## A PARAMETRIC EVALUATION OF THE ULTIMATE SHEAR CAPACITY OF REINFORCED CONCRETE ELEMENTS

Michael Graham Cross

A thesis submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Doctor of Philosophy.

Johannesburg, 1985.

I declare that this thesis is my own, unaided work. It is being submitted for the degree of Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

M G Cross

15th

day of OCTOBER

1985

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#### ABSTRACT

Although the subject of shear performance of reinforced concrete structural elements has been well researched in recent years and is well documented through the medium of a number of sophisticated models, a qualitative and quantitative appreciation of shear failure in a general sense over the broad spectrum of structural types normally encountered in practice oppears to be lacking amongst designers. Each of the current models for shear also appears to have limits of applicability regarding specific zones or types of shear failure, whereas the phenomenon of shear failure itself is of interest to the structural engineer over a wide range of structural types, including structural elements both unreinforced for shear and reinforced for shear.

There thus appears to be a place for a model for shear which will tend to link existing models and which will also concentrate on transition zones and unusual types of shear failure, in order to contribute towards the better qualitative and quantitative assessment by the designer of the phenomenon of shear failure.

This work aims at establishing such a universal model for shear and also at contributing toward a better understanding of shear behaviour of a broad range of structural types.

#### ACKNOWLEDGEMENTS

I gratefully acknowledge the contributions of the following:

Professor Alan Kemp, whose technical and academic ability, leadership qualities and friendship are an inspiration to those working with him.

The Cement Industries Fund, for generous financial backing of this work.

The technical staff of the Civil Engineering Department for their assistance, in particular John Dyer and John Davis.

Mike Stylianakis, for invaluable assistance in aspects of the test programme.

My family, for bearing and being with me.

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#### LIST OF SYMBOLS

a = shear arm of the structural element. (mm) For two-point loading on beams this is traditionally the dimension from the support to the nearer load. In particular reference to the model developed within this work, this is the horizontal dimension of the diagonal shear crack from the compression zone to the point of intersection with the flexural reinforcement, and varies between limits of 0,4d and 2d.

A = cross-sectional area of the flexural reinforcement at the point where it intersects the diagonal shear crack. (mm<sup>2</sup>)  $A_{sh} = cross-sectional area of a horizontal link$ (usually 2 legs) in the shear arm zone. (mm<sup>2</sup>)

a the the

- $A_{sv}$  = cross-sectional area of a vertical link (usually 2 legs) in the shear arm zone. (mm<sup>2</sup>)
- b = width of the structural element under consideration. (mm)
- c = average, axial, compressive prestress
  in a structural element. (MPa)
- d = effective depth of the structural element. (mm)
- D = diagonal compression force in the truss analogy model. (N)

 $E_{c}$  = Elastic Modulus of the concrete. (MPa)

- E\_ = Elastic Modulus of the steel reinforcement. (MPa)
- f = concrete crushing strength in the structural
   element. (MPa)

 $f_{cu}$  = mean concrete cube strength. (MPa)

xi

 $f_v = ave$  ge yield stress of the steel reinforcement. (MPa)

F = average force in a link (usually 2 legs). (N)

 $F_v =$  yield force in a link (usually 2 legs). (N)

- g = lateral dimension of square support column. (mm)
- h = overall depth of the structural element
   (usually with reference to slabs). (mm)

 $k_c$  = subgrade modulus for normal concrete. (N/mm<sup>3</sup>)

1 = span of the structural element. (mm)

- M = ultimate bending moment in a structural element at the section under consideration. (N mm)
- M = plastic moment capacity of a truss analogy model
  structural element at a particular section
  subjected to combined moment and shear. (N mm)

 $M_t$  = transfer moment in flat plate structures. (N mm)

- P<sub>1</sub> = yield force in the lower stringer of the truss analogy model structural element. (N)
- q = hypothetical equivalent UDL acting on the diagonal shear crack, derived from residual principal tensions. (N/mm)
- r = hypothetical equivalent UDL acting on the diagonal shear crack, resulting from the action of the shear reinforcement. (N/mm)

- u = hypothetical span of the diagonal shear crack. (mm)
- UDL = uniformly distributed load (for beam elements the units are N/mm or kN/m and for loads on slabs  $kN/m^2$ )

 $v_a$  = aggregate interlock component of resistant shear

Nr

- ----

 $v_c$  = compression zone component of resistant shea. stress. (MPa)

V = ultimate shear force applied to a structural element at a particular section. (N)

 $V_a$  = aggregate interlock component of resistant shear

- $V_a$ , = enhanced aggregate interlock component of resistant shear force. (N)
- $V_{c}$  = compression zone component of resistant shear force. (N)
- $V_d$  = dowel action component of resistant shear force. (N)
- $V_r$  = total vertical link component ~ resistant shear force, prior to loss te interlock. (N)
- V = total vertical yield force in links crossing
   the diagonal shear crack, after loss of
   aggregate interlock. (N)
- V = ultimate shear capacity of a truss analogy model
   structural element subjected to combined moment
   and shear. (N)
- $V_{po}$  = ultimate shear capacity of a truss analogy model structural element, subjected to zero moment. (N)

- V = ultimate shear capacity of a truss analogy model
  structural element in terms of the mechanical
  link reinforcement ratio. (N)
- $V_{fc}$  = shear force at which web crushing of the truss analogy model structural element occurs. (N)

 $V_{Rp}$  = mean ultimate punching shear resistance. (N)

 $V_p$  = total mean ultimate shear resistance. (N)

- W = width of the shear crack in the Walraven model. (mm)
- w = total maximum half crack width of the diagonal shear crack in the vicinity of the neutral axis at the instant of loss of aggregate interlock. (mm)
- w' = contribution of span deformations to w above. (mm)
- w'' = contribution of spring support deformation to w above. (mm)
- x = postulated average zone of debonding of the flexural reinforcement from the point of intersection of the reinforcement with the diagonal shear crack. (mm)

 $Z_1$  = force in the lower chord of the truss analogy

model structural element. (N)

- $\zeta_{c}$  = mechanical link reinforcement ratio.
- θ = average slope of the diagonal shear crack to the axis of the structural element. (usually to the horizontal) (deg)
- µ = link reinforcement ratio (vertical or horizontal).
- $\xi$  = resistant shear stress enhancement factor for depth.
- p = flexural reinforcement ratio (by common usage usually expressed as a percentage) (%)
- σ<sub>c</sub> = compressive stress in the diagonal compression field of the truss analogy model structural element. (MPa)
- τ = shear stress along a preformed shear crack in the Walraven model (with a/d = 0). (MPa)

#### 1 INTRODUCTION

While it is accepted that a significant amount of research has been conducted in the last few decades in the field of shear strength of reinforced concrete structural elements, complete consensus on this issue has not yet been reached between various researchers or practitioners. It would therefore be somewhat presumptuous for any individual to assume that "the riddle of shear" had been completely solved. In addition, this work in no way intends to repudiate the variety of excellent existing models for shear. Rather, considerable attention has been paid to areas perceived by practicing designers, in South Africa in particular, to be sufficient cause for concern to warrant queries or doubts about current code approaches or practices in the field of the shear evaluation of reinforced concrete structures. In addition, observed shear phenomena are examined in terms of current models and in detecting some limits of applicability of certain models, a generalised model incorporating aspects of existing models and

introducing some new concepts is used to explain these phenomena adequately.

The disquiet expressed by practitioners is amply justified by a relatively recent history of structural collapses on a global scale either attributable to shear failure or where shear failure has been a prime suspect. South Africa has certainly not been guiltless in contributing its share to this record.

Ductility at the ultimate limit state in a reinforced concrete structural element is essential for for most current design approaches. Total collapse can be averted by redistribution of load or removal of an in ulsime load after the absorption of the appropriate quantum of energy by the structure. This form of "benign" partial collapse can only occur if the structure exhibits sufficient ductility for reasonable load redistribution. In contrast, it is generally accepted that shear failure of reinforced concrete structures is one of the most severe inhibitors of the achievement of structural ductility. Thus in cases of abrupt or catastrophic failures of reinforced concrete structures the prime suspect is justifiably often considered to be shear, since this brittle mode of failure is virtually the hallmark of shear collapse.

Even in cases of unserviceability of certain structures, the role played by shear capacity can also be significant, since the imminence of a potentially disastrous situation might well be predictable in terms of the correct interpretation of the nature of the cracking observed in the structure. This may be invaluable information in determining a prognosis for the structure by all the parties concerned.

It is thus appropriate that certain case studies should be identified at this point in order that clarification on the approach adopted in this evaluation of the shear performance of certain structural elements is obtained. Reference to specific cases of structural collapse or unserviceability and to queries raised by practitioners in a more general sense thus forms the framework of this thesis and this approach has identified the following areas as warranting investigation and a large number of both laboratory and field tests have been undertaken on a considerable variety of structural elements as outlined below:

## 1.1 CORBELS

In both precast and in-situ reinforced concrete corbel construction there appears to be a need for a clear interpretation of their structural performance with respect to shear capacity. This need was amply highlighted by a relatively recent collapse

in South Africa of a chimney flue, where the corbel arrangement supporting the flue brickwork was extremely high on the list of possible causes; the mode of failure of the corbels suspected as being a shear failure. The prototype corbel arrangement used in this particular construction was of a precast type which was dowelled to the reinforced concrete flue wall and thus a detailed examination of the mechanics of dowel action itself is warranted in addition to an evaluation of the other modes of shear failure that are likely to occur in corbels.

## 1.2 PUNCHING SHEAR IN REINFORCED CONCRETE SLABS

Queries appear to be relatively commonplace among practitioners as to the punching shear performance of deep reinforced concrete slabs. Pilecaps, raft foundations and even pad foundations are also frequently the source of similar queries with respect to punching shear performance. The reservations expressed by practiticners are often justified as an investigation of current state of the art codes reveals that they can give radically conflicting recommendations in this regard. Even in normally proportioned slabs, in flat plate construction, there appear to be some differences of opinion among designers (and researchers) as to the qualitative assessment of a situation which could be in-

dicative of imminent shear failure. Confusion sometimes exists between an interpretation of purching shear failure or fan yield-line failure.

Evidence of flexural, serviceability cracking on the upper surface of flat plate structures can be mistakenly interpreted as representing imminent punching shear failure of the slab. On the other hand, punching shear failure has indeed been observed in a variety of slab structures, and control of occurrences such as these is of vital importance to the designer.

## 1.3 DEEP BEAMS AND REINFORCED CONCRETE WEBS

Evidence of diagonal serviceability cracking has been observed in the webs of certain bridge structures. This type of cracking is usually associated with either shear or torsional forces in the structure. Since an indication as to the imminence of abrupt shear failure is required in situations such as this, an evaluation of this phenomenon is also given consideration in this thesis. The evaluation of the serviceability implications of cracks such as these, in addition to impending ultimate limit states, is obviously also of considerable importance, especially in aggressive environments with respect to corrosion.

## 1.4 LOCAL BOND

In working load codes the local bond calculation traditionally formed an integral part of the shear check routine. The relevance of such a calculation within the scope of an ultimate limit state approach to shear strength is questionable and it is thus of interest to give attention to an evaluation of this phenomenon.

#### 1.5 FLANGE-WEB INTERFACES

In both general reinforced concrete construction and composite steel and reinforced concrete construction shear forces are usually generated along the flange-web interfaces of T- and box-beams. The capacity of such structural elements to transmit these shear forces across this interface, particularly at the ultimate limit state, obviously demands considerable attention. The achievement of the ultimate limit state with respect to shear capacity in such situations is analogous in a qualitative sense with that of deep corbels and is consequently given consideration on this basis in this thesis.

## 1.6 PRESTRESSED CONCRETE SLABS

Post-tensioned prestressed concrete construction is now being used fairly widely in flat plate structures. A limiting criterion and relatively unknown quantity in the design of these slabs appears to be punching shear capacity at the support columns. This investigation is undertaken in parallel with that for general reinforced concrete flat plate structures in order that clear comparisons and contrasts can be made. Attention is given to unbonded specimens in particular.

#### 1.7 A MODEL FOR SHEAR

In addition to being an essential exercise in terms of academic understanding of shear behaviour, it is also of considerable importance to the designer to be able to make a qualitative assessment of incipient diagonal failure modes, as this will influence the approach adopted in reinforcing for shear. The use of an appropriate model for shear should facilitate this exercise. The evaluation of shear capacity relative to flexural capacity in this regard is also of considerable importance.

It is thus not only on the basis of these perceived areas of general concern that this selection of specimens meriting research was considered. As will become evident in the formulation of a generalised model for the explanation and better understanding of shear behaviour, it is necessary that a suitable variety of types of shear failure be studied in order to establish the overall applicability of the model to this phenomenon.

Many models for shear exist and many researchers have made significant contributions to various types or regimes of shear failure. Each model appears to be well suited to a specific range of specimens or regime of shear failure, yet many seem to have limitations of applicability, especially where the type of shear failure changes substantially. As an example a model based on the truss analogy for reinforced concrete beams with vertical links cannot predict shear behaviour for the equivalent beam unreinforced for shear or for the equivalent beam which may be loaded in very close proximity to a support. In addition, neither the plastic flow truss models nor the current "dowel action, aggregate interlock, compression zone" models provide a satisfactory explanation as to why scale effects occur in the shear evaluation of reinforced concrete structural elements, particularly those unreinforced for shear.

It is evident in testing a wide range of specimen types such as those selected above that specific parameters undeniably affect

the shear performance of structural elements. These parameters affect the shear capacity of reinforced concrete elements to different degrees and those identified within the scope of this thesis are as follows:

- (a) Average shear stress in the element due to ultimate shear force. Resistant shear stress is, however, dependent on a variety of geometric and material parameters. This complex parametric variation, resulting from the use of average stress as the measure of shear performance, has led to some confusion and lack of understanding of shear behaviour historically. The resistant shear stress is currently recognised as being dependent on four major parameters, particularly for elements uareinforced for shear, as follows:
  - (i) Grade of concrete.
  - (ii) Steel ratio of flexural reinforcement, especially structural elements unreinforced for shear.
  - (iii) Depth of section of the structural element, especially for elements unreinforced for shear.

- (iv) The disposition of the shear crack relative to the applied shear force (also referred to as the shear-arm ratio.)
- (b) For sections reasonably lightly reinforced for shear, small diameter bars at closer centres appear to perform better than an equivalent area of large diameter steel at large centres in certain situations.
- (c) For sections reasonably lightly reinforced for shear, the ultimate shear failure mode can still be abrupt with no ductility.
- (d) Reinforcing steel which crosses an interface type shear crack normal to the direction of the applied shear force performs well as shear reinforcement.

In addition to the factors considered above, it is necessary to evaluate whether these principles are also applicable to situations of punching shear in flat slab and flat plate structures and also the effect that prestressing might have on elements of this type.

Any model for shear or conceptual model that intends to foster a better understanding of shear behaviour at the ultimate limit

state should ideally result in all the parameters mentioned above being adequately explained. Current models do not appear to explain all these phenomena simultaneously. The range of specimens selected in this thesis is thus chosen with the intention of concentrating on unexplained peripheral areas in addition to a limited number of the traditional beam specimen types which appear to be well modelled. It is also intended that the model developed here will to some extent explain all the phenomena considered above. After consideration of a variety of existing models for shear, the current "dowel action, aggregate interlock, compression zone" model was adopted as a basis for the formulation of a new, universal model for ultimate shear resistance.

## 2 EXISTING MODELS FOR SHEAR

It is evident that an extensive amount of research in the field of shear has recently been undertaken by eminent researchers on a global scale, and a significant array of models has been forthcoming. Despite the sophistication of these models it is disturbing that a situation of conflict and confusion still persists among designers and in fact even among researchers in evaluating the shear performance of certain structural elements. Code formulations of modern and well respected codes of practice vary to a considerable extent in their treatment of shear capacity while nevertheless remaining reasonably consistent in the evaluation of flexural ultimate limit states.

Before embarking on an attempt to explain these phenomena, it is considered appropriate at this point to examine certain of the better known and more highly regarded model formulations and the more significant attempts at understanding shear behaviour of reinforced concrete structural elements.

## 2.1 MODELS FOR ELEMENTS UNREINFORCED FOR SHEAR

#### 2.1.1 KANI ON SHEAR

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Undoubtedly one of the researchers in the last two decades that has had the most profound impact on the matter of the evaluation of the shear performance of reinforced concrete beams has been Professor Gaspar Kani<sup>24,25</sup> of the University of Toronto.

Kani has been responsible for the development of a theory for the evaluation of shear behaviour in reinforced concrete beams based on a model of the "concrete teeth" which are observed to form in most beams as they are loaded to failure. His most important contribution, however, is probably his conceptual understanding of the behaviour of shear. He identifies initially that shear evaluation in a general sense can only be truly understood by starting with the evaluation of specimens unreinforced for shear, thus giving considerable attention to this aspect of shear behaviour before developing the requirements for reinforcing for shear. He further perceives that shear failure itself is not necessarily always of concern and that it is not always the critical ultimate limit state. The vulnerability of a structural element to shear failure relative to other failure modes is in

fact a function of several geometric and other parameters assoclated with the structural element. This concept is also fundamental to the philosophy of this thesis. In this context it is believed that an appreciation of the importance of shear failure in relationship to flexural failure in particular is in fact a logical extension of the very fundamentals of the concept of reinforcing brittle materials. Although digressing from Kani's work specifically, it is considered appropriate in terms of the objectives of this thesis, to examine briefly the engineering fundamentals of reinforced concrete at this point. Brittle materials, when used in significant structural applications, suffer abrupt failure primarily in regions of tension. Typically, in attempting to use a brittle material such as concrete suitably in a structural application, this tension would invariably in the first instance result from the effects of flexure. Early developers of the technique of reinforcing the brittie material of concrete would thus observe typical, first failures to be rupture in bending tension of the structural element in the zone of maximum bending moment. This tendency would then be remedied by rcinforcing across this crack with an appropriate quantity and quality of steel reinforcement. At the outset, if the quantity of "bending" reinforcement thus added is such that the ultimate moment capacity of the composite reinforced section is less than that which caused the cracking, then little has been achieved as the ultimate limit state is not enhanced and, as important, remains non-ductile. This assessment is of course on purely

flexural grounds and is not intended to reflect the significance of reinforcing for shrinkage or other effects. By the very act of reinforcing for flexural cracking, bending tension failure of the structural element is inhibited, and the ultimate limit state is forced to reflect elsewhere and in a different mode. This would obviously occur at the point in the element which is the most vulnerable to tension after bending tension failure. This could occur in several zones in the element and typically could be either crushing of the concrete in the compression zone (in reality this means tensile splitting due to very large compressive forces and associated Poisson's Ratio effects), or failure due to diagonal tension cracks which are usually observed to occur in zones of high shear force. This second mode of subsequent failure has thus been traditionally referred to as shear failure and early evaluation was related to shear stress, which depended only on concrete grade for its resistant capacity. This approach has probably been responsible to some extent for the lack of understanding of shear behaviour in reinforced concrete structural elements.

The development of the shear diagonal tension cracks results in the first instance from the stress trajectories which occur in a homogeneous element as shown in Figure 2.1. Immediately after the formation of the first diagonal crack, the trajectories are likely to change substantially, however, as the element loses homogeneity. For a variety of reasons, including lack of defin-



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FIGURE 2.1 STRESS TRAJECTORIES IN A LOADED HOMOGENEOUS BEAM IN PLANE STRESS. Itive knowledge of the load that a particular structural element may be subjected to during its design life, and of the plastic compatibility requirements for load redistribution, it is now realised that ductility is of fundamental importance regarding structural design. It is partly this realisation that has led to the introduction and development of limit state codes of practice and has led to the associated concept of structural robustness.

Having thus determined that ductility is virtually a prerequisite for structural design in general and certainly for structures which demand robustness or ductility because of specific design criteria, it is apparent that it is this requirement, along with economic considerations, that prompted the use of reinforcement across the bending tension zone initially. It is equally apparent that in the event of reinforcing the flexural tension crack, the flexural ultimate limit state is not reached and the failure that reflects elsewhere is still abrupt, as in the case of shear failure in general, then adequate overall structural ductility has not yet been attained. The potential forms of failure that still exist must therefore be closely examined and prevented or rendered ductile themselves. Ensuring that the compression zone in the bending region does not fail abruptly is not specifically within the scope of this thesis but the principles involved are certainly similar and it is well known and understood that good flexural design of reinforced concrete is achieved by ensuring that ductile yield of the reinforcement occurs prior to abrupt

compression failure in the flexural zone. Ensuring that the abrupt failure which can potentially occur in the zones of high shear force is controlled is in fact the object of this thesis.

Kani thus made a significant impact on the understanding of shear failure of reinforced concrete beams unreinforced for shear in particular when he related this phenomenon to the ultimate limit state of the beam specimen in flexure. His reasoning was that flexural ductility of the beam is generally readily achieved and that if the shear capacity of the section exceeded the flexural capacity then overall structural ductility would be ensured. This important conceptual realisation led to the development of what Kani termed the "valley of diagonal failure" for reinforced concrete beams unreinforced for shear.

Kani's test results of the ratio of ultimate shear strength to ultimate flexural strength, when plotted against the parameters of steel ratio and shear arm ratio as depicted in Figure 2.2, resulted in the valley indicated. Kani perceived the solution to the riddle of shear in reinforced concrete beams to be the elimination of this valley of diagonal failure. It should be noted that the results referred to in this figure are specifically for rectangular reinforced concrete beams unreinforced for shear with two point loading as indicated the figure. It is also important to realise that the flexural ultimate limit state used in the formulation of this figure was based on the use of mild



FIGURE 2.2 KANI'S VALLEY OF DIAGONAL FAILURE FOR REINFORCED CONCRETE BEAMS SUBJECTED TO TWO POINT LOADING 24 steel flexural reinforcement with an average yield stress of approximately 300MPa.

Changes in the quality and characteristic strength of the flexural reinforcement will consequently affect the extent of the valley shown in the figure. Typically, using high yield flexural steel with a characteristic strength of approximately 425MPa, as is virtually accepted practice in this country, will increase the size of the valley substantially. Nevertheless, in observing the original diagram, it is apparent that certain shear-arm to depth ratios (a/d ratios), typically in the region of 2 to 3, result in a situation which is particularly vulnerable to shear failure relative to flexural failure. Other parameters investigated by Kani included shear in T-beams and and the shear performance of deep rectangular beams. His conclusion regarding T-beams was that they behaved virtually identically to rectangular beams of the web dimensions, with the flange carrying a limited proportion of the shear. He also concluded that the the valley of diagonal failure appeared to increase in magnitude with increase in depth for beams unreinforced for shear as indicated in Figure 2.3. Contrary to the expectations of many other researchers, Kani realised that this phenomenon was not only limited to specimens of smaller depths. In Kani's own words "the reduction in safety factor does not seem to approach a limiting value. An almost constant amount of reduction was observed whenever the leam depth was doubled." It would appear that no current models for shear



# FIGURE 2.3 THE EFFECT OF DEPTH OF SECTION ON KANI'S VALLEY OF DIAGONAL FAILURE.25

explain this phenomenon adequately. It is thus given detailed attention in this thesis as it is considered essential that any model developed which adequately explains all the aspects of shear behaviour should certainly include this phenomenon. Kani's modelling of his observed valley of diagonal failure took the form of a series of concrete teeth located between flexural cracks and a remaining concrete arch component. When these two components are plotted against a/d ratio they form a valley which closely simulates that observed in tests as shown in Figure 2.2. This model is then augmented by a wide variety of choices of shear reinforcement which all have the specific aim of eliminating the valley of diagonal failure and which tend to concentrate on the a/d ratio of the region of 2 to 3. The model representing shear behaviour, once adjusted by the use of suitable shear reinforcement, thus makes no claim to ductility in itself. Rather the object is considered to be that of purely raising the shear capacity of the structural element to a value at least as large as the flexural capacity, and thereby ensuring structural ductility. With respect to the formulation of a model for shear within the scope of this thesis, it is important to note Kani's observation that inclined links and inclined "bent-up" reinforcement (i.e. reinforcement which crosses the diagonal shear crack nearly normally) is generally more efficient in raising the shear capacity of the beam to the flexural capacity than that of reinforcement which crosses the crack obliquely, such as vertical

links, for beams which are reasonably lipting reinforced for shear.

Two aspects of Kani's valley of diagonal failure warrant special consideration. Firstly, it appears that premature shear failure (relative to flexural failure) for structural elements with a/d ratios larger than approximately 5 is extremely unlikely, especially for low steel ratios and thinner sections. It is immediately evident that this set of circumstances is typically applicable to reinforced concrete slabs which are supported normally (thus excluding provisionally the effects of punching) and the remoteness of encountering shear problems in normally proportioned slabs is thus well explained by this observed valley. The important consideration of punching shear, however, is not adequately modelled here and this is therefore perceived as an area requiring special attention within the scope of this work. On the other extreme of the valley (with respect to the a/d ratio) is another area warranting further attention as it is one of considerable controversy and where there is some difficulty in determining thear capacity relative to flexural capacity. Kani observed that premature shear failure in this zone of small a/dratio appeared to be unlikely for the specific tests he undertook. Changing certain parameters, however, can have a marked effect in this zone, and it is suspected that a significant effect could be achieved by changing reinforcement grade. The details of the evaluation of shear behaviour in this zone of small a/d ratios

are considered in the investigation of the shear performance of reinforced concrete corbels later in this work.

Kani has thus presented an invaluable conceptual appreciation of the problem of shear failure in reinforced concrete beams. Areas perceived as warranting closer attention in respect of his model are those for structural elements having a/d ratios less than unity, such as corbels, and the evaluation of punching shear in reinforced and prestressed concrete slabs. It is thus anticipated that some quantitative adjustments to Kani's valley are warranted for changes in specimen type and geometry, because his valley refers specifically to beams subjected to two point loading. The qualitative importance and concept of this valley will, however, be generally applicable.

## 2.1.2 EVALUATION OF THE SHEAR CRACK

Closely allied to the concept of the concrete tooth free body diagram attributable to a large extent to Kani, is the model which considers the free body diagram of the remaining concrete beam after the formation of a diagonal tension crack. Many prominent researchers have contributed in this area and among these are Taylor<sup>40,41</sup>, Fenwick and Paulay<sup>15</sup>. The justification for a model

of this type must rely on the ability of the structural element to accept substantially more load after the initial formation of the diagonal shear crack. This is in fact observed in virtually all test specimens. In considering the free body diagram in Figure 2.4 for an element unreinforced for shear, it is evident that the total resistance to shear force of the beam can be made up of three parts as represented by the following equation:

 $V_{R} = V_{d} + V_{a} + V_{c}$ 

where

 $\boldsymbol{V}_{\boldsymbol{R}}$  is the total ultimate shear resistance of the section. (N)

 $\boldsymbol{V}_{d}$  is the contribution to total ultimate shear resistance by dowel action of the flexural reinforcement. (N)

 $V_a$  is the contribution to total ultimate shear resistance by aggregate interlock. (N)

 $V_{\rm c}$  is the contribution to total ultimate shear resistance by the compression zone. (N)





FIGURE 2.4 TYPICAL DIAGONAL SHEAR CRACK AND FREE BODY DIAGRAM OF THE REMAINING PORTION OF A REINFORCED CONCRETE BEAM AFTER THE FORMATION OF A DIAGONAL TENSION CRACK. In consideration of this model for shear resistance of the reinforced concrete section, it should be noted that initial diagonal cracking is indeed generally observed to occur somewhat before ultimate load is reached, but the occurrence of diagonal cracking load coinciding with ultimate load cannot be discounted entirely. Instances occur, depending on structural parameters, where these do in fact coincide and it is hoped that the general model for shear developed here will adequately explain this phenomenon.

It is important in the understanding of shear behaviour to assess the relative magnitudes of the contributions to the total ultimate shear resistance of dowel action, aggregate interlock and compression zone for any particular structural element.

## 2.1.2.1 DOWEL ACTION

Although it is generally accepted that the contribution of dowel action to total shear resistance for conventional reinforced concrete beams is relatively small, it can have significant contribution in certain structural situations. Typically these might be dowelled joints in a variety of structural applications where the concrete parts are intentionally kept separate, or cases where reinforcement crosses a plane of significant weakness which Author Cross Michael Graham Name of thesis A Parametric Evaluation Of The Ultimate Shear Capacity Of Reinforced Concrete Elements. 1985

#### **PUBLISHER:**

University of the Witwatersrand, Johannesburg ©2013

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