## AN ADAPTIVE OPTIMIZING REGULATOR FOR AN AUTOGENOUS MILL

Mark John Bleloch

A dissertation submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg. in fulfilment of the requirements for the degree of Master of Science in Engineering.

## DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

Mr Bleloch

signed this 12 day of Sept. 1987

Declaration

An adaptive steady-state on-line optimizing regulator that will keep a plant at its optimum, as defined by a measurable objective, in the face of economically significant disturbances is proposed. The concepts and theory underlying the operation of the adaptive optimizing regulator are developed and discussed. A case study of a hypothetical simple ball mill grinding circuit is developed and simulated to test the operation of the regulator. Finally, the adaptive optimizer is proposed and tested by simulation as an integrated approach to the control and optimization of autogenous run-of-mine grinding circuits.

The optimizing regulator continuously estimates an internal model of the controlled process, and based on this, determines mill fresh solids feed rate and mill water flow rate such that a circuit performance objective is optimized. The dual requirements of controlling both the mill pulp load and the load of particles of grinding media size is thus directly and simultaneously addressed.

Results of the simulation study are presented and discussed.

Abstract

## PREFACE

## **ACKNOWLEDGEMENTS**

The supervision, valuable assistance and encouragement given by Prof.I.M. MacLeod is gratefully acknowledged.

I would like to thank my parents, Jean and John for their enthusiastic support. Their wisdom and understanding during the rough patches was much needed and appreciated.

In addition I would like to thank:

- The CSIR's foundation for Research and Development (FRD) for financial assistance.
- The Department of Electrical Engineering, University of the Witwatersrand

Preface

## TABLE OF CONTENTS

1.0 INTRODUCTION	1
1.1 Motivation for this Study	1
1.2 Aim of this Study	2
1.3 Overview of the Organisation of this Report	3
2.0 CONCEPT OF A GLOBAL CONTROL SYSTEM INCORPORATING	
AN ADAPTIVE OPTIMIZING REGULATOR	5
2.1 Problem Formulation	5
2.1.1 General Problem	5
2.1.2 Problem Simplification	7
2.2 Hierarchical Controller Structure	9
2.3 Theoretical Foundations for the Adaptive Optimizing Regulator	11
3.0 DETAILED THEORETICAL AND DESIGN CONSIDERATIONS .	15
3.1 Choice of Lower Level Regulators	15
3.2 On-line Objective Function Identification	16
3.2.1 Choice of Model	16
3.2.2 Estimation of Model Parameters	20
3.3 Making the On-line Objective Function Model Identification Ro-	
bust	23
3.3.1 Coping with Deterministic Disturbances and Plant Noise .	24
3.3.2 Coping with Modelling Error within the Bandwidth of Interest	29
3.4 Extracting the Steady-State Model from the Dynamic Model .	32
3.5 On-line Optimization	33
3.5.1.1 Introduction	33
3.5.1.2 Optimization Algorithm	34
3.6 Summary of Design Procedure and Outline of Algorithm	38

4.0 CASE STUDY: APPLICATION OF ADAPTIVE OPTIMIZER THEORY	
TO A BALL MILL	42
4.1 Introduction	42
4.2 Ball Mill Model	43
4.3 Grinding Circuit Control	48
4.4 Mill Solids Feedrate Regulator	50
4.5 Ball Mili Adaptive Optimizer	52
4.5.1 Problem Formulation	52
4.5.2 Optimizer	52
4.5.3 Simulation	55
4.6 Simulation Results	55
4.7 Conclusions	58
5.0 APPLICATION OF ADAPTIVE OPTIMIZING REGULATOR TO AN	
AUTOGENOUS GRINDING CIRCUIT	60
5.1 Background	60
5.1.1 Milling	60
5.1.2 Autogenous Mill Control	61
5.2 A New Look at Autogenous Mill Control Objectives	64
5.3 Applicability of Adaptive Optimizing Regulator	67
5.4 Simulation	69
3.4.1 Details of Autogenous Mill Model	70
5.4.2 Details of Adaptive Optimizing Regulator	71
5.5 Results of Simulation	77
5.6 Conclusions and Further Work	82
6.0 CONCLUSIONS	83
7.0 REFERENCES	85
APPENDIX A. AN EXAMPLE OF ROBUST MODEL IDENTIFICATION	91

APPENDIX B. EXAMPLE OF A SIMPLE OPTIMIZATION ALGORITHM	94
APPENDIX C. STEADY STATE RELATIONSHIPS FOR THE BALL MILL SIMULATION	
APPENDIX D. BALL MILL ADAPTIVE OPTIMIZER PASCAL PROGRAM LISTING	101
APPENDIX E. FILE CONTAINING PLANT PARAMETERS	112
APPENDIX F. FILE CONTAINING CONTROLLER PARAMETERS .	114
APPENDIX G. AUTOGENOUS MILL ADAPTIVE OPTIMIZER PASCAL SIMULATION	116
APPENDIX H. FILE CONTAINING AUTOGENOUS MILL INITIAL STATES	130
APPENDIX I. FILE CONTAINING GRINDING CIRCUIT CONSTANTS	132
APPENDIX J. FILE CONTAINING AUTOGENOUS MILL SIMULATION SETUP AND CONTROLLER PARAMETERS	134
APPENDIX K. PASCAL INCLUDE FILE MODEL.P	137
APPENDIX L. PASCAL INCLUDE FILE OUTPUT_CALC.P	143
APPENDIX M. PASCAI PROGRAM PLOT.P	146
APPENDIX N. VARIABLES DUMPED DURING A SIMULATION RUN	154

## LIST OF ILLUSTRATIONS

Figure	1.	General structure of a control system incorporating an op-	
		timizing regulator	8
Figure	2.	Hierarchical controller structure	10
Figure	3.	Block diagram of the internal structure of the adaptive	
		optimizing regulator	14
Figure	4.	The Basic Deadzone Function	30
Figure	5.	Functional Structure of Optimizing Regulator	41
Figure	6.	Simplified Ball Mill showing inputs and outputs	43
Figure	7.	Ball Mill Simulation Block Diagram	45
Figure	8.	Relationship between P and F showing the desired operating	
		point and a shift in this point due to a disturbance .	48
Figure	9,	Steady State relationship between P and F showing regions	
		of operation	50
Figure	10.	Simulation Results	57
Figure	11.	Typical Autogenous Grinding Circuit	62
Figure	12.	Autogenous Grinding Circuit incorporating an Adaptive Op-	
		timizing Regulator	72
Figure	13.	Simulation result using setup file: setup_6	80
Figure	14.	Simulation result using setup file: setup.7	81

## LIST OF SYMBOLS

t	discrete time variable, where t = n.time,
	n is a positive integer
<b>x</b> (t)	vector of system states
y(t)	measurable output vector
u(t)	vector of system inputs
<b>z</b> (t)	vector of measurable disturbances
d	r'ant dead-time as a multiple of the sampling time
$q^{-1} \cdot \dots \cdot$	backward shift operator
ξ	general variable representing noise
d	vector of unmeasurable plant disturbances
d <sub>s</sub>	vector of persistent plant disturbances
d <sub>f</sub>	vector of high frequency plant unmeasurable disturbances
r	vector of plant deterministic disturbances
y*	setpoint vector to control outputs $\mathbf{y}(t)$
Ψ	objective function measurement
Ŷ	model prediction of objective function value
θ	matrix of plant parameters
θ	matrix of estimated plant parameters
P	covariance matrix
Φ	regression vector

List of Symbols

ix

## ABBREVIATIONS

SISO Single input single output system

MIMO Multi-input multi-output system

PID Proportional Integral Derivative Controller

RLS Recursive least squares parameter estimation

DARMA Deterministic autoregressive moving average model

ROM Run-of-mine

Abbreviations

#### NOMENCLATURE

Comminution The breaking down of ore into finer material for the

purpose of extracting a mineral.

Autogenous milling Comminution of ore without the use of ferrous grind-

ing media.

Grinding media Components of mill load that cause breakage of small

particles.

Fines Components of mill load that are broken by grinding

media.

Charge Total contents of the mill. Includes grinding media,

fines and water dilution.

Pulp Mixture of fines and water dilution.

Product Ore emitted from the mill.

Coarseness Ratio of grinding media to pulp.

Required grind The mill material that has been ground to an accept-

ably small size.

Nomenclature

#### 1.0 INTRODUCTION

This chapter explains why the research into the theory and operation of an adaptive optimizing regulator was undertaken and defines the aims of the dissertation. It also gives an overview of the report structure.

For background material on digital computer process control the reader is referred to the standard text by Smith(1972). The reader looking for an up to date introduction to adaptive control, and more specifically for this study, on-line dynamic model identification, is faced with a bewildering array of intricate published papers. A fairly gentle, yet comprehensive survey paper (50 borg et al. 1986) is an advisable starting point. Background material on process optimization can be obtained in the references cited in the relevant sections.

## 1.1 MOTIVATION FOR THIS STUDY

The motivation for this research comes directly from the need in the South African mining industry for a solution to the autogenous run-of-mine milling problem. It is a practical problem characterised by the lack of reliable measurements in a harsh environment resulting in the need to use available measurements to their full potential. This involves taking advantage of the computing power now available and using available on-line numerical techniques for input-output modelling. Due to the large quantity of ore processed, a small improvement in mill control and hence a slightly more optimal operation is economically worthwhile.

There is a need for a fresh look at this problem from a more global and co-ordinated point of view to allow the synthesis of a systematic control strategy to achieve a given control objective.

A sensible route to take in tackling this problem with a new approach is first to provide solid theoretical foundations. The theory and design is approached from a general viewpoint because of its applicability to control problems other than autogenous mill control. It is beyond the scope of this research to develop the theory fully. The theory is developed as far as possible and then tested under predictable conditions in a simplified case study. Before actual implementation the controller would need further testing, by simulation, under fairly realistic and variable conditions.

## 1.2 AIM OF THIS STUDY

This work aims to:

- Motivate and define a suitable sub-problem in relation to the global objective of improving plant performance.
- Look at existing methods of solution and identify their shortcomings.
- Develop the theory to solve this sub-problem in the light of new developments in adaptive control.
- Show how the adaptive optimizing regulator fits into a global hierarchizal control system.

- Choose a suitable simple example plant to test the controller under simulation.
- To apply the adaptive optimizing regulator, using simulation, to an example plant.
- Try to solve a real world problem with the adaptive optimizing regulator. The chosen real world problem is o improve the control of an autogenous grinding mill circuit.

## 1.3 OVERVIEW OF THE ORGANISATION OF THIS REPORT

Chapter 2 introduces the concept and puts the adaptive optimizer into a process control context. Starting with a sufficiently general problem, realistic assumptions are introduced to simplify the problem until a solution is plausible within the framework of existing techniques and technology.

Chapter 3 gives detailed theoretical and design considerations for a general adaptive optimizing regulator module. The regulator is broken down into functional entities and each is discussed individually. An outline of the design procedure is summarised at the end of this chapter.

Chapter 4 documents a case study of the application of the adaptive optimizer theory to a simplified ball mill grinding circuit control and optimization problem. It presents a representative simulation and discusses the results.

Chapter 5 represents the culmination of the developments in previous chapters in solving the real world problem of autogenous mill control. Background to the milling problem is given, the autogenous milling control objectives are looked at in a new light, and the applicability of the adaptive optimizer is reinforced. The simulation environment used in this study is discussed as well as the details of the mill model and simulated implementation of the optimizer. Representative simulation runs are given and analyzed.

Finally, chapter 6 draws conclusions from the research. By carefully specifying the limitations and areas that need further research the future direction, namely to achieve the goal of implementation on a mine, is set.

# 2.0 CONCEPT OF A GLOBAL CONTROL SYSTEM INCORPORATING AN ADAPTIVE OPTIMIZING REGULATOR

## 2.1 PROBLEM FORMULATION

## 2.1.1 GENERAL PROBLEM

A very general description of the acceptable operation of a dynamic system consists of a set of n differential equations, a set of q inequality constraints and a set of m output relationships given by:

$$\frac{d}{-x} = f(x, u, d,)$$
 .....(1)

$$g(x,u,d) \leq 0$$
 .....(2)

$$y = h(x,u)$$
 .....(3)

where

x = n-vector of model state variables

u = 1-vector of manipulable control inputs

y ≡ m-vector of measurable outputs

d E r-vector of disturbance variables

Such a model can be used to obtain a current estimate ( at time t=0 ) of the optimum trajectory of the real system by finding control inputs  $\mathbf{u}(t)$  to minimize the integrated total of a specified objective function or

# 2.0 CONCEPT OF A GLOBAL CONTROL SYSTEM INCORPORATING AN ADAPTIVE OPTIMIZING REGULATOR

### 2.1 PROBLEM FORMULATION

## 2.1.1 GENERAL PROBLEM

A very general description of the acceptable operation of a dynamic system consists of a set of n differential equations, a set of q inequality constraints and a set of m output relationships given by:

$$\frac{d}{-x} = f(x, u, d,)$$
 .....(1)

$$g(x,u,d) \leq 0$$
 .....(2)

$$y = h(x,u) \dots \dots (3)$$

where

x ≡ n-vector of model state variables

u = 1-vector of manipulable control inputs

y ≡ m-vector of measurable outputs

d = r-vector of disturbance variables

Such a model can be used to obtain a current estimate ( at time t=0 ) of the optimum trajectory of the real system by finding control inputs  $\mathbf{u}(t)$  to minimize the integrated total of a specified objective function or

performance index,  $\Psi$ , over the time period (0,T). This optimization problem may be written as:

$$\min_{\mathbf{u}} \int_{0}^{T} \Psi(\mathbf{y}, \mathbf{u}, \mathbf{d}) dt \dots (4)$$

such that (1), (2) and (3) are satisfied.

Note that  $\Psi$  is an economic objective function that is possibly measured directly or in the more general case  $\Psi$  could be an arbitrary algebraic function of the control inputs  $\mathbf{U}(t)$ , plant outputs  $\mathbf{Y}(t)$  and measurable disturbances which are a subset of  $\mathbf{d}(t)$ . For particular values of the inputs, outputs and the measurable disturbances,  $\Psi$  is assumed to have an extremum, a minimum or maximum point which drifts due to the unmeasured disturbances. It is desired to keep the system operating at this extremum value while avoiding constraint violations. We call this safe tracking of the optimal operating point, optimizing regulation.

It is well-known that (4) represents an exceptionally difficult non-linear dynamic optimization problem, and excessive amounts of computing are required for its solution. For the case where unmeasurable disturbances do not affect the optimum significantly it may be possible to solve this problem off-line, but it is definitely not feasible for on-line applications. We therefore propose a simplification based on classifying process disturbances according to their frequency spectra.

#### 2.1.2 PROBLEM SIMPLIFICATION

Practical optimization time periods (0,T) are typically large relative to the dominant process time constants, and therefore only persistent disturbances,  $\mathbf{d}_{\mathbf{s}}$ , with periods larger than the process settling time have an important effect on  $\Psi$ . Rapidly varying disturbances,  $\mathbf{d}_{\mathbf{f}}$ , also called process noise, comparable to or faster than the dominant system time constants, are effectively non-existent relative to the optimization period. Also, the influence of these rapidly varying disturbances can be suppressed by using conventional single-variable or multi-variable regulatory control. Therefore the plant can be assumed to be at quasi-steady state during the time period (0,T) for optimization purposes.

In order to implement the conventional regulatory control subsystem, a subset of the inputs  $\mathbf{u}_1$  is selected to control a subset of the outputs,  $\mathbf{y}_1$ , in the face of disturbances,  $\mathbf{d}_f$ . Using the quasi-steady state assumption, optimization problem (4) can then be reformulated as a steady-state or static optimization problem, dependent on persistent disturbances  $\mathbf{d}_c$ :

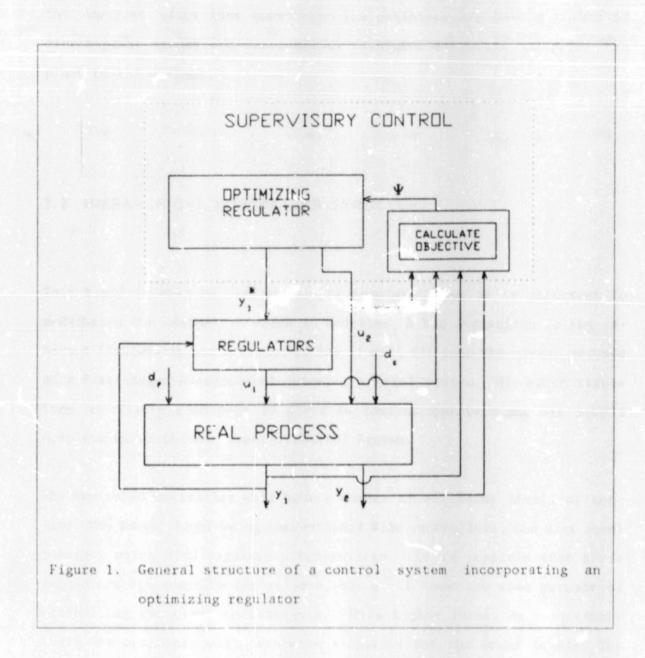
such that

$$g(y_1^*, y_2, u_2, d_s) \le 0$$
 ..... (7)

where

$$y = (y_1, y_2)^T$$
 $u = (u_1, u_2)^T$ 
 $y_1^* = \text{vector of setpoints for the conventional regulators}$ 
 $y_2 = \text{vector of remaining uncontrolled outputs}$ 

The resulting general control structure is shown in Figure 1.



The task of the optimizing regulator is to repeatedly solve (5) for the optimum input variables  $\mathbf{u}_2$  and  $\mathbf{y}_1^*$ . The resulting control inputs,  $\mathbf{u}_2$ , are then applied directly to the plant while control inputs  $\mathbf{u}_1$  have

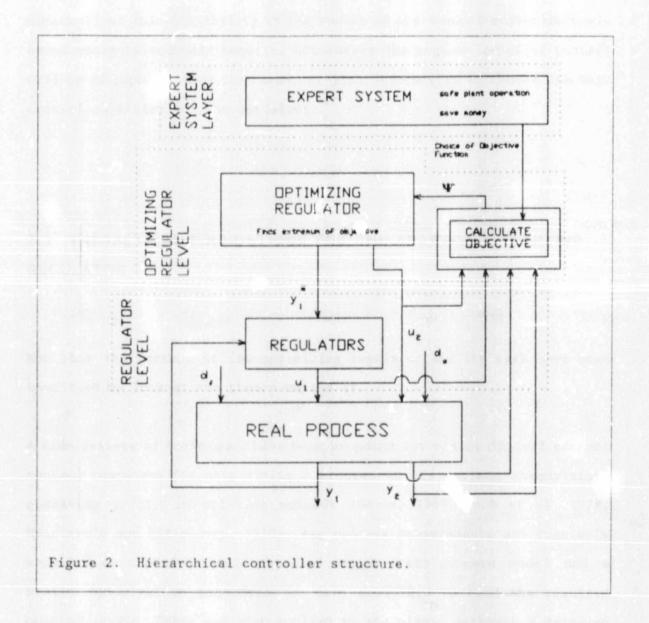
been replaced by the setpoints  $\mathbf{y_1}^*$  of the associated conventional regulators. The solution is conclusive every  $T_0$  seconds where the choice of  $T_0$  depends on the speed of variation of the disturbances,  $\mathbf{d_s}$ .

In summary, the task of the optimizing regulator is to track a shifting optimum that is affected by disturbances that vary slowly compared with the dominant plant time constants. The optimizer may have a number of direct plant inputs and conventional regulator setpoints to guide the plant to the optimum.

#### 2.2 HIERARCHICAL CONTROLLER STRUCTURE

This study focuses on the optimizing regulator, but it is important to understand the context in which it operates. A big attraction is its inherent flexibility and a vision of how it will fit into the global picture of a distributed huerarchical computer control system. The block structure in Figure 2 on page 10 gives an idea of how it might fit into a complete hierarchical computer control system.

The envisaged controller will have a number of different levels of control. The lowest level being conventional SISO controllers, the next level possibly being MIMO regulatory controllers. It is possible that these regulators are adaptive controllers, but still have the same purpose of controlling the plant to setpoints. On a higher level, or supervisory level the optimizer will determine setpoints for the lower levels. The bandwidth of the higher level controller may be orders of magnitude lower than the underlying regulators. This ensures safe operation of the plant while taking the pressure off the higher optimizer level of control.lt



gives the optimizer time to converge to the new settings without the risk of unpredictable operation or constraint violations.

The optimizer level is highly flexible. If the control objective changes then it is a simple matter to re-define the optimization problem. All that needs to be done is to give the estimator new data calculated from the new objective function. The estimator will then adapt the model parameters to the new data and new gradients will be calculated. An example where the objective function may change is when under normal conditions the objective is to maximize the throughput of a plant, but when there is a market surplus of the product one may wan to minimize plant energy

consumption. This flexibility in the choice of the control objective could be of enormous economic benefit. Ultimately the highest level of control  $w^{*1}$  be an expert system that uses off-line information to choose the best control objective for the optimizer.

## 2.3 THEORETICAL FOUNDATIONS FOR THE ADAPTIVE OPTIMIZING REGULATOR

Now that the context of the optimizing regulator and its task have been specified we look at the theory behind it.

A wide variety of techniques have been proposed for use in digital process control computers for maintaining a process at its optimum steady-state operating point. In off-line methods (Savas, 1965, Webb et al. 1978, Maarleveld and Rijinsdorp, 1970), key process measurements are regularly supplied to a predetermined detailed steady-state process model and a static optimization procedure is then performed to find the required control inputs. These are then applied to the plant. Although a detailed non-linear model can be used and fast static optimization algorithms are available, this method suffers from two serious disadvantages. Firstly, most economically important disturbances cannot be measured or modelled exactly. Secondly, even for processes of low complexity, off-line models are difficult to obtain and are always inaccurate owing to the impossibility of modelling all effects. Consequently, it is imperative that the optimizing regulator interacts with the operating process in some way so that all economically important disturbances are detected as soon as they affect the plant outputs, and a detailed fundamental model is not required.

Numerous on-line methods for obtaining steady-state models through direct searches on the operating plant have been proposed. Edler et al. (1970) compare the performance of different techniques. Since steady-state information is required, measurements should be taken only after the process has settled after each change in the control inputs. This results in a very slow search procedure. On the other hand, Sawaragi et al. (1971) have found that very complex stability problems arise if the control inputs are changed before process transients die away. Furthermore, these methods are very sensitive to process noise (Saridis, 1974).

In this research project we have selected an approach based on a two-step procedure of regularly determining the parameters of a steady-state mathematical model and then adjusting the control inputs so that the performance index is at its optimum value. This approach is very closely linked to adaptive control and first attracted attention in the 1960s (Balckman, 1962, Jacobs, 1969), but lack of suitable computing hardware made practical implementation difficult. With the availability of microprocessor there has recently been renewed interest in this approach (Sternby, 1980, Garcia and Morari, 1984). Also encouraging theoretical results concerning the stability and convergence properties of the algorithms incorporating this adaptive two-step optimization procedure are now available (Haimes and Wismer, 1972, Roberts and Williams, 1981).

In order to overcome the problems of having to wait for the plant to reach steady-state after each adjustment to the control inputs and sensitivity to noise, the most promising approach appears to be to determine the steady-state process model parameters by recursively estimating the parameters of a simple dynamic input-output model during the transient response, as suggested by Bamberger and Isermann(1978). It is then a simple matter to extract the corresponding steady-state model. This can be used to solve optimization problem (5) and thereby determine how the control inputs should be varied in order to improve plant economic performance.

The procedure is then repeated at the new operating point with a dynamic process identification followed by an optimization step, and so on until the optimal point is reached. Figure 3 on page 14 shows this closed-loop two step process.

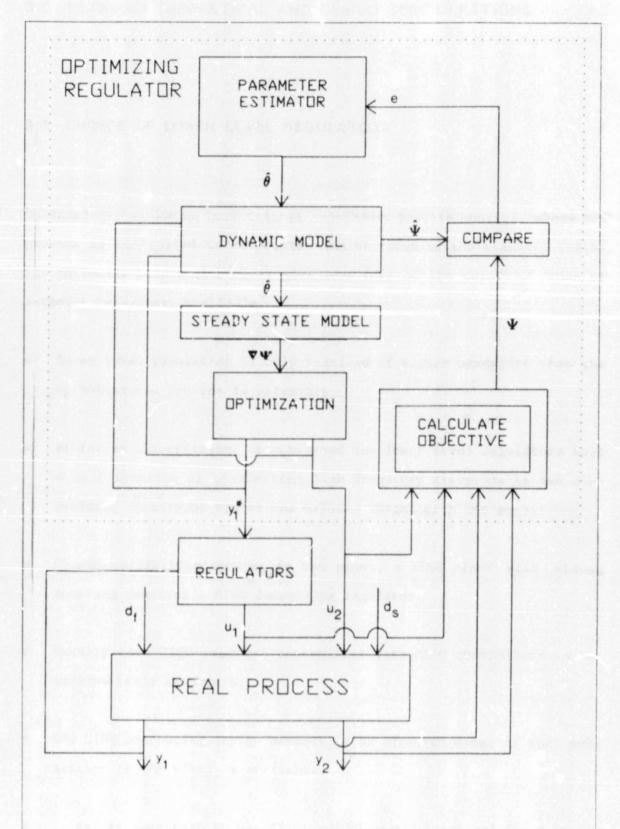


Figure 3. Block diagram of the internal structure of the adaptive optimizing regulator.

#### 3.0 DETAILED THEORETICAL AND DESIGN CONSIDERATIONS

#### 3.1 CHOICE OF LOWER LEVEL REGULATORS

Detailed discussion on conventional regulatory process control, where the process is controlled to setpoints, can be found in the standard texts. The following points do however give some idea of the possible scope of setpoint regulators available.

- Lower level regulation is only required if higher bandwidth than the optimizer can provide is necessary.
- As far as the optimizer is concerned the lower level regulators have a dual function of eliminating high frequency disturbances and also avoiding constraint violations between optimizer plant moves.
- Chosen carefully according to the plant, a MIMO plant with strong coupling requires a MIMO decoupling regulator.
- Usually have SISO cascaded controllers with MIMO controllers to determine their setpoints.
- Use MIMO decoupling design methods if an adequate model of that subsection of the plant is available.
- It may be necessary to use SISO or MIMO adaptive control for a slowly time varying plant.

• If the above two methods are not suitable and very little is known about the plant model then a method due to Garcia and Morari (1983) called Internal Model Control (IMC) might be of use.

#### 3.2 ON-LINE OBJECTIVE FUNCTION IDENTIFICATION

The optimiser needs a dynamic mathematical model of the objective function from which a steady-state model is extracted and gradients are calculated. This puts stringent requirements on the choice of model and the estimation of the model parameters.

### 3.2.1 CHOICE OF MODEL

Here issues are discussed concerning the choice of the form of a dynamic model to represent the objective function. It is not clear how the choice of a dynamic model affects its derived steady state model, so for this section it is assumed that all that is required is a good dynamic objective function model. In a later section we look at the implications of extracting a steady state model from the dynamic model.

For most processes a linear second order model with deadtime is an accurate representation. For the optimizer we need to model the objective function and not just the plant output. Since the objective function is non-linear and the controller operates around the extremum point one has to be more careful when assuming a linear model. If a linear model is assumed it means that at each operating point the estimator must be given

sufficient time to change the me ' parameters and only a new set of plant input moves is given after the estimator has converged. In other words if a general non-linear model is used then less demands are made of the model parameter estimator and/or an increase in controller bandwidth is possible.

If reliable on-line measurements of plant disturbances are available it is imperative that these are included in the model. The models parameters should only have to be updated for the following two reasons:

- o If unmeasurable disturbances change the plant significantly.
- o If the operating point changes and the approximate model no longer fits the plant adequately. For example: a linear model is used on a plant that is non-linear over the operating range.

Unnecessary model adaption will obviously retard the overall performance.

The trade-off between the selection of a more simple linear dynamic model and a non-linear dynamic model, can at this stage, only be evaluated by a simulation study.

All that the standard on-line estimators require is that the model can be written in a certain form. Since the calculated objective is generally a scalar and a plant will typically have multiple inputs the model must be single output multi-input. Using the variables appropriate to the optimizer we write the model in the regressive form required by the parameter estimator:

where

 $t \equiv discrete \ time \ variable \ as \ an \ integral \ multiple \ of \ the \ model \ execution \ time$ 

 $\Phi$  = regression vector or history vector of past  $\Psi(t)$ ,  $\mathbf{y}_1$  (t),  $\mathbf{y}_2$  (t) and  $\mathbf{u}(t)$ .

 $\theta(t-1)$  = parameter vector. This is a vector of unknown model parameters that are estimated.

Writing the model in this form assumes it is linear in its parameters, but note that the regression vector can have non-linear functions of past values of  $\Psi(t)$ ,  $y_2$  (t) and u(t).

The obvious choice for a linear model is the DARMA model. It is widely used in adaptive control because it is the equivalent of an observable discrete state space representation (Goodwin and Sin, 1984:32). Observable and uncontrollable modes are included in the model. The general form of the single output multiple input DARMA model is:

$$\sum_{i=1}^{n} ay(t-i) = \sum_{i=1}^{m} Bu(t-d-i) + \sum_{i=1}^{r} Cz(t-d-i) + c \dots (2)$$

or introducing the backward shift operator  $q^{-1}$ 

$$A(q^{-1})y(t) = B(q^{-1}) u(t-d) + C(q^{-1}) z(t-d) + c$$
 .....(3)

where y(t) = output

u(t) = vector of inputs

z(t) = vector of measurable disturbances

and c is the dc value about which the incremental model operates.

The  $a_i$  coefficients and the B and C coefficient matrices are the unknown model parameter coefficients that need to be estimated using plant data.

n, m and r describe the order of the model, and d is the pure delay.

A general non-linear model that fits a large class of non-linear processes is the Hammerstein model (Haber and Keviczky, 1978). It is linear in its parameters and can be written in regressive form for the estimator. 'In the backward shift operator notation  $q^{-1}$  it is written as:

$$A(q^{-1})y(t) = B_1(q^{-1})u(t-d) + B_2(q^{-1})u^2(t-d) + c$$
 ..... (4)

Note that measurable disturbances can be included in a similar vain to the above DARMA model.

This model can be tested against the DARMA model for a particular plant. The advantage of both the above models is that standard estimators as used in adaptive control can be used, and the models are relatively simple yet highly versatile and very general. The model is only expected to match the plant at a given operating point, such models are succinctly defined as point-parametric (Brdys, 1983).

The choice of the model order depends on the specific plant. For most plants second order will suffice, this means two time constants, one to collect the fast dynamics and the other to collect the slow dynamics. If there are other dominant time constants then a higher order model may be needed.

Pure delay or transport delay d can easily be included in the discrete model but the approximate delay time needs to be known. For this study it is assumed the approximate plant pure delay is known and that it is constant. Work is being done on ways of estimating the dead-time on-line if it is unknown or if it changes with time ('facLeod, 1987).

## 3.2.2 ESTIMATION OF MODEL PARAMETERS

There are numerous on-line recursive parameter estimation methods ( see Seborg et al. 1986 for a comprehensive survey ). The method most widely used and that has received the most attention is the recursive least squares estimator (RLS). This is due to its fast convergence and good statistical properties, it gives unbiased estimates if the noise is uncorrelated. There are modifications to the recursive least squares if the noise in the system is correlated, this could be the case for interconnected subsystems. An example of a modified RLS estimator is the instrumental variables estimator (Wong and Polak, 1967).

The standard RLS estimator (Goodwin and Sin, 1984:49) for a single output multi-input plant has the following form:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + M(t-1)\phi(t-1) . \ e(t) \label{eq:theta}$$
 where

e(t) is the estimation error which is the difference between the measured value and the value the model predicts at time t. It is written as:  $y(t) - \hat{y}(t)$ , where  $y(t) = \phi(t-1)^T\theta(t-1)$ 

0 is the vector or matrix of unknown model parameters.

M is the gain matrix for the estimator

 $\Phi$  is the regression vector.

This estimator has the problem that once it has converged to the set of model parameters it "falls asleep" and will not converge if the plant parameters change again. This loss of sensitivity is due to M being non-increasing.

The RLS estimator can be modified to avoid the above problem. Arious modifications exist (Seborg et al. 1986) and the choice is usually a trade-off between accurate estimates and fast convergence to changing parameters. The convergence theory for the standard RLS can be applied to any modified RLS provided:

- 1. The covariance matrix P (one of the terms in M) is only increasing in magnitude.
- 2. There is an upper bound on P.

We look at some of the better known modified RLS estimator algorithms.

RLS with exponential data weighting: (Goodwin and Sin, 1984:64). For the forgetting factor  $\lambda$ <1 the estimator gives recent plant data a higher weighting so that old data, that may not be accurate if the plant has changed, is forgotten. This is a popular method because  $\lambda$  can be selected according to how fast the plant is changing or what frequency plant changes the estimator must track. It nevertheless has the following pit-fall if  $\lambda$  is not chosen carefully: Consider a plant under regulatory control and in a steady state. If the physical plant does not change (no significant unmeasurable disturbances) and old plant data is being forgotten then there might not be sufficient information content in the plant input to estimate the parameters. This condition of the plant input not being persistently exciting (Goodwin and Sin, 1984:72) leads to deteri-

**Author** Bleloch Mark John **Name of thesis** An Adaptive Optimizing Regulator For An Autogenous Mill. 1987

#### **PUBLISHER:**

University of the Witwatersrand, Johannesburg ©2013

#### **LEGAL NOTICES:**

**Copyright Notice:** All materials on the University of the Witwatersrand, Johannesburg Library website are protected by South African copyright law and may not be distributed, transmitted, displayed, or otherwise published in any format, without the prior written permission of the copyright owner.

**Disclaimer and Terms of Use:** Provided that you maintain all copyright and other notices contained therein, you may download material (one machine readable copy and one print copy per page) for your personal and/or educational non-commercial use only.

The University of the Witwatersrand, Johannesburg, is not responsible for any errors or omissions and excludes any and all liability for any errors in or omissions from the information on the Library website.