# A MACRO-ECONOMIC INDICATOR-BASED RISK MANAGEMENT

# STRATEGY FOR THE SMALL PROPERTY INVESTOR

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A research report submitted to the Faculty of Engineering, Department of Construction Economics, at the University of the Witwatersrand, in partial fulfillment of the requirements for the degree of Master of Science in Property Development and Management.

Declaration

I declare that this research report is my own unaided work. It is submitted in fulfillment of part of the requirements for the Masters Degree in Property Management and Real Estate (Faculty of Engineering) of the University of the Witwatersrand, Johannesburg.

Jan Hendrik Steenkamp

\_\_\_\_\_ day of September 2006

#### DEDICATION

This research is dedicated to all my friends, family and the lecturing and administrative staff of the Department, as well as everyone that donated time and patience during the completion of this research report.

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#### ABSTRACT

Risk to small property investors manifests in the cash flow of the investment and it should thus also be managed in the cash flow. From a practical point of view it is logical that that risk management strategies be incorporated into a property investment at the inception stage of the investment. The cash flow of a property thus needs to include applicable risk management strategies as part of the feasibility study of the investment. The chief manner in which small property investors deal with risk in an investment, is by making conservative allowances in the projected cash flow of the investment. Internal risk is thus managed to a degree, but the small investor is still vulnerable to market risk which originates from outside the investment. Market risk however, is relatively successfully managed by the Institutional Sector of the property market through the application of Modern Portfolio Theory and the use of the portfolio as a vehicle to diversify internal risk. The portfolio vehicle also allows the quantification of external- or market risk, thus creating the opportunity for effective management.

It is however believed that the same principles of Modern Portfolio Theory as applied in the institutional sector of the property market, may be applicable to small direct property investments, to formulate an investment risk management strategy, which is embedded in the conceptual stages of the investment and thus reduce the reliance on often, ineffective, active management and remedial strategies during the holding period.

The main obstacle however, is that the application of Modern Portfolio Theory requires an industry bench mark or index, which is representative of the market and against which performance may be measured measured. The application of such an index to the small direct

property investment is however extremely limited due to a difference between the scale at which small investments function and the scale of the market that an index represents. A substitute for a benchmark to act as a market indicator is thus required, which must be reflective of the market within which the small investment operates.

This report investigates the possibility of deriving an investment-specific benchmark or a hypothetical return curve, based on the relationships that exist between the macro-economy and the property market. If it is indeed possible to establish the credibility of such an alternative market indicator, it would therefore become possible for small property investors to apply the risk management principles inherent to portfolio investing and incorporate these principles in the feasibility cash flows of small direct property investments.

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## **CHAPTER I**

#### **INTRODUCTION**

#### **1.1 BACKGROUND**

The definition of commercial property dictates that the asset earns income and this occurs through either rental income, capital gains (profit) obtained through trade and sale, or both. Effective property valuation methods base the capital value of the commercial property investment on its income or potential income (Ball et al, 1998). Factors influencing the income will thus also affect the value of the investment. The South African economy is characterised by a large sector of small, medium and emerging markets and businesses and the property market is no different. The recent boom in the general property market (2003-2005) was also associated with relatively low interest rates (10.5% prime bank loan rate, SARB, 2006). There was thus incentive for many new entrants into the property market, typified by highly geared investments.

Depending on the gearing of the investment, a fluctuation in the interest rate (or expected return rate with respect to equity financing) will have major impact on the net returns of the investment and hence also on the overall value thereof. It is therefore little wonder that the feasibility of the investment is severely influenced by an increase in interest rate in particularly the long-term. This is especially true of small or first-time investors that have not yet built up significant resources, thus forcing highly geared- and cash flow limited investments.

However, interest rate may be but one of many factors that influence direct property investments. There are additional factors that influence feasibility and value through their impact on the cash flow, which emanate from outside the investment. Property investments, as with the other three investment classes (equity, bonds and cash), operate

within their particular markets but also within the bigger macro-economic environment and similarly are also subject market fluctuation and to movement in the macro-economy. Changes in market conditions result in fluctuations in expected returns from the investment, which is defined as risk that must be managed. It is believed that small direct property investors do not manage their risk, particularly market risk, effectively and that this segment of the property market will be severely affected by adverse market conditions in the future.

#### **1.2 PROBLEM STATEMENT**

Ball et al (1998) indicates that because of the links between the property market and the macro-economy, methods of feasibility analysis and risk management in the commercial property market, are increasingly looking towards the securities market and the risk management strategies that are employed to manage especially market risk. Further, ad hoc adjustments during the lifetime of the investment can be made to compensate for such risk, but that it is more logical to eliminate the risk in the conceptual stages in order for it to be included in the feasibility study of the investment. The relevant methods to manage market risk must therefore have application in the models of feasibility analysis employed by the small investor. One such model is the Discounted Cash Flow (DCF) model used both for feasibility and active management during the holding period of the investment.

In the institutional sector of the property market, Modern Portfolio Theory (MPT) has been used to similar effect as in the securities market as a risk management strategy in investments. Both internal and external risk is managed effectively, predominantly through the use of the portfolio as an investment vehicle and the application of the concept of diversification. The construction of a market portfolio from relevant historical data obtained from property indices, enable the institutional investor to quantify the price of market risk and hence make sufficient allowance for it. The institutional investor is able to more accurately take a view on the market and hence formulate and investment strategy to position the investment effectively in the market from the outset The institutional investor is therefore able to manage both internal- and external risk (market risk) effectively, because the risk management strategy forms part of the feasibility and structuring of the investment and contributes to the investment decision before commitment to the investment is made.

The risk management strategies of the small direct commercial property investor normally revolve around active management and are seated in the cash flow of the investment. The investor therefore has some form of control over the factors that cause internal risk and is therefore able to implement compensatory measures for risk, which normally favour a conservative approach in the estimation of expected future returns and the escalation rates of the key elements of the cash flow.

Small investors however, predominantly because of limited resources and therefore the inability to assimilate a multi-property portfolio as an investment vehicle, manage external or market risk less successfully. For the institutional investor, relevant historical data on market performance is available in the form of property indices, which enables the quantification and hence the effective management of market risk. For the small investor such relevant information on market behaviour is not applicable. Short of conducting costly and time-consuming market research studies, the small investor is not in a position to verify market activity and to quantify the associated market risk.

However, it is believed that the concepts of MPT may still be applicable to single property investments subject to certain conditions and assumptions. In addition, it is also the belief that general property market behaviour in all the sectors of the market are affected by the influence of the macro-economy and that macro-economic indicators may be used to derive general indicators of property market behaviour, thus acting as a substitute for indices and hence enabling the quantification of market risk for small single direct property investments. The influence of the macro-economy on markets of the institutional sector and the small single property market would thus be equal at the macro-scale. This research report therefore seeks to investigate the possibility of a macro-economic indicator-based risk management strategy for small direct property investors.

#### **1.3 SUB-PROBLEMS**

In the attempt to solve the main problem as stated above, the following sub-problems have to be addressed:

#### **1.3.1 Risk Management Theory**

In the background statement it has been indicated that the impact of fluctuating income on the investor results in uncertainty, which is quantified as risk. Risk by definition is a broad concept and is not uniform. This report aims to develop an investment strategy for a specific investor profile (the small investor in direct property with limited resources) in order to manage risk. The investment strategy that has to be developed therefore needs to be specific in dealing with risk from this particular point of view.

The dominant risk management strategy applied in the securities market and the institutional sector of the property market is portfolio investing. Modern Portfolio Theory is applied not only in securities market investments with great success, but also in the institutional sector of the property market. There are however certain fundamental elements which are prerequisites for portfolio investing. What these prerequisites are will be an important factor in the applicability of MPT as an investment strategy to the small property investor. It is therefore essential that MPT, as a risk management strategy, be critically assessed in order to determine the essential prerequisites that have to be met by direct property investments, in order to be used in risk management.

#### 1.3.2 Property Vehicle For Performance Comparison

Modern Portfolio Theory essentially revolves around obtaining diversification in the sources of income. Typically, the information needed for the application of MPT must

include historical figures on income as well as capital value. Thus, in order to successfully apply MPT to direct property investment, a format for a direct property investment is necessary, which must conform to this basic requirement.

In addition, the individual property investment is subject to a number of direct influences from the macro-economy and its indicators of inflation, interest, etc. The aim of this report is then also to develop an investment risk management strategy, which may be based on such economic indicators. The format of the property investment used must therefore also facilitate the incorporation of potential impacts of the macro-economic indicators. It is believed that the Discounted Cash Flow model provides a suitable format for the application of MPT. The format must however be investigated for suitability to the application of MPT, but more importantly, the prerequisites and assumptions necessary to achieve this must be isolated.

#### **<u>1.4 HYPOTHESIS</u>**

Ball et al (1998) indicates that the property market is increasingly becoming aware of the influence of the macro-economy on property investments. Typically, the cash flow is influenced and as result the value of the investment as well. This fluctuation in income and value, are perceived and defined as risk. Small (direct) property investors typically are less capable of managing risk than their institutional sector- or securities market counter-parts (Jaffe and Sirmans, 1995). The small investor is at a disadvantage due to the limited resources at his or her disposal. There is thus a need to develop investment strategies for small investors to manage risk effectively.

It is the main hypothesis of this research report that the principles of Modern Portfolio Theory, as are employed in the securities market and by the institutional sector, may be applied to the small property investor, if:

- The investment vehicle of the individual small property investment (DCF analysis) exhibits suitable characteristics to be used as a vehicle for the possible application of the principles of MPT.
- It is possible to establish an alternative market indicator to property indices to quantify market risk and that it is possible to achieve this through an investigation of the relationship(s) between the property market and the macro-economy.

## **1.5 RESEARCH OBJECTIVES**

The objectives of this research report are:

- Critically assess MPT to clarify the processes and procedures as well as the necessary prerequisites for application.
- Identify the components of the Discounted Cash Flow analysis and assess the possible weaknesses to risk.
- Investigate the application possibilities of the identified principles of MPT to the DCF analysis.
- Establish the critical questions to be answered, which are necessary in the application of MPT principles to the DCF of direct property investments.
- Make recommendations on the application of the outcome of the study as well as on follow-up studies for expansion of the hypothesis.

## **1.6 LIMITATIONS AND SCOPE**

#### 1.6.1 Property Market Analysis

Statistical analysis will be conducted on the property market, through the use of data obtained from a property index. The analysis will be restricted to general market indicators. The justification of the use of general indicators and the use of an index will be addressed theoretically.

#### 1.6.2 Macro-Economic Analysis

The purpose of conducting an analysis on the macro-economy is to determine the relationship between the property market and the macro-economy. The relationship(s) will be tested at the macro-level, where the macro-economic indicators can be used as units of measurement. The analysis will be conducted to determine the nature of the relationships and relevance. More specifically, the analysis will be conducted with the aim of establishing the relevance of the macro-economic indicators to the property market.

#### **1.6.3 Quality of Data**

All data used are from unbiased and authentic sources. The data can be considered primary data, since it was not collected for a particular purpose or particular study. The three main sources of data are Statistics SA, the South African Reserve Bank (various statistical releases) and the IPD/SAPIX South African Property Investors Digest.

#### C. Definitions

- *Small Investor*: For the purposes of this report the term 'small investor' will be applicable to a typical property investor that is investing directly in a single property with limited resources, which prohibits multiproperty investments or developments. The term "small development" may include more than one tenant.
- *Portfolio*: The term portfolio refers to the combination of at least two assets from a single asset class or from multiple asset classes. Investing in more than one property asset is excluded by the definition of the small investor.

#### **D.** Abbreviations

MPT	Modern Portfolio Theory
DCF	Discounted Cash Flow
CAPM	Capital Asset Pricing Model
CML	Capital Market Line
SML	Securities Market Line
NPV	Net Present Value
IRR	Internal Rate of Return
PGI	Projected Gross Income
ESP	Expected Selling Price

#### **1.7 OUTLINE OF REPORT**

The remainder of this report will structure as follows in order to achieve the above research objectives:

Chapter II: Investment Risk

The concept of risk will be evaluated from a general point of view, as well as in terms of MPT. Various theories on portfolio investing will be highlighted with the focus on isolating the main concepts of each, as well as determining the prerequisites of application thereof.

Chapter III: Discounted Cash Flow Analysis

A critical evaluation of the Discounted Cash Flow (DCF) analysis model, as a suitable feasibility model for portfolio-based risk management strategies for direct property investments will be conducted. The focus will be particularly on isolating the elements of the model that are sensitive to risk. A second function of the chapter will be to narrow down the links between the cash flow and the market, which signifies risk to the investor.

Chapter IV: Applying MPT to Direct Property.

The plausibility of applying theories on portfolio investing to direct property will be investigated. The assumptions necessary to achieve this will be isolated.

Chapter V: The Link Between The Investment And The Property Market.

The contemporary link that an index represents between the investment and the market will be investigated in order to determine its role and function. This report seeks an alternative market indicator relevant to small single property investments, which must be able to perform the same function.

The chapter also aims to determine the relationship between the property market and the macro-economy, since one of the main objectives is to determine the possibility of deriving a market indicator for the small investor, which is based on one of the macro-economic indicators, in order to prove the hypothesis.

Chapter VI: Research Methodology

The logical process followed in the research report is outlined. The different modes of analysis that will be used are also discussed for applicability. The theoretical analysis of the hypothesis and sub-problems are narrowed down to the key questions to be answered by the research.

Chapter VII: Data Analysis

The critical questions asked in terms of the research objectives are made subject to the modes of analysis identified in the research methodology.

Chapter VIII Conclusion

Conclusions will be drawn from the investigations of the proceeding chapters, with concluding comments. Recommendations will be made for possible further research.

## **CHAPTER II**

#### **MODERN PORTFOLIO THEORY**

#### 2.1 INTRODUCTION

Feasibility studies of direct property investment require assumptions to be made regarding critical inputs. The assumptions made centre round the variance of the expected escalation rates that are applied to the components of the cash flow. This implies uncertainty and therefore risk that must be accounted for. Risk therefore realises in the cash flow of a direct property investment. For the small property investor, the cash flow also functions as the chief management tool during the holding period. It is therefore paramount that the risk management strategies which are to be employed, must have application in the cash flow.

The property market in general is increasingly looking toward the securities investment market for investment strategies to manage risk. Portfolio investing has been used in this market to manage risk with great success. The use of the principles of Modern Portfolio Theory (MPT) then, has also found favour in especially the institutional sector of the property market, where contemporary security investments are combined with property investments in investment portfolios. However, the resources available to large institutional investors, distinguishes them from the small property investor, who for the purposes of this report, is not able to combine a multitude of investments in a portfolio to deal with risk effectively.

However, the hypothesis of this report is that MPT may have application to single direct property investments, which will enable more informed investment decisions to be made. This chapter will investigate the main theories of portfolio investing through the brief overview of each.

#### 2.2 MARKOWITZ'S DIVERSIFICATION THEORY

According to Markowitz (1952, from Isaac, 1998), risk can be broadly categorized as being either internal or external in nature. Internal risk is identified as risk that results from a variation in the outcome of an event, over which the investor has some form of control. External risk is identified as risk that results from a variation in the outcome of events over which the investor has no control.

#### 2.2.1 External Risk

From a pure economic point of view, Parkin and King (1995) indicates that, all else being equal, a movement of the point of equilibrium between demand and supply along a particular demand curve, is expressed as price, a factor over which the investor has some form of control, but that a cataclysmic change in demand would be a total shift of the demand curve over which the investor has no control. A total shift in demand is incurred (inter alia) by the following factors:

- 1. A change in income
- 2. A change in the price of a substitute
- 3. A change in the expected future price
- 4. A change in the price of a compliment

All of the above events may thus be regarded as factors that will influence the performance of an investment. The investor may or may not have control over these factors. When there is no control over price, a variance in price will thus be deemed external (systematic) risk. A shift in the demand curve may therefore be classified as market risk.

#### 2.2.2 Internal Risk

Markowitz proposed that internal- or unsystematic risk is managed in the portfolio environment through the <u>principle of diversification</u>. Markowitz Diversification theory functions on the premise that the overall risk of the portfolio can be reduced to the extent that the sum of the risks of the underlying assets in their individual capacity, would be more than when they are combined in the one market portfolio. It is argued that in a perfectly efficient portfolio, unsystematic risk can virtually be eliminated to such an extent that all that will remain will be systematic- or external risk (market risk). The net result will be that total risk (comprising both internal and external risk) will be reduced (Goodall, 2005).

According to Markowitz, the investor is to identify an optimal portfolio from an infinite number of portfolios, which could be formed (an optimal portfolio is defined as a broadly diversified portfolio, which represents the market itself). The optimal portfolio is constructed through the specification of a set of securities. The set of securities are analysed in terms of:

- 1. Expected returns
- 2. Variance in returns
- 3. Covariance between securities.

The above is calculated by means of Mean-Variance analysis using historical data. The variance of expected returns constitute risk. Markowitz's formulation of the optimal portfolio is known as Markowitz Diversification.

Markowitz Diversification essentially aims to reduce risk whilst optimising return for a combination of assets. The following example as used by Isaac (1998, p242) will be used to explain the principle of risk reduction through diversification:

Table 2.1

		Investment A	Investment B
Return % on investment i	E <sub>i</sub>	3 or 7	9 or 1
Probability of each return	α	0,5	0,5
Expected Return % (mean)	α	5	5
Variance on investment (%)	$\sigma^2$	4	16
Standard Deviation (%)	σ	2	4
Number of investments	n = 2	-	-

From the example it can be seen that both investments have the same average (mean) return, but that B has a bigger variance and is therefore considered to be more risky.

Variance 
$$(\sigma^2)$$
 is denoted as:  $\underline{\Sigma(\alpha_i - \alpha)^2}$   
n

If the investor elects to invest in both investments, the following assumptions are made:

- a. The two investments are negatively correlated (opposite income/return behaviour)
- b. That there is a proportional allocation of funds (in this case  $\frac{2}{3}$  to  $\frac{1}{3}$  in favour of Investment A based on the fact that B is of a higher risk (bigger variance)
- c. That the probabilities of the expected returns are the same.

The combined expected return for investing in both investments in a portfolio in two scenarios, where the upper expected return for A & B are used respectively with the lower expected return values of the other, would be:

A: High  
ER<sub>P</sub> = 
$$\frac{2}{3}(7) + \frac{1}{3}(1)$$
  
= 5

B: High ER<sub>P</sub> =  $\frac{2}{3}(3) + \frac{1}{3}(9)$ = 5

The formula for Expected Portfolio Return [  $ER_P$  ]is:

$$ER_P = \Sigma \alpha_i(E_i)$$

where  $\alpha_i$  = Proportional allocation of funds in Investment i  $E_i$  = Expected return on Investment i

If the total risk or variance for the portfolio is calculated, then:

$$\sigma^{2} = \underline{\Sigma}(\alpha_{i} - \alpha)^{2}$$

$$n$$

$$= \underline{(5_{A} - 5)^{2}} + \underline{(5_{B} - 5)^{2}}$$

$$2$$

$$= 0$$

where  $\alpha_i = \text{Expected return on investment i}$ 

 $\alpha_A$  = The mean expected return for asset a.

 $\alpha_B$  =The mean expected return for asset a.

n = The number of assets in the portfolio.

#### The total (internal) risk for the portfolio has been reduced to 0.

Looking at Markowitz Diversification Theory, two important factors must be noted which are critical necessities in the application of the principle of diversification:

#### 2.2.3 Correlation

The following important observations are made from Markowitz Diversification Theory and the example above:

- (i) When the two assets are combined the investor will achieve the same return for less risk
- (ii) The two investments have a negative correlation (in this case a correlation coefficient of -1 since the assumption was made that they have perfect negative correlation).
- (iii) The risk (in this case) is diversified away

It can therefore be observed that, when two or more investments are combined in a portfolio, their interactive risk (or co-variance) must be considered. The less synchronised their return patterns are with respect to one another, or the less positively these cycles of return are correlated, the greater the benefit. In fact, negative correlation is central to Markowitz Diversification and is thus essential in achieving a reduction in risk. Negative correlation is therefore a primary requirement and forms the basis of MPT.

In addition, the relationship of two assets is measured over an extended period and is not merely a snapshot view at a specific point in time. The continuity of the relationship is therefore also of great importance. In other words, if the relationship shows a strong underlying connectivity, which proves to be not by mere chance, the interactive risk also becomes predictable as the holding period becomes longer. Obviously the accuracy of the calculation needed to determine the relationship between the return patterns of investment assets, is dependent on reliable historical data, which is usually available for listed securities in the securities market.

The following illustrates perfect positive and perfect negative correlation.

Figure 2.1

-Dubben and Sayce (1991) in Isaac (1998)

Perfectly negative correlation between investments is difficult to achieve. The reason for this is that almost all assets in a particular asset class or similar market respond in a similar manner to external influences, which are normally macro economic or investment class related (Gooddall, 2005). Not all assets have the same response time to external influences and neither do different investment classes. Like products or goods, assets have different price elasticities of demand and hence will respond differently to changes in the market (shift in demand)(Parking and King, 1995, P103). *The response both in magnitude and time is therefore more likely to represent opportunities of negative correlation if the assets are from different investment classes*. It can therefore be seen that a strong negative relationship between two assets will be ideal for a combined investment and that if this relationship proves to be less than coincidental, the risk reduction attributes of the relationship will be even furthered as the holding period extends.

#### 2.2.4 <u>The Efficiency Frontier</u>

From the simple example below it can be seen that an optimum level of efficiency is obtainable between risk and return through portfolio investing. The optimum combinations can be graphically illustrated and is known as the Efficiency Frontier. The line represents the best possible risk-reward combinations.

Figure 2.2

-Isaac (1998), p249

Put alternatively, the "co-variance" (or interactive risk) between two assets are minimised along the frontier. The overall risk is reduced and returns are maximised. The co-variance is thus an important factor in determining the efficiency of the portfolio. The co-variance is a measure of the net risk, or rather, a measure of the combined uncertainty that the variance of the returns of the two assets which are combined in portfolio investment, will culminate to. The correlation between the return curves of the two investments, therefore play a crucial role in determining the overall risk. To be able to invest close to this "limit" in a diversified portfolio (in terms of Markowitz Diversification theory), the interrelationship between the individual investments need to be quantified. The covariance needs to be established. The covariance between assets are defined by the following expression:

 $\text{COV}_{A,B} = \Sigma [a_r - A_r][b_r - B_r] P_r$ 

Where:  $A_r$  = The mean expected return for asset a.

 $a_r$  = The expected return for asset a.  $B_r$  = The mean expected return for asset b.  $b_r$  = The expected return for asset b.  $P_r$  = Combined probability for a and b

Once the covariance is determined it is possible to determine the degree of correlation between two assets. This is known as the correlation coefficient and ideally this value should be as close to -1 as possible, in order to reduce portfolio risk the most effectively. Values close to +1 indicate little or no risk reduction benefits from the combination of two assets being invested in a portfolio. The correlation coefficient is represented by the following expression:

 $Corr_{A,B} = \frac{Cov_{A,B}}{\sigma_A \sigma_B}$ 

where:  $Cov_{A,B}$  = The co-variance between investments A and B

 $\sigma_A$  = The Standard Deviation of investment A

 $\sigma_B$  = The Standard Deviation of investment B

The correlation coefficient between two assets now enables the investor to base a decision on which assets to combine in a combined (portfolio) investment. The purpose of this exercise would be to optimise return / risk characteristics of the cycles within which the individual assets function through the comparison of correlation. An individual property investment can therefore be combined with a securities market investment to optimise return but reduce the overall risk of the combined investment or it may be combined with other property investments to achieve the same results. According to Ball et al (1998) the following expression defines the construction of an investment portfolio:

$$\sigma_{\rm p}^{2} = \Sigma \Sigma w_{\rm i} w_{\rm j} \, \sigma_{\rm i} \, \sigma_{\rm j} \, p_{\rm ij}$$

where:  $\sigma_p^2$  = Total portfolio risk as measured by the variance.

- $w_i w_j$  = The weight of the individual assets i and j
- $\sigma_i \sigma_j$  = The standard deviations on the returns of the individual assets in the portfolio.
- $p_{ij}$  = The correlation coefficient between the returns to assets i and j.

From the above definition an efficient portfolio may be constructed.

#### 2.3 SEPARATION THEOREM

James Tobin (Goodall, 2005) simplified Markowitz's theory on diversification. Unlike Markowitz, who advocated that the only manner to reduce risk is through the combination of risky investments, Tobin proposes that the investor can control risk by deciding what portion of total capital invested should be in a risk-free investment and what portion should be invested in the efficient portfolio (constructed using Markowitz Diversification). There is thus a separation between two decisions being taken.

Isaac (1998) states that if a risk-free investment, such as interest rate instruments like government bonds are introduced into the portfolio, there is a stabilising effect on the portfolio risk-return relationship, since the total return increases with virtually no increase in the risk. As an example, bonds in general as an investment class, provide a relatively stable income stream over the lifetime of the investment (the assumption is made of a long-term investment) and is considered to be a (relatively) risk-free investment.

#### 2.4 THE CAPITAL ASSET PRICING MODEL (CAPM)

Like Tobin, William Sharpe (1963, in Goodall, 2005) developed the CAPM through the simplification of the Markowitz model in the selection of share portfolios. Sharpe advocated that instead of calculating the interactive risk between all the assets in a set of securities, <u>only the relationships between the individual shares and the dominant factor are calculated</u>. For Sharpe the <u>dominant factor was the market itself</u>, which is represented by a "market portfolio". If the price of the individual share is more volatile than the

market portfolio, then the addition of that share would make the portfolio more risky and visa versa.

Sharpe proposed that the volatility of a share is determined by three major factors:

- Volatility attributed to movement of the market as a whole accounted for about a third.
- Movements in the market through factors influencing similar investments in the investment sector or industry would account for about half of the balance.
- The remaining balance relates to factors relating to the individual share itself.

In addition, and relating to Separation Theorem, Sharpe indicated that the investor has the opportunity to increase or decrease earnings by increasing or decreasing risk exposure through borrowing and lending. If more risk is desired, funds may be borrowed at the risk-free rate and invested in the market portfolio, with balance invested in the efficient portfolio. If a reduction in risk is desired, capital is loaned and invested in a risk free asset such as government securities (bonds), with balance invested in the efficient portfolio.

To quote Goodall (2005):

"The market portfolio is a combination of all securities in the market, each in proportion to the market value of all securities in the market (to include property shares and bonds). An efficient portfolio is a broadly diversified portfolio. *The CAPM assumes that investors will not hold different portfolios, but different amounts of the same portfolio*. The portfolio that they hold will be the market portfolio. Each investor will adjust his particular risk/return profile by combining the market portfolio with different amounts of money invested at the risk free rate."

The diagram below illustrates the CAPM.

Figure 2.3

(Goodall, 2005, Part 2, Cp 1,p7)

The CAPM aims to determine how much an asset contributes to the risk of the portfolio as a whole. To Quote Goodaal (2005): "The reward per unit of risk is taken as the difference between the market return less the risk free rate of interest, divided by the unit of risk (as measured by the Standard Deviation of the expected return line representative of the market or the Capital Market Line (CML)". The above is referred to as the Sharpe Ratio.

#### 2.4.1 The Capital Market Line

From the above diagram of the CAPM, investors can invest all funds at the risk-free rate of return (RF) or invest all in the market portfolio (M). The capital Market Line (CML) represents the alternative risk/return combinations obtainable by investing in the market portfolio, with borrowing or lending at the risk free rate. Thus the CML represents the expected return of an efficient portfolio, which represents or emulates the market (market portfolio).

The diagram below illustrates the CML.

Fig 2.4

(Goodall, 2005, Part 2, Cp 2, p8)

The CML is defined by the following expression:

 $E\left(R_{A}\right)=r_{F}+\left[E\left(R_{m}\right)\text{ - }r_{F}\right]\beta_{A}$ 

Where :  $E(R_A) = Expected return from investment A (the optimal portfolio).$ 

 $E(R_m) = Expected return of the market portfolio.$ 

- $r_F$  = The Risk free rate.
- $\beta_A$  =A risk premium for market risk (Beta) for investing in the efficient portfolio.

 $\beta$  is calculated a follows:

$$\beta_{A} = \underline{\sigma_{A}} \, \underline{\phi}_{A,M}$$
$$\sigma_{M}$$

- Where :  $\sigma_A$  = The Standard Deviation (risk) of the efficient portfolio
  - $\varphi_{A,M}$  = The correlation coefficient between the efficient portfolio and the market portfolio.
  - $\sigma_{\rm M}$  = The Standard Deviation (risk) of the market portfolio.

The importance of the CML from an investment, and particularly a cash flow point of view, is that the CAPM can be used to determine the price of risk for the optimal portfolio. The potential effects of market risk, which by definition emanates from the macro-economy which impact on a investment, can be quantified and an adjustment in the investment strategy can therefore be made.

The CAPM reduces risk through diversification until a point is reached where risk can't be diversified any more within the asset class. The risk that remains is termed nondiversifiable- or systematic risk The investor will thus receive no reward taking on additional risk beyond the optimum point allowed by the investment parameters or resources available. This is also termed market risk, since risk will increase only if there is a total shift of demand, thus changing the investment parameters.

#### 2.4.2 The Securities Market Line (SML)

The CAPM may also be used to determine the expected return of a security. If the market as a whole moves in a particular direction, shares or securities will move in the same direction, but at different rates (Goodall, 2005). This attribute of the behaviour of a security or share constitutes a major component of the security's contribution to the overall risk of the portfolio. This relationship is measured by the share/security's beta coefficient ( $\beta$ ). Beta indicates the manner in which a security's return changes systematically with changes in the market's return. Beta above +1 indicates above average risk and -1 indicates below average risk. The investment decision is thus not based on specifics, but rather on whether the particular investment is below or above the risk contained in the market. Figure 2.5

(from Goodall, 2005, Part2, Chp 2, p8)

#### 2.5 SUMMARY

The portfolio environment provides the vehicle for managing internal- or unsystematic risk to investors in the Securities Market. MPT has diversification as its foundation, where by assets that show less than perfect correlation in terms of their risk and return characteristics, are combined in an investment vehicle (portfolio). The desired results would be to obtain better or the same returns in the portfolio compared to the sum of the underlying assets in their individual capacity, but with a reduced total (portfolio) risk. To summarise, the key aspects of Modern Portfolio Theory are as follows:

- 1. Internal risk may be managed through portfolio investing and the concept of diversification.
- 2. Diversification is achieved by constructing an optimum portfolio along the efficiency frontier.
- 3. Negative correlation is important when combining assets and is essential for internal risk reduction.
- 4. Calculating the co-variance between assets allows the quantification of the net risk of the portfolio. It is a measure of the interactive risk between portfolio assets.
- 5. The co-variance may be used to calculate the correlation coefficient between assets, thus enabling the optimisation of asset selection based on their risk/ return cycles.
- 6. Risk / return combinations may be further optimised according to the investor's risk profile through a distinction to be made between the portion of capital invested in the optimum portfolio and the portion invested in a risk-free investment.
- 7. The construction of an efficient portfolio may be based on the profile of the market portfolio, which is representative of the whole market (dominant factor). The introduction of an additional security into the portfolio may be evaluated in terms of the interactive risk between the efficient portfolio and the security.
- 8. Returns may be increased or decreased through the increase and decrease in risk exposure, by including securities in the portfolio based on their risk profile in relation to the market portfolio. This may be achieved through either borrowing funds and investing in the efficient portfolio, or capital may be loaned and invested in a risk-free investment.

In essence the use of MPT enables the investor to manage internal risk through the reduction of the overall combined risk of the portfolio, whilst bettering or at least equalling the sum of returns of the individual securities / assets contained in the portfolio. The use of particularly the Capital Asset Pricing Model and the Capital Market Line enables the determination of a price for market risk (external- or systematic risk). The investor is thus not only in a position to put counter measures for risk in place, but also formulate an investment vehicle to optimise returns. The portfolio investment strategy centres around obtaining returns from income-bearing securities / assets, which are less than perfectly positively correlated to one another with respect to their income patterns.

Contemporary investment theory suggests that favourable correlation to achieve diversification is more likely when assets are combined from different markets as well as from different economic sectors.

In addition, it is also important to note here that the historical information on the returns of securities in the securities market are well documented and are accurate reflections of historical market activity. The use of market indices are therefore considered to be acceptable sources of information for use in the critical calculations required for the implementation of MPT. The indices referred to therefore functions as suitable links between investments and their relevant markets. This may not be necessarily the case with direct property investments, but is a topic which will be further elaborated on in the remainder of this report.

The concepts of Modern Portfolio Theory highlighted in this chapter represent key elements and processes which may be used as criteria to be met in the application of MPT to a direct property investment, in an attempt to facilitate the implementation of MPT as an acceptable risk management strategy. In following chapters, direct property as an investment vehicle will be scrutinised in terms of its ability to fulfil this criteria. The applicability of these principles to a single property investment will logically depend on whether a direct property as an investment vehicle, has the necessary attributes to facilitate this. A suitable format of analysis is needed to act as a vehicle for analysis. The following chapter will investigate direct property as an investment vehicle from a cash flow point of view, with the aim to establish the areas where risk originates in the investment.

# **CHAPTER III**

## DISCOUNTED CASH FLOW ANALYSIS

### 3.1 INTRODUCTION

The value of commercial property, or of a property investment, is rooted in its potential to create an income stream or cash flow (Ball et al, 1998). Property however, is generally accepted to be a long-term investment. The cash flow generated is therefore exposed to the effects of inflation, depreciation and interest over the holding period. The projected cash flow with its contributing elements therefore has to be discounted at an acceptable return rate which includes a compensating factor for the "value" lost over time as well as for interest or returns paid in geared investments – a discount rate. The effects of inflation and interest therefore impact indirectly on the feasibility of a property investment through the discount rate. From this point of view there exists a necessary link between the macro-economic climate with its primary indicators of demand, inflation and interest rate and the cash flow of the investment. The attractiveness of the investment will therefore depend to a degree on the anticipated impact that these indicators will have on the cash flow. Isaac (1998) underlines this by indicating that commercial property investors are increasingly becoming more aware of macro-economic influences on the property market. Property investors are then also looking towards measures to manage the risk associated with the potential impacts of the macro-economy on the investment.

In the previous chapter risk was identified as the uncertainty that arises due to a fluctuation in the expected returns of an investment. Reference was also made to the fluctuations in income as a result of the fluctuation in demand in the market environment, where the investment is exposed to the pressures of inflation and interest rates as well as influences of the macro-economy (and its indicators) on the securities market within which the security or the share representing the underlying business entity is functioning.

However, in the case of the securities market, the behaviour of historical return figures are representative of both income- and equity growth, as a reflection of both the share and the underlying business entity the share represents, meeting their respective price equilibrium points of demand. Any risk contained in the overall performance of both the share and the underlying entity, will thus be reflected in the behaviour of such historical values contained in the relevant index. It is therefore possible to quantify the risk inherent in the historical behaviour of the security investment through the use of index values, since they are representative of income- and capital values where price equilibrium of demand is was met.

The cash flow of a particular investment must therefore be analysed for weaknesses that the external influences of the macro-economy and the market environment may bring to a variance in the projected cash flows and hence, the resultant influence on the value of the investment. Similarly, the deviation in cash flow of a commercial property therefore represents risk to the investor that has to be compensated for during the feasibility analysis and managed during the holding period. For most small, direct property investors, property investments will include active management of the property and therefore also active management of the cash flow. For the direct investor the impact of risk thus realises prominently in the cash flow. It is therefore logical that compensation for risk in the feasibility and in the conceptual stages, as well as possible remedial risk strategies during the holding period must have application in the cash flow.

In the previous chapter, MPT was discussed and the main principles were isolated. In essence, the returns on portfolio investments in the securities market, are similar to the definition of returns for direct property investments, since both include income and capital growth components. For the principles of MPT to be applied to a direct property investment, the DCF must be scrutinised for suitability to the application of these principles. However, reference was made to risk originating from the potential fluctuations in income and capital growth as both the security market investment and the underlying business entity struggles to meet demand in two distinct sub-markets. Similarly, the property market in general may also be segmented into distinct submarkets. There is thus a need to clarify the particular sub-markets being dealt with in the cash flow, in order to pinpoint the origins of risk.

This chapter will discuss risk to a direct property investment, using the discounted cash flow analysis as a framework, but will be preceded with a brief overview of a theoretical structure of the property market using the framework of Four Quadrant model. The cash flow framework will be broken down into its main components and each component will be discussed in terms of the risk it may be subject to.

## 3.2 THE FOUR QUADRANT MODEL OF THE PROPERTY MARKET

Ball et al (1998) states that the property market in general may be regarded as consisting out of four inter-linked sub-markets which are as follows:

- 1. The user market
- 2. The financial asset market
- 3. The development market
- 4. The urban land market

#### a) The User Market

The user market consists out of rentable commercial space (stock) of which there are limited quantities available. Space is theoretically rented at a price where supply of a particular type of rental space meets the demand for a particular market. Rental stock is subject to wear and tear as well as economic obsolescence, all of which influence the attractiveness of space as well as maintenance and holding costs (cost of production)(Ball et al, 1998). It is important to note here that demand for particular space is subject to the demand for business of a particular potential tenant and that the demand for business is exposed to movement in its particular business market as well as movement in the macro-economy. The demand for space is thus a 'derived' demand from the demand for business. The user- or rental market therefore represents a link between the property market and the macro-economy.

#### b) The Financial Asset Market

Ownership of rental (income-bearing) space is regarded as a financial asset, which logically makes property comparable to other investment assets. There is thus incentive to enter the property market for its income-bearing possibilities as well as incentive to exit the market when the opportunity cost of alternative investments are considered. Property as a financial asset may also be owner-occupied and is also subject to depreciation, wear and tear as well as technical- and economic obsolescence. In this market there are also market forces that drive demand. As in the case with the user market, the demand for property as a financial asset is also a derived demand, since the income potential is derived from rental. In the case of owner occupancy, the decision to acquire additional or alternative space is influenced by business economics and market demand and consequently by changes in the macro-economic environment.

### c) The Development Market

The financial incentive for holding property as an income-bearing asset drives the demand for new (or improved) stock/space. Total development activity will thus increase if there is a shortage of supply. Conversely, due to the time frames involved in the re-allocation of resources to develop as well as the time penalty associated with construction, the development market often overshoots supply when there is a general downturn in the demand for space in the property cycle. The development market is therefore logically sensitive to the demand for space and hence also to changes in the macro-economic- and business environments.

In addition, the construction phase of property adds an additional negative dimension to the overall cash flow, since logically this period is associated with, almost exclusively, a cash out-flow. There are thus additional financial handicaps which have to be taken into consideration due to direct influences of particularly inflation and interest rate, in especially highly geared investments.

#### d) Urban Land Market

The user-, capital asset- and development markets are linked to the urban land market. The price of land is a key component in the development process and is thus a fundamental factor of production. Parkin and King (1995) state that the prices of the factors of production used to produce a good or service, influence the supply of the product. Land however is also seen as an asset in its own right and is traded as a commodity is subject to scarcity and abundance. Therefore, if demand dictates that the price of development remains constant and if, hypothetically, the other factors of production (capital and labour) also remain constant, then an increase in the value of land will lead to an increase in the overall cost of production, which will push the price of a product above price equilibrium - thus incurring potential losses. Hyperinflated prices of land in a saturated market environment, therefore represents a strong possibility of low feasibility in overall development – thus discouraging further development in a particular market. In order for developments to be viable, a reduction in the price of land is required or, with land prices stabilising, an increase in potential income. The urban land market is thus dependent on the income potential of the overall development, in which case it is also vulnerable to changes in the macroeconomic environment.

From the above structure of the Four Quadrant model it can be seen that the four submarkets are inter-linked and that there respective demand and supply curves respond to one another. However, the demand in the user market is a derived demand from the demand of business and is thus indirectly related to movement in the market environment of tenants, as well as changes in the macro-economy along with its primary indicators.

The financial asset market also represents a link to the macro-economy, since the income characteristics of property assets offer investment incentives, which is in competition with investments in other asset classes. Movement in the macro-economic climate influences investment decisions in the allocation of capital (Goodall, 2005). The market

of investment property is therefore directly linked to the capital asset investment market and hence also the macro-economy.

For the purposes of this report, the analysis of a typical direct property investment from a cash flow point of view will assume that construction is excluded from the cash flow, but that its inherent value is included as part of the capital value of purchase price. This is necessary in order for the analysis parameters of direct property as an investment vehicle to be compatible to security market investments. Effectively, the cash flow will thus only directly relate to the user market and the financial asset market, in terms of the framework of the Four Quadrant model of the property market. The relevance of the development- and urban land markets are not ignored, but rather seen as reflected in initial purchase price as factors of production. Ultimately the assumption is made that any variance in value due to changes in development costs and land value, will be absorbed in the projected future values of the total package or total asset/ investment value.

### 3.3 DISCOUNTED CASH FLOW MODELS

The DCF model finds application to this report, since it firstly it provides a suitable basis for comparing direct property investments to investments in other assets classes. This is essential, since the main aim here is to establish whether the principles of portfolio investment, as a measure of managing risk in security market and institutional property investments, may be used to manage risk in a single direct property investment.

Secondly, the implied risk resulting from a fluctuation in returns due to changes in the market environment, impacts on a property investment in a number of ways. The cash flow provides both a framework to identify and measure the critical points where this occurs, which in turn would enable the investor to make adjustments or put risk management strategies in place. It provides both an opportunity to incorporate future risks of development into the feasibility analysis, as well as providing a management framework during the holding period. It is this aspect of investment property and of long-

term investments that is affected most by movement in the market as well as movement in the macro-economy through interest rate fluctuation and the effects of inflation.

The DCF analysis provides the investor with two models of investment decision-making criteria; Net Present Value (NPV) and Internal Rate of Return (IRR). This is of particular interest to the investor that needs to compare different types of investments in the other main asset classes (bonds, securities, cash) to property, particularly if it translates into a multi- asset investment portfolio (Ball et al, 1998). According to Jaffe & Sirmans (1995), the basic structure of the Discounted Cash Flow model is as follows:

## A. Expected Cash Flow from Operations

	Projected Gross Income
(minus)	Vacancy and Bad Debts
	Effective Gross Income
<u>(minus)</u>	Operating Expenses
	Net Operating Income
(minus)	Annual Debt Service
	Before Tax Cash Flow
(minus)	Tax from Operations

## After Tax Cash Flow

## **B.** Expected Cash Flow from Sales

**Expected Selling Price** 

(minus) Selling Expenses

Net Sales Proceeds

(minus) Unpaid Mortgage Balance

Before Tax Equity Reversion

(minus) Taxes due on Sale

## **After Tax Equity Reversion**

The After Tax Equity Reversion is then discounted at a suitable rate

From this structure, annual and/or monthly cash flows can be constructed that include both operational cash flows and cash flows from reversion. The ultimate goal of the DCF is to determine the Net Present Value (NPV) and Internal Rate of Return (IRR), which are used as decision-making criteria for investment.

The components of the model will now be scrutinised to determine the origins of risk inherent in the cash. The potential risk present in the key components will be discussed in terms of the established definition of risk as outlined in the previous chapter.

#### 3.4 COMPONENTS OF A TYPICAL CASH FLOW

### 1. Projected Gross Income

Projected Gross income (PGI) in a DCF analysis for property feasibility is an estimate of the growth expected in future income. Income for the purposes of this exercise is defined as obtainable through rental in a commercial property investment. The investment will be exposed to market forces in the relevant market which is being targeted, over the duration of the holding period. PGI therefore has the function of providing a direct link between the investment and its relevant market. The PGI escalation rate must therefore be reflective of changes in the market environment. In essence, the escalation rate should aim to "track" demand in order for rent to remain at equilibrium. The potential fluctuation in demand should thus logically lead to a fluctuation in the escalation rate. The uncertainty caused by such fluctuations implies a risk. The risk being referred to in this sense is related to changes in demand. A robust strategy for managing the risk associated with fluctuation in expected escalation- or growth rates, is the addition of a risk premium to the mean or average rate. This method is more associated with the determination of a discount rate, where the risk premium is representative of risk to the overall investment and the average expected return is the risk free rate (as was discussed in Chapter 2 under the Capital Market Line and will be further discussed under 'discount rates' in this chapter).

However, a distinction must be made here between internal- and external risk associated with variance in expected growth rates of PGI. This stems from the above deduction, where PGI not only functions as a key component in the cash flow, but also acts as an important link between the investment and the market environment. The basic definitions of risk from Chapter 2 and the micro-economic model of supply and demand will be used to illustrate this.

#### Short Run Supply and Demand

Internal risk is defined as risk over which the investor has some form of control (Goodall, 2005). There are many factors that influence the price of a particular good or service in a defined (target) market, over many of which there are no control. However, the decision to increase or decrease price in the short term is under the control of the investor. The micro-economic model of supply and demand dictates that a change in price leads to a movement along a particular curve (Parkin and King, 1995). All else being equal, the increase in price is associated with a decrease in the quantity demanded. Relating this to property, an increase in rent will theoretically lead to a decrease in the amount of space required. Graphically, the model is represented as follows:

Figure 3.1

(Fourie and Van Den Boogaerde, 1994, p7)

However, the above theoretically illustrates the short run scenario and is specific to a particular product (space) and its associated price (rent). From a property perspective, the basic law of demand dictates that the demand for space decreases if the price of space (rent) increases. This however is a static view of supply and demand behaviour.

#### Long Run Supply and Demand

Long run supply and demand dynamics imply that economic activity is dynamic and does not occur in a vacuum, but within the bigger environment of a competitive market. There are thus factors that influence supply and demand from outside the investment over time. Parkin and King (1995) indicate that changes in supply over time (long-run), is associated with changes in demand.

Figure 3.2

(Fourie and Van Den Boogaerde, 1994, p7)

Changes in demand and supply are associated with movements of the curves (as opposed to movements along particular curves) and is a dynamic process which occurs over time (Parkin and King, 1995). Changes in demand are caused *inter alia* by the following:

- 1. Changes in income
- 2. Prices of substitutes and complements
- 3. Expected future prices
- 4. Population
- 5. Preferences

In essence, demand changes when the quantity of a particular good or service can no longer be sold to a willing buyer at any price on a particular demand curve. The curve in totality therefore moves. The investor has no control over the factors above, even though the decision to alter price is controllable. A change (shift) in demand therefore is deemed external risk or market risk.

Like demand, changes in supply are also caused by a number of factors, which include:

- 1. The price of the good
- 2. The prices of the factors of production
- 3. The price of related goods
- 4. Expected future prices
- 5. Number of suppliers
- 6. Technology

Demand and supply do not react to one another immediately. There is thus a period of delay as the one adjusts to the other. The degree to which demand responds is termed the <u>elasticity</u> of demand and is thus indicative of the responsiveness of demand for goods and services with an increase in price due to the above factors. Low elasticity translates into a rapid response of demand and is usually associated with necessities rather than luxury good and services or non-necessities. High elasticity implies that a change in price has a slow response in demand as result.

Typically for property in the user market, such delays translate in to either vacancy or into over supply of rental space, as price moves away from the point of equilibrium. Vacancy and over supply translate into a loss of income at such points during the holding period where these conditions may be encountered. The loss of income over the holding period therefore impacts negatively on the projected escalation rate and represents a down side risk, effectively creating an additional margin over and above the average expected escalation rate which must be taken into account for risk.

## Dealing with Risk

From the discussion in this section, the deduction is therefore made that variance in the expected average escalation of PGI may be attributed to changes in demand as well as changes in supply. In some instances the causes of fluctuation may be under the control of the investor and in other cases not. Active management with respect to price determination, in particularly the short run, will logically play a vital role in reducing PGI risk exposure caused by vacancy and over supply. Reference is made here to management strategies such as longer lease periods, staggered tenant lease renewal dates and careful tenant selection, which in effect aim to ensure more stability in PGI escalation rates through the reduction of vacancy and over supply of space.

In the long-term, more factors potentially influence the investment and therefore there is less control over such factors. Thus, the risk associated with a variance in PGI escalation rates is comprised out of two parts - one part over which control may be exercised and one part over which cannot. Compensating for risk, a premium for risk over which some form of control is possible (in the short term and through active management), may thus be added since it is justifiable. The risk here would be property or asset-specific and would thus deal with potential fluctuations specific to immediate factors relating to rent. Risk associated with shifts of demand and/or supply in the longer term over which there is no control, must be accounted for as part of the risk for the overall investment (the application of which will be discussed further on in this chapter).

It is important to note here that the cash flow generated from the DCF model, assumes that <u>income is derived from a single source</u> or tenant in the user market. Any fluctuation in market demand would thus imply market risk which is applicable to the investment and which is regarded as external in nature.

## 2. <u>Vacancy</u>

Vacancy levels are considered to be a direct consequence of an inefficiency in the price mechanism between supply and demand in especially the short run (as was justified in the previous section). An adjustment to price (rentals) therefore would necessitate an adjustment to the expected and corresponding vacancy level. Active management was however cited as a possible risk management strategy to curb escalating vacancy and possible over-supply of rental space in the short run. It is

therefore possible to counter movements along a particular supply curve through active management.

In the long run however, demand may be less elastic than in the short run. It is therefore reasonable to assume that the potential change in supply will be met with a delay in the response of demand. The response will be largely determined by the elasticity of demand. Vacancy and over supply of space are therefore real possibilities. Vacancy levels will thus escalate if the rental escalation rate being applied, results in rental levels becoming further and further removed from equilibrium rental during the holding period.

Using the demand and supply model, rental escalation- and vacancy rates are considered to be directly related to one another and since it is virtually impossible to predetermine exactly what the degree of sensitivity between the two will be (elasticity of demand) in advance, the assumption is made that relationship is uniform. Thus, if rental reviews are applied on an annual basis, the above assumption may be of use, since the rental review period will be short enough to eliminate the correction of over-or under-adjustment of the escalation rate and therefore neutralise the effect of a potentially disproportionate relationship (non-linear) between vacancy and PGI escalation. There would thus be no need for a risk premium to be included in vacancy escalation rates in the short run. In addition, the potential short-run risk relating to vacancy and over-supply of space may be omitted, since this risk is asset- or property-specific and does not include general market fluctuations (as is the case in the long run)..

In the long run, the potential risk resulting from increases in rental is absorbed in the PGI escalation rate. This stems from the assumption that was justified earlier, where vacancy is not viewed as an independent entity to PGI, but rather seen as a symptom of an inefficiency in the price mechanism between supply and demand.

From a feasibility point of view, vacancy may therefore be accounted for through the use of constant rate. Potential fluctuations in rent is either accounted for in a risk premium for PGI (specific to the property), or compensated for as part of a risk margin applicable to the overall investment due to general market changes in supply and demand.

## 3. **Operating Expenses**

The demand for goods and services is a function of Aggregate Demand and is therefore also sensitive to the pressures of inflation and interest (Parkin and King, 1995). It is therefore logical that operating expenses is escalated at a *growth rate which is related to both those rates*. The DCF analysis provides the opportunity to incorporate this aspect into the individual cash flows of an investment, thus providing an additional link between a direct property investment and the macro economy. Once again there is an opportunity to include an additional margin for risk to compensate for the expected variance in levels of inflation and interest, but investors make use of industry-obtained escalation figures from industry indices (SAPIX, RODE, etc). Figures obtainable from such sources are historical recordings from properties included a particular database. However, the relevance of the information obtained from such recordings or indices are largely irrelevant to small investors due to the "smoothing" effect inherent in compiling the index, which will be discussed further in Chapter 5.

The potential risk that a fluctuation in the escalation rate of operating expenses can impose on the investment is two fold. Firstly, operating expenses has a direct negative influence on the cash flow and constitutes a cash outflow. Any fluctuation in the expected escalation rate therefore can be compensated for by the inclusion of an additional risk margin onto the average expected rate. In this instance, operating expenses are directly related to factors influencing operating expenses specific to the individual property. Secondly, operating expenses applicable to a specific market form part of the factors of production of that market (Parkin and King, 1995) in the supply of rentable space. From the micro economic model of supply and demand (discussed under Projected Gross Income), variance in operating expenses in this scenario, has the potential to cause a change in price and hence a change in supply (movement of the supply curve) in the particular market. The implicit link between operating expenses and income (as justified through the model), dictates that demand will also adjust to a change in supply and vice versa.

From the above it can therefore deduced that the risk associated with a fluctuation in operating expenses also has two components – one relating to the individual property and the other relating to the general market being operated within.

#### Dealing With Risk

The risk imposed on the investment due to the direct influence of operating expenses, may be compensated for by the inclusion of a risk premium in the escalation rate as discussed above. However, the demand and supply model dictates that in the shortrun, operating expenses may be viewed in isolation from income (PGI), since a change of demand can only be expected over time. Demand in the short-terms is therefore regarded as highly elastic to an increase in price. This notion is further strengthened when active management is introduced into the equation, which aims to improve efficiency in operations. From the above argument it is then deduced that the risk being addressed emanating from a fluctuation in operating expenses escalation variance will the portion of risk that is property (asset) specific and virtually minimal due to the positive effect of active management. The risk premium to be added to operating expenses escalation may therefore be conservative (if at all applicable).

In the long-term however, demand is less elastic and responds to changes in price due to changes in supply. The factors of production, of which operating expenses form the majority part during the holding period, will cause a change in supply and hence a corresponding change in demand. The elasticity of demand therefore represents a risk, which is related to operating expenses.

The portion of risk inherent in the fluctuation of operating expenses in the long-term scenario being referred to here, relates to the total investment, because it is relevant to a change in demand. A change in demand logically then implies that operating expenses has an important link to the market environment. The risk imposed by fluctuating operating expense growth rates, is thus not only specific to the particular investment, but is related to the particular market segment of the individual property. Based on this argument, the risk implied by operating expenses thus forms part of market risk, since it would be the same for all other competitors in the same market. The potion of risk specific to the investment has already been compensated for in the short-term.

Therefore, to compensate for the possible risk imposed onto a direct property investment by the fluctuation of operating expenses escalation rates, a single rate for escalation may be used with no margin as a risk premium. The potential margin for risk is transferred to PGI as part of external- or market risk. At best, a conservative margin may be included to compensate for inefficient active management.

#### 4. Debt Service Payment

Perhaps the biggest advantage of direct property ownership, as an investment, is the benefit of financial leverage obtainable from gearing. Debt servicing has a direct negative influence on the cash flow, but the interest portion of an amortized loan is tax deductible (Divaris and Stein, 2002) and has the net effect of boosting annual cash flow significantly through the reduction of taxable income.

The DCF model is able to generate monthly cash flows, which allows rates specific to the cash flow at specific times during the holding period, to be changed as the prime lending rate fluctuates. The risk implied by lending rate fluctuations can therefore be compensated for when the cash flow is managed during the holding period. Ad hoc adjustments strategies are however irrelevant when feasibility studies are formulated and there is an obvious need for a single rate that includes a risk factor which compensates for the average expected fluctuation over the holding period.

Property investments by nature tend to be long-term. There is thus a strong possibility of interest rates fluctuating during the holding period. It is possible to determine a mean value for interest as well as a standard deviation using historical values. The standard deviation in this case could be added to the mean as a risk premium. It is therefore possible to impose an "envelope" of interest rate within which to structure the investment.

The risk being referred to here is specific to the individual characteristics of the financial structure of the investment. The effect of fluctuating interest affects the investment immediately and demand is regarded as being perfectly elastic. The implied risk is however dealt with in a specific manner. There is no proportionate allocation of risk specific to the investment based on the short- or long-term influences. The risk premium necessary to compensate for fluctuation in interest rate is thus representative of the immediate direct influence of interest rate on the cash flow.

However, debt servicing is regarded as a factor of production and hence the risk implied by a fluctuation in interest rates will affect supply. There is thus a possibility that a corresponding change of demand will occur in response to changes in price (from a shift in- or change of demand). This is regarded as an indirect influence of interest rate on the cash flow. In this scenario, as in the case with Projected Gross Income, Vacancy and Operating Expenses, the risk relates to general movements of demand in the market (or market segment) and the compensatory measure therefore forms part of the risk management strategy for the entire cash flow.

#### 5. Expected Selling Price

The expected selling price (ESP) of a property is a subjective issue that is dependent on a multitude of factors that will influence the value thereof at the time of sale. The projection of a future selling price therefore, is a prediction of the market environment in the future. If a comparative sales approach is used for valuation purposes, such an estimate is virtually impossible due to the multitude of factors, which can impact on market activity relating to future developments/ investments of a similar nature.

The value of a property may also be determined using the income approach, where expected selling price may be determined on the basis of the income stream / cash flow, as well as on sector- and area-specific capitalisation rates, usually obtained from indices or in-depth market research (Jaffe and Sirmans, 1995). However, this approach implies a reliance on two variables that originate from outside the investment, which cannot be accounted for directly in the cash flow to eliminate the implied risk internally.

Income (of which rent forms a dominant part) is highly subjected to the forces of supply and demand in the market environment. In the discussion under Projected Gross Income it was established that a margin for risk relating to a potential fluctuation in rental due to factors specific to the property must be included in the escalation rate to compensate for the implied risk. It was also determined that the portion of risk associated with a fluctuation in escalation caused by market variance (changes of demand and supply), is to be transferred to an overall risk margin for the whole investment. It was established that this notion may is also applicable established for Operating Expenses and Vacancy. The portions of risk relating to the individual components of the cash flow are thus separated from the portion of risk relating to the individual elements of the cash flow, is also carried through to the determination of the expected future value of the property. The risk

relating to market fluctuation at the time of sale is therefore part and parcel of price determination, and forms part of the total risk of the overall investment.

Valuing a commercial property on the basis of income and capitalisation rates does provide a level of consistency in the relationship between income and equity value and both variables may be viewed as market specific indicators. However, if both are indicative of the property market environment, their relevance in application must be questioned in terms of the heterogeneous nature of property and the investment parameters associated with the definition of the "small" direct property invesment. Market rentals and capitalisation rates may be obtained from indices, the composition of which are largely based on selective data originating from the institutional sector containing large property investments (Ball et al, 1998). Therefore, if property indices fail to provide a useful link between a direct property investment by the small investor and the property market, an alternative must be found in order to establish a more accurate estimate in expected selling price. The possible links between the property investment and its particular market will be further discussed in Chapter V to determine such a valid link.

## 6. Tax from Operations and Reversion

The tax payable on operations can represent a large portion of the cash outflow of the investment that can reduce the after tax cash flow significantly. The rate at which tax is paid is determined by the legal entity within which business is conducted. As an example, a business (close corporation, company, etc.) pays tax at a rate of 29%. Individuals have a sliding tax rate scale that varies from 28% to 46% depending on annual income (Divaris and Stein, 2002)

From this point of view, tax planning becomes an essential part of financial planning and of cash flow management. From a feasibility point of view though, the level of taxation remains relatively constant and is predetermined when the investment is in the conceptual stage. There is thus variance in income that can result from fluctuating tax rates, but since the investor nor any other competitors in the market, have no control over external risk from this point of view, it is not relevant to the formation of a risk strategy during the feasibility stages of the investment and is viewed as a constant. Since there is a potential risk resulting from a variance in rates of taxation affecting cash flow and since the investor has no control over this element, the risk is deemed systematic in nature and thus may be classified as being part of market risk. In addition, a change in marginal tax rates may be considered to be of application to all participants in a particular market. The implied risk therefore relates to the whole market. Market risk is accounted for elsewhere in the cash flow and pertains to the whole investment.

#### 7. Discount Rate

Broadly speaking, the discount rate used in a cash flow analysis is representative of the required rate of return on the investment (Jaffe & Sirmans, 1995). This is to account for the value of money lost over the holding period (opportunity cost) by entering into the investment. The investor also expects to be compensated for the risk of investing in the particular property investment over and above what a risk free investment may offer. For this reason the expected return rate must include the rate of return that a risk free investment can offer as well as a premium for the risk being referred to. The construction of a discount rate in this manner is similar to the application of the CML as used in the securities market when investments are formulated in portfolios. The difference however is that the risk free rate for property investments is represented by expected return from a real risk free investment such as obtainable from bonds.

Previously in this chapter, Projected Gross Income, Vacancy, Operating Expenses, Debt Servicing and Expected Selling Price were discussed and it was deduced that the potential risk applicable to the individual components may be divided into two portions – one portion relating to a fluctuation in the escalation rates relating specifically to the individual components and another portion relating to the impact of the market environment on those components. Effectively what was stated was that the risk originating externally to the individual components of the cash flow relates to the market and that the risk associated with market fluctuations must be accounted for as part of the overall market risk of the investment. The discount rate therefore must include a premium for risk relating to market fluctuations for the total investment. This includes all portions of risk in the elements of the cash flow that emanate from external influences of the market on the investment.

#### 8. Net Present Value and Internal Rate Of Return

Net Present Value (NPV) and Internal Rate of Return (IRR) are the main decisionmaking criteria for investing. NPV provides a comparison between the present value of the investment compared to the future value thereof. The IRR is defined as the discount rate required for the NPV to be equal to zero (Jaffe and Sirmans, 1995). Thus, the required total return of the investment (discount rate) needs to be equal or greater than the IRR for the investment to lucrative. If this is achieved, and if the discount rate contains an acceptable margin to compensate for the overall risk of the investment, it may be argued that risk is sufficiently compensated for.

#### Dealing with Risk

In the institutional sector, total property return values obtained from indices are used as bench marks against which to measure the performance of individual properties. If one uses the principles of the CAPM (as discussed in Chapter 2), the expected return from the individual investment may be compared to returns obtainable in the (relevant) market. It therefore becomes possible to make adjustments to the critical inputs of the cash flow to match returns obtainable in the market, as portrayed through the index. The investor is therefore in a position to formulate the composition of the individual property to "track" the market – which is not dissimilar to common security market practise in portfolio construction. If this principle is applied to a direct property investment and if the property index is relevant to the individual property, the investor is in a position to determine level of market risk or alternatively determine the margin of additional return required from the investment to compensate for the risk accepted in the investment.

It is also important to note that the IRR represents income growth as well as capital growth (Jaffe and Sirmans, 1995). What the above implies from a cash flow point of view, is that both Projected Gross Income and Expected Selling Price is market related, since they are contained in the expected return rate (discount rate). Any fluctuation in the market (index value) would thus represent market risk applicable to both PGI and ESP. This also extends to the other components (which are considered to be factors of production in producing rental space and investment property) of the cash flow, which are susceptible to risk emanating from a variance in market activity.

These two components in the cash flow thus represent important links between the individual property investment and the market.

#### 3.5 SUMMARY

In this chapter, the Discounted Cash Flow model was used as framework to scrutinise the direct property investment, with the purpose of identifying the types and origins of risk that are inherent in direct property as an investment vehicle. The theoretical analysis was however preceded with a brief discussion of the basic structure of the property market, using the Four Quadrant model as a framework. This exercise was necessary, since the aim of economic activity is for supply to meet demand. The purpose of investment is to obtain returns. The degree of success of the investment thus lies in the ability of the particular investment being able to meet the demand of a particular market. Any variation on the expected returns obtained from the market is deemed an uncertainty, which is classified as risk. Therefore, for direct property to function as a profitable investment vehicle, including putting the necessary measures or strategies in place to manage or compensate for risk, the nature of the market being targeted needed to be established. It was consequently determined that effectively a discounted cash flow encompasses the

user market and the financial asset market. The resultant decision-making criteria emanating from a DCF analysis (IRR and NPV) then also contains both income growthand capital growth components.

An additional purpose of the chapter was to establish a basic format for a direct property investment to which the principles of MPT can be applied. The model was broken down into its critical components and each component was scrutinised in terms of the type of risk it may be exposed to. In addition the aim was also to establish the origins of risk.

It was determined that the norm is to make use of escalation rates and apply them to the components of the cash flow. These rates may be calculated as averages with standard deviations calculated from historical data (if available) or from property specific indices. The escalation rates represent links to the greater investment environment of the macro-economy as well as to the specific market environment of the particular property. This signifies the importance of the link between the macro economic environment and the cash flow of an individual property investment. However, the impact of interest and inflation, two of the main macro-economic indicators, may be regarded as both indirect and direct.

Direct influences relate to the elements of the cash flow, which are directly affected by fluctuations in projected values caused by fluctuation in interest and inflation. Limited interest and inflation rate prediction is possible through the use of probability theory and mean variance analysis, which allows the quantification of the risk that is imposed on the development due to a fluctuation of these rates. The risk referred to here is short-term and relate to factors or circumstances that relate directly to the specific elements of the cash flow. In almost all cases, active management has the potential of eliminating such risk in the short-term.

Indirect influences, and thus indirect links to the macro-economy, relate predominantly to PGI and ESP and are more complex to define and quantify. These influences occur through the market mechanism and are thus considered to be part of external- or market

risk. Market risk in the cash flow is compensated for by the inclusion of a risk premium for the total development in the overall expected return rate or discount rate. The risk premium thus includes compensation for unexpected income growth variance as well as variance on capital value over which the investor has no control. The portions of external risk applicable to the individual components are thus grouped together as a single premium indicative of the overall projected ability of the investment to meet its particular market.

The DCF analysis has Internal Rate of Return (IRR) and Net Present Value (NPV) as its final decision-making criteria. The expected return rate (discount rate) needs to be equal or greater than the IRR in order for the investment to be lucrative. In the institutional sector, property indices are used to obtain benchmarks against which the expected performance of individual properties or property portfolios may be measured to determine market risk. Using the CAPM, the investor is thus able to construct a cash flow for feasibility purposes to emulate market return as portrayed by index values. This makes it possible to for the investor to determine the price of market risk. The market risk determined through the use of a benchmark, is therefore reflective of market risk relating both to PGI and ESP.

One would be forgiven to assume that property indices are suitable as source of reliable market information for use by the small property investor to predict future levels of income and price. However, the relevance of property indices to small direct property investment are questioned for reasons which will be explained in subsequent chapters of this report. Formulating an investment strategy to deal with the long-term risk and allow for the impact of market risk (fluctuation in PGI and ESP) is therefore more complex.

In Chapter 2 the main concepts and requirements of MPTwere isolated and are thus suitable to be used as criteria to be measured up for application. In this chapter, the different components of a generic direct property investment were scrutinised within the framework of the discounted cash flow analysis, to establish the critical points in the investment where risk originates. The following chapter will take the findings of both

chapters, to identify the suitability of direct property as an investment vehicle to the application of the main principles of MPT. The aim is to identify the critical short falls of direct property for application of MPT principles.

# **CHAPTER IV**

## **APPLYING MODERN PORTFOLIO THEORY TO DIRECT PROPERTY**

## 4.1 INTRODUCTION

The hypothesis of this report is that it is possible to make use of the same risk management principles as implemented in the securities market, to formulate a risk management strategy for the small direct property investor to manage market risk. The portfolio structure as an investment vehicle, has been used in the institutional sector of the property market with great success and the applicability of MPT to property as an investment class, is not disputed in this arena. There are however marked differences between the application of MPT in the securities market and the application thereof in the institutional sector of the property market. There would also be anticipated structural differences between the institutional sector of the property market. There would also be anticipated structural differences between the institutional sector of the property market and the small direct single property investment environment.

The definition of the small investor in this report dictates that the investment is limited to a single property. Institutional property investments typically form part of an extensive portfolio containing multiple and often large commercial properties. There is thus a fundamental difference in the scale of the small investor and the institutional sector, when considering the possible application of MPT as an investment strategy to manage risk. However, the current application of MPT in the institutional sector represents the benchmark against which the small individual property investment must be evaluated in order to determine the applicability of the principles of MPT.

In Chapter 2 the main prerequisites and concepts of MPT were isolated and developed as criteria, which may be used to test its applicability to the small direct property investment scenario. This chapter will thus evaluate the small direct property investment vehicle for

suitability to the application of the principles and concepts of MPT, by drawing comparisons with the institutional sector. The shortfalls will be highlighted, with the ultimate aim of developing possible alternative means to achieve successful implementation.

#### **4.2 THE APPLICATION OF MPT TO PROPERTY INVESTMENTS**

In Chapter 2, MPT was examined and critical elements and procedures of portfolio investing were identified, which must be complied with in order to successfully apply MPT. Essentially MPT is used firstly to deal with internal risk through the assimilation of an efficient portfolio and the use of the concept of diversification in the allocation of capital and resources. Secondly, external risk is quantified by determining the price of market risk through the application of the CAPM and the establishment of a market benchmark (CML), thus making it possible to determine a market-related allowance to be included in the overall expected return of the investment. (The CAPM assumes that the efficient portfolio is representative of the market or market portfolio).

The applicability of MPT to small direct property investments will thus also be examined within the framework of firstly managing internal risk and secondly, determining the price of external- or market risk.

#### 4.2.1 Internal risk

#### DCF Approach

In Chapter 3, the DCF model was examined and a distinction was made between internaland external risk, which is present in almost all of the key components of the cash flow. The deduction was also made that internal risk specific to the property and its particular investment characteristics can largely be regarded as being short-term in nature and in most cases active management would play a crucial role in countering the associated risk. It is therefore theoretically possible for internal risk to be virtually eliminated through active management and that the only risk that would remain would be external or market risk as the collective risk of all of the key components of the cash flow, which are associated with fluctuations in the market environment. Practically, this translates to the elimination of risk premiums to compensate for fluctuations in anticipated escalation rates applicable to the individual elements of the cash flow, or at the very least, the inclusion of a substantially smaller premiums. The only risk that would remain would be external- or market risk as the collective risk of the cash flow associated with fluctuations in the market environment.

#### Institutional Sector - Diversification

Modern Portfolio Theory advocates that internal risk may be managed through portfolio investing and the concept of diversification. Diversification is achieved by constructing an optimum portfolio by combining assets that exhibit income characteristics of which the return cycles are less than perfectly correlated to one another. In the institutional sector different commercial properties are combined in a property portfolio. Additional diversification is achieved by not only combing properties from different sectors of the property market (industrial, office and retail), but also from combining properties in different geographical locations. The reasoning for this is as follows:

In Chapter 3, it was eluded to that the demand of property is a derived demand from the demand for business (a topic which will be further explained and elaborated on in the following chapter). The demand for business therefore represents an important link between the individual property and market demand. In Chapter 2 it was stated that assets respond differently to changes or shifts in demand and that this is particularly true when such assets or securities are associated with different economic sectors, since they too have different behavioural patterns in response to changes in the general macro economic climate. Thus, geographic locations have different overall economic growth directions, which are orientated towards the competitive advantages that they hold in particular economic sectors (Ball et al, 1998). Logically therefore, the property market demand in a particular geographic region will lean towards and aim to track the sectoral economic

demand in a particular region where the most growth is experienced. Ball et al (1998) labels this as the location-decision making theory of business. The institutional sector of the property market thus aims to track the demand for business, which the user market (tenants) is focussed on.

However, in the analysis of the DCF model in Chapter 3 an important assumption was made, which is also part of the fundamental differences between the investment environments of the institutional property portfolio sector and the single direct property investment. In the institutional sector, portfolios are constructed by combining a multitude of individual properties, each with their own income- and equity growth figures as part of IRR, which are calculated either from index values or from historical return figures pertaining to the individual investments when held over time. *There is thus diversity in the sources of income*, because income is derived from more than one individual cash flow. This affords the institutional investors the ability to implement the concept of diversification through the construction of an efficient portfolio, thus enabling gains of one investment property to be off-set against the losses of another held in the same portfolio (subject to favourable correlation). This is also similar to the portfolio investment strategy followed in the securities market.

In addition, the backing of large capital reserves affords the opportunity to replace individual properties, which do not perform as desired within the investment targets and overall strategy of the portfolio, with investment properties, which will improve the efficiency of the portfolio towards optimal returns and a further reduction of risk. The DCF analysis of a single property investment makes the assumption that income is derived from only one source – rental income derived from the property itself. The institutional sector therefore has the ability to manage internal risk through the construction of an optimal portfolio, based on the correlation of the return cycles of the individual properties in the portfolio, much in the same manner in which MPT is applied in securities market investment strategies.

For a single direct property investment (in terms of the definition of this report), diversity in sources of income is not obtainable through the combination of different properties. It is however possible to diversify income by catering for a combination of different user markets in the same property development/ investment. In the institutional sector this concept has been employed with particular success in the form of retail shopping centre developments, specifically regional- or super-regional shopping centres, which are mostly owned by the dominant financial institutions. Such facilities are in a position to provide rental space to businesses targeting a wide spectrum of retail markets from different economic sectors, thus building on the benefits from the principle of agglomeration (Ball et al, 1998). This principle also extends to other sectors of the property market, such as office- and industrial park developments. However, the prerequisite of dependable historical information on which market trends can be based, remains. If market performance information is not obtainable from an index, an alternative means of determining market trends is necessary.

From the above it can thus be seen that diversification in the user market may be based not only on the sources of income, but rather on the demand for business and the markets that potential tenants focus on. From this point of view, obtaining diversification in a single direct property investment is therefore theoretically possible, since the origins on income is not based on multiple properties, but rather on the demand for business as dictated by the regional or sub-regional economic sectors applicable to the geographic region pertaining to the particular property market. Logically, active management, with respect to tenant selection, will play an important role to ensure that the desired business markets are purposefully targeted, to ensure that rentals remain at the price equilibrium of demand of the property.

#### **Correlation**

In spite of the fact that agglomeration draws business demand for the user market, the competitive advantage held by regions, cities, precincts, etc, will dictate to a large degree which economic sectors are dominant in a particular location and which ones are

secondary, complementary or supplementary. There is thus a strong possibility that the nature or the profile of the user market is determined by the composition of the particular regional or even sub-regional economy.

From Chapter 2 it was also determined that favourable correlation between assets/securities which are combined in an optimal portfolio, is more likely when they originate from different economic sectors in the macro-economy. From a single property point of view and in light of the above, favourable correlation between sources of income would thus be maximised if the user market that a particular property targets, is not from a single economic sector, but rather from a few economic sectors holding the competitive advantage. Strategies of tenants revolving around business demand of tenants, thus have to focus on the dominant regional / sub-regional economic sectors which would facilitate the highest sustained growth. the demand cycles of the user market are thus targeted, which would theoretically improve sustainable income from stable tenants, since their well-being would be related to only the economic sector holding the competitive advantage in a particular geographic region or sub-region.

There are however limits to the possible application of this concept. Public intervention through planning and urban policy also impacts on the property market and must be noted here, since it serves as an additional framework within which particular types of development in the property market will either be facilitated or discouraged (Ball et al, 1998). In terms of planning legislation and the current land use rights as well possible future rights which could be applied for, only certain land uses will be permitted on the same property or in the same precinct for that matter. There is thus a limitation on the diversity of land uses which will be permitted on a particular property. Implicitly there is also a limitation on the combination of user markets which could be focussed on in a single property investment.

In terms of this section, it does seem theoretically possible to obtain favourable correlation in a direct property investment, by basing the allocation of space on the demand for business and ensuring that demand is met through active management and careful tenant selection. However, it is imperative that reliable and relevant historical property performance data is available, in order to fulfil the role of a strong link between the property investment and market demand. As will be explained in the following chapter, indices ideally should fulfil this role as they do in the institutional sector, but in the case of small single direct property investments, their relevance is questioned. There is thus a need to develop an alternative link between the small, single, direct, commercial property investment and the market.

### 4.2.2 External Risk

### DCF Approach

In Chapter 3, external risk is defined as the uncertainty caused by factors influencing the projected expected overall return of the cash flow, over which the developer has no control and which originate from outside the immediate parameters of the investment. Market risk, from a cash flow point of view, is regarded as external risk, since all the elements in the cash flow are connected to the market environment through inflation, interest rate or price in the form of rental income or expected selling price (as the property's demand is met at price equilibrium). External- or market risk in the DCF framework is normally accounted for by including a risk premium in the expected return-or discount rate.

#### Institutional Sector – Market Benchmark

In the institutional sector, the concept of portfolio investing is possible, because property indices are relevant to this sector. It is therefore possible to use the CAPM in conjunction with a reliable and relevant benchmark against which the performance of not only individual properties may be measured, but also the overall return of the portfolio. Market risk therefore is more accurately accounted for, since it is possible to quantify it.

From Chapter 2 it was determined that reliable and relevant historical data relevant to the return figures of investments must be available in order to perform the calculations necessary to quantify the risk associated with a particular investment. This is true for both the securities market and the institutional sector of the property market, where the sources of data are contained in sector-specific indices (stock market indices and property-specific indices such as Rode, SAPIX, etc). For smaller, single, direct property investments, the relevance of property indices is questioned for reasons which will be further elaborated on in the next chapter of this report.

Property indices, particularly large indices, do however represent prominent links between the property market and the institutional sector of the property market, because they are more reflective of the market environment within which institutional property investments function. Ball et al (1998) under lines this by confirming that larger indices are better suited for use in the construction of property portfolios, because their tracking error is less than what is found in smaller indices. This is mainly attributed to the heterogeneity of the individual property and a larger portion of asset-specific risk being contained in smaller indices. As the sample size of an index increases, so the building or assets-specific risk reduces. Returns obtainable from investing according to the structure of the index would thus approach that which are obtainable in the market. Ball et al (1998) also states that it is impossible for an index to track the market exactly, because in order to do so the index must be representative of all property in the market as well as be reflective of the weighting of different types of property, in order to reduce asset- or building-specific risk completely. Property indices may thus be less accurate with respect to making assumptions of specific indicators relating to the heterogeneous aspects of an individual investment.

What indices do seem to be useful for, is to provide an indication of the general behaviour of the market segment which they are representative of at the macro economic level of the economy. The deduction is made here that it is reasonable to assume that property indices are able to function as a general macro-market indicator of the property market, when broad scoping comparisons are drawn with other macro economic- and
general asset class indicators (equities, bonds and cash), or in other words, useful as a macro market bench-mark indicator.

However, because indices are relevant to institutional sector property investment, institutional investors are able to utilise indices to compile hypothetical market portfolios, thus creating investment / portfolio specific benchmarks, which is utilised in the CAPM to determine the price of market risk. For small direct property investors, index-derived benchmarks are irrelevant. There is thus a need to establish an alternative means of determining a benchmark to be used in conjunction with the CAPM, which will be reflective of a realistic expected market return against which the investment may be compared for performance, as well as to establish the price of market risk.

## 4.3 SUMMARY

The purpose of this chapter was to determine the applicability of MPT to the small, single, direct property investment environment to formulate a risk management strategy to similar effect as is used in the institutional sector of the property market.

There are however marked differences between the two scenarios, the dominant of which being the different scales on which they function. In the institutional sector, direct property investments occur in the portfolio format, where multiple and often large commercial properties are combined in a property portfolio. In terms of the definition of a small direct property investor in this report, the small investor is restricted to a single property. Because of this fundamental difference, there are also distinct structural investment strategy differences, which become important when the objective is to apply MPT as a risk management strategy to single property investments.

The formulation of an efficient portfolio forms the basis of managing internal risk in both the securities market and the institutional sector of the property market. The crux of the strategy is to obtain diversity in the sources of income, which is more likely when they are rooted in different economic sectors. The demand for property is dependent on the demand for business (a topic which will be further discussed in the following chapter). Certain geographic regions (economic regions or sub-regions) hold certain competitive advantages in particular economic sectors. The competitive advantage is reflected in the demand for correlating goods or services – business demand. Therefore the demand for property will lean towards the economic profile of a particular region or sub-region. Based on the above, it is fair to make the deduction that the demand for property must aim to track the particular business demand of a particular region or sub-region. Within a property portfolio it is therefore possible to obtain diversity, because multiple properties afford the possibility of securing income streams from different geographic regions and different economic sectors.

Diversity for a single property investment is still possible when space is provided for more than one user market. There are however legislative limitations placed on the combination of land uses permitted on a single property in the form of Town Planning Schemes and structure plans. As in the case of the institutional sector and multiple property portfolios, it is possible to formulate expected income cycles for different user markets, based on the regional or sub-regional economic profile and the sectoral economic advantages being targeted by business and industry. The diversification in income stream for single property investments is thus obtainable not in multiple properties, but in mixed land use focussed on the regional or sub-regional economic sector holding the competitive advantage. As was also discovered in the analysis of the DCF model, active management will play a crucial role in the reduction of internal risk. In this case, active management must critically extend to careful tenant selection and management to ensure stable and sustainable income.

External risk in the institutional sector and the property portfolio environment is accounted for by quantifying the price of market risk through the formulation of a market benchmark. The CAPM is used to construct such a benchmark in the form of a market portfolio which would be indicative of the expected market return. The portfolio may then be optimised around the benchmark by replacement or the acquisition of new assets (properties) in the portfolio. It is possible for the institutional investor to formulate a market benchmark, because dependable- and above all, relevant historical information is available in the form of property market-specific indices.

In single property investments and as per the analysis of the DCF model, market risk is compensated for by the addition of a risk premium to the expected risk-free return rate. However, the relevance of index values to small scale, single property investments is questioned, inter alia because of the notion that asset specific risk relating to the market, increases as the sample size of the relevant index decreases.

In the case of both internal and external risk management in terms of MPT, there is a dependence on relevant historical performance data, which is used to perform the necessary calculations to quantify risk. This data is usually presented in property market specific indices, which implicitly functions as necessary link between the investment and the market environment. From this chapter it is then observed that the application of MPT to small, single property investments as a risk management strategy is highly dependant on this link. Since the relevance of property indices to small, single property investments is questioned, the need arises to identify an alternative link between the investment and the property market. The following chapter will take a closer look at the current and generally accepted links, with aim of identifying an alternative. If an alternative link is identified, the implication is that expected property returns for small, direct, commercial, property investments can be generated, upon which the necessary calculations to quantify risk may be based.

# **CHAPTER V**

# THE LINK BETWEEN THE INVESTMENT AND THE PROPERTY MARKET

#### 5.1 INTRODUCTION

The initial step towards managing internal investment risk in the institutional sector of the property market is to obtain diversification in the source of income through the construction of an efficient portfolio. Internal risk is normally quantified by making use of historical data on returns to perform the necessary calculations to determine the expected mean, variance and standard deviation on future returns from the securities combined in the portfolio. This is necessary to calculate the interactive risk of the portfolio with the aim to optimise return and reduce internal risk.

External risk in the institutional sector and within the portfolio as investment vehicle is accounted for by the quantification of market risk through the formulation of a market benchmark. The CAPM is used to construct a capital market line (CML) representing a market portfolio, which acts as a benchmark indicative of the expected return obtainable from the market. The portfolio may then be optimised around the benchmark through the replacement or the acquisition of additional properties.

It is thus possible for the institutional investor to manage both internal and external risk by using MPT, because firstly, it is possible to calculate the risk pertaining to the individual properties being considered for inclusion in the efficient portfolio. Secondly, the institutional investor is able to construct a benchmark in the form of a market portfolio, because dependable and above all, relevant historical information is available on in the form of property market-specific indices. From the above and from the previous chapter it is observed that the application of MPT as a risk management strategy is highly dependant the information pertaining to property returns in the market as portrayed by property indices. The property index therefore represents a necessary link between the market and the investment. It is thus critical that the index is both accurate and relevant to the investment and its particular market in order to apply MPT as a risk reduction investment strategy.

In the previous chapter it was eluded to that property indices are not relevant to smaller direct property investments, predominantly because of the difference in scale at which the index is recorded and the level of the small investment where the index should theoretically be applicable. This chapter will firstly aim to justify this statement from a theoretical point of view, before exploring alternative possibilities in deriving an alternative market benchmark applicable to the small investor and the application of MPT.

### 5.2 PROPERTY INDICES

The purpose of an index is to provide a fair representation of the historical behaviour of a particular market. In the securities market, market indices play a critical role particularly in investment decision-making to determine market risk, as had been indicated in the preceding chapter. Implicitly, an index should be a fair reflection of market activity or, the index should be indicative of the price equilibrium, where demand is met by supply in a particular market. The index from a theoretical point therefore, is both an indicator of the returns that may be expected from the market, as well as an indicator of property demand.

However, the relevance to the small, individual, direct, property investor is questioned in this report. The remainder of this section will investigate property indices in order to determine their relevance of this statement. If property indices are irrelevant to the small investor, then the alternative must be able to perform the same function, in order to be useful in the application of MPT. The approach of this chapter will be two-fold in developing such an alternative to index indicators. Firstly, the relevance of the index as a market indicator on expected returns must be justified to be able to use as a test against which the alternative must be measured. Secondly, the alternative market indicator developed, must be able to be reflective of both supply (expected return) and demand in the market.

### 5.2.1 <u>Contentious Issues of Property Indices</u>

The relevance of property indices is a contentious issue. Ball et al (1998) states that the points of contention relate predominantly to the construction of the index and lists the following as the main points of critical importance:

#### a) Defining the Population

Real estate (usually prime office, industrial or retail) is usually owned by large institutions and part of large multi property portfolios, and is the norm for inclusion in an index. The classification of property type in such indices is arguably to the virtual exclusion of all other property, particularly smaller commercial properties. There is thus a strong possibility that smaller portfolios will not find application to a typical property index and even more so in the case of small individual properties.

### b) Sample Selection

Property is heterogeneous in nature and no two properties have the same income- and therefore risk characteristics. *It is therefore very difficult for a particular property index to be a true representation of the market, since much of estimated value revolves around the individual aspects of a particular property or its heterogeneous nature.* According to Ball et al (1998), the property types included in indices will vary from index to index. Even if different indices measured the same property type returns, the total property return would vary according to the composition of the index. For indices to show the same results with respect to overall returns, they must all have the same

weighting of the same property types in the same geographically defined regions, but most importantly, the sampling must take into account the factors that relate specifically to the individual property. Therefore, an index would only be a true reflection of the market (indicative of expected return and of demand) when it is able to consider the individual aspects of property and <u>if it contains all property</u>. The above is impossible to achieve, unless the different indices include the same samples. According to Ball (et al, 1998) the extent of the differences between indices depends on the sample size (number of samples).

Thus it may be argued that an index as a market indicator will provide a more accurate reflection of the general market if it excludes the individual characteristics of property and by implication the figures relating to value and capital growth. The alternative to an index as a market indicator therefore has to be able to perform the same function subject to the same criteria. The subject of the heterogeneity of property and value will be further elaborated on further on in this chapter under the discussion of asset-specific risk.

#### c) <u>Sample Size</u>

In Chapters 2 and 3 of this report it was stated on a number of occasions that the total risk of property includes risk relating to the property type in a market segment (external- or market risk), as well as risk relating to the individual characteristics of the property (internal risk). This distinction is also made in MPT under the CAPM where internal risk is diversified and represented by interactive risk of the portfolio and included in the expected return rate at the risk free rate. Market risk is defined as the difference between the CML (derived from a market portfolio) and the expected return rate of the investment. Ball et al (1998) states that these two components are independent from one another and may be represented by the following expression:

$${\sigma_i}^2 = {\sigma_{mi}}^2 + {\sigma_{si}}^2$$

where  $\sigma_i^2$  = Total risk of the property (variance).

 $\sigma_{mi}^{2}$  = The variance attributable to the market segment or property type.

 $\sigma_{si}^{2}$  = The variance attributable to the individual characteristics of the property (or specific risk).

If MPT for portfolio construction is used to construct a market portfolio or index, then:

$$\begin{split} \sigma_p^{\ 2} &= \Sigma \ \Sigma \ w_i w_j \ \sigma_i \ \sigma_j \ p_{ij} \\ &= \sigma_{mp}^{\ 2} + \Sigma \ w_i^2 \ \sigma_i^{\ 2} \end{split}$$

where

- $\sigma_p^2$  = Total portfolio risk as measured by the variance. w<sub>i</sub>w<sub>i</sub> = The weight of the individual assets i and j
- $\sigma_i \sigma_j$  = The standard deviations on the returns of the individual assets in the portfolio.
- $p_{ij}$  = The correlation coefficient between the returns to assets i and j.

$$\sigma_{mp}^{2}$$
 = The index portfolio's market risk.

 $\Sigma w_i^2 \sigma_i^2$  = The specific risk of the index portfolio.

Assuming that the specific risk for all properties are equal and that all the properties of the index are of equal value (translating into  $\sigma_i = \sigma_s$  and  $w_i = 1/n$  for all i (effectively specific risk and the weighting of the samples contained in the index are ignored)

Then: 
$$\sigma_p^2 = \sigma_{mp}^2 + \underline{\sigma_s}^2$$
  
n

As the sample size increases, the term  $\frac{\sigma_s^2}{n}$  tends to zero and approaches that of the market.

tends to zero and the risk of the index

Practically for indices, this means that the larger the index (the greater the sample size), the lower the portion of asset-specific risk contained in the index. Therefore, it is reasonable to assume that if an index contains all property in a specific market segment, the market portfolio will contain only market risk and that asset-specific risk will be eliminated from the CML under the CAPM (Ball et al, 1998).

For the portfolio investor in the institutional sector this means that the larger the portfolio, the more accurate the pricing of market risk under the application of the CAPM. However, no index can contain returns on all property, which means that a substantial part of the overall risk of the index (market portfolio) is attributable to the risk associated with the specific characteristics of the individual property.

Therefore, if diversification is to be obtained in a single property investment and available space is allocated based on the structure of the index in a similar fashion to the allocation of capital in the acquisition of property in a multi-asset property portfolio (as is practised in the institutional sector), it is fair to assume that a substantial part of the market risk calculated under the application of the CAPM, will be asset-specific (internal risk).

Thus, if internal risk has already been accounted for through active management in the DCF (as was suggested in Chapter 3) and diversification is theoretically obtained based on the allocation of space according to the structure of the market portfolio, the remaining external risk will not be an accurate portrayal of market risk, since the scale at which the principle of diversification is applied is not large enough for asset-specific risk to be eliminated. There would thus be a portion of internal risk not accounted for, since it has not been diversified away. The risk premium applied to the overall expected return of the investment in the DCF in terms of the application of the CAPM would therefore not be accurate enough for the small investor, since a substantial portion of market risk would include asset-specific risk which is unrelated to the individual investment.

#### d) Price Information

An additional problem with the construction of property indices is the lack of price information, which also partially relates to the difficulty in defining the population of an index and very much relates to the heterogeneity of commercial property investments (Ball et al, 1998).

Property is bought and sold infrequently in different lot sizes and there is no central trading market present as a single point where critical information of price may be assimilated in a reliable database, as in the case of the securities market. It must be noted here that the information referred to above relate predominantly to the selling and buying of property. Reference is made to capital value and the relevance and accuracy of the information contained in the index is thus questioned. Information regarding income occurs logically on a more regular basis, since lease contracts are mostly structured for payment to occur in far shorter intervals (monthly, quarterly, annually, etc). The adjustment of the market to fluctuations in supply and demand and hence price equilibrium, occurs naturally on a more regular basis. Thus the relevance of price information is questioned mostly for capital value and less so for income (rental), since there is more opportunity for data entries regarding rent / income than there would be for capital value / selling price. It is therefore believed that capital value contributes more towards asset-specific risk in a market portfolio when included in an index, than income (rental), since value is greatly determined by the individual characteristics of the property.

What the above illustrates is that property indices may prove to be largely non-applicable to small investors, since the information used to construct a particular index or market portfolio may be irrelevant to the particular market segment within which small investors operate. There is thus a need to find an alternative benchmark against which to measure and make forecasts on the performance of small property investments. This is necessary for cash flow construction and for application of MPT in the formulation of a risk efficient investment strategy. It is also recommended that the reliance on the market indicator to derive capital value must also be eliminated, since it implies asset-specific risk which cannot be accounted for.

#### 5.2.1 Validity of Property Indices

Even though the validity of an index to the small investor is questioned in its normal application, it is believed that the index still provides a credible reflection of the behaviour and the general trends of property returns on the macro-level and because asset-specific risk is reduced to relatively small proportions at this level, they may thus still be of use when relationships between the macro-economy and the general property market are sought. Logically, the smaller the scale at which the investment is to function, the more specialised the market will be, but if total returns of an index are considered, the general market behaviour may be brought in comparison with the macro-economy and in particular with its main indicators.

It must be noted that the information of the index for application to the small investment being referred to, refers to figures specific to a particular region and sector, as would be needed to perform the calculations on variance and standard deviation for inclusion in escalation rates in the DCF. It was however suggested in Chapter 3 that asset-specific risk relating to the individual components be managed through the application of risk premiums to such rates and that the external risk considered to be market related (predominantly relating to PGI and ESP), be transferred the overall expected return or discount rate, in the form of a single market risk premium. It is therefore possible to limit the information obtained from the index to only general indicators, which would enable the quantification of market risk.

Essentially property indices function as a necessary link between the property investment and its relevant market. The index therefore should be a reflection of supply meeting demand at equilibrium price, an aspect that was discussed in Chapter 3 as the basis for determining risk from a theoretical point of view. Therefore, if property indices are irrelevant to small investors, the alternative link to be developed must fulfil the same function in representing a reliable link between the market and the investment and be reflective of supply meeting demand. Property demand is thus hereby identified as a determinant of equilibrium price and market behaviour and the first step towards a possible alternative link between the property investment and its market. In order to justify this statement, the links between the property market and the property investment will now serve as a point of departure on a discussion of the relevance of property demand as a possible alternative means of quantifying market behaviour for property returns for the small investor. The discussion will be a theoretical examined form within the framework of a property market model (Four Quadrant model).

# 5.3 LINKS BETWEEN THE PROPERTY MARKET AND THE INVESTMENT

Ball et al (1998) lists a distinctly identifiable property cycle as presenting possible problems to the small investor from an investment strategy formulation-, as well as a risk management point of view. The very nature of the property market seen through the Four Quadrant Model (as was discussed in Chapter 3) of the market implies that the market as a whole will have a delayed reaction to changes in the macro-economic climate.

Briefly, the Four Quadrant Model of the commercial property market comprises the following sub-markets:

- The **Rental Market**, which is driven by the demand for space by business.
- The **Asset Market**, which responds to an increase in rentals and the associated investment opportunity that it represents.
- The **Development Market** responding to an increase in demand for income producing property as an investment.
- The **Land Market** that is driven by a need for vacant space to develop further, as supply is outstripped by demand.

All the components are inter-linked and respond to one another until equilibrium is reached between their respective supply and demand curves.

From the above structure it can be seen that the financial asset market, development market and the urban land market respond in a logical knock-on manner to one another, with the rental market responding to external influences from the macro-economy through the demand for space by business. The capital value for property as a financial asset should also logically be derived from rental or income stream as per the income approach to valuation (Ball et al, 1998).

Deriving capital value from income also finds application in the DCF when feasibility models are constructed. In applying this approach in the determination of value, ESP is directly related to the income characteristics and potential of the property itself. Logically therefore, any potential variation in income with the resultant variation in capital value, is regarded as being specific to the property and is regarded as asset-specific risk.

Thus, by not making use of specific index values on rental and capital value, the contentious point discussed under point 5.2.1 earlier on in this chapter, may avoided. The only impact that the market would have on the investment would thus be related to demand in the form of market risk. Since specific risk is either avoided or accounted for through active management, the relationship between the market and the investment through the cash flow is thus limited to general indicator of the behaviour of the market at the macro-scale – rental escalation rates (as demand is met by supply) and capitalisation rates (used in determining capital value as per the income approach to valuation).

Therefore, in terms of the Four Quadrant model, the primary interface between the property market and the investment is the rental market. The need for space in the useror rental market is however driven by the demand for business. Business activity in turn is driven by the general demand for business. Therefore, the demand for business can therefore be regarded as an evolution of the initial identified link of property demand between the property market and the investment. The demand for property is thus not direct, but a derived demand from the demand for business. A critical question arises from this deduction: What is the means of measurement that may be used to quantify the demand for business? This indicator must be tested for relevance to both income (rental) and overall return, in order to justify the argument that rental is a preferred indicator from an index to be used in the determination of general market behaviour. If income shows a stronger relationship to the business demand indicator than overall return does, it may be used as a valid substitute and indicator of general market behaviour at the macro-scale.

What follows is a discussion on the relationship between the macro-economy and the demand for business.

#### 5.4 MACRO-ECONOMIC THEORY AND THE DEMAND FOR BUSINESS

In order to determine the relationship between the demand for business and the macroeconomy, the theoretical management framework of the macro-economy in South Africa must be clarified, since the logical movement of the macro-economic indicators is largely determined by it. In the development of an alternative benchmark for property returns, the relationships between property index values and the macro-economic indicators will be established.

### 5.4.1 Monetary and Fiscal Policy

Monetarist macro-economic theory centres round an indirect link between macroeconomic growth and investment. This link is known as the transmission mechanism of money supply (Fourie and Van Bogaerde, 1994).

John Maynard Keynes advocated that macro-economic growth is at an optimum when there is a state of full employment in the economy and that one cannot rely on market forces to achieve full employment. He identified Total Expenditure not being at adequate levels, as the crux of the problem. Keynes advocated that active steps are to be taken by government to ensure that this would occur in the form of fiscal policy. However, fiscal policy largely neglects the effects of inflation, which is the primary aim of Monetary Policy. Contemporary schools of thought on macro economic strategy advocate the use of both Fiscal and Monetary Policy as a means of government influencing macro-economic growth. Even though Fiscal Policy plays an important role in economic growth, it is much less dynamic than Monetary Policy in its application and hence doesn't influence the economy in a constant manner to the same degree as Monitory Policy does. Monetary Policy on the other hand focuses on maintaining inflation at acceptable rates in order to facilitate growth, but without leading to macro-economic situations where the levels of debt stunts growth by discouraging investment.

According to Fourie and Van Bogaerde (1994), contemporary Monetary Policy, as is being applied in South Africa, advocates that Total Expenditure is increased when the money supply in the economy allows for sufficient levels of inflation to prevail to stimulate the propensity to invest. Intervention by government in the money supply occurs through the increase or decrease in interest rate (Repo Rate) – the cost of borrowing. If the cost of borrowing is reduced through the lowering of the Interest Rate, marginal investments become more profitable. There is thus theoretically an increase in the demand for firstly capital goods and secondly for expenditure goods. Overall Aggregate Demand (the demand for business) in the economy in other words, is stimulated (increased) if interest rates are lowered, when inflation levels drop to the lower end of the inflation margin. The inflation margin (CPIX) for the South African economy is currently 3% - 6% (South African Reserve Bank, November 2003).

Gross Domestic Product (GDP) is theoretically a measure for Aggregate Demand. This follows from the Circular Flow Model of macroeconomics, where the following components in National Accounts are equivalent and thus inter-changeable (Parkin and King, 1995):

Total Income

- Total Expenditure
- Total Output

The following equation represents the Circular Flow Model:

Y = C + I + G + NX

Where:

Y = Total expenditure
C = Total Consumption
I = Total Income
G = Total Government Expenditure
NX = Net exports

Total Expenditure, Total Consumption, Total Income, Government Expenditure and Net Exports are deemed to be in response to Total Demand. Total Expenditure in the economy occurs in order for Aggregate Supply to meet Aggregate Demand through production (Total Output). Total Output (GDP) is theoretically thus a representation of Aggregate Demand and therefore also the general demand for business.

The above argument theoretically justifies the link between the demand for business and the macro-economy, and since the demand for property is a derived demand from the demand for business, the deduction is made that GDP may therefore be used as a substitute for the demand for property.

# 5.4.2 Economic Indicators

GDP as a measure of demand for business is regarded as a response to a changing economic climate. Monetarist theory dictates that GDP responds to change in inflation (Parkin and King, 1995). Inflation in turn reacts to stimulus from interest rate. There is thus a trilateral relationship between interest, inflation and GDP in the macro-economic environment. The question therefore arises: which indicator shows the closest relationship to the demand for business? It has already been established from the argument above that GDP theoretically must be representative of the demand for business, but the analysis of the individual direct property investment in terms of the DCF framework, had indicated that the cash flow and by implication the feasibility of the investment as a whole, is interest and inflation sensitive as well. Variance in the demand for business and hence the demand for property translates into market risk for the individual property investment. Market risk for property is therefore linked to both inflation and interest Rate, since income levels of business are inflation and interest rate prone.

In the remainder of this report, the most suitable indicator will be determined by establishing the relationships between each of the main economic indicators with overall return and with income return (Net Income Growth) by using data from a typical property index (SAPIX). The relationship between the indicators and overall return will serve as control test, since it is regarded as the norm for application in terms of MPT in the institutional sector where indices are regarded as relevant. The analysis between the indicators and income will be evaluated against the analysis of total return.

### 5.5 SUMMARY

The institutional sector of the property market is able to apply MPT to formulate an investment risk management strategy, because property indices provide relevant and accurate historical data on the performance of property types from which expected future returns may be calculated, as well as risk relating to deviations from the expected mean. The core of the purpose of an index is to provide a fair representation of market activity and price, where demand is met by supply. There is thus a duel function required from the market indicator: a measure of expected return from supplying the market and a measure of demand from the market.

However, the relevance of application of the property index to the small investor for the formulation of an MPT-based risk management strategy is questioned, because of a number of contentious issues regarding the construction of indices. These issues relate to

defining the population of the index, sample selection, sample size and a lack of price information.

Indices are normally constructed from performance figures obtained from investments in the institutional sector, where property assets are usually large in lot size and mostly form part of multi-assts portfolios. The classification of property types therefore is representative of this investment sector of the market to the virtual exclusion of other, mainly smaller, property types.

The heterogeneous nature of property also dictates that no two properties are the same and that the individual characteristics of a property play a large part in the determination of value. It is therefore near impossible for two separate indices to provide similar reflections of a particular market, unless they contain the same property type classification, but most importantly, also include the factors relating to the individual property, which determines value and capital growth. The alternative indicator therefore must be able to perform the same function.

The lack of a central trading facility for property leads to an inconsistency in the determination of price and value. Property is bought and sold infrequently and in varying lot sizes. The individual characteristics of property also greatly determine value, even in the same property type classification. The index figures on total return, which broadly speaking consists out of capital growth and income growth, may therefore be unrelated to one another and not be a true reflective of the market, since asset-specific risk will thus be present in a constructed market portfolio. It is argued that figures relating to income and rent are more reflective of the general property market demand. The adjustment of price to meet market demand occurs systematically on a more frequent basis, since rental review intervals are relatively short compared to the average intervals of sales.

Commonly, the difference between indices depends on sample size. The total risk of a direct property investment comprises market- or external risk as well as asset-specific

(internal) risk. If a market portfolio (index portfolio) is constructed, asset specific risk will thus also be present, but will reduce as sample size increases. If an index therefore includes all property (and property types), all asset specific risk will be eliminated, leaving thus only market risk to remain. Therefore, the larger the portfolio, the less asset-specific risk will be contained in the market portfolio. Similarly, the larger the index from which a market portfolio is constructed, the more true the reflection of risk contained in the market would be. Accuracy of an index with respect to a fair reflection of market risk is therefore highly dependant on scale and size. Thus the larger the investment, the more relevant the index would be in its application of reflecting market behaviour. From this point of view, the necessary calculations and procedures to apply MPT are therefore relevant to large investments or portfolios, but less so for smaller portfolios and small single property investments.

However, if the use of specific index figures is avoided and only general indicators are used, the index is of value at the macro-scale of the property market and may thus be of use when general market behaviour is to be determined. This affords the opportunity of comparison with other macro-economic indicators, which may facilitate the determination of property market activity relative to macro-economic behaviour.

The application of an index as a market indicator of expected return to the small individual property is therefore not acceptable due to the presence of asset-specific risk contained in the index. However, if the income approach to valuation is followed, the dependency on index-derived values for capital growth is eliminated. The effect of assetspecific risk originating from the index will therefore be reduced, since potential variance on income will be more reflective of the market. The capital growth derived from income will thus also be more closely related to the market. The overall market risk (relating to both income and capital growth) will therefore have less asset-specific risk. Thus, if a market portfolio is constructed specific for the individual investment, which is based on income and capital value is derived from rent using the income approach, the expected return from the market will be a hypothetical expected return curve specific for the individual investment, which takes into account the individual characteristics of the property, yet still be reflective of general market behaviour. The risk calculated from potential variance of expected income, will thus be market reflective and will exclude asset-specific risk contained in the market. The income approach to valuation in the determination of capital value also finds suitable application in the DCF, as was suggested in Chapter 3. The necessary calculations to determine market risk for the investment therefore relates to overall performance and are represented by a premium to be added to the discount rate. From the above argument it can therefore be deduced that income (Net Income Growth) from an index may be justifiably used as a single indicator of property market behaviour, since it is also indicative of capital growth and value. The calculated market risk (variance) from rental will thus also include risk pertaining to capital growth and value. It is therefore proposed that the use of rent, as a market indicator from this point of view, may be regarded as an acceptable indicator of the supply side of the market equation and thus perform this function to a similar effect as overall return, except with the elimination of asset-specific risk.

Indices essentially are to function as a necessary link between the investment and the market by providing a historical account of market performance. The information is basically indicative of equilibrium price where supply had been met by demand. The supply side of the intended function of an index for application to the small investor has been clarified. But in order for the alternative indicator (income) to perform the same function as an index, it must also be reflective of market demand. The demand for property may thus be regarded as an initial step towards an alternative property market indicator.

However, the quantification of property demand as a market indicator presents problems, which are related to the structure of the property market. In terms of the Four Quadrant model of the property market, the market consists of four inter-linked sub-markets: useror rental market, financial asset market, development market and urban land market. The sub-markets respond to one another due to demand for the one being created by the other in a knock-on manner, with only the rental market responding to stimulus from out side the property market in the macro-economy. Rental therefore may be considered from a theoretical point of view as a single indicator of property demand and is justified if the income approach to valuation is applied to determine capital value and growth, as had been indicated previously. This is also consistent with the use of rental as an indicator of supply-side market behaviour. The demand for space is driven by the general demand for business in the macro-economy. The demand for property therefore is thus not direct, but a derived demand from the demand for business. The demand for business may therefore be considered as an evolution of the initially identified link of property demand as an alternative link between the property market and the individual investment.

The question arises on how business demand is quantified? Using macro-economic theory, the demand for business is quantified as follows:

The demand for property is a derived demand from the demand for goods and services in business – represented by Aggregate Demand. The measure of Aggregate Demand is GDP, since it is a measure of Aggregate Supply meeting Total Demand in the macroeconomy. GDP may therefore be regarded as a theoretical indicator of the demand for business. However, Monetarist macro-economic theory teaches that inflation is a byproduct of economic activity and escalates during periods of sustained levels of high Aggregate Demand. Monetarist economic theory aims to control the levels of inflation, by regulating levels of Aggregate Demand in the economy, through changes in the domestic money supply, which is achieved by government through the fluctuation of the primary lending rate (Repo Rate) of the South African Reserve Bank. Variance in the demand for business and hence the demand for property constitutes risk. Market risk for property is therefore linked to both inflation and interest rate, since income levels of business and hence strate prone. There is thus merit in the hypothesis that interest rate, inflation and GDP could be used as potential indicators of the demand for business and hence also of the property market.

The question is which of the three indicators (GDP, interest or inflation) has the strongest relationship to the property market, around which to construct an alternative market

indicator to a property index, from which a market portfolio may be constructed for the purposes of applying MPT and therefore to quantify the price of market risk.

# **CHAPTER VI**

# **RESEARCH METHODOLOGY**

#### 6.1 INTRODUCTION

This chapter outlines the methodology followed to meet the objectives of this research report. The main topic of the report reads:

"A macro-economic indicator-based risk management strategy for small commercial property developments".

The preliminary reading that was conducted was wide in scope, but revealed that the subject was general in nature and that the potential data pertaining thereto had application on the macro- and micro-levels of both the macro-economy and the property market. It was initially identified that the nature of the data that would be required would be raw data, which is historical in origin. The data used therefore was not gathered through surveys or through sampling, but is actual recorded data obtained from public institutions and was kept as contemporary as possible.

Leedy (1997) provided guidance on the research methodology that was followed. The general nature of the topic and the origin of the data that is needed in this study, dictates that a quantitative approach to the research needed to be followed, where the conclusions drawn from the analysis of the data aims to qualify the hypothesis.

This chapter outlines the methodology of the research conducted. It will firstly outline the rationale that was followed, providing a basis on which the critical research questions are founded. It will also discuss and qualify the analysis techniques that are employed. The results of the research are outlined in Chapter 7.

#### 6.2 RESEARCH APPROACH

It has been the point of view of this report is that small direct property investors are not able to effectively manage their risk. Although internal risk is debatably managed through active management with reasonable success, external- or market risk is not. The dominant reason for this is that a lack of relevant market information limits the application of Modern Portfolio Theory, an effective risk management strategy employed in the institutional sector of the property market. It is the hypothesis of this report that it is indeed possible to apply the concepts of MPT, but that an alternative market indicator to a property index is needed, which is reflective of the market, but also relevant to the small investor. It is also the contention that such an alternative market indicator may be based on one or more of the macro-economic indicators.

In order to achieve the goal and objectives of this report and thus be in a position to draw conclusions, the research approach was as follows:

- 1. Defining the topic and breaking it down into sub-topics.
- 2. Literature reviews on each of the sub-topics.
- 3. Deriving theoretical conclusions from contemporary literature.
- 4. Identify key questions to be answered in order to prove the hypothesis.
- 5. Selecting methods and procedures to answer the key questions necessary to prove the hypothesis.
- 6. Collection of data for application of the methodology in terms of the theoretical framework.
- 7. Applying theoretical methods of analysis to the data collected.
- 8. Evaluation and verification of deductions in terms of acceptable criteria or margins of error.
- 9. Drawing conclusions from the outcome of the research.

From defining the topic and identifying probable theoretical problems in achieving the objectives of the report, a number of sub-topics were identified, which required

investigation. Literature reviews were conducted on each of the sub-topics in an effort to systematically identify the key questions to be answered.

# 6.3 LITERATURE REVIEWS

The reviews conducted fulfilled two functions apart from providing background knowledge and a theoretical base for arguments in evaluating the hypothesis. They are:

- Identifying critical issues to be evaluated in order to prove the hypothesis, as part of the rational of the research methodology.
- Providing the basis on which the selection of the methods of analysis that are to be applied to the data, are made.

The following section is a summary of the conclusions that were drawn from the literature reviews and forms the basis of the rationale of the research:

### 6.3.1 Investment Theory

In essence the use MPT enables the investor to manage internal risk through the reduction of the overall combined risk of the portfolio, whilst bettering or at least equalling the sum of returns of the individual securities / assets contained in the portfolio. The use of particularly the Capital Asset Pricing Model and the Capital Market Line enables the determination of a price for market risk (external- or systematic risk). The investor is thus not only in a position to put counter measures for risk in place, but also formulate an investment vehicle to optimise returns. The portfolio investment strategy centres around obtaining returns from income-bearing securities / assets, which are less than perfectly positively correlated to one another with respect to their income patterns. Contemporary investment theory suggests that favourable correlation to achieve diversification is more likely when assets are combined from different markets as well as from different economic sectors. In addition, it is also important to note here that the historical information on the returns of securities in the securities market are well documented and are accurate reflections of historical market activity. The use of market indices are therefore considered to be acceptable sources of information for use in the critical calculations required for the implementation of MPT. The indices referred to therefore function as suitable links between investments and their relevant markets. This may not be necessarily the case with direct property investments.

The concepts of Modern Portfolio Theory highlighted in this chapter represent key elements and processes which may be used as criteria to be met in the application of MPT to a direct property investment, in an attempt to facilitate the implementation of MPT as an acceptable risk management strategy.

#### 6.3.2 Discounted Cash Flow Analysis

The Discounted Cash Flow model was used as framework to scrutinise the direct property investment, with the purpose of identifying the types and origins of risk that are inherent in direct property as an investment vehicle. A brief discussion of the basic structure of the property market, within the Four Quadrant model framework, revealed that the degree of success of the investment thus lies in the ability of the particular investment being able to meet the demand of a particular market. Any variation on the expected returns obtained from the market is deemed an uncertainty, which is classified as risk. Therefore, for direct property to function as a profitable investment vehicle, including putting the necessary measures or strategies in place to manage or compensate for risk, the nature of the market targeted needs to be established.

It was also determined that effectively the discounted cash flow predominantly encompasses the user market and the financial asset market. The resultant decisionmaking criteria emanating from a DCF analysis (IRR and NPV) then also contains both income growth- and capital growth components. It was determined that the norm for risk management in the DCF framework is to make use of escalation rates and apply them to the components of the cash flow. These rates may be calculated as averages with standard deviations calculated from historical data (if available) or from property-specific indices. The escalation rates represent links to the greater investment environment of the macro-economy as well as to the specific market environment of the particular property. This signifies the importance of the link between the macro-economic environment and the cash flow of an individual property investment. However, the impact of interest and inflation, two of the main macro-economic indicators, may be regarded as both indirect and direct.

Direct influences relate to the elements of the cash flow, which are directly affected by fluctuations in projected values caused by fluctuation in interest and inflation. The risk referred to here is short-term and relate to factors or circumstances that relate directly to the specific elements of the cash flow. <u>In almost all cases, active management has the potential of eliminating such risk in the short-term</u>.

Indirect influences, and thus indirect links to the market and macro-economy, relate predominantly to Gross Projected Income and Expected Selling Price. These influences occur through the market mechanism and are thus considered to be part of external- or market risk. Market risk in the cash flow is compensated for by the inclusion of a risk premium for the total development in the overall expected return rate or discount rate. The risk premium thus includes compensation for unexpected income growth variance as well as variance on capital value over which the investor has no control. The portions of external risk applicable to the individual components are thus grouped together as a single premium indicative of the overall projected ability of the investment to meet its particular market.

In the institutional sector, property indices are used to obtain benchmarks against which the expected performance of individual properties or property portfolios may be measured to determine market risk. Using the CAPM, the investor is thus able to construct a cash flow for feasibility purposes to emulate market return as portrayed by index values. This makes it possible to for the investor to determine the price of market risk. The market risk determined through the use of a benchmark, is therefore reflective of market risk relating both to PGI and ESP.

One would be forgiven to assume that property indices are suitable as source of relevant market information for use by the small property investor to predict future levels of income and price. However, the relevance of property indices to small direct property investment is questioned.

#### 6.3.3 Link Between The Investment And The Market

The institutional sector of the property market is able to apply MPT to formulate an investment risk management strategy, because property indices provide relevant and accurate historical data on the performance of property types from which expected future returns may be calculated, as well as risk relating to deviations from the expected mean. The core of the purpose of an index is to provide a fair representation of market activity and price, where demand is met by supply. There is thus a duel function required from the market indicator: a measure of expected return from supplying the market and a measure of demand from the market. However, the relevance of application of the property index to the small investor for the formulation of an MPT-based risk management strategy is questioned, because of a number of contentious issues regarding the construction of indices. These issues relate to defining the population of the index, sample selection, sample size and a lack of price information.

It was discovered that the heterogeneous nature of property largely determines value and hence capital growth. Capital growth is a major contributor of asset-specific risk. As a rule therefore, a large portion of the risk contained in property indices is attributable to asset-specific risk and therefore renders indices less relevant to small property investments. The lack of a central trading facility for property leads to an inconsistency in the determination of price and value. The index figures on total return, which broadly speaking consists out of capital growth and income growth, may be unrelated to one another and not be a true reflective of the market, since asset-specific risk will be present in a constructed market portfolio. It is argued that figures relating to income and rent are more reflective of the general property market demand.

It was determined the difference between indices depends on sample size. The total risk of a direct property investment comprises market- or external risk as well as assetspecific (internal) risk. If a market portfolio (index portfolio) is constructed, asset specific risk will thus also be present, but will reduce as sample size increases. If an index therefore includes all property (and property types), all asset specific risk will be eliminated, leaving thus only market risk to remain. Therefore, the larger the portfolio, the less asset-specific risk will be contained in the market portfolio. Similarly, the larger the index from which a market portfolio is constructed, the more true the reflection of risk contained in the market would be. Accuracy of an index with respect to a fair reflection of market risk is therefore highly dependant on scale and size. Thus the larger the investment, the more relevant the index would be in its application of reflecting market behaviour. From this point of view, the necessary calculations and procedures to apply MPT are therefore relevant to large investments or portfolios, but less so for smaller portfolios and small single property investments.

However, if the use of specific index figures is avoided and only general indicators are used, the index is of value at the macro-scale of the property market and may thus be of use when general market behaviour is to be determined. This affords the opportunity of comparison with other macro-economic indicators, which may facilitate the determination of property market activity relative to macro-economic behaviour.

The application of an index as a market indicator of expected return to the small individual property is therefore not acceptable due to the presence of asset-specific risk contained in the index. However, it was discovered that if the income approach to valuation is followed, the dependency on index-derived values for capital growth is eliminated. The effect of asset-specific risk originating from the index will therefore be reduced, since potential variance on income will be more reflective of the market. The capital growth derived from income will thus also be more closely related to the market. The overall market risk (relating to both income and capital growth) will therefore have less asset-specific risk. Thus, if a market portfolio is constructed specific for the individual investment, which is based on income, and capital value is derived from rent using the income approach, the expected return from the market will be a hypothetical expected return curve specific for the individual investment, which takes into account the individual characteristics of the property, yet still be reflective of general market behaviour. The risk calculated from potential variance of expected income, will thus be market reflective and will exclude asset-specific risk. The income approach to valuation in the determination of capital value also finds suitable application in the DCF, as was suggested in Chapter 3.

The necessary calculations to determine market risk for the investment therefore relates to overall performance and are represented by a premium to be added to the discount rate. From the above argument it can therefore be deduced that income from an index may be justifiably used as a single indicator of property market behaviour, since it is also indicative of capital growth and value. The calculated market risk (variance) from rental will thus also include risk pertaining to capital growth and value. It is therefore proposed that the use of income, as a market indicator from this point of view, may be regarded as an acceptable indicator of the supply side of the market equation and thus perform this function to a similar effect as overall return.

Indices essentially are to function as a necessary link between the investment and the market by providing a historical account of market performance. The information is basically indicative of equilibrium price where supply had been met by demand. The supply side of the intended function of an index for application to the small investor has been clarified. But in order for the alternative indicator (rent/income) to perform the same function as an index, it must also be reflective of market demand. The demand for property may thus be regarded as an initial step towards an alternative property market indicator.

However, the quantification of property demand as a market indicator presents problems, which are related to the structure of the property market. In terms of the Four Quadrant model of the property market, only the rental market responds to stimulus from out side the market in the macro-economy. Rental therefore may be considered from a theoretical point of view as a single indicator of property demand and is justified if the income approach to valuation is applied to determine capital value and growth, as had been indicated previously. This is also consistent with the use of rental as an indicator of supply-side market behaviour. The demand for space is driven by the general demand for business in the macro-economy. The demand for property therefore is thus not direct, but a derived demand from the demand for business. The demand for business may therefore be considered as an evolution of the initially identified link of property demand as an alternative link between the property market and the individual investment.

The question arose on how business demand is quantified? Using macro-economic theory the demand for business was quantified as follows:

The demand for property is a derived demand from the demand for goods and services in business – represented by Aggregate Demand. The measure of Aggregate Demand is GDP, since it is a measure of Aggregate Supply meeting Total Demand in the macroeconomy. GDP may therefore be regarded as a theoretical indicator of the demand for business. However, Monetarist macro-economic theory teaches that inflation is a byproduct of economic activity and escalates during periods of sustained levels of high Aggregate Demand. Monetarist economic theory aims to control the levels of inflation, by regulating levels of Aggregate Demand in the economy, through changes in the domestic money supply, which is achieved by government through the fluctuation of the primary lending rate (Repo Rate) of the South African Reserve Bank (Monetary Policy). Variance in the demand for business and hence the demand for property constitutes risk. Market risk for property is therefore linked to both inflation and interest rate, since income levels of business are inflation- and interest rate prone. There is thus merit in the hypothesis that interest rate, inflation and GDP could equally be used as potential indicators of the demand for business and hence also of the property market.

# 6.4 DETERMINING THE RELATIONSHIPS BETWEEN THE MAIN ECONOMIC INDICATORS AND PROPERTY

It has been the hypothesis of this report that the risk management strategy to be used by the small investor must be based on a macro-economic indicator. The question is which of the three indicators (GDP, interest or inflation) has the strongest relationship to the property market, around which to construct an alternative market indicator to a property index, from which a market portfolio may be constructed for the purposes of applying MPT and therefore enabling the quantification of the price of market risk. The relationship between the three indicators and property income must therefore be established at the macro-scale and be compared to their relationships with overall return.

It is proposed in this report that the use of income from the index has application to small direct property investments because it excludes asset-specific risk, the very factor rendering the use of total return from an index irrelevant. However, it is the norm in both the securities market and the institutional sector of the property market to make use of total return figures. The analysis will therefore include the establishment of the relationships between the macro-economic indicators and total return to serve as a control test against which to compare the findings on the relationships between income and the indicators.

In order to determine which of the two indicators will be most suitable, the statistical relationships between the variables described above will be determined. This analysis will be performed using the method of Regression Analysis.

#### 6.4.1 <u>Regression Analysis</u>

Freund and Williams (1975) states that Regression Analysis may be used to determine the statistical relationship between variables. The least Squares Approach to Regression Analysis constructs a linear equation from which a curve – the Regression Curve – may be derived and is denoted by:

$$y = a + bx$$

The data is paired in a set with a determinant and a variable. The assumption is made in this report that net income growth and overall return are the determinants and that GDP, interest and inflation are the variables. The constants "a" and "b" are determined by applying the data set to the following expressions:

$$a = (\underline{\Sigma}\underline{y}) (\underline{\Sigma}\underline{x}^{2}) - (\underline{\Sigma}\underline{x}) (\underline{\Sigma}\underline{x}\underline{y})$$
$$n (\underline{\Sigma}\underline{x}^{2}) - (\underline{\Sigma}\underline{x})^{2}$$
$$b = n (\underline{\Sigma}\underline{x}\underline{y}) - (\underline{\Sigma}\underline{x}) (\underline{\Sigma}\underline{y})$$

$$\frac{1}{n (\Sigma x^2) - (\Sigma x)^2}$$

Obtaining the constants a and b enables the formulation of the Least Squares equation. Using the least squares equation, it is possible to determine a corresponding value for y', using x. It is thus possible to construct a separate hypothetical curve for y' that is based on its statistical relationship to x.

$$y' = ax' + b$$

The Least Squares curve derived from this equation is indicative of the average change in the determinant as a result of the particular variable. Once the Least Squares equation has been established, is possible to derive decision-making criteria from the Regression Curve, which will shed light on the relationship between the determinant and the variable. From the construct a Least Squares curve, it is also possible to determine the strength or "goodness" of the relationship between the two variables by determining the **Regression Coefficient** (r). The Regression Coefficient is denoted by:

$$r = \underline{n (\Sigma x y) - (\Sigma x) (\Sigma y)} \\ \sqrt{n (\Sigma x^2) - (\Sigma x)^2} \sqrt{n (\Sigma y^2) - (\Sigma y)^2}$$

The values of regression Coefficients must lie between -1 and +1. Freund and Williams (1975) state that the closer the regression coefficient is to zero, the less variation of y' (the determinant) is attributable to x' (the variable). Conversely, the closer the regression coefficient is to 1 or -1, the stronger the relationship (correlations) between the determinant and the variable (the sign of the variable indicates the slope of the regression curve – positive or negative and has no bearing on the strength of the relationship).

Using the Regression Coefficient, the underlying portion of the correlation between the two variables, which represents the real relationship between the two variables (Regression Variation, RV) can be determined through the expression:

$$RV = 100r^2$$

The balance of this percentage indicates real underlying relationship between the determinant and the variable attributed to chance or coincidence.

Using Regression Analysis, the statistical relationships between Net Income Growth and GDP / interest / inflation, as well as between overall return and GDP / interest / inflation will be determined as an initial analysis, to identify the strongest link between the property market (demand for property) and the macro-economy. The percentage of the regression relationships due to chance will be determined as a further analysis.

However, using the above-described methodology may produce results that resemble a random relationship between the data of the property market determinant that is used and

the return variables of the macro-economy. The relationship may thus be unique to the samples of data used and not a true representation of the actual relationship that exists between the total populations, which the sets of data represent. There is thus be a need to verify the results obtained using acceptable statistical margins of tolerance and error in prediction. The following method of verification will be employed.

#### 6.4.2 Central Limit Theorem – The Normal Distribution

In determining the Regression Correlation, it is possible that the means that are obtained and used in the calculations, are not fair representations of the total population means, but only relevant to the samples (Freund and Williams, 1975). Theoretically therefore there could be a number of different means, depending on the sample size and composition for one set of data. There is thus an implicit error that occurs when samples are used to determine the mean and standard deviation of a population.

According to Freund and Williams (1975) if a sample size is larger than 30, Chebychev's Central Limit Theorem on distribution may be used to test analysis. If the sample size is less than 30, the alternative means of analysis used is Student–t Distribution. Both modes of analysis provide a measure of evaluating statistical conclusions that are drawn from methods such as Regression Analysis. Regression Analysis assumes the <u>shape</u> of the Normal Distribution curve. This research uses sample sizes of more than 30, since the data collected on the various variables are on a monthly basis over a period of 6 years (1998-2003).

The analysis of the data will thus be conducted using the Central Limit Theorem, where the assumption is made that the data collected has a distribution that follows the Normal Distribution Curve.

The following functions will be used to verify the accuracy of the mean values for the most suitable indicator identified through Regression Analysis (GDP, interest, inflation) as well as the most suited property market return indicator (overall return or rental):

### a. <u>The Maximum Error</u>

The Maximum Error (E) will be used to calculate the maximum difference between the mean obtained from the sample and the "true" mean of the population and is obtained through the expression:

$$\mathbf{E} = \mathbf{z}_{\alpha/2} \cdot \underline{\mathbf{s}}_{\alpha/2}$$

Where:

α = the confidence limit (α-1)
s = the standard deviation of the sample
n = the sample size
z α/2 = Degree of Confidence

### b. <u>Confidence Intervals</u>

The formula for the maximum error does not specify which side of the mean the error is possible. Therefore two margins within which the 'true" population mean will fall is calculated by means of the inequality:

$$x - z_{\alpha/2} \cdot \underline{s} < \mu < x + z_{\alpha/2} \cdot \underline{s}$$
  
 $\sqrt{n} \qquad \sqrt{n}$ 

Where:

x = mean of the sample  $\mu$  = the mean of the population  $\alpha$  = the confidence limit ( $\alpha$ -1) s = the standard deviation of the sample
n = the sample size z  $_{\alpha/2}$  = Degree of Confidence

The Confidence Interval will thus be used as criteria that must be satisfied in order for the calculated values for the mean to be considered valid. The calculated values for the mean will therefore have to fall within the acceptable margin of error within the parameters of the Normal Distribution curve.

### 6.6 DATA COLLECTION

The data utilised was obtained from public and government institutions. The data base used in this research report include the following:

### 1. Interest Rate, Inflation and GDP

The data sources utilised for historical figures on interest rate, inflation and GDP, include numerous statistical releases from Statistics South Africa and the South African Reserve Bank in the form of official government sanctioned internet publications. The data recorded ranged from monthly and quarterly entries to annual figures. The data used were kept as contemporary as possible for the sake of accuracy and relevance

### 2. Total Property Returns and Income/Rental Returns

The information on property returns and net income growth was obtained from a reputable property index: the South African Property Investors Digest (SAPIX/IPD). At the time of assimilating the data, the index included over 2200 investments in its sample size with an estimated value of roughly R98 billion.

# **CHAPTER VII**

# DATA ANALYSIS

### 7.1 INTRODUCTION

This chapter consists out of the analysis of the relevant data as identified in the literature review of this report, in an effort to answer the questions as outlined in the methodology.

The first part of the analysis will aim to determine the relationship between the macroeconomy and the property market at the macro-level. Regression analysis will be used to determine the following relationships:

- Overall property return / interest
- Overall property return / inflation
- Overall property return / GDP
- Rental / interest
- Rental / inflation
- Rental / GDP

The hypothesis is that it is possible to use one of the macro-economic indicators, as a substitute for property demand, to be used in the construction of an alternative benchmark to an index for small investments. The first part of the analysis will be to determine the closest relationship between total return and the each of the indicators.

The second part of the analysis will aim to determine whether rent is a more suitable indicator of property market return behaviour, as opposed to total return. The evaluation will be based on a comparison between the regression relationships of overall return and rental with the macro-economic indicator showing evidence of being the most suitable substitute for property demand.

In the process of Regression Analysis, a number of assumptions are made, including that the mean values of the variables are true. However, this may not necessarily be the case. There is thus a need for verification of the derived figures by subjection to acceptable statistical parameters under Normal Distribution.

### 7.2 ECONOMIC INDICATOR ANALYSIS

The following table illustrates the Regression Correlation relationships between the three main macro-economic indicators, identified as possible substitutes for property demand, and the property market indicators of total return and net income return. The percentage of the Regression Coefficient attributable to chance is also given, along with the Total Regression Variance (Data tables and Regression calculation tables appear in Appendices A and B respectively):

Table	7.	1
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TOTAL	Regression	% Due to	Total Regression	
<b>RETURN</b> /	Coefficient	chance	Variation	
Business	0.40	83.67%	735.06	
Demand (GDP)				
Inflation(CPIX)	-0.15	7.78%	9237.12	
Interest (Repo)	-0.56	69.25%	735.04	

NET INCOME	Regression	% Due to	<b>Total Regression</b>	
GROWTH/	Coefficient	chance	Variation	
Business	-0.51	74.22%	503.22	
Demand (GDP)				
Inflation(CPIX)	-0.29	16.43%	2801.64	
Interest (Repo)	0.37	86.62%	503.22	

From the above summary of the analysis conducted the following observations are made:

- There is a stronger relationship between Net Income Growth and GDP than between Total Return and GDP.
- There is a stronger relationship between Net Income Growth and inflation than between Total Return and inflation.
- The relationship between Total Return and interest is stronger than between Net Income Growth and interest rate.
- The regression analysis shows that when Net Income Growth is used as a determinant as opposed to Total Return, the regression relationships with the respective economic indicators are stronger. The exception to the rule is the relationship between Total Return and interest, which shows stronger correlation than the relationship between Net Income Return and interest.
- In all of the cases, the percentage of the regression relationship attributed to chance seems to increase as the regression coefficient becomes larger. Once again the exception is the relationship between total Return and interest, which has the highest correlation coefficient but not the largest percentage attributable to chance.

### 7.3 CONCLUSIONS FROM THE ANALYSIS

The following conclusions may be drawn from the analysis:

• Net Income Growth shows a closer overall relationship with the macro-economic indicators than Total Return. In terms of the critical questions asked by this

report, Net Income Return is therefore regarded as a more suitable property market indicator than Total Return.

- GDP as an economic indicator shows to be more closely related to the property market than inflation. This conclusion is therefore consistent with the theoretical deduction that was made that the demand for business is closely related and indeed a strong determinant of property demand. GDP as a macro-economic indicator is therefore more suitable as a property market demand indicator.
- The exception to the above deductions, is the relationship between Total Return and interest. The strong correlation coefficient between the two indicators is not surprising, given a number of deductions made in the theoretical analysis of the report. In terms of the DCF, the potential impacts of interest on the investment was identified to be directly related to debt servicing. The overall cost of borrowing would have a significant impact on the cash flow during the holding period as well as at the time of sale. ESP forms a large portion of the cash in-flow of the investment and interest, as outstanding debt, would play a major role in the dilution of the positive impact that ESP may have on the cash in-flow of the investment and therefore on the capital value and growth of the investment as well. The underlying link between interest and capital growth is thus highlighted by the analysis.

From the analysis it was determined that Net Income Growth as a property market return indicator is preferred over Total Return in terms of this report. In addition, GDP has been established as a better property market demand indicator over the other macro-economic indicator. However, the data recording used in the analysis are but samples of the bigger (and possibly infinite) populations. The mean values of the data samples may not be true reflections of the actual mean values contained in the total populations. In order therefore to verify the validity of the calculations performed in the analysis, the distribution of the samples must be representative of the Normal Distribution as may be expected from the total populations.

### 7.4 VERIFICATION THROUGH NORMAL DISTRIBUTION ANALYSIS

To be able to construct an alternative benchmark to an index for expected property returns that would enable the use of the CAPM, it is critical that the credibility of the calculations performed in the analysis, is established.

It has been hypothesised that GDP can be used as an indicator of both macro-economic movement and of the demand for property. It was also determined that Net Income Growth is a better reflection of property market returns as opposed to Total Return.

However, the accuracy of the samples used in the calculations of the analysis needs to be verified, since in essence the data used in the calculations are but samples of infinite populations. The mean rates of the variables must fall within acceptable statistical margins that relate to the actual returns. If so, the conclusions drawn in proof of the hypothesis may be accepted.

Thus, in order to achieve the above verification, the following questions are answered through the application of the related modes of analysis:

# 7.4.1 Is the mean rate of Net Income Growth derived from the sample data a true reflection of the "true" population mean, derived from the Normal Distribution curve?

Estimating the confidence intervals wherein the true population mean will fall, through the use of the Central Limit Theorem, will determine what the criteria is, that the predicted value for Net Income Growth rate must meet. Using the Central limit Theorem

$$\begin{array}{ccc} x & -z \\ \alpha/2 & \cdot \\ & & \sqrt{n} \end{array} & < \mu < x + z \\ \alpha/2 & \cdot \\ & & \sqrt{n} \end{array}$$

It is established that within 95 % certainty that the estimated mean Net Income Growth rate will fall within the confidence intervals of 5.04% and 6.26%. Applying the same test at the 99% degree of confidence, there is a 99 % certainty that the population mean will fall within the confidence intervals of 4.87% and 6.45%. At the 99% degree of confidence, the Maximum Error is estimated to be 0.803%.

Thus, the mean Net Income Growth rate of 5.65% obtained from the sample, falls within the 99% degree of certainty as it does at the 95% degree of certainty in prediction.

The estimated mean Net Income Growth rate of 5.65% may therefore be considered as true reflection of the total population and may thus be used as an acceptable basis for additional analysis.

# 7.4.2 Is the mean rate of Total Property Return derived from the sample data (obtained from the index SAPIX) a true reflection of the "true" population mean, derived from the Normal Distribution curve?

Estimating the confidence intervals wherein the true population mean Total Property Return rate will fall, through the use of the Central Limit Theorem, will determine what the criteria is, that this predicted value must meet. Using the Central limit Theorem

$$\begin{array}{ccc} x & -z_{\alpha/2} & \underline{s} & <\mu < x_{\alpha/2} & \underline{s} \\ & \sqrt{n} & & \sqrt{n} \end{array}$$

It is established that within 95 % certainty that the population mean Total Property Return rate will fall within the confidence intervals of 10.09% and 11.61%. Applying the same test at the 99% degree of confidence, there is a 99 % certainty that the population mean will fall within the confidence interval between 9.85% and 11.85%. At the 99% degree of confidence, the Maximum Error is thus estimated to be 1%.

Thus, the mean Total Property Return rate of 10.85% obtained from the sample falls within the 99% degree of certainty as it does at the 95% degree of certainty in prediction.

The estimated mean Total Property Return rate of 10.85% may therefore be considered as true reflection of the total population and may thus be used as an acceptable basis for additional analysis.

# 7.4.3 Is the mean rate for the derived GDP value, a reflection of the "true" population mean, under the Normal Distribution curve?

Estimating the confidence intervals wherein the true population mean GDP rate will fall, through the use of the Central Limit Theorem, will determine what the criteria is, that this predicted value must meet. Using the Central limit Theorem

$$\begin{array}{cccc} x & -z_{\alpha/2} \ . \ \underline{s} & \langle \mu & \langle x + z_{\alpha/2} \ . \ \underline{s} & \sqrt{n} \end{array}$$

It is established that within 95 % certainty that the population mean GDP rate will fall within the confidence intervals of 2.37% and 2.99%. Applying the same test at the 99% degree of confidence, there is a 99 % certainty that the population mean will fall within the confidence interval between -1.084% and 3.08%. At the 99% degree of confidence, the Maximum Error is thus estimated to be 0.404%.

Thus, the mean GDP rate of 2.68% obtained from the sample falls within the 99% degree of certainty as it does at the 95% degree of certainty in prediction.

The estimated mean GDP rate of 2.68% may therefore be considered as true reflection of the total population and may thus be used as an acceptable basis for additional analysis.

### **Conclusion**

From the distribution analyses of Net Income Return, Total Property Returns and GDP growth, it was established that the estimated mean rates, fall within the derived confidence limits of prediction for Normal Distribution.

## **CHAPTER VIII**

### **CONCLUSION**

The uncertainty in the expected outcome of an event is termed risk. Investment risk therefore is defined as the quantification of the risk brought on by a variance in expected return. Modern Portfolio Theory classifies risk as either internal or external in origin. Internal risk is classified as risk originating from within the investment over which the investor has some form of control. External risk on the other had is classified as risk emanating from outside the investment, mainly from the market environment within which the investment has to function and is thus also termed market risk. For the small direct commercial property investor, the influence of the market, which forms part of the macro-economy, is risk that realises mostly in the cash flow of the investment. The cash flow also forms an integral part of the active management of the investment during the holding period and is therefore is critical in the management of internal risk. Risk management strategies must therefore have application in the cash flow in order to be effective to the small investor.

A Discounted Cash Flow analysis may also be instrumental in investment decisionmaking during the inception and feasibility stages of the investment and since it is prudent to formulate an investment strategy that is oriented towards the management of risk during this stage of the investment, the strategy must have application to the cash flow. Even though small investors are able to manage internal risk with relative effectiveness through the active management of the property and the cash flow, their ability to effectively manage market- or external risk is seriously compromised due a limitation of resources.

The institutional sector of the property market on the other hand, is able to effectively manage both internal and external risk with great success, mostly because of the ability to

spread risk over multiple properties. Multiple property investment allows the construction of an investment portfolio and the application of Modern Portfolio Theory in the formulation of a risk management strategy that is made part of the feasibility analysis. The institutional sector is able to make predictions on market return and therefore also on expected variances in return, because of the availability of property market performance information that is relevant to this sector and this level of investment. Property indices function at a macro-level and are thus relevant to investments that also function and impact on the market at this level. It is therefore possible to draw comparisons between the behaviour of the property market and the greater market environment of the macroeconomy and institutional investors are thus able to position themselves in their markets in relation to the macro-economy. The institutional investor is thus in a position to predetermine market behaviour based on macro-economic movement through the establishment of relationships between market indicators and macro-economic indicators.

The hypothesis of this report is that, the same principles applied with success in the institutional sector of the property market, may be applicable to the small property investor to manage especially market risk and that it is possible to develop a macro-economic indicator-based market indicator, which would allow small investors to position themselves in the market more accurately and thus be able to generate expected market returns for the quantification of external risk.

The investigation needed to determine the prerequisites for the application of MPT as well as the nature of the risk inherent in a typical small single property investment. Theoretical analysis was conducted on the main contributing theories of MPT and it was established that the application thereof critically relies on market performance information, which is relevant to the investment as well as the level of investment. The information is conveyed in the form of market indices, which therefore plays the role of a critical link between the investment and the market.

In the subsequent investigation of indices, it was discovered that the overriding reason for the non-applicability to small investments, is that the scale at which indices are constructed and at which they logically function, is removed from the scale of the small investor. It was also determined that a large part of the risk contained in indices are attributable to the individual characteristics of the properties contained in the index and that such characteristics play an important role in the determination of value. The risk referred to, is known as asset-specific risk and seems to reduce as the size of the index increases. It is therefore a question of scale where the use of general indicators of the index at the macro-scale would be market reflective and specific indicators would not.

A perfect market portfolio constructed from an index containing all property therefore would have no asset-specific risk and contain only market risk. Therefore, logically, the return figures of indices would therefore be more market reflective if capital value is eliminated there from. It was also further discovered that the elimination on the dependency on capital value from an index is possible, if the income approach to valuation is followed in the DCF of the individual property. Therefore, the index would be both market reflective and be applicable to small investments, because the discriminating factor of capital value no longer applies. Thus a theoretical conclusion was made that the general market indicators form an index relating to income, is a preferred indicator of the general property market and may have application to small investments.

In investigating an additional requirement that the market indicator must be reflective of demand, it was determined through the theoretical analysis of the structure of the property market (in terms of the Four Quadrant model) as well as the macro-economy, that income is the primary link and interface between the investment and the macro-economy and that business demand is a theoretical substitute and hence a possible suitable indicator of the demand for property. However, because of the functioning of monetary policy and the direct impacts of inflation and interest on the cash flow of the investment, no outright theoretical conclusion could be drawn on which economic indicator would be the most suitable indicator of property demand. A Statistical analysis was needed to confirm or reject the sub-hypothesis that GDP is the most suitable indicator of property demand. In addition, the validity of income as a general property market indicator needed to be verified. If both were to be confirmed, there would be an

acceptable theoretical basis on which to base the development of a general property market indicator for use by the small investor in formulating an investment strategy, which may encompass the principles of MPT. A general market indicator would not be indicative of specific return which may be expected, but be indicative of the general behaviour and responsiveness of the market in relation to the change in the general macro-economic climate.

### 8.1 STATISTICAL ANALYSIS

A statistical analysis was conducted on the three main macro-economic indicators in relation to Total Return from the market as per an index, as opposed to their relationships with income (Net Income Growth).

The statistical analysis revealed that income has a closer relationship to business demand (GDP) than the other indicators and that the relationships showed a higher correlation when compared to the corresponding relationships between Total Return and the respective indicators. The only exception was the relationship between Total Return and interest, but it is suspected that interest or the cost of borrowing plays a dominant role in the determination of value and capital growth in highly geared investments. The degree in which interest affects the cash flow of an investment is very much dependant on the individual characteristics of property and quite possibly contribute to asset-specific risk.

The findings of the statistical analysis are therefore consistent with the deductions made in the theoretical analysis conducted in this report and has proven that there is merit in the hypothesis that a single macro-economic indicator may be used in conjunction with selective information on property market returns, to generate an alternative market indicator that is relevant to the small investor, for the implementation of the principles of MPT, as part of a risk management strategy for investment.

#### **8.2 RECOMMENDATION FOR FURTHER RESEARCH**

In this research report the analysis of the property market and the economy was conducted at the macro-level to prove the hypothesis. However, the application of a single economic indicator to the small direct property investment environment in the formulation of an investment-specific risk management strategy implies that expected returns from a specific, niche market be predicted. In this report the analysis of the property market was brought in relation to the economy at the same scale – the macro-level. If application is sought to more specific property markets at lower levels, there must be information available on the economic environment at the same corresponding level.

However, regional or sub-regional economic information is not readily available without extensive market research and analysis. In addition, the economic sector performance must be correlated with property type to be able to align property sector analysis with the relevant economic sector business demand. The diversity in land use of property is increasingly causing a blur in the lines distinguishing property types from one another. Therefore, in order to apply the principles and hypothesis of this report at levels closer to the individual property, an increase in the reliance on market indicators, which relate to the individual characteristics of property, may be experienced. Therefore future research on this topic must include an investigation into possible means of property market indicators to include factors relating to the individual property, to enable the investor to account for asset-specific risk.

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# APPENDICES

### **APPENDIX - A - Data tables**

MEAN	VARI	ANCE CALC	ULATIONS OF DAT	4			
REPO	RATE	n = 1998-2003					
Date	Rate	ASCENDING	FREQUENCY	X-mean^2	Rate Class	Frequency	
1998/01	16.00	8.50		7.32	8.50	3	
1998/02	16.00	8.50		7.32	9.50	4	
1998/03	15.00	8.50	3	2.91	10.00	1	
1998/04	14.93	9.50		2.68	10.50	2	
1998/05	18.00	9.50		22.15	11.00	4	
1998/06	20.21	9.50		47.83	11.50	3	
1998/07	21.35	9.50	4	64.90	11.75	9	
1998/08	21.85	10.00	1	73.21	12.00	12	
1998/09	21.86	10.50		73.38	12.19	1	
1998/10	20.72	10.50	2	55.15	12.40	1	
1998/11	19.73	11.00		41.43	12.50	3	
1998/12	19.32	11.00		36.32	13.44	1	
1999/01	18.83	11.00		30.65	13.50	9	
1999/02	17.36	11.00	4	16.53	13.65	1	
1999/03	16.51	11.50		10.34	14.92	1	
1999/04	15.67	11.50		5.65	14.93	1	
1999/05	15.46	11.50	3	4.69	15.00	1	
1999/06	14.92	11.75		2.64	15.46	1	
1999/07	13.65	11.75		0.13	15.67	1	
1999/08	13.44	11.75		0.02	16.00	2	
1999/09	12.40	11.75		0.80	16.51	1	
1999/10	12.19	11.75		1.22	17.36	1	
1999/11	12.00	11.75		1.67	18.00	1	
1999/12	12.00	11.75		1.67	18.83	1	
2000/01	11.75	11.75		2.38	19.32	1	
2000/02	11.75	11.75	g	2.38	19.73	1	
2000/03	11.75	12.00		2.38	20.21	1	
2000/04	11.75	12.00		2.38	20.72	1	
2000/05	11.75	12.00		2.38	21.35	1	
2000/06	11.75	12.00		2.38	21.85	1	
2000/07	11.75	12.00		2.38	21.86	1	
2000/08	11.75	12.00		2.38			
2000/09	11.75	12.00		2.38			
2000/10	12.00	12.00		1.67	<u> </u>		
2000/11	12.00	12.00		1.67	ļ		
2000/12	12.00	12.00		1.67	<u> </u>		
2001/01	12.00	12.00		1.67			
2001/02	12.00	12.00	12	1.67	n =	72	
2001/03	12.00	12.19	1	1.67	Mean =	13.29	

2001/04	12.00	12.40	1	1.67			
2001/05	12.00	12.50		1.67	Variance=	10.20	
2001/06	11.00	12.50		5.26			
2001/07	11.00	12.50	3	5.26	Standard		
2001/08	11.00	13.44	1	5.26	Deviation=	3.19	
2001/09	9.50	13.50		14.39			
2001/10	9.50	13.50		14.39			
2001/11	9.50	13.50		14.39			
2001/12	9.50	13.50		14.39			
2002/01	10.50	13.50		7.81			
2002/02	10.50	13.50		7.81			
2002/03	11.50	13.50		3.22			
2002/04	11.50	13.50		3.22			
2002/05	11.50	13.50	9	3.22			
2002/06	12.50	13.65	1	0.63			
2002/07	12.50	14.92	1	0.63			
2002/08	12.50	14.93	1	0.63			
2002/09	13.50	15.00	1	0.04			
2002/10	13.50	15.46	1	0.04			
2002/11	13.50	15.67	1	0.04			
2002/12	13.50	16.00		0.04			
2003/01	13.50	16.00	2	0.04			
2003/02	13.50	16.51	1	0.04			
2003/03	13.50	17.36	1	0.04			
2003/04	13.50	18.00	1	0.04			
2003/05	13.50	18.83	1	0.04			
2003/06	12.00	19.32	1	1.67			
2003/07	12.00	19.73	1	1.67			
2003/08	11.00	20.21	1	5.26			
2003/09	10.00	20.72	1	10.85			
2003/10	8.50	21.35	1	22.98			
2003/11	8.50	21.85	1	22.98			
2003/12	8.50	21.86	1	22.98			
		957.15		734.75			

MEAN VA	AN VARIANCE CALCULATIONS OF DATA						
CPIX Infla	tion Rate	n = 1998-2003					
Date	Rate	ASCENDING	FREQUENCY	X-mean^2	Rate Class	Frequency	
						. ,	
1998/01	6.60	4.00	1	0.66	4.00	1	
1998/02	6.50	4.10	1	0.83	4.10	1	
1998/03	6.70	4.40	1	0.50	4.40	1	
1998/04	6.70	5.40	1	0.50	5.40	1	
1998/05	7.00	5.80	1	0.17	5.80	1	
1998/06	7.20	5.90	1	0.04	5.90	1	
1998/07	7.00	6.00	1	0.17	6.00	1	
1998/08	7.20	6.30		0.04	6.30	2	
1998/09	7.60	6.30	2	0.04	6.40	3	
1998/10	7.50	6.40		0.01	6.50	4	
1998/11	7.40	6.40		0.00	6.60	3	
1998/12	7.30	6.40	3	0.01	6.70	5	
1999/01	7.40	6.50		0.00	6.80	1	
1999/02	7.30	6.50		0.01	7.00	6	
1999/03	7.20	6.50		0.04	7.10	2	
1999/04	7.00	6.50	4	0.17	7.20	4	
1999/05	7.00	6.60		0.17	7.30	2	
1999/06	7.10	6.60		0.10	7.40	4	
1999/07	7.00	6.60	3	0.17	7.50	2	
1999/08	6.70	6.70		0.50	7.60	1	
1999/09	6.60	6.70		0.66	7.70	7	
1999/10	6.50	6.70		0.83	7.90	3	
1999/11	6.70	6.70		0.50	8.00	1	
1999/12	6.80	6.70	5	0.37	8.10	2	
2000/01	7.00	6.80	1	0.17	8.30	1	
2000/02	7.20	7.00		0.04	8.50	1	
2000/03	7.40	7.00		0.00	8.60	1	
2000/04	7.70	7.00		0.08	9.10	1	
2000/05	7.90	7.00		0.24	9.20	1	
2000/06	7.90	7.00		0.24	9.30	2	
2000/07	7.90	7.00	6	0.24	9.90	1	
2000/08	8.10	7.10		0.48	10.00	1	
2000/09	8.10	7.10	2	0.48	10.80	2	
2000/10	8.00	7.20		0.35	11.30	2	
2000/11	7.70	7.20		0.08			
2000/12	7.70	7.20		0.08			
2001/01	7.70	7.20	4	0.08			
2001/02	7.70	7.30		0.08	n =	72	
2001/03	7.50	7.30	2	0.01	Mean =	7.41	
2001/04	6.70	7.40		0.50			

2001/06 6.40 7.40 1.02	
2001/07 6.40 7.40 4 1.02 <b>Standard</b>	
2001/09 5.80 7.50 2 2.59	
2001/10 5.90 7.60 1 2.28	
2001/11 6.30 7.70 1.23	
2001/12 6.50 7.70 0.83	
2002/01 7.10 7.70 0.10	
2002/02 7.40 7.70 0.00	
2002/03 7.70 7.70 0.08	
2002/04 8.30 7.70 0.79	
2002/05 8.60 7.70 7 1.42	
2002/06 9.20 7.90 3.21	
2002/07 9.10 7.90 2.86	
2002/08 9.90 7.90 3 6.20	
2002/09 10.80 8.00 1 11.49	
2002/10 11.30 8.10 15.13	
2002/11 11.30 8.10 2 15.13	
2002/12 10.80 8.30 1 11.49	
2003/01 10.00 8.50 1 6.71	
2003/02 9.30 8.60 1 3.57	
2003/03 9.30 9.10 1 3.57	
2003/04 8.50 9.20 1 1.19	
2003/05 7.70 9.30 0.08	
2003/06 6.40 9.30 2 1.02	
2003/07 6.60 9.90 1 0.66	
2003/08 6.30 10.00 1 1.23	
2003/09 5.40 10.80 4.04	
2003/10 4.40 10.80 2 9.06	
2003/11 4.10 11.30 10.95	
2003/12 4.00 11.30 2 11.63	
533.50 533.50 143.06	

MEAN V	ARIA/	NCE CALCULA					
GDP		n = 1998-2003					
Date	Rate	ASCENDING	FREQUENCY	X-mean^2	Rate Class	Frequency	
1998/01	1.70	0.20		0.97	0.20	3	
1998/02	1.70	0.20		0.97	0.30	3	
1998/03	1.00	0.20	3	2.84	0.70	3	
1998/04	1.00	0.30		2.84	1.00	6	
1998/05	1.00	0.30		2.84	1.50	3	
1998/06	0.70	0.30	3	3.94	1.70	2	
1998/07	0.70	0.70		3.94	1.90	3	
1998/08	0.70	0.70		3.94	2.00	3	
1998/09	0.30	0.70	3	5.69	2.70	6	
1998/10	0.30	1.00		5.69	2.80	3	
1998/11	0.30	1.00		5.69	2.90	3	
1998/12	0.20	1.00		6.17	3.40	3	
1999/01	0.20	1.00		6.17	3.60	6	
1999/02	0.20	1.00		6.17	3.70	12	
1999/03	1.00	1.00	6	2.84	3.80	3	
1999/04	1.00	1.50		2.84	3.90	4	
1999/05	1.00	1.50		2.84	4.00	3	
1999/06	1.90	1.50	3	0.62	5.20	3	
1999/07	1.90	1.70		0.62			
1999/08	1.90	1.70	2	0.62			
1999/09	2.80	1.90		0.01			
1999/10	2.80	1.90		0.01	n =	72	
1999/11	2.80	1.90	3	0.01	Mean =	2.68	
1999/12	3.70	2.00		1.03	5		
2000/01	3.70	2.00		1.03	Variance=	1.76	
2000/02	3.70	2.00	3	1.03	3		
2000/03	3.60	2.70		0.84	Standard		
2000/04	3.60	2.70		0.84	Deviation=	1.33	
2000/05	3.60	2.70		0.84			
2000/06	3.40	2.70		0.51			
2000/07	3.40	2.70		0.51			
2000/08	3.40	2.70	6	0.51			
2000/09	5.20	2.80		6.33	5		
2000/10	5.20	2.80		6.33	8		
2000/11	5.20	2.80	3	6.33	8		
2000/12	4.00	2.90		1.73	8		
2001/01	4.00	2.90		1.73	3		
2001/02	4.00	2.90	3	1.73	8		

2001/03	3.80	3.40		1.24		
2001/04	3.80	3.40		1.24		
2001/05	3.80	3.40	3	1.24		
2001/06	3.70	3.60		1.03		
2001/07	3.70	3.60		1.03		
2001/08	3.70	3.60		1.03		
2001/09	1.50	3.60		1.40		
2001/10	1.50	3.60		1.40		
2001/11	1.50	3.60	6	1.40		
2001/12	2.00	3.70		0.47		
2002/01	2.00	3.70		0.47		
2002/02	2.00	3.70		0.47		
2002/03	2.90	3.70		0.05		
2002/04	2.90	3.70		0.05		
2002/05	2.90	3.70		0.05		
2002/06	3.70	3.70		1.03		
2002/07	3.70	3.70		1.03		
2002/08	3.70	3.70		1.03		
2002/09	3.60	3.70		0.84		
2002/10	3.60	3.70		0.84		
2002/11	3.60	3.70	12	0.84		
2002/12	3.90	3.80		1.48		
2003/01	3.90	3.80		1.48		
2003/02	3.90	3.80	3	1.48		
2003/03	3.70	3.90		1.03		
2003/04	3.70	3.90		1.03		
2003/05	3.70	3.90		1.03		
2003/06	2.70	3.90	4	0.00		
2003/07	2.70	4.00		0.00		
2003/08	2.70	4.00	3	0.00		
2003/09	2.70	4.00		0.00		
2003/10	2.70	5.20		0.00		
2003/11	2.70	5.20	3	0.00		
2003/12	3.90	5.20		1.48		
	Total	193.30	72	126.75		

MEAN \	/ARIA		ATIONS OF DA	ATA				
PROPER	TY							
RETURN	S	n = 1998-2003						
Date	Rate	ASCENDING	FREQUENCY	X-mean^2		Rate Class	Frequency	
1998/01	5.10	5.10		33.08		5.10	12	
1998/02	5.10	5.10		26.01		9.50	12	
1998/03	5.10	5.10		26.01		9.50	12	
1998/04	5.10	5.10		33.08		11.20	12	
1998/05	5.10	5.10		26.01		13.70	12	
1998/06	5.10	5.10		26.01		15.10	12	
1998/07	5.10	5.10		3.26				
1998/08	5.10	5.10		26.01				
1998/09	5.10	5.10		26.01				
1998/10	5.10	5.10		26.01		n =	72	
1998/11	5.10	5.10		26.01		Mean =	10.85	
1998/12	5.10	5.10	12	26.01				
1999/01	13.70	9.50		90.25				
1999/02	13.70	9.50		90.25		Variance	10.85	
1999/03	13.70	9.50		90.25				
1999/04	13.70	9.50		90.25		Standard		
1999/05	13.70	9.50		90.25		Deviation=	3.29	
1999/06	13.70	9.50		90.25				
1999/07	13.70	9.50		90.25				
1999/08	13.70	9.50		90.25				
1999/09	13.70	9.50		90.25				
1999/10	13.70	9.50		90.25				
1999/11	13.70	9.50		90.25				
1999/12	13.70	9.50	12	90.25				
2000/01	11.20	10.60		112.36				
2000/02	11.20	10.60		112.36				
2000/03	11.20	10.60		112.36				
2000/04	11.20	10.60		112.36				
2000/05	11.20	10.60		112.36				
2000/06	11.20	10.60		112.36				
2000/07	11.20	10.60		112.36				
2000/08	11.20	10.60		112.36				
2000/09	11.20	10.60		112.36				
2000/10	11.20	10.60		112.36				
2000/11	11.20	10.60		112.36				
2000/12	11.20	9.50	12	90.25				
2001/01	10.60	11.20		125.44				
2001/02	10.60	11.20		125.44				

2001/03	10.60	11.20		125.44		
2001/04	10.60	11.20		125.44		
2001/05	10.60	11.20		125.44		
2001/06	10.60	11.20		125.44		
2001/07	10.60	11.20		125.44		
2001/08	10.60	11.20		125.44		
2001/09	10.60	11.20		125.44		
2001/10	10.60	11.20		125.44		
2001/11	10.60	11.20		125.44		
2001/12	10.60	11.20	12	125.44		
2002/01	9.50	13.70		187.69		
2002/02	9.50	13.70		187.69		
2002/03	9.50	13.70		187.69		
2002/04	9.50	13.70		187.69		
2002/05	9.50	13.70		187.69		
2002/06	9.50	13.70		187.69		
2002/07	9.50	13.70		187.69		
2002/08	9.50	13.70		187.69		
2002/09	9.50	13.70		187.69		
2002/10	9.50	13.70		187.69		
2002/11	9.50	13.70		187.69		
2002/12	9.50	13.70	12	187.69		
2003/01	15.10	15.10		228.01		
2003/02	15.10	15.10		228.01		
2003/03	15.10	15.10		228.01		
2003/04	15.10	15.10		228.01		
2003/05	15.10	15.10		228.01		
2003/06	15.10	15.10		228.01		
2003/07	15.10	15.10		228.01		
2003/08	15.10	15.10		228.01		
2003/09	15.10	15.10		228.01		
2003/10	15.10	15.10		228.01		
2003/11	15.10	15.10		228.01		
2003/12	15.10	15.10	12	228.01		
				9206.40		
	Total	781.30	72			

MEAN V	/ARIA	NCE CALCUL	ATIONS OF DA	TA			
Net Inco	ome G	rowth		n = 1998-200	3		
Date	Rate	ASCENDING	FREQUENCY	X-mean^2	Rate Class	Frequency	
1998/01	6.40	6.40		0.56			
1998/02	6.40	6.40		0.56			
1998/03	6.40	6.40		0.56			
1998/04	6.40	6.40		0.56			
1998/05	6.40	6.40		0.56			
1998/06	6.40	6.40		0.56			
1998/07	6.40	6.40		0.56			
1998/08	6.40	6.40		0.56			
1998/09	6.40	6.40		0.56	;		
1998/10	6.40	6.40		0.56	;		
1998/11	6.40	6.40		0.56	;		
1998/12	6.40	6.40	12	0.56	;		
1999/01	10.40	10.40		22.56			
1999/02	10.40	10.40		22.56	;		
1999/03	10.40	10.40		22.56	;		
1999/04	10.40	10.40		22.56	;		
1999/05	10.40	10.40		22.56	;		
1999/06	10.40	10.40		22.56	;		
1999/07	10.40	10.40		22.56	;		
1999/08	10.40	10.40		22.56			
1999/09	10.40	10.40		22.56			
1999/10	10.40	10.40		22.56			
1999/11	10.40	10.40		22.56			
1999/12	10.40	10.40	12	22.56			
2000/01	2.70	2.70		8.70			
2000/02	2.70	2.70		8.70			
2000/03	2.70	2.70		8.70			
2000/04	2.70	2.70		8.70			
2000/05	2.70	2.70		8.70			
2000/06	2.70	2.70		8.70			
2000/07	2.70	2.70		8.70			
2000/08	2.70	2.70		8.70			
2000/09	2.70	2.70		8.70			
2000/10	2.70	2.70		8.70			
2000/11	2.70	2.70		8.70			
2000/12	2.70	2.70	12	8.70			
2001/01	3.70	3.70		3.80			
2001/02	3.70	3.70		3.80	n =	72	
2001/03	3.70	3.70		3.80	Mean =	5.65	
2001/04	3.70	3.70		3.80			

2001/05	3.70	3.70		3.80	Variance=	6.99	
2001/06	3.70	3.70		3.80			
2001/07	3.70	3.70		3.80	Standard		
2001/08	3.70	3.70		3.80	Deviation=	2.64	
2001/09	3.70	3.70		3.80			
2001/10	3.70	3.70		3.80			
2001/11	3.70	3.70		3.80			
2001/12	3.70	3.70	12	3.80			
2002/01	3.60	3.60		4.20			
2002/02	3.60	3.60		4.20			
2002/03	3.60	3.60		4.20			
2002/04	3.60	3.60		4.20			
2002/05	3.60	3.60		4.20			
2002/06	3.60	3.60		4.20			
2002/07	3.60	3.60		4.20			
2002/08	3.60	3.60		4.20			
2002/09	3.60	3.60		4.20			
2002/10	3.60	3.60		4.20			
2002/11	3.60	3.60		4.20			
2002/12	3.60	3.60	12	4.20			
2003/01	7.10	7.10		2.10			
2003/02	7.10	7.10		2.10			
2003/03	7.10	7.10		2.10			
2003/04	7.10	7.10		2.10			
2003/05	7.10	7.10		2.10			
2003/06	7.10	7.10		2.10			
2003/07	7.10	7.10		2.10			
2003/08	7.10	7.10		2.10			
2003/09	7.10	7.10		2.10			
2003/10	7.10	7.10		2.10			
2003/11	7.10	7.10		2.10			
2003/12	7.10	7.10	12	2.10			
		406.80		503.22			

	Total	Y-Mean		X-Mean						Correlation		
	Property	40.96667	REPO	12.20					Degraceion	Chanaa	Totol	
Data	v	10.00007 V A 2	rale v	13.29	YA2	v v	Y' (based (	on v'=a +	(V'-moon)A2		101ai (V-moon)42	
Dale	I	1 2	^		<u>^~2</u>	<u>^.</u> 1	bx)	511 y = u 1	(T-mean) <sup>a</sup> z	(1 - 1 ) = 2	(T-mean) <sup>2</sup> 2	
1009/01	5 10	26.01	16.00		256.00	91.60	, 0.27		2 252	19.20	22.25	
1990/01	5.10	20.01	16.00		256.00	01.00 91.60	9.37		2.233	18.20	33.20	)
1990/02	5.10	20.01	15.00		230.00	76.50	9.37		2.255	23.24	33.20	·
1990/03	5.10	20.01	1/ 03		223.00	76.30	9.92		0.093	23.24	33.25	
1990/04	5.10	26.01	14.33		324.00	91.80	8.26		6.812	9.96	33.25	
1998/06	5.10	26.01	20.21		408.44	103.07	7.03		14.713	3.73	33.25	
1998/07	5.10	26.01	21.35		455.82	108.89	6.40		19.963	1.69	33.25	;
1998/08	5.10	26.01	21.85		477.42	111.44	6.12		22.518	1.04	33.25	
1998/09	5.10	26.01	21.86		477.86	111.49	6.12		22.570	1.03	33.25	
1998/10	5.10	26.01	20.72		429.32	105.67	6.75		16.963	2.72	33.25	5
1998/11	5.10	26.01	19.73		389.27	100.62	7.30		12.741	4.83	33.25	
1998/12	5.10	26.01	19.32		373.26	98.53	7.52		11.170	5.88	33.25	
1999/01	13.70	187.69	18.83		354.57	257.97	7.80		9.427	34.85	8.03	
1999/02	13.70	187.69	17.36		301.37	237.83	8.61		5.086	25.89	8.03	
1999/03	13.70	187.69	16.51		272.58	226.19	9.08		3.182	21.32	8.03	
1999/04	13.70	187.69	15.67		245.55	214.68	9.55		1.737	17.23	8.03	
1999/05	13.70	187.69	15.46		239.01	211.80	9.67		1.443	16.28	8.03	
1999/06	13.70	187.69	14.92		222.61	204.40	9.96		0.813	13.95	8.03	
1999/07	13.70	187.69	13.65		186.32	187.01	10.67		0.039	9.19	8.03	
1999/08	13.70	187.69	13.44		180.63	184.13	10.79		0.007	8.49	8.03	
1999/09	13.70	187.69	12.40		153.76	169.88	11.36		0.246	5.46	8.03	
1999/10	13.70	187.69	12.19		148.60	167.00	11.48		0.375	4.93	8.03	
1999/11	13.70	187.69	12.00		144.00	164.40	11.58		0.515	4.48	8.03	
1999/12	13.70	187.69	12.00		144.00	164.40	11.58		0.515	4.48	8.03	
2000/01	11.20	125.44	11.75		138.06	131.60	11.72		0.733	0.27	0.11	
2000/02	11.20	125.44	11.75		138.06	131.60	11.72		0.733	0.27	0.11	
2000/03	11.20	125.44	11.75		138.06	131.60	11.72		0.733	0.27	0.11	
2000/04	11.20	125.44	11.75		138.06	131.60	11.72		0.733	0.27	0.11	

2000/05	11.20	125.44	11.75	138.06	131.60	11.72	0.733	0.27	0.11	
2000/06	11.20	125.44	11.75	138.06	131.60	11.72	0.733	0.27	0.11	
2000/07	11.20	125.44	11.75	138.06	131.60	11.72	0.733	0.27	0.11	
2000/08	11.20	125.44	11.75	138.06	131.60	11.72	0.733	0.27	0.11	
2000/09	11.20	125.44	11.75	138.06	131.60	11.72	0.733	0.27	0.11	
2000/10	11.20	125.44	12.00	144.00	134.40	11.58	0.515	0.15	0.11	
2000/11	11.20	125.44	12.00	144.00	134.40	11.58	0.515	0.15	0.11	
2000/12	11.20	125.44	12.00	144.00	134.40	11.58	0.515	0.15	0.11	
2001/01	10.60	112.36	12.00	144.00	127.20	11.58	0.515	0.97	0.07	
2001/02	10.60	112.36	12.00	144.00	127.20	11.58	0.515	0.97	0.07	
2001/03	10.60	112.36	12.00	144.00	127.20	11.58	0.515	0.97	0.07	
2001/04	10.60	112.36	12.00	144.00	127.20	11.58	0.515	0.97	0.07	
2001/05	10.60	112.36	12.00	144.00	127.20	11.58	0.515	0.97	0.07	
2001/06	10.60	112.36	11.00	121.00	116.60	12.14	1.618	2.37	0.07	
2001/07	10.60	112.36	11.00	121.00	116.60	12.14	1.618	2.37	0.07	
2001/08	10.60	112.36	11.00	121.00	116.60	12.14	1.618	2.37	0.07	
2001/09	10.60	112.36	9.50	90.25	100.70	12.97	4.427	5.62	0.07	
2001/10	10.60	112.36	9.50	90.25	100.70	12.97	4.427	5.62	0.07	
2001/11	10.60	112.36	9.50	90.25	100.70	12.97	4.427	5.62	0.07	
2001/12	10.60	112.36	9.50	90.25	100.70	12.97	4.427	5.62	0.07	
2002/01	9.50	90.25	10.50	110.25	99.75	12.42	2.401	8.50	1.87	
2002/02	9.50	90.25	10.50	110.25	99.75	12.42	2.401	8.50	1.87	
2002/03	9.50	90.25	11.50	132.25	109.25	11.86	0.990	5.58	1.87	
2002/04	9.50	90.25	11.50	132.25	109.25	11.86	0.990	5.58	1.87	
2002/05	9.50	90.25	11.50	132.25	109.25	11.86	0.990	5.58	1.87	
2002/06	9.50	90.25	12.50	156.25	118.75	11.31	0.194	3.26	1.87	
2002/07	9.50	90.25	12.50	156.25	118.75	11.31	0.194	3.26	1.87	
2002/08	9.50	90.25	12.50	156.25	118.75	11.31	0.194	3.26	1.87	
2002/09	9.50	90.25	13.50	182.25	128.25	10.75	0.013	1.57	1.87	
2002/10	9.50	90.25	13.50	182.25	128.25	10.75	0.013	1.57	1.87	
2002/11	9.50	90.25	13.50	182.25	128.25	10.75	0.013	1.57	1.87	
2002/12	9.50	90.25	13.50	182.25	128.25	10.75	0.013	1.57	1.87	
2003/01	15.10	228.01	13.50	182.25	203.85	10.75	0.013	18.90	17.92	
2003/02	15.10	228.01	13.50	182.25	203.85	10.75	0.013	18.90	17.92	
2003/03	15.10	228.01	13.50	182.25	203.85	10.75	0.013	18.90	17.92	

2003/04	15.10	228.01	13.50	)	182.25	203.85	10.75		0.013	18.90	17.92	2
2003/05	15.10	228.01	13.50	)	182.25	203.85	10.75		0.013	18.90	17.92	2
2003/06	15.10	228.01	12.00	)	144.00	181.20	11.58		0.515	12.36	17.92	2
2003/07	15.10	228.01	12.00	)	144.00	181.20	11.58		0.515	12.36	17.92	2
2003/08	15.10	228.01	11.00	)	121.00	166.10	12.14		1.618	8.77	17.92	2
2003/09	15.10	228.01	10.00	)	100.00	151.00	12.69		3.337	5.79	17.92	2
2003/10	15.10	228.01	8.50	)	72.25	128.35	13.53		7.068	2.48	17.92	2
2003/11	15.10	228.01	8.50	)	72.25	128.35	13.53		7.068	2.48	17.92	2
2003/12	15.10	228.01	8.50	)	72.25	128.35	13.53		7.068	2.48	17.92	2
	Y	Y ^ 2	Х		X^2	Х.Ү						
TOTALS	782.40	9237.12	957.15	5	13458.87	9993.54			225.99	509.05	735.04	L
	612149.7											
	6	Y^	916136.12	X^2								
n=	72.00											
				Solving			REGRES	SION CO	RRELATION			
				"a"	18.24							
				Solving								
				Solving "b"	-0.55							
				~	0.00			0 55 4 4 0	Tat		705.04	
							r =	-0.55449	101	al Correlation =	735.04	6
									Porcontago At	tributable to Ch	anco in tho	
					y' = a + bx			-29339.42	regression rel	ationshin =	ance in the	
									regressionrei			
					y' = 18.24 ·	- 0.55x		230.00508	3			
											69.25	5
								230.04973	3			

	Total	Y-Mean		X-Mean						Correlation	n –
	Property	40.05	Inflation	7.44					Democriem	Oh an a a	Tatal
Data	Returns	10.85		7.41	V 4 2	v v	V' (based)	on v' – a +	Regression		
Date	Y	¥^2	Χ		Χ^Ζ	Χ.Υ	bx)	011 y = a +	(1 <sup>-</sup> -mean)^2	(Y - Y) ^2	(Y-mean)^2
1998/01	5.10	26.01	6.60		43.56	33.66	11.14		124.060	36.46	26.01
1998/02	5.10	26.01	6.50		42.25	33.15	11.17	7	124.808	36.87	26.01
1998/03	5.10	26.01	6.70		44.89	34.17	11.10	)	123.314	36.06	26.01
1998/04	5.10	26.01	6.70		44.89	34.17	11.10	)	123.314	36.06	26.01
1998/05	5.10	26.01	7.00		49.00	35.70	11.00	)	121.090	34.86	26.01
1998/06	5.10	26.01	7.20		51.84	36.72	10.94	-	119.618	34.07	26.01
1998/07	5.10	26.01	7.00		49.00	35.70	11.00	)	121.090	34.86	26.01
1998/08	5.10	26.01	7.20		51.84	36.72	10.94		119.618	34.07	26.01
1998/09	5.10	26.01	7.60		57.76	38.76	10.80	)	116.702	32.52	26.01
1998/10	5.10	26.01	7.50		56.25	38.25	10.84		117.427	32.91	26.01
1998/11	5.10	26.01	7.40		54.76	37.74	10.87	7	118.155	33.29	26.01
1998/12	5.10	26.01	7.30		53.29	37.23	10.90	)	118.885	33.68	26.01
1999/01	13.70	187.69	7.40		54.76	101.38	10.87	7	118.155	8.01	187.69
1999/02	13.70	187.69	7.30		53.29	100.01	10.90	)	118.885	7.82	187.69
1999/03	13.70	187.69	7.20		51.84	98.64	10.94		119.618	7.63	187.69
1999/04	13.70	187.69	7.00		49.00	95.90	11.00	)	121.090	7.27	187.69
1999/05	13.70	187.69	7.00		49.00	95.90	11.00	)	121.090	7.27	187.69
1999/06	13.70	187.69	7.10		50.41	97.27	10.97	7	120.353	7.45	187.69
1999/07	13.70	187.69	7.00		49.00	95.90	11.00	)	121.090	7.27	187.69
1999/08	13.70	187.69	6.70		44.89	91.79	11.10	)	123.314	6.74	187.69
1999/09	13.70	187.69	6.60		43.56	90.42	11.14		124.060	6.56	187.69
1999/10	13.70	187.69	6.50		42.25	89.05	11.17	7	124.808	6.39	187.69
1999/11	13.70	187.69	6.70		44.89	91.79	11.10	)	123.314	6.74	187.69
1999/12	13.70	187.69	6.80		46.24	93.16	11.07	*	122.570	6.91	187.69
2000/01	11.20	125.44	7.00		49.00	78.40	11.00	)	121.090	0.04	125.44
2000/02	11.20	125.44	7.20		51.84	80.64	10.94		119.618	0.07	125.44
2000/03	11.20	125.44	7.40		54.76	82.88	10.87	*	118.155	0.11	125.44
2000/04	11.20	125.44	7.70		59.29	86.24	10.77	,	115.978	0.19	125.44

2000/05	11.20	125.44	7.90	62.41	88.48	10.70	114.538	0.25	125.44
2000/06	11.20	125.44	7.90	62.41	88.48	10.70	114.538	0.25	125.44
2000/07	11.20	125.44	7.90	62.41	88.48	10.70	114.538	0.25	125.44
2000/08	11.20	125.44	8.10	65.61	90.72	10.64	113.107	0.32	125.44
2000/09	11.20	125.44	8.10	65.61	90.72	10.64	113.107	0.32	125.44
2000/10	11.20	125.44	8.00	64.00	89.60	10.67	113.821	0.28	125.44
2000/11	11.20	125.44	7.70	59.29	86.24	10.77	115.978	0.19	125.44
2000/12	11.20	125.44	7.70	59.29	86.24	10.77	115.978	0.19	125.44
2001/01	10.60	112.36	7.70	59.29	81.62	10.77	115.978	0.03	112.36
2001/02	10.60	112.36	7.70	59.29	81.62	10.77	115.978	0.03	112.36
2001/03	10.60	112.36	7.50	56.25	79.50	10.84	117.427	0.06	112.36
2001/04	10.60	112.36	6.70	44.89	71.02	11.10	123.314	0.25	112.36
2001/05	10.60	112.36	6.50	42.25	68.90	11.17	124.808	0.33	112.36
2001/06	10.60	112.36	6.40	40.96	67.84	11.21	125.558	0.37	112.36
2001/07	10.60	112.36	6.40	40.96	67.84	11.21	125.558	0.37	112.36
2001/08	10.60	112.36	6.00	36.00	63.60	11.34	128.582	0.55	112.36
2001/09	10.60	112.36	5.80	33.64	61.48	11.41	130.108	0.65	112.36
2001/10	10.60	112.36	5.90	34.81	62.54	11.37	129.344	0.60	112.36
2001/11	10.60	112.36	6.30	39.69	66.78	11.24	126.311	0.41	112.36
2001/12	10.60	112.36	6.50	42.25	68.90	11.17	124.808	0.33	112.36
2002/01	9.50	90.25	7.10	50.41	67.45	10.97	120.353	2.16	90.25
2002/02	9.50	90.25	7.40	54.76	70.30	10.87	118.155	1.88	90.25
2002/03	9.50	90.25	7.70	59.29	73.15	10.77	115.978	1.61	90.25
2002/04	9.50	90.25	8.30	68.89	78.85	10.57	111.685	1.14	90.25
2002/05	9.50	90.25	8.60	73.96	81.70	10.47	109.569	0.94	90.25
2002/06	9.50	90.25	9.20	84.64	87.40	10.27	105.397	0.59	90.25
2002/07	9.50	90.25	9.10	82.81	86.45	10.30	106.086	0.64	90.25
2002/08	9.50	90.25	9.90	98.01	94.05	10.03	100.632	0.28	90.25
2002/09	9.50	90.25	10.80	116.64	102.60	9.73	94.668	0.05	90.25
2002/10	9.50	90.25	11.30	127.69	107.35	9.56	91.433	0.00	90.25
2002/11	9.50	90.25	11.30	127.69	107.35	9.56	91.433	0.00	90.25
2002/12	9.50	90.25	10.80	116.64	102.60	9.73	94.668	0.05	90.25
2003/01	15.10	228.01	10.00	100.00	151.00	10.00	99.960	26.03	228.01
2003/02	15.10	228.01	9.30	86.49	140.43	10.23	104.709	23.69	228.01
2003/03	15.10	228.01	9.30	86.49	140.43	10.23	104.709	23.69	228.01

2003/04	15.10	228.01	8.50	)	72.25	128.35	10.50		110.272	21.15	228.01	
2003/05	15.10	228.01	7.70	)	59.29	116.27	10.77		115.978	18.75	228.01	
2003/06	15.10	228.01	6.40	)	40.96	96.64	11.21		125.558	15.17	228.01	
2003/07	15.10	228.01	6.60	)	43.56	99.66	11.14		124.060	15.70	228.01	
2003/08	15.10	228.01	6.30	)	39.69	95.13	11.24		126.311	14.91	228.01	
2003/09	15.10	228.01	5.40	)	29.16	81.54	11.54		133.186	12.67	228.01	
2003/10	15.10	228.01	4.40	)	19.36	66.44	11.88		141.039	10.39	228.01	
2003/11	15.10	228.01	4.10	)	16.81	61.91	11.98		143.439	9.76	228.01	
2003/12	15.10	228.01	4.00	)	16.00	60.40	12.01		144.243	9.55	228.01	
TOTALS	782.40	9237.12	533.50	)	4096.15	5749.39			8518.17	718.95	9237.12	
			284622.2									
	612149.76	Y^2	5	X ^ 2								
	70.00											
n=	72.00											
				Solving "a"	13.35		REGRES	SION CC	RRELATION	1		
				Solving "b"	-0.34							
							r =	-0.15	Total	Correlation =	9237.12	
									Dene en 1 e m e . A 1	tribustable te O	hanas in the	
					y' = a + b	X		-3454.32	Percentage At	tributable to C ationshin –	nance in the	
									regression ren			
					y' = 8.36	-0.94x		101.4916				
											7.78	
								230.0497				
												_

	Total	Y-Mean		X-Mean					C	orrelation	
	Property	10.95	CDB	2 69					Pogrossion	Chanco	Total
Data	v	10.05 V A 2	GDF V	2.00	¥^2	x v	 Y' (base	d on v' =	(V'-moon)^2		101ai (V-moon)^2
Dale	I	1 2	^		A**2	A.I	 a + bx	aony –		(1 - 1) 2	(T-mean) <sup>a</sup> z
1000/01	E 40	00.04	4 70		2.00	0.07	,		0.000	22.42	22.0025
1998/01	5.10	26.01	1.70		2.89	8.67	9.91		0.886	23.12	33.0625
1996/02	5.10	20.01	1.70		2.89	0.07	9.91		0.000	23.12	33.0625
1996/03	5.10	20.01	1.00		1.00	5.10	9.23		2.032	17.04	33.0625
1990/04	5.10	20.01	1.00		1.00	5.10	9.23		2.032	17.04	33.0025
1008/06	5.10	20.01	0.70		0.49	3.10	 9.23		2.032	17.04	33.0025
1008/07	5.10	20.01	0.70		0.49	3.57	8.0/		3.665	14.71	33.0625
1998/08	5.10	26.01	0.70		0.49	3.57	8 94		3.665	14.71	33.0625
1998/09	5.10	26.01	0.70		0.49	1 53	8 55		5 306	11.88	33.0625
1998/10	5 10	26.01	0.30		0.09	1.53	8.55		5 306	11.88	33.0625
1998/11	5.10	26.01	0.30		0.09	1.53	8.55		5.306	11.88	33.0625
1998/12	5.10	26.01	0.20		0.04	1.02	8.45		5.764	11.22	33.0625
1999/01	13.70	187.69	0.20		0.04	2.74	8.45		5.764	27.57	8.1225
1999/02	13.70	187.69	0.20		0.04	2.74	8.45		5.764	27.57	8.1225
1999/03	13.70	187.69	1.00		1.00	13.70	9.23		2.632	20.00	8.1225
1999/04	13.70	187.69	1.00		1.00	13.70	9.23		2.632	20.00	8.1225
1999/05	13.70	187.69	1.00		1.00	13.70	9.23		2.632	20.00	8.1225
1999/06	13.70	187.69	1.90		3.61	26.03	10.10		0.558	12.94	8.1225
1999/07	13.70	187.69	1.90		3.61	26.03	10.10		0.558	12.94	8.1225
1999/08	13.70	187.69	1.90		3.61	26.03	10.10		0.558	12.94	8.1225
1999/09	13.70	187.69	2.80		7.84	38.36	10.98		0.017	7.40	8.1225
1999/10	13.70	187.69	2.80		7.84	38.36	10.98		0.017	7.40	8.1225
1999/11	13.70	187.69	2.80		7.84	38.36	10.98		0.017	7.40	8.1225
1999/12	13.70	187.69	3.70		13.69	50.69	11.85		1.009	3.41	8.1225
2000/01	11.20	125.44	3.70		13.69	41.44	11.85		1.009	0.43	0.1225
2000/02	11.20	125.44	3.70		13.69	41.44	11.85		1.009	0.43	0.1225
2000/03	11.20	125.44	3.60		12.96	40.32	11.76		0.823	0.31	0.1225
2000/04	11.20	125.44	3.60		12.96	40.32	11.76		0.823	0.31	0.1225

2000/05	11.20	125.44	3.60	12.96	40.32	11.76	6	0.823	0.31	0.1225		
2000/06	11.20	125.44	3.40	11.56	38.08	11.56	6	0.508	0.13	0.1225		
2000/07	11.20	125.44	3.40	11.56	38.08	11.56	6	0.508	0.13	0.1225		
2000/08	11.20	125.44	3.40	11.56	38.08	11.56	6	0.508	0.13	0.1225		
2000/09	11.20	125.44	5.20	27.04	58.24	13.31		6.071	4.47	0.1225		
2000/10	11.20	125.44	5.20	27.04	58.24	13.31		6.071	4.47	0.1225		
2000/11	11.20	125.44	5.20	27.04	58.24	13.31		6.071	4.47	0.1225		
2000/12	11.20	125.44	4.00	16.00	44.80	12.15	5	1.681	0.90	0.1225		
2001/01	10.60	112.36	4.00	16.00	42.40	12.15	5	1.681	2.39	0.0625		
2001/02	10.60	112.36	4.00	16.00	42.40	12.15	5	1.681	2.39	0.0625		
2001/03	10.60	112.36	3.80	 14.44	40.28	11.95	5	1.214	1.83	0.0625		
2001/04	10.60	112.36	3.80	 14.44	40.28	11.95	5	1.214	1.83	0.0625		
2001/05	10.60	112.36	3.80	 14.44	40.28	11.95	5	1.214	1.83	0.0625		
2001/06	10.60	112.36	3.70	 13.69	39.22	11.85	5	1.009	1.57	0.0625		
2001/07	10.60	112.36	3.70	 13.69	39.22	11.85	5	1.009	1.57	0.0625		
2001/08	10.60	112.36	3.70	 13.69	39.22	11.85	5	1.009	1.57	0.0625		
2001/09	10.60	112.36	1.50	 2.25	15.90	9.71		1.290	0.78	0.0625		
2001/10	10.60	112.36	1.50	 2.25	15.90	9.71		1.290	0.78	0.0625		
2001/11	10.60	112.36	1.50	 2.25	15.90	9.71		1.290	0.78	0.0625		
2001/12	10.60	112.36	2.00	 4.00	21.20	10.20	)	0.422	0.16	0.0625		
2002/01	9.50	90.25	2.00	 4.00	19.00	10.20	)	0.422	0.49	1.8225		
2002/02	9.50	90.25	2.00	 4.00	19.00	10.20	)	0.422	0.49	1.8225		
2002/03	9.50	90.25	2.90	 8.41	27.55	11.08	3	0.051	2.48	1.8225		
2002/04	9.50	90.25	2.90	8.41	27.55	11.08	3	0.051	2.48	1.8225		
2002/05	9.50	90.25	2.90	 8.41	27.55	11.08	3	0.051	2.48	1.8225		
2002/06	9.50	90.25	3.70	 13.69	35.15	11.85	5	1.009	5.54	1.8225		
2002/07	9.50	90.25	3.70	 13.69	35.15	11.85	5	1.009	5.54	1.8225		
2002/08	9.50	90.25	3.70	 13.69	35.15	11.85	5	1.009	5.54	1.8225		
2002/09	9.50	90.25	3.60	 12.96	34.20	11.76	6	0.823	5.09	1.8225		
2002/10	9.50	90.25	3.60	 12.96	34.20	11.76	6	0.823	5.09	1.8225		
2002/11	9.50	90.25	3.60	12.96	34.20	11.76	5	0.823	5.09	1.8225		
2002/12	9.50	90.25	3.90	15.21	37.05	12.05	5	1.438	6.50	1.8225		
2003/01	15.10	228.01	3.90	15.21	58.89	12.05	5	1.438	9.31	18.0625		
2003/02	15.10	228.01	3.90	15.21	58.89	12.05	5	1.438	9.31	18.0625		
2003/03	15.10	228.01	3.70	13.69	55.87	11.85	5	1.009	10.53	18.0625		
2003/04	15.10	228.01	3.70		13.69	55.87	11.85		1.009	10.53	18.0625	
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2003/05	15.10	228.01	3.70		13.69	55.87	11.85		1.009	10.53	18.0625	
2003/06	15.10	228.01	2.70		7.29	40.77	10.88		0.001	17.80	18.0625	
2003/07	15.10	228.01	2.70		7.29	40.77	10.88		0.001	17.80	18.0625	
2003/08	15.10	228.01	2.70		7.29	40.77	10.88		0.001	17.80	18.0625	
2003/09	15.10	228.01	2.70		7.29	40.77	10.88		0.001	17.80	18.0625	
2003/10	15.10	228.01	2.70		7.29	40.77	10.88		0.001	17.80	18.0625	
2003/11	15.10	228.01	2.70		7.29	40.77	10.88		0.001	17.80	18.0625	
2003/12	15.10	228.01	3.90		15.21	58.89	12.05		1.438	9.31	18.0625	
TOTALS	782.40	9237.12	193.30		645.71	2223.85			120.01	615.05	735.06	
	612149.7											
	6	۲^	37364.89	X^2								
n=	72.00											
				Solving "a"	8.25		 REGRE	SSION CO	RRELATION			
				Solving "b"	0.97							
							r =	0.40	Tota	Correlation =	735.06	
					y' = a + bx			8879.28	Percentage Att	ributable to Ch	ance in	
									the regression	relationship =		
					y' = 8.36 + 0	094x		95.5313				
											83.67	
								230.0497				

	Income	Y-Mean		X-Mean					(	Correlation		
	Growth	5.65	REPO Rate	13.29					Regression	Chance	Total	
Date	Y	Y ^ 2	х		X^2	Х.Ү	Y' (base	d on y' = a	(Y'-mean)^2	(Y - Y') ^2	(Y-mean)^	2
							+ bx)					
1998/01	6.40	40.96	16.00		256.00	102.40	6.47	,	0.671	0.00	0.56	j
1998/02	6.40	40.96	16.00		256.00	102.40	6.47	,	0.671	0.00	0.56	j
1998/03	6.40	40.96	15.00		225.00	96.00	6.17	7	0.267	0.05	0.56	į
1998/04	6.40	40.96	14.93		222.90	95.55	6.15		0.245	0.06	0.56	į
1998/05	6.40	40.96	18.00		324.00	115.20	7.07	,	2.029	0.46	0.56	į
1998/06	6.40	40.96	20.21		408.44	129.34	7.74	ļ	4.383	1.81	0.56	i
1998/07	6.40	40.96	21.35		455.82	136.64	8.09	)	5.947	2.85	0.56	į
1998/08	6.40	40.96	21.85		477.42	139.84	8.24	ŀ	6.708	3.39	0.56	į
1998/09	6.40	40.96	21.86		477.86	139.90	8.24	ŀ	6.724	3.40	0.56	į
1998/10	6.40	40.96	20.72		429.32	132.61	7.90	)	5.053	3 2.24	0.56	į
1998/11	6.40	40.96	19.73		389.27	126.27	7.60	)	3.796	i 1.44	0.56	į
1998/12	6.40	40.96	19.32		373.26	123.65	7.47	7	3.327	' 1.15	0.56	,
1999/01	10.40	108.16	18.83		354.57	195.83	7.33	3	2.808	9.45	22.56	į
1999/02	10.40	108.16	17.36		301.37	180.54	6.88	6	1.515	12.38	22.56	,
1999/03	10.40	108.16	16.51		272.58	171.70	6.62		0.948	14.26	22.56	,
1999/04	10.40	108.16	15.67		245.55	6 162.97	6.37	7	0.517	16.25	22.56	,
1999/05	10.40	108.16	15.46		239.01	160.78	6.31		0.430	16.76	22.56	,
1999/06	10.40	108.16	14.92		222.61	155.17	6.14	L	0.242	18.13	22.56	,
1999/07	10.40	108.16	13.65		186.32	141.96	5.76	ò	0.012	21.55	22.56	į
1999/08	10.40	108.16	13.44		180.63	139.78	5.69	)	0.002	22.14	22.56	į
1999/09	10.40	108.16	12.40		153.76	128.96	5.38	3	0.073	25.21	22.56	į
1999/10	10.40	108.16	12.19		148.60	126.78	5.32		0.112	25.85	22.56	į
1999/11	10.40	108.16	12.00		144.00	124.80	5.26	5	0.153	26.44	22.56	į
1999/12	10.40	108.16	12.00		144.00	124.80	5.26	5	0.153	26.44	22.56	į
2000/01	2.70	7.29	11.75		138.06	31.73	5.18	8	0.218	6.16	8.70	,
2000/02	2.70	7.29	11.75		138.06	31.73	5.18	8	0.218	6.16	8.70	ļ
2000/03	2.70	7.29	11.75		138.06	31.73	5.18	8	0.218	6.16	8.70	
2000/04	2.70	7.29	11.75		138.06	31.73	5.18	8	0.218	6.16	8.70	/
2000/05	2.70	7.29	11.75		138.06	31.73	5.18	8	0.218	6.16	8.70	

2000/06	2.70	7.29	11.75	138.00	6 31.73	3 5.18	0.21	18 6.16	8.70
2000/07	2.70	7.29	11.75	138.00	6 31.73	5.18	0.21	6.16	8.70
2000/08	2.70	7.29	11.75	138.00	6 31.73	5.18	0.21	6.16	8.70
2000/09	2.70	7.29	11.75	138.00	6 31.73	5.18	0.21	6.16	8.70
2000/10	2.70	7.29	12.00	144.00	32.40	5.26	0.15	53 6.55	8.70
2000/11	2.70	7.29	12.00	144.00	32.40	5.26	0.15	53 6.55	8.70
2000/12	2.70	7.29	12.00	144.00	32.40	5.26	0.15	53 6.55	8.70
2001/01	3.70	13.69	12.00	144.00	0 44.40	5.26	0.15	53 2.43	3.80
2001/02	3.70	13.69	12.00	144.00	0 44.40	5.26	0.15	53 2.43	3.80
2001/03	3.70	13.69	12.00	144.00	0 44.40	5.26	0.15	53 2.43	3.80
2001/04	3.70	13.69	12.00	144.00	0 44.40	5.26	0.15	53 2.43	3.80
2001/05	3.70	13.69	12.00	144.00	0 44.40	5.26	0.15	53 2.43	3.80
2001/06	3.70	13.69	11.00	121.00	40.70	4.96	0.48	32 1.58	3.80
2001/07	3.70	13.69	11.00	121.00	40.70	4.96	0.48	32 1.58	3.80
2001/08	3.70	13.69	11.00	121.00	40.70	4.96	0.48	32 1.58	3.80
2001/09	3.70	13.69	9.50	90.2	5 35.15	5 4.50	1.31	0.64	3.80
2001/10	3.70	13.69	9.50	90.2	5 35.15	5 4.50	1.31	0.64	3.80
2001/11	3.70	13.69	9.50	90.2	5 35.15	5 4.50	1.31	0.64	3.80
2001/12	3.70	13.69	9.50	90.2	5 35.15	5 4.50	1.31	0.64	3.80
2002/01	3.60	12.96	10.50	110.2	5 37.80	4.80	0.71	1.45	4.20
2002/02	3.60	12.96	10.50	110.2	5 37.80	4.80	0.71	1.45	4.20
2002/03	3.60	12.96	11.50	132.2	5 41.40	5.11	0.29	95 2.27	4.20
2002/04	3.60	12.96	11.50	132.2	5 41.40	5.11	0.29	95 2.27	4.20
2002/05	3.60	12.96	11.50	132.2	5 41.40	5.11	0.29	95 2.27	4.20
2002/06	3.60	12.96	12.50	156.2	5 45.00	5.41	0.05	58 3.28	4.20
2002/07	3.60	12.96	12.50	156.2	5 45.00	5.41	0.05	58 3.28	4.20
2002/08	3.60	12.96	12.50	156.2	5 45.00	5.41	0.05	58 3.28	4.20
2002/09	3.60	12.96	13.50	182.2	5 48.60	5.71	0.00	)4 4.46	6 4.20
2002/10	3.60	12.96	13.50	182.2	5 48.60	5.71	0.00	)4 4.46	6 4.20
2002/11	3.60	12.96	13.50	182.2	5 48.60	5.71	0.00	)4 4.46	6 4.20
2002/12	3.60	12.96	13.50	182.2	5 48.60	5.71	0.00	)4 4.46	6 4.20
2003/01	7.10	50.41	13.50	182.2	5 95.85	5 5.71	0.00	)4 1.93	3 2.10
2003/02	7.10	50.41	13.50	182.2	5 95.85	5 5.71	0.00	)4 1.93	3 2.10
2003/03	7.10	50.41	13.50	182.2	5 95.85	5 5.71	0.00	)4 1.93	3 2.10
2003/04	7.10	50.41	13.50	182.2	5 95.85	5 5.71	0.00	)4 1.93	3 2.10

2003/05	7.10	50.41	13.50		182.25	95.85	5.71		0.004	1.93	2.10	
2003/06	7.10	50.41	12.00		144.00	85.20	5.26		0.153	3.39	2.10	
2003/07	7.10	50.41	12.00		144.00	85.20	5.26		0.153	3.39	2.10	
2003/08	7.10	50.41	11.00		121.00	78.10	4.96		0.482	4.60	2.10	
2003/09	7.10	50.41	10.00		100.00	71.00	4.65		0.994	5.99	2.10	
2003/10	7.10	50.41	8.50		72.25	60.35	4.20		2.106	8.42	2.10	
2003/11	7.10	50.41	8.50		72.25	60.35	4.20		2.106	8.42	2.10	
2003/12	7.10	50.41	8.50		72.25	60.35	4.20		2.106	8.42	2.10	
TOTALS	406.80	2801.64	957.15	5	13458.87	5630.31			67.32	435.90	503.22	
	165486.24	۲^	916136.12	X^2								
	70.00											<u> </u>
n=	72.00											
				Solving "a"	1.63		REGRES		RELATION			
				Solving "b"	0.30							
							r =	0.37	Total	Correlation =	503.22	
					v' – 2 + I	hy		16012.24	Percentage At	tributable to C	Chance in	
					y – a + 1	57		10013.34	the regression	relationship	=	-
					y' = 1.63	3 - 0.3x		230.00508	5			
											86.62	
								190.34663				

	Income	Y-Mean	Inflation	X-Mean				(	Correlation	
	Growth	5.65	CPIX	7.41				Regression	Chance	Total
Date	Y	Y^2	х		X^2	X.Y	Y' (based on y' =	(Y'-mean)^2	(Y - Y') ^2	(Y-mean)^2
							a + bx)			
1998/01	6.40	40.96	6.60		43.56	42.24	6.09	37.139	0.09	40.96
1998/02	6.40	40.96	6.50		42.25	41.60	6.15	37.810	0.06	40.96
1998/03	6.40	40.96	6.70		44.89	42.88	6.04	36.473	0.13	40.96
1998/04	6.40	40.96	6.70		44.89	42.88	6.04	36.473	0.13	40.96
1998/05	6.40	40.96	7.00		49.00	44.80	5.87	34.513	0.28	40.96
1998/06	6.40	40.96	7.20		51.84	46.08	5.77	33.236	0.40	40.96
1998/07	6.40	40.96	7.00		49.00	44.80	5.87	34.513	0.28	40.96
1998/08	6.40	40.96	7.20		51.84	46.08	5.77	33.236	0.40	40.96
1998/09	6.40	40.96	7.60		57.76	48.64	5.55	30.754	0.73	40.96
1998/10	6.40	40.96	7.50		56.25	48.00	5.60	31.365	0.64	40.96
1998/11	6.40	40.96	7.40		54.76	47.36	5.66	31.983	0.55	40.96
1998/12	6.40	40.96	7.30		53.29	46.72	5.71	32.606	0.48	40.96
1999/01	10.40	108.16	7.40		54.76	76.96	5.66	31.983	22.51	108.16
1999/02	10.40	108.16	7.30		53.29	75.92	5.71	32.606	21.99	108.16
1999/03	10.40	108.16	7.20		51.84	74.88	5.77	33.236	21.48	108.16
1999/04	10.40	108.16	7.00		49.00	72.80	5.87	34.513	20.48	108.16
1999/05	10.40	108.16	7.00		49.00	72.80	5.87	34.513	20.48	108.16
1999/06	10.40	108.16	7.10		50.41	73.84	5.82	33.871	20.98	108.16
1999/07	10.40	108.16	7.00		49.00	72.80	5.87	34.513	20.48	108.16
1999/08	10.40	108.16	6.70		44.89	69.68	6.04	36.473	19.02	108.16
1999/09	10.40	108.16	6.60		43.56	68.64	6.09	37.139	18.54	108.16
1999/10	10.40	108.16	6.50		42.25	67.60	6.15	37.810	18.07	108.16
1999/11	10.40	108.16	6.70		44.89	69.68	6.04	36.473	19.02	108.16
1999/12	10.40	108.16	6.80		46.24	70.72	5.98	35.814	19.50	108.16
2000/01	2.70	7.29	7.00		49.00	18.90	5.87	34.513	10.08	7.29
2000/02	2.70	7.29	7.20		51.84	19.44	5.77	33.236	9.39	7.29
2000/03	2.70	7.29	7.40		54.76	19.98	5.66	31.983	8.73	7.29
2000/04	2.70	7.29	7.70		59.29	20.79	5.49	30.149	7.79	7.29
2000/05	2.70	7.29	7.90		62.41	21.33	5.38	28.956	7.19	7.29

2000/06	2.70	7.29	7.90	62.41	21.33	5.38	28.956	7.19	7.29
2000/07	2.70	7.29	7.90	62.41	21.33	5.38	28.956	7.19	7.29
2000/08	2.70	7.29	8.10	65.61	21.87	5.27	27.787	6.61	7.29
2000/09	2.70	7.29	8.10	65.61	21.87	5.27	27.787	6.61	7.29
2000/10	2.70	7.29	8.00	64.00	21.60	5.33	28.369	6.90	7.29
2000/11	2.70	7.29	7.70	59.29	20.79	5.49	30.149	7.79	7.29
2000/12	2.70	7.29	7.70	59.29	20.79	5.49	30.149	7.79	7.29
2001/01	3.70	13.69	7.70	59.29	28.49	5.49	30.149	3.21	13.69
2001/02	3.70	13.69	7.70	59.29	28.49	5.49	30.149	3.21	13.69
2001/03	3.70	13.69	7.50	56.25	27.75	5.60	31.365	3.61	13.69
2001/04	3.70	13.69	6.70	44.89	24.79	6.04	36.473	5.47	13.69
2001/05	3.70	13.69	6.50	42.25	24.05	6.15	37.810	6.00	13.69
2001/06	3.70	13.69	6.40	40.96	23.68	6.20	38.488	6.27	13.69
2001/07	3.70	13.69	6.40	40.96	23.68	6.20	38.488	6.27	13.69
2001/08	3.70	13.69	6.00	36.00	22.20	6.42	41.259	7.42	13.69
2001/09	3.70	13.69	5.80	33.64	21.46	6.53	42.680	8.03	13.69
2001/10	3.70	13.69	5.90	34.81	21.83	6.48	41.966	7.72	13.69
2001/11	3.70	13.69	6.30	39.69	23.31	6.26	39.172	6.55	13.69
2001/12	3.70	13.69	6.50	42.25	24.05	6.15	37.810	6.00	13.69
2002/01	3.60	12.96	7.10	50.41	25.56	5.82	33.871	4.93	12.96
2002/02	3.60	12.96	7.40	54.76	26.64	5.66	31.983	4.22	12.96
2002/03	3.60	12.96	7.70	59.29	27.72	5.49	30.149	3.58	12.96
2002/04	3.60	12.96	8.30	68.89	29.88	5.16	26.643	2.44	12.96
2002/05	3.60	12.96	8.60	73.96	30.96	5.00	24.971	1.95	12.96
2002/06	3.60	12.96	9.20	84.64	33.12	4.67	21.790	1.14	12.96
2002/07	3.60	12.96	9.10	82.81	32.76	4.72	22.305	1.26	12.96
2002/08	3.60	12.96	9.90	98.01	35.64	4.28	18.353	0.47	12.96
2002/09	3.60	12.96	10.80	116.64	38.88	3.79	14.367	0.04	12.96
2002/10	3.60	12.96	11.30	127.69	40.68	3.52	12.363	0.01	12.96
2002/11	3.60	12.96	11.30	127.69	40.68	3.52	12.363	0.01	12.96
2002/12	3.60	12.96	10.80	116.64	38.88	3.79	14.367	0.04	12.96
2003/01	7.10	50.41	10.00	100.00	71.00	4.23	17.886	8.24	50.41
2003/02	7.10	50.41	9.30	86.49	66.03	4.61	21.281	6.18	50.41
2003/03	7.10	50.41	9.30	86.49	66.03	4.61	21.281	6.18	50.41
2003/04	7.10	50.41	8.50	72.25	60.35	5.05	25.522	4.19	50.41

2003/05       7.10       50.41       7.70       59.29       54.67       5.49       30.149         2003/06       7.10       50.41       6.40       40.96       45.44       6.20       38.488         2003/07       7.10       50.41       6.60       43.56       46.86       6.09       37.139         2003/08       7.10       50.41       6.30       39.69       44.73       6.26       39.172         2003/09       7.10       50.41       5.40       29.16       38.34       6.75       45.595         2003/10       7.10       50.41       4.40       19.36       31.24       7.30       53.304         2003/11       7.10       50.41       4.40       19.36       31.24       7.30       55.734         2003/12       7.10       50.41       4.00       16.81       29.11       7.47       55.734         2003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         7       7       533.50       4096.15       2935.80       2341.47       4         165486.24       Y^2       284622.25       X ^ 2       1       1       1         n=	2.59       50.41         0.80       50.41         1.01       50.41         0.71       50.41         0.12       50.41         0.04       50.41         0.13       50.41         0.18       50.41         460.17       2801.64	2.59 0.80 1.01 0.71 0.12 0.04 0.13 0.18 460.17	30.149 38.488 37.139 39.172 45.595 53.304 55.734 56.556 2341.47 40	30.       38.       37.       39.       39.       45.       53.       55.       56.       234.	49 20 29 26 75 30 47 52	5.49 6.20 6.09 6.26 6.75 7.30 7.47 7.52	54.67 45.44 46.86 44.73 38.34 31.24 29.11 28.40 2935.80	59.29 40.96 43.56 39.69 29.16 19.36 16.81 16.00 <b>4096.15</b>	Image: state	7.70 6.40 6.60 6.30 5.40 4.40 4.10 4.00	50.41 50.41 50.41 50.41 50.41 50.41 50.41 50.41 2801 64	7.10 7.10 7.10 7.10 7.10 7.10 7.10 7.10	2003/05 2003/06 2003/07 2003/08 2003/09 2003/10 2003/11 2003/12
2003/06       7.10       50.41       6.40       40.96       45.44       6.20       38.488         2003/07       7.10       50.41       6.60       43.56       46.86       6.09       37.139         2003/08       7.10       50.41       6.30       39.69       44.73       6.26       39.172         2003/09       7.10       50.41       5.40       29.16       38.34       6.75       45.595         2003/10       7.10       50.41       4.40       19.36       31.24       7.30       53.304         2003/11       7.10       50.41       4.10       16.81       29.11       7.47       55.734         2003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         0       0       16.00       28.40       7.52       56.556       0       0         105486.24 Y^2       284622.25       X ^ 2       0       0       0       0       0       0       0       0       0       0       0       0       0       0       0       0       0<	0.80 50.41 1.01 50.41 0.71 50.41 0.12 50.41 0.04 50.41 0.13 50.41 0.18 50.41 460.17 2801.64	0.80 1.01 0.71 0.12 0.04 0.13 0.18 460.17	38.488 37.139 39.172 45.595 53.304 55.734 56.556 2341.47 40	38.       37.       39.       45.       53.       55.       55.       56.       234.	20 09 26 75 30 47 52	6.20 6.09 6.26 6.75 7.30 7.47 7.52	45.44 46.86 44.73 38.34 31.24 29.11 28.40 2935.80	40.96 43.56 39.69 29.16 19.36 16.81 16.00 <b>4096.15</b>	Image: state	6.40 6.60 6.30 5.40 4.40 4.10 4.00 533.50	50.41 50.41 50.41 50.41 50.41 50.41 50.41 2801 64	7.10 7.10 7.10 7.10 7.10 7.10 7.10 7.10	2003/06 2003/07 2003/08 2003/09 2003/10 2003/11 2003/12
2003/07       7.10       50.41       6.60       43.56       46.86       6.09       37.139         2003/08       7.10       50.41       6.30       39.69       44.73       6.26       39.172         2003/09       7.10       50.41       5.40       29.16       38.34       6.75       45.595         2003/10       7.10       50.41       4.40       19.36       31.24       7.30       53.304         2003/11       7.10       50.41       4.10       16.81       29.11       7.47       55.734         2003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         TOTALS       406.80       2801.64       533.50       4096.15       2935.80       2341.47       4         n=       72.00       REGRESSION CORRELATION	1.01       50.41         0.71       50.41         0.12       50.41         0.04       50.41         0.13       50.41         0.18       50.41         460.17       2801.64	1.01 0.71 0.12 0.04 0.13 0.18 460.17	37.139 39.172 45.595 53.304 55.734 56.556 2341.47 40	37.       39.       39.       45.       53.       55.       55.       56.       234.	09 26 75 30 47 52	6.09 6.26 6.75 7.30 7.47 7.52	46.86 44.73 38.34 31.24 29.11 28.40 2935.80	43.56 39.69 29.16 19.36 16.81 16.00 <b>4096.15</b>	>       >       >       >       >       >       >	6.60 6.30 5.40 4.40 4.10 4.00	50.41 50.41 50.41 50.41 50.41 50.41 2801 64	7.10 7.10 7.10 7.10 7.10 7.10 7.10	2003/07 2003/08 2003/09 2003/10 2003/11 2003/12
2003/08       7.10       50.41       6.30       39.69       44.73       6.26       39.172         2003/09       7.10       50.41       5.40       29.16       38.34       6.75       45.595         2003/10       7.10       50.41       4.40       19.36       31.24       7.30       53.304         2003/11       7.10       50.41       4.10       16.81       29.11       7.47       55.734         2003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         7003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         700       50.41       4.00       16.00       28.40       7.52       56.556         700       7.10       50.41       4.00       16.00       28.40       7.52       56.556         700       7.10       533.50       4096.15       2935.80       2341.47       4         165486.24 Y^2       284622.25 X ^ 2       7       1       1       1       1         n=       72.00       7       7       7       7       1       1       1         Norriginal       9.71	0.71 50.41 0.12 50.41 0.04 50.41 0.13 50.41 0.18 50.41 460.17 2801.64	0.71 0.12 0.04 0.13 0.18 460.17	39.172 45.595 53.304 55.734 56.556 2341.47 40	39       45       53       55       55       55       56       234	26 75 30 47 52	6.26 6.75 7.30 7.47 7.52	44.73 38.34 31.24 29.11 28.40 2935.80	39.69 29.16 19.36 16.81 16.00 <b>4096.15</b>	Image: state	6.30 5.40 4.40 4.10 4.00	50.41 50.41 50.41 50.41 50.41 2801 64	7.10 7.10 7.10 7.10 7.10 7.10	2003/08 2003/09 2003/10 2003/11 2003/12
2003/09       7.10       50.41       5.40       29.16       38.34       6.75       45.595         2003/10       7.10       50.41       4.40       19.36       31.24       7.30       53.304         2003/11       7.10       50.41       4.10       16.81       29.11       7.47       55.734         2003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         707ALS       406.80       2801.64       533.50       4096.15       2935.80       2341.47       4         165486.24 Y^2       284622.25       X ^ 2                 n=       72.00         Solving "a"       9.71       REGRESSION CORRELATION	0.12 50.41 0.04 50.41 0.13 50.41 0.18 50.41 460.17 2801.64	0.12 0.04 0.13 0.18 460.17	45.595 53.304 55.734 56.556 2341.47 40	6 45. 53. 55. 56. 2 56. 2 34	75 30 47 52	6.75 7.30 7.47 7.52	38.34 31.24 29.11 28.40 2935.80	29.16 19.36 16.81 16.00 <b>4096.15</b>		5.40 4.40 4.10 4.00 533.50	50.41 50.41 50.41 50.41 2801 64	7.10 7.10 7.10 7.10	2003/09 2003/10 2003/11 2003/12
2003/10       7.10       50.41       4.40       19.36       31.24       7.30       53.304         2003/11       7.10       50.41       4.10       16.81       29.11       7.47       55.734         2003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         Image: Constraint of the state of the	0.04 50.41 0.13 50.41 0.18 50.41 460.17 2801.64	0.04 0.13 0.18 460.17	53.304 55.734 56.556 2341.47 40	2 53. 2 55. 2 56. 2 234	30 47 52	7.30 7.47 7.52	31.24 29.11 28.40 <b>2935.80</b>	19.36 16.81 16.00 <b>4096.15</b>	) )   	4.40 4.10 4.00	50.41 50.41 50.41 2801.64	7.10 7.10 7.10	2003/10 2003/11 2003/12
2003/11       7.10       50.41       4.10       16.81       29.11       7.47       55.734         2003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         Image: Constraint of the state of the stat	0.13 50.41 0.18 50.41 460.17 2801.64	0.13 0.18 460.17	55.734 56.556 2341.47 40	2 55. 56. 234	47 52	7.47	29.11 28.40 2935.80	16.81 16.00 <b>4096.15</b>		4.10 4.00 <b>533.50</b>	50.41 50.41 2801.64	7.10	2003/11 2003/12
2003/12       7.10       50.41       4.00       16.00       28.40       7.52       56.556         TOTALS       406.80       2801.64       533.50       4096.15       2935.80       2341.47       4         165486.24 Y^2       284622.25 X ^ 2	0.18 50.41 460.17 2801.64	0.18 460.17	2341.47 4	2 56. 234	52	7.52	28.40 2935.80	16.00 4096.15	) 	4.00	50.41	7.10	2003/12
TOTALS       406.80       2801.64       533.50       4096.15       2935.80       2341.47       4         165486.24 Y^2       284622.25 X ^ 2                  4	460.17 2801.64	460.17	2341.47 4	234			2935.80	4096.15		533.50	2801 64	406.90	τοτλι ε
165486.24 Y^2       284622.25 X ^ 2       Image: Constraint of the second secon											2001.04	400.00	IUTALS
n= 72.00 Solving "a" 9.71 REGRESSION CORRELATION									X ^ 2	284622.25	(^2	165486.24	
n= 72.00 Solving "a" 9.71 REGRESSION CORRELATION													
Solving "a" 9.71 REGRESSION CORRELATION												72.00	n=
					ESSION C	REGRES		9.71	Solving "a"				
Solving "b" -0.55								-0.55	Solving "b"				
r =-0.29 Total Correla	elation = 2801.64	orrelation =	Total Correlat	- <b>0.29</b> Tot	=-0.29	r =							
y' = a + bx -5650.20 Percentage Attributa	outable to Chance	tributable to C	Percentage Attributa	-5650.20 Percentag	-5650.2		x	y' = a + b					
In the regression rel	relationship =	on relationshi	in the regression rela	in the regi									
y' = 9.71 -0.55x 101.492				101.492	101.49		-0.55x	y' = 9.71					
	16.43												
190.347				190.347	190.34								
				1 1	1			1	1				

	Income	Y-Mean		X-Mean						Correlation	
	Growth	5.65	GDP	2.68					Regression	Chance	Total
Date	Y	Y ^ 2	х		X^2	X.Y	Y' (based	on y' = a	(Y'-mean)^2	(Y - Y') ^2	(Y-mean)^2
							+ bx)				
1998/01	6.40	40.96	1.70		2.89	10.88	6.65		0.992	0.06	0.5625
1998/02	6.40	40.96	1.70		2.89	10.88	6.65		0.992	0.06	0.5625
1998/03	6.40	40.96	1.00		1.00	6.40	7.35		2.905	0.91	0.5625
1998/04	6.40	40.96	1.00		1.00	6.40	7.35		2.905	0.91	0.5625
1998/05	6.40	40.96	1.00		1.00	6.40	7.35		2.905	0.91	0.5625
1998/06	6.40	40.96	0.70		0.49	4.48	7.66		4.031	1.58	0.5625
1998/07	6.40	40.96	0.70		0.49	4.48	7.66		4.031	1.58	0.5625
1998/08	6.40	40.96	0.70		0.49	4.48	7.66		4.031	1.58	0.5625
1998/09	6.40	40.96	0.30		0.09	1.92	8.06		5.820	2.76	0.5625
1998/10	6.40	40.96	0.30		0.09	1.92	8.06		5.820	2.76	0.5625
1998/11	6.40	40.96	0.30		0.09	1.92	8.06		5.820	2.76	0.5625
1998/12	6.40	40.96	0.20		0.04	1.28	8.16		6.318	3.11	0.5625
1999/01	10.40	108.16	0.20		0.04	2.08	8.16		6.318	5.00	22.5625
1999/02	10.40	108.16	0.20		0.04	2.08	8.16		6.318	5.00	22.5625
1999/03	10.40	108.16	1.00		1.00	10.40	7.35		2.905	9.28	22.5625
1999/04	10.40	108.16	1.00		1.00	10.40	7.35		2.905	9.28	22.5625
1999/05	10.40	108.16	1.00		1.00	10.40	7.35		2.905	9.28	22.5625
1999/06	10.40	108.16	1.90		3.61	19.76	6.44		0.630	15.65	22.5625
1999/07	10.40	108.16	1.90		3.61	19.76	6.44		0.630	15.65	22.5625
1999/08	10.40	108.16	1.90		3.61	19.76	6.44		0.630	15.65	22.5625
1999/09	10.40	108.16	2.80		7.84	29.12	5.53		0.014	23.68	22.5625
1999/10	10.40	108.16	2.80		7.84	29.12	5.53		0.014	23.68	22.5625
1999/11	10.40	108.16	2.80		7.84	29.12	5.53		0.014	23.68	22.5625
1999/12	10.40	108.16	3.70		13.69	38.48	4.62		1.055	33.37	22.5625
2000/01	2.70	7.29	3.70		13.69	9.99	4.62		1.055	3.70	8.7025
2000/02	2.70	7.29	3.70		13.69	9.99	4.62		1.055	3.70	8.7025
2000/03	2.70	7.29	3.60		12.96	9.72	4.72		0.857	4.10	8.7025
2000/04	2.70	7.29	3.60		12.96	9.72	4.72		0.857	4.10	8.7025
2000/05	2.70	7.29	3.60		12.96	9.72	4.72		0.857	4.10	8.7025

2000/06	2.70	7.29	3.40	11.56	9.18	4.93	0.524	4.96	8.7025
2000/07	2.70	7.29	3.40	11.56	9.18	4.93	0.524	4.96	8.7025
2000/08	2.70	7.29	3.40	11.56	9.18	4.93	0.524	4.96	8.7025
2000/09	2.70	7.29	5.20	27.04	14.04	3.11	6.474	0.16	8.7025
2000/10	2.70	7.29	5.20	27.04	14.04	3.11	6.474	0.16	8.7025
2000/11	2.70	7.29	5.20	27.04	14.04	3.11	6.474	0.16	8.7025
2000/12	2.70	7.29	4.00	16.00	10.80	4.32	1.770	2.62	8.7025
2001/01	3.70	13.69	4.00	16.00	14.80	4.32	1.770	0.38	3.8025
2001/02	3.70	13.69	4.00	16.00	14.80	4.32	1.770	0.38	3.8025
2001/03	3.70	13.69	3.80	14.44	14.06	4.52	1.273	0.68	3.8025
2001/04	3.70	13.69	3.80	14.44	14.06	4.52	1.273	0.68	3.8025
2001/05	3.70	13.69	3.80	14.44	14.06	4.52	1.273	0.68	3.8025
2001/06	3.70	13.69	3.70	13.69	13.69	4.62	1.055	0.85	3.8025
2001/07	3.70	13.69	3.70	13.69	13.69	4.62	1.055	0.85	3.8025
2001/08	3.70	13.69	3.70	13.69	13.69	4.62	1.055	0.85	3.8025
2001/09	3.70	13.69	1.50	2.25	5.55	6.85	1.436	9.91	3.8025
2001/10	3.70	13.69	1.50	2.25	5.55	6.85	1.436	9.91	3.8025
2001/11	3.70	13.69	1.50	2.25	5.55	6.85	1.436	9.91	3.8025
2001/12	3.70	13.69	2.00	4.00	7.40	6.34	0.480	6.98	3.8025
2002/01	3.60	12.96	2.00	4.00	7.20	6.34	0.480	7.52	4.2025
2002/02	3.60	12.96	2.00	4.00	7.20	6.34	0.480	7.52	4.2025
2002/03	3.60	12.96	2.90	8.41	10.44	5.43	0.047	3.36	4.2025
2002/04	3.60	12.96	2.90	8.41	10.44	5.43	0.047	3.36	4.2025
2002/05	3.60	12.96	2.90	8.41	10.44	5.43	0.047	3.36	4.2025
2002/06	3.60	12.96	3.70	13.69	13.32	4.62	1.055	1.05	4.2025
2002/07	3.60	12.96	3.70	13.69	13.32	4.62	1.055	1.05	4.2025
2002/08	3.60	12.96	3.70	13.69	13.32	4.62	1.055	1.05	4.2025
2002/09	3.60	12.96	3.60	12.96	12.96	4.72	0.857	1.26	4.2025
2002/10	3.60	12.96	3.60	12.96	12.96	4.72	0.857	1.26	4.2025
2002/11	3.60	12.96	3.60	12.96	12.96	4.72	0.857	1.26	4.2025
2002/12	3.60	12.96	3.90	15.21	14.04	4.42	 1.511	0.67	4.2025
2003/01	7.10	50.41	3.90	15.21	27.69	4.42	 1.511	7.18	2.1025
2003/02	7.10	50.41	3.90	15.21	27.69	4.42	 1.511	7.18	2.1025
2003/03	7.10	50.41	3.70	13.69	26.27	4.62	 1.055	6.14	2.1025
2003/04	7.10	50.41	3.70	13.69	26.27	4.62	1.055	6.14	2.1025

2003/05	7.10	50.41	3.70	)	13.69	26.27	4.62		1.055	6.14	2.1025	
2003/06	7.10	50.41	2.70	)	7.29	19.17	5.63		0.000	2.15	2.1025	
2003/07	7.10	50.41	2.70	)	7.29	19.17	5.63		0.000	2.15	2.1025	
2003/08	7.10	50.41	2.70	)	7.29	19.17	5.63		0.000	2.15	2.1025	
2003/09	7.10	50.41	2.70	)	7.29	19.17	5.63		0.000	2.15	2.1025	
2003/10	7.10	50.41	2.70	)	7.29	19.17	5.63		0.000	2.15	2.1025	
2003/11	7.10	50.41	2.70	)	7.29	19.17	5.63		0.000	2.15	2.1025	
2003/12	7.10	50.41	3.90	)	15.21	27.69	4.42		1.511	7.18	2.1025	
	Y	Y ^ 2	х		X^2	X.Y						
TOTALS	406.80	2801.64	193.30	)	645.71	963.92			129.71	373.51	503.22	
	165486.24	۲ <b>۰</b>	37364.89	X^2								
n=	72.00											
				Solving "a"	8.37		REGRES	SION COR	RELATION			
				Solving "b"	-1.01							
							r =	-0.51	Total	Correlation =	503.22	
					y' = a +	bx		-9232.20	Percentage Attr	ibutable to C	hance in the	
					-				regression relat	ionship =		
					y' = 8.37	7 -1.01x		95.5313				
					-						74.22	
								190.3466				
L												

## 6.6.1 Is the mean rate of Net Income Growth derived from the sample data a true reflection of the "true" population mean, derived from the Normal distribution curve?

Estimating the confidence intervals wherein the true population mean will fall through the use of the Central Limit Theorem:

$$\begin{array}{cccc} x & -z_{\alpha/2} \centerdot & \underline{s} \\ & \sqrt{n} \end{array} < \mu < x + z_{\alpha/2} \centerdot & \underline{s} \\ & \sqrt{n} \end{array}$$

Where:

x = mean of the sample = 5.65%  $\mu$  = the mean of the population  $\alpha$  = the confidence limit ( $\alpha$ -1) = 0.95, 0.99 s = the standard deviation of the sample = 2.64% n = the sample size = 72 z  $_{\alpha/2}$  = Degree of Confidence at certainty level: (95% = z  $_{0.05}$  = 1.96) (99% = z  $_{0.01}$  = 2.58)

5.65 - 1.96 . 2.64 < 
$$\mu$$
 < 5.65 +1.96. 2.64  
 $\sqrt{72}$   $\sqrt{72}$   
5.04% <  $\mu$  < 6.26%

There is thus a 95 % certainty that the population mean will fall within the confidence interval between 5.04% and 6.26%. Applying the same test at the 99% degree of confidence:

5.65 - 2.58 . 2.64 < 
$$\mu$$
 < 5.65 + 2.58 . 2.64  
 $\sqrt{72}$   
 $\sqrt{72}$   
 $4.87\%$  <  $\mu$  < 6.45\%

There is a 99 % certainty that the population mean will fall within the confidence interval between 4.87% and 6.45%.

At the 99% degree of confidence, the Maximum Error is thus estimated to be:

$$\mathbf{E} = \mathbf{z}_{\alpha/2} \cdot \underline{\mathbf{s}}_{\alpha/2}$$

Where:

 $\alpha$  = the confidence limit ( $\alpha$ -1) = 0.01 s = the standard deviation of the sample n = the sample size z  $_{\alpha/2}$  = Degree of Confidence (2.58)

$$E = 2.58 \cdot 2.64$$
  
 $\sqrt{72}$   
 $= 0.803\%$ 

Thus, the mean Net Income Growth rate of 5.65% obtained from the sample thus falls within the 99% degree of certainty as it does at the 95% degree of certainty in prediction and may therefore accepted as a credible reflection of the population mean.

## 6.6.2 Is the mean rate of Total Property Return derived from the sample data (obtained from the index SAPIX) a true reflection of the "true" population mean, derived from the Normal Distribution curve?

Estimating the confidence intervals wherein the true population mean will fall through the use of the Central Limit Theorem:

$$\begin{array}{ccc} x & -z_{\alpha/2} \cdot \underline{s} & <\mu < x + z_{\alpha/2} \cdot \underline{s} \\ & \sqrt{n} & & \sqrt{n} \end{array}$$

Where:

x = mean of the sample = 10.85%  $\mu$  = the mean of the population  $\alpha$  = the confidence limit ( $\alpha$ -1) =95% and 99% s = the standard deviation of the sample = 3.29% n = the sample size = 72 z  $_{\alpha/2}$  = Degree of Confidence at certainty level: (95% = z  $_{0.05}$  = 1.96) (99% = z  $_{0.01}$  = 2.58) 10.85% -1.96 . 3.29 <  $\mu$  < 10.85% +1.96. 3.29

0.85% -1.96. 
$$\frac{5.29}{\sqrt{72}}$$
 < µ < 10.85% +1.96.  $\frac{5.29}{\sqrt{72}}$   
√ 72  $\sqrt{72}$   
10.09% < µ < 11.61%

There is thus a 95 % certainty that the population mean will fall within the confidence interval between 10.09% and 11.61%. Applying the same test at the 99% degree of confidence:

10.85% - 2.58 . 3.29 
$$< \mu <$$
 10.85% +2.58 . 3.29  $\sqrt{72}$   $\sqrt{72}$   $\sqrt{72}$  9.85%  $< \mu <$  11.85%

There is a 99 % certainty that the population mean will fall within the confidence interval between 9.85% and 11.85%.

At the 99% degree of confidence, the Maximum Error is thus estimated to be:

$$\mathbf{E} = \mathbf{z}_{\alpha/2} \cdot \underline{\mathbf{s}}_{\alpha/2}$$

Where:

 $\alpha$  = the confidence limit ( $\alpha$ -1) = 0.01 s = the standard deviation of the sample n = the sample size  $z_{\alpha/2}$  = Degree of Confidence (2.58) E = 2.58 • <u>3.29</u>  $\sqrt{72}$ 

Thus, the mean Total Property Return rate of 10.85% obtained from the sample thus falls within the 99% degree of certainty as it does at the 95% degree of certainty in prediction and may therefore be accepted as a true representation of the total population mean.

6.6.3 Is the mean rate of GDP derived from the sample data a true reflection of the "true" population mean, derived from the Normal Distribution curve?

Estimating the confidence intervals wherein the true population mean will fall through the use of the Central Limit Theorem:

$$\begin{array}{ccc} x & -z_{\alpha/2} \cdot \underline{s} & <\mu < x + z_{\alpha/2} \cdot \underline{s} \\ & \sqrt{n} & & \sqrt{n} \end{array}$$

Where:

x = mean of the sample = 2.68%  $\mu$  = the mean of the population  $\alpha$  = the confidence limit ( $\alpha$ -1) =95% and 99% s = the standard deviation of the sample = 1.33% n = the sample size = 72 z  $_{\alpha/2}$  = Degree of Confidence at certainty level: (95% = z  $_{0.05}$  = 1.96) (99% = z  $_{0.01}$  = 2.58)

2.68% - 1.96 . 1.33 
$$< \mu < 2.68\% + 1.96$$
. 1.33  $\sqrt{72}$   $\sqrt{72}$   $\sqrt{72}$  2.37%  $< \mu < 2.99\%$ 

There is thus a 95 % certainty that the population mean will fall within the confidence interval between 2.37% and 2.99%. Applying the same test at the 99% degree of confidence:

2.68% - 2.58 . 1.33 < 
$$\mu$$
 < 2.68% + 2.58. 1.33  
 $\sqrt{72}$   $\sqrt{72}$   
-1.084% <  $\mu$  < 3.08%

There is a 99 % certainty that the population mean will fall within the confidence interval between -1.084% and 3.08%.

At the 99% degree of confidence, the Maximum Error is thus estimated to be:

$$\mathbf{E} = \mathbf{z}_{\alpha/2} \cdot \underline{\mathbf{s}}_{\alpha/2}$$

Where:

 $\alpha$  = the confidence limit ( $\alpha$ -1) = 0.01 s = the standard deviation of the sample n = the sample size z  $_{\alpha/2}$  = Degree of Confidence (2.58)

$$E = 2.58 \cdot 1.33$$
  
 $\sqrt{72}$   
 $= 0.404\%$ 

Thus, the mean Total Property Return rate of 10.85% obtained from the sample thus falls within the 99% degree of certainty as it does at the 95% degree of certainty in prediction and therefore may be accepted as a true representation of the total population mean.