

Forecasting return of commercial property in South Africa using macroeconomic factors

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A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, in partial fulfilment of the requirements for the degree of Master of Science in Property Development and Management.

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Declaration

I declare that this research report, unless otherwise acknowledged, is my own, unaided work. It is being submitted in partial fulfilment for the degree of Master of Science in Building in the field of Property Development and Management at the University of the Witwatersrand, Johannesburg. It has not been submitted for any other degree or examination at any other university.

Signed this _____ day of _____ 2011

Abstract

The forecasting ability of the macroeconomic factors upon South African commercial property return is investigated in this research. Such research is still very novel in South Africa and only Brooks and Tsolacos (2003) has recently investigated this relationship with several European markets. In this research, both direct property returns (IPD) and indirect property returns (J255 and J256) are investigated. The macroeconomic factors that are identified to have some influence on commercial property return are term structure, gilt-equity ratio, employment index, building plan passed and changing inflation rate (CPIX index). Four different types of models were investigated, namely the univariant ARMA model, the univariant GARCH model, the VAR model and the MLP neural network model. The optimal model for each type is identified using AICc and BIC information criterion techniques. The optimal models are then used in long-term forecasting and short-term forecasting. The ARMA model and the neural network were identified to best predict indirect and direct property returns, respectively.

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Foreword

This is a research report presented for the degree of Master of Science in Building, by coursework and research, at the University of the Witwatersrand, Johannesburg, South Africa.

The research report is titled “Forecasting return of commercial property in South Africa using macroeconomic factors”, which investigates the relationship between commercial property return in South Africa and various macroeconomic variables.

The research commences with a literature survey, identifying the macroeconomic variables that have influence on commercial property return and the models that are previously used in predicting property return. The degree of relationship between each macroeconomic variable and property return is calculated and those that are strongly related are used for forecasting. The forecasting models are optimised and the performances of the models in predicting retail, office and industrial property return are compared.

The research report is presented in the format of a thesis that contains the essential analysis and results of the research. The appendices, which are digitised in EXCEL and WORD formats, and the rest of the information associated with this research can be found in the associated CD.

The first appendix (Appendix A) contains tables of input and output data, which are the investigated macroeconomic factors and the property return, used in the research.

The second appendix (Appendix B) contains graphs of all the macroeconomic factors, the indirect and direct property returns and their deviations and the correlograms of the indirect and direct property returns and their deviations.

The third appendix (Appendix C) contains the result from the Granger causality analysis where the degree of relationship between each macroeconomic variable and property return are tabulated.

The fourth appendix (Appendix D) contains a background on the Matlab software and the development of the models investigated in this research.

The fifth appendix (Appendix E) contains the result of the optimisation process for each type of model investigated in this research.

The sixth appendix (Appendix F) contains graphs from the impulse analysis of the optimal models for long-term predictions.

The seventh appendix (Appendix G) contains the schedule of the M-file developed in this research.

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1 Introduction

It is an established theory that the performance of various investment markets, such as that of stocks, commodities and bonds, is related to the macroeconomic environment and its components. When investigating the macroeconomic environment of an economy, the focus tends to be placed on aggregate demand and supply (Ball et al., 1998: 159-161). Aggregate demand is usually defined by the total expenditure flowing through the economy, which is the sum of the total consumption expenditure in the economy, total investment, government expenditure and net export (Ball et al., 1998: 159-161). The investment in property is considered as either government expenditure, if the properties are invested for the use and operation of government, or is considered to be part of the total investment through the private sector. Thus one can assume that there is an expected relationship between the property market and the macroeconomy. As discussed in Ball et al. (1998: 220), it is essential for investors and portfolio managers to forecast property return as it provides a prediction of the expected target return, which consequently assists in making accurate investment decisions. This research presents models that determine the effect on the return of commercial property arising from changes in the macroeconomic environment on the South African property market.

1.1 Problem Statement

This research is based on the observation that there is a relationship between various macroeconomic factors and the return of commercial property markets. Such relationship is investigated extensively in developed markets such as in the USA, UK, Singapore and Australia. However, in South Africa where the market is not as well developed as those of the US and the UK, this relationship has received little attention. Therefore, this research is largely concerned with the relationship between property returns and expected returns in the property sector.

1.2 Research Hypothesis and Objective

This research hypothesis is concerned with the return of commercial property investments in South Africa, which can be forecasted using advanced time-series modelling techniques and macroeconomic factors. In considering this research hypothesis, three main research objectives are addressed. The first objective is to identify the macroeconomic factors that affect the return of commercial property in South Africa. The second is to identify various types of time-based models used previously in forecasting the property sector and using them to predict property return. The third is to compare the predictive ability of models from which an optimal predictive model is identified.

1.3 Scope of Research

The scope of the research is limited to the South African property market and its macroeconomy. Furthermore, the research is focused on the returns of South African commercial and industrial properties only. This is based on the view that commercial

properties are more directly related to the variation of macroeconomic factors than, for instance, the residential sector. The research is also focused on existing models used for forecasting such relationships and thus little consideration is given to other predictive models.

1.4 Research methodology

The quantitative research methodology adopted by this research project also determines the sequence of chapters of the thesis. It is divided according to the following sections:

1. Literature reviews made in this field of research
2. Evaluation of the characteristic of the commercial property return investigated and the various macroeconomic factors considered
3. Discussion of the implementation of the models in the simulation software
4. Evaluation of the relationships between commercial property return and various macroeconomic factors
5. Identification of the optimal parameters for each type of models investigated
6. Evaluation of the performance the models used in the research
7. Conclusion to findings

2 Literature Review

The relevant literature is evaluated and divided based on three essential criteria, namely the macroeconomic input variables of the proposed model, the type of return that the model predicts, and the type of model used for prediction.

The review focused on several significant international research studies, namely the works of McCue and Kling (1994), Brook and Tsolacos and West and Worthington (2004), as well as existing local studies that are somewhat related to the field investigated in this research. Ball et. al (1998: 245) highlighted that it is very difficult to predict yield or return since this factor is relatively stable in established market. Return is determined by the sum of the risk free rate and the risk premium of the investment. There are two different approaches in predicting return, namely regression methods and the cash flow method. Since the focus in this research is on macroeconomic scale, only regressive methods are investigated.

2.1 Macroeconomic factors

McCue and Kling (1994) conducted some of the earliest significant research on the subject. Their research is focused around the effect of prices, short-term nominal interest rate, economic output, and investment as the macroeconomic factors. They concluded that there is a strong relationship between short-term nominal interest rate and property returns and a weak relationship between economic output and property returns. Prior to this work, they had also researched the macroeconomic factors affecting office investment (McCue and Kling, 1987). The research found that nominal interest rate significantly affects the volume of office construction, which coincides with their 1994 findings.

Ling and Naranjo (1997) and (1998) are two other research studies that investigate the macroeconomic factors that affect the risk premium of property. Ling and Naranjo (1997) identified the growth rate in real per capita consumption, the real Long bond rate (T-bill rate), which reflects the real short term interest rate calculated by deducting the inflation rate (measured by consumer price index) from the 3-month bond rate (Ling and Naranjo, 1998), the term structure of interest rates, and the unexpected inflation rate; which is the difference between the actual inflation value (defined by the consumer price index) and the expected inflation rate (predicted using the Box-Jenkins process) as influencing factors.

In their later work, Ling and Naranjo (1998) included the stock market performance, which is quantified using the excess return of a value-weight portfolio of stocks trading on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX) and the NASDAQ. The research identified that growth rate in real per capita consumption is a significant factor for all types of return. Furthermore, the change in real short-term interest rate and interest rate term structure are negatively correlated with property returns. The interest rate term structure in this research is defined as the difference between the average annualised yield of the 10-year Long bond (Treasury bond) and the 3-month

bond (Treasury bill). Returns were found to be most sensitive towards the changes in real short-term interest rate and unexpected inflation.

Brook and Tsolacos (1999), (2001), (2001a) and (2003) have extensively researched the impact of the macroeconomy on the property market in the United Kingdom. They have published three research papers on the topic. The earliest work was Brook and Tsolacos (1999) where the investigated economic factors were previous property return as a dependant variable, the rate of unemployment, nominal short-term interest rates, term spread (term structure) of interest rate, unanticipated inflation and dividend yield as the independent variable. Their analysis identified that there are strong relationships between the unexpected inflation and term structure of interest rate and the property return in the UK.

Term structure of interest rate is defined as the difference between the yields on long-term bonds and short-term bonds (Brook and Tsolacos, 2003). The term structure is said to determine the future expectation of the interest rate and the economic condition as mentioned in Brook and Tsolacos (2001). As discussed in Investopedia (2009), the term structure is generally positive under normal economic conditions. When the value is close to zero, the short-term rate is high and the long-term rate is low and is an indication that the market is sending mixed signals. In the situation where the term structure value is negative, the long-term rate is lower than the short-term rate and thus the future interest rate is expected to decline. Brook and Tsolacos (2001) also cited the fact that the short-term bond rate determines the rate of inflation of the economy while the long-term bond rate reflects future economic growth, activities and probably inflationary tendencies, factors that affect both short-term and long-term investment in the economy.

Unanticipated inflation is defined as the difference between the realized inflation rate and an estimated series of expected inflation (Brook and Tsolacos, 1999). Unexpected inflation is obtained by fitting an (ARIMA) model to the inflation data with a one period lag and extracting the mean from the model, which is the resultant expected inflation.

In the research by Brook and Tsolacos (Brook and Tsolacos, 2001 and 2001a), the number of macroeconomic factors (independent variables) were narrowed down to two. In Brook and Tsolacos (2001), the term structure of interest rate and gilt equity yield ratio, along with the indirect property index (dependant variable), were selected for the analysis, while in Brook and Tsolacos (2001a), the effect of both short-term and long-term interest rates and the term spread of interest rate on property returns were examined. The term spread of interest rate is the difference between the long-term interest rate and the short-term interest rate.

Gilt-equity yield ratio, according to Brook and Tsolacos (2003), is the ratio of the income yield of long-term government bond to the dividend yield on equities. When the dividend yield is low, the ratio is high. In such a situation, equity becomes more expensive than bond. Conversely, the ratio is low when the dividend yield is high. In both cases, the income yield of the bond will have to be adjusted so that equilibrium state is reached.

Other macroeconomic factors previously investigated, such as the rate of unemployment, nominal short-term interest rate and inflation, were not investigated in this research because those factors were considered to have inferior predictive power in Brook and Tsolacos (2001).

Brook and Tsolacos (2001) conclude that the term-spread of interest rate and the gilt-equity yield ratio can improve the accuracy of short-term property return prediction. In the work of Brook and Tsolacos (2001a), it was found that the term spread of interest rate is co-integrated with property return but both the term spread and the short-term interest have relatively small significance on the variation of property return.

Brooks and Tsolacos (2003) once again investigated the relationship between the gilt-equity yield ratio and the term structure of interest rates and their impact on property returns. However, they had also introduced the dividend yield of the property index as an additional macroeconomic factor. Dividend yield was introduced in the research as it reflects the future growth, profitability and dividend of the investment.

The economic factors researched in all of the above works by Brook and Tsolacos was derived from the significant work of Qi and Maddala (1999). Qi and Maddala (1999) identified that there is a non-linear relationships between various economic factors and stock market return and conversely a non-linear relationship between excess stock market return and certain economic factors. The economic factors were dividend yield, short term interest rate, variations in short-term interest rate, growth rate of industrial production, inflation rate and money growth rate. The growth rate of industrial production was calculated based on the logarithmic differences of the 12-month average of the industrial production index between two successive periods. The inflation rate was also calculated based on the logarithmic differences of the annual average of producer price index on finished goods between two successive periods.

One of the most recent research studies on the topic is West and Worthington (2004). They employed previous general market return as dependant variable and interest rate and inflations as independent variables. Furthermore, they introduced construction activities, industrial production and employment index into their model, which were factors previously examine by McCue and Kling (1994) and Brook and Tsolacos (1999).

They calculated the inflation rate based on the Consumer Price Index (CPI) in the housing sector. Unexpected as well as expected inflation is calculated using the Box-Jenkin ARIMA model where the trend extracted from the model represents the expected inflation and the error movement remaining represents the unexpected inflation. The level of construction activity, which indicates the level of supply in the market, is represented by the number of building plans approved for non-residential buildings; while the indices for manufacturing, which indicate the level of demand in the market, represent the level of industrial production. Lastly, the employment index is used to represent the level of growth in various industries. The result of the research indicated that inflation, industrial production, employment index and interest rate are all significant factors affecting commercial property return.

Several research studies were published focusing on specific factors, such as stock market performance, employment growth rate and inflation, affecting the property return and market performance. Lizieri and Satchell (1997) investigated the relationship between property market return and stock market performance and found that lagging equity return affected the property market return. Liang and McIntosh (1998) investigated the relationship between employment growth rate and property return. The employment growth rate data gathered for this research was from 46 different metropolitan areas across the US. They have found that the relationship is positively correlated and is significant only for short-term property return.

The research on the relationship between inflation and property return has been the most focused topic in this field and is the most conflicted. Chan et al. (1990), Stevenson and Murray (1999), Onder (2000) and almost all of the findings in Liu et al. (1997) (except for French index for short-term return) found that such relationship is negative correlated. While Hartzell et al. (1987), Hoesli (1997), Bond and Seiler (1998) and Quan and Titman (1999), found that for a long-term investment, such a relationship is positively correlated. Liu et al. (1997) and Hoesli (1997) argued that the cause of such discrepancy is due to the fact that some of the investigated indirect property returns, which are indexes from REITs and various other listed property stocks and trusts, behave more like stocks than an individual property asset. The work from Onder (2000) contradicts such finding as the property return data used was direct house prices from various metropolitan areas in Turkey, an economy with highly volatile inflation.

The following is a summary of other related research that has bearing on this work:

1. Chan et al. (1990) investigated the effects of changes in risk and term structure of interest rate, unexpected inflation and the discount on closed-end stock funds on the return of some REITs. The research identified that REIT return is negatively correlated to unexpected inflation.
2. Karolyi and Sanders (1998) employed the weight-index of NYSE, Amex and NASDAQ, the risk premium of high-yield corporate bonds, the term spread of interest rate and unexpected inflation rate as the examining economic factors. The research identified that the risk premium of high-yield corporate bonds and the stock market have little influence on the return of the property index.
3. Further to the investigation of the relationship between inflation and residential property return, Bond and Seiler (1998) evaluated other variables that are also positively correlated to the return, namely the ratio of household to the total population, the real disposable income and its rate, GDP level and its growth rate.
4. Liow (2004) identified that there is a link between office and retail excessive return and five macroeconomic factors, namely growth rate of GDP, growth rate of industrial production output, unexpected inflation, short-term interest rate and market portfolio.
5. In Ball et al. (1998), the only related research study identified is the work from Hetherington (1988). The research proposed a model that predicts initial yield based on the yield of long-dated gilt (long term bond rate), which relates to the

risk free rate, and the average investment in property and bank lending rate, which relates to the risk premium.

The research of the affecting factors on property return is summarised in the table below.

Research	Macroeconomic factors
McCue and Kling (1994)	short-term nominal interest rate, property price, economic output, level of investment
Ling and Naranjo (1997) and (1998)	growth rate in real per capita consumption, the real Long bond rate, term structure of interest rates, unexpected and expected interest rate, stock market performance,
Brook and Tsolacos (1999)	previous property return, rate of unemployment, nominal short-term interest rates, term spread of interest rate, unanticipated inflation and dividend yield
Brook and Tsolacos (2001)	the term structure of interest rate, gilt equity yield ratio, indirect property index, rate of unemployment, nominal short-term interest rate, inflation
Brook and Tsolacos (2001a)	short-term interest rate, long-term interest rates, the term spread of interest rate
Brook and Tsolacos (2003)	gilt-equity yield ratio, term structure of interest rates, dividend yield of the property index
Qi and Maddala (1999)	dividend yield, short term interest rate, variations in short-term interest rate, growth rate of industrial production, inflation rate and money growth rate
West and Worthington (2004)	previous property return, interest rate, inflations, construction activities, industrial production and employment index
Lizieri and Satchell (1997)	equity return
Liang and McIntosh (1998)	employment growth rate
Stevenson and Murray (1999), Onder (2000), Liu et al. (1997), Hartzell et al. (1987), Hoesli (1997) and Quan and Titman (1999)	Inflation
Chan et al. (1990)	changes in risk and term structure of interest rate, unexpected interest rate and the discount on closed-end stock funds
Karolyi and Sanders (1998)	stock market index, the risk premium of high-yield corporate bond, the term spread of interest rate and unexpected inflation
Bond and Seiler (1998)	inflation, ratio of household to the total population, the real disposable income and real disposable income rate, GDP level and GDP growth rate
Liow (2004)	growth rate of GDP, growth rate of industrial production output, unexpected inflation, short-term interest rate and market portfolio
Hetherington (1998)	long term bond rate, level of property investment and bank lending

Table 1.1: Summary of factors influencing property return in previous research

The most investigated macroeconomic factors are inflation, interest rate (in particular term structure of interest rate) and general macroeconomic data such as GDP, production level and employment rate. In some research, the performance of the stock market is also considered.

2.2 Predicted returns

There are two different types of property return investigated, namely direct and indirect property returns. Direct property return reflects returns from direct investment in properties, i.e. directly held property investment. While indirect property return refers to returns from indirect property investment, i.e. purchasing listed stocks of companies and trusts that own and invest in properties.

Bond and Seiler (1998), Liang and McIntosh (1998), Quan and Titman (1999), Stevenson and Murray (1999), Onder (2000), Liow (2000) and Liow (2004), researched the effect on direct property returns. The data has been sourced from US (in Liang and McIntosh, 1998, and Bond and Seiler, 1998), Singapore (in Liow, 2000 and 2004) and Turkey (in Onder, 2000).

The returns are generally related to certain geographical locations or types of property. For example, Bond and Seiler (1998), Liang and McIntosh (1998) and Onder (2000) researched on property returns in specific suburbs; Onder (2000) utilised only residential properties return and Liow (2004) utilised office, retail and industrial property returns.

Chan et al. (1990), Liu and Mei (1992), McCue and Kling (1994), Liu et al. (1997), Hoesli (1997), Karolyi and Sander (1998), Brook and Tsolacos (1999; 2003) researched the effect on indirect property returns. Many of the works are based on the US REITs index, such as Chan et al. (1990), Liu and Mei (1992), McCue and Kling (1994) and Karolyi and Sander (1998). The other data are derived from the Swiss real estate mutual fund (Hoesli, 1997), FTSE Property Total return index (Brook and Tsolacos, 1999) and property stock index from London, Amsterdam, Brussel, Paris and Milan exchange (Brook and Tsolacos, 2003).

Ling and Naranjo (1998) was one of the earliest works to investigate both indirect and direct property returns. The indirect property return data was calculated from REIT and returns from listed construction, property management, hospitality and other property-related companies. The direct property return data was the appraised-based return obtained from the National Council of Real Estate Investment Fiduciaries (NCREIF), which is an organisation based in America that collects direct property investment data, and return calculated from the capitalisation rate of an insurance company property portfolio. The authors divided the return data into geographical, regional and property type categories. The author then deduced the short-term interest rate from the return data to obtain the risk premium data.

Brook and Tsolacos (2001) and (2001a) used UK property index for indirect property return, which consisted of a market value-weighted index based on the top 26 property stocks traded in London Stock Exchange, and were the first research to use IPD (UK) property return data for direct property return.

West and Worthington (2004) commented that both types of returns should be investigated and compared. For direct return, they used the direct commercial property indices from Australia Property Council, which is an appraisal based accumulated indices that measures total returns of 70% of commercial properties held by institutions in Australia. For indirect return, they derived the return from the Australian Stock Exchange Listed Property Trust (ASX/LPT) 300 Index, which is derived based on logarithmic changes between two consecutive indices.

The research of the type of returns investigated is summarised in the table below:

Type of return	Research
Indirect	Chan et al. (1990), Liu and Mei (1992), McCue and Kling (1994), Liu et al. (1997), Hoesli (1997), Karolyi and Sander (1998), Brook and Tsolacos (1999; 2003)
Direct	Bond and Seiler (1998), Liang and McIntosh (1998), Quan and Titman (1999), Stevenson and Murray (1999), Onder (2000), Liow (2000; 2004)
Both indirect and direct	Ling and Naranjo (1998), Brook and Tsolacos (2001; 2001a), West and Worthington (2004)

Table 1.2: Summary of the type of returns investigated in previous research

Having reviewed the previous research, most use only one type of return. Only a few in recent times compare the performance of both type of return. The research here moves on with a discussion of the types of models used in forecasting the property return.

2.3 Models

The research is focused around four types of models, namely the Vector Autoregression (VAR) model, the Autoregressive Moving Average (ARMA) model, the General Autoregressive Conditional Heteroskedasticity (GARCH) model and the neural network model. In this section, the theories behind these types of models as well as other types of models and their application in the field of commercial property return are discussed.

2.3.1 Autoregressive moving average (ARMA)

This is regarded as one of the most useful and widely-used time-series models. The output of the model is dependent on the previous output (known as the dependent variables) and the previous value of other variables (known as independent variables). The model is a combination of two separate models, namely the autoregressive (AR) model and the moving average (MA) model as defined in Chatfield (2001: 59-64).

The AR(R) model is defined by the following equation:

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} \dots + a_{R-1} X_{t-R+1} + a_R X_{t-R} + Z_t \quad (1)$$

Where:

- X_t the value of the variable X at time t (predicting variable)
- X_{t-1} the value of the variable X at time $t-1$ (predictor variable)
- X_{t-R} the value of the variable X at time $t-R$
- a_1 the degree of influence of X_{t-1} on X_t
- a_R the degree of influence of X_{t-R} on X_t
- Z_t variable of random process determining the error term of the equation
- R the degree of lags, which determines the number of previous X values that influence the current X value

The MA(M) model is defined by the following equation:

$$X_t = b_0 Z_t + b_1 Z_{t-1} \dots + b_{M-1} Z_{t-M+1} + b_M Z_{t-M} \quad (2)$$

Where:

- X_t the value of the variable X at time t (predicting variable)
- Z_t variable of random process at time t (predictor variable)
- b_0 the degree of influence of Z_t on X_t
- Z_{t-M} variable of random process at time $t-M$
- b_M the degree of influence of Z_{t-M} on X_t
- Z_t variable of random process at time t
- M the degree of lags, which determines the number of Z values that influence the current X value

The two equations above for both of the models assume that the mean is zero. If the mean is not zero, it will be of the following form:

$$X_t - m = a_1 (X_{t-1} - m) + a_2 (X_{t-2} - m) \dots + a_{R-1} (X_{t-R+1} - m) + a_R (X_{t-R} - m) + Z_t \quad (3.1)$$

$$X_t = m + b_0 Z_t + b_1 Z_{t-1} \dots + b_{M-1} Z_{t-M+1} + b_M Z_{t-M} \quad (3.2)$$

In both of the equations, the mean is deduced from the predicting variable series X so that the actual fluctuation caused by the predictor is investigated. It must be noted that the mean (m) must be a constant.

The combination of the equations of the two models is the following:

$$X_t - m = a_1(X_{t-1} - m) + a_2(X_{t-2} - m) \dots + a_{R-1}(X_{t-R+1} - m) + a_R(X_{t-R} - m) \quad (4)$$

$$+ Z_t + b_1 Z_{t-1} \dots + b_{M-1} Z_{t-M+1} + b_M Z_{t-M}$$

The notation of the above equation is ARMA(R, M) where the variable R and M determine the lags of each model. The degree of lags R and M is calculated using selection techniques to be covered in later section. After identifying the degree of lags, the parameters are then calculated. Prior the calculation of the parameter, the above equation should be converted in terms of Z_t , which is as follows:

$$Z_t = X_t - m - a_1(X_{t-1} - m) - a_2(X_{t-2} - m) \dots - a_{R-1}(X_{t-R+1} - m) - a_R(X_{t-R} - m) \quad (5)$$

$$- b_1 Z_{t-1} \dots - b_{M-1} Z_{t-M+1} - b_M Z_{t-M}$$

Chatfield (2001: 64-65) define the following iterative procedure for calculating the parameters $a_1, a_2, \dots, a_R, b_1, \dots, b_M$:

1. Estimate a suitable value for all of the parameters
2. Calculate the value of Z_t using the above equation for all of the values of X_t in the series. Take the previous value of Z in place of Z_{t-1} and so on. For the initial values in the series, assume the previous value as zero. For example, if one is calculating Z_1 and only has the value of X_1 , then assume all of the previous values of X and Z to be zero.
3. Calculate the residual sum of square for Z_t , using an equation similar to the one below:

$$RSS_{Z_t} = \frac{\sum Z_t^2}{n} \quad (6)$$

Where:

n = number of samples in the series

4. Repeat step 2 and 3 by adjusting the values of the parameters accordingly so that the residual sum of square for Z_t reaches a satisfying level close to zero, usually a predefined significant level.

Such iterative procedures are usually performed using multivariable optimisation techniques by means of a computer.

The use of this technique to examine the relationships between current property return and previous property return was introduced in Brook and Tsolacos (2001) and used again in Brook and Tsolacos (2003).

A variation of this type of time series model is the autoregressive integrated moving average (ARIMA) model. This type of model is widely used in many econometric problems where the investigated data is non-stationary, i.e. the mean of the data is continuously increasing or decreasing, and there is a presence of random or seasonal fluctuation in the data along the mean. This type of model is defined by a similar equation to the ARMA model defined in Chatfield (2001: 66):

$$X_t = a_1 \nabla^d X_{t-1} + a_2 \nabla^d X_{t-2} \dots + a_{R-1} \nabla^d X_{t-R+1} + a_R \nabla^d X_{t-R} \quad (7.1)$$

$$+ Z_t + b_1 Z_{t-1} \dots + b_{M-1} Z_{t-M+1} + b_M Z_{t-M}$$

Where:

∇^d differential of X_{t-1} to the order of d , where:

$$\nabla^d X_{t-1} = \nabla^{d-1} X_{t-1} - \nabla^{d-1} X_{t-2} \quad (7.2)$$

For example if $d = 1$, then:

$$\nabla X_{t-1} = X_{t-1} - X_{t-2} \quad (7.3)$$

If $d = 2$, then:

$$\nabla^2 X_{t-1} = \nabla X_{t-1} - \nabla X_{t-2} \quad (7.4)$$

The model is defined by the notation of ARIMA(R,d,M) where R and M are the lags of the autoregressive (AR) part and the moving average (MA) part of the model respectively and d represents the order of differencing required for data X . The AR part of the model forecasts the seasonal or random fluctuations about the mean of the data, hence the requirement of differencing, whilst the MA part forecasts the mean movement of the data. This type of model is widely used in McCue and Kling (1994) and Brook and Tsolacos (1999; 2003) to extract the unexpected and the expected inflation from the provided inflation data, where the mean (Moving Average part) represents the expected inflation while the fluctuation about the mean (Autoregressive part) represents the unexpected inflation. This model is also used in the research of Ling and Naranjo (1997; 1998).

The models covered in this section are generally regarded as univariate time-series model, which means that the model predicts the outcome of a variable using values from one variable, which could be the predicting variable. The model is useful in identifying the degree of influence of the current state of the property return due to its previous state.

2.3.2 Vector autoregression (VAR)

This type of model is an extended form of the univariate autoregressive (AR) model described in the previous section and is one of the most widely used models in this subject as the model allows for multiple variables to develop multiple simultaneous equations. This model has been used throughout Brook and Tsolacos (1999; 2001; 2003).

The model is defined by the following equation (Chatfield, 2001: 246):

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} \dots + \alpha_{p-1} X_{t-p+1} + \alpha_p X_{t-p} + Z_t \quad (8.1)$$

Where:

- X_t vector of variables X at time t
- X_{t-1} vector of variables X at time $t-1$ (predictor variable)
- X_{t-p} the value of the variable X at time $t-p$
- α_1 the degree of influence of X_{t-1} on X_t
- α_p the degree of influence of X_{t-p} on X_t
- Z_t variable of random process determining the error term of the equation
- p the degree of lags, which determines the number of previous X value that influence the current X value

Another variation of this equation defined in McCue and Kling (1994) and Brook and Tsolacos (1999) is as follows:

$$X_t = \alpha_0 + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} \dots + \alpha_{p-1} X_{t-p+1} + \alpha_p X_{t-p} + Z_t \quad (8.2)$$

Where:

- α_0 constant term
- Z_t error term of the equation

For m variables with one equation for each variable, there are m equations in the model. The variable X_t will then be a vector of $m \times 1$, the parameter α_0 will be a vector of $m \times 1$ and the parameter $\alpha_1, \dots, \alpha_p$ are all vectors of $m \times m$.

The parameters $\alpha_0, \alpha_1, \dots, \alpha_p$ is calculated using ordinary least square equation or the Yule-Walker equation. The use of the iterative method mentioned in the previous section is not required.

The model has been used to find the interaction between variables within a system. It was first introduced by McCue and Kling (1994) to identify the relationships between various macroeconomic factors and the return of REITs. Brook and Tsolacos (1999; 2001; 2001a; 2003) all used this type of model to evaluate such relationship. Generally the researchers use a simple VAR model for their analysis. Exceptions apply to Brook and Tsolacos (1999) and Brook and Tsolacos (2001a). Brook and Tsolacos (1999) employed a simplified version of the model where the lagged values of the variables on the left-hand side of the equation were not used, i.e. the lagged values calculated from the model were not used. In their subsequent work (Brook and Tsolacos, 2001a), they employed a bivariate VAR model to analyse the relationships of the interest rate and its spread on property return.

2.3.3 General Autoregressive Conditional Heteroskedasticity (GARCH)

This model is derived from autoregressive conditional heteroskedasticity (ARCH) model, which was developed by Engle (1982). The model is designed for series where volatility and conditional variance is particularly significant. Generally according to West and Worthington (2004), the model is used in financial application where expected return is directly related to expected risk. Pena et al. (2001: 307-327) claims that volatility requires the following characteristics in order to apply this model:

1. Volatility must be clustered, i.e. high at certain time period and low at other time period
2. Volatility evolves continuously with time
3. Volatility is stationary, i.e. it varies within certain fixed range
4. Volatility reacts differently with positive and negative outcome

The equation for the ARCH model is as follows:

$$Z_t = \sqrt{h_t} \varepsilon_t \quad (9.1)$$

and

$$h_t = a_0 + a_1 Z_{t-1}^2 + \dots + a_p Z_{t-p}^2 \quad (9.2)$$

Where:

Z_t	calculated output at time t
a_0, \dots, a_p	parameters measuring the affect of previous output on current output
ε_t	sequence of independent and identically distributed (iid) random variable
P	degree of lag of the model
h_t	variance of Z_t

Pena et al. (2001: 307-327) described the above relationship between successive values of Z_t as serially uncorrelated, but dependant on its previous value by a simple quadratic equation. The random variable ε_t should be normally or t-distributed about a mean of zero with a variance of 1.

The GARCH model is very similar to the ARCH model with the exception that h_t is defined by the following equation:

$$h_t = a_0 + a_1 Z_{t-1}^2 + \dots + a_p Z_{t-p}^2 + b_1 h_{t-1} + b_2 h_{t-2} + \dots + b_Q h_{t-Q} \quad (9.3)$$

Where:

h_{t-1}	previous variance, i.e. variance value at $t-1$
b_0, \dots, b_Q	parameters measuring the affect of previous variance on current variance
Q	degree of lag of variance

The notation for the above model is GARCH(P, Q) where P and Q defines the lags affecting the current variable and can be optimised.

Pena et al. (2001: 307-327) summarised the approach for building a GARCH or ARCH model:

1. Remove all seasonal and non-stationary movements in the data and deduce the mean of the data to zero
2. Check for conditional heteroskedasticity by checking the distribution of the sum of residual square
3. Identify the optimal order P and Q for the model

4. Calculate the value of the parameters

The use of the GARCH model in this application is very recent, namely that of West and Worthington (2004) and Liow (2004). West and Worthington (2004) employed the GARCH in mean (GARCH-M) model in their research while Liow (2004) employs a typical GARCH (1,1) model and a general method of moment (GMM) is used to analysed the relationship between the variances of the macroeconomic factors and property returns. West and Worthington (2004) commented that the benefit of using such model is to allow risk to vary so that account can be made for conditional covariance of returns with the market. Furthermore, the model accounts the effect of volatility clustering, where a large variation will lead to a larger variation in future predictions and likewise a small variation will lead to a smaller variation in future predictions.

2.3.4 Neural Network

Neural network is a black box modelling techniques that emulates the structure of a brain according to Siganos and Stergiou (1996). The network comprised of neurons, which are simple model defining a simple mathematic equation. An illustration of this model is as follows (Demuth and Hagan, 1999):

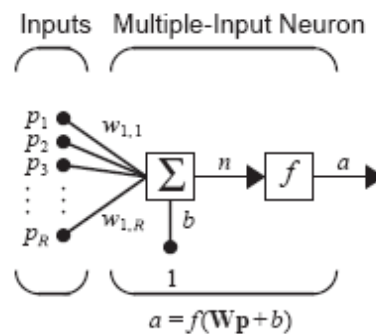


Figure 1: A block diagram illustration of a neuron in a neural network

The above is a simple multiple input neurons where p_1 to p_R are inputs to the neuron. Each of these inputs are multiplied by a constant weight, which determines the significance of each input. The inputs are then summed together with a constant b and feed into a transfer function. The equation defining the model is as follows (Demuth and Hagan, 1999).

$$a = f(\mathbf{W}_p + b) \quad (10.1)$$

Where:

$$\mathbf{W}_p + b = n = w_{1,1}p_1 + w_{1,2}p_2 + \dots + w_{1,R}p_R + b \quad (10.2)$$

The typical transfer function $F(x)$ employed is a log-sigmoid transfer function where equation is as follows (Demuth and Hagan, 1999).

$$F(x) = \frac{1}{1 + e^{-x}} \quad (11)$$

However, one can choose to deploy another transfer function, but such transfer function is ideal for the general purpose of approximating a model.

The simplest type of neural network is called the Multi-layered Perceptron (MLP) network. The network consisted of three layers, namely input layer, hidden layer and an output layer. The input layer consisted of the input to the network and the output layer consisted of the last layer of neurons that produces the output of the network. In between these two layers is the hidden layer, where most of the manipulation and calculation occurs.

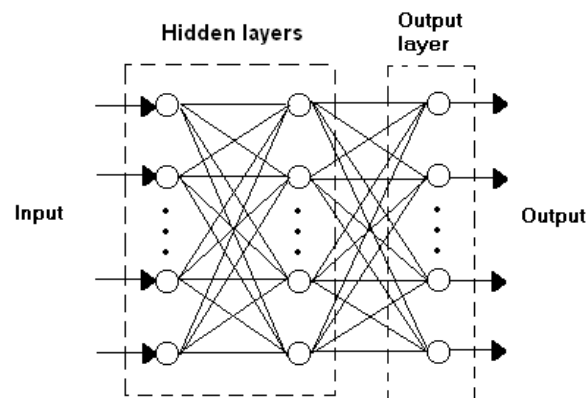


Figure 2: Diagram illustrating the structure of a MLP neural network

This type of neural network is ideal for defining very complex regression and classification as discussed in Demuth and Hagan (1999). The parameters within the network such as weight and bias need to be calculated before the system is operational. Such process is called training. Existing input and output data and a method of optimizing the parameters of the network is required before training commences. Generally, the initial parameters set for the network is estimated. The input set is then fed into the network and the calculated output from the network is then compared with the actual output, the expected output from the data set. The difference of the outputs, which is defined as the error of the network is then used to adjust the parameters of the network. The input set is then fed into the network again and the calculated output is then compared with the actual output again. This process is repeated until the predefined satisfactory conditions from the user are met (usually the number of iterations or error level of the output). The calculation of the parameters of the network requires the use of multiple variable optimization techniques such as genetic algorithm (GA) and particle

swarm optimization (PSO). The most popular technique is the backpropagation algorithm, where the parameters are optimized using gradient-descent technique.

The advantages of using a neural network in this application are that it does not require complex mathematical modelling and understanding (Brook and Tsolacos, 2003), all that the model required is a set of input and output data, and it is very robust, thus the effect of many macroeconomic factors on the return can be investigated.

The work of Brook and Tsolacos (2003) was the only published work that has employed neural network in comparing property return and macroeconomic factors, where they employed a simple MLP model with one hidden unit and one lag for each variable. This neural network model was identified to be most successful for short-term prediction. The work of Brook and Tsolacos (2003) is based on the work of Qi and Maddala (1999). Qi and Maddala (1999) compare the ability of a neural network model and a linear regression model in predicting the relationship between macroeconomic factors and the stock market.

In the wide field of property studies, neural network was mainly used in the valuation of property. Neural network has mainly been employed to model effects on the valuation of the property due to the sale price and date of the house (Do and Grudnitski, 1992, and Rossini, 1997), dimension and layout of the house (Worzala et al., 1995, Do and Grudnitski, 1992, and Rossini, 1997), material used for the house (Rossini, 1997), macroeconomic data and geographical information system (GIS) data (Ge and Runeson, 2004) and the effect of aircraft noise (Collins and Evans, 1994). Other property related research studies found using neural networks are the forecasting of construction demands (Hua, 1996), mass appraisal techniques (Borst and McCluskey, 1997) and selection of property portfolio (Ellis and Wilson, 2005).

From the review above, the most popular model used in this application appears to be the VAR model, followed by the ARIMA model, the GARCH model and the neural network. The review also indicated that in the earlier works by McCue and Kling (1994) and Brook and Tsolacos (1999), the VAR model was used and only in recent works do researchers employ the GARCH model and the neural network model. The argument for such a trend is that the VAR model is more established than the GARCH and neural network model. Further to this argument, the VAR model is mathematically less complex than the GARCH and neural network model and thus demands less computation power than the latter model.

2.3.5 Other models

In much of the other research studies, the researchers employed a simple multiple linear regression model, namely in Ling and Naranjo (1998), Hoesli (1997) and Karolyi and Sanders (1998). Bond and Seiler (1998) employed a method called the Added Variable Regression Model (AVRM) - a slightly more complex multiple linear regression model, in their research. The problem with employing such model, as discussed in Chatfield

(2001: 245), is that the output of the model is only defined by the specified input of the model and there is no relationship between previous outputs and the current output.

2.4 Related research in South Africa

In terms of similar research done in South Africa, there were three significant research studies, namely Njuguna (2002), Poensgen (2000) and May (2004).

The earliest research by Poensgen (2000) investigated the macroeconomic factors that affect residential property prices. The author uses the ABSA and Rode house price index as a benchmark for measuring residential property prices and has identified that the business confidence index (BCI) is highly correlated to the index. Based on this finding, a stepwise multiple regression model was developed based on four variables namely the residential property index that consisted of BCI, investment level of residential houses, investment level of infrastructure construction and the value of real estate transaction.

Further to the model developed by the author, two simple regression models from industry that predict the effect of certain macroeconomic factors on specific property market characteristics were identified. The first model was a regression model predicting the return of the property unit trusts (PUTs) in Southern Africa based on the repo rate, inflation rate and yield of long-term (30 year US) bonds. The US bond data was used as international benchmark. The second model was a five-variable model predicting the ABSA index for residential property. The five variables were net migration, consumer price index for housing, personal saving, real building cost and real PDI per capita.

Njuguna (2002) investigated the macroeconomic factors that drive the movement of the CBD Property Fund, a private fund that was established by Sage Property Trust Managers Limited in 1981. The fund had a market capitalisation of over R 2 billion in 2002 of which 32% of the value is properties in the Johannesburg and Pretoria CBDs. The model developed is similar to the four variables model developed by Poensgen (2000) consisting of the R150 10 year long bond index, the producer price index, the CPI for housing and the JSE real estate share price index. The model explains 69% of the price index of the fund, i.e. $R^2 = 0.69$.

May (2004) investigates the effects of macroeconomic variables on the changes to the stock market return, which is the JSE All Listed Share Index Return, between January 1990 and December 2003. The macroeconomic variables investigated in the research were the change in the real industrial production, the change in the real term structure of interest rate and the change in the real effective exchange rate. In this research, the term structure of the interest rate is the difference between the Long-term government bond and the Treasury-bill rate, which is the 3 month short term rate. Using the Chen Roll and Ross (CRR) model, which is a univariant regressive model relating the rate of return in the market with various macroeconomic factors. The research identified the following findings:

1. the growth rate of real industrial output positively influenced the change to the stock market return; as production rate rises, the return increases
2. the term structure of interest rate is inversely related to the rate of return, the term structure value is positive when the business cycle reaches a low point, then decrease to zero when the business cycle reaches expansion stage and finally becomes negative when the business cycle reaches the peak of the business cycle
3. the depreciating change in the real effective exchange rate affects positively to the rate of return

All of the above-mentioned South African research used simple stepwise multivariable regression model for their work, which has no autoregressive mechanism.

2.5 Summary of Literature Review

The macroeconomic factors investigated in most of the research studies, which seem to have an influence on property returns, are that of interest rates and the inflation rate. In recent research, the term structure of interest rate was investigated in place of interest rate, in particular in the works of Ling and Naranjo (1998) and Brook and Tsolacos (1999; 2001; 2001a; 2003). Both expected and unexpected inflation were investigated, but unexpected inflation appears to be more useful in the forecast as identified in the works of Ling and Naranjo (1998) and Brook and Tsolacos (1999; 2001; 2001a; 2003). Various research studies focusing on inflation, mainly identify the usefulness of property investment in hedging against inflation and in such a case both unexpected and expected inflation is examined. The investigation of interest rate and inflation corresponds to the finding of Ball et al. (1998: 160), where the use of monetary policies and tools are required to control the demand of money in the economy.

Further to interest rate and inflation, another significant factor that previous research has investigated is previous (or lagged) property return data. Such factors must be evaluated, as most of the investment decision is based upon the performance of the investment in the previous periods. In most of the previous works, this is a significant input factor for the developed model.

Economic factors such as industrial production (McCue and Kling, 1994, West and Worthington, 2004, and Liow, 2004), employment growth rate (Liu and Mei, 1992, and West and Worthington, 2004), and GDP growth rate (Ling and Naranjo, 1998, Bond and Seiler, 1998, and Liow, 2004) were also identified as significant factors on property return. These factors are related to the aggregate supply (industrial production and GDP growth rate) and aggregate demand (employment growth rate) of the macroeconomic activities as expressed in Ball et al. (1998: 161). As previously researched in May (2005), where the growth rate of real industrial production is related to the return of the stock market, such factors should be significant in the investment market of South Africa.

Brook and Tsolacos (2001a) and (2003) introduced gilt-equity yield ratio and identified it as a significant factor affecting property return. This factor is only introduced in recent times in the work of Qi and Maddala (1999) and further investigation is required.

The property return investigated is divided mainly between indirect and direct returns. The indirect return is based on the return of listed property stocks and funds and direct return is based on the return of property portfolios of property companies. Most of the previous work on indirect return focused heavily on established and regulated property market, in particular the US and the UK market. Furthermore, the funds and stocks in these markets control a very significant portion of the property market. There are few research studies that investigated direct property return, and the data is mainly based on property portfolios from independent evaluators such as NCREIF and IPD. Ling and Naranjo (1998), Brook and Tsolacos (2001) and West and Worthington (2004) are recent investigations that investigated both types of returns. Such comparison is required as they react differently to macroeconomic variables (Stringer, 2001), especially with indirect return as it is influenced by the performance of the stock market.

The models used are distinguished between an univariate model, where the model only depend on the previous value of the predicted variable, and a multivariate model, where the model depend on the previous value of the predicted variable as well as other variables. In this research, both types of models are employed. An univariate model is useful in to establish the relationship between the current property return and the previous property return and is simple to implement. A multivariate model is used for mainly identifying the effect of other factors affecting the property return. The most widely use model is the VAR model, where it is use to identify relationships between macroeconomic factors and predicted return. The ARMA model was used in the works of Brook and Tsolacos (2001; 2003) in identifying the autoregressive nature of property return. The GARCH model was only introduced in recent research in West and Worthington (2004) and Liow (2004) where the return of the property is considered as a highly volatile variable. Lastly, Brook and Tsolacos (2003) investigated the use of neural networks to predict property return due to the ability of neural networks in predicting non-linear relationships. In terms of complexity, the GARCH model is the most complex model implement as it requires one to first implement a VAR or a VARMA model before one can employ the GARCH model.

There are three research studies in South Africa that are of importance to this research. The earliest research by Poensgen (2000) investigated the effect of business confidence level, investment level and house prices on ABSA residential house price index. The author also discussed two other simple models used in industry, of which a significant model was developed by ABSA, where the return of property unit trust was forecasted based on the repo rate, inflation rate and the yield of the US 30 year long-bond. Njuguna (2002) developed a model to predict the performance of the Johannesburg CBD fund using macroeconomic factors such as the long bond index, production price index, consumer price index (CPI) for housing and the JSE share price index for property. May (2005) is the most recent related research where a model predicting the return on the stock market was developed. The macroeconomic factors identified as having an effect

were industrial production, expected and unexpected inflation, risk premium and term structure of interest rate. The factors used to predict the results in all of these research studies are similar to international studies, such as inflation rate and interest rate. Factors such as business confidence level and investment levels that reflects the macroeconomic environment were used in the work of Poensgen (2000). Generally, the model used in all of these research studies was a simple multivariable regression model with no autoregressive component, which indicates that the use of complex model in predicting property-related factors in South Africa has yet to be investigated.

3 Data specification and analysis

3.1 Data specification

The macroeconomic factors examined in this research are property return, inflation, term structure of interest rate, the gilt-equity yield ratio, industrial production, employment growth rate and GDP growth rate based on the conclusion of the literature review. The term structure of the interest rate is the difference between the yield of 10-year government bond and 3-year government bond, as discussed in Brook and Tsolacos (2003). As discussed in the literature review of the research of Brook and Tsolacos (2001), Ling and Naranjo (1997; 1998) and of May (2004), the term structure is an indication of the business cycle that the economy is in at a specific period and is related to the future trend of interest rate. Therefore, it is more worthwhile using the term structure of the interest rate in the model rather than using long term and short term interest rate in isolation. Inflation is determined by the consumer price index (CPIX). Gilt-equity yield ratio is calculated based on the ratio of the yield of the 10-year government bond (long-term bond) to the dividend yield of the JSE All Listed Share Index (ALSI), which is the equivalent overall stock market yield in South Africa. The calculation method of the gilt-equity yield ratio is in accordance with the method used in Brook and Tsolacos (2001; 2003).

$$GER = \frac{LTBR}{DY} \quad (12)$$

Where:

LTBR long term bond rate
DY dividend yield

The effect of the manufacturing index, rate of employment in the construction sector and the GDP of the country are also examined in the research, as recommended by West and Worthington (2004). West and Worthington (2004) also examined the affect of the level of construction on the property returns in their research. They argued that the level of construction is useful in determining the level of supply of new properties and consequently affecting the aggregate supply of the macroeconomy. In this research, the number of building plans passed, which represents the number of new construction works approved, is examined.

Furthermore, the prime lending rate from financial institutions is introduced in this research, which is defined as the interest rate that financial institutions charge when lending their money to the public and is relate to the ability of property investors to access loans to purchase properties (Liberta, 2011). Repossession rate, which is the rate at which financial institutes borrow money from the reserve bank and directly related to the prime lending rate (Liberta, 2011a), was not considered. This is because there is

insufficient historical data available in the market; data is only available from 1999 onwards. The movement of the prime lending rate, measured by the change in prime lending rate, is also investigated in this research.

The indirect and direct property returns are examined in the research. The direct return is the IPD property return of retail, office and industrial properties in South Africa. The indirect returns examined are the J255 property trust index and the J256 property loan stock index. The observation period for indirect property return is between 1st quarter of 1989 and 4th quarter of 2007 for J255 property trust index and 3rd quarter of 1991 and 4th quarter of 2007 for J256 property trust index. The observation period for direct property return is between 1st quarter of 1995 and 4th quarter of 2007.

The analysed return data are divided into two further parts, namely the actual return value and the return deviation value. The actual return value is the average return in a specific quarter and the return deviation value is the standard deviation of the return in a specific quarter. The return deviation is valuable for determining the degree of volatility in the market in a specific quarter. The standard deviation of indirect return is calculated based on the annual return recorded monthly. The standard deviation of direct return is calculated based on the annual return recorded from different types of properties in different regions of South Africa.

3.2 Data Analysis

The data analysis in this research is divided into two different sections, namely graphical analysis and analytical analysis. Although most previous research studies have employed analytical analysis, graphical analysis is also selected as it assists in understanding the trends and movement of the data, as discussed in Chatfield (2001: 13-20) and Ebert et al. (2008).

3.2.1 Graphical analysis

The trends of the factors affecting property return (input variables) and the property returns (output variables) are plotted below against time, in Appendix B, and are examined and discussed. Similar to the case of Brook and Tsolacos (2001) and Ling and Naranjo (1997), unexpected inflation is calculated by determining the difference between the actual inflation and the simulated (anticipated) inflation. The simulated inflation is calculated based on an ARIMA (1,1,1) model developed using actual inflation data.

Referring to appendix B, generally all of the input variables appear to follow a cyclic pattern with the exception of the GDP index, the CPIX index and the building plans passed index. The GDP and the CPIX index are non-stationary and increase constantly with time. In order to obtain useful data that can be use in forecasting, stationary data is required as defined by Chatfield (2001: 15-20). First order differencing is therefore implemented for these two variables, which represents the rate of GDP index and the rate

of CPIX index. The graphs for these two indices display a more random pattern. The building plans passed index followed a skewed pattern, where the index increased from approximately 120 point in 2003 to over 200 point thereafter. This is an indication of the property boom between 2003 and 2007, where building activities have increased drastically. The prime lending rate also follows a cyclic pattern and the changing prime lending rate displays a more random pattern.

The direct and indirect property returns (output variables) and their deviations are plotted and presented in Appendix B. The graphs indicate that the indirect returns displayed a more volatile and stochastic trend in comparison to the direct returns, this suggest that the indirect market is influenced by movements on the stock exchange. The trend of the direct returns appear to move upward between 2004 and 2007, an indication of the property boom during these years.

Similarly, the indirect return deviations are more volatile than the direct return deviations. The indirect return deviations oscillate around an average value and generally spike at points where indirect returns are at their greatest, which is an indication of increasing risk and volatility. The direct return deviations again fluctuate very gently from a peak during 1998 to a dip between 2002 and 2003 and gradually increase to another peak from 2006 onward.

The difference in the trend between the indirect and direct property data is also due to the nature of the data. The direct return data from IPD is based on the performance of a group of commercial properties on an annual basis, while the indirect return data is based on the performance of portfolios on the listed sector on a monthly basis. This is evident in the rapid increase in direct property deviations from 2006 onward, where the number of evaluated samples (properties) has drastically increase and has lead to greater variation in the data.

3.2.2 Autocorrelation of the return

The investigation into the autocorrelation relationship of the return (output) is essential as it allows one to understand the significance of the historic returns (output) on the current return as defined in Chatfield (2001: 55-59). As evident in the models investigated in the literature survey above, the autocorrelation component is an essential component of the models and such analysis will assist one with a basic understanding of the models under investigation and in identifying some initial parameters for these models. Referring to previous research, only Brook and Tsolacos (2001) have investigated the autocorrelation relationships of each return (output). As suggested in Chatfield (2001: 55-59), one of the best methods in analysing the autocorrelation relationship is to use the correlogram. The correlogram illustrates the relationships between the current value of a variable and its previous values. The correlogram of each output is presented in Appendix B. Due to the different data sizes between direct and indirect returns, a lag of 15 is investigated for the indirect return data and a lag of 10 is investigated for the direct return data.

The analysis of the indirect return indicates that the autocorrelation between the samples declines rapidly with increasing lag value as referred to the correlogram in appendix B. For J255 and for J256, the autocorrelations were only significant up to 2 lags and 3 lags respectively, which is an indication that historic indirect returns have minimal effect on current and future indirect return. Unlike the indirect return samples, the autocorrelation between direct return samples deteriorates gradually with increasing lag value. For all three different types of commercial properties, the autocorrelations were significant up to 10 lags, an indication that the historic direct returns have very strong influence on current and future direct return. According to Chatfield (2001: 55-59), this usually suggest a non stationary trend. The correlograms of the indirect return also indicate that the relationships between current and historic values is slightly cyclical, which is an indication of the influence on the return due to the stock market.

The correlogram of the indirect return deviations fluctuates randomly about the zero value, which indicates that the current return deviation is unlikely to be affected by historic return deviations. Similar to the result discussed above, the autocorrelation between direct return deviations is higher than the autocorrelation between indirect return deviations. For all three different types of commercial properties, the autocorrelations were significant in the first few lags.

3.2.3 Analytical analysis

Instead of graphical analysis, nearly all of the previous research studies employed analytical analysis in determining the characteristic of the data. Data is evaluated based on the distribution of the sample sets and their movement.

The mean, range, standard deviation (or variance), skewness and kurtosis of the sample sets are investigated. The first three characteristics are self-explanatory and are widely investigated in any statistical problems. The skewness of a sample set determines the degree of deviation of the samples from the arithmetic mean of the set and is defined by the following equation (Spiegel and Boxer, 1972: 91):

$$S = \frac{E(x - \bar{x})^3}{\sigma^3} \quad (13.1)$$

Where:

x	variable under investigation
\bar{x}	mean of x
σ	standard deviation of x

and

$$E(x - \bar{x})^3 = \frac{\sum_{k=1}^n (x_k - \bar{x})^3}{n} \quad (13.2)$$

The distribution is known to be negative skew or left-skewed if a negative skewness value is calculated. In this case, most of the samples lie above or to the right of the arithmetic mean. Conversely, the distribution is known to be positive skew or right-skewed if a positive skewness value is calculated. In this case, most of the samples lie below or to the left of the arithmetic mean. The skewness of a normal distribution is zero based on the above equations.

The kurtosis of a sample set determines the shape of its distribution based on the effect of its outliers. It is defined by the following equation (Spiegel and Boxer, 1972: 91):

$$K = \frac{E(x - \bar{x})^4}{\sigma^4} \quad (14.1)$$

Where:

x	variable under investigation
\bar{x}	mean of x
σ	standard deviation of x

and

$$E(x - \bar{x})^4 = \frac{\sum_{k=1}^n (x_k - \bar{x})^4}{n} \quad (14.2)$$

A high kurtosis value indicates that the distribution has a sharper peak and a longer tail while a low kurtosis value indicates that the distribution has a round peak and a shorter tail. Typically, a sample kurtosis is calculated and is determined by the following equation (Wikipedia, 2009a).

$$K = \frac{E(x - \bar{x})^4}{\sigma^4} - 3 \quad (14.3)$$

The constant at the end represents the kurtosis value of a normal distribution, which is 3. Resultantly, the above equation (Equation 14.3) measures the distortion of the analysed distribution with respect to a normal distribution.

Following previous research studies by Brook and Tsolacos and West and Worthington (2004), the distribution of the data is benchmarked against the normal distribution. The

Jarque-Bera method is used in these studies when analysing normality of distribution (Wikipedia, 2009b). The method first requires the sample skewness and kurtosis of the data set, as defined by equation 15 below. The test statistic of the sample set (known as JB statistic) is first calculated and then compared with the values on the JB statistical table, as defined in Bera and Jarque (1981). For a normal distribution, the JB statistical value is zero. In practice, the researcher usually defines a significant level, where if the calculated JB value is below this level, the data set is deemed to be normally distributed. Brook and Tsolacos defined a 5% and 1% significant level in their works. In this research, a 5% significant level is selected.

$$JB = \frac{n}{6} \left(S^2 + \frac{(K-3)^2}{4} \right) \quad (15)$$

Where:

S	sample skewness of the data set
K	sample kurtosis of the data set
n	sample size of data set

The final analysis of the examined variables is the stationary test. In the graphical analysis above, it was identified that the GDP and CPIX indices were highly non-stationary and first order differencing was used to obtain significant data. For analytical analysis, the augmented Dickey-Fuller (ADF) test is widely used. Brook and Tsolacos (2003) used such technique in their work to determine whether a variable is stationary. The augmented Dickey-Fuller test is derived from the Dickey Fuller test (Dickey and Fuller, 1979), which is a hypothesis test that examine whether a set of data is modelled by an autoregressive time series with a unit root, i.e. the following simple autoregressive equation with $\rho = 1$.

$$Y_t = \rho Y_{t-1} + e_t \quad (16)$$

The augmented Dickey-Fuller test utilised a more sophisticated model than the one defined by Equation 16 above. It utilises an ARMA model that accommodates unknown orders, similar to the ARMA model equation presented in section 2.3.1 (Equation 4). The result of the simulated model is then analysed and compared with a predefined table, which is called the Dickey-Fuller table. Once again, the researcher usually defines a significant level where, if the calculated value is below this level, the sample set is deemed to be stationary.

The table below summarised the analytical analysis of the examined explanatory data.

Properties	Term Structure	CPIX index	Gilt-equity ratio	Manufacturing index	GDP	Employment index
Mean	-0.7279	92.171	461.72	81.700	906690	154.570
Min	-3.4467	36.767	246.26	76.950	735580	95.040
Max	2.0100	151.800	739.91	86.470	1265000	213.490
Standard deviation	0.9853	32.73	134.11	2.598	152710	34.467
Skewness	0.2695	0.0619	0.3822	0.2864	0.8300	-0.2249
Kurtosis	3.3467	1.8257	2.1511	2.0591	2.5489	2.0059
Jarque-Bera Test						
p-value	0.55883	0.10315	0.11238	0.11167	0.00726	0.12050
JB test result	1.16380	4.54310	4.37180	4.38440	9.85000	4.23230
Critical value at 5%	5.99150	5.99150	5.99150	5.99150	5.99150	5.99150
Augmented Dickey Fuller Test						
Alpha value	0.75536	4.22770	0.90936	0.60611	8.75560	0.71720
Adf test value	-1.32990	0.58206	-0.20258	-1.14330	3.29540	-0.72949
Critical value	-3.43910	-3.56630	-3.43910	-3.43910	-3.43910	-3.43910
	-2.91520	-2.93700	-2.91520	-2.91520	-2.91520	-2.91520
	-2.58410	-2.61520	-2.58410	-2.58410	-2.58410	-2.58410
	-0.40460	-0.43928	-0.40460	-0.40460	-0.40460	-0.40460
	-0.04810	-0.04988	-0.04810	-0.04810	-0.04810	-0.04810
	0.53845	0.69424	0.53845	0.53845	0.53845	0.53845

Properties	Building plans index	Changing GDP	Changing CPIX index	Unexpected changing CPIX	Prime interest rate	Changing prime interest rate
Mean	127.360	6517.9	1.6202	0.0692	16.416	0.0188
Min	83.867	-8689.7	0	-2.9846	10.5	-2.5
Max	238.120	18549.0	4.0333	1.9121	23.5	4
Standard deviation	45.922	6347.8	0.7633	0.9813	3.4611	1.0620
Skewness	1.4750	-0.2048	0.5626	-0.4929	-0.1282	0.2498
Kurtosis	3.7063	2.4526	3.6700	3.1966	1.9191	5.2267
Jarque-Bera Test						
p-value	3.12×10^{-7}	0.41233	0.10324	0.24791	0.1077	4.4×10^{-4}
JB test result	29.95800	1.77190	4.54140	2.78940	4.4569	15.457
Critical value at 5%	5.99150	5.99150	5.99150	5.99150	5.9915	5.9915
Augmented Dickey Fuller Test						
Alpha value	0.62376	0.57021	0.39339	0.38341	0.70492	0.27424
Adf test value	-0.88985	-2.23930	-4.99310	-5.08330	-0.77902	-6.74560
Critical value	-3.43910	-3.43910	-3.56630	-3.56630	-3.43910	-3.43910
	-2.91520	-2.91520	-2.93700	-2.93700	-2.91520	-2.91520
	-2.58410	-2.58410	-2.61520	-2.61520	-2.58410	-2.58410
	-0.40460	-0.40460	-0.43928	-0.43928	-0.40460	-0.40460
	-0.04810	-0.04810	-0.04988	-0.04988	-0.04810	-0.04810
	0.53845	0.53845	0.69424	0.69424	0.53845	0.53845

Table 2.1: Analytical analysis of the input (explanatory) variables

The mean, minimum, maximum and standard deviation values calculated correspond to the graphs plotted for each variable.

The variables are positively skewed with the exception of the employment index, changing GDP, unexpected inflation (unexpected change in CPIX) and prime lending rate, which means that the samples are generally higher than the mean. The skewness of the CPIX index is the lowest while the skewness of the building plans passed is the highest. This corresponds to the graphical analysis above for the variables where the graph of the CPIX index approximate to a linear curve, an indication of a normally distributed data set, and the graph of the building plans passed index remained at a very low level between 1988 and 2002 and spiked to a very high level from 2003 onward, which is an indication of a distribution with two means.

The building plans passed index, the GDP value at market price and the changing prime lending rate have all failed the Jarque-Bera test at a 5% significant level, which is a value of 5.9915. This is because the Jarque-Bera test result calculated for these variables is higher than the defined critical level. The Jarque-Bera test result for the CPIX index and the building plans passed index is consistent with previous finding as they have the two highest skewness values, which is related to the normality of the data set, as defined in the Jarque-Bera equation (Equation 15). However, the changing prime lending rate did not pass the Jarque-Bera test even though it has an average skewness value. The cause of this phenomenon is the sudden variation in the sample values between 1998 and 2000.

The kurtoses of term structure, building plans passed index, changing inflation (CPIX index), unexpected inflation (unexpected change in CPIX) and changing prime lending rate are higher than the normal distribution level, which is 3. The result indicates that the data samples of these variables are less spread out than the normal distribution, i.e. they have distribution curves with sharper peak. These data sets also have a “longer and fatter” tail (Wikipedia, 2009a) and hence a higher variance value. The kurtoses of other variables are lower than the normal distribution level of 3, which indicate that the data samples of these variables are more spread out than the normal distribution, i.e. they have distribution curves with more rounded peak. These data sets also have a “shorter and thinner” tail (Wikipedia, 2009a) and hence a lower variance value. The changing prime lending rate has the highest kurtosis value (5.2267) while inflation has the lowest kurtosis value (1.826).

With the exception of the changing inflation value (changing CPIX), the unexpected inflation and the changing prime lending rate, the calculated augmented Dickey Fuller test values of the other variables are all higher than the 10% significance level and thus have failed the augmented Dickey Fuller (ADF) hypothesis of a stationary zero-order series. The CPIX and GDP indices have the highest Dickey Fuller test values, which is consistent with previous graphical analysis where these two variables are highly non-stationary.

Properties	J255 total return	J256 total return	IPD return - retail	IPD return - office	IPD return - industrial
Mean	0.18527	0.20196	0.19324	0.14471	0.17658
Min	-0.15719	-0.15799	0.09008	0.01388	0.02074
Max	0.58246	0.55825	0.32719	0.35162	0.35264
Standard deviation	0.15964	0.18229	0.06739	0.08830	0.10476
Skewness	0.15313	0.10781	0.36741	0.83760	0.38569
Kurtosis	2.45370	2.10270	1.84390	2.53450	1.69440
Jarque-Bera Test					
p-value	0.47673	0.25994	0.11291	0.04035	0.07123
JB test result	1.48160	2.69460	4.36230	6.42060	5.28380
Critical value at 5%	5.99150	5.99150	5.99150	5.99150	5.99150
Augmented Dickey Fuller Test					
Alpha value	0.68424	0.89115	1.41870	2.83270	2.02280
Adf test value	-1.63500	-0.46478	0.50532	1.73310	0.70452
Critical value	-3.43910	-3.56630	-3.56630	-3.56630	-3.56630
	-2.91520	-2.93700	-2.93700	-2.93700	-2.93700
	-2.58410	-2.61520	-2.61520	-2.61520	-2.61520
	-0.40460	-0.43928	-0.43928	-0.43928	-0.43928
	-0.04810	-0.04988	-0.04988	-0.04988	-0.04988
	0.53845	0.69424	0.69424	0.69424	0.69424

Properties	J255 return deviation	J256 return deviation	IPD return deviation - Retail	IPD return deviation - Office	IPD return deviation - Industrial
Mean	0.04597	0.04136	0.02932	0.03418	0.02399
Min	0.00628	0.00227	0.01840	0.00558	0.00548
Max	0.15507	0.13382	0.07291	0.06860	0.05011
Standard deviation	0.02652	0.02578	0.01194	0.01566	0.01123
Skewness	1.01560	1.32890	2.06360	0.07303	0.41953
Kurtosis	5.39060	5.31450	7.10400	2.19600	2.26730
Jarque-Bera Test					
p-value	5.07×10^{-7}	1.22×10^{-7}	3.33×10^{-15}	0.41673	0.23543
JB test result	28.9910	31.8390	66.6560	1.75060	2.89260
Critical value at 5%	5.99150	5.99150	5.99150	5.99150	5.99150
Augmented Dickey Fuller Test					
Alpha value	-0.10543	-0.05163	2.71940	0.78423	1.10400
Adf test value	-9.73830	-8.58080	3.40140	-0.29888	0.47419
Critical value	-3.43910	-3.56630	-3.56630	-3.56630	-3.56630
	-2.91520	-2.93700	-2.93700	-2.93700	-2.93700
	-2.58410	-2.61520	-2.61520	-2.61520	-2.61520
	-0.40460	-0.43928	-0.43928	-0.43928	-0.43928
	-0.04810	-0.04988	-0.04988	-0.04988	-0.04988
	0.53845	0.69424	0.69424	0.69424	0.69424

Table 2.2: Analytical analysis of the output (return)

The mean of the returns are generally between 14% and 20%, the indirect return of J256 property loan stock, which corresponds to the graphs above. The standard deviation for the indirect return is higher than the direct return. The deviation of the indirect return

deviation is also higher than the standard deviation of the direct return deviation, an indication that indirect return is more volatile than direct return.

All of the returns and return deviations are positively skewed, i.e. the sample values are generally higher than the mean value, which correspond with the trend of increasing return and volatility over the years.

Similar to the analytical analysis of the explanatory variables, the data sets with the highest skewness levels, the direct office return, the indirect return deviations and the direct retail return deviations, failed the Jarque-Bera test. For direct return and return deviation, the result corresponds to the sudden increase in the data from 2006 onwards identified in the graphical analysis, which skewed the data significantly. While for indirect return deviation, a possible cause is the sharp spikes during periods where indirect return peaks, which can skew the mean significantly.

With the exception of the indirect return deviations and the direct retail return deviation, all of the data sets have a lower kurtoses value than the normal distribution. In other words, these data sets have higher variance and deviation level. The high kurtosis value of the indirect return deviations and the direct retail return deviation also indicate that the deviation are more likely to be at a specific level and thus it is more likely to predict the levels of deviation for these data sets.

Similar to the analysis of the variables, all of the return and return deviation series, with the exception of indirect return deviation series, failed the ADF test with 10% significance level and thus the series are non-stationary at zero-order. This is an indication of increasing average return of commercial properties over the years.

3.3 Summary

The explanatory (input) variables selected for this research are term structure of interest rate, gilt-equity yield ratio, manufacturing index, employment index, building plans passed index, prime lending rate, GDP index and CPIX index, which is inflation. Furthermore, the changing GDP index, changing CPIX index, changing prime lending rate and the unexpected inflation, which is the unexpected CPIX index change, were extracted from the input data for analysis. Indirect property return (output variables) derived from the J255 and J256 property indices and direct property return derived from IPD data were selected for this research.

All of the data under investigation for this research is first graphically analysed. The graphical analysis of the input indicates that the GDP and CPIX indices increases linearly with time and is highly non-stationary. The indices are then subjected to first order differencing and thus the changing GDP and CPIX indices trends are obtained. Furthermore, based on previous research, the unexpected inflation trend is extracted from the CPIX indices using an ARIMA model. The graphical analysis of the output indicates

a cyclic movement over time and the trends of the indirect return are much more volatile than the trends of the direct return.

The autocorrelations of the outputs are also analysed by means of correlograms. The autocorrelation relationships of the indirect returns deteriorated much quicker than the direct returns. The relationships between the indirect return samples become insignificant after the 3rd lag and the relationships between the direct return samples become insignificant after the 7th lag. A similar trend is identified between indirect return deviation and direct return deviation, where the autocorrelation relationships between indirect return deviation samples is not present and the relationships between the direct return samples become insignificant after the 2nd lag.

Lastly, the data was analysed using analytical techniques from which the calculated results generally correspond to what was observed in the graphical analysis. The ADF stationary test was also performed and the test indicates that most of the data sets are slightly non-stationary, an indication of a slight increase in these trends over time.

4 Relationships between macroeconomic factors and return

The relationships between macroeconomic factors and return are analysed, after an analysis of the nature of the input and output data in the previous section. Here, a technique is used in identifying the factors that have the most impact on the return. This process is essential in eliminating the factors that have little effect on the change of property return and isolating the significant factors that can be use for the development of simpler and concise models at a later stage.

4.1 Analysis of causality

The analysis used, which is previously used in the works of Brook and Tsolacos (2001; 2001a; 2003), is called the Granger Causality Analysis. The initial model, developed by Clive Granger (Granger, 1969), was designed for the examination of the relationships for two time-series at a time. The basis of the analysis is to examine the effect on the changes to the dependant variable due to changes to the explanatory variable and quantify such effect by means of statistical F-test. Wikipedia (2009c) discussed that this technique does not apply when a relationship involves more than two variables. For relationships that involved multiple variables, the normal procedure is to develop a general VAR model with all of the variables and use the F-test to identify the relationships between the variables. In this research, VAR models combining the macroeconomic variables and the returns or the return deviations are developed and the relationships between the two are identified using a typical F-test, as discussed in LeSage (1998: 216-218). Such a method is also used in Brook and Tsolacos (2003). The following is the result from the VAR models developed based on a lag of 4 for indirect property return series and a lag of 3 for direct property return series. The short lag length defined in the models is due to a fundamental limitation imposed by VAR model, as discussed in LeSage (1998: 218-219). The limitation stated that given a specific sample size, the model is only permitted to have a maximum number of parameters, which is less than the number of samples available, before the model become inaccurate. Since the available sample size is so small, the number of parameters permitted for investigation and the lag values for the models investigated is limited to a low value. The tabulated results are the Granger causality probability test where the lower the value, the greater the relationship between the variables. For probability value higher than the defined cut off point, which is set as 0.5, "NaN" is indicated.

Variables	J255 return	J256 return
Term Structure	0.13	0.42
Gilt-equity ratio	NaN	NaN
Manufacturing index	0.25	0.09
Employment index	0.04	0.07
Building plans passed index	0.02	0.01
Changing GDP	0.47	0.48
Changing CPIX index	0.21	0.20
Unexpected changing CPIX	0.43	NaN
Prime lending rate	0.44	0.39
Changing prime lending rate	0.44	0.44
J255 return	0.12	0.10
J256 return	0.19	0.40

Table 3.1: Granger causality probabilities of indirect property returns

Variables	J255 return deviation	J256 return deviation
Term Structure	0.08	0.04
Gilt-equity ratio	0.03	0.23
Manufacturing index	0.44	0.24
Employment index	0.15	0.24
Building plans passed index	0.20	0.09
Changing GDP	0.22	NaN
Changing CPIX index	NaN	0.42
Unexpected changing CPIX	NaN	NaN
Prime lending rate	0.46	NaN
Changing prime lending rate	0.33	NaN
J255 return deviation	0.02	0.44
J256 return deviation	NaN	NaN

Table 3.2: Granger causality probabilities of indirect property return deviations

Variables	Retail return	Office return	Industrial return
Term Structure	0.19	NaN	0.24
Gilt-equity ratio	0.09	0.25	0.28
Manufacturing index	0.28	NaN	0.11
Employment index	NaN	NaN	0.38
Building plans passed index	NaN	0.42	NaN
Changing GDP	0.38	0.25	NaN
Changing CPIX index	0.05	0.05	0.19
Unexpected changing CPIX	0.16	NaN	0.35
Prime lending rate	NaN	NaN	NaN
Changing prime lending rate	NaN	NaN	NaN
Retail return	0	NaN	0.24
Office return	0.21	0	0.28
Industrial return	NaN	NaN	0

Table 3.3: Granger causality probabilities of direct property returns

Variables	Retail return deviation	Office return deviation	Industrial return deviation
Term Structure	NaN	0.31	NaN
Gilt-equity ratio	0	0.03	0.23
Manufacturing index	0.34	0.31	0.18
Employment index	NaN	NaN	NaN
Building plans passed index	0.05	0.09	NaN
Changing GDP	0.02	0.19	NaN
Changing CPIX index	0.01	0.01	0.4
Unexpected changing CPIX	0.26	0.12	0.3
Prime lending rate	0.14	0.50	0.02
Changing prime lending rate	0.39	0.03	NaN
Retail return deviation	0	NaN	0.27
Office return deviation	0.19	0	0.32
Industrial return deviation	0.35	0.08	0

Table 3.4: Granger causality probabilities of direct property return deviations

From the test result above, the relationships between the explanatory variables and the output are investigated further if the probability of the relationships is less than 0.2. The result is summarised in the table below. The summary table below does not take into account the autocorrelation relationship, which will also be incorporated in the models.

Output	Variables
J255 return	Term structure, employment index in construction, building plans passed index, J256 return
J256 return	Manufacturing index, employment index in construction, building plans passed, changing CPIX index, J255 return
J255 return deviation	Term structure, gilt-equity ratio, employment index in construction, building plans passed index
J256 return deviation	Term structure, building plans passed index
Retail return	Term structure, gilt-equity ratio, changing CPIX index, unexpected changing CPIX
Office return	Changing CPIX index
Industrial return	Manufacturing index, changing CPIX index
Retail return deviation	Gilt-equity ratio, building plans passed index, changing GDP, changing CPIX index, office return deviation, prime lending rate
Office return deviation	Gilt-equity ratio, building plans passed index, changing GDP, changing CPIX index, unexpected changing CPIX, industrial return deviation, changing prime lending rate
Industrial return deviation	Manufacturing index, prime lending rate

Table 3.5: Explanatory variables with significant causality on return series

4.2 Summary

The Granger Causality method, which is based on the F-test for VAR models, is used to identify the relationships between the explanatory variables and the return. The factors most significant to each specific returns and return deviations are tabulated in Table 3.5 above. The analysis identified that the three most influential factors on indirect return and return deviation are the term structure, employment index and building plans passed index.

Both the term structure and employment index factors were identified to have significant influence in indirect property returns in previous research, term structure in the works of May (2005), Ling and Naranjo (1998) and Brook and Tsolacos (1999; 2001; 2001a; 2003) and employment index factor in the works of Liu and Mei (1992) and West and Worthington (2004). Since the term structure is directly related to interest rate, the result also corresponds to the general consensus that interest rate is strongly related to the indirect property return. However, this relationship was not identified in the test above. The strong relationship between the building plans passed index and indirect return is a new finding and unique to this research. This finding indicates that there is a link between direct property market, in terms of actual level of building activity, and indirect property market, in terms of indirect property return. Previously only West and Worthington (2004) investigated the effect of building activities, related to the building plans passed index, on indirect property return and no significance was found in the relationship between the two.

The factors that influence the movement of direct return and direct return deviation are the changing CPIX index and the gilt-equity ratio. With the exception of the industrial return deviation, the changing CPIX index significantly affects direct return and return deviations. This observation confirms the results from previous investigations where direct property investment has hedging ability against inflation, or in this case the movement of inflation. The effect of gilt-equity ratio on any property return has only previously been identified in Brook and Tsolacos (2001a) and (2003) where this factor did have an effect on indirect property return. The strong relationship between the gilt-equity ratio and direct return is again a new finding and is unique to this research. Lastly, it is of interest to note that industrial return and return deviations are strongly related to the manufacturing index. This finding corresponds to the findings of McCue and Kling (1994) and May (2005).

Most of the abovementioned relationships identified corresponds to the finding of previous research, which are predominantly conducted in well established property markets with extensive property resource - namely the European, US and Australian market. The behaviour of the South African commercial property market is therefore related to the global commercial property market.

5 Development of simulated models

The application used for developing and simulating the models is Matlab version 7.0, which is designed for sophisticated mathematical calculation and modelling. The software is designed particularly to operate with matrices and big data sets with designed modules of sophisticated equations and calculations. Existing modules for the required models in this research are available in this software. Appendix D is dedicated to the theory of the respective models and one can refer to this section if further understanding of the models is required. It must be noted that the ARMA and the GARCH models in this software are limited to one output for each model and a model is designed for each output. The VAR and neural network models in this software can accommodate multiple outputs and a model is designed for each type of return, i.e. the outputs of the model developed for a specific type of return will accommodate for the return and its deviation.

6 Optimising models

In developing an optimal model for a problem, the trade off between the complexity of the model and the accuracy of the model must be considered. Traditionally, one would employ an ad-hoc triads and error approach in order to identify an optimal model. Information criterion techniques provide a more logical and scientific way of finding an optimal model. Two different information criterions methods are used in this research, which are also used in the works of Brooks and Tsolacos (1999; 2001; 2001a; 2003) and West and Worthington (2004). The information criterion methods are Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC), which is also known as the Bayesian Information Criterion.

6.1 Akaike Information Criterion (AIC)

This information criterion technique is the most widely used information criterion in this field and was developed by Hirotugu Akaike (1974). The method is based on the approximately unbiased estimator of the expected Kullback-Leibler information theory for a fitted model, as discussed in Hurvich and Tsai (1989) and Bedrick and Tsai (1994), and is defined by the following equation.

$$\Delta(\theta, \sigma^2) = E_F \{-2 \log L\} \quad (17)$$

Where:

L maximum likelihood function of the approximating model

The general equations for AIC, based on the above equation, are as follow (Hurvich and Tsai (1989)).

$$AIC = 2k - 2 \log(L) \quad (18.1)$$

OR

$$AIC = 2k - 2 \ln(L) \quad (18.2)$$

Where:

k number of parameters in the model

L maximum likelihood of the approximating model

In the case where the log maximum likelihood of the approximating model is not defined, under further assumption that the errors are normally and independently distributed, the equation above (Equation 18.1) is equated to the following (Hurvich and Tsai, 1989).

$$AIC = 2k + n[\log \hat{\sigma}^2 + 1] \quad (18.3)$$

Where:

$\hat{\sigma}^2$ variance of the error function of the approximating model
 n number of samples used for estimating the model

The variance of the error function of the approximating model can be calculated based on the residual sum of squares derived from the approximating model. The best model examined is the model that produces the lowest AIC value as the equation merits the model with the lowest residual sum of square error and the lowest number of parameters, which is essential in avoiding overfitting. There is a variation to the above equation (Equation 18.3), which is defined in Egriolgu et al. (2008).

$$AIC = \frac{2k}{n} + [\log \hat{\sigma}^2] \quad (18.4)$$

The calculated value from Equation 18.3 and 18.4 will not be the same as the two equations are different. However, it must be noted that the AIC value calculated by the equation are merely an indication of the optimal model for a set of data. Provided that the same equation is used in the evaluation, the use of either of the equations has no impact on the evaluation process of the optimal model.

In the case where the sample set is small, Burnham and Anderson (2004) and Hurvich and Tsai (1989) suggested that a constant factor should be added to the calculated AIC value in the above equation. This modified form of AIC is known as the modified AIC (AICc).

$$AICc = AIC + \frac{2k(k+1)}{n-k-1} \quad (19)$$

Where:

k number of parameters in the model
 n number of samples used for estimating the model

The additional constant in the above equation (Equation 19) imposes a more severe penalty on the complexity of the model.

When optimising models with multiple outputs, such as the VAR model and the neural network model, the only difference in the AIC equations listed above (Equation 18.3 and 18.4) is that the determinant of the error covariance matrix of the approximating model is used in place of the variance of the error function ($\hat{\sigma}^2$), as defined in Bedrick and Tsai

(1994), who derived the following general AIC equation for a model with multivariate regression.

$$AIC = 2k + n[\log|\hat{\sigma}| + p] \quad (20)$$

Where:

k	number of parameters in the model
$ \hat{\sigma} $	determinant of the error covariance matrix of the approximating model
p	number of output in the model

6.2 Schwarz Information Criterion (SIC)

The name of this information criterion is derived from Gideon E Schwarz, who developed this information criterion technique in 1978. It is also called the Bayesian Information Criterion (BIC) as the technique was developed based on Bayesian argument (Wikipedia, 2009e). The general equation for BIC, which is similar to the AIC equation, is as follows (Wikipedia, 2009e):

$$BIC = k \log(n) - 2 \log(L) \quad (21)$$

Where:

k	number of parameters in the model
L	maximum likelihood of the approximating model
n	number of samples used for estimating the model

The main difference between the BIC and the AIC equation is the constant term. For BIC, the constant term considers the data distribution to be exponential, which is more suitable for data sets with a higher kurtosis value, i.e. those sets with longer tail and larger mean (Wikipedia, 2009e). Furthermore, the inclusion of the sample size of the data set penalises models with higher sample value more severely.

Similarly, assuming that the error is normally distributed, the model can be derived to the following form (Laio et al., 2009, and Egriolgu, Aladag and Gunay, 2008).

$$BIC = k \log(n) + n \log \hat{\sigma}^2 \quad (22.1)$$

OR

$$BIC = \frac{k \log(n)}{n} + [\log \hat{\sigma}^2] \quad (22.2)$$

Where:

- k number of parameters in the model
- $\hat{\sigma}^2$ variance of the error function of the approximating model
- n number of samples used for estimating the model

Similar to the AIC technique, the best model is one with the lowest BIC value. This model again merits the model with the lowest residual sum of square error and the lowest number of parameters. For a model that has multiple outputs, the determinant of the error covariance matrix of the approximating model is again used in place of the variance of the error function ($\hat{\sigma}^2$), as defined in Bedrick and Tsai (1994).

$$BIC = k \log(n) + n \log |\hat{\sigma}| \quad (23)$$

Where:

- k number of parameters in the model
- $|\hat{\sigma}|$ determinant of the error covariance matrix of the approximating model
- p number of output in the model

6.3 Identification of optimal models

The optimising model parameters need to be defined prior to the implementation of the abovementioned information criterion techniques. In the works of Brook and Tsolacos and West and Worthington (2004), the lag of the autoregressive part (R) and the lag of the moving average part (M) are the optimising parameters for the univariant ARMA model. Likewise, the lag of the autocorrelative variance (P) and the error variance (Q) are the optimising parameters for the univariant GARCH model. Further to these parameters, the lag of the explanatory factors is an additional parameter incorporated in this optimisation process. This parameter is essential in this research as it provides an indication of the degree of influence that the historic macroeconomic factors have on current and future property returns.

The optimising parameters investigated for the VAR model are the number of lags in the model, which is used in Brook and Tsolacos (2003), and the number of explanatory factors to be used in the model. For the neural network model, the optimising parameters considered are the number of lags in the model, the number of explanatory factors, the

number of neurons in the middle layer and the type of transfer function used by the neurons in the output layer. Once again, the incorporation of the number of explanatory factors as an optimising parameter in these two models provides an indication of the degree of influence that the macroeconomic factors have on current and future property returns.

In numerous literatures, the two information criteria are compared and BIC is generally preferred to AIC in selecting an optimal model. This is because BIC imposes a more severe penalty for model complexity (Qi and Zhang, 2001) when the sample set is larger than 7. This is also evident in Brook and Tsolacos (2003), where the selection using BIC technique is preferred to the selection using AIC technique on occasions where the selections using the two techniques do not agree. However, Hurvich and Tsai (1989) and Bedrick and Tsai (1994) proved that with the inclusion of the constant factor, AICc is more powerful in identifying the optimal model than BIC. Consequently, both BIC and AICc techniques are used in identifying the optimal model. Input explanatory variables are selected for each model based on the result from the causality test in section 5 and are incorporated in the models alongside previous output values.

The following is a summary of the input explanatory variables selected for the univariant ARMA and GARCH models, which are variables identified in the causality test in section 4 that are significantly related to the respective output, as referred to Table 3.5 above.

Output	Univariant ARMA and GARCH models
J255 return	Term structure, employment index in construction, building plans passed index, J256 return
J256 return	Manufacturing index, employment index in construction, building plans passed index, changing CPIX index, J255 return
J255 return deviation	Term structure, gilt-equity ratio, employment index in construction, building plans passed index
J256 return deviation	Term structure, building plans passed index
Retail return	Term structure, gilt-equity ratio, changing CPIX index, unexpected changing CPIX
Office return	Changing CPIX index
Industrial return	Manufacturing index, changing CPIX index
Retail return deviation	Gilt-equity ratio, building plans passed index, changing GDP, changing CPIX index, office return deviation, prime lending rate
Office return deviation	Gilt-equity ratio, building plans passed index, changing GDP, changing CPIX index, unexpected changing CPIX, industrial return deviation, changing prime lending rate
Industrial return deviation	Manufacturing index, prime lending rate

Table 4.1: Explanatory variables selected for ARMA and GARCH models

The maximum lag investigated for both parts of the univariant ARMA model, the autoregressive part and the moving average part, is limited to 8. Similarly for the

GARCH model, the lag for the autocorrelative variance (P) and the error variance (Q) is limited to 8. The lag of the explanatory factors for both type of models investigated are 1, 2 and 4 respectively. Since the log likelihood function is available for both of these models, Equation 18.1 and Equation 21 are used to calculate the AIC (AICc) and BIC values respectively. The parameters of the optimal univariant models for each output are tabulated below.

	Univariant ARMA		Univariant GARCH	
	AICc	BIC	AICc	BIC
J255 Return	(6,2,1)	(6,2,1)	(1,1,2)	(1,1,2)
J256 Return	(1,3,1)	(1,3,1)	(1,1,1)	(1,1,1)
Retail Return	(1,4,1)	(1,4,1)	(2,5,2)	(2,5,2)
Office Return	(1,3,4)	(1,3,4)	(1,1,1)	(1,1,1)
Industrial Return	(1,3,2)	(1,3,2)	(1,4,4)	(1,4,4)
J255 Return Deviation	(6,1,1)	(6,1,1)	(3,1,1)	(3,1,1)
J256 Return Deviation	(1,1,2)	(1,1,1)	(1,4,1)	(1,2,1)
Retail Return Deviation	(1,3,1)	(1,3,1)	(3,5,1)	(3,5,1)
Office Return Deviation	(1,2,1)	(1,2,1)	(1,6,1)	(1,6,1)
Industrial Return Deviation	(3,1,1)	(1,1,1)	(3,1,1)	(3,1,1)

Table 4.2: Result of the information criterion test for the univariant models

The first two parameters represent the lag of the autoregressive part (R) and the lag of the moving average part (M) of the optimal ARMA model. Similarly, the first two parameters represent the lag of the autocorrelative variance (P) and the error variance (Q) of the optimal GARCH model. The final parameter represents the lag of the explanatory variable. Generally, the tests result from both methods yield the same optimal model with the exception of the ARMA and GARCH model for J256 Return Deviation and the ARMA model for direct industrial return deviation. In these cases, AICc method selects a slightly more complex model than the BIC method, which contradicts the finding in the work of Hurvich and Tsai (1989). Consequently, further evaluation of models selected by both methods is made.

The following is a summary of the input explanatory variables selected for the multivariate VAR and neural network models.

Output	VAR and Neural Network
Four (4) explanatory variables	
J255 Return and Deviation	Term structure, gilt-equity ratio, employment index in construction, building plans passed index
J256 Return and Deviation	Manufacturing index, employment index in construction, building plans passed index, term structure
Retail Return and Deviation	Gilt-equity ratio, changing CPIX index, changing GDP, building plans passed index
Office Return and Deviation	Gilt-equity ratio, changing CPIX index, building plans passed index, changing prime lending rate
Industrial Return and Deviation	Manufacturing index, gilt-equity ratio, prime lending rate, changing CPIX index
Two (2) explanatory variables	
J255 Return and Deviation	Gilt-equity ratio, building plans passed index
J256 Return and Deviation	Building plans passed index, term structure
Retail Return and Deviation	Gilt-equity ratio, changing CPIX index
Office Return and Deviation	Gilt-equity ratio, changing CPIX index
Industrial Return and Deviation	Manufacturing index, prime lending rate

Table 4.3: Explanatory variables selected for VAR and neural network models

The maximum lag investigated for the VAR model is 6. For each lag, a model with four explanatory variables (two of the best performed explanatory variables for each output) and a model with two explanatory variables (the best performed explanatory variable for each output) are investigated. These variables are summarised in Table 4.3 above. The number of lags investigated for neural network models are 1, 2 and 4 respectively. Similar to the VAR model, for each number of lag, a model with four explanatory factors and a model with two explanatory factors are investigated. The maximum number of neurons investigated for the model is 50 and the transfer function investigated are the linear function, the logistic function and the softmax function (Equation 6.1 to 6.3 in Appendix E) defined in the software. For these models, equation 20 and equation 23 above are used to calculate the AIC (AICc) and BIC values respectively, since log likelihood functions are not available and both types of models produce multiple outputs.

	VAR		Neural Network	
	AICc	BIC	AICc	BIC
J255 Return and Deviation	(4,2)	(1,2)	(4,4,2,Linear)	(2,2,1,Linear)
J256 Return and Deviation	(4,2)	(1,2)	(4,4,2,Linear)	(2,2,1,Linear)
Retail Return and Deviation	(3,2)	(1,2)	(4,2,2,Linear)	(2,2,2,Linear)
Office Return and Deviation	(3,2)	(1,2)	(4,2,2,Linear)	(2,2,2,Linear)
Industrial Return and Deviation	(3,2)	(1,2)	(4,2,2,Linear)	(2,2,1,Logistic)

Table 4.4: Result of the information criterion test for VAR and neural network

For VAR models, the first parameter represents the lag of the model and the second parameter represents the number of explanatory factor to be use in the model. For neural network models, the first parameter represents the number of neurons in the middle layer, the second parameter represents the number of explanatory factors in the model, the third parameters represents the lag of the explanatory factors and the last parameter represents the type of transfer function selected for the neurons. Unlike the univariant models, the test results from both methods do not yield the same optimal model. The AICc method tends to select a more complex model that the BIC method, in particular for VAR model, BIC selected the simplest model as the optimal model for all of the output. As discussed in the work of Bedrick and Tsai (1994), the result from both methods should be considered and thus further investigation is made of the models from both methods

The result of all the models evaluated in the optimisation process using the information criterion tests is outlined in Appendix E.

6.4 Summary

In this section, the model optimisation process is investigated. The process involves the implementation of information criterion test. Two established tests were consider, namely the AICc and the BIC test. The BIC test is widely used in previous research and the AICc test is an enhanced method derived from the AIC test, which is also widely used in previous research.

Thereafter, the optimising parameters for each type of models are defined. For the ARMA, GARCH and VAR models, the parameters are the lag variables in each respective model. For the neural network model, the parameters are the number of neurons and the type of transfer functions on the output layer. In addition, the number of explanatory variables (macroeconomic factors) and their lags are also optimised.

The optimal models calculated using the AICc test and BIC test are generally identical for the ARMA and GARCH models. The lag for the autoregressive (AR) component does

not exceed 3, with the exception of the ARMA models for J255 return and return deviation. This finding corresponds with the observation in section 3.2.2, as the lag increases, the correlation between current output (return or return deviation) and previous outputs decreases. In the case of the ARMA models for J255 return and return deviation, the lag for the autoregressive (AR) component is 6, which correlated to the finding in the correlograms of the two outputs. In both cases, the correlogram indicates a significant correlation level between the current value and the value 6 periods (lags) ago. The tests also identified that simple models with low lag values and fewer parameters are optimal for this application.

The result from the optimisation process of the VAR and neural network is quite different in comparison to the result above. The models calculated from the two tests do not agree with one another. The optimal models identified by AICc are more accurate than the models identified by BIC. However, the optimal models identified by BIC are simpler than the models identified by AICc. Once again, the tests identified simple models with low lag values and fewer parameters for this application. The general lag for the explanatory variables in the optimal models is 1 or 2 periods. This is an indication that the effect of the macroeconomic factors on the outputs (return or return deviation) is only significant up to 2 periods.

7 Model comparison and evaluation

In this final section, the performances of each type of models with the optimal configuration identified in section 6 are compared and evaluated. Each model is evaluated under two main scenarios, namely the long-term prediction scenario and the short-term prediction scenario. The predicted result from each models is then compared with the actual result both graphically (for long-term prediction scenario only) and analytically, by means of a set of comparison tools used in previous research.

7.1 Performance comparison tools

As previously discussed, two types of performance comparison methods are employed in this research. The first method is the graphical method, where the predicted results are plotted against the actual results. This method is the simplest method of evaluating the performances of the models but is highly subjective, as discussed in Ebert et al. (2008). Consequently, the analytical method is also introduced in this research, which provides information regarding the performance of a model that one might overlook or is not apparent using the graphical method. The following benchmarks are used for the analytical method, as employed in Qi and Maddala (1999), Egrioglu et al. (2008) and Qi and Zhang (2001). In the research of Brook and Tsolacos (2001; 2003) and West and Worthington (2004), only the first four criterions are considered.

1. Mean squared error,

$$MSE = \frac{\sum (y - \hat{y})^2}{n} \quad (24)$$

Where:

y	predicted output by the model
\hat{y}	actual output
n	number of samples used for estimating the model

2. Root mean squared error,

$$RMSE = \sqrt{\frac{\sum (y - \hat{y})^2}{n}} \quad (25)$$

3. Mean Absolute error,

$$MAE = \frac{\sum |y - \hat{y}|}{n} \quad (26)$$

4. Mean absolute percentage error,

$$MAPE = \frac{\sum \left| \frac{y - \hat{y}}{y} \right|}{n} \quad (27)$$

For these four criteria, a low error value is preferred as a low error value indicates an accurately predicted value by the model under evaluation.

5. The Pearson correlation coefficient between the actual and the fitted or predicted return (only applies for 2-step or more forecast), which is introduced in Qi and Maddala (1999).

$$r_{y\hat{y}} = \frac{\text{cov}(y, \hat{y})}{\sigma_y \sigma_{\hat{y}}} = \frac{\sum_{i=1}^n (y - \bar{y})(\hat{y} - \bar{\hat{y}})}{\sqrt{\sum_{i=1}^n (y - \bar{y})^2 \sum_{i=1}^n (\hat{y} - \bar{\hat{y}})^2}} \quad (28)$$

Where:

y	predicted output by the model
\hat{y}	actual output
\bar{y}	mean of the actual output samples
$\bar{\hat{y}}$	mean of the predicted output samples

The range of the coefficient varies from 1 to -1 where 1 indicates that the predicted return directly correlates to the actual value and -1 indicates that the predicted return correlates to the inversed actual value. A high absolute value indicates a strong correlation between the two.

6. Direction accuracy, which measures the number of times that the model accurately predicts the movement of the return.

$$DA = \frac{\sum_{i=1}^n a_i}{n} \quad (29.1)$$

Where

$$a_i = \begin{cases} 1 & \text{if } (y_{i+1} - y_i)(\hat{y}_{i+1} - y_i) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (29.2)$$

y_{i+1} the $i+1$ -th predicted sample by the model

\hat{y}_{i+1} the $i+1$ -th sample in the actual output

n number of samples used for estimating the model

The numerator in the above equation increments when the trends of the predicted output and the actual output move in the same direction. Consequently, a high value is preferred, since it indicates a more accurately predicted trend.

7. Modified direction accuracy, which is a modified version of the abovementioned test, as defined in Egrioglu et al. (2008). Unlike the calculation above, the count only increment when the trend of the predicted result differ to the trend of the actual result. Therefore, a low value is preferred, since it indicates a more accurately predicted trend.

$$MDA = \frac{\sum_{i=1}^{n-1} D_i}{n-1} \quad (30.1)$$

Where:

$$D_i = (A_i - F_i)^2 \quad (30.2)$$

and

$$A_i = 1 \quad \text{if } y_{i+1} - y_i \leq 0$$

$$A_i = 0 \quad \text{if } y_{i+1} - y_i > 0$$

$$F_i = 1 \quad \text{if } \hat{y}_{i+1} - \hat{y}_i \leq 0$$

$$F_i = 0 \quad \text{if } \hat{y}_{i+1} - \hat{y}_i > 0$$

8. Sign, which compare the number of time where the predicted value and the actual value have the same sign, i.e. where both the predicted and the actual value are

positive or negative. A high value indicates that most of the predicted value follows the same sign as the actual value.

$$Sign = \frac{\sum_{i=1}^n z_i}{n} \quad (31.1)$$

Where:

$$z_i = \begin{cases} 1 & \text{if } y_{i+1} \cdot \hat{y}_{i+1} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (31.2)$$

The analytical method is used for both long-term and short-term predictions, while the graphical method will only be used in the long-term prediction scenario, as it is not conducive to use such technique in the short-term prediction scenario.

7.2 Long term prediction

The long term prediction scenario require the model to predict the respective output over a long period of time (horizon) as stated in the title. The data set is manipulated using the method defined in Qi and Zhang (2001) and Halekoh (2007). The output data is divided into two sets of equal length, a set for training and optimising the model, which is called the in-sample period set, and a section for verifying the performance of the model, which is called the out-of-sample period set. As discussed in section 7.1, both of the evaluation methods are used for this scenario. The result produced by all of the models under investigation is plotted against the actual values below.

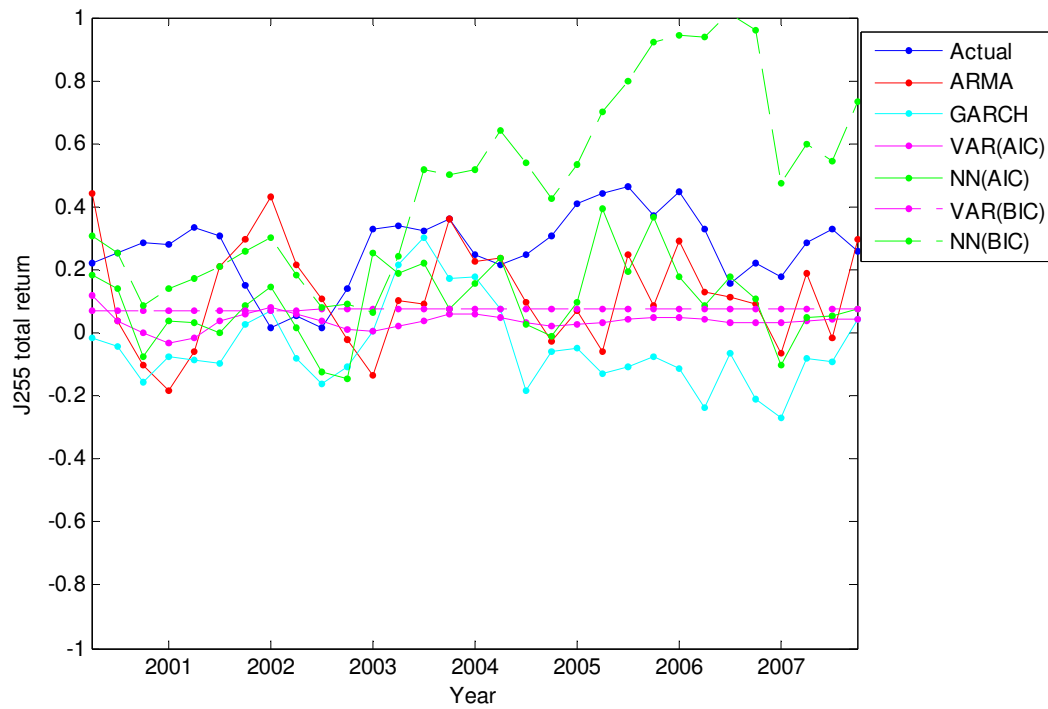


Figure 3.1: Long term prediction of J255 total return

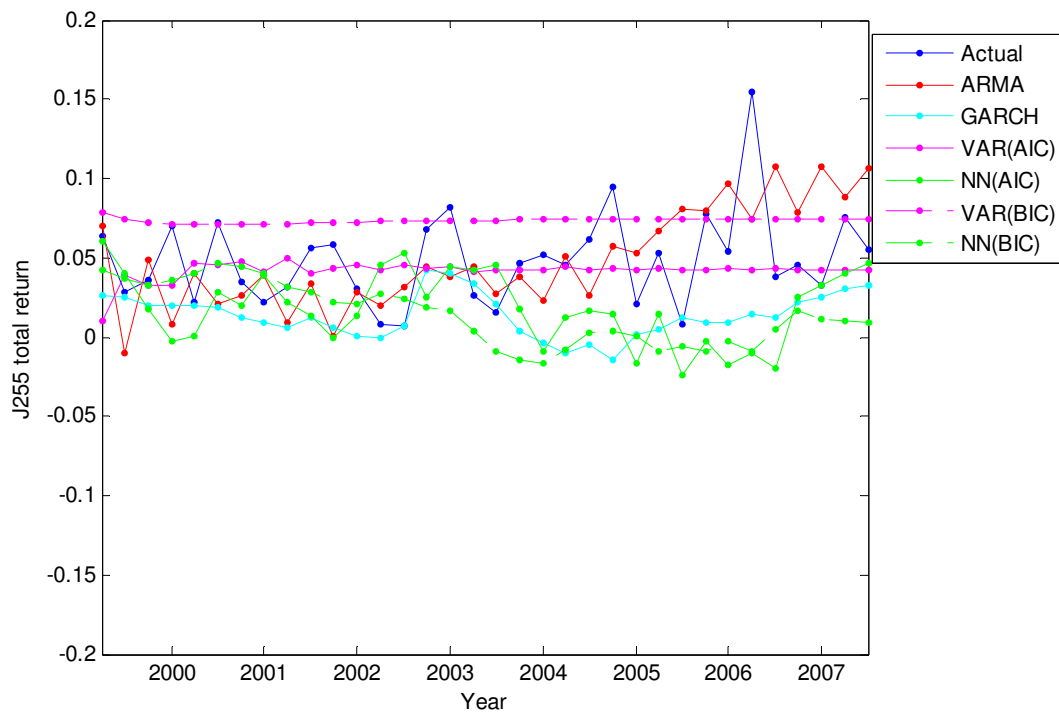


Figure 3.2: Long term prediction of J255 total return deviation

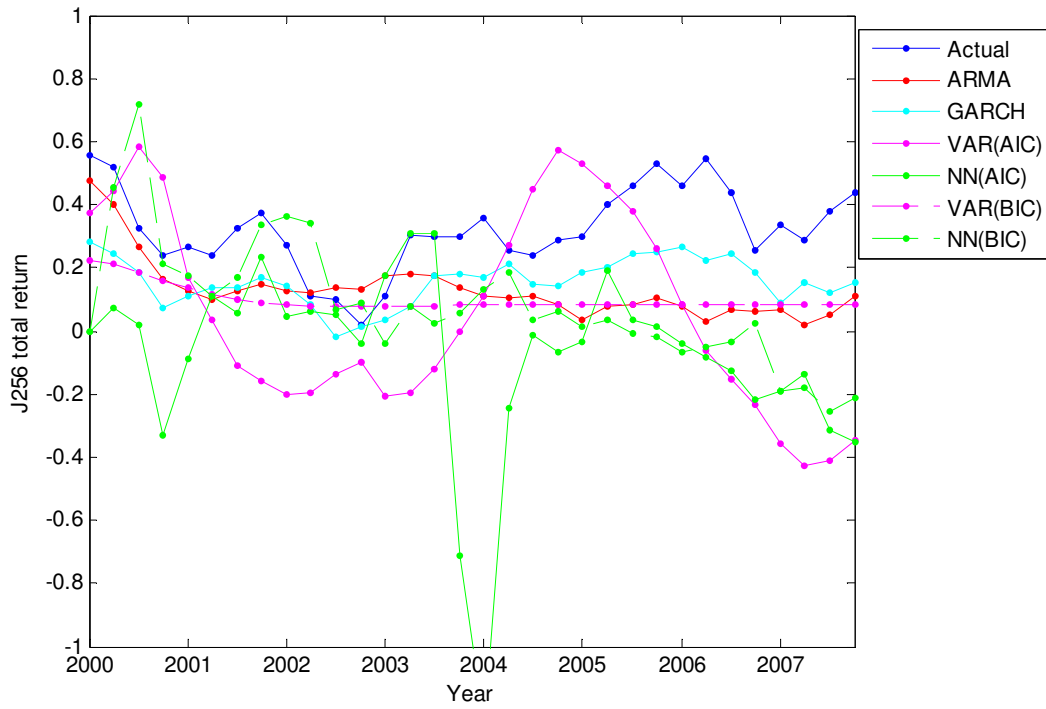


Figure 3.3: Long term prediction of J256 total return

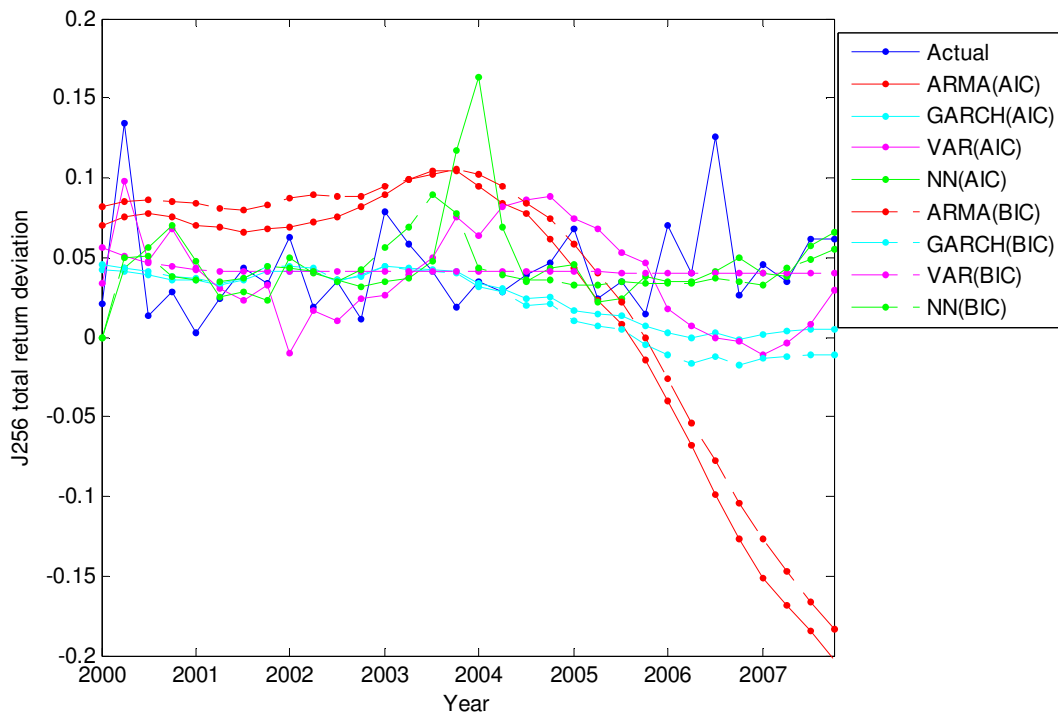


Figure 3.4: Long term prediction of J256 total return deviation

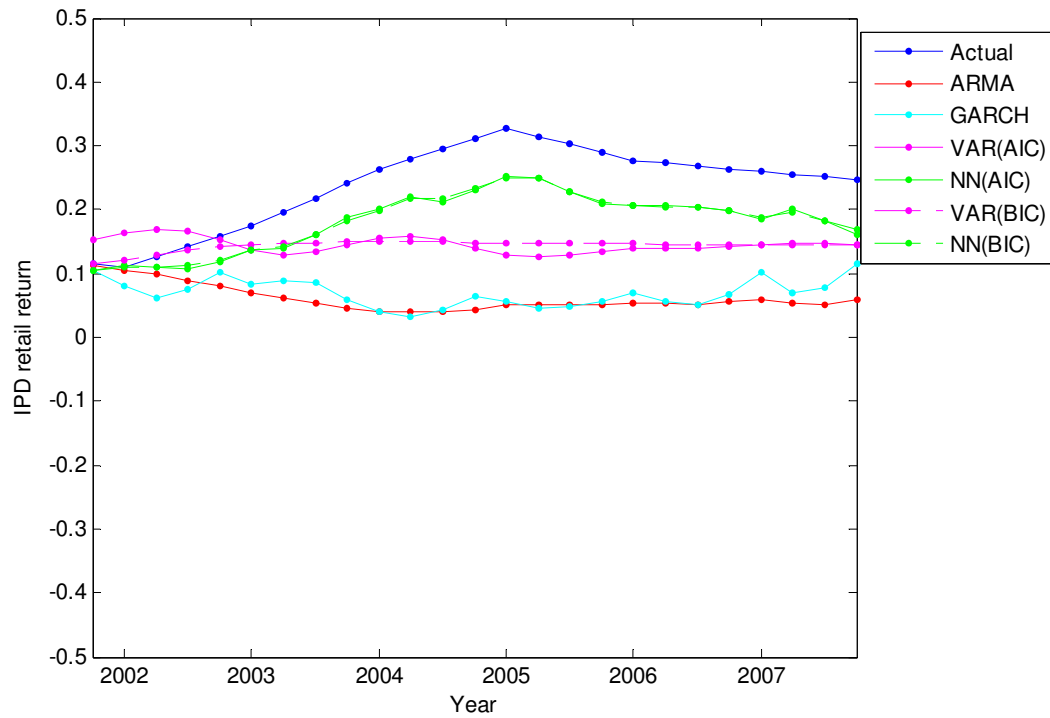


Figure 3.5: Long term prediction of IPD retail return

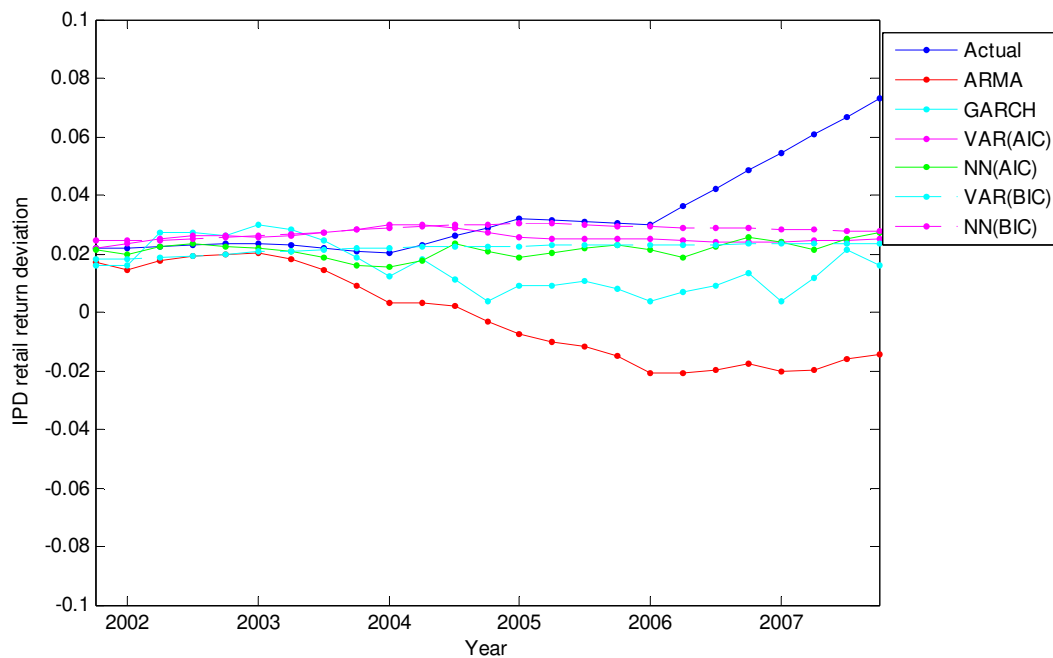


Figure 3.6: Long term prediction of IPD retail return deviation

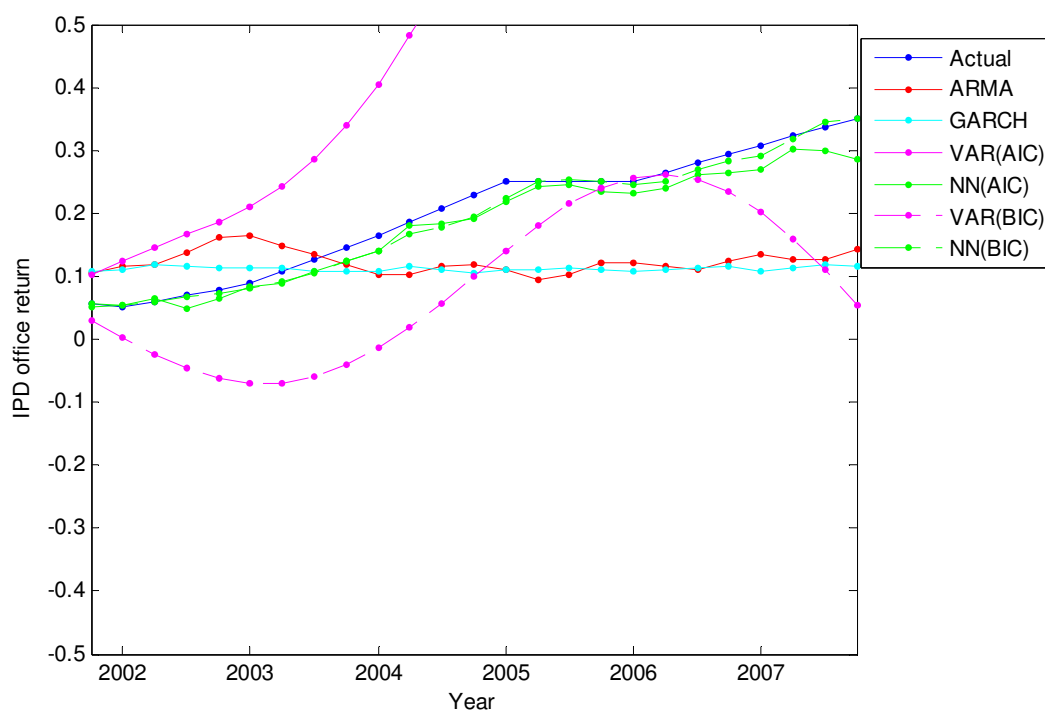


Figure 3.7: Long term prediction of IPD office return

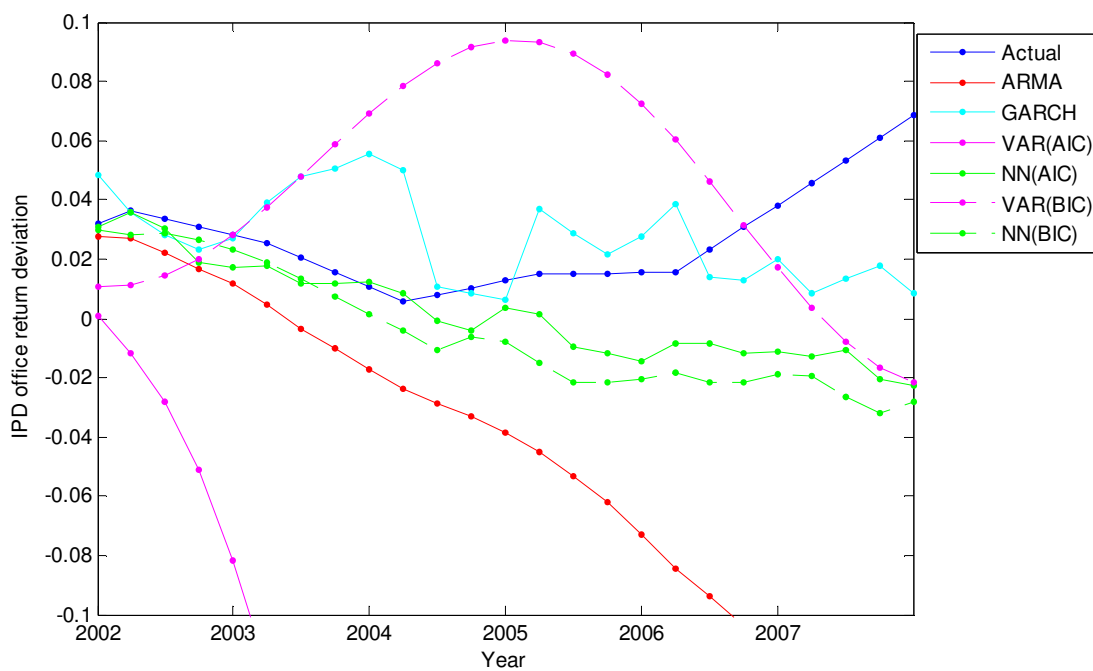


Figure 3.8: Long term prediction of IPD office return deviation

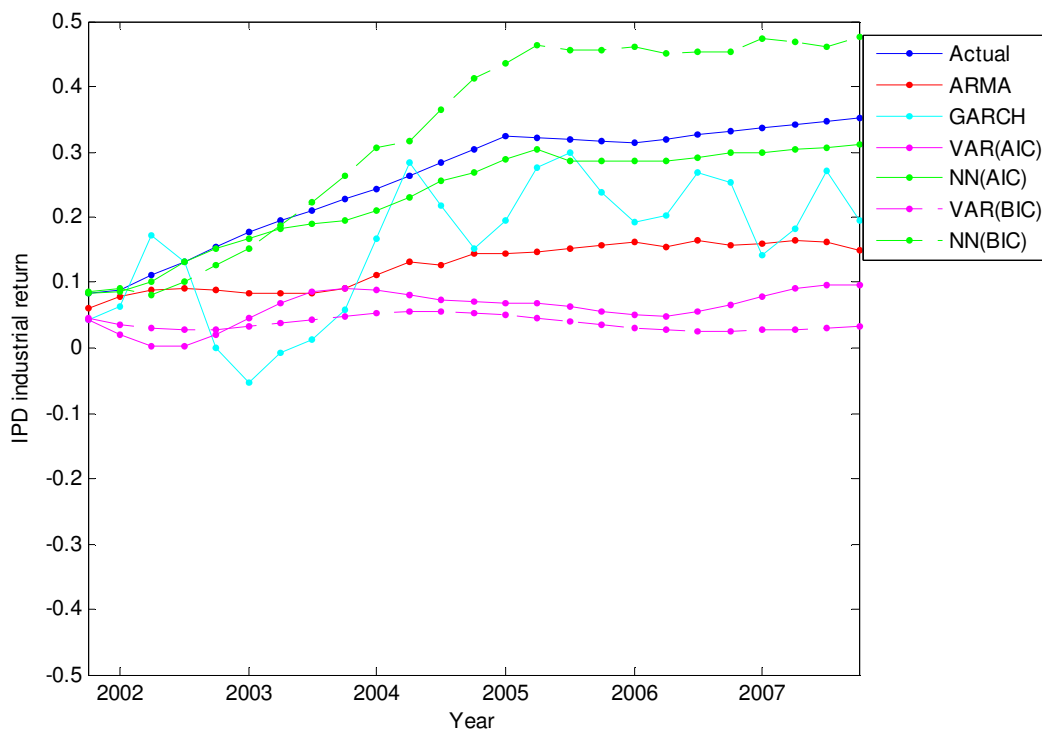


Figure 3.9: Long term prediction of IPD industrial return

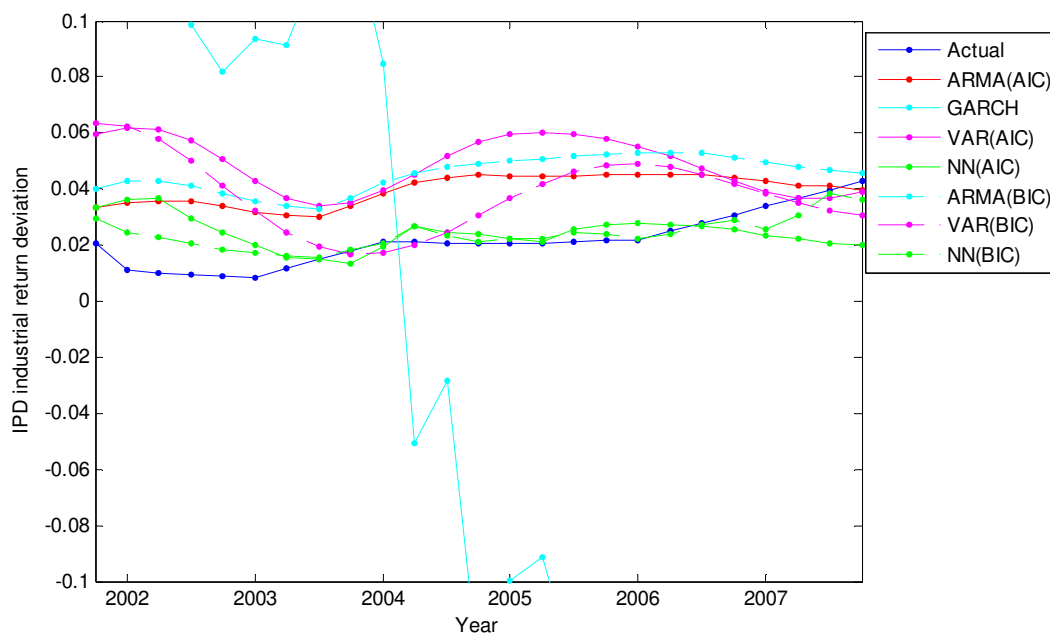


Figure 3.10: Long term prediction of IPD industrial return deviation

Using the performance comparison tools discussed in section 7.1, the performance of the model evaluated using the analytical method is summarised in the tables below.

	MSE	RMSE	MAE	MAPE	Pearson Coefficient	DA	MDA	Sign
J255 Return								
ARMA	0.054661	0.233800	0.191780	1.162100	0.198720	0.437500	0.612900	0.687500
GARCH	0.125090	0.353680	0.316220	1.576600	0.158340	0.500000	0.419350	0.312500
VAR (AICc)	0.070639	0.265780	0.239040	0.982310	0.555550	0.500000	0.405410	0.815790
VAR (BIC)	0.070639	0.265780	0.239040	0.982310	0.555550	0.500000	0.405410	0.815790
NN (AICc)	0.048270	0.219700	0.190720	0.959460	0.561740	0.558820	0.424240	0.794120
NN (BIC)	0.099887	0.316050	0.255690	1.342500	0.362110	0.617650	0.484850	0.970590
J256 Return								
ARMA	0.057704	0.240220	0.207020	0.770760	0.198480	0.515150	0.281250	0.969700
GARCH	0.033597	0.183290	0.165750	0.509700	0.838540	0.545450	0.468750	0.939390
VAR (AICc)	0.173220	0.416200	0.356310	1.399200	0.214020	0.515150	0.656250	0.424240
VAR (BIC)	0.173220	0.416200	0.356310	1.399200	0.214020	0.515150	0.656250	0.424240
NN (AICc)	0.266310	0.516050	0.402350	1.301400	-0.085614	0.548390	0.433330	0.419350
NN (BIC)	0.116380	0.341150	0.281670	0.977390	-0.148930	0.483870	0.500000	0.645160
IPD Retail Return								
ARMA	0.036715	0.191610	0.170620	0.658800	-0.902260	0.538460	0.640000	0.961540
GARCH	0.033271	0.182400	0.163150	0.640940	-0.603470	0.538460	0.560000	0.961540
VAR (AICc)	0.013306	0.115350	0.102450	0.407440	-0.542380	0.538460	0.760000	0.961540
VAR (BIC)	0.013306	0.115350	0.102450	0.407440	-0.542380	0.538460	0.760000	0.961540
NN (AICc)	0.003792	0.061576	0.057485	0.230930	0.976880	0.520000	0.208330	0.960000
NN (BIC)	0.003731	0.061083	0.057077	0.228530	0.985680	0.520000	0.208330	0.960000
IPD Office Return								
ARMA	0.015308	0.123720	0.108450	0.594820	-0.057087	0.384620	0.560000	0.961540
GARCH	0.016202	0.127290	0.107780	0.558360	0.002002	0.384620	0.560000	0.961540
VAR (AICc)	11.664000	3.415200	1.903300	6.591400	0.780230	0.730770	0.280000	0.961540
VAR (BIC)	11.664000	3.415200	1.903300	6.591400	0.780230	0.730770	0.280000	0.961540
NN (AICc)	0.000663	0.025739	0.021369	0.114040	0.990780	0.400000	0.250000	0.960000
NN (BIC)	0.000250	0.015801	0.012027	0.069639	0.992790	0.640000	0.083333	0.960000
IPD Industrial Return								
ARMA	0.019650	0.140180	0.127000	0.468070	0.944020	0.153850	0.560000	0.961540
GARCH	0.014757	0.121480	0.102600	0.446760	0.673760	0.307690	0.440000	0.884620
VAR (AICc)	0.041785	0.204410	0.189150	0.739150	0.627140	0.153850	0.480000	0.961540
VAR (BIC)	0.041785	0.204410	0.189150	0.739150	0.627140	0.153850	0.480000	0.961540
NN (AICc)	0.000829	0.028789	0.025324	0.088808	0.996980	0.440000	0.166670	0.960000
NN (BIC)	0.009368	0.096788	0.081477	0.278370	0.986590	0.680000	0.291670	0.960000

Table 5.1: Comparison between different models for long-term return prediction

	MSE	RMSE	MAE	MAPE	Pearson Coefficient	DA	MDA	Sign
J255 Return Deviation								
ARMA	0.001386	0.037233	0.030135	0.972280	0.200910	0.567570	0.638890	0.945950
GARCH	0.002000	0.044723	0.033441	0.630450	0.086779	0.567570	0.472220	0.837840
VAR (AICc)	0.000926	0.030433	0.022992	0.769440	-0.132350	0.657890	0.621620	0.973680
VAR (BIC)	0.000926	0.030433	0.022992	0.769440	-0.132350	0.657890	0.621620	0.973680
NN (AICc)	0.002472	0.049722	0.039247	1.134100	-0.169750	0.500000	0.545450	0.705880
NN (BIC)	0.002573	0.050726	0.039442	0.889140	-0.195480	0.588240	0.545450	0.705880
J256 Return Deviation								
ARMA (AICc)	0.013381	0.115670	0.084794	3.102800	-0.255890	0.545450	0.625000	0.666670
ARMA (BIC)	0.011837	0.108800	0.084019	3.340300	-0.260370	0.545450	0.687500	0.666670
GARCH (AICc)	0.001621	0.040265	0.029856	1.121700	-0.180550	0.636360	0.406250	0.909090
GARCH (BIC)	0.002202	0.046922	0.035452	1.287700	-0.197530	0.636370	0.375000	0.666670
VAR (AICc)	0.001811	0.042553	0.035193	1.466000	-0.020984	0.666670	0.593750	0.818180
VAR (BIC)	0.001811	0.042553	0.035193	1.466000	-0.020984	0.666670	0.593750	0.818180
NN (AICc)	0.001793	0.042347	0.028133	1.473500	-0.098766	0.806450	0.433330	0.967740
NN (BIC)	0.000961	0.031003	0.022035	1.099700	0.086208	0.709680	0.633330	0.967740
IPD Retail Return Deviation								
ARMA	0.002015	0.044883	0.034977	0.908800	-0.706830	0.320000	0.250000	0.440000
GARCH	0.000692	0.026297	0.020318	0.523070	-0.283200	0.400000	0.333333	0.960000
VAR (AICc)	0.000301	0.017349	0.011098	0.262430	-0.308050	0.500000	0.560000	0.961540
VAR (BIC)	0.000301	0.017349	0.011098	0.262430	-0.308050	0.500000	0.560000	0.961540
NN (AICc)	0.000328	0.018111	0.012199	0.284730	0.630740	0.440000	0.500000	0.960000
NN (BIC)	0.000247	0.015705	0.009570	0.223690	0.242010	0.560000	0.500000	0.960000
IPD Office Return Deviation								
ARMA	0.009032	0.095038	0.073153	3.119100	-0.456870	0.320000	0.708330	0.200000
GARCH	0.000687	0.026219	0.020492	1.122500	-0.381120	0.480000	0.625000	0.960000
VAR (AICc)	24.851000	4.985100	2.815400	85.552000	-0.812120	0.307690	0.720000	0.038462
VAR (BIC)	24.851000	4.985100	2.815400	85.552000	-0.812120	0.307690	0.720000	0.038462
NN (AICc)	0.001262	0.035518	0.024953	0.911680	-0.231960	0.400000	0.500000	0.440000
NN (BIC)	0.001768	0.042052	0.031395	1.296900	-0.167820	0.360000	0.458330	0.320000
IPD Industrial Return Deviation								
ARMA (AICc)	0.000377	0.019410	0.017833	1.131000	0.484480	0.576920	0.520000	0.961540
ARMA (BIC)	0.000609	0.024673	0.023164	1.428600	0.464040	0.615380	0.560000	0.961540
GARCH	0.032478	0.180220	0.163190	8.027500	-0.673500	0.307690	0.560000	0.384620
VAR (AICc)	0.000973	0.031194	0.027566	1.813300	-0.354950	0.538460	0.680000	0.961540
VAR (BIC)	0.000973	0.031194	0.027566	1.813300	-0.354950	0.538460	0.680000	0.961540
NN (AICc)	0.000148	0.012177	0.009120	0.596480	-0.18221	0.360000	0.625000	0.960000
NN (BIC)	0.000038	0.006183	0.004562	0.331100	0.831450	0.520000	0.375000	0.960000

Table 5.2: Comparison between different models for long-term deviation prediction

The following is observed for each output based on the two performance comparison approaches.

7.2.1 J255 return and return deviation

The ARMA and the neural network models optimised using the AICc method produces the most accurate J255 return forecast, particularly between 2005 and 2008 where the volatility in the market was accurately predicted. This is confirmed in the evaluation using the analytical method, where these two models produce the lowest error. The models predicted the return fairly accurately as the actual trend is not too volatile and quite predictable. However, referring to Figure 3.1, the reaction time of the predicted result to directional changes is delayed by 2 to 3 periods. Term structure, employment index and building plans passed index are variables that appear to provide accurate indications on the trend of the return as they are found in both of the models, and are identified in the causality test to be strongly related to the J255 return.

The trend of the J255 return deviation appears to be highly volatile and it is much more difficult to forecast this output. This is particularly evident in the analysis using the graphical method where the movement of the output trends from the models deviates significantly from the actual trend. Referring to Figure 3.2, the neural network model optimised using the AICc method once again produces the most accurate forecast, followed by the GARCH model and the neural network model optimised using the BIC method. The result from the analytical method however contradicts this finding. It indicated that the VAR models produces the most accurate J255 return deviation forecast, with the least error, where the output trend forecasted from these two models is constant. A possible cause of this difference could be that the predicted trends from the VAR models hovers around the average of the actual trend and doesn't fluctuates significantly around the actual trend, which leads to a higher mean square error value. Gilt-equity ratio and building plans passed index are the explanatory variables found in all of the optimal models, of which gilt-equity ratio is the significant variable in the forecast of the deviation.

7.2.2 J256 return and return deviation

The ARMA and the GARCH models produce the most accurate J256 return forecast, as refer to Figure 3.3. The analysis using the analytical method verified this finding. The actual trend of the J256 return is more volatile than the J255 return but it can still be accurately forecasted using these two models throughout the forecast period between 2002 and 2008. Manufacturing index, employment index and building plans passed index are variables that appear to provide accurate indications of the trend of the return, as they are found in both of the models and are identified in the causality test to be strongly related to the J256 return.

Once again, the trend of the J256 return deviation appears to be highly volatile and is very difficult to forecast. With the exception of the neural network model optimised using the BIC method, Figure 3.4 shows that none of the models appear to forecast the actual trend. This result coincides with the evaluation using the analytical method where it produces the lowest mean square error and a relatively high directional accuracy. The reaction time of the model to directional change is slightly delayed as referred to in Figure 3.4. Term structure and building plans passed index are explanatory variables used in all of the models and their influence on predicting the return deviation is minimal.

7.2.3 Retail return and return deviation

Both of the neural network models produce the most accurate forecast of the retail return. This is evident in the analysis using both the graphical and the analytical methods. Since the actual trend is not volatile, the models can produce a highly accurate forecast. The explanatory variables that contribute to the forecast are the gilt-equity ratio and the changing CPIX index, which are significantly related to retail return, as evident in the causality test above.

The actual trends of the retail return deviation remain fairly constant between 2002 and 2005 and rise sharply between 2006 and 2008. The VAR models and the neural network models predict the constant period fairly accurately but none of the models are able to predict the sudden sharp rise between 2006 and 2008 (last 8 periods). The result from the analytical method indicates that the neural network model optimised with the BIC method predicted the trend with the highest accuracy. Similar to the retail return model, the gilt-equity ratio and the changing CPIX index are the two variables that contribute to the forecast.

7.2.4 Office return and return deviation

The result from the graphical method again indicates that both of the neural network models produce the most accurate forecast of the office return. The result from the analytical method indicates that the most accurate model is the neural network model optimised using the BIC method. The actual trend is not volatile and the models can produce highly accurate forecasts. The explanatory variables that contribute to the forecast are the gilt-equity ratio and the changing CPIX index.

The neural network models are able to predict the office return deviation between 2002 and 2004 and none of the models are able to predict the fluctuation in the deviations thereafter. The result from the analytical method indicates that the GARCH model produces the best forecast but this is due to the fact that the GARCH model predicted a trend that oscillates around the actual trend between 2002 and 2006. Similar to the retail return model, the gilt-equity ratio and the changing CPIX index are the two variables that contribute to the forecast.

7.2.5 Industrial return and return deviation

The neural network model optimised using the AICc method produce the most accurate forecast of the industrial return. This is evident in the analysis using both the graphical and the analytical methods. The actual trend once again is not volatile and the model can produce a highly accurate forecast. The explanatory variables that contribute to the forecast are the manufacturing index and the prime lending rate, which contribute highly to the retail return, as evident in the causality test above.

Both of the neural network models produce fairly accurate prediction of the industrial deviation trend. However, the neural network optimised using the BIC method is the only model able to predict the gradual rise in the trend between 2006 and 2008. This is substantiated in the result from the analytical method, where the data indicates that this model best predicts this trend. Similar to the retail return model, the manufacturing index and the prime lending rate are the two variables that contribute to the forecast.

7.2.6 Variance analysis

So far in this section, only the predicted mean is analysed. Since the variance component of the GARCH model is time-varying, the variance part of the predicted return or return deviation should also be analysed. With the exception of the GARCH model, the variances of all of the other models are constant. The tables below summarized the standard deviation of these models.

	J255 Return	J256 Return	Retail Return	Office Return	Industrial Return
Actual	0.159640	0.182290	0.067392	0.088302	0.104760
ARMA	0.075587	0.079617	0.012041	0.012462	0.011655
VAR (AICc)	0.037300	0.316300	0.011600	3.000000	0.026700
VAR (BIC)	0.009500	0.038400	0.009900	0.117900	0.010900
NN (AICc)	0.129000	0.298900	0.045500	0.087700	0.075700
NN (BIC)	0.295200	0.203600	0.044800	0.097500	0.151600

Table 6.1: Standard deviations of all the models with constant variance for return

	J255 Return Deviation	J256 Return Deviation	Retail Return Deviation	Office Return Deviation	Industrial Return Deviation
Actual	0.026516	0.025782	0.011943	0.015662	0.011229
ARMA (AICc)	0.020551	0.018079	0.001539	0.004455	0.005400
ARMA (BIC)	0.020551	0.018118	0.001539	0.004455	0.006400
VAR (AICc)	0.007200	0.030600	0.002100	4.200000	0.009700
VAR (BIC)	0.000700	0.003300	0.001700	0.036600	0.013900
NN (AICc)	0.022900	0.028100	0.002900	0.016600	0.005900
NN (BIC)	0.019200	0.013600	0.001800	0.020500	0.005700

Table 6.2: Standard deviations of all models with constant variances for return deviation

The standard deviations of the accurately predicted indirect return trends (from the ARMA and the NN (AICc) models) tend to be slightly lower than the actual trends, which is a reflection of the predicted trends being less volatile than the actual trends. Similarly, the standard deviations of the accurately predicted direct returns trends (from neural networks) also tend to be slightly lower than the actual trends. For both indirect and direct return, the standard deviations of the predicted trends from the VAR model are either very high or very low. This is an indication that the predicted trends from this model either fluctuate excessively or remain constant, which corresponds with the result from the graphical method. A similar result was identified in the analysis of the standard deviations of the accurately predicted return deviation trends, where they are lower than the actual value.

Since the actual outputs are presumed to have a constant variance, it is not helpful to directly compare the variance trends of the GARCH model to the variance of the actual trends. In order to understand the impact on the variance due to the returns and return deviations, the predicted variance is therefore plotted against the actual returns and return deviations.

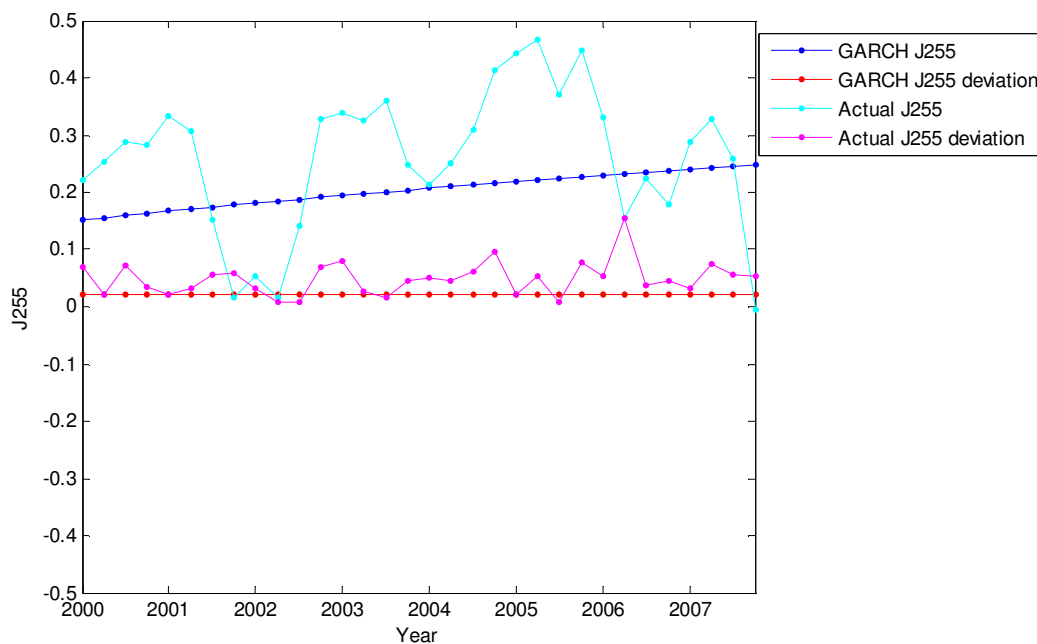


Figure 4.1: Relationship between GARCH variance function and actual J255 output

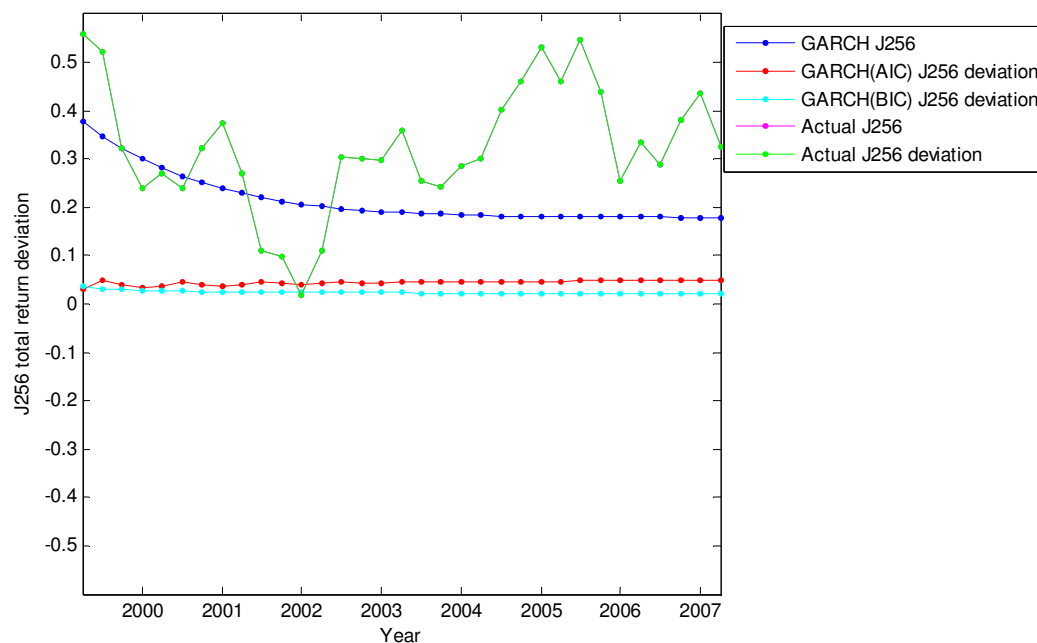


Figure 4.2: Relationship between GARCH variance function and actual J256 output

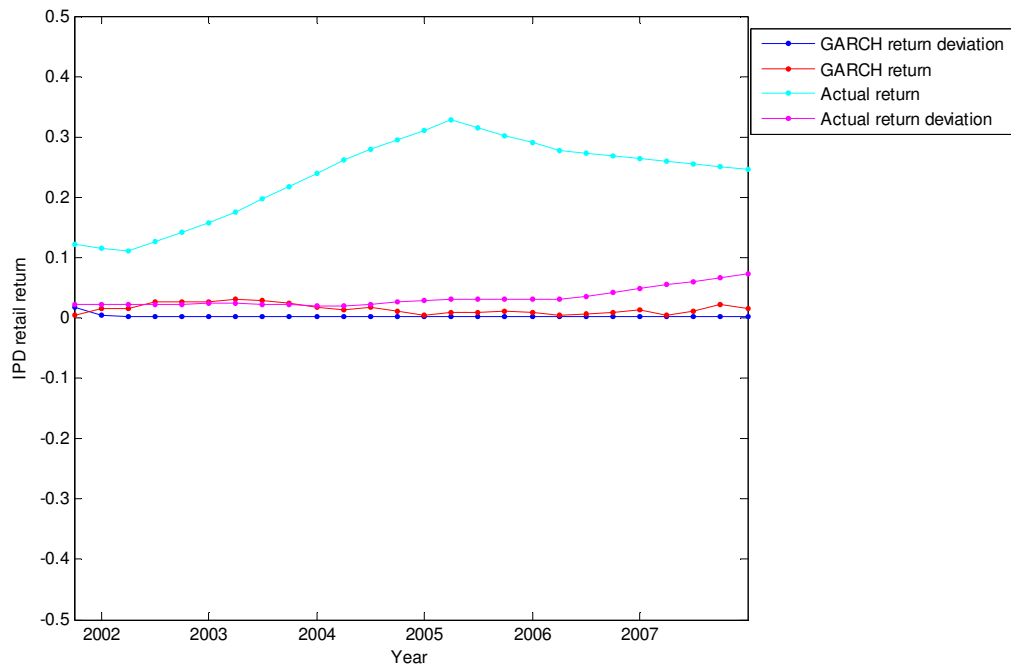


Figure 4.3: Relationship between GARCH variance function and actual retail return output

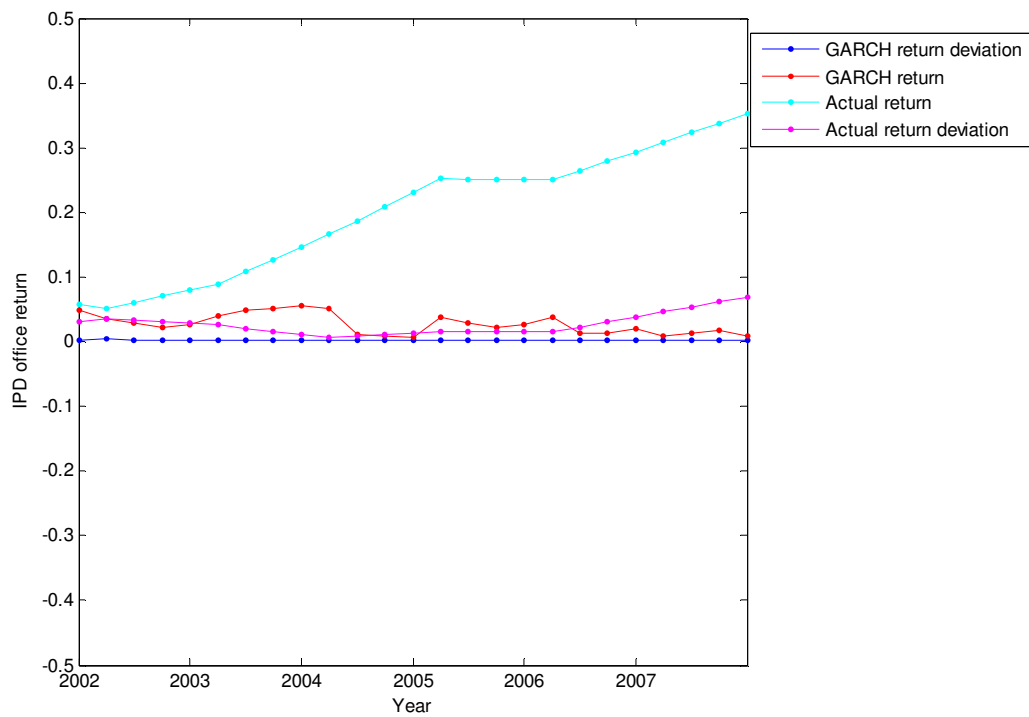


Figure 4.4: Relationship between GARCH variance function and actual office return output

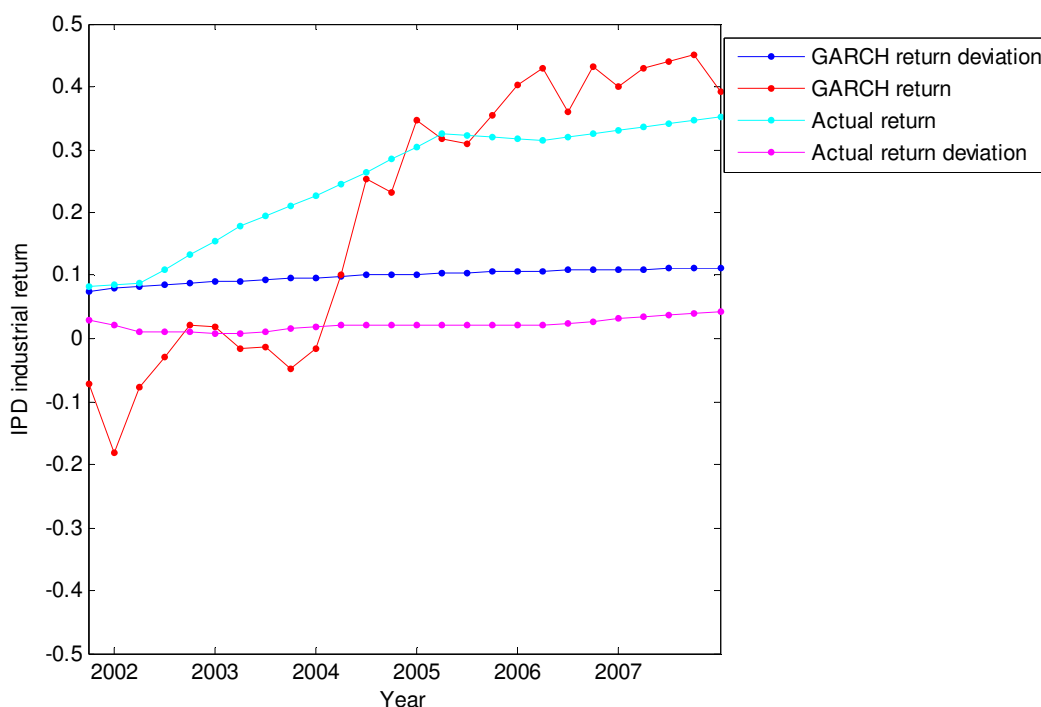


Figure 4.5: Relationship between GARCH variance function and actual industrial return output

The results above indicate that the GARCH variance generally remains constant or increases linearly with time, with the exception of J256, where the trend decreases from the beginning and remains constant. In all of the outputs, the variance function does not follow the trends of the actual output. It can be concluded that there is very little correlation between the actual output and the movement of the variance in a GARCH model, and that a variance based forecast model, such as the GARCH model, does not improve on the forecasting of return and return deviation.

7.3 Impulse response of long-term prediction models

Based on the optimal models identified in the long-term prediction section above (section 7.2), the effects of each explanatory (input) variables on the output of the model are investigated. The impulse response technique is a widely used technique used in many previous investigations, namely the works of Brooks and Tsolacos (1999; 2003), West and Worthington (2004) and McCue and Kling (1994). In this research, a shock of 1 standard deviation is injected into each of the inputs separately and the (output) response is investigated. The effect on the response is measured in terms of a standard deviation unit. For example, if the predicted output resulting from the injected impulse is one deviation higher or lower than the predicted output with normal inputs, then there is 1

unit change in the result. The response change is plotted over 20 periods for all of the optimal models and is located in appendix F.

Besides the response for the ARMA model predicting J255 return and the industrial return, the effect is due to impulse shock deviated to zero within 10 periods. In the J255 return and the industrial return, the effect decreases gradually to approximately 0.2 and 0.5 units respectively at the 20th period. Unlike the ARMA model, the graphs in appendix F indicate that the impulse shock has virtually no effect whatsoever on the GARCH model and the neural network model, as these models absorb the shock within 2 periods. Generally, the shock with the biggest impact on the predicted output of the ARMA model is the autocorrelative variable, i.e. the AR component. While for the GARCH model and the neural network model, the explanatory variables can cause the biggest shock to the output. The impact on the response due to the shocks injected into the GARCH model and neural network are investigated further. The magnitude of the response due to the shocks injected into the GARCH model and neural network are tabulated below.

	J255	J255 deviation
J255	1.00000	-0.13217
Term structure	0.35841	-0.46998
Gilt equity	-1.37300	-4.84453
Employment	2.65298	3.06900
Building plans	0.15632	-0.67204
J255 deviation	0.17942	1.00000

Table 7.1: Magnitude of impulse response of NN(AICc) model for J255 return

	J256
J256	1.00000
Manufacturing index	0.06088
Employment	0.01512
Building plans	-0.02557
Changing CPIX	-0.03017
J255	-0.17028

Table 7.2: Magnitude of impulse response of GARCH model for J256 return

	J256 deviation
J256 deviation	1.00000
Term structure	22.12524
Building plans	-1.80131
J256	-3.65365

Table 7.3: Magnitude of impulse response of NN(BIC) model for J256 return deviation

	Retail return
Retail return	1.00000
Gilt equity	0.26958
Changing CPIX	0.00088
Retail return deviation	-0.02632

Table 7.4: Magnitude of impulse response of NN(BIC) model for retail return

	Office return	Office return deviation
Office return	1.00000	-24.24876
Gilt equity	-0.09026	43.69154
Changing CPIX	0.08582	-11.42289
Office return deviation	0.21864	1.00000

Table 7.5: Magnitude of impulse response of NN(BIC) model for office return

	Retail return deviation
Retail return deviation	1.00000
Gilt-equity	1.18390
Changing CPIX	0.97150
Retail return	1.66690

Table 7.6: Magnitude of impulse response of NN(BIC) model for retail return deviation

	Office return deviation
Office return deviation	1.00000
Gilt-equity	2.60380
Building plans	-6.06120
Changing GDP	0.56474
Changing CPIX	-1.75810
Unexpected CPIX	0.11791
Changing prime lending rate	-0.68379
Industrial return deviation	8.18940

Table 7.7: Magnitude of impulse response of GARCH model for office return deviation

	Industrial return
Industrial return	1.00000
Gilt-equity	0.04180
Manufacturing index	0.04968
Industrial return deviation	-0.24367

Table 7.8: Magnitude of impulse response of NN(AICc) model for industrial return

	Industrial return deviation
Industrial return	0.05510
Prime lending rate	-0.09644
Manufacturing index	0.09766
Industrial return deviation	1.00000

Table 7.9: Magnitude of impulse response of NN(BIC) model for industrial return deviation

In most of the results above, the shock with the biggest impact on the predicted output of the models is the autocorrelative variable. However, there are some variables that have significant impact on a few of the outputs, such as the gilt-equity ratio, the employment rate index, the term structure, the changing CPIX and the building plans passed index. The change in the gilt-equity ratio has significant impact on the J255 output and the office return deviation. The employment rate index also has significant impact on the J255 output. Similarly, the building plans passed index and the industrial return deviation has significant impact on the office return deviation. The J256 return deviation is very sensitive to the change in the term structure.

For the GARCH model, the impact due to autocorrelative shock, i.e. shock in the return or return deviation on the variance, is also investigated. The percentage change in standard deviation with and without the shock is plotted in appendix F over 20 periods.

The result indicates that the shock applied to J256 return and the office return deviation has significant impact on the variance of the respective GARCH models. The impact of the shock on the variances of all three models decreases exponentially with time to less than 10% at the 20th period.

7.4 Short term prediction

Lastly, as previously discussed, the short term prediction scenario is investigated. This investigation is employed in the works of Brook and Tsolacos (1999; 2001; 2003). In this research, three different forecasts are investigated, namely the 1-step, 2-steps and 4-steps ahead forecast. Once again, the data are separated into two sets, where one set (in-sample period set) is for training and optimising the initial model. However, the second set (out-of-sample period set) is used for both verification and optimisation. In the second set, the samples are first used for verification and performance evaluation. When the model forecasts in subsequent period, the actual samples are used for updating and retraining the model. This forecasting method is known as the recursive forecasting method and it allows the model to evolve with the environment that it is forecasting. This technique mimics closely the real life situation where the user only requires the developed model to perform in the short term forecast, a matter of few quarters ahead forecast. The results of all three scenarios are summarised in the table below. The Pearson Coefficient is only calculated for 2-step and 4-step ahead forecasts, as it requires at least two predictions or more to calculate the value. The presented values are the mean of the values calculated after each forecast.

	MSE	RMSE	MAE	MAPE	DA	MDA	Sign
J255 Return							
ARMA	0.009854	0.083651	0.083651	0.704860	0.593750	0.406250	1.000000
GARCH	774.2200	6.396800	6.396800	367.8900	0.483870	0.516130	0.806450
VAR (AICc)	0.029779	0.132740	0.132740	1.024500	0.552630	0.447370	0.947370
VAR (BIC)	0.028528	0.131130	0.131130	1.382000	0.552630	0.447370	0.947370
NN (AICc)	0.032236	0.140130	0.140130	1.289800	0.500000	0.500000	0.970590
NN (BIC)	0.079996	0.149470	0.149470	0.785230	0.441180	0.558820	0.970590
J256 Return							
ARMA	0.007522	0.070169	0.070169	0.408400	0.696970	0.303030	0.969700
GARCH	5270.900	23.88800	23.88800	136.4600	0.727270	0.272730	0.848480
VAR (AICc)	0.034246	0.142660	0.142660	0.528030	0.454550	0.545450	0.939390
VAR (BIC)	0.026509	0.126080	0.126080	0.587170	0.484850	0.515150	1.000000
NN (AICc)	0.027113	0.132580	0.132580	0.582300	0.451610	0.548390	1.000000
NN (BIC)	0.015560	0.095989	0.095989	0.492410	0.483870	0.516130	1.000000
IPD Retail Return							
ARMA	0.000983	0.018833	0.018833	0.083135	0.615380	0.384620	1.000000
GARCH	0.025719	0.123090	0.123090	0.607890	0.461540	0.538460	0.961540
VAR (AICc)	0.000611	0.018570	0.018570	0.090941	0.500000	0.500000	1.000000
VAR (BIC)	0.000787	0.023255	0.023255	0.108720	0.384620	0.615380	1.000000
NN (AICc)	0.000198	0.011623	0.011623	0.056107	0.760000	0.240000	1.000000
NN (BIC)	0.000160	0.009911	0.009911	0.046859	0.760000	0.240000	1.000000
IPD Office Return							
ARMA	0.000283	0.013162	0.013162	0.113130	0.730770	0.269230	1.000000
GARCH	0.016547	0.109480	0.109480	0.565960	0.346150	0.653850	1.000000
VAR (AICc)	0.000583	0.021332	0.021332	0.153610	0.269230	0.730770	1.000000
VAR (BIC)	0.000520	0.018836	0.018836	0.120150	0.307690	0.692310	1.000000
NN (AICc)	0.000091	0.007955	0.007955	0.052945	0.760000	0.240000	1.000000
NN (BIC)	0.000126	0.009217	0.009217	0.057107	0.800000	0.200000	1.000000
IPD Industrial Return							
ARMA	0.000306	0.009217	0.009217	0.052799	0.769230	0.230770	1.000000
GARCH	0.003958	0.048553	0.048553	0.215190	0.423080	0.576920	1.000000
VAR (AICc)	0.000529	0.019007	0.019007	0.115090	0.192310	0.807690	1.000000
VAR (BIC)	0.000488	0.019378	0.019378	0.099219	0.269230	0.730770	1.000000
NN (AICc)	0.000107	0.008237	0.008237	0.034334	0.640000	0.360000	1.000000
NN (BIC)	0.000095	0.008063	0.008063	0.035733	0.560000	0.440000	1.000000

Table 8.1: Comparison between different models for 1-step return forecast

	MSE	RMSE	MAE	MAPE	DA	MDA	Sign
J255 Return Deviation							
ARMA	0.001021	0.025437	0.025437	0.874630	0.595240	0.404760	1.000000
GARCH	0.000906	0.024277	0.024277	0.809450	0.595240	0.404760	1.000000
VAR (AICc)	0.001015	0.025011	0.025011	0.876580	0.657890	0.342110	0.973680
VAR (BIC)	0.000803	0.021108	0.021108	0.778140	0.657890	0.342110	1.000000
NN (AICc)	0.001288	0.026879	0.026879	0.915940	0.647060	0.352940	1.000000
NN (BIC)	0.001375	0.027788	0.027788	0.806630	0.617650	0.382350	0.970590
J256 Return Deviation							
ARMA (AICc)	0.000942	0.023518	0.023518	0.944340	0.727270	0.272730	1.000000
ARMA (BIC)	0.000958	0.022742	0.022742	0.994010	0.696970	0.303030	1.000000
GARCH (AICc)	0.004791	0.031298	0.031298	1.166200	0.666670	0.333330	0.969700
GARCH (BIC)	9.128200	0.592300	0.592300	22.833000	0.696970	0.303030	1.000000
VAR (AICc)	0.001537	0.031482	0.031482	1.604900	0.636360	0.363640	0.969700
VAR (BIC)	0.000921	0.022838	0.022838	1.253700	0.666670	0.333330	1.000000
NN (AICc)	0.001248	0.026659	0.026659	1.482400	0.709680	0.290320	1.000000
NN (BIC)	0.001122	0.024169	0.024169	1.196500	0.677420	0.322580	1.000000
IPD Retail Return Deviation							
ARMA	0.000947	0.017031	0.017031	0.468220	0.600000	0.400000	0.960000
GARCH	1.414921	0.469245	0.469245	20.88453	0.714286	0.700000	0.961540
VAR (AICc)	0.000025	0.003572	0.003572	0.094210	0.307690	0.692310	1.000000
VAR (BIC)	0.000034	0.004406	0.004406	0.119440	0.230770	0.769230	1.000000
NN (AICc)	0.000175	0.008028	0.008028	0.172440	0.360000	0.640000	1.000000
NN (BIC)	0.000220	0.009074	0.009074	0.198130	0.400000	0.600000	1.000000
IPD Office Return Deviation							
ARMA	0.000019	0.003474	0.003474	0.180061	0.720000	0.280000	0.840000
GARCH	0.005781	0.033960	0.033960	1.499500	0.280000	0.720000	0.880000
VAR (AICc)	0.000059	0.006307	0.006307	0.298420	0.230770	0.769230	0.961540
VAR (BIC)	0.000092	0.008451	0.008451	0.401270	0.115380	0.884620	0.961540
NN (AICc)	0.000181	0.010338	0.010338	0.427680	0.360000	0.640000	0.960000
NN (BIC)	0.000069	0.006536	0.006536	0.337870	0.440000	0.560000	0.960000
IPD Industrial Return Deviation							
ARMA (AICc)	0.000500	0.010675	0.010675	0.579575	0.520000	0.480000	0.884620
ARMA (BIC)	0.000022	0.003307	0.003307	0.206420	0.423077	0.576923	0.875000
GARCH	99.89423	2.598345	2.598345	125.9842	0.333333	0.666667	1.000000
VAR (AICc)	0.000073	0.005569	0.005569	0.315070	0.461540	0.538460	1.000000
VAR (BIC)	0.000098	0.005934	0.005934	0.322950	0.461540	0.538460	1.000000
NN (AICc)	0.000132	0.009611	0.009611	0.505990	0.200000	0.800000	1.000000
NN (BIC)	0.000121	0.009898	0.009898	0.495740	0.240000	0.760000	1.000000

Table 8.2: Comparison between different models for 1-step return deviation forecast

	MSE	RMSE	MAE	MAPE	Pearson Coefficient	DA	MDA	Sign
J255 Return								
ARMA	0.016414	0.116120	0.108010	1.096900	0.161290	0.483870	0.935480	0.500000
GARCH	8041.900	17.96200	15.18200	276.8500	-0.200000	0.450000	1.100000	0.433330
VAR (AICc)	0.040206	0.166420	0.158590	1.273600	0.243240	0.527030	0.837840	0.418920
VAR (BIC)	0.034177	0.151340	0.144060	1.551000	0.081081	0.554050	0.918920	0.472970
NN (AICc)	0.043783	0.166570	0.153250	1.423600	0.030303	0.484850	0.939390	0.484850
NN (BIC)	0.122150	0.166430	0.154260	0.949520	-0.030303	0.424240	1.121200	0.500000
J256 Return								
ARMA	0.019544	0.117460	0.106730	0.533540	0.062500	0.531250	0.843750	0.468750
GARCH	6721.100	27.74600	27.07400	116.2100	0.062500	0.625000	0.750000	0.406250
VAR (AICc)	0.047821	0.186030	0.173830	0.761370	0.125000	0.515630	1.000000	0.453130
VAR (BIC)	0.033359	0.158910	0.149460	0.804370	0.000000	0.421880	1.031300	0.500000
NN (AICc)	0.056528	0.161110	0.151640	0.818270	0.066667	0.450000	1.033300	0.483330
NN (BIC)	0.018276	0.121980	0.110210	0.539500	0.066667	0.450000	1.033300	0.500000
IPD Retail Return								
ARMA	0.000524	0.018967	0.017397	0.084245	0.040000	0.580000	0.720000	0.500000
GARCH	0.013929	0.098995	0.098008	0.379230	0.040000	0.560000	0.920000	0.500000
VAR (AICc)	0.001256	0.026225	0.024972	0.121060	0.280000	0.440000	0.880000	0.500000
VAR (BIC)	0.001353	0.031124	0.029965	0.137820	-0.120000	0.340000	1.200000	0.500000
NN (AICc)	0.000177	0.011309	0.010708	0.050644	0.666670	0.791670	0.291670	0.500000
NN (BIC)	0.000355	0.015505	0.014550	0.075514	0.583330	0.708330	0.458330	0.500000
IPD Office Return								
ARMA	0.000710	0.022180	0.020368	0.178670	0.440000	0.580000	0.560000	0.500000
GARCH	0.016130	0.108880	0.108370	0.561020	0.280000	0.360000	1.000000	0.500000
VAR (AICc)	0.001150	0.030674	0.029353	0.218030	-0.040000	0.240000	1.240000	0.500000
VAR (BIC)	0.000926	0.025992	0.024917	0.162360	0.120000	0.280000	1.160000	0.500000
NN (AICc)	0.000105	0.008673	0.007916	0.058484	0.750000	0.750000	0.333330	0.500000
NN (BIC)	0.000206	0.011998	0.011458	0.072258	0.916670	0.625000	0.375000	0.500000
IPD Industrial Return								
ARMA	0.000227	0.012245	0.011185	0.055472	0.440000	0.700000	0.400000	0.500000
GARCH	0.004532	0.055542	0.053808	0.242860	0.120000	0.420000	1.040000	0.500000
VAR (AICc)	0.001201	0.027582	0.026305	0.157490	-0.200000	0.200000	1.400000	0.480000
VAR (BIC)	0.000839	0.025780	0.024917	0.126370	0.200000	0.220000	1.120000	0.500000
NN (AICc)	0.000093	0.007538	0.006966	0.031235	0.750000	0.666667	0.416667	0.500000
NN (BIC)	0.000054	0.017572	0.016783	0.069085	0.750000	0.666667	0.416667	0.500000

Table 8.3: Comparison between different models for 2-step return forecast

	MSE	RMSE	MAE	MAPE	Pearson Coefficient	DA	MDA	Sign
J255 Return Deviation								
ARMA	0.001034	0.028529	0.025677	0.887930	0.222220	0.611110	0.805560	0.500000
GARCH	0.682230	0.174840	0.171560	7.764500	-0.121950	0.597560	0.951220	0.541670
VAR (AICc)	0.000979	0.026280	0.023704	0.848940	0.027027	0.662160	0.810810	0.500000
VAR (BIC)	0.000856	0.024993	0.022107	0.826620	-0.297300	0.648650	0.972970	0.500000
NN (AICc)	0.001225	0.031146	0.027444	0.838510	-0.151520	0.621210	0.969700	0.500000
NN (BIC)	0.001584	0.031703	0.028389	0.798770	0.212120	0.606060	0.787880	0.484850
J256 Return Deviation								
ARMA (AICc)	0.000891	0.024891	0.022776	0.899060	0.625000	0.812500	0.343750	0.484380
ARMA (BIC)	0.000912	0.025612	0.022874	0.927920	0.250000	0.734380	0.656250	0.500000
GARCH (AICc)	0.000892	0.024507	0.021032	0.989580	0.062500	0.734380	0.687500	0.500000
GARCH (BIC)	11.75500	0.703030	0.693170	22.49700	0.187500	0.734380	0.687500	0.484380
VAR (AICc)	0.001500	0.034289	0.030947	1.466300	0.187500	0.625000	0.750000	0.484380
VAR (BIC)	0.001041	0.026715	0.023918	1.430300	-0.312500	0.687500	0.968750	0.500000
NN (AICc)	0.001205	0.030330	0.026447	1.317100	-0.266670	0.650000	0.966670	0.483330
NN (BIC)	0.000981	0.026959	0.023752	1.314400	-0.066667	0.700000	0.833330	0.500000
IPD Retail Return Deviation								
ARMA	0.000207	0.010622	0.009749	0.340195	0.083333	0.456522	0.958333	0.500000
GARCH	307.1200	3.700100	2.720100	87.75500	-0.360000	0.340000	1.240000	0.440000
VAR (AICc)	0.000042	0.004990	0.004736	0.129730	0.040000	0.300000	1.160000	0.500000
VAR (BIC)	0.000055	0.005686	0.005506	0.149400	-0.120000	0.260000	1.320000	0.500000
NN (AICc)	0.000023	0.004021	0.003811	0.114200	0.583330	0.375000	0.875000	0.500000
NN (BIC)	0.000194	0.008847	0.008686	0.197620	0.333330	0.500000	0.791670	0.500000
IPD Office Return Deviation								
ARMA	0.000346	0.010071	0.008530	0.494123	0.250000	0.645800	0.625000	0.437500
GARCH	0.018862	0.050777	0.047877	2.686600	0.083333	0.250000	1.208300	0.416670
VAR (AICc)	0.000111	0.008607	0.008236	0.387690	-0.040000	0.180000	1.280000	0.460000
VAR (BIC)	0.000158	0.011150	0.010797	0.520100	-0.280000	0.100000	1.520000	0.440000
NN (AICc)	0.000113	0.009111	0.008899	0.546860	0.416670	0.312500	0.958330	0.416670
NN (BIC)	0.000037	0.004717	0.004482	0.267040	0.500000	0.520830	0.708330	0.479170
IPD Industrial Return Deviation								
ARMA (AICc)	0.001310	0.061149	0.056270	4.500000	-0.333000	0.500000	1.110000	0.400000
ARMA (BIC)	0.000059	0.005475	0.005091	0.354212	-0.250000	0.416667	1.166667	0.420000
GARCH	0.010947	0.038429	0.037864	1.857500	-0.120000	0.240000	1.320000	0.420000
VAR (AICc)	0.000121	0.007423	0.007089	0.438890	-0.120000	0.440000	1.080000	0.500000
VAR (BIC)	0.000153	0.007596	0.007415	0.447660	-0.280000	0.440000	1.160000	0.500000
NN (AICc)	0.000030	0.004545	0.004416	0.253580	0.166667	0.416667	1.041700	0.500000
NN (BIC)	0.000031	0.004446	0.004331	0.246040	0.250000	0.395830	1.000000	0.500000

Table 8.4: Comparison between different models for 2-step return deviation forecast

	MSE	RMSE	MAE	MAPE	Pearson Coefficient	DA	MDA	Sign
J255 Return								
ARMA	0.028194	0.149270	0.134150	1.489100	0.264490	0.491380	0.632180	0.715520
GARCH	7878.100	24.76900	16.19500	202.6000	0.113380	0.464290	0.642860	0.598210
VAR (AICc)	0.060972	0.222190	0.203850	1.354700	0.319930	0.535710	0.609520	0.600000
VAR (BIC)	0.041166	0.176660	0.161480	1.466600	0.115430	0.557140	0.628570	0.721430
NN (AICc)	0.038495	0.163430	0.145840	1.931100	0.365090	0.500000	0.634410	0.741940
NN (BIC)	0.061474	0.172500	0.147040	1.002300	0.132280	0.362900	0.677420	0.709680
J256 Return								
ARMA	0.022613	0.130030	0.115000	0.696280	0.251750	0.600000	0.544440	0.716670
GARCH	5920.000	25.54000	23.75400	93.42900	0.346560	0.600000	0.544440	0.633330
VAR (AICc)	0.067988	0.232180	0.211950	0.942030	0.153530	0.525000	0.666670	0.608330
VAR (BIC)	0.043644	0.190320	0.174020	0.953560	0.047916	0.466670	0.700000	0.750000
NN (AICc)	0.027599	0.147980	0.132100	0.845890	0.072827	0.464290	0.702380	0.705360
NN (BIC)	0.016930	0.126410	0.106730	0.546970	0.286430	0.464290	0.642860	0.732140
IPD Retail Return								
ARMA	0.002083	0.037059	0.032566	0.139960	-0.064986	0.478260	0.565220	0.750000
GARCH	0.016081	0.111590	0.108160	0.417870	-0.316970	0.500000	0.768120	0.750000
VAR (AICc)	0.003362	0.041689	0.038408	0.174520	0.189720	0.402170	0.565220	0.750000
VAR (BIC)	0.002883	0.046032	0.043075	0.190070	-0.304830	0.217390	0.884060	0.750000
NN (AICc)	0.000480	0.020054	0.018031	0.083562	0.729760	0.636360	0.318180	0.750000
NN (BIC)	0.000329	0.015267	0.013815	0.068563	0.884510	0.681820	0.227270	0.750000
IPD Office Return								
ARMA	0.002106	0.040388	0.035976	0.267860	0.062300	0.467390	0.492750	0.750000
GARCH	0.015615	0.108540	0.106810	0.544940	-0.179400	0.391300	0.753620	0.750000
VAR (AICc)	0.003103	0.051579	0.047464	0.330590	-0.169620	0.206520	0.753620	0.728260
VAR (BIC)	0.002375	0.043468	0.039876	0.260670	-0.073473	0.239130	0.840580	0.739130
NN (AICc)	0.000228	0.014365	0.013177	0.084172	0.757450	0.534090	0.212120	0.750000
NN (BIC)	0.000379	0.017673	0.016176	0.107480	0.844560	0.477270	0.257580	0.750000
IPD Industrial Return								
ARMA	0.001138	0.029671	0.025517	0.113060	0.188350	0.456520	0.521740	0.750000
GARCH	0.005379	0.063277	0.059379	0.252060	0.254650	0.423910	0.608700	0.750000
VAR (AICc)	0.003700	0.047666	0.042964	0.228590	-0.179790	0.228260	0.898550	0.717390
VAR (BIC)	0.002112	0.041147	0.038061	0.183200	0.250750	0.260870	0.652170	0.750000
NN (AICc)	0.000135	0.010023	0.008948	0.037517	0.827470	0.647730	0.227270	0.750000
NN (BIC)	0.000929	0.024416	0.022129	0.086449	0.746040	0.647730	0.242420	0.750000

Table 8.5: Comparison between different models for 4-step return forecast

	MSE	RMSE	MAE	MAPE	Pearson Coefficient	DA	MDA	Sign
J255 Return Deviation								
ARMA	0.001063	0.030676	0.025566	0.941130	-0.170420	0.632350	0.617650	0.735290
GARCH	0.684800	0.185020	0.179690	6.322200	0.005448	0.583330	0.649570	0.711540
VAR (AICc)	0.000947	0.028003	0.023922	0.836210	0.085902	0.642860	0.580950	0.750000
VAR (BIC)	0.000844	0.026785	0.022947	0.849000	-0.063641	0.657140	0.714290	0.750000
NN (AICc)	0.001338	0.034211	0.028498	0.942110	-0.058416	0.596770	0.612900	0.733870
NN (BIC)	0.001165	0.031610	0.025947	0.843340	-0.072486	0.604840	0.709680	0.733870
J256 Return Deviation								
ARMA (AICc)	0.000963	0.027814	0.023860	1.006700	0.417460	0.758330	0.400000	0.725000
ARMA (BIC)	0.001049	0.029386	0.024484	1.121500	0.230910	0.700000	0.522220	0.725000
GARCH (AICc)	0.000761	0.024659	0.019062	0.981060	0.285100	0.725000	0.555560	0.750000
GARCH (BIC)	14.44500	0.837030	0.818120	40.87900	0.259310	0.691670	0.566670	0.741670
VAR (AICc)	0.001431	0.036198	0.029492	1.389800	-0.059904	0.641670	0.577780	0.708330
VAR (BIC)	0.000915	0.027592	0.022377	1.284900	-0.140590	0.708330	0.711110	0.750000
NN (AICc)	0.001393	0.033232	0.028334	1.683800	0.116330	0.660710	0.595240	0.750000
NN (BIC)	0.000760	0.025064	0.020493	1.268100	0.068113	0.750000	0.678570	0.750000
IPD Retail Return Deviation								
ARMA	0.001321	0.025339	0.023071	0.700934	0.076068	0.431818	0.651510	0.663040
GARCH	0.156954	0.226910	0.216806	8.922188	0.010136	0.466667	0.577775	0.717390
VAR (AICc)	0.000088	0.007532	0.006772	0.190640	0.287240	0.358700	0.637680	0.750000
VAR (BIC)	0.000102	0.007682	0.007067	0.194140	-0.078331	0.304350	0.826090	0.750000
NN (AICc)	0.000174	0.008888	0.008379	0.201740	0.409500	0.568180	0.484850	0.750000
NN (BIC)	0.000175	0.008530	0.007988	0.186030	0.319980	0.522730	0.545450	0.750000
IPD Office Return Deviation								
ARMA	0.001825	0.021958	0.019613	1.142830	-0.088966	0.488636	0.621205	0.725000
GARCH	0.110070	0.095859	0.084771	5.095500	-0.133420	0.238640	0.772730	0.568180
VAR (AICc)	0.000309	0.014652	0.013147	0.617810	-0.254000	0.119570	0.884060	0.630430
VAR (BIC)	0.000346	0.016761	0.015408	0.771530	-0.451570	0.130430	1.058000	0.576090
NN (AICc)	0.000305	0.014043	0.013282	0.626380	0.216820	0.352270	0.606060	0.556820
NN (BIC)	0.000380	0.017189	0.016253	0.873530	-0.001110	0.136360	0.757580	0.727270
IPD Industrial Return Deviation								
ARMA (AICc)	0.003731	0.029680	0.025851	1.492360	-0.095481	0.363636	0.772719	0.541300
ARMA (BIC)	0.000128	0.008746	0.008079	0.606275	-0.034685	0.409091	0.742418	0.492000
GARCH	0.003891	0.028854	0.027794	1.861078	0.120250	0.483333	0.577778	0.717390
VAR (AICc)	0.000236	0.011414	0.010454	0.725600	-0.088390	0.358700	0.739130	0.750000
VAR (BIC)	0.000237	0.010193	0.009686	0.672640	-0.285700	0.347830	0.826090	0.750000
NN (AICc)	0.000041	0.005531	0.004994	0.259800	0.440500	0.590910	0.545450	0.750000
NN (BIC)	0.000028	0.004724	0.004404	0.253140	0.525710	0.465910	0.500000	0.750000

Table 8.6: Comparison between different models for 4-step return deviation forecast

The best and the worst performing models from the above forecasts are summarised below.

	1-step		2-steps		4-steps	
	Best	Worst	Best	Worst	Best	Worst
J255 Return	ARMA	GARCH	ARMA	GARCH	ARMA	GARCH
J255 Return Deviation	VAR(BIC)	NN(BIC)	VAR(AICc)	GARCH	VAR(AICc)	GARCH
J256 Return	ARMA	GARCH	ARMA	GARCH	NN(BIC)	GARCH
J256 Return Deviation	ARMA	GARCH	ARMA(AICc)	GARCH(BIC)	GARCH(AICc)	GARCH(BIC)
Retail Return	NN(BIC)	GARCH	NN(AICc)	GARCH	NN(BIC)	GARCH
Retail Return Deviation	VAR(AICc)	GARCH	NN(AICc)	GARCH	NN(BIC)	GARCH
Office Return	NN(AICc)	GARCH	NN(AICc)	GARCH	NN(AICc)	GARCH
Office Return Deviation	NN(BIC)	GARCH	NN(BIC)	GARCH	NN(AICc)	GARCH
Industrial Return	NN(Both)	GARCH	NN(AICc)	GARCH	NN(AICc)	GARCH
Industrial Return Deviation	ARMA(BIC)	GARCH	NN(BIC)	GARCH	NN(BIC)	ARMA(AICc)

Table 8.7: Summary of the best and the worst performing models for short-term forecast

7.4.1 J255 return and return deviation

The ARMA model is the best performing model and the GARCH model is the worst performing model for all short-term forecasts of the J255 return. This result corresponds with the finding in the long term prediction. The performance of the neural network model optimised using the AICc method, which is the optimal model for long term prediction, is average.

The VAR model proves to be the most accurate in predicting the J255 return deviation with the lowest error and highest directional accuracy for the three short-term forecasts, VAR model optimised using the BIC method for 1-step forecast and VAR model optimised using the AICc method for 2-steps and 4-steps forecasts. This finding corresponds with the findings of the analytical method for long term prediction and even in the work of Brook and Tsolacos (2001). However, as evident in the result of the graphical method for long term prediction, VAR model predicts the trend of the actual return deviation poorly and can produce highly misleading results. The neural network model optimised using the BIC model is the worst performing model for 1-step forecast and the GARCH model is the worst performing model for 2-step and 4-step forecasts, which are fairly accurate models for long-term forecast.

7.4.2 J256 return and return deviation

The ARMA model is the best performing model for 1-step and 2-steps forecasts of the J256 return and the neural network model optimised using the BIC model is the best performing model for 4-step forecasts of the J256 return. Likewise, the ARMA model is also one of the best performing models for long-term forecast. The GARCH model is

once again identified as the worst performing model for all short-term forecasts of the J256 return. On the contrary, this model is identified to be one of the best performing models for long-term prediction.

The ARMA model is the best performing model for 1-step and 2-steps forecasts of the J256 return deviation and the GARCH model optimised using the AICc model is the best performing model for 4-step forecasts of the J256 return deviation. The forecasting power of the neural network model optimised using the BIC method, which is the best performing model for long term prediction, improves with increasing number of forecasting steps. The GARCH model is the worst performing model for 1-step and 2-steps forecasts of the J256 return deviation and the GARCH model, optimised using the BIC model, is the worst performing model for 4-step forecasts of the J256 return deviation.

7.4.3 Retail return and return deviation

Similar to the long term prediction analysis, the neural network models produce the most accurate forecasts of the retail return. The GARCH model is the worst performing model for all three forecasts of the retail return, which concur with the finding of the analytical method for long term prediction.

The VAR model is the best performing model for 1-step forecast of the retail return deviation and the neural network model is the best performing model for 2-steps and 4-steps forecast of the retail return deviation. The GARCH model is again the worst performing model for all three forecasts of the retail return deviation, which concurs with the findings of the analytical method for long term prediction.

7.4.4 Office return and return deviation

Similar to previous return and the long term prediction analysis result, the neural network models produce the most accurate forecast of the office return and office return deviations. The GARCH model is the worst performing model for all of the forecasts.

7.4.5 Industrial return and return deviation

The neural network models produce the most accurate forecasts of the industrial return and the GARCH model produces the least accurate forecasts of the industrial return.

The ARMA model, optimised using the BIC method, is the best performing model for 1-step forecast of the industrial return deviation and the neural network model optimised using the BIC model is the best performing model for 2-steps and 4-steps forecasts of the industrial return deviation. Likewise, the neural network model optimised using the BIC model is the best performing model for long term prediction. The GARCH model is again

the worst performing model for 1-step and 2-steps forecasts of the industrial return deviation and the ARMA model optimised using the AICc model is the worst performing model for 4-steps forecasts of the industrial return deviation.

7.5 Summary

The long-term and short-term prediction performances of the ARMA, GARCH, VAR and neural network models with optimal parameters and explanatory variables are evaluated in this section. Two different methods are used in the evaluation, namely the graphical method and the analytical method.

The long-term indirect return prediction indicates that the ARMA model and the neural network models are the preferred model, while for the long-term direct return predictions the neural network significantly outperformed other models.

However, for long-term predictions of return deviation, the result is not so clear-cut, mainly because the trends are so much more difficult to predict. The ARMA, GARCH and neural network models are able to predict return deviations with similar accuracy. Even though the result from the analytical method indicates that the VAR model performed better than the other models in specific cases, the result from the graphical method indicates that the output produced by the model is insensitive to the changes in the actual trend and can produce misleading results. Lastly, it is evident that the accuracy of the predictions of direct return deviation decreased significantly from 2006 onward. This is due to the increase in the gap between the properties with the highest and the lowest return. For example, the differences in return for retail and industrial property in 2006 are 13.4% and 5.2% respectively, which are higher than in previous years.

For short-term return predictions, the result is similar where the ARMA model and the neural network optimised using the AICc method are the best performing models. The neural network model is the preferred choice for direct return predictions. Again, it is much more difficult to define the best models in predicting short-term return deviation as various models are able to predict return deviations with similar accuracy. In general, the GARCH model is inferior in producing short term forecast. The result of the analysis is tabulated below.

	Long term	Short term
J255 Return	ARMA(6,2,1)/NN(4,4,2,Linear)	ARMA(6,2,1)
J256 Return	ARMA(1,3,1)/GARCH(1,1,1)	ARMA(1,3,1)/NN(2,2,1,Linear)
Retail Return	NN(both models)	NN(both models)
Office Return	NN(both models)	NN(both models)
Industrial Return	NN(both models)	NN(both models)
J255 Return Deviation	NN(4,4,2,Linear)	VAR(4,2)/ARMA(6,1,1)
J256 Return Deviation	NN(2,2,1,Linear)	ARMA(1,1,2)/GARCH(1,4,1)
Retail Return Deviation	NN(2,2,2,Linear)	NN(both models)
Office Return Deviation	NN(4,2,2,Linear)	NN(both models)
Industrial Return Deviation	NN(2,2,1,Logistic)	NN(2,2,1,Logistic)/ARMA(1,1,1)

Table 9: Summary of analysis for the long-term and the short-term prediction

The following is a summary of the accuracy of the prediction of the optimal model for each return and return deviation and the associated explanatory variables used in the models.

Output	Forecast accuracy	Explanatory Variables
J255 Return	Fair	Term structure, employment index, building plans passed index
J256 Return	Poor	Manufacturing index, employment index, building plans passed index
Retail Return	Good	Gilt-equity ratio, changing CPIX index
Office Return	Very Good	
Industrial Return	Very Good	Manufacturing index, prime lending rate
J255 Return Deviation	Poor	Gilt-equity ratio, building plans passed index
J256 Return Deviation	Very Poor	Term structure, building plans passed index
Retail Return Deviation	Fair	Gilt-equity ratio, changing CPIX index
Office Return Deviation	Very Poor	
Industrial Return Deviation	Very Good	Manufacturing index, prime lending rate

Table 10: Summary of accuracy of predictions and variables used by optimal models

Referring to the table above, the models are able to predict the trends of the J255 indirect returns fairly accurately. However, the models cannot accurately predict the trends of the J256 indirect property return and the indirect return deviations. The models are able to predict both the trends and the actual value of the direct return and the industrial return deviation accurately. The retail return deviation is predicted by the models fairly accurately, except in the last 8 periods, where none of the models predicted the sudden rise and the office return deviation cannot be predicted by any of the models.

The impulse response of the optimal long-term prediction models is analysed. For the most part, the biggest influence on the output of the models is the autocorrelative components. However, the output of some of the GARCH and neural network models are more susceptible to changes in the explanatory variables such as gilt-equity ratio, term structure and changing CPIX. This finding corresponds to the result in the causality analysis in section 4, where these explanatory variables are closely related to the outputs of the model under investigation.

The significance of the conditional variance from the GARCH model on the prediction of the actual output trend is also investigated, and no relationship between the two was found.

While the models examined in this research are far more sophisticated than previous models used in the South African property market environment, there is a debate on whether to deploy a complex model with few explanatory variables, or a simple model with multiple explanatory variables, to forecast the return. In this section, it is clear that a simple model with multiple explanatory variables such as the ARMA model is suitable for predicting indirect returns, which are volatile and slightly stochastic in nature. Conversely, a complex non-linear model with few explanatory variables such as the neural network model is suitable for predicting direct returns, which slopes gradually. However, a simple model with few explanatory variables, such as the VAR model, is not suitable for this application.

Further to the debate above, cognisance must be taken of the limitations and shortfall of these models. The general shortfall of these models is that their accuracies are limited to the number of samples available, as discussed in work of Brook and Tsolacos (1999). Since the quantity of data available for this research is fairly scarce, there is still potential in the near future to increase the accuracy of these models. There are also technical limitations to these models, as each of the models selected was designed to serve specific scenarios, different from each other.

The ARMA model and the GARCH model are designed for a single output (univariate). Resultantly, separate models are required for the return and its deviation and the number of models required to simulate a set of return doubled, which increased the amount of time required to develop models for a specific set of outputs. Even though there is a multivariable derivative for these two types of model, they are highly complex and difficult to implement in software. Furthermore, the GARCH model is designed for a volatile environment and thus it performs much better in predicting indirect returns and return deviations where the trend is volatile.

The VAR model and the neural network model are different from the ARMA and the GARCH model in that they are designed to produce multiple outputs. While the VAR model has the ability to correlate a set of input and output variables, it is an extension of a multiple linear regression model with multiple outputs and is only suitable for linear relationships. In a highly non-linear environment such as the one investigated in this research, this model failed comprehensively in predicting the trend of the return.

For non-linear relationships, the neural network model is employed, which operates like a black box (Brook and Tsolacos, 2003). In comparison to the previous models, several extra steps are required in order to implement a neural network model, such as normalisation of all input and output variables and separating the data into training and validating set. Consequently, the accuracy of this type of model relates significantly to the size of the training data set. Furthermore, as discussed in Brook and Tsolacos (2003), there is an underlying problem with neural network in determining direct relationships between a variable and an output. This is because it operates in a black box manner and there is no theory to link the relevance in the values of the weights to the relationship between an input and an output variable.

Finally, the reason behind the introduction of the return deviation in the forecast is that the prediction of the return deviation assists one in determining the risk of achieving the forecast return.

8 Conclusion

The macroeconomic factors that influence commercial property return are first identified and investigated. These factors are inflation, term structure of interest rate, gilt-equity yield ratio, manufacturing index, employment growth rate, building plans passed, prime lending rate and GDP growth rate. Both indirect and direct property return are investigated in this research. The direct property return are gathered from the IPD commercial property data while the indirect property returns are gathered from the J255 property trust index and the J256 property loan stock index. The deviation of the return, which is essential in determining the risk of the return value, is also extracted from the data for evaluation. Using Granger causality technique, the term structure, employment index and building plans passed index were identified to have significant influence in indirect return. While the changing CPIX index and gilt-equity ratio were the factors identified to have significant influence on direct return.

The ARMA, GARCH, VAR and MLP neural network models are used in this research to predict and forecast the returns. The ARMA and GARCH models investigated in this research are univariant, i.e. the model only produces one output at a time. While the VAR and neural network models investigated are multivariant, i.e. the model can produce multiple outputs at a time. The parameters for each type of model are identified and optimised by means of information criterion techniques. The optimised parameters are the number of lags, input variables and functions to be used in a neuron for neural network.

The performances of the optimal models are then evaluated. The models are required to perform long-term forecasting and short-term forecasting. The ARMA model predicted the indirect return most accurately, while the neural network predicted the direct return most accurately. This is because the indirect return trend is a volatile trend and is suitable for the ARMA model to predict such a trend, while the direct return is less volatile and slightly non-linear and is suitable for the neural network to predict such a trend. There was no particular model that is preferable in predicting all of the return deviation. Consequently, a combination of models is required in order to predict the return deviation accurately.

The result from the performance evaluation indicates that the South African commercial property return can be forecast, in particular the direct return. However, further investigation is required to refine the forecast model, while this research should serve as a stepping stone in the investigation of the relationship between macroeconomic factors and the South African property market.

9 References

- Akaike, H. (1974). A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control*, 19(6): 716-723.
- Ball, M., Lizieri, C. and MacGregor, B.D. (1998). *The Economics of Commercial Property Market*. New York: Routledge.
- Bedrick, E.J. and Tsai, C. (1994). Model Selection for Multivariate Regression in Small Samples. *Biometrics*, 50(1): 226-231.
- Bera, A.K. and Jarque, C.M. (1980), Efficient Tests for Normality, Homoscedasticity and Serial Independence of Regression Residuals. *Economics Letters*, 6(3): 255-259.
- Bera, A.K. and Jarque, C.M. (1981), Efficient Tests for Normality, Homoscedasticity and Serial Independence of Regression Residuals: Monte Carlo Evidence. *Economics Letters*, 7(4): 313-318.
- Bollerslev, T. (1986), Generalized Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics*, 31(3): 307-327
- Bond, M.T. and Seiler, M.J. (1998). Real Estate Returns and Inflation: An Added Variable Approach. *Journal of Real Estate Research*, 15(3): 327-338.
- Borst, R.A. and McCluskey, W.J. (1997). An Evaluation of MRA, Comparable Sales Analysis, and ANNs for the Mass Appraisal of Residential Properties in Northern Ireland. *Assessment Journal*, 4(1): 47-55.
- Brooks, C. and Tsolacos, S. (1999). The Impact of Economic and Financial Factors on UK Property Performance. *Journal of Property Research*, 16(2): 139-152.
- Brooks, C. and Tsolacos, S. (2001). Forecasting Real Estate Returns Using Financial Spreads. *Journal of Property Research*, 18(3): 235-248.
- Brooks, C. and Tsolacos, S. (2001a). Linkage between Property Assets Returns and Interest Rates: Evidence for the UK. *Applied Economics*, 33(6): 711-719.
- Brooks, C. and Tsolacos, S. (2003). International Evidence on the Predictability of Returns to Securitized Real Estate Assets: Econometric Models versus Neural Networks. *Journal of Property Research*, 20(2): 133-155.
- Burnham, K.P. and Anderson, D.R. (2004). Multimodel Inference: Understanding AIC and BIC in Model Selection. *Sociological Methods and Research*, 33(2): 261-304.

- Chan, K.C., Hendershott, P.H. and Sanders, A.B. (1990). Risk and Return on Real Estate: Evidence from Equity REITs. *AREUEA Journal*, 18(4): 431-452.
- Chatfields, C. (2004). *The Analysis of Time Series – An Introduction*. 6th Edition. USA: Chapman and Hall CRC.
- Collins, A. and Evans, A. (1994). Aircraft Noise and Residential Property Value. *Journal of Transport Economic and Policy*, 28(2): 175-197.
- Demuth, H.B. and Hagan, M.T. (1999). Neural Network for Control. Invited Tutorial, 1999. *American Control Conference*, June, 1999, San Diego, pp. 1642-1656.
- Dickey, D.A. and Fuller, W.A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association*, 74(366): 427-431.
- Dickey, D.A. and Said, S.E. (1984). Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order. *Biometrika*, 71(3): 599-607.
- Do, A.Q. and Grudnitski, G. (1992). A Neural Network Approach to Residential Property Appraisal. *The Real Estate Appraiser*, 58(3): 38-45.
- Ebert, B. et al. (2008). Forecast Verification – Issues, Methods and FAQ. 4th *International Verification Methods Workshop*. Retrieved 5th August 2009 from the World Wide Web: http://www.bom.gov.au/bmrc/wefor/staff/eee/verif/verif_web_page.html.
- Egriolgu, E., Aladag, C.H. and Gunay, S. (2008). A New Model Selection Strategy in Artificial Neural Networks. *Applied Mathematics and Computation*, 195(2): 591-597.
- Ellis, C. and Wilson, P. (2005). Can a Neural Network Property Portfolio Selection Process Outperform the Property Market? *Journal of Real Estate Portfolio Management*, 11(2): 105-121.
- Engle, R.F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of Variance of United Kingdom Inflation. *Econometrica*, 50(4): 987-1008.
- Granger, C.W.J. (1969). Investigating Causal Relations by Econometric Models and Cross-Spectral Methods. *Econometrica*, 37(3): 424-438.
- Ge, X.J. and Runeson, G. (2004). Modeling Property Prices Using Neural Network Model for Hong Kong. *International Real Estate Review*, 7(1): 121-138.
- Hetherington, J. (1998) Forecasting of rents, in MacLeary, A. And Nathakumaran, N. (Eds.), *Property Investment Theory*, London: E & F N Spon (pp. 97-107).

- Hoesli, M. (1994). Real Estate as a Hedge against Inflation: Learning from the Swiss Case. *Journal of Property Valuation and Investment*, 12(3): 51-59.
- Hua, G.B. (1996). Residential Construction Demand Forecasting using Economic Indicators: A Comparative Study of Artificial Neural Networks and Multiple Regression. *Construction Management and Economics*, 14(1): 10-34.
- Hurvich, C.M. and Tsai, C. (1989). Regression and Time Series Model Selection in Small Samples. *Biometrics*, 76(2): 297-307.
- Halekoh, U. (2007). *Model Selection*. Retrieved 21st November 2009 from the World Wide Web: <http://gbi.agrsci.dk/statistics/courses/phd07/material/Day6/modelSelection-handout.pdf>.
- Investopedia. (2009). *Term Structure of Interest Rates*. Retrieved 28th February 2010 from the World Wide Web: <http://www.investopedia.com/terms/t/termstructure.asp>.
- Laio, F., Di Baldassare, G. and Montanari, A. (2009). Design flood estimation using model selection criteria. *Physics and Chemistry of the Earth*, 34: 606-611.
- LeSage, J.P. (1999). *Spatial Econometrics*. Retrieved 21st November 2009 from the World Wide Web: <http://www.spatial-econometrics.com/wbook.pdf>.
- Liang, Y. and McIntosh, W. (1998). Employment Growth and Real Estate Return: Are They Linked? *Journal of Real Estate Portfolio Management*, 4(2): 125-168.
- Ling, D.C. and Naranjo, A. (1997). Economic Risk Factors and Commercial Real Estate Returns. *Journal of Real Estate Finance and Economics*, 14(3): 283-307.
- Ling, D.C. and Naranjo, A. (1998). The Fundamental Determinants of Commercial Real Estate Returns. *Real Estate Finance*, 14(4): 13-24.
- Lizieri, C. and Satchell, S. (1997). Interactions Between Property and Equity Markets: An Investigation of Linkages in the United Kingdom 1972-1992. *Journal of Real Estate Finance and Economics*, 15(1): 11-26.
- Liow, K.H. (2000). The Dynamics of the Singapore Commercial Property Market. *Journal of Property Research*, 17(4): 279-291.
- Liow, K.H. (2004). Time-Varying Macroeconomic Risk and Commercial Real Estate: An Asset Pricing Perspective. *Journal of Real Estate Portfolio Management*, 10(1): 47-57.
- Liu, C.H. and Mei, J. (1992). The Predictability of Returns on Equity REITs and Their Co-Movement with Other Assets. *Journal of Real Estate Finance and Economics*, (5): 401-418.

- Liu, C.H., Hartzell, D.J. and Hoesli, M.E. (1997). International Evidence on Real Estate Securities as an Inflation Hedge. *Real Estate Economics*, 25(2): 193-221.
- Liberta (2011). Prime Interest Rate in South Africa. Retrieved 15th January 2011 from the World Wide Web: <http://liberta.co.za/blog/prime-interest-rate-in-south-africa-current-and-historical>.
- Liberta (2011a). What is the Repo Rate. Retrieved 15th January 2011 from the World Wide Web: <http://liberta.co.za/blog/what-is-the-repo-rate/>.
- May, C. (2004). "An empirical analysis of the macroeconomic variables that affect stock market return", University of the Witwatersrand: M.Comm thesis.
- Mathwork (2004). "Full Product Family Help". Matlab Version 7 Release 14.
- McCue, T. and Kling, J.L. (1987). Office Building Investment and the Macroeconomy: Empirical Evidence, 1973-1985. *Real Estate Economics*, 15(3): 234-255.
- McCue, T. and Kling, J.L. (1994). Real Estate Returns and the Macroeconomy: Some Empirical Evidence from Real Estate Investment Trust Data 1972-1991. *Journal of Real Estate Research*, 9(2): 277-287.
- Njuguna, P.K. (2002). "Macroeconomic value drivers for Johannesburg CBD property fund units", University of the Witwatersrand: MBA thesis.
- Nabney, I. (2004). *Netlab: Algorithm for Pattern Recognition*, 4th Edition. UK: Springer.
- Onder, Z. (2000). High Inflation and Returns on Residential Real Estate: Evidence from Turkey. *Applied Economics*, 32: 917-931.
- Pena, D., Tiao, G.C. and Tsay, R.S. (2001). *A Course in Time Series Analysis*. USA: Wiley Series.
- Poensgen, R. (2000). "The Influence of Macroeconomic Factors on Residential Property Values", University of the Witwatersrand: MBA thesis.
- Qi, M. and Maddala, G.S. (1999). Economic Factors and the Stock Market: A New Perspective. *Journal of Forecasting*, 18: 151-166.
- Qi, M. and Zhang, G.P. (2001). An Investigation of Model Selection Criteria for Neural Network Time Series Forecasting. *European Journal of Operational Research*, 132(2001): 666-680.
- Quan, D.C. and Titman, S. (1999). Do Real Estate Prices and Stock Prices Move Together? An International Analysis. *Real Estate Economics*, 27(2): 183-207.

Rossini, P. (1997). *Application of Artificial Neural Networks to the Valuation of Residential Property*. Paper Presented in the Third Annual Pacific-Rim Real Estate Society Conference, Palmerston North, New Zealand.

Schwarz, G. (1978). Estimating the Dimension of a Model. *Annals of Statistics*, 6: 461-464.

Siganos, D. and Stergiou, C. (1996). Neural Networks. Retrieved 31st May 2006 from the World Wide Web:
http://www.doc.ic.ac.uk/~nd/surprise_96/journal/vol4/cs11/report.html.

Spiegel, M. and Boxer, R.W. (1972). *Schaum's Outline of Theory and Problems of Statistics in SI Units*. New York: McGraw-Hill.

Stevenson, S. and Murray, L. (1999). An Examination of the Inflation Hedging Ability of Irish Real Estate. *Journal of Real Estate Portfolio Management*, 5(1): 59-69.

Stock, J.H. and Watson, M.W. (2001). Vector Autoregressions. *Journal of Economic Perspectives*, 15(4): 101-115.

West, T. and Worthington, A. (2004). *Macroeconomic Risk Factors in Australian Commercial Real Estate, Listed Property Trust and Property Sector Stock Returns: A Comparative Analysis using GARCH-M*, Proceedings of the Pacific Rim Real Estate Society Conference, Bangkok, Thailand.

Wikipedia (2009a). Kurtosis. Retrieved 1st December 2009 from the World Wide Web:
<http://en.wikipedia.org/wiki/Kurtosis>.

Wikipedia (2009b). Jarque-Bera test. Retrieved 1st December 2009 from the World Wide Web: http://en.wikipedia.org/wiki/Jarque%E2%80%93Bera_test.

Wikipedia (2009c). F-test. Retrieved 1st December 2009 from the World Wide Web:
<http://en.wikipedia.org/wiki/F-test>.

Wikipedia (2009d). Hyperbolic function. Retrieved 1st December 2009 from the World Wide Web: http://en.wikipedia.org/wiki/Hyperbolic_function.

Wikipedia (2009e). Bayesian Information Criterion. Retrieved 1st December 2009 from the World Wide Web: http://en.wikipedia.org/wiki/Bayesian_information_criterion.

Worzala, E., Lenk, M. and Silva, A. (1995). An Exploration of Neural Networks and its Application to Real Estate Valuation. *The Journal of Real Estate Research*, 10(2): 185-201.

Appendix A

Tables of input and output data

Macroeconomic Variables in Quarter Basis

Date	Term Structure	CPIX Index	Gilt-equity Ratio	Manufacturing Index	GDP at Market P/Es	Employment Rate Index in Construction	Building Plan Passed Index	Changing GDP	Changing CPIX Index	Unspecified Inflation	Anticipated Inflation from ARIMA Model	Prime Interest Rate (in %)	Changing Prime Interest Rate (in %)
1985/01	-3.45			82.83	743650.50	184.54	104.22					13.00	0.50
1985/02	-2.31			83.10	748800.13	185.78	110.68	5279.63				15.00	2.00
1985/03	-2.37			84.28	758901.25	186.85	105.61	10071.12				16.00	1.00
1985/04	-1.77			84.39	768027.19	187.23	104.54	7125.94				19.00	2.00
1989/01	-1.47		367.50	84.19	770771.31	187.89	100.34	4744.12				19.00	0.00
1989/02	-0.67		384.56	85.00	774115.38	187.17	103.33	3344.07				20.00	2.00
1989/03	-1.01		411.55	83.94	774737.31	186.46	114.55	621.93				20.00	0.00
1989/04	-0.69		419.71	83.05	769343.00	186.85	105.79	-4794.31				21.00	1.00
1990/01	0.01		413.45	81.52	770569.38	189.67	102.88	616.38				21.00	0.00
1990/02	-0.70		444.19	81.38	769823.50	186.50	109.91	-636.88				21.00	0.00
1990/03	-0.86		452.63	81.48	769276.44	184.50	115.62	-647.06	1.37	0.42	1.79	21.00	0.00
1990/04	-0.23	36.77		81.69	769990.44	183.05	110.30	174.00	1.33	0.45	1.78	21.00	0.00
1991/01	-0.05	41.20	441.43	80.66	769865.75	179.17	109.97	-6124.69	1.73	0.15	1.89	21.00	0.00
1991/02	-0.42	44.70	412.44	81.01	762142.19	174.80	115.10	-1723.62	1.60	-0.09	1.51	21.00	0.00
1991/03	-0.54	46.43	439.29	80.83	761848.50	171.30	112.09		1.90	0.17	2.07	20.00	-1.00
1991/04	-0.68	48.77	471.67	80.74	760535.83	171.86	96.59	-1310.87	1.73	-0.26	1.48	20.00	0.00
1992/01	-0.82	60.60	468.46	80.01	755205.19	171.04	104.04	-5239.44	2.33	0.53	2.86	20.25	0.25
1992/02	-1.18	62.53	488.29	78.95	750673.94	166.78	96.68	-4632.25	1.83	0.48	2.21	20.25	0.00
1992/03	-2.30	64.37	449.06	76.95	741884.25	163.43	111.32	-8699.69	1.93	-0.55	1.39	19.25	-1.00
1992/04	-2.29	66.43	429.00	77.27	736590.38	162.09	99.53	-6303.87	1.83	0.88	1.94	18.25	0.00
1993/01	-1.94	66.77	424.57	77.48	742048.98	162.26	86.74	6516.50	1.07	0.16	1.49	17.25	-1.00
1993/02	-1.83	69.43	440.61	77.40	750236.69	167.21	88.50	8139.81	2.67	-1.60	1.07	16.25	-1.00
1993/03	-1.36	71.99	428.28	77.99	751036.44	169.37	92.89	10796.75	0.87	0.72	1.58	16.25	0.00
1993/04	-1.34	61.30	415.65	78.27	768874.56	165.66	90.51	5638.12	1.00	-0.09	0.91	15.25	-1.00
1994/01	-1.51	62.87	447.04	78.53	766682.13	162.36	95.03	-1092.43	1.57	0.08	1.65	15.25	0.00
1994/02	-1.36	64.10	522.47	79.93	773887.63	163.57	90.87	8105.50	1.23	0.08	3.15	15.25	0.00
1994/03	-2.09	66.53	642.79	79.66	782691.94	164.57	106.42	8704.31	2.43	-0.92	1.91	15.25	0.00
1994/04	-1.00	67.73	732.41	82.27	796654.31	161.47	116.12	13062.37	1.20	0.68	1.88	16.25	1.00
1995/01	-1.01	69.20	739.91	83.58	798628.38	164.54	133.10	2874.07	1.47	1.42	2.89	16.25	0.00
1995/02	-1.05	70.63	727.37	83.33	802074.06	160.71	122.37	3545.68	1.43	-0.85	0.78	17.50	1.25
1995/03	-0.81	70.90	671.01	82.76	806636.19	164.70	121.46	9462.07	0.17	-0.09	0.07	18.50	1.00
1995/04	-0.60	71.39	600.74	82.45	808715.19	163.29	116.99	3179.06	0.53	1.27	1.80	18.50	0.00
1996/01	-0.63	73.03	596.20	80.95	820063.38	148.08	115.45	3179.06	0.53	0.08	1.62	18.50	0.00
1996/02	-0.91	74.20	687.84	81.10	835423.56	146.23	123.26	15900.18	1.17	0.72	1.89	20.50	2.00
1996/03	-0.61	75.73	683.25	81.39	844933.56	145.04	115.72	9480.00	1.53	0.95	2.48	19.50	-1.00
1996/04	-0.40	77.60	711.87	81.48	852917.25	141.88	111.84	8019.69	1.87	0.40	2.26	19.25	-0.25
1997/01	-0.20	79.70	683.66	81.09	863687.56	140.69	118.76	2450.31	2.10	0.59	2.69	20.25	1.00
1997/02	-0.12	81.40	633.69	81.33	880699.50	141.10	105.91	5231.94	1.70	0.73	2.43	20.25	0.00
1997/03	-0.08	82.27	693.91	81.00	882397.31	139.42	103.14	1797.81	0.87	1.67	2.53	20.25	0.00
1997/04	0.02	83.17	579.10	81.15	883699.44	136.71	105.39	1302.19	0.90	0.25	1.15	19.25	-1.00
1998/01	-0.02	84.63	545.34	80.46	884791.00	135.13	104.61	1091.56	1.47	-0.34	1.13	19.25	0.00
1998/02	-0.24	80.37	664.26	80.37	889014.00	128.00	100.36	1223.00	2.00	-0.29	1.71	18.25	-1.00
1998/03	1.07	87.80	680.35	79.26	884116.00	125.81	101.20	-1686.00	1.17	-0.74	0.42	22.25	4.00
1998/04	0.57	88.87	739.39	79.39	884951.00	113.80	96.14	895.00	1.07	0.30	1.37	23.50	1.25
1999/01	0.08	90.50	539.35	78.66	872905.00	111.23	116.83	7954.00	1.63	-0.47	1.16	21.00	-2.50
1999/02	-0.54	92.30	510.72	78.26	879852.00	104.62	104.62	6947.00	1.80	0.39	2.19	19.00	-2.00
1999/03	-1.39	92.40	532.57	78.59	883461.00	102.58	93.87	9609.00	1.80	0.57	1.67	16.50	-2.50
1999/04	-1.20	94.73	545.14	79.32	885242.00	99.08	89.20	9781.00	1.33	0.20	1.53	15.50	-1.00
2000/01	-1.31	96.87	568.82	79.81	909357.00	100.51	93.82	10115.00	2.13	-0.01	2.12	14.50	-1.00
2000/02	-1.60	99.70	610.04	79.11	917724.00	100.61	99.63	8967.00	2.83	-1.89	0.95	14.50	0.00
2000/03	-2.01	101.23	577.94	79.48	928813.00	99.08	96.09	9089.00	1.53	-1.84	-0.11	14.50	0.00
2000/04	-1.78	102.17	547.72	79.89	934638.00	99.80	106.25	7885.00	0.93	0.10	1.03	14.50	0.00
2001/01	-1.49	104.23	476.28	80.48	940768.00	97.90	101.97	6070.00	2.07	-0.93	1.14	14.50	0.00
2001/02	-1.11	106.03	451.76	79.97	946471.00	101.60	88.45	4703.00	1.80	-0.12	1.68	14.50	0.00
2001/03	-1.40	106.80	401.58	78.62	947982.00	98.76	85.10	2511.00	0.77	1.55	2.31	13.50	-1.00
2001/04	-1.46	107.97	381.71	79.92	955271.00	95.04	86.83	7289.00	1.17	1.65	2.83	13.00	-0.50

Date	Term Structure	CPIX Index	Gilt-equity Ratio	Manufacturing Index	GDP at Market Prices	Employment Rate Index in Construction	Building Plan Passed Index	Changing GDP	Changing CPIX Index	Unexpected Inflation	Anticipated Inflation from ARIMA Model	Prime Interest Rate (in %)	Changing Prime Interest Rate (in %)
2002/01	-1.41	111.53	410.78	80.50	965346.00	96.26	105.63	10175.00	3.57	-1.02	2.55	14.00	1.00
2002/02	-0.34	114.67	400.85	81.15	977694.00	96.61	107.13	12249.00	3.13	-1.95	1.19	15.00	1.00
2002/03	0.45	116.97	369.25	81.10	986359.00	157.61	97.83	11105.00	2.20	-0.50	1.70	16.00	1.00
2002/04	0.80	119.30	361.07	80.69	996985.00	155.86	107.01	9226.00	2.43	-1.30	1.13	17.00	1.00
2003/01	0.95	121.20	309.44	80.71	1003734.00	160.36	109.90	6849.00	1.90	-0.66	1.24	17.00	0.00
2003/02	0.69	122.00	275.13	80.28	1006919.00	133.76	110.20	5585.00	0.80	0.94	1.74	17.00	0.00
2003/03	-0.69	122.73	267.89	80.67	1016432.00	129.70	120.62	6113.00	0.73	0.84	1.57	14.50	-2.50
2003/04	-1.15	123.37	246.26	82.14	1022567.00	129.24	125.45	7135.00	0.63	0.81	1.44	12.00	-2.50
2004/01	-0.83	125.13	259.69	84.04	1037914.00	120.63	147.45	15347.00	2.77	-0.49	2.28	11.50	-0.50
2004/02	-0.94	127.47	302.03	83.75	1054530.00	123.68	143.60	16616.00	1.33	-0.61	1.50	11.50	0.00
2004/03	-1.49	127.47	306.02	85.19	1073079.00	126.11	163.88	18549.00	0.00	1.22	1.22	11.25	-0.25
2004/04	-1.45	128.47	295.20	84.82	1082595.00	165.42	172.94	9506.00	1.00	1.65	2.65	11.00	-0.25
2005/01	-0.73	129.63	281.96	84.71	1084599.00	169.53	184.22	12108.00	1.17	1.10	2.26	11.00	0.00
2005/02	-0.81	131.77	296.00	85.28	1109279.00	195.68	204.00	14685.00	2.13	0.08	2.21	10.50	-0.50
2005/03	-0.65	133.37	296.32	85.89	1122778.00	196.60	221.09	13500.00	0.60	0.17	1.77	10.50	0.00
2005/04	-0.34	133.97	297.32	86.23	1132891.00	200.32	235.44	11113.00	0.80	0.47	2.07	10.50	0.00
2006/01	-0.25	135.40	283.97	85.95	1151045.00	205.66	226.26	17154.00	1.43	0.81	2.24	10.50	0.00
2006/02	-0.34	137.63	314.66	86.24	1168364.00	206.13	238.12	17319.00	2.23	-0.68	1.55	10.50	0.00
2006/03	0.03	140.23	361.93	85.84	1182000.00	205.64	230.57	13635.00	2.60	-1.33	1.27	11.50	1.00
2006/04	0.80	140.60	344.67	86.22	1199465.00	204.53	233.40	17455.00	0.27	1.09	1.36	12.00	0.50
2007/01	1.16	142.33	331.57	86.26	1214573.00	212.49	233.38	15119.00	1.83	-1.27	0.56	12.50	0.50
2007/02	1.29	143.37	341.40	86.47	1227577.00	213.49	235.26	13004.00	4.03	-2.98	1.05	12.50	0.00
2007/03	1.43	145.97	367.12	85.76	1242075.00	212.39	225.04	14498.00	2.60	-0.79	1.91	13.00	0.50
2007/04	2.01	151.80	358.53	85.96	1258283.00	214.56	204.56	15209.00	2.83	-0.98	1.86	14.00	1.00
2008/01				85.41	1284983.00		232.69	6700.00				14.50	0.50

Table A1.1 : Table of Input Data

Property Return in Quarter Basis

Date	Return				Return Deviation					
	J255	J256	IPD Retail	IPD Office	IPD Industrial	J255	J256	IPD Retail	IPD Office	IPD Industrial
1989/01	0.10177					0.03838				
1989/02	0.22264					0.04474				
1989/03	0.38856					0.09790				
1989/04	0.41495					0.06795				
1990/01	0.39540					0.02631				
1990/02	0.26730					0.06713				
1990/03	0.02315					0.04500				
1990/04	0.04064					0.03025				
1991/01	0.01113					0.06126				
1991/02	0.14200					0.00705				
1991/03	0.22062	0.21077				0.01846	0.00623			
1991/04	0.22486	0.22225				0.04466	0.03821			
1992/01	0.17174	0.14846				0.04744	0.01720			
1992/02	0.06519	0.02904				0.00628	0.02472			
1992/03	0.05414	-0.02948				0.01762	0.01199			
1992/04	0.06755	-0.06286				0.01484	0.02134			
1993/01	0.04044	-0.04263				0.01005	0.03245			
1993/02	0.01057	-0.06307				0.03696	0.03080			
1993/03	0.07506	0.05648				0.02896	0.03170			
1993/04	0.06573	0.11035				0.07196	0.05896			
1994/01	0.21210	0.2827				0.00664	0.03464			
1994/02	0.14414	0.34541				0.03971	0.03036			
1994/03	0.13451	0.34172				0.01360	0.05121			
1994/04	0.01772	0.28050				0.08951	0.06906			
1995/01	-0.15719	0.11964	0.17224	0.14271	0.13580	0.01426	0.02372	0.01840	0.03899	0.04600
1995/02	-0.06511	0.04635	0.17201	0.13182	0.14627	0.06597	0.04258	0.01888	0.03964	0.03587
1995/03	-0.08585	-0.04314	0.17178	0.12092	0.15675	0.06844	0.04299	0.01937	0.04030	0.02574
1995/04	0.12894	0.06113	0.17154	0.11003	0.16722	0.03564	0.02363	0.01985	0.04095	0.01561
1996/01	0.12326	0.07934	0.17131	0.09914	0.17770	0.06168	0.01620	0.02033	0.04161	0.00548
1996/02	0.05957	0.00972	0.18512	0.10629	0.17497	0.05404	0.04720	0.02314	0.04079	0.01017
1996/03	0.05709	0.02795	0.19893	0.11343	0.17224	0.05808	0.00853	0.02596	0.03998	0.01486
1996/04	-0.08261	-0.06368	0.21273	0.12058	0.16952	0.03422	0.04888	0.02877	0.03916	0.01956
1997/01	0.02023	-0.00995	0.22654	0.12773	0.16679	0.10640	0.04396	0.03158	0.03835	0.02425
1997/02	0.10055	0.06151	0.19242	0.09927	0.13028	0.03598	0.03074	0.03118	0.04146	0.02871
1997/03	0.13863	0.07742	0.15831	0.07081	0.09376	0.03493	0.01507	0.03079	0.04457	0.03317
1997/04	0.14991	0.05492	0.12420	0.04234	0.05725	0.04084	0.04517	0.03039	0.04768	0.03764
1998/01	0.17482	0.14752	0.09008	0.01388	0.2074	0.02976	0.01770	0.03000	0.05080	0.04210
1998/02	0.11649	0.07192	0.11210	0.03392	0.03591	0.06203	0.06927	0.03130	0.05341	0.03390
1998/03	-0.14060	-0.15799	0.13412	0.05397	0.05109	0.05776	0.07055	0.03261	0.05602	0.02571
1998/04	0.00660	-0.09072	0.15614	0.07401	0.06626	0.05825	0.03040	0.03391	0.05863	0.01752
1999/01	0.06153	-0.07168	0.17815	0.09405	0.08144	0.04069	0.09729	0.03522	0.06124	0.00932
1999/02	0.29902	0.18499	0.15973	0.10216	0.07863	0.06348	0.08020	0.03113	0.05507	0.01150
1999/03	0.58245	0.53852	0.14131	0.11027	0.07581	0.02852	0.07409	0.02704	0.04891	0.01368
1999/04	0.50410	0.55825	0.12288	0.11838	0.07300	0.03612	0.02104	0.02294	0.04274	0.01586

Date	Return			Return Deviation								
	J255	J256	IPD Retail	IPD Office	IPD Industrial	IPD Retail	J256	J255	J256	IPD Retail	IPD Office	IPD Industrial
2000/01	0.42727	0.52024	0.10446	0.12650	0.07019	0.07054	0.13382	0.07054	0.13382	0.01885	0.03657	0.01804
2000/02	0.22238	0.32317	0.11125	0.11437	0.07196	0.02173	0.01350	0.02173	0.01350	0.01958	0.03212	0.02606
2000/03	0.25273	0.24072	0.11804	0.10225	0.07252	0.07187	0.02800	0.07187	0.02800	0.02031	0.02766	0.03407
2000/04	0.28761	0.26878	0.12483	0.09013	0.07369	0.03465	0.00227	0.03465	0.00227	0.02104	0.02321	0.04209
2001/01	0.28252	0.23039	0.13162	0.07801	0.07485	0.02241	0.02436	0.02241	0.02436	0.02176	0.01876	0.05011
2001/02	0.33431	0.32319	0.12652	0.07125	0.07812	0.03111	0.04355	0.03111	0.04355	0.02184	0.02322	0.04031
2001/03	0.30711	0.37391	0.12142	0.06448	0.08138	0.05595	0.03326	0.05595	0.03326	0.02192	0.02767	0.03051
2001/04	0.15276	0.26884	0.11632	0.05772	0.08464	0.05784	0.06255	0.05784	0.06255	0.02200	0.03212	0.02072
2002/01	0.01567	0.11172	0.11123	0.05096	0.08790	0.03096	0.01830	0.03096	0.01830	0.02208	0.03657	0.01092
2002/02	0.05437	0.09940	0.09641	0.06041	0.11030	0.00810	0.03427	0.00810	0.03427	0.02248	0.03384	0.01025
2002/03	0.01731	0.01888	0.14275	0.05985	0.13269	0.00738	0.01142	0.00738	0.01142	0.02288	0.03111	0.00958
2002/04	0.14079	0.11054	0.15851	0.07931	0.15508	0.06828	0.07998	0.06828	0.07998	0.02329	0.02837	0.00892
2003/01	0.32748	0.30393	0.17427	0.08876	0.17747	0.08132	0.05811	0.08132	0.05811	0.02369	0.02564	0.00825
2003/02	0.33875	0.29936	0.19630	0.10796	0.19414	0.02593	0.04160	0.02593	0.04160	0.02280	0.02063	0.01152
2003/03	0.32463	0.29827	0.21833	0.12716	0.21081	0.01585	0.01875	0.01585	0.01875	0.02191	0.01561	0.01480
2003/04	0.36123	0.35834	0.24037	0.14536	0.22748	0.04655	0.03427	0.04655	0.03427	0.02102	0.01060	0.01807
2004/01	0.24679	0.25349	0.26240	0.16556	0.24414	0.05153	0.02797	0.05153	0.02797	0.02013	0.00558	0.02134
2004/02	0.21463	0.24187	0.27860	0.20874	0.26419	0.04575	0.03871	0.04575	0.03871	0.02309	0.00796	0.02112
2004/03	0.24964	0.28588	0.29479	0.20874	0.28425	0.05101	0.04657	0.05101	0.04657	0.02604	0.01033	0.02089
2004/04	0.30890	0.30053	0.31099	0.23033	0.30430	0.09475	0.06809	0.09475	0.06809	0.02899	0.01270	0.02066
2005/01	0.41229	0.40184	0.32719	0.25192	0.32435	0.02033	0.02399	0.02033	0.02399	0.03194	0.01508	0.02043
2005/02	0.44164	0.46153	0.31460	0.25147	0.32189	0.05319	0.03504	0.05319	0.03504	0.03145	0.01516	0.02081
2005/03	0.46547	0.53191	0.30201	0.25102	0.31942	0.00787	0.01467	0.00787	0.01467	0.03096	0.01524	0.02120
2005/04	0.37099	0.46135	0.28943	0.25057	0.31695	0.07713	0.07002	0.07713	0.07002	0.03047	0.01533	0.02158
2006/01	0.44760	0.54693	0.27684	0.25012	0.31448	0.05418	0.04011	0.05418	0.04011	0.02998	0.01541	0.02196
2006/02	0.33076	0.43802	0.27252	0.26462	0.31993	0.15507	0.12514	0.15507	0.12514	0.03611	0.02301	0.02492
2006/03	0.15460	0.25393	0.26820	0.27912	0.32538	0.03843	0.02567	0.03843	0.02567	0.04224	0.03051	0.02789
2006/04	0.22369	0.33566	0.26388	0.23962	0.33083	0.04569	0.04558	0.04569	0.04558	0.04838	0.03821	0.03086
2007/01	0.17745	0.28993	0.25956	0.30812	0.33629	0.03315	0.03487	0.03315	0.03487	0.05451	0.04581	0.03382
2007/02	0.28742	0.38016	0.25524	0.32262	0.34174	0.07563	0.06150	0.07563	0.06150	0.06064	0.05341	0.03679
2007/03	0.32785	0.43697	0.25092	0.33712	0.34719	0.05540	0.05137	0.05540	0.05137	0.06678	0.06101	0.03975
2007/04	0.25919	0.32557	0.24660	0.35162	0.35264	0.05201	0.06826	0.05201	0.06826	0.07291	0.06860	0.04272
2008/01	-0.00413	0.05716				0.05307	0.04929	0.05307	0.04929			

Table A1.2 : Table of Output Data

Appendix B

Graphs of the input variables and property returns

1.1 Introduction

This document presents the graphs of the input variables and output variables (property returns and their deviation) for the forecasting models in this research. The autocorrelation graphs of the output variables are also included in this document.

1.2 Graphs of the input variables

The input variables investigated in this research are the term structure of interest rate, gilt-equity ratio, manufacturing index, employment index, building plans passed index, nominal GDP index, changing nominal GDP index, CPIX index, changing CPIX index, prime interest rate and changing prime interest rate. The graphs below present the trend of input variables between 1988 and 2007.

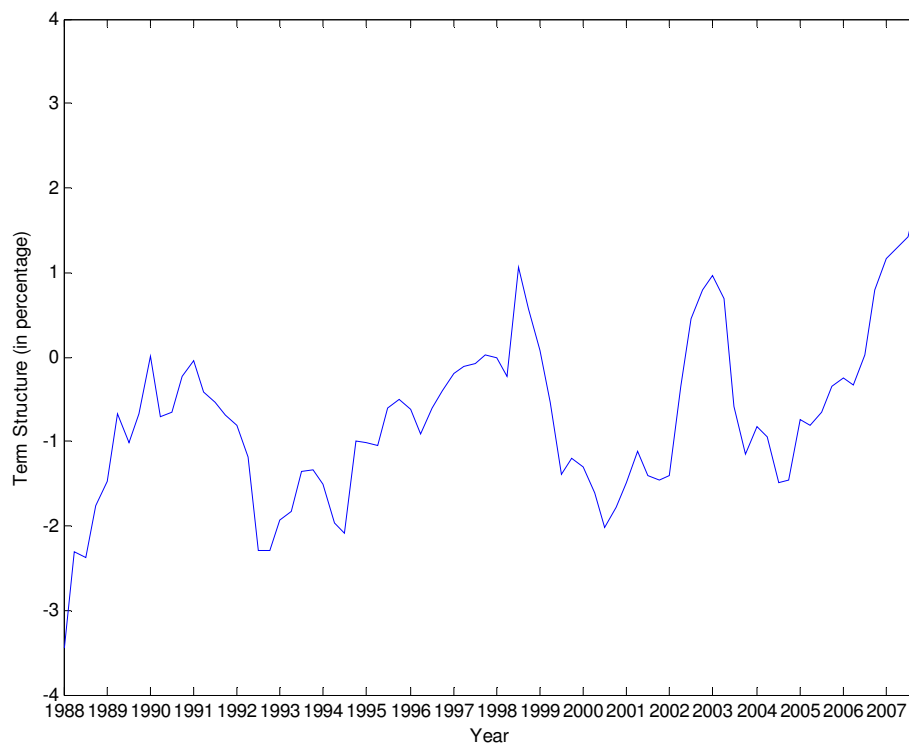


Figure B1.1: Term Structure (in percentage point) between 1988 and 2007

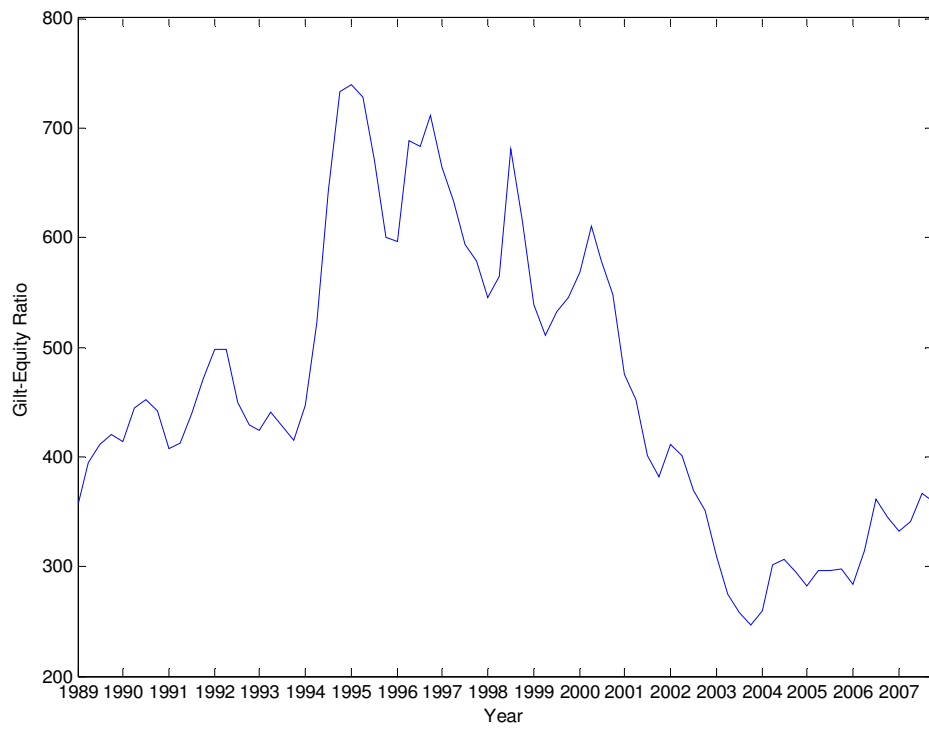


Figure B1.2: Gilt-Equity Ratio between 1989 and 2007

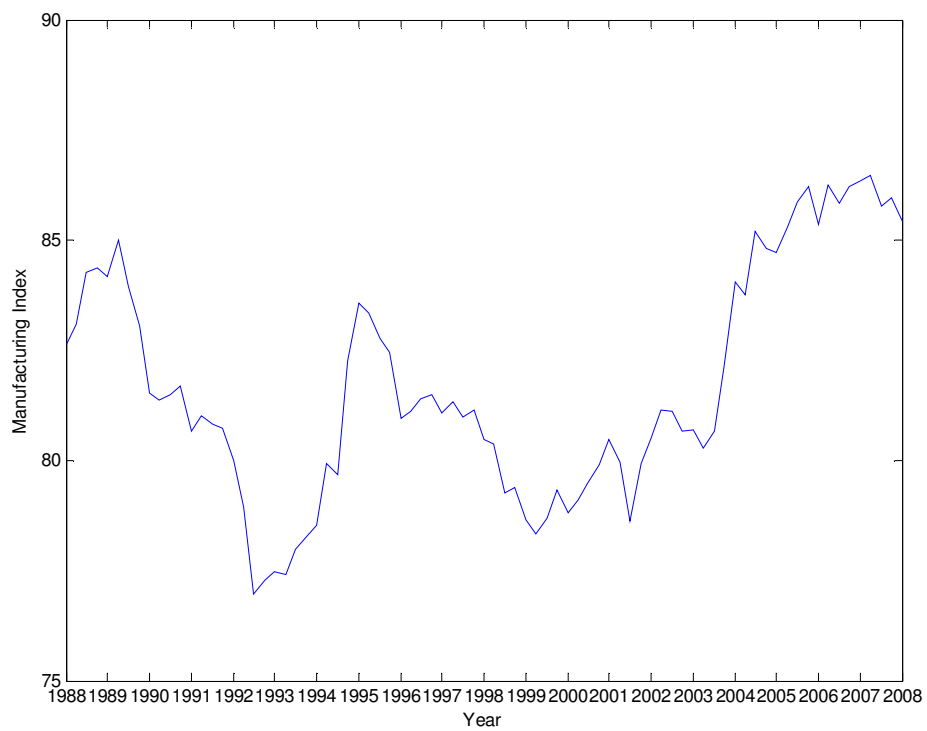


Figure B1.3: Manufacturing index between 1988 and 2008

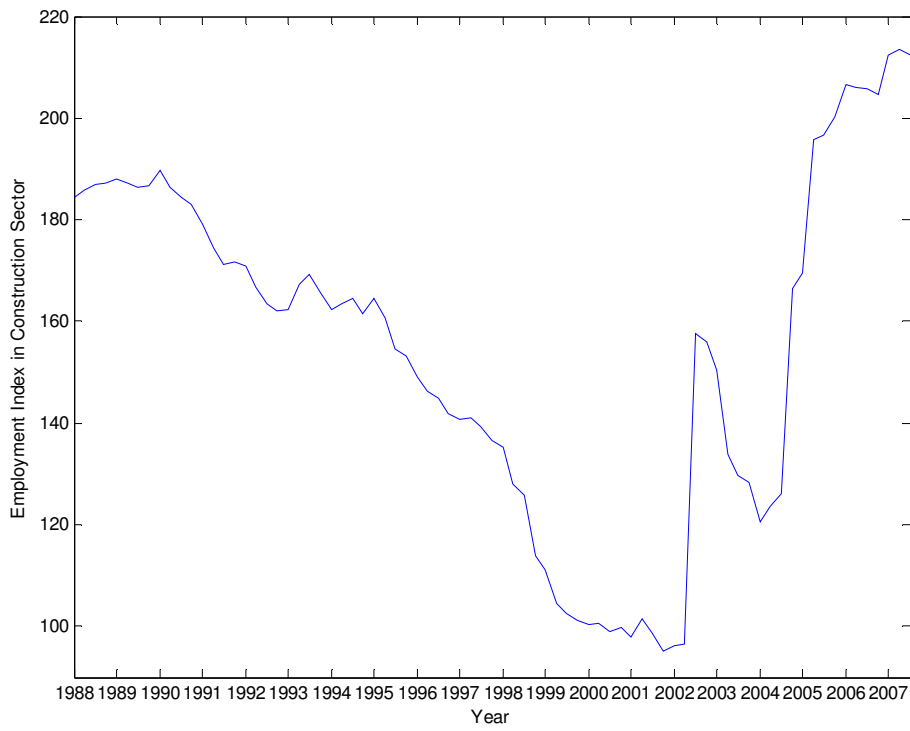


Figure B1.4: Employment index in the construction sector between 1988 and 2007

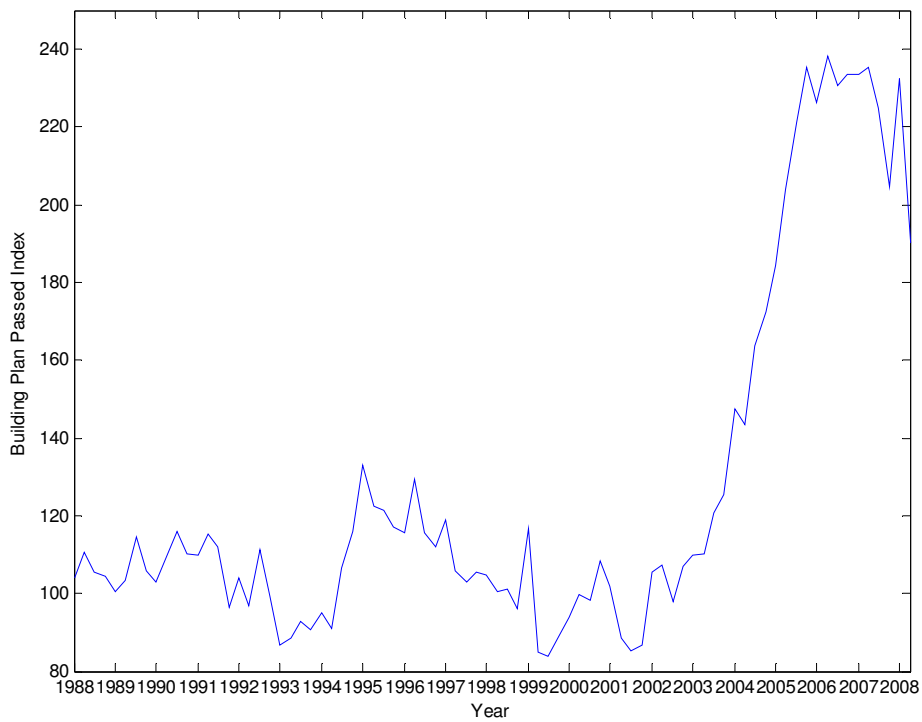


Figure B1.5: Building plans passed index between 1988 and 2008

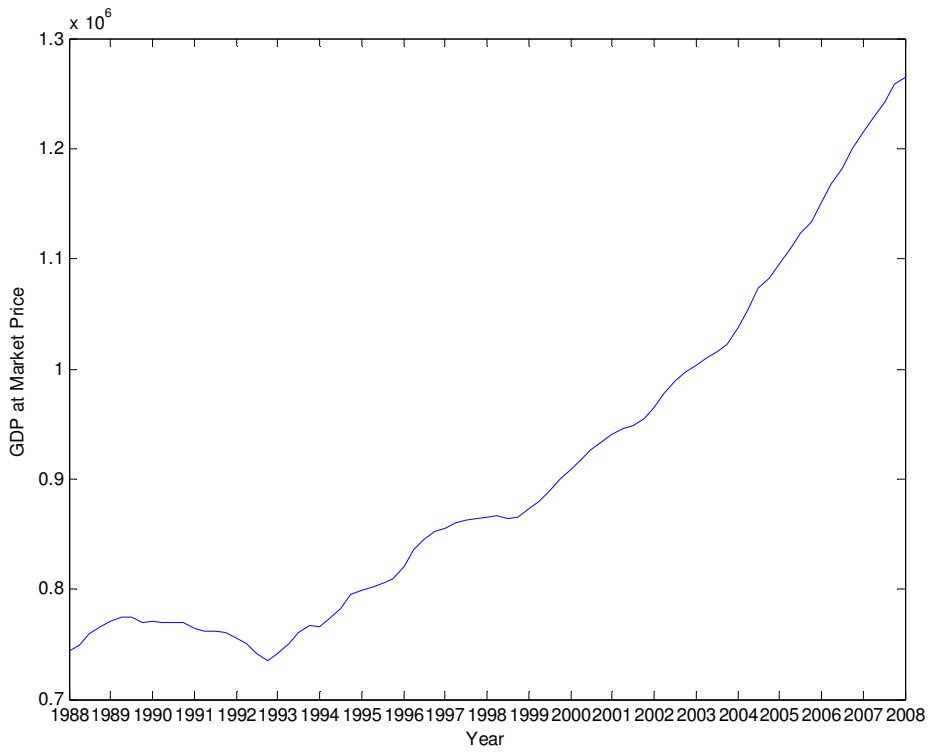


Figure B1.6: Nominal GDP Index between 1988 and 2008

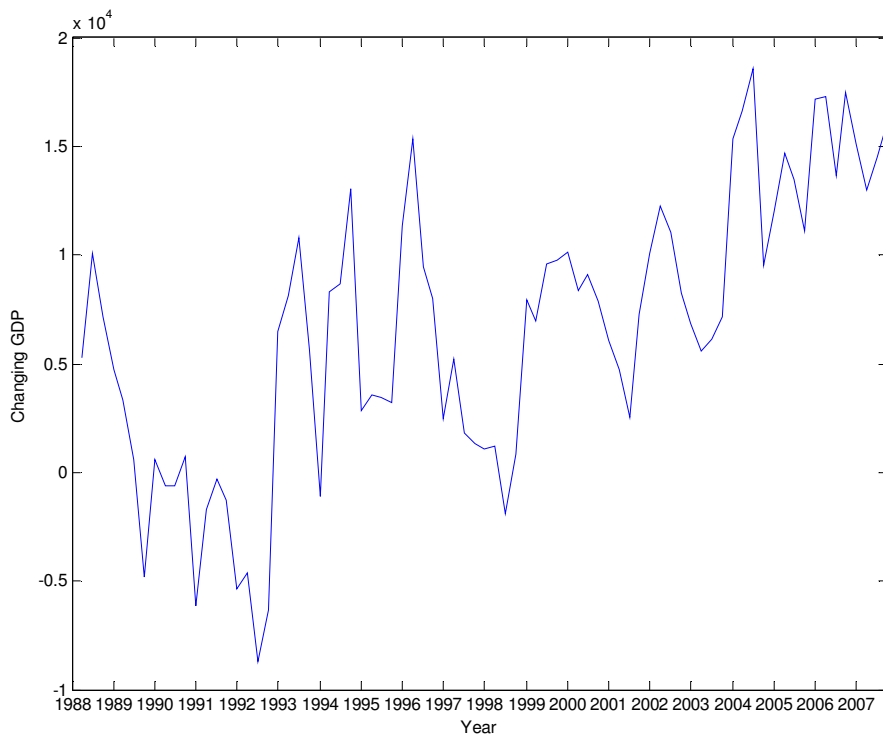


Figure B1.7: Changing nominal GDP Index between 1990 and 2007

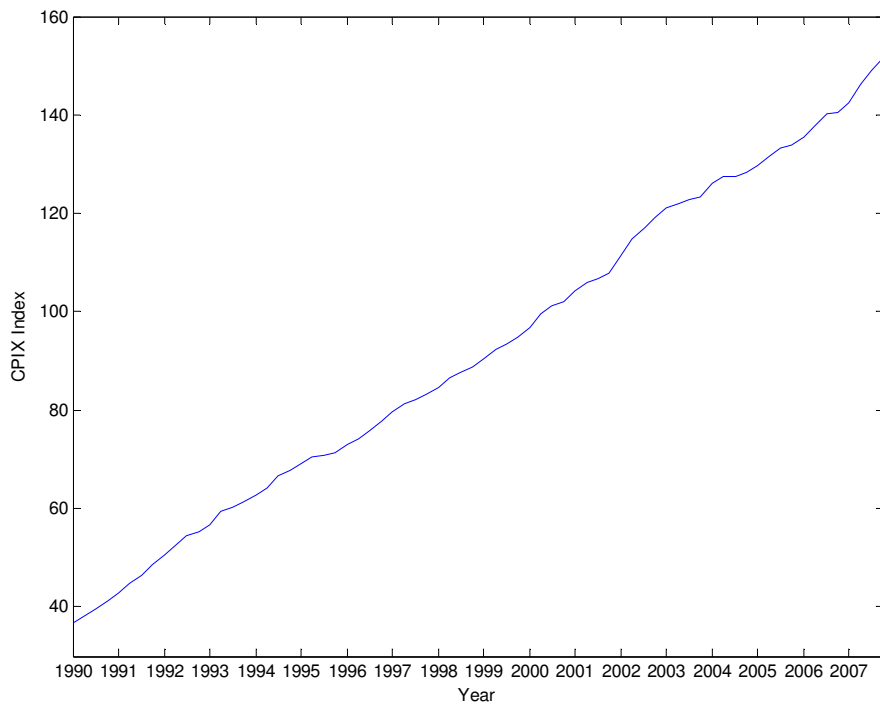


Figure B1.8: CPIX Index between 1990 and 2007

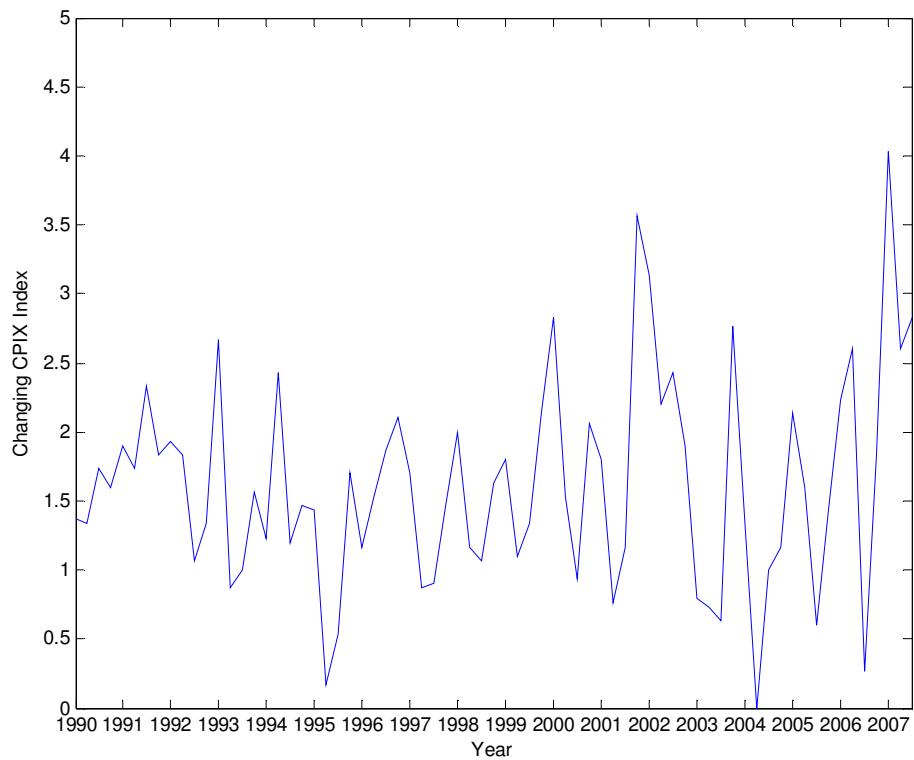


Figure B1.9: Changing CPIX Index between 1990 and 2007

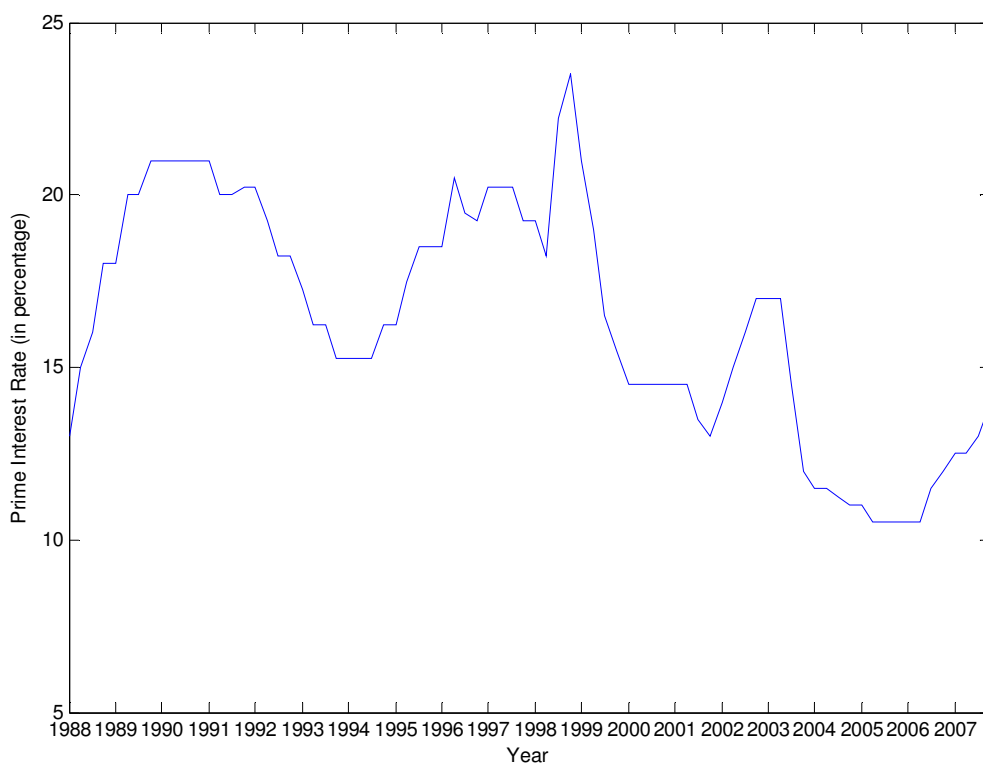


Figure B1.10: Prime Interest Rate (in %) between 1988 and 2008

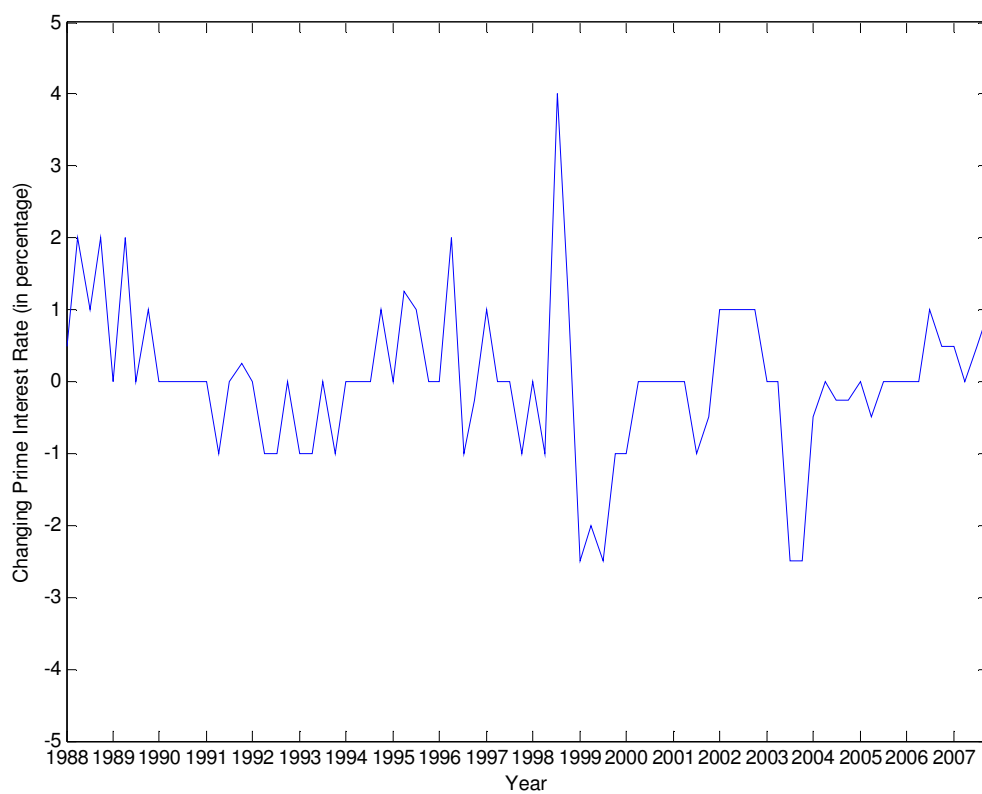


Figure B1.11: Changing Prime Interest Rate (in %) between 1988 and 2008

1.3 Graphs of the output variables

The output variables investigated in this research are the indirect J255 total property return, the indirect J256 total property return, the direct IPD retail property return, the direct IPD office property return and the direct IPD industrial property return and their return deviations.

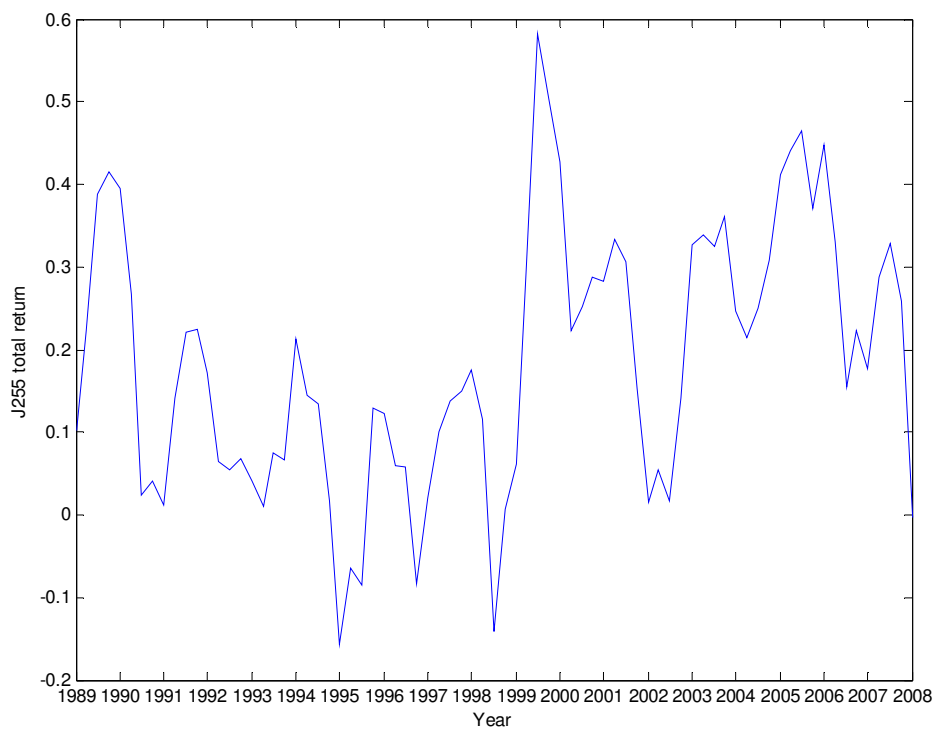


Figure B2.1: J255 total return between 1989 and 2008

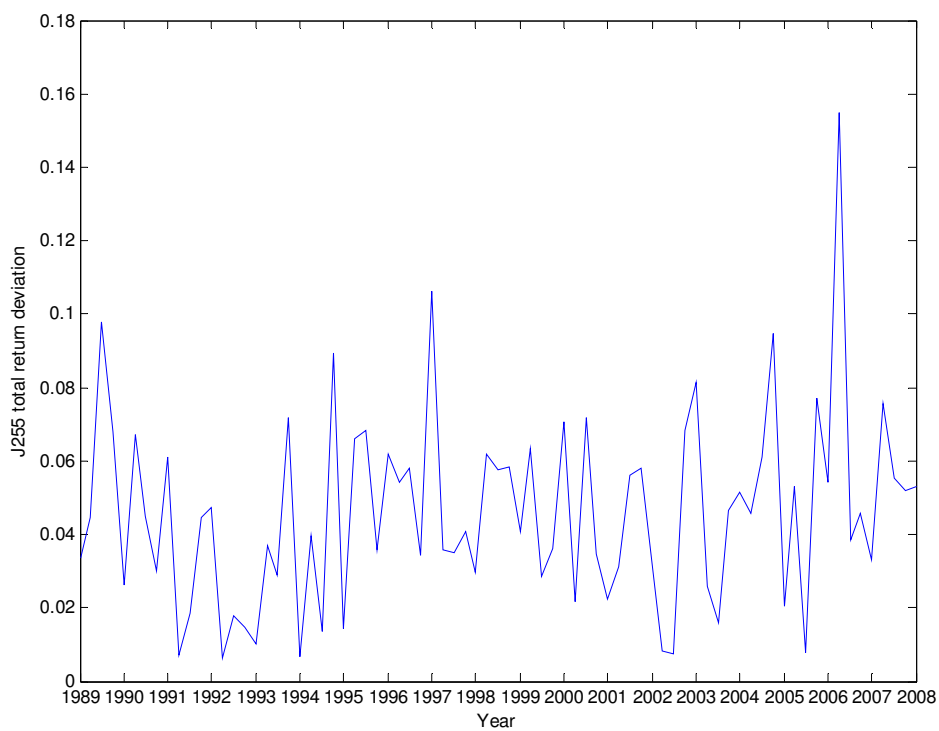


Figure B2.2: J255 total return deviation between 1989 and 2008

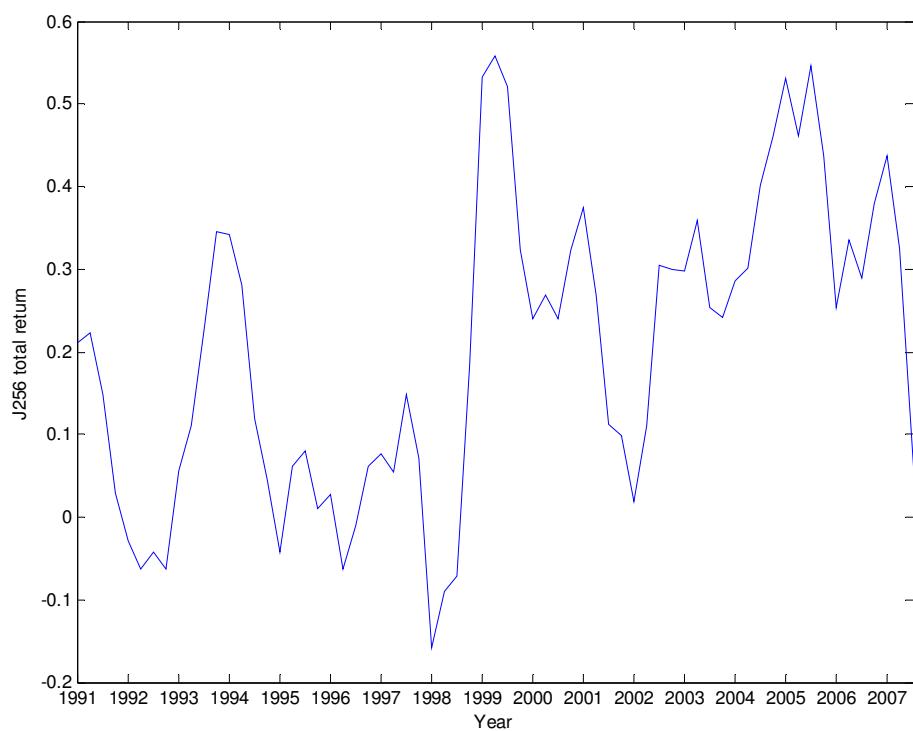


Figure B2.3: J256 total return between 1991 and 2007

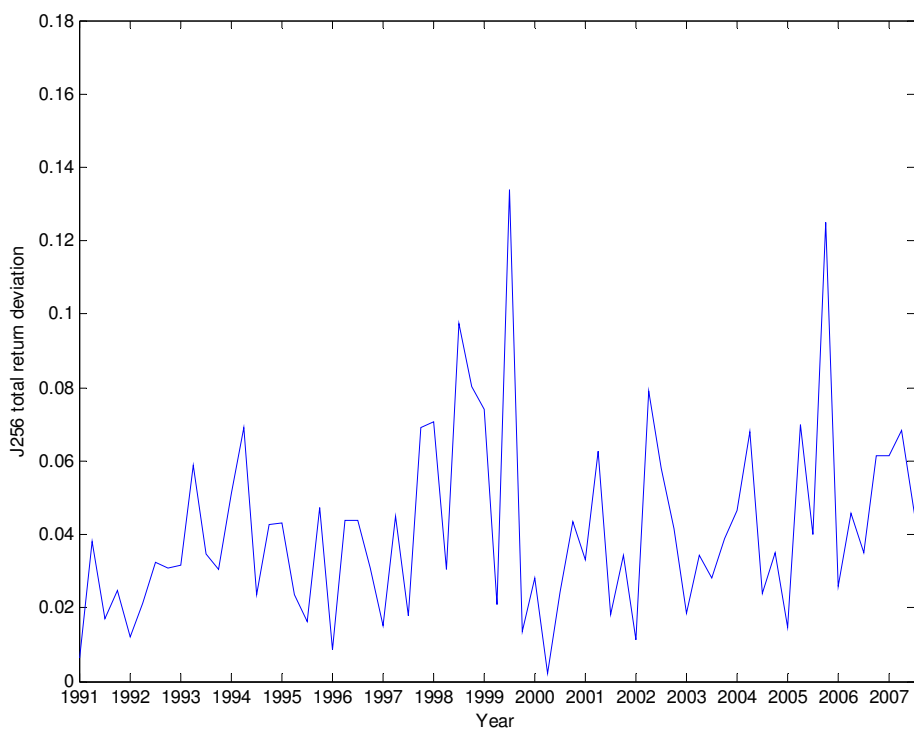


Figure B2.4: J256 total return deviation between 1991 and 2007

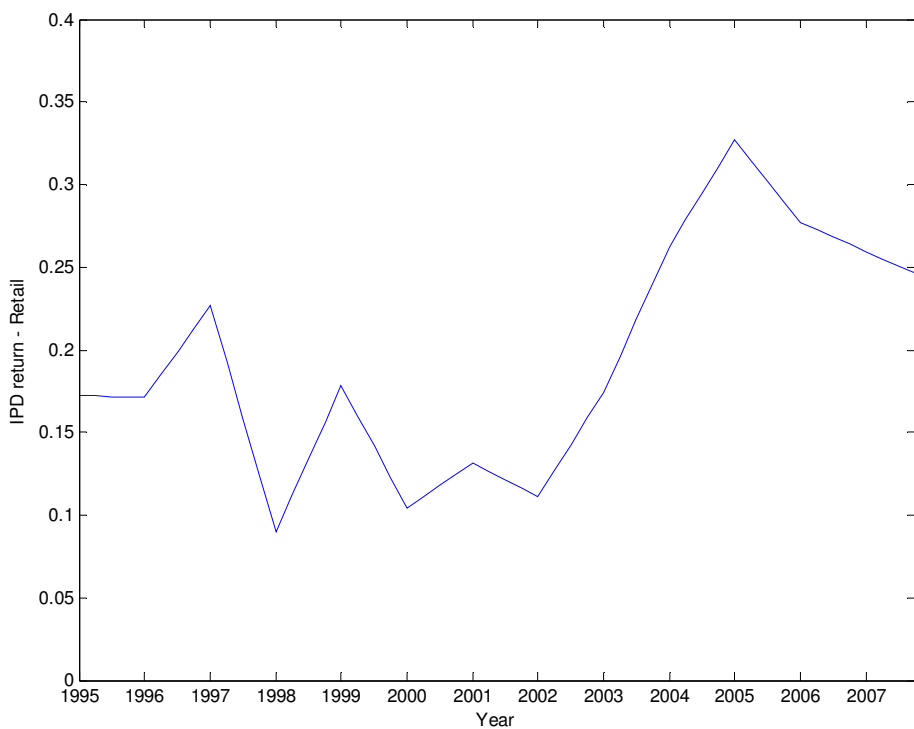


Figure B2.5: IPD retail property return between 1995 and 2007

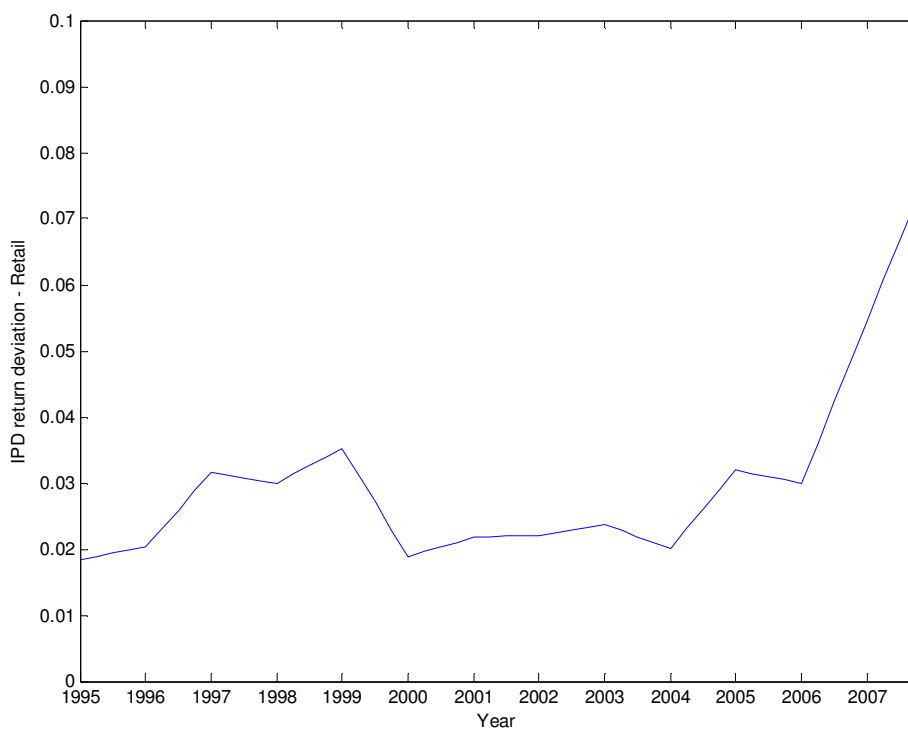


Figure B2.6: IPD retail property return deviation between 1995 and 2007

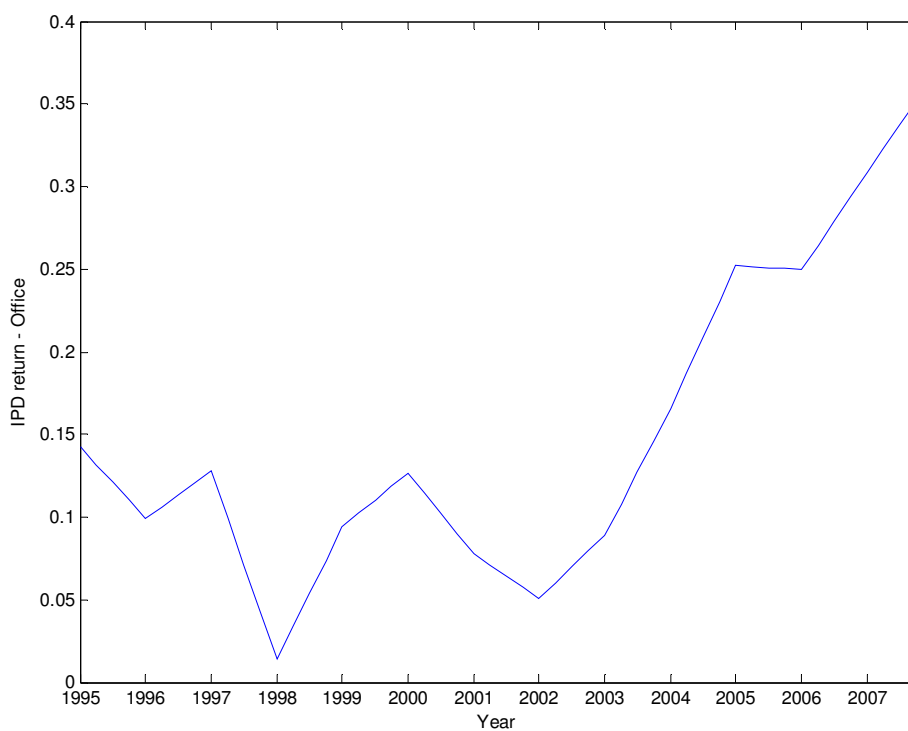


Figure B2.7: IPD office property return between 1995 and 2007

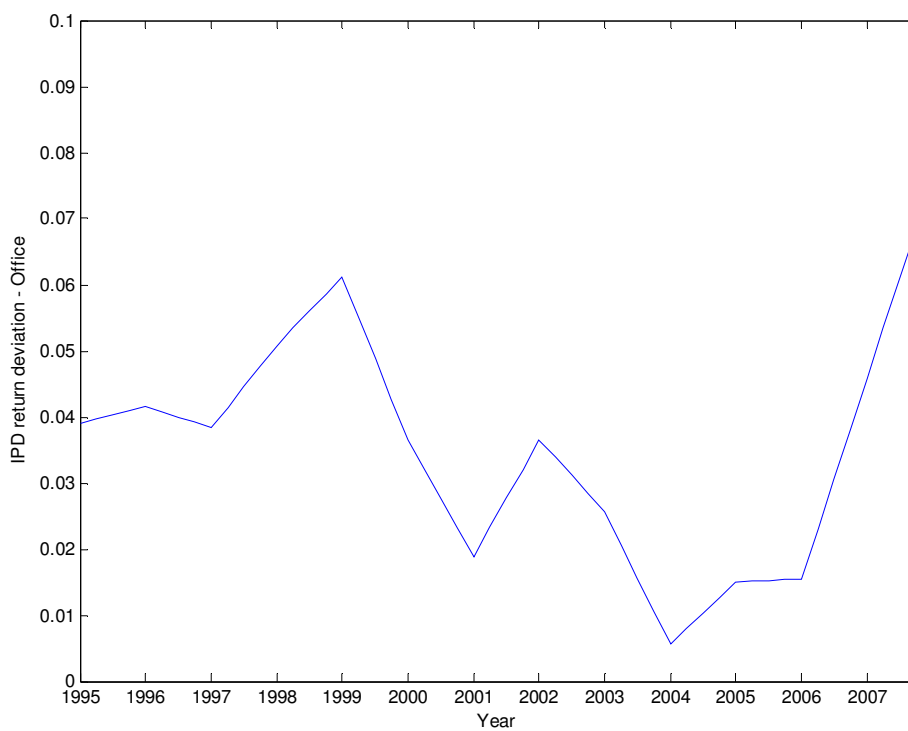


Figure B2.8: IPD office property return deviation between 1995 and 2007



Figure B2.9: IPD industrial property return between 1995 and 2007

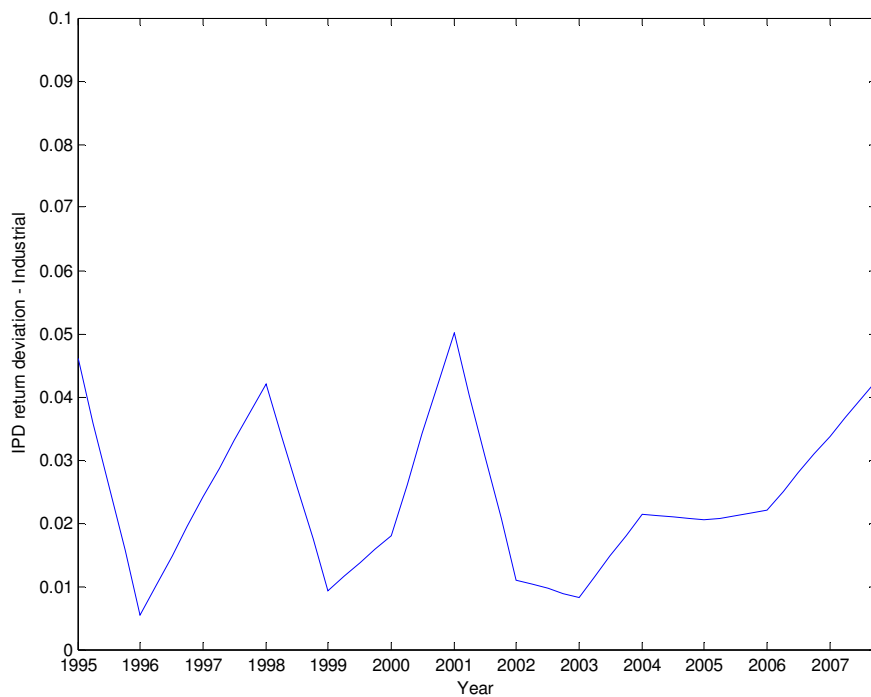


Figure B2.10: IPD industrial property return deviation between 1995 and 2007

1.4 Correlogram of the outputs

Hereunder are the correlograms of each output. A lag of 15 (k-value) is investigated for the indirect return data and a lag of 10 (k-value) is investigated for the direct return data. The discrepancy is due to the different data sizes between the two types of return, where there are more samples for indirect returns than direct returns. The bar graphs indicate the sample autocorrelation function values, which indicate the degree of correlation between the current sample and the sample in the set k^{th} period previous. The red dotted lines indicate the critical sample error level where there is no correlation between the two values if the calculated sample autocorrelation function value is below this level.

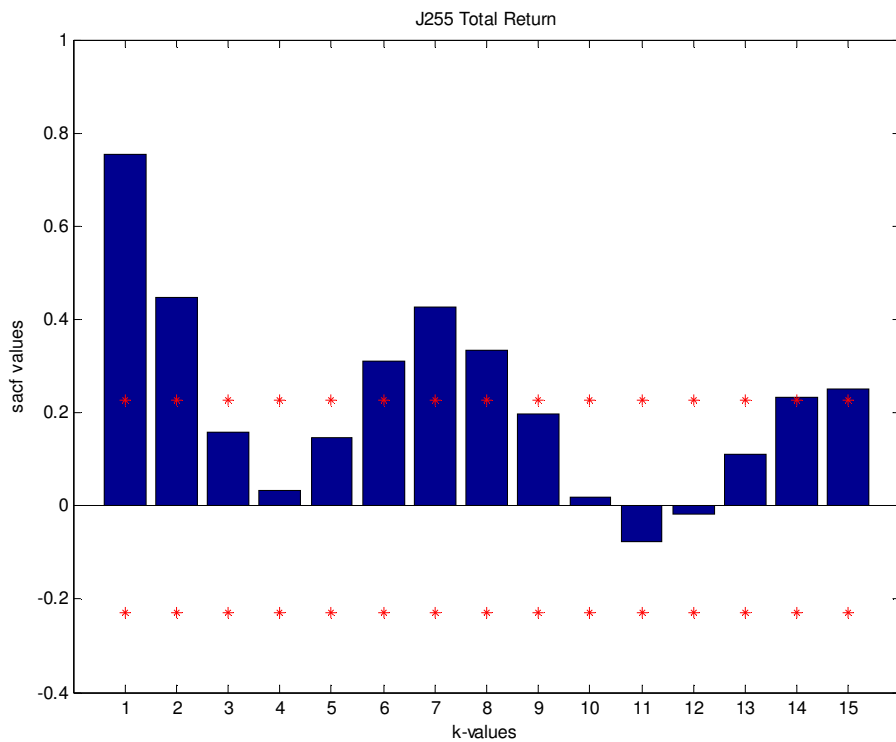


Figure B3.1: Correlogram of J255 total return

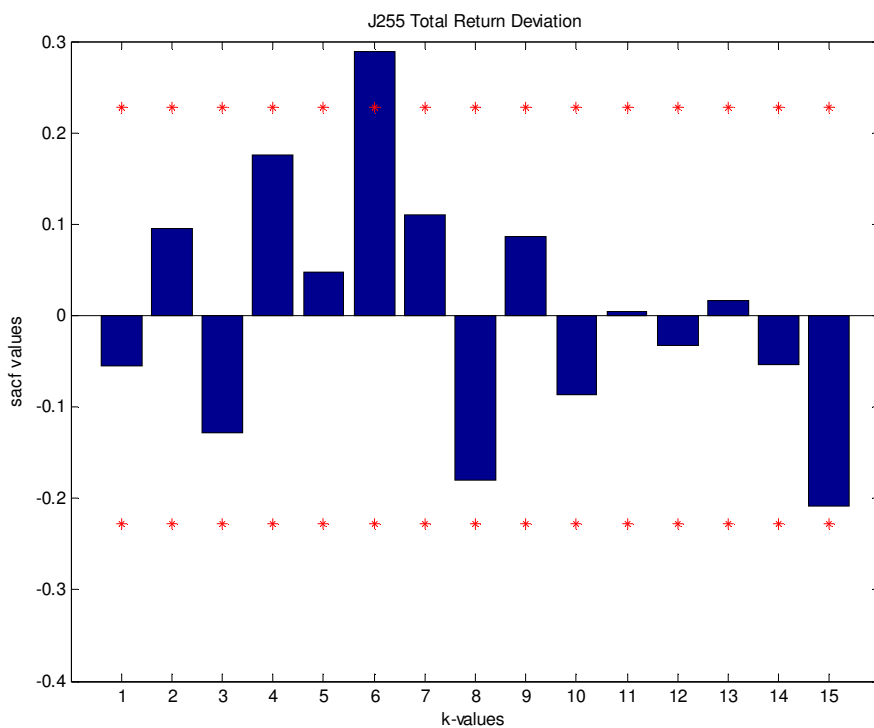


Figure B3.2: Correlogram of J255 total return deviation

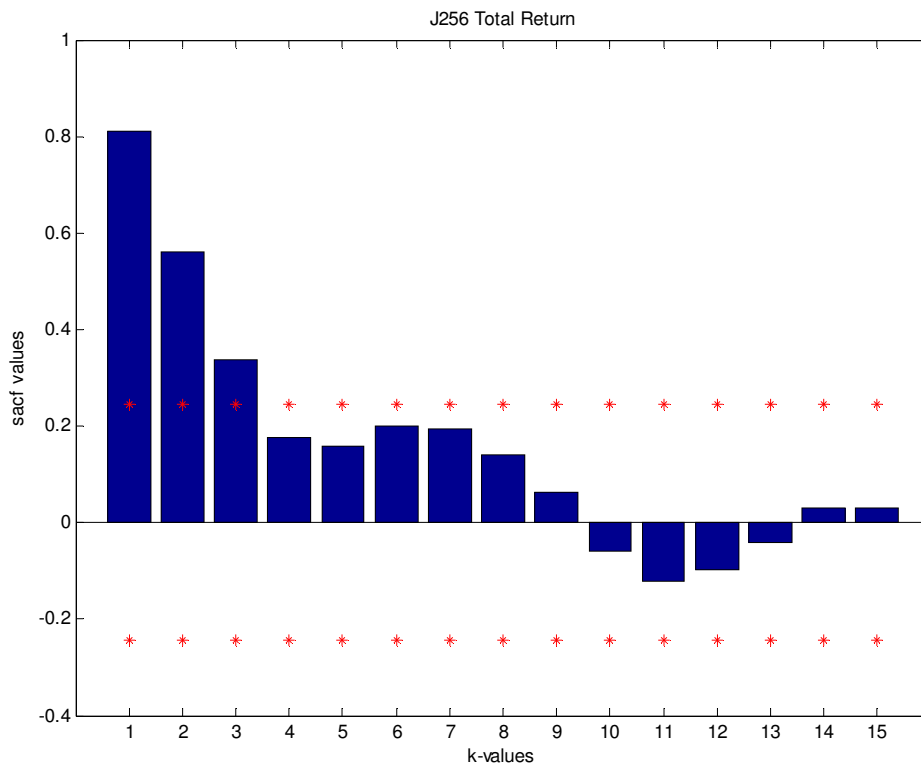


Figure B3.3: Correlogram of J256 total return

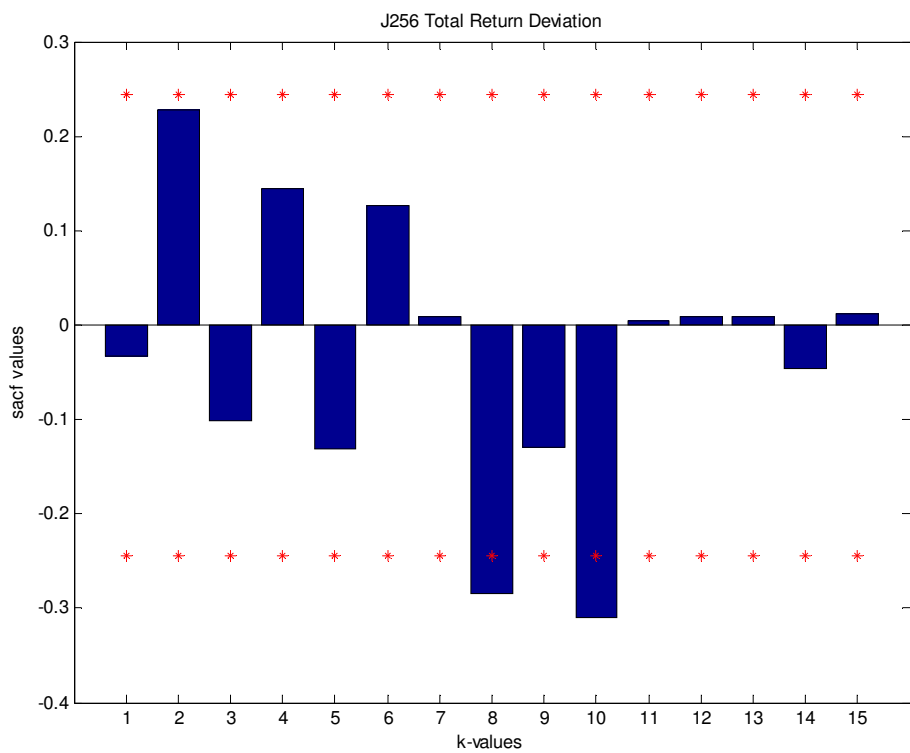


Figure B3.4: Correlogram of J256 total return deviation

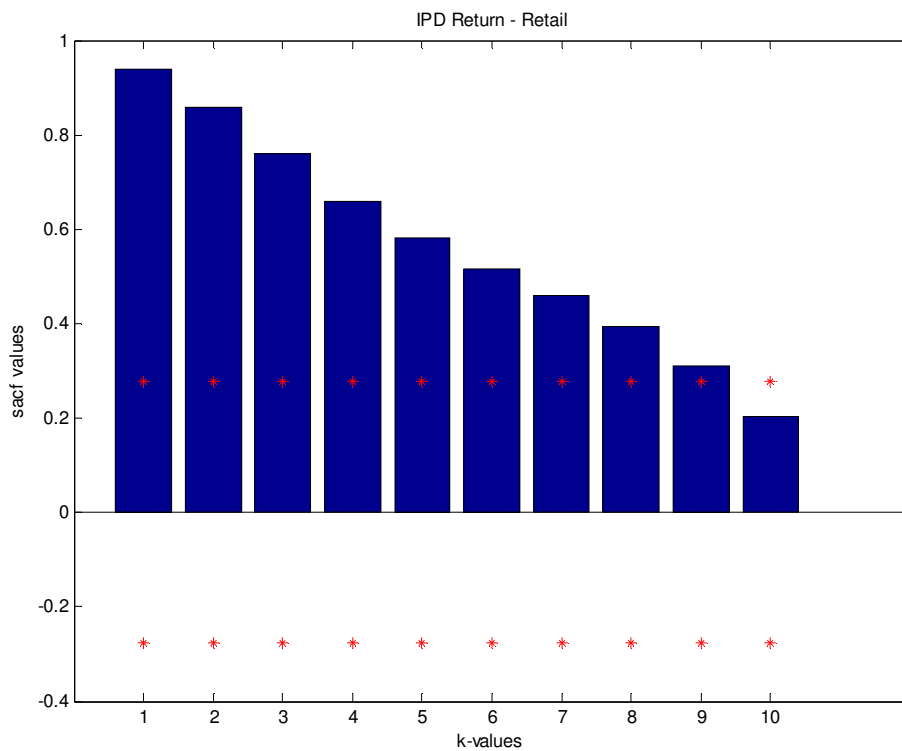


Figure B3.5: Correlogram of IPD retail property return

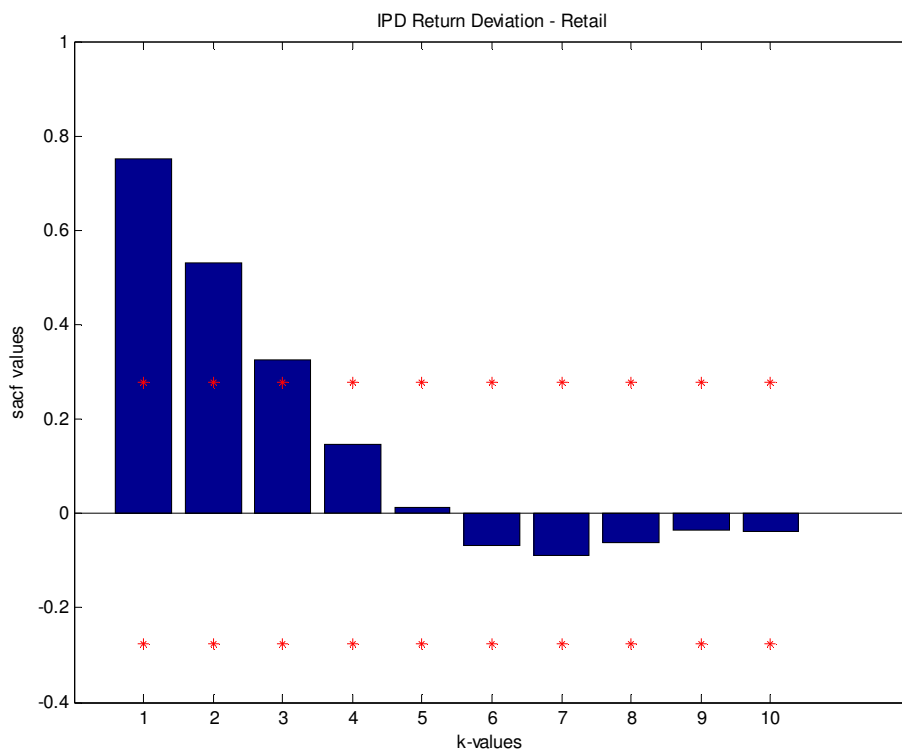


Figure B3.6: Correlogram of IPD retail property return deviation

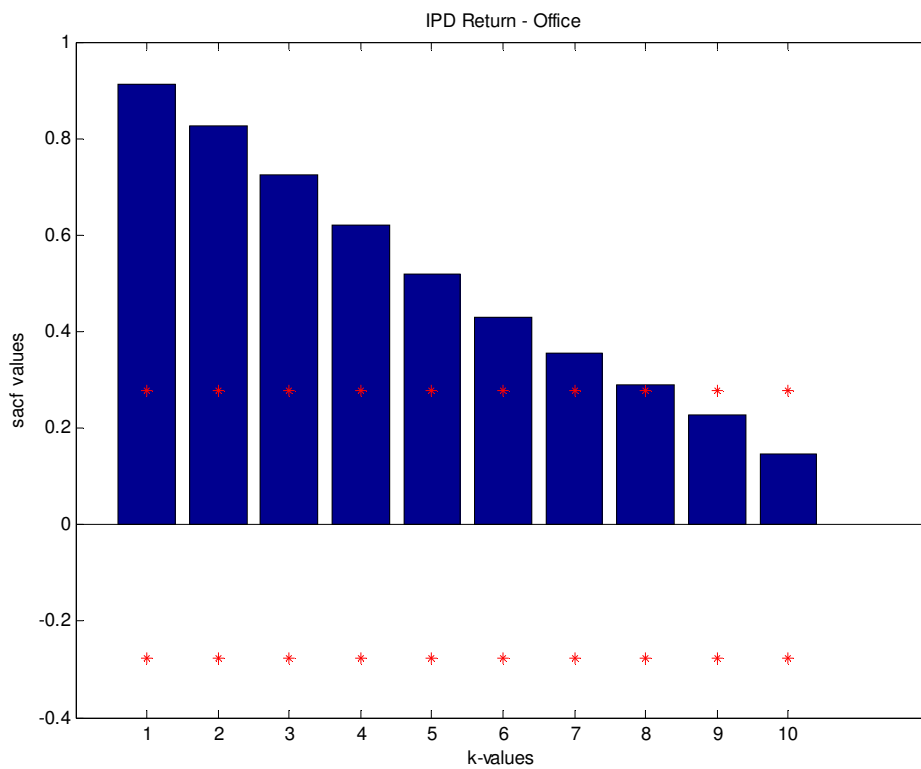


Figure B3.7: Correlogram of IPD office property return

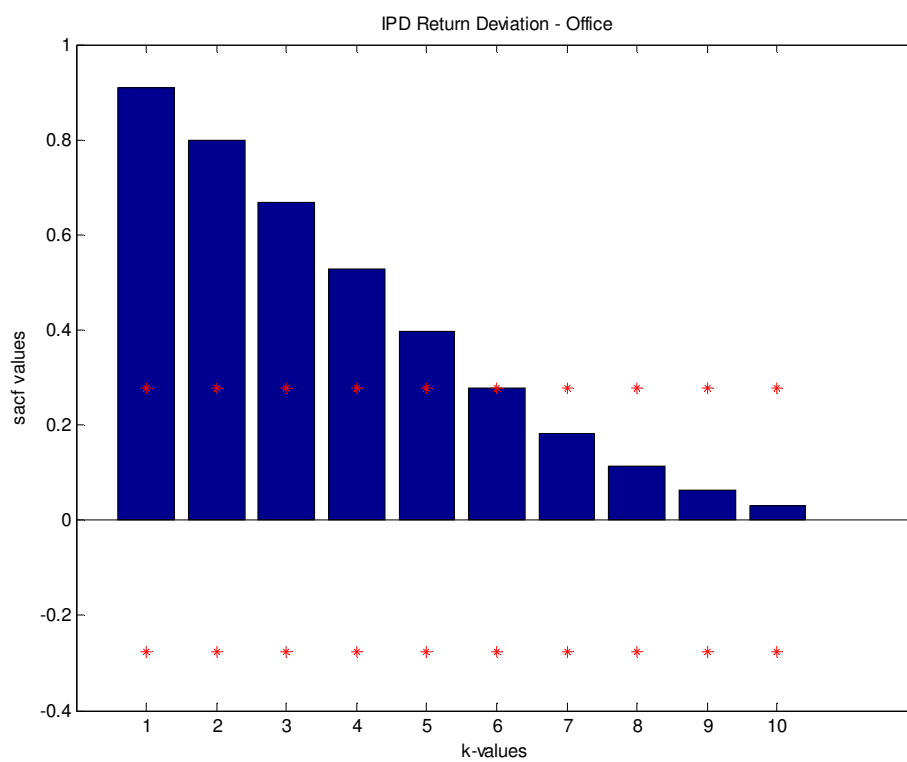


Figure B3.8: Correlogram of IPD office property return deviation

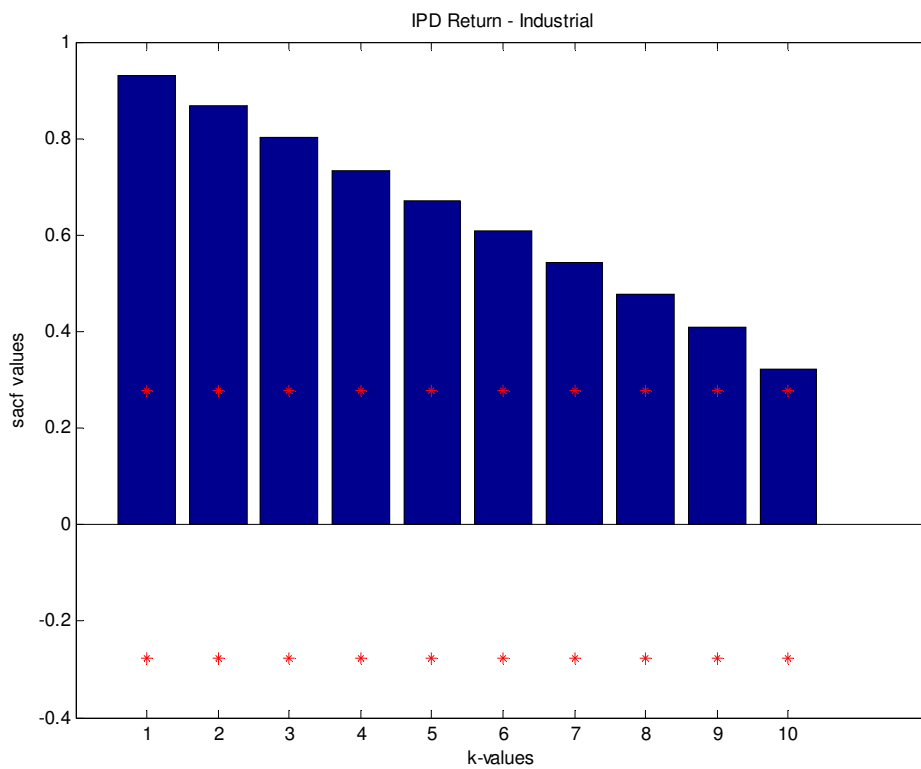


Figure B3.9: Correlogram of IPD industrial property return

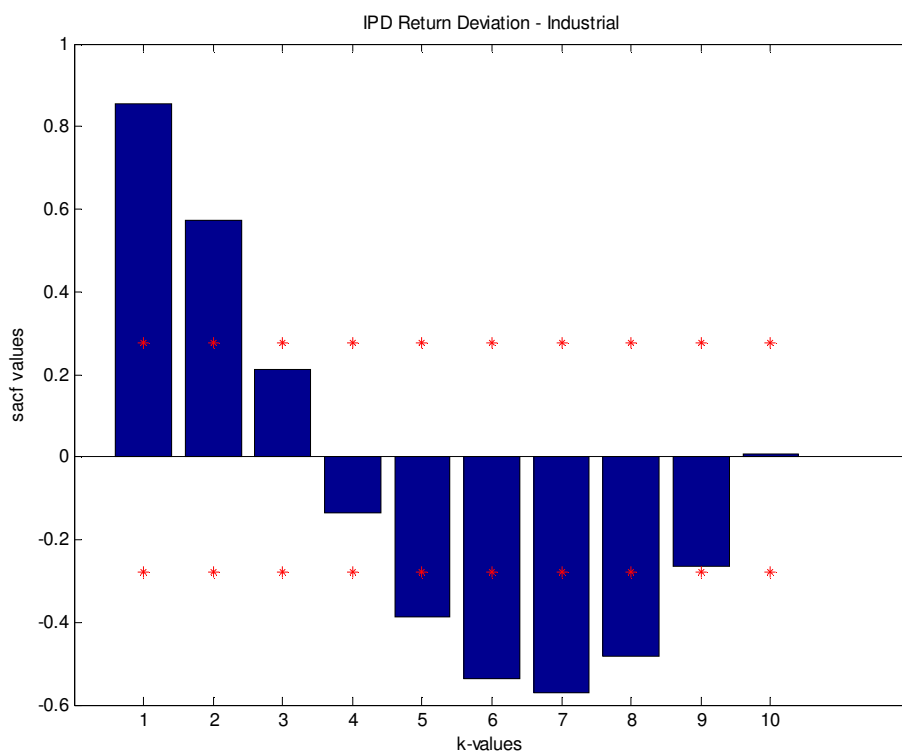


Figure B3.10: Correlogram of IPD industrial property return deviation

Appendix C

Tables of result from Granger causality test

Granger Causality Analysis
Granger Causality Probabilities

Note: Cut off probability point set at 0.5, "NaN" represents a cut off probability of higher than 0.5

1. Indirect Return

Variable	Term Structure	GM Equity Ratio	Manufacturing Index	Employment Index	Building Plan Index	Changing GDP	Changing CPIX	Unexpected Inflation	Prime Rate	Changing Prime Rate	J255 Return	J256 Return
Term Structure	0.00	0.39	NaN	0.26	NaN	NaN	0.35	0.28	0.04	0.08	0.02	0.05
GM Equity Ratio	NaN	0.00	NaN	NaN	0.22	0.34	0.04	0.11	0.16	NaN	0.27	0.10
Manufacturing Index	NaN	0.49	0.17	NaN	NaN	0.20	NaN	NaN	0.15	0.34	NaN	NaN
Employment Index	0.01	0.03	0.03	0.02	0.11	0.23	0.11	0.04	0.19	0.20	0.10	NaN
Building Plan Index	0.41	0.26	0.30	0.43	0.15	NaN	0.41	NaN	0.08	0.30	NaN	NaN
Changing GDP	NaN	0.14	0.29	NaN	0.31	0.09	0.27	0.49	NaN	NaN	0.09	0.17
Changing CPIX	0.11	NaN	0.03	0.10	NaN	NaN	0.37	0.43	NaN	NaN	0.23	0.05
Unexpected Inflation	0.07	NaN	0.15	0.31	NaN	0.49	NaN	NaN	NaN	0.48	NaN	0.26
Prime Rate	0.04	0.03	0.09	NaN	0.12	0.45	0.23	NaN	0.34	NaN	0.16	0.02
Changing Prime Rate	0.32	0.31	0.44	NaN	NaN	NaN	NaN	NaN	NaN	0.18	0.10	0.04
J255 Return	0.13	NaN	0.25	0.04	0.02	0.47	0.21	0.43	0.44	0.44	0.12	0.19
J256 Return	0.42	NaN	0.09	0.07	0.01	0.48	0.20	NaN	0.39	0.34	0.10	0.04

Table CL.1: Granger Causality Probability for Indirect Return

2. Indirect Return Deviation

Variable	Term Structure	GM Equity Ratio	Manufacturing Index	Employment Index	Building Plan Index	Changing GDP	Changing CPIX	Unexpected Inflation	Prime Rate	Changing Prime Rate	J255 Return Deviation	J256 Return Deviation
Term Structure	0.00	0.40	NaN	NaN	0.42	NaN	0.27	0.43	0.41	NaN	NaN	NaN
GM Equity Ratio	NaN	0.00	NaN	0.38	0.34	NaN	0.03	0.12	0.19	NaN	NaN	NaN
Manufacturing Index	NaN	0.43	0.00	NaN	NaN	0.41	NaN	0.42	0.25	0.46	0.48	NaN
Employment Index	0.15	0.36	0.25	0.00	NaN	NaN	NaN	NaN	0.02	NaN	0.19	NaN
Building Plan Index	0.32	0.00	0.38	0.42	0.00	NaN	NaN	NaN	0.06	0.08	NaN	0.20
Changing GDP	NaN	NaN	NaN	NaN	NaN	0.21	0.49	NaN	0.37	0.37	NaN	0.39
Changing CPIX	0.26	NaN	0.01	0.13	NaN	0.27	NaN	NaN	NaN	NaN	0.29	0.20
Unexpected Inflation	0.14	NaN	0.07	0.41	NaN	NaN	0.29	0.10	0.45	NaN	0.24	0.04
Prime Rate	0.15	0.15	NaN	NaN	0.23	0.26	0.25	0.41	0.31	NaN	0.46	0.33
Changing Prime Rate	0.15	0.40	0.36	NaN	0.48	NaN	0.18	0.27	NaN	NaN	0.46	0.49
J255 Return Deviation	0.08	0.03	0.44	0.15	0.20	0.22	NaN	NaN	0.46	0.33	0.02	NaN
J256 Return Deviation	0.04	0.23	0.24	0.24	0.09	NaN	0.42	NaN	NaN	NaN	0.44	NaN

Table CL.2: Granger Causality Probability for Indirect Return Deviation

3. Direct Return

Variable	Term Structure	GI& Equity Ratio	Manufacturing Index	Employment Index	Building Plan Index	Changing GDP	Changing CPIX	Unexpected Inflation	Prime Rate	Changing Prime Rate	Retail Return	Office Return	Industrial Return
Term Structure	0.31	0.23	0.18	0.15	NaN	0.43	0.23	0.47	NaN	NaN	0.28	0.20	NaN
GI& Equity Ratio	0.26	0.00	0.25	0.18	0.08	0.01	0.05	0.15	NaN	NaN	NaN	0.30	0.07
Manufacturing Index	0.46	NaN	NaN	NaN	NaN	0.10	0.18	0.40	NaN	NaN	0.23	NaN	0.17
Employment Index	0.15	0.00	0.02	0.00	0.00	0.06	0.05	0.02	0.35	NaN	0.01	0.01	0.01
Building Plan Index	0.27	0.02	NaN	0.11	0.12	0.26	NaN	NaN	0.05	0.05	0.01	0.46	0.41
Changing GDP	NaN	0.35	0.30	NaN	0.28	0.22	NaN	NaN	NaN	0.31	0.34	NaN	0.11
Changing CPIX	0.20	NaN	0.23	0.12	0.37	NaN	0.49	0.14	0.04	NaN	0.04	NaN	NaN
Unexpected Inflation	0.17	NaN	0.26	0.19	NaN	0.17	NaN	0.33	0.50	0.43	0.05	0.10	0.44
Prime Rate	NaN	0.11	NaN	0.34	NaN	0.35	NaN	0.34	NaN	NaN	0.32	0.48	0.26
Changing Prime Rate	NaN	0.37	0.48	NaN	NaN	0.32	NaN	NaN	NaN	NaN	0.32	NaN	0.11
Retail Return	0.19	0.09	0.28	NaN	NaN	0.38	0.05	0.16	NaN	NaN	0.00	0.21	NaN
Office Return	NaN	0.25	NaN	NaN	0.42	0.25	0.05	NaN	NaN	NaN	NaN	0.00	NaN
Industrial Return	0.24	0.28	0.11	0.38	NaN	NaN	0.19	0.35	NaN	NaN	0.24	0.28	0.00

Table C2.1: Granger Causality Probability for Direct Return

4. Direct Return Deviation

Variable	Term Structure	GI& Equity Ratio	Manufacturing Index	Employment Index	Building Plan Index	Changing GDP	Changing CPIX	Unexpected Inflation	Prime rate	Changing Prime Rate	Retail Return Deviation	Office Return Deviation	Industrial Return Deviation
Term Structure	0.40	NaN	0.20	0.28	NaN	NaN	0.45	NaN	0.20	0.09	0.25	0.04	0.33
GI& Equity Ratio	0.26	0.00	0.31	0.26	0.29	0.04	0.06	0.20	NaN	0.43	0.32	0.09	0.03
Manufacturing Index	0.02	0.16	0.00	0.10	0.29	0.07	0.07	0.17	0.10	NaN	0.02	0.11	0.20
Employment Index	0.17	0.32	0.01	0.19	0.18	NaN	0.44	0.33	0.27	0.20	0.27	0.01	0.12
Building Plan Index	0.01	0.02	NaN	0.02	0.04	0.03	0.01	0.06	NaN	0.22	0.02	0.12	0.02
Changing GDP	0.02	0.45	0.25	0.20	0.16	0.04	0.44	0.39	NaN	NaN	0.28	0.44	0.01
Changing CPIX	0.38	0.18	0.25	NaN	NaN	0.41	0.23	0.15	0.14	NaN	0.12	NaN	NaN
Unexpected Inflation	0.16	0.28	NaN	NaN	NaN	NaN	0.27	NaN	0.00	0.27	0.08	0.30	NaN
Prime Rate	NaN	0.31	NaN	NaN	0.33	0.23	NaN	NaN	0.08	NaN	0.04	0.12	0.07
Changing Prime Rate	NaN	0.18	NaN	0.31	0.36	0.17	NaN	0.27	NaN	NaN	0.01	0.07	0.04
Retail Return Deviation	NaN	0.00	0.34	NaN	0.05	0.02	0.01	0.26	0.14	0.39	0.00	NaN	0.35
Office Return Deviation	0.31	0.03	0.31	NaN	0.09	0.19	0.01	0.12	0.50	0.03	NaN	0.00	0.08
Industrial Return Deviation	NaN	0.23	0.18	NaN	NaN	NaN	0.40	0.30	0.02	NaN	0.27	0.32	0.00

Table C2.2: Granger Causality Probability for Direct Return Deviation

Appendix D

Background on econometric model algorithms in software

1.1 Introduction

This document presents the background on the algorithms of the models developed in this research, which are the ARMA model, the GARCH model, the VAR model and the MLP neural network model. A brief introduction to the background of Matlab, the software used to simulate the model in this research, is presented. The theory behind each model investigated in this research is discussed. A section summarising the theory and characteristics of each model is also presented.

1.2 Algorithms in software

The models developed in Matlab are based on structures and functions as defined by Mathwork (2004). Structure is similar to an object where each structure consists of a set of attributes defined by the user. For example, if a user need to define a structure for student records, then the possible attributes of this structure are name, student number, standards, class code and subjects selected etc. Within these attributes, they store the information that the user previously entered. In this case, the structures are used to store information regarding the model such as the residual calculated, the values of the respective parameters, the information of the model and the result of some test conducted. Function defines a group of code that utilises the information stored in the attribute of a structure in order to perform a specific task with it. The result from the specific task calculated using the function is then in return stored in the attributes of other structures or in the case where the task is repetitive, the original attributes are updated with the new set of information. The general procedure for developing the models is defined by the flow diagram below (Figure D1). Generally, the first step requires the input of specific parameters from the user and the last step produces the result in the desirable format predefined by the user or the functions under operation.

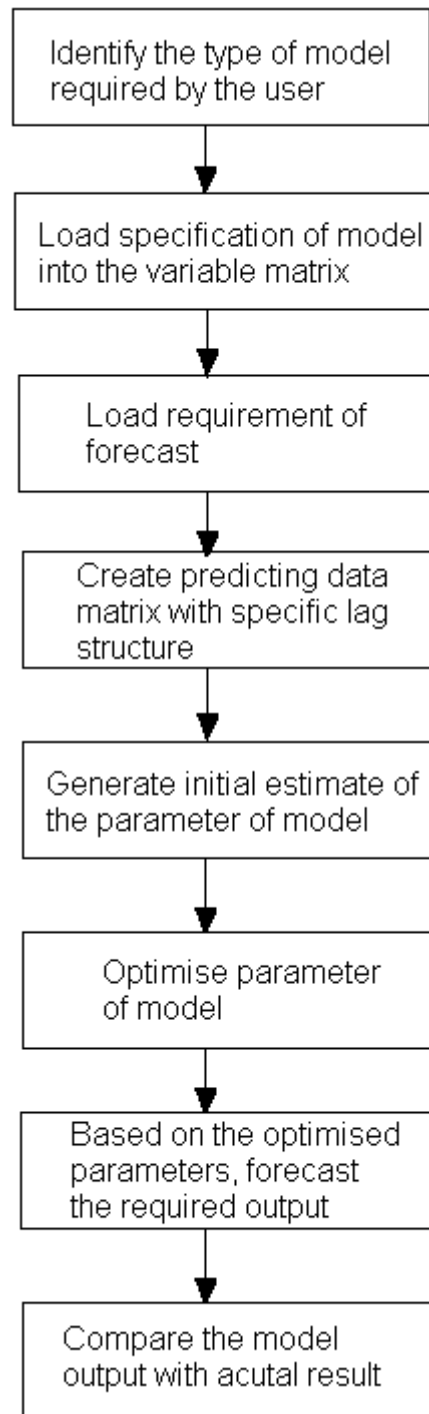


Figure D1: Flow chart of the basic structure of the code for the models

1.3 Univariate ARMA Model

The algorithm for this model is integrated into the GARCH Toolbox of Matlab because the GARCH and other non-constant conditional variance models also require one to implement such models first before implementing the required scheme for the conditional variance part of the model, as defined by Mathwork (2004). In such case where only the ARMA model is required, the conditional variance of the model is kept constant, i.e. the variance range of the

output is fixed. The model is defined by the following equations and is called the ARMAX model, as it incorporates a component for explanatory variables.

$$y_t = C + \sum_{i=1}^R \phi_i y_{t-i} + \varepsilon_t + \sum_{j=1}^M \theta_j \varepsilon_{t-j} + \sum_{k=1}^{N_x} \beta_k X(t, k) \quad (1.1)$$

Where:

C	constant bias term
ϕ_i	autoregressive (AR) coefficient for i -th lag
y_{t-i}	output variable y at time $t-i$
R	autoregressive (AR) lag parameter
θ_j	moving average (MA) coefficient for j -th lag
ε_{t-j}	variable of random process (innovation) at time $t-j$
M	moving average (MA) lag parameter
β_k	coefficient for k -th explanatory variable
$X(t, k)$	value of the k -th explanatory variable at time t
N_x	total number of explanatory variables, including lags

The above equation (Equation 1.1) is interpreted as the following in the algorithm.

$$y(t) = C + AR(1)y(t-1) + \dots + AR(R)y(t-R) + e(t) + MA(1)e(t-1) + \dots \quad (1.2)$$

$$+ MA(M)e(t-M) + B(1)X(t,1) \dots + B(N_x)X(t, N_x)$$

Where:

$AR(R)$	autoregressive (AR) coefficient for R -th lag
$MA(M)$	moving average (MA) coefficient for M -th lag
$B(N_x)$	coefficient for N_x -th explanatory variable

As discussed in previous section, the user has to specify an initial value for the parameters where the algorithm examines them, before proceeding any further. Amongst other examinations, the algorithm examines two essential conditions, namely that the autoregressive part of the equation must be stationary and the moving average part must be invertible. The algorithm achieves this by calculating the eigenvalues of the AR and MA parameters and determines whether the calculated eigenvalues lie within the unit roots circle. If the eigenvalues lie with the circle, then the autoregressive part is stationary and the moving average part is invertible.

Once all of the required conditions have been satisfied, the algorithm first commences with the calculation of an initial estimation of the parameters based on the input from the user. The initial estimation is performed in two steps. The first step is to calculate the parameters of the autoregressive (AR) part of the model and the parameters of the explanatory variables using ordinary least square regression. The second step is to extract the residual of the OLS regression and use it to estimate the coefficient of the moving average (MA) part, where each coefficient of the moving average part is based on its auto-covariance with the autoregressive

part. Thereafter, the parameters are then optimised based on minimising the following linear equation, derived from Equation 1.2, with the initial estimated values substituted in the parameters.

$$e(t) = -C + y(t) - AR(1)y(t-1) - \dots - AR(R)y(t-R) - MA(1)e(t-1) - \dots \quad (2)$$

$$- MA(M)e(t-M) - B(1)X(t,1) \dots - B(N_x)X(t, N_x)$$

This equation is an extension to Equation 1.2 and defines the differences between the actual result and the predicted result based on the generated model at present time t , which is the error term $e(t)$.

1.4 Univariate GARCH Model

As previously discussed in the ARMA model section, the algorithm of the model used in the analysis is found in the GARCH Toolbox of Matlab and can be easily extended from the previously developed ARMA model. The model used is an univariate GARCH model, which is similar to the model used in the work of West and Worthington (2004). The model is defined by two different components, namely the conditional mean component and the conditional variance component. The conditional mean component is the ARMAX equation defined by Equation 1.1 and 1.2. The conditional variance component is defined by the equation below.

$$\sigma_t^2 = \kappa + \sum_{i=1}^P G_i \sigma_{t-i}^2 + \sum_{j=1}^Q A_j \varepsilon_{t-j}^2 \quad (3.1)$$

Where:

- K constant standard deviation term
- σ_{t-i}^2 standard deviation at time $t-i$
- G_i autocorrelative coefficient of standard deviation for i -th lag
- P autocorrelative component (GARCH) lag parameter
- ε_{t-j}^2 variable of random process (innovation) at time $t-j$
- A_j coefficient for innovation for j -th lag
- Q innovation component (ARCH) lag parameter

Matlab defined the following constraint to the above equation (Equation 3.1):

$$\sum_{i=1}^P G_i + \sum_{j=1}^Q A_j < 1 \quad (3.2)$$

$$\kappa > 0 \quad (3.3)$$

$$G_i \geq 0 \quad i = 1, 2, \dots, P \quad (3.4)$$

$$A_j \geq 0 \quad j = 1, 2, \dots, Q \quad (3.5)$$

Once again, the above equation (Equation 3.1) is interpreted to the following in the algorithm:

$$h(t) = K + GARCH(1)h(t-1) + \dots + GARCH(P)h(t-P) + ARCH(1)e(t-1)^2 + \dots \quad (3.6)$$

$$+ ARCH(Q)e(t-Q)^2$$

Where:

$GARCH(P)$ autocorrelative coefficient for P -th lag
 $ARCH(Q)$ innovation coefficient for Q -th lag

Similar to the ARMA model, the algorithm inspects whether the initial parameters set by the user satisfy the above requirements. Once again, the algorithm calculates an initial estimation of the parameters based on the input from the user. However, the algorithm employs an ad hoc approach for this estimation. Based on the condition defined in Equation 3.2, the algorithm proportion 0.85 to all of the G coefficients and 0.05 to all of the A coefficients, from which the parameters are optimised by means of minimising the error between the actual standard deviation values and the standard deviation values calculated using the above equation (Equation 3.6) with the initial parameters.

1.5 Vector Autoregressive (VAR) Model

The algorithm is developed in the Econometric Toolbox by James LeSage (1999). There is various form of VAR models developed in the toolbox but in this study the simple VAR model is employed. The simple VAR model with 1 lag is based on the following equation as defined in LeSage (1999: 214). Unlike the previous two models discussed, this model is designed for multiple outputs and thus it is also known as multivariant model.

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{nt} \end{bmatrix} = \begin{bmatrix} A_{11}(l) & \dots & A_{1n}(l) \\ \vdots & \ddots & \vdots \\ A_{n1}(l) & \dots & A_{nn}(l) \end{bmatrix} \begin{bmatrix} y_{1(t-1)} \\ y_{2(t-1)} \\ \vdots \\ y_{n(t-1)} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{bmatrix} \quad (4)$$

Where:

$y_{1(t-1)}$ 1st variable in the matrix at time $t-1$
 $y_{n(t-1)}$ nth variable in the matrix at time $t-1$
 $A_{nm}(l)$ coefficient for the nth variable for the nth equation
 C_n constant for the nth equation

ε_{nt} error term for the n^{th} equation

Once again, initial values are defined for the parameters and the parameters are optimised. As discussed in LeSage (1999:214), the parameters are optimised using ordinary least square (OLS) regression method.

1.6 Neural Network

The Netlab Toolbox developed by Ian Nabney (2004) is used for the development of the neural network model. Unlike the Neural Network Toolbox provided in Matlab, the Netlab Toolbox is simple to use and provides greater flexibility in developing a neural network model. In this research, only the multi-layered perceptron (MLP) model is investigated. The algorithm models the neural network based on Equation 10.1 and Equation 10.2 in section 2.3.4 of the thesis. The algorithm implements the simplest form of MLP network, which has an input layer, a hidden layer and a output layer. The transfer function used for the neurons in the input and the hidden layer is the tanh function, which is defined by the following equation (Wikipedia, 2009d).

$$f(W_p + b) = a^{(1)} = \tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \quad (5)$$

Once again, the algorithm first requires some basic inputs from the users on various parameters, such as the number of neurons in each layer (the input layer, the hidden layer and the output layer) and the transfer function to be used in the output layer. There are three choices of transfer functions available from the toolbox, namely the linear, the logistic and the softmax functions, as defined in Nabney (2004: 151).

$$\text{Linear:} \quad y_k = a_k^{(2)} \quad (6.1)$$

$$\text{Logistic:} \quad y_k = \frac{1}{1 + e^{-a_k^{(2)}}} = \frac{1}{1 + \exp(-a_k^{(2)})} \quad (6.2)$$

$$\text{Softmax:} \quad y_k = \frac{e^{a_k^{(2)}}}{\sum_{j=1}^n e^{a_j^{(2)}}} = \frac{\exp(a_k^{(2)})}{\sum_{j=1}^n \exp(a_j^{(2)})} \quad (6.3)$$

Where:

$a_k^{(2)}$ the sum of all weighted inputs and bias, for neuron k in the output layer

N number of samples used for estimating the model

In addition to defining some basic parameters, the input data need to be normalised. The process of normalisation is essential as the neural network operation is dependent on the consistent range of the input data. The reason behind this is to prevent the weight and bias parameters of neurons from undergoing saturation caused by data sets that have higher values than others. Furthermore, the process ensures that the movements in the data sets are detected without any bias. Each set of data is normalised by dividing all of the samples by the

maximum value (in terms of magnitude) in the set, i.e. the normalised value of the maximum sample will be 1.

The next step is for the algorithm to initialise and optimise the parameters of the model. It defines the initial weights and biases of each neuron in the network with random values. The weights and biases are then optimised using the optimisation method defined by the user, which also requires the set of input and output data (the data set of the predicting variable and the predictors), the information of the neural network and the terminating criteria of the optimisation. The terminating criteria are usually the threshold error value between the actual and the predicted output, the number of iterations of the optimisation algorithm or a combination of both. Finally, the user can utilise this model to evaluate its performance.

1.7 Summary and discussion

The models are developed using Matlab version 7.0, which is designed for sophisticated calculations and modelling. The general procedure of the algorithm in developing the model is to obtain initial parameters from the user, examine the input parameters from the user, optimise the parameters, implement the model using the optimised parameter and then compare it with the actual values. The four models under investigation in this research, namely the univariant ARMA model, the univariant GARCH model, the VAR model and the MLP neural network model, have all been previously developed in Matlab.

The univariant ARMA model and GARCH model are both integrated into the GARCH toolbox where Matlab permits the user to develop a model with both conditional mean and conditional variance. Both of the models provide one output with functions that allow the inclusion of explanatory variables in the conditional mean equation (Equation 1.1). The conditional variance is considered as a constant when implementing the ARMA model and the factors influencing the outputs (return and return deviation) are considered as the explanatory variables in the equation. The parameters for the ARMA model are set to zero, i.e. the autocorrelation terms are removed from the equation, for the development of a pure GARCH model. As a result, the conditional mean is dependently solely on explanatory variables, while the conditional variance for the GARCH model will be a function of the standard deviations of the outputs (return and return deviation). In this case, the conditional mean equation mimics that of a multiple linear regression (MLR) model, where the output depends on other factors or independent variables.

The VAR models and the neural network models are multivariant models, in other words, the model can produce multiple outputs. In the case of VAR model, there is a limitation, where the size of the output variable matrix has to correspond to the size of the input (explanatory) variables matrix (referring to Equation 4). As a consequence, the output or predicting variables (the return or return deviations) and the explanatory variables are combined together into the variable matrix. While in the neural network model, such problems do not exist, as there is no relation between the size of the output and the size of the input.

Since the ARMA and the GARCH models are limited to one output for each model, a model is designed for each output (return and return deviations). While for the VAR and neural network models, a model is developed for each type of return and its deviation, i.e. the outputs of the model developed for J255 return will be the J255 return and the J255 return deviation.

1.8 References

LeSage, J.P. (1999). *Spatial Econometrics*. Retrieved 21st November 2009 from the World Wide Web: <http://www.spatial-econometrics.com/wbook.pdf>.

Mathwork (2004). "Full Product Family Help". Matlab Version 7 Release 14.

Nabney, I. (2004). *Netlab: Algorithm for Pattern Recognition*, 4th Edition. UK: Springer.

West, T. and Worthington, A. (2004). *Macroeconomic Risk Factors in Australian Commercial Real Estate, Listed Property Trust and Property Sector Stock Returns: A Comparative Analysis using GARCH-M*, Proceedings of the Pacific Rim Real Estate Society Conference, Bangkok, Thailand.

Wikipedia (2009d). Hyperbolic function. Retrieved 1st December 2009 from the World Wide Web: http://en.wikipedia.org/wiki/Hyperbolic_function.

Appendix E

Tables of result from optimisation process

J255 Return

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-71.446	-66.155	-70.115	-60.218	19.566	30.191	110.372	119.908
2	-71.085	-62.349	-63.200	-57.831	38.614	62.237	125.626	96.567
3	-46.073	-59.364	-27.146	-31.191	89.083	73.392	127.272	133.214
4	-76.739	-70.298	-55.643	54.829	83.033	93.931	136.366	125.036
5	-79.567	-73.099	-45.632	70.559	94.432	101.274	152.133	159.719
6	-73.696	-66.146	-43.841	80.792	116.536	120.704	146.681	158.418
7	-69.164	-61.913	-54.957	91.444	128.746	122.253	158.009	161.989
8	-72.052	-60.257	-50.234	101.538	-6.673	-18.979	168.602	155.750
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-52.920	-48.458	-53.514	-45.001	33.084	41.668	119.436	126.151
2	-53.388	-45.747	-47.983	-44.313	50.092	71.301	131.870	99.542
3	-29.472	-44.148	-13.628	-19.714	98.148	79.635	130.248	132.429
4	-61.522	-56.780	-44.165	63.894	89.277	96.907	135.582	119.948
5	-66.049	-61.621	-36.567	76.802	97.407	100.490	147.045	149.727
6	-62.218	-57.081	-37.597	83.767	115.751	115.616	136.690	142.855
7	-60.099	-55.670	-51.982	90.659	123.659	112.261	142.446	140.110
8	-65.808	-57.282	-51.018	96.450	-16.664	-34.542	146.722	126.720

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-87.498	-83.306	-85.966	-105.104	-23.267	-2.034	64.691	67.917
2	-95.061	-91.656	4.894	-85.860	-26.197	19.377	69.530	59.037
3	-91.592	-80.803	5.637	-82.900	-9.300	-23.976	74.505	61.512
4	-101.950	-101.603	11.854	-76.364	37.543	33.208	79.697	84.074
5	-109.432	-102.588	-20.546	-95.864	-23.155	42.942	88.593	90.257
6	-105.753	-98.959	21.556	-22.538	39.682	48.181	80.451	90.511
7	-105.978	-103.415	27.434	-87.842	50.452	38.840	91.730	102.231
8	-108.353	-100.794	-0.049	1.369	-91.404	-79.445	94.835	104.781
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-69.912	-64.995	-67.081	-85.804	-3.726	17.565	84.150	87.023
2	-76.750	-72.770	24.194	-66.319	-6.597	38.836	88.636	77.562
3	-72.707	-61.504	25.178	-63.301	10.160	-4.869	93.030	79.209
4	-82.650	-82.061	31.453	-56.905	56.650	51.733	97.394	100.675
5	-89.890	-82.989	-1.087	-76.758	-4.630	60.639	105.195	105.474
6	-86.154	-79.500	40.663	-4.013	57.379	64.782	95.668	104.029
7	-86.519	-84.308	45.959	-70.145	67.053	54.056	105.248	113.709
8	-89.247	-82.269	17.648	17.970	-76.187	-65.927	106.312	113.846

ARMA model (Input variables with 1 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-89.988	-84.174	-88.985	-106.975	-111.745	-75.081	-15.284	54.403
2	-99.558	-86.478	-87.313	-95.437	-102.486	-65.586	-23.383	58.181
3	-96.925	-109.424	-100.907	-106.443	-63.884	-67.597	-33.612	62.989
4	-105.700	-51.540	-106.096	-33.388	-54.437	-93.505	-13.652	65.280
5	-111.840	-79.357	-102.611	-88.080	-34.667	-53.100	-2.639	71.100
6	-109.709	-113.634	-101.850	-97.895	-86.085	-65.999	-11.622	72.004
7	-108.791	-106.406	-94.885	-96.143	-73.660	-82.222	0.377	73.246
8	-108.881	-104.595	-27.030	-98.541	-58.067	-85.500	31.671	74.656
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-76.631	-69.576	-73.264	-90.255	-94.160	-56.770	3.602	73.703
2	-84.961	-70.757	-70.593	-77.851	-84.175	-46.700	-4.084	77.722
3	-81.204	-92.705	-83.321	-88.132	-44.998	-48.298	-14.070	82.588
4	-88.981	-33.954	-87.785	-14.502	-35.138	-73.964	5.947	84.739
5	-94.254	-61.047	-83.725	-68.780	-15.126	-33.500	16.820	90.207
6	-91.399	-94.748	-82.551	-78.354	-66.485	-46.540	7.485	90.529
7	-89.905	-87.107	-75.344	-76.544	-54.201	-63.115	18.902	90.943
8	-89.582	-85.054	-7.431	-79.082	-38.960	-66.975	49.368	91.258

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-0.996	231.691	-59.232	-13.319	-45.047	154.405	265.100	1.481
2	563.075	-36.088	-45.399	-47.560	-46.653	428.219	223.327	521.808
3	734.470	116.638	-44.093	-38.138	-36.952	-31.293	129.741	107.182
4	-41.801	-31.435	402.784	-28.048	-28.382	213.354	368.805	320.145
5	50.331	-39.495	-33.974	-24.362	-30.281	-11.650	481.808	6.883
6	74.389	-34.004	439.475	-25.265	-18.152	-5.425	126.052	623.525
7	258.401	-27.498	-19.633	-11.914	-10.903	0.144	1.118	369.923
8	19.992	-19.533	-11.861	-5.386	-0.434	13.188	14.625	18.208
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	17.529	249.388	-42.631	1.898	-31.529	165.883	274.164	7.724
2	580.772	-19.486	-30.182	-34.042	-35.175	437.283	229.570	524.783
3	751.071	131.855	-30.575	-26.660	-27.887	-25.049	132.717	106.398
4	-26.584	-17.917	414.261	-18.984	-22.139	216.330	368.020	315.057
5	63.849	-28.017	-24.909	-18.118	-27.305	-12.435	476.720	-3.109
6	85.867	-24.939	445.718	-22.290	-18.937	-10.513	116.060	607.961
7	267.465	-21.255	-16.657	-12.699	-15.991	-9.848	-14.445	348.044
8	26.235	-16.557	-12.645	-10.474	-10.426	-2.375	-7.254	-10.822

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-74.095	351.029	-69.816	-66.024	-62.295	-58.242	-57.811	-53.347
2	43.386	-69.861	-66.164	-62.294	-58.241	-53.991	-53.347	-48.655
3	-66.378	-65.029	-60.758	-58.594	-53.991	-46.440	-48.655	-43.716
4	116.202	-59.743	-56.704	-53.991	-50.620	-45.928	-43.716	-38.510
5	64.039	-46.478	72.328	-52.172	-47.865	-42.962	-39.070	-33.574
6	-50.427	-51.557	-48.871	-28.992	-39.004	-35.783	-33.574	-27.765
7	-52.333	-51.374	-44.179	-42.009	-33.798	-31.308	-30.432	-24.281
8	278.387	-21.666	-39.240	-37.723	-32.227	-25.498	-24.281	-17.757
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-56.509	369.340	-50.930	-46.724	-42.754	-38.643	-38.352	-34.241
2	61.697	-50.976	-46.865	-42.753	-38.642	-34.532	-34.241	-30.130
3	-47.493	-45.730	-41.216	-38.995	-34.532	-27.334	-30.130	-26.019
4	135.501	-40.202	-37.105	-34.532	-31.514	-27.403	-26.019	-21.908
5	83.580	-26.878	91.787	-33.065	-29.340	-25.265	-22.468	-18.357
6	-30.828	-32.097	-29.765	-10.467	-21.307	-19.181	-18.357	-14.246
7	-32.873	-32.268	-25.654	-24.313	-17.197	-16.091	-16.914	-12.803
8	297.493	-3.141	-21.543	-21.121	-17.011	-11.980	-12.803	-8.692
GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-66.661	526.548	-62.424	-58.936	-56.067	-52.681	-51.085	-47.388
2	-62.460	-62.347	-59.303	-56.067	-52.681	-49.145	-47.388	-43.520
3	74.244	-59.084	527.709	-52.681	-49.145	-45.448	-43.520	-39.466
4	-59.234	-52.084	-51.376	-49.145	-45.448	-41.579	-39.466	-35.216
5	-55.668	-52.412	-45.563	-45.448	-41.579	-37.526	-35.178	-30.752
6	57.654	-50.991	-44.829	-43.940	-39.887	-35.636	-31.172	-26.480
7	-50.242	185.622	-43.940	-39.887	-35.636	-31.172	-26.480	-21.541
8	-43.558	-42.091	-38.520	-35.636	-31.172	-26.480	-21.541	-16.335
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-53.304	541.146	-46.703	-42.216	-38.481	-34.370	-32.200	-28.089
2	-47.862	-46.626	-42.583	-38.481	-34.370	-30.259	-28.089	-23.978
3	89.965	-42.364	545.295	-34.370	-30.259	-26.149	-23.978	-19.867
4	-42.515	-34.498	-33.065	-30.259	-26.149	-22.038	-19.867	-15.756
5	-38.082	-34.101	-26.678	-26.149	-22.038	-17.927	-15.719	-11.645
6	75.964	-32.106	-25.529	-24.398	-20.288	-16.177	-12.066	-7.955
7	-31.357	204.922	-24.398	-20.288	-16.177	-12.066	-7.955	-3.844
8	-24.259	-22.549	-18.921	-16.177	-12.066	-7.955	-3.844	0.267

Table E1: ARMA and GARCH Models for J255 return

J256 Return

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-96.981	-83.008	-51.125	-50.685	126.285	62.742	126.076	110.836
2	-88.959	-78.140	-31.358	90.780	132.504	-26.514	156.156	155.945
3	-84.196	22.728	-21.979	101.896	142.425	121.865	166.673	171.807
4	-78.441	-60.509	-35.788	110.963	151.471	136.821	177.754	168.505
5	-67.685	-59.605	-20.084	116.678	160.619	148.769	190.497	195.691
6	-63.251	49.179	3.330	127.949	171.330	156.434	194.050	205.472
7	-56.012	-29.874	-26.413	139.520	190.044	172.511	202.464	219.938
8	-54.364	-34.673	-5.324	150.690	73.618	120.899	220.674	235.684
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-83.463	-71.530	-42.060	-44.441	129.261	61.958	120.989	100.844
2	-77.481	-69.076	-25.115	93.755	131.719	-31.602	146.164	140.382
3	-75.131	28.972	-19.004	101.111	137.337	111.874	151.110	149.928
4	-72.197	-57.533	-36.572	105.876	141.479	121.258	155.874	139.475
5	-64.709	-60.390	-25.172	106.686	145.056	126.889	161.467	158.572
6	-64.035	44.092	-6.662	112.385	149.450	127.403	156.931	159.203
7	-61.100	-39.866	-41.976	117.640	161.014	135.391	156.195	163.313
8	-64.355	-50.236	-27.203	121.660	36.498	74.629	164.049	167.326

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-99.473	-96.154	-89.567	-44.693	33.537	-48.345	68.743	78.795
2	-96.369	-93.034	-101.928	-18.793	39.106	14.903	77.125	83.764
3	-93.640	-88.929	-52.049	-52.587	40.458	37.495	52.531	87.578
4	-70.602	-95.159	-97.309	12.669	55.064	31.223	89.459	96.206
5	-100.196	-94.328	-87.106	5.163	61.863	67.179	100.571	106.293
6	-96.793	-91.960	-76.705	-68.593	65.394	73.186	86.209	108.373
7	-92.538	-45.630	-82.568	18.673	71.901	81.177	107.548	116.484
8	-99.312	-96.985	-74.100	33.880	-67.762	-62.408	115.765	119.937
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-80.587	-76.854	-70.025	-25.094	52.996	-29.239	87.268	96.492
2	-77.070	-73.492	-82.329	0.666	58.213	33.428	94.822	100.365
3	-74.098	-69.330	-32.590	-33.480	58.983	55.192	69.133	102.795
4	-51.003	-75.700	-78.202	31.194	72.761	47.825	104.676	109.724
5	-80.737	-75.222	-68.581	22.860	78.464	82.396	114.089	117.770
6	-77.686	-73.435	-59.008	-51.991	80.610	86.704	97.686	117.437
7	-74.013	-27.933	-65.967	33.890	85.419	92.655	116.612	122.727
8	-81.615	-80.384	-58.883	47.398	-56.284	-53.343	122.009	122.913

ARMA model (Input variables with 1 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-101.889	-87.611	-118.840	-99.148	-99.940	-68.733	-94.352	57.349
2	-106.254	-94.433	-48.813	-83.686	-78.427	-88.181	-23.942	62.714
3	-103.846	-110.246	-37.642	-46.109	-51.190	-52.962	-49.805	66.374
4	-107.954	-105.395	-100.974	-109.779	-52.501	-24.350	-47.024	71.045
5	-108.597	-115.350	-13.185	-80.724	-18.205	-2.920	-38.626	77.269
6	-105.835	-104.339	-92.241	-67.139	-9.042	3.162	-29.882	79.131
7	-102.803	-100.847	-89.250	-40.650	5.820	61.580	-12.113	83.656
8	-109.046	-115.030	-66.142	-17.027	6.653	-85.449	-9.654	87.823
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-87.291	-71.890	-102.121	-81.562	-81.629	-49.847	-75.052	76.890
2	-90.533	-77.713	-31.228	-65.375	-59.542	-68.881	-4.401	82.313
3	-87.127	-92.660	-19.332	-27.224	-31.891	-33.420	-30.205	85.833
4	-90.368	-87.084	-82.088	-90.479	-32.959	-4.751	-27.565	90.151
5	-90.286	-96.464	6.115	-61.182	1.394	16.539	-19.519	95.794
6	-86.950	-85.039	-72.700	-47.540	10.417	22.268	-11.356	96.828
7	-83.504	-81.305	-69.650	-21.191	24.927	80.105	5.584	100.257
8	-89.504	-95.431	-46.683	2.080	25.178	-67.752	6.947	103.040

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	1024.700	475.700	401.700	58.400	-47.800	-39.100	-30.327	-24.758
2	-17.200	11.500	42.500	-46.300	-38.900	-32.000	-25.753	-23.166
3	349.400	29.900	459.800	-38.700	-34.100	-25.900	-18.232	-15.159
4	232.400	111.700	45.900	-30.400	-15.200	-22.100	364.335	-6.596
5	988.200	140.600	786.200	71.500	-18.200	-7.300	3.860	8.400
6	163.900	203.800	-22.800	-16.800	-7.200	152.400	18.373	21.484
7	517.800	134.600	-13.700	-5.800	4.300	11.700	27.221	37.915
8	451.700	-12.800	-1.700	48.900	14.900	25.000	44.104	50.042
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	1038.200	487.200	410.700	64.600	-44.900	-39.900	-35.414	-34.749
2	-5.800	20.600	48.700	-43.400	-39.700	-37.100	-35.744	-38.729
3	358.400	36.100	462.700	-39.500	-39.100	-35.900	-33.796	-37.039
4	238.700	114.700	45.100	-35.500	-25.200	-37.700	342.456	-35.627
5	991.200	139.800	781.100	61.500	-33.800	-29.200	-25.170	-28.719
6	163.100	198.800	-32.800	-32.300	-29.100	123.300	-18.746	-24.786
7	512.700	124.600	-29.300	-27.700	-24.700	-25.400	-19.049	-18.710
8	441.700	-28.300	-23.500	19.900	-22.200	-21.300	-12.521	-18.316

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-40.819	-11.469	-72.313	-77.579	-64.358	-71.895	-77.876	-64.044
2	655.474	-65.122	-70.139	-63.498	-64.256	-63.913	-65.703	-71.880
3	-23.279	-64.056	-13.441	-60.892	-55.569	-62.625	-41.948	-64.678
4	-68.729	-57.834	-59.554	-56.318	-55.158	-60.557	-60.496	-59.605
5	27.884	-58.676	12.246	-64.083	-40.475	-58.404	-58.350	-51.409
6	-60.501	-59.212	-54.458	-60.201	-44.457	-54.897	-33.693	-47.458
7	9.520	-55.055	-44.840	-43.338	-37.973	-51.431	-26.953	-24.079
8	316.001	-43.264	-19.988	-46.725	139.700	-45.232	-38.700	-16.700
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-21.934	7.831	-52.772	-57.980	-44.899	-52.789	-59.351	-46.347
2	674.773	-45.581	-50.539	-44.039	-45.150	-45.388	-48.007	-55.279
3	-3.737	-44.457	6.019	-41.786	-37.044	-44.929	-25.346	-49.461
4	-49.129	-38.374	-40.447	-37.793	-37.461	-43.956	-45.279	-46.087
5	47.343	-39.570	30.771	-46.386	-23.874	-43.187	-44.831	-39.931
6	-41.394	-40.687	-36.762	-43.600	-29.240	-41.379	-22.215	-38.393
7	28.045	-37.358	-28.239	-28.121	-24.455	-39.954	-17.889	-17.835
8	333.698	-26.663	-4.771	-33.207	151.178	-36.168	-32.457	-13.725

GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-94.904	-91.717	-91.141	-88.706	-85.881	-82.345	-79.818	-76.051
2	365.240	-89.702	-89.267	-82.705	-82.345	-78.648	-74.485	-71.997
3	60.559	-85.078	-85.881	-82.345	-78.648	-74.779	-71.895	-67.747
4	775.303	-80.600	-82.345	-78.648	-74.779	-70.726	-67.808	-63.373
5	674.664	-78.349	-77.106	-74.779	-70.726	-66.475	-63.344	-58.681
6	-68.028	-76.636	-74.779	-70.722	-66.475	-62.012	-58.652	-53.742
7	-20.650	-71.223	-70.726	-66.475	-62.012	-57.319	-53.713	-47.395
8	271.629	-64.723	-63.233	-62.012	-57.319	-52.380	-49.193	-41.900
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-80.306	-75.996	-74.421	-71.121	-67.570	-63.459	-60.518	-56.509
2	380.961	-72.982	-71.681	-64.394	-63.459	-59.349	-54.944	-52.398
3	77.279	-67.492	-67.570	-63.459	-59.349	-55.238	-52.296	-48.287
4	792.889	-62.289	-63.459	-59.349	-55.238	-51.127	-48.349	-44.267
5	692.975	-59.464	-57.807	-55.238	-51.127	-47.016	-44.238	-40.156
6	-49.143	-57.336	-55.238	-51.123	-47.016	-42.905	-40.127	-36.045
7	-1.350	-51.682	-51.127	-47.016	-42.905	-38.794	-36.016	-30.794
8	291.170	-45.124	-43.774	-42.905	-38.794	-34.683	-32.592	-26.683

Table E2: ARMA and GARCH Models for J256 return

J255 Return Deviation

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-245.398	-242.482	-73.7961	73.6533	87.6195	42.6401	129.1468	143.2357
2	-243.420	-191.345	-184.506	110.3048	125.8979	145.1622	115.8258	169.9173
3	-215.138	-137.549	-18.1539	116.6441	133.9578	125.4451	153.5577	182.6359
4	-245.803	-214.706	-139.145	123.5611	146.1396	172.1956	150.6097	188.2158
5	-186.401	-149.815	7.5567	118.7633	150.8112	133.2612	181.5245	192.0952
6	-225.555	-212.667	2.2727	145.5011	173.4663	138.5316	145.0924	119.8495
7	-130.080	-16.9027	-164.959	151.107	155.8931	149.8576	148.8963	108.5252
8	-229.478	41.7787	60.733	142.7549	100.1415	108.87	174.1902	83.556
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-223.399	-220.915	-52.8864	93.6642	106.4714	60.0524	144.816	156.833
2	-221.853	-170.435	-164.496	129.1567	143.3102	160.8314	129.4232	181.0858
3	-194.228	-117.538	0.6981	134.0565	149.6271	139.0425	164.7262	190.9873
4	-225.792	-195.854	-121.732	139.2303	159.7369	183.3642	158.961	193.3262
5	-167.549	-132.403	23.2259	132.3607	161.9797	141.6125	186.6349	193.5012
6	-208.143	-196.998	15.8701	156.6697	181.8176	143.642	146.4985	117.0428
7	-114.41	-3.3054	-153.79	159.4584	161.0035	151.2636	146.0896	100.9466
8	-215.881	52.9473	69.0844	147.8653	101.5475	106.0633	166.6117	70.5886

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-268.992	-264.588	-255.115	-92.739	34.484	-149.444	96.454	70.788
2	-246.194	-256.050	-258.480	-212.851	-16.160	-3.566	-5.652	74.811
3	-263.002	-260.430	-257.551	-196.423	4.613	-156.085	115.025	74.512
4	-270.690	-259.678	-257.596	34.554	55.990	18.419	122.605	87.983
5	-267.019	-38.341	-254.035	14.519	59.019	38.792	133.105	93.326
6	-267.138	-261.220	-254.158	44.009	68.417	52.263	62.576	11.109
7	-218.010	-260.407	-177.247	46.318	87.248	67.197	55.882	39.298
8	-118.569	-181.094	-64.916	53.547	15.252	-218.186	72.664	-2.334
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-250.036	-244.721	-234.469	-71.456	56.253	-127.349	118.704	93.011
2	-226.327	-235.405	-237.198	-191.082	5.935	18.685	16.571	96.810
3	-242.357	-239.148	-235.782	-174.328	26.863	-133.862	137.024	96.079
4	-249.407	-237.909	-235.501	56.804	78.213	40.418	144.171	108.893
5	-245.250	-16.246	-231.785	36.741	81.019	60.359	154.015	113.337
6	-245.043	-238.970	-231.936	66.008	89.984	73.173	82.587	29.961
7	-195.760	-238.184	-155.248	67.885	108.157	87.208	74.734	56.711
8	-96.346	-159.095	-43.349	74.457	35.263	-199.334	90.076	13.335

ARMA model (Input variables with 1 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-274.825	-279.032	-249.928	-125.685	-226.022	-49.347	-0.040	18.429
2	-274.228	-273.311	-97.770	-79.724	-15.025	-39.468	48.875	49.173
3	-283.739	-115.123	-148.754	-262.962	15.650	-43.817	59.297	50.580
4	-273.293	-271.303	-261.648	-164.571	21.639	13.960	42.006	45.266
5	-271.616	-262.872	-265.699	-98.515	26.102	22.904	12.482	68.451
6	-287.568	-262.468	-186.979	-214.041	13.340	15.732	-238.626	70.921
7	-270.537	-263.165	-262.027	-9.990	33.762	42.837	54.377	64.195
8	0.510	-261.497	-262.916	-9.810	-155.203	-192.362	-238.304	52.858
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-260.702	-263.536	-233.165	-107.767	-207.067	-29.480	20.605	39.712
2	-258.733	-256.548	-79.851	-60.769	4.842	-18.822	70.158	70.942
3	-266.976	-97.204	-129.799	-243.095	36.295	-22.534	81.066	72.675
4	-255.375	-252.348	-241.781	-143.926	42.921	35.729	64.101	67.516
5	-252.661	-243.005	-245.054	-77.233	47.871	44.999	34.732	90.673
6	-267.701	-241.822	-165.696	-192.273	35.435	37.982	-216.404	92.920
7	-249.892	-241.883	-240.258	12.105	56.012	65.059	76.377	85.762
8	21.793	-239.728	-240.821	12.440	-132.981	-170.363	-216.737	73.768

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	123.800	394.400	663.100	-163.000	-168.900	-172.100	-202.119	-182.115
2	641.100	140.300	97.800	-153.800	-163.800	137.300	-194.746	-181.065
3	283.600	384.400	83.800	-195.400	-174.600	48.400	-202.220	-186.385
4	216.600	54.400	-187.200	-202.300	-141.300	-197.800	-173.642	-165.851
5	930.900	631.300	-208.300	-138.600	-191.600	-145.900	315.018	-173.467
6	1022.600	41.600	94.700	-168.400	-156.500	241.400	-166.531	-148.369
7	1034.300	483.800	309.100	-191.700	-148.900	-146.100	68.645	-157.686
8	1048.900	210.100	-145.400	-171.400	-178.000	-165.500	-150.154	-151.469
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	145.800	416.000	684.000	-143.000	-150.000	-154.700	-186.450	-168.518
2	662.700	161.200	117.800	-134.900	-146.400	152.900	-181.149	-169.897
3	304.500	404.400	102.700	-178.000	-158.900	62.000	-191.052	-178.033
4	236.600	73.300	-169.700	-186.700	-127.700	-186.600	-165.291	-160.741
5	949.800	648.700	-192.700	-125.000	-180.500	-137.500	320.129	-172.061
6	1040.100	57.300	108.300	-157.200	-148.200	246.500	-165.125	-151.176
7	1050.000	497.400	320.300	-183.400	-143.800	-144.700	65.839	-165.264
8	1062.500	221.300	-137.100	-166.300	-176.600	-168.300	-157.733	-164.436

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-209.500	159.100	-265.200	-261.700	-258.000	-254.200	-250.100	146.800
2	-256.000	-82.300	-190.900	-258.000	-254.200	-250.100	-245.900	-241.400
3	-241.600	-75.000	-258.300	-172.700	-250.100	-246.700	-241.500	-238.200
4	683.700	-245.400	-254.200	-250.100	-245.900	-241.500	-236.900	-233.100
5	857.100	-246.900	-227.100	-248.100	-243.800	-237.900	-234.400	-229.300
6	346.700	-236.800	744.900	-244.400	-239.800	-234.900	-229.900	-224.000
7	1112.400	-38.400	-242.900	-238.900	-47.800	-228.900	-184.400	-219.000
8	441.200	220.600	-238.800	-232.000	-229.900	-223.600	-219.000	-213.100
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-190.500	179.000	-244.600	-240.400	-236.200	-232.100	-227.900	169.000
2	-236.200	-61.600	-169.600	-236.200	-232.100	-227.900	-223.700	-219.400
3	-221.000	-53.700	-236.500	-150.600	-227.900	-224.500	-219.600	-216.600
4	705.000	-223.600	-232.100	-227.900	-223.700	-219.600	-215.400	-212.200
5	878.900	-224.800	-204.900	-225.900	-221.800	-216.300	-213.500	-209.300
6	368.800	-214.600	767.100	-222.400	-218.200	-214.000	-209.900	-205.200
7	1134.600	-16.100	-220.900	-217.300	-26.900	-208.900	-165.500	-201.600
8	463.400	242.600	-217.200	-211.100	-209.900	-204.800	-201.600	-197.400
GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-267.449	-179.912	-273.760	-271.028	-267.890	328.785	-261.232	194.093
2	385.514	-269.462	-270.988	-267.890	-256.612	-260.841	-257.694	-256.476
3	-217.680	-265.048	-265.567	-264.478	-261.232	-259.232	-255.193	-217.286
4	93.670	-263.113	-259.208	231.766	104.331	-255.543	-251.695	-247.676
5	170.344	-253.109	-259.458	-259.083	-255.781	-251.695	-247.676	-241.656
6	-254.344	-144.688	-256.738	-257.682	-247.554	-249.814	-245.612	-241.215
7	-257.225	-251.567	-256.548	-247.467	-249.814	-245.612	-241.215	-235.641
8	-252.987	-249.602	-253.573	-248.503	362.460	-241.115	-234.520	-229.688
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-253.327	-164.417	-256.998	-253.110	-248.935	348.652	-240.586	215.376
2	401.009	-252.699	-253.070	-248.935	-236.745	-240.196	-236.412	-234.707
3	-200.918	-247.130	-246.611	-244.611	-240.586	-237.949	-233.424	-195.191
4	111.588	-244.157	-239.341	252.411	125.613	-233.774	-229.600	-225.426
5	189.299	-233.242	-238.812	-237.800	-234.012	-229.600	-225.426	-219.434
6	-234.477	-124.042	-235.456	-235.913	-225.459	-227.564	-223.390	-219.216
7	-236.580	-230.284	-234.780	-225.372	-227.564	-223.390	-219.215	-214.074
8	-231.704	-227.833	-231.478	-226.253	384.682	-219.115	-212.953	-208.779

Table E3: ARMA and GARCH Models for J255 return deviation

J256 Return Deviation

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-275.140	-264.109	-196.423	-252.802	-62.687	-3.889	52.329	65.045
2	-269.392	-271.673	-233.751	-245.607	3.391	-18.623	-195.248	72.554
3	-245.397	-95.330	-176.598	-257.048	-73.367	12.299	76.358	78.455
4	-130.883	-271.402	-243.010	-249.639	10.185	27.906	83.623	74.850
5	-262.619	-257.712	-239.204	-132.555	21.182	35.193	81.869	91.640
6	-259.582	-94.547	-173.702	-8.104	22.204	50.873	-65.173	71.381
7	-80.431	-230.158	-170.257	-47.152	35.184	54.238	91.822	88.739
8	-197.152	-255.987	-111.727	0.028	-226.916	-188.609	93.751	90.805
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-256.184	-244.242	-175.778	-231.520	-40.918	18.206	74.579	87.268
2	-249.525	-251.028	-212.468	-223.838	25.486	3.627	-173.026	94.554
3	-224.752	-74.047	-154.829	-234.953	-51.117	34.521	98.357	100.022
4	-109.600	-249.633	-220.915	-227.389	32.408	49.905	105.190	95.759
5	-240.850	-235.617	-216.954	-110.332	43.182	56.759	102.779	111.651
6	-237.487	-72.297	-151.480	13.895	43.770	71.783	-45.162	90.233
7	-58.181	-207.936	-148.258	-25.586	56.093	74.248	110.674	106.151
8	-174.929	-233.988	-90.161	20.938	-206.905	-169.757	111.163	106.474

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-283.601	-277.951	-224.763	-269.611	-208.962	-164.748	5.958	29.849
2	-278.926	-104.610	-94.371	-222.773	-268.560	-240.172	46.035	38.961
3	-275.966	-91.418	-88.375	-267.413	-79.417	-191.874	54.012	57.697
4	-277.157	-99.446	-87.676	-270.000	-149.917	-113.782	59.819	60.403
5	-274.819	-95.747	-110.210	-264.896	-173.658	-220.128	62.792	61.569
6	-272.569	-164.673	-51.466	-266.663	-74.159	33.957	12.423	68.623
7	-268.117	-87.591	-85.096	-251.814	-171.255	-35.214	54.899	68.656
8	-262.673	-106.154	-61.011	-259.463	-205.752	-250.667	69.568	81.087
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-269.478	-262.455	-208.000	-251.693	-190.007	-144.881	26.604	51.131
2	-263.431	-87.847	-76.452	-203.817	-248.693	-219.526	67.317	60.729
3	-259.204	-73.500	-69.420	-247.546	-58.771	-170.591	75.781	79.792
4	-259.239	-80.491	-67.809	-249.355	-128.635	-92.013	81.914	82.653
5	-255.864	-75.880	-89.564	-243.613	-151.889	-198.033	85.042	83.792
6	-252.702	-144.027	-30.184	-244.894	-52.064	56.207	34.646	90.622
7	-247.471	-66.309	-63.327	-229.719	-149.004	-12.991	76.899	90.223
8	-241.390	-84.385	-38.916	-237.213	-183.529	-228.668	91.135	101.997

ARMA model (Input variables with 1 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-282.610	-87.466	-277.864	-279.853	-238.220	-271.136	39.350	24.995
2	-278.067	-253.226	-275.082	-275.902	-201.565	-253.110	44.293	39.790
3	-280.149	-275.899	-279.062	-87.589	-264.201	-252.105	47.641	44.934
4	-277.492	-264.787	-273.359	-279.914	-268.219	-247.195	50.903	41.378
5	-276.389	-61.495	-265.316	-111.797	-176.583	-254.538	50.427	56.618
6	-271.929	-260.534	-258.778	-261.314	-184.860	-259.897	-224.379	-18.274
7	-273.644	-272.561	-266.502	-267.494	-152.523	-257.888	-153.328	64.762
8	-275.885	-218.935	-267.127	-259.087	-260.769	-255.821	42.407	72.183
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-271.529	-74.817	-263.742	-264.357	-221.457	-253.217	58.305	44.862
2	-265.418	-239.103	-259.586	-259.139	-183.647	-234.155	64.160	60.436
3	-266.026	-260.403	-262.299	-69.671	-245.245	-232.238	68.287	66.217
4	-261.996	-248.024	-255.441	-260.959	-248.352	-226.549	72.185	63.147
5	-259.626	-43.577	-246.360	-91.930	-155.937	-233.256	72.196	78.713
6	-254.010	-241.579	-238.911	-240.669	-163.578	-238.128	-202.284	3.976
7	-254.689	-252.694	-245.857	-246.211	-130.754	-235.793	-131.078	86.985
8	-256.018	-198.290	-245.845	-237.318	-238.674	-233.570	64.629	94.183

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	261.732	317.351	-277.552	-276.838	-270.327	-266.479	-265.494	-259.020
2	203.264	-160.786	-274.015	-270.327	-266.479	-265.283	-259.020	-256.895
3	-251.196	-260.026	-270.327	-269.273	-265.283	-258.258	-256.895	-252.288
4	318.636	-244.513	-255.423	-253.368	-258.877	-254.480	-250.016	-247.456
5	-237.827	-253.894	-263.079	-261.081	-256.683	-249.873	-247.456	-242.383
6	107.318	-247.164	138.752	-256.683	-249.873	-245.041	-240.111	-237.050
7	-241.165	-247.202	-254.480	-249.873	-246.027	-239.968	-234.778	-231.436
8	63.141	77.726	-250.054	-245.220	-240.954	-234.275	-231.436	-223.390
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	280.688	337.218	-256.907	-255.556	-248.558	-244.384	-243.244	-236.798
2	223.131	-140.140	-252.733	-248.558	-244.384	-243.033	-236.798	-234.895
3	-230.550	-238.743	-248.558	-247.178	-243.033	-236.035	-234.895	-230.721
4	339.919	-222.744	-233.328	-231.118	-236.655	-232.480	-228.449	-226.546
5	-216.058	-231.799	-240.829	-238.858	-234.684	-228.306	-226.546	-222.372
6	129.413	-224.914	160.974	-234.684	-228.306	-224.132	-220.100	-218.198
7	-218.915	-224.980	-232.480	-228.306	-225.117	-219.957	-215.926	-214.023
8	85.363	99.726	-228.487	-224.311	-220.943	-215.423	-214.023	-207.721

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-275.340	-154.241	-273.389	-280.281	-272.069	-273.881	-270.487	286.143
2	-272.998	-275.104	-277.949	-277.144	-262.792	-270.485	-266.950	131.431
3	-270.786	-267.812	-274.812	-265.736	-264.489	260.759	56.835	-47.347
4	-267.767	-268.900	-256.313	-270.487	-265.680	260.578	-259.414	-191.389
5	296.815	-265.747	-268.195	-266.949	-225.534	35.732	-255.394	403.531
6	196.003	-259.687	-255.234	-255.178	-259.413	-249.705	-251.192	390.010
7	11.845	-199.271	-258.769	-259.841	-254.855	-251.192	-243.302	-227.851
8	282.254	-245.969	-257.461	-256.543	-252.341	-247.943	-243.336	-218.372
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-261.218	-138.746	-256.626	-262.363	-253.113	-254.014	-249.841	307.425
2	-257.503	-258.341	-260.030	-258.188	-242.925	-249.840	-245.667	153.199
3	-254.024	-249.894	-255.856	-245.869	-243.844	282.041	78.604	-25.252
4	-249.849	-249.945	-236.446	-249.841	-244.397	282.347	-237.319	-169.139
5	315.770	-245.880	-247.550	-245.667	-203.765	57.827	-233.144	425.754
6	215.870	-239.041	-233.952	-233.409	-237.318	-227.455	-228.970	412.010
7	32.491	-177.988	-237.000	-237.746	-232.605	-228.969	-221.302	-206.284
8	303.537	-224.200	-235.366	-234.293	-230.118	-225.944	-221.769	-197.463

GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-280.759	-282.670	18.695	-284.890	-281.983	-278.964	-275.827	26.127
2	-278.303	-280.085	-275.384	-281.983	-278.964	-275.827	-267.061	168.912
3	-277.679	670.901	-271.837	-278.964	-275.714	-272.564	-259.546	-262.285
4	-273.073	-270.674	-276.977	-275.827	-266.789	-263.393	-264.145	-256.445
5	-262.539	-270.925	-273.840	-264.920	-269.168	-265.602	-256.175	-258.095
6	-266.724	-268.334	-265.440	-263.642	-265.631	33.111	-257.588	-244.937
7	52.646	-171.827	-261.749	-265.631	-60.013	-258.095	108.357	-248.778
8	-262.236	-261.750	-264.292	-261.897	-252.368	-254.090	89.437	-245.491
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-269.678	-270.020	32.818	-269.395	-265.220	-261.046	-256.872	45.995
2	-265.654	-265.963	-259.888	-265.220	-261.046	-256.872	-247.194	189.557
3	-263.557	686.397	-255.074	-261.046	-256.759	-252.697	-238.900	-241.003
4	-257.578	-253.912	-259.059	-256.872	-246.922	-242.748	-242.862	-234.676
5	-245.776	-253.007	-254.885	-245.053	-248.523	-244.320	-234.406	-236.000
6	-248.806	-249.379	-245.572	-242.996	-244.349	54.880	-235.493	-222.686
7	71.601	-151.960	-241.104	-244.349	-38.245	-236.000	130.607	-226.556
8	-242.369	-241.105	-243.010	-240.128	-230.273	-231.840	111.660	-223.491

Table E4: ARMA and GARCH Models for J256 return deviation

IPD Retail Return

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-254.857	-245.933	-209.386	70.168	104.488	72.842	113.809	137.406
2	-242.308	-234.816	-191.956	104.647	115.213	96.292	136.673	155.283
3	-176.427	-184.598	-171.967	112.363	131.751	130.504	145.866	165.148
4	-118.098	-203.429	4.489	120.544	133.502	98.663	162.902	169.274
5	-224.501	-191.217	-139.837	136.516	149.369	118.681	182.367	179.626
6	18.706	-169.258	-123.591	134.376	171.551	154.326	133.800	203.217
7	-58.457	-158.711	-40.199	145.210	184.604	178.481	202.928	218.795
8	-102.045	81.950	2.707	148.608	-60.479	50.134	225.800	242.362
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-248.082	-241.508	-207.775	68.446	98.851	62.637	98.297	115.747
2	-237.882	-233.205	-193.678	99.011	105.009	80.780	115.014	126.517
3	-174.816	-186.320	-177.603	102.158	116.240	108.845	117.099	128.171
4	-119.820	-209.065	-5.715	105.032	111.843	69.897	125.925	122.808
5	-230.138	-201.421	-155.349	114.857	120.603	81.704	135.901	122.180
6	8.502	-184.770	-145.250	105.610	134.574	107.860	76.353	133.036
7	-73.968	-180.370	-68.965	108.233	138.138	121.034	132.747	133.795
8	-123.704	53.184	-34.270	102.142	-117.926	-20.047	140.800	140.044

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-279.638	-275.947	-163.316	-251.024	-217.120	-32.644	95.220	71.040
2	-209.621	-273.196	-100.776	-163.989	-153.802	28.178	-243.262	82.543
3	-249.906	-272.606	-99.136	13.497	37.731	42.504	111.495	86.343
4	-274.076	-53.984	-88.824	-118.295	-2.359	-70.027	-81.651	102.332
5	-109.740	-269.766	-26.326	-5.153	62.599	52.075	-183.599	105.476
6	-239.518	-195.666	-64.756	40.231	69.101	63.228	78.483	108.375
7	-86.266	-217.794	-94.350	40.077	76.362	72.964	-103.061	116.914
8	-143.163	-112.531	-41.769	46.070	-238.604	-229.929	57.194	126.251
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-266.294	-262.500	-149.985	-238.047	-204.756	-21.178	105.477	79.744
2	-196.174	-259.865	-87.799	-151.625	-142.336	38.434	-234.558	89.318
3	-236.575	-259.629	-86.773	24.963	47.988	51.209	118.270	90.769
4	-261.098	-41.620	-77.358	-108.038	6.346	-63.252	-77.225	103.944
5	-97.376	-258.300	-16.069	3.552	69.374	56.500	-181.987	103.754
6	-228.052	-185.409	-56.051	47.006	73.527	64.839	76.761	102.738
7	-76.009	-209.089	-87.576	44.503	77.973	71.242	-108.697	106.710
8	-134.458	-105.757	-37.343	47.681	-240.326	-235.565	46.990	110.739

ARMA model (Input variables with 1 lag)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-286.106	-253.732	-277.213	-301.744	-220.609	-66.864	-248.416	26.119
2	-284.129	-207.902	-287.419	-294.449	-134.370	-23.229	-255.374	59.907
3	-290.035	-128.273	-228.443	-243.085	-100.454	-10.620	-50.377	54.793
4	-264.573	-242.534	-205.639	-248.976	18.008	24.839	-243.639	67.148
5	-222.978	-181.840	-48.238	-234.884	29.986	36.719	-257.026	72.427
6	-70.277	-282.915	-268.960	-9.118	27.815	37.972	-240.419	80.001
7	-252.973	-210.259	-283.823	-2.740	34.050	40.141	-255.001	86.116
8	-142.816	-28.515	-192.657	5.960	-63.186	-202.957	16.613	92.245
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-275.042	-241.845	-264.664	-288.705	-207.265	-53.417	-235.085	39.097
2	-272.243	-195.353	-274.380	-281.105	-120.923	-9.897	-242.396	72.270
3	-277.486	-115.234	-215.099	-229.638	-87.123	2.357	-38.013	66.259
4	-251.534	-229.190	-192.192	-235.645	30.985	37.202	-232.173	77.405
5	-209.634	-168.393	-34.906	-221.907	42.349	48.185	-246.769	81.131
6	-56.830	-269.584	-255.983	3.246	39.281	48.228	-231.714	86.775
7	-239.642	-197.282	-271.460	8.726	44.307	48.846	-248.227	90.542
8	-129.839	-16.152	-181.191	16.216	-54.481	-196.182	21.039	93.857

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-29.262	-109.051	-100.596	-131.752	-132.507	-120.108	-104.255	355.075
2	523.426	-111.322	633.018	-119.242	-118.673	-114.931	-91.814	441.312
3	831.949	-81.676	9.443	152.321	-110.638	-103.537	-89.028	90.904
4	170.340	85.695	-39.846	-112.588	-104.123	-83.494	-85.021	-62.175
5	-64.649	-88.111	-103.496	-105.920	-94.676	-76.952	-74.763	-40.949
6	202.230	586.529	-91.058	-84.805	-73.064	-65.619	-59.863	-40.587
7	812.801	233.039	-86.188	-82.777	-72.438	-56.497	-34.720	-22.254
8	64.314	-94.959	-70.945	-68.338	-65.430	-29.441	-24.300	9.048
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-22.487	-104.626	-98.985	-133.473	-138.143	-130.312	-119.767	333.416
2	527.852	-109.711	631.296	-124.878	-128.877	-130.443	-113.472	412.546
3	833.561	-83.398	3.807	142.117	-126.150	-125.195	-117.794	53.927
4	168.619	80.059	-50.050	-128.099	-125.782	-112.260	-121.998	-108.642
5	-70.285	-98.315	-119.008	-127.579	-123.443	-113.929	-121.230	-98.396
6	192.026	571.018	-112.717	-113.571	-110.041	-112.085	-117.310	-110.769
7	797.290	211.380	-114.954	-119.754	-118.905	-113.944	-104.902	-107.254
8	42.655	-123.725	-107.923	-114.805	-122.877	-99.623	-109.300	-93.270

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-120.666	-136.820	-147.092	-155.144	-151.430	-160.263	-141.456	-143.575
2	-129.300	-139.135	-146.708	-158.726	-161.728	-153.410	-135.936	-145.162
3	-132.478	-132.346	-135.080	-151.328	-139.059	-151.157	-130.176	-124.528
4	-129.830	-147.740	-134.230	-153.683	-102.649	-145.307	-139.014	-132.275
5	-129.424	-137.459	-141.520	-139.024	-130.519	-123.951	-132.265	-125.005
6	-103.714	-135.128	166.230	-142.961	-139.013	-132.268	-110.050	-117.156
7	-132.820	-105.632	-100.041	-138.184	-117.112	-80.290	-75.905	-108.660
8	-109.190	-139.606	-129.700	-132.266	-79.596	-117.154	-91.483	-99.418
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-107.322	-123.373	-133.761	-142.166	-139.066	-148.797	-131.199	-134.870
2	-115.853	-125.804	-133.730	-146.362	-150.262	-143.153	-127.231	-138.388
3	-119.147	-119.369	-122.716	-139.862	-128.802	-142.453	-123.402	-120.103
4	-116.853	-135.377	-122.764	-143.427	-93.945	-138.533	-134.589	-130.664
5	-117.060	-125.993	-131.263	-130.319	-123.744	-119.526	-130.654	-126.727
6	-92.248	-124.871	174.935	-136.187	-134.588	-130.657	-111.772	-122.793
7	-122.563	-96.927	-93.266	-133.758	-115.500	-82.011	-81.541	-118.865
8	-100.486	-132.831	-125.274	-130.655	-81.318	-122.790	-101.687	-114.930

GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-154.515	-152.558	-147.017	-150.348	-153.042	-142.317	-145.166	-140.880
2	-145.066	-157.073	-153.632	-146.707	-149.213	-138.269	-140.880	-136.335
3	42.690	-151.657	-135.927	-146.175	-145.166	-133.983	-136.335	-124.608
4	37.524	-148.029	-94.015	-144.554	-140.880	-136.335	-131.505	-126.364
5	244.013	-142.873	-114.035	-140.268	-136.335	-131.505	-126.364	-120.880
6	-125.599	-123.752	-130.771	-135.723	-131.505	-126.364	-120.880	-115.018
7	-144.554	-114.190	-135.723	-130.893	-119.467	-120.880	-108.121	-101.840
8	54.378	-117.187	-130.893	-125.752	-120.880	-115.018	-101.840	-101.991
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-143.451	-140.672	-134.468	-137.309	-139.698	-128.869	-131.835	-127.903
2	-133.179	-144.524	-140.592	-133.362	-135.766	-124.938	-127.903	-123.971
3	55.239	-138.617	-122.583	-132.728	-131.835	-121.006	-123.971	-113.142
4	50.564	-134.685	-80.568	-131.222	-127.903	-123.971	-120.039	-116.107
5	257.357	-129.426	-100.704	-127.291	-123.971	-120.039	-116.107	-112.175
6	-112.152	-110.420	-117.794	-123.359	-120.039	-116.107	-112.176	-108.244
7	-131.222	-101.213	-123.359	-119.427	-109.210	-112.175	-101.347	-97.415
8	67.355	-104.823	-119.427	-115.495	-112.175	-108.244	-97.415	-100.380

Table E5: ARMA and GARCH Models for retail property return

IPD Office Return

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-306.683	-303.792	-334.716	-331.057	-252.037	-231.001	-289.883	-13.081
2	-303.880	-300.234	-247.178	-319.456	-276.124	-244.417	-266.599	-76.280
3	-310.343	-306.963	-327.904	-322.860	-256.123	-284.567	-212.720	-53.267
4	-307.083	-278.986	-307.232	-34.787	-306.143	-284.019	-264.703	-144.614
5	-308.013	-306.804	-305.674	-319.554	-297.241	-303.289	-246.924	-79.649
6	-308.283	-299.982	-310.371	-312.698	-305.275	-297.172	-281.567	-242.864
7	-307.347	-303.129	-313.188	-304.445	-292.319	-239.752	-286.455	67.200
8	-286.001	-279.072	-308.205	-281.431	-301.068	-300.643	-295.290	-270.878
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-295.619	-291.906	-322.167	-318.018	-238.692	-217.554	-276.552	-0.103
2	-291.994	-287.685	-234.138	-306.112	-262.677	-231.086	-253.621	-63.916
3	-297.794	-293.923	-314.560	-309.413	-242.792	-271.590	-200.356	-41.801
4	-294.043	-265.642	-293.785	-21.456	-293.166	-271.655	-253.237	-134.357
5	-294.669	-293.357	-292.343	-306.577	-284.877	-291.823	-236.667	-70.945
6	-294.836	-286.651	-297.394	-300.334	-293.809	-286.915	-272.862	-236.090
7	-294.016	-290.152	-300.825	-292.979	-282.062	-231.047	-279.680	71.626
8	-273.023	-266.709	-296.739	-271.174	-292.364	-293.868	-290.864	-269.266

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-312.285	53.622	-262.758	-77.730	23.839	12.788	-284.827	-10.597
2	-307.890	-243.645	81.554	64.360	-91.603	-37.300	57.330	-22.040
3	31.288	-296.697	-295.147	46.648	-229.346	46.667	60.606	-81.841
4	-306.995	-97.168	-311.492	12.820	70.962	69.017	64.161	-65.464
5	-317.345	-308.134	-293.910	52.559	-186.583	-208.402	48.227	-24.527
6	-293.069	-308.563	-284.414	-303.401	-277.064	-223.796	12.619	-30.709
7	-164.395	-304.153	-277.319	-289.977	-234.205	-289.043	34.601	-161.001
8	-249.236	-307.051	-307.589	-301.606	-278.020	-255.895	-266.259	-263.709
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-303.299	63.717	-251.693	-65.843	36.388	25.827	-271.483	2.851
2	-297.796	-232.580	93.440	76.909	-78.564	-23.956	70.777	-8.708
3	42.353	-284.811	-282.598	59.687	-216.001	60.114	73.937	-68.864
4	-295.108	-84.619	-298.453	26.164	84.409	82.348	77.139	-53.100
5	-304.796	-295.094	-280.566	66.006	-173.251	-195.425	60.590	-13.061
6	-280.029	-295.219	-270.967	-290.070	-264.087	-211.432	24.085	-20.452
7	-151.051	-290.706	-263.988	-277.000	-221.841	-277.577	44.858	-152.297
8	-235.789	-293.719	-294.612	-289.243	-266.554	-245.639	-257.554	-256.935

ARMA model (Input variables with 1 lag)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-312.463	144.174	-320.692	-144.241	74.093	3.688	-262.169	-163.192
2	63.477	-225.791	67.943	52.402	71.005	-112.674	-277.803	-148.362
3	48.899	-307.015	62.959	53.508	72.248	-322.181	-71.560	-77.627
4	-177.011	-313.052	-290.891	-301.309	-161.554	69.092	-153.612	-180.073
5	42.707	-142.043	-233.079	51.189	-240.909	23.782	31.269	-276.643
6	-316.159	-296.833	-306.042	-294.310	-282.803	-250.907	-219.778	-181.133
7	-317.903	-312.826	-318.946	40.041	-258.356	-195.012	-241.059	19.311
8	-137.562	-310.063	-294.095	-284.759	19.879	-284.750	-181.048	-233.378
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-304.713	153.160	-310.598	-133.177	85.980	16.237	-249.129	-149.848
2	72.463	-215.697	79.007	64.288	83.554	-99.635	-264.459	-134.915
3	58.993	-295.950	74.846	66.057	85.288	-308.837	-58.113	-64.296
4	-165.946	-301.165	-278.342	-288.269	-148.210	82.539	-140.280	-167.096
5	54.594	-129.494	-220.039	64.533	-227.462	37.113	44.247	-264.279
6	-303.610	-283.793	-292.698	-280.863	-269.472	-237.929	-207.414	-169.667
7	-304.863	-299.482	-305.499	53.372	-245.379	-182.648	-229.593	29.568
8	-124.218	-296.616	-280.763	-271.782	32.243	-273.284	-170.792	-224.673

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-157.377	-154.268	-150.998	-147.557	-143.930	-140.101	-136.053	-131.768
2	-154.268	-150.998	-147.557	-143.929	-140.101	-136.053	-131.768	-127.222
3	-150.998	-147.557	-143.930	-140.101	-136.053	-131.768	-127.222	-122.393
4	-147.557	-143.930	-140.101	-136.053	-131.768	-127.222	-122.393	-117.251
5	-143.930	-140.101	-136.053	-131.768	-127.222	-122.393	-117.251	-111.768
6	-140.101	-136.053	-131.768	-127.222	-122.393	-117.251	-111.768	-105.906
7	-136.053	-131.768	-127.222	-122.393	-117.251	-111.768	-105.906	-99.625
8	-131.768	-127.222	-122.393	-117.251	-111.768	-105.906	-99.625	-92.879
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-146.313	-142.381	-138.449	-134.518	-130.586	-126.654	-122.722	-118.790
2	-142.381	-138.449	-134.518	-130.585	-126.654	-122.722	-118.790	-114.858
3	-138.449	-134.518	-130.586	-126.654	-122.722	-118.790	-114.858	-110.927
4	-134.518	-130.586	-126.654	-122.722	-118.790	-114.858	-110.927	-106.995
5	-130.586	-126.654	-122.722	-118.790	-114.858	-110.927	-106.995	-103.063
6	-126.654	-122.722	-118.790	-114.858	-110.927	-106.995	-103.063	-99.131
7	-122.722	-118.790	-114.858	-110.927	-106.995	-103.063	-99.131	-95.199
8	-118.790	-114.858	-110.927	-106.995	-103.063	-99.131	-95.199	-91.267

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-160.514	-157.690	-154.728	-151.618	-148.349	-144.908	-141.280	-137.452
2	-157.690	-154.728	-151.618	-148.349	-144.908	-141.280	-137.452	-133.404
3	-154.728	-151.618	-148.349	-144.908	-141.280	-137.452	-133.404	-129.118
4	-151.618	-148.349	-144.908	-141.280	-137.452	-133.404	-129.118	-124.573
5	-148.349	-144.908	-141.280	-137.452	-133.404	-129.118	-124.573	-119.743
6	-144.908	-141.280	-137.452	-133.404	-129.118	-124.573	-119.743	-114.568
7	-141.280	-137.452	-133.404	-129.118	-124.573	-119.743	-114.602	-109.118
8	-137.452	-133.404	-129.118	-124.573	-119.743	-114.602	-109.118	-103.256
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-151.527	-147.596	-143.664	-139.732	-135.800	-131.868	-127.936	-124.005
2	-147.596	-143.664	-139.732	-135.800	-131.868	-127.936	-124.004	-120.073
3	-143.664	-139.732	-135.800	-131.868	-127.936	-124.005	-120.073	-116.141
4	-139.732	-135.800	-131.868	-127.936	-124.005	-120.073	-116.141	-112.209
5	-135.800	-131.868	-127.936	-124.005	-120.073	-116.141	-112.209	-108.277
6	-131.868	-127.936	-124.005	-120.073	-116.141	-112.209	-108.277	-104.312
7	-127.936	-124.005	-120.073	-116.141	-112.209	-108.277	-104.345	-100.414
8	-124.005	-120.073	-116.141	-112.209	-108.277	-104.345	-100.414	-96.482

GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-163.093	-160.398	-157.574	-154.612	-151.503	-148.233	-144.792	-141.165
2	-160.392	-157.574	-154.612	-151.503	-148.233	-144.792	-141.165	-137.336
3	-157.574	-154.612	-151.503	-148.233	-144.792	-141.165	-137.336	-133.288
4	-154.612	-151.503	-148.233	-144.792	-141.165	-137.336	-133.288	-129.003
5	-151.503	-148.233	-144.792	-141.165	-137.336	-133.288	-129.003	-124.457
6	-148.233	-144.792	-141.165	-137.336	-133.288	-129.003	-124.457	-119.628
7	-144.792	-141.165	-137.336	-133.288	-129.003	-124.457	-119.628	-114.486
8	-141.165	-137.336	-133.288	-129.003	-124.457	-119.628	-114.486	-109.003
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-155.343	-151.412	-147.480	-143.548	-139.616	-135.684	-131.752	-127.821
2	-151.406	-147.480	-143.548	-139.616	-135.684	-131.752	-127.821	-123.889
3	-147.480	-143.548	-139.616	-135.684	-131.752	-127.821	-123.889	-119.957
4	-143.548	-139.616	-135.684	-131.752	-127.821	-123.889	-119.957	-116.025
5	-139.616	-135.684	-131.752	-127.821	-123.889	-119.957	-116.025	-112.093
6	-135.684	-131.752	-127.821	-123.889	-119.957	-116.025	-112.093	-108.161
7	-131.752	-127.821	-123.889	-119.957	-116.025	-112.093	-108.161	-104.230
8	-127.821	-123.889	-119.957	-116.025	-112.093	-108.161	-104.230	-100.298

Table E6: ARMA and GARCH Models for office property return

IPD Industrial Return

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-272.174	-32.1627	-278.601	-276.068	-22.8232	-279.175	-33.1609	67.6894
2	-262.285	-49.8802	-236.542	-166.739	-13.852	-223.224	-11.2302	79.3809
3	-277.047	-205.842	-150.105	-66.4894	6.171	-265.676	-15.2756	86.6251
4	-69.251	-60.6815	-214.738	-203.471	-263.718	-249.648	-19.2012	93.0629
5	-141.989	-137.658	7.1151	2.7107	-246.645	-260.08	-18.8707	101.7384
6	-33.596	-27.6692	-130.305	1.5019	-262.692	-221.645	69.0477	108.5349
7	-67.128	-8.9037	8.4703	0.9375	-214.304	-168.298	90.5177	117.048
8	-17.885	23.0261	-15.7585	23.5471	-0.1559	63.2349	116.1187	126.3094
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-258.830	-18.716	-265.270	-263.091	-10.459	-267.709	-22.904	76.394
2	-248.838	-36.549	-223.564	-154.375	-2.386	-212.968	-2.526	86.155
3	-263.715	-192.865	-137.742	-55.023	16.428	-256.971	-8.501	91.051
4	-56.274	-48.318	-203.272	-193.214	-255.013	-242.873	-14.776	94.674
5	-129.625	-126.192	17.372	11.415	-239.871	-255.655	-17.259	100.017
6	-22.130	-17.412	-121.600	8.276	-258.266	-220.033	67.326	102.899
7	-56.871	-0.199	15.245	5.363	-212.692	-170.020	84.882	106.844
8	-9.180	29.801	-11.333	25.158	-1.878	57.599	105.914	110.798

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-283.349	-267.554	-302.751	-37.328	-59.030	-46.646	-39.067	52.054
2	-279.979	-265.908	-106.079	-235.919	-39.389	-28.265	-150.917	57.541
3	-298.844	-161.890	-285.774	-44.698	-31.042	-32.795	-7.721	64.376
4	-237.249	-204.331	-180.033	-265.260	-48.593	-1.896	14.062	67.524
5	-42.242	-290.078	-41.852	-217.802	-251.382	-43.528	-75.092	74.459
6	-156.561	-26.803	-252.407	-206.232	-234.502	-6.529	-64.809	79.677
7	-28.141	-232.370	-64.901	-12.461	-106.225	-5.913	31.389	73.756
8	-58.584	-40.406	-99.131	-252.770	-126.427	41.312	47.694	93.344
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-272.284	-255.668	-290.202	-24.288	-45.686	-33.199	-25.736	65.031
2	-268.092	-253.359	-93.039	-222.575	-25.942	-14.934	-137.939	69.905
3	-286.295	-148.850	-272.429	-31.251	-17.710	-19.817	4.643	75.842
4	-224.209	-190.987	-166.586	-251.928	-35.616	10.468	25.528	77.781
5	-28.898	-276.631	-28.520	-204.825	-239.018	-32.062	-64.836	83.164
6	-143.114	-13.472	-239.429	-193.868	-223.036	3.728	-56.104	86.451
7	-14.810	-219.392	-52.537	-0.995	-95.968	2.792	38.164	78.181
8	-45.607	-28.042	-87.665	-242.513	-117.722	48.086	52.119	94.955

ARMA model (Input variables with 1 lag)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-284.007	-290.040	-292.321	-288.795	-193.144	-246.630	-257.001	-18.599
2	-287.601	-277.026	-139.293	-281.596	-59.192	-191.982	-255.668	-1.484
3	-296.535	-293.612	-264.200	-290.223	-271.061	-157.114	-257.357	-7.320
4	-293.108	-286.499	-257.025	-291.521	-93.471	-92.320	10.401	66.091
5	-136.410	-277.923	-272.534	-271.851	-84.475	-248.570	-270.699	-5.586
6	-172.103	-282.214	-218.744	-278.280	-28.984	-43.082	-85.568	13.167
7	-254.558	-264.518	-41.788	-196.120	-54.397	-83.418	-86.985	18.943
8	-105.859	-205.122	-265.910	-205.515	-136.630	-7.544	-22.924	-60.189
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-275.021	-279.945	-281.257	-276.909	-180.595	-233.590	-243.657	-5.152
2	-277.507	-265.962	-127.406	-269.047	-46.153	-178.638	-242.221	11.847
3	-285.470	-281.725	-251.651	-277.184	-257.717	-143.667	-244.025	5.657
4	-281.222	-273.950	-243.985	-278.177	-80.024	-78.989	23.379	78.455
5	-123.861	-264.884	-259.190	-258.404	-71.144	-235.593	-258.335	5.880
6	-159.063	-268.870	-205.297	-264.949	-16.007	-30.718	-74.102	23.424
7	-241.214	-251.071	-28.457	-183.142	-42.033	-71.952	-76.728	27.648
8	-92.412	-191.791	-252.933	-193.151	-125.164	2.712	-14.219	-53.415

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-189.413	-101.677	-170.451	-201.846	-197.325	-199.251	-187.334	-181.867
2	256.621	-171.856	-187.243	-197.301	-192.482	-187.350	-188.585	-182.911
3	44.513	-186.141	182.135	-192.497	-187.335	-188.407	-175.987	-169.716
4	-193.775	-154.934	-179.391	-187.333	-188.615	-175.993	-176.444	-169.219
5	13.717	-151.571	-143.404	-188.609	-182.831	-169.707	-162.993	-155.721
6	236.467	-146.702	-181.880	-182.771	-176.432	-169.889	-162.454	-154.630
7	18.160	-132.368	-176.024	-169.736	-169.568	-162.500	-147.856	-139.370
8	-108.184	107.958	-169.714	-162.962	-162.473	-147.874	-139.402	-130.109
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-176.068	-88.229	-157.120	-188.868	-184.961	-187.785	-177.078	-173.162
2	270.068	-158.524	-174.266	-184.937	-181.016	-177.093	-179.880	-176.136
3	57.845	-173.163	194.498	-181.031	-177.078	-179.702	-169.212	-165.291
4	-180.798	-142.570	-167.924	-177.076	-179.910	-169.218	-172.019	-167.608
5	26.081	-140.105	-133.147	-179.904	-176.056	-165.282	-161.382	-157.443
6	247.933	-136.445	-173.175	-175.997	-172.006	-168.277	-164.176	-160.266
7	28.417	-123.664	-169.249	-165.310	-167.957	-164.222	-153.492	-149.575
8	-99.479	114.732	-165.288	-161.351	-164.195	-153.510	-149.606	-145.621

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	7.632	-188.649	-185.379	-181.938	-178.311	-174.558	-175.716	-171.782
2	-188.413	-161.692	-181.938	-178.311	-174.482	-170.510	-171.430	-167.236
3	41.905	-176.193	-178.311	-174.482	-170.434	-166.225	-166.884	-162.407
4	-145.861	-151.499	-174.482	-170.434	-166.149	-161.679	-161.099	-157.266
5	153.894	-129.641	-170.500	-166.214	-161.668	-156.850	-156.914	-151.781
6	79.785	-162.648	-168.532	-163.986	-159.157	-154.016	-151.420	-147.048
7	-159.490	-155.611	-164.147	-159.317	-154.176	-148.692	-145.581	-137.963
8	-131.695	-164.090	-159.317	-154.176	-148.692	-142.830	-139.291	-134.018
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	18.697	-176.762	-172.830	-168.898	-164.967	-161.111	-162.384	-158.804
2	-176.527	-149.143	-168.898	-164.967	-161.035	-157.179	-158.453	-154.872
3	54.454	-163.153	-164.967	-161.035	-157.103	-153.247	-154.521	-150.941
4	-132.822	-138.155	-161.035	-157.103	-153.171	-149.316	-149.633	-147.009
5	167.238	-116.194	-157.168	-153.236	-149.305	-145.384	-146.658	-143.077
6	93.232	-149.317	-155.554	-151.623	-147.691	-143.759	-142.715	-140.273
7	-146.159	-142.634	-151.783	-147.851	-143.920	-139.988	-138.807	-133.538
8	-118.718	-151.727	-147.851	-143.920	-139.988	-136.056	-134.865	-132.406

GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-144.114	-158.857	-137.136	-177.526	-174.257	-170.815	-167.188	-163.359
2	-30.838	-153.586	-177.526	-174.257	-170.815	-167.188	-163.359	-159.312
3	359.562	-177.526	-174.257	-170.815	-167.188	-163.359	-159.312	-155.026
4	-177.526	-174.257	-170.815	-167.188	-163.359	-159.312	-155.026	-150.481
5	-174.257	-170.815	-167.188	-163.359	-159.312	-155.026	-150.481	-145.651
6	-170.815	-167.188	-163.359	-159.312	-155.026	-150.481	-145.651	-140.510
7	-165.833	-163.359	-159.312	-155.026	-150.481	-145.651	-141.587	-135.026
8	-163.363	-159.316	-155.030	-150.484	-145.655	-140.514	-135.030	-129.168
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-135.128	-148.762	-126.071	-165.640	-161.708	-157.776	-153.844	-149.912
2	-20.744	-142.521	-165.640	-161.708	-157.776	-153.844	-149.912	-145.980
3	370.626	-165.640	-161.708	-157.776	-153.844	-149.912	-145.980	-142.049
4	-165.640	-161.708	-157.776	-153.844	-149.912	-145.980	-142.049	-138.117
5	-161.708	-157.776	-153.844	-149.912	-145.980	-142.049	-138.117	-134.185
6	-157.776	-153.844	-149.912	-145.980	-142.049	-138.117	-134.185	-130.253
7	-152.489	-149.912	-145.980	-142.049	-138.117	-134.185	-131.330	-126.321
8	-149.916	-145.984	-142.052	-138.121	-134.189	-130.257	-126.325	-122.393

Table E7: ARMA and GARCH Models for industrial property return

IPD Retail Return Deviation

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	70.000	143.400	153.800	246.800	346.200	341.200	392.600	377.800
2	58.100	161.600	174.000	271.400	372.400	384.800	427.100	425.300
3	121.900	182.100	198.200	295.800	408.800	425.000	479.400	480.300
4	175.300	206.100	225.900	369.700	446.400	498.800	539.500	672.200
5	210.700	233.300	259.200	396.100	481.900	565.800	612.400	664.600
6	238.300	265.500	298.000	422.000	500.700	584.100	681.500	2715.400
7	265.600	309.800	351.600	474.000	621.100	744.900	874.100	1017.300
8	305.600	352.600	406.500	551.800	614.500	767.200	985.200	1484.600
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	19.400	80.800	77.100	153.300	232.700	203.900	226.400	176.200
2	-4.500	84.800	80.500	158.000	235.200	218.600	225.400	179.400
3	45.100	88.600	84.800	158.500	242.600	223.300	233.600	178.300
4	81.800	92.700	88.600	203.600	244.700	253.000	237.400	296.700
5	97.300	96.100	93.000	194.400	236.100	263.800	237.000	190.100
6	101.000	99.300	96.300	176.100	198.700	208.700	207.000	2100.600
7	99.500	108.100	105.800	171.900	245.700	270.400	259.300	190.100
8	104.000	106.800	104.500	176.400	140.000	152.500	158.100	301.000

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-268.846	23.235	-38.097	-20.891	141.307	159.408	146.935	138.663
2	-270.624	28.955	-13.400	-145.237	149.316	168.254	68.897	158.182
3	-214.541	35.048	2.566	219.004	158.829	177.822	171.397	168.176
4	-14.807	41.300	48.457	151.918	163.868	185.206	193.874	182.691
5	-148.363	49.186	56.731	163.730	174.749	196.187	175.662	196.304
6	-10.956	56.696	67.406	150.017	167.572	188.129	217.348	200.201
7	-118.679	65.399	78.665	171.279	174.289	202.797	230.009	212.986
8	60.552	73.818	86.350	166.357	133.339	173.572	159.299	178.600
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-260.347	30.259	-32.943	-18.046	141.350	156.095	139.639	126.671
2	-263.600	34.110	-10.555	-145.194	146.003	160.957	56.905	140.681
3	-209.387	37.893	2.609	215.691	151.532	165.830	153.896	144.230
4	-11.962	41.343	45.143	144.622	151.876	167.704	169.927	151.216
5	-148.320	45.872	49.434	151.738	157.248	172.241	144.187	156.036
6	-14.269	49.400	55.414	132.516	143.626	156.653	177.080	149.652
7	-125.975	53.407	61.163	147.332	142.814	162.529	179.460	150.390
8	48.560	56.317	62.404	134.882	93.070	123.023	96.703	101.838

ARMA model (Input variables with 1 lag)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-130.000	-146.971	-333.357	90.027	79.914	30.897	86.999	106.248
2	-42.477	-77.506	-332.043	94.266	82.744	87.106	22.972	88.518
3	-195.116	-211.024	-308.735	102.551	89.757	94.266	88.025	93.695
4	-126.377	14.105	-205.726	66.231	97.092	82.430	98.450	91.532
5	-58.440	-28.727	-135.498	83.286	102.072	25.696	114.260	102.185
6	-123.440	12.415	-178.811	79.877	100.350	114.134	115.463	107.282
7	20.340	21.435	12.185	89.117	109.983	110.252	118.185	114.975
8	-113.683	23.517	-169.168	91.281	-98.577	84.850	78.956	96.211
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-119.041	-135.796	-322.186	100.954	90.332	40.516	95.498	113.272
2	-31.302	-66.334	-321.116	104.684	92.363	95.605	29.996	93.673
3	-183.945	-200.097	-298.316	112.170	98.256	101.290	93.180	96.540
4	-115.450	24.523	-196.107	74.730	104.116	87.585	101.295	91.575
5	-48.022	-19.108	-127.000	90.310	107.226	28.541	114.303	98.872
6	-113.821	20.914	-171.787	85.032	103.195	114.177	112.149	99.985
7	28.839	28.459	17.340	91.963	110.026	106.938	110.889	102.983
8	-106.659	28.671	-166.323	91.324	-101.891	77.553	66.964	78.710

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	218.200	272.800	114.800	85.900	200.400	111.400	126.000	546.400
2	272.900	336.100	218.800	80.600	168.900	118.900	168.600	237.800
3	384.600	321.800	312.100	281.900	382.600	154.800	235.600	272.400
4	722.400	742.600	136.100	262.500	414.000	236.500	291.500	414.800
5	277.800	176.600	144.000	286.200	799.400	342.800	457.600	573.100
6	526.900	427.900	130.200	369.200	392.800	431.300	569.700	667.100
7	789.000	669.800	328.000	386.400	451.500	556.400	672.200	927.600
8	484.700	603.100	381.100	421.100	547.200	674.000	894.700	1258.800
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	167.679	210.186	38.023	-7.583	86.999	-25.882	-40.141	344.756
2	210.299	259.371	125.253	-32.776	31.620	-47.251	-33.074	-8.012
3	307.883	228.330	198.683	144.610	216.441	-46.882	-10.275	-29.691
4	628.929	629.235	-1.144	96.276	212.342	-9.316	-10.514	39.433
5	164.355	39.293	-22.160	84.604	553.526	40.749	82.222	98.600
6	389.580	261.680	-71.403	123.367	90.805	55.909	95.168	52.306
7	622.820	468.151	82.123	84.339	76.086	81.856	57.402	100.411
8	283.064	357.242	79.010	45.685	72.685	59.249	67.509	75.162

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-80.739	-9.883	-320.094	-105.469	25.828	-224.247	-204.925	-196.970
2	424.566	417.920	-186.711	-229.037	-223.242	-40.785	60.347	-247.848
3	80.978	268.651	173.148	182.699	72.006	-198.612	60.103	-158.496
4	685.564	54.167	-8.556	-171.251	-220.037	101.971	-258.339	-248.450
5	204.352	348.022	43.377	358.488	-232.857	64.707	-176.767	-170.282
6	115.937	-192.839	420.332	-201.998	276.120	-230.146	237.267	-217.417
7	578.207	685.794	-43.935	18.435	-243.469	-222.292	-217.546	-187.449
8	191.523	-223.739	379.660	-18.632	-198.330	-176.815	-155.031	462.376
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-72.240	-2.859	-314.939	-102.624	25.871	-227.560	-212.222	-208.962
2	431.590	423.075	-183.866	-228.994	-226.556	-48.082	48.355	-265.350
3	86.132	271.496	173.191	179.385	64.710	-210.604	42.602	-182.442
4	688.409	54.210	-11.870	-178.548	-232.029	84.469	-282.285	-279.925
5	204.395	344.708	36.080	346.496	-250.359	40.760	-208.243	-210.550
6	112.623	-200.136	408.340	-219.499	252.174	-261.621	196.999	-267.966
7	570.910	673.802	-61.436	-5.511	-274.944	-262.560	-268.094	-250.045
8	179.531	-241.240	355.713	-50.107	-238.598	-227.364	-217.627	385.614

GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-161.998	-207.342	-266.487	-353.235	-232.863	80.158	-360.432	-75.671
2	381.410	94.686	-204.784	-42.985	-301.811	15.064	10.559	-292.800
3	60.167	-204.460	-66.244	210.436	-358.709	-15.011	2.495	-338.006
4	687.896	117.248	32.015	-103.663	-31.659	-36.370	-344.781	-142.493
5	746.605	154.262	266.900	-193.861	107.943	-241.275	-50.503	98.070
6	90.347	-112.156	-245.898	-351.655	-318.414	-339.086	-330.929	-323.973
7	-286.350	-166.147	73.447	194.801	123.508	309.465	21.532	-307.374
8	-64.037	-336.307	-341.869	-220.250	-238.715	-323.290	-315.412	-304.177
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-151.039	-196.166	-255.316	-342.308	-222.444	89.777	-351.933	-68.647
2	392.585	105.857	-193.857	-32.567	-292.192	23.563	17.582	-287.646
3	71.339	-193.533	-55.826	220.055	-350.210	-7.987	7.650	-335.160
4	698.823	127.666	41.634	-95.164	-24.635	-31.215	-341.936	-142.450
5	757.023	163.881	275.399	-186.837	113.098	-238.430	-50.460	94.756
6	99.965	-103.657	-238.874	-346.500	-315.569	-339.043	-334.243	-331.270
7	-277.851	-159.123	78.602	197.647	123.551	306.151	14.235	-319.366
8	-57.013	-331.152	-339.024	-220.207	-242.029	-330.587	-327.404	-321.678

Table E8: ARMA and GARCH Models for retail property return deviation

IPD Office Return Deviation

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	125.100	232.100	262.700	379.200	488.600	546.600	600.200	649.400
2	155.900	265.200	302.500	418.100	545.800	625.800	701.800	793.300
3	213.600	304.600	348.500	477.000	625.700	737.300	843.300	1011.400
4	302.800	352.900	409.900	585.000	726.300	900.700	1063.100	1435.000
5	317.100	413.000	488.300	687.400	870.200	1101.400	1416.100	2092.000
6	397.700	490.900	590.200	817.000	1039.200	1428.700	2130.400	4337.900
7	491.800	594.500	734.500	1028.300	1438.300	2124.600	4299.400	Inf
8	596.800	737.900	949.600	1382.100	2055.200	4216.700	Inf	4349.900
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	11.652	94.867	96.526	177.507	242.779	244.522	224.829	174.892
2	18.658	99.045	100.905	172.290	243.729	250.392	227.259	178.506
3	47.373	102.971	102.625	174.972	250.278	262.788	228.519	184.220
4	101.171	107.057	107.826	209.550	251.826	285.938	235.998	251.395
5	71.264	110.994	112.875	212.900	255.404	274.207	232.424	191.597
6	95.613	115.494	115.687	202.229	212.049	245.085	229.935	279.336
7	116.399	120.029	119.741	201.169	254.627	224.184	240.767	207.332
8	122.255	123.108	122.455	198.519	154.735	158.145	168.986	247.211

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-196.172	36.099	41.722	-38.496	152.045	150.508	186.011	149.381
2	-206.155	7.291	44.484	159.619	162.023	168.946	189.334	162.128
3	-87.873	46.713	54.624	253.074	173.902	179.811	183.597	172.179
4	-124.592	56.585	64.140	172.296	186.337	188.417	215.170	187.991
5	-77.671	64.964	-25.005	184.713	190.680	192.308	233.941	202.710
6	-52.871	74.456	84.962	174.533	186.167	238.153	242.661	220.867
7	48.617	83.656	96.881	181.496	224.298	219.501	264.200	242.184
8	82.002	93.846	113.693	200.374	163.401	181.718	200.236	222.678
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-191.017	38.944	41.765	-41.810	144.748	138.515	168.510	125.435
2	-203.310	7.334	41.171	152.322	150.031	151.445	165.387	130.653
3	-87.830	43.399	47.328	241.082	156.401	155.864	152.122	131.911
4	-127.905	49.288	52.147	154.795	162.391	156.942	174.902	137.442
5	-84.968	52.972	-42.506	160.767	159.205	152.039	183.392	140.115
6	-64.863	56.955	61.016	143.058	145.899	187.604	180.065	144.105
7	31.116	59.709	65.406	141.228	173.749	156.905	187.438	148.682
8	58.055	62.370	73.425	149.825	100.805	104.955	106.733	109.267

ARMA model (Input variables with 1 lag)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-372.569	-392.296	-314.876	87.938	86.302	78.532	103.844	50.056
2	-376.121	-181.476	-306.437	96.348	90.598	71.956	109.379	81.582
3	-28.808	-237.934	-118.262	105.542	97.740	86.532	116.044	45.694
4	-105.835	-45.067	-97.249	78.389	103.738	108.517	81.607	108.863
5	-90.865	17.119	15.690	97.197	107.513	111.864	105.489	120.788
6	-20.890	22.774	15.253	103.598	110.492	113.174	102.398	115.089
7	-261.891	28.116	27.195	94.639	123.701	129.622	139.791	125.405
8	28.266	34.219	24.490	97.823	91.218	94.359	93.813	106.292
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-361.394	-381.125	-303.949	98.357	95.921	87.031	110.868	55.210
2	-364.950	-170.549	-296.019	105.967	99.097	78.980	114.533	84.427
3	-17.881	-227.516	-108.643	114.041	104.764	91.687	118.889	45.737
4	-95.417	-35.448	-88.750	85.413	108.893	111.362	81.650	105.549
5	-81.246	25.618	22.714	102.352	110.358	111.907	102.175	113.491
6	-12.391	29.798	20.407	106.443	110.535	109.860	95.101	103.097
7	-254.867	33.270	30.040	94.682	120.388	122.325	127.799	107.903
8	33.420	37.064	24.533	94.510	83.921	82.367	76.311	82.345

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	290.200	435.600	264.200	273.800	258.300	344.600	389.300	773.400
2	334.400	437.400	267.100	267.900	605.600	399.400	510.400	683.400
3	368.600	600.100	517.900	416.800	681.000	493.200	669.400	839.800
4	1042.500	848.000	517.700	773.400	589.100	612.900	939.900	1271.500
5	422.200	568.200	433.900	1074.100	720.900	924.500	1294.200	1996.600
6	999.000	518.300	696.400	730.700	945.400	1308.800	2045.500	4525.200
7	741.100	1714.000	703.000	1234.100	1287.400	1996.800	4171.800	Inf
8	1199.300	1040.600	933.600	1283.400	1984.300	4178.200	Inf	4460.100
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	176.800	298.300	98.000	72.100	12.400	42.500	13.900	298.900
2	197.100	271.200	65.500	22.000	303.600	24.000	35.900	68.600
3	202.400	398.400	272.000	114.800	305.600	18.700	54.600	12.600
4	840.900	602.100	215.700	398.000	114.600	-1.900	112.800	87.900
5	176.400	266.200	58.400	599.600	106.100	97.400	110.600	96.100
6	697.000	142.900	221.900	115.900	118.200	125.200	145.000	466.600
7	365.700	1239.500	88.200	406.900	103.800	96.400	113.200	120.600
8	724.800	425.800	106.400	99.700	83.800	119.700	102.900	137.000

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-116.647	-21.896	-78.041	-78.029	-34.699	-199.602	-166.548	-161.444
2	27.326	4.249	172.301	-94.382	54.432	-171.961	202.299	145.430
3	89.578	362.402	104.480	-87.094	232.380	-14.573	-115.416	-154.265
4	68.071	465.885	837.522	391.275	36.828	142.518	179.862	-127.097
5	66.487	126.977	20.868	-168.237	-11.334	-144.573	440.191	450.516
6	67.404	-129.571	69.314	-155.023	187.999	-141.008	-105.016	-112.832
7	360.888	393.085	202.161	462.277	327.504	-123.820	188.664	-55.562
8	383.073	102.287	-3.270	-135.720	315.009	-110.160	349.391	407.577
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-111.493	-19.051	-77.998	-81.342	-41.996	-211.594	-184.049	-185.390
2	30.172	4.291	168.988	-101.679	42.440	-189.462	178.352	113.955
3	89.621	359.089	97.183	-99.086	214.879	-38.519	-146.891	-194.533
4	64.757	458.588	825.530	373.774	12.882	111.043	139.594	-177.646
5	59.190	114.985	3.367	-192.183	-42.809	-184.841	389.642	387.920
6	55.412	-147.072	45.368	-186.498	147.731	-191.557	-167.612	-189.594
7	343.387	369.139	170.686	422.009	276.956	-186.416	111.902	-149.065
8	359.126	70.812	-43.538	-186.269	252.414	-186.922	255.888	294.166

GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	200.905	254.236	143.968	-146.871	171.431	-264.865	-98.801	-58.782
2	-239.651	-62.026	-188.553	-253.526	-239.006	-34.251	73.237	-251.879
3	-174.898	-76.622	203.291	-242.673	-253.392	488.717	-127.434	-138.935
4	-167.312	170.798	185.118	-244.364	592.374	-226.926	-245.177	115.490
5	-106.607	-199.555	138.989	-210.365	-11.771	134.179	-55.624	-225.660
6	316.034	-102.703	-254.071	-241.133	-111.682	-238.778	-238.921	170.462
7	-215.294	-223.879	-243.583	53.441	139.789	-219.247	-217.165	279.456
8	-234.799	26.370	-204.040	-221.847	-135.674	284.260	-205.534	-196.948
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	212.080	265.408	154.894	-136.453	181.050	-256.366	-91.777	-53.627
2	-228.480	-51.099	-178.135	-243.907	-230.507	-27.227	78.392	-249.034
3	-163.971	-66.203	212.910	-234.174	-246.368	493.872	-124.589	-138.892
4	-156.894	180.417	193.617	-237.340	597.528	-224.081	-245.134	112.176
5	-96.989	-191.056	146.013	-205.210	-8.926	134.222	-58.937	-232.957
6	324.533	-95.680	-248.917	-238.288	-111.639	-242.092	-246.217	158.470
7	-208.270	-218.724	-240.738	53.484	136.475	-226.544	-229.158	261.955
8	-229.644	29.216	-203.997	-225.161	-142.971	272.268	-223.035	-220.894

Table E9: ARMA and GARCH Models for office property return deviation

IPD Industrial Return Deviation

ARMA model (Input variables with 4 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-348.668	-164.166	-76.837	-355.396	-43.027	-72.841	45.157	71.021
2	-300.947	-185.399	-66.908	-91.536	26.559	-16.425	67.043	78.685
3	-23.429	-79.207	-53.404	-207.344	29.489	35.933	74.957	82.238
4	-312.787	-94.282	-25.660	2.091	29.783	-73.759	85.920	92.547
5	-333.795	-83.472	-35.658	-46.335	48.397	0.205	88.835	100.079
6	-328.480	-94.205	-17.726	-143.795	48.389	31.182	100.583	107.165
7	-221.990	-86.272	-14.564	14.932	60.762	49.530	108.091	116.228
8	-18.663	-68.283	-1.472	26.671	-58.853	81.080	116.978	125.991
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-335.324	-150.719	-63.506	-342.419	-30.663	-61.375	55.413	79.725
2	-287.500	-172.067	-53.930	-79.172	38.025	-6.168	75.747	85.459
3	-10.097	-66.229	-41.040	-195.878	39.746	44.638	81.731	86.664
4	-299.810	-81.919	-14.193	12.347	38.488	-66.984	90.346	94.158
5	-321.431	-72.006	-25.401	-37.630	55.172	4.631	90.446	98.357
6	-317.014	-83.948	-9.021	-137.021	52.814	32.793	98.861	101.529
7	-211.733	-77.567	-7.790	19.358	62.373	47.808	102.455	106.024
8	-9.959	-61.508	2.953	28.282	-60.575	75.444	106.774	110.479

ARMA model (Input variables with 2 lags)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-366.916	-137.868	-59.143	-324.342	-28.327	-354.828	48.874	50.024
2	-339.019	-57.518	-55.334	-69.597	-10.264	-357.246	53.246	31.938
3	-341.754	-52.433	-47.688	-121.037	7.717	8.060	56.103	59.309
4	-112.852	-51.603	-36.766	-194.783	17.230	-347.087	63.434	67.379
5	-145.712	-42.522	-32.429	-288.402	19.294	-208.434	62.484	73.699
6	-34.154	-48.658	-43.491	-172.186	23.429	-215.447	74.130	80.196
7	-34.519	-34.026	-37.182	-27.902	32.111	-316.390	79.650	85.783
8	-61.893	-32.344	-24.789	-52.868	-242.116	-62.282	86.102	92.421
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-355.851	-125.981	-46.594	-311.302	-14.983	-341.381	62.205	63.001
2	-327.133	-44.969	-42.295	-56.253	3.183	-343.915	66.223	44.302
3	-329.205	-39.394	-34.344	-107.590	21.048	21.038	68.467	70.775
4	-99.813	-38.259	-23.319	-181.452	30.207	-334.723	74.900	77.635
5	-132.368	-29.075	-19.097	-275.424	31.658	-196.968	72.741	82.403
6	-20.707	-35.327	-30.513	-159.822	34.895	-205.191	82.834	86.970
7	-21.188	-21.048	-24.818	-16.435	42.368	-307.686	86.425	90.209
8	-48.915	-19.980	-13.323	-42.612	-233.411	-55.508	90.527	94.032

ARMA model (Input variables with 1 lag)								
AICc								
R	M							
	1	2	3	4	5	6	7	8
1	-370.364	-57.193	-253.141	-191.363	-33.845	-379.768	36.977	39.531
2	-370.412	-349.457	-313.113	-144.876	-17.258	-189.848	41.711	46.775
3	-371.028	-330.605	-50.180	-259.012	-56.000	-41.135	49.363	48.991
4	-346.057	-208.391	-46.850	-68.750	-0.106	-40.268	50.238	54.380
5	-259.979	-142.303	-107.938	-34.413	-18.296	-109.853	57.536	57.649
6	-34.920	-32.242	-70.241	-39.328	13.736	5.751	62.386	68.585
7	-232.143	-358.544	-32.850	-24.247	-32.975	163.644	68.836	68.412
8	-250.958	-26.271	-23.646	-22.221	28.943	-272.052	72.091	77.540
BIC								
R	M							
	1	2	3	4	5	6	7	8
1	-361.378	-47.098	-242.077	-179.476	-21.296	-366.729	50.321	52.978
2	-360.318	-338.393	-301.226	-132.327	-4.218	-176.504	55.158	60.106
3	-359.964	-318.719	-37.631	-245.973	-42.656	-27.688	62.694	61.968
4	-334.170	-195.842	-33.810	-55.406	13.341	-26.937	63.215	66.744
5	-247.430	-129.264	-94.594	-20.966	-4.964	-96.875	69.900	69.115
6	-21.880	-18.898	-56.794	-25.997	26.713	18.115	73.852	78.842
7	-218.799	-345.097	-19.519	-11.270	-20.611	175.110	79.093	77.116
8	-237.511	-12.940	-10.669	-9.857	40.409	-261.795	80.796	84.314

GARCH model (Input variables with 4 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-162.936	-302.068	-294.635	-309.120	171.334	-246.555	177.163	-47.795
2	-84.895	-302.649	-308.839	-288.791	27.752	-51.119	-248.479	214.656
3	-260.795	-225.673	-292.627	121.079	-81.693	-280.482	37.279	313.322
4	247.854	25.163	-231.141	-271.565	-195.685	149.735	-167.126	82.024
5	-280.283	122.138	-267.667	30.629	4.926	-125.027	171.952	195.566
6	-258.717	-158.570	-183.160	86.750	-88.041	-259.748	-235.540	-125.965
7	566.401	-51.473	-273.936	-238.145	-264.399	241.408	115.545	207.406
8	-24.842	-197.779	169.060	-168.787	-133.245	153.598	93.911	239.248
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-176.281	-288.621	-281.303	-296.143	158.970	-258.021	166.906	-39.091
2	-71.448	-289.318	-295.862	-276.427	39.218	-40.862	-239.774	221.430
3	-247.464	-212.696	-280.264	132.545	-71.436	-271.777	44.053	317.748
4	260.831	37.526	-219.675	-261.308	-186.980	156.510	-162.700	83.635
5	-267.920	133.604	-257.410	39.333	11.701	-120.601	173.563	193.844
6	-247.251	-148.313	-174.456	93.524	-83.615	-258.136	-237.262	-131.602
7	576.657	-42.768	-267.161	-233.719	-262.787	239.686	109.909	197.202
8	-16.137	-191.004	173.485	-167.176	-134.967	147.962	83.707	223.737

GARCH model (Input variables with 2 lags)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-322.838	-312.512	-327.513	102.151	-316.878	-286.605	-279.639	-304.056
2	-308.546	-328.765	21.315	-306.892	96.082	-303.383	-274.961	-292.889
3	-314.666	-202.967	-324.658	71.526	-288.115	-287.029	-304.957	-280.350
4	-290.831	125.321	-321.291	154.532	-312.492	-286.422	218.722	293.317
5	-315.648	19.154	-308.325	-307.694	8.356	-279.954	-286.844	-261.498
6	-310.917	-317.149	-302.875	-307.389	-302.533	207.419	-233.050	-264.069
7	763.224	-291.739	-280.985	-305.569	-300.041	-294.960	-249.422	105.793
8	-304.341	-288.599	-265.179	-300.986	-295.501	-256.424	-249.955	-242.855
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-311.774	-300.625	-314.964	115.191	-303.534	-273.158	-266.307	-291.078
2	-296.659	-316.216	34.355	-293.548	109.529	-290.052	-261.983	-280.525
3	-302.117	-189.928	-311.314	84.973	-274.783	-274.051	-292.594	-268.884
4	-277.791	138.665	-307.844	167.863	-299.514	-274.058	230.188	303.574
5	-302.304	32.601	-294.993	-294.717	20.720	-268.488	-276.587	-252.793
6	-297.470	-303.818	-289.898	-295.025	-291.067	217.675	-224.346	-257.294
7	776.556	-278.761	-268.621	-294.103	-289.784	-286.255	-242.647	110.219
8	-291.364	-276.235	-253.713	-290.730	-286.796	-249.650	-245.530	-241.244

GARCH model (Input variables with 1 lag)								
AICc								
P	Q							
	1	2	3	4	5	6	7	8
1	-308.238	-200.153	-334.246	-331.320	-325.415	-321.967	-321.928	-305.709
2	168.057	-322.880	-331.134	-327.856	-322.229	-319.315	-314.509	-315.812
3	-334.503	-169.809	-328.412	-296.207	-321.438	-314.507	-310.463	-311.622
4	-331.670	-320.721	-324.963	-321.329	-317.502	-277.827	-309.823	-307.079
5	-328.399	-324.957	-321.321	-291.731	-313.460	-308.761	-305.274	-302.247
6	-321.751	-305.203	-318.134	-276.559	-309.805	-305.276	-299.677	-297.051
7	-323.209	-319.377	-315.110	-312.294	-306.686	-301.915	-287.736	-291.598
8	-317.540	-316.935	-314.964	-306.450	-303.609	-298.468	-294.815	-289.278
BIC								
P	Q							
	1	2	3	4	5	6	7	8
1	-299.252	-190.059	-323.182	-319.434	-312.866	-308.927	-308.583	-292.262
2	178.151	-311.815	-319.247	-315.307	-309.189	-305.971	-301.062	-302.481
3	-323.438	-157.923	-315.863	-283.168	-308.094	-301.060	-297.132	-298.645
4	-319.783	-308.172	-311.924	-307.985	-304.055	-264.496	-296.846	-294.715
5	-315.850	-311.918	-307.977	-278.284	-300.129	-295.783	-292.910	-290.781
6	-308.712	-291.859	-304.687	-263.228	-296.827	-292.912	-288.211	-286.795
7	-309.865	-305.930	-301.779	-299.317	-294.322	-290.449	-277.480	-282.893
8	-304.093	-303.604	-301.986	-294.087	-292.143	-288.211	-286.111	-282.504

Table E10: ARMA and GARCH Models for industrial property return deviation

J255 Index

Four (4) Input Variables			
Number of lags	Return	Deviation	Covariance
AICc			
1	352.203	183.138	-119.845
2	-520.156	-676.873	-985.523
3	-207.296	-373.249	-684.757
4	-153.238	-331.902	-644.715
5	-145.856	-320.201	-647.033
6	-140.241	-366.913	-697.704
BIC			
1	-130.324	-299.389	-602.372
2	12.538	-144.179	-452.829
3	157.271	-8.681	-320.190
4	301.833	123.169	-189.645
5	433.005	258.659	-68.172
6	572.451	345.779	14.988
Two (2) Input Variables			
AICc			
1	18.100	-154.400	-452.400
2	133.500	-27.800	-328.900
3	764.800	600.500	299.200
4	-996.000	-1160.400	-1461.200
5	-469.100	-623.000	-936.100
6	-353.800	-551.400	-860.800
BIC			
1	-216.770	-389.179	-687.209
2	-153.840	-315.166	-616.257
3	-88.115	-252.376	-553.652
4	-23.100	-187.504	-488.303
5	29.094	-124.779	-437.866
6	97.112	-100.453	-409.884

Table E11: VAR Model for J255 Return and J255 Return Deviation

J256 Index

Four (4) Input Variables			
Number of lags	Return	Deviation	Covariance
AIC			
1	355.566	178.797	-121.304
2	-527.005	-683.720	-999.273
3	-217.503	-369.767	-694.071
4	-169.023	-315.879	-649.238
5	-186.279	-294.983	-670.748
6	-186.579	-296.156	-680.025
BIC			
1	-126.961	-303.730	-603.831
2	5.689	-151.026	-466.579
3	147.065	-5.199	-329.504
4	286.048	139.192	-194.168
5	392.581	283.878	-91.887
6	526.113	416.536	32.667
Two (2) Input Variables			
AIC			
1	18.900	-158.400	-456.400
2	127.800	-35.700	-342.300
3	762.000	599.500	296.000
4	-997.800	-1155.000	-1457.800
5	-479.600	-612.900	-936.400
6	-370.400	-505.800	-831.400
BIC			
1	-215.930	-393.184	-691.173
2	-159.516	-322.985	-629.671
3	-90.875	-253.375	-556.861
4	-24.965	-182.117	-484.878
5	18.612	-114.640	-438.217
6	80.497	-54.932	-380.461

Table E12: VAR Model for J256 Return and J256 Return Deviation

IPD Retail

Four (4) Input Variables			
Number of lags	Return	Deviation	Covariance
AIC			
1	595.800	384.100	-54.400
2	-411.400	-624.200	-1091.300
3	-327.600	-547.000	-1012.300
4	-310.600	-523.900	-986.000
5	-335.500	-515.900	-1011.200
6	-341.600	-521.600	-1022.500
BIC			
1	-254.544	-466.262	-904.723
2	-150.062	-362.889	-830.003
3	-9.896	-229.323	-694.623
4	122.064	-91.204	-553.293
5	223.777	43.354	-451.869
6	347.645	167.629	-333.343
Two (2) Input Variables			
AIC			
1	-145.700	-342.200	-774.400
2	74.900	-120.300	-586.200
3	-1104.600	-1295.100	-1756.000
4	-535.800	-721.300	-1174.300
5	-465.400	-652.700	-1105.700
6	-437.400	-616.500	-1070.400
BIC			
1	-340.503	-536.928	-969.126
2	-308.587	-503.880	-969.725
3	-245.308	-435.788	-896.747
4	-178.023	-363.440	-816.533
5	-130.337	-317.559	-770.563
6	-75.297	-254.341	-708.300

Table E13: VAR Model for Retail Return and Retail Return Deviation

IPD Office

Four (4) Input Variables			
Number of lags	Return	Deviation	Covariance
AIC			
1	571.100	-439.100	-4.400
2	-437.700	-572.700	-1052.000
3	-356.700	-506.100	-986.200
4	-329.300	-478.200	-951.900
5	-323.500	-470.700	-949.300
6	-350.100	-526.900	-1032.500
BIC			
1	-279.277	-411.301	-854.811
2	-176.426	-311.390	-790.685
3	-38.997	-188.418	-668.483
4	103.445	-45.520	-519.225
5	235.751	88.618	-390.024
6	339.124	162.284	-343.272
Two (2) Input Variables			
AIC			
1	-169.200	-286.400	-727.300
2	54.500	-72.000	-539.300
3	-1122.000	-1246.100	-1708.100
4	-552.100	-682.100	-1136.200
5	-471.000	-600.600	-1057.300
6	-434.400	-569.200	-1020.900
BIC			
1	-363.921	-481.160	-922.070
2	-328.988	-455.520	-922.851
3	-262.675	-386.845	-848.835
4	-194.319	-324.272	-778.391
5	-135.881	-265.450	-722.230
6	-72.290	-207.060	-658.743

Table E14: VAR Model for Office Return and Office Return Deviation

IPD Industrial

Four (4) Input Variables			
Number of lags	Return	Deviation	Covariance
AIC			
1	577.000	477.300	27.200
2	-438.200	-533.500	-1018.500
3	-354.600	-449.200	-931.300
4	-332.000	-423.900	-903.800
5	-345.800	-430.300	-929.500
6	-393.600	-443.400	-991.800
BIC			
1	-273.365	-373.023	-823.157
2	-176.935	-272.183	-757.231
3	-36.941	-131.490	-613.616
4	100.712	8.788	-471.125
5	213.475	129.005	-370.261
6	295.553	245.851	-302.586
Two (2) Input Variables			
AIC			
1	-161.000	-258.400	-705.800
2	55.800	-43.100	-518.100
3	-1117.200	-1217.000	-1682.300
4	-549.300	-645.600	-1107.000
5	-465.500	-555.500	-1017.700
6	-471.200	-544.600	-1032.500
BIC			
1	-355.724	-453.208	-900.596
2	-327.731	-426.594	-901.664
3	-257.890	-357.706	-822.962
4	-191.473	-287.741	-749.202
5	-130.423	-220.419	-682.574
6	-109.095	-182.500	-670.314

Table E15: NN Model for Industrial Return and Industrial Return Deviation

J255 Index

Four (4) Input Variables and Four (4) Lags									
No. of Neurons	Linear Function			Logistic Function			Softmax Function		
	Return	Deviation	Covariance	Return	Deviation	Covariance	Return	Deviation	Covariance
AICc									
2	2115.200	1985.300	1664.300	2189.300	2004.900	1747.400	2381.500	2216.700	1805.300
4	-498.400	-587.600	-939.200	-469.000	-586.500	-908.500	-201.400	-369.800	-801.900
6	-450.400	-534.500	-912.500	-333.900	-508.000	-768.800	-178.700	-188.500	-660.200
8	-497.300	-501.300	-949.100	-380.300	-482.400	-821.900	-113.600	-238.600	-650.300
10	-451.800	-522.000	-935.700	-384.800	-480.200	-826.900	-119.600	-187.000	-663.500
12	-440.000	-503.300	-913.400	-372.900	-474.000	-820.800	-88.100	-242.100	-637.200
14	-406.500	-506.700	-886.600	-362.800	-462.400	-808.100	-92.800	-207.700	-648.600
16	-391.000	-491.100	-858.900	-341.700	-477.600	-796.400	-135.500	-129.400	-620.900
18	-422.500	-509.800	-913.700	-334.800	-458.700	-772.700	-70.600	-259.600	-647.900
20	-417.500	-523.700	-922.200	-372.000	-473.200	-826.300	-81.000	-207.200	-610.800
22	-399.600	-494.800	-877.100	-274.300	-441.700	-731.700	-77.300	-219.000	-636.600
24	-392.100	-499.200	-875.200	-331.700	-472.300	-801.700	-127.800	-125.300	-639.000
26	-387.500	-513.900	-887.800	-323.500	-474.900	-787.900	-68.100	-239.600	-633.800
28	-450.800	-531.600	-968.700	-327.400	-470.800	-786.700	-69.900	-230.700	-652.100
30	-419.200	-517.300	-923.200	-338.000	-454.900	-780.000	-66.800	-237.600	-642.800
32	-403.800	-532.700	-923.600	-337.500	-478.700	-803.900	-57.300	-280.100	-639.900
34	-384.900	-494.500	-866.900	-280.500	-451.900	-734.800	-47.200	-359.900	-647.100
36	-409.700	-527.900	-926.700	-330.600	-473.600	-794.400	-90.700	-164.000	-598.400
38	-422.700	-527.300	-938.500	-338.700	-482.900	-814.800	-60.400	-254.000	-641.900
40	-410.500	-531.300	-931.000	-319.900	-477.100	-787.800	-72.700	-202.900	-590.400
42	-419.000	-529.400	-939.500	-266.300	-381.500	-655.000	-52.500	-291.800	-622.500
44	-393.800	-506.100	-889.700	-355.400	-496.900	-843.200	-85.400	-166.200	-598.000
46	-388.300	-513.500	-892.900	-351.900	-478.800	-821.600	-68.600	-214.500	-630.500
48	-409.000	-522.800	-922.800	-269.400	-449.500	-723.600	-79.600	-182.200	-634.300
50	-412.100	-506.700	-911.500	-327.900	-478.600	-798.100	-88.100	-163.500	-646.600
BIC									
2	-92.600	-222.500	-543.500	-18.400	-202.900	-460.400	173.700	8.900	-402.500
4	99.300	10.100	-341.500	128.700	11.300	-310.700	396.300	227.900	-204.100
6	295.500	211.400	-166.600	412.000	237.900	-22.900	567.200	557.400	85.700
8	447.000	442.900	-4.900	563.900	461.800	122.300	830.600	705.600	293.900
10	702.700	632.600	218.800	769.800	674.400	327.600	1035.000	967.600	491.000
12	929.600	866.300	456.200	996.700	895.600	548.800	1281.500	1127.400	732.300
14	1180.400	1080.300	700.400	1224.100	1124.500	778.800	1494.100	1379.200	938.400
16	1414.700	1314.600	946.700	1464.000	1328.100	1009.300	1670.100	1676.300	1184.700
18	1602.700	1515.300	1111.500	1690.400	1566.400	1252.500	1954.600	1765.600	1377.200
20	1827.700	1721.600	1323.000	1873.200	1772.100	1418.900	2164.300	2038.000	1634.500
22	2066.200	1971.000	1588.600	2191.400	2024.000	1734.000	2388.500	2246.800	1829.100
24	2294.400	2187.200	1811.300	2354.800	2214.200	1884.800	2558.700	2561.100	2047.400
26	2519.900	2393.500	2019.600	2583.900	2432.500	2119.500	2839.300	2667.800	2273.600
28	2677.700	2596.900	2159.800	2801.200	2657.700	2341.800	3058.600	2897.900	2476.400
30	2930.600	2832.500	2426.500	3011.700	2894.900	2569.800	3283.000	3112.100	2707.000
32	3167.200	3038.400	2647.500	3233.600	3092.400	2767.200	3513.800	3290.900	2931.200
34	3407.600	3298.000	2925.600	3512.000	3340.600	3057.700	3745.300	3432.600	3145.400
36	3604.300	3486.100	3087.200	3683.400	3540.400	3219.600	3923.300	3850.000	3415.600
38	3812.800	3708.200	3297.000	3896.800	3752.600	3420.700	4175.200	3981.500	3593.600
40	4046.600	3925.800	3526.200	4137.200	3980.000	3669.300	4384.400	4254.200	3866.800
42	4259.800	4149.400	3739.300	4412.400	4297.200	4023.800	4626.200	4386.900	4056.300
44	4506.600	4394.400	4010.700	4545.000	4403.500	4057.200	4815.000	4734.200	4302.400
46	4733.800	4608.600	4229.200	4770.200	4643.300	4300.500	5053.600	4907.600	4491.600
48	4934.800	4821.100	4421.000	5074.400	4894.400	4620.200	5264.300	5161.600	4709.500
50	5153.500	5058.900	4654.100	5237.600	5086.900	4767.500	5477.400	5402.100	4918.900
Four (4) Input Variables and Two (2) Lags									
AICc									
2	-45.500	-170.400	-483.200	40.300	-161.600	-391.500	197.900	95.100	-315.100
4	-2768.700	-2891.900	-3216.700	-2748.600	-2884.500	-3190.900	-2495.400	-2671.100	-3051.100
6	-548.900	-673.000	-996.600	-537.900	-669.200	-982.000	-282.900	-430.000	-826.700
8	-445.100	-568.900	-895.600	-421.200	-552.600	-862.600	-180.300	-316.400	-713.400
10	-396.000	-534.300	-849.300	-392.300	-530.800	-841.500	-138.100	-299.000	-713.100
12	-389.000	-492.400	-818.000	-379.600	-507.300	-824.800	-121.300	-270.400	-667.000
14	-381.300	-510.800	-841.800	-376.700	-498.100	-823.000	-136.300	-193.700	-649.700
16	-377.300	-492.900	-826.600	-281.400	-401.800	-642.000	-141.900	-165.100	-649.700
18	-366.000	-490.000	-818.300	-339.900	-470.500	-774.800	-96.400	-243.300	-677.500
20	-365.300	-489.100	-820.900	-341.100	-479.400	-787.700	-88.900	-252.800	-669.900
22	-357.400	-496.300	-823.100	-352.800	-479.000	-801.100	-107.000	-178.700	-549.800
24	-365.800	-487.200	-825.100	-333.900	-473.800	-787.500	-80.500	-247.400	-582.200
26	-376.500	-483.700	-836.900	-346.600	-476.800	-798.600	-96.400	-196.100	-641.800
28	-365.600	-484.900	-828.400	-343.800	-474.100	-795.300	-98.000	-187.200	-619.500
30	-361.400	-510.000	-849.300	-340.400	-471.500	-789.600	-123.300	-137.500	-577.300
32	-343.200	-486.300	-808.400	-328.500	-469.100	-779.700	-108.100	-160.500	-634.700
34	-366.500	-480.200	-827.200	-324.100	-456.700	-761.000	-75.000	-235.900	-610.200
36	-357.100	-487.600	-826.900	-331.200	-463.700	-778.300	-71.300	-246.600	-616.500
38	-348.400	-480.500	-811.400	-303.100	-465.200	-751.300	-71.000	-243.800	-644.500
40	-375.100	-482.800	-841.600	-284.100	-432.700	-700.700	-115.800	-140.100	-572.300
42	-364.200	-490.600	-839.000	-334.600	-462.900	-781.700	-73.000	-227.100	-631.600
44	-358.800	-483.600	-826.700	-285.400	-434.100	-715.700	-70.800	-235.300	-671.500
46	-363.800	-487.600	-836.900	-298.600	-445.200	-735.800	-69.600	-235.700	-623.800
48	-335.400	-470.500	-791.800	-317.500	-468.300	-771.200	-70.800	-226.500	-596.800
50	-361.500	-478.200	-827.200	-342.300	-464.700	-794.800	-67.100	-242.800	-643.800

BIC									
2	-183.100	-308.000	-620.800	-97.300	-299.200	-529.100	60.300	-42.400	-452.700
4	-71.800	-195.000	-519.800	-51.700	-187.600	-494.100	201.500	25.800	-354.200
6	53.100	-71.000	-394.500	64.200	-67.200	-379.900	319.100	172.000	-224.700
8	174.600	50.800	-275.900	198.600	67.100	-242.900	439.400	303.300	-93.600
10	309.600	171.300	-143.700	313.300	174.800	-136.000	567.500	406.600	-7.600
12	420.700	317.300	-8.300	430.100	302.400	-15.100	688.400	539.300	142.700
14	540.000	410.500	79.600	544.700	423.300	98.400	785.000	727.700	271.600
16	659.600	543.900	210.200	755.500	635.100	394.900	894.900	871.700	387.100
18	788.500	664.500	336.300	814.700	684.000	379.800	1058.200	911.200	477.000
20	908.300	784.500	452.800	932.500	794.200	485.900	1184.700	1020.800	603.700
22	1036.300	897.300	570.500	1040.800	914.600	592.500	1286.600	1215.000	843.800
24	1148.500	1027.100	689.100	1180.400	1040.500	726.800	1433.800	1266.900	932.100
26	1259.000	1151.800	798.600	1288.800	1158.600	836.800	1539.100	1439.400	993.700
28	1391.400	1272.100	928.600	1413.100	1282.900	961.700	1658.900	1569.700	1137.400
30	1517.300	1368.700	1029.500	1538.400	1407.200	1089.100	1755.400	1741.300	1301.400
32	1657.500	1514.400	1192.300	1672.300	1531.600	1221.000	1892.600	1840.200	1366.100
34	1756.500	1642.800	1295.700	1798.800	1666.200	1361.900	2048.000	1887.100	1512.800
36	1888.200	1757.600	1418.300	1914.000	1781.600	1466.900	2174.000	1998.700	1628.700
38	2019.300	1887.200	1556.300	2064.600	1902.500	1616.400	2296.700	2123.900	1723.200
40	2115.100	2007.400	1648.600	2206.100	2057.500	1789.600	2374.500	2350.100	1918.000
42	2248.600	2122.200	1773.900	2278.300	2149.900	1831.200	2539.800	2385.800	1981.300
44	2376.700	2252.000	1908.900	2450.100	2301.400	2019.800	2664.700	2500.200	2064.000
46	2494.400	2370.700	2021.400	2559.700	2413.100	2122.500	2788.600	2622.600	2234.500
48	2645.700	2510.600	2189.300	2663.600	2512.700	2209.900	2910.300	2754.600	2384.300
50	2742.400	2625.800	2276.700	2761.600	2639.200	2309.100	3036.800	2861.100	2460.100
Four (4) Input Variables and One (1) Lags									
AICc									
2	-112.500	-259.600	-556.000	-60.800	-253.700	-501.400	90.400	53.800	-419.800
4	42.000	-101.200	-404.000	66.800	-96.700	-376.000	285.100	143.200	-309.900
6	2131.100	1986.100	1690.700	2156.700	2002.800	1715.400	2382.400	2214.400	1798.900
8	-795.200	-937.400	-1241.200	-775.500	-932.600	-1218.900	-543.800	-720.000	-1115.500
10	-533.200	-677.800	-985.200	-523.000	-660.400	-957.800	-294.400	-400.800	-840.500
12	-444.200	-601.300	-898.200	-385.700	-575.200	-815.100	-239.800	-279.000	-727.800
14	-419.100	-556.000	-865.200	-389.800	-527.600	-813.300	-178.200	-286.800	-717.200
16	-398.700	-542.900	-853.600	-317.200	-509.800	-739.100	-140.400	-310.700	-693.000
18	-387.100	-528.800	-842.900	-315.600	-518.100	-761.000	-131.800	-280.000	-695.600
20	-380.000	-514.400	-831.000	-329.700	-487.000	-763.600	-117.400	-280.600	-645.500
22	-361.900	-509.400	-815.600	-336.200	-487.000	-767.400	-122.500	-231.400	-660.700
24	-350.300	-502.400	-803.300	-330.400	-487.300	-772.900	-146.700	-172.700	-612.400
26	-350.200	-500.500	-805.500	-318.900	-478.100	-757.100	-94.900	-285.800	-674.700
28	-347.200	-495.500	-801.300	-330.100	-467.900	-757.200	-120.000	-194.300	-691.000
30	-339.900	-489.200	-791.000	-341.500	-478.800	-796.200	-87.500	-281.900	-686.200
32	-341.600	-492.600	-798.900	-325.900	-460.400	-778.400	-90.900	-251.500	-652.600
34	-331.900	-487.700	-786.500	-320.900	-451.300	-740.400	-132.100	-156.500	-653.300
36	-342.300	-489.100	-800.300	-281.000	-441.400	-704.700	-143.000	-139.900	-638.200
38	-333.500	-461.600	-765.700	-297.600	-439.600	-711.500	-132.100	-146.700	-640.600
40	-337.000	-466.100	-775.200	-285.800	-453.300	-713.600	-78.600	-257.100	-628.400
42	-325.200	-483.400	-782.200	-325.300	-482.200	-781.400	-107.000	-176.700	-649.700
44	-329.800	-467.800	-772.300	-316.800	-481.400	-773.000	-79.400	-238.600	-577.300
46	-338.300	-467.800	-781.700	-304.800	-462.100	-745.800	-120.300	-143.900	-517.900
48	-323.700	-472.200	-772.400	-304.900	-470.300	-753.900	-85.900	-215.800	-669.100
50	-331.900	-479.000	-788.800	-305.400	-456.500	-741.300	-87.600	-205.200	-608.200
BIC									
2	-216.000	-363.100	-659.500	-164.300	-357.200	-604.900	-13.100	-49.700	-523.300
4	-148.400	-291.500	-594.400	-123.600	-287.100	-566.400	94.700	-47.100	-500.300
6	-76.700	-221.700	-527.100	-51.100	-205.000	-492.300	174.600	6.600	-408.900
8	-1.000	-143.200	-447.000	18.700	-138.400	-424.600	250.400	74.200	-321.300
10	68.800	-75.800	-383.200	79.000	-58.400	-355.800	307.600	201.300	-238.500
12	153.500	-3.600	-300.500	212.000	22.500	-217.400	357.900	318.700	-130.100
14	215.100	78.300	-230.900	244.500	106.700	-179.000	456.100	347.500	-82.900
16	287.600	143.400	-167.200	369.200	176.500	-52.800	545.900	375.600	-6.700
18	358.800	217.100	-97.000	430.300	227.800	-15.100	614.100	465.900	50.300
20	429.700	295.200	-21.300	480.000	322.700	46.000	692.300	529.100	164.200
22	514.100	366.700	60.400	539.900	389.000	108.700	753.600	644.700	215.400
24	593.900	441.800	140.900	613.800	456.900	171.300	797.600	771.500	331.800
26	663.300	513.000	208.000	694.600	535.500	256.500	918.600	727.700	338.800
28	736.500	588.200	282.400	753.600	615.800	326.500	963.700	889.400	392.700
30	814.600	665.300	363.500	813.000	675.700	358.300	1067.100	872.600	468.300
32	884.200	733.200	427.000	899.900	765.400	447.400	1134.900	974.300	573.300
34	965.700	809.800	511.000	976.700	846.300	557.200	1165.400	1141.100	644.300
36	1027.300	880.500	569.300	1088.500	928.200	664.900	1226.600	1229.700	731.300
38	1108.300	980.300	676.100	1144.200	1002.200	730.300	1309.700	1295.100	801.200
40	1177.300	1048.200	739.100	1228.500	1061.000	800.600	1435.700	1257.200	885.900
42	1261.700	1103.500	804.800	1261.700	1104.700	805.600	1479.900	1410.200	937.300
44	1329.900	1192.000	887.400	1342.900	1178.300	886.700	1580.300	1421.100	1082.400
46	1394.400	1264.800	950.900	1427.800	1270.500	986.800	1612.300	1588.700	1214.800
48	1481.900	1333.400	1033.200	1500.700	1335.400	1051.700	1719.800	1589.800	1136.500
50	1546.800	1399.700	1090.000	1573.400	1422.200	1137.400	1791.100	1673.500	1270.600

Two (2) Input Variables and Four (4) Lags									
AICc									
2	52.100	-55.400	-387.300	110.700	-55.400	-330.700	317.200	193.500	-276.800
4	-721.700	-837.600	-1172.200	-685.700	-815.900	-1117.800	-473.400	-531.900	-1040.800
6	-484.200	-585.600	-938.200	-413.200	-573.700	-856.400	-211.700	-284.800	-792.000
8	-437.400	-534.000	-890.500	-388.400	-525.800	-833.500	-138.800	-286.600	-717.900
10	-389.800	-516.600	-849.200	-380.700	-498.700	-821.400	-163.800	-173.700	-691.300
12	-383.700	-512.200	-852.800	-384.100	-488.400	-830.000	-135.200	-183.100	-654.200
14	-412.600	-505.600	-880.100	-350.000	-477.800	-796.500	-93.200	-257.900	-669.100
16	-372.100	-507.100	-848.100	-325.300	-473.600	-775.700	-92.400	-236.700	-648.700
18	-365.600	-498.200	-836.500	-304.100	-473.300	-749.900	-84.600	-244.600	-660.900
20	-362.900	-495.600	-832.200	-350.000	-476.600	-800.300	-85.400	-230.800	-661.200
22	-360.600	-500.900	-838.500	-344.300	-472.400	-794.700	-86.500	-218.000	-637.500
24	-347.200	-491.200	-818.400	-321.300	-432.800	-733.700	-122.700	-141.900	-641.400
26	-368.600	-489.500	-838.200	-345.800	-479.900	-806.000	-111.000	-152.600	-561.800
28	-351.200	-502.400	-836.100	-335.000	-469.300	-785.400	-89.800	-191.100	-666.400
30	-359.600	-490.900	-834.900	-344.300	-469.900	-797.300	-85.800	-194.900	-610.300
32	-374.800	-496.700	-856.200	-333.500	-447.700	-767.800	-72.900	-233.500	-669.500
34	-356.300	-498.000	-839.700	-327.900	-466.400	-779.700	-81.800	-201.100	-669.400
36	-347.500	-491.000	-825.000	-323.300	-475.200	-785.800	-114.900	-136.200	-641.200
38	-360.200	-483.200	-830.600	-338.200	-446.400	-772.500	-74.200	-213.600	-607.100
40	-352.900	-493.800	-837.500	-286.700	-453.300	-725.700	-98.500	-158.200	-635.800
42	-342.800	-483.900	-813.400	-341.900	-476.700	-805.200	-107.700	-139.900	-593.600
44	-354.000	-487.100	-831.700	-289.400	-430.400	-714.400	-66.000	-237.600	-615.700
46	-362.300	-500.900	-851.900	-334.300	-470.800	-792.800	-94.500	-158.700	-589.100
48	-351.500	-487.300	-829.500	-340.800	-469.000	-797.600	-116.200	-128.300	-635.800
50	-349.500	-487.400	-826.800	-320.600	-462.500	-779.400	-53.900	-298.700	-655.800
BIC									
2	-166.700	-274.200	-606.200	-108.100	-274.300	-549.500	98.400	-25.300	-495.600
4	-13.700	-129.700	-464.300	22.200	-108.000	-409.900	234.600	176.000	-332.900
6	123.100	21.700	-330.900	194.100	33.500	-249.100	395.600	322.500	-184.700
8	274.800	178.100	-178.400	323.800	186.300	-121.400	573.300	425.600	-5.800
10	456.500	329.800	-2.900	465.700	347.700	24.900	682.600	672.700	155.000
12	606.600	478.100	137.500	606.200	501.900	160.300	855.200	807.300	336.100
14	726.200	633.100	258.700	788.700	660.900	342.200	1045.500	880.800	469.600
16	917.500	782.500	441.400	964.200	815.900	513.800	1197.100	1052.900	640.800
18	1076.200	943.600	605.300	1137.800	968.600	691.900	1357.200	1197.300	781.000
20	1232.100	1099.400	762.800	1245.100	1118.400	794.700	1509.700	1364.300	933.800
22	1388.200	1247.900	910.300	1404.600	1276.400	954.200	1662.300	1530.800	1111.400
24	1555.900	1411.900	1084.700	1581.900	1470.300	1169.400	1780.400	1761.300	1261.800
26	1689.200	1568.200	1219.600	1712.000	1577.800	1251.700	1946.800	1905.200	1495.900
28	1861.400	1710.200	1376.500	1877.600	1743.300	1427.200	2122.800	2021.500	1546.200
30	2008.000	1876.800	1532.800	2023.400	1897.800	1570.400	2281.900	2172.900	1757.400
32	2148.100	2026.200	1666.700	2189.400	2075.200	1755.200	2450.100	2289.400	1853.400
34	2322.000	2180.300	1838.500	2350.400	2211.900	1898.600	2596.400	2477.200	2008.900
36	2486.200	2342.800	2008.700	2510.400	2358.500	2048.000	2718.800	2697.500	2192.600
38	2629.100	2506.100	2158.700	2651.100	2542.900	2216.800	2915.100	2775.700	2382.200
40	2792.000	2651.100	2307.400	2858.200	2691.600	2419.200	3046.400	2986.700	2509.100
42	2957.800	2816.600	2487.200	2958.700	2823.900	2495.400	3192.900	3161.300	2707.000
44	3102.300	2969.200	2624.600	3166.900	3025.900	2741.900	3390.300	3218.700	2840.600
46	3249.800	3111.200	2760.100	3277.800	3141.300	2819.200	3517.600	3453.400	3023.000
48	3416.400	3280.600	2938.400	3427.100	3298.900	2970.300	3651.700	3639.600	3132.100
50	3574.200	3436.400	3096.900	3603.100	3461.200	3144.300	3869.800	3625.100	3268.000
Two (2) Input Variables and Two (2) Lags									
AICc									
2	-93.800	-235.300	-534.200	-28.400	-234.000	-469.700	147.900	-3.700	-447.400
4	255.200	112.500	-191.600	321.400	130.500	-120.000	492.800	378.300	-90.300
6	-1116.200	-1263.400	-1562.400	-1112.500	-1257.400	-1554.500	-872.000	-1031.900	-1437.500
8	-548.100	-686.500	-994.500	-527.400	-685.400	-973.600	-326.900	-375.100	-857.500
10	-452.200	-590.600	-901.000	-431.200	-585.600	-877.200	-195.600	-365.100	-775.200
12	-409.800	-547.700	-856.000	-378.400	-530.700	-810.800	-188.100	-241.100	-760.600
14	-384.500	-528.600	-833.800	-382.400	-508.500	-813.800	-165.100	-219.500	-709.800
16	-370.500	-518.900	-824.300	-346.700	-488.200	-770.000	-126.300	-262.000	-705.800
18	-364.000	-507.600	-816.200	-341.100	-470.100	-760.300	-109.200	-280.600	-688.200
20	-355.400	-498.300	-804.900	-345.700	-483.200	-783.200	-103.400	-267.600	-680.600
22	-355.500	-497.700	-809.800	-323.000	-463.600	-748.800	-102.500	-248.000	-692.300
24	-344.200	-492.000	-797.200	-313.000	-474.000	-754.400	-129.000	-174.800	-669.300
26	-342.500	-488.000	-794.600	-329.700	-463.300	-757.900	-102.900	-213.200	-652.200
28	-347.600	-491.900	-806.700	-339.300	-470.900	-777.400	-127.500	-160.800	-649.600
30	-334.300	-488.900	-792.500	-322.900	-464.700	-764.000	-102.400	-194.700	-618.000
32	-326.300	-480.600	-778.200	-298.500	-440.100	-711.200	-115.900	-168.100	-666.700
34	-331.600	-479.100	-783.700	-336.700	-443.800	-755.900	-82.700	-242.300	-660.200
36	-339.300	-481.900	-795.800	-321.900	-467.400	-765.200	-81.800	-239.400	-675.800
38	-326.100	-472.900	-774.700	-313.300	-464.500	-757.100	-88.200	-211.400	-674.000
40	-328.100	-472.900	-778.000	-308.900	-464.900	-751.600	-75.000	-254.100	-652.200
42	-327.900	-466.600	-772.500	-318.700	-452.400	-751.100	-98.300	-182.200	-670.200
44	-329.400	-474.500	-782.900	-308.200	-466.200	-755.300	-80.500	-223.100	-640.000
46	-329.600	-474.700	-784.300	-322.900	-469.400	-772.900	-110.600	-152.300	-615.000
48	-331.400	-477.700	-789.600	-315.400	-444.900	-744.300	-67.800	-264.600	-649.000
50	-319.000	-470.200	-770.400	-327.100	-470.100	-778.900	-63.600	-286.800	-658.700

BIC									
2	-202.200	-343.800	-642.700	-136.900	-342.500	-578.200	39.400	-112.200	-555.800
4	-116.800	-259.400	-563.600	-50.600	-241.400	-492.000	120.800	6.400	-462.300
6	-21.400	-168.600	-467.500	-17.600	-162.600	-459.600	222.900	62.900	-342.600
8	60.100	-78.400	-386.400	80.700	-77.300	-365.500	281.200	233.000	-249.400
10	148.300	9.900	-300.500	169.400	15.000	-276.700	404.900	235.400	-174.700
12	240.700	102.800	-205.500	272.100	119.700	-160.300	462.400	409.300	-110.100
14	334.200	190.200	-115.000	336.400	210.200	-95.100	553.600	499.200	8.900
16	424.800	276.400	-29.100	448.500	307.000	25.200	669.000	533.300	89.500
18	512.100	368.500	59.900	535.000	406.000	115.800	766.800	595.500	187.800
20	604.200	461.200	154.600	613.800	476.300	176.300	856.200	691.900	278.900
22	689.100	547.000	234.800	721.600	581.100	295.900	942.100	796.600	352.300
24	786.700	638.800	333.600	817.800	656.900	376.400	1001.800	956.100	461.600
26	875.400	729.900	423.300	888.200	754.600	460.000	1115.000	1004.700	565.700
28	957.900	813.600	498.900	966.200	834.600	528.100	1178.000	1144.800	655.900
30	1059.300	904.700	601.200	1070.700	929.000	629.600	1291.200	1198.900	775.600
32	1155.800	1001.500	703.900	1183.500	1042.000	770.900	1366.100	1313.900	815.400
34	1239.200	1091.700	787.000	1234.100	1127.000	814.900	1488.100	1328.400	910.600
36	1320.400	1177.800	864.000	1337.800	1192.300	894.500	1578.000	1420.300	983.900
38	1422.800	1276.000	974.200	1435.600	1284.400	991.700	1660.600	1537.400	1074.800
40	1510.000	1365.200	1060.100	1529.200	1373.200	1086.600	1763.200	1584.000	1185.900
42	1599.600	1460.900	1155.000	1608.800	1475.100	1176.400	1829.200	1745.400	1257.300
44	1687.600	1542.500	1234.100	1708.900	1550.800	1261.700	1936.500	1793.900	1377.000
46	1777.100	1631.900	1322.300	1783.700	1637.300	1333.800	1996.000	1954.400	1491.600
48	1864.900	1718.600	1406.700	1880.900	1751.400	1452.000	2128.500	1931.700	1547.300
50	1967.000	1815.800	1515.600	1958.900	1815.900	1507.200	2222.500	1999.300	1627.400
Two (2) Input Variables and One (1) Lags									
AICc									
2	-120.200	-273.600	-560.100	-108.700	-262.900	-540.200	104.500	-29.200	-482.700
4	-38.300	-203.500	-490.800	13.700	-106.100	-428.100	188.400	49.900	-332.100
6	193.500	36.700	-253.800	210.800	52.600	-220.700	400.300	338.100	-166.200
8	7021.200	6868.200	6573.300	7063.300	6881.600	6623.000	7251.000	7131.400	6688.800
10	-859.900	-1013.900	-1306.700	-845.600	-1004.200	-1283.100	-630.500	-757.800	-1180.700
12	-569.200	-727.100	-1021.200	-527.700	-707.000	-962.100	-328.400	-498.400	-895.900
14	-477.800	-634.100	-929.500	-413.700	-486.600	-730.800	-260.500	-338.100	-823.700
16	-428.500	-587.200	-878.600	-394.100	-434.300	-759.400	-182.400	-397.500	-778.400
18	-406.200	-562.100	-858.300	-325.600	-485.200	-754.600	-203.400	-238.800	-731.700
20	-387.100	-543.500	-838.400	-352.200	-532.500	-793.900	-165.400	-257.800	-714.300
22	-370.800	-529.400	-821.200	-347.000	-507.400	-790.300	-140.200	-277.200	-727.500
24	-364.300	-521.500	-816.100	-319.100	-502.700	-753.200	-130.500	-262.500	-612.800
26	-353.300	-510.200	-801.000	-302.500	-442.400	-703.800	-138.300	-221.700	-695.000
28	-354.300	-508.900	-806.500	-277.200	-256.400	-574.000	-135.800	-202.100	-636.000
30	-346.900	-503.800	-798.900	-324.200	-450.600	-752.600	-105.500	-273.600	-690.400
32	-340.200	-499.700	-792.300	-289.400	-457.600	-730.300	-105.800	-252.000	-622.900
34	-339.600	-496.800	-792.100	-320.300	-484.900	-765.600	-99.000	-264.900	-679.100
36	-334.700	-492.600	-785.900	-295.600	-358.000	-655.900	-99.600	-247.300	-646.100
38	-330.500	-490.900	-782.500	-309.000	-447.800	-725.800	-99.700	-237.800	-673.200
40	-332.200	-491.100	-786.500	-292.400	-442.700	-706.200	-99.600	-219.900	-601.300
42	-330.400	-485.900	-781.500	-261.400	-464.400	-714.900	-134.800	-155.800	-651.800
44	-326.000	-484.400	-777.400	-315.300	-471.200	-756.500	-137.100	-150.700	-664.600
46	-323.000	-479.000	-770.700	-302.600	-436.100	-727.300	-101.700	-199.600	-637.400
48	-325.200	-482.200	-777.300	-315.200	-477.900	-763.800	-88.900	-235.700	-646.100
50	-323.600	-479.400	-774.100	-281.800	-460.000	-714.200	-51.800	-413.900	-683.400
BIC									
2	-222.700	-376.100	-662.600	-211.100	-365.300	-642.700	2.100	-131.700	-585.200
4	-165.400	-330.600	-617.800	-113.400	-233.200	-555.200	61.300	-77.200	-459.200
6	-111.600	-268.400	-558.900	-94.300	-252.500	-525.800	95.100	33.000	-471.300
8	-58.400	-211.400	-506.300	-16.300	-197.900	-456.500	171.500	51.900	-390.700
10	1.200	-152.800	-445.600	15.500	-143.200	-422.100	230.500	103.200	-319.600
12	57.400	-100.400	-394.600	98.900	-80.400	-335.400	298.300	128.300	-269.200
14	113.800	-42.500	-337.900	177.900	105.100	-139.100	331.200	253.500	-232.100
16	175.300	16.500	-274.900	209.600	169.400	-155.700	421.300	206.200	-174.700
18	228.100	72.200	-224.000	308.600	149.100	-120.300	430.900	395.500	-97.400
20	286.900	130.400	-164.500	321.800	141.400	-120.000	508.500	416.200	-40.400
22	347.900	189.300	-102.400	371.800	211.400	-71.500	578.500	441.600	-8.800
24	402.500	245.300	-49.300	447.700	264.100	13.600	636.300	504.300	154.000
26	463.700	306.700	16.000	514.500	374.600	113.100	678.600	595.300	122.000
28	514.300	359.700	62.100	591.400	612.200	294.600	732.800	666.600	232.600
30	574.500	417.500	122.500	597.200	470.700	168.800	815.800	647.700	231.000
32	634.700	475.200	182.600	685.500	517.300	244.600	869.100	722.900	352.000
34	689.400	532.300	237.000	708.700	544.100	263.400	930.000	764.100	349.900
36	749.000	591.100	297.800	788.100	725.700	427.800	984.100	836.400	437.600
38	808.200	647.800	356.200	829.700	690.900	412.900	1039.000	900.900	465.600
40	861.900	703.000	407.600	901.700	751.300	487.900	1094.500	974.200	592.800
42	919.300	763.800	468.200	988.300	785.300	534.800	1114.900	1093.900	597.900
44	979.500	821.100	528.100	990.200	834.300	549.000	1168.400	1154.900	640.900
46	1038.500	882.500	590.900	1058.900	925.500	634.300	1259.800	1161.900	724.100
48	1092.500	935.500	640.400	1102.500	939.800	653.900	1328.900	1182.000	771.600
50	1150.400	994.600	699.900	1192.200	1014.000	759.800	1422.200	1060.100	790.600

Table E16: NN Model for J255 Return and J255 Return Deviation

J256 Index

Four (4) Input Variables and Four (4) Lags									
No. of Neurons	Linear Function			Logistic Function			Softmax Function		
	Return	Deviation	Covariance	Return	Deviation	Covariance	Return	Deviation	Covariance
AIC									
2	2086.100	1986.000	1635.600	2173.700	1990.300	1727.000	2378.600	2220.200	1806.400
4	-496.400	-596.200	-945.600	-454.500	-586.400	-898.900	-204.500	-364.200	-792.200
6	-465.400	-529.700	-922.600	-324.900	-519.300	-772.800	-168.900	-202.700	-724.900
8	-498.500	-504.000	-956.600	-369.800	-499.900	-823.600	-113.700	-244.000	-643.000
10	-464.200	-518.200	-947.100	-355.300	-483.500	-801.800	-123.300	-184.200	-672.500
12	-447.900	-519.400	-937.000	-317.900	-480.900	-767.800	-88.900	-246.200	-630.500
14	-440.100	-519.800	-935.500	-330.500	-477.300	-783.900	-94.800	-205.800	-620.200
16	-422.100	-507.800	-912.700	-310.500	-457.500	-756.500	-74.100	-268.000	-672.600
18	-449.200	-525.100	-957.200	-315.700	-470.800	-766.200	-71.800	-264.200	-645.400
20	-418.400	-526.500	-931.700	-318.400	-470.700	-771.300	-75.200	-239.400	-651.900
22	-429.000	-518.600	-933.100	-361.700	-485.900	-830.400	-83.400	-206.000	-638.200
24	-428.100	-512.100	-926.100	-337.700	-473.100	-797.700	-68.100	-256.700	-657.500
26	-420.400	-518.300	-927.100	-361.000	-466.400	-812.400	-70.000	-240.300	-635.900
28	-445.000	-534.800	-970.400	-275.800	-422.300	-684.000	-73.200	-226.200	-658.500
30	-438.300	-538.600	-967.000	-320.300	-461.700	-770.500	-69.100	-236.400	-646.200
32	-384.200	-483.600	-855.700	-351.900	-470.300	-814.700	-61.900	-265.500	-643.400
34	-409.300	-533.100	-932.200	-350.300	-472.900	-817.500	-50.600	-343.300	-633.600
36	-438.200	-546.100	-976.100	-342.500	-474.900	-805.500	-96.200	-155.000	-574.200
38	-413.300	-523.100	-927.200	-337.900	-477.700	-805.100	-63.000	-252.600	-651.400
40	-410.200	-515.500	-917.100	-310.800	-445.500	-745.300	-82.700	-181.800	-606.200
42	-433.800	-539.700	-966.500	-314.300	-476.800	-781.500	-59.000	-266.500	-628.900
44	-429.700	-518.800	-939.700	-339.400	-484.200	-816.000	-87.500	-166.200	-590.800
46	-410.200	-528.600	-932.100	-284.200	-451.800	-732.900	-66.600	-230.500	-644.700
48	-402.500	-512.600	-905.900	-320.200	-475.700	-786.800	-77.700	-193.300	-646.400
50	-429.100	-516.000	-939.100	-292.900	-460.400	-748.000	-92.000	-159.300	-627.300
BIC									
2	-121.700	-221.800	-572.200	-34.100	-217.500	-480.800	170.800	12.400	-401.400
4	101.300	1.500	-347.900	143.200	11.300	-301.200	393.200	233.500	-194.500
6	280.500	216.200	-176.700	420.900	226.600	-26.900	577.000	543.200	21.000
8	445.700	440.300	-12.300	574.400	444.300	120.600	830.500	700.200	301.200
10	690.300	636.300	207.400	799.300	671.000	352.800	1031.300	970.300	482.000
12	921.700	850.200	432.500	1051.700	888.700	601.800	1280.700	1123.300	739.100
14	1148.900	1067.100	651.400	1256.500	1109.700	803.000	1492.200	1381.100	966.700
16	1383.600	1297.800	892.900	1495.200	1348.200	1049.200	1731.500	1537.600	1133.000
18	1576.000	1500.000	1067.900	1709.400	1554.400	1258.900	1953.400	1760.900	1379.800
20	1826.900	1718.800	1313.600	1926.800	1774.500	1473.900	2170.100	2005.900	1593.400
22	2036.700	1947.100	1532.700	2104.000	1979.800	1635.400	2382.300	2259.700	1827.500
24	2258.400	2174.300	1760.300	2348.800	2213.400	1888.700	2618.400	2429.800	2028.900
26	2487.000	2389.100	1980.300	2546.400	2441.000	2095.000	2837.400	2667.100	2271.500
28	2683.500	2593.700	2158.200	2852.700	2706.200	2444.500	3055.400	2902.300	2470.000
30	2911.500	2811.200	2382.700	3029.400	2888.000	2579.300	3280.700	3113.300	2703.500
32	3186.900	3087.500	2715.400	3219.200	3100.800	2756.400	3509.100	3305.600	2927.700
34	3383.200	3259.400	2860.300	3442.200	3319.600	2975.000	3741.900	3449.200	3158.900
36	3575.800	3467.900	3037.900	3671.500	3539.100	3208.500	3917.800	3859.000	3439.800
38	3822.200	3712.400	3308.400	3897.700	3757.800	3430.400	4172.500	3982.900	3584.100
40	4046.900	3941.600	3540.000	4146.300	4011.700	3711.800	4374.500	4275.300	3850.900
42	4245.000	4139.100	3712.300	4364.500	4202.000	3897.200	4619.800	4412.200	4049.800
44	4470.700	4381.600	3960.700	4561.000	4416.200	4084.400	4812.900	4734.200	4309.600
46	4711.900	4593.500	4190.000	4837.900	4670.300	4389.200	5055.500	4891.600	4477.400
48	4941.400	4831.200	4437.900	5023.600	4868.100	4557.000	5266.100	5150.500	4697.400
50	5136.500	5049.600	4626.500	5272.600	5105.200	4817.600	5473.600	5406.300	4938.300
Four (4) Input Variables and Two (2) Lags									
AIC									
2	-39.300	-179.800	-486.200	56.700	-175.200	-389.700	196.200	95.300	-301.800
4	-2774.000	-2898.700	-3228.800	-2741.600	-2891.000	-3191.600	-2495.800	-2680.400	-3058.600
6	-546.900	-679.500	-1000.700	-546.900	-673.400	-999.700	-284.200	-431.700	-821.200
8	-461.700	-580.800	-925.300	-401.500	-565.200	-848.500	-184.700	-308.300	-706.600
10	-412.000	-553.100	-882.400	-362.300	-494.300	-808.100	-143.000	-288.200	-711.600
12	-409.100	-542.200	-888.500	-369.400	-511.800	-818.000	-122.100	-274.800	-665.400
14	-392.800	-515.700	-857.500	-334.800	-496.600	-781.300	-116.500	-243.500	-684.600
16	-387.900	-526.500	-871.500	-281.700	-476.900	-715.000	-101.600	-258.800	-684.500
18	-370.100	-511.100	-843.800	-317.800	-482.600	-763.300	-97.600	-246.600	-686.900
20	-413.700	-505.900	-890.100	-360.800	-483.300	-810.200	-93.500	-242.800	-674.900
22	-401.300	-491.300	-863.300	-373.100	-486.700	-830.800	-101.000	-201.500	-568.400
24	-375.900	-505.900	-854.800	-360.700	-479.000	-817.200	-91.600	-215.300	-572.200
26	-415.900	-504.300	-899.800	-356.500	-476.100	-807.100	-95.900	-201.700	-633.300
28	-397.000	-506.100	-879.500	-348.500	-477.100	-806.400	-98.600	-190.000	-608.400
30	-366.600	-520.400	-865.400	-329.000	-476.500	-786.100	-113.600	-156.500	-595.700
32	-352.700	-503.100	-835.000	-299.800	-470.100	-754.200	-104.900	-170.900	-643.000
34	-377.000	-507.400	-864.300	-351.700	-478.600	-813.200	-75.700	-239.600	-593.600
36	-379.700	-509.400	-870.000	-350.400	-468.600	-803.900	-76.600	-234.400	-623.600
38	-362.600	-485.400	-830.700	-293.700	-456.600	-734.300	-75.900	-230.200	-628.700
40	-391.000	-491.000	-867.600	-342.700	-478.900	-804.500	-62.600	-292.900	-646.700
42	-377.400	-507.800	-869.400	-332.200	-465.300	-784.100	-76.400	-222.000	-632.900
44	-358.300	-497.500	-840.200	-353.700	-473.800	-812.000	-74.700	-226.200	-649.100
46	-387.600	-504.700	-879.400	-328.100	-466.800	-784.500	-75.700	-221.600	-627.300
48	-368.200	-507.800	-861.800	-368.800	-479.200	-835.500	-75.300	-218.000	-601.200
50	-381.900	-504.400	-874.900	-339.800	-483.400	-810.400	-71.500	-232.900	-635.000

BIC									
2	-176.800	-317.400	-623.800	-80.800	-312.800	-527.200	58.600	-42.300	-439.300
4	-77.100	-201.800	-532.000	-44.800	-194.200	-494.700	201.000	16.500	-361.700
6	55.100	-77.500	-398.700	55.100	-71.400	-397.700	317.800	170.300	-219.200
8	158.000	38.900	-305.600	218.200	54.500	-228.700	435.100	311.500	-86.900
10	293.500	152.500	-176.800	343.300	211.200	-102.500	562.600	417.400	-6.100
12	400.600	267.500	-78.800	440.300	297.800	-8.300	687.600	534.800	144.300
14	528.500	405.700	63.900	586.600	424.800	140.000	804.900	677.900	236.700
16	649.000	510.400	165.300	755.100	559.900	321.900	935.300	778.000	352.300
18	784.400	643.500	310.700	836.700	671.900	391.300	1057.000	907.900	467.600
20	859.900	767.700	383.500	912.900	790.300	463.400	1180.100	1030.800	598.700
22	992.300	902.300	530.300	1020.500	906.900	562.800	1292.600	1192.100	825.200
24	1138.400	1008.400	659.500	1153.600	1035.300	697.100	1422.700	1299.000	942.100
26	1219.600	1131.100	735.700	1278.900	1159.400	828.300	1539.500	1433.800	1002.200
28	1360.000	1250.900	877.400	1408.500	1279.900	950.600	1658.400	1566.900	1148.600
30	1512.100	1358.300	1013.300	1549.700	1402.300	1092.700	1765.100	1722.200	1283.000
32	1648.000	1497.700	1165.800	1700.900	1530.700	1246.600	1895.900	1829.900	1357.800
34	1745.900	1615.600	1258.600	1771.300	1644.400	1309.800	2047.200	1883.400	1529.300
36	1865.600	1735.800	1375.200	1894.900	1776.700	1441.300	2168.700	2010.900	1621.600
38	2005.100	1882.300	1537.000	2073.900	1911.100	1633.300	2291.800	2137.500	1739.000
40	2099.300	1999.300	1622.600	2147.500	2011.300	1685.700	2427.600	2197.300	1843.500
42	2235.500	2105.100	1743.400	2280.700	2147.500	1828.700	2536.400	2390.900	1980.000
44	2377.300	2238.000	1895.400	2381.800	2261.700	1923.600	2660.800	2509.400	2086.500
46	2470.700	2353.600	1978.800	2530.100	2391.500	2073.800	2782.600	2636.700	2231.000
48	2612.900	2473.200	2119.300	2612.300	2501.800	2145.600	2905.800	2763.100	2379.900
50	2722.000	2599.500	2229.100	2764.100	2620.600	2293.500	3032.400	2871.000	2468.900
Four (4) Input Variables and One (1) Lags									
AIC									
2	-104.200	-262.800	-551.700	-83.300	-262.600	-529.900	85.200	58.400	-427.600
4	50.600	-103.900	-399.000	72.100	-97.200	-373.700	282.900	144.100	-308.400
6	2137.100	1981.600	1682.700	2226.100	1994.300	1782.900	2382.900	2202.100	1785.800
8	-784.000	-951.400	-1243.400	-760.700	-940.600	-1209.600	-543.600	-732.700	-1137.600
10	-525.300	-685.200	-986.100	-487.200	-664.600	-931.000	-297.500	-398.000	-829.900
12	-444.500	-606.100	-903.200	-367.500	-405.300	-627.300	-216.600	-326.600	-786.100
14	-414.100	-565.400	-869.600	-374.700	-558.000	-822.700	-182.300	-281.900	-708.600
16	-386.700	-543.300	-842.000	-288.000	-511.500	-718.600	-141.100	-317.900	-699.100
18	-377.200	-531.700	-835.300	-330.400	-516.800	-778.200	-133.200	-283.000	-711.500
20	-366.500	-519.000	-822.500	-335.800	-473.700	-746.300	-116.000	-296.500	-656.700
22	-357.000	-514.200	-815.400	-331.000	-497.400	-776.100	-124.000	-233.300	-672.800
24	-347.400	-500.200	-797.600	-316.300	-484.200	-752.800	-152.700	-166.200	-608.900
26	-343.000	-505.300	-803.000	-348.100	-491.600	-795.500	-101.900	-261.400	-668.200
28	-340.900	-505.400	-805.000	-316.400	-488.600	-763.800	-123.600	-189.600	-657.900
30	-335.800	-498.800	-796.300	-358.000	-494.300	-814.400	-90.100	-278.900	-683.100
32	-340.700	-502.600	-807.700	-340.700	-486.800	-792.800	-91.500	-258.900	-663.300
34	-328.300	-487.200	-782.600	-294.900	-473.500	-743.800	-136.000	-153.800	-630.000
36	-340.100	-502.200	-810.900	-347.300	-480.100	-796.400	-114.000	-181.600	-683.500
38	-328.800	-493.300	-792.700	-286.300	-468.200	-728.000	-130.200	-152.000	-638.600
40	-332.000	-490.400	-794.300	-330.000	-480.900	-783.200	-80.300	-258.000	-620.000
42	-327.800	-492.800	-794.700	-317.000	-471.200	-763.100	-103.300	-189.600	-672.300
44	-330.400	-488.900	-793.700	-280.100	-367.200	-622.700	-82.200	-235.200	-570.600
46	-326.000	-483.800	-785.400	-310.300	-465.500	-752.600	-120.200	-147.900	-510.900
48	-316.700	-486.500	-780.000	-310.500	-440.000	-727.400	-86.100	-221.400	-674.200
50	-317.800	-486.600	-781.900	-321.900	-410.200	-713.200	-87.800	-212.000	-614.500
BIC									
2	-207.700	-366.300	-655.200	-186.800	-366.100	-633.400	-18.300	-45.100	-531.100
4	-139.700	-294.200	-589.400	-118.200	-287.600	-564.100	92.600	-46.300	-498.800
6	-70.700	-226.200	-525.100	18.400	-213.500	-424.900	175.200	-5.700	-422.000
8	10.300	-157.200	-449.200	33.600	-146.400	-415.400	250.600	61.500	-343.400
10	76.700	-83.200	-384.100	114.800	-62.600	-329.000	304.500	204.000	-227.900
12	153.200	-8.300	-305.500	230.200	192.400	-29.600	381.100	271.200	-188.300
14	220.100	68.900	-235.300	259.600	76.300	-188.400	452.000	352.400	-74.400
16	299.600	143.100	-155.700	398.300	174.800	-32.300	545.300	368.400	-12.800
18	368.700	214.200	-89.400	415.500	229.100	-32.300	612.700	462.900	34.400
20	443.200	290.700	-12.800	473.900	336.000	63.400	693.700	513.200	153.000
22	519.100	361.900	60.700	545.100	378.600	100.000	752.100	642.800	203.300
24	596.800	444.000	146.700	627.900	460.000	191.400	791.500	778.000	335.300
26	670.500	508.200	210.500	665.400	521.900	218.000	911.600	752.100	345.300
28	742.800	578.300	278.700	767.300	595.100	319.900	960.100	894.100	425.800
30	818.700	655.700	358.200	796.600	660.300	340.100	1064.400	875.600	471.400
32	885.100	723.300	418.200	885.100	739.100	433.000	1134.400	967.000	562.500
34	969.300	810.300	514.900	1002.600	824.000	553.800	1161.600	1143.700	667.600
36	1029.500	867.400	558.700	1022.200	889.400	573.100	1255.600	1187.900	686.100
38	1113.100	948.500	649.200	1155.600	973.600	713.800	1311.600	1289.800	803.200
40	1182.300	1023.900	720.000	1184.300	1033.400	731.100	1434.000	1256.300	894.300
42	1259.100	1094.100	792.200	1270.000	1115.800	823.800	1483.600	1397.300	914.600
44	1329.300	1170.800	866.000	1379.700	1292.500	1037.000	1577.500	1424.500	1089.100
46	1406.600	1248.800	947.200	1422.400	1267.100	980.000	1612.400	1584.700	1221.700
48	1489.000	1319.100	1025.600	1495.200	1365.700	1078.200	1719.500	1584.200	1131.400
50	1561.000	1392.100	1096.900	1556.800	1468.500	1165.600	1790.900	1666.700	1264.300

Two (2) Input Variables and Four (4) Lags									
AIC									
2	58.900	-67.000	-389.800	106.900	-65.100	-340.200	310.000	207.400	-248.500
4	-752.200	-839.000	-1202.200	-656.000	-836.300	-1103.500	-478.900	-526.000	-1036.800
6	-510.900	-583.800	-963.400	-354.100	-578.700	-801.400	-204.200	-303.700	-721.000
8	-483.500	-541.500	-945.400	-353.900	-529.000	-804.200	-129.900	-330.300	-689.200
10	-487.500	-534.300	-966.800	-307.400	-487.600	-741.100	-138.800	-210.700	-638.500
12	-448.400	-546.000	-949.000	-307.700	-495.900	-757.700	-144.400	-171.000	-581.800
14	-457.000	-553.100	-975.600	-278.700	-436.500	-680.600	-115.500	-194.800	-559.700
16	-438.200	-512.200	-917.600	-292.800	-473.500	-732.800	-121.300	-173.200	-561.300
18	-438.800	-528.700	-940.300	-278.300	-466.600	-720.800	-116.300	-169.500	-563.800
20	-451.100	-538.500	-963.800	-309.600	-452.200	-744.300	-79.200	-259.500	-616.800
22	-452.000	-506.600	-934.400	-249.800	-463.300	-694.400	-85.700	-226.200	-627.300
24	-436.800	-511.200	-927.500	-278.100	-454.600	-724.800	-121.500	-144.100	-587.300
26	-453.700	-515.700	-948.900	-233.800	-343.000	-562.200	-99.300	-173.100	-534.000
28	-430.000	-526.300	-937.300	-328.900	-463.000	-780.400	-113.100	-150.500	-576.000
30	-456.800	-525.800	-968.600	-279.200	-468.500	-730.800	-83.800	-205.100	-608.600
32	-429.500	-521.000	-938.100	-242.400	-463.900	-698.700	-83.900	-200.700	-590.000
34	-434.000	-535.800	-956.500	-276.300	-451.200	-712.000	-72.600	-231.800	-571.500
36	-433.700	-518.500	-937.800	-321.800	-475.700	-782.400	-79.600	-208.800	-667.900
38	-436.800	-530.400	-957.100	-289.200	-454.400	-737.300	-70.600	-233.900	-575.300
40	-447.800	-531.200	-966.500	-310.500	-471.300	-770.300	-79.500	-200.900	-604.500
42	-417.500	-527.000	-933.300	-275.500	-463.700	-726.400	-63.300	-265.300	-638.200
44	-437.500	-530.300	-966.300	-305.200	-438.400	-733.700	-63.600	-255.600	-605.200
46	-454.500	-527.900	-976.100	-259.400	-323.200	-571.400	-93.900	-163.200	-585.000
48	-427.300	-528.600	-944.400	-264.000	-431.500	-687.500	-62.800	-259.500	-666.100
50	-436.000	-543.900	-971.200	-285.600	-456.400	-739.100	-67.300	-233.000	-587.400
BIC									
2	-159.900	-285.800	-608.600	-111.900	-284.000	-559.100	91.200	-11.400	-467.300
4	-44.300	-131.100	-494.300	51.900	-128.400	-395.600	229.000	181.900	-328.800
6	96.400	23.400	-356.100	253.100	28.600	-194.200	403.100	303.600	-113.700
8	228.600	170.600	-233.300	358.300	183.100	-92.100	582.300	381.900	23.000
10	358.800	312.000	-120.500	538.900	358.700	105.200	707.500	635.600	207.900
12	541.900	444.300	41.300	682.600	494.400	232.600	845.900	819.300	408.500
14	681.700	585.700	163.100	860.100	702.300	458.200	1023.200	943.900	579.000
16	851.400	777.400	371.900	996.800	816.000	556.800	1168.200	1116.400	728.300
18	1003.000	913.100	501.500	1163.500	975.300	721.000	1325.500	1272.400	878.100
20	1143.900	1056.500	631.200	1285.400	1142.800	850.700	1515.800	1335.500	978.200
22	1296.900	1242.300	814.400	1499.100	1285.600	1054.500	1663.100	1522.600	1121.500
24	1466.400	1391.900	975.700	1625.000	1448.600	1178.400	1781.600	1759.000	1315.800
26	1604.100	1542.000	1108.800	1824.000	1714.700	1495.500	1958.400	1884.700	1523.700
28	1782.600	1686.300	1275.400	1883.700	1749.600	1432.200	2099.500	2062.100	1636.700
30	1910.900	1841.900	1399.100	2088.500	1899.200	1636.900	2283.900	2162.600	1759.100
32	2093.400	2001.900	1584.800	2280.500	2059.000	1824.200	2439.000	2322.200	1932.900
34	2244.300	2142.500	1721.700	2401.900	2227.100	1966.300	2605.700	2446.500	2106.800
36	2400.000	2315.200	1895.900	2511.900	2358.000	2051.300	2754.100	2625.000	2165.800
38	2552.500	2458.900	2032.200	2700.000	2534.900	2252.000	2918.700	2755.400	2414.000
40	2697.100	2613.700	2178.400	2834.400	2673.600	2374.600	3065.400	2944.000	2540.400
42	2883.000	2773.600	2367.300	3025.100	2836.800	2574.100	3237.200	3035.300	2662.400
44	3018.800	2926.100	2490.000	3151.100	3017.900	2722.600	3392.700	3200.700	2851.100
46	3157.600	3084.200	2636.000	3352.700	3288.900	3040.600	3518.200	3448.900	3027.100
48	3340.600	3239.300	2823.500	3503.900	3336.400	3080.400	3705.100	3508.400	3101.800
50	3487.800	3379.800	2952.600	3638.200	3467.300	3184.700	3856.500	3690.800	3336.300
Two (2) Input Variables and Two (2) Lags									
AIC									
2	-109.700	-244.200	-559.300	-0.100	-238.200	-445.800	130.800	46.300	-300.800
4	237.500	108.000	-213.600	353.700	123.200	-89.200	486.700	388.800	-57.200
6	-1132.200	-1275.200	-1590.100	-1046.200	-1261.900	-1506.100	-897.700	-968.200	-1372.900
8	-561.800	-693.700	-1015.700	-491.500	-682.500	-936.100	-288.900	-494.200	-890.000
10	-461.500	-603.100	-922.600	-352.500	-585.700	-805.900	-199.400	-356.300	-766.900
12	-462.400	-565.700	-926.600	-342.700	-537.400	-782.700	-196.300	-231.600	-710.100
14	-440.600	-549.000	-911.500	-367.700	-528.500	-822.500	-180.400	-198.700	-651.900
16	-392.400	-534.700	-861.900	-339.700	-515.200	-791.300	-124.400	-272.900	-669.400
18	-392.800	-530.300	-869.300	-334.300	-501.600	-787.000	-115.900	-261.500	-651.600
20	-382.500	-515.300	-849.400	-339.100	-506.900	-798.600	-107.600	-257.900	-660.700
22	-400.100	-509.700	-866.300	-332.700	-497.200	-788.700	-114.800	-217.000	-627.500
24	-366.000	-514.500	-841.200	-334.900	-455.100	-781.500	-138.600	-163.600	-637.600
26	-370.600	-511.200	-846.900	-253.200	-413.200	-637.600	-93.800	-235.700	-548.400
28	-380.900	-498.900	-846.600	-288.600	-477.700	-753.200	-97.800	-223.800	-680.000
30	-370.100	-499.500	-838.900	-325.900	-486.000	-784.600	-103.500	-195.700	-591.400
32	-359.100	-503.400	-834.500	-326.500	-480.000	-780.600	-96.700	-207.700	-599.400
34	-366.900	-507.800	-847.600	-333.000	-479.900	-787.000	-93.000	-212.200	-583.100
36	-384.300	-505.200	-864.900	-296.400	-446.300	-761.500	-94.800	-205.400	-621.800
38	-373.900	-502.900	-852.700	-313.800	-471.200	-769.800	-105.000	-177.000	-602.700
40	-380.100	-510.500	-867.900	-316.900	-475.100	-771.200	-80.500	-239.200	-632.500
42	-365.900	-489.800	-836.500	-329.800	-475.800	-786.700	-73.300	-252.200	-558.900
44	-371.400	-502.500	-853.000	-314.600	-455.900	-749.800	-106.400	-166.000	-561.700
46	-367.600	-506.900	-854.200	-323.500	-489.200	-797.900	-121.100	-143.500	-615.900
48	-354.300	-497.500	-832.300	-320.800	-461.900	-775.000	-74.200	-241.700	-606.700
50	-372.600	-497.700	-851.800	-323.900	-388.200	-696.200	-75.000	-238.100	-598.900

BIC									
2	-218.200	-352.700	-667.800	-108.600	-346.600	-554.200	22.300	-62.100	-409.300
4	-134.500	-263.900	-585.600	-18.300	-248.800	-461.200	114.700	16.800	-429.200
6	-37.300	-180.300	-495.200	48.700	-167.000	-411.200	197.200	126.600	-278.000
8	46.300	-85.600	-407.600	116.600	-74.400	-328.000	319.200	113.900	-281.900
10	139.000	-2.600	-322.000	248.000	14.900	-205.400	401.100	244.300	-166.400
12	188.100	84.700	-276.200	307.800	113.000	-132.300	454.200	418.900	-59.700
14	278.200	169.800	-192.800	351.100	190.300	-103.700	538.400	520.000	66.800
16	402.800	260.600	-66.600	455.600	280.100	4.000	670.900	522.400	125.800
18	483.300	345.800	6.800	541.800	374.500	89.000	760.200	614.600	224.400
20	577.000	444.200	110.200	620.400	452.600	160.900	851.900	701.600	298.800
22	644.600	534.900	178.400	711.900	547.400	256.000	929.800	827.600	417.100
24	764.800	616.400	289.700	795.900	675.800	349.300	992.200	967.200	493.300
26	847.300	706.700	371.000	964.700	804.700	580.300	1124.100	982.200	669.500
28	924.700	806.700	458.900	1016.900	827.800	552.400	1207.800	1081.700	625.500
30	1023.500	894.100	554.700	1067.800	907.700	609.000	1290.100	1197.900	802.200
32	1123.000	978.700	647.500	1155.600	1002.100	701.500	1385.300	1274.400	882.700
34	1203.900	1063.000	723.200	1237.700	1090.900	783.800	1477.800	1358.600	987.700
36	1275.400	1154.500	794.800	1363.300	1213.500	898.200	1564.900	1454.400	1037.900
38	1374.900	1245.900	896.200	1435.100	1277.600	979.000	1643.800	1571.900	1146.100
40	1458.000	1327.600	970.300	1521.200	1363.100	1066.900	1757.600	1598.900	1205.600
42	1561.700	1437.700	1091.100	1597.700	1451.700	1140.800	1854.200	1675.300	1368.600
44	1645.600	1514.600	1164.000	1702.500	1561.100	1267.200	1910.600	1851.000	1455.300
46	1739.000	1599.700	1252.400	1783.100	1617.500	1308.800	1985.500	1963.100	1490.700
48	1842.000	1698.800	1364.000	1875.500	1734.500	1421.300	2122.100	1954.600	1589.600
50	1913.500	1788.300	1434.200	1962.200	1897.800	1589.800	2211.100	2048.000	1687.200
Two (2) Input Variables and One (1) Lags									
AIC									
2	-123.600	-285.400	-576.000	-99.800	-280.300	-546.400	110.200	-54.400	-481.600
4	-46.900	-202.100	-498.100	103.300	-156.600	-334.000	190.700	33.400	-350.200
6	190.400	32.800	-261.700	216.800	38.300	-228.900	410.500	313.200	-160.800
8	7016.800	6864.100	6564.100	7048.100	6871.600	6603.700	7235.800	7163.900	6767.400
10	-862.800	-1020.400	-1317.400	-831.900	-1007.000	-1277.400	-628.900	-769.200	-1181.900
12	-572.400	-727.600	-1025.000	-552.500	-723.900	-1001.700	-326.700	-514.900	-909.200
14	-485.900	-642.600	-946.200	-401.300	-525.000	-794.200	-265.500	-332.600	-824.700
16	-434.900	-597.300	-895.300	-355.000	-572.000	-791.200	-205.000	-314.500	-667.600
18	-412.700	-568.300	-870.900	-383.800	-554.000	-830.200	-163.200	-340.900	-708.800
20	-393.200	-550.900	-852.200	-361.700	-531.400	-801.000	-152.800	-296.500	-690.000
22	-378.400	-539.300	-838.400	-356.900	-520.800	-801.400	-131.800	-315.000	-714.800
24	-369.000	-530.600	-829.800	-344.600	-516.800	-792.200	-123.600	-281.900	-562.900
26	-357.000	-523.500	-818.300	-343.900	-504.900	-792.300	-117.800	-287.600	-707.300
28	-350.500	-515.200	-809.700	-333.200	-490.400	-771.500	-107.300	-301.200	-698.900
30	-348.600	-510.400	-807.200	-325.900	-488.500	-763.000	-116.200	-242.200	-634.000
32	-354.600	-517.200	-824.100	-315.300	-457.800	-739.600	-96.600	-301.300	-673.600
34	-344.900	-505.500	-806.100	-322.100	-445.900	-736.400	-98.600	-275.200	-692.300
36	-339.900	-503.300	-801.900	-318.600	-476.600	-756.300	-135.700	-169.000	-531.900
38	-334.000	-502.300	-797.900	-323.300	-486.200	-771.100	-89.600	-283.400	-663.400
40	-334.900	-502.000	-800.200	-305.600	-435.600	-711.400	-131.300	-168.000	-593.000
42	-332.500	-502.500	-800.600	-301.400	-480.900	-747.600	-90.800	-257.300	-661.000
44	-334.300	-497.700	-799.100	-289.600	-463.600	-721.200	-136.000	-155.000	-666.600
46	-335.700	-500.900	-805.000	-262.700	-400.700	-709.500	-75.100	-313.900	-623.400
48	-327.700	-466.000	-763.500	-295.000	-477.100	-742.800	-84.000	-261.200	-646.400
50	-329.700	-491.700	-792.500	-297.800	-431.600	-702.900	-82.000	-249.800	-546.700
BIC									
2	-226.000	-387.800	-678.500	-202.200	-382.700	-648.800	7.700	-156.800	-584.000
4	-174.000	-329.200	-625.200	-23.800	-283.700	-461.100	63.700	-93.700	-477.300
6	-114.700	-272.300	-566.800	-88.300	-266.800	-534.100	105.400	8.100	-465.900
8	-62.800	-215.400	-515.500	-31.500	-208.000	-475.900	156.200	84.300	-312.200
10	-1.800	-159.300	-456.400	29.100	-145.900	-416.300	232.200	91.800	-320.900
12	54.200	-100.900	-398.300	74.100	-97.300	-375.000	300.000	111.700	-282.500
14	105.700	-51.000	-354.600	190.300	66.600	-202.600	326.100	259.100	-233.100
16	168.800	6.400	-291.500	248.700	31.700	-187.500	398.700	289.300	-63.900
18	221.600	65.900	-236.700	250.400	80.300	-196.000	471.100	293.400	-74.500
20	280.800	123.100	-178.300	312.300	142.500	-127.100	521.100	377.400	-16.100
22	340.400	179.500	-119.600	361.800	198.000	-82.600	586.900	403.700	3.900
24	397.800	236.200	-63.000	422.200	250.000	-25.400	643.200	484.900	203.900
26	459.900	293.400	-1.400	473.000	312.000	24.600	699.200	529.400	109.600
28	518.100	353.400	58.900	535.500	378.200	97.100	761.400	567.400	169.700
30	572.800	410.900	114.200	595.400	432.900	158.400	805.100	679.100	287.400
32	620.200	457.700	150.800	659.600	517.100	235.300	878.300	673.600	301.300
34	684.100	523.600	223.000	707.000	583.200	292.700	930.500	753.800	336.800
36	743.800	580.400	281.800	765.100	607.100	327.400	948.000	914.700	551.800
38	804.700	636.400	340.800	815.400	652.600	367.700	1049.200	855.400	475.400
40	859.200	692.100	393.800	888.500	758.500	482.700	1062.800	1026.100	601.100
42	917.200	747.200	449.100	948.300	768.800	502.100	1158.900	992.400	588.700
44	971.300	807.800	506.400	1016.000	841.900	584.400	1169.500	1150.500	638.900
46	1025.900	860.700	556.600	1098.900	960.800	652.100	1286.400	1047.600	738.100
48	1090.000	951.700	654.200	1122.700	940.600	674.900	1333.700	1156.600	771.300
50	1144.300	982.400	681.500	1176.200	1042.400	771.100	1392.000	1224.200	927.300

Table E17: NN Model for J256 Return and J256 Return Deviation

IPD Retail

Four (4) Input Variables and Four (4) Lags									
No. of Neurons	Linear Function			Logistic Function			Softmax Function		
	Return	Deviation	Covariance	Return	Deviation	Covariance	Return	Deviation	Covariance
AIC									
2	-752.200	-839.400	-1196.600	-645.800	-825.500	-1083.300	-441.300	-561.800	-1041.700
4	-533.600	-513.300	-976.600	-319.200	-498.900	-756.800	-112.400	-243.700	-730.200
6	-554.800	-529.600	-1044.800	-302.000	-468.500	-741.300	-112.200	-142.600	-674.900
8	-510.700	-511.700	-994.000	-339.800	-457.500	-768.700	-77.400	-179.600	-626.300
10	-477.800	-504.900	-960.400	-360.100	-460.800	-798.200	-101.100	-118.100	-619.200
12	-547.700	-530.000	-1058.400	-356.400	-455.300	-792.400	-67.700	-169.700	-591.600
14	-451.800	-502.700	-939.500	-460.600	-464.700	-915.200	-62.900	-175.200	-636.000
16	-560.600	-523.700	-1069.800	-368.300	-449.900	-803.800	-55.800	-194.000	-684.800
18	-466.000	-496.100	-948.300	-338.800	-444.400	-771.200	-66.900	-151.800	-638.900
20	-518.600	-520.000	-1026.000	-452.700	-487.300	-928.300	-83.800	-115.300	-545.600
22	-508.800	-515.600	-1013.600	-408.600	-456.600	-864.300	-62.400	-154.600	-600.000
24	-486.800	-518.600	-994.900	-509.000	-504.600	-1003.600	-48.900	-203.400	-650.300
26	-471.200	-498.200	-959.800	-344.400	-452.500	-786.900	-69.800	-133.100	-539.300
28	-484.300	-523.900	-1001.100	-441.300	-465.400	-896.900	-65.900	-141.200	-598.300
30	-506.000	-522.100	-1019.800	-390.600	-442.200	-824.600	-58.700	-157.300	-568.900
32	-463.100	-504.600	-959.600	-415.300	-456.300	-864.600	-51.400	-181.600	-615.300
34	-505.300	-520.500	-1017.000	-463.500	-510.300	-965.600	-35.600	-298.700	-668.900
36	-539.100	-527.700	-1064.200	-510.300	-496.000	-1003.100	-57.200	-156.500	-603.100
38	-464.800	-513.200	-970.100	-525.600	-566.700	-1086.400	-67.500	-131.400	-557.300
40	-486.500	-511.500	-989.900	-402.200	-530.100	-934.400	-60.500	-142.500	-563.400
42	-535.500	-544.300	-1074.400	-499.500	-491.300	-991.700	-73.000	-118.500	-512.200
44	-473.700	-511.400	-978.100	-487.400	-513.200	-993.600	-45.500	-198.700	-602.900
46	-462.100	-509.000	-964.500	-502.500	-516.900	-1012.600	-55.700	-158.900	-600.300
48	-503.100	-528.400	-1024.100	-522.300	-519.000	-1034.900	-68.300	-128.100	-575.600
50	-489.500	-512.200	-995.500	-506.300	-511.400	-1011.300	-68.100	-127.900	-601.800
BIC									
2	-141.500	-228.700	-585.900	-35.100	-214.800	-472.600	169.400	48.900	-431.000
4	-41.600	-21.200	-484.600	172.900	-6.900	-264.700	379.600	248.400	-238.200
6	115.000	140.200	-375.000	367.800	201.300	-71.500	557.500	527.200	-5.100
8	355.700	354.600	-127.700	526.500	408.800	97.600	788.900	686.700	240.000
10	590.500	563.400	107.900	708.200	607.500	270.100	967.200	950.200	449.100
12	724.800	742.500	214.100	916.100	817.200	480.100	1204.900	1102.900	680.900
14	1026.200	975.300	538.500	1017.300	1013.200	562.700	1415.000	1302.700	842.000
16	1123.500	1160.400	614.300	1315.800	1234.100	880.300	1628.300	1490.100	999.200
18	1424.700	1394.600	942.300	1551.900	1446.200	1119.400	1823.700	1738.800	1251.800
20	1578.900	1577.500	1071.500	1644.800	1610.200	1169.100	2013.700	1982.200	1551.800
22	1795.700	1788.900	1291.000	1895.900	1847.900	1440.200	2242.200	2150.000	1704.600
24	2024.900	1993.200	1516.900	2002.700	2007.200	1508.200	2462.800	2308.900	1861.500
26	2247.800	2220.800	1759.300	2374.700	2266.500	1932.100	2649.300	2585.900	2179.700
28	2442.200	2402.600	1925.400	2485.200	2461.100	2029.600	2860.600	2785.300	2328.200
30	2627.900	2611.900	2114.200	2743.400	2691.800	2309.400	3075.200	2976.700	2565.000
32	2878.400	2836.900	2381.900	2926.200	2885.200	2476.900	3290.100	3159.900	2726.200
34	3043.800	3028.500	2532.100	3085.500	3038.800	2583.500	3513.500	3250.400	2880.200
36	3217.600	3229.000	2692.500	3246.400	3260.700	2753.600	3699.500	3600.200	3153.600
38	3499.500	3451.100	2994.200	3438.800	3397.700	2878.000	3896.900	3832.900	3407.100
40	3685.600	3660.500	3182.100	3769.800	3642.000	3237.600	4111.600	4029.600	3608.600
42	3844.200	3835.400	3305.400	3880.200	3888.400	3388.000	4306.700	4261.200	3867.500
44	4113.800	4076.000	3609.400	4100.100	4074.300	3593.800	4542.000	4388.800	3984.500
46	4333.100	4286.200	3830.700	4292.700	4278.300	3782.600	4739.400	4636.200	4194.900
48	4499.900	4474.500	3978.800	4480.700	4483.900	3968.000	4934.700	4874.900	4427.400
50	4721.200	4698.500	4215.200	4704.400	4699.300	4199.400	5142.600	5082.800	4609.000
Four (4) Input Variables and Two (2) Lags									
AIC									
2	-43.000	-115.500	-495.500	80.000	-99.800	-357.600	277.600	185.600	-284.400
4	-666.000	-747.400	-1157.100	-677.200	-694.700	-1114.200	-323.500	-371.100	-807.300
6	-541.100	-577.600	-1023.800	-484.400	-532.100	-921.400	-143.700	-248.900	-698.100
8	-510.700	-543.000	-993.500	-474.800	-492.900	-907.700	-114.400	-194.500	-619.600
10	-525.300	-540.500	-1021.900	-412.400	-482.400	-855.100	-94.900	-189.000	-634.900
12	-496.400	-539.000	-1000.700	-380.200	-497.100	-845.900	-79.700	-199.600	-624.600
14	-483.700	-528.600	-982.700	-417.800	-473.300	-866.300	-93.800	-144.100	-556.200
16	-500.800	-545.100	-1020.800	-274.600	-454.300	-712.200	-70.900	-187.900	-681.100
18	-477.800	-528.600	-983.600	-309.200	-450.700	-749.200	-82.400	-146.100	-569.900
20	-459.600	-508.500	-948.100	-387.800	-462.600	-830.000	-81.200	-141.800	-565.500
22	-486.200	-519.300	-986.800	-487.400	-514.300	-983.900	-96.200	-114.300	-543.400
24	-462.000	-490.100	-934.600	-443.700	-475.700	-904.300	-48.600	-252.000	-655.300
26	-451.300	-520.500	-955.800	-433.000	-476.800	-895.500	-75.200	-141.900	-609.000
28	-441.900	-529.500	-960.200	-392.200	-475.200	-853.300	-59.000	-181.500	-618.100
30	-485.300	-522.100	-992.900	-434.100	-488.100	-913.200	-50.500	-217.300	-668.200
32	-487.000	-573.700	-1050.500	-427.500	-460.900	-881.100	-73.800	-133.000	-586.100
34	-453.900	-497.800	-938.700	-475.100	-508.600	-975.000	-47.600	-225.300	-650.700
36	-446.800	-513.800	-948.700	-438.400	-484.200	-911.300	-61.700	-160.600	-568.700
38	-473.300	-529.700	-990.800	-445.200	-470.800	-904.900	-54.300	-184.900	-674.600
40	-442.900	-519.300	-951.600	-447.000	-504.400	-940.100	-46.800	-219.500	-657.200
42	-496.400	-577.400	-1062.800	-458.200	-499.100	-947.400	-72.900	-128.800	-524.100
44	-466.600	-516.500	-974.900	-449.000	-468.300	-909.500	-57.300	-168.300	-651.600
46	-440.700	-482.000	-916.400	-413.400	-508.900	-911.700	-75.600	-123.200	-538.500
48	-451.300	-500.400	-943.100	-479.800	-504.300	-974.600	-57.500	-161.500	-560.400
50	-443.400	-510.200	-943.600	-445.700	-493.700	-937.900	-62.700	-148.800	-586.300

BIC									
2	-250.400	-322.900	-702.900	-127.500	-307.200	-565.000	70.200	-21.800	-491.800
4	-170.900	-252.300	-662.000	-182.100	-199.600	-619.200	171.600	124.000	-312.200
6	-92.900	-129.400	-575.600	-36.200	-83.900	-473.200	304.500	199.300	-249.900
8	17.200	-15.100	-466.700	53.100	35.000	-379.900	413.500	333.400	-91.700
10	102.400	87.100	-394.300	215.200	145.300	-227.400	532.800	438.600	-7.300
12	237.900	195.300	-266.400	354.100	237.200	-111.600	654.600	534.700	109.700
14	360.400	315.500	-138.600	426.300	370.800	-22.100	750.400	700.000	287.900
16	454.900	410.600	-65.100	681.100	501.400	243.500	884.800	767.800	274.500
18	590.500	539.700	84.700	759.100	617.600	319.100	985.900	922.100	498.400
20	722.000	673.100	233.500	793.800	719.000	351.500	1100.300	1039.700	616.000
22	809.200	776.000	308.500	807.900	781.000	311.400	1199.100	1181.000	751.900
24	947.400	919.300	474.800	965.700	933.700	505.100	1360.800	1157.400	754.100
26	1072.400	1003.200	567.900	1090.700	1046.900	628.200	1448.500	1381.800	914.700
28	1196.300	1108.800	678.100	1246.000	1163.000	784.900	1579.200	1456.700	1020.200
30	1267.600	1230.800	759.900	1318.800	1264.800	839.700	1702.400	1535.600	1084.700
32	1380.700	1294.000	817.200	1440.100	1406.800	986.500	1793.900	1734.600	1281.600
34	1528.700	1484.800	1043.900	1507.500	1473.900	1007.500	1934.900	1757.200	1331.900
36	1650.700	1583.700	1148.700	1659.100	1613.200	1186.200	2035.800	1936.900	1528.800
38	1739.200	1682.800	1221.700	1767.200	1741.700	1307.600	2158.200	2027.600	1537.800
40	1884.600	1808.300	1376.000	1880.600	1823.200	1387.500	2280.800	2108.000	1670.400
42	1946.200	1865.300	1379.800	1984.400	1943.600	1495.300	2369.700	2313.800	1918.600
44	2091.200	2041.300	1582.900	2108.800	2089.600	1648.300	2500.500	2389.500	1906.300
46	2232.300	2191.000	1756.600	2259.600	2164.100	1761.300	2597.400	2549.800	2134.400
48	2336.900	2287.800	1845.100	2308.400	2283.900	1813.600	2730.600	2626.700	2227.800
50	2460.000	2393.300	1959.800	2457.800	2409.700	1965.500	2840.700	2754.600	2317.100
Four (4) Input Variables and One (1) Lags									
AIC									
2	-212.800	-271.700	-655.300	-206.200	-263.100	-638.600	104.800	53.800	-519.800
4	212.700	136.400	-264.600	238.000	179.300	-196.500	558.300	480.800	-14.300
6	-797.300	-859.200	-1262.100	-645.800	-825.500	-1083.300	-450.500	-534.200	-996.700
8	-557.700	-648.900	-1053.600	-558.500	-602.100	-1007.500	-208.100	-286.600	-796.400
10	-497.500	-577.100	-978.700	-440.400	-527.700	-876.000	-150.800	-229.900	-713.100
12	-487.100	-598.900	-1018.200	-321.400	-498.800	-758.900	-123.300	-209.100	-691.400
14	-474.900	-557.000	-975.900	-452.500	-487.700	-884.500	-130.000	-156.800	-668.800
16	-467.600	-576.600	-998.400	-374.100	-487.500	-814.900	-101.600	-178.000	-569.200
18	-445.800	-553.300	-960.000	-432.600	-478.700	-871.100	-103.000	-157.000	-576.800
20	-448.400	-545.500	-959.600	-448.900	-511.200	-924.700	-77.700	-207.800	-641.600
22	-435.600	-519.200	-922.900	-428.600	-469.700	-866.700	-110.900	-125.400	-573.400
24	-451.600	-543.500	-966.300	-442.000	-503.500	-916.600	-94.300	-141.200	-626.400
26	-440.700	-536.900	-955.000	-428.200	-495.800	-897.300	-73.300	-182.300	-608.300
28	-448.400	-548.700	-972.800	-414.900	-463.600	-853.800	-90.500	-136.500	-638.900
30	-436.700	-525.400	-941.700	-415.800	-473.500	-867.000	-66.700	-189.700	-649.200
32	-423.600	-511.400	-914.400	-425.300	-499.700	-903.400	-83.600	-139.900	-571.000
34	-434.300	-537.800	-953.100	-429.600	-493.300	-902.900	-95.500	-119.500	-605.900
36	-444.500	-529.400	-955.600	-411.800	-470.800	-863.600	-82.400	-134.600	-610.700
38	-441.000	-556.700	-979.500	-397.800	-453.500	-833.100	-89.300	-119.600	-541.000
40	-450.100	-519.900	-952.400	-426.200	-511.900	-921.500	-79.200	-137.300	-601.500
42	-427.700	-513.200	-926.400	-421.300	-480.800	-885.400	-85.100	-125.700	-614.200
44	-439.000	-540.100	-964.900	-405.400	-474.300	-869.100	-52.300	-216.900	-622.200
46	-416.700	-497.100	-899.400	-430.500	-486.900	-902.200	-85.100	-119.000	-506.900
48	-431.700	-524.800	-941.700	-427.400	-469.600	-885.000	-67.200	-156.100	-671.200
50	-421.900	-523.200	-931.100	-420.700	-471.700	-878.200	-82.300	-121.300	-513.300
BIC									
2	-306.800	-365.700	-749.300	-300.200	-357.100	-732.500	10.800	-40.200	-613.800
4	-256.700	-333.000	-734.000	-231.400	-290.100	-665.900	88.900	11.400	-483.700
6	-186.600	-248.500	-651.400	-35.100	-214.800	-472.600	160.200	76.500	-386.000
8	-121.700	-212.900	-617.600	-122.500	-166.000	-571.500	227.900	149.400	-360.400
10	-49.300	-128.900	-530.500	7.900	-79.500	-427.800	297.400	218.300	-264.900
12	5.000	-106.800	-526.100	170.600	-6.800	-266.900	368.800	282.900	-199.300
14	72.000	-10.000	-429.000	94.400	59.200	-337.600	417.000	390.200	-121.900
16	139.300	30.400	-391.400	232.800	119.400	-208.000	505.300	428.900	37.700
18	224.000	116.500	-290.200	237.200	191.000	-201.300	566.700	512.800	93.000
20	285.900	188.800	-225.400	285.400	223.000	-190.400	656.600	526.500	92.700
22	364.300	280.700	-123.000	371.300	330.200	-66.800	689.000	674.500	226.500
24	414.700	322.800	-100.000	424.300	362.800	-50.200	772.000	725.100	240.000
26	492.500	396.400	-21.700	505.100	437.500	35.900	860.000	751.000	324.900
28	552.300	452.000	27.800	585.700	537.000	146.800	910.100	864.100	361.700
30	631.600	542.900	126.600	652.400	594.800	201.300	1001.600	878.600	419.100
32	712.600	624.800	221.800	710.800	636.500	232.700	1052.600	996.300	565.200
34	770.000	666.500	251.200	774.700	711.000	301.400	1108.800	1084.800	598.400
36	828.100	743.100	316.900	860.700	801.800	409.000	1190.200	1137.900	661.900
38	899.900	784.200	361.400	943.100	887.400	507.800	1251.600	1221.300	799.900
40	959.300	889.500	457.000	983.200	897.400	487.800	1330.200	1272.100	807.900
42	1050.300	964.700	551.600	1056.600	997.200	592.500	1392.900	1352.200	863.700
44	1107.600	1006.600	581.700	1141.200	1072.300	677.500	1494.300	1329.700	924.400
46	1198.600	1118.300	715.900	1184.800	1128.500	713.100	1530.200	1496.300	1108.400
48	1252.400	1159.300	742.400	1256.700	1214.400	799.100	1616.900	1528.000	1012.900
50	1331.000	1229.700	821.800	1332.200	1281.200	874.700	1670.600	1631.600	1239.600

Two (2) Input Variables and Four (4) Lags									
AIC									
2	424.700	421.000	-15.100	447.900	429.000	16.000	807.700	722.400	183.100
4	-551.400	-579.900	-996.800	-447.900	-563.100	-878.100	-209.000	-235.500	-805.200
6	-483.500	-520.400	-939.300	-335.500	-494.100	-767.100	-130.300	-178.300	-753.400
8	-495.500	-507.900	-960.100	-461.100	-480.000	-902.300	-98.900	-176.400	-629.200
10	-497.000	-519.700	-983.800	-163.400	-429.700	-568.300	-68.700	-240.400	-712.900
12	-499.800	-528.000	-1003.100	-489.300	-467.500	-933.600	-105.500	-122.000	-616.900
14	-472.000	-506.700	-955.700	-456.500	-459.700	-893.800	-70.200	-177.800	-631.600
16	-482.300	-506.700	-971.300	-433.800	-461.600	-879.500	-74.700	-155.000	-606.300
18	-464.600	-519.200	-973.600	-349.000	-451.100	-785.300	-68.400	-163.300	-598.800
20	-476.700	-533.800	-997.600	-435.000	-455.500	-875.600	-64.600	-169.700	-638.500
22	-444.700	-485.500	-915.800	-280.600	-444.900	-710.400	-69.700	-150.600	-610.600
24	-491.900	-537.000	-1024.300	-352.400	-445.300	-783.300	-100.300	-100.700	-645.400
26	-453.900	-504.600	-950.100	-434.200	-451.400	-872.900	-98.300	-100.400	-552.800
28	-472.500	-537.800	-1000.700	-378.100	-462.700	-828.000	-70.500	-139.500	-593.900
30	-450.500	-530.000	-969.900	-456.200	-453.000	-905.100	-71.400	-136.100	-593.900
32	-486.400	-526.300	-1003.100	-415.000	-451.800	-855.100	-57.800	-168.800	-599.000
34	-474.000	-526.900	-997.200	-458.900	-464.700	-913.500	-64.900	-148.000	-671.300
36	-447.300	-508.900	-947.800	-425.400	-470.300	-885.000	-61.800	-154.100	-677.400
38	-455.800	-504.300	-949.900	-453.100	-455.000	-899.500	-59.600	-157.100	-555.800
40	-469.300	-495.000	-955.200	-459.600	-461.700	-912.800	-82.300	-112.400	-605.900
42	-486.600	-523.400	-1002.500	-434.100	-453.000	-884.900	-54.200	-172.200	-578.100
44	-459.800	-496.200	-948.800	-454.900	-450.700	-901.500	-55.900	-166.200	-616.700
46	-455.900	-525.300	-973.800	-442.300	-450.600	-887.600	-81.700	-109.700	-531.700
48	-441.900	-499.100	-933.900	-447.000	-449.900	-888.100	-53.000	-174.300	-624.500
50	-463.400	-498.800	-953.600	-370.300	-453.500	-817.200	-45.100	-207.300	-625.500
BIC									
2	-284.100	-287.800	-723.900	-260.900	-279.800	-692.800	98.900	13.600	-525.700
4	-118.100	-146.600	-563.500	-14.500	-129.700	-444.800	224.400	197.900	-371.800
6	26.000	-10.900	-429.700	174.000	15.400	-257.600	379.200	331.200	-243.900
8	139.100	126.700	-325.500	173.500	154.600	-267.700	535.700	458.200	5.400
10	273.600	251.000	-213.100	607.300	340.900	202.300	702.000	530.300	57.800
12	411.100	382.900	-92.200	421.600	443.400	-22.700	805.500	788.900	294.000
14	581.200	546.500	97.600	596.700	593.500	159.400	983.100	875.400	421.700
16	714.400	690.000	225.400	763.000	735.100	317.300	1122.000	1041.700	590.400
18	876.300	821.700	367.300	991.900	889.900	555.600	1272.500	1177.600	742.100
20	1008.900	951.800	488.000	1050.600	1030.100	610.000	1421.000	1315.900	847.100
22	1185.900	1145.100	714.800	1350.000	1185.700	920.200	1560.900	1480.000	1020.000
24	1283.900	1238.900	751.500	1423.500	1330.500	992.500	1675.500	1675.100	1130.400
26	1467.400	1416.600	971.200	1487.100	1469.900	1048.400	1823.000	1820.800	1368.500
28	1594.400	1529.000	1066.100	1688.700	1604.100	1238.800	1996.300	1927.400	1472.900
30	1762.000	1682.500	1242.600	1756.300	1759.500	1307.400	2141.100	2076.400	1618.600
32	1871.800	1831.900	1355.100	1943.200	1906.500	1503.100	2300.400	2189.500	1759.300
34	2030.100	1977.200	1506.900	2045.100	2039.400	1590.600	2439.100	2356.000	1832.800
36	2202.600	2141.000	1702.100	2224.600	2179.700	1765.000	2588.100	2495.900	1972.600
38	2340.100	2291.600	1846.000	2342.800	2340.800	1896.400	2736.300	2638.800	2240.100
40	2472.600	2446.800	1986.700	2482.200	2480.100	2029.000	2859.600	2829.500	2336.000
42	2601.300	2564.500	2085.300	2653.700	2634.800	2203.000	3033.700	2915.600	2509.800
44	2774.100	2737.700	2285.100	2779.000	2783.200	2332.400	3178.000	3067.700	2617.100
46	2924.000	2854.600	2406.100	2937.600	2929.300	2492.300	3298.200	3270.200	2848.300
48	3084.100	3026.900	2592.100	3079.000	3076.100	2637.900	3473.000	3351.700	2901.500
50	3208.700	3173.300	2718.500	3301.800	3218.600	2854.900	3627.000	3464.800	3046.600
Two (2) Input Variables and Two (2) Lags									
AIC									
2	-229.500	-236.300	-667.700	-47.100	-226.700	-485.200	153.400	51.400	-451.300
4	-4759.800	-4772.500	-5204.700	-4716.400	-4768.200	-5158.300	-4383.600	-4460.100	-4999.100
6	-616.800	-691.200	-1120.000	-588.400	-623.300	-1019.700	-236.300	-355.000	-785.600
8	-552.200	-561.100	-1014.900	-463.100	-529.500	-894.300	-141.000	-278.200	-763.200
10	-508.600	-567.700	-1009.100	-466.000	-498.100	-896.700	-115.200	-224.100	-672.500
12	-479.800	-510.200	-939.400	-403.700	-484.200	-836.600	-125.700	-154.000	-686.500
14	-472.200	-577.600	-1012.900	-490.500	-482.100	-931.500	-117.200	-142.000	-607.400
16	-488.700	-567.400	-1027.500	-435.700	-472.000	-871.100	-94.900	-162.500	-624.000
18	-463.200	-565.900	-1006.000	-434.500	-470.200	-872.700	-78.300	-188.000	-618.100
20	-478.700	-495.700	-947.900	-444.200	-468.900	-885.100	-76.000	-181.100	-643.200
22	-458.200	-564.600	-1002.800	-431.600	-461.800	-868.000	-75.100	-173.700	-656.400
24	-469.100	-573.600	-1025.900	-426.800	-460.200	-863.800	-100.500	-119.600	-625.800
26	-471.100	-560.700	-1018.000	-416.300	-460.300	-854.800	-57.900	-220.600	-591.800
28	-464.700	-539.300	-986.600	-473.200	-469.100	-929.300	-81.100	-141.900	-642.400
30	-458.900	-545.200	-990.100	-439.400	-460.300	-881.500	-81.300	-137.400	-594.700
32	-461.000	-556.300	-1004.200	-411.400	-456.100	-849.700	-90.500	-120.700	-669.500
34	-466.000	-549.900	-1003.400	-435.300	-459.000	-878.000	-65.100	-168.800	-627.000
36	-452.000	-548.400	-985.900	-449.600	-458.500	-893.200	-63.100	-171.700	-660.800
38	-446.900	-516.100	-948.800	-395.300	-458.500	-840.600	-67.400	-156.600	-631.500
40	-469.900	-556.400	-1013.000	-434.400	-466.400	-886.300	-59.900	-176.000	-608.200
42	-445.200	-520.100	-951.000	-437.800	-466.900	-890.500	-76.000	-134.100	-618.900
44	-459.200	-522.300	-969.100	-441.100	-458.800	-887.400	-65.300	-155.100	-604.500
46	-461.200	-516.300	-965.400	-438.400	-458.900	-884.300	-49.000	-217.100	-648.000
48	-448.000	-527.600	-963.800	-432.900	-460.000	-880.200	-55.800	-181.600	-617.600
50	-458.900	-547.000	-996.600	-415.700	-480.300	-883.500	-51.000	-200.200	-610.900

BIC									
2	-341.000	-347.800	-779.200	-158.600	-338.200	-596.700	41.900	-60.100	-562.800
4	-256.700	-269.400	-701.600	-213.300	-265.100	-655.200	119.600	43.000	-496.000
6	-165.000	-239.400	-668.200	-136.600	-171.500	-567.800	215.500	96.800	-333.800
8	-107.600	-116.500	-570.300	-18.400	-84.900	-449.700	303.600	166.400	-318.600
10	-10.900	-89.900	-511.400	31.700	-0.300	-399.000	382.500	273.700	-174.700
12	86.700	56.300	-372.900	162.800	82.300	-270.100	440.800	412.500	-120.000
14	169.400	64.000	-371.400	151.100	159.500	-289.900	524.400	499.600	34.200
16	231.200	152.400	-307.700	284.100	247.900	-151.300	624.900	557.300	95.900
18	336.700	234.000	-206.100	365.500	329.800	-72.800	721.600	611.900	181.800
20	402.500	385.500	-66.700	437.000	412.300	-3.900	805.200	700.000	238.000
22	505.000	398.600	-39.700	531.600	501.400	95.200	888.000	789.500	306.800
24	576.600	472.100	19.800	618.900	585.500	181.900	945.200	926.100	419.900
26	657.500	567.900	110.700	712.300	668.300	273.900	1070.800	908.000	536.900
28	747.200	672.500	225.200	738.600	742.700	282.600	1130.800	1069.900	569.400
30	836.400	750.100	305.200	856.000	835.000	413.800	1214.000	1157.900	700.600
32	918.000	822.600	374.700	967.500	922.800	529.200	1288.500	1258.300	709.500
34	996.700	912.800	459.300	1027.400	1003.800	584.800	1397.600	1294.000	835.700
36	1094.600	998.200	560.700	1097.000	1088.100	653.400	1483.500	1374.900	885.800
38	1183.700	1114.500	681.800	1235.300	1172.100	790.000	1563.200	1474.000	999.100
40	1244.700	1158.200	701.700	1280.300	1248.200	828.300	1654.700	1538.700	1106.400
42	1353.600	1278.700	847.800	1361.000	1331.900	908.300	1722.800	1664.700	1179.900
44	1423.800	1360.700	913.900	1441.900	1424.200	995.500	1817.700	1727.800	1278.500
46	1506.000	1450.900	1001.900	1528.800	1508.300	1082.900	1918.200	1750.100	1319.200
48	1603.500	1523.900	1087.700	1618.600	1591.500	1171.300	1995.700	1869.900	1433.900
50	1676.900	1588.800	1139.200	1720.200	1655.500	1252.300	2084.800	1935.600	1524.900
Two (2) Input Variables and One (1) Lags									
AIC									
2	-243.600	-298.900	-688.300	-230.600	-293.400	-669.100	93.400	3.300	-497.300
4	-117.100	-259.000	-668.000	68.900	-99.600	-354.900	231.400	148.000	-240.000
6	3911.900	3761.200	3343.200	3940.300	3886.900	3506.900	4258.900	4198.800	3648.900
8	-743.400	-856.100	-1268.700	-717.700	-770.300	-1152.100	-393.700	-464.200	-908.300
10	-559.000	-643.200	-1039.500	-381.400	-581.300	-800.100	-216.700	-303.600	-773.700
12	-525.200	-672.200	-1093.300	-488.300	-545.400	-924.200	-155.600	-267.000	-709.700
14	-494.300	-591.900	-1006.600	-455.000	-521.900	-897.100	-148.700	-194.500	-707.400
16	-477.000	-610.300	-1025.300	-436.200	-497.600	-867.200	-104.200	-259.700	-704.500
18	-463.800	-609.500	-1030.500	-458.100	-491.800	-893.800	-107.100	-199.600	-675.600
20	-460.500	-622.700	-1046.200	-445.300	-474.300	-871.700	-111.700	-165.400	-636.200
22	-452.300	-591.000	-1006.100	-435.200	-483.500	-876.500	-93.600	-187.700	-683.200
24	-456.900	-598.800	-1021.900	-419.900	-478.600	-860.100	-74.100	-244.000	-713.400
26	-441.100	-583.900	-1000.600	-428.000	-477.300	-871.100	-87.600	-174.800	-648.700
28	-441.000	-580.100	-995.400	-440.900	-473.700	-882.900	-79.300	-187.800	-689.100
30	-432.600	-579.100	-994.400	-404.300	-466.800	-841.300	-76.100	-187.300	-651.600
32	-431.400	-545.600	-953.500	-425.100	-469.100	-866.700	-83.000	-157.400	-532.500
34	-433.700	-581.900	-996.600	-420.300	-477.800	-872.000	-72.500	-183.800	-650.100
36	-425.000	-586.900	-1001.200	-426.900	-464.000	-867.100	-77.700	-160.300	-561.300
38	-436.200	-574.800	-993.300	-412.400	-462.900	-852.300	-76.100	-162.700	-645.200
40	-430.800	-564.100	-980.000	-414.900	-469.300	-863.300	-104.500	-112.700	-648.600
42	-430.500	-562.200	-977.000	-424.300	-459.400	-862.800	-70.500	-168.200	-577.900
44	-432.100	-578.600	-1000.600	-415.700	-454.200	-850.100	-56.400	-222.500	-702.300
46	-426.300	-548.100	-960.300	-401.400	-460.200	-842.100	-66.400	-174.500	-622.700
48	-428.200	-582.100	-1001.100	-431.200	-473.600	-886.000	-70.500	-159.800	-602.600
50	-424.500	-573.400	-989.700	-416.300	-467.900	-866.600	-52.400	-228.600	-622.600
BIC									
2	-329.100	-384.400	-773.800	-316.100	-378.900	-754.600	7.900	-82.200	-582.800
4	-287.800	-429.700	-838.800	-101.900	-270.400	-525.600	60.700	-22.700	-410.800
6	-240.700	-391.400	-809.400	-212.300	-265.700	-645.700	106.300	46.200	-503.600
8	-185.500	-298.200	-710.800	-159.800	-212.300	-594.200	164.200	93.700	-350.400
10	-119.600	-203.800	-600.200	58.000	-141.900	-360.700	222.600	135.800	-334.300
12	-86.600	-233.600	-654.700	-49.700	-106.800	-485.600	283.000	171.600	-271.100
14	-28.700	-126.300	-541.000	10.700	-56.300	-431.400	317.000	271.100	-241.800
16	26.500	-106.700	-521.800	67.300	6.000	-363.600	399.400	243.800	-200.900
18	83.100	-62.500	-483.600	88.800	55.100	-346.900	439.800	347.300	-128.700
20	132.800	-29.400	-452.900	148.000	119.000	-278.400	481.600	427.900	-42.900
22	189.300	50.600	-364.500	206.400	158.100	-234.900	548.000	453.900	-41.600
24	234.300	92.400	-330.800	271.200	212.500	-169.000	617.000	447.100	-22.300
26	300.500	157.600	-259.000	313.500	264.300	-129.500	653.900	566.700	92.800
28	351.600	212.500	-202.800	351.700	318.900	-90.300	713.300	604.800	103.500
30	411.600	265.000	-150.300	439.800	377.300	2.800	768.100	656.800	192.600
32	464.600	350.500	-57.500	470.900	426.900	29.300	813.000	738.600	363.500
34	514.500	366.300	-48.300	527.900	470.400	76.200	875.800	764.400	298.100
36	575.600	413.700	-0.600	573.700	536.600	133.600	922.900	840.300	439.300
38	617.000	478.400	60.000	640.900	590.300	201.000	977.200	890.500	408.000
40	675.200	541.900	126.000	691.100	636.600	242.700	1001.500	993.300	457.300
42	728.400	596.600	181.800	734.600	699.500	296.000	1088.400	990.700	581.000
44	779.800	633.300	211.300	796.200	757.700	361.800	1155.500	989.300	509.600
46	838.600	716.800	304.700	863.500	804.700	422.800	1198.600	1090.400	642.200
48	889.900	736.000	317.000	886.900	844.600	432.100	1247.600	1158.300	715.500
50	946.800	798.000	381.700	955.000	903.400	504.800	1318.900	1142.800	748.800

Table E18: NN Model for Retail Return and Retail Return Deviation

IPD Office

Four (4) Input Variables and Four (4) Lags									
No. of Neurons	Linear Function			Logistic Function			Softmax Function		
	Return	Deviation	Covariance	Return	Deviation	Covariance	Return	Deviation	Covariance
AIC									
2	-847.300	-852.200	-1302.400	-835.900	-824.100	-1263.000	-435.800	-553.900	-965.900
4	-547.100	-552.000	-1029.500	-542.600	-609.200	-1081.700	-106.900	-235.800	-655.900
6	-532.000	-525.600	-1021.900	-561.600	-593.000	-1115.500	-105.600	-135.900	-589.000
8	-506.400	-544.600	-1032.900	-565.300	-594.000	-1130.500	-87.400	-136.500	-546.600
10	-519.600	-535.400	-1039.100	-539.800	-588.900	-1108.200	-66.600	-165.100	-622.900
12	-540.800	-526.100	-1049.200	-567.800	-609.100	-1161.600	-61.000	-166.000	-550.500
14	-507.300	-570.900	-1068.700	-568.700	-584.200	-1136.500	-51.900	-188.200	-603.900
16	-501.100	-548.800	-1046.600	-582.100	-599.300	-1166.400	-73.300	-120.700	-536.100
18	-525.300	-552.800	-1086.600	-621.500	-604.300	-1219.300	-45.700	-200.200	-607.500
20	-527.100	-578.100	-1103.800	-550.500	-576.600	-1115.900	-71.800	-120.400	-559.000
22	-501.800	-536.600	-1033.600	-554.600	-578.600	-1124.100	-49.200	-174.200	-581.700
24	-534.900	-547.100	-1071.600	-597.400	-606.600	-1197.700	-46.400	-180.800	-582.200
26	-543.200	-573.400	-1110.100	-570.200	-588.100	-1150.000	-47.700	-165.600	-494.300
28	-516.000	-532.900	-1049.000	-558.200	-599.800	-1148.000	-43.500	-187.800	-576.100
30	-495.100	-561.900	-1059.700	-593.400	-620.200	-1208.200	-44.700	-180.800	-586.500
32	-510.700	-577.300	-1079.500	-576.700	-598.300	-1167.200	-45.800	-169.400	-529.500
34	-540.400	-559.200	-1091.100	-542.300	-598.900	-1132.300	-86.200	-89.400	-515.000
36	-514.500	-563.200	-1077.300	-596.600	-588.800	-1182.500	-43.000	-181.700	-605.900
38	-568.700	-607.700	-1169.900	-562.500	-607.800	-1162.700	-48.300	-161.000	-566.500
40	-532.900	-571.200	-1103.600	-590.000	-582.400	-1165.200	-72.400	-104.400	-511.100
42	-565.000	-596.300	-1156.900	-576.300	-600.800	-1169.800	-45.500	-166.500	-519.500
44	-534.600	-578.300	-1122.300	-592.100	-593.400	-1182.300	-43.800	-175.400	-589.500
46	-576.800	-588.700	-1167.700	-566.300	-582.100	-1141.100	-70.200	-110.000	-555.600
48	-563.400	-584.200	-1145.800	-567.600	-592.500	-1155.400	-44.300	-169.000	-525.900
50	-596.000	-623.400	-1212.500	-584.900	-597.100	-1176.100	-67.700	-113.200	-577.000
BIC									
2	-236.600	-241.500	-691.700	-225.200	-213.400	-652.300	174.900	56.800	-355.200
4	-55.100	-59.900	-537.500	-50.600	-117.200	-589.700	385.200	256.300	-163.800
6	137.700	144.100	-352.100	108.100	76.700	-445.700	564.200	533.800	80.800
8	359.900	321.800	-166.600	301.000	272.400	-264.200	778.900	729.800	319.700
10	548.700	532.900	29.200	528.500	479.400	-39.900	1001.700	903.200	445.400
12	731.700	746.500	223.300	704.700	663.400	110.900	1211.600	1106.600	722.000
14	970.700	907.100	409.300	909.300	893.800	341.400	1426.100	1289.800	874.100
16	1183.000	1135.200	637.500	1102.000	1084.800	517.700	1610.800	1563.400	1147.900
18	1365.300	1337.900	804.000	1269.200	1286.300	671.300	1844.900	1690.400	1283.100
20	1570.400	1519.400	993.700	1547.000	1520.800	981.600	2025.700	1977.100	1538.500
22	1802.700	1767.900	1270.900	1749.900	1726.000	1180.400	2255.300	2130.400	1722.800
24	1976.800	1964.700	1440.100	1914.300	1905.100	1314.100	2465.300	2330.900	1929.600
26	2175.900	2145.600	1609.000	2148.800	2130.900	1569.000	2671.300	2553.500	2224.800
28	2410.500	2393.600	1877.500	2368.300	2326.700	1778.500	2883.000	2738.600	2350.300
30	2638.900	2572.100	2074.200	2540.600	2513.700	1925.800	3089.200	2953.100	2547.400
32	2830.800	2764.200	2262.000	2764.800	2743.200	2174.300	3295.700	3172.100	2812.000
34	3008.700	2989.800	2458.000	3006.700	2950.100	2416.700	3462.900	3459.700	3034.100
36	3242.100	3193.500	2679.400	3160.100	3167.900	2574.200	3713.700	3575.000	3150.800
38	3395.600	3356.600	2794.500	3401.800	3356.600	2801.700	3916.000	3803.400	3397.900
40	3639.100	3600.800	3068.400	3582.000	3589.600	3006.900	4099.600	4067.600	3660.900
42	3814.700	3783.400	3222.800	3803.500	3778.900	3209.900	4334.300	4213.200	3860.300
44	4052.800	4009.200	3465.200	3995.300	3994.000	3405.100	4543.700	4412.000	3998.000
46	4218.400	4206.500	3627.500	4228.900	4213.100	3654.100	4725.000	4685.200	4239.600
48	4439.600	4418.800	3857.100	4435.300	4410.400	3847.600	4958.700	4833.900	4477.000
50	4614.700	4587.300	3998.200	4625.800	4613.700	4034.600	5143.000	5097.500	4633.700
Four (4) Input Variables and Two (2) Lags									
AIC									
2	-119.00	-100.30	-550.20	-112.40	-69.60	-520.60	290.30	169.80	-266.80
4	-702.30	-699.50	-1144.30	-689.40	-779.70	-1212.80	-304.70	-392.30	-841.50
6	-568.10	-643.00	-1115.40	-553.80	-626.10	-1084.00	-132.40	-259.40	-684.50
8	-523.10	-555.60	-1019.20	-534.30	-598.00	-1073.00	-96.20	-228.70	-678.10
10	-516.50	-537.80	-1010.50	-519.80	-585.80	-1061.80	-80.60	-211.80	-671.20
12	-465.50	-524.50	-960.30	-515.10	-558.50	-1038.90	-86.30	-158.90	-607.10
14	-505.10	-564.30	-1040.40	-528.60	-555.90	-1054.90	-65.40	-198.70	-615.90
16	-473.90	-540.00	-996.80	-512.00	-550.60	-1039.30	-57.30	-216.20	-650.30
18	-505.10	-553.20	-1040.10	-521.40	-562.60	-1061.60	-63.00	-176.00	-594.60
20	-469.70	-538.00	-987.20	-510.40	-562.50	-1052.80	-73.10	-134.50	-548.50
22	-499.00	-551.10	-1031.00	-509.90	-555.00	-1046.20	-60.90	-164.80	-584.50
24	-497.20	-543.30	-1024.70	-494.40	-555.90	-1033.80	-54.40	-182.40	-607.50
26	-477.40	-535.00	-996.40	-488.30	-548.50	-1024.70	-65.70	-143.40	-575.00
28	-495.70	-528.30	-1016.50	-520.90	-549.10	-1055.30	-52.20	-180.10	-589.60
30	-501.50	-553.30	-1043.60	-483.80	-543.80	-1014.60	-68.70	-131.40	-578.50
32	-500.70	-539.40	-1027.70	-468.50	-532.10	-988.10	-50.10	-178.60	-555.00
34	-504.50	-576.50	-1070.10	-488.20	-541.40	-1018.90	-67.00	-128.80	-531.20
36	-490.90	-551.40	-1034.30	-493.60	-543.20	-1024.90	-63.90	-130.30	-511.20
38	-490.30	-538.60	-1018.70	-477.40	-537.90	-1004.60	-68.30	-122.80	-526.00
40	-485.60	-555.80	-1034.20	-475.10	-514.00	-979.40	-83.00	-102.90	-623.30
42	-482.10	-550.90	-1023.70	-502.40	-550.60	-1042.10	-51.70	-162.20	-558.90
44	-475.50	-534.00	-999.70	-500.80	-553.50	-1043.70	-36.40	-229.60	-538.80
46	-502.60	-537.00	-1032.20	-501.40	-546.40	-1039.20	-75.80	-108.40	-530.80
48	-497.50	-552.90	-1047.40	-505.40	-547.90	-1042.90	-44.10	-187.40	-576.50
50	-477.70	-535.00	-1005.20	-491.20	-552.40	-1034.50	-90.50	-90.90	-600.60

BIC									
2	-326.400	-307.700	-757.600	-319.800	-277.000	-728.000	-82.800	37.600	-474.200
4	-207.200	-204.500	-649.300	-194.400	-284.600	-717.700	190.400	102.700	-346.400
6	-119.900	-194.800	-667.200	-105.500	-177.900	-635.800	315.800	188.800	-236.300
8	4.700	-27.800	-491.400	-6.500	-70.100	-545.100	431.700	299.200	-150.200
10	111.100	89.800	-382.800	107.800	41.800	-434.200	547.100	415.800	-43.500
12	268.800	209.800	-226.000	219.200	175.700	-304.700	648.000	575.400	127.200
14	339.100	279.800	-196.200	315.500	288.200	-210.800	778.700	645.400	228.200
16	481.800	415.700	-41.100	443.700	405.100	-83.600	898.400	739.500	305.400
18	563.200	515.000	28.200	546.900	505.700	6.600	1005.200	892.300	473.700
20	711.900	643.500	194.300	671.200	619.100	128.800	1108.500	1047.100	633.000
22	796.300	744.200	264.300	785.400	740.300	249.100	1234.500	1130.500	710.800
24	912.200	866.100	384.600	915.000	853.500	375.600	1355.000	1227.000	801.900
26	1046.300	988.800	527.300	1035.400	975.200	499.000	1458.000	1380.300	948.700
28	1142.500	1109.900	621.700	1117.300	1089.100	582.900	1586.100	1458.100	1048.700
30	1251.400	1199.600	709.300	1269.100	1209.000	738.300	1684.200	1621.500	1174.400
32	1367.000	1328.300	839.900	1399.200	1335.500	879.500	1817.600	1689.100	1312.700
34	1478.000	1406.100	912.400	1494.400	1441.100	963.600	1915.600	1853.800	1451.300
36	1606.600	1546.100	1063.200	1603.900	1554.200	1072.500	2033.600	1967.200	1586.300
38	1722.200	1673.800	1193.700	1735.100	1674.600	1207.900	2144.200	2089.700	1686.500
40	1842.000	1771.700	1293.300	1852.500	1813.500	1348.200	2244.600	2224.700	1704.200
42	1960.500	1891.800	1418.900	1940.300	1892.100	1400.500	2390.900	2280.500	1883.800
44	2082.300	2023.800	1558.100	2057.000	2004.300	1514.100	2521.400	2328.200	2019.000
46	2170.400	2135.900	1640.800	2171.600	2126.600	1633.800	2597.200	2564.600	2142.200
48	2290.700	2235.300	1740.800	2282.800	2240.300	1745.300	2744.100	2600.800	2211.700
50	2425.700	2368.400	1898.200	2412.300	2351.100	1868.900	2812.900	2812.500	2302.800
Four (4) Input Variables and One (1) Lags									
AIC									
2	-248.200	-257.800	-676.200	-236.600	-230.500	-645.200	133.200	-0.200	-450.300
4	196.800	84.500	-332.900	201.900	96.800	-315.300	564.400	488.700	68.700
6	-819.300	-939.100	-1361.300	-826.100	-913.900	-1343.000	-449.400	-513.400	-921.300
8	-575.400	-680.700	-1103.200	-581.400	-673.800	-1102.200	-206.300	-272.000	-745.100
10	-521.300	-627.200	-1052.600	-529.600	-604.500	-1039.000	-143.600	-226.000	-693.200
12	-499.300	-608.000	-1036.700	-501.900	-567.700	-999.100	-143.300	-159.200	-707.200
14	-486.400	-580.800	-1011.200	-490.900	-572.400	-1007.500	-115.200	-161.500	-582.600
16	-466.600	-561.400	-981.200	-486.300	-565.600	-1005.500	-94.500	-176.400	-583.200
18	-463.200	-573.500	-996.600	-476.600	-567.900	-1005.600	-105.600	-135.300	-568.200
20	-466.700	-586.500	-1017.900	-468.900	-552.900	-986.200	-69.700	-212.100	-650.900
22	-464.200	-580.300	-1013.000	-470.300	-544.600	-983.500	-68.400	-196.500	-584.400
24	-463.300	-578.500	-1012.700	-465.800	-545.900	-982.900	-67.400	-187.200	-593.200
26	-457.900	-575.200	-1006.500	-459.700	-539.600	-972.600	-60.700	-200.500	-603.300
28	-458.500	-569.400	-1003.400	-450.300	-541.400	-967.200	-59.800	-196.800	-640.300
30	-457.000	-574.200	-1008.200	-459.300	-543.500	-979.700	-55.400	-207.600	-620.700
32	-455.300	-577.400	-1011.300	-455.100	-556.200	-989.800	-73.000	-143.000	-550.400
34	-455.200	-570.800	-1005.700	-453.900	-549.000	-982.600	-59.800	-175.800	-609.400
36	-449.000	-548.100	-978.000	-465.400	-544.000	-990.200	-66.200	-153.300	-641.500
38	-454.700	-562.400	-998.600	-458.300	-545.600	-986.500	-66.300	-148.800	-582.200
40	-450.700	-567.500	-1000.800	-451.900	-543.800	-978.300	-50.800	-197.000	-584.400
42	-445.600	-549.800	-978.500	-457.900	-544.100	-985.200	-66.000	-145.000	-592.400
44	-443.300	-551.700	-979.000	-448.700	-555.500	-988.800	-68.100	-134.700	-532.200
46	-458.900	-528.300	-971.700	-444.400	-553.200	-982.100	-78.700	-116.500	-619.300
48	-451.100	-552.900	-989.000	-448.500	-545.600	-979.000	-64.600	-142.600	-623.400
50	-447.900	-549.700	-983.100	-440.600	-529.300	-955.400	-62.500	-145.500	-585.400
BIC									
2	-342.100	-351.800	-770.200	-330.500	-324.500	-739.200	39.300	-94.100	-544.200
4	-272.600	-384.900	-802.400	-267.600	-372.600	-784.700	95.000	19.300	-400.700
6	-208.600	-328.400	-750.600	-215.400	-303.200	-732.300	161.300	97.300	-310.600
8	-139.400	-244.700	-667.100	-145.400	-237.800	-666.200	229.700	164.000	-309.100
10	-73.100	-179.000	-604.400	-81.400	-156.300	-590.700	304.600	222.200	-245.000
12	-7.300	-115.900	-544.700	-9.900	-75.700	-507.100	348.700	332.800	-215.200
14	60.500	-33.900	-464.200	56.100	-25.500	-460.600	431.800	385.500	-35.600
16	140.400	45.500	-374.200	120.700	41.300	-398.600	512.500	430.600	23.700
18	206.500	96.200	-326.900	193.100	101.900	-335.800	564.100	534.500	101.500
20	267.600	147.800	-283.600	265.400	181.400	-251.900	664.600	522.200	83.400
22	335.700	219.600	-213.100	329.600	255.300	-183.600	731.600	603.400	215.500
24	403.100	287.900	-146.400	400.600	320.400	-116.600	798.900	679.200	273.100
26	475.400	358.100	-73.200	473.600	393.700	-39.400	872.500	732.700	329.900
28	542.100	431.200	-2.800	550.300	459.200	33.400	940.800	803.800	360.300
30	611.300	494.100	60.100	609.000	524.800	88.600	1012.900	860.700	447.600
32	680.900	558.800	124.900	681.100	580.000	146.400	1063.200	993.200	585.800
34	749.100	633.500	198.600	750.400	655.300	221.700	1144.500	1028.400	594.900
36	823.600	724.400	294.500	807.100	728.500	282.400	1206.300	1119.200	631.000
38	886.300	778.500	342.300	882.600	795.300	354.400	1274.600	1192.100	758.700
40	958.700	841.900	408.600	957.500	865.600	431.100	1358.500	1212.400	825.000
42	1032.400	928.200	499.400	1020.000	933.800	492.800	1412.000	1333.000	885.600
44	1103.300	994.900	567.600	1097.900	991.100	557.800	1478.500	1411.900	1014.400
46	1156.500	1087.000	643.700	1170.900	1062.100	633.200	1536.600	1498.800	996.000
48	1233.000	1131.100	695.100	1235.600	1138.500	705.100	1619.500	1541.500	1060.700
50	1305.000	1203.200	769.800	1312.300	1223.600	797.500	1690.400	1607.400	1167.500

Two (2) Input Variables and Four (4) Lags									
AIC									
2	406.600	428.600	-26.000	442.500	473.000	54.300	816.100	721.900	246.200
4	-593.600	-568.500	-1028.900	-536.900	-527.200	-934.400	-198.300	-236.300	-758.000
6	-521.300	-502.300	-959.000	-434.500	-482.600	-858.900	-120.500	-179.400	-720.400
8	-521.700	-510.200	-990.700	-450.600	-455.000	-864.900	-90.000	-178.500	-636.300
10	-520.900	-498.600	-986.500	-265.900	-434.000	-669.300	-77.400	-174.200	-652.900
12	-517.600	-533.500	-1024.100	-467.700	-454.300	-896.800	-94.500	-122.600	-595.700
14	-501.500	-502.300	-982.900	-449.400	-438.700	-864.800	-61.700	-180.000	-587.400
16	-496.900	-514.200	-990.600	-448.200	-414.900	-842.800	-66.600	-154.400	-570.000
18	-484.800	-522.800	-989.300	-420.300	-412.900	-815.100	-60.200	-163.600	-558.800
20	-494.500	-503.400	-986.100	-424.200	-425.600	-837.300	-57.100	-169.700	-633.900
22	-481.500	-495.600	-963.700	-373.600	-438.600	-800.300	-60.200	-153.900	-595.400
24	-498.300	-534.900	-1018.900	-433.000	-407.800	-828.700	-86.700	-103.200	-615.500
26	-494.900	-546.600	-1031.400	-452.300	-421.700	-862.700	-86.500	-101.400	-547.000
28	-497.500	-529.800	-1021.200	-388.400	-443.900	-822.500	-62.500	-138.200	-560.000
30	-486.700	-541.400	-1017.600	-416.100	-425.600	-834.200	-62.500	-136.900	-582.900
32	-491.000	-533.800	-1013.300	-404.500	-425.900	-832.300	-50.100	-169.600	-585.400
34	-502.300	-543.800	-1034.900	-455.800	-446.700	-891.800	-56.600	-148.700	-653.900
36	-496.500	-546.100	-1032.100	-470.800	-459.700	-920.200	-53.600	-154.700	-618.700
38	-476.500	-512.100	-979.100	-477.500	-427.800	-895.000	-53.800	-151.800	-544.900
40	-497.900	-533.500	-1023.800	-454.700	-453.400	-898.800	-69.900	-116.300	-600.400
42	-497.300	-548.900	-1037.900	-419.400	-493.400	-903.500	-47.500	-170.000	-566.000
44	-484.600	-522.700	-998.500	-456.100	-470.000	-919.200	-47.900	-168.300	-614.200
46	-500.100	-556.700	-1048.700	-478.000	-434.400	-904.600	-71.100	-110.900	-518.900
48	-502.300	-535.900	-1032.900	-463.900	-421.800	-877.100	-45.600	-174.500	-618.100
50	-476.300	-522.500	-995.100	-433.200	-442.100	-867.300	-45.000	-174.300	-589.400
BIC									
2	-302.200	-280.200	-734.800	-266.300	-235.800	-654.500	107.300	13.100	-462.600
4	-160.300	-135.100	-595.500	-103.500	-93.900	-501.100	235.000	197.100	-324.700
6	-11.800	7.200	-449.500	75.000	27.000	-349.400	389.000	330.100	-210.800
8	112.900	124.400	-356.100	184.000	179.600	-230.400	544.600	456.000	-1.700
10	249.700	272.100	-215.900	504.700	336.600	101.400	693.200	596.500	117.700
12	393.300	377.400	-113.200	443.300	456.600	14.100	816.400	788.300	315.200
14	551.700	550.900	70.300	603.800	614.600	188.400	991.500	873.200	465.800
16	699.800	682.500	206.100	748.500	781.800	353.900	1130.100	1042.300	626.700
18	856.100	818.100	351.600	920.600	928.000	525.800	1280.700	1177.300	782.100
20	991.100	982.200	499.500	1061.300	1060.000	648.300	1428.500	1315.900	851.700
22	1149.100	1135.000	666.900	1257.000	1192.000	830.300	1570.400	1476.700	1035.100
24	1277.600	1240.900	756.900	1342.900	1368.000	947.100	1689.100	1672.600	1160.400
26	1426.400	1374.700	889.800	1468.900	1499.600	1058.500	1834.700	1819.900	1374.200
28	1569.300	1537.000	1045.600	1678.500	1622.900	1244.300	2004.400	1928.600	1506.800
30	1725.700	1671.100	1194.900	1796.400	1786.900	1378.300	2150.000	2075.600	1629.600
32	1867.300	1824.500	1345.000	1953.800	1932.300	1525.900	2308.100	2188.700	1772.900
34	2001.800	1960.300	1469.200	2048.300	2057.400	1612.200	2447.500	2355.300	1850.100
36	2153.400	2103.900	1617.800	2179.200	2190.300	1729.800	2596.300	2495.300	2031.300
38	2319.400	2283.700	1816.700	2318.300	2368.100	1900.800	2742.100	2644.100	2250.900
40	2443.900	2408.400	1918.100	2487.100	2488.500	2043.000	2871.900	2825.600	2341.500
42	2590.500	2538.900	2049.900	2668.400	2594.400	2184.300	3040.300	2917.900	2521.800
44	2749.300	2711.100	2235.400	2777.800	2763.900	2314.700	3186.000	3065.600	2619.700
46	2879.900	2823.200	2331.200	2901.900	2945.600	2475.300	3308.800	3269.000	2861.000
48	3023.700	2990.200	2493.100	3062.100	3104.200	2648.900	3480.400	3351.500	2907.900
50	3195.800	3149.600	2677.000	3238.900	3230.000	2804.800	3627.100	3497.800	3082.700
Two (2) Input Variables and Two (2) Lags									
AIC									
2	-243.500	-281.400	-728.800	-25.900	17.900	-211.700	161.800	49.700	-395.600
4	-4779.900	-4751.900	-5204.100	-4728.700	-4699.600	-5114.800	-4374.300	-4462.200	-4942.100
6	-648.900	-711.800	-1169.000	-602.000	-584.800	-996.300	-228.700	-354.800	-745.700
8	-565.600	-606.600	-1072.300	-493.400	-491.600	-885.900	-133.600	-279.700	-747.200
10	-528.700	-576.200	-1036.700	-479.200	-460.600	-871.400	-107.100	-225.800	-641.300
12	-503.600	-546.000	-997.200	-455.700	-443.000	-849.900	-115.600	-154.100	-661.300
14	-492.000	-555.900	-1005.100	-463.700	-469.000	-903.000	-106.200	-143.300	-582.000
16	-496.300	-555.900	-1016.400	-452.800	-431.200	-847.800	-86.500	-162.800	-633.100
18	-489.500	-546.800	-1004.600	-452.300	-429.200	-850.600	-70.700	-187.800	-590.100
20	-490.600	-544.200	-1008.400	-462.900	-426.800	-871.000	-67.300	-183.900	-606.700
22	-478.500	-541.800	-995.400	-446.900	-420.600	-841.800	-67.600	-172.800	-618.400
24	-479.100	-540.000	-996.100	-445.200	-419.500	-841.100	-90.000	-119.900	-600.300
26	-502.100	-539.100	-1019.600	-440.000	-417.100	-835.800	-63.500	-168.600	-566.300
28	-490.300	-538.900	-1008.800	-458.300	-505.700	-951.100	-71.800	-143.100	-649.000
30	-479.000	-538.600	-998.600	-449.500	-419.600	-863.600	-72.400	-137.400	-576.200
32	-478.500	-538.300	-999.500	-427.500	-439.000	-853.800	-80.500	-120.800	-674.800
34	-479.900	-530.900	-994.400	-465.800	-475.100	-924.300	-56.900	-170.600	-602.700
36	-474.500	-543.400	-1001.800	-450.800	-476.000	-918.600	-55.800	-170.500	-605.500
38	-464.900	-524.100	-973.400	-452.800	-447.700	-886.000	-59.500	-156.200	-599.900
40	-489.700	-557.200	-1032.700	-452.900	-489.700	-930.000	-52.600	-175.400	-584.800
42	-474.600	-538.200	-998.500	-435.600	-434.800	-866.500	-66.900	-134.800	-618.000
44	-476.100	-530.700	-993.400	-458.800	-463.200	-913.400	-57.900	-152.900	-561.100
46	-472.100	-524.500	-983.700	-445.800	-434.300	-871.900	-46.600	-191.600	-583.400
48	-469.100	-534.000	-991.700	-461.700	-516.300	-965.100	-48.300	-181.600	-586.600
50	-469.300	-529.800	-986.900	-423.700	-463.900	-875.300	-50.400	-172.700	-580.100

BIC									
2	-355.000	-392.900	-840.300	-137.400	-93.600	-323.200	50.300	-61.800	-507.100
4	-276.800	-248.800	-701.000	-225.500	-196.500	-611.700	128.800	40.900	-439.000
6	-197.100	-259.900	-717.200	-150.200	-133.000	-544.500	223.100	97.000	-293.800
8	-121.000	-162.000	-627.700	-48.800	-47.000	-441.300	311.000	164.900	-302.600
10	-31.000	-78.400	-539.000	18.500	37.100	-373.700	390.700	272.000	-143.600
12	62.900	20.500	-430.700	110.800	123.500	-283.400	450.900	412.400	-94.800
14	149.600	85.700	-363.600	177.900	172.600	-261.400	535.400	498.300	59.600
16	223.600	164.000	-296.500	267.000	288.600	-128.000	633.300	557.100	86.800
18	310.400	253.100	-204.700	347.600	370.700	-50.700	729.200	612.100	209.900
20	390.600	337.000	-127.300	418.300	454.400	10.200	813.800	697.300	274.500
22	484.600	421.400	-32.200	516.300	542.600	121.400	895.600	790.400	344.800
24	566.600	505.700	49.600	600.500	626.200	204.600	955.700	925.800	445.400
26	626.500	589.600	109.000	688.600	711.600	292.900	1065.200	960.100	562.300
28	721.600	673.000	203.000	753.500	706.200	260.800	1140.000	1068.700	562.900
30	816.300	756.700	296.700	845.800	875.700	431.700	1222.900	1157.900	719.100
32	900.500	840.600	379.400	951.400	939.900	525.100	1298.400	1258.100	704.200
34	982.900	931.800	468.300	996.900	987.600	538.400	1405.900	1292.100	860.000
36	1072.100	1003.200	544.800	1095.800	1070.700	628.000	1490.800	1376.100	941.200
38	1165.700	1106.500	657.200	1177.800	1182.900	744.600	1571.100	1474.400	1030.700
40	1225.000	1157.500	681.900	1261.800	1224.900	784.700	1662.100	1539.200	1129.800
42	1324.200	1260.600	800.300	1363.200	1364.000	932.300	1731.900	1664.000	1180.700
44	1406.900	1352.200	889.600	1424.200	1419.800	969.600	1825.000	1730.000	1321.900
46	1495.100	1442.700	983.500	1521.400	1532.900	1095.300	1920.600	1775.600	1383.800
48	1582.400	1517.400	1059.800	1589.800	1535.200	1086.400	2003.200	1869.900	1464.900
50	1666.500	1606.000	1149.000	1712.100	1671.900	1260.600	2085.500	1963.100	1555.700
Two (2) Input Variables and One (1) Lags									
AIC									
2	-270.600	-275.200	-691.100	-234.500	-252.100	-631.700	101.800	2.000	-485.700
4	-138.300	-230.000	-652.500	-83.900	-104.300	-472.700	238.200	150.900	-228.300
6	3890.300	3791.500	3361.300	3936.500	3918.800	3535.100	4266.800	4200.500	3677.000
8	-764.200	-866.400	-1294.100	-719.200	-749.800	-1132.900	-384.600	-465.500	-877.600
10	-589.600	-694.700	-1120.300	-541.600	-555.200	-933.100	-210.300	-299.600	-762.200
12	-536.400	-638.100	-1065.000	-501.200	-514.000	-906.500	-147.000	-270.900	-699.400
14	-515.700	-613.900	-1048.200	-496.800	-481.800	-902.300	-139.300	-194.800	-674.200
16	-502.300	-601.400	-1037.900	-444.000	-455.900	-834.100	-100.800	-241.300	-647.700
18	-488.200	-588.500	-1021.000	-444.300	-453.600	-841.900	-99.700	-198.700	-667.100
20	-477.900	-595.200	-1024.700	-431.600	-445.700	-828.600	-102.700	-165.500	-615.700
22	-466.700	-571.500	-995.200	-451.100	-452.600	-871.200	-84.800	-190.600	-684.700
24	-469.100	-574.800	-1006.000	-439.600	-433.200	-834.600	-76.900	-198.600	-704.000
26	-463.100	-559.200	-986.300	-445.000	-437.700	-899.800	-79.600	-175.000	-644.800
28	-456.800	-573.800	-998.600	-446.200	-452.100	-866.400	-71.400	-189.100	-689.400
30	-458.700	-570.400	-999.200	-422.400	-426.200	-818.800	-67.800	-189.400	-609.600
32	-456.300	-560.800	-990.100	-424.400	-479.200	-877.000	-74.100	-159.700	-525.400
34	-454.200	-560.600	-989.300	-440.500	-500.200	-914.500	-65.200	-183.000	-623.600
36	-449.000	-567.300	-991.800	-440.200	-437.800	-855.200	-70.100	-158.500	-531.500
38	-454.100	-562.300	-993.000	-440.100	-425.500	-843.300	-67.600	-164.100	-647.000
40	-455.800	-570.200	-1004.100	-435.100	-472.100	-885.000	-95.100	-110.700	-599.900
42	-455.400	-564.700	-999.000	-428.100	-425.600	-833.000	-63.000	-167.700	-565.000
44	-450.800	-544.000	-974.600	-415.400	-441.000	-836.200	-55.400	-192.100	-635.200
46	-446.200	-542.500	-969.300	-432.500	-418.100	-835.400	-59.900	-171.600	-626.500
48	-449.600	-556.200	-987.200	-451.800	-463.700	-898.100	-64.000	-156.400	-592.700
50	-443.400	-554.300	-979.800	-439.200	-461.000	-882.800	-53.100	-187.900	-570.700
BIC									
2	-356.100	-360.700	-776.600	-320.000	-337.600	-717.200	16.300	-83.500	-571.200
4	-309.000	-400.800	-823.300	-254.600	-275.000	-643.400	67.400	-19.900	-399.100
6	-262.300	-361.100	-791.300	-216.100	-233.800	-617.500	114.200	47.900	-475.600
8	-206.300	-308.500	-736.200	-161.300	-191.900	-575.000	173.300	92.400	-319.600
10	-150.200	-255.400	-680.900	-102.200	-115.800	-493.800	229.000	139.700	-322.800
12	-97.800	-199.500	-626.400	-62.600	-75.400	-467.900	291.500	167.700	-260.800
14	-50.100	-148.300	-582.600	-31.200	-16.200	-436.700	326.300	270.900	-208.600
16	1.300	-97.800	-534.300	59.600	47.700	-330.500	402.800	262.300	-144.200
18	58.800	-41.600	-474.000	102.600	93.300	-295.000	447.300	348.200	-120.200
20	115.400	-1.900	-431.400	161.700	147.600	-235.400	490.600	427.800	-22.400
22	174.900	70.100	-353.600	190.500	189.000	-229.600	556.800	450.900	-43.100
24	222.000	116.400	-314.900	251.500	257.900	-143.500	614.200	492.500	-12.900
26	278.400	183.400	-244.800	296.600	253.800	-158.300	661.900	566.500	96.800
28	335.800	218.800	-206.000	346.400	340.500	-73.900	721.200	603.500	103.200
30	385.400	273.700	-155.100	421.700	418.000	25.300	776.300	654.700	234.500
32	439.700	335.200	-94.100	471.600	416.800	19.100	821.900	736.300	370.700
34	494.000	387.600	-41.100	507.700	448.000	33.700	883.000	765.200	324.600
36	551.600	433.300	8.800	560.400	562.800	145.400	930.500	842.100	469.200
38	599.200	490.900	60.200	613.100	627.700	209.900	985.600	889.100	406.300
40	650.200	535.800	101.900	670.900	633.900	221.000	1010.900	995.300	506.100
42	703.500	594.200	159.800	730.800	733.300	325.900	1095.900	991.200	593.900
44	761.100	667.900	237.300	796.500	770.900	375.600	1156.500	1019.800	576.700
46	818.800	722.400	295.700	832.400	846.800	429.600	1205.000	1093.300	638.400
48	868.500	761.900	330.900	866.300	854.400	420.000	1254.100	1161.700	725.400
50	927.900	817.000	391.500	932.200	910.300	488.600	1318.200	1183.400	800.600

Table E19: NN Model for Office Return and Office Return Deviation

IPD Industrial

Four (4) Input Variables and Four (4) Lags									
No. of Neurons	Linear Function			Logistic Function			Softmax Function		
	Return	Deviation	Covariance	Return	Deviation	Covariance	Return	Deviation	Covariance
AIC									
2	-848.200	-867.300	-1318.500	-877.800	-823.600	-1305.800	-434.900	-569.200	-1009.600
4	-546.700	-551.200	-1030.200	-552.800	-612.300	-1098.800	-115.500	-217.400	-713.400
6	-523.500	-525.500	-1009.400	-520.400	-556.100	-1037.100	-105.800	-143.800	-627.700
8	-509.200	-527.800	-1008.500	-508.800	-534.300	-1022.000	-81.800	-155.000	-635.700
10	-505.000	-497.900	-980.800	-500.700	-527.900	-1008.900	-69.800	-164.900	-652.700
12	-494.100	-495.300	-970.800	-513.900	-578.100	-1073.200	-64.500	-165.500	-554.800
14	-484.100	-530.000	-997.300	-517.700	-568.000	-1069.600	-58.500	-174.200	-611.700
16	-484.700	-532.300	-1003.100	-501.000	-549.500	-1035.500	-91.400	-103.400	-575.300
18	-496.800	-532.100	-1015.200	-510.700	-559.600	-1056.800	-59.300	-158.500	-600.600
20	-500.100	-555.700	-1046.700	-476.200	-528.800	-993.400	-66.800	-134.700	-570.700
22	-485.400	-525.600	-999.100	-509.200	-544.800	-1042.200	-46.500	-197.400	-584.900
24	-488.300	-517.600	-995.600	-499.100	-543.800	-1031.800	-52.600	-169.200	-643.200
26	-496.900	-527.800	-1014.200	-524.800	-566.600	-1081.200	-43.300	-203.500	-600.300
28	-487.400	-516.300	-994.000	-509.000	-558.300	-1058.600	-41.100	-218.100	-631.900
30	-490.900	-536.800	-1018.400	-504.900	-559.600	-1056.900	-46.800	-184.000	-593.300
32	-491.500	-535.100	-1017.300	-521.000	-557.800	-1070.300	-43.100	-199.900	-630.400
34	-489.500	-548.600	-1033.200	-494.700	-571.300	-1060.500	-37.800	-232.100	-632.500
36	-494.800	-527.800	-1014.500	-511.900	-568.200	-1071.500	-43.900	-192.900	-630.100
38	-498.800	-544.800	-1035.700	-482.200	-537.400	-1011.400	-46.000	-181.300	-620.300
40	-507.400	-556.800	-1061.300	-506.400	-551.300	-1050.800	-38.700	-220.500	-620.900
42	-497.100	-556.800	-1046.400	-519.900	-566.200	-1078.100	-38.100	-221.300	-600.900
44	-498.400	-546.400	-1037.800	-498.900	-539.200	-1031.300	-41.100	-201.400	-607.700
46	-501.100	-547.600	-1042.500	-508.100	-553.000	-1055.100	-37.300	-225.400	-631.600
48	-489.500	-548.500	-1032.300	-501.200	-550.100	-1044.100	-47.300	-167.200	-562.700
50	-483.500	-546.900	-1024.000	-488.300	-530.500	-1013.600	-67.200	-118.700	-592.700
BIC									
2	-237.500	-256.600	-707.800	-267.100	-212.900	-695.100	175.800	41.500	-398.900
4	-54.600	-59.200	-538.100	-60.700	-120.300	-606.800	376.500	274.700	-221.400
6	146.300	144.300	-339.700	149.400	113.600	-367.300	563.900	526.000	42.100
8	357.100	338.500	-142.200	357.500	332.100	-155.700	784.500	711.300	230.600
10	563.300	570.400	87.500	567.600	540.400	59.400	998.500	903.300	415.600
12	778.400	777.300	301.700	758.700	694.400	199.300	1208.000	1107.000	717.800
14	993.900	947.900	480.700	960.300	910.000	408.300	1419.500	1303.800	866.300
16	1199.400	1151.700	680.900	1183.100	1134.500	648.600	1592.700	1580.700	1108.800
18	1393.800	1358.600	875.400	1379.900	1331.100	833.900	1831.400	1732.100	1290.000
20	1597.400	1541.800	1050.800	1621.300	1568.700	1104.100	2030.700	1962.800	1526.700
22	1819.100	1779.000	1305.400	1795.300	1759.800	1262.300	2258.100	2107.100	1719.600
24	2023.500	1994.100	1516.100	2012.600	1967.900	1479.900	2459.200	2342.500	1868.500
26	2222.200	2191.300	1704.900	2194.200	2152.500	1637.800	2675.800	2515.600	2118.800
28	2439.100	2410.200	1932.500	2417.500	2368.200	1867.900	2885.400	2708.400	2294.600
30	2643.000	2597.100	2115.500	2629.100	2574.400	2077.100	3087.100	2949.900	2540.700
32	2850.000	2806.400	2324.200	2820.500	2783.700	2271.200	3298.400	3141.600	2711.100
34	3059.500	3000.500	2515.900	3054.400	2977.800	2488.500	3511.200	3317.000	2916.600
36	3261.900	3228.900	2742.200	3244.800	3188.500	2695.200	3712.800	3563.800	3126.600
38	3465.600	3419.600	2928.600	3482.100	3427.000	2953.000	3918.300	3783.000	3344.100
40	3664.700	3615.300	3110.700	3665.600	3620.700	3121.200	4133.400	3951.500	3551.100
42	3882.600	3822.900	3333.400	3859.900	3813.500	3301.600	4341.600	4158.400	3778.800
44	4099.000	4041.100	3549.600	4088.600	4048.300	3556.200	4546.400	4386.100	3979.800
46	4294.100	4247.600	3752.700	4287.000	4242.200	3740.100	4757.900	4569.800	4163.600
48	4513.400	4454.500	3970.600	4501.800	4452.900	3958.800	4955.600	4835.800	4440.300
50	4727.200	4663.800	4186.700	4722.400	4680.300	4197.100	5143.500	5092.000	4618.000
Four (4) Input Variables and Two (2) Lags									
AIC									
2	-129.100	-116.700	-574.400	-135.200	-97.900	-562.200	289.100	163.600	-288.000
4	-716.300	-729.600	-1187.600	-706.600	-788.000	-1236.500	-305.600	-398.300	-871.200
6	-560.400	-639.100	-1105.700	-558.300	-606.300	-1069.200	-139.100	-248.700	-683.600
8	-514.800	-569.700	-1024.300	-490.600	-569.600	-1000.700	-100.200	-222.900	-659.000
10	-506.800	-551.900	-1014.300	-512.700	-572.500	-1041.300	-85.200	-205.600	-640.700
12	-485.900	-513.100	-967.900	-489.200	-537.400	-991.500	-96.800	-145.100	-615.500
14	-490.500	-555.800	-1017.300	-482.600	-543.200	-999.800	-67.500	-202.600	-658.700
16	-486.400	-525.200	-987.500	-494.700	-538.100	-1007.000	-57.000	-234.700	-661.700
18	-480.400	-534.500	-992.400	-471.800	-550.700	-1000.900	-68.100	-168.500	-613.500
20	-483.100	-543.800	-1006.500	-462.400	-526.800	-969.600	-77.600	-137.900	-596.300
22	-482.000	-542.400	-1006.000	-467.600	-540.600	-989.700	-66.900	-157.500	-587.300
24	-475.800	-539.300	-998.600	-479.000	-556.700	-1019.700	-54.400	-195.300	-645.300
26	-479.200	-541.100	-1005.200	-468.100	-534.700	-987.900	-61.600	-163.600	-585.900
28	-476.500	-538.600	-999.800	-468.400	-542.400	-999.000	-51.100	-198.300	-612.000
30	-469.100	-523.900	-978.500	-462.100	-538.200	-985.900	-66.000	-143.900	-607.600
32	-475.400	-537.400	-1002.000	-464.700	-532.300	-986.800	-70.500	-132.200	-620.800
34	-470.100	-543.600	-1002.500	-467.700	-545.700	-1002.500	-81.300	-113.600	-586.100
36	-475.600	-540.900	-1005.700	-450.500	-523.400	-964.000	-69.700	-129.200	-568.300
38	-474.400	-545.800	-1008.600	-460.500	-525.400	-977.400	-70.700	-126.900	-537.000
40	-472.600	-550.300	-1012.800	-461.800	-549.000	-1004.600	-86.000	-103.000	-624.400
42	-471.700	-534.900	-997.900	-467.300	-548.900	-1009.400	-53.800	-163.700	-575.400
44	-474.400	-543.900	-1009.700	-465.400	-536.500	-995.300	-30.900	-316.600	-594.500
46	-470.200	-520.700	-981.200	-468.300	-538.900	-998.300	-64.500	-132.200	-568.800
48	-471.000	-549.600	-1014.200	-457.300	-532.400	-979.900	-52.000	-165.100	-565.700
50	-472.100	-540.700	-1003.900	-469.100	-562.400	-1022.600	-91.100	-93.400	-564.100

BIC									
2	-336.500	-324.100	-781.800	-342.600	-305.300	-769.700	81.700	-43.800	-495.400
4	-221.200	-234.600	-692.500	-211.500	-293.000	-741.400	189.500	96.800	-376.100
6	-112.200	-190.900	-657.500	-110.100	-158.100	-621.000	309.100	199.500	-235.400
8	13.100	-41.800	-496.400	37.300	-41.800	-472.800	427.600	305.000	-131.100
10	120.800	75.800	-386.700	115.000	55.100	-413.700	542.400	422.000	-13.100
12	248.400	221.200	-233.600	245.100	196.900	-257.200	637.400	589.100	118.800
14	353.600	288.300	-173.100	361.600	306.900	-155.700	776.600	641.500	185.500
16	469.200	430.500	-31.900	461.000	417.600	-51.300	898.700	721.000	294.000
18	587.900	533.800	75.800	596.500	517.600	67.400	1000.100	899.800	454.800
20	698.400	637.800	175.100	719.100	654.800	212.000	1103.900	1043.600	585.200
22	813.300	752.900	289.800	827.700	754.700	305.600	1228.400	1137.900	708.000
24	933.600	870.100	410.800	930.400	852.700	389.700	1355.000	1214.100	764.000
26	1044.500	982.600	518.500	1055.600	989.000	535.800	1462.100	1360.100	937.900
28	1161.800	1099.700	638.500	1169.800	1095.800	639.200	1587.100	1439.900	1026.200
30	1283.800	1228.900	774.400	1290.800	1214.700	767.000	1686.900	1609.000	1145.300
32	1392.200	1330.300	865.700	1402.900	1335.400	880.900	1797.200	1735.500	1246.800
34	1512.500	1438.900	980.000	1514.800	1436.800	980.000	1901.200	1868.900	1396.400
36	1621.800	1556.600	1091.700	1646.900	1574.100	1133.500	2027.800	1968.300	1529.100
38	1738.100	1666.700	1203.900	1752.000	1687.100	1235.100	2141.800	2085.600	1675.500
40	1855.000	1777.200	1314.800	1865.800	1778.600	1323.000	2241.500	2224.600	1703.200
42	1971.000	1907.700	1444.800	1975.400	1893.800	1433.300	2388.800	2279.000	1867.300
44	2083.500	2013.900	1548.100	2092.400	2021.300	1562.500	2526.900	2241.200	1963.300
46	2202.700	2152.200	1691.800	2204.700	2134.100	1674.600	2608.500	2540.800	2104.200
48	2317.200	2298.500	1774.000	2330.900	2255.800	1808.300	2736.100	2623.100	2222.500
50	2431.400	2362.700	1899.500	2434.300	2341.000	1880.900	2812.300	2810.000	2339.300
Four (4) Input Variables and One (1) Lags									
AIC									
2	-250.100	-256.000	-677.200	-256.000	-256.800	682.600	128.900	-5.900	452.500
4	196.000	102.200	-324.900	183.200	105.500	-336.700	557.000	496.100	12.200
6	-815.500	-904.200	-1329.900	-836.000	-917.700	-1380.600	-446.900	-530.900	-1029.400
8	-574.100	-665.900	-1095.900	-584.500	-669.400	-1117.700	-209.700	-271.900	-774.100
10	-514.400	-605.800	-1032.000	-537.100	-605.800	-1066.500	-149.300	-220.600	-733.900
12	-490.200	-590.800	-1026.200	-511.200	-591.700	-1046.800	-105.400	-256.400	-698.700
14	-477.400	-570.500	-999.100	-501.000	-576.200	-1036.700	-127.900	-151.000	-582.700
16	-463.900	-557.100	-986.500	-483.100	-561.600	-1015.600	-90.200	-197.200	-626.000
18	-459.900	-552.700	-982.300	-472.700	-560.600	-1006.500	-74.300	-230.800	-677.300
20	-455.400	-554.200	-984.900	-481.300	-559.000	-1014.600	-72.100	-212.100	-619.000
22	-449.500	-548.500	-976.000	-475.500	-550.900	-1010.700	-67.500	-213.100	-639.200
24	-447.300	-544.500	-976.200	-471.100	-540.600	-996.100	74.600	-174.900	-605.200
26	-446.100	-543.600	-979.000	-465.800	-540.300	-994.000	-69.900	-178.800	-656.900
28	-447.600	-543.800	-980.800	-464.900	-538.900	-994.100	-66.600	-180.300	-616.600
30	-440.800	-534.700	-963.100	-468.600	-543.500	-1000.600	-57.900	-208.400	-635.300
32	-443.900	-536.200	-968.600	-470.900	-534.300	-991.600	-63.800	-177.300	-651.600
34	-437.700	-533.200	-960.700	-458.400	-528.500	-980.100	-61.100	-181.900	-638.700
36	-437.500	-532.200	-957.300	-453.300	-536.200	-985.700	-76.200	-137.500	-614.000
38	-435.100	-529.400	-955.200	-453.300	-534.000	-982.300	-74.500	-135.500	-558.100
40	-438.900	-528.800	-974.900	-457.700	-534.300	-990.100	-51.300	-208.600	-621.900
42	-435.900	-533.800	-962.800	-463.300	-531.100	-984.200	-64.300	-156.700	-618.300
44	-434.600	-528.500	-957.200	-464.700	-537.000	-997.800	-72.600	-132.400	-550.800
46	-433.000	-532.800	-962.100	-458.100	-533.300	-990.800	-83.000	-117.000	-611.800
48	-432.800	-528.300	-955.100	-451.800	-532.200	-985.600	-65.700	-147.400	-607.000
50	-433.400	-536.800	-967.200	-447.700	-521.600	-967.300	-63.500	-150.700	-580.700
BIC									
2	-344.000	-350.000	-771.200	-350.000	-350.800	-776.500	34.900	-88.000	-546.400
4	-273.400	-367.200	-794.300	-286.200	-363.900	-806.100	87.600	26.700	-457.200
6	-204.800	-293.600	-719.200	-225.300	-307.000	-769.900	163.800	79.800	-418.700
8	-138.100	-229.900	-659.900	-148.500	-233.400	-681.700	226.300	164.100	-338.100
10	-66.200	-157.600	-583.800	-88.900	-157.600	-618.300	298.900	227.600	-285.700
12	1.800	-98.700	-534.200	-19.200	-99.700	-554.800	386.600	235.700	-206.700
14	69.500	-23.600	-452.200	45.900	-29.200	-489.800	419.000	395.900	-35.700
16	143.000	49.800	-379.500	123.800	45.400	-408.600	516.700	409.700	-19.100
18	209.900	117.100	-312.500	197.100	109.100	-336.700	595.400	439.000	-7.500
20	278.900	180.100	-250.600	252.900	175.300	-280.300	662.100	522.100	115.300
22	350.500	251.400	-176.100	324.400	249.000	-210.700	732.400	586.800	160.700
24	419.000	321.800	-109.800	395.200	325.700	-129.800	791.800	691.400	261.100
26	487.200	389.700	-45.800	467.500	393.000	-60.700	863.400	754.500	276.400
28	553.000	456.900	19.800	535.800	461.700	6.600	934.100	820.300	384.000
30	627.500	533.600	105.200	599.700	524.800	67.700	1010.400	859.900	433.000
32	692.300	600.000	167.600	665.300	601.900	144.600	1072.400	958.900	484.600
34	766.600	671.100	243.600	745.900	675.800	224.200	1143.100	1022.400	565.500
36	835.000	740.400	315.300	819.300	736.300	286.800	1196.300	1135.000	658.500
38	905.800	811.500	385.700	887.600	806.900	358.600	1266.500	1205.400	782.800
40	970.500	880.600	434.500	951.700	875.100	419.300	1358.100	1200.800	787.500
42	1042.100	944.200	515.100	1014.700	946.800	493.800	1413.700	1321.300	859.600
44	1112.000	1018.100	589.400	1081.900	1009.600	548.800	1474.000	1414.200	995.800
46	1182.300	1082.500	653.200	1157.200	1082.000	624.500	1532.400	1498.300	1003.500
48	1251.300	1155.800	729.000	1232.300	1151.900	698.400	1618.400	1536.600	1077.100
50	1319.500	1216.100	785.700	1305.200	1231.300	785.600	1689.400	1602.200	1172.200

Two (2) Input Variables and Four (4) Lags									
AIC									
2	401.900	423.900	-35.000	397.600	434.400	-30.100	815.700	714.200	223.800
4	-576.500	-630.800	-1073.400	-586.200	-607.000	-1063.100	-189.600	-257.900	-767.200
6	-475.500	-527.000	-942.800	-495.500	-527.800	-958.900	-104.500	-229.400	-670.600
8	-480.900	-537.100	-974.800	-497.400	-513.200	-969.000	-126.000	-126.000	-669.500
10	-474.700	-503.600	-945.500	-473.000	-499.300	-939.600	-103.100	-130.600	-639.100
12	-460.500	-516.900	-950.400	-465.000	-490.700	-928.400	-57.700	-240.600	-659.000
14	-455.600	-500.600	-934.600	-469.300	-496.400	-945.000	-66.300	-177.100	-657.500
16	-461.700	-517.200	-959.800	-462.300	-480.900	-924.500	-59.300	-189.400	-654.600
18	-428.200	-504.400	-915.100	-434.600	-483.900	-900.300	-66.500	-153.300	-625.000
20	-462.100	-520.600	-967.700	-456.200	-498.600	-940.900	-46.600	-232.900	-653.900
22	-450.300	-521.500	-959.900	-433.900	-490.100	-909.600	-71.600	-135.500	-647.100
24	-458.500	-523.300	-969.300	-463.000	-481.400	-930.000	-43.700	-235.200	-637.900
26	-446.400	-525.000	-961.300	-466.500	-502.500	-957.400	-43.400	-231.000	-647.700
28	-452.000	-518.900	-959.000	-453.100	-485.400	-927.200	-44.700	-215.300	-648.400
30	-453.100	-522.200	-965.800	-461.800	-505.200	-956.500	-42.700	-222.800	-643.700
32	-456.900	-525.100	-971.700	-442.400	-503.900	-934.900	-75.700	-117.500	-641.400
34	-453.000	-514.600	-959.100	-454.900	-497.100	-940.800	-43.100	-213.800	-645.700
36	-456.400	-520.900	-968.900	-455.100	-488.400	-934.800	-41.400	-221.800	-639.900
38	-456.500	-514.600	-962.600	-455.200	-496.900	-941.800	-54.500	-159.800	-630.000
40	-455.900	-526.500	-975.800	-446.700	-483.500	-920.200	-37.500	-240.200	-645.100
42	-457.300	-523.300	-972.400	-441.500	-480.600	-912.600	-82.100	-102.400	-620.500
44	-437.300	-492.100	-922.000	-451.200	-504.500	-947.100	-41.500	-211.200	-643.400
46	-458.800	-531.800	-985.800	-454.900	-494.300	-940.100	-39.700	-221.500	-635.800
48	-446.900	-505.300	-945.500	-426.400	-480.700	-898.500	-50.200	-168.800	-614.100
50	-454.100	-531.400	-978.500	-434.100	-477.900	-903.300	-82.000	-100.100	-632.600
BIC									
2	-306.900	-284.800	-743.800	-311.200	-274.400	-738.900	106.900	5.400	-485.000
4	-143.100	-197.500	-640.000	-152.900	-173.600	-629.700	243.700	175.400	-333.800
6	34.000	-17.500	-433.300	14.000	-18.200	-449.400	405.000	280.100	-161.100
8	153.700	97.500	-340.200	137.200	121.400	-334.400	508.600	508.600	-34.900
10	295.900	267.000	-174.800	297.700	271.300	-168.900	667.600	640.000	131.600
12	450.500	394.100	-39.500	445.900	420.200	-17.500	853.200	670.400	251.900
14	597.600	562.600	118.700	583.900	556.900	108.200	987.000	876.200	395.700
16	735.000	679.500	236.900	734.400	715.800	272.200	1137.500	1007.300	542.100
18	912.700	836.500	425.800	906.300	857.000	440.600	1274.400	1187.700	715.900
20	1023.500	965.000	517.900	1029.400	987.000	544.700	1439.000	1252.700	831.700
22	1180.300	1109.100	670.700	1196.700	1140.500	721.000	1559.000	1495.100	983.500
24	1317.300	1252.500	806.500	1312.800	1294.400	845.900	1732.100	1540.700	1138.000
26	1474.800	1396.300	959.900	1454.800	1418.800	963.800	1877.900	1690.200	1273.600
28	1614.800	1548.000	1107.900	1613.800	1581.500	1139.600	2022.200	1851.500	1418.500
30	1759.400	1690.300	1246.700	1750.700	1707.200	1256.000	2169.800	1989.700	1568.800
32	1901.400	1833.100	1386.500	1915.900	1854.300	1423.300	2282.500	2240.800	1716.800
34	2051.100	1989.500	1544.900	2049.200	2007.000	1563.300	2461.000	2290.300	1858.400
36	2193.500	2129.100	1681.000	2194.900	2161.600	1715.200	2608.500	2428.200	2010.000
38	2339.300	2281.300	1833.200	2340.700	2299.000	1854.100	2741.400	2636.100	2165.800
40	2486.000	2415.400	1966.100	2495.100	2458.400	2021.700	2904.300	2701.700	2296.800
42	2630.500	2564.600	2115.500	2646.300	2607.200	2175.200	3005.700	2985.400	2467.300
44	2796.600	2741.800	2311.800	2782.600	2729.400	2286.700	3192.300	3022.700	2590.500
46	2921.200	2848.200	2394.200	2925.000	2885.600	2439.800	3340.200	3158.400	2744.200
48	3079.200	3020.700	2580.500	3099.600	3045.300	2627.500	3475.800	3357.200	2911.900
50	3218.100	3140.700	2693.600	3238.000	3194.200	2768.800	3590.100	3572.100	3039.500
Two (2) Input Variables and Two (2) Lags									
AIC									
2	-242.700	-258.100	-704.900	-223.900	-224.400	-650.200	163.900	30.000	-432.300
4	-4769.100	-4817.300	-5258.700	-4773.700	-4799.400	-5245.200	-4368.700	-4488.200	-4930.400
6	-635.500	-654.300	-1097.800	-636.900	-673.100	-1119.100	-227.600	-373.900	-819.600
8	-527.100	-564.900	-993.100	-549.200	-554.600	-1004.400	-187.200	-177.600	-718.000
10	-516.700	-567.000	-1018.000	-511.700	-529.500	-972.900	-97.700	-289.500	-691.800
12	-499.700	-566.300	-1015.100	-511.900	-536.200	-996.000	-107.000	-175.800	-689.300
14	-490.700	-522.500	-971.100	-501.200	-518.100	-976.600	-72.800	-259.800	-665.900
16	-482.700	-549.700	-998.200	-498.900	-524.200	-986.900	-73.200	-212.300	-672.400
18	-474.200	-532.400	-977.600	-483.700	-514.100	-966.200	-64.300	-229.600	-643.000
20	-476.600	-518.200	-967.600	-477.500	-503.600	-952.800	-70.800	-184.000	-647.000
22	-467.600	-517.100	-959.500	-461.100	-503.100	-938.800	-101.800	-113.800	-625.400
24	-469.500	-549.400	-1002.100	-458.200	-490.000	-924.800	-52.100	-249.100	-651.600
26	-463.100	-519.100	-960.600	-465.000	-497.900	-941.500	-53.500	-225.900	-653.300
28	-460.100	-526.900	-972.500	-478.500	-507.300	-966.200	-98.900	-106.600	-653.600
30	-466.200	-516.100	-965.600	-463.500	-495.300	-940.700	-46.500	-251.900	-651.200
32	-453.900	-516.400	-952.400	-458.200	-485.500	-925.800	-84.300	-120.400	-638.200
34	-463.500	-527.700	-978.100	-464.500	-500.900	-948.800	-86.700	-114.500	-630.700
36	-461.500	-521.800	-967.900	-450.500	-477.100	-913.600	-80.700	-120.400	-666.000
38	-464.100	-509.400	-962.600	-451.200	-495.200	-931.400	-58.500	-168.800	-639.300
40	-455.400	-532.000	-981.100	-453.300	-491.600	-930.900	-44.400	-233.800	-645.600
42	-457.400	-513.000	-958.400	-451.900	-481.200	-919.000	-58.600	-163.800	-617.100
44	-456.400	-507.200	-951.300	-455.500	-488.600	-930.700	-74.300	-124.400	-653.300
46	-462.800	-523.900	-979.700	-449.100	-497.800	-934.500	-59.100	-157.500	-630.100
48	-454.100	-520.100	-961.900	-447.400	-491.600	-926.300	-76.300	-118.200	-645.400
50	-458.000	-522.900	-971.300	-447.100	-495.400	-930.300	-72.700	-122.500	-608.100

BIC									
2	-354.200	-369.600	-816.400	-335.400	-335.900	-761.700	52.400	-81.500	-543.800
4	-266.000	-314.200	-755.600	-270.600	-296.300	-742.100	134.400	14.900	-427.300
6	-183.700	-202.500	-646.000	-185.100	-221.300	-667.300	224.200	77.900	-367.800
8	-82.500	-120.300	-548.500	-104.600	-110.000	-559.800	257.400	267.000	-273.400
10	-18.900	-69.200	-520.300	-14.000	-31.800	-475.200	400.000	208.300	-194.100
12	66.800	0.200	-448.600	54.600	30.300	-429.500	459.500	390.700	-122.800
14	150.900	119.100	-329.500	140.400	123.500	-335.000	568.800	381.800	-24.300
16	237.200	170.100	-278.400	220.900	195.700	-267.100	646.600	507.600	47.500
18	325.700	267.500	-177.700	316.200	285.900	-166.300	735.600	570.300	157.000
20	404.600	362.900	-86.400	403.600	377.600	-71.600	810.400	697.100	234.200
22	495.600	446.100	3.700	502.000	460.100	24.400	861.400	849.400	337.700
24	576.200	496.300	43.600	587.500	555.700	120.900	993.600	796.600	394.100
26	665.600	609.500	168.100	663.700	630.700	187.100	1075.200	902.800	475.400
28	751.800	685.000	239.400	733.400	704.600	245.700	1113.000	1105.200	558.300
30	829.100	779.200	329.700	831.900	800.000	354.700	1248.800	1043.400	644.100
32	925.100	862.600	426.600	920.700	893.400	453.100	1294.600	1258.600	740.800
34	999.200	935.000	484.600	998.200	961.900	514.000	1376.000	1348.200	832.000
36	1085.100	1024.800	578.700	1096.100	1069.500	633.000	1465.900	1426.200	880.600
38	1166.500	1121.200	668.000	1179.400	1135.400	699.200	1572.100	1461.800	991.300
40	1259.300	1182.700	733.500	1261.400	1223.000	783.700	1670.200	1480.900	1069.100
42	1341.400	1285.800	840.400	1346.800	1317.600	879.700	1740.200	1635.000	1181.700
44	1426.600	1375.800	931.700	1427.500	1394.400	952.300	1808.700	1758.600	1229.700
46	1504.400	1443.300	987.500	1518.100	1469.400	1032.800	1908.100	1809.700	1337.100
48	1597.400	1531.400	1089.600	1604.100	1559.900	1125.200	1975.200	1933.300	1406.100
50	1677.800	1612.900	1164.500	1688.700	1640.400	1205.500	2063.100	2013.300	1527.700
Two (2) Input Variables and One (1) Lags									
AIC									
2	-261.700	-314.200	-726.000	-270.400	-317.900	-734.100	116.700	-83.000	-489.000
4	-119.500	-160.900	-575.300	-138.400	-200.500	-632.100	233.100	153.000	-347.400
6	3912.700	3828.400	3412.300	3904.900	3869.200	3454.100	4271.600	4182.300	3693.600
8	-744.300	-836.900	-1257.400	-764.800	-828.900	-1264.900	-401.000	-442.000	-963.500
10	-571.300	-656.200	-1073.800	-593.200	-643.500	-1077.900	-187.700	-446.700	-793.100
12	-518.300	-609.800	-1033.800	-537.500	-591.400	-1026.400	-143.100	-300.200	-732.900
14	-494.400	-582.900	-1010.400	-510.100	-565.900	-998.600	-160.800	-169.600	-695.500
16	-473.400	-568.200	-986.000	-483.700	-536.200	-956.500	-96.600	-282.600	-701.500
18	-462.400	-557.200	-976.300	-490.000	-540.400	-979.200	-91.800	-238.000	-683.900
20	-455.200	-547.500	-964.700	-478.300	-528.200	-964.500	-101.100	-175.800	-619.900
22	-453.400	-545.200	-968.200	-457.400	-515.000	-939.400	-81.600	-211.400	-672.500
24	-445.400	-538.800	-958.600	-466.300	-514.200	-944.900	-74.600	-219.400	-683.800
26	-444.200	-536.600	-958.000	-463.900	-517.800	-954.200	-64.400	-255.700	-670.600
28	-439.800	-533.500	-953.300	-464.300	-520.100	-957.300	-74.400	-188.500	-655.300
30	-439.500	-530.700	-952.300	-461.000	-515.600	-953.500	-98.500	-122.900	-583.900
32	-435.600	-528.800	-950.700	-461.500	-514.200	-955.400	-55.400	-260.800	-652.100
34	-435.200	-527.100	-950.200	-461.500	-517.600	-960.800	-55.200	-247.900	-649.600
36	-431.500	-527.200	-947.100	-468.100	-518.900	-969.300	-67.800	-176.100	-641.100
38	-437.500	-524.800	-953.800	-459.500	-513.300	-957.100	-55.200	-225.700	-656.300
40	-439.600	-524.700	-954.200	-460.900	-515.100	-961.200	-85.200	-130.200	-652.200
42	-437.000	-517.700	-947.600	-437.500	-504.300	-927.200	-47.600	-267.600	-657.200
44	-428.900	-522.600	-942.000	-443.300	-487.800	-915.600	-53.000	-218.500	-651.400
46	-428.500	-518.800	-942.700	-443.100	-499.800	-927.000	-54.000	-207.000	-647.600
48	-434.100	-520.700	-949.900	-443.600	-506.700	-941.000	-56.900	-190.200	-638.300
50	-429.500	-522.100	-946.400	-437.500	-508.100	-937.100	-85.600	-119.100	-564.800
BIC									
2	-347.200	-399.700	-811.500	-355.900	-403.400	-819.600	31.200	-168.500	-574.500
4	-290.300	-331.700	-746.000	-309.100	-371.300	-802.900	62.300	-17.700	-518.100
6	-239.900	-324.200	-740.300	-247.700	-283.300	-698.500	119.000	29.700	-459.000
8	-186.400	-279.000	-699.500	-206.900	-271.000	-707.000	157.000	115.900	-405.500
10	-131.900	-216.900	-634.500	-153.800	-204.100	-638.600	251.700	-7.400	-353.700
12	-79.700	-171.300	-595.200	-98.900	-152.900	-587.900	295.500	138.400	-294.300
14	-28.700	-117.300	-544.800	-44.500	-100.300	-533.000	304.800	296.100	-229.900
16	30.200	-64.600	-482.500	19.900	-32.700	-453.000	407.000	221.000	-198.000
18	84.600	-10.300	-429.400	56.900	6.500	-432.300	455.100	309.000	-137.000
20	138.100	45.800	-371.400	115.000	65.100	-371.200	492.200	417.500	-26.600
22	188.200	96.300	-326.600	184.200	126.600	-297.800	559.900	430.200	-30.900
24	245.700	152.400	-267.500	224.800	176.900	-253.800	616.600	471.700	7.300
26	297.300	204.900	-216.500	277.600	223.700	-212.700	677.200	485.800	70.900
28	352.800	259.100	-160.700	328.300	272.500	-164.700	718.100	604.100	137.300
30	404.600	313.400	-108.100	383.100	328.500	-109.300	745.600	721.200	260.200
32	460.400	367.200	-54.700	434.600	381.800	-59.400	840.600	635.200	243.900
34	513.000	421.100	-2.000	486.700	430.600	-12.600	893.000	700.300	298.600
36	569.100	473.400	53.500	532.500	481.700	31.400	932.800	824.600	359.500
38	615.700	528.500	99.500	593.700	540.000	96.100	998.000	827.500	397.000
40	666.400	581.300	151.800	645.100	590.900	144.800	1020.800	975.800	453.800
42	721.900	641.200	211.300	721.400	654.600	231.700	1111.300	891.300	501.700
44	783.000	689.300	269.900	768.600	724.100	296.300	1158.900	993.400	560.500
46	836.500	746.200	322.300	821.900	765.100	337.900	1210.900	1058.000	617.400
48	884.000	797.400	368.200	874.500	811.400	377.100	1261.300	1127.900	679.900
50	941.900	849.300	424.900	933.800	863.200	434.200	1285.700	1252.200	806.500

Table E20: NN Model for Industrial Return and Industrial Return Deviation

Appendix F

Impulse response of optimal models for each return

1.1 Introduction

This document presents the impulse response of the optimal model for returns and return deviations. The plotted result is the output of the model subjected to a shock of 1 standard deviation in each input and the effect is measured terms of a standard deviation unit.

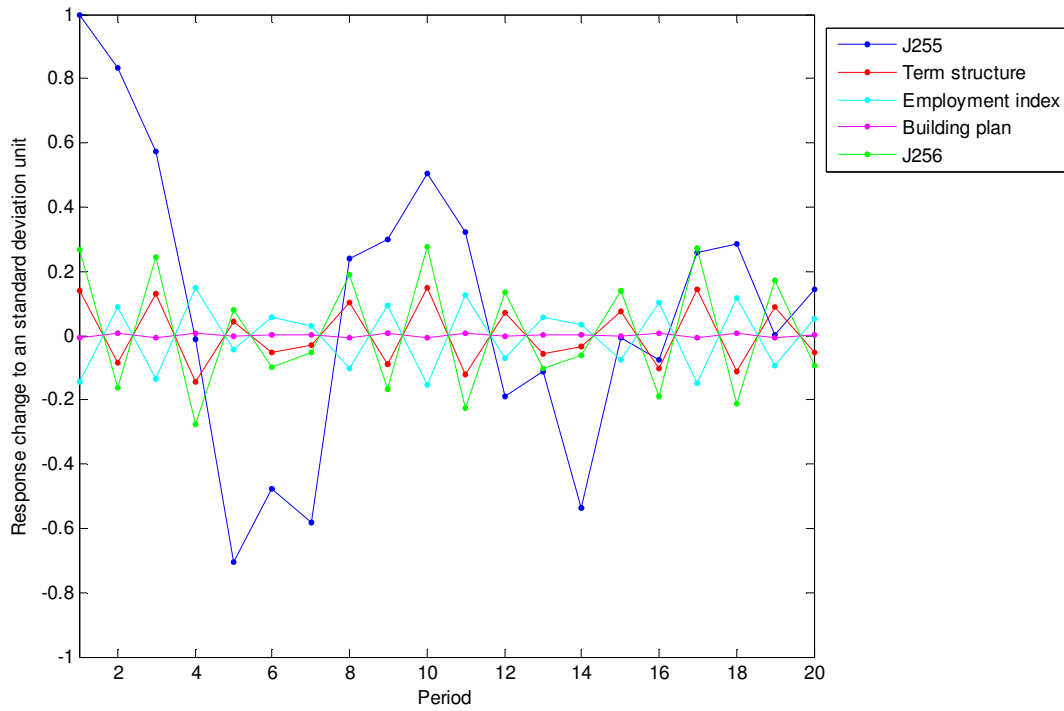


Figure F1: Impulse response of ARMA model for J255 return

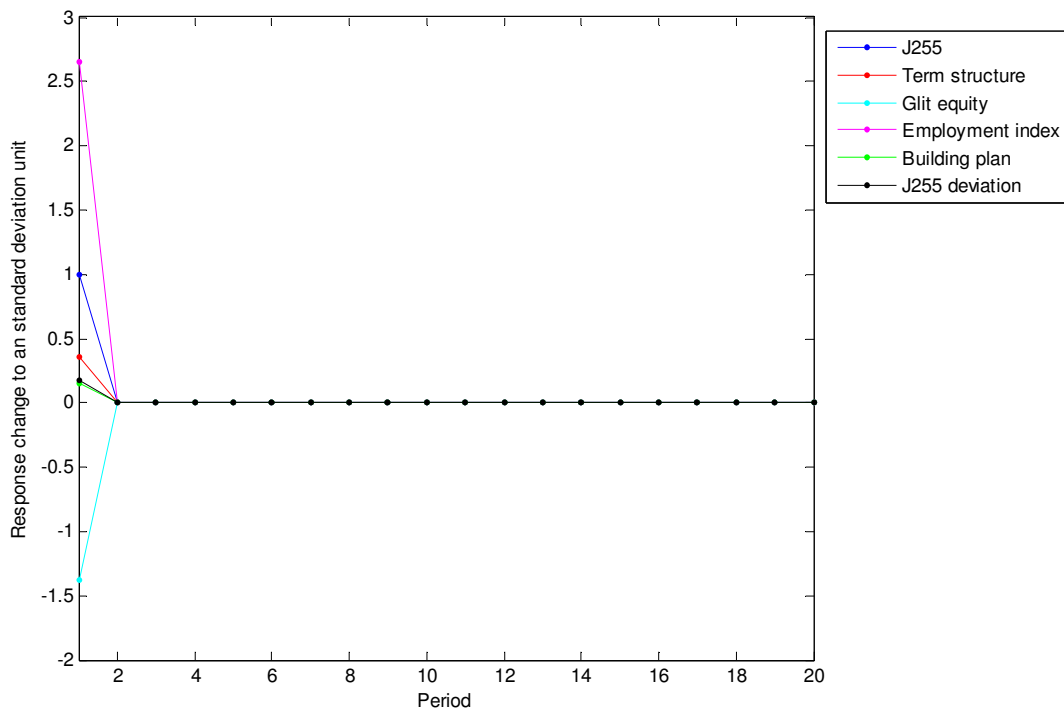


Figure F2: Impulse response of NN(AICc) model for J255 return

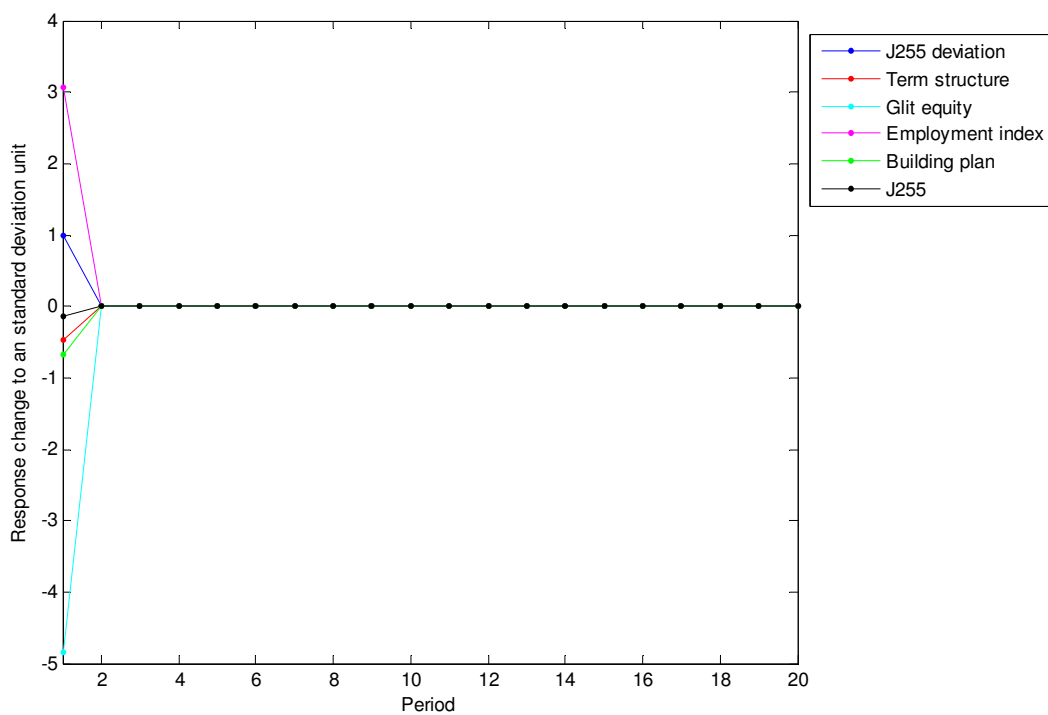


Figure F3: Impulse response of NN(AICc) model for J255 return deviation

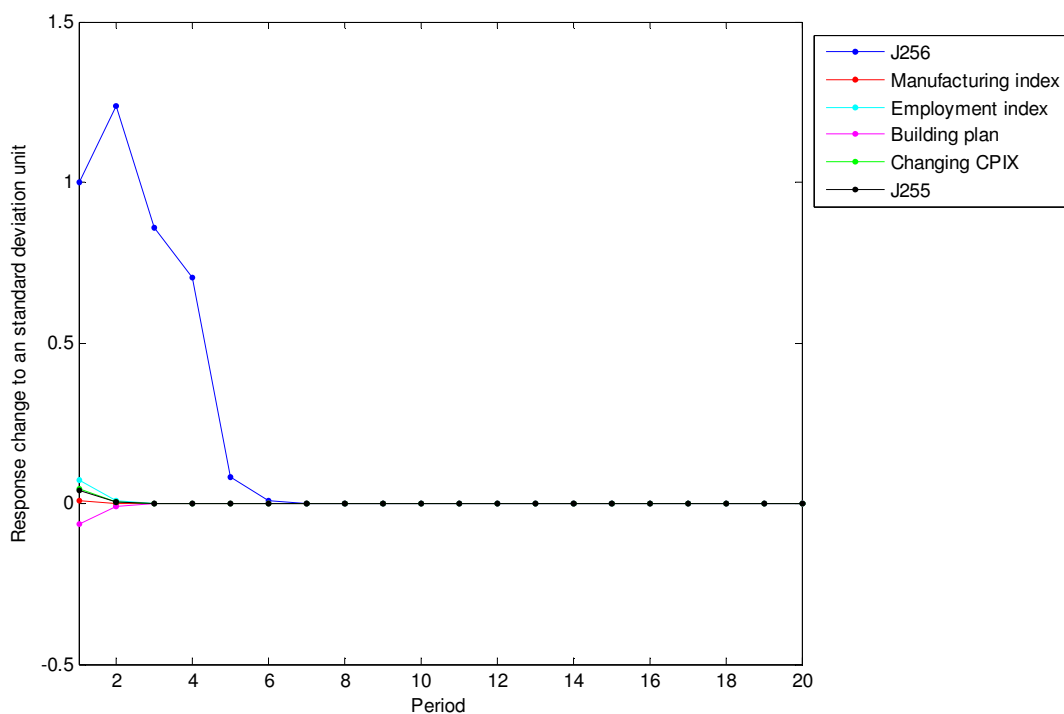


Figure F4: Impulse response of ARMA model for J256 return

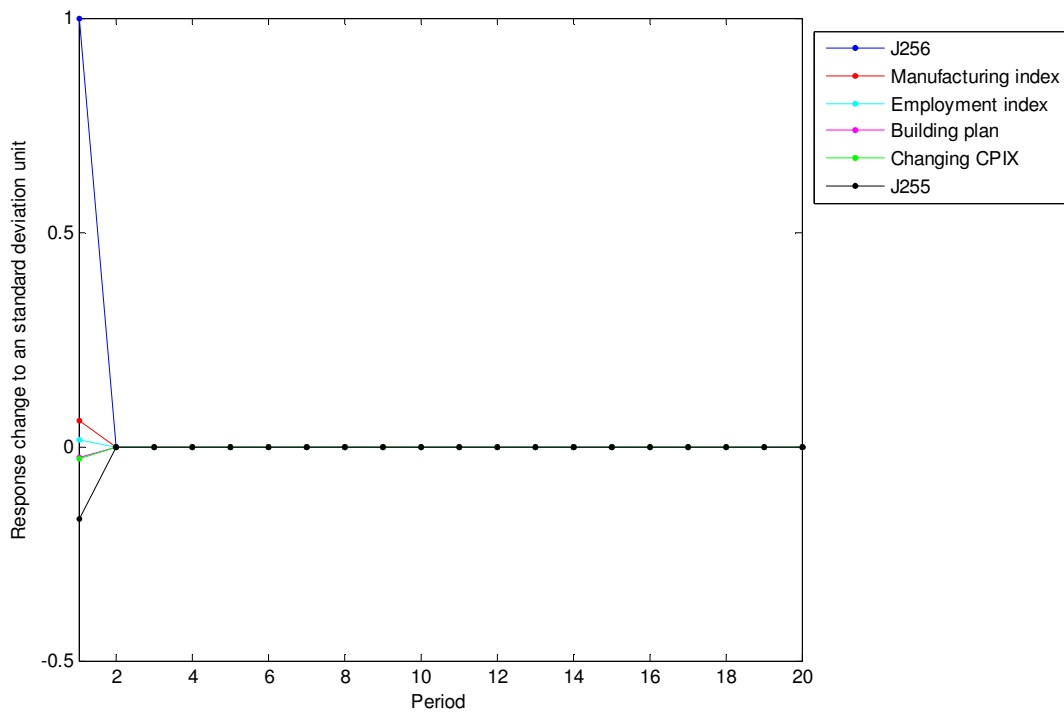


Figure F5: Impulse response of GARCH model for J256 return

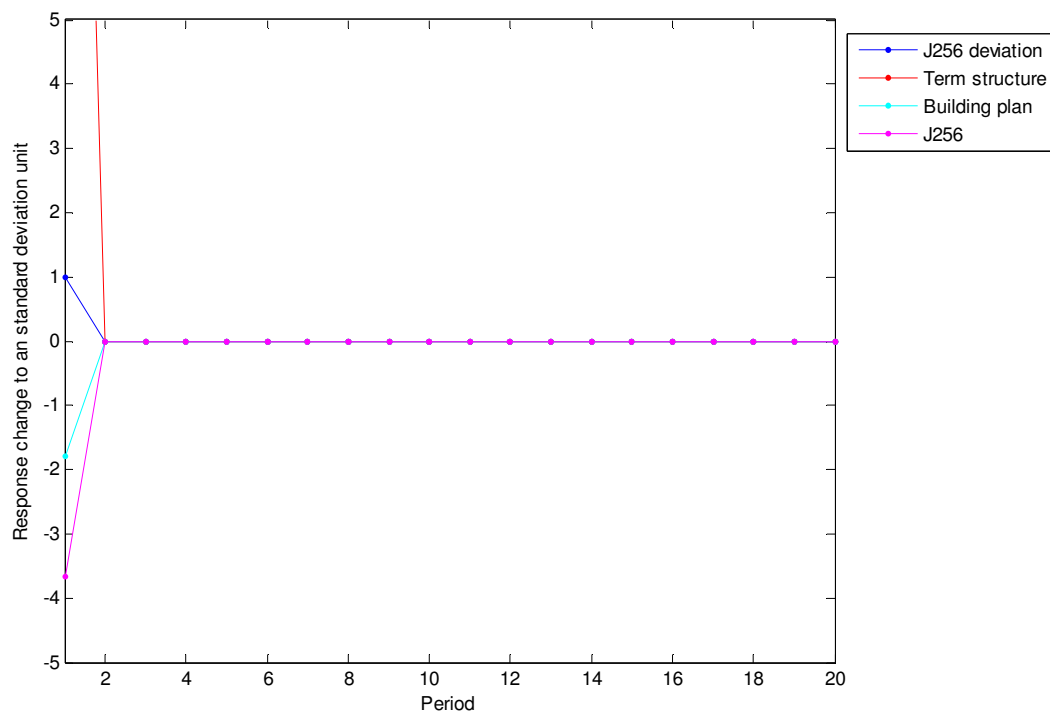


Figure F6: Impulse response of NN(BIC) model for J256 return deviation

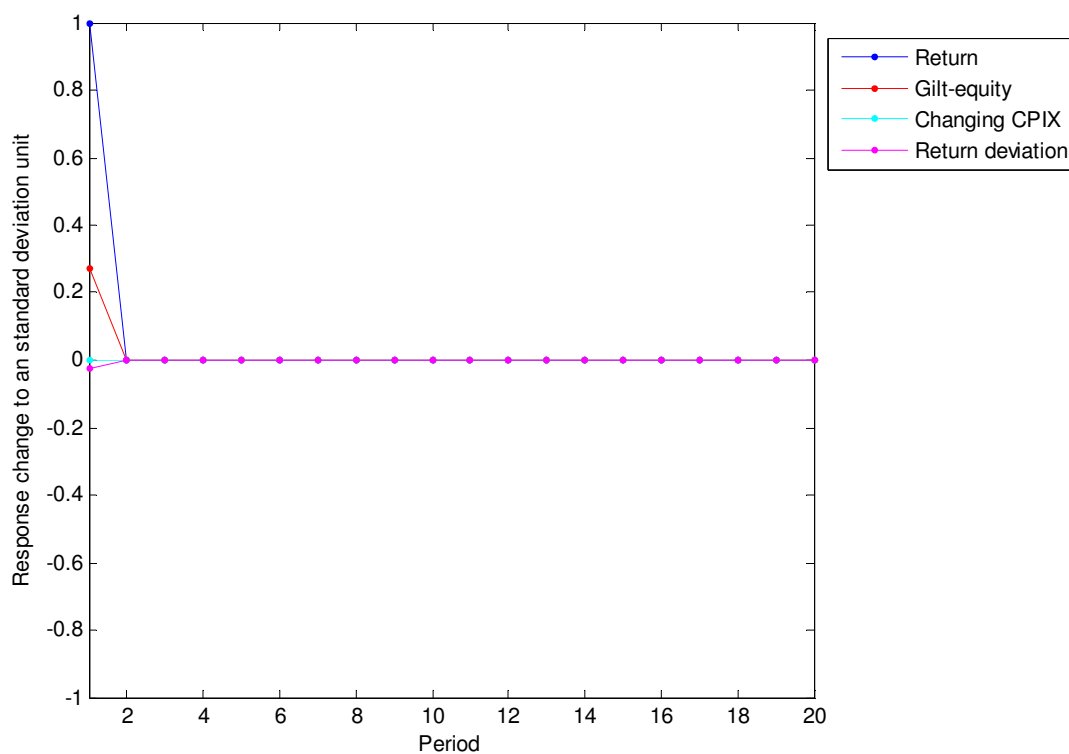


Figure F7: Impulse response of NN(BIC) model for retail return

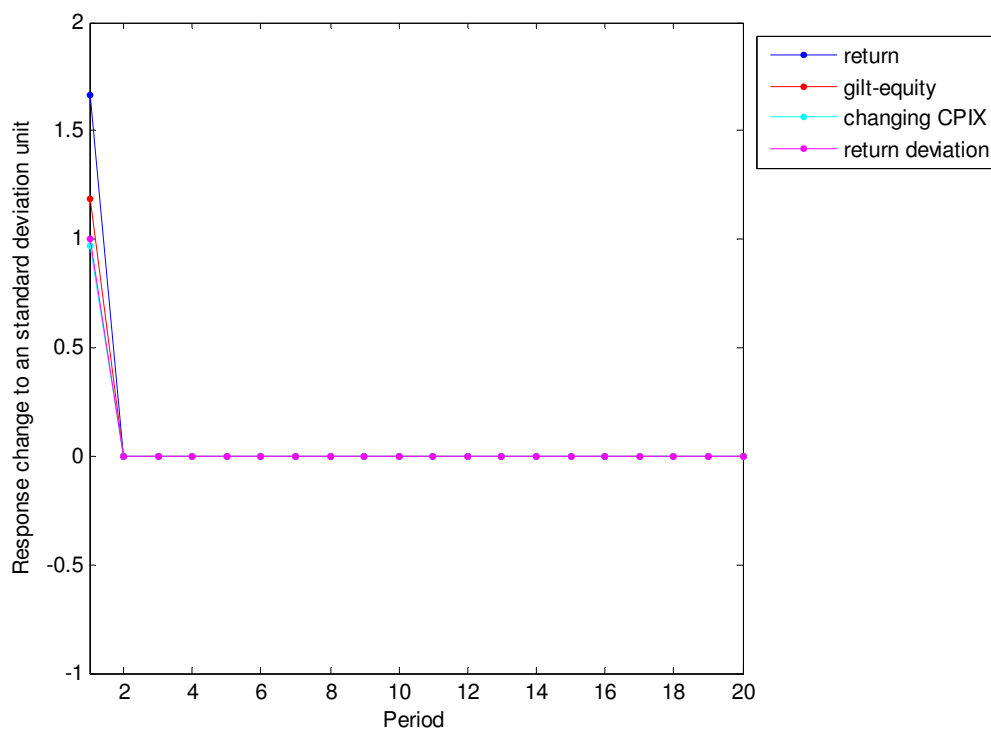


Figure F8: Impulse response of NN(BIC) model for retail return deviation

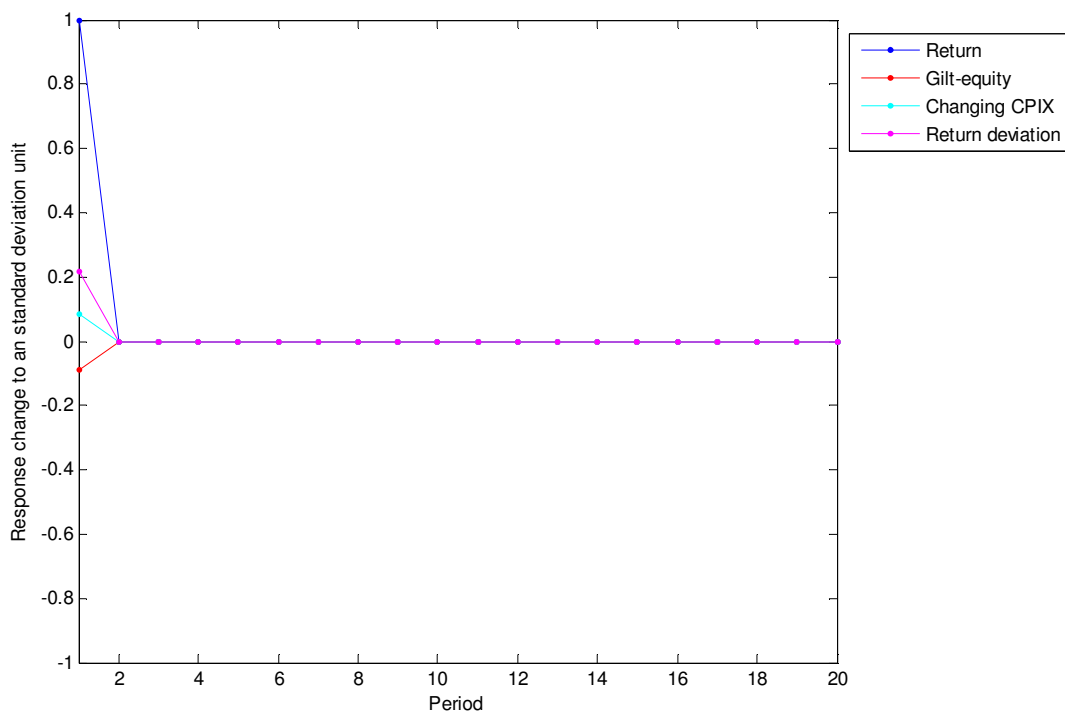


Figure F9: Impulse response of NN(BIC) model for office return

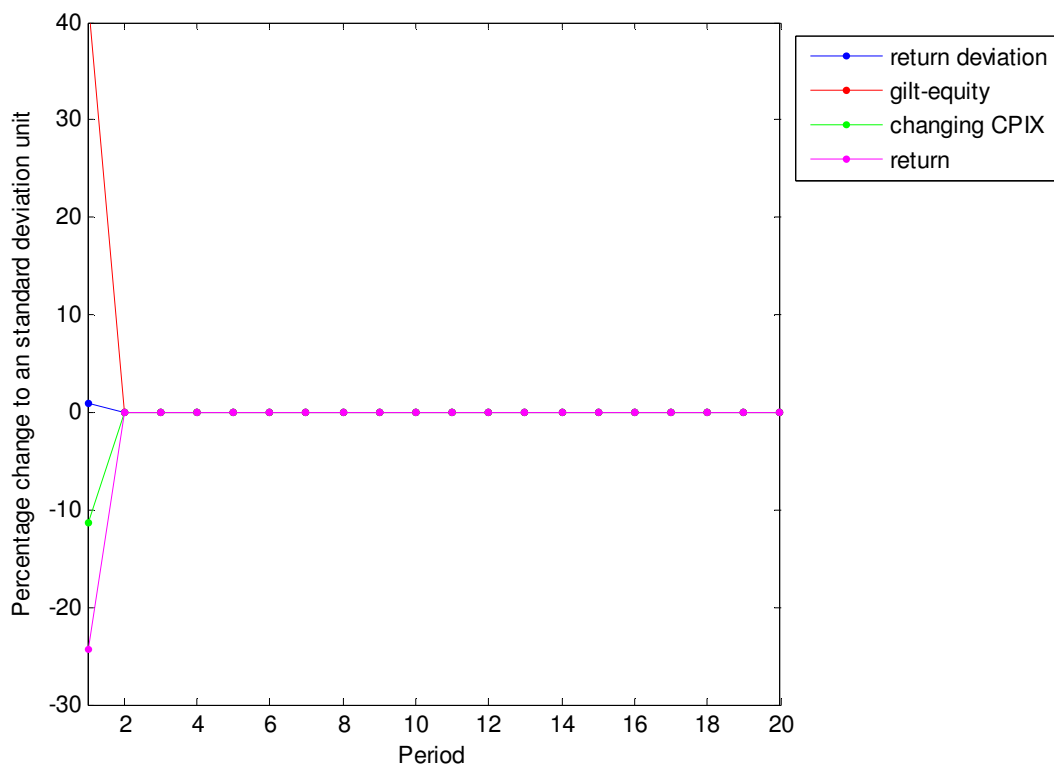


Figure F10: Impulse response of NN(BIC) model for office return deviation

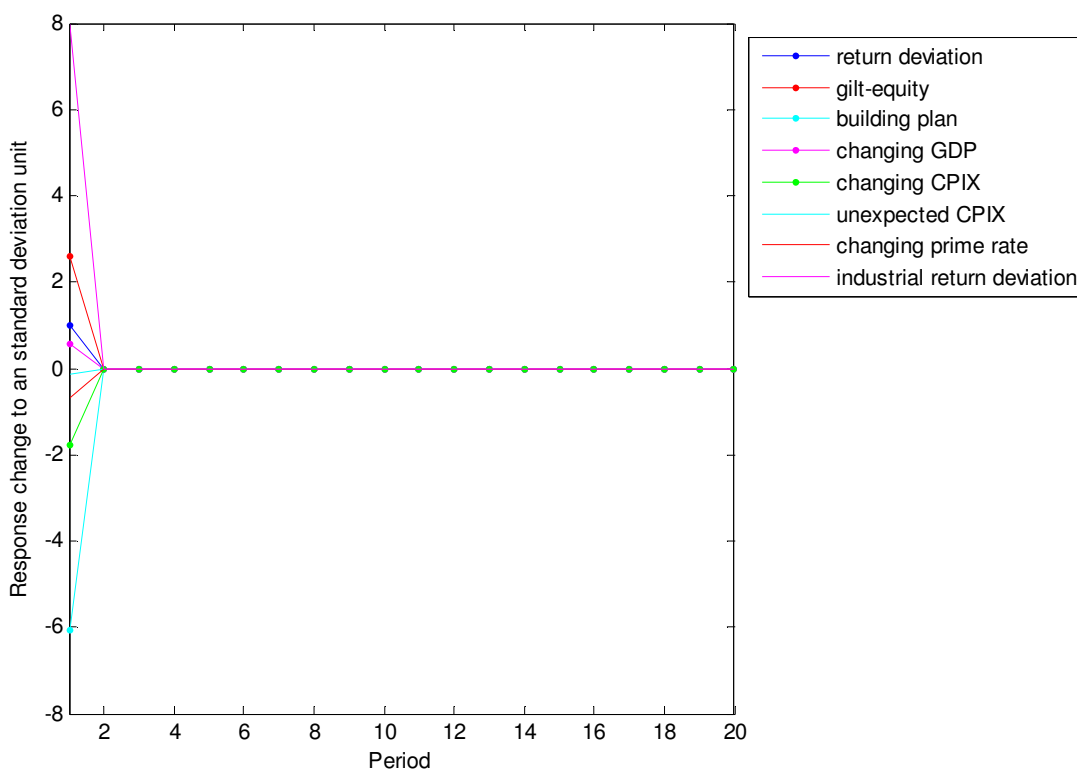


Figure F11: Impulse response of GARCH model for office return deviation

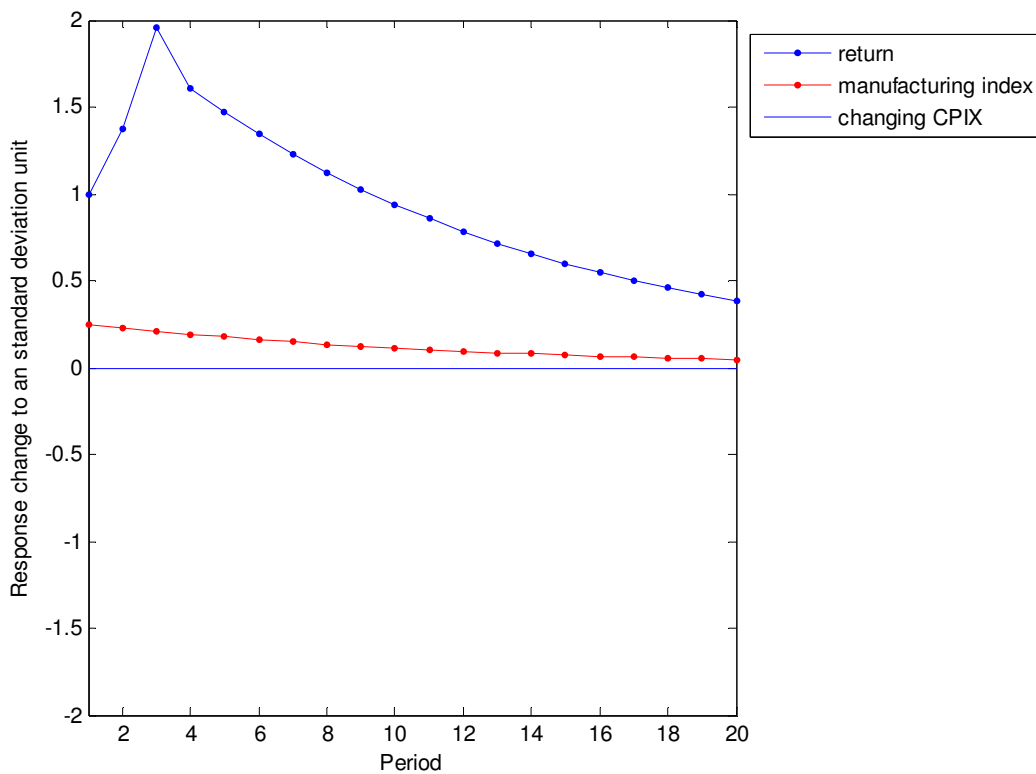


Figure F12: Impulse response of ARMA model for industrial return

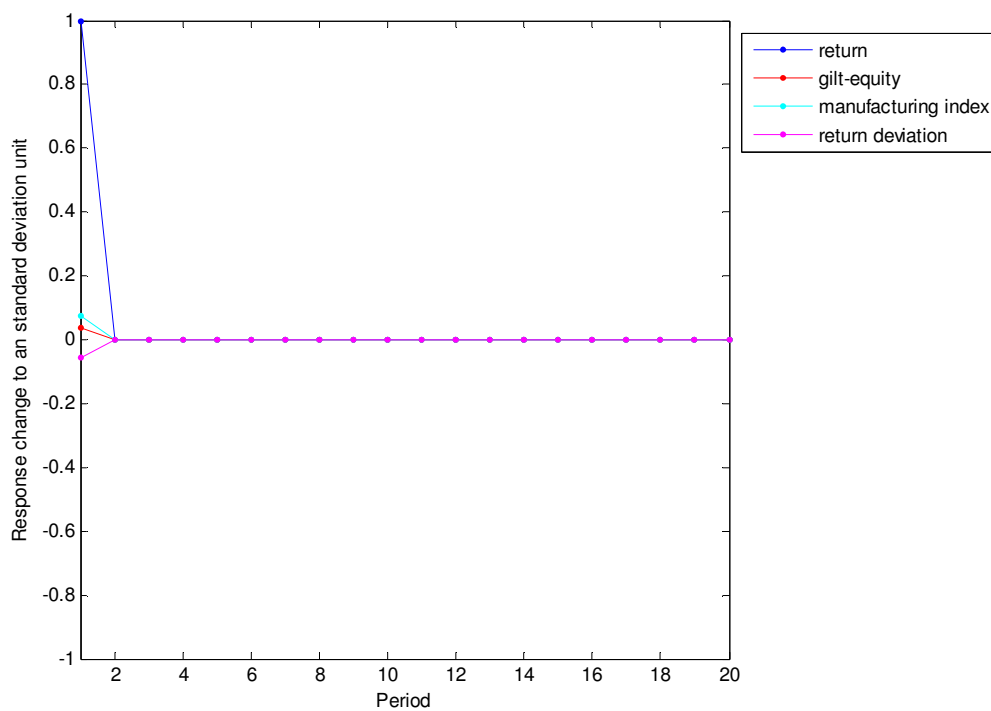


Figure F13: Impulse response of NN model for industrial return

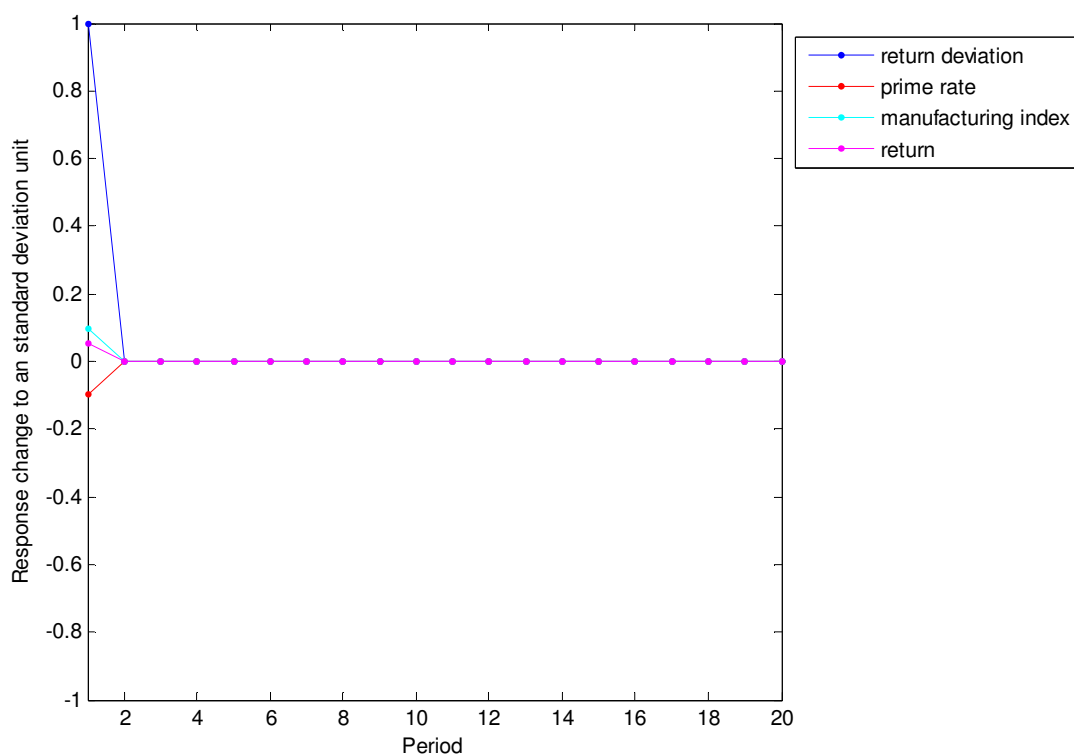


Figure F14: Impulse response of NN(BIC) model for industrial return deviation

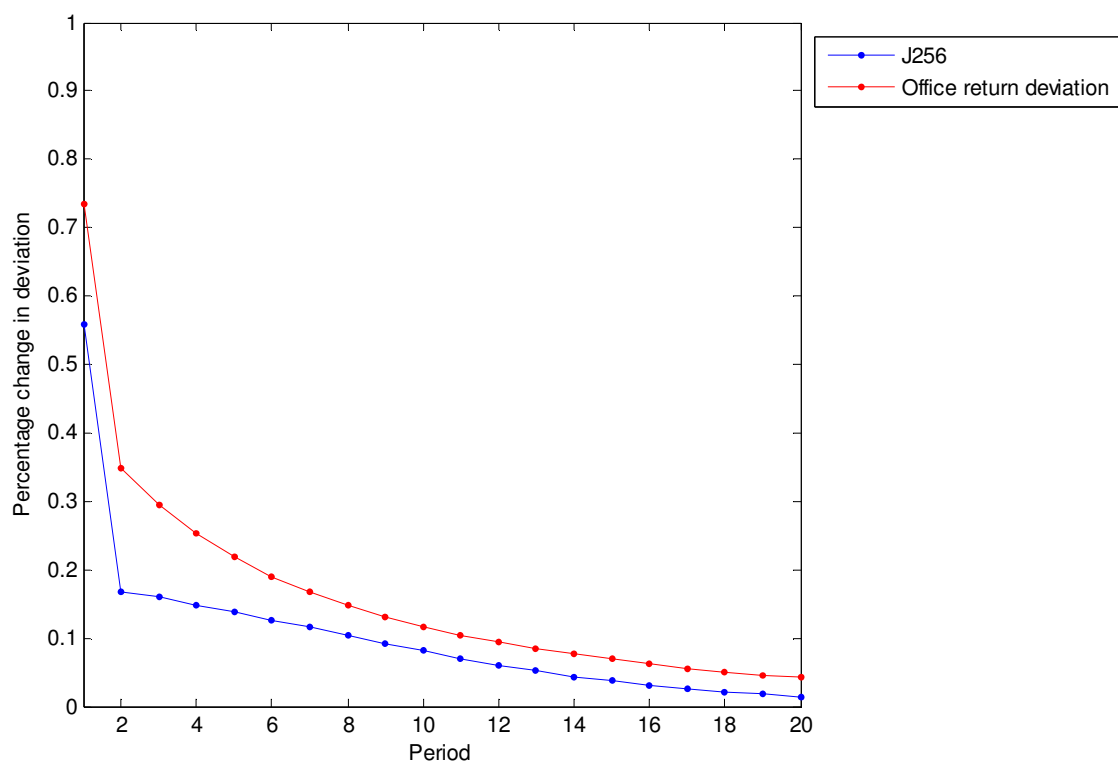


Figure F15: Impulse response of variance function of GARCH models

Appendix G

Schedule of M-files developed

Appendix G
Schedule of M-file

M-file name	Description
ARMA and GARCH model	
anaret.m	Load the table with the return and return deviation data and calculates the statistical properties of each return/return deviation
anavar.m	Loads the table with the independant input variable data and calculates the statistical properties of each variable
ARMAana.m	Implements the optimal ARMA model for each output, calculates the result of the test and load the result of the long prediction
ARMAana1.m	Implements the optimal ARMA model for each output, calculates the result of the test and load the result of the 1-step ahead forecast
ARMAana2.m	Implements the optimal ARMA model for each output, calculates the result of the test and load the result of the 2-step ahead forecast
ARMAana3.m	Implements the optimal ARMA model for each output, calculates the result of the test and load the result of the 4-step ahead forecast
ARMAeval.m	Calculated the mean of the error of each set of ARMA simulation and load the result for a 1-step ahead prediction.
ARMAeval1.m	Calculated the mean of the error of each set of ARMA simulation and load the result for a 2-step ahead prediction.
ARMAeval2.m	Calculated the mean of the error of each set of ARMA simulation and load the result for a 4-step ahead prediction.
ARMAimp.m	Perform the impulse analysis of each ARMA model set of simulation and load the result
ARMAVD.m	Perform the impulse analysis of each ARMA model set of simulation and load the result
autocoret.m	Loads the output data from the table and plot the autocorrelation function of each output
causedev.m	Performs the granger analysis for direct and indirect return deviation and produce a table with probability value of the output and the investigated independant input variables.
causeret.m	Performs the granger analysis for direct and indirect return and produce a table with probability value of the output and the investigated independant input variables
GARCHana.m	Implements the optimal GARCH model for each output, calculates the result of the test and load the result of the long prediction
GARCHana1.m	Implements the optimal GARCH model for each output, calculates the result of the test and load the result of the 1-step ahead forecast
GARCHana2.m	Implements the optimal GARCH model for each output, calculates the result of the test and load the result of the 2-step ahead forecast
GARCHana3.m	Implements the optimal GARCH model for each output, calculates the result of the test and load the result of the 4-step ahead forecast
GARCheval.m	Calculated the mean of the error of each set of GARCH simulation and load the result for a 1-step ahead prediction.
GARCheval1.m	Calculated the mean of the error of each set of GARCH simulation and load the result for a 2-step ahead prediction.
GARCheval2.m	Calculated the mean of the error of each set of GARCH simulation and load the result for a 4-step ahead prediction.
GARCHimp.m	Perform the impulse analysis of each GARCH model set of simulation and load the result
GARCHVD.m	Perform the impulse analysis of each GARCH model set of simulation and load the result
infoARMA.m	Perform the information criterion test for the ARMA model for each output when the lag of the independant variables is 4
infoARMA2.m	Perform the information criterion test for the ARMA model for each output when the lag of the independant variables is 2
infoARMA3.m	Perform the information criterion test for the ARMA model for each output when the lag of the independant variables is 1
infoGARCH.m	Perform the information criterion test for the GARCH model for each output when the lag of the independant variables is 4
infoGARCH2.m	Perform the information criterion test for the GARCH model for each output when the lag of the independant variables is 2
infoGARCH3.m	Perform the information criterion test for the GARCH model for each output when the lag of the independant variables is 1
loaddata.m	Load the independant variables, return and return deviation data onto a "mat" file for Matlab consisting of input and the ouput tables.
PerformUniana.m	Perform the error analysis of a multi step ahead forecast model based on the actual and predicted output
PerformUniana1.m	Perform the error analysis of a 1-step ahead forecast model based on the actual and predicted output
plotimp1.m	Plot the impulse response of the ARMA model for J255 return
plotimp2.m	Plot the impulse response of the NN(AIC) model for J255 return
plotimp3.m	Plot the impulse response of the NN(AIC) model for J255 return deviation
plotimp4.m	Plot the impulse response of the ARMA model for J256 return
plotimp5.m	Plot the impulse response of the GARCH model for J256 return
plotimp6.m	Plot the impulse response of the NN(BIC) model for J256 return deviation
plotimp7.m	Plot the impulse response of the NN(BIC) model for retail return
plotimp8.m	Plot the impulse response of the NN(BIC) model for retail return deviation
plotimp9.m	Plot the impulse response of the NN(BIC) model for office return
plotimp10.m	Plot the impulse response of the NN(BIC) model for office return deviation
plotimp11.m	Plot the impulse response of the GARCH model for office return deviation
plotimp12.m	Plot the impulse response of the ARMA model for industrial return
plotimp13.m	Plot the impulse response of the NN model for industrial return
plotimp14.m	Plot the impulse response of the NN(BIC) model for industrial return deviation
plotimp15.m	Plot the comparison of the impulse response of the GARCH model for J256 return and office return deviation
plotres.m	Plot the predicted output of each model for J255 total return
plotres2.m	Plot the predicted output of each model for J255 and J256 total return
plotres3.m	Plot the predicted output of each model for J255 total return deviation
plotres4.m	Plot the predicted output of each model for J256 total return deviation
plotres5.m	Plot the predicted output of each model for retail, industrial and office return
plotres6.m	Plot the predicted output of each model for retail return deviation
plotres7.m	Plot the predicted output of each model for office return deviation
plotres8.m	Plot the predicted output of each model for industrial return deviation
plotres11.m	Plot the predicted output of the GARCH model for J255 return and deviation
plotres12.m	Plot the predicted output of the GARCH model for J256 return and deviation
plotres13.m	Plot the predicted output of the GARCH model for retail return and deviation
plotres14.m	Plot the predicted output of the GARCH model for office return and deviation
plotres15.m	Plot the predicted output of the GARCH model for industrial return and deviation
plotret.m	Plot the return and return deviation of the analysis
plotvar.m	Plot the independant input variables (determinants) of the analysis

VAR model	
VARAna.m	Implements the optimal VAR model for each output, calculates the result of the test and load the result of the long prediction
VARAna1.m	Implements the optimal VAR model for each output, calculates the result of the test and load the result of the 1-step ahead forecast
VARAna2.m	Implements the optimal VAR model for each output, calculates the result of the test and load the result of the 2-step ahead forecast
VARAna3.m	Implements the optimal VAR model for each output, calculates the result of the test and load the result of the 4-step ahead forecast
VARtest1.m	Perform the information criterion test for VAR models for each output when the number of variables is 4
VARtest2.m	Perform the information criterion test for VAR models for each output when the number of variables is 2
VARic.m	Calculate the AIC and BIC constant for a VAR model with 4 variables
VARic2.m	Calculate the AIC and BIC constant for a VAR model with 2 variables
VARImp.m	Perform the impulse analysis of each VAR model set of simulation and load the result
PerformUniana.m	Perform the error analysis of a multi step ahead forecast model based on the actual and predicted output
PerformUniana1.m	Perform the error analysis of a 1-step ahead forecast model based on the actual and predicted output
NN (neural network) model	
NNAna.m	Implements the optimal NN model for each output, calculates the result of the test and load the result of the long prediction
NNAna1.m	Implements the optimal NN model for each output, calculates the result of the test and load the result of the 1-step ahead forecast
NNAna2.m	Implements the optimal NN model for each output, calculates the result of the test and load the result of the 2-step ahead forecast
NNAna3.m	Implements the optimal NN model for each output, calculates the result of the test and load the result of the 4-step ahead forecast
NNtest1.m	Perform the information criterion test for NN models for each output where there are 4 independant variables and a lag of 4.
NNtest2.m	Perform the information criterion test for NN models for each output where there are 4 independant variables and a lag of 2.
NNtest3.m	Perform the information criterion test for NN models for each output where there are 4 independant variables and a lag of 1.
NNtest4.m	Perform the information criterion test for NN models for each output where there are 2 independant variables and a lag of 4.
NNtest5.m	Perform the information criterion test for NN models for each output where there are 2 independant variables and a lag of 2.
NNtest6.m	Perform the information criterion test for NN models for each output where there are 2 independant variables and a lag of 1.
NNic.m	Calculate the AIC and BIC constant for a NN model
NNImp.m	Perform the impulse analysis of each NN model set of simulation and load the result
NNVD.m	Perform the impulse analysis of each NN model set of simulation and load the result
PerformUniana.m	Perform the error analysis of a multi step ahead forecast model based on the actual and predicted output
PerformUniana1.m	Perform the error analysis of a 1-step ahead forecast model based on the actual and predicted output

Table G1: Schedule of developed m-files

Note: These files need to be loaded in conjunction with the associated file for these models in the work directory of the Matlab program, i.e. When simulating the ARMA model, one need to load the developed file and the associated file in the work directory