac{M_i}{k_i^{(p)}} \left(B_i^{req} - \frac{p_i}{2c_2 k_i^{(p)}}\right)} \tag{37}$$

From equation (5) (page 23), the market-equilibrium is defined as the price, p_i^* at which the spectrum supply function equals the spectrum demand function,

$$\boldsymbol{\mathcal{S}_i(p_i^*) = \mathcal{D}_i(p_i^*), \forall i} \tag{38}$$

where p^* is a vector consisting of the market-equilibrium prices for all service providers.

As a primary service provider is unaware of the existence of other primary service providers, it can be seen that the supply function is independent of the prices offered by other primary service providers.[27].

4.4.1 Algorithm for Market-Equilibrium Pricing Strategy

1. *Initialisation:* Primary service provider i, where $i = 1,2 \ldots N$
2. *Determine the demand*: Using the demand function (29), find the demand for each primary service providers' set price, p_i
3. *Determine the revenue and cost functions:* Using the revenue (32) and (33) and cost functions (34), calculate the revenue and cost discount for each primary service providers' set price, p_i with the bandwidth requirement for each of the primary users.
4. *Calculate the profit:* Using the profit function (35), calculate the profit for each primary service providers' at set price, p_i

5. *Derive supply function:* Determine the supply function (37) by taking the derivative of the profit function (35). The supply function represents the supply of services for each set price assigned by the primary service provider.
6. *Determine equilibrium price where the supply function is equal to the demand function:* Solve the function where supply is equal to demand (38) simultaneously to obtain the price at equilibrium. This price is the best strategy for market-equilibrium.
7. *Confirmation:* Confirm if this equilibrium price is the best strategy to ensure there is no excess supply.

4.5 Competitive Pricing Strategy

In the competitive pricing model, each of the primary service providers is aware of the competition amongst each other and each of the primary service providers aim to maximise their own profit [27]. The primary service providers compete through price adjustment, i.e. given the spectrum prices offered by other primary service providers, one primary service provider will choose the price for its own spectrum such that its individual profit is maximised [27].

To model the competition for price among the primary service providers, a non-cooperative game model is used where the players (sellers in an oligopoly market) are the primary service providers, the strategy of each player is the price offered by unit of spectrum (non-negative) and the payoff for each player is the individual profit due to spectrum trading under competition to the secondary users [27].

The profit ($\mathcal{P}_i$) for each primary service provider, i, can be expressed by,

$$\boldsymbol{\mathcal{P}_i(p) = \mathcal{R}_i^l + \mathcal{R}_i^s - \mathcal{C}_n} \tag{39}$$

where p is a vector containing all the prices offered per unit of spectrum and $\mathcal{R}_i^l, \mathcal{R}_i^s, \mathcal{C}_n$ as defined in equations (32), (33) and (34) [27].

The solution of this game is Nash Equilibrium and this can be obtained by using the best response function ($\mathcal{B}_i$) of the players, which is the best strategy of one player given the other's strategies. The best response function of primary service provider i, given a set of prices offered by all other primary services (p_{-i}) is defined by,

$$\boldsymbol{\mathcal{B}_i(p_{-i}) = arg \max_{p_i} \mathcal{P}_i(p_i, p_{-i})} \tag{40}$$

The Nash equilibrium of this game is denoted by the set p^* if and only if,

$$\boldsymbol{p^* = \mathcal{B}_i(p^*_{-i}), \forall i} \tag{41}$$

where p^*_{-i} denotes a vector of best responses for player j and $j \neq i$. To obtain this Nash equilibrium, the derivative of the profit is required with respect to p_i and setting this to zero, to obtain the price at Nash equilibrium (i.e. $\frac{d\mathcal{P}_i}{dp_i} = 0\ \forall i$) [27].

In the competitive pricing strategy, the size of the shared bandwidth, b_i, in the profit function ($\mathcal{P}_i$) can be replaced with the spectrum demand, $\mathcal{D}_\mathrm{i}(\mathrm{p})$ (Equation (29), page 57). Therefore the profit function can be expressed as,

$$\boldsymbol{\mathcal{P}_i(p) = c_1 M_i + p_i \mathcal{D}_i(p) - c_2 M_i \left(B_i^{req} - k_i^{(p)} \frac{W_i - \mathcal{D}_i(p)}{M_i} \right)^2} \tag{42}$$

To obtain Nash equilibrium solve equation (22) for p_i^*,

$$\boldsymbol{\frac{d\mathcal{P}_i}{dp_i} = 0 = 2c_2 k_i^{(p)} \mathcal{D}_2 \left(B_i^{req} - k_i^{(p)} \frac{W_i - (\mathcal{D}_1(p_{-i}) - \mathcal{D}_2 p_i}{M_i} \right) + \mathcal{D}_1(p_{-i}) - 2\mathcal{D}_2 p_i} \tag{43}$$

where $\mathcal{D}_1(p_{-i})$ and $\mathcal{D}_2$ is per equation (30) and (31). Once the price at Nash equilibrium is found, it can be substituted into $\mathcal{D}_\mathrm{i}(\mathrm{p}^*)$ to obtain the size of the shared spectrum.

For the special case of two primary service providers ($i = 1\ and\ j = 2$), equation (43) can be represented by equations (44) and (45). [40]

$$\boldsymbol{-\left(\frac{k_i^{(s)} - v k_j^{(s)}}{1 - v^2} + \frac{2c_2 k_i^{(p)}}{1 - v^2} \left(B_i^{req} - \frac{k_i^{(p)} W_i}{M_i} + \frac{k_i^{(p)} k_i^{(s)}}{M_i(1 - v^2)} - \frac{k_i^{(p)} v k_j^{(s)}}{M_i(1 - v^2)} \right) \right)}$$

$$\boldsymbol{= p_i \left(\frac{2c_2 (k_i^{(p)})^2}{M_i(1 - v^2)} + \frac{2}{1 - v^2} \right) + p_j \frac{v}{1 - v^2} \left(1 + \frac{2c_2 (k_i^{(p)})^2}{M_i(1 - v^2)} \right)} \tag{44}$$

$$\boldsymbol{-\left(\frac{k_j^{(s)} - v k_i^{(s)}}{1 - v^2} + \frac{2c_2 k_j^{(p)}}{1 - v^2} \left(B_j^{req} - \frac{k_j^{(p)} W_j}{M_j} + \frac{k_j^{(p)} k_j^{(s)}}{M_j(1 - v^2)} - \frac{k_j^{(p)} v k_i^{(s)}}{M_j(1 - v^2)} \right) \right)}$$

$$\boldsymbol{= p_j \left(\frac{2c_2 (k_j^{(p)})^2}{M_j(1 - v^2)} + \frac{2}{1 - v^2} \right) + p_i \frac{v}{1 - v^2} \left(1 + \frac{2c_2 (k_j^{(p)})^2}{M_j(1 - v^2)} \right)} \tag{45}$$

By solving equations (44) and (45) simultaneously, the price at Nash equilibrium, p^*, can be determined for each primary service provider.

4.5.1 Algorithm for Competitive Pricing Strategy

1. *Initialisation:* Primary service provider i, where $i = 1,2 \ldots N$
2. *Determine the demand*: Using the demand function (29), find the demand for each primary service providers' set price, p_i
3. *Determine the revenue and cost functions:* Using the revenue (32) and 33) and cost functions (34), calculate the revenue and cost discount for each primary service providers' set price, p_i with the bandwidth requirement for each of the primary users.
4. *Calculate the profit:* Using the profit function (42), calculate the profit for each primary service providers' at set price, p_i
5. *Solve for Nash Equilibrium:* Solve the derivative of the profit (43) for the special case of two primary service providers, p_i^* and p_j^*. These prices are the best strategy for Nash-equilibrium
6. *Determine the profit at equilibrium:* Use p_i^* and p_j^* as found in 5 in the profit function (42) to determine the profit attained at Nash-equilibrium
7. *Confirmation:* Confirm if this equilibrium price is the best strategy to maximise one players profit given the other players strategies.

4.6 Cooperative Pricing Strategy

In the cooperative pricing model, the primary service providers collude with each other to attain the highest total profit by selling spectrum to secondary users. All the service providers are aware of each other and fully cooperate with each other.

To model the cooperative pricing strategy, an optimization problem is formulated where the highest profit can be achieved through an optimal price. The problem can be formulated by,

$$\boldsymbol{maximise}: \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \boldsymbol{\mathcal{P}_i(p)} \tag{46}$$

$$\boldsymbol{subject\ to}: \boldsymbol{W_i \geq b_i \geq 0} \tag{47}$$

$$\boldsymbol{p_i \geq 0} \tag{48}$$

where equation (46) is the total profit for all the primary service providers and if b_i is replaced by $\mathcal{D}_\mathrm{i}(\mathrm{p})$, the Lagrangian ($\mathcal{L}(p)$) is represented by,

$$\mathcal{L}(\boldsymbol{p}) = \sum_{i=1}^{N} \mathcal{P}_i(\boldsymbol{p}) - \sum_{j=1}^{N} \lambda_j(-p_j) - \sum_{k=1}^{N} \mu_k(\mathcal{D}_k(\mathbf{p}) - W_k) - \sum_{l=1}^{N} \sigma_l(-\mathcal{D}_l(\mathbf{p})) \quad (49)$$

where $\lambda_{j,}\mu_k$ and σ_l are Lagrange multipliers for the constraints in equation (47) and (48) respectively. With the use of Karush-Kuhn-Tucker (KKT) conditions, the vector of optimal prices, p^* can be obtained such that to maximise the total profit of all the primary service providers. [27]

4.7 Summary

This chapter presents the market-equilibrium pricing strategy and the competitive pricing strategy in theoretical details. Both algorithms are applied to a simple scenario, given the same conditions and assumptions and a number of test conditions are applied.

Both the market-equilibrium pricing and competitive pricing strategies have been explained in detail and the mathematical frameworks have been presented. A step by step description of a generalised operational procedure for each scheme is presented. The concluding subsections describe the third pricing strategy, cooperative pricing strategy.

The methodology to implement the framework discussed in this chapter as well as the simulation results are discussed in the ensuing chapter. The next chapter also provides an analysis of spectrum trading regulations and policy in South Africa and the key findings from the simulation results and interview sessions.

Chapter 5

5 Methodology, Simulation Results and Findings

5.1 Introduction

This chapter deals with the methodology and simulation results for two pricing strategies, namely the market-equilibrium pricing strategy and the competitive pricing strategy as well as the methodology and findings from in depth interviews. The second section defines the approaches adopted, the scenarios set-up (test-cases), the assumptions made and the methods used. The third section presents comprehensive simulation results derived from using MATLAB to simulate the two pricing strategies. The properties and performance of the two pricing strategies, based on the two different simulation results, are then compared and analysed in the next section. The concluding sections discuss the research methodology and findings for the regulatory component of this research report.

5.2 Methodology for the Simulation of the Pricing Models

The formulation of the market-equilibrium pricing strategy and the competitive pricing strategy were performed using the step-by-step approach, as outlined in Chapter 4. The simulations were conducted in two phases using MATLAB software. In the first phase, the market-equilibrium pricing strategy was simulated and results were derived. In the second phase, the competitive pricing strategy was simulated. The two phases of the simulations are explained in detail below.

5.2.1 Market-Equilibrium Pricing Strategy

The simulation for the market-equilibrium pricing strategy is set up in MATLAB, according to the procedure outlined in the flow chart in Figure 5-1 . The simulation is based on two primary services offering spectrum to group of secondary users while each primary service services a fixed number of primary users. Each primary service has a set total frequency spectrum available to them. The following outlines all parameters and assumptions used for the simulation.

1. *Initialisation:* The number of primary services is set to two ($N = 2$). The total frequency spectrum available to each primary service is 20 MHz ($Wi = 20$). Each primary service serves 10 primary users ($M1 = M2 = 10$). The target Bit-Error-Rate (BER) for the secondary users is $BER^{tar} = 10^{-4}$. The bandwidth requirement of each of the primary users is 2 Mbps ($B_i^{req} = 2$) . The

constants $c1 = 5$ and $c2 = 10$ (where $c1$ is the constant weight for the revenue earned by primary users, and $c2$ is the constant weight for the cost discount, these are chosen as per [31]). The channel quality for secondary users can vary between 9 and 22 dB and is initialised to 9 dB. The spectrum substitutability, v is set at 0.7. The price offered by each primary service, p_i and p_j, are both varied from 1.0 to 2.0 in increments of 0.1. p_j forms the outer loop for the calculations, while p_i forms the inner loop.

2. *Determine the demand*: Using the demand function (29), find the demand for each primary service providers' set price, p_i. The demand is based on both p_i and p_j. Firstly the values for spectral efficiency, k, and sum function in the demand, $\sum_{i \neq j}(k_i^{(s)} - p_j)$ are calculated.
3. *Determine the revenue and cost functions:* Using the revenue (32) and (33) and cost functions (34), calculate the revenue and cost discount for each primary service providers' set price, $p_i$and p_j with the bandwidth requirement for each of the primary users.
4. *Calculate the profit:* Using the profit function (35), calculate the profit for each primary service providers' at set price, p_i and p_j for Market Equilibrium.
5. *Derive supply function:* Determine the supply function (37) by taking the derivative of the profit function (35). The supply function represents the supply of services for each set price assigned by the primary service provider.
6. *Determine equilibrium price where the supply function is equal to the demand function:* Solve the function where supply is equal to demand (38) simultaneously to obtain the price at equilibrium. This is done graphically in the simulation and is indicated by the point where the supply function crosses the demand function. This price is the best strategy at market-equilibrium.
7. *Confirmation:* Confirm there is a Market-Equilibrium price and that it is the best strategy to ensure there is no excess supply for varying values of price and the bandwidth requirement of each of the primary users.

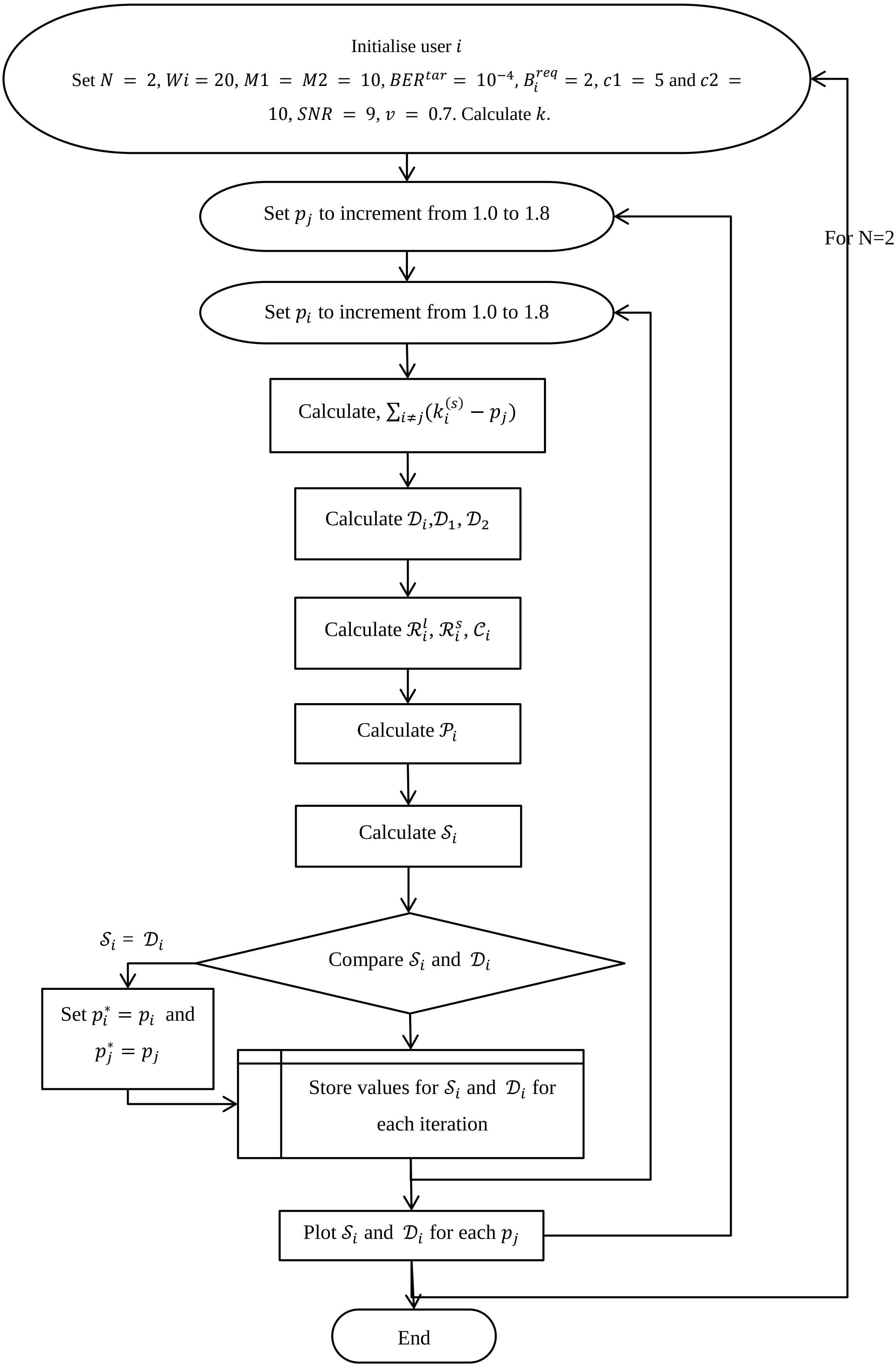


Figure 5-1: Flow-chart of the market-equilibrium pricing algorithm

5.2.2 Competitive Pricing Strategy

The simulation for the competitive pricing strategy is set up in MATLAB, according to the procedure outlined in the flow chart in Figure 5-2. The simulation is based on two primary services offering spectrum to group of secondary users while each primary service services a fixed number of primary users. Each primary service has a set total frequency spectrum available to them. The following outlines all parameters and assumptions used for the simulation.

1. *Initialisation:* The number of primary services is set to two ($N = 2$). The total frequency spectrum available to each primary service is 20 MHz ($Wi = 20$). Each primary service serves 10 primary users ($M1 = M2 = 10$). The target Bit-Error-Rate (BER) for the secondary users is $BER^{tar} = 10^{-4}$. The bandwidth requirement of each of the primary users is 2 Mbps ($B_i^{req} = 2$). The constants $c1 = 5$ and $c2 = 10$ (where $c1$ is the constant weight for the revenue earned by primary users, and $c2$ is the constant weight for the cost discount, these are chosen as per [31]). The channel quality for secondary users can vary between 9 and 22 dB and is initialised to 9 dB. The spectrum substitutability, v is set at 0.7. The price offered by each primary service, p_i and p_j, are both varied from 1.0 to 2.0 in increments of 0.1. p_j forms the outer loop for the calculations, while p_i forms the inner loop.
2. *Determine the demand*: Using the demand function (29), find the demand for each primary service providers' set price, p_i. The demand is based on both p_i and p_j. Firstly the values for spectral efficiency, k, and sum function in the demand, $\sum_{i\neq j}(k_i^{(s)} - p_j)$ are calculated.
3. *Determine the revenue and cost functions:* Using the revenue (32) and (33) and cost functions (34), calculate the revenue and cost discount for each primary service providers' set price, $p_i$and p_j with the bandwidth requirement for each of the primary users.
4. *Calculate the profit:* Using the profit function (42), calculate the profit for each primary service providers' at set price, p_i and p_j using the demand function for each set price, p_i and p_j
5. *Solve for Nash Equilibrium:* Solve the derivative of the profit (43) for the special case of two primary service providers, p_i^* and p_j^* . Solve the equations simultaneously to get p_i^* and p_j^*, this can be seen graphically from the highest

point of the competitive profit curve (42). These prices are the best strategy for Nash-equilibrium

6. *Determine the profit at equilibrium:* Use p_i^* and p_j^* as found in 5 in the profit function (42) to determine the profit attained at Nash-equilibrium
7. *Confirmation:* Confirm if this equilibrium price is the best strategy to maximise one players profit given the other players strategies. The profit attained by each primary service can be compared at each iteration for the given set of prices, p_i and p_j.

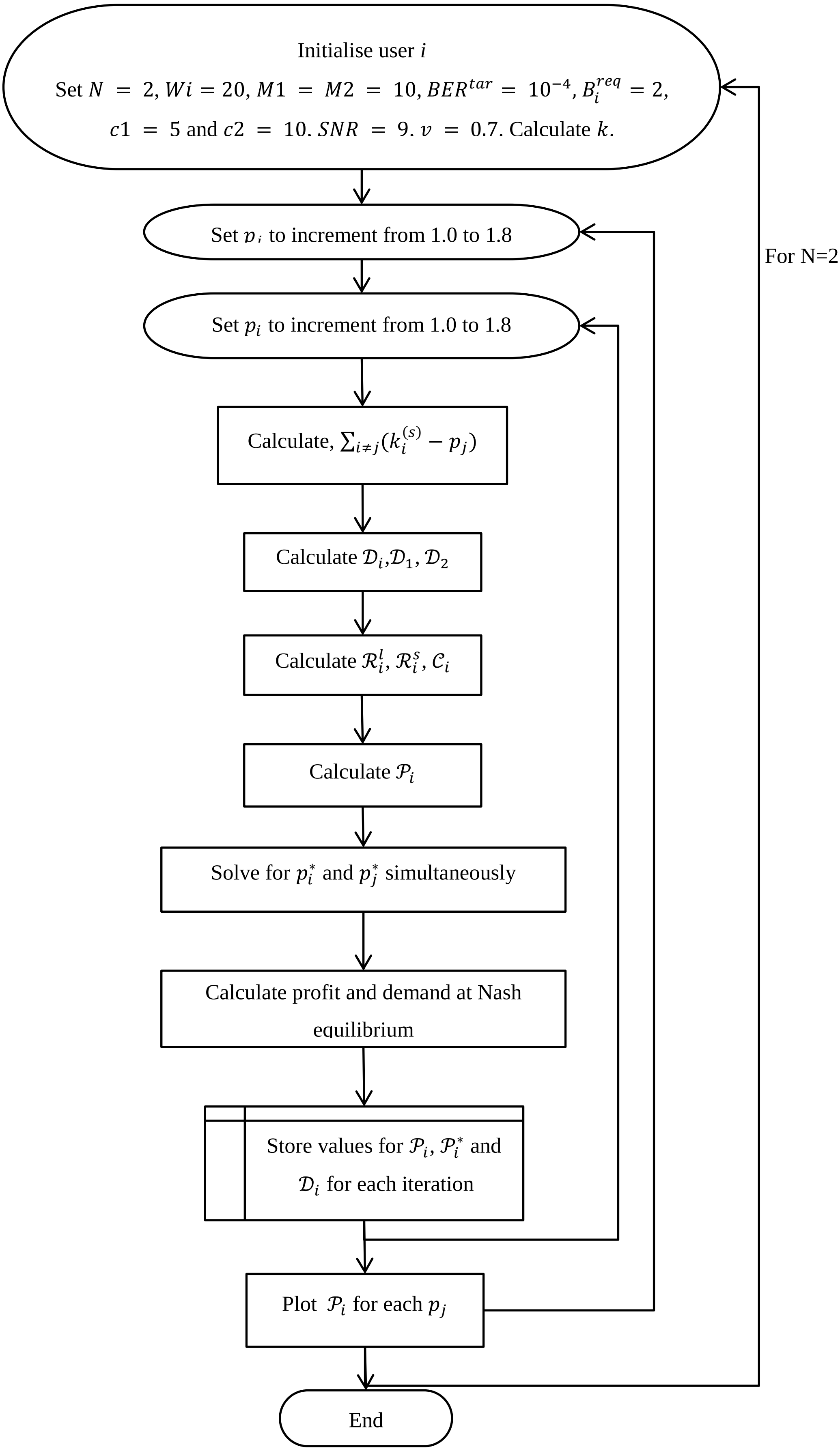


Figure 5-2: Flow-chart of the competitive pricing algorithm

5.3 Test cases

To test the performance of the pricing strategies, the following test cases are used. Each test case requires a variation of the initial parameters.

5.3.1 Efficiency of the pricing strategies

The total profit of both primary services achieved with market-equilibrium pricing and competitive pricing is shown with varying rates of the bandwidth requirement for each of the primary users.

5.3.2 Existence of pricing solutions

The existence of pricing solutions is shown by varying the value of the bandwidth requirement for each of the primary users, and plotting demand and supply functions for the market-equilibrium pricing strategy and the best responses for the competitive pricing strategies to determine the if there is a market-equilibrium and Nash equilibrium respectively. The best response of one primary service is a linear function of price offered by the other primary service (40). The market-equilibrium is the point where the spectrum supply and spectrum demand curves meet and Nash-equilibrium is at the point where the best response functions intersect.

5.3.3 Variations of primary services profit with offered price

By varying the price offered by primary service one and two, the relationship of the profit to the price can be determined.

5.3.4 Variations of profit under different channel qualities

By varying the channel qualities, the relationship between the channel quality and the profit of primary service 1 and 2 can be determined as well as the relationship between the demand for primary service 1 and 2 under different channel qualities.

5.3.5 Impact of spectrum substitutability factor

By varying the channel qualities, the relationship between the substitutability factor and the profit of primary service 1 and 2 can be determined as well as the relationship between the demand for primary service 1 and 2 under different substitutability factor.

5.4 Simulation Results

This section details the computer simulation results of the two pricing strategies, market-equilibrium and competitive.

5.4.1 Efficiency of the pricing strategies

Figure 5-6, Figure 5-7, Figure 5-8 shows the total profit achieved for both pricing models, competitive pricing model and market-equilibrium pricing model, for varying rates of the bandwidth requirement for each of the primary users.

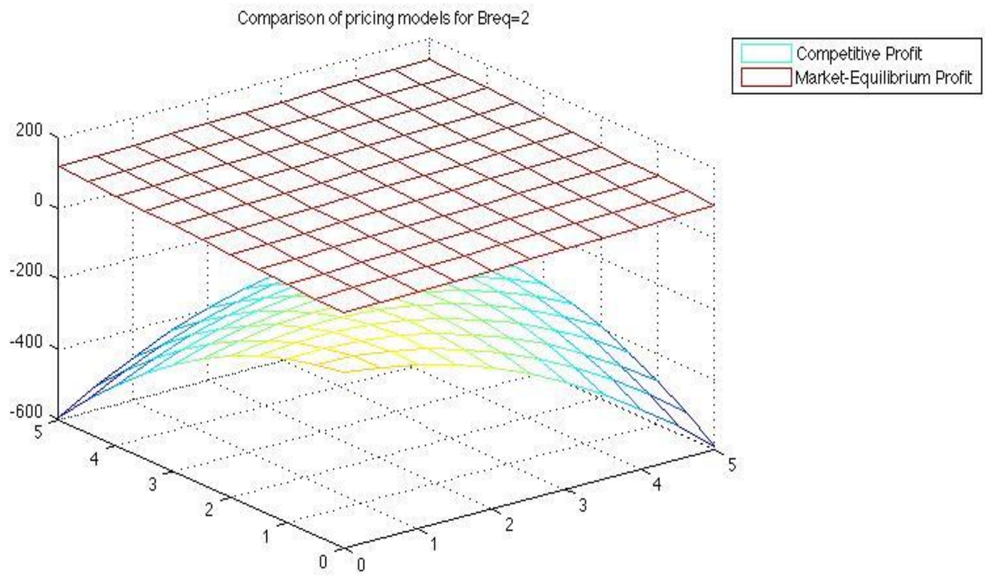


Figure 5-3: The total profit for both pricing strategies with the bandwidth requirement per user set to 2 Mbps

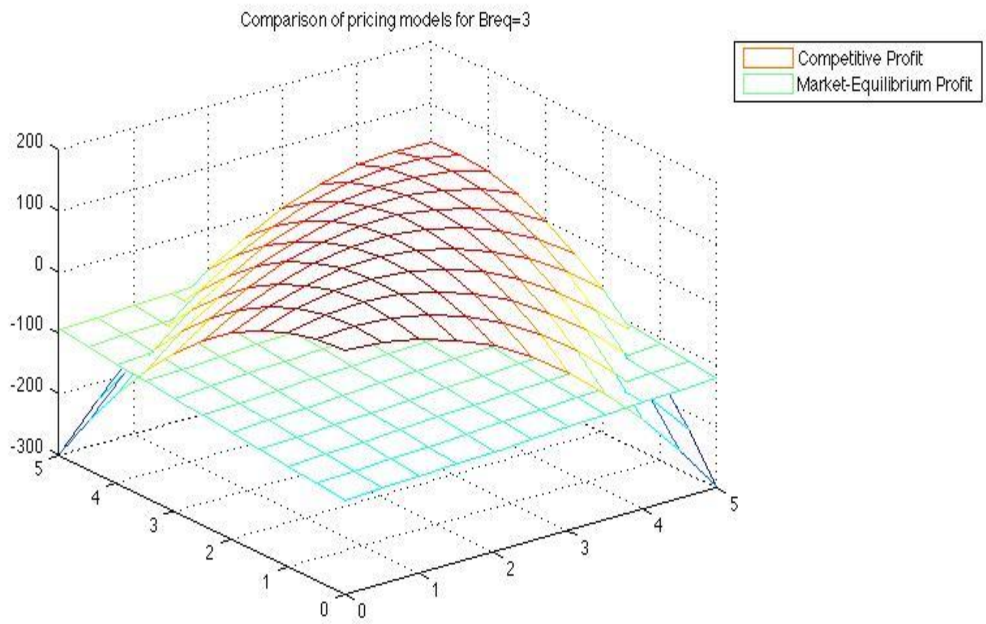


Figure 5-4: The total profit for both pricing strategies with the bandwidth requirement per user set to 3 Mbps

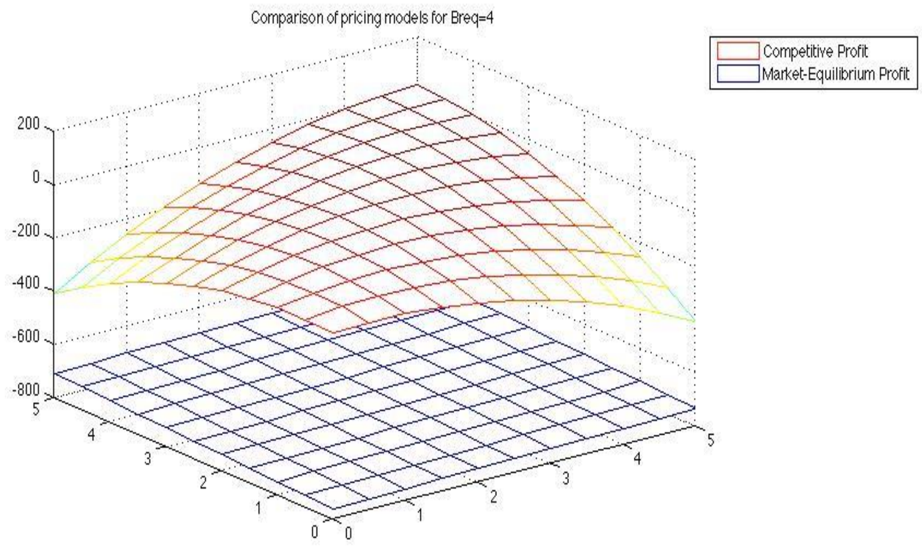


Figure 5-5: The total profit for both pricing strategies with the bandwidth requirement per user set to 4 Mbps

It can be seen the higher the bandwidth requirement for each of the primary users, the higher the revenue earned from the competitive pricing model, alternatively, the revenue earned from market-equilibrium pricing model decreases as the bandwidth requirement for each of the primary users increases.

5.4.2 Existence of pricing solutions

Figure 5-6, Figure 5-7, Figure 5-8 and Figure 5-9 shows the effect on the market-equilibrium point for varying values of B_i^{req}, the bandwidth requirement for each of the primary users. The scenario shows the effect of on primary service one, with a fixed offered price for primary service two and a varying offered price for primary service two.

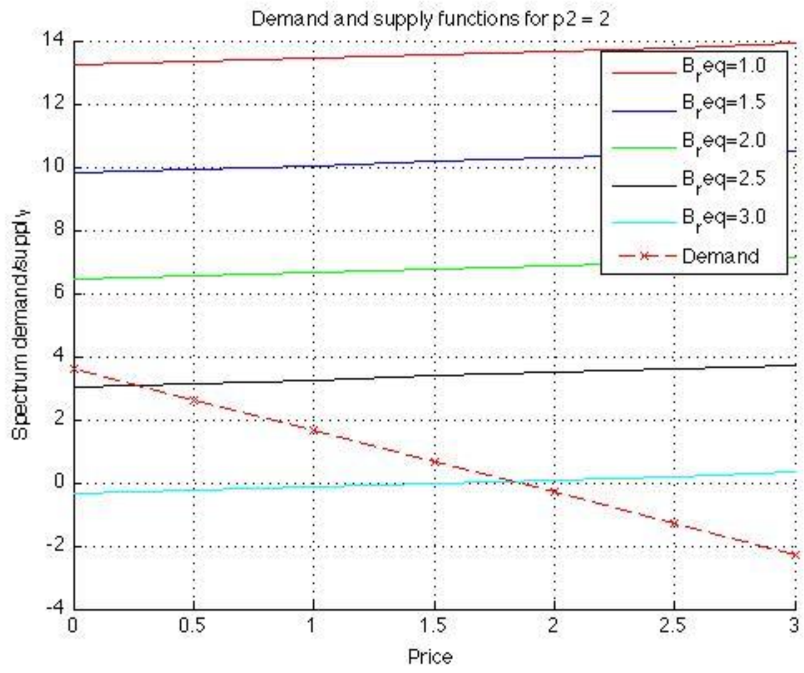


Figure 5-6: The spectrum supply and demand functions for a varying price offered by primary service provider 1 and the price offered by primary service provider 2 set to 2

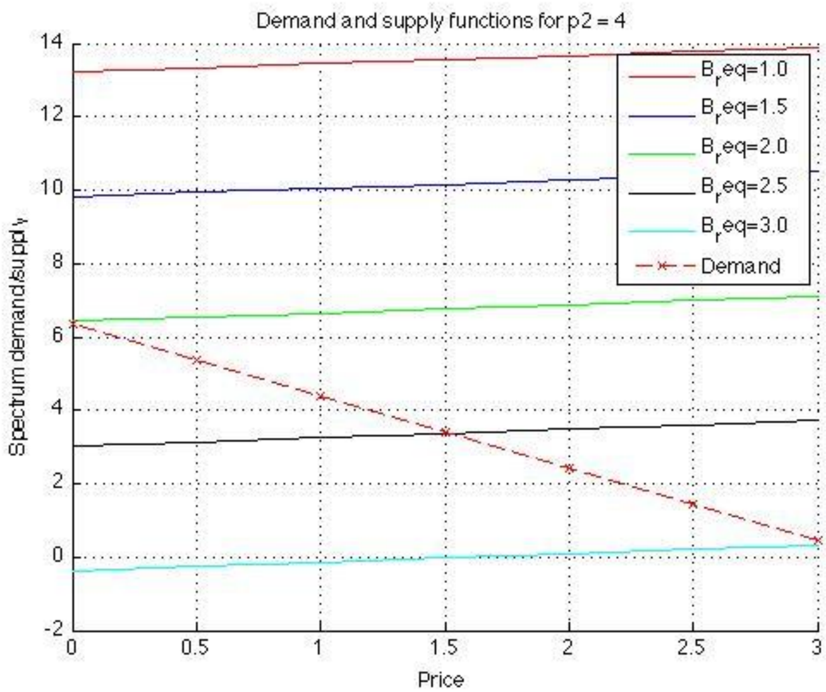


Figure 5-7: The spectrum supply and demand functions for a varying price offered by primary service provider 1 and the price offered by primary service provider 2 set to 4

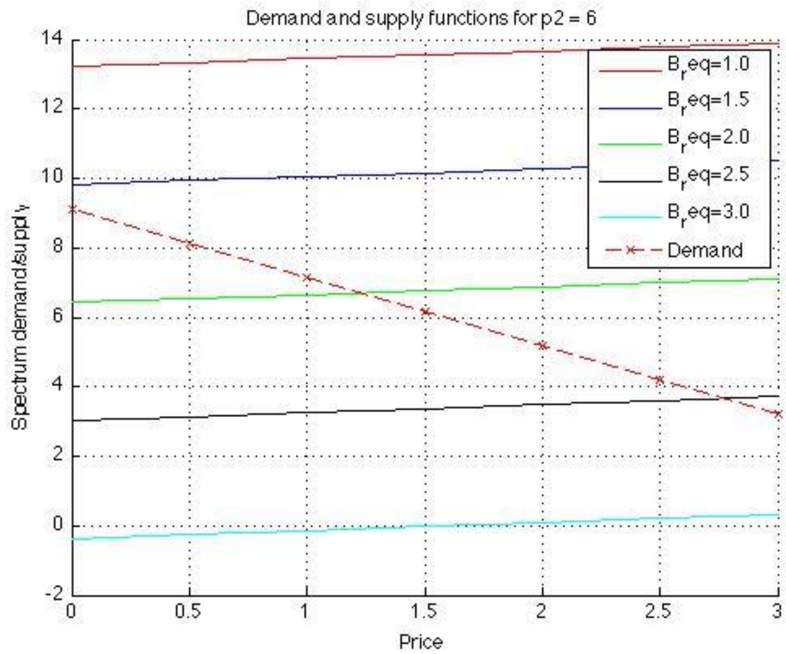


Figure 5-8: The spectrum supply and demand functions for a varying price offered by primary service provider 1 and the price offered by primary service provider 2 set to 6

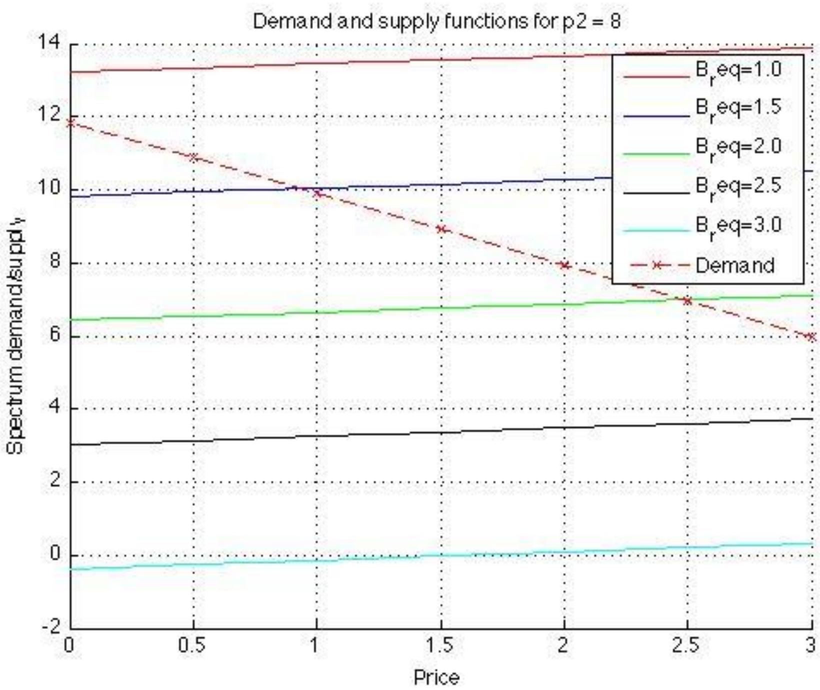


Figure 5-9: The spectrum supply and demand functions for a varying price offered by primary service provider 1 and the price offered by primary service provider 2 set to 8

It can be seen that as the bandwidth requirement of each of the primary users increases, the spectrum supply decreases and as the offered price for primary service one increases, so does the spectrum supply. Spectrum demand on the other hand is a decreasing function of offered price. These figures show that spectrum supply depends largely on the number of primary users and their bandwidth requirements. The market-equilibrium point is located at the point where the spectrum demand meets the spectrum supply; it is observed that market-equilibrium exists only for certain values of offered prices and certain ranges of bandwidth requirement (B_i^{req}).

For the competitive pricing model, the best responses were determined by the best response function, which is a linear representation of price offered by the other primary service. Figure 5-10, Figure 5-11 and Figure 5-12 shows the existence of Nash equilibrium is dependent on the number of primary users and their bandwidth requirements and only exists for certain ranges of bandwidth requirement and offered prices.

As the profit of primary service two increases, the profit of primary service one decreases, this is due to the effect of the offered price on the demand. If the price is too low, spectrum demand from the secondary service becomes high and the performance of the primary service degrades (causing a loss of profit due to the cost discount), however as profit from selling spectrum to secondary users is higher than the cost discount due to the performance degradation and the profit of the service increases. If the price is too high, the demand for bandwidth becomes low and the profit decreases [31].

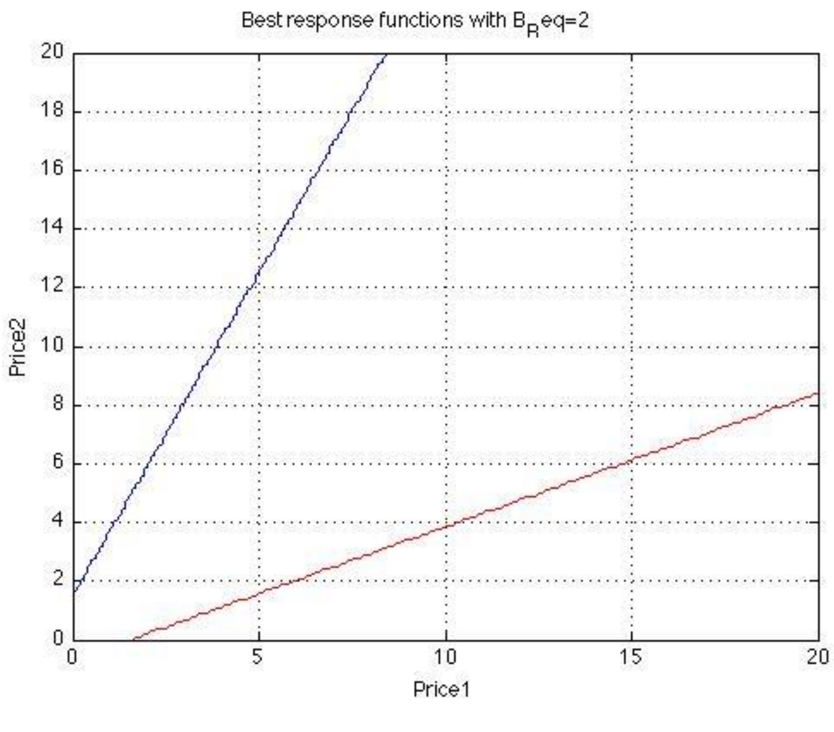


Figure 5-10: The best response functions for a fixed price offered by both primary service providers respectively with the bandwidth requirement per user set to 2

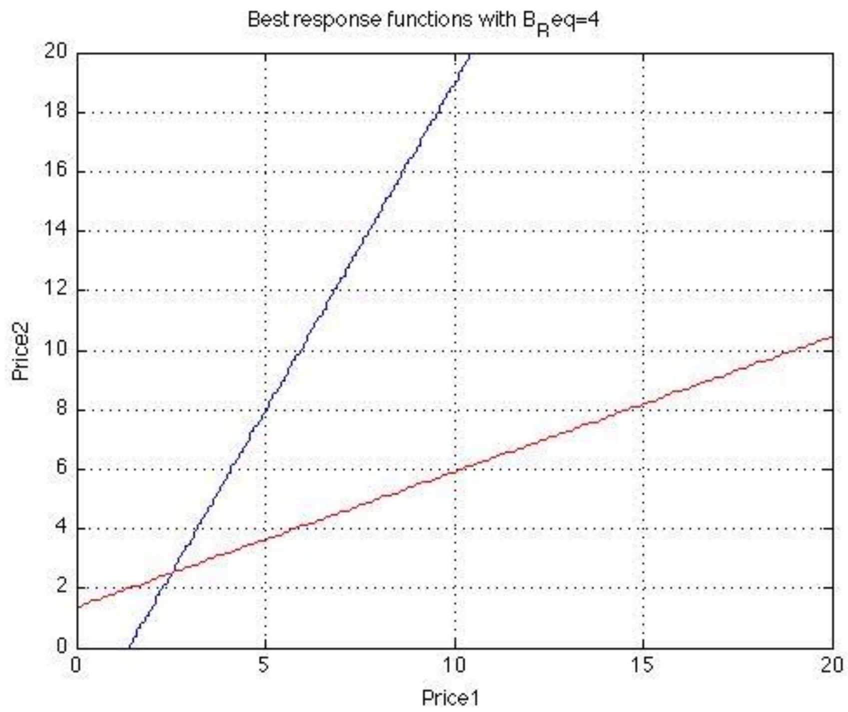


Figure 5-11: The best response functions for a fixed price offered by both primary service providers respectively with the bandwidth requirement per user set to 4

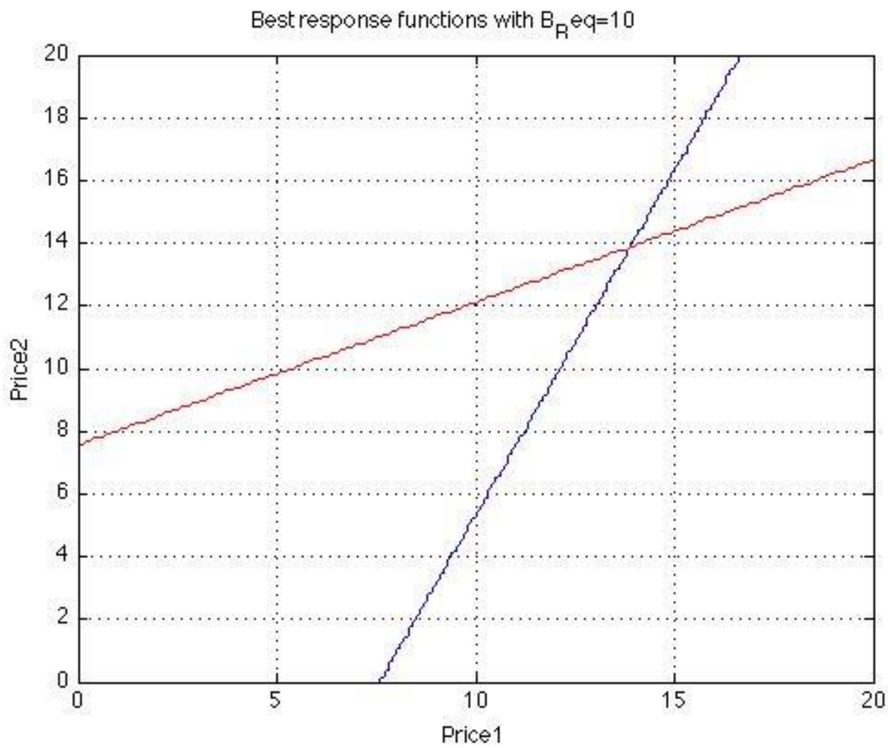


Figure 5-12: The best response functions for a fixed price offered by both primary service providers respectively with the bandwidth requirement per user set to 10

It can be seen that optimal values for the prices offered by primary service provider 1 and 2 are when they are the same. This way, both primary service providers earn the same revenue.

5.4.3 Variations of primary services profit with offered price

The effects on the profit of primary service one by varying the offered prices for primary service one and two are shown in Figure 5-13 below, where $p1$ is the price offered for primary service one, and $p2$ is the price offered to primary service two. The maximum profit attained for primary service one is at the highest point of the curve.

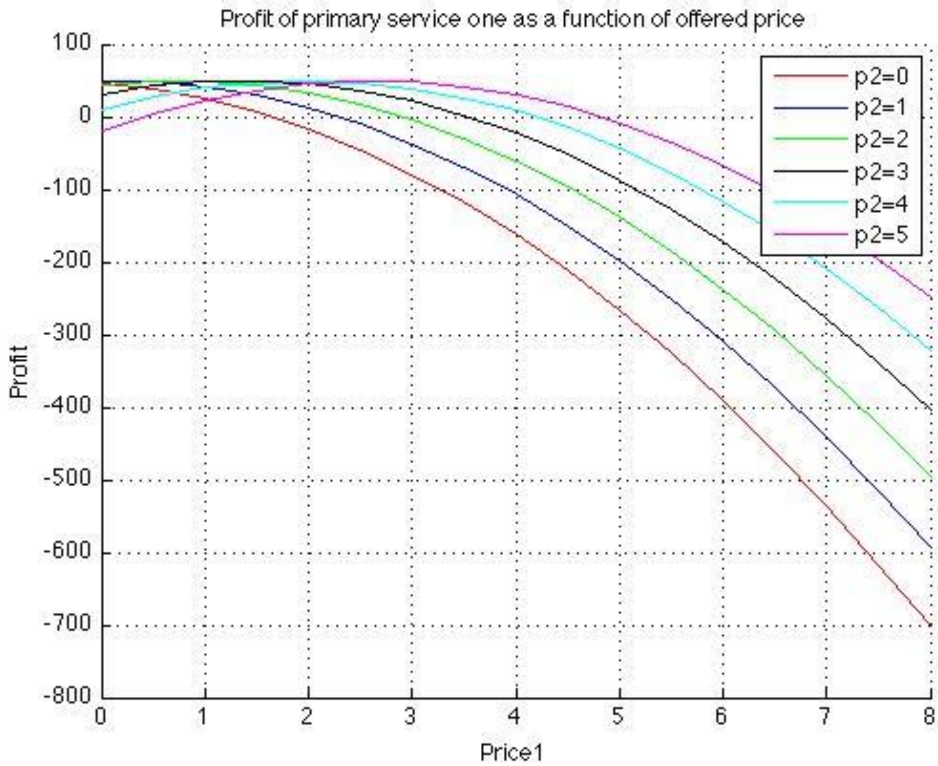


Figure 5-13: The profit of primary service 1 for a varying price offered by both primary service providers

As the offered price for primary service one increases, the cost due to the QoS degradation to the primary user's increases, resulting in a negative profit for one primary service. It can be seen that as the offered price increases, the profit increases to a point until the demand from secondary users decreases causing the profit to decrease. The point of the highest profit is known as the best response

5.4.4 Variations of profit under different channel qualities

Figure 5-14, Figure 5-15 and Figure 5-16 below show the effect of a varying channel quality on the profit in a market-equilibrium pricing strategy of primary service one with a set offered price. It can be seen that profit is a decreasing function of SNR and does not change with the offered price.

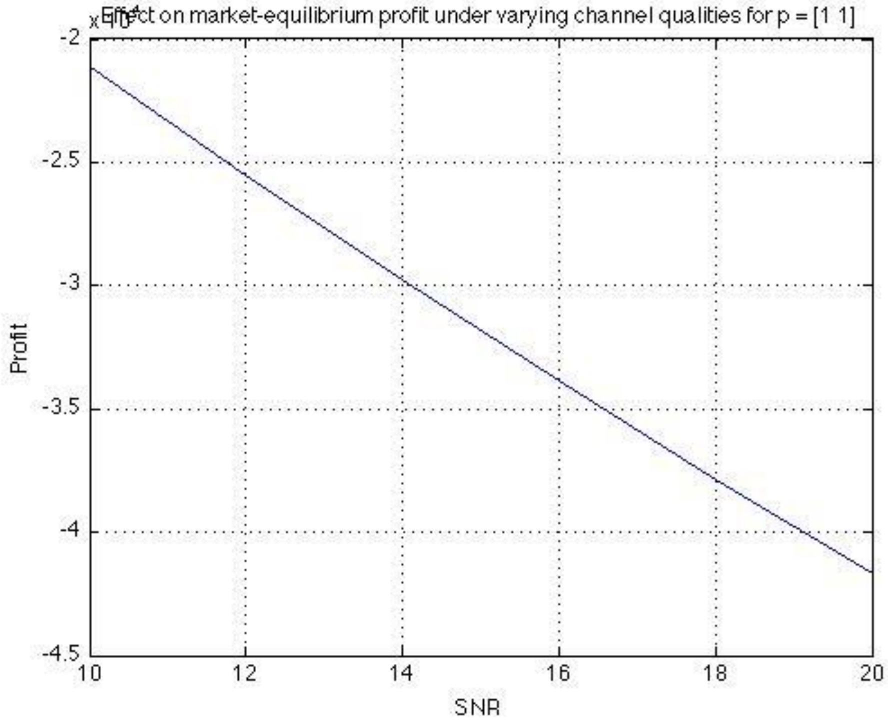


Figure 5-14: The profit of primary service provider 1 for a varying SNR and offered prices set to [1 1]

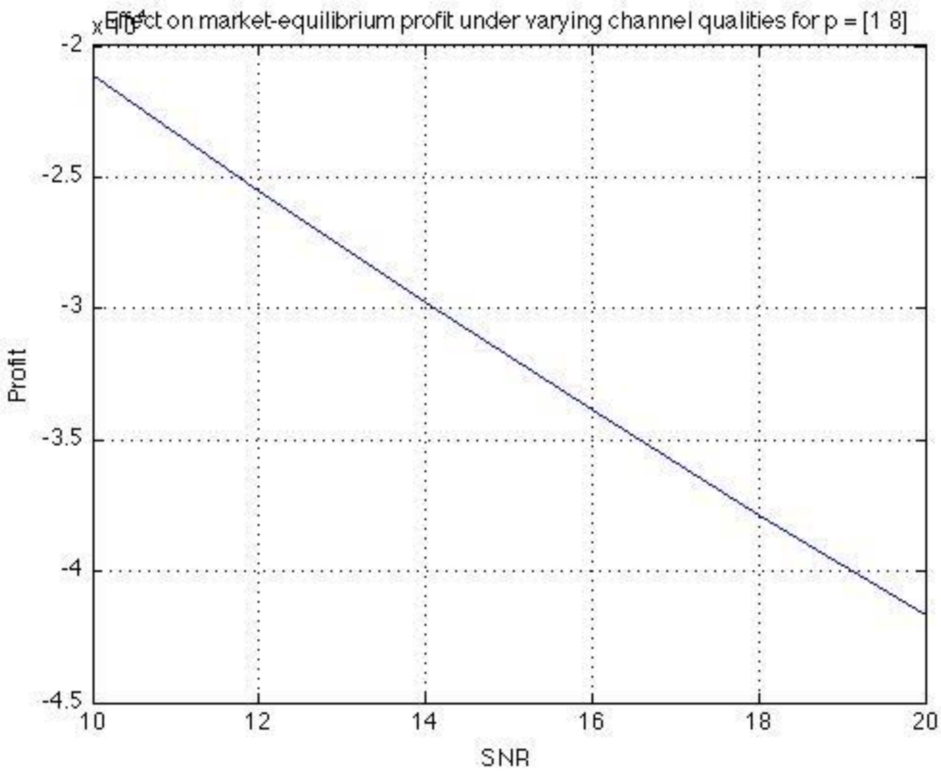


Figure 5-15: The profit of primary service provider 1 for a varying SNR and offered prices set to [1 8]

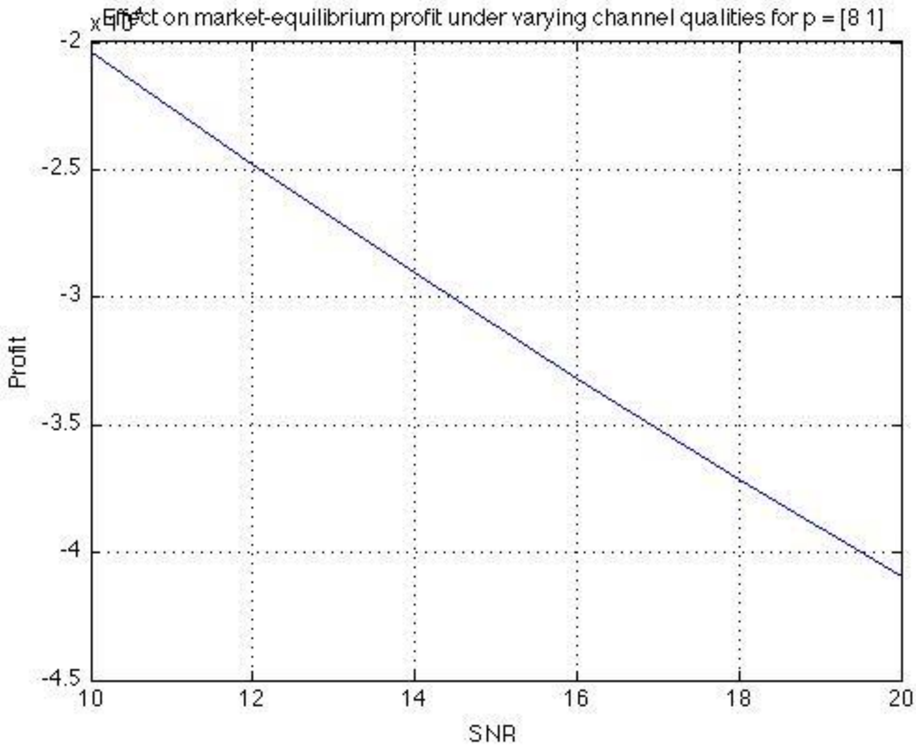


Figure 5-16: The profit of primary service provider 1 for a varying SNR and offered prices set to [8 1]

Figure 5-17, Figure 5-18 and Figure 5-19 below show the effect of a varying channel quality on the profit in a competitive pricing strategy of primary service one with a set offered price.

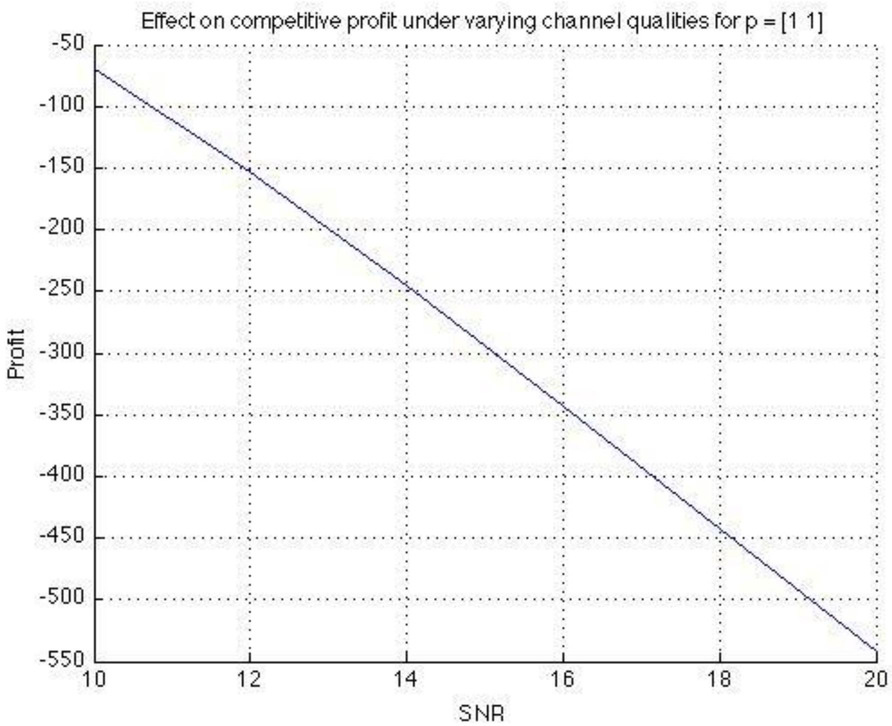


Figure 5-17: The profit of primary service provider 1 for a varying SNR and offered prices set to [1 1]

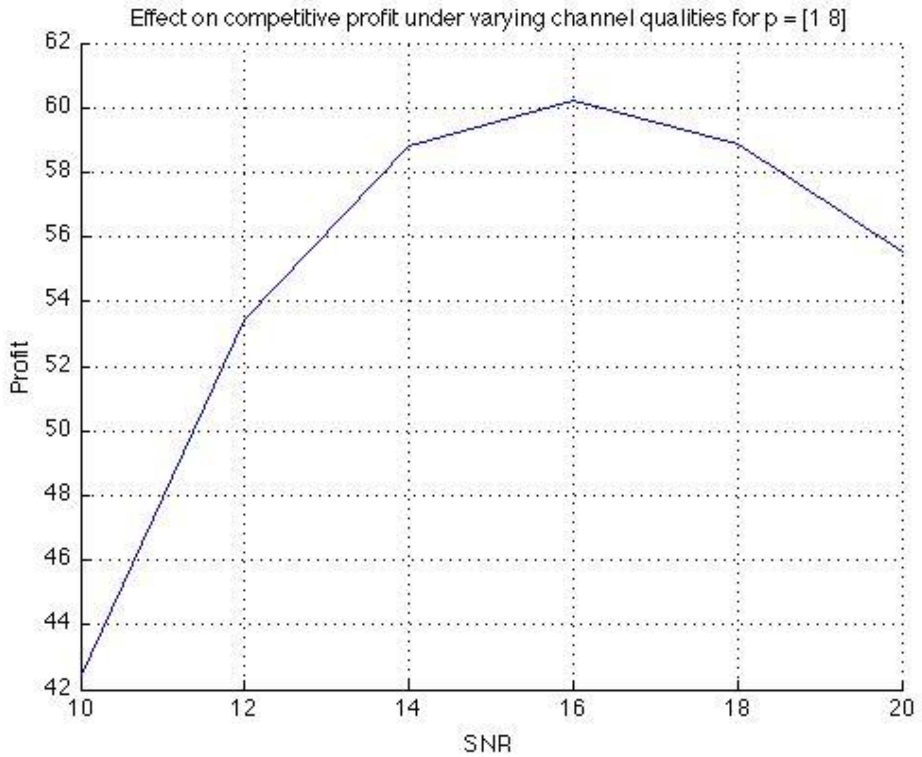


Figure 5-18: The profit of primary service provider 1 for a varying SNR and offered prices set to [1 8]

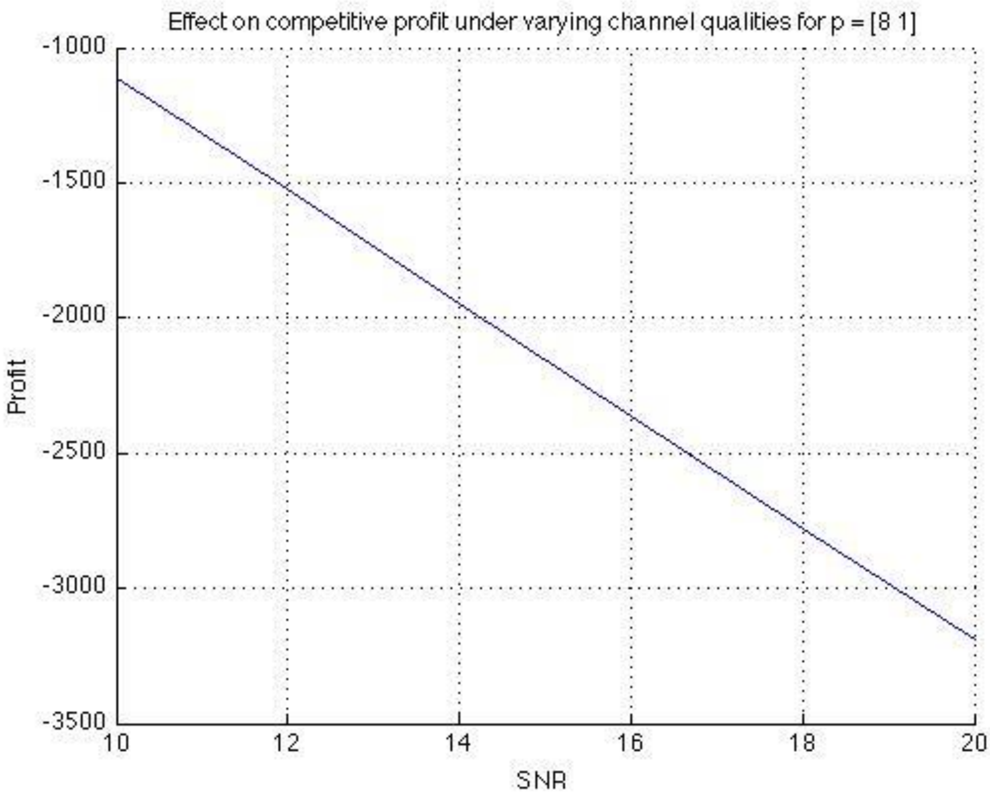


Figure 5-19: The profit of primary service provider 1 for a varying SNR and offered prices set to [8 1]

It can be seen that as the SNR increases, the profit decreases except for when there is a large difference between the offered prices, the profit increases until the point where the demand decreases. As the SNR increases, the demand increases but due to the set price in Figure 5-18 and Figure 5-19, the secondary users would rather buy the service from primary service two, hence causing a decrease in profit for primary service one.

5.4.5 Impact of spectrum substitutability factor

The spectrum substitutability, v is where, $v = 0$, a secondary user cannot switch among the frequency spectra, while for $v = 1$, a secondary user can switch among the operating frequency spectra freely. Figure 5-20, Figure 5-21 and Figure 5-22 shows the effect on profit of primary service one by varying v with a set offered price for primary service two. It can be seen that $v = 0.8$ provides the most flexibility and allows secondary users to switch between operating frequencies freely showing that as the offered price for service one becomes too high, the secondary user would rather move to primary service two, causing a decrease in profit for primary service one.

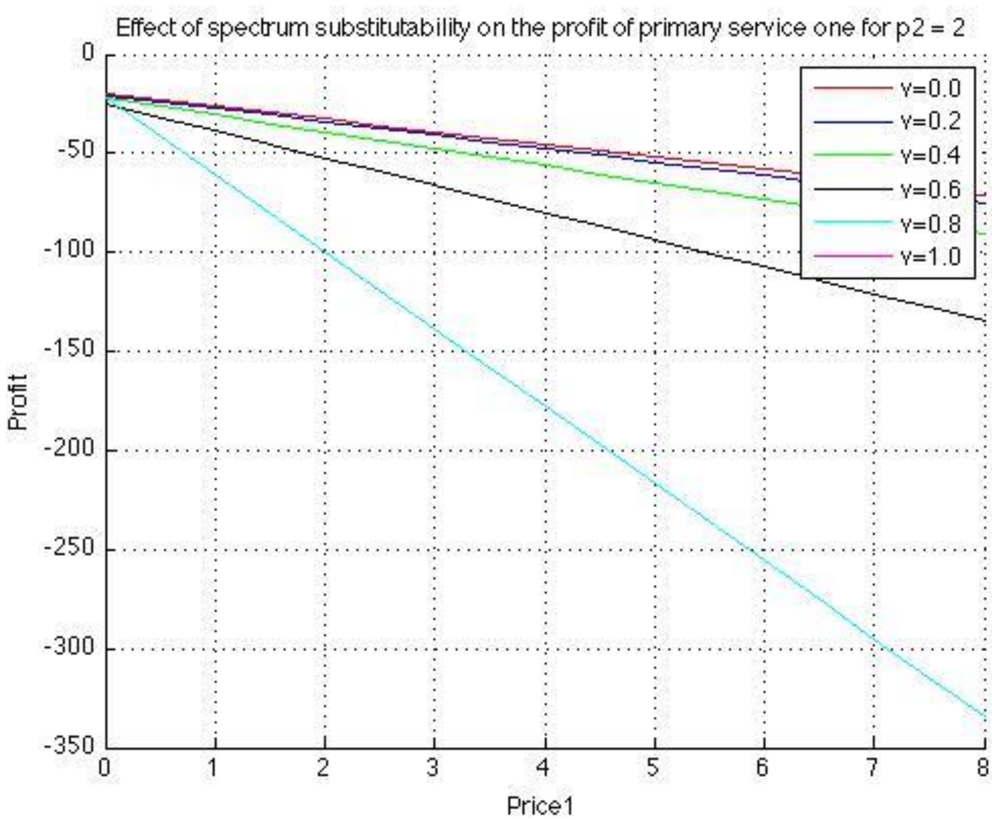


Figure 5-20: The profit of primary service provider 1 for varying spectrum substitutability and offered price by service provider 1 with the offered price of service provider 2set to 2

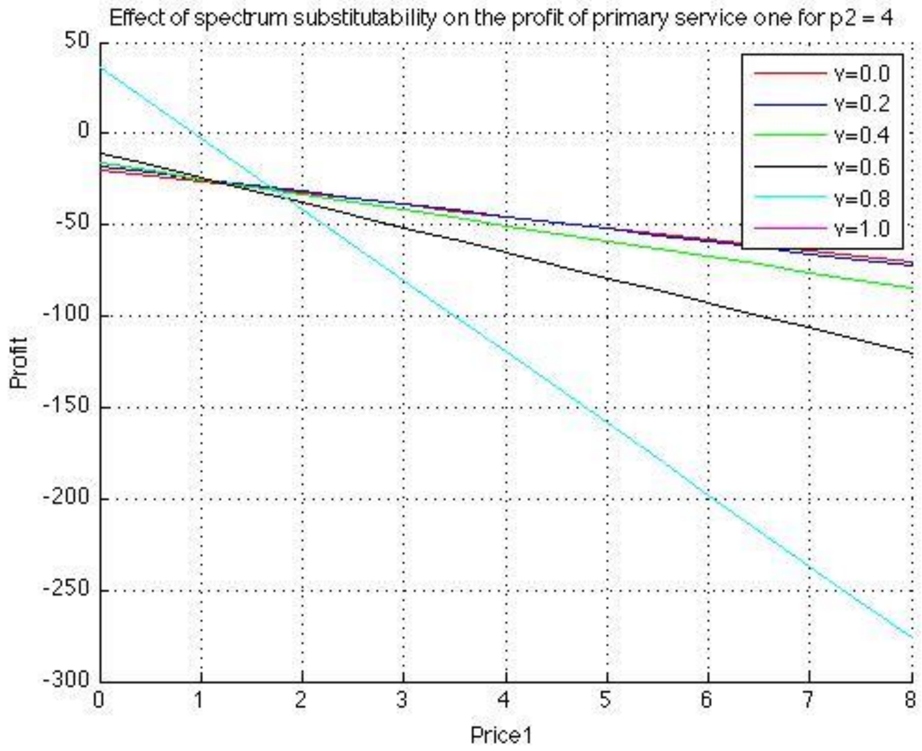


Figure 5-21: The profit of primary service provider 1 for varying spectrum substitutability and offered price by service provider 1 with the offered price of service provider 2 set to 4

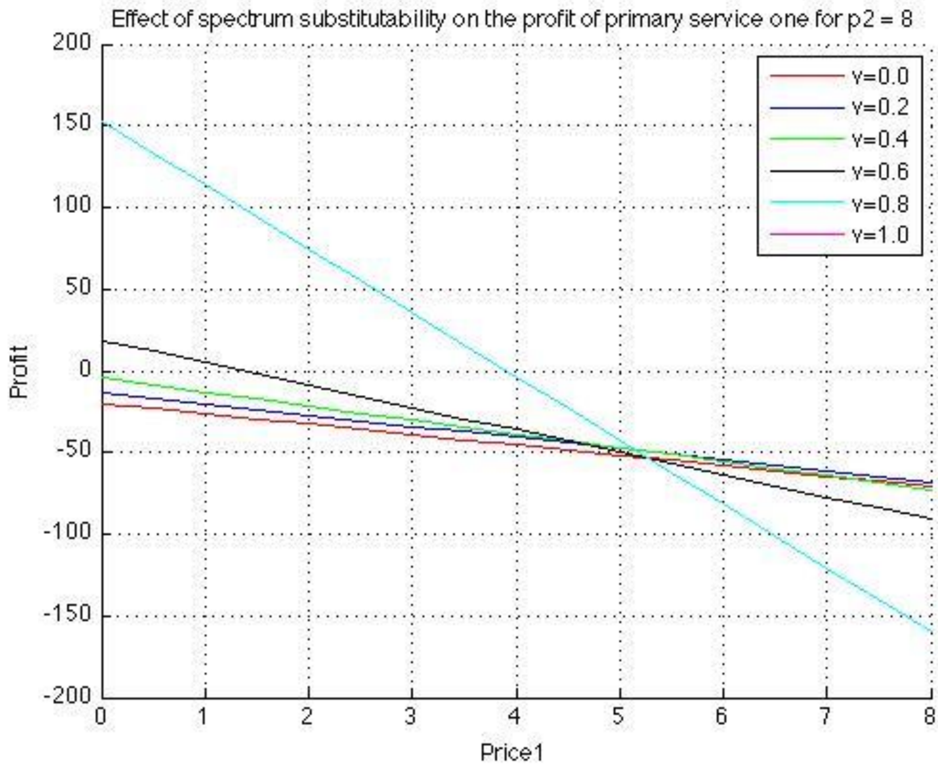


Figure 5-22: The profit of primary service provider 1 for varying spectrum substitutability and offered price by service provider 1 with the offered price of service provider 2 set to 8

5.4.6 Analysis of Simulation Results

From the simulations it can be seen that when comparing the competitive and market-equilibrium pricing models, at equilibrium, the competitive pricing model earns a higher revenue than the market-equilibrium. For the case of the market-equilibrium pricing model spectrum supply depends largely on the number of primary users and their bandwidth requirements and that market-equilibrium exists only for certain values of offered prices and certain ranges of bandwidth requirement. For the case of the competitive pricing model the optimal values for the prices offered by primary service provider 1 and 2 are when they are the same, this way, both primary service providers earn the same revenue, if one price is lower. The competitive pricing model exists when the bandwidth requirement is neither too high nor too low and is dependent on the number of primary users and their bandwidth requirements. In the competitive model, the offered price increases the profit increases to a point until the demand from secondary users decreases causing the profit to decrease, this is due to the cost discount offered to primary users which causes a loss of revenue.

The varying channel quality has no effect on the revenue earned with a market-equilibrium pricing model, however in the competitive pricing model; the varying SNR varies the demand, hence changing the profit. From the simulations, $v = 0.8$ provides the most flexibility and allows secondary users to switch between operating frequencies freely showing that as the offered price for service one becomes too high, the secondary user would rather move to primary service two, causing a decrease in profit for primary service one.

5.5 Methodology for Analysis of Spectrum Trading Regulations and Policy

As spectrum management is seen as a specialised field worldwide, it is often neglected in developing countries as these countries only participate in International Telecommunications Union (ITU) World Radiocommunication Conferences' (WRC) at a very high level [33]. In South Africa, the research in the spectrum management field is limited to a few discussion papers and opinions and the spectrum management roles appear to be a combination of institutional arrangements that exist elsewhere.

Due to the limited available documentation, the data collected is performed through document analysis of material that could be obtained through one-on-one interviews with decision makers of the regulator, policy maker and the entities listed in Appendix A which have been purposely selected for this research and may not be in the public domain. The interviewees were presented in advance with the questionnaire in Appendix B. The reviews of published and internal records and reports from regulators and policy makers as discussed by Peter Zimri in [33] are made reference too and discussed.

The interviewees were selected, due to their expertise in the field of spectrum management and the organisation they represented as major spectrum holders [33]. It should be noted that a few individuals directly involved in spectrum management within in the Regulator, the Policy maker and licensees have turned down interviews and the questionnaire. Interview candidates highlighted the sensitivities around both spectrum decisions and potential license applications, which may have compromised them personally and the organisations they represent. The interviews took place around 12/2013 and 01/2014.

5.6 Findings on Spectrum Trading Regulatory Approaches

The findings in this section are from analysis of relevant documentation and interviews with key informants. According to the all the interviewees [46][47][48][50] the market would be welcoming to sharing underutilised white spaces in spectrum with secondary users provided it does not cause interference, if there is interference, this would cause degraded service resulting in a lower revenue as well as unsatisfied customers and the main aim of the operator is to provide a guaranteed QoS.

The behaviour of the market if spectrum trading were allowed would be reflective on the market, as South Africa is not a mature market, the operators would act competitively to outdo the other operators and earn the most revenue based on the Competition Act in South Africa [46]. As spectrum is seen as a scarce resource due to the current spectrum assignment model, any operator with spectrum can have a monopoly over the underutilised spectrum in the band and the more important this band is the higher competition for it to be traded [47].

If operators cooperate with one another, jointly they can earn more revenue, however this is dependent on the market and the operators, it is believed that it is possible that the primary service providers would cooperate with the smaller role players, however the larger operators could try to eliminate the smaller role players from the market. [48] It is also believed every business would need to make a profit and the Competition Act and ICASA will ensure this is carried out fairly [46]. In the future, when the market matures, cooperation can be a viable option.

As operators pay very high rates for the spectrum licenses, they agree [46][47][48][49][50][51] it cannot be given to secondary users for free and this would distort their economic model. On the other hand, for the market with no spectrum, this would be a positive response.

In terms of considering methods of assigning spectrum, the regulator is considering a beauty contest with a closed bid auction [46]. Some operators [47] [48] feel auctioning could work well however the adverse effect could be that you end up paying more for spectrum than it is actually worth. It is good to include a social responsibility segment that the bidders need to submit in their applications.

The market considered for spectrum trading could include provision of backhaul links, rural sectors and point to multipoint sites for access networks [49][50]. In rural areas, operators [46][47][48] agree it would be good to provision limited free services as the economy is taking strain on what it can afford, however at the same time building network in rural areas come at a high cost and returns on these investments need to be recovered. These services to such areas can be offered as a premium service with reduced rates however a detailed market research will need to be carried out.

The suggested bands for this service are the freed up spectrum after the digital migration, 700 MHz and 800 MHz as well as the 2.6 GHz band [46][47][48]. It is imperative spectrum management is technology agnostic however the economy of scale of devices also need to be taken into account, e.g. devices are usually built supporting specific spectrum bands and if the TVWS is to be utilised, the devices will only be available in the future [50].

If the underutilised spectrum is part of the frequency spectrum that the operator could use to provide additional services, then they would buy back the additional spectrum, however the cost effectiveness of this will need to be investigated as well as the return of investment [48]. It is thought that it would be better to sell the underutilised spectrum directly to the secondary users as you make more revenue and it is better to keep the regulator involved as little as possible as this could cause delays [46]. However in the current scheme, the spectrum will need to be given back to ICASA as they are responsible to manage the assignments.

The regulator [51] agrees fully that spectrum needs to be used more efficiently and believes that all bands can be used for spectrum trading as every band is underutilised dependent on geographical areas. Dense areas can be seen to have scarce spectrum. Spectrum trading is long overdue and current legislation does not prevent it except to the main telecommunications operators. Small players in the industry could be trading spectrum and the regulator would be unaware.

The regulator [51] feels the best pricing model for the market is a cooperative scheme as with infrastructure sharing, the highest profit can be attained. Infrastructure sharing assists in minimising the costs associated with installing and maintaining new infrastructure. The regulator can be involved in enforcing a cooperative pricing model, however if they are involved, they cannot select who the spectrum is awarded too and the bids may appear unfair. The regulator [51] also agrees that spectrum should be given to secondary users for free as it distorts the economic model. With regards to the market for spectrum trading, if spectrum trading is used for backhaul, there may be an issue of who the primary user is and who the secondary user is and if spectrum trading is used for rural areas, who covers the costs.

5.7 Summary

This chapter presents the methodology used in this work, the simulation results obtained and the findings on regulations and policy in South Africa. An economic duopoly game is presented showing the effects of the variation in offered price, channel quality, spectrum substitutability, bandwidth requirement of primary users and effect of the offered price on the primary service. The effect on the profit of each pricing strategy is identified and noted. The findings from the survey show the market is ready is ready for spectrum trading for a fee charged to the secondary users for the service and the best band currently is TVWS band as it will be freed up in the near future. The concluding chapter discusses the key findings, conclusions, recommendations and future work.

Chapter 6

6 Conclusions and Recommendations

6.1 Introduction

In order to improve the utilization of the radio frequency spectrum, intelligent wireless communication systems such as cognitive radio technologies is utilized while accommodating the exponential growth in wireless services and applications. Enormous research challenges stand in the way of the implementation of cognitive networks. This research report provides theoretical and experimental solutions to one of such challenges, which are secondary user pricing strategies.

Chapter 1 of this research reports provided a brief introduction into the subject of spectrum sharing for cognitive radio networks and discussed the motivation and problem definition behind on whether spectrum trading is viable in South Africa; and if it is, which spectrum band is it most suited for. An extension of this research is would the payoffs received from secondary user spectrum trading would be worth charging for or is it acceptable for it to be offered for free? The objectives of this research are discussed.

Chapter 2 introduced the cognitive radio environment, its components, cognitive radio applications and dynamic spectrum access. An introduction is given to game theory and economic theories in dynamic spectrum access, which presented a method to model the relationship and interactions among primary and secondary users competing for spectrum and pricing strategies to assign the price. An optimal and stable solution for spectrum trading in terms of price and allocated spectrum is imperative to maximise the revenue of the seller and utility of the buyer while still satisfying both the seller and buyer and their solutions.

Chapter 3 extended the literature survey in Chapter Two to a review of specific application scenarios considered in some research papers from the perspective of contextual assessment of spectrum sharing, game theory and economic theory models applied in cognitive networks. The concluding section provides comparative approaches to telecommunication regulations with respect with spectrum trading and spectrum licensing in South Africa.

Chapter 4 presented the frameworks for two of the pricing strategies presented for cognitive radio networks in sufficient theoretical detail for a duopoly.

Chapter 5 presented the methods used for simulation and an interview on policy and regulation and the results obtained thereof. Test cases were applied to visualise the effects under different constraints.

6.2 Research Findings

The analysis and simulation results presented in this research report give insights into the two pricing strategies, market-equilibrium and competitive pricing. The succeeding subsection outlines these in perspective.

The solution of a market-equilibrium pricing strategy is market equilibrium, whereas for a competitive pricing strategy is Nash equilibrium. The primary services compete for revenue in a competitive pricing strategy and there is neither competition nor cooperation in a market-equilibrium strategy. The revenue attained in competitive pricing strategy is higher than that in a market-equilibrium strategy. The existence of a solution for both models is when the bandwidth requirement is neither too high nor too low. The spectrum substitutability can greatly influence the profit as if it is too high, users can move around the frequency spectrum bands freely changing depending on the offered price and if it is too low, users would not want to choose that service. This highlights the importance of the variables in the pricing strategies and shows the impact they have on the profit. From [31], it is seen that the cooperative model has the highest profit; however it is also the least stable for a distributed implementation (which is the most realistic) and market-equilibrium pricing model has the lowest profit of the three models. One of the downfalls of a competitive pricing model is that there is decrease in profit when there are more primary services competing. The regulator feels a cooperative pricing model is the most desirable for a developing country.

From the survey it is seen that the market would welcome sharing of underutilised white spaces in spectrum with secondary users provided it does not cause interference; however operators do not agree for it to be given to secondary users for free as this would distort their economic model as they pay very high rates for the spectrum licenses. The suggested bands for this service are the freed up spectrum after the digital migration, 700 MHz and 800 MHz (TVWS) as well as the 2.6 GHz band. Therefore it can be said that the best band for spectrum sharing is the TVWS band, however the regulator feels all the bands can be used for spectrum trading.

The behaviour of the market if spectrum trading were allowed would be reflective on the market, as South Africa is not a mature market, the operators would act competitively to outdo the other operators and earn the most revenue based on the Competition Act. If operators cooperate with one another, jointly they can earn more revenue, however this is dependent on the market and the operators, it is believed that it is possible that the primary service providers would cooperate with the smaller role players, however the larger operators could try to eliminate the smaller role players from the market. From this, it can be seen that the best pricing strategy currently suited to primary services is the competitive pricing strategy where the primary services compete with each other to make a profit.

The market considered proposed for spectrum trading could include provision of backhaul links, rural sectors and point to multipoint sites for access networks. In rural areas, this comes at high costs and operators feel these services to such areas can be offered as a premium service with reduced rates however not for free. If spectrum trading is used for backhaul, there may be an issue of who the primary user is and who the secondary user is.

6.3 Recommendations

In wireless communication, cognitive radio technology is generally perceived as a disruptive technology, because of its ability to autonomously adapt to changing network conditions in order to ensure a more flexible and spectrally efficient wireless network. The flexibility of cognitive radio comes with the downfall of complicated spectrum management and hence pricing.

In recent years, there have been many research studies that have investigated different methods for pricing and spectrum allocation. However, many are not from an economic and regulatory perspective.

Based on the background analyses presented in this research report, the following recommendations can be made:

- It is recommended that distributed algorithms be used to allow the primary service to learn the behaviour of other entities from the history
- It can also be recommended that a joint spectrum allocation and spectrum bidding model be proposed as it encompasses the cognitive radio environment as a whole
- ICASA should consider a competitive pricing model for the interaction between primary services to determine the best price
- ICASA need to undertake detailed market research on the benefits of providing secondary trading to the market, which market and the cost effectiveness to the operator
- ICASA need to weigh up the pros and cons of the different auction models before deciding on a 'beauty contest' with a closed bid auction and the impact it has on secondary trading

6.4 Future Work

The idea behind distributed algorithms is a good place to start as in a practical cognitive radio environment; a primary service may not have the complete network information. Distributed algorithms allow the primary service to learn the behaviour of other entities from the history and a distributed price adjustment algorithm is required to reach the final solution. To achieve distributed algorithms, an information exchange protocol is required for signalling. It would be of interest to determine a stability analysis of each algorithm.

Future work in allocation of spectrum and pricing models in joint spectrum bidding and pricing would be useful in the field, for e.g. using a double auction to assign spectrum and then a pricing strategy thereafter to charge for the spectrum.

Open research issues in this field include, a spectrum trading model for a large number of users, spectrum pricing under time-varying demand and supply, a risk-return model of dynamic spectrum access and the proposed market for spectrum trading.

From a regulation and policy view, it would be good to perform future work on which market spectrum trading can be used in as well as what level of service an operator would like to provide. The benefits of spectrum trading over other technologies such as the satellite band, fibre and 3G picocells need to be investigated.

6.5 Conclusion

In this research report, solutions to the problem of secondary user pricing strategies in a cognitive radio environment have been approached from the perspective of game and economic theory.

In terms of game theory, an attempt has been made to characterise the resolution of conflict among multiple cognitive radio users involved in selfish interaction. In terms of economic theory, an attempt has been made to characterise the conflict among primary services and the competition between them to maximise their revenue by selling spectrum to secondary users.

Two strategies, the competitive pricing strategy and the market-equilibrium strategy, which have derived from economic theory and game theory jointly, have been introduced and represented in sufficient theoretical details. The results from the simulations are compared to the regulatory and policy views in South Africa.

A survey of background technical details, a review of existing research works, comprehensive simulation and survey results have been presented. The simulation results presented indicate the competitive pricing strategy produces a higher profit the market-equilibrium pricing strategy and the competitive pricing strategy describes the behaviour of primary service providers in South Africa provided regulations are imposed to allow spectrum trading. Until ICASA revise the frequency plan and decide on a method of assigning spectrum, spectrum trading is just a thought for the future.

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Appendix A –List of Interviewees

Table 1: List of Interviewees

INSTITUTION	ORGANISATION
Major Spectrum Licence Holders	Telkom SA
	Neotel
	Vodacom SA
	Cell C
	MTN
Academia	University of Witwatersrand
	University of Namur
Portfolio Organisations	Ellipsis Regulatory Solutions
	Independent Communications Authority of South Africa

Appendix B – Questionnaire

1 December 2013

Telephone: 082 615 9310

Ref: Participation in Research Study

Dear Participant.................................

I Elicia Naidu, am a MSc in Electrical Engineering -Telecommunications student, at the University of Witwatersrand. I wish to conduct a survey on the regulatory side of "Secondary user pricing strategies in the cognitive radio environment". My academic supervisor is Professor Rex Van Olst and my co-supervisor is Ms. Lucienne Abrahams.

Strict measures will be taken in order to protect your anonymity and confidentiality to avoid any harm that may result as a result of your participation on this survey. Your participation in this study is voluntary, and you have the right to withdraw your participation at any stage of the research should you wish to do so. The benefits are that you will have the opportunity to share your experiences with your peer group. The research results may even be made available to you on request.

Yours Sincerely

E. Naidu

Table 1: Interview Questions

1.	Opinion on spectrum trading concepts	
	a.	How do you feel about sharing underutilised white spaces in spectrum with secondary users?
	b.	Would you be happy with partially degraded service for a profit
	c.	How do you think the other operators would behave if spectrum trading were allowed? (Would they act competitively with each other or cooperate to attain the highest total profit by selling spectrum to the secondary service?)
2.	Is it possible that the primary service providers would ever be cooperative and collude to attain the highest total profit from selling spectrum to the secondary service or collude to provide the best service to the public sector and rural areas?	

3.	Reaction to secondary users piggybacked on primary service provider’s underutilised spectrum for free?	
4.	Have you considered methods of assigning spectrum, e.g. auctioning of spectrum? If so, what are your views?	
5.	If you had to allow spectrum trading, what markets would you propose this to in South Africa or where would you want this service to be used (for both cases where spectrum is traded for free and at a cost to secondary users)	
6.	What type of footprint/impact would primary service providers like to have on the South African Economy?	
	a.	Would you as an operator like to better the economy by providing free access to public sectors or rural areas where access is poor or would you prefer to offer it a paid for service to those who can afford it?
	b.	If you had to price this service, would you offer it as a premium service or with reduced rates?
7.	What spectrum band do you feel would be the best suited for spectrum trading?	
8.	Have you considered pricing strategies as a primary service provider? If so, what have you considered?	
9.	Would you as a primary service provider buy back underutilised spectrum if you had that opportunity?	
10.	Would you prefer to sell the underutilised spectrum directly to secondary users or would it be better to sell it back to the regulator (ICASA)?	