USING CONCEPT MAPPING TO EXPLORE GRADE 11 LEARNERS' UNDERSTANDING OF THE FUNCTION CONCEPT

SELVAN NAIDOO

A research report submitted to the School of Science Education in the Faculty of Science, University of the Witwatersrand, Johannesburg, in the partial fulfilment of the requirements for the degree of Master of Science.

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Declaration

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

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Selvan Naidoo

_______ day of ____________ 2006

Abstract

This study used concept mapping to explore South African Grade 11 learners' understanding of the function concept. Learners' understanding of the function concept was investigated by examining the relationships learners made between the function concept and other mathematical concepts. The study falls within a social constructivist framework and is underpinned by the key educational notion of understanding. The research method employed was a case study. Data for the study was collected through a concept mapping task, a task on functions and individual learner interviews. In the analysis four key issues are identified and discussed. They are concerned with (a) learners who make most connections; (b) issues related to learners' omission and addition of concepts; (c) learners' use of examples in concept mapping and (d) the nature of connections learners made. The study concludes that concept mapping is an effective tool to explore learners' understanding of the function concept. The report concludes with recommendations for classroom practice, teacher education and further research, particularly given the context of school mathematics practice in the South African curriculum where concept mapping (i.e. use of metacogs) has recently been incorporated as an assessment tool.

Keywords

Understanding

Function concept

Concept mapping

Mathematical relationships

Conceptual understanding

Procedural understanding

To

my wife Krishnaveni

and our children

Caylen and Kelsey

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CHAPTER ONE: INTRODUCTION

1.1 Background

Understanding is an important outcome for education since it is argued to be a higher outcome of learning than the rote acquisition of knowledge. In the South African mathematics curriculum there is a deliberate drive to promote conceptual understanding of mathematical concepts among students (Department of Education (DoE), 2001). Conceptual understanding is characterized by a rich network of linking information. However, a lack of conceptual understanding in mathematics among learners has been reported in various studies that have been presented in mathematics education literature (Williams, 1998). Many studies have explored learners' understanding in mathematics in various ways. However, only a few of these have researched learners' understanding of specific mathematical concepts. Furthermore, the focus of these studies has been on learners' errors and misconceptions. However, there is a need to consider deeper learning issues associated with understanding in mathematics. Hence this study explored the use of concept mapping as a tool to investigate Grade 11 learners' understanding of the concept of function. Function is one of the key concepts in the South African mathematics curriculum at the secondary school level. Several other concepts in mathematics are linked to the concept of function.

1.2 Statement of purpose

The purpose of the study was to investigate Grade11 learners' understanding of the function concept by examining the relationships that learners make between the function concept and other mathematical concepts. A concept mapping task was given to the learners to elicit these relationships.

1.3 Rationale

There are three key notions at the centre of this study, namely, *understanding*; the *function concept* and *concept mapping*. I elaborate on each of these key notions to provide a rationale for this study.

1.3.1 Understanding

Almost all statements of aims for education systems worldwide include *understanding* as an important outcome. This is based on the assumption that understanding is a higher form of learning than rote acquisition of knowledge (White and Gunstone, 1992:1). It is not surprising to note that in mathematics education in South Africa, the notion of understanding is also seen as critically important. This is evident from Revised National Curriculum Statement (RNCS) Grades R-9 (Schools): Learning area statement for mathematics (DoE, 2001). According to the RNCS mathematics is conceived as a conceptual domain in which “mathematical ideas and concepts build on one another creating a coherent whole” (DoE, 2001:16). This conceptual nature of mathematics is emphasised in the curriculum. There is a push to focus learners’ learning towards understanding mathematics concepts and making sure that learners are able to establish relationships between concepts. The RNCS advocates that learners acquire conceptual understanding rather than merely achieve proficiency in mathematical procedures.

The RNCS document also advocates that teaching and learning of mathematics should enable learners to “develop deep conceptual understanding in order to make sense of mathematics” (DoE, 2001:17). Much emphasis is placed on developing learners who have a thorough knowledge of mathematical relationships. The Grade 10-12 National Curriculum Statement on Mathematical Literacy (DoE, 2003a) also requires learners to be able to “demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation” (p.2).

From the above discussion it is clear that understanding is considered to be a key notion of mathematics education generally and mathematics education in South Africa specifically. From this perspective, it is critical to explore learners’ understanding of specific mathematical concepts.

1.3.2 Location of the function concept in the mathematics curriculum

In the previous mathematics curriculum in South Africa, the concept of function did not enjoy much emphasis and focus (see for example, the Interim Core Syllabus for Mathematics Standards 8 – 10, DoE, 1997). For example, in standard 8 the concept of function is merely mentioned as one of ten syllabus topics to be taught. In the new South African mathematics curriculum, however, the concept of function is at the core of mathematics education. It is a critical concept because of its prevalence across the different domains of mathematics in the curriculum. For example, the concept of function is found in the algebraic and geometric domains. The concept of function is important because it is given a space of its own in both the learning outcomes and the assessment standards of school mathematics in South Africa. Assessment is an important consideration in any curriculum. In fact, in some cases it drives curriculum, for example, Curriculum 2005, which is an umbrella term for the principles governing education at different levels in post-apartheid South Africa.

1.3.2.1 Learning outcomes

An entire Learning Outcome is dedicated to the concept function in three mathematics curriculum documents in South Africa. These documents are: (i) the RNCS Grades R-9 (Schools): Learning area statement for mathematics (DoE, 2001); (ii) the Grade 10-12 National Curriculum Statement on Mathematical Literacy (DoE, 2003a) and (iii) the Grade 10-12 National Curriculum Statement on Mathematics (DoE, 2003b). Learning Outcome 2 in the RNCS document places emphasis on the learner being able to recognize, describe and represent patterns and *relationships*. Learning Outcome 2 in the Mathematics Statement (DoE, 2003b) requires that learners be able to investigate, analyse, describe and represent a wide range of *functions* and solve related problems. In this outcome the language of algebra is used as a vehicle to "study the nature of the relationships between specific variables in a situation" (DoE, 2003b:12). It is envisaged in this learning outcome that learners understand and are able to describe various patterns and functions and also investigate the effect of changing parameters on the graphs of functions.

Learning Outcome 2 in the Mathematical Literacy Statement (DoE, 2003a) requires learners to be able to recognise, interpret, describe and represent various *functional relationships* to solve problems in real and simulated contexts. The purpose of this Learning Outcome is to provide learners with opportunities to investigate functional relationships in other subjects and in real life situations. In this outcome, rate of change is emphasised as an effective way to describe the behaviour of functions (p.12). Learners are also required to "reflect on relationships between variables" (p.12).

1.3.2.2 Assessment standards

The concept of function is given primary attention in the assessment standards in the new mathematics curriculum. The Grades 10-12 Mathematics National Curriculum Statement (DoE, 2003b:7) defines Assessment Standards as "the criteria that collectively describes what a learner should know and be able to demonstrate at a specific grade. Assessment Standards embody the knowledge, skills and values required to achieve the Learning Outcomes." An analysis of the Assessment Standards for Learning Outcome 2 across all the grades reveals that learners are required to have had exposure to an intensive course of study of functions by the time they complete Grade 12. Learners are required to demonstrate that they are able to identify various characteristics of functions such as zeros, average gradient, shape and symmetry. Learners are also required to work with various functions and to draw inverses for linear; quadratic and absolute value functions. In particular, at the Grade 12 level, learners need to be able to demonstrate knowledge of the formal definition of a function; determine which inverses is function and know how to restrict the domain of the original function so that the inverse is also a function. Learners also need to be able to sketch, interpret and solve power, logarithmic and cubic functions.

Clarke (1996:327) argues that "assessment is at the heart of education endeavour". This being the case and the fact that the function concept is given a space of its own in the assessment standards, it is important to get to understand more about functions and investigate how learners experience this concept.

1.3.3 Concept mapping

National policy documents (DoE, 2001; 2003a; and 2003b) place much emphasis on the establishment of relationships among concepts as an indicator of understanding in the area of mathematics. However, the provincial policy document: Guideline Document Portfolio: Mathematics (Gauteng Department of Education, (GDE), 2005) concretises the idea of relationships much more. In this document, metacogs are suggested as one of the tools that learners and educators need to use in displaying relationships among concepts in a topic. Metacogs are similar to concept maps. Novak (1998) point out that concept maps are tools for organizing and representing knowledge components graphically. They include concepts that are enclosed in circles or boxes called nodes. These nodes are connected by lines with linking words, called propositions, which specify the relationship between the nodes. Williams (1998) points out that "concept maps are a direct method of looking at the organization and structure of an individual's knowledge within a particular domain." This organization and structure of one's knowledge components within a particular domain are indicative of one's thinking and understanding in that particular domain. With concept maps, "the connections are explicitly depicted and are visible to the person constructing the map as well as to anyone who observes the map" (Bartels, 1995:548). This suggests that concept mapping is potentially an effective tool for researching student understandings.

Concept mapping has been introduced very recently, only four years ago, in the South African school curriculum. It is introduced as a Grade 12 Continuous Assessment (CASS) component under the name of "metacogs". A metacog has a central concept and all other information branches out from that concept. Metacogs may be used at any stage of the learning process; however it is often used at the end of a section so that learners can get an overall picture and an opportunity to reflect on the section (GDE, 2005:4).

Metacogs are being promoted as a new way of working in mathematics where learners are encouraged to organize and reflect on their knowledge by establishing relationships between concepts. The introduction of metacogs as a CASS component creates an opportunity to use concept mapping both as a pedagogical and a research tool to explore learners' mathematical understandings.

Other documents such as new mathematics textbooks for learners (e.g., van Rensburg et al., 2001) have also incorporated the use of concept mapping in various ways. "Mind maps", similar to concept maps, are included in reflection sections at the end of each chapter in van Rensburg et al. (2001). The purpose of this reflective feature (although not articulated by the authors) appears to be aimed at steering learners to think back on what they have learnt. In this particular textbook, mindmaps (see, for example, Figure 1 below) consist of nodes that are linked by lines.

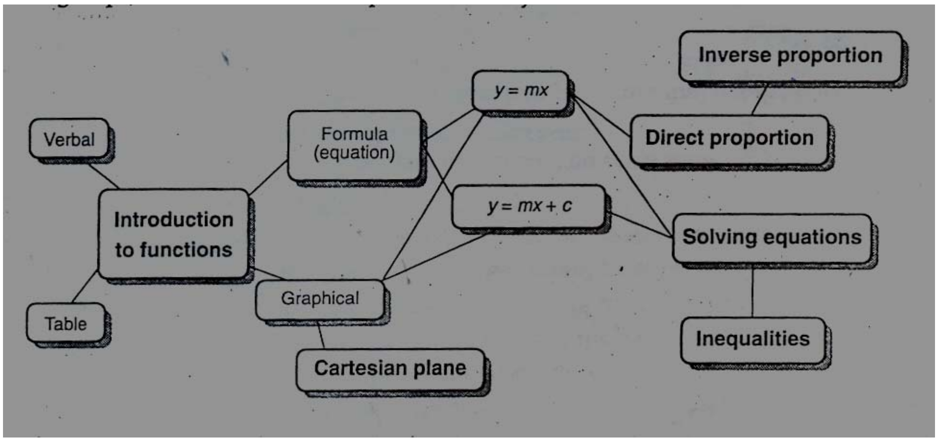

Figure 1: An example of a mindmap

As seen in the figure above, the links between nodes are not made explicit. However, what is significant here is that the authors have organised, structured and related the important concepts of the section concerned. This is an indication that the authors are placing emphasis on mathematical relationships and hence understanding. A notable aspect in the book is that learners are not encouraged to produce their own maps to demonstrate their understanding of the chapter. Nevertheless, this is an important departure from older mathematics textbooks for learners (e.g., Laridon et al., 1995) which are still widely used in South Africa, but do not have reflective sections that specifically incorporate use of mind maps or concept maps.

In the above sections, I have described the importance of the notion of understanding, the importance of the concept of function in the mathematics curriculum in South Africa, and have also indicated that concept mapping is a new tool that is being advocated for use in learning and assessing mathematics. The introduction of this new tool in the curriculum presents an opportunity for the researcher to investigate how concept mapping can be used for researching understanding of specific mathematical concepts and relationships among them.

1.4 Research questions

The aim of the study was to use concept mapping to explore Grade 11 learners' understanding of the function concept. The study sought to answer the following research questions:

- What relationships do Grade 11 learners make between the concept of function and other mathematical concepts?
- How do learners explain through interviews the relationships they make between function and other concepts?
- What is the effectiveness of concept maps as a tool for exploring South African Grade 11 learners' understanding of the concept of function?

In this introductory chapter I have pointed to the prominence that functions enjoy in the present mathematics curriculum in South Africa; the emphasis that is placed on understanding mathematics concepts and establishing relationships between them and the emergence of concept mapping in mathematics in the form of metacogs and mindmaps. In light of these three current "issues" in South African mathematics curriculum, the above research questions were pertinent because learners' understanding of the critical concept of function was explored through the relationships they made between function and other mathematical concepts. Furthermore the study employed the relatively new tool of concept mapping to enable learners to make explicit their relationships between concepts.

In the next chapter the theoretical framework and literature that underpinned this study is explored.

CHAPTER TWO: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Theoretical framework

This study falls within a social constructivist framework and is underpinned by the key educational notion of understanding.

According to Ernest (1991) cited in Jaworski (1994:24), there are two key features to social constructivism:

> First of all there is the active construction of knowledge, typically concepts and hypotheses, on the basis of experiences and previous knowledge. These provide the basis for understanding and serve the purpose of guiding future actions. Secondly there is the essential role played by experience and interaction with the physical and social worlds, in both physical action and speech modes.

A social constructivist framework is adopted due to the importance of social interactions in a classroom in shaping learner's understanding.

2.1.1 What is understanding?

It is important to answer this question since, as pointed out by White and Gunstone (1992), almost all statements of aims for education include understanding as an important outcome. This is also evident in curriculum documents, for example, the Revised National Curriculum Statement: Mathematics (Department of Education, 2001:17), has one of its purposes, "to develop deep conceptual understandings so as to make sense of Mathematics". It is clear that understanding is pivotal to education. However, as Nickerson (1985:215) observes, "What it means to understand is a disarmingly simple question to ask but one that is likely to be anything but simple to answer". White and Gunstone (1992) observe that to understand a concept, for example, democracy, just a definition of it is "hardly likely to be sufficient for you to feel that you understand the term to any substantial degree. To understand a concept you must have in your memory some information (knowledge) about it" (p.3). Gagne and White (1978) (in White and Gunstone, 1992) give the elements of memory which are relevant for understanding. They are propositions, strings, images, episodes, intellectual and motor skills.

White and Gunstone (1992) incorporate the above elements of memory to provide a definition of understanding a concept, for example, democracy.

> Our definition of a person's understanding of democracy is that it is a set of propositions, strings, images episodes and intellectual and motor skills that the person associates with the label 'democracy' (p.5).

The better the memory elements are integrated, the greater will be the understanding of the concept. Furthermore the understanding of a concept is over a continuum since "everyone understands to some degree anything they know something about" (p.5). Hence understanding is never complete because we can always add more knowledge or see new links between things we know already. Commentators on the notion of understanding have also regarded "description" and "talk" as important elements of understanding concepts. Martin (1970) has pointed out that "if we want to be precise in talking about understanding something, then we must describe it" (p.155)

2.1.2 Forms of understanding

2.1.2.1 Conceptual understanding

Hiebert and Lefevre (1986) point out that conceptual understanding is characterised by a rich network of linking between pieces of information. Conceptual understanding is developed by the construction of relationships between pieces of information that are stored in memory or between an existing piece of knowledge and a newly learned one. When previously unrelated pieces of information are suddenly seen has being related in some way then there is an increase in conceptual understanding. A second way in which conceptual understanding increases is through the establishment of connections between existing knowledge and new knowledge that is being encountered. The Piagetian concept of assimilation may be used here to describe the appropriate linking of new knowledge to existing knowledge networks.

2.1.2.2 Procedural understanding

Hiebert and Lefevre (1986) see procedural understanding in mathematics as consisting of two parts. The first is the "symbol representation system of mathematics" (p.6). In this part the individual is familiar with mathematical symbols and is aware of "syntactic rules for writing symbols in acceptable form" (p.6). For example, individuals who possess this form of knowledge would know that 6,25 ÷ □ = 1,25 is syntactically acceptable, although they may not know the answer and that 10 + = □ 5 is not acceptable. However in general, being knowledgeable about symbols and syntax does not imply knowledge of their meanings.

The second part of procedural understanding in mathematics assumes the individuals' familiarity with rules, algorithms or procedures used to solve mathematical tasks. These procedures are essentially "step-by-step instructions that prescribe how to complete mathematical tasks" (p.6). A central feature of procedures is that they are "executed in a predetermined linear sequence" (p.6).

Hiebert and Lefevre (1986) distinguish between two kinds of procedures depending on the objects on which they operate. A distinction is made between objects that are standard written symbols (e.g., 5, - , +) and non-symbolic objects such as concrete objects or mental images. Hiebert and Lefevre (1986) point out that the objects students encounter at school are mainly symbols. They work with problems that are in the form of symbol expressions, such as solving algebraic equations and adding fractions (e.g., 3/4 + 2/3 = أ). Students transform the symbol expression in successive steps until they arrive at a number which they recognize as an answer. Such symbol manipulation procedure makes up the dominant part of school mathematical activity and its "importance should not be underestimated" (p.7).

Hiebert and Lefevre (1986) identify a second kind of procedure which is a "problem-solving strategy that operates on concrete objects, visual diagrams, mental images or other objects that are not standard mathematical symbols" (p.7). Such procedures are widely used by pre-school children and by older children on "new school" tasks. Carpenter in Hiebert and Lefevre (1986) cites the example of young children using a variety of counting strategies to solve verbally presented addition and subtraction problems.

Procedural understanding has the important feature of being structured and hierarchical with some procedures being embedded in others as sub-procedures. A sequence of sub-procedures gives rise to a super procedure. Creating super procedures has the distinct advantage since all sub-procedures may be "assessed by retrieving a single super procedure" (Hiebert and Lefevre, 1986:7). Hiebert and Lefevre (1986) cite the following example of a super procedure: multiply two decimal numbers, 3,81 x 0,43. To produce an answer, three sub-procedures are applied: write the problem in appropriate vertical format; determine the numerical part of the answer and finally insert the decimal point in the answer. All three sub-procedures are assessed sequentially in the super procedure.

2.1.2.3 Relational and instrumental understanding

Skemp (1976) observes that there are in current use two meanings for the word understanding. He identifies them as "relational understanding" and "instrumental understanding". By relational understanding it is meant "knowing what to do and why", and instrumental understanding is described as "rules without reasons" (p.20). He adds that for many learners and their teachers, the possession and ability to use a rule constitutes understanding.

Skemp (1976) points out that much of the mathematical activity that takes place in schools is instrumental mathematics. He offers three reasons why so many teachers teach instrumental mathematics:

- Instrumental mathematics is usually easier to understand.
- The rewards are more immediate and more apparent.
- One can often get the right answer more quickly.

Skemp (1976) also identifies situational factors that seem to promote instrumental mathematical activity:

- The backwash effect of examinations

- The overburdened syllabi.
- Difficulty of assessment
- The great psychological difficulty for teachers of re-structuring their existing and longstanding schemas.

Hiebert and Lefevre's (1986) elaboration on conceptual understanding point out that the construction of relationships between knowledge components is at the heart of conceptual understanding. This notion dovetails with Skemp's relational understanding. Also Hiebert and Lefevre's (1986) view on procedural understanding ties in with Skemp's instrumental understanding.

2.1.3 Understanding and relationships

Relationships between concepts may be established on different bases and therefore concepts may be understood on different levels.

Martin (1970) writes that there are different types of relationships, hence understandings, depending on "the sort of thing (concept) being understood" and whether the concept to be understood is "treated as unity (that is, a simple whole) or as a composite (that is, as a complex whole)" (p.154). These relationships could be temporal, spatial, casual, logical, linear or organic, "depending on ones interests, purposes and points of view" (p.154).

Hiebert and Lefevre (1986) have also distinguished between two levels at which relationships between mathematical concepts may be established. These are the primary and reflective levels. At the primary level the "relationship connecting the information is constructed at the same level of abstractness at which the information itself is represented" (p.4). At this level the relationships that are being made are closely tied to the specific contexts. At the reflective level the relationships are less tied to specified contexts. The "relationships are constructed at a higher, more abstract level than the pieces of information they connect" (p.5). The assumption here is that the higher the level of relationships, the more likely the learner is able to "see much more of the mathematical terrain" (p.5). Hence, relationships at the reflective level necessitate one to step back and reflect on the mathematical concepts being connected.

2.1.4 Understanding a particular mathematical concept?

Mathematical understanding entails the establishment of "significant, fundamental relationships between conceptual and procedural knowledge" (Hiebert and Lefevre, 1986:9). Learners are not fully competent in mathematics if either conceptual or procedural knowledge is not fully developed or if both have been developed but remain separate. When conceptual and procedural mathematical knowledge is not significantly related, learners "may have a good intuitive feel for mathematics, however they may not solve problems fully or they may produce answers but not fully understand what they are doing." (Hiebert and Lefevre, 1986:9)

Based on White and Gunstone's (1992) notion of understanding, a person's understanding of a mathematical concept may be considered a "set of propositions, strings, images, episodes and intellectual and motor skills" (p.5) that the person associates with the concept. Furthermore the individual should be able to make valid connections between related facts and the particular mathematical concept.

It is clear that relationships between items of knowledge cannot be established if the knowledge does not exist (Hiebert and Lefevre, 1986). Hence it is imperative to establish a learner's prior knowledge in a field in which a learner will be engaging a particular concept. This will ensure that if the base knowledge is deficient then appropriate measures may be put in place to develop this "lack" of knowledge. Hence, a particular concept may be integrated into a learner's existing schemas. The more relationships the learner is able to make with other pieces of existing information, the better the understanding will be of the concept (p.6). However if these relationships are inadequate or incomplete then it will be indicative that the learner has a partial understanding of the concepts in that particular domain.

2.1.5 Understanding of the function concept

Referring to the definition of understanding a concept (see section 2.1.4), there are certain elements of the definition that are particularly related to the understanding of the function concept such as images and episodes.

According to Tall (1992), during mathematical activity, mathematical notions are not only used in accordance with formal definitions but also through mental representations which may vary for different individuals. According to Thompson (1994:4), Vinner, Tall and Dreyfus define concept image as “comprising the visual representations, mental pictures, experiences and impressions evoked by the concept name”. All these aspects will not necessarily come to the fore at the same time and probably not be coherent at all times. That portion of the concept image that is activated at the particular time is called the evoked concept image. Concept definition “is a customary or conventional linguistic formulation that demarcates the boundaries of word’s or phrase’s application” (Thompson, 1994: 4).

People understand words through the imagery that is generated in their minds when they hear particular words. People operate largely from the imagery we have in our minds and not “from the basis of conventional constraints adopted by a community” (Thompson, 1994:4). Thompson (1994) writes that Vinner, Tall and Dreyfus arrived at the distinction between concept image and concept definition after puzzling over students’ misuse and misapplication of mathematical terms like function, limit, tangent and derivative. For example, if a learner’s experience of tangents was only limited to circles then it is quite possible for the learner’s concept image of a tangent to be one of where a tangent only touches a curve at a single point. So it is understandable for a learner who holds this concept image to be perplexed when he draws tangents to a cubic graph and finds that the tangent touches the curve at more than one point. Thompson (1994) adds that “a predominant image evoked in students by the word ‘function’ is of two written expressions separated by an equal sign. This image of function is not restricted to novices but extends to some university students as well.” Thompson (1994) makes the point that “mal-formed concept images are insidious and show up in the strangest places” (p.6). However one could not function intellectually without having concept images. The "experts" use concept image and concept definition dialectically. Hence over time students' images become refined and consistent with "conventionally accepted concept definition" (p.6).

Vinner and Dreyfus (1989) points out that students do not necessarily use the definition of function to decide whether a given mathematical object is an example or non-example of the concept. In most cases they decide on the basis of their concept images. According to Thompson (1994) concept images of functions may be evident in one of three forms, i.e. function as an action, as a process or as an object.

White and Gunstone (1992) define episodes "as memories of events that you think happened to you or that you witnessed, such as recollection of voting or of doing an experiment" (p.4). What is being pointed out here is that the understanding of a concept is also tied to the context in which the concept was encountered.

Mwakapenda's (2004) study found links between concepts and contexts. His findings showed that concepts are not seen as entities on their own but are linked to contexts such as diagrams and theorems. These findings also agree with those of Hasemann and Mansfield (1995), who found that some students had made "reference to actions". Students included on their maps the notion that there is something "to do" (p.51), which is indicative of procedural understanding.

2.1.6 Assessing understanding of a particular concept?

Understanding of a concept is a continuum, i.e. there are different degrees of understanding. However Nickerson (1985:230) point out that there are various things that we may accept as evidence of understanding such as:

- the ability to communicate effectively with people who are knowledgeable in that domain;
- the ability to apply a principle consistently in a variety of contexts;
- the ability to carry out a process in such a way so as to obtain the desired results consistently.

In other words one should be able to demonstrate that one understands in a variety of ways.

Understanding of a concept is also dependent on the complexity of the concept. Complexity may be thought of in terms of "the breadth and depth of a concept's connectedness" (Nickerson 1985:231). That is, the concept may be related to other concepts and more deeply to some of them. Therefore one needs to have a "broad knowledge base" of a concept. Understanding of this concept may be assessed in terms of whether the individual is capable of "forming, testing and revising hypotheses" (p.232). If this can be done, it shows that the individual's understanding has gone beyond the "surface structure" of the concept to the "deep structure trace" (p.232), which incorporates aspects of a concept that are not obvious in its surface representation. For example, assessing one's understanding of a solution to a mathematical equation necessarily entails one to be aware of the legitimacy of each step in the solution and of the reasons the problem solver had for developing the solution along those particular lines (Nickerson 1985). From the above example, it is clear that understanding requires an adequate knowledge base and it is an active process. Assessing such understanding necessarily involves one to be able to establish that the understander has been able to connect related facts and relate new information to prior knowledge in order to develop an integrated knowledge system.

2.2 Literature review

2.2.1 Relevant studies on communicating understanding

How have other researchers used concept mapping as a tool to help learners communicate their understandings? As noted earlier, concept maps are tools for organizing and representing knowledge. Concept mapping is conceived to be a tool that enables one to externalise some aspects of one's understanding with regard to a particular domain. Shavelson, Lang and Lewin (1993) (in Chinnappan, Lawson and Nason, 1999) refer to linking of nodes with labelled lines as propositions, and have argued that concept maps represent some important elements of a learner's knowledge in a domain. Viewed in this way, concept maps can be said to have been used as a tool to help learners communicate their understanding of concepts.

However, it is critical to realize that a concept map is a partial representation of one's understanding in a domain. This is because it can be argued that it is problematic to assume that "something which is inside the mind can be mapped to the outside" (Tergan, 1988, in Hasemann & Mansfield, 1995:45). The "mapping" cannot be completely one to one; hence understanding cannot be seen as complete. In order to attempt to obtain a fuller picture of a learner's understanding in a particular domain, other dimensions of understanding such as describing and talking about that domain, and applying knowledge of that domain to different contexts need to be considered.

Studies have been conducted (Williams, 1998; Mwakapenda, 2004) that have used concept mapping as a research tool to assess student mathematical understandings. In Williams' study, which used two groups of students and eight professors, she found that concept mapping was an effective tool to assess conceptual knowledge of the function concept. According to Williams (1998:414), "concept maps are a direct method of looking at the organization and structure of an individual's knowledge within a particular domain". Her study revealed that many of the students from both groups had connections that were trivial and irrelevant. Their maps were algorithmic in nature: procedures were given instead of concepts with relationships between them. Students from both student groups generally had procedural knowledge with regard to the function concept. The professors' maps differed significantly from students' maps with regard to “general homogeneity” and had no hint of being algorithmic (Williams 1998:420). This sharp difference indicates that concept maps can differentiate fairly well between different levels of conceptual understanding.

In Mwakapenda’s study, which involved 22 first year students who obtained differing mathematics results in the Senior Certificate Examination, he found that concept mapping not only revealed connection to other concepts, but also to the contexts in which these concepts were learned such as theorems and diagrams. Students who included contextual connections were those whose mathematical knowledge had not been adequately developed. Furthermore, Mwakapenda’s analyses of students’ maps revealed that most of the students “displayed a limited integrated knowledge of concepts.” (Mwakapenda 2004: 33).

Mwakapenda (2004) also conducted interviews with students to augment information about their maps. It became clear that students’ lack of fluency in explaining their connections were due to their lack of expertise in expressing mathematics and an inadequate understanding of mathematical language.

Chinnappan et al. (1999) used concept mapping to describe the quantity and quality of organization of a teacher’s knowledge of geometry. By counting the number of nodes and the number of mathematically acceptable links they arrived at a quantitative score for the teacher’s map. The dimensions of integrity and connectedness were used to assess the quality of the map. To assess the integrity of the map they focussed on completeness and accuracy. In assessing connectedness they concentrated on depth, branching, cross-linking and quality of relationships. Chinnappan et al. point out that their study went further than that of Williams (1998) because Williams did not attempt to represent the quality of conceptual understanding. Chinnappan’s et al. study had a common methodology element with that of Williams (1998). Williams (1998) used experts, professors of mathematics, in her study. Chinnappan et al. had also recruited an academic mathematician who specialized in geometry to be part of the project information team. Chinnappan’s et al. methodology also included interviews with the teacher. In one interview the teacher was asked to talk about a list of focus schemas in the areas of geometry, trigonometry, and coordinate geometry and in another interview the teacher was asked to expand on these ideas.

Chinnappan et al. (1999) found that the teacher, an experienced teacher of mathematics for at least 15 years, showed a "rich set of connections with evidence of complex differentiation in some instances" (p.174). They also identified areas where the quality of the teacher's knowledge could be improved. These areas were determined by their low levels of cross-linking. Chinnappan et al.'s findings show that the quantitative and qualitative analyses of an individual's concept map is revealing with regard to their "qualitative features of a knowledge network" (p.175).

Roberts (1999) did a study in the form of an action research that used concept maps to measure statistical understanding in tertiary students. Students were asked to draw a concept map from a given list of terms that was seen to be fundamental to the definition of statistical analysis. Mwakapenda (2004) also adopted the same methodological approach when he provided his participants with a list of mathematical concepts that were fundamental to high school mathematics from which they drew their concept maps. Another common methodological feature between the two studies was that both sets of tertiary students were inducted into the process of concept mapping. Roberts (1999) found that students who scored better in their concept maps, which pertained to defining a statistical problem, tended to score higher on a traditional assignment based on the same area. Hence, she concluded that concept maps can be used to assess understanding. Roberts (1999) adds that concept maps are useful to provide "valuable information on some fundamental aspects of student knowledge which could not be obtained from traditional assessment tools" (p.716). This finding concurs with Williams (1998), Mwakapenda (2004) and Chinnappan et al. (1999). Roberts (1999) suggests that the scoring method proposed for these concept maps could be included in a student's summative assessment.

The above studies reveal that concept mapping is an effective tool to assess mathematical understandings. Doing follow-up interviews in conjunction with analyses of maps is very revealing with regards to reasons students give for making connections between particular concepts.

2.2.2 Implications for the study

The research conducted in particular by Williams (1998) and Mwakapenda (2004) have implications for my study. Their studies are related to my research questions because I also investigated what relationships were made by learners between the function concept and other mathematical concepts. Furthermore I want to establish to what extent these relationships were conceptual in nature. Both studies have shown that the links in concept maps are indicators of the degree of conceptual understanding. In my study I used concept maps as one of the research tools to establish the extent of conceptual understanding of the function concept by learners. Mwakapenda's interview with students shed light on their concept maps and on the links they made. In my study I conducted interviews with learners to get a deeper understanding of their maps and any contextual influences that may be pertinent to the production of the maps.

Williams found that many of the students had connections that were trivial and irrelevant. In my induction of learners into the activity of concept mapping, I did an example of a concept map, highlighting the use of relevant concepts and connections. I made available to them a variety of concept maps with them deciding which type to draw. In the analysis of their maps I sought to identify what was the predominant view that learners held for the function concept and how they defined it (if they did). Their view and definition may have implication for teaching practices.

This study builds particularly on Williams' (1998) study since it focussed primarily on learners' understanding of the function concept. However, in contrast to Williams' study where students were not given specific concepts apart from the concept of function in order to draw concept maps, in my study, learners were given specific concepts (see section 3.6.2.1) in addition to function as a focus concept. The other point of departure concerns the fact that in order to obtain confirmatory evidence for understanding (or lack thereof), learners were given a mathematical task to solve in addition to a concept mapping task. The aim here was to establish the extent to which learners who drew “detailed” concept maps were also able to “perform” well on the mathematical task on functions.

2.2.3 Distinction between assessing and researching understanding

Assessing understanding in a particular domain pertains to establishing whether one is able to link related facts and assimilate new information with previous knowledge to create an integrated knowledge system.

Researching understanding involve conducting a review of relevant studies that focussed on understanding. It includes discussing and contrasting the framework, methodology and findings of the different studies and pointing out any omissions or short-comings in the studies.

CHAPTER THREE: METHODOLOGY

3.1 Introduction

In this chapter I provide a justification for the research approach that was adopted in the study. Secondly, I detail the steps taken to ensure that the study was trustworthy in terms of validity and reliability. Thirdly, I explain how ethical issues that arose in the study were addressed. Finally, I make explicit the design of the study.

3.2 Research approach

This research is informed by a social constructivist framework which focuses on learners' understanding of the function concept. This study particularly explores the relationships that learners make between function and other mathematical concepts.

Creswell (1994:2) in Leedy (1997) defines a qualitative study as "inquiry process of understanding a social or human problem, based on building a complex, holistic picture, formed with words, reporting detailed views of informants and conducted in a natural setting". As this study aims to be exploratory and interpretative in nature, it falls within a qualitative approach. In particular the overall design of the study was that of a case study. Opie (2004) describes a case study as "an in-depth study of interactions of a single instance in an enclosed system" (p.74). Opie (2004) also points out the essential features of a case study: data collection is systematic; the focus is on a real situation with real people; the researcher is familiar with the environment; provide a picture of an activity in a particular setting. The researcher will show that the above features of a case study are evident in this study.

In this research the participants of the study were a single Grade 11 class of 36 learners. The researcher was a mathematics educator who had taught Grade 11 mathematics for 10 years and hence is familiar with the environment of the study. The researcher initially inducted the learners in the process of concept mapping in two 1 hour periods.
The first part of the data collection process was the production of the concept maps by learners. The second part of the process involved interviews carried out with a sample

of the students based on the "completeness" (Nickerson, 1985) of their maps. This data collection process was clearly systematic. In other words data was collected sequentially. In this study there was an induction process to the task, the task was carried out and interviews were conducted. Detailed analyses of the maps were done including getting learners to explain, through interviews, the cross links that they drew. These analyses provided a holistic picture of the learners' understanding of the function concept.

A case study design was an appropriate research approach in this study as it allowed the researcher to explore the learners' relationships between concepts and hence gave insight into their understandings of function concept.

3.3 Validity

"Validity refers to the degree to which a method, test or research tool actually measures what it is supposed to measure" (Wellington, 2000: 201 cited in Opie 2004: 68). This definition is essentially in keeping with quantitative research. Hammersley (1992) in Cohen, Manion and Morrison (2000) suggests that validity in qualitative research, the approach that was adopted in my study, replaces certainty with, "confidence in our results, and that as reality is independent of the claims made for it by researchers, our accounts will only be representatives of that reality rather than reproductions of it" (p.107). For my study I needed to ensure that I saw how effective concept mapping was as a tool for exploring learners' understanding of the concept of function. To do this I made use of Ruiz-Primo and Shavelson (1996) definition of validity. "Validity refers to the extent to which inferences to student's cognitive structures, on the bases of their concept map scores, can be supported logically and empirically" (p.592). This implied that learner's maps should show content and concurrent validity. To have content validity the learner's concept maps must show comprehensive links between the concepts provided. Concurrent validity will be demonstrated if there was a correlation between the learner's concept map scores and other "measures of student achievement" (Ruiz-Primo and Shavelson 1996:592). To evaluate content validity Ruiz-Primo and Shavelson (1996) suggest using experts in the particular domain to score the maps. In this regard a criterion map will be appropriate. To check for concurrent validity I gave learners a task to do which tests understanding in the area of functions. The researcher decided on the suitability of the task for this purpose. As a means of moderation the researcher first assessed the task and then gave it to another mathematics educator to assess the task independently.

"External validity refers to the degree to which the results can be generalised to the wider population, cases or situations" (Cohen et al., 2000:109). Since the study was conducted at one school, using one class of Grade11 learners, the external validity of the findings is limited.

3.3.1 Validity and interviews

One of the instruments used in the study was interview. Cannell and Kahn (1968) cited in Cohen et al. (2000) observes that in studies where interview was used, "validity was a persistent problem" (p.120), due to bias. Bias is defined as "to overstate or understate the true value or attribute" (Lansing, Ginsberg and Braaten, 1961, in Cohen et al., 2000). According to Cohen et al. validity and researcher bias are closely linked. Cohen et al. suggest that the "most practical way of achieving greater validity is to minimise the amount of bias as much as possible" (p.121). Cohen et al. identify sources of bias as the characteristics of interviewer, characteristics of respondent and the substantive content of questions. Since the purpose of the interview in the study was for learners to explain the relationships between concepts in their maps, it was clear that overcoming the above limitations of interviews would be difficult especially if a learner had difficulty in explaining their relationships. This situation necessitated that the researcher's probes be more detailed than may have been otherwise necessary. When interviews were conducted I endeavoured to be as "neutral" as possible and did not ask any leading questions.

3.4 Reliability

Bell (1999:103) in Opie (2004:66) define reliability as "the extent to which a test or procedure produces similar results under constant conditions." According to LeCompte and Preissle (1993) cited in Cohen et al. (2000), a typical definition for reliability like the one above may not be workable for qualitative research. This is due to the fact one of the premises of qualitative studies is the "uniqueness and idiosyncrasy of situations such that the study cannot be replicated – that is their strength rather that their weakness" (p.119).

Ruiz-Primo and Shavelson (1996) offer a definition of reliability as it pertains specifically to concept mapping. Reliability refers "to the consistency of scores assigned to student's concept maps" (p.585). The assumption here is that assigning a numerical score will be the primary strategy for analysing maps. In the study scoring of maps was minimal. It was limited to counting the number of valid and / or invalid links made between concepts, the number of concepts used from the given list and the number of new concepts added.

3.5 Ethical considerations

"Ethics have to do with the application of moral principles to prevent harming or wrongdoing others, to promote the good, to be respectful and to be fair" (Sieber, 1992:14, cited in Opie 2004:25). The above definition point out clearly that the researcher must accord the informants in the study the utmost respect and to value their dignity and privacy. The researcher must also realize that educational institutions are hierarchical and when access is required to conduct research in them, there is a protocol that must be adhered to.

Setati (2000) extends this point by pointing out that since schools are hierarchical there is "a Power structure that controls them" (p.353). Setati uses Power with a capital 'P' to indicate hierarchical power. In schools the Power relations are at many levels: education department, the School Governing Body (SGB), the principal, the teachers and the learners. This differentiated level of Power gave direction to the researcher when negotiating access.

I sought permission from the University of the Witwatersrand to conduct this study as a masters student of the university. To ensure that my research was ethical, I firstly approached Gauteng Department of Education to request permission to conduct research at one of its schools. A proposal of the research and the prescribed documentation was submitted to its offices. I also approached my principal for permission to conduct the research, (the study was conducted at the secondary school where I taught since I had access to Grade 11 learners and it would be convenient to do the interviews during my non-teaching time) and explained to him the purpose of the study. The principal and the other informants were informed that the research is in partial fulfilment of the requirements for a masters degree. A commitment was made that in my report no reference will be made to the school or the educator whose Grade 11 class I used. However the school will be referred to by a pseudonym.

The study involved one class of Grade 11 learners. This meant that I needed to negotiate access with the teacher and the learners. At my school the Grade 11 mathematics learners were taught by another educator and myself. I taught at the standard grade level and the other educator at the higher grade level. I made an appointment to meet with the educator and after explaining the purpose of the study requested her participation. The educator was given a letter to show in writing her agreement to participate in the study.

Learners were briefed on the study and were given an information letter which explained the purpose of the study. Learners were invited to be part of the research project. Keeves (1988:181) states that "school children do not always understand the distinction between data which are being gathered anonymously for research purposes and assessments which are being made of them personally." In light of Keeve's statement I emphasized that the concept maps they drew were for research purposes only and did not have anything to do with their reports or the marks that they received for mathematics. Furthermore, I requested their teacher to reinforce this point.

Frankfort – Nachmias and Nachmias (1992) cited in Cohen et al. (2000) state, "the obligation to protect the anonymity of research participants and to keep research data confidential is all - inclusive. It should be fulfilled at all costs unless arrangements to the contrary are made with the participants in advance" (p.61). Considering the above quote, I assured learners and the educator that their responses would be treated as confidential. In my report they would be referred to by pseudonyms unless they request that their real names be used. Learners were also informed that some of them would be selected for an interview, which focused on the concept maps they drew. Learners were not forced to answer any questions.

Learner's permission was sought to tape- record the interviews, so that I could make an accurate record of what was said. Diener and Crandall (1978) cited in Cohen et al. (2000:51) defined informed consent as, "the procedures in which individuals choose whether to participate in an investigation after being informed of the facts that would likely influence their decisions." In light of the above definition learners were informed that if they wanted to participate in the study they would need their parents' / guardian's consent to do so. This was necessary since the participants were minors. Thereafter the learners together with their parents / guardians were asked to sign a consent form agreeing to their children's participation in the study.

Patti Lather (1986) in Opie (2004:29) talks about 'rape research', i.e. where the researcher conducts research and do not report back to the informants. Setati (2000) mentions four parties to whom feedback should be given: research community, teachers involved in the study, principals and District office. To further ensure that my study is ethical I will provide feedback in terms of the findings of the study, to research participants after completion of the study. This will be done to strengthen the 'relationship of trust' (Setati 2000:358) that was accorded to me when they decided to participate in the study.

3.6 Design of study

3.6.1 Context of study and sample

3.6.1.1 Context of study

The research was conducted at Kelcay[1] Secondary where I taught since it was easily accessible to me. I had taught at the school for 16 years and I was familiar with the setting and the sample. The principal and the mathematics teaching staff were receptive to the request to be part of the study. Also, the learners that formed the sample were generally known to me from day to day school interaction. My familiarity with the

[1] pseudonym

sample served as a basis for the establishment of a “relationship” to facilitate an effective implementation of the study. The classroom culture is focused on teaching and learning. Hence learners were also receptive to the study. The school is located in an urban area. It is a multi-racial, co-educational school with a population of 1150. Approximately 85% of the learners are Indian and the remaining learners hail from Black and Coloured communities. The vast majority of learners come from a working class socio-economic background. The school is well resourced in terms of educators, learning support materials, furniture, laboratories, technical centres and library resource centre. At the time the study was conducted, the computer centre was not fully operational due to many computers not being in working order. The educators at the school are all qualified with a minimum M+3 qualification, i.e. matric followed by three years of tertiary education. In the mathematics department there were six educators, two of whom have a post graduate mathematics education qualification, a BSc. (Hons) and an MSc. Four of the six mathematics educators had been teaching mathematics for more than 12 years with the remaining two educators having each been teaching mathematics for six years. In the last four years the school has achieved a pass rate of over 90% in the Senior Certificate Examinations.

3.6.1.2 Sample

The study involved one class of 36 Grade 11 learners who offered mathematics on the higher grade. The Grade 11 learners were chosen because they had at least two years of explicit instruction on functions and it would be enlightening to see what kinds of connections they made between the concept of function and other mathematical concepts. At Kelcay Secondary the Grade 11 mathematics learners were taught by another educator and I. I taught at the standard grade level and the other educator taught at the higher grade level. The higher grade learners were chosen because it is my opinion and since I did not teach them that they would be more “open” in their responses, especially in the follow up interviews. I negotiated to involve a class that was ahead of their work schedule since the time they spent on the research project would not negatively impact on the completion of their work.

3.6.2 Data collection

Data for this study was collected through a concept mapping task, a task on functions and individual learner interviews. Two one hour periods were used to introduce learners to drawing of concept maps. The reason for this extended induction session was that the learners in the sample had no experience in constructing concept maps. Bartels (1995:542) observes that "concept mapping is a useful tool for explicitly stressing mathematical connections" and that "mathematical connections are important because they link mathematical concepts to each other and to the real world". Curriculum 2005 places strong emphasis in developing learners who have a thorough knowledge of mathematical relationships as is clear from "learners have a critical awareness of how mathematical relationships can be responsibly used in different situations" (RNCS: Mathematics, DoE, 2001:18). The above quotation shows that Curriculum 2005 places much emphasis on the establishment of relationships among concepts as an indicator of understanding in the area of mathematics. In light of the above quotations, I used the first period to firstly point out the importance of making connections in everyday experiences and specifically when doing mathematics. Secondly, I introduced learners to drawing concept maps by using Bartels first suggested method of providing learners with a list of terms and an incomplete map. In the second introductory period, I used the second method suggested by Bartels to introduce learners to concept mapping, i.e. to provide learners with a list of concepts but without a map. Learners were also shown examples of concepts maps. The importance of making explicit the links between concepts was emphasised.

3.6.2.1 Concept Mapping Task

Subsequent to the introductory lessons the learners were asked to draw a concept map on the topic functions using the following concepts: equation; zero; variables; gradient; inverse; parallel; perpendicular and function. The topic function was chosen because of its prevalence across the mathematics curriculum from Grade 9 to Grade 12. Learners encounter functions explicitly in the form of linear, quadratic, cubic, logarithmic, trigonometric, power and absolute value functions. The topic functions also appear extensively as assessment items in National Senior Certificate Mathematics Examinations (Guidelines Study Aids – Mathematics, 2004). The topic function has also been the subject of research (see Williams, 1998; Stein et al., 1990; Leinhardt et al., 1990). The eight concepts, listed above, were chosen because they cut across the algebraic and geometric domains of the school mathematics curriculum.

3.6.2.2 Task on functions

The task on functions was given to gather evidence on learners' mathematical competency in the domain of functions (see 2.2.2). The task was adapted from Laridon et al. (1995). The task was based on interpreting a quadratic and a linear function drawn on the same set of axes. For learners to engage effectively with the questions of the task an understanding of the mathematical concepts mentioned in 3.6.2.1 was necessary. The questions that were given in the task tests procedural knowledge (e.g., 2.1 and 2.3); conceptual knowledge (e.g., 2.2) and both procedural and conceptual knowledge (e.g., 2.4).

3.6.2.3 Interviews

Interviews were conducted with learners after a preliminary analysis of their maps. Allchin (2002:146) cited in Mwakapenda (2004) has argued that concept maps are "inherently selective. They can only represent selectively based on the mapmaker's purpose." Roth and Roychoudhury (1992) also observe that concept maps only reveals a part of an individual's thoughts. In an attempt to address these shortcomings with regard to concept mapping, interviews sought to identify and clarify the propositional links on the maps. This process also afforded learners an opportunity to reflect on the connections they made and to even refine them, if necessary.

3.6.3 Administration of instrument

The instrument which comprised a concept mapping task and a task on functions was typed; duplicated and stapled one week prior to its implementation to minimize any potential disruption to the docketing process. The participants required one hour to complete the instrument. I used a double mathematics period i.e. 2 x 30 minute periods for this purpose. The double period was chosen so that it finished just before a break. The reason for doing this was to offer participants a few minutes extra time, if necessary, to complete the task. The learners worked individually with the instrument in their mathematics classroom. The participants were asked to first complete the task on function and thereafter to do the concept mapping task. My reasoning for this particular order was that learners were familiar with the test on functions and they would therefore be able to proceed quite readily with the task. On the other hand concept mapping was a new assessment for the learners and they may not proceed with this task as smoothly. I made prior arrangements with the educator that taught the sample group to teach my classes during the double period concerned. I was in attendance with the sample group during this time. My learners were informed about this arrangement in advance. The sample group were informed a week earlier with regards to the date for the concept mapping task and the task on functions.

3.6.4 Piloting

The data collection instruments were piloted with 6 higher grade learners from a Grade 12 class at the school. The motivation for conducting the pilot with Grade 12 learners rather than Grade 11s was that at the time the pilot was conducted, the Grade 11 learners did not cover sufficient work on functions of the grade 11 syllabus. The purpose of the pilot was to establish whether the learners were able to construct "meaningful" concept maps involving the given concepts. Also, the Grade 12 learners had some familiarity with concept mapping methods since they had some experiences of drawing metacogs as part of their continuous assessment in mathematics. The learners selected for the pilot were of mixed ability which reflected the ability levels in the sample group.

3.6.5 Method of analysis

Williams (1998) points out that many researchers using concept maps as research instruments frequently analyse their data quantitatively, which essentially involves allocating a numerical score to each map. Aspects of the maps that are usually scored include valid propositions; levels of hierarchy; cross-links and examples used to elaborate the links. Bartels (1995) provides an example of a scoring rubric for concept maps which embraces the above quantitative principles of analysis. The rubric has three categories: concepts and terminology; knowledge of the relationships among concepts and the ability to communicate through concept maps. Each of the three categories is given equal weight with the scores ranging from 0 to 3 points based on a continuum of criteria. For example, scoring the category, knowledge of the relationships among concepts: 3 points were allocated if all the important concepts for the particular domain are identified and an understanding of the relationships are shown among the concepts; 2 points were given if important concepts are identified but some incorrect connections are made; 1 point was given if many incorrect connections are made and 0 points was assigned if there was a failure to use any appropriate concepts or appropriate connections.

Chinnappan, (1999) also offer a strategy to analyse concept maps. Maps are scored based on quantity and quality. Chinnappan et al. consider quantity as comprising the number of nodes and links in a map; quality relates to the integrity and connectedness of the map. Integrity was analysed in terms of completeness and accuracy while connectedness involved depth; branching; cross-linking and quality of relationships in the map. Chinnappan et al.'s strategy was developed "to generate a comprehensive description" (p.170) of experts' maps.

Formal scoring of maps, such as the two examples provided above, does provide insights into the mapmaker's connections between concepts, however Novak and Musonda (1991:127) cited in Mwakapenda (2004) have argued that "any map scoring procedure reduces some of the richness and detail of information contained in a concept map". Considering Novak and Musonda's point of view; and that the participants in the study were novices in constructing concept maps, no formal scoring techniques was used for the analysis of maps in a large part of this study. In the analysis I employed some counting "only for the purpose of showing the degree of inter-linking displayed in maps". (Mwakapenda, 2004:4) I recorded the number of concepts used or omitted from the list (see appendix 1A) provided; any additional concepts that were used and the number of valid/ invalid propositions between concepts. The maps were largely analysed in terms of "organisational factors" (Prawat, 1989:5) which involves determining the use of central concepts with which links are made to other concepts. A critical component of data analysis involved interviews. In the interviews learners were asked to make explicit their understandings of central concepts, if used, and to elaborate their meanings of any connection that I was not clear about.

The task on functions was marked using a memorandum prepared by myself. Another educator also marked the task as a form of moderation. The number of correct responses that participants provided served as another source of evidence of their understanding of the function concept. The aspects of function that were assessed in this task included zeros; perpendicular; parallel, determining the equation of a linear function. In the analysis of the task, I was alert to any misunderstandings on the part of the participants. For example, the gradient relationship between linear functions that are parallel or perpendicular to each other. The participants' performance in the functions task contributed to the triangulation of the findings of this study.

CHAPTER FOUR: RESEARCH PROCESSES - DATA COLLECTION AND ANALYSIS

4.1 Piloting

The instrument was piloted with six Grade 12 learners. The purpose of the pilot was to check whether the instructions and questions were clear. The pilot was conducted in July 2005 during the school vacation. The learners made themselves available for a two and a half hour session in which the pilot was completed. In the first part of the session the learners were introduced to the activity of concept mapping and in the second they drew a concept map and carried out the task on functions.

In the induction of learners to concept mapping, they were shown three examples of concept maps by Bartels (1995) and Novak (1998). They were then given an incomplete map i.e. a map with empty ovals and propositions, to complete using a list of concepts. The learners worked in pairs. Finally they drew three concept maps, the first two in pairs, and the third individually. This was done to prepare them for the concept mapping task which required the learners to draw maps individually. In the second part of the session the learners were given one hour to complete the two tasks.

Five learners returned completed maps and written responses to the task on functions. The sixth learner had to be excused from the session due to an emergency at his home.

4.2 Main study

I conducted the main study with a sample of 36 Grade 11 learners in the third week of August 2005. I had planned to use two one-hour periods to induct learners in the activity of concept mapping. However thirty minutes of the first period was lost due to some unforeseen administrative office duties that I had to attend to. The third one-hour period was used to draw the concept maps and do the task on functions. The learners were introduced to concept mapping in a similar manner to the pilot group i.e. they were shown examples of concept maps; worked in pairs on incomplete maps and finally drew concept maps individually from a list of concepts. Learners exchanged maps that they had completed in class. The purpose of exchanging maps was two-fold: firstly, learners could see how the same concepts could be linked in a variety of ways. Secondly, learners were given the opportunity to ask for clarification of any links they did not fully understand in their friends' maps. This clarification may have helped to increase the network of concepts for some learners who may have not realised a particular link between a pair or pairs of concepts. Figure 2 below shows an example of a map drawn by learners in the induction session.

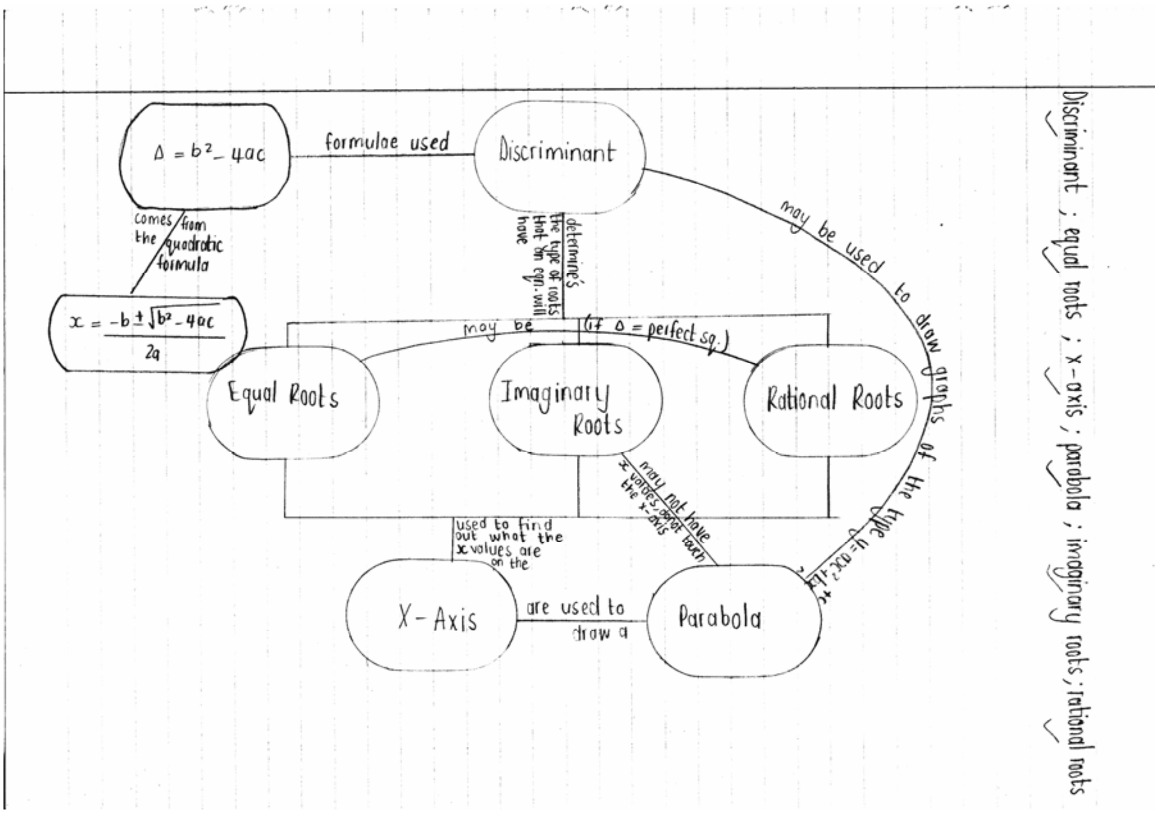

Figure 2: An example of a map drawn during induction

Due to loss of time in the first period, I asked learners to complete the last concept map of the induction session in their own time. Having seen the maps learners had completed in their own time, it became clear that they had become familiar with the process of drawing concept maps. Hence, my concern that the loss of time would adversely affect the induction process was unfounded.

Thirty six concept maps and written responses to the task on functions were returned, 34 of which were used for analysis. The other two learners were not considered because they did not return their consent forms. However each time this was requested they responded that the forms had been signed but that they had forgotten to bring them to school.

4.3 Analysis

4.3.1 Concept maps

A preliminary analysis of the 34 concept maps was done. The maps were coded L1 to L34 (i.e. learner 1 to learner 34) to preserve anonymity of the learners. The concept maps were analysed with regard to the number of concepts used/ omitted from the list; number of additional concepts used and the nature of the propositions i.e. whether the propositions were valid or invalid. Valid propositions were further categorised as trivial or non-trivial.

Valid non-trivial propositions are propositions that are mathematically acceptable and link two concepts in a substantative way. For example, the concepts gradient and parallel may be linked in a in a substantative way with the proposition: lines that are parallel have equal gradients.

Valid trivial propositions are propositions that do not meaningfully contribute to the mathematical weight of the map. For example, equations made up of numbers i.e. zero; equation has variables; parabola is an example of graphs; graphs are sketched on a set of axes; etc.

Tables 4.1a and 4.1b below display a summary of the preliminary analysis of the 34 concept maps.

Table 4.1a: Preliminary analysis of concept maps for L1 – L20

Learner ID	No. of concepts used	No. of additional concepts used	No. of concepts omitted	Nature and number of propositions used		
				Valid – Non trivial	Valid – trivial	Invalid
L1	12	4 (x- axis; y-axis; graphs; axis of symmetry)	0	0	8	1
L2	8	0	0	0	4	2
L3	8	0	0	0	4	2
L4	8	0	0	2	6	0
L5	8	0	0	2	10	0
L6	7	0	1 (zero)	1	4	0
L7	8	0	0	2	9	1
L8	10	2 (Straight line graph; parabola)	0	0	15	0
L9	10	2 (Straight line graph; parabola)	0	1	6	2
L10	8	0	0	0	3	2
L11	10	2 (Intercepts; length)	0	0	8	1
L12	10	4 (Straight line graph; parabola; absolute value graph; x-axis)	2 (Variables ; perpendic ular)	0	9	1
L13	7	1 (parabola)	2 (Parallel; perpendic ular)	0	8	0
L14	8	0	0	2	6	0
L15	8	0	0	2	6	1
L16	8	0	0	1	2	1
L17	11	3 (Graphs; parabola; straight line graphs)	0	0	6	0
L18	10	2 (Parabolas; straight line graphs)	0	0	8	2
L19	8	0	0	1	8	0
L20	7	0	1 (inverse)	0	1	0

Table 4.1b: Preliminary analysis of concept maps L21 – L34

Learner ID	No. of concepts used	No. of additional concepts used	No. of concepts omitted	Nature and number of propositions used		
				Valid – Non trivial	Valid – trivial	Invalid
L21	13	5 (Absolute value graphs; parabolas; straight line graphs; x- intercepts; y- intercept)	0	0	10	3
L22	10	2 (Dual intercept, methods)	0	0	5	1
L23	7	0	1 (variables)	0	4	0
L24	10	2 (Straight line graph; parabola)	0	1	6	0
L25	10	4 (Parabola; straight line graphs; graphs; absolute value graphs)	2 (Parallel; perpendicular)	0	16	1
L26	13	5 (Parabolas; hyperbolas; straight line ; y – form; positive)	0	2	16	0
L27	10	2 (y- intercept; turning point)	0	0	7	2
L28	11	3 (Hyperbola; parabola; straight line graph)	0	1	5	0
L29	10	2 (y- axis; x- axis)	0	0	9	0
L30	10	2 (Lines; parabola)	0	0	13	0
L31	11	2 (Parabola; straight line graphs)	0	1	12	2
L32	9	1 (graphs)	0	0	11	2
L33	8	0	0	0	4	2
L34	6	1 (quadrilateral)	3 (Variables; inverse; gradient)	0	6	1

Table 4.2 below shows the learners' score for the functions task. The total score for the task was out of 20 marks.

Table 4.2: Learners' score out of 20 marks in the functions task

Learner's ID	Score	Learner's ID	Score
L 1	4	L 18	2
L 2	11	L 19	12
L 3	7	L 20	5
L 4	15	L 21	11
L 5	20	L 22	4
L 6	15	L 23	11
L 7	17	L 24	16
L 8	16	L 25	12
L 9	6	L 26	20
L 10	9	L 27	10
L 11	10	L 28	12
L 12	11	L 29	12
L 13	11	L 30	10
L 14	18	L 31	7
L 15	15	L 32	13
L 16	5	L 33	2
L 17	7	L 34	10

The maps were analysed with regard to its comprehensiveness. Comprehensiveness refers to whether the learner used all 8 concepts that were provided; were the propositions mainly valid or trivial; and the prevalence of cross links, i.e. were there many or few. Depending on the comprehensiveness of the map, it was categorised as good, average or poor.

Table 4.3 below shows the criteria for the classification of the maps with regard to its comprehensiveness.

Table 4.3: Criteria used to categorise maps and task

MAP	Good map	Used all 8 concepts; mostly valid propositions; many cross links
	Average map	Used 5-8 concepts; valid propositions; few cross links
	Poor map	Used 0-5 concepts; invalid (or a few valid) propositions; minimal cross links
TASK	Good performance	Score of 14 - 20
	Average performance	Score of 8 - 13
	Poor performance	Score of 0 -7

4.3.2 Task on functions

The task on function was marked by myself and then by another educator to ensure that the marking was consistent with the marking memorandum. The tasks were then categorised according to the learner's performance. The tasks were categorised as good, average or poor performance. Table 4.3 above shows the criteria for the categorisation of the tasks.

Each learner's response (i.e. the concept map together with the task on functions) was thereafter categorised according to the comprehensiveness of the map and the performance in the task. Table 4.4 below displays the matrix for the comprehensiveness of map versus performance in task. Nine categories were established for learner's responses. For example, a learner's response was categorised as "A" if the learner drew a good concept map and the learner's performance in the task on functions was good.

Table 4.4: Comprehensiveness of map versus performance in task

	Good map	Average map	Poor map
Good performance	A	B	C
Average performance	D	E	F
Poor performance	G	H	I

Learner’s responses fell into six of the above nine categories. None of the learner’s responses were categorised as C; D; or G. Table 4.5 below displays the categories into which learners’ responses were placed.

Table 4.5: Categorisation of learners’ responses according to comprehensiveness of maps and performance in the task.

Learner ID	A	B	C	D	E	F	G	H	I
1									*
2						*			
3									*
4					*				
5	*								
6		*							
7	*								
8		*							
9									*
10						*			
11						*			
12					*				
13						*			
14	*								
15	*								
16						*			
17						*			
18									*
19					*				
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22									*
23						*			
24		*							
25					*				
26	*								
27					*				
28					*				
29		*							
30									*
31								*	
32					*				
33						*			
34						*			
TOTALS	5	4	NIL	NIL	8	9	NIL	1	7

I selected one learner from each of the six (A, B, E, F and I) categories to be interviewed. In addition a second “reserve” learner from each category was also chosen in the event the first learner was not available for the interview.

4.3.3 Interviews

The interviews were scheduled to take place in the last week of the third term. However the Gauteng Department of Education (GDE) had rescheduled RNCS training for educators during that week. Eighteen of the 35 educators at the school were involved in the training and this resulted in the disruption of normal schooling activities. Many learners were absent during this period and I had to take on additional administrative responsibilities. Hence, I rescheduled interviews for the first week of the fourth term.

At the beginning of the fourth term I took up a promotional post at the District office of the GDE. I sought permission (which was granted) again from the principal to interview the learners. I interviewed six learners. Five of the learners were from the original group of selected learners and the sixth learner was one of the reserves (a learner from the original group was absent on the day of the interviews). Prior to the interview I contacted each of the six learners telephonically to inform them about the date for the interview. I also informed them that their mathematics teacher would give them a copy of their concept map, on my behalf, to read through so that they could refresh themselves on what they did in the concept mapping task.

The map of each learner in each of the six categories were analysed to determine the relationships learners made between the concept of function and the other concepts. The primary purpose of the interview was two-fold: firstly, learners were asked to make explicit the links that they had made between concepts. Secondly, they were given another opportunity to link concepts that might not have been linked or/and to add concepts to their map that might have been omitted from the list from the concept-mapping task. I drew up an interview schedule for each of the interviewees (see appendix 12). The schedule was largely based on their concept maps. For example, I asked learners to elaborate on certain links that were unclear. The schedule also included some generic questions, such as "what does the term function mean to you?"

I sought permission again from learners to record the interview, to which they agreed. The interviews were recorded using a digital recorder. I purchased this particular recorder because it facilitated the transcription of the interviews. This recorder interfaces with a personal computer. Some of the facilities it has include reducing the playback speed by up to fifty percent and playing back a selected segment repeatedly. Hence, I was able to transcribe the interviews quite fluidly. Transcribing the interviews from a tape recorder would have been time consuming.

At the beginning of each interview the learner was asked to elaborate on how they had started to draw their concept map. Some learners, for example, L34 and L8, spoke briefly about which concepts they started with, while others such as L31 and L25 gave more detail about the concepts they started with. I then specifically asked for clarification of certain links and for the learner to provide examples to substantiate the links. At the conclusion of each interview I asked learners how they felt about drawing the concept map.

The interview transcription process was carried out as follows: I first listened to the whole recording to get an overview of the interview and then I transcribed the entire interview. I denoted parts of the interview that had a long pause with …. Inaudible segments were denoted by (inaudible). There were very few inaudible segments across all the interviews.

The interview data was analysed qualitatively. This involved reading through the transcript to obtain an overview of the interview. Key words or phrases were underlined or highlighted in different colours (see appendix 13 A1) to code certain trends or patterns that were emerging in the interview. I also wrote comments on the transcripts with regard to the underlined or highlighted keywords or phrases (see appendix 13 A1 and 13 A2). For example, phrases were highlighted in yellow if they reflected procedural understanding. A typical phrase here would be:

when you trying to do an inverse all the x values from the normal graph will change to the y values of the inverse and the y values of the normal graph will change to the x values of the inverse.

When the learner wants to "do an inverse", in other words draw an inverse of a parabola, he follows certain steps i.e. change x- intercepts to y- intercepts and y-intercept to x- intercept (x-values and y-values refer to x-intercept and y-intercept respectively). This sentence was thus categorised as relating to procedural understanding (see section 5.2.3.1).

I highlighted phrases or words in purple if the phrase or word actually referred to something else other than what the learner meant or was pointing out on his/ her concept map. For example, L8 pointed out on her concept map: *here is your x-intercept and your y-intercept* (see appendix 13 A1). In fact, L8 was referring here to the x-axis and the y-axis.

Key words which indicated learners' views on concept mapping were underlined in blue For example,

> *It made me understand certain things that I didn't know and I did not know that certain things can link up with each other and know I understand it better.*

In the next chapter I present detailed analysis of each of the six interviewees' concept maps.

CHAPTER FIVE: DATA ANALYSIS

5.1 Pilot study

As stated earlier the purpose of the pilot study was to check whether the instructions and the questions in the instrument were clear. After marking the task on functions it was clear that learners understood the questions and answered accordingly. Therefore no changes were made to the tasks. Four of the five learners had all their answers correct in the functions task. The fifth learner had one problem incorrect.

An analysis of the five concept maps revealed that the learners drew adequate concept maps. By adequate maps it is meant that the learner's maps contained the concepts within ovals and these nodes were linked with other nodes with propositions. Figure 3 below shows an example of a learner's concept map. The fact that learners were able to draw adequate concept maps indicated that the induction process to concept mapping was successful. This was also an indication that learners were ready to do the concept mapping task.

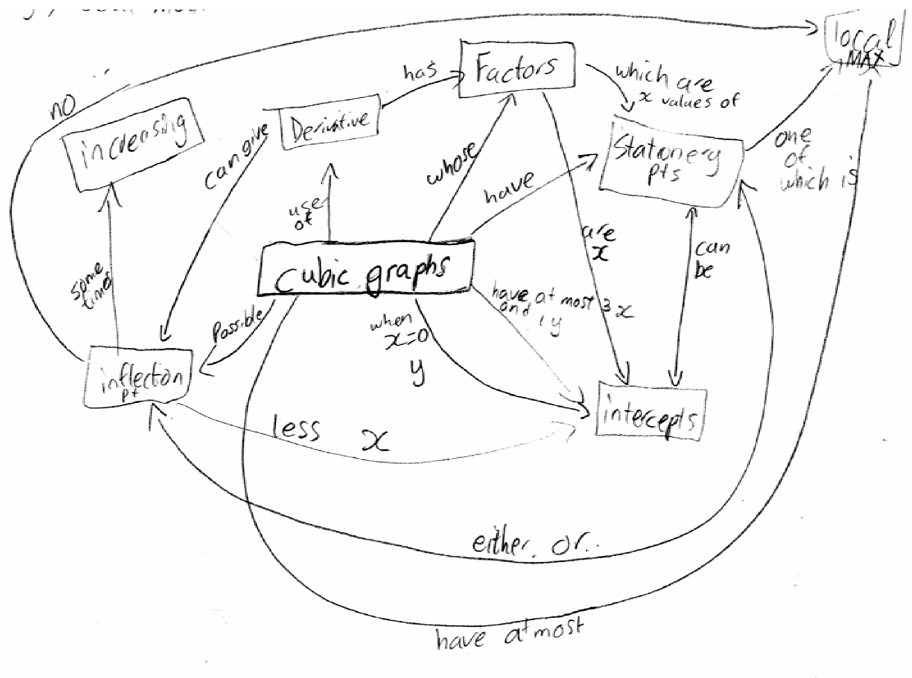

Figure 3: Concept map drawn by a learner after induction

5.2 Main study

The main study was carried out with 36 Grade 11 learners at Kelcay Secondary in Gauteng. Thirty-four learner responses were analysed as explained in the previous chapter. Analysis of maps is focused on six learners (L8; L25; L26; L30; L31 and L34) since they were interviewed.

Tables 4.1a and 4.1b show that 7 of the learners made between 13 and 18 connections, 16 learners made between 7 and 12 connections, and 11 learners made between 4 and 6 connections. Focusing on the 7 learners who made the most connections i.e. L8; L21; L25; L26; L30; L31and L32, it is observed that 2 learners (L21 and L26) have used 13 concepts i.e. 5 more than in the original list; L31 used 11 concepts; 3 learners (L8, L25 and L30) have used 10 concepts and L32 used 9 concepts. An important question that needs to be asked here is:

Issue 1: Does making more connections mean better understanding?[2]

From Table 4.5 it can be seen that 5 learner responses were categorised as A and 4 as B. The responses that fall within categories A and B suggest that the comprehensiveness of one's concept map is indicative of one's performance in the functions task. Eight out of 34 responses were categorised as E which shows that these learners who produced an average map also had an average performance in the functions task. Similarly, 9 learners who had an average performance in the functions task drew a poor map. The responses that were categorised as E and F suggest that the relationships that learners establish between concepts may be indicative of their performances in a written task in that domain. It is interesting to note that none of the responses were categorised as C; D or G. In other words none of the learners who produced a poor map, i.e. a map with minimal connections performed well in the task.

Table 4.1a 4.1b also reveals that two learners (L13 and L25) omitted the concepts perpendicular and parallel. Learner L12 also omitted the concept perpendicular. It would be interesting to see how the above three learners dealt with the concepts of

[2]Issue 1 and subsequent issues will be discussed in the next chapter.

parallel and perpendicular as they were found in the functions task. Questions 2.2 and 2.4 of the task (see appendix 1B) on functions dealt with the concepts perpendicular and parallel respectively. In both of these questions, learners were required to make reference to the equation of a line.

L12 who had omitted the concept of parallel did not answer 2.2 fully. Adequate response to this question required learners to make reference to the equation of the line of symmetry, which was drawn as a dotted line to the parabola (see appendix 1B). L12 gave her answer as $x = –b/2a$. Her answer refers in general to the equation of the axis of symmetry of a parabola.

L13 wrote down the correct answer for question 2.2, which was $x = 2$. Question 2.4 required the equation of a line that was parallel to another line. L13 wrote an incorrect solution. In fact L13 provided an answer that represented a parabola instead of a straight line.

L25, like L12, gave the answer for question 2.2 as $x = –b/2a$. L25 did not attempt a solution to question 2.4. L25 was one of the learners chosen to be interviewed. A detailed analysis of his concept map is presented later in the chapter (see section 5.2.2).

Discussion around Issue 1 will also take into consideration learner performances in the functions task.

5.2.1 An analysis of L26's concept map

Figure 4 below is L26's concept map (see appendix 10A). She had used all 8 concepts from the given list and included an additional 5 concepts. The structure of her map may be classified as spider web. She had organised her map around the central concept of function. Her map was comprehensive with many cross-links and examples. Some of the links and examples needed to be clarified and to this end follow up interviews were done.

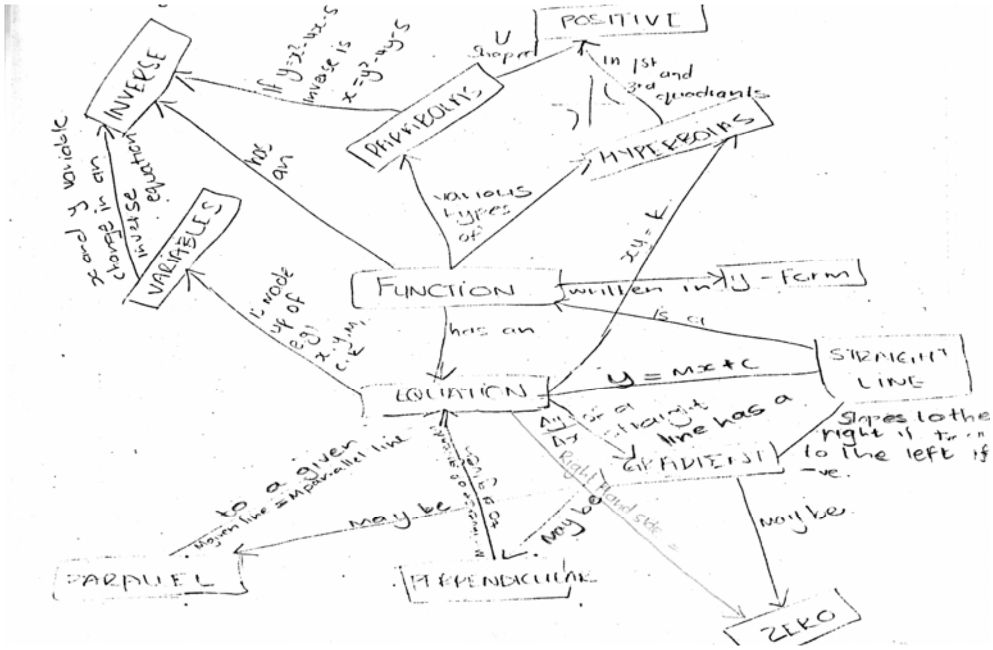

Figure 4: Concept map drawn by L26

5.2.1.1 Links made with the concept function

From Figure 4 it can be seen that L26 has given three explicit examples of functions. They are parabolas, hyperbolas and straight line. In the interview L26 explained the links that she had made with the concept function as follows (SN – researcher):

SN: Please explain what makes parabolas and hyperbolas functions.

L26: A function as like for every x value has one y value and you look at the hyperbolas and parabolas and you draw vertical lines down like you see like one x value and one y value. It won't be a relation because then there will be like for every x value there may be like two y values or more. That's what I thought.

SN: Why would you draw a vertical line?

L26: It is to test to see like if any point like for any x value how many y values there are, for any y value how many x value there are.

SN: Ok. What is it that makes you say that parabolas and hyperbolas are examples of functions?

L26: I just think that because that for every x value there one y value, so they are functions.

In the above excerpt it is clear that L26 is capable of providing examples of functions. However there seems to be some uncertainty as to exactly why they are functions when she uses the term relation. In other words she indicates that a function is not a relation. She is also able to justify why certain graphs are graphs of functions by means of the vertical line test. In her justification she indicates that there is something to be done before she can identify the graph as a function. Her explanation also alludes to a definition of function i.e. "for every x value there is only one y value." It is interesting to note that none of the other 33 learners gave a definition of function either on their concept maps or during interviews.

Three of the other interviewees i.e. L8, L30 and L32 also gave parabola as an example of function. An important question that needs to be asked here is:

Issue 2: What is the reason for learners giving examples of a concept but not providing its definition?

5.2.1.2 Links made between the concepts parabola and inverse

L26 also linked function to the concept of inverse in her concept map with the proposition: "Function has an inverse". She also linked parabola and inverse with the link: "if $y = x^2 - 4x - 5$ inverses is $x = y^2 - 4y - 5$". Implied in this link is the procedure for determining an inverse, i.e. by swapping the x and y. In order to find out what she actually knew about the links between function and inverse, I probed L26 as follows:

> SN: How is a graph of a parabola related to the graph of its inverse?
> L26: If you draw like two graphs you get like the axis symmetry in between and may be the different points at which they meet also indicate something. I don't know.
> (She drew a sketch to illustrate her explanation. See Figure 5 below.)
> She continued:
> L26: I think it is where y = x at the points where they meet …. (inaudible.)

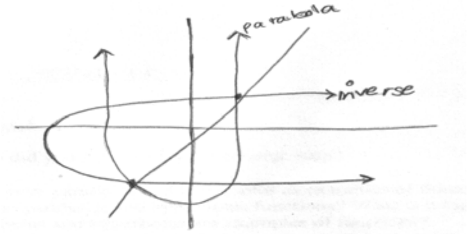

Figure 5: L26's illustration of a parabola, its inverse and the line of symmetry

In her illustration (shown in Figure 5 above), she firstly drew a set of axis, which was not labelled, and then a sketch of a parabola which she labelled "parabola". Thereafter she drew a sketch of the inverse of the parabola. The sketch of the inverse was consistent with the sketch of the parabola. L26 then proceeded to mark the points of intersection of the parabola and the inverse in the first and third quadrants. She joined these points of intersection in a line that resembled the graph of $y = x$.

L26 demonstrates understanding of the concept of inverse. Her sketches were appropriate. Furthermore there were two other points of intersection of the parabola and the inverse, i.e. in the second and fourth quadrants, which she did not mark. This indicates that she probably knew that the other two points of intersection were not significantly related to the parabola, its inverse and its line of symmetry. However L26 did not seem entirely convinced about the significance of the points of intersection when she said, "I don't know." From her illustration she has correctly related a parabola, an inverse and the line $y = x$. Hence, her illustration demonstrated that the relationships she drew between inverse and function is conceptual to an extent. It may be the case that the proposition "if $y = x^2 - 4x - 5$ inverse is $x = y^2 - 4y - 5$" is tied to the line of symmetry $y = x$.

In my opinion, to have a conceptual understanding of a parabola and its inverse, the following aspects should be demonstrated by a learner:

- determining the equation of the inverse correctly.
- being aware that the inverse is composed of two arms each with its own equation.
- the conditions under which the inverse is a function.
- the notation to be used when writing an inverse that is a function or non-function.
- the role of symmetry about the line $y = x$ in determining the co-ordinates of critical points.

L26 has demonstrated that she can find the equation of the inverse of a parabola when she wrote "if $y = x^2 - 4x - 5$ inverse is $x = y^2 - 4y - 5$". However I cannot conclude from this that she would be able to correctly write the equation of the inverse in the y-form. Hence, I cannot be certain that L26 realizes that the inverse she drew actually consist of two separate graphs. In fact I did not probe her with regard to the last three bullets above, so I am not sure whether she understood them.

The topic of inverse as presented in Laridon et al. (1995: 146), which is the textbook used by the learners who participated in this research, incorporates the above aspects. It is my opinion that learners should be exposed to the topic in its entirety at Grade 11 because the curriculum in Grade 12 does not address this topic again. Hence, learners who are not exposed to a rigorous treatment of this topic in Grade 11 will probably not get another opportunity to deal with the various aspects of the inverse of a parabola. There is no evidence in the concept maps to suggest that this topic was given rigorous treatment.

L26 needed to be probed further with regard to whether the inverse will be a function or not and under what restrictions will it be a function.

L25 was also asked how the graph of a parabola and its inverse were related. He replied as follows with the aid of two sketches (see Figures 6 & 7):

> L25: Parabolas will have the diagrams either if it is positive then it is u shaped going down like this (learner draws on an additional page) down the axis and the inverse is n shape in that manner and if it is an inverse well it is related in the manner that if you change the x and y points then you will do your inverse then you will either going to end up with a parabola such or in the opposite way.

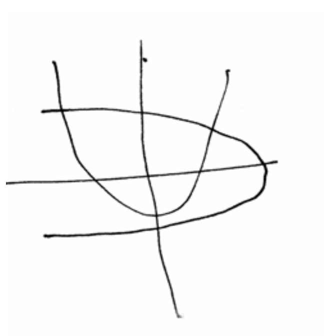

Figure 6: L25's u shaped parabola and "inverse"

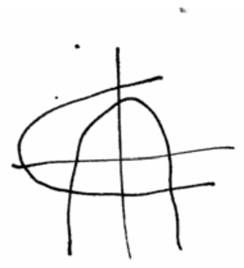

Fig 7: L25's n shaped parabola and "inverse"

L31 provided two sketches as well to show a parabola and its inverse. Figures 8 and 9 shows the parabola and the "inverse" respectively.

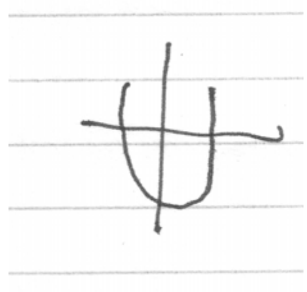

Figure 8: L31's parabola

Figure 9: L31's "inverse"

It can be seen from Figures 6 and 7 that L25 knows the general shape of the inverse of a parabola. However his sketches for the inverse are inconsistent with the parabolas he has drawn. He has "switched the inverses around." When probed further about the inverse drawn in Figures 6 and 7 he indicated that the inverse may not be as he had sketched it. However he explained that the inverse will depend on the equation of the parabola and he would need to work out the points, i.e. the coordinates of the x-intercept, y-intercept and the turning point, and then sketch the inverse. L25 seemed to imply that Figures 6 and 7 may not represent the correct inverses but that he knew how to go about drawing the inverse if he had to work with an equation of a parabola.

L31 seemed convinced that her sketches were correct as can be seen from the following excerpt:

> L31: They both have the y intercept and the x intercept … except in the parabola, not the inverse the other one … the y intercept is negative but in the inverse the y intercept will be positive.
> SN: Is there any changes with x intercept?
> L31: Umm… no not really.

Clearly Figure 9 does not represent the inverse of the parabola in Figure 8. L31 seems to have "inverted" the parabola to get the inverse because she maintained that the x intercepts remained the same but the y-intercepts are opposite in sign.

The above three learners have expressed quite differing views concerning a parabola and its inverse. L26 was clear in her expression of her ideas and her sketch reinforced this clarity. Although L25 drew inconsistent sketches for the inverses, he had a general idea about the relationship between a parabola and its inverse. In other words the sketches were of inverses but they were switched around. The extent to which he understood the relationship between a parabola and its inverse could have been explored further had he actually been given an equation of a parabola and asked to sketch its inverse. L31 has an inadequate understanding of a parabola and its inverse, as her sketch in Figure 9 does not depict an inverse. Another learner, L30, was able to draw correctly a sketch of a parabola and its inverse (see Figure 10 below).

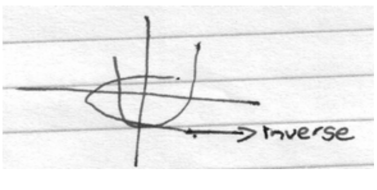

Figure 10: L30's illustration of a parabola and its inverse

5.2.1.3 Links made between the concepts function and equation

L26 has also linked the concepts of function and equation in her map with the proposition that "a function has an equation." She further linked the concept straight line with those of equation and function by indicating that a straight line has an equation y = mx + c and a straight line is a function. In addition to providing examples of functions, L26 seemed to relate a function with an equation. She was not alone in taking this view.

L25 also drew a link between function and equation. He gave the following explanation when asked to elaborate on the links he had made.

.

> SN: You say that function can be called an equation. Please explain what you mean by that and give an example.
> L25: When we studied the chapter functions and eh with graphs our teacher told us and I remembered and I put it down on the concept map as well. To give you an example, as I said functions can also be called an equation, has variables. Can take anything, for example, eh f: x maps or f x is equal to umm... 3x plus 5 something, for an example. The same function written in functional notation can also be written as an equation where fx changes to y where y = 3x + 5.

In the above extract L25 seems to indicate that since an expression written in f(x) notation can be rewritten as an equation then the equation represents a function. However, the equation may not necessarily represent a function. In the above example given by L25 is it a coincidence that the equation represents a function?

L30 was also asked to expand on the links she made between the concepts function and parabola, which brought in the concept of equation.

> SN: What makes a parabola a function?
> L30: The equation making it got a zero.
> SN: Can you give me an example.
> L30: Any numbers (learner writes an example of a linear equation), y = 3x + 1
> SN: So that is the equation of the parabola?
> L30: Yes. So you must make like the one side equal to zero.

What is implied here is that the terms 3x + 1 must be transposed i.e. y – 3x – 1 = 0. Now the equation has a zero. From the above extract, L30 seems to indicate that a function is an equation. It is interesting to observe that L30, like L25 has used an equation that is a function. A question that needs to be asked here is the following: “Were these learners exposed to equations that are non–functions and functions that are not equations?” For example, will these Grade 11 learners recognise that the equation $x^2 + y^2 = 9$ is a non-function? Or that the following diagram represents a function?

OPEL	→	KELS GP
CHRYSLER	→	VINO GP
SENTRA	→	CAAY GP

What may be possible reason(s) for learners to view a function as an equation? Let’s consider how the section on functions is presented in the textbook, *Classroom Mathematics Grade 11* (Laridon et. al, 1995), the textbook that is used by these learners.

The topic on functions is treated mainly in two parts. They are sketching graphs using given equations (see p.92) and answering questions based on graphical representations of functions (see p.100). Hence, when learners are engaged in this section of functions they are typically given an equation and are required to sketch its graph. For example, in Laridon et al. (1995, p. 92, 99) learners are required to “sketch the graph of each function on a separate system of axes”:

(a) $y = (x + 2)^2 - 1$ (b) $y = 2\,(x - 1)^2 + 3$

(c) $f\colon x \to (x - 2)^2 + 1$ (d) $f(x) = (x - 2)^2 - 1$

(e) $y = x^2 - 2x - 3$ (f) $y = 2x^2 + 8x + 10$

Another significant aspect of the topic functions in the Grade 11 is: completing the square, (see Laridon et al., 1995:93). This aspect requires learners to manipulate a given equation until it is in the required form. For example: in Laridon et al. (1995: 93)

Write $y = -2\,x^2 + 4x - 6$ in the form $y = a\,(x - p)^2 + q$

Solution: $y = -2(x^2 - 2x + 3)$, is eventually manipulated to

$$Y = -2(x - 1)^2 - 4$$

Learners would have experienced a similar treatment of linear functions when they were in Grade 10. For example, they are required to "sketch graphs for the following lines: (a) $y = 2x + 1$; (b) $x - 2y = 4$, and manipulate equations into the form $y = mx + c$" (See Laridon et al., 1984:128).

So learners' experiences of functions would have included much work with equations. Therefore it is not surprising that some learners come to see functions as equations. This may have been the case with L25 and L30.

Also, when sketching graphs, the zeros of the graph are determined by letting $y = 0$ and the equation becomes, for example, $x^2 - 2x - 1 = 0$ (see Laridon et al., 1995: 97). This could possibly be the reason why L30 wanted to make one side of the equation $y = 2x + 1$ equal to zero.

L25's association of the f(x) notation with equations and functions could have stemmed from the presentation of textbook problems such as the one given earlier, for example, $f : x \rightarrow (x - 2)^2 + 1$.

For these three learners (L25, L26 and L30) who have made links between function and equation, it seems that their experiences with function have resulted in them having an "equation model" (Martinez-Cruz, 1995) of function. Thompson (1994) observes that the equation view of function is the predominant view of function that is held by learners.

5.2.1.4 Links made between the concepts function and graphs

As mentioned earlier, L26 linked functions with graphs by giving three examples of graphs that are functions. Two of the six interviewees, i.e. L8 and L31 have also linked functions to graphs.

SN: You say that straight lines and parabolas are types of functions. Please explain what makes straight lines and parabolas functions?
L8: Um firstly, because a function is a where you take a equation and from there you draw graphs. So that's why it is a graph and that's why it takes from there we take an equation and make a graph which makes a function.

SN: What does the term function mean to you?
L31 Function are types of things that can be represented on a graph. (pause) Like a straight line or a hyperbola

From the above excerpts it is clear that L8 and L31 see a function as a graph. By graph it is meant a sketch of a straight line, a parabola or a hyperbola drawn on the Cartesian plane. A similar question to the one asked earlier in relation to function and equations could be asked here: "Were these learners exposed to graphs that are non-functions?"

Again a possible reason for L8 and L31 taking the view that functions are graphs could stem from the way these concepts are presented in textbooks. Considering Laridon et al. (1995), drawing and interpreting graphs forms the major part of the section on functions. Purely in terms of the space devoted to graphs in this section, it covers 16 of 24 pages. A typical problem in this section is one that has a diagram followed by a series of questions based on the diagram. The diagram usually consists of a parabola and straight line(s) graphs. For example, see Figure 11 below (Laridon et al., 1995:101).

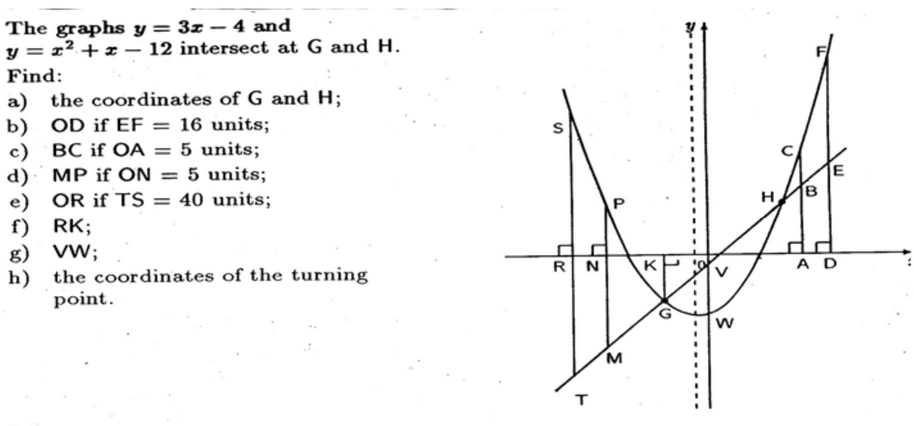

Figure 11: A typical problem on functions from learners' textbook: Classroom Mathematics Grade 11

Since the major part of functions is presented as in Figure 11, and that textbooks are used frequently as a source of problems, it is inevitable that some learners will take the view that functions are graphs. These learners' (L8 and L31) "concept image" (Thompson, 1994) of function is that of a graph. Martinez-Cruz (1995) points out that a "graph model" of function is one of the multiple representational views of function held by some learners.

5.2.1.5 Clarification of links: gradient relationships

L26 had written the link: "gradients may be parallel". Clearly gradients cannot be parallel, since gradients refer to a ratio. However when L26 was asked to explain what she meant by this link, she responded:

L26: like even in a case of the straight line they give you a given straight line and they ask you what will the gradient be of the line parallel to the straight line and it will be like the same thing the gradient.
SN: Would you make an example of such a straight line?
L26: Umm… like in the form $y = mx + c$ they tell you that m is 3 maybe then the gradient of the parallel line will also be 3.

It is clear that L26 may not have expressed the link correctly between the concepts gradient and parallel but when she was asked to elaborate on the link she was able to do so. Further her knowledge regarding the concepts gradient, parallel, equation and perpendicular is conceptual. The reason for this is that she has been able to draw relationships between the above concepts and link them into a network. For example, from her concept map, the concepts of equation and perpendicular are linked by the words, "m – inverse of given m." What is probably meant by this link is that the gradient of a given line is equal to the negative reciprocal of the perpendicular line. She has also linked the concepts equation and gradient with the proposition "$\Delta Y/\Delta X$." In other words the equation for the gradient of a straight line is $\Delta Y/\Delta X$. L26 was able to transfer the conceptual knowledge that she demonstrated on her map to the task on functions since she was able to solve the two problems, i.e. questions 2.3 and 2.4 which dealt with gradient, correctly. In fact, L26 correctly answered all the problems in the functions task.

L25 could not remember the gradient relationship between lines that are parallel or perpendicular. To my probes he responded, "It is very vague I can't think of it very clear now." His inability to recall the gradient relationships between lines that are parallel or lines that are perpendicular was also evident in his task where he failed to attempt question 2.4 which made reference to the equation of a line parallel to another line (see appendix 11B).

There was no evidence of gradient relationships on L8's concept map. In the interview, as can be seen in the excerpt below, she gave an example to demonstrate that she knew the gradient relationship between perpendicular lines.

> L8: Sir isn't perpendicular draw perpendicular then it means that the gradients of each add up and you multiply them and they come to -1. So it means that both the gradients so if the gradient is 2 over 3 in the one equation and the other equation it will be 3 over 2 and the signs will be negative.

However she was unable to give the gradient relationship between lines that are parallel. L8 failed to answer question 2.4 correctly which made reference to parallel lines. Similarly L30 response to my probe on gradient relationships was inadequate and she also got question 2.4 incorrect. However L31 and L34 were able to give the correct gradient relationship between lines that are parallel (see excerpt below) but they were unable to transfer this knowledge to the task on functions. They probably remember the facts but are unable to apply or identify that particular aspect in the task.

> L34: Lines that are parallel to each other, the gradients are equal.

Four of the six interviewees (L25, L30, L31 and L34) were unable to give the correct gradient relationship between lines that are either parallel or perpendicular to each other. For these learners recalling the gradient relationships proved problematic. Skemp (1978) observes that difficulty in recalling previously learned mathematics is indicative of instrumental understanding in that domain.

5.2.1.6 Links with the concept zero

All six learners used the concept zero in their map. The linkages included "point of origin is zero", "gradient maybe zero", "when solving an equation the RHS/LHS should be zero" and "x and y equal to zero", i.e. when determining the x-intercept (or y-intercept) let y = 0 (or x = 0) intercept. It is interesting to note that all six learners correctly determined the x-intercept in their functions task but none of them related the concept zero to the x- intercept in their maps. In fact none of the 34 learners made this connection in their maps. Perhaps these learners as a group did not encounter the concept zero in relation to x-intercepts of graphs.

L26 made the links "gradient maybe zero" and "gradient of straight line slopes to the right if +ve and to the left if –ve" on her concept map. She is able to describe the behaviour of a line under two conditions, i.e. when the gradient is either positive or negative. However she is not able to describe a line when it has a gradient of zero. When probed she said:

> L26: Maybe it is like a vertical line or something that there isn't really a
> slope so maybe there won't be a gradient.

From the above excerpt, L26 seemed to indicate that a line with a gradient of zero may be a "special case" of some sort when she indicated the line may be vertical. However it is interesting to note that she can describe a line with a positive or negative gradient but a line with a gradient value "in between these two" (i.e. zero) eludes her. Also she does not seem to be able to grasp the "gradient" of a vertical line. A question that is pertinent here: "How will L26 represent a line whose gradient is undefined?"

During the interview she was also given an opportunity to link the concept zero to any of the other concepts.

> SN: If you were to think of a link between zero and any other concept what might
> that link be?
> L26: May be equation , because every equation ends up like equal to ***zero,*** or normally
> If you have an equation you bring all the terms to the left hand side so that you have
> A zero. Maybe you have like a trinomials or things that you might be able to factorize
> and so you bring everything on to that side so therefore in an equation you will
> always have something that will say equal to zero.

The proposition that L26 added to her map was "right hand side equal to zero:" In explaining her link she indicates that there is "something that needs to be done". For example, she said, "You have an equation *you bring* all terms to the left hand side"

L8 was another learner who correctly described the behaviour of a line when the gradient was either positive or negative. However like L26 she also had difficulty in representing a line with zero gradient. When probed she offered a sketch as well in her response.

> L8: OK, here is your x-intercept and your y- intercept and usually your graph goes like this. But if it has a zero as a gradient then I can't go straight then it means it is getting on the intercept.

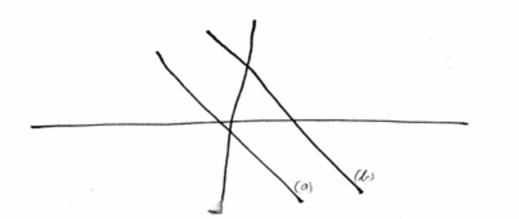

Figure 12: L8's illustration of a line with zero gradient i.e. line (a)

In the above excerpt, L8 is referring to line (a) which she indicates have a gradient of zero. By saying: "getting on the intercept", she probably means that the line (a) is passing through the origin. L8 does not seem to have realised that both the lines she have drawn have negative gradients. Hence, her diagram is contradicting her explanation that line (a) has a gradient of zero. Furthermore it does not seem that she has mixed her y-intercept with the gradient i.e. the m and c values in $y = mc + c$, of line (a). In other words line (a) has a y-intercept of zero and in her response she probably could not have been referring to the y-intercept of zero. I arrived at this conclusion because a little later in the interview she refers to an equation of a line and explicitly points out the gradient and correctly describes the behaviour of the line. She said, *"Then if you have a graph $y = -2x$, then this -2 tells you that it is a negative gradient. So when you draw your graph the line will slope to the left."* From the above quote, it is clear that L8 knows which coefficient in an equation of a line refers to the gradient.

The links made with the concept zero and other given concepts were essentially trivial. For example, the RHS of an equation may be zero. L8 and L26 were unable to describe a line that had a gradient of zero.

5.2.2 An analysis of L25's concept map

Figure 13 below shows L25's concept map. He has used 6 of the 8 concepts from the given list. His map may be classified as radial with the concept graphs being the central concept. His map has mainly trivial links such as "parabola is an example of a graph" and a few cross- links. His map is characterised by many examples.

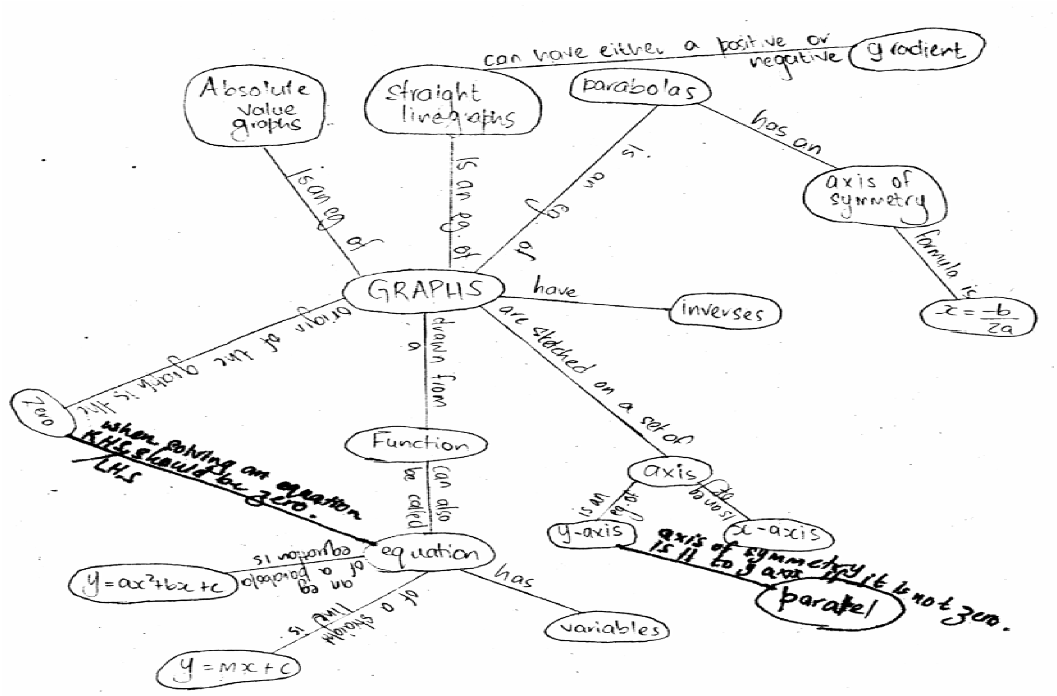

Figure 13: Concept map drawn by L25

5.2.2.1 Central concept

The central concept of his map is graphs and this was one of the four additional concepts that he had used in his map. His justification for using graphs as a central concept is that there are many links that can be made with graphs to the concepts from the given list. He made seven links with graphs and three of them were examples of graphs such as parabolas and straight lines. It is interesting to note that L25 did not choose his central concept from the given list. Did he perhaps not identify any concept from the list to which links to other concepts could be made?

5.2.2.2 Links with the concept parallel and perpendicular

As mentioned in section 5.2.1.5, L25 could not explicitly indicate the gradient relationships of lines that are parallel or perpendicular to each other. However he did elaborate on what he understood by "gradient of lines being perpendicular" with the aid of two sketches (see figure 14 and figure 15 below).

> L25: If you have your set of axis and you have your two gradients going parallel to each other (learner draws on page) something like that.
>
> L25: Perpendicular to each other. One gradient will go this way and the other … It will form a ninety degree angle

Figure 14: L25's illustration of lines with "gradients going parallel"

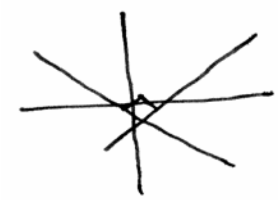

Figure 15: L25's illustration of perpendicular lines

In the above excerpts, L25's reference to "gradients going parallel" and "one gradient will go this way", is to lines that are parallel in figure 14 and perpendicular in figure 15. L25 does not seem to be able to differentiate between the relationship between the gradients of parallel lines and parallel lines themselves. Similarly, he could not differentiate between perpendicular lines and the gradient relationship that exist between perpendicular lines.

5.2.3 An analysis of L8's concept map

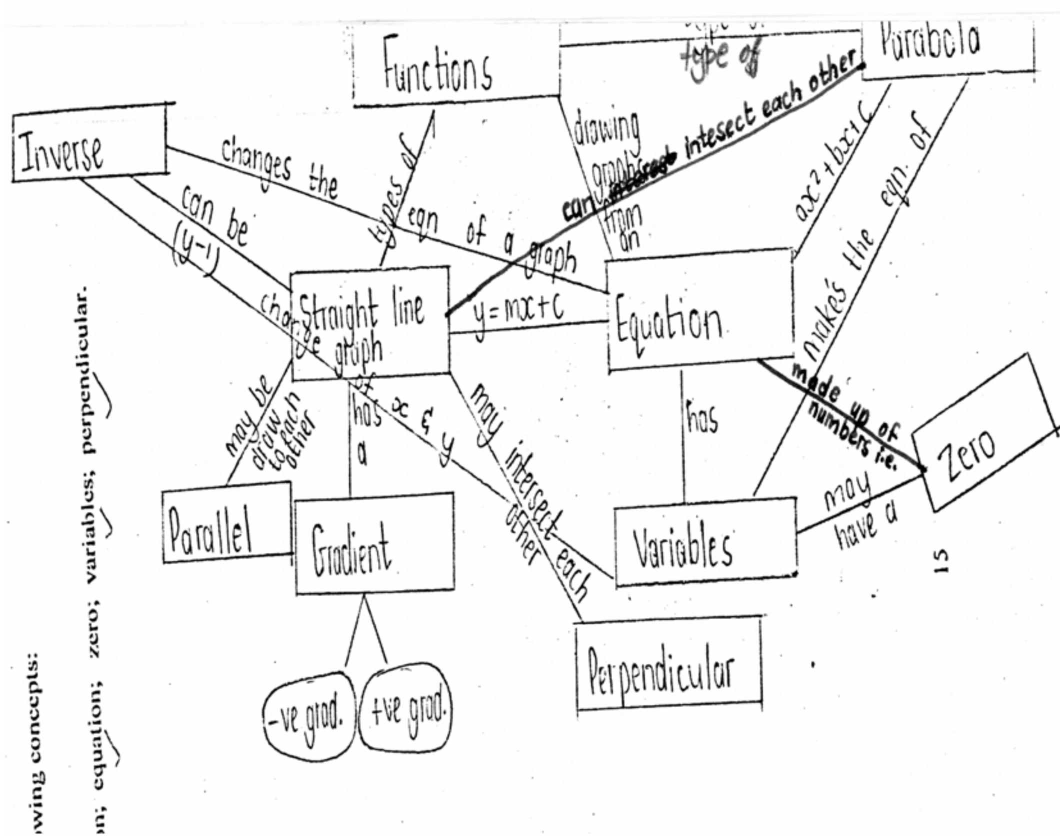

Figure 16: Concept map drawn by L8

Figure 16 above shows L8's concept map. She used all 8 concepts from the given list as well as two additional concepts i.e. parabola and straight line graph. The structure of her map may be classified as spider web with many cross-links. However, like L25, many of her links are trivial. For example, "equation has variables."

5.2.3.1 Clarification of links: "Inverse changes the equation of a graph"

L8 was asked to clarify what she meant by the link: "inverse changes equation of graphs". She used an example to explain the link.

> L8 : Like maybe for a straight line graph we say that y = 2x + 3. Then when we do an inverse graph you have to take the 2x, you have to make x the product of the whole equation. So it will be x is equal to umm 2y over 3 or something like that

For L8 to arrive at the equation of the inverse for the straight line y = 2x + 3 she indicates that there is "something to do." This is clear when she says, "…. when *we do* an inverse graph you will *have to take* the 2x, you *have to make* x the product …." (emphasis added). [L8 used the word "product", however I think that she meant "subject" since she ended by saying that x is equal to 2y over 3.]

Another learner, L25, similarly indicated some steps that he would follow when determining the equation of an inverse graph, as may be seen from the following excerpt:

> *L25: …*that *when you do a graph*, when you *trying to do an inverse* all the x values from the normal graph will change to the y values of the inverse and the y values of the normal graph will change to the x values of the inverse. *In this manner you will work out* the inverse graph.
>
> (emphasis added)

L25 also indicates that there is "something to be done" sequentially. In other words, he starts by first swapping the intercepts and then proceeds to determine (although he does not mention it) the turning point. I make this assumption since he uses the words "In this manner" which may imply that he is following a sequence of steps. An important question here is:

Issue 3: Does the nature of connections made by learners reveal the kind of understanding that they may have?

As mentioned earlier in the interview, L8 gave a correct example illustrating the gradient relationships between lines that are perpendicular. However on her map she does not link the concepts gradient and perpendicular. Similarly, L8 has got question 2.4 correct which pertains to determining the equation of a line parallel to another. This question requires one to know and apply the gradient relationship between parallel lines. However L8 does not link the concepts gradient and parallel on her map. An obvious question that arises here is: "Why don't L8 link concepts on her map while she was able to correctly answer questions involving these concepts in mathematical tasks? Could it be that the problem brings to the foreground the relationship that exists between these concepts? Or could it be that the mathematical problem "forces" the learner to seek out the relationship between these concepts, whereas in the concept mapping activity it is not imperative for the learner to link all concepts?

In a conventional mathematical problem one aspect at a time is being tested. For example, find the gradient of line 1 which is perpendicular to line 2. So the question directly channels the learner to seek and use the particular relationship between concepts "perpendicular" and "gradient." On the other hand, in a concept mapping task it is conceivable that the learner may know the relationship between two concepts but possibly may not be able to communicate this relationship as a proposition between two concepts. L8's map and interview may be used as an example to illustrate this point. In her map she does not link the concepts "perpendicular" and "gradient." However, in the interview it is clear that she knows the relationship that exists between these concepts as can be seen in the following extract:

> L8: Sir isn't perpendicular draw perpendicular then it means that the gradients of each add up and you multiply them and they come to -1. So it means that both the gradients so if the gradient is 2 over 3 in the one equation and the other equation it will be 3 over 2 and the signs will be negative.

L8, like L25, has also linked the concepts equation and zero by indicating that an equation can have a zero. What is meant here is that the RHS or LHS of the equation is equal to zero. The equations the interviewees are referring to seem to be quadratic equations. Their focus on quadratic equations probably stem from the topic of functions of which quadratic equations is an integral part at Grade 11 level. Instances were learners may have the RHS or LHS of a quadratic equal to zero is when they are determining the zeros of a parabola.

5.2.4 An analysis of L34's map

Figure 17 below shows L34's map. He initially used 4 of the 8 concepts from the given list, omitting the concepts variables, inverse; functions and gradient. The concept gradient was added to the map when the learner was given an opportunity to draw further links to his map.

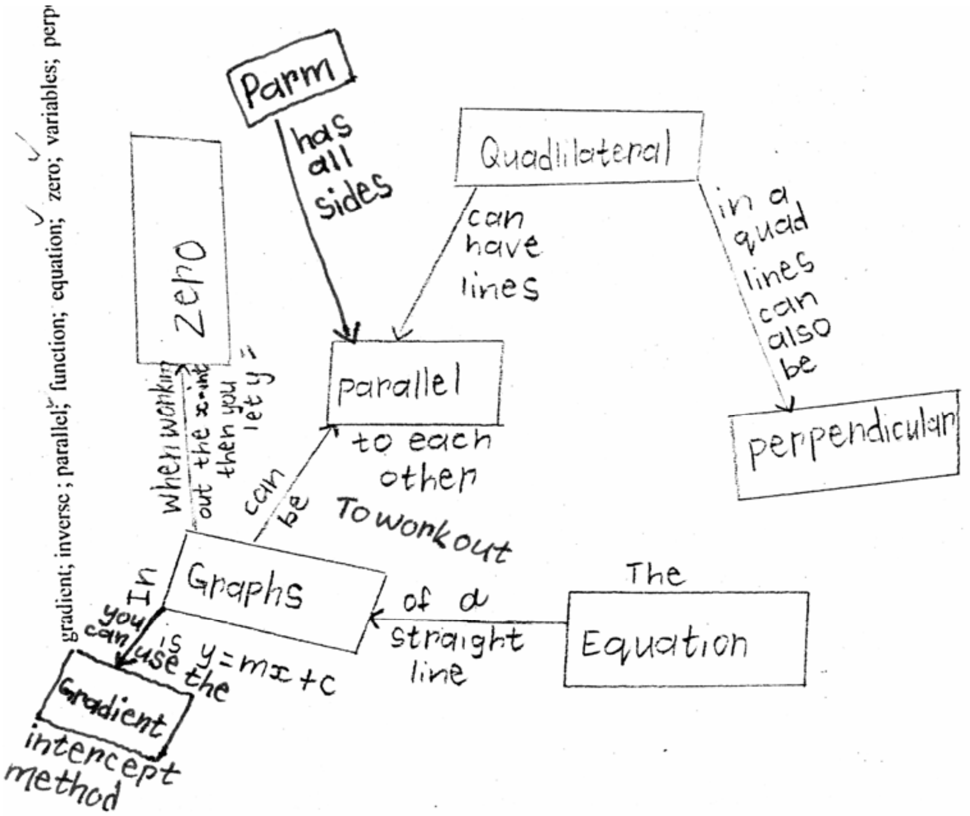

Figure 17: Concept map drawn by L34

5.2.4.1 *Use of additional concepts*

L34's map may be classified as hierarchical. His key concept was quadrilateral, a concept that was not part of the given list. His motivation for using the concept quadrilateral was that both parallel and perpendicular lines may be found in a quadrilateral. Hence, links between the concepts parallel and perpendicular may be made.

L34's map seems to have a geometric and an algebraic perspective to it. The four concepts: quadrilateral, parallel, perpendicular and "parm" (meaning parallelogram) have geometric links. For example, one of the links that L34 made was, "in a "quad" (meaning quadrilateral) lines can also be perpendicular and parm has all sides parallel".

What he probably meant is that the opposite sides of a parallelogram are parallel. The other four concepts: equation, graphs, zero and gradient have an algebraic perspective. For example, L34 wrote "the equation of a straight line graph is y = mx + c." The concept of parallel seems to link the two parts of his map.

L34's map is very interesting because it is the only map from the six interviewees that includes a geometric perspective to it. The other five learners took an exclusively algebraic view of the concepts from the given list. The rationale for the choice of concepts in the list was that they cut across the algebraic and geometric domains of school mathematics and it would be interesting to see the kinds of relationships that were drawn between these concepts. A possible reason for the other five learners drawing their maps from an algebraic perspective is that textbooks (e.g., Laridon et al., 1995) present algebra and geometry as separate branches of mathematics with very little or no integration between the two domains. Hence, learners assimilate these concepts as being either algebraic or geometric.

L34, like the other interviewees, were given an opportunity to add new links to his map. L34 added the concepts "parm" and gradient. He linked the concepts gradient and graphs with the proposition "to workout graphs you can use the gradient intercept method." What he meant here is that to draw a straight line graph one could use the gradient y-intercept method. Prior to adding new links to his map, L34 had just given the correct gradient relationship, as mentioned earlier (section 5.2.1.5), between lines that are parallel. However he did not link the concepts of gradient and parallel with that specific relationship i.e. parallel lines have equal gradients. Also, he had question 2.4 incorrect (see appendix 11D), a question that made reference to parallel lines. The question here is: Why doesn't L34 correctly apply the gradient relationship between parallel lines (a relationship that he knows) in mathematical tasks? Possibly L34 just know the fact that parallel lines have equal gradients, but does not know how to use this fact in a task. What does this then imply about his understanding in this area of gradients and parallel lines? It seems that his understanding in this area is instrumental to an extent since he only knows the fact and not its application.

Tables 4.1a and 4.1b show that 20 learners used additional concepts. These "concepts" were mainly examples of functions and terms commonly used in mathematics classrooms such as x-axis, y-axis, x-intercept and y-intercept.

5.2.4.2 Omitted concepts

L34 explained that the reason for omitting some of the concepts from the given list was that he could not make a link with them and the other concepts. Could this failure to make links to the other concepts imply that he did not fully understand the omitted concepts such as inverse or variables? Tables 4.1a and 4.1b show that 6 other learners omitted concepts. The most frequently omitted concepts were perpendicular and variables. A pertinent question here is: why did some learners add or omit certain concepts?

Issue 4: What is the implication of learners omitting or adding concepts to their maps?

5.2.5 An analysis of L31's map

Figure 18 below shows L31's map (see appendix 10 E). Her map is hierarchical with the central concept of function. She used the 8 concepts from the list as well as three additional concepts: parabola, straight line and graph. L31's map is characterised by many examples. The links in her map are mainly trivial. For example, "straight lines can be parallel" and "the x and y axes are perpendicular."

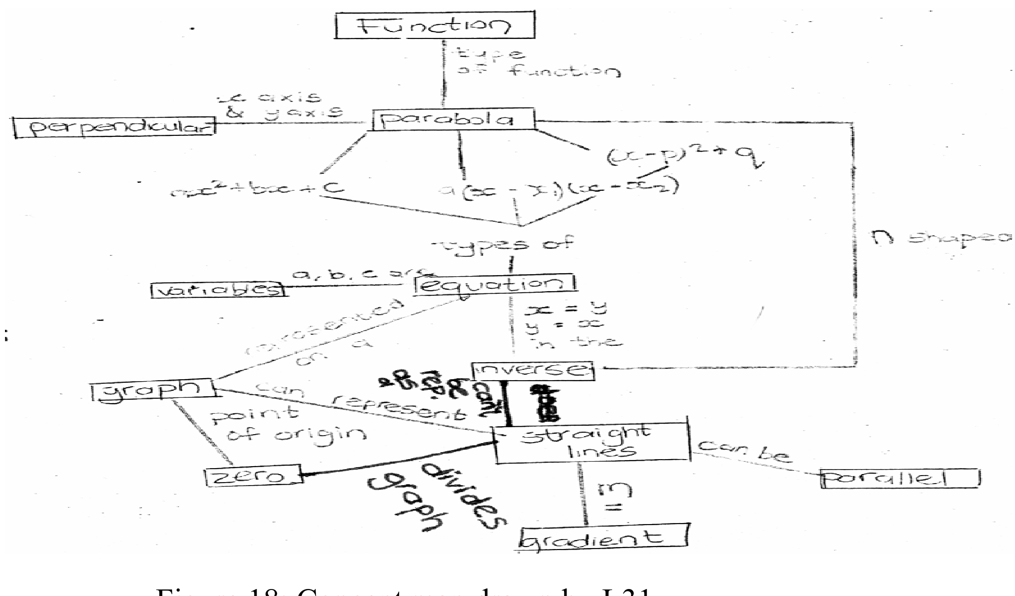

Figure 18: Concept map drawn by L31

5.2.5.1 Clarification of links

5.2.5.1.1 Straight line and its inverse

L31: A straight line doesn’t have an inverse. No straight line inverse.

L31 drew a sketch (see figure 19 below) to elaborate on what she meant by “a straight line doesn’t have an inverse.”

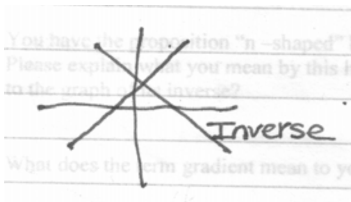

Figure19: L31’s illustration of a straight line “not having an inverse”

In the above figure, L31 has drawn a line which she labels as the inverse. However she did not consider this line to be an inverse since “there won’t be a different shape.” She gave an example to explain what she meant here. She had previously mentioned in the interview that the inverse of a “U” shaped parabola is an “∩” shaped parabola i.e. the graph of the inverse has a different shape. However, in the case of a straight line, the “shape” remains the same and the “inverse” just slopes in a different direction. Therefore she maintains that a straight line does not have an inverse.

L31 is looking for a change in the shape of a graph, for the graph to have an inverse. She did not seem to consider the inverse in terms of symmetry about the line $y = x$. If she had viewed inverse in these terms then she would have probably realised that the “shape” of a straight line cannot change but just its inclination will change.

5.2.5.1.2 Straight line passing through zero

L31: When a straight line passes through zero it divides the graph into two equal halves.

What L31 means is that if a straight line graph is drawn through the origin then the Cartesian plane seems to be divided into halves. L31 (and L25) refer to the origin as zero and to the x and y axes as graphs. The reference to the Cartesian plane as graphs probably stem from learners’ first experiences with drawing graphs in which they start by drawing a set of axes. Hence, axes become synonymous with graphs.

5.2.5.1.3 Equation represented on a graph

L31 had linked the concepts equation and graphs with the linking words "represented on a" (see figure 18 above). What L31 means by this link is that the graph, for example, a parabola, is labelled with an equation.

5.2.6 An analysis of L30's concept map

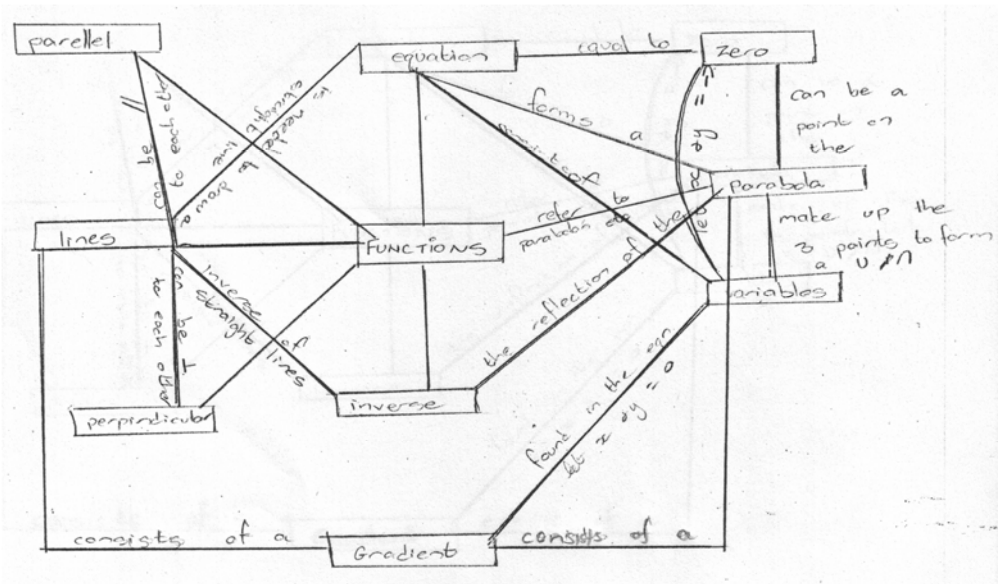

Figure 20: Concept map drawn by L30

L30's map is presented above. Her map may be described as radial with functions as the central concept. She has used all 8 concepts from the given list and included two additional concepts: lines and parabola. Her map has many cross-links. However the propositions are primarily trivial. For example, "lines can be perpendicular to each other." Some concepts are linked with a line but there are no linking words or proposition accompanying the line. For example, linking lines between the concepts parallel and function; or perpendicular and function, do not have any linking words. This is despite the fact that I emphasised the importance of propositions to concept mapping during the induction process.

5.2.6.1 Clarification of links: "Lines consist of a gradient"

L30 was requested to elaborate on the above connection: *Lines consist of a gradient*. In her elaboration she pointed out that the gradient of a line is the c value in the equation

$y = mx + c$. She arrived at this conclusion by substituting $x = 0$ in the equation which resulted in $y = c$, as is evident in the excerpt below

> L30: It is like saying letting your $x = 0$ and if you let $x = 0$ then it is going to be
> $y =$ to the c value, which is your gradient.

L30's elaboration on gradient is in direct contrast to her solution to question 2.3. This question required the learner to find the equation of a line. In her solution (see appendix 11F) L30 used a formula and correctly determined the gradient of a line, although she had used incorrect co-ordinates. She then wrote the correct y–intercept for the line which was -5 as $c = -5$ and gave the equation of the line as $y = 4/3\ x - 5$. What is clear from the above is that L30 could correctly determine and distinguish between the gradient and the y–intercept of a line in the functions task. However in the interview her elaboration on gradient was completely different to what she did in the functions task and it was also incorrect.

L30 also wrote the proposition "parabola consists of a gradient." Similarly, she elaborated that if $x = 0$ in $y = ax^2 + bx + c$ then $y = c$ is the gradient of the parabola. In her elaboration she used a linear equation, $y = 2x + 1$, and referred to it as an equation for a parabola.

L30 confuses between the y-intercept and gradient of a line when she has to elaborate verbally between them. However, she is quite capable of doing calculations which involve y-intercept and gradient. A possible reason for this dichotomy is that L30 knows procedures for determining the gradient and y-intercept of a line but only have a limited understanding of their meanings. In other words, L30 has instrumental understanding (Skemp, 1976) in this area.

5.3 Summary of data analysis

The analysis of data presented in chapter 4; sections5.1 and 5.2 have shown firstly, that the number of connections made by learners varied from 5 to 18 and the marks learners obtained on the functions task ranged from 2 to 20 out of a total mark of 20. Secondly, learners provided many examples of concepts such as function, equation and variables in their maps. Furthermore, learners omitted and /or added concepts to the given list. Finally, the nature of connections are primarily trivial.

Four key issues have emerged from the data analysis:

- Do learners who make more connections necessarily have a better understanding of the concepts involved?
- What are the reason(s) for learners being able to provide many examples of concepts such as function, without knowing what is meant by the concept function?
- Why do learners omit and/or add concepts and what is the nature of these additional concepts?
- Does the nature of the connections made by learners reveal the kind of understanding they have?

The above issues are the subject of discussion in the next chapter.

CHAPTER SIX: DISCUSSION OF FINDINGS

It is important that students make connections. It is even more important for students to make as many connections as they possibly can. Even of more critical importance is that the connections that learners make are valid. As seen from the data analysis, concept mapping provides a tool for learners to make these connections.

6.1 Discussion issue one: Learners making most connections

As indicated in 5.2 there were only 7 learners who made the most connections. What does the fact that only 7 learners had made the most connections as compared to the other learners mean? According to Williams (1998) this means that these 7 learners showed evidence of a better understanding than the other learners because they made more connections than the others. However the claim that Williams makes should not be a general one. According to the data analysis presented in chapter 4, L31, who had made the third most connections among the 34 learners, did not do as well as the others in the functions task. As can be seen from Table 4.2 (p.39) she only obtained 7 out of 20 marks. This implies that one needs to look beyond the number of connections that are made as an indicator of better understanding. So perhaps one needs to also consider the quality of the connections made.

As seen from Table 4.1b, of the 15 connections that L31 made 13 were valid and 2 were invalid. Also of the 13 valid connections 12 were trivial and 1 was non-trivial. It is evident from the data that making many connections may not necessarily mean better understanding since many of these connections, as in the case of L31, may be trivial. Perhaps one may also need to perform well in a task to demonstrate a better understanding.

L31's poor performance in the task in contrast to the many connections she made in her concept map may be explained in terms of the nature of the two tasks. The nature of the two tasks is different. The functions task is limited. For example, a typical question in the task may be: determine the equation of a line. There isn't anything further that the learner can do after determining the equation of the line. However for learners such as

L31 who indicate more links it may be because the concept mapping task provided the opportunity to look for other possibilities rather than just the answer as it is in the functions task.

Considering the 10 learners who made the least number of connections i.e. between 4 and 6 connections, 4 of these learners, L1, L10, L23 and L28 had an average performance in the functions task. Their scores were between 9 and 12 out of 20. L6 who had made just five connections obtained a score of 15 out of 20 in the task. So for these five learners, although they are not able to make many connections between the given concepts they do perform adequately in the task. From the perspective of the data it is clear that doing a concept mapping task is different from a computational functions task. Hence, it is not surprising to find that there are learners who are able to make more connections than other learners but under perform these learners in the functions task and vice versa. Also, it could signify that students may be unfamiliar with the concept mapping task because it is not a method of "assessing" that they are used to.

For these learners who made many connections but under performed in the functions task, concept mapping being a metacognitive tool (Williams, 1998), helped them to think about what they were doing and it offered them the opportunity to think in varied ways. This is in contrast to the functions task which seeks just a correct answer. For these learners the data shows that engaging them in concept mapping gives them an opportunity to think in mathematics. This is a dimension of mathematics that learners need to be made aware of rather than mere engagement in computational tasks.

When learners are being assessed it is crucial that a range of assessments is looked at. It is evident that some learners will perform better in one assessment than in another. Therefore it is imperative that learners be given many avenues to express their knowledge and understanding.

6.2 Discussion issue two: Use of examples

Looking at the sample all 34 learners made valid propositions. Fifteen of these 34 learners have written propositions that tend to be about examples of given concepts. For example, a parabola or a straight line is an example of a function. The fact that so many of these learners have used examples in their maps is not uncommon because if one considers the conventional practice in mathematics teaching the use of examples is quite extensive. Does this then say something about the role of examples in learners' learning? Zaslavsky and Louise (undated) point out that the use of "instructional examples" is an "integral part of teaching mathematics that has a great influence on students' learning" (p.1). They further elaborate that instructional examples play a central role in mathematics and mathematical thinking.

The data can possibly be explained by acknowledging that the use of examples by learners is possibly coming from classroom practice in which they are exposed to, both in the teaching and use of textbooks. For example, Laridon et al. (1995) utilise examples to illustrate a concept. While it is important that learners are given examples to illustrate the concept, they are not pushed further to get a deeper understanding of the concept. Possibly for these learners their knowledge of examples has not helped them to know what these concepts mean and what they are. For example, what is a function? In the interviews learners were given an opportunity to elaborate on this concept. However they were not able to do so in any significant way as illustrated by the following excerpt.

> SN: What does the term function mean to you?
>
> L31: Function are types of things that can be represented on a graph. (Pause) Like a straight line or a hyperbola.

While these learners are able to give examples of concepts such as function, they are not able to elaborate what these concepts actually are. This finding does not seem to be unexpected. As Mwakapenda (2006) argues, a close look at various school mathematics textbooks (e.g., Laridon et al., 1995) in use in schools shows that a central feature of the presentation of function concepts involves explorations and exercise problems on types of functions. A form of knowing that is predominant here concerns knowing what examples of functions one has had many experiences with. This is also consistent with an acknowledged fact that much of school mathematics practice involves working with examples of problems relating to specific concepts (Bills, 1996). Aspects of functions such as “what do we mean by function?” are not a regular activity of school practice.

6.3 Discussion issue three: Nature of connections

6.3.1 Procedural connections

Interviews revealed the nature of some of the connections made by learners such as L25, and L8. Some of the connections made by these learners indicated that there was “something to do.” For example, (see the first two excerpts in section 5.2.3.1 p.64) L25 said, “*…when you do a graph, to do an inverse...”* and L8 said, “*…take the 2x, make x the* ***product*** (she meant subject) … x is equal to 2y over 3…” (See appendices 13 A1 and 13 B). The above excerpts indicate that these learners have procedural understanding since their responses have elements that are common to those of procedural understanding. For example, both the learners had indicated that there was “something to be done” when determining the graph (in L25’s case) or the equation (in L8’s case) of the inverse. Furthermore L8 transformed the equation ($y = 2x + 3$) by following a sequence of steps (although this was not explicitly articulated) i.e. interchange x and y and thereafter make x the subject. I inferred that she interchanged x and y since her transformed equation included 2y.

On the other hand the learners’ concept maps did not show evidence of relationships that were conceptual in nature, with the exception of L26 (see sections 5.2.1.2, p.49 and 5.2.1.5, p.58). As mentioned earlier, the connections generally involved examples of concepts with mainly trivial connections. This lack of conceptual relationships may be attributed to traditional mathematics practices that involve little integration of the different mathematical domains.

Tatolo (2005) identified concept relationships indicating “existence” and “possibility” in his study involving Grade 12 learners. I also found these types of relationships in my study as well and it is to these that I now turn.

6.3.2 Relationship indicating existence

Three of the six learners interviewed seem to have the view that some mathematical concepts are embedded in others. This existential relationship was evident in the concept maps of L8, L25 and L26. For example, L8 wrote the proposition "Equation has variable" and "equation made up of zero". L25 wrote that "graphs have inverses". This view shared by these learners of mathematical concepts, concurs with the view that "mathematical concepts do not exist as isolated entities having a life on their own but are related to each other so as to form a coherent structure" (Vygotsky, 1962, in Mwakapenda, 2004).

6.3.3 Relationships indicating possibility

Five of the six interviewees had propositions on their maps which used the word "can". For example, L8 wrote "straight lines can be parallel." Tatolo referred to this type of relationship as indicating "possibility". This possibility relationship was also found by Williams (1998:418) who wrote: "the student viewed graphed and polynomial as being connected in the same way to function and that the connections indicated possibility rather than necessity." The prevalence of the word "can" in the learners' propositions is suggestive of the classroom practice that these learners are part of and its influence on their map constructions. Such practices leave learners with many "possibilities" in their learning of mathematics which could also manifest as "uncertainties" for the learner.

6.3.4 Relationships with no explicit propositions

Only one learner, L30, of the six learners interviewed had drawn a linking line between concepts but had not written how the concepts were related. For example, the concepts function and inverse were linked without an explicit proposition (see figure 20, p.71). Although L30 had encountered these concepts it is clear that she lacks confidence in relating them. This is evident in the following excerpt.

> SN: Can you write a proposition between these two concepts, function and inverse?
>
> L30: I don't think so.

Why did L30 seem hesitant to link these two concepts even though she had drawn a sketch of an inverse of parabola? (see figure 10, p.53). Possibly she had a partial understanding of the concepts, and was therefore reluctant to write a proposition. I arrived at this conclusion because earlier in the interview she had asked "Sir, what exactly is a function?" This question conveys uncertainty. Hence the absence of a proposition between concepts may stem from a learner's partial understanding of the concepts involved.

6.4 Discussion issue four: Omission and addition of concepts

6.4.1 Omission of concepts

For some learners, for example L31, concept mapping is an enabling tool because it allows them to communicate their ideas and knowledge. However for other learners, for example L34, concept mapping seems to be a constraining tool because it does not allow them to show all possible relationships that they know but cannot put them on a concept map.

Learners may omit concepts because they do not see how a particular concept relates to the others at the time of constructing the map. To illustrate this point L25 may be used as an example. In the interview he was asked to give reasons why he did not use the concepts parallel and perpendicular in his map. His response was "the link did not come to mind." It is interesting to note that in his response he did not say that he didn't know a link but that he just could not think of the connection at that particular time. However during the interview he made a connection with one of these two concepts, i.e. he linked parallel and y-axis (see appendix 10B). So he was capable of making a connection with the concept parallel but had not done so at the time he was constructing his map.

It is also reasonable to suspect that concept mapping may be a constraining tool for a particular learner at some of the time because he may be able to communicate a relationship in some other way but not on his map. For example L34 said in the interview that parallel lines have equal gradients. However this relationship was not put into his concept map even after he was given another opportunity to make links on his map. Looking at his concept map (see appendix 10D) it is possible to have made a connection between the two relevant concepts i.e. parallel and gradient. So the question here is: Why didn't he make the connection? Possibly the answer lies in the structure of the map construction.

In concept map construction learners don't usually write the proposition fully. They often have a limited space to write the proposition between the two nodes. Possibly if they are unable to fit in the proposition then they just leave it out. In this way concept mapping could be a constraining tool for certain learners but not in totality. Furthermore the nature of concept mapping is such that the propositions are usually short phrases. Now if a learner has much more to write as a proposition, for example, a sentence or two which cannot fit between two nodes then the proposition may be omitted as well.

Taber (1994) suggests another way of constructing a concept map to overcome the problem of limited space between nodes that some learners may experience. He suggests that learners construct maps by linking concepts with lines but without writing in the proposition. The linking lines should then be numbered. Below the map, learners could use a key corresponding to the numbers on the linking lines to write out their propositions in full sentences. In addition, to overcoming the problem of limited space between nodes, Taber's suggestion has the added benefit of affording learners an opportunity to explain their connections fully. Their propositions could then be assessed more effectively because there will be less room for ambiguity. Bolte (1999) in her research requested learners to write an accompanying interpretive essay to their concept maps which explained the organisation and clarified the relationships in their maps. Bolte's research design may also be useful in encouraging learners to produce a comprehensive a map as possible. One of the ways that Wilcox and Lanier (2000) used to "enhance the power of the maps to reveal student understanding" (p.138) was to ask students the following two questions related to their maps:

- Select a [concept] on your map that you think you know a lot about and write all you know about that [concept]
- Select a [concept] on your map that you think you don't know a lot about and tell why you think you are having trouble with that [concept]

Wilcox and Lanier (2000) found that learners' written responses to the above questions provided insights into student understanding in terms of what the students knew and on what they still needed clarification.

6.4.2 Use of additional concepts

It is important that learners made links not just with the concepts that they were given but that they also saw opportunities to think about and bring in other concepts that could be linked to the given set. The question here is: How different are these additional concepts from those that were from the given set? This is important because in mathematics there are other concepts that are different from those of the given set. Can these learners make links between those concepts and the ones already given? A learner who is able to make such links is different from a learner who is just able to stay within the boundaries of the given concepts.

Table 4.1a and 4.1b shows that there are 20 learners who used additional concepts. Four of these learners did not use all 8 concepts from the given set, whilst 16 of them did. The vast majority of these additional concepts were just examples of functions, such as parabola, hyperbola, straight lines and absolute value graphs. In my analysis I have characterised these concepts as additional concepts but they are not necessarily new or different concepts. They are just qualifiers of the given concepts.

Only one learner, L34, used a new concept i.e. a concept that is distanced from the given concepts. He used the concept of quadrilateral in his map. This concept is from the geometric domain of mathematics. L34 showed that he was able to make some connection between the algebraic and geometric domains. Being able to see connections among different mathematical domains is important for learners as they may be able to recognise the same concept being used in various contexts and in equivalent forms. Learners who stay within the boundaries of the given concepts my not necessarily see and realise the inter-linking of different mathematical areas. As pointed out in section 1.3.1 the RNCS Mathematics (DoE, 2001) and the NCS Mathematical Literacy (DoE, 2003a) demand that teaching and learning of mathematics develops in learners a sound knowledge of mathematical relationships i.e. learners who have a "critical awareness of how mathematical relationships are used" in various contexts (DoE, 2001, p.17).

In this chapter I elaborated on four key issues to explain the data. In the next chapter, I conclude the research report by providing a summary of the findings of the study as well as presenting recommendations for classroom practice; teacher education and further research. Finally I reflect on the research process.

CHAPTER SEVEN: CONCLUSION

7.1 Summary of findings

The purpose of the study was to investigate Grade 11 learners' understanding of the function concept by exploring the relationships that they made between mathematical concepts related to functions. A concept mapping task was given to elicit these relationships.

7.1.1 Critical research questions

a) What relationships do Grade 11 learners make between the concept function and other mathematical concepts? To what extent are these relationships conceptual in nature?

b) How do learners explain through interviews the relationships they make between function and other concepts?

c) What is the effectiveness of concept maps as a tool for exploring Grade 11 learners' understanding of the function concept?

a) Relationships made by Grade 11 learners

The Grade 11 learners at Kelcay Secondary are able to make relationships between the function concept and other mathematical concepts. Two thirds of the sample was able to make between 7 and 18 connections. The nature of the relationships made by the learners was largely valid with only two learners having made three invalid connections each (see tables 4.1a and 4.1b). However the vast majority of the valid connections were trivial with the highest number of valid non-trivial connections for any one learner being two. Most learners did not have any non-trivial connections amongst their valid connections. The relationships made by some of the learners (see section 6.3) were procedural in nature. The learners (with the exception of L26) did not make relationships that could be regarded as conceptual. Many of the relationships consisted of providing examples of concepts (see section 6.2).

b) Learners' explanations through interviews

The interviews provided an opportunity for the learners to make explicit their relationships. The interviewees made use of examples to substantiate the connections that they made. On occasion, the examples were incorrect as was the case with L25. Most of the learners drew diagrams to explain their connections. Furthermore some of the learners (e.g., L26) restated their links to make them mathematically sound. They knew a valid connection between the concepts but had used inappropriate linking words.

c) Effectiveness of concept maps

The study found that concept mapping was an enabling tool for some learners whilst for other learners it could be considered a constraining tool (See 6.4.1.) Furthermore it was also found that learners who made many (e.g., L31) connections did not perform well in the functions task. However for most of the learners, such as L26, L8 and L14, who produced a good or an average map, they also had a good or an average performance in the functions task. Hence concept mapping in this regard may be considered an effective tool to explore Grade 11 learners' understanding of the function concept.

Five of the six learners interviewed, with the exception of L34, expressed positive responses with regard to how they felt about constructing concept maps. L34 did not express any feelings towards the task. In other words it was just a mathematical task that he carried out. Other interviewees, for example, L25 and L26 (see appendices 13B and 13C) expressed responses which showed that they had enjoyed concept mapping. L8 and L25 said that concept mapping had given them a better understanding of the chapter on functions (see appendices 13 A2 and 13B). Taking into consideration these learners' responses to this activity, concept mapping may be considered an effective tool to explore Grade 11 learners' understanding of the function concept.

7.1.2 Reflection on learners' understanding

What can be said about these learners' understanding of the concept function in view of the above findings? Considering section 2.1.4 it may be said that these learners have a partial understanding of the concept function since they have not established "significant relationships between conceptual and procedural knowledge." Although they had made valid connections between function and related concepts, most of these connections were trivial. Hence, their understanding of the function concept was partial.

Taking section 2.1.5 into account and the lack of reference to the definition of function by these learners, it may be concluded that these learners' understanding is largely based on their concept image of function. This conclusion is consistent with Vinner and Dreyfus (1989) who pointed out that students do not necessarily use the definition of function to make decisions on whether a mathematical object is an example or non-example of a function. In most instances they decide on the basis of their concept images.

Learners' understanding of the concept function is also linked to the context in which they learned them. For example, they illustrated their understanding by drawing diagrams of graphs to depict functions. Some learners also demonstrated their understanding of the concept function by the equation view they held of function.

7.2 Recommendations

7.2.1 Recommendations for classroom practice

"Metacogs" are one of the forms of assessment at Grade 12 level in school mathematics in South Africa. Educators could introduce the concept of metacogs, which essentially is a radial concept map, to Grades 8 to 11 as well. Educators in their practice could evolve metacogs to concept maps which is inclusive of nodes and propositions. Concept mapping will provide learners with opportunities to display relationships among concepts in a topic, between topics and between domains of mathematics such as algebra, geometry, trigonometry and analytic geometry. The importance of writing propositions with sufficient detail that clearly define the relationship between concepts should be emphasised to learners in the induction process to concept mapping. If learners write such proposition then it will be clear for the educator to "look at the organisation and structure of a learner's knowledge within a particular domain" (Williams, 1998:414). Information gathered from learners' concept maps such as scope of concepts used, concepts omitted and the validity of propositions could inform the educator's subsequent teaching.

Educators could also use concept mapping as a tool for assessment (Al-Kunifed and Wandersee, 1990, cited in Taber, 1994). Concept mapping will give learners an opportunity to demonstrate their understanding in an alternate way as compared to computational assessments that they are usually engaged in. Considering the positive responses expressed by the interviewees (e.g., L25, L26 and L30) with regards to how they felt doing their concept maps, concept mapping may offer learners an interesting and an enjoyable way of working in mathematics.

Function is a critical concept in the mathematics curriculum. Educators should endeavour to get learners beyond the point of merely knowing examples of functions to the stage where they know what a function is. Furthermore, according to NCS: Mathematics (2003b), learners in Grades 11 and 12 should know the definition of function.

7.2.2 Recommendation for teacher education

Recommendations are given for teacher education that could take the form of in service training or educator developmental workshops at either school, cluster or district level. Concept mapping can be developed and presented as a module. From my interactions with mathematics educators and mathematics facilitators at Grades 11 and 12 levels, awareness of concept mapping is limited to metacogs. Mathematics educators teaching at Grade 8 to 10 levels that I have interacted with have almost no knowledge of concept mapping. Hence, there is a potential demand from mathematics educators for workshops and in-service training in concept mapping.

Theoretical principles that underpin concept mapping (e.g., Novak, 1998) can be presented to educators. Educators can also then be inducted into concept map construction using, for example, the strategies suggested by Bartels (1995). Educators may be exposed to a variety of ways of evaluating concept maps such as those suggested by Chinnappan et al. (1999) and Prawat (1989).

7.2.3 Recommendations for further research

- If interviews are being conducted to explore relationships between concepts, the researcher can explore, with the learners, whether there are multiple connections that could be made between two concepts. For example, in the concept mapping task L34 linked the concepts parallel and graphs with the proposition "can be to each other." What is meant here is that straight line graphs can be parallel to each other. L34 may then be probed with a question such as: You have linked these two concepts with this proposition; can you now link these same two concepts in another way?

  A learner may make a connection between two concepts but this may not necessarily be the only way the learner knows how these concepts are related. In order that a more complete picture may be established of the relationships that the learner can make between concepts, it is important to expose learners to the construction of multiple connections during the induction process of concept mapping.

- Learners may also be probed with regards to the omission of concepts at the time of constructing the map. It may be the case that the learners cannot think of a connection at the time of constructing the map, or it may be even related to ineffective teaching and learning practices or a lack of teaching at a particular site (Mwakapenda and Adler, 2003, cited in Mwakapenda, 2004).

7.3 Reflections: Disruptions to research process

Extensive planning was undertaken for this study. However, I discovered that due to extenuating circumstances flexibility in the preparation of the study is essential.

The educator that I initially approached to participate in the study, i.e. whose learners were to be the participants in the study, resigned from Kelcay Secondary at short notice. A second educator was approached to participate in the study.

The schooling schedule was disrupted in the week that I had planned to conduct interviews since 18 educators were called at short notice to undergo RNCS training. As explained in section 4.3.3 interviews were subsequently rescheduled. During the study access to the school and participants also had to be renegotiated since I had taken up a promotion post at district level. These disruptions to the research process had placed additional pressure on me with regard to the timeframes within which I was operating. Although I had effectively managed the situation and coped with the disruptions, the disruptions afforded me the opportunity to reflect more on the ideas of my research. As Vithal (1998) explains, disruptions need to be considered a positive aspect of mathematics education research.

Reflecting on the research process, planning for every eventuality was not possible yet being flexible in my approach allowed me to manage the process to completion.

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APPENDICES

Appendix 1A

Concept mapping task

Appendix 1B

Task on functions

Appendix 1C

Marking memorandum for task on functions

APPENDIX 1A

CONCEPT MAPPING TASK AND

TASK ON FUNCTIONS

INSTRUCTIONS TO PARTICIPANTS:

Write your name in the space provided on both pages.
You will be given an hour to draw the concept map and to complete the task on functions.

NAME: ______________________________________

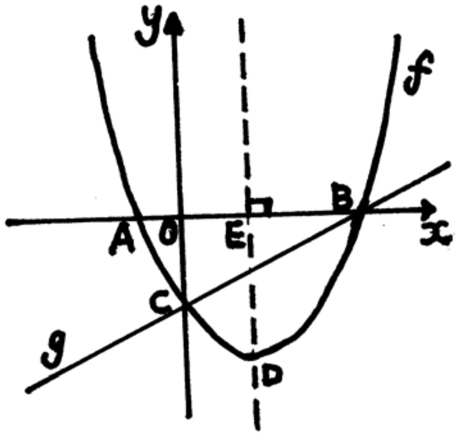
1. TASK: CONCEPT MAPPING

Draw a concept map using the following concepts:
gradient; inverse; parallel; function; equation; zero; variables; perpendicular.

APPENDIX 1B

NAME: ____

2. TASK ON FUNCTIONS



The sketch above shows the graphs of

f : $x \to x^2 - 4x - 5$ and

g : $x \to mx + c$

Answer the following questions on the double page, showing all your calculations:

2.1 Calculate the lengths of OA; OB; OC; and ED.

2.2 Write down the equation of the broken line?

2.3 Determine the equation of g.

2.4 Determine the equation of a straight line passing through A and parallel to g.

Thank you for your time and participation in the project.

Appendix 1C

Marking memorandum for task on functions.

	Mark allocation
2. $f : x \rightarrow x^2 - 4x - 5$ $g: x \rightarrow mx + c$	
2.1 $x^2 - 4x - 5 = 0$ $(x - 5)(x + 1) = 0$ $x = 5$ or $x = -1$	√ equating to zero √ both factors correct √ both solutions correct
OA = 1, OB = 5 and OC = 5 [6]	√ √√ one mark for each correct length of line segment.
$x = -b/2a$ $= -(-4) / 2(1)$ $= 2$	√ correct substitution into correct formula √ correct simplification
y coordinate of D = $(2)^2 - 4(2) - 5$ $= -9$ ED = 9 [5]	√ correct substitution √ correct simplification √ correct length of line segment
2.2 $x = 2$ [1]	√ correct answer only
2.3 $y = x - 5$ [4]	√√ correct m √√ correct c OR any alternate solution
alternate solution : $m_{BC} = \frac{\Delta Y}{\Delta x}$ $= \frac{0 - (-5)}{5 - 0}$ $= \frac{5}{5}$ $= 1$ $y = 1x - 5$	√ correct substitution √ correct simplification √√ correct equation √ correct gradient

2.4 $m = 1$ (parallel to g) and going through (-1; 0) $y = mx + c$ $0 = 1(-1) + c$ $c = 1$ $y = x + 1$ [4] MAX [20]	√ correct substitution for y √ and x values √ writing of correct equation

Appendix 2

Information letter to principal

Appendix 2 Letter to the school principal
Mathematics Education Research Report

Research Title: Using concept mapping to explore Grade 11 learners' understanding of the function concept.

I, Selvan Naidoo, am currently registered as a post-graduate student at the University of the Witwatersrand under the supervision of Dr Mwakapenda. As part of my studies I am doing a research project that focuses on learners' understanding of mathematical concepts. I would like to request permission to conduct my research at this school, which will involve one Grade 11 mathematics class.

The aim of my research is to use concept maps to explore Grade 11 learners' understanding of the function concept. This study is important for two reasons. Firstly, it tries to get insights about how learners actually understand specific concepts in mathematics and whether they are able to describe what they understand or know about such concepts.

Secondly, metacogs, which are similar to concept maps, have just been introduced in Grade 12 mathematics curriculum. Metacogs are being promoted as a new way of assessment in mathematics where learners are encouraged to organise and reflect on their knowledge by indicating links between mathematical concepts. Presently, there is little understanding among learners about what metacogs are. Also little is known how educators actually use metacogs to make sense of learners' mathematical experiences in this new way of assessing mathematics.

The research will be conducted over a period of 1 week using 3 double mathematics periods. Two of the double periods will be used to introduce the learners to the activity of concept mapping. In the third period the learners will be involved in a concept mapping task and also do one mathematics task based on their knowledge of functions. Some learners will be selected for an interview so that they describe more fully their understanding of concepts. Interviews will be tape-recorded so that I obtain an accurate record of what learners say. Each interview will last about 30 minutes.

All data collected in the study will be kept confidential and will be used only for research purposes. In my final report, learners will be referred to by pseudonyms and no details of the school will be reported. At the end of the study I will provide the school with a summary of the research findings. The results of the research may be reported in educational journals and at conferences.

I will be delighted to be granted permission to conduct the study at the school. If permission is granted, you will still be free to withdraw the school from the study at any time. If this happens, any data collected from the learners will not be used.

If you have any questions about the research project you may speak to me or contact my supervisor, Dr Mwakapenda, on (011) 717 3410.

Thank you
Selvan Naidoo.

Appendix 3

Information letter to educator

Mathematics Education Research Report

Letter to the educator

Research Title: Using concept mapping to explore Grade 11 learners' understanding of the function concept.

Hi! I am currently registered as a post-graduate student at the University of the Witwatersrand under the supervision of Dr. Mwakapenda. As part of my studies I am doing a research project that focuses on learners' understanding of mathematical concepts. I would like to request your permission to involve your Grade 11 higher grade mathematics class in my research.

The aim of my research is to use concept maps to explore Grade 11 learners' understanding of the function concept. This study is important for two reasons. Firstly, it tries to get insights about how learners actually understand specific concepts in mathematics and whether they are able to describe what they understand or know about such concepts.

Secondly, metacogs, which are similar to concept maps, have just been introduced in Grade 12 mathematics curriculum. Metacogs are being promoted as a new way of assessment in mathematics where learners are encouraged to organise and reflect on their knowledge by indicating links between mathematical concepts. Presently, there is little understanding among learners about what metacogs are. Also little is known how educators actually use metacogs to make sense of learners' mathematical experiences in this new way of assessing mathematics.

The research will be conducted over a period of 1 week using 3 double mathematics periods. Two of the double periods will be used to introduce the learners to the activity of concept mapping. In the third period the learners will be involved in a concept mapping task and also do one mathematics task based on their knowledge of functions. Some learners will be selected for an interview so that they describe more fully their understanding of concepts. Interviews will be tape-recorded so that I obtain an accurate record of what learners say. Each interview will last about 30 minutes.

All data collected in the study will be kept confidential and will be used only for research purposes. In my final report, learners will be referred to by pseudonyms, no details of the school will be reported and no reference will be made to the learners' mathematics educator.

At the end of the study I will provide the school with a summary of the research findings. The results of the research may be reported in educational journals and at conferences.

I'm hopeful that you will give permission for your learners to be involved in this project as they will be contributing to the learning of mathematics. If permission is given you will still be free to withdraw your class from the project at any time. If this happens, any data collected from your class will not be used.

If you have any questions about the project, please speak to me or you may contact my supervisor on 011 717 3410.

Thank you.

Selvan.

Appendix 4

Information letter to learners' parents

MATHEMATICS RESEARCH PROJECT

Letter to learners' parents.

Research Title: Using concept mapping to explore Grade 11 learners' understanding of the function concept.

Hello! My name is Selvan Naidoo, a member of the mathematics department at Liverpool Secondary. I am currently registered as a post-graduate student at the University of the Witwatersrand under the supervision of Dr. Mwakapenda. As part of my studies I am doing a research project that focuses on learners' understanding of mathematical concepts. I would like to request your permission to involve your child in my research.

The aim of my study is to use concept maps to explore Grade 11 learners' understanding of the function concept. Concept maps are similar to mindmaps and flow diagrams that your child may have used in other subjects, especially in biology.

I envisage your child's participation in the project will be beneficial to him/her. Metacogs, which are similar to concept maps, have just been introduced in the grade 12 mathematics curriculum. Metacogs are being promoted as a new way of working with mathematics which encourages learners to organise and reflect on their knowledge by making links between mathematical concepts. Participation in the study will increase your child's exposure to this new way of working in mathematics.

If you give permission for your child to participate in the project, he/she will be asked to draw a concept map and answer one mathematics question based on their knowledge of function. This activity will take no more than one hour to complete and will be done during their mathematics period. Your child may be selected for an interview to describe more fully his/her understanding of concepts. The interview will last about 30 minutes and will be conducted immediately after school. I would like to tape record the interview so that I have an accurate record of your child say.

In my final report, learners will be referred to by pseudonyms. All data provided by your child will be securely kept under lock and key. If you consent to your child's participation in the study, and decide at a later stage to withdraw your child, you will be free to do so. If your child is withdrawn, I will not use any information provided by your child.

I will be delighted if you grant permission for your child to participate in the project as he/she will be contributing to the learning of mathematics. If you have any questions about the project, please contact me or my supervisor at the following telephone numbers:
Selvan Naidoo (011 849 2316); Dr Willy Mwakapenda (011-717 3410)

Appendix 5

Consent letter for learners

Appendix 5

MATHEMATICS EDUCATION RESEARCH PROJECT

Consent letter for learners.

Research Title: Using concept mapping to explore Grade 11 learners' understanding of the function concept.

Hello! My name is Mr S. Naidoo, a member of staff at this school. I am currently registered as a post-graduate student at the University of the Witwatersrand. My supervisor is Dr. Mwakapenda. As part of my studies I am doing a research project aimed at exploring grade 11 learners' understanding of the function concept. I am writing this letter to invite you to participate in this project.

As part of my research, I will introduce you to an activity called concept mapping. Concept maps are similar to mindmaps that you may have drawn in other subjects, especially in biology. I will use two double periods to introduce you to drawing concept maps. This introduction will take place in your mathematics classroom. I'm hopeful that my findings will be used to improve the teaching and learning of mathematics at our school.

Participation in the project will be beneficial to you. Metacogs, which are similar to concept maps, have just been introduced in the grade 12 mathematics curriculum. Metacogs are being promoted as a new way of working in mathematics where learners are encouraged to organise and think about their knowledge of mathematics. At the moment, there is very little research that has explored how mathematics learners in South Africa use metacogs or concept maps in their learning. This means that your participation in this research will be contributing to a new way of working in mathematics.

If you agree to participate in this research project you and all the other learners in your class will be asked to be present during the lessons when you will be introduced to concept maps. In a subsequent lesson you will be asked to draw a concept map and answer one mathematics question based on your knowledge of functions. This activity will take no more than one hour to complete. This activity will not be used for your class marks. There is a possibility that you might be selected for an interview. This interview will last for 30 minutes and will focus on the concept maps you have drawn. With your permission, interviews will be tape-recorded so that I can ensure that I make an accurate record of what you say. If you are selected for the interview, you will not be forced to answer any questions. If you do not know an answer to a question that is being asked you will not be penalized. The answers you give to the questions asked will only be seen by me.

I will not use your real names in the final report. I will remove any reference to personal information that might allow someone to identify you. If however, for any reasons you would like your real name to be used in the report you will need to make this known to me.

Your responses to the activity and the tape recording of the interview will be securely kept under lock and key for a period of five years. Thereafter this data will be destroyed.

Remember that you do not have to take part unless you want to. You will be free to withdraw from the project at any time. If you choose to leave, I will not use any information obtained from you.

I am hopeful that you will choose to participate in the project. Your participation will make an important contribution to this study and hence to the learning of mathematics. If your have any questions about this research project you may speak to me or my supervisor about it.

Mr S. Naidoo
Telephone No. 011 849 2316.

Dr W. Mwakapenda (Supervisor), Wits University, Telephone No. 011 7173410

Appendix 6

Consent form for learners

Appendix 6

Consent form for learners

Research Title: Using concept maps to explore Grade 11 learners' understanding of the function concept.

Researcher: MR S. Naidoo

Name of learner: ____

1. I agree to participate in the research project named above, whose details have been explained to me. A written information letter has been given to me to keep.
2. The project is for purposes of research and the results will be reported in a research report and may be reported in scientific and academic journals and conferences.
3. I have been informed that the information I provide will be kept secure under lock and key for a period of 3 years after which it will be destroyed.
4. If I am selected for an interview, I agree that the researcher will tape-record the interview.
5. I have been informed that my participation or non-participation in this research project will have no effect on the marks that I get for mathematics.
6. I am free to withdraw my consent at any time during the study, in which case my participation in the study will immediately cease and any information obtained from me will not be used.

_________________________________ ____________________

Signature of learner Date

Appendix 7

Consent form for educator

Appendix 7

Consent form for educator.

Research Title: Using concept maps to explore Grade 11 learners' understanding of the function concept.

Researcher: MR S. Naidoo

Name of educator____

7. I give permission for my class to participate in the research project named above, whose details have been explained to me. A written information letter has been given to me to keep.
8. The project is for purposes of research and the results will be reported in a research report and may be reported in scientific and academic journals and conferences.
9. I have been informed that the information my class provide will be kept secure under lock and key for a period of 3 years after which it will be destroyed.
10. I am free to withdraw my class at any time from the project, in which case my class's participation in the study will immediately cease and any information obtained from them will not be used.

___________________________________ ___________________

Signature of educator Date

Appendix 8

Consent form for parent/ guardian

Appendix 8

Consent form for parent/ guardian.

Research Title: Using concept maps to explore Grade 11 learners' understanding of the function concept.

Researcher: MR S. Naidoo

Name of parent / guardian ____

Name of learner: ____

11. I agree for my child to participate in the research project named above, whose details have been explained to me. A written information letter has been given to me to keep.

12. The project is for purposes of research and the results will be reported in a research report and may be reported in scientific and academic journals and conferences.

13. I have been informed that the information provided by my child will be kept securely under lock and key for a period of 3 years after which it will be destroyed.

14. If my child is selected for an interview, I agree that the researcher will tape-record the interview.

15. I am free to withdraw my consent at any time during the study, in which case my child's participation in the study will immediately cease and any information obtained from him/ her will not be used.

_____________________________________ ____________________

Signature of parent /guardian Date

Appendix 9

Letter of approval from the Department of education to conduct the research

UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Appendix 9

Date:	12 July 2005
Name of Researcher:	Naidoo Selvan
Address of Researcher:	27 Oak Street
	Northmead Ext 4
	Benoni, 1501
Telephone Number:	(011) 4213853
Fax Number:	(011) 4213853
Research Topic:	Using Concept mapping to explore Grade 11 Learners' Understanding of the Function Concept
Number and type of schools:	1 Secondary School
District/s/HO	Ekurhuleni East

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*

Office of the Senior Manager – Strategic Policy Research & Development
Room 525, 111 Commissioner Street, Johannesburg, 2001 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0488 Fax: (011) 355-0286

4. A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
5. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
6. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Senior Manager (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
7. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.
8. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
9. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
10. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
11. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
12. On completion of the study the researcher must supply the Senior Manager: Strategic Policy Development, Management & Research Coordination with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.
13. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
14. Should the researcher have been involved with research at a school and/or a district/head office level, the Senior Manager concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

ALBERT CHANEE
ACTING DIVISIONAL MANAGER: OFSTED

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	
Date:	

Appendix 10

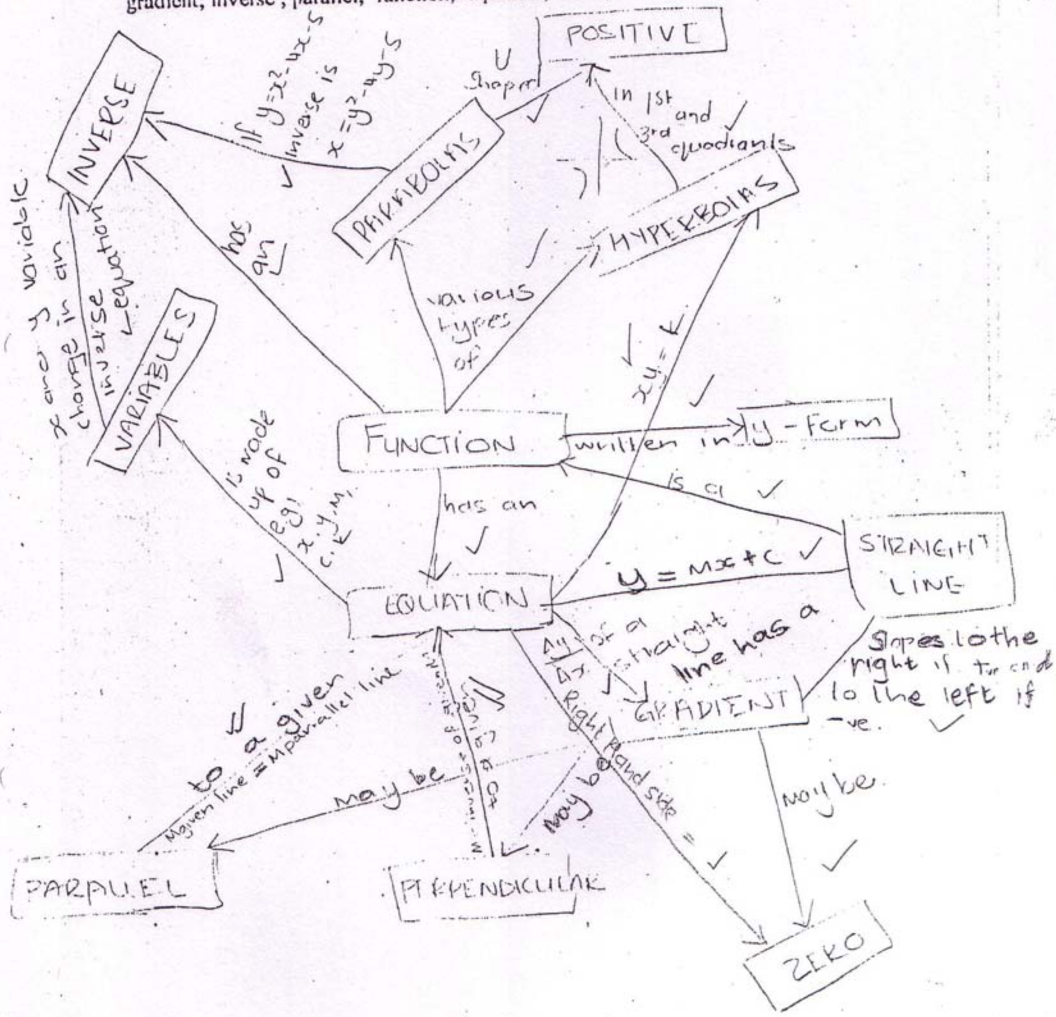
Learners' concept maps

1. TASK : CONCEPT MAPPING. Appendix 10 A

L26

Draw a concept map using the following concepts:

gradient; inverse ; parallel; function; equation; zero; variables; perpendicular.



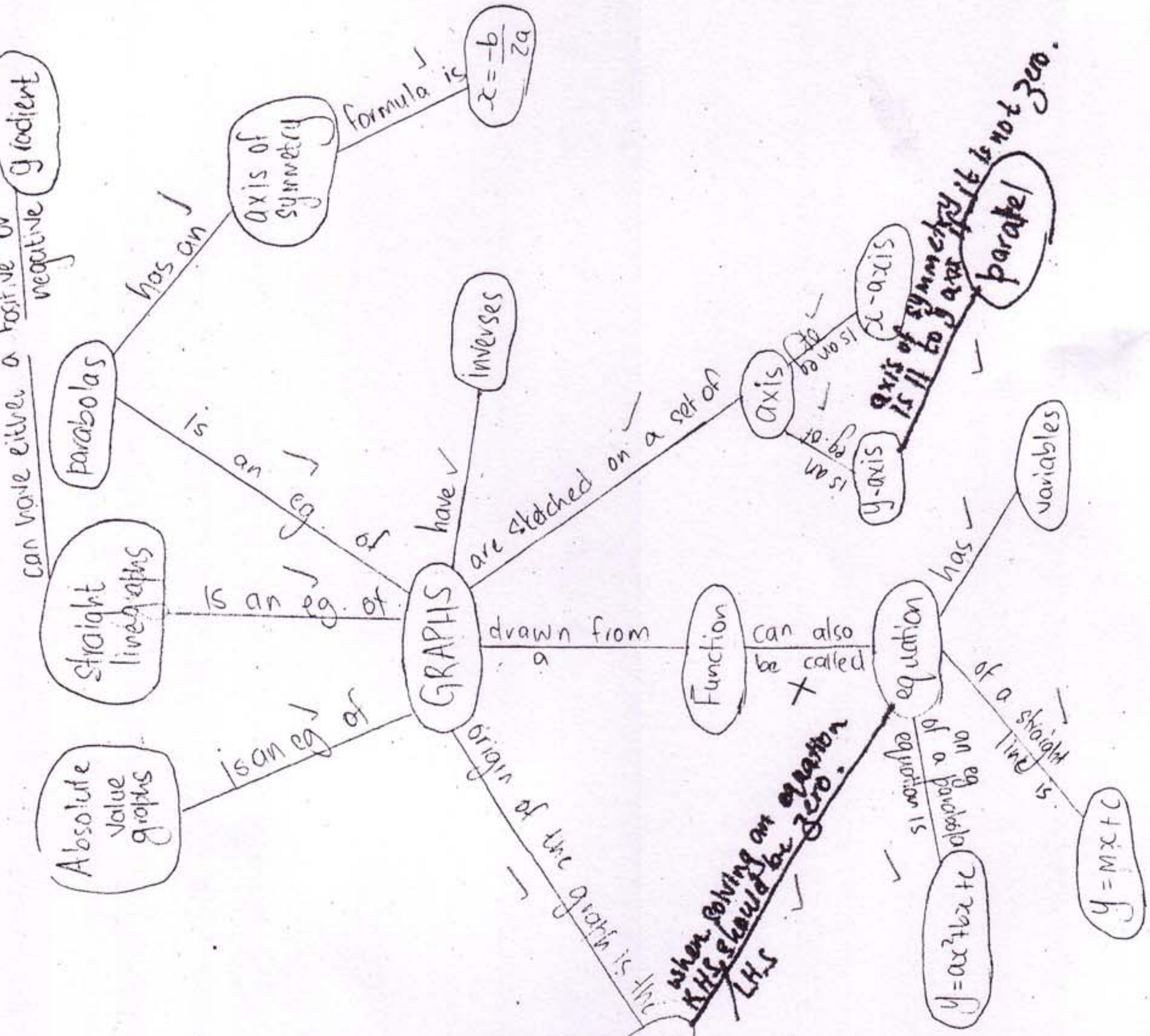
1. TASK : CONCEPT MAPPING.

Appendix 10 B

L25

Draw a concept map using the following concepts:

gradient; inverse ; parallel; function; equation; zero; variables; perpendicular.



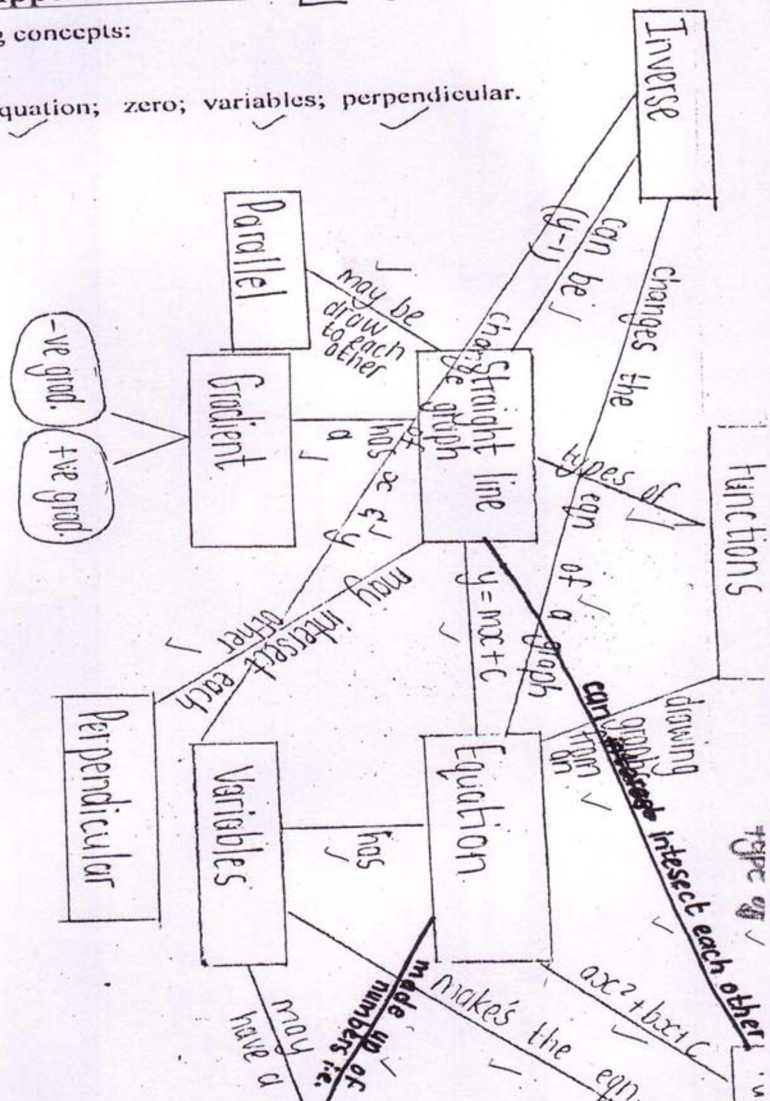
1. TASK : CONCEPT MAPPING.

Appendix 10 C

L 8

Draw a concept map using the following concepts:

gradient; inverse ; parallel; function; equation; zero; variables; perpendicular.



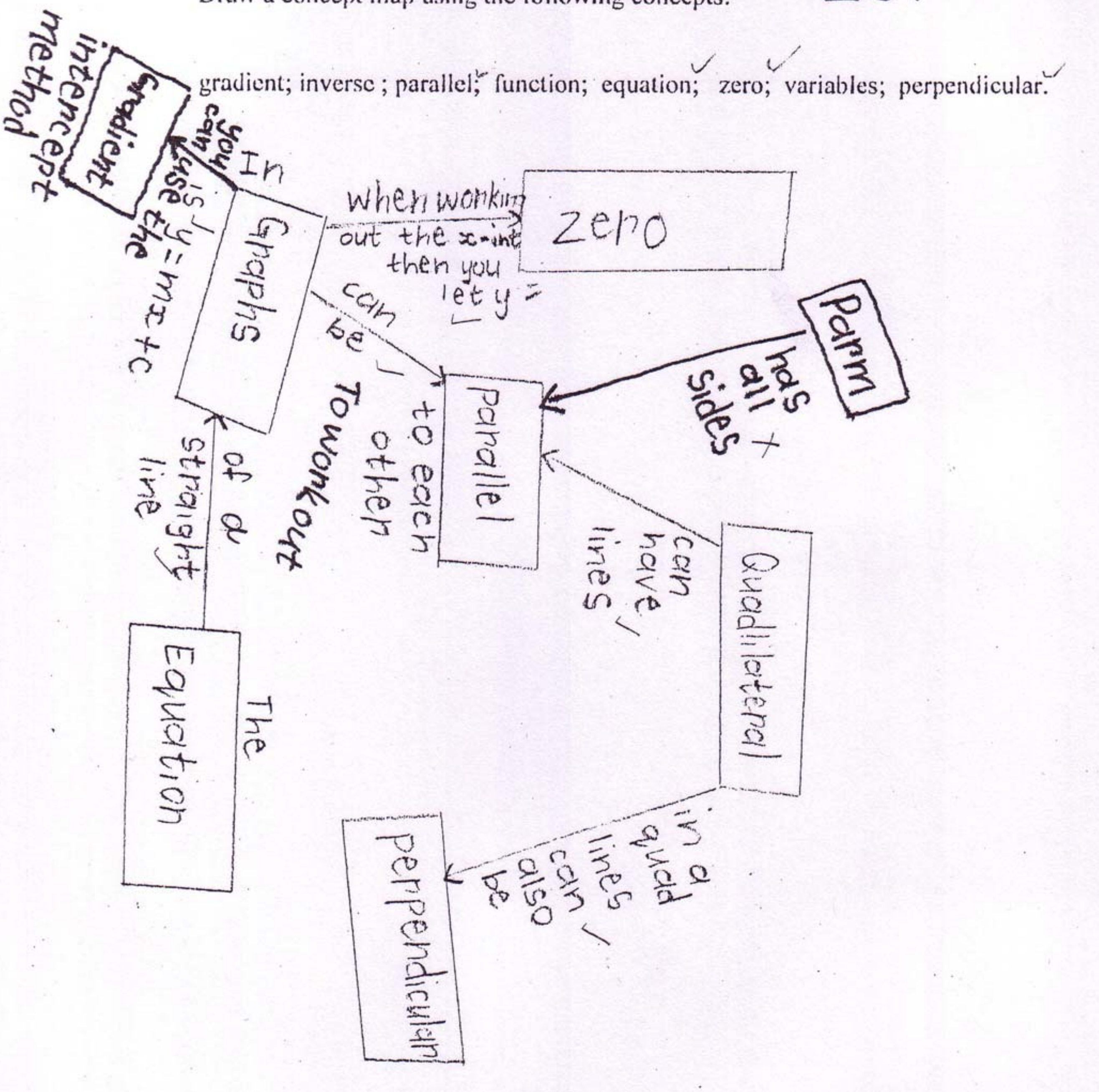
1. TASK : CONCEPT MAPPING.

Appendix 10 D

L34

Draw a concept map using the following concepts:

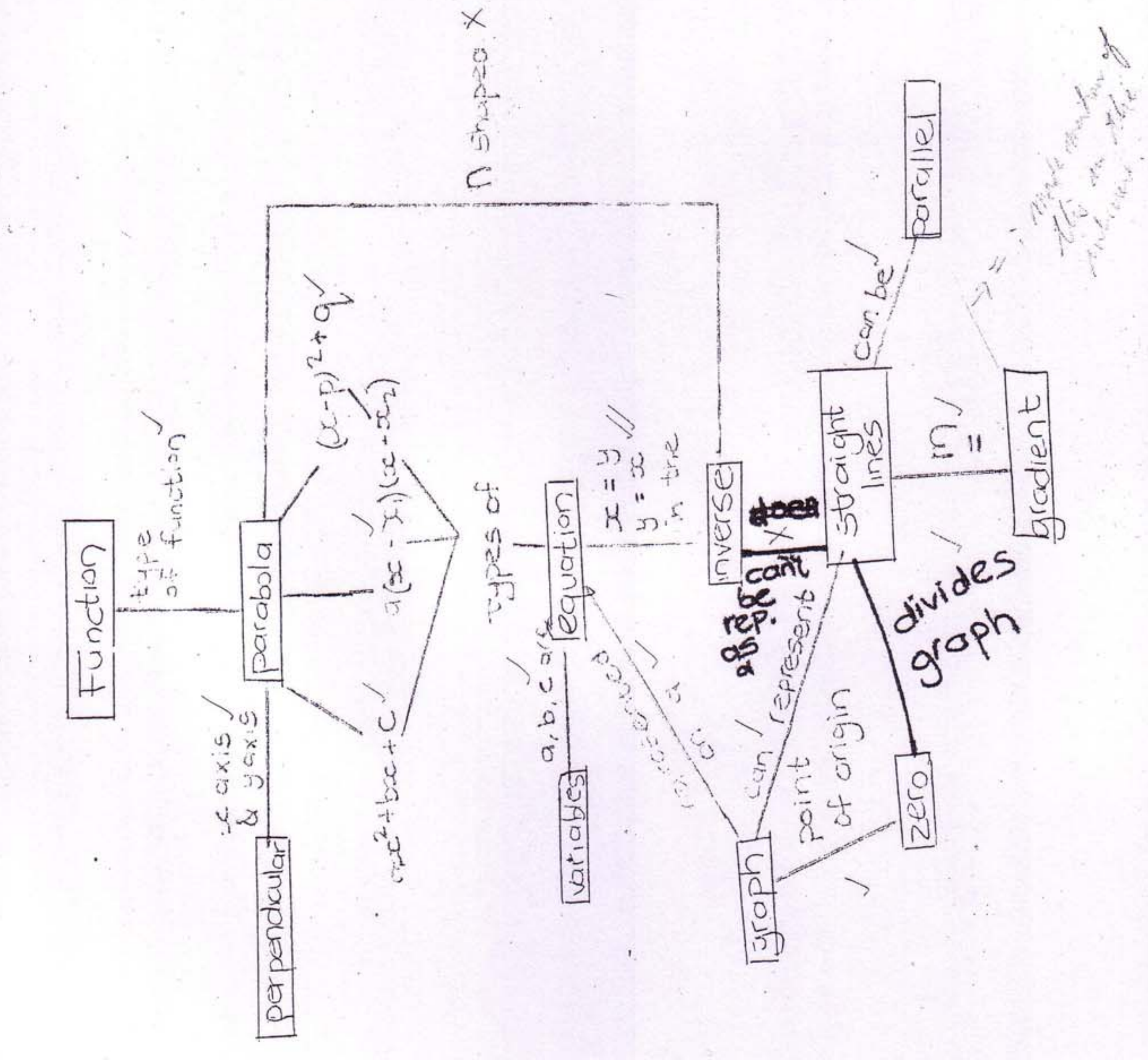
gradient; inverse ; parallel; function; equation; zero; variables; perpendicular.



1. **TASK : CONCEPT MAPPING.** **Appendix 10 E** L 31

Draw a concept map using the following concepts:

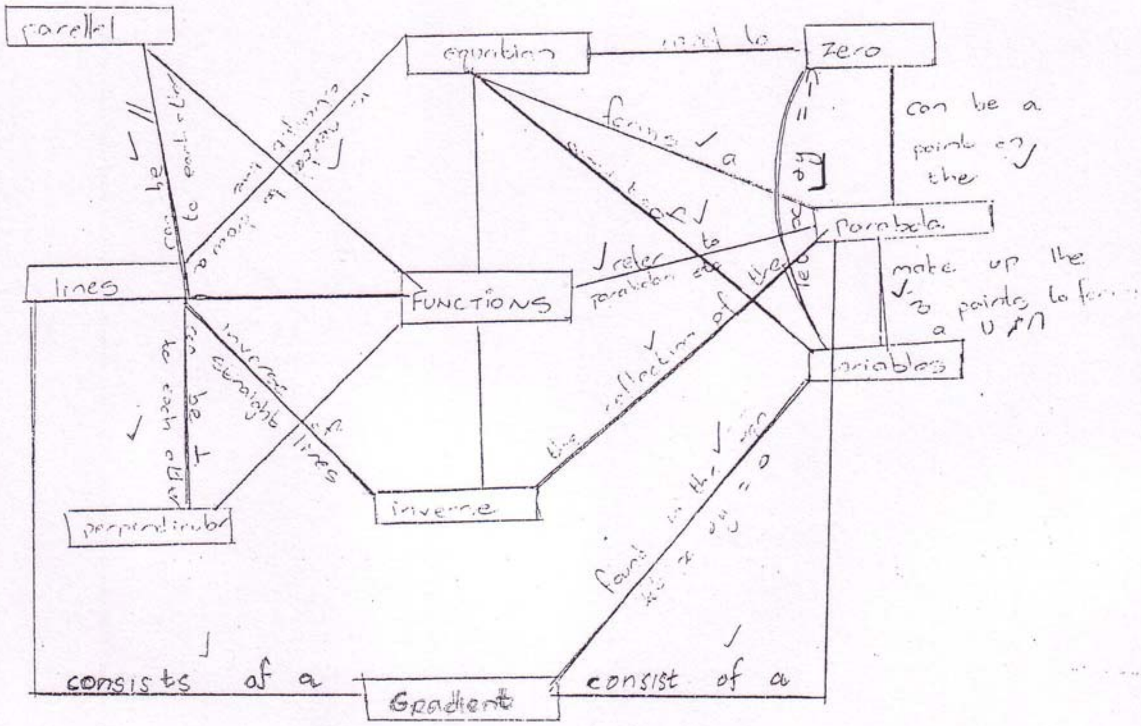
gradient; inverse ; parallel; function; equation; zero; variables; perpendicular.



1. TASK : CONCEPT MAPPING. **Appendix 10 F** L30

Draw a concept map using the following concepts:

gradient; inverse; parallel; function; equation; zero; variables; perpendicular., lines, parabola



Appendix 11

Learners' solutions to the task on functions

2

Appendix 11 A

L26

20/20

2.1 OA and OB are the x-int of the parabola

$y = x^2 - 4x - 5$

x int: let $y = 0$

$0 = x^2 - 4x - 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or $x = -1$

$\therefore$ OA = 1 unit

OB = 5 unit.

OC is the y-int

y int: let $x = 0$

$y = x^2 - 4x - 5$

$y = (0)^2 - 4(0) - 5$

$y = -5$

$\therefore$ OC = 5 units.

ED is the axis of symmetry

AS: $= \frac{-b}{2a}$

$= \frac{-(-4)}{2(1)}$

$x = 2$

TP: $y = x^2 - 4x - 5$

$= (2)^2 - 4(2) - 5$

$= -9$ $(2; -9)$

FD = 9 units

2.2. $x = 2$ ✓ L26 1 1

2.3. $y = mx + c$

y int is -5

$y = mx - 5$

The straight passes thro' $(\overset{x}{5};\overset{y}{0})$

$0 = m(5) - 5$

$5m = 5$

$m = 1$

$m = \frac{\Delta y}{\Delta x}$

$= \frac{5}{5}$

$= 1$

The required equation is

$y = x - 5$ ✓ 4 4

2.4. $M_{\text{given line}} = 1$

$M_{\text{parallel line}} = 1$ ✓

$y = x + c$

The line passes thro' A $(\overset{x}{-1}, \overset{y}{0})$

$0 = -1 + c$

$c = 1$

The required equation is ✓

$y = x + 1$ 4 4

Appendix 11 B L25 $\frac{12}{20}$ S.N.

1. $f: x \to x^2 - 4x - 5$ & $g: x \to mx + c$

$f(x) \mid y = x^2 - 4x - 5$ $\quad$ $g(x) \mid y = mx + c$

$y = x^2 - 4x - 5$

x int: let $y = 0$

$0 = x^2 - 4x - 5$

$x^2 - 4x - 5 = 0$ 1

$(x-5)(x+1) = 0$ 1

$x = 5$ or $x = -1$ 1

$\therefore$ OA $= -1$ 0

& OB $= 5$ ✓ 1

y int: let $x = 0$

$y = (0)^2 - 4(0) - 5$

$y = -5$

$\therefore$ OC $= -5$ 0

ED / Turning Point: $x = \frac{-b}{2a}$

$= \frac{-(-4)}{2(1)}$ 1

$= \frac{4}{2}$

$= 2$ ✓ 1

Sub $x = 2$ in $y = x^2 - 4x - 5$

$y = (2)^2 - 4(2) - 5$ 1

$= 4 - 8 - 5$

$= -9$

ED $= -9$ 1

2. Equation of dotted line is $x = \frac{-b}{2a}$ 0

3. $y(x) = mx + c$

$y = mx + c$

$c = -5$

$m = \frac{\Delta y}{\Delta x}$

$= \frac{5}{5}$

$= 1$

Equation is $y = x - 5$ ✓

L25	
4	4
0	c

4. ———

L 8

16/20 B S.N. 16/20

Task on Functions

2.1. $f : x \to x^2 - 4x - 5$

$y = x^2 - 4x - 5$

x-int : let $y = 0$

$0 = x^2 - 4x - 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ OR $x = -1$

$\therefore$ OA = 1 unit ✓

$(-1; 0)$

OB = 5 units ✓

$(5; 0)$

y-int : let $x = 0$

$y = (0)^2 - 4(0) - 5$

$= -5$

$\therefore$ OC = 5 units ✓

$(0; -5)$

Axis of symmetry : $x = \frac{-b}{2a}$

$x = \frac{-(-4)}{2(1)}$

$= \frac{4}{2}$

$= 2$ ✓

~~$\therefore$ ED = 2 units~~

~~$(2; 0)$~~

sub $x = 2$ in $x^2 - 4x - 5$

$y = (2)^2 - 4(2) - 5$

$= 4 - 8 - 5$

$= -9$ ✓

$\therefore$ ED = 9 units

$(2; -9)$

5

2.2. Eqn of dotted line : $\underline{x = 2}$ ✓ | 1 | 1

2.3 $g : x \rightarrow mx + c$

$y = mx + c$

$\underline{c = -5}$

$y = mx - 5$

the graph is passing throu' $(\underset{x}{5};\underset{y}{0})$

$0 = m(5) - 5$

$0 = 5m - 5$

$5m = 5$

$\underline{m = 1}$

$\therefore \underline{y = x - 5}$ ✓ | 4 | 4

2.4 Straight line = Parabola − Straight line

$= x^2 - 4x - 5 - (x - 5)$

$= x^2 - 4x - 5 \mp x + 5$

$\underline{y = x^2 - 5x}$ | 0 | 0

2.1. $y = x^2 - 4x - 5$ **Appendix 11 D** F L34 S.N.

x int : let $y = 0$

$0 = x^2 - 4x - 5$ 1 1

$= (x - 5)(x + 1)$ 1 1

$= x = 5$ OR $x = -1$ 1 1

OA = 1 unit ✓ 1 1

OB = 5 units ✓ 1 1

Axis of symmetry : $y = x^2 - 4x - 5$ 2

$x = \frac{-b}{2a}$ TP: sub $x = 2$ in $y = x^2 - 4x - 5$

$= \frac{-(-4)}{2(1)}$ $= (2)^2 - 4(2) - 5$

$= \frac{4}{2}$ $= 4 - 8 - 5$

$= 2$ $= -9$ 2

ED = 9 units ✓ 1 1

OC = 2 units 0 0

2.2. $x = \frac{-b}{2a}$ 0 0

2.3. $y = mx + c$

$m = \frac{\Delta y}{\Delta x}$

$m = \frac{-2}{5}$

$m = \frac{2}{5}$

$y = \frac{2}{5}x + c$

2.4. $y = mx + c$

$m = \frac{\Delta y}{\Delta x}$

$m = \frac{5}{1}$

$m = 5$

$y = 5x + c$

Appendix 11 E $\angle 31$ S.N. $\frac{1}{20}$

2.1 $f(x) = x^2 - 4x - 5$

x int let $y = 0$

$0 = x^2 - 4x - 5$ 1 1

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0$ 1 1

$x = 5$ OR $x = -1$ 1 1

y int Let $x = 0$

$y = (0)^2 - 4(0) - 5$

$y = -5$

$\therefore OA = -1$ 0 0

$OB = 5$ ✓ 1 1

$OC = -5$ 0 0

$A.O.S = \frac{-b}{2a}$

$= \frac{-(-4)}{2(1)}$ 1 1

$= 2$ 1 1

$T.P = (2)^2 - 4(2) - 5$ 1 1

$= 4 - 6 - 5$ 0 0

$= -7$ $(2;-7)$ 0 0

$\therefore ED = -7$

2.2 $A.O.S = \frac{-b}{2a}$

$= \frac{-(-4)}{2(1)}$

$= 2$ $(2;0)$

$TP = (2)^2 - 4(2) - 5$

$= -7$ $(2;-7)$ 0 0

L31

$y = mx + c$

~~The str line is passing thro' (5; -5)~~

$m = \frac{\Delta y}{\Delta x}$

$= \frac{-5}{5}$

$= -1$

$\therefore$ The eqn is $y = -1x + c$

The str line is passing thro' $(5; -5)$

$-5 = -1(5) + c$

$-5 = -5 + c$

$-5 + 5 = c$

$c = 0$

$\therefore$ The eqn is $y = -1x$

4. $y = mx + c$

$m_{given} = -1$

$m_{\parallel} = 1$

$\therefore$ The eqn is $y = 1x + c$

The str line is passing thro' $(-1; 1)$

$1 = 1(-1) + c$

$1 = -1 + c$

$1 + 1 = c$

$c = 2$

$\therefore$ The eqn is $y = 1x + 2$

$\therefore y = x + 2$

0 0

0 0

0

0 0

L30 I

S.N.

2.1. $y = x^2 - 4x - 5$.

x int. let $y = 0$

$0 = x^2 - 4x - 5$ | 1 | 1

$(x - 5)(x + 1)$ | 1 | 1

$x = 5$ or $x = -1$ | 1

$\overrightarrow{OA} = \underline{-1}$ | 0

$\overrightarrow{OB} = 5$ ✓ | 1 | 1

y int. let $x = 0$

$y = (0)^2 - 4(0) - 5$

$y = -5$

$\overrightarrow{OC} = \underline{-5}$ | 0 | 0

$\overrightarrow{OD} = \underline{-9}$

Sub. A.O.S (2) into $x^2 - 4x - 5$ | 2 | 2

$(2)^2 - 4(2) - 5$

$= -9$ | 2 | 2

2.2. $x = \frac{-b}{2a}$

$= \frac{-(-4)}{2(1)}$

$= \frac{4}{2}$

$= 2$ ✓

T.P. sub. 2 into $x^2 - 4x - 5$

$= (2)^2 - 4(2) - 5$

$= 4 - 8 - 5$

$= -9$ ✓ $(2; -9)$ | 0 | 0

L30

3. $y = mx + c$

The str. line is passing through --- $(\overset{x_1}{5}; \overset{y_1}{-5})$ & $(\overset{x_2}{2}; \overset{y_2}{-9})$ 0

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-9 - (-5)}{2 - 5}$

$= \dfrac{-4}{-3}$

$= \dfrac{4}{3}$ ✓

$m = 4/3$

$C = -5$ 2 2

$\therefore y = 4/3\, x - 5$

2.4. ie str. line will pass thro'

5 0 0

Appendix 12

Learners' interview schedule

Appendix 12 A

INTERVIEW SCHEDULE

Learner 26

- How did you start to do your concept map?
- You gave parabolas and hyperbolas as examples of functions. Please explain what makes parabolas and hyperbolas functions? What is it that makes you say that parabolas and hyperbolas are examples of functions?
- You say that “gradient may be parallel”, what do you mean by that? Please give an example to show what you mean. What does the term gradient mean to you?
- You have also indicated that “gradient may be zero”. Please explain what you mean by that and give an example.
- You have provided an example of an equation of a parabola and its inverse in this link. How is the graph of a parabola related to the graph of its inverse?
- This link states that a straight line slopes to the right then its gradient is positive and if it slopes to the left then its gradient is negative. What if the line is vertical, what is its gradient?
- If you were to think of a link between ZERO and another concept, what might that link be?
- Looking at your concept map do you have anything else that you want to add to the way you made links between concepts?
- How did you feel about drawing the concept map?

Appendix 12 B

<u>INTERVIEW SCHEDULE</u>

<u>Learner 25</u>

- How did you start to do your concept map?
- You say that “function can be called equation”. Please explain what you mean by that and give an example?
- You have also indicated that “graphs have inverses”. Please explain how you will determine the equation of an inverse? How is the graph of a parabola related to the graph of its inverse?
- You did not include the concepts parallel and perpendicular in your map. Any reasons for that? If you were given another opportunity to draw your concept map, how would you link the concepts parallel and perpendicular to the other concepts?
- What does the term gradient mean to you?
- Please can you describe the link between the gradients of lines that are parallel to each other? Perpendicular to each other?
- You say that the “origin of a graph is ZERO”. Please explain what you mean by that and give an example?
- If you were to think of a link between ZERO and another concept, what might that link be?
- Looking at your concept map do you have anything else that you want to add to the way you made links between concepts?
- How did you feel about drawing the concept map?

Appendix 12 C

INTERVIEW SCHEDULE

Learner 8

- How did you start to do your concept map?
- You say that straight line and parabola are types of functions. Please explain what makes straight line graphs and parabolas functions?
- You have also indicated that variables may be zero? Please explain what you mean by that and give an example.
- You say that "inverse changes the equation of the graph." Please explain what you mean by that and give an example.
- You say that a straight line graph has a gradient that may be positive or negative. Please explain how does a straight line graph with a positive gradient differ from one with a negative gradient? What does the term gradient mean to you? Could a line have a gradient of zero? Please can you describe such a line?
- Can you describe the link between the gradients of lines that are parallel to each other? Perpendicular to each other?
- If you were to think of a link between ZERO and another concept, what might that link be?
- Looking at your concept map do you have anything else that you want to add to the way you made links between concepts?
- How did you feel about drawing the concept map?

Appendix 12 D

INTERVIEW SCHEDULE

Learner 34

- How did you start to do your concept map?

- You did not use the concept of gradient in your map. Any reasons for that? If you were given another opportunity to draw your concept map, how would you relate the concept gradient to other concepts in your map? What does the term gradient mean to you? { variables ; function and inverse were also omitted --- same line of questioning}

- Please can you describe the link between the gradients of lines that are parallel to each other? Perpendicular to each other?

- You included the concept "quadrilateral" in your map. Please explain what your reasons for doing that were.

- If you were to think of a link between ZERO and another concept, what might that link be?

- Looking at your concept map do you have anything else that you want to add to the way you made links between concepts?

- How did you feel about drawing the concept map?

Appendix 12 E

INTERVIEW SCHEDULE

Learner 31

- How did you start to do your concept map?
- You say that a parabola is a type of function. Please explain what makes a parabola a type of function?
- Please can you explain this link "point of origin" between the concepts graph and zero?
- If you were to think of a link between ZERO and another concept, what might that link be?
- You have the proposition "n –shaped" between the concepts parabola and inverse. Please explain what you mean by this link. How is the graph of a parabola related to the graph of its inverse?
- What does the term gradient mean to you?
- Please can you describe the link between the gradients of lines that are parallel to each other? Perpendicular to each other?
- Between the concepts equation and graph you have the link "represented on a" – please can you explain the link to me?
- Looking at your concept map do you have anything else that you want to add to the way you made links between concepts?
- How did you feel about drawing the concept map?

Appendix 12 F

INTERVIEW SCHEDULE

Learner 30

- How did you start to do your concept map?
- The link " inverse the reflection of the parabola." What do you mean by this? How is the graph of the parabola related to the graph of its inverse?
- You say that functions relate parabola (and equation, inverse, lines). Could you please explain how are they related? Could you fill in a proposition between the concepts function and equation, function and inverse?
- How do you determine the equation of an inverse of a parabola?
- You have said that lines consist of a gradient. Please explain what you mean by this and give an example? You have also indicated that parabolas consist of a gradient. Please explain what you mean by that and give an example?
- What does the term gradient mean to you?
- Please describe the link between the gradients of lines that are parallel to each other? Perpendicular to each other?
- The link" found in the equation let x and y = 0" between the concepts variables and gradient. Please explain what you mean by this.
- You have also indicated that zero can be a point on a parabola. Please give me an example of such a point
- Looking at your concept map do you have anything else that you want to add to the way you made links between concepts?
- How did you feel about drawing the concept map?

Appendix 13

Learners' interview transcripts

Appendix 13 A1 : Preliminary analysis of L8's interview transcript

Interview transcript L 8

SN: How did you start to draw your concept map?

L8: Ok, I started with the concept function and I think it will be the easiest step because I can link everything up to that because I have a concept that seems to be part of the function topic so I think I can start with the concept function and link everything up to it. *justification for central concept*

SN: Ok.

You say that straight lines and parabolas are types of functions. Please explain what makes straight lines and parabolas functions?

L8: Um firstly, because a function is a where you take a equation and from there you draw graphs. So that's why it is a graph and that's why it takes from there we take an equation and make a graph which makes a function. *• all eqns are functions* *incorrect conclusion. It seems that fns are synonyms for graphs* *Thompson view on graphs.*

SN: In the case of a parabola?

L8: Ja, it is the same thing. (silence)

SN: Is there any other eh .. is there anything else you want to add why they may be functions? Take the case of the parabola. *• (L) can give example of functions ?? but don't know defn / why it is a fn.*

L8: (silence) No.

SN: No.

You also indicated that variables may be zero. Please explain what you mean by that and may be you can give me an example.

L8: I think I was supposed to link it to an equation . Because actually in an equation you can have a zero and passes and the zero can be passed as a variable as well. Actually I think it supposed to be linked up to equation *Suppose to link Equation & Zero*

SN: Oh, actually you supposed to put a link between zero and equation .Do you want to put the link and write the proposition for me?

L8: (Learner writes proposition) *See concept map*

SN: So this link between variables and zero, is that not what you wanted to say? Would you be able to, if I gave you an opportunity now to put another link between variables and the concept zero or equation? You want to find a link now?

L8: (silence) can't think of anything now. *⇒ as opposed to not knowing*

SN: Can't think of anything now.

You also say that an inverse changes the equation of a graph. Please explain what you mean by that?

L8: Um, sir, what I mean by that is that the equation of a normal graph always have, it is in the form y equals to the equation

SN: Ok

L8: But when you do inverse graph you change the y into the x is equal to that equation *traditional*

SN: Can you give me an example of that?

L8: Like maybe for a straight line graph we say that y = 2x + 3. Then when we do an inverse graph you have to take the 2x, you have to make x the product of the whole equation . So it will be x is equal to umm 2y over 3 or something like that *means : Subject* *x = 2y + 3*

SN: Ok.

You say that a straight line graph has a gradient that may be positive or negative. Please explain how does a straight line graph with a positive gradient differ from a straight line graph with a negative gradient?

L8: Firstly, in your equation you will determine what gradient it is. So if the equation is in a negative form then it means it is a negative gradient. If it is in a positive form then it is a positive gradient. And when you draw the graph, the graph it looks different from each other and the …

SN: What do you mean by positive form and negative form?

L8: The equation , sir like ..

SN: Do you want to write an example? → writes: $y = 2x + 1$ on the additional page

L8: (learner writes an example)

SN: Could you just say what you are writing, for the record.

L8: y = 2x + 2 means it is a positive gradient. When you draw your graph it will have a positive gradient (underlines the 2 in $y = 2x + 1$, on her page). Then if you have a graph $y = - 2x$. then this -2 tells you that it is a negative gradient. So when you draw your graph the line will slope to the left

Traditional

SN: Then how will the line slope if you have a positive gradient?

L8: Towards the right

SN: What does the term gradient mean to you?

L8: It shows the (pause)

SN : You can think about it a little while. What does the term gradient mean to you?

L8 : (pause) The way in which the graph lies and from the graph you read of the certain thing. Ja.

Can differentiate between +ve & -ve grad. but doesn't know a defn. of grad

SN: What things were you referring to?

L8: (pause) I have no idea how to explain it.

SN: You spoke about a gradient being positive and being negative .Could a line have a gradient of zero?

L8: I don't think so. If it has a gradient that is zero then it will be on your x and y intercept → Meaning the origin

SN: Could you draw a sketch perhaps to explain that?

L8: Sir I don't know if I can , ok, here is your x intercept and your y intercept (draws on the page) and usually the graph goes like this. But if it has a zero has a gradient then it can't go straight then it means it is getting on the intercept. → THE ORIGIN

→ meaning axes

FIG 1

SN: Would you be able to describe the line?

L8: Umm… won't the gradient be undefined if the gradient is zero?

Common errors (from exp.) with vertical & horizontal lines

SN: You think it is perhaps undefined

L8: Because your whole equation will be zero well because I mean your intercepts will be zero . Therefore it will be an undefined function.

SN: Alright. Can you describe the link between gradients of lines that are parallel to each other?

L8: Sir , that is when you take two equations from the graph that you are drawing you take two equations and you join them simultaneously then the when you draw them they are parallel to each other.

SN: What is the link between the gradients of each of these two parallel lines?

L8: They may be positive or negative.

don't know the grad. Relationship for parallel lines

SN: Ok. Can you describe the link between gradients of two lines that are perpendicular to each other?

L8: They will be opposite of each other the gradients. One will be positive and one will be negative.

SN: Would you perhaps give me an example to show me how they will be opposite to each other?

L8: Umm…Sir isn't perpendicular draw perpendicular then it means that the gradients of each add up and you multiply them and they come to -1. So it means that both the gradients so if the gradient is 2 over 3 in the one equation and the other equation it will be 3 over 2 and the signs will be negative. knows the relationship between gra of perp. lin

SN: Ok. If you were to think of a link between zero and any other concept what might such a link be?

L8: (pause) Cannot think of it

SN: At the moment

or don't know

L8: Ja. (pause)

SN: Looking at your concept map do you have anything else that you want to add to the way you made links between the concepts?

L8: (pause)

SN: In other words is there any other links that you may want to fill in between the concepts you have on your map?

L8: (pause) You can say that the straight line and the parabola may intersect each other.

SN: You want to eh

L8: If you draw

SN: You can draw in the link for me and fill in the proposition

L8: (learner fills in proposition)

SN: Can you say what you are doing just for the recorder.

L8: What I mean by this is that if you get to draw a, if you are given an equation to draw a parabola and a straight line at one time then they may intersect each other at one point. See concept map

SN: What makes you say eh that they may intersect at one point?

L8: Because a straight line and (pause) it can also stem from the equation because maybe for your straight line graph maybe the your y – intercept may go higher than your parabola so maybe they could also intersect at two points. (attempts a sketch on the additional page) FIG 3

SN: Ok. How did you feel about drawing this concept map?

L8: Well eh it made me understand certain things that I didn't and eh and I did not know that certain things can link up with each other and now I understand it better and umm. → 3rd Research Question

SN : Are there any examples of things that you didn't know that may be able to linked to each other.

L8: Ja the zero. Ja. → $ linked to equation

SN: Are there any examples of aspects that you understood better?

L8: Ja, the inverse graphs I understood better about the change of the y and the x variables, even the parabola that it can intersect the straight line graph and the ja.

SN: Ok. Thanks very much for your time.

L8: Pleasure

SN: And thanks for your contribution towards the research project.

<u>Interview transcript L 8</u>	**<u>Comment</u>**
SN: How did you start to draw your concept map?	Metacognitive
L8: Ok, I started with the concept function and I think it will be the easiest step because I can link everything up to that because I have a concept that seems to be part of the function topic so I think I can start with the concept function and link everything up to it. SN: Ok. You say that straight lines and parabolas are types of functions. Please explain what makes straight lines and parabolas functions?	Justification for central concept
L8: Um firstly, because a function is a where you take a equation and from there you draw graphs. So that's why it is a graph and that's why it takes from there we take an equation and make a <u>graph which makes a function.</u> SN: In the case of a parabola? L8: Ja, it is the same thing. (silence) SN: Is there any other eh .. is there anything else you want to add why they may be functions? Take the case of the parabola. L8: (silence) No. SN: No. You also indicated that variables may be zero. Please explain what you mean by that and may be you can give me an example.	All equations are functions? Incorrect conclusion. It seems functions are synonymous with graphs. Thompson- their dominant view of graphs. L8 can gives examples of functions but don't know the definition of function or why the parabola is a function.
L8: I think I was supposed to link it to an equation. Because actually in an equation you can have a zero and passes and the zero can be passed as a variable as well. Actually I think it supposed to be linked up to equation SN: Oh, actually you supposed to put a link between zero and equation .Do you want to put the link and write the proposition for me?	L8 was supposed to link the concepts equation and zero and not the concepts variables and zero.
L8: (Learner writes proposition) SN: So this link between variables and zero, is that not what you wanted to say? Would you be able to, if I gave you an opportunity now to put another link between variables and the concept zero or equation? You want to	See concept map.
find a link now? L8: (silence) can't think of anything now. SN: Can't think of anything now. You also say that an inverse changes the equation of a graph. Please explain what you mean by that? L8: Um, sir, what I mean by that is that the equation of a normal graph always have, it is in the form y equals to the equation	As opposed to not knowing
SN: Ok L8: <u>But when you do inverse graph you change the y into the x is equal to that equation</u>	Procedural understanding?

SN: Can you give me an example of that? L8: Like maybe for a straight line graph we say that y = 2x + 3. Then when we do an inverse graph you have to take the 2x, you have to make x the <u>product of the whole equation.</u> <u>So it will be x is equal to umm 2y over 3 or something like that</u>	Product? Perhaps mean subject? Procedural understanding?
SN: Ok. You say that a straight line graph has a gradient that may be positive or negative. Please explain how does a straight line graph with a positive gradient differ from a straight line graph with a negative gradient? L8: Firstly, in your equation you will determine what gradient it is. <u>So if the equation is in a negative form then it means it is a negative gradient.</u> If it is in a positive form then it is a positive gradient. And when you draw the graph, the graph it looks different from each other and the …	Meaning the coefficient of x is negative?
SN: What do you mean by positive form and negative form? L8: The equation , sir like .. SN: Do you want to write an example? L8: (learner writes an example) SN: Could you just say what you are writing, for the record.	Writes y = 2x + 1, on the elaboration page.
L8: <u>y = 2x + 2</u> , means it is a positive gradient. When you draw your graph it will have a positive gradient (underlines the 2 in y = 2x + 1 , on her page). Then if you have a graph y = - 2x <u>. then this -2 tells you that it is a negative gradient. So when you draw your graph the line will slope to the left</u> SN: Then how will the line slope if you have a positive gradient?	Procedural understanding?
L8: Towards the right SN: <u>What does the term gradient mean to you?</u> L8: It shows the (pause) SN : You can think about it a little while. What does the term gradient mean to you? L8 : (pause) <u>The way in which the graph lies and from the graph you read of the certain thing.</u> Ja. SN: What things were you referring to? L8: (pause) I have no idea how to explain it.	L8 can differentiate between positive and negative gradients but does not know a definition of gradient.
SN: You spoke about a gradient being positive and being negative .Could a line have a gradient of zero? L8: I don’t think so. <u>If it has a gradient that is zero then it</u>	Meaning the origin.
<u>will be on your x and y intercept.</u> SN: Could you draw a sketch perhaps to explain that?	Meaning the x and y axes
L8: Sir I don’t know if I can , <u>ok, here is your x intercept and your y intercept (draws on the page)</u> and usually the	Referring to the origin.

graph goes like this. But if it has a zero has a gradient then it can't go straight then <u>it means it is getting on the intercept.</u> SN: Would you be able to describe the line? L8: Umm… <u>won't the gradient be undefined if the gradient is zero?</u> SN: You think it is perhaps undefined L8: Because your whole equation will be zero well because I mean your intercepts will be zero. Therefore it will be an undefined function. SN: Alright. Can you describe the link between gradients of lines that are parallel to each other? L8: <u>Sir, that is when you take two equations from the graph that you are drawing you take two equations and you join them simultaneously then the when you draw them they are parallel to each other</u>. SN: What is the link between the gradients of each of these two parallel lines? L8: They may be positive or negative. SN: Ok. Can you describe the link between gradients of two lines that are perpendicular to each other? L8: They will be opposite of each other the gradients. One will be positive and one will be negative. SN: Would you perhaps give me an example to show me how they will be opposite to each other? L8: Umm…Sir isn't perpendicular draw perpendicular then it means that the gradients <u>of each add up</u> and you <u>multiply them and they come to -1.</u> So it means that both the gradients so if the gradient is 2 over 3 in the one equation and the other equation it will be 3 over 2 and the signs will be negative. SN: Ok. If you were to think of a link between zero and any other concept what might such a link be? L8: (pause) Cannot think of it SN: At the moment L8: Ja. (pause) SN: Looking at your concept map do you have anything else that you want to add to the way you made links between the concepts? L8: (pause) SN: In other words is there any other links that you may want to fill in between the concepts you have on your map? L8: (pause) You can say that the straight line and the parabola may intersect each other. SN: You want to eh L8: If you draw SN: You can draw in the link for me and fill in the proposition	Common error (from experience) with vertical and horizontal lines. Don't know the gradient relationship for lines that are parallel? Perhaps cancel off with each other? L8 knows the relationship between gradients of perpendicular lines. See concept map.

L8: (learner fills in proposition)	
SN: Can you say what you are doing just for the recorder.	
L8: What I mean by this is that if you get to draw a, if you are given an equation to draw a parabola and a straight line at one time then they may intersect each other at one point .	
SN: What makes you say eh that they may intersect at one point?	See page where elaborations were done.
L8: Because a straight line and (pause) it can also stem from the equation because maybe for your straight line graph maybe the your y – intercept may go higher than your parabola so maybe they could also intersect at two points. (attempts a sketch on the additional page)	Implications for third research question?
SN: Ok. How did you feel about drawing this concept map?	
L8: Well eh it made me understand certain things that I didn't and eh and I did not know that certain things can link up with each other and now I understand it better and umm .	Linking the concept zero to equation
SN : Are there any examples of things that you didn't know that may be able to linked to each other.	Implications for third research question?
L8: Ja the zero. Ja .	
SN: Are there any examples of aspects that you understood better?	
L8: Ja, the inverse graphs I understood better about the change of the y and the x variables, even the parabola that it can intersect the straight line graph and the ja.	
SN: Ok. Thanks very much for your time.	
L8: Pleasure	
SN: And thanks for your contribution towards the research project.	

Interview transcript: L 25

S: How did you start to do your concept map?
L25: Firstly, I tried eh to look for a main concept so I thought that graphs will be eh a perfect one to start of with. From graphs there are many different eh links or other types of concepts that you can bring out. So I chose graphs and then, we studied different types of graphs. Absolute value graphs, straight line graphs, etc. So I tried to bring all of that out of the main concept graphs. The main graph concepts.
SN: What about graphs that makes it the main concept for you?
L25: Umm …It is the first thing, there are many different things that you can write about graphs because there are so many different steps that you have to go about making the graphs. Like firstly, the axes and then depending on the graph, how you plot your points. Your, if it is a parabola and stuff, your turning point etc. So these are different things that make it up.
SN: Fine. You say that function can be called an equation. Please explain what you mean by that and give an example.
L25: Umm. When we studied the chapter functions and eh with graphs our teacher told us and I remembered and I put it down on the concept map as well. To give you an example, as I said functions can also be called an equation, has variables. Can take anything, for example , eh f : x maps or f x is equal to umm.. 3x plus 5 something, for an example. The same function written in functional notation can also be written as an equation where fx changes to y where y = 3x + 5.
SN: Ok. You have also indicated that graphs have inverses. Please explain how you determine the equation of an inverse?
L25: From what we studied eh in this chapter graphs eh according to my knowledge, that when you do a graph, when you trying to do an inverse all the x values from the normal graph will change to the y values of the inverse and the y values of the normal graph will change to the x values of the inverse. In this manner you will work out the inverse graph.
SN: How is the graph of the parabola related to the graph of its inverse?
L25: mmm…Well parabolas you will have it eh the the
SN: You may make a sketch if you want to.
L25: Parabolas will have the diagrams either if it is positive then it is u shaped going down like this (learner draws on additional page) down the axis and the inverse is n shape in that manner and if it is an inverse well it is related in the manner that if you change the x and y points then you will do your inverse then you will either going to end up with a parabola such or in the opposite way
SN: Ok. So if you have a u shaped parabola
L25: Right
SN: Then the this will be the inverse of the u shaped parabola.
L25: Well depending on the equation of the parabola well it depends on the equation.
SN: Ok. Do you have an example of hand that for a parabola that
L25: It might take me a while to work out an equation and work out the points etc. and then we sketch the graph.
SN: Ok. You did not include the concepts parallel and perpendicular in your map umm.. any reasons for that?
L25: I the concepts were there , I could have attached it to the straight line graph but the eh eh mmm

SN: the link
L25: the link did not come to mind .So I could have attached it but I could not find a proper link to link the straight line graph eh to the parallel and the perpendicular.
SN: Ok. At that time
L25: At that time
SN: If you were given another opportunity to draw your concept map how will you link your concept parallel and perpendicular to the other concepts?
L25: mmm
SN: You could write the link with that
L25: I could put the axis of symmetry on the y axis eh when you drawing a graph as such eh parabola for example you can join your equation if your axis of symmetry is not on zero not on your y axis and if it is any other number for example for example 1 and when you draw your axis of symmetry , your axis of symmetry is parallel to the y axis . That could be eh I will link parallel to my axis and
SN: Would you mind drawing that link onto the concept map what you just said now
L25: (Learner writes in the link on the concept map) parallel to the y axis mmm .. (silence)
SN: What won't be zero there
L25: If the axis of symmetry is not zero and if it is any other number besides zero
SN: Yes
L25: Then it will be parallel to the y axis
SN: Will you be able to link the concept perpendicular to any other concept on your map?
L25: mmm ..I can't think of anything right now
SN: Earlier you mentioned eh .. you were talking about the straight line graph
L25: The straight line graph eh I do have a vague idea but it is not very clear eh I never went over the perpendicular aspect of the straight graph. I won't be able to link perpendicular.
SN: Ok . What does the term gradient mean to you?
L25: Gradient I understand it to the direction in which your mmm. For example, if we are doing a straight line graph and if your your your graph is sloping to the right then your gradient is positive.
SM: Yes
L25: Meaning that ok then …(inaudible) you get from your x value x value your m your m value is positive your coefficient of x in your equation while you got your mx + c.
SN: What does the m represent?
L25: It represents your gradient
SN: Ok
L25: On the contrary side if your graph is sloping to the left then your gradient is negative. Your m value your gradient.
SN: Is there anything further that you want to add about gradient?
L25: That is far as I know about gradient.
SN: Please can you describe the link between gradients of lines that are parallel to each other?
L25: (silence) Gradients of lines
SN: Gradients of lines that is parallel to each other. It is not in your concept map.
L25: Alright

SN: I want to know in eh do you see any relationship between gradients of lines that are parallel to each other?
L25: mmm I remember doing it but it is not very clear. I think it got something to do with the gradients of two lines being perpendicular. I'm not very sure about it.
SN: What do you mean gradients of two lines
L25: If you got two equations and the gradients are perpendicular that is what the assignment asked. It is very vague I cant think of it very clear now.
SN: Would you be able to make an example perhaps to show what you mean?
L25: Ok. If you have your set of axis and you have your two gradients going parallel to each other (learner draws on page) something like that.
SN: The relationship between gradient of lines that are perpendicular to each other.
L25: Perpendicular to each other. One gradient will go this way and the other mmm… it will form a ninety degree angle.
SN: Is there anything further that you want to add to those?
L25: As I mentioned they are not very clear right now. This is as much as I can come up with right now.
SN: Fine. You say that the origin of a graph is zero.
L25: Ja
SN: Please explain what you mean by that and give an example.
L25: The centre point of the axis your y axis and your x axis is the zero point also known as your origin.
SN: Sorry, can you just repeat that
L25: You have your y axis and your x axis the centre point. The centre point your zero it is also known as the origin. That is what I remember.
SN: Where is your graph on that origin?
L25: Your origin is just the centre point but if you had to plot a graph depending on the equation then what type of graph you will plot then it will be around the origin the centre point
SN: Ok. If you were to link zero to another concept what might that link be?
L25: (silence)
SN: If you thought of some link, what might that link be?
L25: I could link zero to an equation and say when solving an equation the right hand side or left hand side should be zero to make your solving easier.
SN: Would you mind making that link and writing the proposition
L25: (Learner writes on map)
SN: Your proposition is when solving an equation rhs or
L25: Lhs which ever side from the point of view of the person solving should be zero. It makes the solving of the equation easier.
SN: Is there any other link perhaps you want to add to zero and another concept?
L25: Zeroo no nothing
SN: Looking at your concept map do you have anything else that you want to add to the way you made links between these concepts?
L25: (silence)
SN: In other words is there perhaps any other links that you may want to..
L25: Nothing.
SN: Nothing. How did you feel about drawing this concept map?

L25: It gives you a better understanding of the chapter and shows what you know on paper. Ja it felt nice. It shows about it brought about what you know on the chapter on functions and graphs. It felt good.
SN: Thank you very much for your time
L25: My pleasure.
SN: and thank you for the input you made into this research project. Thanks.

Interview transcript : L 26

SN : Where did you start to do your concept map?
L26: I started with the main concept as functions because there are lots of other things that may branch out of it.
SN : From all these concepts you recognize that perhaps function is the main concept. Alright. Umm. In your concept map you gave parabolas and hyperbolas as examples of functions. Please explain what makes parabolas and hyperbolas functions.
L26 : A function as like for every x value has one y value and you look at the hyperbolas and parabolas and you draw vertical lines down like you see like one x value and one y value. It won't be a relation because then there will be like for every x value there may be like two y values or more. That's what I thought.
SN : Why would you draw a vertical line?
L26 : It is to test to see like if any point like for any x value how many y values there are, for any y value how many x value there are.
SN : Ok. What is that makes you say that parabolas and hyperbolas are examples of functions?
L26 : I just think that because that for every x value there one y value , so they are functions.
SN : Alright. You say as well , eh, gradient may be parallel. What do you mean by that?
L2 :Umm. Like examples of graphs for instance they may tell you like, what will the gradient be of a line that will be parallel.
SN : Sorry umm I was saying you said that gradient may be parallel. What did you mean by that?
L26 :In some graphs may be you do have like ,for instance the straight line graph, they ask you like, there might be a line parallel to that graph and for that you will say that the gradient for the given graph is equal to the gradient of the parallel line ….. (inaudible) I would think.
SN : Please would you be able to give an example to show what you mean.
L26 : Yeh, like even in a case of the straight line they give you a given straight line and they ask you what will the gradient be of the line parallel to the straight line and it will be like the same thing the gradient .
SN : Would you make an example of such a straight line?
L26 : Umm… like in the form y = mx + c they tell you that m is 3 maybe then the gradient of the parallel line will also be 3.
SN : What does the term gradient mean to you?
L26 : The slope at which the graph goes like the direction it like slopes to the right its positive if it slopes to the left its negative,
SN : Is there anything further of gradient you want eh… your understanding of it
L2 : Like…. It is like the change in y over like the change in x like the way the you see from a certain point how many units down or how many units right or left you see that how it goes.
SN: You have also indicated the gradient may be zero. Please explain what you mean by that and maybe give an example.
L26: Maybe it came about one day like a mistake …laughs…. Aah .. it could be zero maybe may be I'm not too sure about it
SN: You have an example maybe .

L26: Maybe it is like a vertical line or something that there isn't really a slope so maybe there won't be a gradient
SN: So you perhaps think that a vertical line will not have a gradient.
L26: I think it will but I'm not too sure , but
SN: You have indicated ….Umm.. You have provided an example of an equation of a parabola and its inverse .
L26: Umm
SN: How is the graph of a parabola related to the graph of its inverse… at this point here (pointing on the concept map)…(inaudible) it seems that you were busy with.
L26: Umm …If you draw like the two graphs then you get like the axis symmetry in between and maybe the different points at which they meet also indicate something. I don't know
SN: What will the axis of symmetry be for the graph and the graph of the equation of the inverse ?
L26: Won't it be like your (draws on the additional page provided) …something like this .SN: Would you be able to write down the equation of the axis of symmetry that you are referring to on your sketch?
L26: I think it is where $y = x$, at these points where they meet …(inaudible)
SN: Could you write down which is the inverse of the graph. { Learner indicates the inverse on the page)
Alright. If you were to think of a link between ***zero*** and any other concept… . and what might that link be?
L26: May be equation , because every equation ends up like equal to ***zero, or*** normally if you have an equation you bring all the terms to the left hand side so that you have a zero. Maybe you have like a trinomials or things that you might be able to factorize and so you bring everything on to that side so therefore in an equation you will always have something that will say equal to zero.
SN: So you will link equation to zero?
L26: Where ever possible.
SN: Could you link those two concepts for me and maybe write a proposition for it
L26: Mmm…. (learner writes on additional page) .. . could say right hand side equal to zero.
SN: Right hand side may be equal to zero
L26: Like in solving equations and everything, normally you do have a zero on the right hand side.
SN: Ok. Looking at your concept map, do you have anything else that you want to add to the way you made links between the concepts?
L26: Pause….there might be a few, I don't know .I think when I did it like whatever came into my mind drew them all and thought of what like all these to say what I thought of at that time try to mmm
SN: Would you be able to see any additional links between the concepts now?
L26: Pause …..
SN: And if you see those links, what would those links be?
L26: Pause …(inaudible)
SN: May be you should think about it
L26: Not at the moment.
SN: Not at the moment. Ok.
How did you feel about drawing this map, this concept map?

L26: Interesting ‘cos when you do the work you don’t realize how much they , like how things link to each other like when you sit and really think about it , it is really amazing to see that so many things actually does have a link with each other. Like over the years what ever you learn like in maths and won’t think that it will link up with so many other things. So it just shows that like maths goes on and on and it is very interesting. I enjoyed drawing concept maps, I really did.
SN: O k. Thank you so much for your time.
L26: Ok, sir.
SN: And thank you for giving us input into the research project.
L26: Ok. Thank you.
SN: Thanks.

Interview transcript : L 34

SN: How did you start to do your concept map?
L34: I started by I started by looking at what I know and how I can link on with other things and and I felt that
SN: What do you mean by link the other things?
L34: I took quadrilateral as my main concept and I saw parallel and I link it and I said that quads can have lines parallel to each other and I saw perpendicular so I said in a quad lines can be perpendicular and I added zero to that then I said how lines can be parallel and I said and I can encounter x intercepts and let y equal to zero. Then I linked equation and I said equation of a straight line graph is y = mx + c.
SN: You included the concept quadrilateral in your map .Please explain what were your reasons for doing that?
L34: When I first looked at all the concepts then I thought that if there are any other concepts that I can add on .Then I looked at it then I said that a quad can have parallel, lines parallel to each other. Then I saw that we have other concepts like graphs so I said that I can link that to that . Then I also saw perpendicular then I thought that quads have lines that can be perpendicular to each other and then I just carried on from there.
SN: Ok. You did not use the concept gradient in your map. Any reasons for that?
L34: I first of all could not make a link with it at that time
SN: At that time. If you were given another opportunity to draw your concept map, how will you relate the concept gradient to the other concepts in your map?
L34:Mmm, ok, …(inaudible)
SN: Would you mind drawing in the link for gradient and any other concept of your map?
If you can please explain what you are doing while you are doing the link.
L34: Working out graphs to work out graphs you can use the gradient intercept method
SN: Ok. Complete the link for me.
L34: (learner fills in the link on the concept map)
SN: Could you just explain what you have done with the concept gradient and the link you made to graphs.
L34: In graphs there are many ways to work out graphs. In graphs you can use the gradient intercept method to work out different parts of your graphs.
SN: Ok. What does the term gradient mean to you?
L34: Gradient (silence)
SN: Any thoughts come to mind when you see the term gradient?
L34: (silence) No.
SN: You also did not make use ,did not include the concept function into your map. Any reasons for that?
L34: No. I don't….
SN: The concepts inverse and variables were omitted as well. Any reasons why you left those concepts out?
L34: No I can't make a link with it. Nothing comes to mind .
SN: Sorry.
L34: Nothing comes to mind when I , I can't make a link for any of the other
SN: At that time you could not make a link.
L34: Ja, I don't know about now.

SN: And if I give you another opportunity now to draw some links between the concepts you have and the concepts you omitted ,for example variables or functions or inverse.
L34: No. I don't
SN: Don't think so. Ok fine.
Please can you describe the link between gradients of lines that are parallel to each other?
L34: Lines that are parallel to each other , the gradients are equal.
SN: Can you describe the link that exists between lines that are perpendicular to each other? The link between gradients of lines that are perpendicular to each other.
L34: Aah. ..gradients will be eh
SN: Will you be able to give an example to show what you mean?
L34: (silence) No.
SN: If you were to think of a link between zero and any other concept, what might such a link be?
L34: Umm, (silence)
SN: Or even the links between zero and the concepts that were omitted.
L34: (silence) No.
SN: No. Looking at your map do you have anything else to add to the way you made links to the other concepts?
L34: (silence)
SN : In other words now would there be any other links that you may want to fill in between the concepts you have or even with the concepts you may have omitted?
L34: I can maybe add one.
SN: What might that link be?
L34: Parm. Parm can have. A parm has , a figure with all sides. A parallelogram is a figure with all sides parallel to each other .
SN: Would you mind filling in the link for me with the concept parm.
L34: (Learner fills in the link)
SN: Fine. How did you feel about drawing this concept map?
L34: It was something different than what you normally do. It was a bit of…it was ok but it was different when you look at things. To make different links to all the other concepts and think about different things and how they are linked. It is a good way to getting to know different things and how they are … how different concepts can be linked to each other
SN: Ok thank you. Thanks very much for your time and thanks for your input into this research project. Thanks.

Interview transcript L 31

SN: How did you start to do your concept map?

L31: Ok I first took the function like I choose one from here and because those things were about functions and I started with functions as my main heading and then I just linked everything with what I know. A parabola is a type of function and then I took equation and then I thought of the x and y axis and then I said it is perpendicular to use the word because I didn't know how and then I could add that one .And then I said eh types of equation. And the variables I said they are a, b, and c and for the inverse of the parabola I said that x is equal to y and y is equal to x .And I said an equation can be represented on a graph and a graph can also represent a straight line.

SN: Ok. Some of the things that you mentioned I am going to get into detail a little later in the interview.

You say that a parabola is a type of function. Please explain what makes a parabola a type of function?

L31: Umm..Why is it a function?

SN: Yes.

L31: Umm..I am not sure

SN: What does the term function mean to you?

L31: Function are types of things that can be represented on a graph. (pause) . Like a straight line or a hyperbola

SN: Ok. Please can you explain this link "point of origin" between the concepts graph and zero?

L31: Ok . Umm…

SN: What does the link mean?

L31: You know when you draw the graph

SN: Ok you may draw it (learner draws on the additional page)

L31: When you draw the graph like that , you call this the point of origin, which is zero. So I linked it like that.

SN: Why is it referred to as the origin?

L31: I think it is because where it separates the negative and the positive and the y axis umm.. form the y axis it separates the positive and the negative and also on the x axis so it is like the middle of the whole graph.

SN: Ok. If you were to think of a link between ZERO and another concept on the map, what might that link be?

L31: Mmm..(pause) zero .. when…umm ..when a straight line passes through zero it umm..(pause)

SN: Will you be able to draw the link in and fill in the proposition?

L31: When a straight line passes through zero it separates the umm.. graph into two equal halves.

SN: Would you mind putting in the link for me on your concept map?

L31: Ok

SN: If you can just say what you are doing , so I have a record of it.

L31: Umm..a straight line is zero .It divides the graph into two separate halves (learner fills in proposition on concept map.

SN: You have the proposition "n –shaped" between the concepts parabola and inverse. Please explain what you mean by this link.

L31: Ok when a parabola is negative , the inverse is negative then the shape goes n shaped. When the.. when the .. a … (learner draws on page) when it goes like that , u shape goes like that.
SM: That will be a u shaped parabola. Yes.
L31: When it is in the inverse it goes upside down the n shaped That is (learner draws n shaped parabola on page)
SN: So if you got a u shaped parabola
L31: Ja it’s negative . The equation is negative
SN: Can you give an example to show where the equation is negative?
L31: Ok like one of the equation is eh $y = ax^2 + bx + c$, now if a is negative then than x^2 … then an n shaped
SN: Then an n shaped. The n shaped…
L31: Is the inverse of the parabola.
SN: How is the graph of a parabola related to the graph of its inverse?
L31:Umm..They both have the y intercept and the x intercept … except in the parabola , not the inverse the other one … the y intercept is negative but in the inverse the y intercept will be positive .
SN: Is there any changes with x intercept?
L31: Umm… no not really.
SN: What does the term gradient mean to you?
L31: Umm… In the straight line? You know the equation for a straight line is $y = mx + c$ so the gradient is m but x… eh…
SN: You can make an example.
L31: Like $y = 2x + 1$. Then the gradient will be 2 because the x ..
SN: Is there any other description that you perhaps have of the gradient?
L31: The gradient eh eh (pause)
SN: Please can you describe the link between the gradients of lines that are parallel to each other?
L31: The gradient that is parallel is equal to the … they are equal. The parallel lines gradient are equal.
L31: Ok. can you describe the link between the gradients of lines that are perpendicular to each other?
L31: Umm… can I use m_1 and m_2 ?
SN: you can write an example
L31: $m_1 = 3$ then m perpendicular will be equal to m_1 divided by 1 (learner writes on page). No. This is the m_2 (learner referring to m perpendicular) .So $m_2 = m_1 \times 1$, ja
SN: So you have written your $m_1 = 3$. What will be your value for m_2 ?
L31: 3 also 3 x 1
SN: $m_1 = 3$ and also $m_2 = 3$.
L31: Ja
SN: Then what will be the relationship between the lines?
L31: they are equal.
SN: The gradients will be equal. Then how will those lines look with equal gradients?
L31: Oh…. then they won’t be parallel….oh then….umm..
SN: Do you want to take a moment to clarify that in your mind?
L31: Ja (pause)
SN: Just to repeat my question, I wanted to know what was the link between the gradients of two lines that are perpendicular to each other.

L31: Umm…
SN: And you can make an example to show what you mean
L31: The perpendicular line is equal to m_1 x m_2 divided by 1, I think, I'm not sure.
SN: Ok. Between the concepts equation and graph you have the link "represented on a" – please can you explain what this link means.
L31: Umm… an equation can be represented on a graph form. Like one of these equations will be represented in a graph form.
SN: You are referring to…
L31: $ax^2 + bx + c$ and a is equal to (x – x1)(x – x2) and (x – p)2 + q .Those are represented on a graph. Like on a graph form. The equation is represented.
SN: If you had to draw the graph would you , represented on a, does it mean that you will label the graph represented by the equation.
L31: Like after you have worked out the x intercepts and the y intercepts. Then you will draw your parabola , for example, then you would like write the equation on the side like
SN: Labelling the graph?
L31: To say that graph, what is represented on the graph, that equation
SN: Ok. Looking at your concept map do you have anything else that you want to add to the way you made links between concepts?
L31: Mmm…A straight line doesn't have a inverse. No straight line inverse
SN: Would you mind putting the link and writing down the proposition for me? Could you just say what you are doing, so I have a record of it?
L31: A straight line doesn't have a inverse. Can't be represented as a inverse. (learner writes the proposition on the concept map).
SN: What will be your reasons for saying that a straight line cannot be represented as an inverse?
L31: Because a straight line there is no inverse. The … the… x will never be negative. It can be negative but then … or is it ? There is no inverse to say that the shape will change. Only the line will slope a different way. Like in a parabola the shape changes from a u to a n . Now for this the line will just slope a different way. There won't be a different shape.
SN: Would you mind drawing an example of a line and its inverse?
L31: Ok so umm… the line that is not … The inverse of a line will be , if it slopes to the … right then it will be positive and if it slopes to the left then it will negative
SN: Can you draw that for me
L31: (learner draws on the page) If it goes to the right then it will be positive and if it goes to the left then it will be negative.
SN: Which of the two lines will be the inverse?
L31: Eh… eh
SN: If you can just write the word inverse for me.
Are there any other links that you can make?
L31: Not that I can think of.
SN: How did you feel about drawing the concept map?
L31: Some of them were easy to link but some of them I really had to think about. Something different than what we do everyday in class.
SN: Can you show me perhaps which were the concepts that were easy to link?
L31: Parabola and equation. Function and parabola and the inverse and the graph and zero.

SN: You found were easy.
L31: Ja.
SN: And which concepts you found…
L31: perpendicular, gradient to the zero, and the variable.
SN: But you have made links to the…
L31: Ja , after thinking about it.
SN: Alright. Is there anything else that you want to add to your concept map?
L31: No.
SN: Thanks very much for your time
L31: Thank you
SN: and thanks for participating in the project. It has been appreciated.
L31: Thank you for teaching us something new.
SN: You welcome.

Interview transcript L 30

SN: How did you start to do your concept map?
L30: I started by function then according to these terms I started linking them. From the parabola how to get another, where does the equation comes from. From the equation I will get your points on the parabola.
SN: Is there any particular reason why you started of with that concept of function?
L30: I thought it was like more of an important term and almost everything is linked to functions. If you were to start of with parabola or something it won't work or it may work. I don't know.
SN: Ok. In this link you say " inverse the reflection of the parabola." Please explain what do you mean by that?
L30: I thought of it as for a absolute value graph. Like you got one line and you link it . If you draw that section , it is symmetrically equal. That's what I think.
SN: Perhaps draw a sketch for me.
L30: (learner draws on the additional page) Say like you got a graph here and it got a straight line, a positive straight line or something
SN: Yes
L30: Then if you draw the inverse of the negative then it might come up like that (indicates on the page)
SN: Ok.
L30: For an absolute value graph it will come up this way.
SN: Would you mind labelling which line is the inverse for me there?
L30: Sir, I am not to sure.
SN: The one you drew there (indicating on the page) . Which is the inverse?
L30: I think this one.
SN: Do you think that will be the inverse?
L30: Umm… I'm not to sure
SN: You are not to sure. Ok that is fine.
How is the graph of the parabola related to the graph of its inverse?
L30: Sir, when you draw a absolute value graph and you got a positive side and your negative side
SN: mm.. mm
L30: and (pause) from your line sir if you drawing like a straight line… from a straight line graph
SN: Yes
L30: I will take some of the straight line graph but it is not exactly linked to a parabola. If you say it is linked to a parabola , if you got a graph and the one is positive and the one is then the one will go this way (learner drawing on the page)
SN: Will that be a u shaped?
L30: And another will go this way maybe
SN: Ok. Would you mind writing which one is the inverse for me on that graph?
L30: (learner labels the sketch)
SN: Is there any other relationships that you see between the graph of the parabola and the graph of the inverse?
L30: No, sir.
SN: You have also said that functions refer to parabolas etc. in this link. Can you explain to me what you mean by that?

L30: Functions you can draw up your different graphs and one of them is parabola and you can draw up your straight line graph or hyperbola or something.
SN: What makes a parabola a function?
L30: The equation making it got a zero.
SN: Can you give me an example
L30:Umm…
SN: You can write down the equation if you wanted to.
L30: (learner writes an example of a linear equation , 2y = 3x + 1)
Sir, do I have to work it out?
SN: No you can just
L30: Any numbers
(learner writes an example of a linear equation)
y = 3x + 1
SN: So that is the equation of the parabola?
L30: Yes. So you must make like the one side equal to zero.
SN: What about the equation that makes it a function?
L30: Mmm…Sir isn't it like an equation and then in order to work out the points on the parabola that's why its vertex will be a function.
SN: Ok . How do you determine the equation of an inverse for a parabola?
L30: Mmm …Sir I'm not to sure.
SN : Not to sure. Ok.
In your concept map you have linked functions to equations and functions to inverses but you did not have a proposition for it. Any reasons for that?
L30: Mmm… I don't know sir
SN: If I had to give you an opportunity to eh think of a link between functions and equation, what might that link be?
L30: Sir, what exactly is a function? Isn't like some numbers and that? And from there you get your equation and your inverse.
SN: How do you see a function?
L30: As some numbers.
SN: As an example which numbers are you referring?
L30: No, the points the normal numbers.
SN: Can you make an example perhaps?
L30: Sir, I don't know.
SN: Not sure?
L30: mm mm
SN: Between function and inverse, if you had an opportunity to put in a link what would that be?
L30: (silence) Sir, between function and inverse umm…comes from an equation. I think it comes from an equation to draw a inverse.
SN: Can you write a proposition between those two concepts function and inverse?
L30: I don't think so.
SN: Ok. You have said that lines consist of gradients . Please explain what you mean by that?
L30: Sir, isn't it like your mx + c ? The c value that you get on your y axis. Isn't it like a point on the y axis, you draw a line from the y axis to the x axis?

SN: Ja ,so you are joining the…
L30: Line to the gradient that is where you can get your point from to join the x and y to form a line
SN: What does the term gradient mean to you?
L30: From your c value. It will be your y umm…It is like saying letting your x =0 and if you let x = 0 then it is going to be y = to the c value , which is your gradient.
SN: Ok. Is there anything else that comes to mind when you come across the term gradient?
L30: Umm…. No sir
SN: You got in this link between gradient and parabola " gradient consist of a parabola" in that link. What is meant by that?
L30: It is something like a gradient . It is like a y value also in a parabola's equation.
SN: Can you give me an example of that?
L30: Sir if you got an equation maybe y = 2x + 1 and let your x value equal to zero . Then 2 x 0 is 0 then y = 1.
SN: What does that tell you that y = 1?
L30: That the y value on the graph on the parabola maybe that is like one of the points>
SN : Where is the gradient on the parabola?
L30: Sir I'm not to sure.
SN: Not sure of that ok.
If you have two lines that are parallel , you said in this link that lines can be parallel, so if you have two lines that are parallel, what is the relationship between their gradients? What is the link between their gradients?
L30: Between the two lines?
SN: Yes
L30: Mmm…mmm. Sir just say that lines can be parallel to each other and you got a graph and you draw two straight lines … the two equations
SN: So you have these two lines that are parallel to each other with their two equations. Is there any link between their two gradients?
L30: Their gradients might be different sir, or it can be the same thing, passing through the same point
SN: Can you draw a sketch maybe to show me what you mean?
L30: (learner draws a sketch) Ok sir you got a graph and maybe this line here is parallel to this line here or they can't be passing through each other unless they are perpendicular to each other
SN: Ok. What will be the link between the gradients of the two lines that are perpendicular to each other?
L30: It is going to be on the same point.
SN: They will pass through the same point.
L30: Ja
SN: And the link between the two gradients?
L30: Eh…eh… I'm not to sure
SN: Ok that's fine.
In this link here you got between gradient and variables "found in the equation let x and y = 0". What do you mean by that link?
L30: Ok sir, your variables are like 2a or 3a or something so I link it together because you got like y = 2a + 1
SN: Yes

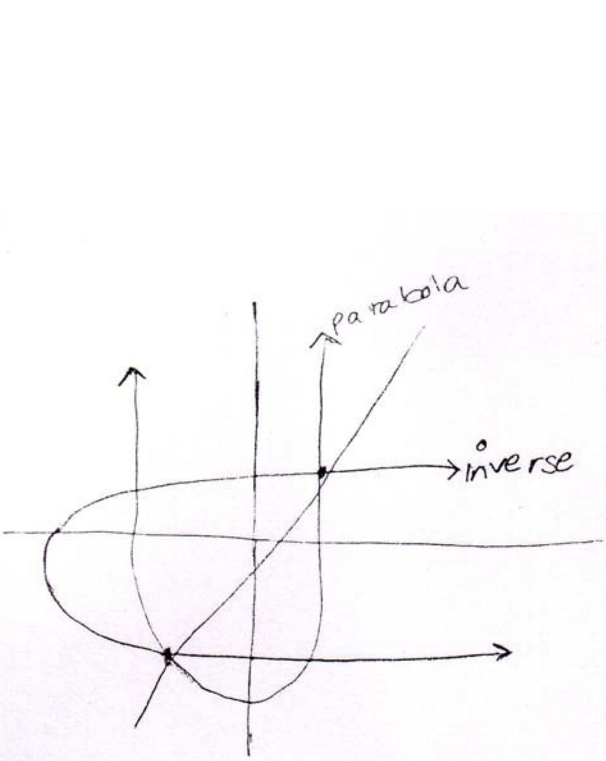
L30: And if you let x eh … or maybe 2x + 1. Then if you let x = 0 , then y = 1.
SN: What does that mean when you have y = 1?
L30: (silence)… Sir I'm not to sure.
SN: Ok. Here you have said that zero can be a point on the parabola. Can you give an example of such a point?
L30: Sir like if you got a graph and you got your parabola and the turning point is passing through zero.
SN: Ok that's fine.
Can you think of any other links that you can have between zero and any other concepts on your map?
L30: Mmm…(silence) no sir
SN: No. Looking at your concept map do you have anything else that you want to add to the way you made links between concepts?
L30: (silence)
SN: In other words are there any additional links that you want to put in between the concepts that you may not have done?
L30: No sir
SN: How did you feel about drawing the concept map?
L30: Mmm… sir it might have been like a simpler way like putting all the knowledge that I know and linking it to each other. The way you say it is like understanding graphs better. Or to see if I understand my graphs.
SN: What will tell you whether you knew your graphs or perhaps you didn't know your graphs?
L30: Sir this interview
SN: What do you mean by that?
L30: Sir like some of the questions you asked me. Sir I do not know why I linked like gradient and variables. I don't know why I did some of the links.
SN: You can't recall why you had some of the links.
L30: I think so
SN: Is there any other point that you want to make with regard to your concept map?
L30: No sir.
SN : Thanks so much for your time
L30 : Ok
SN: and thanks for being part of this research project. I really appreciate that. Thank you.

Appendix 14

Diagrams drawn by learners during the interview to illustrate their propositions

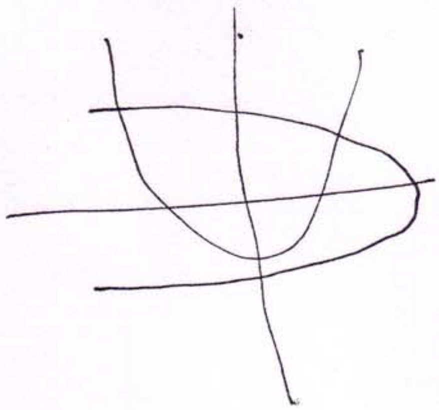
Appendix 14 A

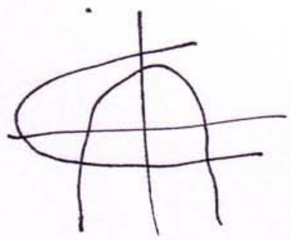
L 26

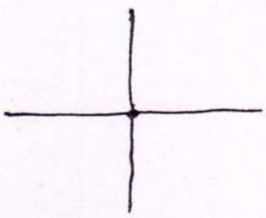


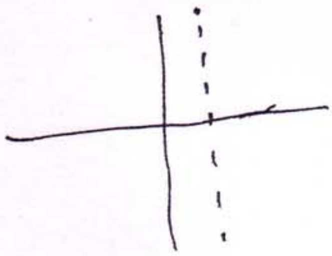
Appendix 14 B

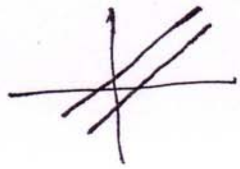
L25

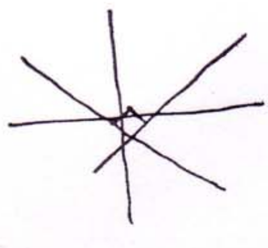






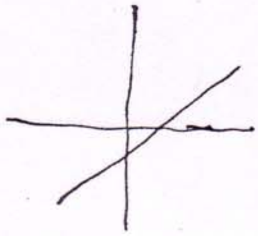






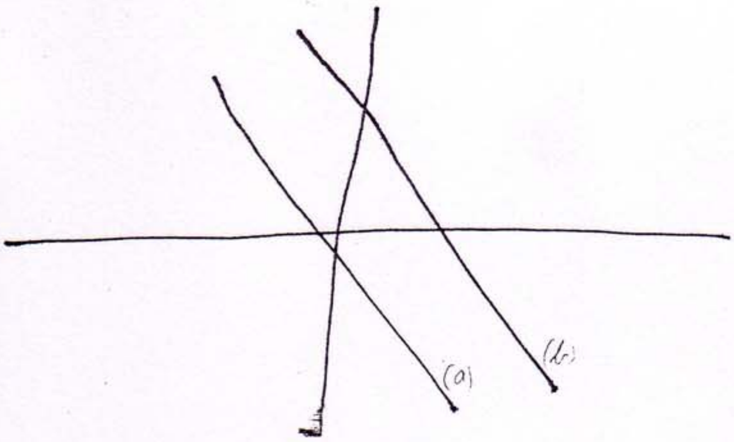
$y = \underline{m}x + c$

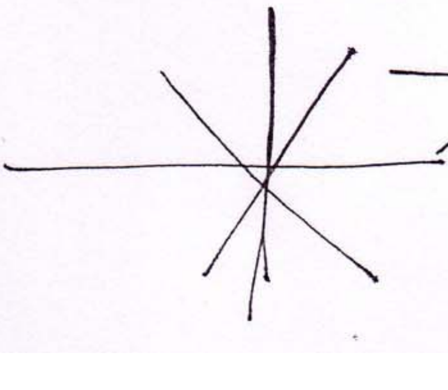
M+

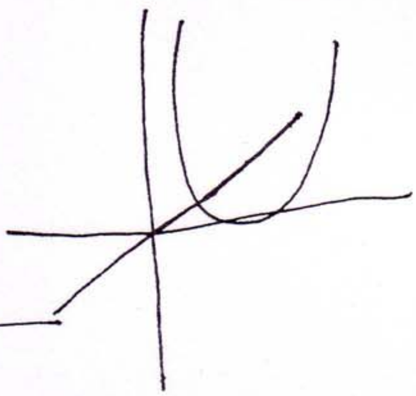


$y = \underline{2}x + 1$ $\qquad \frac{2}{3}$ $\qquad$ L8

$y = \underline{-}2x + 1$ $\qquad -\frac{3}{2}$





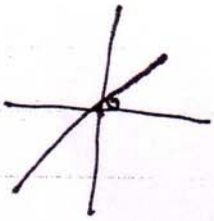


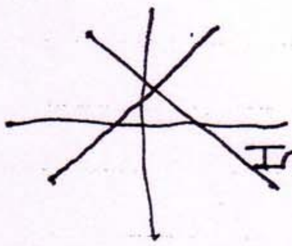
Appendix 14 D

∠ 31

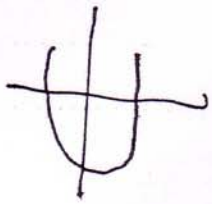
$m_1 = 3$

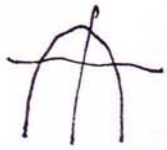
$m_{2\perp} = m_1 \cancel{\div} 1$





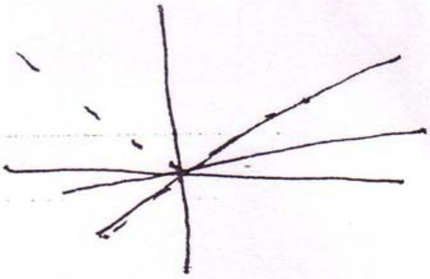
Inverse

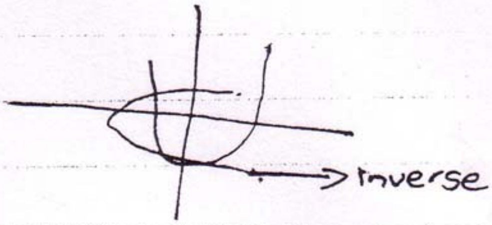




Appendix 14 E

L30





$2y = 3x + 1$

$y = 2x + 1$

$y = 1$