STUDENT TEACHERS' KNOWLEDGE AND UNDERSTANDING OF ALGEBRAIC CONCEPTS: THE CASE OF COLLEGES OF EDUCATION IN THE EASTERN CAPE AND SOUTHERN KWAZULU NATAL, SOUTH AFRICA

BY

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ABSTRACT
This study is aimed at investigating the knowledge and understanding of algebra amongst final year College of Education students in and around Transkei region of Eastern cape, South Africa. Triangulation methods were used to gather data for the study, which included an algebra test instrument, adapted from the CDMTA and Kaur and Sharon (1994) test instruments, interviews and classroom observations. Six Colleges of Education with a total number of 212 students constituted the sample from the Eastern Cape province and South KwaZulu Natal. Data were collected from August 1997 to July 1998.

The motivation for the study was that such an exploratory investigation could contribute significantly to the understanding of some of the principal reasons underlying the poor results in the final schooling examination (the “matric”) of the teaching and learning of mathematics in rural areas of South Africa. Algebra forms a big proportion of the final matric examination in mathematics. The overall results of the study indicate that the conceptual algebraic knowledge and understanding of these College students is weak and fragile.

In analysing the algebraic knowledge and understanding of students as evidenced by the data, factors such as language, the nature of mathematics, the philosophy underpinning teaching and learning and textbooks were seen to have played important roles in the conceptions and misconceptions which many of the subjects of the study portrayed.

My research clearly shows that College of Education students have misconceptions, poor learning and teaching of algebraic concepts. This suggests that these prospective teachers do not have well developed concepts in algebra. The participants’ knowledge and understanding of algebraic concepts are therefore not good enough to assist learners as far as learning for conceptual understanding is concerned at schools.

The results show that much of the knowledge and understanding of algebra came from previous knowledge and understanding gained during high school. Little change had happened during the years spent at the Colleges. The conceptions and misconceptions arose out of the traditional framework of knowledge acquisition rather than through approaches advocated by the newly implemented South African curriculum (Curriculum 2005), which has been revised to a National Curriculum Statement similar to what NCTM (1991) Standards might have envisaged.

The lack of pedagogical content knowledge in algebra, shown by these student teachers during their teaching practice lessons reflects a deeper problem pertaining to

their future teaching after completion of the courses at Colleges. This is bound to have a cascading effect on teaching and learning in schools, perpetuating the cycle of misconceptions in algebra shown by this study, unless something radical is done about the teaching and learning of algebra at Colleges.

The study concludes with recommendations arising out of the results and a number of suggestions for education departments, curriculum implementers, lecturers and future researchers. Strategies are suggested for improving the existing poor state of affairs in the learning and teaching of algebra at Colleges of Education and at secondary schools. These include improvement in algebraic competencies like multiple representations; understanding of basic principles such as: one cannot add unlike terms, checking solutions of equations and inequations. Real life examples should be used to give meaning to the algebraic concept they want to teach where possible. College of Education lecturers should place emphasis on conceptual understanding and correct usage of algebraic notations and symbols. Curriculum developers should include history and relevance of certain algebraic topics in order to create interest and meaning for some of the concepts. Instructional materials in school algebra should be designed specifically for local contexts. Researchers should investigate the cause of misconceptions and misunderstandings of algebraic concepts at high schools and try to address them before they are carried through to the College of Education level.

DECLARATION

I declare that this thesis is my own unaided work. It is submitted for the degree of DOCTOR OF PHILOSOPHY at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination in any other University.

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DEDICATION

This thesis is dedicated to my late Father, Opayin Kwadwo Nsoah, may his soul rest in perfect peace.

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CONTENTS

ABBREVIATIONS AND ACRONYMS

CDMTA Chelsea Diagnostic Mathematics Test in Algebra.

PTD Primary Teachers' Diploma.

STD Senior Teachers' Diploma

SSTD	Senior Secondary Teacher's Diploma
COE	College of Education
C2005	Curriculum 2005- South Africa
MLMMS	Mathematical Literacy, Mathematics and Mathematical Sciences Learning Area in C2005
PCK	Pedagogical Content Knowledge
OBE	Outcome-Based Education
SMK	Subject-Matter Knowledge
H.G	Higher grade (In the current South African senior secondary school curriculum).
S.G	Standard Grade (In the current South African senior secondary school curriculum).
S.A	South Africa
NQF	National Qualifications Framework
VAT	Value Added Tax
SO	Specific Outcome
AC	Assessment Criterion
NCS	National Curriculum Statement
SEK	STD Edgewood College of Education
STK	STD Transkei College of Education
SIK	STD Ndumiso College of Education
SRK	STD Rubusana College of Education

PUE	PTD Umzimkulu College of Education
PME	PTD Maluti College of Education

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DATA REFERENCE

The data analysed in this thesis was recorded during the written algebra test, algebra test interview and classroom observation in the period between 1997 and 1998. The data were compiled from audio recordings, answer scripts from College of Education students and written field notes. All the original transcriptions, scripts and field notes are not included in this thesis, but are available for verification if required.

CHAPTER ONE

INTRODUCTION AND MOTIVATION FOR THE STUDY

1.1 INTRODUCTION

This study investigates final year South African College of Education (COE) students' subject matter knowledge and its relationship to pedagogical content knowledge in the context of school algebra. The first phase of the study is aimed at getting a general picture of the prospective teachers' subject matter knowledge. This phase entails a determination of the level of algebraic thought of the student teachers using a modified Chelsea Diagnostic Mathematics Test in Algebra (CDMTA) and some test items from Kaur and Sharon (1994). The second phase is aimed at clarifying this picture through interviews on the responses to the test items. Lastly, the third phase is a follow up study in the classroom to compare the students' knowledge of algebra with the manner in which they approach the teaching of school algebra.

1.1.1 **Historical Knowledge of Mathematics**

The historical knowledge of topics or subjects is likely to give an insight into the way these have developed over time. If one finds the history of a topic or subject to be interesting the tendency of the person to develop a positive attitude towards it might be high. If, on the other hand, the history, for example of algebra, is boring and valueless the tendency of having a negative attitude to the study might be high and therefore may have an effect on the knowledge and understanding of it. The history of algebra really seems necessary to this research because it can enhance the knowledge and understanding of the subject by College of Education students. Confrey (1987:89) states that historical development of a concept acts as a rich source for:

" 1. *Describing some of the potential misconceptions*
2. *For demonstrating at least one developmental sequence which leads to the current concepts and*
3. *As a source for a variety of problems which provoke considerations of alternative frameworks"*.

Fauvel (1991:4) takes the discussion further by proposing that the use of the history of mathematics in teaching has the following outcomes:

- *“ Helps to increase motivation for learning*

  - *Makes mathematics less frightening*
  - *Pupils derive comfort from knowing they are not the only ones with problems*
  - *Gives mathematics a human face*
  - *Changes pupil’s perceptions of mathematics”.*

Knowledge of the historical development provides the teacher with a field of investigation from which he/she can better understand the learner’s difficulties and, more generally from which they can put some “meat” to the “bare bones”. The knowledge of the historical development is a necessity in order to have an epistemological control, either when analysing existing teaching projects or when experiencing new ways of teaching algebra.

The history of mathematics is seen to be advantageous for the student learning any aspect of mathematics like algebra which this study is about. It paves the way for learners of algebra to think of algebra as a continuous effort of reflection and improvement by man, rather than as finished and irrefutable and unchangeable facts and truths. The history of algebra under consideration provides one with access to different types of algebra to the development of the concept and how they have been used over some years now. The conceptual understanding cannot be seen as being divorced from the social context of learning from which it emanates, that is, the linguistic propaganda of mathematics both inside and outside the classroom has to be considered. As Ernest (1991:18) states *“mathematics is seen as embedded in history and in human practice”*. According to Ernest it has been proposed that the proper concern of philosophy of mathematics (See 2.1.4) should include external questions as to the historical origins and social context of mathematics. I found it necessary to discuss the history of mathematics in this study.

Conceptual development of algebra (See1.4) is also discussed as a way of explaining the difficulties learners and educators are likely to face if the concepts or topics are not understood well. It is therefore necessary for me to discuss the history of algebra, the types of algebra (See 1.2) and the conceptual development of algebra as they contribute to the knowledge and understanding of algebra by College of Education students.

Bromme and Steinbring (1994) also support the idea that teacher’s beliefs concerning the pedagogy of his/her discipline contributes to the subject matter knowledge, which they assert are dependent on the teacher’s mathematical philosophy. The pedagogy and philosophy of mathematics are therefore discussed to address some of the issues, which make teachers teach the way they do in the classrooms. Language as a means of shaping the learners cognitive processes is also discussed.

1.2 ALGEBRA

Algebra is an important branch of mathematics; today it is studied not only in high schools and colleges but also in junior schools. Algebra is useful like the other branches of mathematics. Algebra by nature has different types and since the study is about school algebra it is necessary to differentiate school algebra from the other types. The knowledge of algebra is useful in other mathematical topics like calculus, engineering and science and technology. For example in geometry, algebraic methods are used when solving geometric problems, especially when synthetic techniques become cumbersome, e.g., the synthetic proof of the Pythagoras Theorem which an average geometry student usually finds somewhat difficult to follow. Similarly, deductive reasoning of vertical, adjacent, complementary, supplementary angles in geometry also makes use of algebra. Likewise, deriving sum of angles, interior and exterior angles. In analytical geometry, algebraic concepts are also applied. Again, in school algebra substitution plays an important role in problems involving calculus, sequences and series and nature of roots. Factorisation of algebraic expressions is also of importance in school algebra such as in solving of quadratic equations. In school algebra, the knowledge and understanding of algebra contributes a lot to problem solving and drawing of graphs.

Algebra is one of the oldest branches of mathematics. There is historical evidence that the Babylonians were versed in its methods 4000 years ago. In 2000 B.C the Babylonians used algebraic methods in solving problems. However, they used no mathematical symbols other than primitive numerals. This lack of symbolism in algebra continued for many centuries. Gradually, some of the more common words used in mathematics were abbreviated, which led to a syncopated algebra. Symbolic algebra however, did not begin to emerge until 1500 A.D. One person who can be credited with the early development of symbolic algebra is the French mathematician Viete (1540 - 1603) in about 1600 A.D (Van Reeuwijk, 1995:144).

Classical algebra was introduced in about 830 A.D. by al-Khwarizmi in the Middle East. The name algebra comes from the Arabic al-jabr, which was the title of an algebra text written by al-Khwarizmi. It was presented as a list of rules and procedures needed to solve specific linear and quadratic equations. Until the end of the eighteenth century, algebra could roughly be described as the branch of mathematics, which dealt with the solution of equations. The nineteenth century marks the beginning of modern algebra. Modern algebra, in addition to its concern with solving equations, supplies the language and patterns of reasoning used in other branches of mathematics.

Problems involving quadratic equations were solved more than 3500 years ago. Such problems and their solutions have been deciphered from ancient Egyptian and

Babylonian tablets. Although a general method is not given, the solutions involved completion of squares. Wheeler (1996) accounts for the long period of time it took to develop algebra. He states that the full development took at least 1000 years. Algebra was seen as a completion of arithmetic. Arithmetic, he notes, appears to need the real numbers for its trouble-free functioning, and these could not be fully developed without the aid of algebra. Alan Bell (1996) however states that there is a multiplicity of algebras, not just a single one. This confirms that algebra is not restricted to the world of numbers and may therefore not be inextricably tied to arithmetic.

An understanding of the fundamental concepts of algebra and of how these concepts may be applied is necessary in most technical careers. In the 18th century there existed two substantially different, but mutually supplementary concepts of algebra. One of these considered algebra to be a science of equations and their solutions; the other a science of quantities in general. The latter concept is the "calculation with letters". There are different types of algebra, namely, modern (abstract) algebra, Boolean algebra, linear algebra and school algebra. This research focuses on school algebra.

Modern algebra developed between 1770-1870. Modern algebra deals with the theory of groups and fields. The concepts of modern algebra have been found to be very useful in other branches of mathematics as well as in the physical and social sciences. A chemist may use modern algebra in a study of the structure of crystals; a physicist may use modern algebra in the design of electronic computers, and a mathematician may use modern algebra in the study of logic.

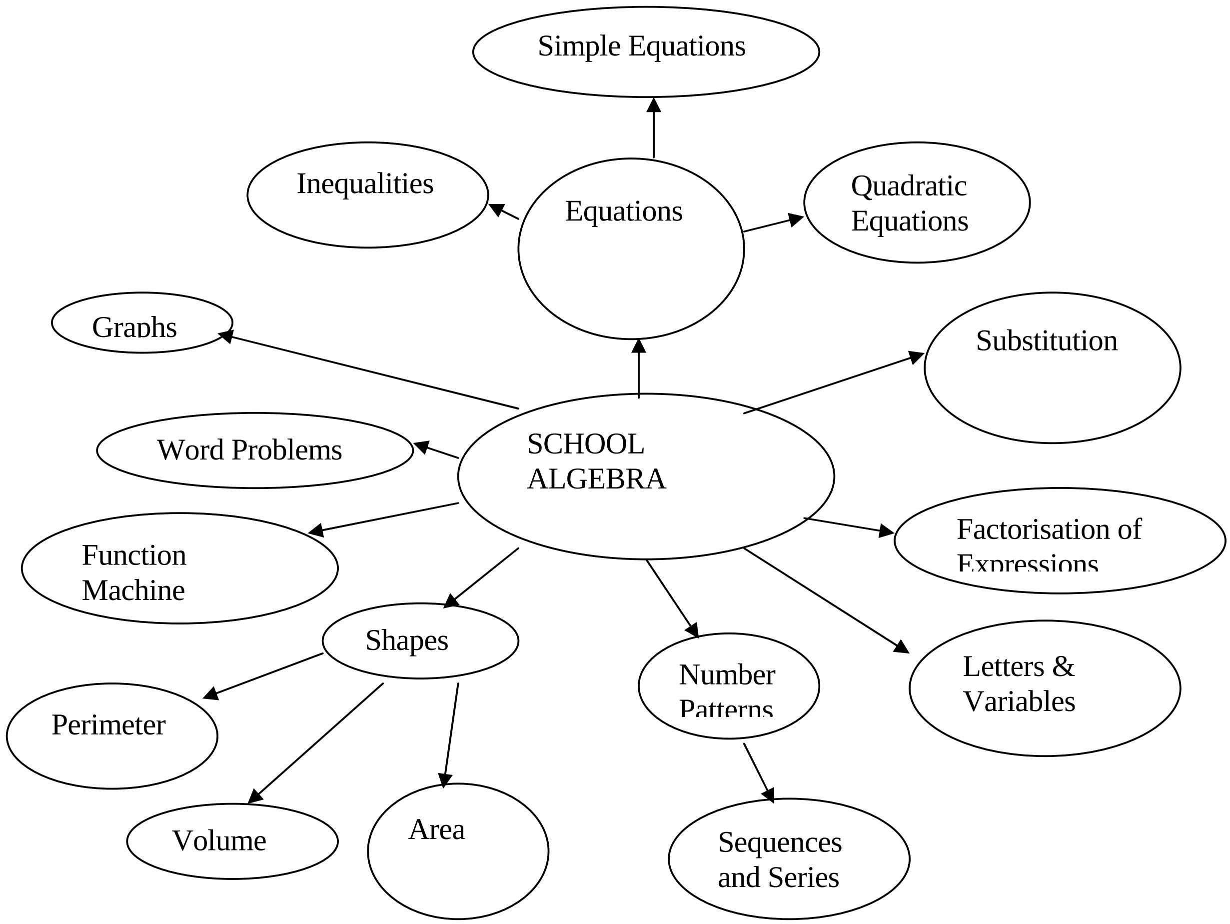
Boolean algebra is a branch of algebra named after George Boole (1815-64). It combines algebraic methods and logic. The basic principles of Boolean algebra relate to logic. Knowledge of Boolean algebra is very useful in fields requiring the application of mathematics and logic. Electronic-computer programming and the construction of electronic circuits are examples of such fields.

Linear algebra is the branch of mathematics concerned with the study of vectors, vector space or (linear space), linear transformation, and surface linear equations. Linear algebra is the earlier of the two mathematical disciplines devoted to the study of that broad and useful notion called linearity: the study of lines and planes in analytic geometry and the system of linear algebraic equations. Linear algebra has a concrete representation in analytic geometry. The history of modern linear algebra dates back to the years 1843 and 1844. Most mathematical problems encountered in scientific or industrial applications involve solving a linear system. For example systems of linear equations and reduction of matrices to standard forms are applications that belong to everyone. Linear systems arise in applications to areas like business, economics, sociology, ecology, genetics, electronics, engineering and physics.

1.3 ALGEBRA IN SOUTH AFRICAN SCHOOLS.

School algebra is seen to focus on manipulative skills of simplifying, factoring, solving equations, functions and graphs, variables, word problems, and patterns (See fig 1.1). Algebra is introduced to pupils in South Africa around the ages of 13 and 14 years. The place of school algebra in the mathematics curriculum of schools in South Africa has been debated for a long time. The central pedagogical problem in teaching algebra is finding convincing intrinsic reasons for the value of the study. Van Reeuwijk (1995) claims that the place and role of algebra in schools have changed. Algebra in the traditional curriculum has been presented as a language and a fixed structure. Students are made to copy rules and tricks of algebra without a real understanding of the subject matter. Students are not given the opportunity and time to develop their own schemes. The traditional algebra course according to Romberg & Spence (1993) is seen as sterile, disconnected from other mathematics and from the " real world". At present the learners in South African schools will be studying algebra with conceptual understanding and reasoning. It is encouraging to observe that outcomes-based education (OBE) which is one of the principles underpinning Curriculum 2005, is consistent with this paradigm (See 1.3.1). This system of education allows for active learners, learners who will think and reason critically. It involves integration of knowledge, relevant learning, and is connected to real-life situations. It is about learner-centred (See 2.1.3) education where the teacher is the mediator. The teacher usually is required to use group work and teamwork to consolidate knowledge. The main aim of OBE is to develop sense-making ways for learning and teaching of mathematics. The learners' experiential real world is used as the base to start the development of concepts and skills (See fig 1.1). Learners take charge of their own learning with guidance from the teacher who is supposed to be innovative and creative in designing programmes, which will help to achieve the expected outcome.

Figure 1.1 Concept map of South African School Algebra



According to the National Department of Education policy document (1997) on OBE (See the Mathematical Literacy, Mathematics and Mathematical Sciences [MLMMS] learning area), the definition of mathematics includes conceptual understanding, which is supposed to be expressed, developed and contested through language, symbols and social interaction. College of Education students' knowledge and understanding therefore need to develop a proper sense of algebra in order to face the demands and challenges of Curriculum 2005. Teachers and student mathematics teachers play an important role in building algebraic mathematical understanding through the type of classroom environment they create and the teaching practices they employ, and through the activities they select.

The society in which we live has also changed where people have to be educated to understand complex situations. The point of discussion is that an education system should be designed in relation to the reality of the society it serves and must be sensitive to changing needs. Teachers should be knowledgeable enough to initiate, sustain and guide learning under these changing conditions. Capps & Vocke (as cited in Barras-Baker,1993: 24) state:

> *" Our society has moved into what is frequently referred to as the information age. Such an age is characterised by the explosion of advanced technology and leaves one unable to master all the information available in a given field. Thus, attention should be given to providing skills which will enable the student to process the ongoing flow of information".*

These skills should be seen as evaluation and synthesis skills, critical thinking, problem-solving strategies, organization and reference skills, application abilities, creativity, decision-making based on incomplete information, and communication skills through a variety of modes.

The curriculum shift from the traditional way of teaching manipulative algebra to an OBE approach where the learner is engaged in activities to enable him/her to think creatively thereby laying greater emphasises on conceptual understanding necessitates that student teachers are provided with a powerful aid to concept and strategy development in algebraic thinking. This shift will demand that teachers of algebra develop in learners the appreciation of the meaningfulness and purpose of algebra and its use as a thinking tool. In order for prospective teachers to be able to help their learners to be flexible in their approach to algebra and make good choices between available approaches, it is essential that the prospective teachers themselves should have the knowledge and understanding of algebra. Knowing what algebraic concepts are and being able to work with them in alternative ways, different representations and using appropriate methods are important for the prospective teacher. These will lead them to have a good grasp themselves to handle algebraic lessons and hence, make the topic interesting and relevant for the learners to appreciate the existence of algebra in the school curriculum.

Words are not manipulatable in the way symbols are. It is this manipulating which makes algebra very important. Algebra has the power to express relationships in clear and concise ways, which are manageable for further purposes. For example, if one is asked to find the number of hand shakes there will be if in a party of 25 guests each one has to shake hands with each other. It will be easier to use arithmetic but as the number of guests increases it will be difficult to determine the number easily. It will be easy to use algebra to generalise and get a formula, which can be used to determine the number of handshakes for any number of guests.

School algebra begins when an attempt is made to find an unknown number on which a given operation is preferred and a given result is obtained by making use of symbols or letters. To many people studying algebra, symbols appear to have little or no meaning. Yet, algebraic symbols are very important in mathematics, not only as a means to express generalisations but also as a means of thinking and drawing conclusions about and manipulating problems, which might be difficult to answer. It is therefore very important and necessary to assist learners to develop meaning for symbols used in algebra and to make them see their usefulness in communicating and expressing mathematical ideas. One way of making symbols meaningful is to introduce symbols as a natural development of learner's own attempts to record their own generalisation patterns. According to Costello (1991) the most profitable way to introduce algebra is not probably through 'missing number' problem but rather through number patterns, relationships and generalisations. This is what is expected in Curriculum 2005 for MLMMS (Mathematical Language Mathematics and Mathematical Sciences). Specific Outcome number 2 addresses number patterns and requires that learners " *Manipulate numbers and number patterns in different ways"* (National Department of Education, 1997:7).

1.3.1 **Background to OBE Curriculum 2005**

Curriculum 2005 is based on 7 critical outcomes and 5 developmental outcomes related to the principles of *"cooperative, critical thinking and social responsibility"* for all the citizens of South Africa (National Department of Education, 1997). These principles are based on the premise that if learners were to be prepared to participate in all aspects of life then that was the way to approach learning. Curriculum 2005 was introduced for the first time to grade one learners in January 1998, as part of the National Qualification Framework (NQF), a framework which aims at promoting an integrated approach to lifelong education and training in South Africa.

The two main challenges envisaged for mathematics teachers who have been using the traditional approach to teaching are the methodology and the method of assessment. The methodology has to shift from a teacher-centred to learner-centred approach (See 2.1.3). Assessment in OBE is based on a continuous assessment policy and the use of different types of assessment, which include, self, peer and group assessments, portfolios, journals, projects, investigations, tests and assignments, class work and homework, and the usual examinations.

For the current teachers of mathematics the main challenge will be adopting the view that mathematics is not based on absolute knowledge but that mathematical knowledge is fallible (Von Glasersfeld, 1990). Also that mathematics is a tool to understand and solve social problems (Ernest, 1991). The challenge therefore, is to move away from the traditional way of teaching to the learner-centred approach. The notion that algebra is based on a set of rules and principles which can be manipulated using symbols and letters should change to a learner-centred approach which embraces change in thinking about criteria for assessment, conceptual understanding and method of teaching, the learner-centred approach. Curriculum 2005 is seen to have adopted some of the reform efforts developed by National Council of Teachers of

Mathematics standards documents (1989, 1991).

1.3.2. **Reform in Mathematics Education**

Reform in Mathematics Education revolves around the nature of mathematics, mathematics learning, and mathematics teaching. The Curriculum and Evaluation Standards for Mathematics (National Council of Teachers of Mathematics, 1989: 3) defines the reform goals as follows:

" All students should:

1. *learn to value mathematics,*
2. *become confident in their ability to do* mathematics*,*
3. *become mathematical problem solvers,*
4. *learn to communicate mathematically,*
5. *learn to reason mathematically"*.

This vision of reform is a shift from the traditional method of teaching (See 2.1.2) mathematics. One aspect of the reform is the integration of mathematics content and pedagogy. It also provides teachers of mathematics with experiences that conform to the constructivists' mode of teaching (See 2.1.5.1), which can be useful to knowledge and understanding. It provides teachers with experiences of active participation of learners in the mathematics classrooms. This shift is summarised by The Professional Standards for Teaching Mathematics (National Council of Teachers of Mathematics, 1991:3) into five visions as follows:

"1. Towards classroom as mathematics communities- away from classrooms as simply a collection of individuals;
2. Towards logic and mathematical evidence as verification- away from the teacher as the sole authority for right answers;
3. toward mathematical reasoning-away from merely memorising procedures;
4. toward conjecturing, inventing, and problem solving- away from an emphasis on mechanistic answer finding;
5. toward connecting mathematics, its ideas, and its applications- away from treating mathematics as a body of isolated concepts and procedures".

The goals of mathematics education as expressed by National Council of Teachers of Mathematics (1989, 1991) have been embraced by the South African National Department of Education hence the introduction of Curriculum 2005. I believe with this approach to school algebra effective teaching can be done in the classroom by the prospective teachers. The learners on the other hand, will be able to participate actively in the school algebra lessons delivered by these prospective teachers and might lead to conceptual knowledge and understandi*ng.*

1.4 CONCEPT DEVELOPMENT

In traditional algebra classrooms, algebraic concepts have been introduced to students by their teachers. The teachers apply these concepts and expect the students to understand them and also apply them successfully. Most of the misconceptions pupils have are partly due to this type of attempt at transmission of knowledge (See 2.7.1). Concept development is both important and challenging; it is important because understanding a concept allows it to be used correctly and recognised in new situations, challenging because of the difficulty of enabling students to "see" clearly that which is confusing and vague. Nonroutine and open-ended problems are normally used to develop concepts. Open-ended problems have more than one solution; indeed there are usually many solutions to them. Considering that an individual may construct an extensive collection of ideas related to a concept, it is clear that such a schemata of concepts, has the potential to represent a substantial amount of knowledge. Some of the individual's ideas may be contradictory. It is such contradictory ideas, when evoked simultaneously, that cause cognitive conflict to occur.

Concept schemata can be developed through a student-centred instructional approach. The role of the teacher is to create a classroom environment conducive to exploration and risk taking. By grouping the students and sharing their ideas and methods, they are able to develop concepts on their own. Group work encourages and cultivates communication. Because students feel safe in their groups, they gain confidence and willingness to share ideas and thoughts.
Vygotsky (1986:148) states that

> *" concept is more than the sum of certain associative bonds formed from memory, more than a mere habit; it is a complex and genuine act of thought that cannot be taught by drilling, but can be accomplished only when the child's mental development itself has reached the requisite level"*

According to Vygotsky there are two types of concepts, the spontaneous and the scientific. The spontaneous concept is the child's everyday concept like "brother". The child works them out for himself/herself. Scientific concepts evolve from the systematic cooperation between the child and the teacher. Vygotsky further states that scientific concepts develop more accurately than the spontaneous concepts because of the benefit from systematic of instruction and cooperation from teachers. Scientific concept development is what is seen in schools where the teacher helps the learner to absorb ready-made concepts through a process of negotiation and assimilation. But a teacher who tries to teach concepts directly by telling will embark on a fruitless and impossible journey towards making learners understand and assimilate. This is impossible because for conceptual change to occur the new information to be assimilated or accommodated into the existing conceptual system must according to Hewson and Thorby (1991) be:

1. Intelligible: it must have meaning and make sense to the learner.
2. Plausible: it has to appear from the outset to be used for solving problems that could be solved by the concept it replaces, while still being consistent with its predecessor.
3. Fruitful: it must be extendable, suggest wider possibilities and open wider doors.
4. Adequate: it must render its predecessor inadequate and cause the judging student to feel dissatisfied over it.

Conceptual change is therefore likely to occur when the conditions above are satisfied. Assimilation normally occurs when the new information of the learner fits into the existing knowledge. Where the new information does not fit into or is incompatible with the existing knowledge the learner rearranges or reorganises his/her knowledge in order to accommodate the new information.

According to Vygotsky (1986) instruction is seen as one of the principal ways of helping school children to develop conceptually. Instruction, he further states, determines the fate of the child's total mental development and acts as a powerful source in directing conceptual development and evolution. He claims " instruction creates the zone of proximal development" (See 2.1.5.1). As Vygotsky argues, it is necessary for the teacher to help learners attain the scientific knowledge. The knowledge and understanding of algebraic concepts acquired by College of Education students will render these students a service to their students in the classroom. These students should be able to transform the subjective knowledge of their learners in algebra to objective knowledge, which is accepted by the mathematics community (Ernest, 1991). A learner in algebra might have a different understanding of roots. The "nature of roots" in algebra has different interpretation to the ordinary meaning of roots. It is therefore the responsibility of a teacher of algebra to explain to the learners the meaning of the "nature of roots" in algebra, which involve the formula, the meaning of the symbols in the formula and how to substitute these symbols with values from the quadratic equations.

Ernest (1985) also argues that mathematical knowledge is a social construct, therefore the teacher or the student teacher in algebra has the social responsibility to help learners in algebra to transform their subjective mathematical knowledge to objective mathematical knowledge. Ernest however, states that the new teachers (prospective teachers) joining the teaching profession are likely to bring along undiluted theoretical views of mathematics, which might determine the way the subjective mathematical knowledge is transformed into mathematical objective knowledge. For this reason it is necessary to understand the philosophical views of College of Education students, which might contribute to the ways and manner College of Education students help learners in algebra to learn new concepts
(See 2.1.4).

1.5 MOTIVATION FOR THE STUDY

In the classroom situation in many parts of South Africa, the teacher has the sole responsibility to decide about the style of presentation of the subject matter to his students. The teacher's own subject matter knowledge thus plays a major role. The teacher's decision about whether a certain response is correct is based on the teacher's subject matter knowledge. As a consequence, I feel that one of the main reasons why school students fail to understand mathematical concepts is the insufficient and often unsuitable training and preparation of teachers. This leads to teachers resorting to transmission of knowledge to students as discussed in section 2.7.1.

Ball (1988) claims that there is an emerging consensus that students should experience and practice how mathematics is developed and communicated as opposed to encountering mathematics in its completed form, that is, as the 'finished product' presented in mathematics textbooks. The view of what it means to know and do mathematics is very different from how present prospective teachers have themselves learned mathematics.

The teacher's own experience, both as a learner and as a teacher, is likely to influence pedagogical content knowledge. Shulman (1986) refers to pedagogical content knowledge (PCK) as the ways of representing and formulating the content that makes it comprehensible to others (See 2.3). For example, for the concept of functions students can be asked to find Value Added Tax (VAT) on a wide range of appliances so that they develop intuition about the relationship between cost of the item and the amount of VAT instead of asking a question like; " What is the Value Added Tax on a car which costs R42 000?". Carpenter, Fennema, Petterson, Cavey (1988:396) also view PCK to include

> *"knowledge of the conceptual and procedural knowledge that students bring to the learning of a topic, the misconceptions about the topic that they may have developed, and the stages of having little understanding of the topic to enable mastery of it. It also includes knowledge of techniques for assessing students' understanding and diagnosing their misconceptions, knowledge of instructional strategies that can be used to enable students to connect what they are learning to the knowledge they already possess, and knowledge of instructional strategies to eliminate the misconceptions they may have developed"*.

Another source of PCK is the nature and depth of teachers' own subject matter knowledge of the material they teach. Pedagogical content knowledge of a specific piece of mathematics includes more than conceptual and procedural knowledge. It also includes knowledge about the nature of mathematics. Many teachers, particularly in junior schools, have such shaky mathematical understanding and misconceptions themselves that this may lead to not laying proper foundations for their pupils. A teacher who has a good understanding of certain mathematical ideas and techniques is more likely to be able to apply that learning to contexts that might be very different from contexts in which the mathematics was originally learned.

Freudenthal (1991) proposed a sense making way of teaching and learning mathematics called Realistic Mathematics Education (RME). This approach considers mathematics as a human activity and therefore builds on the learners' experiential real world and is used as a meaningful way to develop mathematical concepts and skills.

Teachers' PCK is not to be gauged by the number of courses they have taken or their success on standardised tests. Analysing what it means to know mathematics has some contributions to make to the improvement of PCK preparation for teachers and therefore the quality of teaching and learning. According to Bromme and Steinbring (1994) students are better able to understand and remember concepts and principles if they have meaning. There is therefore the need to analyse the PCK of College of Education students with respect to the development of meaning.

In order that effective learning may take place, the teacher has to help pupils have proper conceptual understanding, identify the pupils' misconceptions in the subject and employ appropriate teaching techniques to reduce them to the minimum. Educators and facilitators often find that identification of misconceptions is of great help to all those who are concerned with education. The existence of other cognitive obstacles such as inadequate language skills (on the part of second language learners) as pertaining to rural Colleges of Education also needs to be identified. All these will help alleviate the problems of learners as well as of teachers in the learning of mathematics, which has its own language.

For the purpose of this research the conceptual understanding of algebra is chosen to illustrate the difficulties and misconceptions caused by a lack of understanding related to several bodies of knowledge. Regarding the position and importance of algebra in the undergraduate mathematics programme, the teaching of algebra and the creation of meaningful algebra curricula have been a major concern of educators as well as students and a major concern of mathematics educators in South Africa as we embrace Curriculum 2005. There have been efforts to reform the teaching of algebra using technology and other specific curriculum changes such as OBE in Curriculum 2005. These are to improve students' understanding of algebra concepts as well as to execute algorithmic procedures and skills in more meaningful ways other than memorising the procedures and techniques and applying them without conceptual understanding. Research on College of Education students' understanding of algebra can also help the development of more efficient algebra curricula. The study of calculus at undergraduate and graduate level programmes also needs a solid conceptual understanding of algebra for its successful completion, since algebra forms part of the foundation to the learning of calculus.

Since the understanding of mathematical content in general and algebraic content in particular (facts, concepts, principles, algorithms) is determined by cognitive factors, I find it appropriate to construct the theoretical framework within which to investigate students' understanding and knowledge, in terms of such cognitive factors. The theoretical framework is therefore developed according to the principles of the theory of constructivism (See 2.1.5.1) and the investigative approach (See 2.1.6) to teaching. The central idea of constructivist theory is that mathematical knowledge cannot be transferred ready-made from person to person, but is reconstructed by each individual learner (von Glasersfeld, 1990). According to Piaget (1973) pupils acquire mathematical knowledge not by internalising rules imposed from outside but by construction from inside through their own thinking abilities. When errors are committed, these arise because of the pupils' alternative conceptions on what they are basing their thinking and not necessarily because they are careless. The task therefore of a teacher is to create an environment conducive to self correction (See 2.1.5.1).

An investigative teaching style is a pedagogical approach to the mathematics curriculum. The investigative approach according to Frobisher (1994) is the application of communication, reasoning, operational and recording processes to the study of the core topics, which make up the content of a mathematics curriculum. This approach according to Frobisher encourages pupils to put their thinking and their conjectures to the test and if necessary modify their first thoughts. This entails that the teacher makes pupils work in groups to see whether they produce solutions, which are similar or different from one another (See 2.1.6). Using this type of approach requires the teacher to have explored the problem at first hand, to have experience of frustration and the joys, which accompany the process of solving the problem. This again brings the College of Education (COE) students' knowledge and conceptual understanding of algebra into the question of having the necessary pedagogical knowledge to help solve the problems they require their pupils to solve.

I have been involved in the teaching of mathematics both at the matric[1] and College of Education level for several years (from 1984 to 2000) in the former Transkei, now part of the Eastern Cape province of South Africa. Personal observations during the above mentioned periods of interaction have indicated to me that pupils and college students do show more interest in the teaching of algebra than geometry. One would expect the College of Education students to do well in algebra. In spite of algebra being allotted a higher proportion of marks than geometry in final examinations, results from several colleges I surveyed, indicate that students perform worse in mathematics than in other subjects. Table 1.1 and Table 1.2 are six-yearly and four-yearly summaries of

[1] Matric in the South African educational system is equivalent to grade 12 in other educational systems.

percentage scores of final year examinations per subjects offered by Primary Teachers Diploma (PTD) and Senior Teachers Diploma (STD) colleges in the former Transkei.

I observed again that students were exposed to excessive teacher talk. Most of the time the learners sat passively and silently, listening to what the teacher said without participating in the lesson. Interaction between teacher-learner, learner-learner, was minimal. Students usually resorted to memorising lecture notes or contents of the textbooks, without understanding what is contained in them. During class tests and examinations, students often wrote verbatim accounts of what they had memorised without showing any sign of critical thinking and understanding. Because most of the teachers in Transkei were trained in Colleges of Education, I became interested and curious to know what happens to the knowledge and understanding of algebra among some of the final year students in these colleges.

YEAR							
	Maths	Geog.	Xhosa	Gen. Sc	Rel.Ed	Educ.	Com.Sk
1986	**55.4**	68.8	93.6	89.2	98.7	92.6	95.7
1987	**48.9**	83.4	94.5	88.5	98.3	94.8	97.9
1988	**50.9**	67.1	90.8	88.3	97.0	89.9	94.0
1989	**71.8**	91.0	94.8	91.6	97.9	60.4	98.4
1990	**47.5**	93.7	89.1	92.1	97.6	74.9	85.9
1991	**63.7**	63.3	92.4	72.6	94.6	85.3	72.0
Ave %	**56.4**	77.9	92.5	87.1	97.4	83.0	90.7

Table 1.1 Six-Year (1986-1991) summary of results (mean %) for final year PTD Colleges in Transkei. Source: Transkei Department of Education. (Com Sk-Communication Skills, Rel. Ed- Religious Education, Gen. Sc- General Science)

YEAR									
	Acc	Agric	Bio	Econs	Geo	His	**Maths**	P.Sc	Eng
1995	68	15	51	51	70	63	**46**	65	80
1996	63	43	45	31	72	77	**34**	53	76
1997	11	30	17	0*	37	23	**24**	17	36
1998	50	38	30	100	33	29	**13**	36	50
Ave.%	48	32	36	61	53	48	**29**	43	61

Table 1.2 Four-Year (1995-1998) summary of results (mean %) for final year STD Colleges in Transkei. (Acc- Accounting, P.Sc- Physical Science, Eng- English)

From the Tables 1.1 and 1.2, the percentage scores in the final examinations fluctuate year after year for mathematics (maths), but the result for mathematics is almost always lower than for the other subjects.

As a requirement to offer mathematics at a College of Education, prospective students should have obtained at least an E symbol at Higher Grade or a D symbol at the Standard Grade in mathematics at matric level. Regardless of their symbols there seems to be a gap in their background in algebraic knowledge and understanding. Most of the students I have taught over the years have very little conceptual knowledge in algebra. For example, if learners are asked to simplify (x + 2)/2 the answer some come out with is x + 1. This type of an answer is not expected from someone going to teach children in future. Their reading and drawing of algebraic graphs, are of much concern to educators. Even when the student appears to have understanding of certain topics there seems to be misconceptions in some aspects of the topic. Some students find it difficult to give an answer to a situation where one is asked to subtract (e.g., 3 from t) or add 5 to an unknown quantity. There also seems to be a problem when a student is asked to solve for x in the following equation 2(x + 2) = x + 4 and comes out with an answer of (2 = 1) This is supported by Blubaugh (1988) who suggested that the rampant use of the word "cancel" during mathematics instruction could lead to students' misconceptions. He attributed this to the unclear use of the word, which they are unable to distinguish between the different meanings associated with cancel as they apply to different mathematical topics. Solution to inequalities is also a problem to many students studying algebra. For example, to solve (x - 5)/(3-x) > 1 the first thing most students do is to multiply both sides of the equation by 3-x to get x - 5 > 3 - x. Kaur and Sharon (1994) investigated this type of problem with first year students at a junior college in Singapore and about 72% of them did not see anything wrong with multiplying both sides of the inequality by 3-x.

In my sixteen years (1984-2000) as a teacher of mathematics in secondary schools and as a lecturer in three Colleges of Education in Transkei, my experience of teaching mathematics and observation of College of Education students shows that the "exposition and practice" approach to the teaching of mathematics, algebra included; is general. The teacher after introducing the topic and giving one or two examples on a topic, asks pupils to do similar exercises. College Students often take what appears in mathematics textbooks as given and the mathematics they teach is often not different from what they themselves have learnt. My experience with an educator who asked me to moderate his examination questions for a final year examination argued with me when I said a question was wrongly phrased. The educator's argument was that the question was copied from a textbook. This is likely to be a person who holds the philosophy of mathematics as absolute and not fallible. There are instances where teachers are tempted to use their lesson plans from the college years without any amendments to them especially when teaching the same topic. Another point, which is often the case with many teachers, is that they find it difficult to change their teaching styles and strategies. As Steffe (1990) puts it, teachers who are mathematically inactive usually present mathematics as static, dualist (either right or wrong), and as consisting of routine procedures. Solving a quadratic equation, for example, might be demonstrated as a sequence of steps that, if followed, would give a correct answer.

Many College of Education students like many teachers of mathematics have formalist/structuralist view of mathematics and they find it difficult to change their belief about the teaching and learning of mathematics. They see algebra as a topic full of rules, procedures, tricks for solving problems and therefore find it unnecessary to have conceptual understanding, which normally takes a longer time to achieve. It is therefore necessary for College of Education students to change their knowledge and understanding of mathematics/algebra. Adopting a view that mathematics is a human activity and that mathematical meaning is constructed as a result of activity would possibly root out the formalist and abstract symbolic presentations of algebraic rules and procedures. This belief might have far reaching consequences for algebra teaching in schools.

The most important problem facing a mathematics teacher in algebra is to foster the development of mathematical understanding and meaning in the pupils they teach. As Thom (as cited in Steffe, 1990) rightly puts it, "*the teacher's role is that of a midwife-to free it from the mother-structure which engenders it*" (p.167). The College of Education students with their conceptual understanding of algebra should be able to guide their pupils to conceptualise, accommodate and assimilate algebraic knowledge and understanding.

The Specific Outcome number 9 (SO9) of the MLMMS learning area in the Outcomes-Based Education Curriculum 2005 deals with communication in mathematics through, for example, the use of symbols in algebra. It states, " *Use mathematical language to communicate mathematical ideas, concepts, generalizations, and thought processes*" (National Department of Educational policy, MLMMS, 1997:29). The College of Education students should have knowledge and understanding to facilitate the achievement of this SO9. I am currently assisting in the implementation of OBE in schools in grade 7 to 9 (senior phase). In one of the training sessions with practising teachers when we were unpacking this SO9 the problem of adding unlike terms came up. For the addition of 7a to 5b, some of the educators came up with the answer of 12ab. This illustrates the lack of knowledge and understanding many of the teachers have of this type of algebra.

Because of the above, several educators and researchers have become involved in the search for solutions to problems of a pedagogical nature in the learning of algebra.

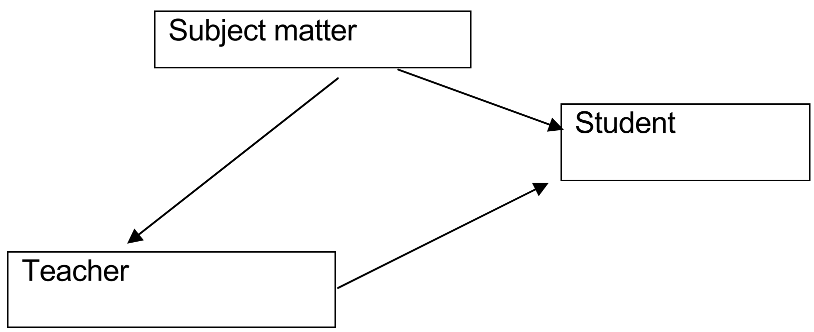
These efforts are mainly due to dissatisfaction with the results of algebra teaching. Meaningful research on students' knowledge and understanding of algebraic concepts can indeed have far-reaching consequences regarding the more efficient ways of teaching algebra to pupils. Taking into consideration the importance of algebra in the teaching of calculus and physical science and other school subjects, a thorough understanding of key concepts should be the main purpose of studying the topic.

Driscoll (1982) attributes difficulties involved in developing a deep understanding of algebra, to algebra having several different faces. Algebra in one face is seen as a kind of generalized arithmetic, with rules of addition, subtraction, multiplication, and division. On the other hand, it is a structured system for formulating and manipulating variables and formal mathematical statements. According to Driscoll because of misconceptions or slow cognitive development, young people may succeed in some applications but fail to connect algebra to its broader mathematical application. He gives an example of students mastering the mechanical skills of factorising the expression $a^2 + 2ab + b^2$, but not recognising that it can be applied to $a^4 + 2a^2b^2 + b^4$. What contributes to this might be the lack of conceptual understanding on the part of students, due to the way the concept was taught by the teacher who was once a college student. This however creates an overwhelming demand on the cognitive resources of the teacher in the school classroom. Since the process of learning is likely to be influenced by the teacher, it is necessary to understand how College of Education students explain algebra concepts to learners. What they emphasize and what they do not, what resources they use and what ways they choose to help learners understand.

The specification of the knowledge and understanding of algebra for student teachers, the investigation of the practice of the student teachers in the subject content of algebra and of the problems of learning and teaching algebra at school could contribute considerably to alleviate some of the problems teachers have in teaching school algebra. With proper attention in an appropriate curriculum the College of Education (COE) students' pedagogical knowledge and understanding of algebraic concepts could in the future put such school learners into a position where they will be able to overcome such difficulties. Kubinova, Mares and Novatna (2000) support this view when discussing the traditional scheme “subject matter-teacher-students” (See fig 1.2). They argue that the subject matter serves as an intermediary which enables development or modification of already existing concepts and the creation of new ones. The SMK is referred to as intermediary because both the teacher and the learner need to have it. The teacher needs to have a sound and a higher knowledge to be able to assist a learner who is grappling with a certain concept. Intermediary implies a common ground of knowledge acquisition whereby the teacher has a superior knowledge than the learner but working toward common understanding thereby helping to explain, direct or make the subject matter clear to the learner. This scheme goes to support the notion that the subject matter knowledge of COE students in algebra will help learners of algebra to develop and modify their existing algebra knowledge.

As fig 1.2 indicates, students are capable of constructing their own subject matter knowledge from their experiential world. This is in line with the second principle of radical constructivists (Von Glasersfeld, 1990). This principle states that ideas and thoughts cannot be transferred from one person to the other. Implying that as students experience something new they internalise it through their past experiences or knowledge constructs they previously established. This is shown in the diagram where the arrow goes straight from subject matter to student. On the other hand, students at a certain stage of knowledge construction needs the help of the adult (the teacher). This stage is what Vygotsky (1986) terms Zone of Proximal Development (ZPD). At this stage the teacher will have to act as a mediator in the knowledge construction of the learners. The arrow from the subject matter via the teacher to the student represents this stage of knowledge construction by the learner.

Figure 1.2: The traditional scheme of interactions.



1.6 RESEARCH FOCUS

The major purpose of this research is to identify some common misconceptions and other cognitive problems in algebra in so far as they are prevalent among COE students in and around the Transkei area of South Africa. College of Education students come into colleges with a knowledge of school algebra and an expectation that the courses at colleges will prepare them to teach school algebra and thus initiate them into pedagogical content knowledge (PCK) in school algebra. As discussed in 2.3 the PCK of COE students plays a very important role in the duties of a mathematics teacher as a mediator and a guide to the teaching of algebra to learners in schools. The research therefore may help mathematics teachers in South Africa to modify their methods of teaching. In the light of the above discussion, the purpose of this study is firstly to investigate the subject knowledge and PCK of COE students. Secondly, the investigation endeavours to find out whether conceptual understanding of algebra affects their instructional practices.

1.7. RESEARCH QUESTIONS

The questions, which this research will investigate are:

1.7.1 To what extent do South African College of Education students understand basic algebraic concepts?

1.7.2 What are the common misconceptions that South African COE students acquire in algebra?

1.7.3 How does the subject matter knowledge of South African College of Education students affect their instructional practice?

1.8 PROBLEMS AND LIMITATIONS

There are, of course, problems and limitations to this study. Some of the College students had not studied algebra as a major topic for matriculation. Some who might have studied algebra did so at the standard grade instead of the higher grade. The teaching and learning of algebra again at the matriculation level is geared towards passing the final examination, which does not ask for the understanding of the topic but rather emphasises rote manipulative learning. Lack of qualified teachers at the matriculation level, where many teachers have to rely on one textbook, can also be a contributing factor towards the poor understanding of certain topics.

The test scores in the research did not count towards the COE students' final examination result. Students might have not given their best in responding to research instruments. Some students' responses to the test did not have reasons to support their answers. It was, therefore, difficult to find the roots of misunderstanding and errors in some cases.

There is also the problem of the small number of students offering mathematics who were available as participants in the colleges. Again the problem of following the students up when teaching because they fail to get employed in schools due to the current policy of the Department of Education in not employing more teachers. I therefore had to use school practice periods of current students.

The practice periods could not help much since there were only few times I could see the student teachers because of the time schedules of the schools and the Colleges of Education. Hence the findings could not be generalised.

These timetable problems did not allow me to observe lessons in which topics were being taught which related to some of the misconceptions that student teachers had, as shown by the algebra test.

Most of the students involved in the study were from disadvantaged African backgrounds. The sample was not fully representative of the South African population so generalisation to other population groups of South Africa should be done with care and caution.

Because College of Education students knew about the algebra test beforehand, there was the likelihood of trying to prepare in a special way in advance before the test. To overcome such biases, I requested the subject lecturers to explain the purpose of the test. That the aim of the test was to get what they know and understand in algebra. That meant all the algebra they have studied from the junior school up to the College level. This in essence would make trying to prepare for in all the topics very cumbersome and unnecessary.

With these limitations the study depicts the trend of what Kaur and Sharon (1994) had to say about knowledge and understanding of College students, which is weak and fragile coupled with conceptual misunderstanding and misconceptions.

1.9 OUTLINE OF THE STUDY

The structure of the study is as follows:

Chapter 1 Introduction and motivation for the study

Chapter 2 Literature review

Chapter 3 Research Design and Pilot

Chapter 4 Analysis of data and preliminary discussions

Chapter 5 General Analysis of data for all Colleges of Education in the study

Chapter 6 Summary and findings

Bibliography

Appendices

CHAPTER TWO

LITERATURE REVIEW

2.1 METHODS OF TEACHING ALGEBRA

This chapter reviews the literature about the knowledge, the nature and the understanding of mathematics. The teaching, the philosophy and language and thought

of mathematics in general is also reviewed. It is devoted to methods of teaching algebra,
understanding of algebra, difficulties of students learning algebra, pedagogical content knowledge, preservice teachers' subject matter knowledge, misconceptions, causes of misconception and other cognitive problems. These are all related to the research questions in 1.7 of this study.

2.1.1 **Mathematics as a body of Knowledge**

According to Lerman (1983:62) *"the philosophy of the teacher determines the choice of the syllabus content, the teaching style*". Lerman describes two types of methods of teaching: the Euclidean approach and the Heuristic approach (problem-solving). He explains the Euclidean approach to mean the teaching of mathematics as a process, which will lead to students seeing the deductive nature of it. The Euclidean approach he sees as the means or 'method' to arrive at a solution to a problem as the ultimate aim of teaching. According to Lakatos (1976), this style starts with stated lists of axioms, lemmas and or definitions. They are taken as given and cannot be questioned by learners of mathematics. In deductivist style, all propositions are true and all inferences valid. Mathematics is presented as a set of eternal, immutable truths and facts. Counterexamples, refutations, criticism are not easily accepted. Deductivist development and presentation hide the struggle, hide the adventure and the end result is that mathematics knowledge is considered to be infallible. Concerning this view of teaching mathematics he states

> " *one must learn methods first and understand uses, applications or relevance afterwards…Mathematics is a steadily accumulated body of knowledge, linear or hierarchical, dependable, reliable and value-free. Concepts do not develop they are discovered" (Lerman, 1983:62).*

This view of mathematics teaching, if adopted by COE students, might lead them to show learners how and not why certain algebraic solutions are arrived at. Lakatos views mathematics as intrinsically fallible and speculative becoming more like the Popperian view of science, which attributes science to pattern of conjectures and criticisms.

The heuristic approach which has been associated with the 'conceptual change theory of mathematics knowledge' advocated by Blaire (cited in Lerman,1983:63) states that,

> *" students must be encouraged to propose ideas and suggest methods. They must be led to test hypotheses themselves, to try to generalize their methods, compare them with other possibilities and search out other problems of similar nature that may have previously been solved".*

Lerman (1983) argues with this theory that the degree of involvement and participation of students is likely to increase with the heuristic approach when mathematics is made relevant to the problem set. This approach by Lerman emphasises the recognition of knowledge of mathematical systems, their applications, meaning and relevance on the part of learners of mathematics. He further reiterates the responsibilities learners have to take for the construction of their own knowledge. Lerman claims the heuristic approach may lead to both conceptual growth of mathematical knowledge and also the nature of the learning process. This approach is similar to what curriculum 2005 is advocating for the learners of South Africa (Learner-centred, see 2.1.3). With reference to the teaching of algebra for conceptual understanding the COE student should endeavour to use the heuristic approach instead of the axiomatic-deductive approach.

The shift in approach requires individual teachers to change their approaches to the teaching of mathematics. A shift to either the investigative (See 2.1.6) or constructivist approach (See 2.1.5.1) to teaching requires deep changes. This may depend on the teacher's belief system, and in particular on the teacher's conception of the nature of mathematics and mental models of teaching and learning, and the level of thought processes and reflection. Subject matter knowledge is important, but it alone is not enough to account for the differences among mathematics teachers. Two teachers can have similar knowledge of, for example, a topic in algebra, but their approach to teaching might be different. One may prefer the constructivist or investigative approach while the other may prefer the traditional approach (See 2.1.2). These belief systems can be related to a philosophy of mathematics (See 2.1.4). Do the knowledge and understanding of algebra affect the way COE students approach the teaching of algebra in the classrooms either by the deductive approach or the heuristic approach? Mathematics as a body of knowledge is fundamental to my research in this study of prospective teachers' knowledge and understanding of algebra. In what follows, I paint a fuller picture of the encounter between prospective teachers and their learners. This is methodologically important to the study.

2.1.2 **The Traditional Method of Teaching Algebra**

In the traditional approach students are made to learn rules and tricks to solve algebra problems, to the detriment of their understanding. The teacher is the possessor of knowledge and the students is the recipient. Govender (1986:34) gives the basic conditions for the traditional method which involve:

"(i) Repetition, as pupils are drilled to memorise rules and procedures without understanding

(ii) Extrinsic motivation, where the child learns not because he/she wants to but because he/she has to satisfy his/her educators, avoid punishment, competition in the classroom etc

(iii) Streaming children into homogeneous ability classes, to facilitate uniform teaching methods and

(iv) application of standard techniques of computations and setting out of work".

In the classroom, the teacher and the textbook are the authorities, and mathematics is not a subject to be created or explored. According to Lampert (1990) in the school situation, the truth is given in the teacher's explanations and the answer book, there is no zigzag between conjectures and arguments for their validity, and one could hardly imagine hearing the words "maybe" or "perhaps" in a lesson. A student who does not have proper understanding of algebra is likely to read algebraic expressions without, according to Blais (1988) "perceiving their essence". Blais further puts it that students, without a proper understanding of algebra fail to retain entire algorithms and associations with the proper cue that call for their use. The students therefore according to Blais are left with partial algorithms that are no longer paired with the proper cues. With time therefore the lack of conceptual understanding becomes apparent, since it is no longer mastered by correct performances. The near total collapse of performance occurs because according to Blais (1988:626) *"shallow knowledge is difficult to retain*".

MacGregor and Stacey (1997) suggest that poor performance in algebra is caused by factors such as different approaches to beginning algebra, teaching material (e.g., interpreting **r** to mean "red pencils", or 6r means "six red pencils") or learning environment. They state that the use of letters as abbreviated words and labels was traced to the use of some textbooks. They further attributed a variety of misuse of algebraic notations after year 7 in school to interference from new learning. They are of the opinion that, if algebraic concepts and methods are not used in other parts of the mathematics curriculum, learners forget them and forget the notation for expressing them. This they acknowledged leads to the failure of learners to link new concepts and notation, or differentiate them from the previously taught concepts and notations (See 1.4).

College of Education students without proper a understanding of algebra are likely to rely on conventional instruction, which permits and allows algorithmic activity. The conventional activity is when the teacher explains the topic of the day and works out examples. This top-down teaching of algebra is usually clear, well organised, logical and easy for pupils to follow. This however creates a listener-follower role for pupils.

Such a role, according to Blais, contributes to dependence, eliminates the need to think for oneself, and fosters the growth of learned helplessness. The traditional behaviourist approach to teaching algebra is giving way gradually to a constructivist approach as will be argued below (See 2.1.5.1).

2.1.3 Learner-centred Education.

Learner-centred education embraces ‘activity-based learning’ as learners are led by activities designed by their teachers to discover things for themselves. The philosophy of learner-centred is “ putting learners first”. This approach assumes that the learner comes to the class with ideas, beliefs, and opinions that need to be modified or changed by the teacher who mediates this change by devising tasks or questions that challenge the learner to construct his/her own knowledge. Characteristic instructional practices include “discovery learning”, and hands-on activities, such as using manipulative learner tasks that challenge existing concepts and thinking processes, and questioning techniques that probe learners, beliefs and encourage examination and testing of beliefs.

Learners are placed in a position where they do most of the talking in the classroom while the teacher mediates. Teachers using the principle of mediation are expected to engage with learner’s ideas and knowledge.   The engagement in learner’s ideas and knowledge involves listening to learners and constructively trying to follow and challenge their line of reasoning and arguments. The assumption that teaching is based on ideas from the learner’s experiential world, where cultural context and values are taken into consideration is what is expected in a learner-centred classroom. In this way learners are allowed to explain and justify their reasoning and their way of thinking. Teachers or college of education students who use this approach in the algebra classroom have a crucial part to play. They have to have knowledge and understanding of the topic, the conception and the misconception of algebra held by learners so as to be able to mediate in the classroom. College of Education students’ responses to learners’ ideas are very important to sustain the learner’s progress in algebra knowledge in the classroom. How the student teacher is able to respond to learners’ ideas and allow learners to progress depends very much on the way the COE student is able to introduce ideas, plan activities for the learners, make available the necessary resources to learners. One way to achieve a learner-centred type of teaching is by the investigative approach, which is discussed in section 2.1.6. According to Malcolm (1997) in addition to what the Department of Education curriculum framework termed as learner-centredness, the learner-centred curriculum should allow teachers to have knowledge of and respect students’ beliefs, interests and learning strategies and that power has to be shared in curriculum so that students can contribute to choices about what is worth learning as well as contexts and methods.

According to the National Department of Education (NDE, 1996) curriculum materials/programmes should be designed by recognising and building on learners’ knowledge and experience and responding to their needs. Cultural values and lifestyles should be incorporated in the development and implementation of learning programs. The College of Education students or teachers of algebra should design the lesson having the learners as the main focus of the teaching process. This type of approach is what constructivism envisages.

In the process of mediating in learner-centred classrooms, teachers/COE students should design the lesson to include knowledge, skills, attitudes and values (SKAV a term used in South Africa in OBE). For example, in (SO9) Specific Outcome nine in MLMMS (National Department of Education, 1997), which is about algebra, the Assessment Criteria talks about mathematical notations and symbols. The knowledge required is that of mathematical expression, terminology, conventions and mathematical language. The skills expected to be achieved include manipulation, solving of equations, negotiation skills, etc. Attitudes and values can include positive attitudes towards algebra, appreciation, cooperation, respect of each other in the algebra classroom, etc. A College of Education student with the proper understanding and knowledge of algebra is likely to mediate in the lesson about symbols and notations to achieve such goals and outcomes.

To summarise what student-centred instruction means it will be appropriate to cite Cuban (1993). According to him students who work together more freely around the classroom, and determine classroom tasks for themselves, do not have to adhere to fixed classroom routines.

> *"Student-centred instruction means that students exercise a substantial degree of responsibility for what is taught, how it is learned, and for movement within the classroom. Observable measures of student-centred instruction are:*
>
> - *Student talk about learning tasks is at least equal to, if not greater than, teacher talk.*
> - *Most instruction occurs individually, in small groups (2 to 6 students), or in moderate-sized groups (7 to 10) rather than being directed at the entire class.*
> - *Students help choose and organise the content to be learned.*
> - *Teachers permit students to determine, partially or wholly, rules of behaviour, classroom rewards and penalties, and how they are to be enforced.*
> - *Varied instructional materials (e.g., activity centres, learning stations, interest centres) are available in the classroom so that students can use them independently or in small groups.*
> - *Use of these materials is scheduled, either by the teacher or in consultation with students, for at least half of the academic time available.*
> - *The classroom is usually arranged in a manner that permits students to work together or separately, in small groups or in individual work spaces; no dominant pattern in arranging classroom furniture exists, and desks, tables, and chairs are realigned frequently." (Cuban 1993:7)*

These observable measures will help one to classify a lesson to be teacher-centred or learner-centred when observing algebra lessons in the classrooms (See 3.2.1.3).

2.1.4 PHILOSOPHIES OF MATHEMATICS

" *The philosophy of mathematics is that branch of philosophy whose task is to reflect on, and account for the nature of mathematics*" (Ernest, 1991:3). Furthermore, according to Even (1990) the nature of mathematics goes beyond the conceptual and procedural knowledge of mathematics. The knowledge about the nature of mathematics Even asserts, might influence the knowledge and understanding of a specific piece of mathematics and in this study I make reference to algebra. The College of Education students may have different perspectives and belief systems, even different epistemologies. The philosophical differences by these students concerning such issues as the nature of mathematics and the foundations of mathematical knowledge may lead to conflict in teaching and learning of algebra by College of Education students. The belief system adopted by a teacher may again hinder the successful implementation of a constructivist approach in the classrooms in South Africa. Lerman (1983) contends teachers' view of mathematics influences the methodology employed in the classrooms. Many teachers because of their own experiences as pupils when being taught through teacher explanation, makes them feel that by being able to explain well will make them good teachers. According to Ernest (1985:603) " *the philosophy is of value for the teaching of mathematics".* He argues that prospective teachers entering the teaching profession for the first time carry along with them undigested theoretical views of mathematics from the philosophical schools. For this reason he states that there is need for explicit discussion of the views of the schools and their educational significance to these new entrants. He reiterates that no matter how the philosophical views are viable, some may have educational connotations, which are less acceptable.

Ernest (1995) states that the foundation of mathematical knowledge has been the fundamental problem of philosophy of mathematics. He acknowledges that there are two main movements in the philosophy of mathematics. Firstly, the absolutist philosophies of mathematics including logicism, formalism, intuitionism and Platonism, which assert that mathematics is a body of absolute and certain knowledge. Absolutists believe that mathematical truths are universal, independent of humankind and culture- and value-free. In contrast there is the second version, the conceptual change philosophies which assert that mathematics is corrigible, fallible and a changing social product, which include constructivism. Constructivism is an epistemology that offers an explanation of the nature of knowledge and how human beings learn. With the above statements and arguments of the philosophy of mathematics, I intend discussing the five important philosophies/approaches to mathematics. They are, logicism, formalism, intuitionism, Platonism, and fallibilism.

2.1.4.1. Logicism

Logicism is the view that pure mathematics is part of logic. According to the supporters of this view mathematical concepts can be reduced to logical concepts and that all mathematical truths can be proved from the axioms of logic alone (Ernest, 1985). The logicists hold that mathematics is a body of truths that are not about anything. As far as the teaching of algebra in South Africa is concerned little or no logic is used in its teaching, however the influence of logic is felt at the tertiary levels in the form of proofs. Ernest argues that the treatment of logic does not contribute to the acquisition of knowledge of the subject matter, he reiterates that it is counter- productive to the development of the topic. Ernest explains that a logicist approach to teaching does not rely on the historical development of the topic, nor does it aim at developing the cognitive structures relating to the topic in the learners, hence does not help in the learning of mathematics. Being supportive of the views expressed by Ernest I feel college students need not take a rigorous approach to the teaching of algebra.

2.1.4.2. **Formalism.**

"Formalism is the view that mathematics is a meaningless game played with marks on paper, following rules" Ernest (1985:606). Formalists according to Ernest, favour rules as against meaning making. Formalism can be equated to rote learning, which is about learning without understanding. If prospective teachers require conceptual understanding of algebra, then this philosophy is not suited to teaching as it only makes learners learn rules in algebra without understanding the underlying concepts. This type of learning again is similar to instrumental understanding (Skemp, 1976), where learning takes place without the necessary understanding in relation to the previous knowledge. Rote learning is seen to have a very low retention period as compared to learning with meaningful understanding. The formalist philosophy at this level is not going to help a teacher who wants to teach algebra for conceptual understanding since only rules and procedures for manipulating symbols will be learnt.

2.1.4.3. **Platonism**

Ernest (1985:607) states that, *"Platonism is the view that objects of mathematics have an objective existence in some ideal realism*". Platonism discards the notion that human beings are creators of mathematical knowledge. Platonists emphasise a static body of knowledge as compared to the dynamic nature of mathematical knowledge. Accordingly the Platonists are saying mathematics is a product and not a process. This view, if adopted by a teacher of mathematics, may lead teachers to ignore learner's mistakes and not probe to find out why the learner made a mistake. Negotiation is therefore not encouraged with a teacher who has this type of philosophy. Platonists have similar belief systems to the absolutists.

2.1.4.4. **Fallibilism**

This is a philosophy, which sees history as an important part in the development of mathematics knowledge. This view of teaching and learning has valuable educational parallels which can be directly transferred to the classroom and is likely to be used for conceptual understanding since it views human beings as creators of mathematics. It also embraces the discovery method of teaching and learning to justify conclusions made. History and problem solving are seen as important in the learning of mathematics.

Fallibilists view mathematics as culture-laden and that mathematics cannot be viewed in isolation from its history, its sociology and its applications in the sciences and elsewhere. This epistemology embraces the social constructivist's mode of thinking. This epistemology's central themes are societal and personal development, particularly critical thinking. Curriculum 2005 is virtually based on this philosophy. COE students teaching mathematics/algebra should understand this philosophy to be able to address such issues in the classrooms in which they teach.

2.1.4.5. **Intuitionism**

This philosophy is the opposite of Platonism. According to Ernest (1991) the best known intuitionists are Brouwer and Heyting. These mathematicians assert that classical mathematics may be unsafe and that both mathematical truth or existence must be established through constructive methods. Ernest (1991: 12) states that the intuitionists claim

> "*that mathematics takes place primarily in the mind and that written mathematics is secondary. …provide a certain foundation for their version of mathematical truth by deriving it (mentally) from intuitively certain axioms, using intuitively safe methods of proof.*"

Their view thus bases mathematical knowledge on subjective belief. Intuitionism

> "*acknowledges that human mathematical activity as fundamental in the construction of proofs or mathematical objects, the creation of new knowledge, and acknowledges that axioms of intuitionistic mathematical theory (and logic) are fundamentally incomplete, and need to be added to as more mathematical truth is revealed informally or by intuition" (*Ernest, 1991:29)

Intuitionism denies the existence of any mathematical reality external to the mathematician, or even of any mathematical truth beyond what the mathematician has actually proved or could actually prove (Ernest, 1991). Adopting intuitionism implies the rejection of public validated truth which comes about as the result of interaction with others, and coming to understand and agree with each other about that truth.

Intuition also gives the impression that our inner experiences are the only source of knowledge available to mankind whereby, rejecting external knowledge influences. Mathematicians therefore, have no access to other knowledge construction apart from their own construction, which makes that knowledge subjective to the person possessing it. Mathematical truth however, is a social product. It is created and developed by many minds coming together. It is therefore, important that individual subjective knowledge is published and reacted to by other mathematicians in order for it to become an objective, justified and accepted knowledge.

2.1.5 CONSTRUCTIVISM

2.1.5.1 An Introduction to Constructivism

Constructivism according to Jaworski (1994), is a philosophical perspective on knowledge and learning, it has indeed gained international recognition as a theory, which has much to offer to mathematics education. The NCTM Mathematics Standards (NCTM, 1989), which is the basis of many mathematics curricula in the USA, upholds most of the pedagogical implications of constructivist methodology. Constructivism has also significant implications for the Outcomes-Based Education (OBE), which is now being introduced to the South African educational system. This policy of education in South Africa lays emphasis on learners' empowerment. More importance is attached to what learners do with the knowledge they acquire than to whether they know all facts off by heart.

Radical constructivism has been described e.g. Kilpatrick (cited in Lerman 1989:211) as consisting of two hypotheses:

> "*(1). Knowledge is actively constructed by the cognising subject, not passively received from the environment.*
>
> *(2). Coming to know is an adaptive process that organises one's experiential world, it does not discover an independent, pre-existing world outside the mind of the knower".*

According to the radical constructivist von Glasersfeld (1990) these two principles view knowledge construction and adaptation as the consequences of cognitive structuring. Thus he says, knowledge is as a result of individual construction by modification of experiences. The radical constructivist deviates from the traditional position of realism to adopt the relativist position that knowledge is something which is actively constructed by the individual person. As von Glasersfeld (1990:37) states *" knowledge is the result of an individual subject's constructive activity, not a commodity that somehow resides outside the knower and can be conveyed or instilled by diligent perception or linguistic communication".* The radical constructivists imply that knowledge cannot be simply transferred from the teacher to the learner. The role of the teacher then is to act as a mediator (See 2.1.5.2) and have in mind as von Glasersfeld (1989:136) puts it that

> "*verbally explaining a problem does not lead to understanding, unless the concepts the listener has associated with the linguistic components of the explanation are compatible with those the explainer has in mind. Hence it is essential that the teacher has an adequate model of the conceptual network within which the student assimilates what he or she is being told. Without such a model as basis, teaching is likely to remain a hit-or-miss affair. From the constructivist perspective 'learning' is the product of self-organisation"*

The radical constructivist position focuses on the individual's construction of concepts, thus taking a cognitive perspective. This may lead to constructing mathematical concepts in such a way that they fit in with our real-world experiences. Bodner (1986) views construction as a process in which knowledge is both built and continually tested. It is, however important that students are led to express their thoughts to each other, to the teacher, or to both, since their knowledge must be viable, must "work", not only for the individual but also for the people around them.

Constructivists make a sharp distinction between teaching and training. The first aims at generating understanding and the second at competence in performance. Training leads to replication of the behavioural response, and teaching aims at generating autonomous conceptual understanding (Von Glasersfeld, 1990). According to Macnad and Cummine (1986) the central idea of the constructivist is that, the experience a pupil gains from learning activities cannot be predetermined by the teacher, since the experience depends both on how the child relates the learning activities to previous experience and also on his affective/emotional attitudes; thus the teacher cannot cause a pupil to construct specific knowledge. At the heart of the constructivist approach is sensitivity, on the part of the teacher, to be able to feel what the learner feels; to put himself in the learner's shoes, not only in the cognitive sense but also in the emotional sense.

For cognitive development Piaget and Vygotsky as constructivists have different ways of perceiving it. Piaget's theory is based on the notion that knowledge construction is a dynamic and continuous process, whereby the individual interacts with the environment, thus bringing about cognitive development. For Piaget, individuals operate within the system of schemes or structures, which is determined by advances through the stages. Gredler (1992:225) summaries Piaget's theory of stages as follows:

- *sensori-motor period: This period is characterised by " practical schemes" such as reaching, touching, smelling, grasping and pulling.*
- *Pre-operational period: the cognitive activities at this stage are not*

absolutely logical.

- *Concrete operations: At this stage, the child begins to reflect on his behaviour and has rudimentary knowledge of the physical world.*
- *Formal operations: This is the stage where thinking is at the highest level. According to Piaget, this is also the stage of advanced forms of logical thinking, which is often associated with the adolescent and adult years .*

Piaget's theory contributes to the knowledge of the child's cognitive development, for it demonstrates that in cognitive development through interaction with the environment, assimilation and accommodation function together in an interdependent way. It further establishes that children have the capacity to develop, depending on the cognitive structures available to them at each stage. Vygotsky on the other hand, states that cognition develops as a result of human interaction. He argues that symbol systems of the culture and the interactions with members of the culture are essential factors of cognitive development.

Vygotsky's theory (Gredler, 1992: 270) suggests that the interaction with the members of a particular culture gives rise to higher forms of mental development. The differences between the two theories is therefore that with the Piagetian approach cognitive development is dependent of the mental structures governing cognition at a particular stage rather than on social interaction. While Piaget believes that a person's knowledge differs according to different ages, Vygotsky however associates different kinds of knowledge as existing within different cultures.

According to Simon and Schifter (1991) the constructivist does not prescribe explicit instructional strategies, however the sense of learning and understanding does imply a new set of goals for the classroom. Teaching mathematics is to be understood as providing students with the opportunity and stimulation to construct powerful mathematical ideas for themselves and to come to know their own power as mathematics thinkers and learners. They further state that in classroom discussions, the teacher is the mediator asking questions, requesting, paraphrases ideas, managing and focusing the discussion as needed, but avoiding comment on the correctness or the value of particular ideas. The constructivists make distinction between mediator and facilitator. According to the radical constructivist a facilitator is the person who provides an environment conducive for the learner to learn on his/her own. This supports the radical notion that minds work by themselves. The mediator according to the social constructivist is the person who guides the learner to achieve a certain prescribed goal.

In terms of the new education policy of Curriculum 2005, facilitator refers to mediator in that sense of the word. The term facilitator as is used in OBE supports Vygotsky's notion of placing great emphasis on social and linguistic influences on learning and the role the teacher plays. He introduced a concept known as "the zone of proximal development" (ZDP), this is where the more competent help the less competent to reach higher conceptual levels than they would have been able to reach naturally, on their own. The ZDP is

> "*the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978:86).*

What is implied here is that with the correct and appropriate help from the teacher/mediator, (See 2.1.5.2) learners may attain a higher level of development than they would have on their own. The higher level of development, which results from the assistance given to the child, enables the child independently to solve problems at that new level and becomes a foothold which can lead to further development. The ZPD actually distinguishes the distance between what the learner can do on his/her own and what he/she can do with the assistance of a more capable other be it mathematics or playing games. The ZPD should not be understood as a discrete zone which an individual possess at a particular stage of their development but that it can occur at any stage of development. Vygotsky's ZPD helps to identify the gap in understanding or capability in which a teacher may give assistance to pupils so that they will be able to increase their ability to understand and be able to perform some tasks on their own. The importance of prior knowledge of the pupils is often there to assist the pupils in their ZPD. The prior knowledge, which normally forms part of the introduction to a lesson that is designed to link the new content to be taught with what the pupil already know. The teacher normally uses verbal means to identify the prior knowledge of the pupils in a lesson. The teacher based on the response from the pupils is able to assist in establishing a ZPD. Implicit in Vygotsky's concept of the ZPD, is the notion that learning should be a joint activity between the teacher and the learner based on mutual cooperation and agreement. Both Piaget and Vygotsky argue that the child is very active in the construction of knowledge.

Constructivists go on to say that the teacher, instead of looking for a simple, short, straightforward path to student success, encourages the exploration of the potential pitfalls and misconceptions with the aim of developing broader, more resilient concepts. College of Education students' pedagogical knowledge is therefore very important as far as the new curriculum is concerned so that the mediators will have the necessary knowledge and understanding to guide the learners to achieve the specific outcomes outlined in the policy document. The role of the facilitator, according to the radical constructivist, is to initiate constructive activity.

A teacher as a mediator should be able to establish a sound emotional environment conducive to interaction between mediator and the learners. The mediator's duty is to develop a great deal of trust and encourage confidence and openness. These can be achieved through respect on the part of the mediator. The mediator must show empathy with his learners as far as the learning of algebra is concerned. He must take into consideration the background of the learners. When a learner has misconstructed some algebraic knowledge, it is the role of the mediator to organise the learning environment in such a way that it will encourage the individual learner to construct meaning in a way that agrees with the algebraic knowledge that is seen as legitimate within the school context.

To me the constructivist perspective is more appropriate than other learning theories since it facilitates the development of understanding and meaning in students. According to Von Glasersfeld (1989) the constructivist teacher would tend to explore how students see the problem and why their path towards a solution seemed promising to them. The social constructivist Ernest (1991) refers to the knowledge that resides in the mind of an individual as subjective knowledge. The subjective knowledge of an individual is a unique creation of that individual. The subjective mathematics knowledge however becomes objective knowledge when an individual's subjective knowledge fits the socially accepted knowledge of mathematics.

According to Robinson, Even, & Tirosh (1994) students often make sense of the subject matter in their own ways, which is not always isomorphic with the structure of the subject matter or the instruction. The teacher's role as a mediator or leader of discussion about strategies and processes rather than that of dispenser of knowledge is to be looked into. Teachers should be able to initiate and sustain discussion. It is also believed that the teacher's subject matter knowledge generally aids the problem solving of the learner.

Noddings (1993) supports the effectiveness of constructivism as it leads us to think critically and imaginatively about the teaching-learning process. Believing in the premises of constructivism he states that, we no longer look for simple solutions and that we have a powerful set of criteria by which to judge our possible choices of teaching method.

From the constructivist perspective the teacher cannot transmit knowledge ready made and intact to the pupil. Adopting the constructivist theoretical notion has much to offer the prospective mathematics teacher. The suggestions made by Von Glasersfeld support this. Von Glasersfeld (1990) suggests:

" 1. There will be a radical separation between educational procedures that aim at generating understanding ("teaching") and those that merely aim at the repetition of behaviours ("training"),

2.The researcher's and to some extent also the educator's interest will be focused on what can be inferred to be going on inside the student's head, rather than on overt "responses".

3.The teacher will realise that knowledge cannot be transferred to the student by linguistic communication but that language can be used as a tool in the process of guiding the student's construction.

4. The teacher will try to maintain the view that students are attempting to make sense in their experiential world. Hence he or she will be interested in student's "errors" and indeed, in every instance where students deviate from the teacher's expected path because it is these deviations that throw light on how the students, at that point in their development, are organising their experiential world". (von Glasersfeld as cited in Jaworski 1994:27)

Von Glasersfeld (1990) in 4 above is no different from Piaget. According to Piaget, children acquire mathematical knowledge not by internalising rules imposed from outside but by construction from inside through their own natural thinking abilities. When errors are committed, it is said, these arise because the children are thinking and not because they are careless. Thus the task of the teacher is not to try to correct from outside, but to create a situation in which the children will inevitably correct themselves.

The College of Education students have been exposed mostly to the traditional ways of teaching where the lecturer with limited time probably shows the ways of arriving at solutions to algebraic problems by working out a few examples and expecting students to do similar examples. The final examinations in mathematics and algebra in particular do not test conceptual knowledge. These students do not, in this case, acquire the necessary conceptual knowledge of the subject matter to teach and therefore rely mostly on the traditional approaches of teaching algebra. This is likely to lead to laying improper foundations at the junior school level where most of them go to teach. Misconceptions in the learners at the schools they teach are likely to be as a result of lack of appropriate subject matter knowledge on the part of the teacher.

Pirie and Kieren (1992:507) have however suggested four ways of creating a constructivist environment which can be good for mathematics learning and understanding. They are:

- *"Although a teacher may have the intention to move students towards particular algebraic goals, he will be well aware that such progress may not be achieved by some of the students and may not be achieved as expected by others.*
- *In creating an environment or providing opportunities for children to modify their algebraic understanding, the teacher will act upon the belief that there are different pathways to similar algebraic understanding.*
- *The teacher will be aware that different people will hold different algebraic understandings*

- *The teacher will know that for any topic there are different levels of understanding, but these are never achieved "once and for all".*

The problem not addressed by these statements from the Vygotskian point of view is that the mental structures, which are developed, are dependent on the learner's individual activity and are not necessarily the desired result of the mutuality of society, represented by the teachers or competent peers, and the individual. From the above implications for teaching are as follows:

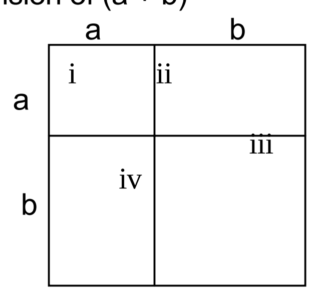
1. Teachers should teach central concepts in algebra instead of just facts.
2. Teachers should use materials and ideas relevant to students' life.
3. Teachers should give learners opportunity to solve problems and reason scientifically.
4. Teachers should understand learners' thinking.
5. Teachers should match strategies against abilities of learners.
6. Teachers should actively engage learners of algebra during the learning process.
7. Teachers should make sure learners are developmentally ready for concepts to be learnt.

2.1.5.2 **Mediation**

Vygotsky (1978) called the process through which assistance is given, mediation. He asserts that mediation is an important facet of the learner's learning experience. The mediator is the person who stands between the point of incompetence and that of competence of the learner in order to assist him/her to competence. Vygotsky's theory rests on the assumption that the cognitive process is a product of social and cultural interaction. Mediation occurs in the zone of proximal development, where the child cannot understand some concepts on his/her own but has to do so through the help of an adult/teacher. It is upon this that in mediational teaching the teacher should target the critical gap of potential development so that the child's understanding is shifted to a new level. It is very clear that if the teacher presents an algebra concept and facts, which are beyond the comprehension of the learner, there is bound to be a misconception or misunderstanding. It is also good to note that if the teacher presents a concept, which is already known by the learners, it may lead to loss of interest and the learners developing negative attitude towards the subject. During mediated learning, the child internalises new information or knowledge and adapts them to fit what is understood and accepted by the community at large. The teacher during mediational learning is required to present the lesson by using descriptions and examples that suit the level of comprehension of the learner. This therefore, demands from the teacher of mathematics/algebra a higher level of competency in the subject. According to Vygotsky (1978) during mediational teaching, the teacher acts as the cultural agent who guides instruction so that the learners will master and internalise the skills that permit higher cognitive functioning.

It is through this mediation that there is support from the teacher. The teacher acts as a guide in the learning situation. If mediated learning is assisted learning then it relates to the idea of scaffolding. Scaffolding implies that the teacher provides the learner with a great deal of support during the learning process. Scaffolding is used as a metaphor for the method, which is designed by the teacher to help the learners during the learning process to develop. To build a house the builder uses scaffolding as assistance to reach the top of the building because the top of the building is not easily accessible to him. Scaffolding is used as a temporary measure until the job is done and completed. In algebra the example below in figure 2.1 is used to help the learners get the conceptual understanding of the expansion of $(a + b)^2$. From fig 2.1, the expansion of $(a + b)^2$ is found by adding the areas of the four quadrilaterals i, ii, iii, and iv, formed out of the big figure drawn to represent $(a + b)^2$,i.e., Area i = a^2 , area ii = ab, area iii = ab and area iv = b^2. The addition of the four areas gives the result of the expansion $(a + b)^2$, which is $a^2 + 2ab + b^2$. According to Vygotsky (1978) scaffolding is used by the teacher for interactional support, often in the form of adult-learner dialogue that is structured by the adult to maximise the growth of the learner's intra-psychological functioning. The teacher uses scaffolding continuously as far as the lesson difficulty increases. The teacher creates a lesson format, provides scaffolding, within which she/he promotes emerging skill, allows the learner to work with the familiar, introduces the unfamiliar in a measured way, and deals constructively with errors and misconceptions. Within the classroom environment, the teacher is required to create a conducive atmosphere whereby an appropriate period of time is allowed to provide support for the learner to produce an answer, give regular feedback, give praise when necessary and acknowledge achievement.

Figure 2.1 the expansion of $(a + b)^2$



In practical terms, scaffolding may include giving learners more structure at the beginning of a lesson and later shifting responsibility over to them. The teacher might suggest questions, model the type of questions the learner may ask and later allow the learners to generate their own questions around the given topic. Scaffolding is constructed on the foundation of the learner's prior knowledge. For example, to help the learner understand the expansion of $(a + b)^2 = a^2 + 2ab + b^2$ in algebra, the teacher can use the geometric approach when the learners know how to find the area of a rectangle or a square. The figure 2.1 will help the learner use area formula to arrive at the expansion $(a + b)^2 = a^2 + 2ab + b^2$

2.1.5.3 **Constructivism and Fallibilism**

Constructivists argue that knowledge and reality do not have an objective or absolute value or that we have no way of knowing this reality. Von Glasersfeld (1995: 7) indicates in relation to the concept of reality that: " *it is made up of the network of things and relationships that we rely on in our living, and on which, we believe, others rely on too"*. Von Glasersfeld, rather than thinking of truth in terms of a match to reality, focuses rather on the notion of viability. He argues that " *to the constructivists, concepts, models, theories, and so on are viable if they prove adequate in the contexts in which they were created"* (Von Glasersfeld, 1995: 7).

Early epistemology emphasised knowledge as being awareness of objects that exist independent of any subject. From an epistemological stance, objectivism and constructivism would represent opposite extremes. Ernest (1995) points out that there are as many varieties of constructivism as there are researchers. According to Ernest (1995) the first version of constructivism emanated from the work of Piaget which holds that knowledge is actively constructed by the learner and not passively transmitted by the teacher. The second version is the radical constructivism of von Glasersfeld (1990), in which cognition is considered adaptive in the sense that the learner's experiential world influences knowledge construction. Radical constructivism does not deny an objective reality but simply states that we have no way of knowing what that reality might be (Ernest, 1995).Thirdly, the social constructivist version which stresses the primary role of language, thought and social interaction in meaning making and cognition. Social constructivism is traced back to Vygotsky (1978) a pioneering theorist in psychology who focussed on the roles that are socially played in the development of an individual. The social constructivist thesis is a social construction, a cultural product, fallible like any other branch of knowledge. Ernest (1995) writes that social constructivists place special emphasis on cultural and sociological processes through which knowledge is formulated. According to him there are two claims to this view: firstly, origins of mathematics are social and cultural; secondly, the justification of mathematical knowledge rests on its quasi-empirical basis.

Theories of Piaget, Ernest and von Glasersfeld fall under educational (psychological) constructivism while Brouwer (Intuitionism) rather talks of mathematical constructivism. In my study I refer to constructivism in the educational sense. Constructivism equates mathematics with a constructive process where the learner is allowed to build on the prior knowledge and make use of the environment to acquire new knowledge (See

2.1.5.1). Constructivism is seen as an alternative to absolutist knowledge, a very important theoretical perspective in mathematics education, which holds promise to the existing paradigm of mathematics education. According to Ernest (1995) this philosophical view of mathematics focuses on human activities, for example problem solving and modelling in the learning and teaching processes.

The implication for mathematical knowledge is that, it takes human understanding, activity and experience to make or justify it. In terms of algebraic knowledge acquisition, the topic of my research, this theory implies that learners cannot be seen to have mastered algebra without educator or peer assessment, formal or informal, and feedback to that knowledge. On the other hand, the educator's decision to accept mathematical responses in a learner's work depends on the educator's professional judgement, which is influenced by the educator's philosophical view and belief about mathematics.

The implication again is that the previously traditional methods of teaching should give way to that which could be more productive and humanistic (constructivism) to the learner and community at large. In the classroom, what the educator knows should therefore be fused with the sense of purpose as an educator of mathematics, the philosophy of teaching and learning and the sense of responsibility given the community in which she/he teaches. It is however important for College of Education students to acknowledge the importance of intuition, curiosity, and reasoning in teaching algebra.

It seems that if we are to address the problem of being able to use knowledge, we must teach our learners how to access and use knowledge that is already present. Based on the constructivist perspective which asserts that learners construct their own reality or at least interpret it based upon their perceptions of experiences, the primary role of the teacher is therefore, to create an environment for the learners to help them make the necessary mental constructions. It therefore suggests that mathematics/algebra knowledge will result from learners constructing models in response to the question and challenges that come from actively engaging mathematics/algebra problems not just simply taking information from the teacher. The teachers' role is to create the environment/experiences that engage the learners and support their experiences. From this premise a teacher of school algebra should look for different approaches to improve the teaching of the subject, develop a rich environment for exploration, prepare coherent algebra problems to elicit and communicate learners' perceptions and interpretations. As discussed in 2.1.5.1, some of the practices needed in a constructivist classroom should include: Group learning using cooperative learning strategies, active cognitive involvement, learner-centred classrooms (See 2.1.3), integration of subject matter to convey connections to the experiential world, interaction, discussions and reflections. Inquiry and investigations should be the premise upon which constructivist teaching should be based and as mathematics is seen as fallible and of human construction the teaching of it should reflect that mathematical truth can be challenged. Learners of algebra can construct their personal understanding through effective teaching by the educator, but the educator should be able to shape his/her algebraic performances and representations through the knowledge and understanding of the philosophy of mathematics. It is imperative that the nature of algebraic knowledge the educator possesses, influences the way in which knowledge is held and the ability to use that knowledge in a reflective, adaptive way. As discussed above, instructional activities by the educator of algebra should encourage learners to reflect, to explain and justify solutions, to agree and disagree with one another and to question alternatives to problems and solutions.

2.1.5.4 **Language and Thought**

Language is viewed as a medium through which mathematics is communicated in a specific cultural milieu. Language as a tool of communication plays an important role in mathematics education. Vygotsky (1962) argues that language is a very powerful tool for the development of thought. He explains how language shapes the learners cognitive processes and enables/disables the learner in his/her thinking about mathematics and appropriate procedural knowledge. In South Africa the connection had been made between language and thought but is being adopted at a slow pace. Thinking is likely to be a bigger problem for many learners in South Africa hence the advocating of Curriculum 2005, which is intended to lead learners to be creative and innovative thinkers. In South Africa thinking is expected to relate to mathematics development and secondly to the ability to generate conceptualisation as is expected in the algebra skills of College of Education students. There is evidence that the language that we speak has an influence on our thought patterns (Brodie, 1989:43). Brodie further states that learning in a foreign language can result in serious cognitive difficulties. Brodie again emphasises what others have said that instruction in a weaker language (poor written and spoken competency) can result in decreased academic progress. Brodie contends that one of the problems facing developing countries, which includes South Africa, is caused by the learners having to learn mathematics in a language other than the mother-tongue. The need to communicate is an inborn one. A learner, who finds him/herself in an environment where s/he cannot understand, cannot relate well to that environment, is doomed to fail academically. The language of instruction in many African schools in South Africa up to grade 4 is the mother-tongue. From grade 5 the majority of learners receive instruction through the medium of a second language, English or Afrikaans. If the language, in this case, English is not spoken outside the classroom and rarely heard, it could be a problem trying to use and understand it. The majority of rural African children will seldom come across English speakers outside the classroom or access to electronic media such as the TV and radio. The debate between mother-tongue or English as an initial language of instruction is outside the scope of this research.

The confusion which exists in African schools at present is not solely due to the crisis in the language issue, but a learner who cannot fully understand what is being said to him/her and thus cannot cope with external examinations because s/he does not fully understand what is being asked, cannot be said to be unintelligent but rather disadvantaged in the language of instruction. This problem is likely to surface with the participants in this research as some of these students were taught in a second language and have to teach in a second language (English). The medium of instruction in and around Transkei Junior schools where most of these College of Education students are likely to teach after completion of their diploma course, is English. There appears to be a problem in having to teach learners who do not understand English well enough and thus have to revert to the use of the mother-tongue of the learner to clarify some concepts in algebra. This gives rise to the idea of Code-Switching (Setati, 1998).

Berry (1985) deals with language and cultural influences on Botswana children learning mathematics in his study of learning mathematics in a second language (some cross-cultural issues). The focus of the study was on aspects of learning and teaching mathematics in which the "distance" between the mother- tongue and the language of the curriculum plays a major role. "Distance" is used here to show the disparity between the mother-tongue and language of instruction which are often incompatible. In the study reference was made to two types of classifications depending on how language affects the learning process. The first category, which was termed A, was about when the language of instruction is not the learner's mother-tongue. The type B category is about when the learner is taught at the early age of education in the mother-tongue and later switch to a foreign language of instruction (English). Berry argues that the type A problem is not only linguistic but also mathematical which he said could be remedied. He suggested a modification in the curriculum and the methodologies to build on learner's natural mode of cognition. He referred to the category B problem as cultural rather than linguistic. He cites an example of Botswana society where number words do not go beyond twenty. He recounts how children or people in Botswana can keep record of their cattle with one to one counting, since it is a taboo against enumeration of objects, and reservation of larger numerals for mystical or ceremonial purposes, which dictate against their use by children. These, according to Berry, have had an adverse effect on Botswana children in learning mathematics. The participants of this study fall into these two categories and hence are likely to have similar adverse effects in their learning of mathematics.

This argument is further supported by Setati (1998) who attributes the problem faced in the multilingual classrooms in South Africa to language where she used the term "*code-switching*" (Setati, 1998:34). She defines code-switching as the use of more than one language in a single speech act. This mode of teaching is very common in and around the Transkei where the study was done. Some teachers normally argue that after all there are 11 official languages in South Africa therefore one has the right to use whatever language one finds suitable and accommodatable to the situation. This argument is correct and sound but at the end of the child's study the final examination is written in English or Afrikaans. At this point the learner is at a disadvantage as one has to read the question in either English or Afrikaans, think of the problem in the mother-tongue and later translate into English or Afrikaans. This is likely to take a longer time than may be stipulated. Secondly, there may be some concepts, which have no vernacular names and therefore may make the translation difficult.

From Setati's point of view code-switching performs three types of functions: reformulating, switching for content activity and regulating which I believe are good for the learning and teaching of mathematics. The danger, which is foreseen, is whether code-switching will help the learner answer the questions in the final examination, which is normally in English. This type of approach does not attract foreign expert teachers to the lower classes to lay solid foundations for the learning of critical subjects like mathematics/algebra at the intermediate phase of the schooling process of the learner. With a lot of trained South Africans to teach mathematics I think this will be the best way to solve language problems faced by most of the African learners learning mathematics as a second language.

Brodie (1989) emphasises the need to use English in the teaching of mathematics due to the lack of learner support materials in the vernacular and the shortage of qualified teachers who cannot teach in vernacular. She also supports the use of English as most of the vernacular languages do not have a scientific and mathematical vocabulary. The knowledge and understanding of algebra will therefore mean the teacher should know the language of instruction well to be able to read and interpret and formulate algebraic statements.

However, in the Transkei region of South Africa cattle rearing is the main occupation of its peoples. Once a question was put to a colleague about how the illiterate locals know that all their cattle return from grazing in the field. The answer was that the tally system was used where stones represented the cattle. They put down stones when the cattle leave and take away the stones when the cattle come in. This shows every culture has its own mathematics but the accepted mathematics is the Western type, which goes with its language, which we should strive hard, to master in order to teach our children the critical subject mathematics.

Multilingualism is however entrenched in the South African constitution, where it is stated that,

> *" In terms of the new constitution of the Republic of South Africa the government, and the Department of Education, have to promote multilingualism, the development of the official languages and respect for all languages used in the country. This shift is in line with the fact that multilingualism is the norm today, especially on the African continent"* (National Department of Education*, 1997:22).*

The above quote emphasises the use of language in multilingual classrooms, which is becoming increasingly the case, even in ex-Model C schools in South Africa. According to Vygotsky (1986:160) *" the acquisition of a foreign language differs from acquisition of a native one precisely because it uses the semantics of the native language as its foundation".* He relates foreign and native language to algebra and arithmetic. Vygotsky states that the knowledge of algebra stands to gain from the knowledge of arithmetic, enhancing its understanding and turning it into a concrete application of the general algebraic laws. He contends that the child's language becomes more abstract and generalised in algebra. He refers to the ways algebra liberates the child from the domination of concrete figures to generalised laws. A noted example is the number pattern, which can generate into general algebraic laws.

Bilingualism has been seen by Cummins (as cited in Brodie, 1989) to have an effect on cognitive development. It is seen as a potential handicap. The effect is that the acquisition of second language skills can lead to a decrease in the first language skills. Reed (cited in Brodie 1989:46) distinguishes three types of activities in which learners engage themselves while learning mathematics. Firstly, they must understand the language of the problem or the text. Secondly, the learners must formulate the mathematical concept or concepts required, and lastly, must be able to translate the concepts into mathematical symbolism and work with them in this form. Conceptual understanding cannot be seen as being divorced from the social context of learning from which it emanates, the linguistic propaganda both inside and outside the classroom has to be considered. This means the texts of mathematics problems can only be mapped onto other mathematics relationships once the linguistic aspects of language and mathematics are qualitatively recognised.

According to Ernest (1991) school mathematics reflects the nature of mathematics as social construction that is human creation and decision-making. Contribution to knowledge is achieved through language, social life and interaction with different cultural groups. As the philosophy of constructivism does not prescribe but suggests a teaching method, it will be unwise to exclusively advocate a particular approach in the diversity of classrooms in the South African school context. Ernest again emphasises that teaching should entail mathematical activities not passive reception of information or knowledge. Constructivism (See 2.1.5.1) in general does not rule out that learners cannot progress by themselves, but that the role of the teachers to mediate where necessary is important.

On the other hand, Cobb, Wood, & Yackel (1991 in Wain, 1994) assert that social interaction between partners might influence their mathematical activity and may give rise to learning opportunities and hence the need to group learners is advocated by OBE Curriculum 2005. Orton (1994) supports this notion by stating that the social constructivists claim children construct mathematical knowledge more solidly when they are encouraged to defend their ideas within a group or even a whole class. The knowledge, which is acquired, is then that which the group agrees should be accepted. This now suggests the need of classroom teachers in the process of mediating, to group the learners in small groups so that they can argue, debate and share ideas. Large classes can still generate debate and discussions but the tendency that some of the learners will be passive cannot be ruled out. As the radical constructivists argue that there is no absolute knowledge, the socio-constructivists argue that through discussion and sharing knowledge a public agreement is reached on the meaning and understanding of certain concepts.

In the algebra classroom should the COE students accept the above philosophy about the nature of mathematics and develop an understanding of the type of learners in and around Transkei classrooms, these COE students will approach teaching in a manner that embraces all the learners in the classrooms. The COE students should not take mathematics as culture-free but rather treat mathematics as culture-laden. If learning should be taken as shared knowledge, then learners should be guided to conform to the majority decision to accept some agreed upon knowledge without necessarily having to understand it. For example a learner who expands $(a + b)^2$ as $a^2 + b^2$ may have to agree to the majority including the teacher that the expansion should be $a^2 + 2ab + b^2$ without initially understanding it. It is therefore the duty of the COE teacher to assist the learner to understand from within. Especially allowing the learner to substitute numerical values in both expressions to satisfy him/herself that $a^2 + b^2$ is in fact not a correct expansion of the binomial expression but rather $a^2 + 2ab + b^2$ is correct.

The discussion above implies that COE students should understand their learners as they perform their duties in the classroom. As a teacher one should better understand the learners and their social groups, which form the class. The teacher has the responsibility to interact with learners in a dialogue situation where they are considered and treated as equals. The COE teacher should provide the necessary platform for learners to use their own reality as a basis for learning mathematics/algebra. This includes the use of appropriate language e.g. mother-tongue which can enhance the motivation to come out clear in their ways of thinking through assimilation and accommodation of concepts. Von Glasersfeld (1990) emphasises that each individual needs to have proper meaning of words if clear transference of concepts is to be achieved between two people or from a teacher to promote successful learning, teachers should know well what the learner's ideas already are that will help the learners to construct the desired understanding.

Language of instruction or language spoken in the process of teaching algebraic concepts is, however, affected by means of communication. Vygotsky (1962) states that the culture of students who are taught needs to be understood since in teaching appropriate thought is carried by the words of the language. Language is intertwined with traditional beliefs among students in a cross-cultural instructional setting. Second language used in instruction, is therefore affected by socio-cultural issues such as traditional beliefs, social orientations, political persuasions and religious beliefs. It is therefore noted that to successfully communicate algebraic concepts, cultural factors must be considered.

2.1.6 **THE INVESTIGATIVE APPROACH.**

Investigative teaching is about "opening-up" mathematics; about asking questions, which are open-ended; about encouraging student enquiry rather than straightforward "learning" of facts and procedures (Jaworski, 1994:2). According to Jarworski mathematical investigation seems to involve the students in a loosely defined problems, asking their own questions, following their own interests and inclinations, setting and achieving their own goals and above all having fun while delving into the mathematics. Students owning their own mathematics are usually able to retain and apply it when necessary. Students learning algebra through the investigative approach are likely to acquire the conceptual knowledge and understanding expected of them.

According to Jaworski (1994:4) the purposes of investigations in a mathematics classroom are:

"*(1) To promote more truly mathematical behaviour in students than a diet of traditional topics and exercises.*
(2) To promote the development of mathematical processes, which could be applied in other mathematical work.
(3) Seen as an alternative means of bringing students up against traditional mathematical topics". .

An investigative approach to algebra should therefore be in line with the policy of Outcomes-Based Education of Curriculum 2005 where learners are given tasks in order to achieve a certain specific outcome by their facilitators.

Mathematics will be fun and meaningful to learners. This can however be achieved with the conceptual knowledge and understanding on the side of the facilitators who are the student teachers in this study. To be a good facilitator in algebra will therefore mean setting contextualised activities which have meaning and understanding, encouraging student autonomy, independence, and self-direction and persistence. For example, learners can be given the: Hand shake" problem to investigate (Find out how many handshakes there will be if each person shakes hands with each other in a party of twenty guests). Such a problem will generate curiosity and interest among the learners to look for the solution. This question is practical and meaningful instead of teaching permutation and combination for the sake of teaching them. Learners may choose to use practical method starting with 2,3,4… people until a general trend is found. Algebraic method of substitution can then be applied to deal with any given number of guests. These are types of problems which are expected to be dealt with in Curriculum 2005.

Jaworski (1994) summarises investigational work to mean the encouragement of critical construction of knowledge. She states that investigational work in the teaching of mathematics, literally investigates the most appropriate ways in which a teacher can enable concept development in students. Jaworski sees the investigative approach as encouraging mathematical exploration, enquiry, and discovery on the part of the students and also exploring the role of the teacher in respect of how the teacher's own knowledge and understanding of mathematics supports students' learning. She further elaborates that investigational work through an emphasis on process, is likely to be an effective way of approaching the content of the mathematics/algebra curriculum.

Working in groups encourages the learners to see whether they have produced any results which are the same or different across groups. College of Education mathematics/algebra students are expected in the methodology course, to create a learning environment in which learners' contributions are valued and learners are encouraged to ask questions without risk of embarrassment. Within this environment the learners will feel comfortable sharing their solutions and ideas and seeking clarification.

2.1.7 SUMMARY

The literature in this field (constructivism) shows that constructivism advocates discovery and enquiry-based learning incorporating opportunities for discussion and negotiation, sharing and exchange of ideas. The teaching method, which tries to put constructivism into practice is more likely to be the investigative approach. This approach however has a lot of problems judging from the class sizes of between 35 and 100 in some African schools where the teacher is working with a prescribed syllabus. Traditional methods are most often used in these African classrooms, giving the impression that the entire class of learners form a homogeneous unit and in terms of cognitive development all the learners move at the same pace towards the acquisition of knowledge and understanding. Adopting the constructivist approach may reveal that things are not what are perceived to be in the traditional teaching approach where all are classified as equally developed cognitively to understand and learn the same concept.

Group discussions in the classrooms as advocated by the social constructivist can assist learning but the role of the teacher is very sensitive. The teacher in the mathematics/algebra classroom should play the part of designing and encouraging and guiding learners' discussion and debate about mathematics/algebra problems to enhance knowledge and understanding of the concepts involved.

In multicultural post apartheid South Africa where eleven official languages are enshrined in the constitution the learning of algebra may be hampered as some teachers and learners learn in a second or third language. Most of the algebra textbooks are written in English or Afrikaans with examples drawn from those backgrounds. These may have a detrimental effect on knowledge and understanding on the part of college students who have to draw examples from the environments in which they teach. One of the aims of Curriculum 2005 in South Africa is about equity and the students right to learn. But most teachers including final year College of Education students may not be sure of what this means and how to approach learners in the classroom to bring about this equity and the right to knowledge and understanding of mathematics/algebra. Language, culture and pedagogy must be the driving forces towards achieving the goals of OBE. The realities of the past and the need to redress imbalances calls for a change based on an appropriate philosophy and pedagogy of mathematics.

Wain (1994) attributes problems faced in the teaching of mathematics to culture. This suggests that the constructivist approach in the classroom will need to be interpreted according to cultural diversity. I am of the opinion that the pedagogy based on constructivism and a fallibilist epistemology will enable teachers/ final year College of Education students and learners to address the imbalances caused by apartheid through the knowledge and understanding of algebra, and the belief systems. On the other hand, Ernest (1989) emphasised that mathematical deduction is needed to establish validity. By construction the whole problem becomes meaningful to the learner. Similarly, COE students by mediating in the construction of algebra concepts are likely to contribute to making the learning of algebra meaningful and relevant to learners they may teach after the completion of their diploma course. It seems from the literature that nothing is more important for effective teaching of algebra than a thorough knowledge and understanding of the topic and the learners. Ernest asserts that constructivism places enormous responsibility on the teacher. The teacher has to define goals, design tasks, assignments, problems, projects and other forms of learning that stimulate thought and mental activity.

2.2 DIFFICULTIES IN THE LEARNING of ALGEBRA.

2.2.1 Research on Specific Difficulties

The difficulties of students' learning of algebra have generated quite a number of studies. Herscovics and Linchevski (1994) approach the problem in terms of the cognitive gap between arithmetic and algebra. Kieran (1991) and Arzarello (1991) also approached the problem in terms of a dialectic between procedural and relational thought. Fischbein and Barash (1993) centred their attention on the significance of errors made by students learning algebra. Kieran (1992), Sfard (1991), Sfard and Linchevski (1994) highlighted the important sources of students' difficulties with the introduction to algebra. They revealed in these studies that the students often seem to have a limited view of algebraic expressions, their notion of the solution of algebraic equations seems to be associated more with the ritual of the solution process rather than the numerical solution obtained, and they fail to grasp the meaning of the operations to be performed on the literal symbols, the algebraic expressions or the equations.

Herscovics and Linchevski (1992) gave special attention to the students' procedures of solving equation prior to a formal instruction in algebra. Filloy and Rojano (1984) emphasised the importance of the acquisition of algebraic language and thought. Orton (1983) in his study also mentions the difficulties students experienced with elementary algebra, which appeared to obscure the fundamental ideas in calculus. MacGregor and Stacey (1997) presented evidence of difficulties in learning to use algebraic notation which included among others:

- intuitive assumptions and sensible, pragmatic reasoning about an unfamiliar notation system;
- analogies with symbol systems used in everyday life, in other parts of mathematics or in other school subjects;
- interference from new learning in mathematics;
- poorly-designed and misleading teaching materials.

Booth (1988) also identified some of the root causes of students' difficulty in learning algebra as: The algebraic activity to perform, the nature of answers, the use of algebraic notations and conventions, and the meaning of letters and variables. The above difficulties could be as a result of teaching deficiencies, learning deficiencies and probably the textbooks, the social background, the curriculum and examination influences also. All these mainly centre on the teacher as the mediator and a guide (See 2.1.5.2).

With adequate pedagogical content knowledge teachers are likely to present algebra in a manner which may enhance learning by pupils and may serve as a cure to the learning difficulties mentioned above. College of Education students involved in the study and teaching of algebra may be able to interpret the textbooks so that they become meaningful to their learners. Student teachers in South Africa, as prospective teachers, are therefore required to have considerable mathematical knowledge as well as the necessary pedagogical skills to enable them to overcome the deficiencies stated above. How can teachers find out about learning deficiencies and problems with textbooks if they do not know enough mathematics or algebra in this case? To teach algebra with understanding student teachers studying mathematics/algebra should have the necessary pedagogical content knowledge and understanding to deliver effective practice in the classroom they teach. This should enable them to translate their knowledge into teaching to overcome the difficulties mentioned above.

2.2.2 General Obstacles in Learning Algebra

Wheeler (1996) attributes one of the obstacles to the learning of algebra to language (See 2.1.5.4). He states that algebra as a language employs the words and symbols that students have already met in arithmetic. It encourages the assumption that the common words and symbols bear exactly the same significance as they do in arithmetic.

As formal mathematics has more Western influence South African (African) students might have problems with some of the concepts portrayed by this mathematics. Haris (as cited in Zevenbergen, 1996) states, studies show that the Anglo-Australians and the Aborigines' concept of time differs, so this is expected also to be in South African schools which have a school population with diverse home languages. The experiences, which African students in South Africa will have prior to formal education in algebra might cause different knowledge and meaning from the European-South African. This should be the background upon which new knowledge should be incorporated in the educational system. For the students whose culture is different from that which is represented by the formal schooling, the construction of meaning will be more demanding when it is expected that they construct meaning similar to that represented in the formal school context. The constructivists acknowledge the fact that the African student learning algebra is less likely to construct concepts properly when compared to students in Western schooling. It fails to acknowledge that it is the Western concept of algebra, which conveys power, status and therefore of economic reward. These are some of the problems OBE is about to address, where learners at the foundation-phase will be taught in their mother-tongue (See 2.1.5.4) to address these differences. For example, 3 x 4 can be expressed in different ways by the use of language. It can be said the product of 3 and 4, or 3 multiply by 4 or 3 times of or the sum of four threes. As was evident in the pilot study some COE students were confused with the phrase "more than" with "multiply by". This is likely to be associated with the use of language as all the students study English as a second language at the matric level.

Shifter and Fosnot (1993:125) argue that

> "*overcoming an obstacle does not mean switching to another system of beliefs or another persistent and believed universal scheme of thinking but rather in changing the status of these things to 'one possible way of seeing things', 'one possible attitude', or 'a locally valid method of approaching problems."*

Herscovics (1989) however classifies three types of distinct sources of obstacles:

(i) obstacles of an epistemological nature,
(ii) obstacles induced by instruction, and
(iii) obstacles associated with the process of accommodation in learners.

2.2.3. Obstacles of an Epistemological Nature.

The epistemological obstacles have their parallel in the cognitive obstacles encountered by the individual. Wheeler talks of cognitive obstacle, where Piaget's theory of equilibration provides a suitable framework. In Piagetian theory, acquisition of knowledge is a process involving a constant interaction between the learning subject and its environment. This process of equilibration involves not only assimilation (the integration of the things to be known into some existing cognitive structure) but also accommodation (changes in the learner's cognitive structure necessitated by the acquisition of new knowledge). However the learner's existing cognitive structures are difficult to change significantly, their very existence becoming cognitive obstacles in the construction of new structures. This difficulty is related to the distinction between arithmetic addition and algebraic addition. In arithmetic 2+5 is viewed as the problem or question, and 7 is the answer, whereas x+3 is viewed as adding 3 to x as well as the answer.

2.2.4. The Obstacles Induced by Instruction

The cognitive obstacles induced by instruction are often due to a formalistic presentation of the subject matter. In such cases, Herscovics (1989) is of the opinion that the learner is not able to relate the notion being introduced to their existing knowledge. Schools do not accept the individuals' construction of knowledge. Learners are rewarded by the knowledge accepted from their teachers. If knowledge has to be accepted then it should be socially constructed.

Algebraic knowledge construction is a shared knowledge of the individual and the social group (teachers). If the individuals' knowledge is not compatible with the instruction given then there is bound to be a problem of assimilation. The construction of the new knowledge is required by a problem situation, which disturbs the individual's existing knowledge. This disturbance leads to mental activity and a modification of previously held views of the new knowledge. Cognitive reorganisation is prompted by the instruction given which might be an obstacle to the individual.

2.2.5 Obstacles Associated with the Learner's Accommodation Process

The third set of obstacles, which Wheeler associates with the learner's process of accommodation, are pedagogically the most challenging. Here the structural changes cannot be done by mere transmission of information, since the learner is constrained to alter the mental structure in his/her own mind. Dorier (1990) also points out the difficulties in the learning of algebra. He states that students have elements of knowledge, which initially are not well enough distributed in different settings (geometrical, analytical, logical, formal settings mainly). These analyses of students' practice also reveal the lack of ability to change setting and point of view.

The statement by Dubinsky (as cited in Breidenbach, Dubinsky, Hawks & Nicholas,1992:249) that,

> *" A person's mathematical knowledge is her or his tendency to respond to certain kinds of perceived problem situations by constructing, reconstructing and organising mental processes and objects to use in dealing with the situations"*

shows evidence of how the learner struggles to make meaning or acquires new knowledge .

Robinson, Even, Tirosh, (1994:129) acknowledges cognitive obstacle by stating that

> " *The incomplete nature of algebraic expressions is considered one of the main cognitive obstacles in learning to simplify expressions, eg 3m + 7 = 10m. This is an example of misconception which comes as a result of applying an arithmetic rule to algebraic expression"* .

Von Glasersfeld (1992) links this cognitive obstacle to misconceptions on the part of teaching and learning. He argues that if a conception has satisfied the demand of a learner, the learner has no earthly reason to change. This is likely to result in misconceptions. It is therefore the duty of the teacher to make them change the concepts developed by showing them that there are situations where the conceptions do not apply. For example $C + O_2 = CO_2$ in chemistry does not imply 2 + m = 2m in algebra and similarly 25 = 2 tens + 5 units does not mean a + b = ab.

2.3 PEDAGOGICAL CONTENT KNOWLEDGE

Stein, Baxter, & Leinhardt (1990:640) have shown that, although theory and "common sense" suggest that teachers' pedagogical content knowledge will influence their instructional activities, fine-grained empirical work linking teacher knowledge to classroom instruction is in its infancy. This statement focuses the intended research I wish to do with regards to algebra teaching and learning in the rural South African context. This research supposedly may highlight some of the issues of learning and teaching of algebra in this part of South Africa. Nonetheless, one cannot ran away from the fact that correct content knowledge is basic to pedagogical content knowledge and hence making the effort to explore the content knowledge of prospective teachers, just before they go out to teach.

In order to build a solid understanding of how teacher knowledge relates to instructional practice, we need to develop and draw upon detailed, qualitative descriptions of how teachers know, understand and communicate their subject matter. Shulman (1986) describes pedagogical content knowledge to include two types of knowledge. One is 'knowing' what makes understanding a particular concept difficult or easy and the conceptions or preconceptions students normally bring to the frequently taught topics in the discipline. The second is 'knowing' representations of regularly taught topics that provide teachers with a *"veritable armamentarium of alternative forms of representations"* (p.9).

Shulman (as cited in McEwan and Bull 1991:317), has drawn attention to the other major dimension of the stereotyped view of the teacher's knowledge. It is, according to him, *"no longer reasonable to suppose that the teacher's knowledge of the content is identified with ordinary scholarly knowledge*". The common sense belief that good scholars are not necessarily good teachers and the research finding (Ball, 1991) that there is apparently little relationship between teachers' scores on standardised subject matter tests and rating of their instructional effectiveness suggest not only what teachers know about children, classroom, schools, and teaching processes but also that they know something special about the subject that they teach.

Shulman's framework, while attending to alternative representations and ways of making the subject comprehensible to others, does not reflect what is found in most Colleges of Education in and around the Transkei. Lecturers are made to follow a prescribed curriculum, which is examination-bound. The time allotted to actual teaching of the content is limited. While the lecturers are faced with an enormous task of completing the syllabus within a specific time for the end of year examination, much is not done towards teaching for conceptual understanding. The teaching of algebra is no exception to these limitations. If learners are to have conceptual understanding of algebra, they require teachers/College of Education students to be able to help them recognise the validity, generalisability and efficacy of their own solution methods. Methods that may not be those the teachers expect. Such teaching places demands on the College of Education students' content knowledge, knowledge of representations and attitude towards teaching and learning. If College of Education students fail to provide alternative paths to the understanding of algebraic concepts, they may leave some of the algebra learners without understanding.

Similarly, pedagogical content knowledge is believed to promote the learning of knowledge in appropriate and meaningful ways. Furthermore, pedagogical content knowledge of a teacher can enhance motivation and interest on the part of students. Teachers will be able to connect or to link a particular topic to everyday life, which will assist to create understanding and meaning of the mathematics for students. Mathematics will therefore not be seen as an abstract and isolated subject reserved for a few talented students.

The pedagogical content knowledge which a teacher develops in algebra enables a mathematics teacher to be actively involved with pupils thinking in a way that otherwise would not be possible. By knowing what each pupil is thinking and being able to predict some ways that the child's thinking will develop, teachers are able to plan instructional activities that keep the child mentally active and develop his/her mathematical knowledge in an accurate and comprehensive manner
(Fennema and Carpenter, 1988)

Baturo and Nason (1996) in their study of student teachers understanding of area, redefined pedagogical content knowledge to include the substantive knowledge, discourse knowledge, mathematical cultural knowledge and dispositions towards the subject. By substantive knowledge they meant the correctness of the knowledge, their understanding of the underlying meanings of concepts and processes, and the degree of connectedness between concepts and processes. In this study I will investigate the College of Education students' correctness of knowledge as well as their understanding of the underlying meaning of algebraic concepts and processes.

Knowledge about the nature and discourse of mathematics, has a focus on:

(a) what counts as an "answer" in mathematics and also to realise that in mathematics discourse, justification is as much a part of the answer itself;
(b) how the truth or reasonableness of an answer is established;
(c) that doing mathematics is not "figuring columns of sums or performing long division, but instead consists of creative activities such as examining patterns, formulating and testing generalisations, and
(d) what can be derived logically versus what could be defined as mathematical conventions.

This study focuses on College of Education students' pedagogical content knowledge and on their notions of correctness and what it means to do algebra problems.

2.3.1 **Knowledge About Mathematics in Culture and Society.**

Mathematics as well as algebra plays an important role in our societies as discussed in chapter one (See 1.3). If teachers are to show the importance of mathematics and algebra in our societies then College of Education students need to understand these various roles played by mathematics and algebra. In this study College of Education students' understanding of how mathematics in general and algebra in particular is used in everyday life is looked into.

2.3.2 **Dispositions Towards Mathematics.**

According to Baturo and Nason (1996), learners dispositions towards mathematics is greatly influenced by their teachers' tastes and distastes for particular topics and activities and their propensities to pursue certain questions and kinds of mathematical investigations. Because of this College of Education students' dispositions towards algebraic concepts, processes and investigations are looked into.

Even and Tirosh (1995) did research on pedagogical content knowledge and knowledge about students as sources of teacher presentations of the subject-matter. They based their research on two important sources of pedagogical content knowledge: content knowledge and knowledge about the students. With the subject-matter knowledge they discriminated between "*knowing that*" and “*knowing why*". They defined *"knowing that*" to mean the declarative knowledge of rules, algorithms, procedures and concepts related to specific mathematical topics in the school curriculum. They further stated the importance of "*knowing that*" which is the basis for adequate pedagogical knowledge related to asking questions and suggesting activities. They found that teachers by letting their students explore and raise questions, find themselves caught up in situations which lead to providing students with responses which were mathematically inadequate. "*Knowing why*" was defined as the knowledge, which pertains to the underlying meaning and understanding of why things are the way they are, which enables better pedagogical decisions. This enables teachers to understand the reasoning behind students' conceptions and anticipate sources for common mistakes. They concluded that the participants in most cases did not "*know that*": they did not know the definitions or incorrectly solved problems presented to them. When it comes to " *knowing why*", students knew why a specific case was set in a certain way, but could not solve problems related to unusual cases.

From the above statements the teachers' subject matter knowledge and understanding in algebra is important as it may have an impact on the pupils' learning in the classrooms they teach. This means the algebra teacher must know the nature of the current algebra knowledge base of the learner, and must have insights into the knowledge, which can grow from such a base. Furthermore, the teacher must at least be in touch with the relevant and systematic algebra knowledge, which will provide him with the ideas, activities, insights, and feedback to help his learners build their algebraic knowledge.

2.4 PRESERVICE TEACHERS' SUBJECT-MATTER KNOWLEDGE

Preservice teachers' knowledge and understanding in mathematics has been investigated by many researchers as related concerns have been expressed by many as part of the problems facing the learning of mathematics. Ball (1990) investigated first year prospective primary and secondary student teachers' substantive knowledge and understanding of division in three different mathematical contexts: division with fractions; division by zero; and division with algebraic equation. She concluded by saying that the students' division seemed to be procedural and rule-bound instead of providing conceptual understanding.

Even (1993) investigated prospective secondary teachers' subject-matter knowledge and its interrelations with pedagogical content knowledge in the context of teaching the concept function, and concluded that many of the subjects did not have the modern conception of function. Tirosh and Graeber (1990) investigated elementary preservice teachers misconceptions and beliefs about multiplication and division and concluded that the students relied on the saying: "*multiplication always makes bigger and division makes smaller*". It was also found that the subjects relied on procedural methods. In yet another study by Baturo and Nason (1996:263) on student teachers' understanding of area the researchers concluded by saying that:

> " *the impoverish nature of the students' area measurement subject-matter knowledge would extremely limit their ability to help their learners develop integrated and meaningful understanding of mathematical concepts and processes".*

The above studies all point to the preservice teachers lack of adequate knowledge and understanding, which makes the teaching of mathematics procedural rather than conceptual. It is therefore not surprising to hear mathematics educators calling for preservice teacher education to include more opportunities for students to develop a deeper and more integrated understanding of the mathematics they are required to teach (Vithal, 1991).

Graeber (1999) notes an obvious connection between preservice teachers' pedagogical content knowledge as it relates to alternative representations and success in helping learners to achieve conceptual understanding. Graeber further states that, *“if preservice teachers fail to provide alternative paths to understanding, they are apt to leave learners without understanding” (p.203).* It is therefore obvious that knowing different models and representations to topics places demands on College of Education students’ knowledge and understanding of a subject. According to Graeber in South Africa, to teach algebra well, College of Education students should acquire adequate subject-matter knowledge to face the real problems in the classroom. This means the algebra course should be structured in line with constructivist views on teaching and learning as discussed earlier (See 2.1.5.1).

2.5 MISCONCEPTIONS

Students’ errors made in mathematics classes are the frustration of both mathematics teachers and students. According to Kaur and Sharon (1994) many errors occur due to carelessness. A major cause is the lack of understanding of mathematical concepts. Such misunderstanding poses a grave concern as it leads to the formation of misconceptions and false generalisations, which in turn hinder the learning of mathematics. Leinhardt; Zaslavsky; & Stein (1990) define misconception as incorrect features of student knowledge that are repeatable and explicit. They attribute the misconception in functions and graphs to previous formal learning. They also associate it with the lack of variety of instructional examples, or a translation, which may be performed inaccurately because of the confusion over symbolic notation. According to the constructivist position, a misconception

> *“identified when a relatively stable and functional set of beliefs held by an individual comes into conflict with an alternative position held by the community of scholars, experts or teachers as a whole" (Confrey, 1987:96).*

According to the constructivists (See 2.1.5.1), learning is the interaction between the individuals' past experience and the individual's current experience of the world around him. Olivier (1992) maintains that misconceptions play a very important role in learning and teaching, because misconceptions form part of the pupils' conceptual structure that will interact negatively with new concepts, which then lead to further misconceptions or alternative conceptions.

Misconceptions differ from errors in several ways. Olivier (1992) states that errors are wrong answers due to planning, and that they are systematic and are applied regularly in the same circumstance while misconceptions are symptoms of shared cognitive structures that could in turn cause errors. From the constructivist perspective, it is noted that students draw on previous and concurrent learning from other areas to work with algebraic symbols. They make parallels with the notation systems, such as in writing fractions where conjoining represents addition (e.g. 3 + m = 3m), or in chemistry where CO_2 is produced by adding oxygen to carbon. It is also evident to Resnick (1980) that the more incomplete the students' knowledge base, the greater the likelihood that the student will generate incorrect inferences, develop misconceptions and produce inaccurate problem solutions.

Although algebra seems to be as simple as the computational algorithm suggests, previous research (e.g. Robinson et al., 1994) indicated that both precollege and college students have many misconceptions about algebraic concepts. The misconceptions are not due to students' lack of procedural knowledge of algebra, rather due to their conceptual understanding (e.g. 3m + 2n has been reported by Robinson et al. that some students often give an answer of 5mn).

Errors as part of misconception have been classified into five main categories by Radatz (1979) as consisting of errors due to pupils':

(1) language difficulties;
(2) difficulties in obtaining spatial configuration;
(3) deficient mastery of prerequisite facts and concepts;
(4) incorrect associations or rigidity of thinking;
(5) application of irrelevant rules or strategies.

Most teachers and curriculum developers are unaware of the preconceptions of children and this has a detrimental effect on the learning of algebra. The background knowledge, theories, beliefs, ideas, etc., called preconceptions can be of advantage to the learner provided an understanding and sympathetic teacher, aware of such preconceptions, is prepared and ready to help the learner. On the other hand, these preconceptions can severely impede the learning ability of learners in the absence of the teacher. Everyday life experiences of the learner and their common wisdom are also partially responsible for the presence of misconceptions in algebra. Identifying the misconceptions of our students in the classroom is very important in the sense that it will help us to see the shortcomings, deficiencies and inadequacies of our method of teaching. Reading about and discussing of a number of prominent misconceptions can be one way of helping preservice teachers realise the models, curriculum experiences, and the use of language that promote misconceptions such as $(a + b)^2 = a^2 + b^2$. Such reading and discussion are also a relevant way of helping some COE students discover their own misconceptions.

2.6 CAUSES OF MISCONCEPTION AND OTHER COGNITIVE PROBLEMS

There are various ways misconceptions and other cognitive problems can occur in mathematics. These can be broadly grouped under:

(i) the cognitive level of the pupils;
(ii) the pupils' background knowledge and preconceptions;
(iii) the influence of the teacher;

(i) Cognitive level of the pupils;

Piaget and Inhelder (as cited in Kurian 1990) assert that the learner assimilates new information into existing cognitive schemata (See 2.1.5.1). They further state that Ausubel et al.(as cited in Kurian,1990) conceptualise learning of new information as a process of subsumption by preconceptions already possessed by the learner. The learner (regardless of age) continually retrieves the earlier learnt concepts in order to internalise or interpret the new information for himself/herself.

(ii) Learners' background knowledge and preconceptions;

Students, because of their background, normally come to the class with other meanings for some of the words, which are commonly used in mathematics. These alternative meanings occur because of the children's culture. For example, the normal use of the word volume, which is associated with the loudness of a radio or a TV etc, is different from the mathematical use which is about space.

Pincback (1991) analysed errors exhibited by students in schools in a remedial intermediate algebra mathematics course, and found two major classes of incorrect responses: conceptual errors and prerequisite errors. Conceptual errors are committed when

> *"the student attempts to apply the appropriate procedures as required for the concept but makes errors in carrying through the necessary steps" (p.55). Prerequisite errors occur when "the student attempts to solve the problem but made the first error in a deficient mastery of a concept previously discussed" (p.56*).

(iii) The influence of the teacher;

Pupils' errors and misconceptions have been traced to teachers' explanation. Blubagh (1988) suggests that the rampant use of the word "cancel" during mathematics lesson could have led to student misconception. Students' interpretations of the word "cancel" have led to confusion in the minds of the students which have made it impossible for them to distinguish between different meanings associated with cancel as they apply to different mathematical topics. These have resulted in errors and misconceptions as students formulate and generalise "cancel".

2.6.1 **Misunderstanding Perpetuated by Mathematics Teachers**

Many research studies have shown that some teachers harbour some misunderstanding, which is eventually passed on to the learners they teach. These are mainly as a result of language. Language can either help learners to understand a concept or hinder their understanding. Arzarello (1998) reports that many learners do not understand algebraic language correctly, and as a result, their thinking and performance are badly affected. Representation in algebra and symbols or the language of algebra, in general are likely to be a major factor affecting misunderstanding. Inappropriate information, or lack of it, by teachers is likely to be some of the major contributing factors affecting misunderstanding of algebra in schools.

2.7 TEACHING

According to Lambert (1990:34) teachers and students from communities of discourse, come to agree on working definitions of what counts as knowledge and the processes whereby knowledge is assumed to be acquired. When classroom culture is taken into consideration, it becomes clear that teaching is not only teaching what is conventionally called content, but includes social, political and cultural entities of learning (See 2.1.5.1). From the activities the teacher sets for them, students learn what counts as knowledge and what kind of activities constitute legitimate academic tasks. The teacher has more power over how acts and utterances get interpreted, being in a position of social and intellectual authority, but these interpretations are finally the result of negotiation with students about how activity is to be regarded. To challenge conventional assumptions then, teachers and students need to do different sorts of activities together. Teaching she further states is a cognitive skill and as such, is amenable to analyses in ways similar to other cognitive skills.

According to Leinhardt & Smith (1985), the overall cognitive system of a teacher is represented in two organised knowledge bases. One consists of general teaching skills and strategies, the other consists of domain-specific information necessary for content representation. This second body of information has as resources the text material, teachers' manuals, and elements of experience that identify what to teach. It also includes algorithmic competence and, at some level, implicit understanding of how the goals, subgoals and constraints of the tasks being taught. It is believed, according to Leinhardt et al.(1990) that teaching is more than the logical extension of facilitation of learning by individuals- it involves guiding and presenting (See 2.3). Guiding and presenting, in turn, involve selecting and transforming the teacher's own knowledge of algebra and other mathematical ideas as well as the supporting system of knowledge from text.

> *"Teaching is a complex and imprecise activity involving multiple objectives, many*

> *different types of tasks, and the possible use of a large variety of different materials. The teacher operates on the front line, constantly guiding and making decisions in an environment that is dynamic and, to a significant extent, unpredictable" (Hannawy, 1992:5).*

Teaching involves helping the learner to construct knowledge. Teaching involves facilitating the acquisition of knowledge and skills by the learners*. The aim of teaching hence, is to make learners' learning possible. Teaching is undoubtedly a craft that demands creativity. Constructive mode of teaching hence places the role of the educator in a way to create a context conducive to learning. Learning however, is a dynamic active and cumulative process of knowledge construction that takes place through understanding and interpretation.* With the constructivist approach to learning the teaching process commences in the teaching-learning situation, with the information that is presented in lessons as facts, concepts, principles, rules and ideas. It is the learner's cognitive processing of the information that is transformed into knowledge.

The subject matter knowledge (See 2.4) of the teacher can help achieve some of the things Cockcroft states in his report. The Cockcroft Report (Cockcroft, 1982), stresses that teachers should encourage students to think in an investigative way. The idea of investigation is fundamental both to the study of mathematics itself and also to the understanding of the ways in which mathematics can be used to extend knowledge and solve problems in many fields. It also indicates that the teacher must be willing to pursue questions raised by students and follow some false trails.

As a teacher one needs to enjoy the teaching, one needs to grow in his/her teaching. One needs to reflect on his/her teaching in such a way that he or she is constantly changing and improving his/her practice. The way in which the teacher views algebra has important implications for his/her instructional goals. A teacher who believes that algebra is primarily about manipulating symbols will teach algebra differently from someone who sees algebra as a language for generalising arithmetic.

Again, as teachers, we must be concerned with how learners come to understand concepts. According to Day (1996), there are two crucial components of the learning environment, the teachers' behaviour with learners and teachers expectations of the learners. Because teachers play a vital role in shaping the learning environment, they ought to reflect on their behaviour and on their expectations for the appropriate use of support learning/teaching materials in the classroom. Teachers have the responsibility to provoke learners through questioning and modelling algebraic concepts. For example, the error normally found in $(x + 2)^2 = x^2 + 4$, the learners should be encouraged to predict whether this conjecture is valid. The learners will need to explore the conjecture to see whether it is true or false for all values of x, by testing the conjecture with different values of x. The same can be done for $(x + 2)^2 = x^2 + 4x + 4$.

Shulman (1986) asserts that the teaching of mathematics is an extremely complex task which, to be successful, requires considerable breadth and depth of knowledge of both a mathematical and pedagogical nature (See 2.3), as well as the ability to organise this knowledge in a useful way. By posing, problem and using investigations we help learners to value the processes used in concept explorations and to facilitate their understanding of the concepts and connections within problem settings in algebra. The question now is what is the role of the teacher in teaching school algebra? Kieran (1984: 187) used this metaphor to answer the question.

> *"An architect must know the site or ground on which a structure will be built and must know in some detail the nature of the structure to be built and the theories which underlie the soundness of such structures."*

In the case of teaching algebra these student teachers must know the nature of current algebra knowledge base of their learners, and must have insights into the knowledge structures, which can grow from such a base. These teachers therefore, provide ideas, activities, insights, and feedback which should help the learners build this knowledge in algebra.

This approach is supported by what NCTM (1991:128) states that:

> *"Mathematics and mathematics education instruction should enable all learners to experience mathematics as a dynamic engagement of solving problems. These experiences should be designed deliberately to help teachers rethink their conceptions of what mathematics is learned. Instruction should be organised around searching for solutions to problems and should include continuing opportunities to talk about mathematics. Working in groups is an excellent way for learners to explore, develop mathematical arguments, conjectures, validates possible solution and identify connections among mathematical ideas"*

2.7.1 **Transmission Teaching**

It is worth talking about the transmission view of teaching (traditional method of teaching, see 2.1.2) since most of our teaching is based on that. Transmission view of teaching considers the teacher as the dominant transmitter of information to passive and submissive learners. Learners are taken as empty vessels, which have to be filled by the expert, the teacher. The current observation of teaching was characterised by mere memorisation and repetition of transmitted algebra content. The transmission teacher views knowledge as part of his/her claim to authority and expertise. The teacher has to provide knowledge and therefore is much concerned about the mastery of the subject matter. The learner is given no opportunity to think since it becomes the duty of the teacher to make sure the information is supplied to the learners. Question and answer method is mostly used to check whether what has been transmitted has been correctly memorised by the learners. Transmission teaching is characterised by the teacher doing the talking most of the time, lecturing, instructing and demonstrating while the learner remains silent, observant, obedient, submissive, passive, attentive and concentrating. Assessment is done by means of tests and examinations and learners are supposed to give response to the questions inline with the teacher expectations, criteria and standards. In transmission teaching, the teacher's views himself/herself as being responsible for handing over ready-made knowledge from texts to the learners as a ticket for promotion or certificate. Learners are therefore, rewarded by the kind of knowledge that meets the teacher's approval and standards. This view of teaching does not create the type of citizens the South African government is looking for hence the introduction of the Outcomes-Based Education, curriculum 2005, which should take the form of learner-centred approach (See 2.1.3). With this approach the learner is supposed to participate in whatever happens as far the learning is concerned and is supposed to be an active participant in the learning process. Transmission method of teaching has contributed to the way school algebra (See 1.3) had been taught previously and the traditional method of teaching algebra (See 2.1.2).

Based on the preceding literature analysed, I conclude that College of Education students do not enter the teacher education programme in mathematics with a solid background from schools thus resulting in transmission teaching. I am of the conviction that with the appropriate subject matter knowledge (See 2.4) and understanding the student teachers, through proper instruction obtained from their various Colleges of Education, will be in a position to attain or fulfil the objectives mentioned above where teaching will not only be based on transmission method but through the constructivist (See 2.1.5.1) approach. The art of teaching algebra will therefore mean to be able to select examples that exemplify or challenge algebraic knowledge that can critically elucidate conceptual thinking. The teaching approaches, such as in the emphasises on active learning and cooperative group work, can help in creating more responsive learning environments, where learners can be treated as individuals whilst at same time taking part in experiences that encourage higher success
and achievement.

2.7.2 Integrated Teaching

The notion of integrated teaching features prominently in the conceptualisation of classroom teaching under Curriculum 2005 in South Africa. According to Van Reeuwijk (1995) during integrated teaching, school mathematics become a subject where different mathematics topics are put together and taught as one unit. This creates a situation where one finds no separate courses for algebra, geometry, statistics, calculus and so on. With this approach all learners get the opportunity to learn about all topics of mathematics at the same time. With integrated teaching and learning, mathematics is developed as a whole, and the connections between the different sub-domains are constantly made explicit because these connections are implicitly present in the materials development. With curriculum 2005, integrated teaching is to have educators within the same phase plan the same lessons together and teach in a coordinated effort, by so doing the same topic or concept is not duplicated in different subjects. According to Van Reeuwijk (1995) integration involves development of materials with assessment, instruction, and with teacher education and teacher support.

2.8 THE MATHEMATICS CURRICULUM AND UNDERSTANDING

There has been much talk about reform in mathematics and much centres around the development of understanding. In the "new math" programmes of the 1960s the emphasis was on mathematical structure and understanding, whereas the back-to-basics movement of the 1970s and 80s focused on teaching procedural skills. During these times curriculum development viewed the teacher's role as that of implementing an expert made curriculum. As Even and Tirosh (1995:2) put it:

> *" teacher-proof curricula, the extreme outcomes of this process, assumed that children could learn directly from ready-made curriculum materials while the teacher, instead of teaching, would adopt a role of the manager and facilitator"* .

According to Even and Tirosh, the mid 1980s marked a change in conceptions of the teacher's role in promoting learning; which now came to include setting mathematical goals and creating classroom environments to pursue them; helping students understand the subject-matter by representing it in appropriate ways; asking questions, suggesting activities and conducting discussions. Subject-matter knowledge (See 2.4) is much more critical for this "new" role of the teacher. The recent introduction of OBE (Outcomes-Based Education) in South Africa is also laying emphasis on education with meaning and understanding.

According to Sierpinska (1994), the word understanding in ordinary language has many Interpretations (See 2.1.5.4). Understanding is used in many forms and expressions in an informal speech. We say that a person "understands" something, we speak of personal "understanding" of something, and of the various "understandings" people may have. We also speak of "mutual understanding", understanding a word, an expression, and a concept. We qualify understanding as "good", "deep", "poor", "incomplete", "intuitive", or "wrong" etc. For this study the researcher implies "understanding why". Schifter & Fosnot (1993) refer to understanding as an ongoing process, deepening as conceptions expand. Understanding is sometimes an elusive goal in mathematics. Although we may believe we have a complete understanding of a concept, another approach to this same concept may bring us additional insight. This mode of understanding requires that, the pedagogical content knowledge (See 2.3) of College of Education students should be sound enough to meet such situations. Understanding algebra derives from the view that learning is primarily a process of concept construction and active interpretation as opposed to the absorption and accommodation of received items of information. To achieve conceptual understanding Schifter & Fosnot suggest a structure for the class. Students organised in small groups, facilitated learning, working in a setting in which they verbalize their own ideas, students can clarify their understanding and identify confusion.

Hiebert (1986b) defined understanding as *" the process of creating relationships between pieces of knowledge. Students understand something as they recognise how it relates to other things they know already*" (p.2). This definition of understanding involves establishing relationships between segments of knowledge, which means in this case the learner establishing relationships between the knowledge possessed and knowledge that the learner is acquiring. The College of Education students' understanding of algebra will then mean the cognitions and behaviours, which will influence their classroom instruction on the teaching of the topic. This classroom instruction is determined by the decisions they make which are directly influenced by their knowledge and beliefs.

It is believed that, since understanding involves the ability to apply a particular concept, skill, or procedure to unfamiliar situations, an individual who has a good understanding of certain mathematical ideas and techniques is likely to be able to apply that learning to contexts, other than those in which mathematics was originally learned.

Upon the argument put forward by Hiebert (1986b) if understanding means to apply the knowledge acquired in new situation then apprenticeship learning is also part of understanding. Apprenticeship learning or learning-in-practice as used by Lave (1990) emphasizes the situated character of the problem- solving activity while focusing on learning in doing. According to Lave, *" learning takes place on the assumption that knowing, thinking, and understanding are generated in practice, in situations whose specific characteristics are part of practice as it unfolds" (p19*).

She states further that "*apprentices learn to think, argue, act, and interact in increasing knowledgeable ways, with people who do something well, by doing it with them as legitimate, peripheral participants" (p19).*

Apprenticeship learning, according to Lave, is possible if the learner knows from the beginning that after the apprenticeship he or she will likely get a job or practise his/her acquired profession. This serves as an intrinsic motivation for the apprentice hence the student spends more time in learning. The learners know clearly the curriculum in practice. What makes the learners work hard in most cases is the fact that they own and bear the problems, which arise. Apprenticeship curriculum brings to conceptual development, the learning curriculum and dilemma-motivated curriculum.

Understanding learners' current understanding is important according to Graeber (1999). It is imperative that understanding and supporting the learners' reasoning about algebra may lead to successful instruction. Graeber however, acknowledged the fact that it is for preservice teachers to know what is understood by student thinking in an area but it seemed reasonable to ask that preservice teachers know what is understood about students' thinking in some areas and understood how assessing students' thinking can give direction to instruction (See 2.3). If college of education students enter the classroom without valuing student understanding of algebra, they are not apt to assess understanding or use knowledge of students' current understanding to make instructional decisions.

If the Understanding of algebra is to be achieved by College of Education students, then they should own and understand the problems they encounter at their colleges. As Lave (1990) puts it, "*what is learned "out of context" is in danger of being suspended* in vacuo' (p28). The OBE Curriculum 2005 with its outcomes, and activity based teaching approaches is likely to achieve understanding of algebra since the learners are going to be motivated, and are going to own the contextual problems they will have to deal with. They will thus hopefully work harder to solve their envisaged problems.

2.9 PROCEDURAL AND CONCEPTUAL UNDERSTANDING

These are two forms of mathematical knowledge. Procedural knowledge refers according to Eisenhart; Underhill; Brown; Jones; and Agard (1993 :9) to

> *"mastery of computational skills and knowledge procedures for identifying mathematical components, algorithms, and definitions, i.e., knowing how to identify a problem, in its broadest and most routine sense, and how to solve correctly".*

They distinguish two types of procedural knowledge.

> " *(a) Knowledge of the format and syntax of the symbol representation system and*
> *(b) knowledge of rules and algorithms, some of which are symbolic, that can be used to complete mathematical tasks " (Eisenhart et al., 1993:9).*

For example, if one is asked to solve a quadratic equation using the formula, the first thing is to remember the formula, then to know what each letter in the formula stands for, do the substitution, simplify and get the two answers. This type of knowledge leads to Skemp (1976) instrumental understanding.

Conceptual knowledge refers to the relationships and interconnections of ideas that explain and give meaning to mathematical procedures. Conceptual knowledge is based on many relationships formed in the mind. It is acquired when learners are able to make the connection between incoming information and existing knowledge. According to Schifter and Fosnot (1993) if the creation of conceptual algebraic understanding is the product of constructive and interpretive activity, then it follows that no matter how carefully and patiently teachers explain to their students (See 2.7.1), they cannot understand for their students. Once one accepts that the learner must herself/himself actively explore algebraic concepts in order to build the necessary structures of understanding, it follows that teaching algebra must be reconceived as the provision of meaningful problems designed to encourage and facilitate the constructive process (See 2.1.5.1). The algebraic classroom designed for conceptual understanding, rather than on computational drill, promotes students' confidence in their own mathematical abilities. For example, as we move sets of objects around, put them together, we acquire implicit understanding of commutativity, associativity, and reversibility. This type of knowledge can be associated with what Skemp (1976) calls relational understanding.

It is important for College of Education students to understand that executing an algorithm, or getting the right answer does not imply conceptual understanding. For an example if a student is able to factorise a quadratic equation $x^2 - 2x + 1 = 0$, but fails to solve $x^2 - 2x = 0$, then it shows lack of understanding on the part of the particular student. I will therefore differentiate between knowledge and understanding by stating that knowledge is the ability to remember, recall, identify, define, describe, list, name, match, state principles, facts and concepts. Knowledge is simply the ability to remember or recall material already learned. Knowledge constitutes the lowest level of learning. Understanding is however, the ability to explain, summarise, translate, rewrite, paraphrase, give examples, generalise, estimate or predict consequences based on trend. Understanding is generally the ability to grasp the meaning of some material that may be verbal, pictorial, or symbolic. It also involves the ability to use conventional methods, algorithms or step by step procedures in solving problems. Understanding also means being able to explain concepts in several different ways and in real world situations.

2.10 CONCLUSION

To summarise, it is important for College of Education students to attain a proper understanding of algebra so that they will be able to translate their knowledge of algebra into effectively teaching algebra to the pupils they teach. They must be able to go beyond their personal and preferred ways of teaching to generate different ways of presenting and explaining the acquired knowledge of algebra to their pupils. It is important that College of Education students identify the preconceptions of algebra pupils bring along with them to mathematics classrooms. Only after identifying the preconceptions will the teacher be able to take steps to reduce if not to remove misconceptions completely from the minds of their pupils. According to Jaworski (1994:29)

> *"the teacher's construction of a student's mathematical understanding, no less than students' constructions of mathematics, needs supportive or constraining feedback. This can be provided potently by students' errors or apparent misconceptions, which can be the basis for diagnosis by the teacher and subsequent modification of the teacher's vision of the* students' conception".

In order to diagnose a pupils' misconception in algebra the teacher should be able to interact with the pupil by talking to them, setting tasks and analysing the outcome of the tasks. According to Vygotsky (1986) conceptual change can happen as a result of dialogue between the teacher and the learner. Student teachers' pedagogical knowledge is very important at this point. To help learners to remove some of the misconceptions and errors in algebra, one has to adopt the constructivist philosophy of teaching and learning which is compatible with the current vision of change in mathematics education in South Africa, the Outcomes-Based Education. The philosophical constructivists (See 2.1.4) assert that it is we who construct (not discover) the known world on the basis of our experiences and active processes of developing knowledge. Student teachers should therefore, use dialogue as a means of negotiating conceptual change in the teaching and learning of school algebra.

The literature reviewed here and the findings provide a rationale for requiring College of Education students to make learners in algebra explain their thinking and justify their procedures and answers to meaningful algebra questions. It is however, noted that with regard to teaching for understanding in algebra, the literature on conceptual change suggests that, "teaching (only) by telling", will not result in the kind of understanding required of the new curriculum 2005. It is based on these findings that the framework of the study included a follow-up in the classroom to ascertain the pedagogical content knowledge of COE students in algebra.

The literature also suggests different sources of students' errors and misconceptions in algebra. Students' tendency to conjoin open expressions, Tirosh and Graeber (1990) give an explanation for this, stating that students' face cognitive difficulty in accepting lack of closure. What has also come out of the literature review is that the process of teaching and learning involves interaction among the teacher, the learner and the materials used by both the teacher and the learner. It is also evident in the literature that the success of teaching for conceptual understanding depends on the appropriate teaching strategies the teacher adopts, especially constructivist approaches.

The problems learners face in the classrooms when learning mathematics and algebra in particular are also reviewed. Invariably, concerns have been raised (MacGregor and Stacey,1997, Blais,1988, Shifter and Fosnot,1993) about the concepts, the nature and organisation of algebra content within the classroom which requires a certain amount of knowledge and understanding on the part of teachers of school algebra. I witnessed as a lecturer in one of the rural Colleges of Education, the lack of facilities, the poor background of the study of mathematics as a result of poor teaching by the previous mathematics teachers, as some of the factors contributing to these concerns raised. The study investigates the extent to which reports by researchers about the knowledge and understanding of algebra affect College of Education students from in and around the Transkei in South Africa. These students are known in South Africa to have come from the so-called disadvantaged communities.

The study furthermore, finds out whether the new era in the study of mathematics by the constructivists has dawned on these College of Education students in and around the Transkei, as one of the methods applied in the teaching of mathematics. As has been stated in chapter 3 (See 3.1.2.5) the study of mathematics content in the Colleges of Education in the Transkei does not differ much from what is used at the level grade 12 in the Transkei. This study will find out whether going through the same or slightly more advanced algebra content at the College of Education level with that of the grade 12 students will have any impact on the level of knowledge and understanding as per their instructional practices. The study again will look into the relationships between the knowledge, understandings, and misconceptions of algebra with the pedagogy of teaching and learning of algebra.

The topic, algebra, was chosen to explore knowledge, understanding and instructional practice, because algebra forms a major part of the mathematics curriculum in South African schools. The literature reveals that high school students have found mathematics difficult and have received it badly due to the poor teaching of mathematics. My experience as a mathematics teacher in the high school showed that only few students offered mathematics, the trend has not changed at the college level. Only a few of these students study to teach mathematics. Algebra being a major component of mathematics might have contributed to the difficulties reported. For the academically disadvantaged College of Education students studying mathematics in and around the Transkei, algebra might have become a severe obstacle in the acquisition of the necessary knowledge for instructional practice in the classroom.

I believe that a study has to be done on these prospective College of Education students to identify problems, understandings as well as misconceptions they carry along in the process of studying to become teachers of school algebra. It is my belief that through the algebra test, the interviews and the classroom observations that the student teachers' knowledge and understanding will be unfolded and might lead to identification of problems, obstacles and misconceptions in algebra. This can lead to creating a course or a change in the methodology in the Colleges, or re-inspect the existing curriculum in the Colleges that will engage the final year College of Education students to acquire kinds of knowledge that will be needed in the teaching of algebra in the school classrooms.

Most of the studies and findings mentioned in the literature emanate from research done in different parts of the world. The problem with College of Education students in and around Transkei may not be the same as elsewhere. I am interested in investigating the knowledge and understanding of College of Education students in this part of South Africa (Transkei and neighbouring regions) with reference to algebra, the conceptual understanding and the pedagogical aspect of it. Determining what student teachers know about algebra, how they interpret it, and how they apply their knowledge in the classroom is an important element of a complete understanding of the knowledge, understanding and teaching of school algebra. It is upon this premise that I investigate the subject matter knowledge as well as pedagogical content knowledge of some final year College of Education inside and around the Transkei in the Eastern Cape.

CHAPTER 3

RESEARCH DESIGN AND PILOT

3.1 METHOD

3.1.1 INTRODUCTION.

As mentioned before, the purpose of this study is to examine the subject matter and pedagogical content knowledge and understanding of final year College of Education students. For this purpose, the algebra test instrument is adapted from the Chelsea Diagnostic Test in Algebra: CDMTA (Brown, M., Hart, K., and Kuchemann, D., 1984) and a test constructed by Kaur and Sharon (1994). CDMTA was designed as a diagnostic instrument that was used to ascertain a child's level of understanding and misconception in algebra. Kaur and Sharon's were used to investigate algebraic misconceptions of first year College students in a junior college in Singapore.

These instruments were applied to the final year College students from a selection of colleges, which were representative of the types of Colleges in the South African (S.A) system. One College from a more advantaged group and five from ex- Department of Education and Training (DET) or “Homelands Colleges”. Two Primary Teachers Diploma (PTD) Colleges were involved in the study. These two Colleges were taken for two reasons. One reason was to see whether there was a difference in algebraic knowledge acquired after matric between Senior Teachers' Diploma (STD) students and PTD students from these colleges. The second reason is that only a few colleges are offering STD while most of the colleges are offering PTD courses, and due to the shortage of mathematics teachers, many PTD teachers find themselves teaching in the junior secondary schools where they are compelled to teach algebra. A pilot study was conducted to ascertain the validity and reliability of this test instrument adapted from CDMTA, Kaur and Sharon.

The main study followed after the test instrument had been piloted and modified where necessary. My work differs from that of the CDMTA in that, I am concerned with older students in the design implementation, and evaluation of instructional treatment. Kaur and Sharon's work differs from this research since the instrument was mainly given to final year students. Kaur and Sharon's test on the other hand was given to students from the Science stream (students with an aptitude to read mathematics and science courses) and had enrolled for the advanced level mathematics course. The situation in Kaur and Sharon's research however, was similar in nature to my research. There most probably was a difference in curriculum and resources, since Singapore is an economically strong, small state in Asia while South Africa has a relatively large population many of whom live in impoverished rural communities.

The research essentially falls into the qualitative paradigm with a triangulated design (See 3.1.1.1). Triangulation was achieved by making use of both quantitative and qualitative data. Merriam (1998) asserts that in qualitative research the researcher is the primary instrument for data collection and analysis. She argues that in qualitative research, data is collected by the use of human beings as instrument, instead of through some inanimate inventory, questionnaire, or computer. Qualitative research again focuses on process meaning and understanding. In order to arrive at judgements regarding the quality of a teacher' PCK I would need *"richly descriptive data"* (Cohen and Manion 1980: 29). Words and pictures were therefore used instead of numbers to convey the outcomes of the research findings. Data are also in the form of the participants' own words, direct citations from documents are used to support findings. The procedure was to collect and analyse data on knowledge and understanding of algebra from three sources as mentioned earlier: The algebra test, interviews of participants and classroom observation of participants.

3.1.1.1 **TRIANGULATION**

Triangulation is defined by Cohen and Manion (1980: 208) as

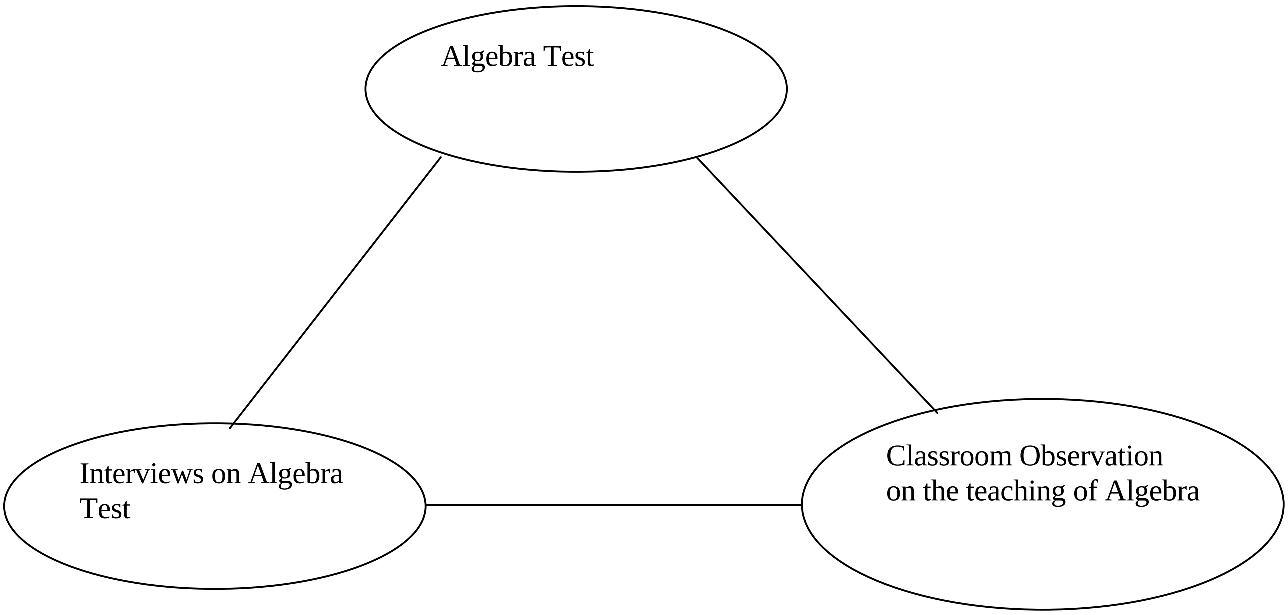
> *" the use of two or more methods of data collection in the study of some aspects of human behaviour.... Triangulation techniques in social sciences attempt to map out, or explain more fully the richness and complexity of human behaviour by studying from more than one standpoint and, in so doing, by making use of both quantitative and qualitative data".*

According to Cohen and Manion if triangulation is used in interpretive research the more the methods agree with each other the greater the researcher's confidence. In this study, the more the interview and observation data results of participants agree with the algebra test results, the more I became confident with the findings.

Triangulation has different types, Cohen and Manion make mention of six types: Time triangulation, Space triangulation, Combined levels of triangulation, theoretical triangulation, Investigative triangulation and Methodological triangulation. This study made use of the Methodological triangulation technique where more than one method was used in pursuit of a given objective of knowledge and understanding of algebra. For validity however, a large sample was needed but as a result of the problems encountered (See 1.8) a small sample was used.

The process of gathering data from three distinct standpoints has an epistemological justification. As shown in figure 3.1 below each corner of the triangle represents a unique epistemological data source on knowledge and understanding of school algebra.

Figure 3.1 Triangulation



The algebra test helped to gather information on the knowledge and understanding of algebra expressed in the form of written answers to questions in algebra. At this stage individual knowledge and understanding of participants were gathered without the intervention or mediation of their lecturers or colleagues. Secondly, the interview process helped me to get the verbal explanations of the knowledge and understanding the participants had towards school algebra. Thirdly, the classroom observations helped me to observe the impact the algebra knowledge and understanding had on the instructional practices of the teachers.

3.1.2 METHODOLOGY

3.1.2.1 RESEARCH DESIGN

The research design is shown in fig 3.2. The design entailed the following steps: Design Instruments, Pilot, Test, Interviews, Field Observations and Data Analysis.

Design Instruments

Refers to the preliminary stage where I had to plan to undertake this study. This included obtaining permission from DOE and the colleges included in the study to allow me the use of their mathematics students. The type of tools to use to gather data which included: the Algebra Test, interview plan and classroom observations schedules drafted.

Pilot

Refers to the pilot study I conducted prior to the main study to ascertain the validity and reliability of the instruments I used, which included the Algebra Test and the Interview plan (See 3.2).

Test

This is the stage where the finalised Algebra Test instrument was sent to the various colleges for the participants to answer.

Interviews

This part of the study involved the interview by me of some selected students from various Colleges of Education. These were students who had shown some misconceptions, misinterpretations or had shown an extraordinary way of answering some of the questions from the algebra test instrument I had given them (See 3.1.3.3).

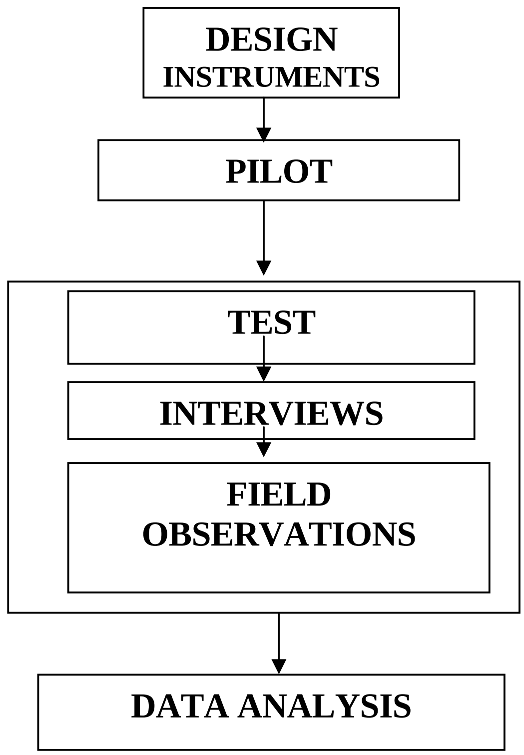
Field Observation

At this stage I conducted classroom observations to verify how this algebra Knowledge and understanding observed from the written test and the interviews were put into instructional practice (See 3.2.1.3).

Data Analysis

This was the stage were the data collected through triangulation were analysed to arrive at conclusions, suggestions and recommendations.

Figure 3.2 Research Design



3.1.2.2 **SUBJECTS**.

Table 3.1 is a summary of the participants from various Colleges. The Colleges with * are PTD and the remaining ones are STDs.

NAME OF COLLEGE	YEAR1	YEAR2	YEAR3	YEAR4	TOTAL
PUE*	29	0	24	0	53
PME*	0	0	27	0	27
SRK	0	0	19	0	19
SIK	0	0	23	0	23
STK	24	0	20	13	57
SEK	0	0	19	14	33

TOTAL	53	0	132	27	212

Table 3.1 Summary of participating colleges.

Subjects were drawn from a total of 212 participants in this study in various ways (See 3.1.1), which included 132 year three students and 27 year four students totalling 159 final year College of Education students. 53 year one students also participated in the study making the overall total of 212 participants. Although the participating Colleges were not randomly selected, the subjects include colleges from two provinces in South Africa: KwaZulu-Natal province and the Eastern Cape province. Two are from KwaZulu-Natal: Edgewood College of Education and Ndumiso College of Education. The other four namely, Transkei College of Education, Umzimkulu College of Education, Maluti College of Education and Rubusana College of Education from the Eastern Cape. The subjects were mainly third and fourth year students except for the two Colleges, Umzimkulu College of Education and Transkei College of Education, where the first, third and fourth year students were used. The reason for using these two Colleges is to get some insight into the knowledge and understanding of students at these levels of education. Two Colleges as shown above included 4th-year final students. Thirty-one students from two Colleges of Education participated in the first phase (pilot) in which they were given the draft algebra test. Two subjects each from these two colleges participated in the pilot interview as well as observation lessons in the classrooms. All subjects were enrolled in mathematics and mathematics method classes. All of the students who were present on the day of the test were included. There were no specific criteria used in the selection of this sample other than the proximity to my place of work and the co-operation and help from the lecturers from those colleges.

3.1.2.3 **ACCESS AND ACCEPTANCE TO COLLEGES OF EDUCATION, LECTURERS AND STUDENTS.**

The main aim of this study is to investigate the knowledge and understanding of algebraic concepts of final year college students. As part of the study I had to test, interview and observe these college students. To have access to these students, I had to go to the Colleges involved. Colleges in the Eastern Cape and KwaZulu-Natal are however, semi-autonomous, which means access can be gained by direct contact with the Colleges involved. Semi-autonomous in the sense that, funding is done by the provincial Department of Education. This department sees to it that the funds are put to the daily running of those institutions, like paying for electricity bills, water and the tuition of the students. Lecturers' salaries are also paid by the Department of Education. The management and administration are however, done internally by various Colleges under the leadership of their Rectors. Below the Rectors come the Vice-Rectors, the Heads of Departments, the Senior Lecturers and lastly the subject lecturers at the bottom of the ladder of administration and duties. To have access to the subjects for the study therefore, I had to write a formal request to the Rectors accompanied with a brief proposal of what the aims of the research were (See appendix K). The Rectors in turn referred me to the Vice-Rectors. I was finally referred to the Heads of Departments of mathematics or the Senior Lecturers in mathematics. My proposal and request were attended to and permission was granted for the study in the colleges in this way.

I was then introduced to the subject lecturer in charge of mathematics. From then on all communications with the participants were done through the subject lecturers. I was introduced to subjects on the day of the Algebra Test, otherwise all arrangements for the date of the test, the interview and the classroom observations were done by the subject lecturers. On ethical grounds I wrote two letters to thank the Colleges involved and the subjects respectively.

3.1.2.4 **ACCESS TO STATISTICAL INFORMATION.**

To gain access to the final year results analysis of Eastern Cape Colleges of Education, I had to write to the Director of Collegiate Education [2]with a brief summary of my research proposal attached. This Collegiate Education is the one, which gives accreditation to Colleges of Education in Transkei and therefore keeps these results. The only condition attached was to give the department a copy of my final research proposal. I was however, referred to the Examinations office at the University of Transkei, Umtata in the Eastern Cape where these records are kept. I had good assistance from the officer in charge of those documents. This officer released these documents to me a day after I had met with the Director.

3.1.2.5 **ACADEMIC BACKGROUND**

According to the College of Education entry requirements to gain admission into the mathematics stream one is supposed to have completed matric and have at least a D symbol in mathematics at Standard Grade level or an E symbol at Higher Grade level. Except for the PTD Colleges, which required a matric plus and at least grade 9 mathematics. Subjects from four of these Colleges of Education included final year students who had completed at least two years of mathematics. The other two Colleges included year 1 students. Year 1 students were in the process of completing the first year College of Education mathematics.

The PTD Colleges from the Eastern Cape follow the syllabus of the Department of Education (see appendix I) up to standard eight in the High School and the STD's had the syllabus slightly above matric from both provinces. The two PTD Colleges selected were from the Transkei region, the reason being that the PTD Colleges from KwaZulu–Natal were far from my place of work while the Transkeian PTDs were close to my

[2] The Department of Collegiate Education at University of Transkei ceased to exist in 2000

place of work.

3.1.3 INSTRUMENTATION

3.1.3.1 RATIONALE

As has been noted earlier, this study had three phases. The first phase (Test) of the study aimed at getting a general picture of College of Education students' subject matter knowledge in school algebra. The second phase (Interview) aimed at clarifying this picture. Finally, the third phase (Observation) aimed at the pedagogical aspect of teaching school algebra in the classroom by a selected sample of college students. The intention however, was to observe these teachers teaching in schools the following year (+ 6 months) after graduation from these colleges to look at pedagogical content knowledge of algebra.

Classroom observation however, took place during their practicum period due to the fact that the final year College of Education students failed to get teaching posts after completing their courses, because of the policy of the government of South Africa not to employ new teachers.

3.1.3.2 ALGEBRA TEST

The algebra test brings together questions adapted from the Chelsea Diagnostic Mathematics Test in Algebra (CDMTA) and Kaur and Sharon test in algebra. The CDMTA test questions addressed the conceptual understanding of algebra by beginners and the Kaur and Sharon's test addressed fundamental concepts and ideas that College of Education students would have encountered many times at the secondary school level and were expected to understand.

3.1.3.2.1 CDMTA TEST

This test was developed by the mathematics team of the United Kingdom Social Science Research Council Programme "Concepts in Secondary Mathematics and Science", the programme was based at the centre for Science Education, Chelsea College, University of London, during the period 1974-79. It was a diagnostic test aimed at ascertaining a child's level of understanding and at identifying misconceptions (Brown et al, 1984).

There were four levels involved, according to Hart, Brown, Kuchemann, Kerslake, Ruddock, and McCartney (1981).

Level 1.
The items at this level are purely numerical or they can have a simple structure and can be solved by using letters as objects. For example: " What can you say about a if a + 5 = 8 ?"

Level 2.
The difference between level 2 items and level 1 items is the increase in complexity, however the letters still have to be evaluated or used as objects. Children at this level can still not consistently cope with specific unknowns, generalised numbers or variables. For example: " what can you say about u if u = v + 3 and v = 1 ?"

Level 3.
At this level children are willing to accept answers, which are "incomplete" or "lack of closure". For example, children at this level are able to regard answers like 7+ g, 2n + 3, p = 2n as meaningful, even though the letters represent numbers and not objects. For example: " e + f = 8, what is e + f + g = ?"

Level 4
Children at this level are able to cope with items that require specific unknowns and which have a complex structure. For example: " which is larger, 2n or n + 2? Explain."

According to Hart et al. (1981) items at levels 1 and 2 can be solved without having to operate on letters as unknowns, whereas at levels 3 and 4 the letters have to be treated at least as specific unknowns and in some cases as generalised numbers or variables.

3.1.3.2.2. **Kaur and Sharon Test**

The Kaur and Sharon (1994) test was used to identify some algebraic misconceptions among first year students at a Junior College in Singapore. There were sixteen items included in the test, which covered algebraic topics such as Equations, Inequalities, Quadratic Equations and Indices, Logarithms and Surds. The test also assessed fundamental concepts and ideas that students are expected to have mastered during their secondary education course in algebra.

The issues surveyed in the test included:
(a) A knowledge of major conventions like |x| and √x, and the capacity to interpret them in context and with consistency;
(b) The effective use of counter-examples;
(c) The ability to interpret the precise meaning of statements, and judge whether specific criteria are adequate or have been met;
(d) The capacity not only to apply rules, but to habitually consider whether it is legitimate

to do so;
(e) Awareness of appropriate mathematical terminology.

In all I used a 20 modified item test instrument (See appendix C), which included all the four levels of the CDMTA and all the issues expressed by Kaur and Sharon. The questions demanded that the participants show how they arrived at the answers.
Items 1-12 of this algebra test instrument were selected from CDMTA (See appendix G).
This included one question each from level 1 and level 2 respectively, three questions from level 3 and seven from level 4 as shown in Table 3.2.

Algebra test	CDMTA Level
1	1
2-4	3
5	4
6	2
7-12	4

Table 3.2 Level of algebra items

The pilot study guided the choice of the questions. It also helped to improve the conceptual framework, design, instrumentation and procedural requirements for the main study. For example, the responses from the open-ended questions included in the pilot helped to develop to standardise questions matching the level of algebraic knowledge and understanding of these college students. These necessitated an increase in the scope of the test by including more questions from Kaur and Sharon. It was noticed from the pilot that level 1 and level 2 questions were well answered and posed no problem to the subjects, hence did not help in what the research was all about. I therefore decided to use only one each from level 1 and level 2 respectively. I again used three from level 3 because they were also not very challenging. I used seven questions from level 4, which were a little challenging and were up to the task of unveiling the problems and misconceptions these subjects had. As has been discussed in 3.1.3.2.1 the CDTMA was meant for beginners of algebra whereas the subjects in this study are adult post matric students.

3.1.3.2.3 **Explanation of Choice of Questions From CDMTA (see appendix G).**

Questions 1-4 from the CDMTA were not selected because they involve direct easy substitutions and additions. Questions 5a and 5c were selected. Question 5b was rejected because it was similar to 5c. Question 6 was also not selected because it is a level 1 question. Question 7d was selected because the rest of the questions were level 1 and level 2 questions. In question 9, preference was given to 9c as a question in level 2 over 9a and 9b because, 9a is a level 1 question and 9b is similar to 9c.

Question 9d was not used because I had enough level 3 questions. Question 10 was selected for a level 4 question. Question 11 was rejected because it belongs to level 2. Question 12 was however, selected as against the rejection of question 13, which had a similar question 11 already selected for the final algebra test instrument. Question 14 was selected because it was suitable for the study. Question 15 was not selected because it belongs to levels 2 and 3.
Question 16 was selected because it involves an inequality, which is one of the areas of misconceptions the study intends to investigate. Question 17 was similar to one of the questions already selected therefore was rejected. Questions 18 and 19 were rejected because they belong to levels 1 and 2 respectively. Question 20 was rejected because a similar one has been selected. Question 21 was selected and question 22 was rejected because a similar question has been selected. Question 23 was replaced by questions on functions, because I wanted to test for the knowledge and understanding of functions. Functions have been treated as part of the algebra curriculum in South Africa and I therefore decided to include two questions on functions in this study.

3.1.3.2.4 **Explanation of Choice of Questions From Kaur and Sharon (1994) Test**

Items 15-20 were selected from Kaur and Sharon's test (See appendix H). In section A of their test, item1 was selected because it involves the √ sign, which relates to one of the misconceptions in algebra as reported by Kaur and Sharon. Item 2 was selected because it involves the use of inequalities, which has been reported by Kaur and Sharon to pose difficulties to students of algebra. Item 3 was selected due to the nature of confusion students have over solutions of inequalities where most of them apply the principles of equations exactly to inequalities. Item 4 was selected to investigate the knowledge of expansion of binomial expressions, which is also a problem to algebra students as reported by Kaur and Sharon. Item 5 was chosen to investigate how students use only positive numbers to verify statements involving positive and negative numbers as solutions. Item 6 from was however not selected because it involves surds, which is not part of the topics the study wanted to investigate.

In section B of Kaur and Sharon test, item 7 was not selected because it is about inequalities, which have been catered for already in the instrument. Item 8 was not selected because it tests for the laws of indices. Item 9 similarly was not selected because it involves logarithms. Item 10 was selected to investigate the knowledge and understanding of the difference between quadratic inequation and quadratic expression. Items 11-16 were not selected because the required number of questions needed for the study has been attained and the concepts entailed have been addressed already in the instrument.

3.1.3.3 **INTERVIEW**

The purpose of interview method as a data collection instrument is for me to have face to face interaction with the person to be interviewed. This enabled me to obtain valid and reliable information to check against facts, in this case verbal or descriptive knowledge and understanding of the algebra test, which was given to the final year College of Education students.

Semi-structured open-ended questions were developed out of the results of the test. The duration of the interview was largely dependant on the responses given during the interview but lasted no longer than forty-five minutes. The interview method was preferred here. According to Cohen and Manion (1994) interviews are used to, for example, follow up unexpected results, or to validate other methods, or to go deeper into the motivations of respondents and their reasons for responding as they do. Open-ended questions and unstructured interviews were used, Cohen and Manion (1994:297) suggest a number of advantages of open-ended questions:

> *"they are flexible; they allow the interviewer to probe so that he may go into more depth if he chooses, or clear up misunderstandings; they enable the interviewer to test the limits of the respondent's knowledge; they encourage co-operation and help establish rapport; and they allow the interviewer to make a truer assessment of what the respondent really believes".*

Cohen and Manion (1980:243) define unstructured interview as an interview which the content and procedures are based on: *"an opened situation, having greater flexibility and freedom".* For example, what are the subtopics in algebra? Instead of the more structured way, name four subtopics in algebra. Cobb (1986) acknowledges that it is a type of interview, which allows the interviewer to negotiate meaning during the process of the interview. Each interview was audiotaped and later transcribed and analysed qualitatively. According to Hitchcock and Hughes (1989) there exists structured interview in an unstructured interview. The only difference they make for structured and unstructured interview is the degree of negotiation between the interviewer and the interviewee. The distinction is made to the extent that, in the unstructured interview there is flexibility in scope to allow for the interviewer to introduce new material into the discussion as the discussion progresses. The structured interview on the other hand, does not allow for flexibility, content and procedures are organised in advance. Structured within unstructured interview however, implies that unstructured interview should be carefully planned in order not to loose focus of the purpose of the interview. Hitchcock and Hughes again assert that the aim of an unstructured interview is to provide for the interviewer and the interviewee a greater and freer flow of information. This allows for the interviewer to move forward and backward in the process of interviewing to provide the platform to clarify points, the opportunity to go over earlier points and to raise fresh questions. They view unstructured interview with the overall goal of creating an atmosphere where the interviewee will feel free to come out with subjective and sometimes highly personal information to the interviewer.

The unstructured interview is more free-flowing, with its structure limited only by the focus of the research. This type of interview is conducted more like a normal conversation, but with a purpose. With this type of interview the interviewer may start with a broad opening statement like: ‘ how do you feel about...’. Depending on the response the interviewer invites the interviewee to clarify the response with further probes. According to Brink (1996), unstructured interviews will produce more in-depth information on subjects’ beliefs and attitudes than can be obtained through any other data-gathering procedure.

3.1.3.4. **OBSERVATION**

Observation is a means of studying the events, behaviours, and artifacts in the social setting. The social setting in question in this study is the classroom. I had to check for the knowledge and understanding of Algebra for the College of Education students and the way this knowledge and understanding is translated into instructional practice in the classroom. Once the students leave the College and start to teach at the schools, a follow up study was conducted on at least three students from each of the selected colleges identified for the study. The students selected seemed to have misconceptions and lack of algebraic knowledge as seen on their scripts. The students who were identified, were notified by their lecturers on the date of the visit by me and what was required of them to do and to bring along on the day of the interview. The lecturers were requested to tell their students to come with their lesson notes and should be ready for an interview after the lesson was over. Again the lecturers were asked to tell their students to accommodate me in their classrooms.

Three types of data were collected: 1) lesson plans- each teacher was expected to present a lesson plan after the lesson had been delivered. 2) observations- all lessons were observed by me, field notes were taken during the observations and were supplemented by audio-taped records, and 3) post-lesson interviews- the interview questions were for the most part, related to specific events of the lesson. These interviews were different from the one done in 3.1.3.3 since it only dealt with what happened in the classroom that day. The lessons were audio-taped for two purposes: to secure a record of the lesson for later analysis and to be used during the post lesson interviews as an aid to stimulating the teacher's recall of the lesson's events. Video-taping would have been more appropriate but since most rural schools do not have electricity it became impossible to use that instrument.

3.1.4. **PROCEDURE**

The main study followed after the instruments had been modified where necessary from the pilot (See 3.2). Data collection for this test was conducted from May 1998 to

August 1999. I administered the test instrument with the assistance of the College lecturers concerned at various colleges.

All the test items for the pilot were analysed quantitatively to determine the frequency of correct, incorrect, misconceptions, misinterpreted and unattempted answers (See 3.2.2). This helped in the selection of the subjects for interviewing. Data for the interview were collected a week or two weeks after the test. This gave me enough time to mark the test and make a possible selection of subjects for the interview. The short time of one or two week's gap made it possible for the subjects to recall their answers to the questions and the reasoning behind them. The subjects were told in advance about the test. They wrote the test during one of their normal double periods for mathematics and in some cases after the normal lecturing time. An interview and observation were done with twelve students who showed unusual misconceptions and errors in the test.

3.1.5 SCORING

The test scripts were analysed for learners' errors and misconceptions. The item was marked right or wrong, but comments were made along side to indicate whether the student showed a misconception, a misinterpretation or a peculiar way of answering the question. The question was scored correct if the student gave the correct answer and followed the correct procedure. In the case where the answer was correct but the procedure wrong, I had to look for misconception or misinterpretation, which might have cropped up. Where the student got the question wrong I looked at the process of getting the answer to see whether there was a misconception or misinterpretation.

3.2 THE PILOT STUDY.

3.2.1 DESIGN OF THE STUDY.

The pilot study provided an opportunity to assess the appropriateness and practicality of the data collection instruments. The pilot used algebra test instrument, the unstructured interview, and classroom observations to ascertain the instruments for the main study. The pilot colleges were different from the colleges in the main study.

3.2.1.1 DATA COLLECTION

Data were collected in two Colleges, one from an urban area and one from a rural area. The urban College was a STD College and the rural College a PTD College. Twelve students from the PTD College were used and nineteen from the STD College. The STD College was from an advantaged community and the PTD from a previously disadvantaged community. The test items consisted of fifteen questions, which were

composed of variables, algebraic word problems, functions, equations and inequalities, and absolute values (See appendix A). Fifteen test items were initially used to cater for both the STD and the PTD College students to finish within the intended time of one hour. The PTD students according to the entry requirement should have a lower knowledge and understanding of algebra than their counterparts in the STD stream. Hence, I felt that fifteen questions would be enough, but from the responses I found out that the two groups finished almost at the same time and earlier than expected. Also, some of the questions were too elementary for even the PTD College students. Most of the questions had actually been selected from the CDMTA test, which comprised of levels 1 and 2 questions, which were originally meant for beginners of algebra in the elementary schools. The items were then increased and modified to twenty, making use of similar items from the Kaur and Sharon test as those items were meant for post matric
students like the subjects for this study. The test items were administered to participants in their classrooms in May 1998. They were asked to provide some biographical information.

The pilot study demonstrated the adequacy of the research procedures and the instruments that had been selected for the variables. This pilot study allowed students to identify and comment on items that were ambiguous or contained language problems. These "problem" items were either modified or excluded from the final instrument. For example,

1. The question, fill in the gaps: x → x+2
   6 →
   r →
   4x →

2. Which is the larger, 2n or n+2?
   Explain

3. Which of the following relations are functions?

   (a) {(-10;10), (-5;10), (0;10), (5;0), 10;0)}
   (b) {(-9;8), (-8;9), (-7;6), (-6;7), (7;-6), (-7;-6)}
   (c) {(4x, 2x-3) | x an integer}
   (d) {(-4;10), (-3;10), (-2; 10), (-1;10), (0;10)}
   (e) {(|x|, x)| x an integer

The above questions were excluded not because they posed any problems but because they were well answered, hence were not going to help in the study. The results of the test were categorised into five categories: correct answer, incorrect answer, misconception, misinterpretation and unattempted (See 3.2.2).

From this pilot study I assessed the feasibility of the study and was able to see some of the problems students encountered in answering some of the questions, which were amended in the final study. During the pilot it was found out that some of the questions were too elementary and therefore necessitated more challenging questions from Kaur and Sharon (1994).

3.2.1.2. **PILOT INTERVIEW**

Interviews were conducted on two students each from both the two Colleges to clarify some of the answers, which were given by the students, especially when there were some elements of misconceptions portrayed by them. Participants were probed during the interview. Probing during the interview was designed to give a more accurate and detailed picture of the participants' subject matter knowledge and pedagogical content knowledge. The probing focused on asking participants to explain what they did in the test and why, asking questions related to the test but requiring more general, longer, or more thoughtful responses.

The interview consisted of two parts. In the first part subjects were presented with items designed by me, the Biographical Questionnaire (See appendix E), these required personal responses, which only the subject could answer. In the second part of the interview the participants were asked to reflect on their thinking and to explain and clarify their answers to the test. Each participant was interviewed individually and the same procedures were followed for all subjects.

From the analysis of the interviews additional statements were added (See appendix B). For example a question like: " Did you do HG or S.G at matric?". This became evident in one of the interviews due to the differences in curriculum for S.G and H.G. There are some topics, which only the H.G students are required to do. Qu.15, a quadratic inequation is one of such questions. It is an example of a topic meant for H.G. students only. The PTD Students due to the nature of their syllabus and the grades they are supposed to teach after graduation do not necessary have to know or learn some of the topics meant for the STD's and these schools. One therefore has to pardon a PTD student who never did H.G mathematics. As has been discussed earlier on (See 3.1.2.5), the PTD syllabus does not go beyond the standard 8 mathematics syllabus. This confirms the response from some of the participants.

3.2.1.2.1 **Pilot Interview Format**

As can be seen in the Biographical Questionnaire (See appendix E) it was necessary to have the name and the name of the College to be able to identify students for observation purpose. The highest standard passed in mathematics was there to find out whether the participant had really passed matric mathematics and therefore supposed to have covered the syllabus required of a matric student. The names of standard 5, 6 and 10 mathematics teachers were required to find out if they had many different teachers in preparing for matric. This information was required to find out whether, teachers had influence on the attitude of the subjects to mathematics. The approach to teaching and the attitude of a teacher can have an effect on a student's like or dislike of mathematics.

3.2.1.3 **CLASSROOM OBSERVATIONS**

The classroom observations were intended to elicit understanding and pedagogical content knowledge of algebra, as the student teachers used their acquired knowledge in algebra to bring about conceptual understanding on the part of the learners in their classes. According to Day (1996) a teacher in the algebra classroom requires two crucial components in the learning environment. The behaviour with learners and expectations of them are required. The teacher is expected to pose problems that will provoke learners to use their previously acquired conceptual knowledge and understanding in algebra in their learning. The teacher is expected to prompt learners to refine their strategies and consider alternatives when tackling algebraic problems. For example, the misconception towards the expression $(x + 3)^2 = x^2 + 9$. He asks the question: does the teacher encourage learners to explore this conjecture using different values of x to discuss and argue the validity of the conjecture? Again, does the teacher encourage learners to explore the expression $(x + 3)^2 = x^2 + 6x + 9$ in its correct form to satisfy themselves? In this way he will make learners able to distinguish between the wrong and the right as far as this problem is concerned.

Day (1996) asserts again that there is expectation from those who study algebra, that is the more one learns about algebra, the more their cognitive structures are expected to developed. The parameters of understanding are supposed to widen, and the level of sophistication of conceptual understanding is also expected to deepened, and become more integrated. For example, one should see the connection between (a-b)(a+b) and (r-s+t)(r+s-1). The classroom observation was to look for the appearance of new algebraic concepts not previously used by learners. Observing how algebraic concepts are defined and noting how different concepts are related to each other. Classroom observations were done on these four students from these two colleges. These helped me to find out how they imparted their algebraic knowledge to learners at schools. The observations were only possible during their Block Teaching Practice period in August 1998 (See 3.2.1.1). The observation schedule was adapted and modified from that used by Maluti College of Education, Matatiele, Eastern Cape Province of South Africa (See appendix F).

3.2.1.4 **FIELD NOTES**.

Field notes according to Patton (1980:164) "*contain the ongoing data that are being* *collected. They consist of descriptions of what is being experienced and observed, quotations from the people observed, the observer's feeling and reactions to what is observed and field-generated insights and interpretations"*. During the course of my observation I observed and experienced the following:

1. Different types of representation
2. Time allotted to learner to respond to a given question
3. The response of teacher when a learner gives a wrong answer to a question.
4. Knowledge of algebra problem areas.
5. Knowledge of solution strategies to algebra problem.
6. How content of algebra was transmitted
7. Where and when teaching and learning aids were used.

3.2.2. **ANALYSIS AND RESULTS OF DATA FROM PILOT STUDY**

3.2.2.1 **Algebra Test Analysis**

Item Number	Category1 Correct answer	Category2 Incorrect answer	Category3 Misconception	Category4 Misinterpretation	Category5 Unattempted	Total % Correct items
1	19	0	0	0	0	100
2	19	0	0	0	0	100
3	17	2	0	0	0	89
4	1	2	0	15	1	5
5	3	13	0	0	3	16
6	19	0	0	0	0	100
7	0	4	0	15	0	0
8	11	3	0	5	0	57
9	6	1	4	7	1	31
10	5	4	2	4	4	26
11	7	1	10	0	1	37
12	18	1	0	0	0	95
13	6	1	12	0	0	31
14	0	17	0	0	2	0
15	1	4	13	0	1	5
Total No	132	53	41	46	13	
% Total	46	19	14	16	5	46

Table 3.3 Results of STD College E.

The analysis consists of:

1. The identification of the students' strength in algebra.
2. The identification of misconceptions in algebra.
3. The identification of strengths in presentation of the subject matter (algebra).
4. The identification of conceptual knowledge, which could facilitate teaching of certain topics in algebra.

Item Number	Category1 correct answer	Category2 incorrect answer	Category3 misconception	Category4 misinterpreted	Category5 unattempted	Total % Corr. Items
1	12	0	0	0	0	100
2	11	1	0	0	0	94
3	12	0	0	0	0	100
4	2	0	10	0	0	17
5	1	6	0	0	5	8
6	6	4	0	0	0	50
7	2	7	0	0	3	17
8	4	7	0	0	1	33
9	3	5	1	0	3	25
10	0	4	1	4	3	0
11	4	3	5	0	0	33
12	8	2	4	0	0	67
13	0	6	0	0	6	50
14	2	8	0	0	2	17
15	0	4	7	0	1	0
Total No.	67	57	28	4	24	
% Total	43	32	10	2	13	41

Table 3.4 Results for PTD College M.

Data for two colleges were analyses as indicated in Tables 3.3 and 3.4.
Algebra Test for the STD College E
Number of students:- 19
Number of male students:- 7
Number of female students:- 12

Algebra Test for PTD College M
Number of students:- 12
Number of males:- 0
Number of females:- 12
The results of the participants were grouped according to gender. This was a way to make sure that both male and female participants were selected for interview and

observation purposes.

The participants’ knowledge and understanding of algebra were analysed by categorizing the answers into:

category1 (correct answer),
category2 (incorrect answer),
category3 (misconception),
category4 (misinterpretation),
category5 (unattempted). The frequency of the answers is given in Table 3.3 and Table 3.4

I will now discuss the analyses of some items to illustrate how the answers were categorised. Item1 the answer is 37 so if one gives an answer of 31, there is some element of misconception shown as the learner instead of adding rather subtracts. Such a case is placed under category 3. Otherwise I had to go through the scribbles on the paper to find out the intention of the students and then categorise the question according to the set categories. Item 2, the answer is 8 + g, again if one gives an answer of 12 or 15 , the response is placed under category 3. This student does reveal one of the misconceptions reported as “incomplete”. Olivier (1992) in this very question reports that pupils’ arithmetic schema is retrieved and hence resorted to giving numerical answers. He reports that 58% of standard six pupils supplied numerical answers to the question. He said the most common responses were “12(4 + 4 + 4), 15(from 3 + 5 + 7, introducing a relationship between algebraic order and number order) and 15(from 8 + g, and g is the 7^{th} letter of the alphabet.). In item 4 the answer is $c < 5$, $c \in R$ if one gives an answer of c = 10 - d, the response is placed under category 4, misinterpretation. It is assumed that the student did not interpret the question well in order to see that not one value was required but rather more values. If one gives an answer of c = 1,2,3,4 she or he is classified under category 3 (misconception) because this person does not include all real numbers less than 5 as answers. The question never asked for natural numbers only therefore 0 should be included as one of the answers. In item 9 the answer is, (Sum of ages = 2y + 5), if one continues to solve for y, then classified under category 4, misinterpretation because this student had interpreted to mean solve for. In item10 failure by the student to write the correct symbolic form of the question implies incorrect interpretation, therefore classified under category 4. For “more than” if a student had written 3x, it is classified as a misconception. Continuing to solve for x also implied misinterpretation, that is category 4.

In item11 the answer is total cost = 50a + 40p, if one gives an answer of 50 cents + 40 cents, then the response is placed under category 3. This is evidence of a cognitive obstacle. This has been termed by Clement (1982) as a translation problem. Clement in his investigation of 150 freshmen engineering students at a major state University had problems with similar question. The students were asked to write an equation using the variable s and p to represent the following statement, "There are six times as many students as professors at this University'. Use s for the number of students and p for the number of professors. She recorded 63% correct responses. 68% of the errors she attributes to reversals, 6s = p instead of 6p = s. In item 12 similar categorisation like item 11 is applied. In item 13, if the student defined function as a relation, then it is classified as a misconception (category 3), otherwise the student got the answer correct, incorrect or unattempted according to the definition given. In item14, the answer could be given either in words or in symbolic form to be classified correct. Any other answer is analysed and placed under the correct category. Item 15 the answer is categorised under misconception if the student cross-multiplied the inequality.

The percentage average scores for the STD and PTD were 46% and 41% respectively. This according to the Algebra Test instrument shows that there was not much between the PTD and STD students results. The results are however, not satisfactory which supports the aim of this study to find out more about the knowledge and understanding of some of this algebraic concepts. There were minor changes made to the pilot test as the results above show few problems with the questions. For an example, item 13 and item 15 had to be replaced since many of the students failed to answer correctly as is shown by the table above. Item 13 was replaced with the definition of a function. Instead of identifying functions from the previous item, I wanted to find out whether the participants knew the definition of a function. According to Sfard and Linchevski (1994) before one learns a certain concept, s/he must know all the supportive concepts. The function machine is seen as a supportive mechanism in the learning of algebra for beginners as found in many textbooks in South Africa. This is the more reason why I found it necessary to include functions in the study. According to Sfard and Linchevski (1994:201), "*the idea of function constitutes a conceptual bond between numerical calculations and formal algebraic manipulations*". They went on to say that functions act as a link through which new algebraic knowledge is tied to the system of arithmetical concepts. Concepts according to Laridon (1992:397) takes time to mature. He argues that "*some misconceptions in concept formation are due to syllabuses not going far enough to allow a learner to form a complete, meaningful equilibration of the concepts involved*" (Laridon, 1992:400), which he labels, "*lack of closure*".

According to Even (1993), the concept of a function has changed over the years, because new knowledge in mathematics has called for those changes. The participants are required therefore, to give a modern definition of function. If the participant gives a modern definition of function (Univalence), the participant according to Even (1993), could make knowledgeable decisions about the place of the function concept in the curriculum and the emphasises they should put on the univalence definition of function.

If however, the participants appeared to hold "*a linear prototypic image of functions*" (Even, 1993:96), i.e., they expect functions to be reasonable, representable etc., and so forget about other types of functions like the constant functions, inverse and composite functions, they may not be able to present functions to their learners acceptably.

Item 15 was replaced by a question from Kaur and Sharon's test on inequalities. According to them misconceptions surrounded this item in their study in Singapore. I intended to verify this misconception around quadratic inequalities among my subjects here in South Africa. Item 5 was retained but the pilot result showed that many of the participants failed to answer the question correctly. This question is one of the questions taken from CDMTA, and therefore reasons should be found from participants as to why they failed to answer this question. A further five questions from Kaur and Sharon had to be added to cater for the academic level of the students since many of the previous questions from CDMTA were originally meant for primary school learners. Again CDMTA mainly tests the conceptual understanding of letter, symbols and variables in algebra. Item 16 catered for the ability to interpret the precise meaning of statements, and judge whether specific criteria are adequate or have been met. The results showed that the participants in this pilot study did not do well in the area of items involving functions.

3.2.2.2 **Interview Analysis**

This involved the identification of misconceptions and misinterpretations on the part of the participants. The following excerpts from two participants illustrate how misconceptions and misinterpretations were identified. In the extracts quoted below, **I** stands for interviewer (myself) and **S** stands for the College student. S1, 2, 3 etc, refer to student 1, 2 and so on. Only two episodes incorporating misconceptions and misinterpretations are dealt with here.

1. I. In your answer to item 4, you gave an answer of c =10- d.
2.S1. Yes
3. I. Did you understand the question well?
4. S1. Yes
5. I. What did the question say?
6. S1. To find the values of c
7. I. Values of c, does this require one value or more than one value?
8. S1. Require more values
9. I. Does c = 10-d give more than one answer?
10. S1. Oh!, I am wrong

The above excerpt shows misinterpretation as this student could now interpret the question which required c to have more than one value. In item 9 there was some element of misconception. The excerpt below shows this.

11. I. In item 9 you gave an answer of 6y.
12. S3. Yes
13. I. What do you understand by 5 more than y?
14. S3. 5y
15. I. Why 5y?
16. Because 5 more than y is 5 times.
17. I. Let us use numbers to verify the answer. Suppose Sipho is 18 years old. If Mpho is 5 years older than Sipho, what will be Mpho's age?
18. S3. 18 + 5 = 23
19. I. Then what will be their total ages?
20. S3. Total ages = 23 + 18 = 41
21. I. Let us verify your answer by putting 18 in place of y in 6y.
22. S3. 6y = 6 x 18 = 108
23. I. Do you see your mistake?
24. S3. Yes
25. I. What is Mpho's age in terms of y?
26. S3. It will be y + 5
27. Can you write the total ages of the two?
28. S3. Yes
29. Write it for me
30. S3. She writes on paper y + y + 5 = 2y + 5

I added an interview question to the pilot interview questions to cater for students who did either H.G or S.G at the matric level. This was as a result of students failing to answer some questions because they did S.G at matric level. This in essence reveals that the content syllabus at the College level is not sufficiently comprehensive.

3.2.2.3 **Classroom Observation.**

It was difficult to have access to a student who was interviewed, when teaching a topic she or he showed misconception or misinterpretation. This is because the students had to follow the scheme the particular school had given them. Below is an observation of a lesson given by one of the participants who was interviewed.

The student teacher used the traditional approach to the teaching and learning of algebra (Teacher-centred approach). The emphasis was on procedures and algorithms. The lesson lasted 40 minutes. Student teacher S2's lesson was about solution to equations involving difference of two squares. The lesson started with the teacher explaining the difference between an expression and an equation. From the observation it became clear that the teacher had shown the learners two types of expressions, binomial and trinomial with exponents. Again the question of the meaning of equation, the teacher led learners to know that an equation is an expression equated to zero. In the same lesson it was interesting to notice how the teacher asked learners to find out the difference between square and square-root by making them use their calculators. It was interesting to see how the teacher reacted to a wrong answer given by a learner. The teacher did not say the answer was wrong or right, but rather decided to lead the learner to the correct answer, by making the learners realise the mistake made earlier on.

It was amazing to hear in the same lesson when a learner was asked to give an answer to the question X divided by X, that the answer "X" was given. This resulted in the teacher asking learners to give answers to, X x X, which they said was X^2, X + X which they answered 2X, and X - X, which they said zero. Upon reflection, the teacher expressed disappointment in the way that learner has answered the question of X divided by X. He was however pleased with the way he was able to make the learner know that the answer to that question was one.

3.3 COLLECTION OF DATA FOR FINAL STUDY

Data collection for the final study was conducted from May 1998 to August1999. The administration of the test was given to three colleges during the students' double mathematics lesson period, and at their normal mathematics-teaching period. The other three colleges were given the test during a special period arranged by the subject lecturer.

The test items were given to every final year mathematics student present that day. It took approximately 40 minutes to complete the test. I had to explain some of the questions students came up with in answering the items of the test. For example one student asked why they have to write their names on the answer sheet, meanwhile, instructions made clear on the question papers.

The interviews took place after normal teaching time. Students were entertained to a small snack to compensate for their wait. These students were notified to make themselves available for lesson observation when they went out for practice teaching as it was not possible to observe them when they would secure permanent appointments, which they were not sure of as explained earlier.

3.3.1. SAMPLE FOR TEST

The modified test, which consisted of 20 items in algebra, was given to the final year student participants from four Senior Teachers Diploma (STD) Colleges of Education and two Primary Teachers Diploma (PTD) Colleges of Education. These Colleges were selected because they were in and around the Eastern Cape Province and were not part of the pilot study. The four STD Colleges were among the few which offer STD courses. The PTD Colleges were selected because they were close to where I was operating and the subject lecturers from those Colleges were interested in the research. These lecturers were prepared to render their services where and when needed. These tests were administered in May, 1999 by me. In two Colleges, I was assisted by the mathematics lecturers from those colleges. These were, a College of Education in Durban and a College of Education in former Ceskei in the Eastern Cape. In all other instances the mathematics lecturers were present to introduce me and also talked to the participants about the importance of the research. There were a total of 212 participants as indicated earlier.

3.3.2. SAMPLE FOR INTERVIEWS

Five students who showed some misconceptions and also showed strange or peculiar ways of answering some of these questions were identified from each of the participating Colleges. These names were submitted to the subject lecturer to get them ready for interview a week after the test. Three identified College students from the list submitted to the lecturers concerned were used for analysis purposes. The list of five names was to cater for any absenteeism on the part of the College participants. Three students from each of the Colleges were to be involved at this stage, which should have given 18 participants, but two Colleges had completed their block teaching, which made it impossible to observe their students. Twelve participants were left for the test-interview.

3.3.3. SAMPLE FOR OBSERVATION

Nine students out of twelve interviewed were observed in the classrooms in August, 1999 during their teaching practice time (3 of them had refused to be observed). The subject lecturers were contacted by telephone to indicate to me the day any of the nine students was going to teach a topic in algebra. The subject lecturer and myself went to observe the student teacher teaching. The subject lecturer was also asked to fill in the observation form, which had been designed by me (See appendix F). I had to wait after the lesson to discuss the lesson plan, which was used by the student teacher.

3.4 VALIDITY

According to Anastasi (1968: 134) the validity of a test concerns what the test measures and how well it does so. Content validity involves the systematic examination of the test content to determine whether it covers a representative sample of the behaviour domain to be measured. Anastasi (1968:139) notes that, face validity pertains to whether the test " looks valid" to the examinees who take it, the administrative personnel who decide on its use, and other technically untrained observers. Face validity is whether the question asked looks as if they are measuring what they claim to measure. To determine the content validity of the modified CDMTA test, which is acknowledged in 3.1.1 to explore misconceptions and depth of understanding of algebra, the test was given to the three subject lecturers of the selected colleges and the subject adviser from the department of collegiate education, University of Transkei. The position adopted in the research is that validity is not about whether methods or data are valid or not. Validity is about the extent to which the claims to knowledge made on the basis of data collected through different methods are “trustworthy”. According to Cohen and Manion (1994) convergent validity is normally use for interviews and classroom observation. In this study various data were used to enhance validity by making use of triangulation method. Triangulation of evidence provide for validity and reliability. This method allowed the use of algebra test, interviews and observation to supplement each other to provide evidence for judgement.

CHAPTER 4

ANALYSIS OF DATA AND PRELIMINARY DISCUSSIONS.

4.1 INTRODUCTION

In this chapter the findings of the three phases outlined in chapter 3 are reported and discussed. The process of analysis entails three activities: data reduction, data display, and conclusion drawing/verification. Analysis of the data contributes to an understanding of some aspects of algebra and the reasoning involved in the construction of such understanding, particularly with respect to algebra. The students' solutions and explanations of their answers helped to identify reasoning behind them. The analysis that follows is an attempt to develop useful ways to identify important components of knowledge and understanding of algebra and the resulting contribution to teaching and learning of algebra.

The chapter also deals with the findings of this study in terms of the data collected from the final year College students. Students' written tests were analysed script by script, question-by-question and College-by-College. Frequencies of students answering questions according to the five categories were calculated. Calculating percentages allows for comparisons within and across student teachers and Colleges of Education. Interviews and observations were analysed to supplement the information about misconception and misunderstandings of algebra and also to understand how College of Education students help develop some concepts in algebra in the classroom.

4.1.1 Identification of misconceptions in algebra.

Misconceptions held by participants involved in the study were identified from the written test, interviews and observations. The responses to the test items were both quantitatively and qualitatively analysed. The modified test instruments were analysed by categorising the data into five groups as shown below.

4.1.2 Test

The participants' knowledge and understanding of algebra was analysed by categorising the answers into:

1. category1 - correct answer
2. category2 - incorrect answer
3. category3 - misconceptions
4. category4 - misinterpretation
5. category5 - unattempted.

This entailed the counting of frequencies of each of the responses according to the categorisations. Attention was however given to the final answer, the method used to get to that answer, mistakes made, and different ideas shown.

4.1.3. Interviews

The algebra test instrument provided important yet limited information. In order to understand better how the participants answered the questions, a sample of three participants from each of the colleges were interviewed. During the interview the participants were probed to clarify some of the answers they had given. Some of the probes were standard- "why?" "what do you mean by that"?, and " can you give me an example?". The interviews were audiotaped and later transcribed. The tape-recorder plays an important and crucial part of the interview as information is captured unambiguously and faithfully. The tape-recorder was essential as it recorded information on the spot during the period of the interview. Human beings are fallible, and inevitable notes made after the event are likely to have distortions particularly where there were intervening events. It is therefore necessary to have such an instrument, which can guarantee the authenticity of the human recall. The transcripts were analysed with reference to student's conceptions, misconceptions and errors to some of the questions. The analysis began by listening to the taped interviews and editing the transcripts. The interview transcripts were analysed by person, and by item.

4.1.4. Lesson observations/lesson plans

The lessons of those interviewed were observed in the actual classroom situation, some were video-taped. These observations provided additional information on the pedagogical content knowledge of the participants, their beliefs, dispositions and difficulties. The observation naturally provided additional support for conclusions arrived at later. The different forms of presentation of certain topics in algebra that the college student teachers made available to their learners to see different ways and to arrive at the solution to a problem are also highlighted. It is in the context of presentation that the teacher introduces new concepts, reviews learned materials, and assists learners to understand the subject matter. It is also in this context that the teacher relies mostly on his/her subject matter knowledge. I was therefore on the lookout for such statements like, " Let p = apples", rather than the more correct way of saying let p = number of apples. The college students' pedagogical content knowledge would be observed by way of anticipated ways of creating and maintaining an open and informal classroom atmosphere to ensure the learners' freedom to ask questions and express their ideas. Probing for potential misconceptions in the learners by using carefully chosen examples and non-examples and planned suitable teaching strategies to encourage learners to guess and conjecture and to allow them to reason things on their own rather than the student teacher showing them how to arrive at the solutions or the answers are another focus of my observation. From the college students' lesson plans I asked them to explain and describe their thoughts as they developed the lesson for the class. They were asked the following questions:

a). Please explain the context in which your plans were made, e.g., the type of class, the type of student.

b). What were your areas of concern as you constructed the lesson?

c). What were your main goals for the lesson?

d). What plans or procedures did you use to achieve those goals?

4.2 THIRD YEAR COLLEGE SEK STUDENTS

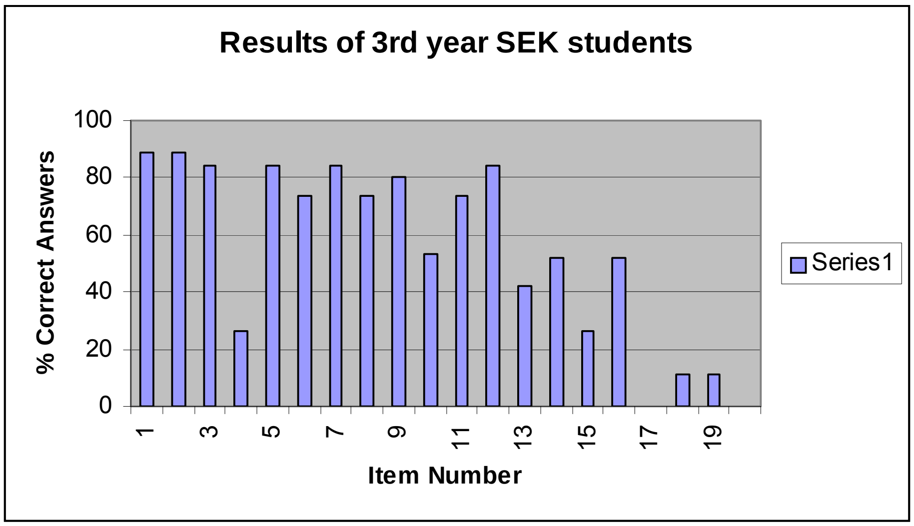
4.2.1 Category frequencies

Table 4.1 shows the results of 3rd year College students. Student No. = 19

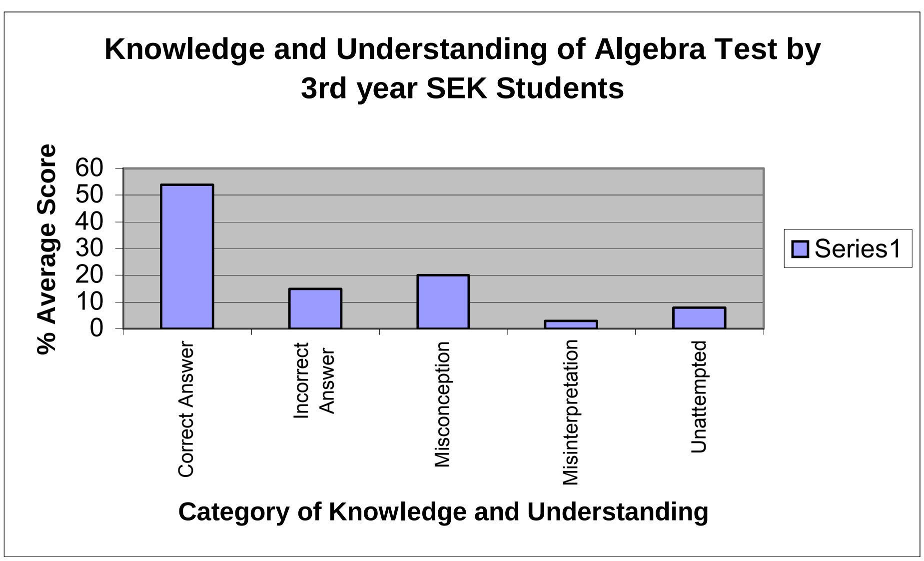
Item number	Correct Answer	Incorrect Answer	Miscon ception	Misinter pretation	Unattem pted	% Correct Answers
1	17	2	0	0	0	89
2	17	2	0	0	0	89
3	16	2	0	0	1	84
4	5	4(21%)	10(52%)	0	0	26
5	16	2	0	0	1	84
6	14	4	0	0	1	74
7	16	2	1(5%)	0	0	84
8	14	2	2(11%)	0	1	74
9	15	0	2(11%)	2(11%)	0	80
10	10	0	2(11%)	7(37%)	0	53
11	14	4	0	0	1	74
12	16	0	0	0	3	84
13	8	3(16%)	0	0	8	42
14	10(53%)	6(32%)	0	0	3	52
15	5	5	5(26%)	1(5%)	3	26
16	10	4	5(26%)	0	0	52
17	0	0	16(84%)	0	3	0
18	2	1	16(84%)	0	0	11
19	2	8(42%)	8(42%)	0	1	11
20	0	7	9(47%)	0	3	0
Total	207	57	76	11	29	

Score						
Total (%) Percentage	54	15	20	3	8	54

Table 4.1 Results of 3rd year SEK students.



Graph 4.1 Algebra test results of 3rd year SEK students



Graph 4.2 Knowledge and understanding of algebra by 3rd year SEK students.

The summary of Table 4.1 shows that in the overall analysis 54% of the total possible responses to items (20x19) were correct. This analysis also shows that 20% of the answers were classified under misconception, 3% under misinterpretation and 8% were unattempted (See Graph 4.2). Table 4.1 and graph 4.1 show that for items 1 and 2, 89% got each of them correct, which shows good understanding of the two questions. In item 1, two of the responses were incorrect but could not be classified because of the nature of the answer. In item 2, two of the responses were also placed under category 2 because they could not be classified under misconception or misinterpretation, however, one of the incorrect responses was ***e + g + f* = 12**. This student was not among the participants who were interviewed, however one can deduce from the answer that he assumed ***e*** to be equal to **g** and therefore divided 8 by 2 arriving at 4 as a value for **e** and **g**. Similarly taking **g** as equal to **e** and **f** gave him the total answer of 12 (4 + 4 + 4). There was no response, which was categorised under misconception or misinterpretation for item 3, but two (11%) of the responses were placed under category 2 and one (5%) under unattempted which means the item was left unanswered. Item 4 was poorly done, as only 26% of the responses were correct, 10(53%) of the responses were placed under misconception and 4(21%) responses under incorrect answer. Item 5 shows 84% categorised under correct answer. These students seemingly replaced x in x = 6 by 5x getting 5x = 6, hence arriving at an answer of 6/5. Two students tried to expand and got it wrong in the process of expanding and one student did not attempt it. (See excerpt I3SEK1, I- Algebra Test Interview, 3- Third year student, SEK- Name of College and 1- Student number)

I. I saw from your script that you were trying to expand $(5x + 1)^3$
S. Yes, but I could not expand
I. What was the problem?
S. The expansion became too tedious and I got frustrated
I. Did you think there was an easier way to the problem?
S. I suspected so but could not reason it out.
I. Do you know how to solve it now?
S. Yes my friends showed me and it was a matter of replacing x by 5x in x = 6.

(I- Interviewer and S-Student)

Item 6 was satisfactorily answered with 4 responses, which could not be classified under misconception or misinterpretation, one student left it unanswered. Item 7 was well answered as the percentage responses showed 84%. Two (11%) of the responses were placed under category 2 and 5% under misconception. Item 8 had 14(74%) of the responses correct, 2(11%) incorrect, 11% under misconception and 5% unattempted. Item 9 had 80% correct answers but showed that 11% of the responses were misinterpreted and 11% were having misconception. Item 10 was fairly done, 11% of the responses were placed under misconception, 7(37%) responses misinterpreted and 10(53%) correct answers. Items 11 and 12 were well answered with 74% and 84% correct answers respectively. While there were incorrect answers none of the students showed any misconception or misinterpretation of these questions. In item 11, 21% of the students gave incorrect answers and one student failed to attempt it. Item 12 on the contrary 3(16%) students did not attempt it while 16(84%) of them got the answer correct. Item 13 was on the other hand, not well answered with 42% correct, 42% unattempted which shows many of these students did not know the definition of a function. The variety of incorrect responses to this item indicates the low level of conceptual understanding that final year College of Education students have of functions as a topic. What was gathered from the answer is that functions are not treated as a concept in its own right but as a function “machine”. For item 13, majority of the students did not attempt it. 42% of the responses were placed under unattempted, 42% correct answers and 16% incorrect answers.

Item 14, ten (53%) of the responses were correct, as many as 6(32%) were incorrect and 16% did not attempt it. This supports the idea that many of the college students do not understand functions. Item 15 was poorly done only five (26%) of the students got the answer correct. Five (26%) of the responses could not be classified as misconception or misinterpretation, 5(26%) of the responses were however placed under misconception, 5% under misinterpretation and three (16%) unattempted. 10(53%) of the students for item 16 got the answer correct, 4(21%) incorrect answers and as many as 5(26%) misconceptions. Item 17 was very poorly answered none of the students got it right. As many as 16(84%) of the responses were placed under misconception and the rest failed to attempt it. For item 18 as much as it was poorly answered 2(11%) of the responses were correct, one (5%) incorrect and 16(84%) of them placed under misconception. Item 19 also had one 2(11%) responses correct but 8(84%) of them were placed under incorrect answers, 8(42%) again under misconception and one (5%) failed to attempt it. Finally, in item 20 there was no correct answer given by the students, 7(37%) of the responses were incorrect, while 9(47%) of the responses had misconceptions. Three (16%) of the students did not answer this item.

4.2.2 **Misconceptions**

For item 4 students seemed to have a misconception. Students who gave responses of the type c = 1,2,3,4 did not realise that $c < 5$, $c \in R$ is the solution, hence misconception.

52 % gave such responses and hence had this misconception. The solution of the item was not restricted to only natural numbers, therefore all real numbers less than 5 were supposed to be answers.

For item seven, 84% got the question correct but 5% of the students seemed to have a misconception, where the students wrote CB = $3x$ cm for the problem which states that AC = x cm and CB is 3 cm more than AC (See appendix A). These students arrived at an answer of $4x$ instead of $2x + 3$. These students seemed to have a problem with "3 more than x", which should be $x + 3$, but wrote $3x$, hence arrived at ($x + 3x$) cm giving a total of $4x$ cm. A similar situation occurred in item 9 with the same students. In this item, it was stated that Mpho was 5 years older than Sipho. Instead of the total ages being $y + (y + 5) = 2y + 5$ these students wrote $y + 5y = 6y$. An interview with one of these students revealed that she had a problem with the meaning of $3x$ and $x + 3$. She confused " 3 times bigger than" with "three more than". The following transcript (I3SEK2) excerpt for this particular student illustrates how this misconception was identified. (In this extract quoted below I– Algebra Test Interview, 3- Third year student, SEK -Name of the college and 2- Student number 2).

I. In item 9 you gave an answer of $6y$.
S. Yes
I. What do you understand by 5 more than y?
S. $5y$
I. If Sipho is 20 years old and Mpho is 5 years older than Sipho. What will be Mpho's age?
S. 20 + 5 = 25
I. What will be their total ages?
S. 45 years.
I. Why is Mpho's age not 5 x 20 = 100?
S. It cannot be, because Mpho is only five years older than Sipho
I. What do you understand by $6y$ then?
S. 6 multiply by y.
I. Then it means Mpho and Sipho's ages together should be 6 x 20 years if Sipho is 20 years old and Mpho is 5 years older.
S. No
I. Can you substitute 20 for y in $6y$ and tell me the answer according to the question on the test paper?
S. Yes, 6 x 20 = 120 that is not the same as 45.
I. Can you now see your problem?
S. Yes, I see.
I. If Sipho is y years old, what will Mpho's age be, if Mpho is 5 years older than Sipho?
S. (Scratched the head) I think $y + 5$ years.
I. Yes, $y + 5$, then what should their total ages be?
S. He writes $y + y + 5 = 2y + 5$

I. Good that is what was required from you.

Mathematics terminology can be an impediment to second language users. There are words, whose meanings depend on the text in which it is used. For example “by” can be used to mean multiply or divide, multiply **a** by **b** gives an answer of **ab**. On the other hand if you divide $x^2 + 2x$ by x you arrive at an answer of x + 2. During the pilot study a similar case was detected, where students wrote 5y instead of 5 + y when it was said Mpho was 5 years older than Sipho whose age was given as y (See 3.2.2.2). Meaning conveyed by some prepositions and connectives must be a concern for COE students as they prepare to teach algebra as is shown here. This concern is likely to be addressed if the COE student’s knowledge and understanding is built on a sound note.

In item 8 as I expected, most the students used the formula Perimeter = 2*(length+ breadth) instead of the straight application of the definition of a perimeter (i.e. adding all the sides). One student however, might have some misconception about the use of the formula of a perimeter of a rectangle, she wrote perimeter = 2*(length x breadth) arriving at 28h as the answer. Here this student is combining the perimeter formula with that of area for a rectangle. One other reason might be, according to Macgregor and Stacey (1997:10) that older students had more opportunities of making mistakes than younger students because of the interference from new schemas only partly learned or because of their expectations of being able to use more advanced knowledge, this means the area formula, which is normally learnt after the perimeter of a rectangle might have interfered in this instance. The answer 28h might also be associated with conjoining (Macgregor and Stacey, 1997) of algebra terms, where 2(14 + h) is not seen to be completed. This item was however, well answered with 74% correct answers. This misconception of combining the perimeter formula for the rectangle with the area formula may be as a result of teachers giving the formula for the perimeter of a rectangle instead of making the learners add all the sides of the rectangle to get the perimeter.

In item 10, 53% of the participants got the answer correct. One student however, wrote 3(x + 4) instead of 3x + 4 = 31 which was categorised as misconception. In the expansion of the bracket, this student failed to multiply the two terms inside. A similar mistake was found by the same student, when he failed to expand p = 2(h + 9) to get p = 2h +18 but rather arrived at an answer of p = 2h + 9. Poor conceptual understanding also did not warn this student when he got a negative number when he tried to solve for h (solving 2h + 18 = 0 to get h = -9). Getting breadth to be a negative number does not make sense at all but lack of conceptual understanding did not act on this student to know that the answer was wrong. This student’s lack of the connections, which characterise conceptual knowledge, makes his knowledge of algebra inaccessible to real life situations. Items 15, 17, and 19 showed very low percentages of 26%, 0%

and 11% correct answers respectively. A very high incidence of multiplying both sides of the equation in item 15 by (3 - x) was noticed, which was categorised as a misconception.

For item 15, 26% of the participants multiplied both sides of the inequality by (3 - x) as in the case of solving an equation, getting $x - 7 > 3 - x$. From the interview it was noticed that the students had the knowledge for multiplying an inequality through by a negative number, where the inequality sign reverses. They however had a problem when dealing with unknowns in the solving of inequalities with variables.

The following excerpt (I3SEK3) comes from an interview from one of the participant from this College. (I3SEK3, I- Algebra Test Interview, 3- Third year student, SEK- Name of the College and 3- Student number three)

I. Did you solve this problem and get an answer of $x > 5$?
S. Yes
I. Can you show me how you arrived at the answer?
S. Yes, wrote on a paper and multiply both sides of the equation by 3 - x and arriving at an answer of $x > 5$
I. Did you check your answer?
S. No
I. Check your answer.
S. How?
I. Substitute a number using your solution.
S. He puts 5 and arrived at $1 > 1$, oh there seems to be a problem
I. Where is the problem? (He was quiet, did not see the problem) I told him to substitute a number more than 5, which I asked him to suggest the number.
S. 6 he said
I. Can you do the substitution in the original inequality?
S. He puts 6 in place of x giving $(6 - 7)/(3 - 6) > 1$ (wrote on a piece of paper and got an answer of $1/3 > 1$)
I. Does your answer satisfy the conditions of the problem?
S. No but the solution is correct, he points to $x > 5$ on the paper.
I. I think you have a problem with solving inequality. You are solving it like using equal sign.
S. Oh, I remember now, we have solved this type of a problem before but I have forgotten how to solve it now, can you show me?
I. Yes, (the interviewer showed this student how this was solved to arrive at an answer of $3 < x < 5$. (We checked the answer with the value of 4 and it satisfied the inequation).

Again some of the students did not offer Higher Grade (H.G) mathematics at matric level where this topic is treated, as was gathered from the interviews (see excerpt I3SEK2 , I- Algebra Test Interview, 3- Third Year student, SEK- Name of College, 2-

Student number two).

I. You seemed to have difficulty with item 15
S. Yes
I. What was the problem?
S. I know how to solve inequality problems but this is not the type I am used to.
I. What type are you used to?
S. The linear inequality
I. Does it mean you have never come across this type of inequality problem?
S. Yes
I. But this type is in the matric syllabus.
S. Yes, but it is for the Higher Grade students
I. Does the college syllabus not include this topic?
S. It is there but we have not treated it.
I. Why did you not leave the question altogether?
S. I thought I could use my previous knowledge of solving equations for this too.

Notwithstanding, the syllabus for the STD course at the College caters for this. The fact that these students failed to check the reasonableness of their answers indicated deficient conceptual knowledge and understanding of this particular problem.

In item 17, 84% showed misconception. Here again the participants solved this inequality in a manner that normally pertains to using an equal sign. Majority of the students ticked "b" ($y < 5$) as the answer, as if $y^2 = 25$. Item19 showed similar responses to item 17 with 42% categorised under misconception. This is a case where students mostly used positive numbers to ascertain the validity of statements forgetting about negative numbers. This was evident by the scribbling along the edges of their scripts where they checked the answer only with positive integers. These again are indications of how the College students understand inequalities. This student is likely to be holding a misconception about real numbers, which agrees with why only positive numbers were used to verify the answer to this question. Having a wrong conception of real numbers might lead to a wrong understanding of algebra. Investigation of a situation by checking specific cases is very powerful in algebra, but drawing conclusions which are based on the checking of some examples without making sure that all possible cases are covered leads to a lack of understanding.

In item16, 26% of some of the students seemed to have a misconception, in the sense that they forgot that two numbers whose squares are equal can either be equal or opposite in sign. Thus 26% of the students ticked $x = y$ as the answer to $x^2 = y^2$ forgetting that $x = -y$ also satisfies that equation, which meant the answer was $x = \pm y$. Item18 was also poorly answered with a high 84% categorised under misconception. The misconception started with the first statement (See appendix C). The first statement was seen to be correct by those students who ticked it, forgetting that by squaring a binomial you get three terms, which does not give that answer which was given. This is based on the misconception that when squaring both sides of a binomial you square the individual terms, e.g. $(a + b)^2 = c^2$ might result in $a^2 + b^2 = c^2$. This misconception was also noticed in item 6 when a student tried to expand $(5x + 1)^3 + 5x = 349$. In a similar fashion this student had expanded as follows:

$$
\begin{aligned}
(5x + 1)^3 + 5x &= 349 \\
15x + 3 + 5x &= 349 \\
20x &= 346 \\
x &= 17{,}3
\end{aligned}
$$

The expansion of $(5x + 1)^3$ to $15x + 3$ was a confusion which might have been brought about by the previous knowledge of removing brackets where 3 was used to multiply. This was brought about probably because the student did not know what to do; hence he retrieved the knowledge of removing brackets, which might have not been learnt well due to the incomplete conceptual understanding. Item 20 required students to identify two answers describing what an expression is, but none of the students were able to identify the two answers, which described an expression. Those who identified the two statements added another statement. The most common inclusion was (d), (See appendix C). After identifying (b) and (c) as some of the characteristics of an expression, few went further on to include (d). This may be attributed to the difficulty students have with differentiating between an expression and an equation.

4.2.3 Misinterpretation

37% misinterpreted item 10 by trying to solve the equation, e.g. instead of leaving the answer at $3x + 4 = 31$ they went further to solve for x, thus having an answer of $x = 9$. This can be ascribed to the belief that one has to get a numerical answer for an unknown in an equation. These students had written the correct expression but had tried to use routine manipulation techniques. It is therefore likely that the recent learning of procedures for simplifying algebraic expressions and solving of equations have caused the retrieval of schemas related to that learning Macgregor and Stacey (1997). In item 9 the 11% misinterpretation was attributed to students who tried to solve for y to arrive at $y = -5/2$, when the question did not ask for the numerical value of y. With such an answer the student should have also known that ages are always positive and not negative. This goes to say that this student is somehow lacking with the conceptual understanding of algebra.

4.2.4 Classroom observation

The students who were observed used the Initiation-Response-Feedback (I-R-F) (Brodie, 1988) style in their teaching (See LO3SEK2, LO- Lesson observation, 3- Third year, SEK- Name of the college and 2- Student number 2) approach, which was teacher-centred with the learners seated in rows facing the chalkboard. Though the lesson was lively, most of the talking was done by the student teacher while the learners answered the questions, sometimes in chorus. Notably the lesson of one of the three students who was interviewed from this College of Education on the topic "Collection of like terms" will illustrate this. This lesson was for a grade 8 class, which was made up of 42 learners and the time allotted to the lesson was 35 minutes. This student teacher was showing them how to add 2x ÷ 3 and 3x ÷ 4. The excerpt (LO3SEK2) below will illustrate the teacher-centred approach of the student teacher. (S, stands for student teacher and, L, stands for learner).

S: What is the common denominator of 3 and 4?
L: 12 (chorus)
S: 3 goes into 12?
L: 4 (chorus)
S: 4 multiply by 2x
L: 8x (chorus)
S: 4 goes into 12?

This went on until he arrived at an answer of 17x/12. This meant the learners were involved in the lesson by just answering the questions of this student teacher. There was no new algebraic concept introduced at this stage of the lesson. This student teacher started this lesson by giving the rule for collecting like terms. He said to the learners, "to collect like terms together you will have to add the signs of the terms, which are the same and then maintain the same sign". To collect unlike terms with different signs, he gave the rule that they will have to subtract the smaller number from the bigger one and retain the sign of the bigger number. This student teacher had not used any informal or concrete object to illustrate why this rule works. He nonetheless used correct arithmetic procedure to show the learners how to multiply fractions. Two different ways of doing multiplication of fractions were illustrated, i.e., multiplying the numerators and denominators separately before looking for the common factors to cancel. Alternatively reducing the numerators and denominators by cancelling with the common factors.

At the end of the lesson by student LO3SEK2, I could deduce that the student teacher was not teaching for conceptual understanding but rather supplying the learners with rules and showing them how to apply them. This student teacher had not led the learners to think for themselves. There was no time for discussions with the learners. Learners who had answered questions wrongly were not led to correct them by themselves but rather waited for the correct answers from other learners or the teacher. This student teacher did not give any hint to the learners of any misconception different researchers had come across in their studies, which was likely to arise when teaching this topic and the way one can try to overcome them. Such as the tendency of adding **a** and **b** to get **ab** (Robinson et al.,1994)

4.2.5 Pedagogical content knowledge

Carpenter, Fennema, Peterson, & Carey (1988) have a notion of PCK that refers to the concepts and the misconceptions of a topic, techniques of understanding and diagonising these misconceptions of the topic and the knowledge of instructional strategies to eliminate these misconceptions. These students who were observed did not make reference to any misconception likely to be encountered in the lessons they taught; neither did they come up with different representations of the lesson. The student's tendency to conjoin expressions should have been mentioned during the lesson on 'collection of like terms'. For example drawing the learners' attention to this misconception by explicitly mentioning to learners that some research literature has revealed that some students do add for example 6x + 5 to get 11x or 11, Robinson et al.(1994), which is not the case with such expressions. The teacher could have asked some questions like what happens if you add x^2 to 2x? The teacher should have probed further to find out if some learners had this misconception. Different representations as part of PCK of the student teacher discussed earlier in 4.2.4 the teacher failed to come out with not more than one way of colleting like terms. When it comes to alternative ways of approaching this topic, it became imperative when this student tried to multiply fractions in two ways. Alternative approaches to teaching are very important as they can assist learners in their approach to problems in algebra. The understanding of algebra requires the teacher to be able to relate the application of the concept under study in variety of situations. In item six of the algebra test, students who tried to solve the problem by expanding ended up not getting the correct answer as it became too tedious and complicated. Other students who solved the same question by merely replacing x = 6 by 5x = 6 could easily arrive at the answer required. The student teacher in the 4.2.4 episode allowed chorus answers and therefore did not have time to identify and attend to individuals who did not understand or had different reasoning or approaches to the topic.

The tendency to provide rules for the learners in order to arrive at the correct answers was revealed from the observation. The following excerpt will illustrate this (See observation LO3SEK2, S = student teacher, L = Learner).

S: To collect like terms you add terms with the same sign and maintain the sign and you subtract in the case of unlike terms the smaller number from the bigger one keeping the sign of the bigger term. (The student teacher should have said "like" or "unlike" signs)

L: Yes teacher (in chorus)
S: What is $- 2x^2 - x^2$ bearing in mind the rule that, you add like terms and you subtract

unlike terms.

For conceptual understanding it was expected that this teacher (See LO3SEK2) would use different examples like the fruit or a set of different objects or models to assist in making meaning to the learners and also to make learners see the link between terms and signs. For example, to differentiate between 2*x* and x^2, 3*x* and x^3 etc. The difference between 3*x* and x^3 will come out clearly if the learners are asked to substitute different numbers in place of the *x* in both terms.

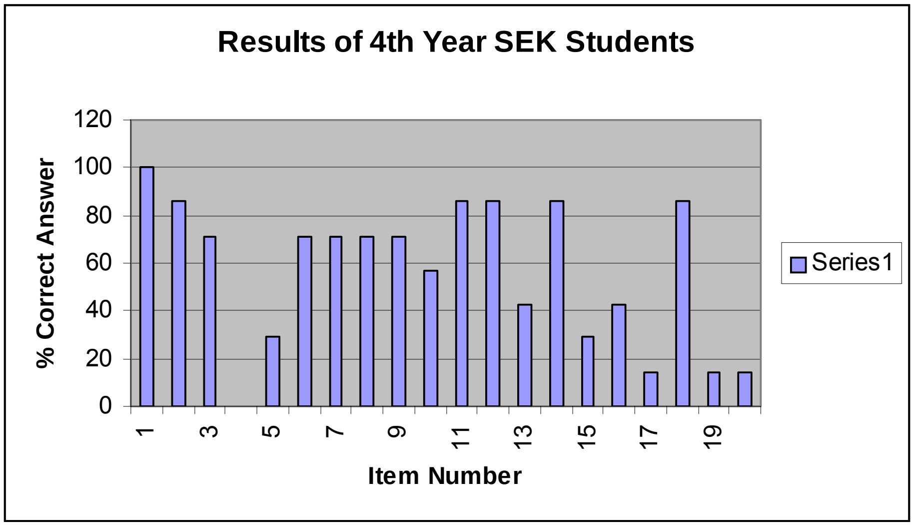
4.3 FOURTH YEAR COLLEGE SEK STUDENTS

Table 4.2 shows the analysis for 4th year SEK students. Number of students =14

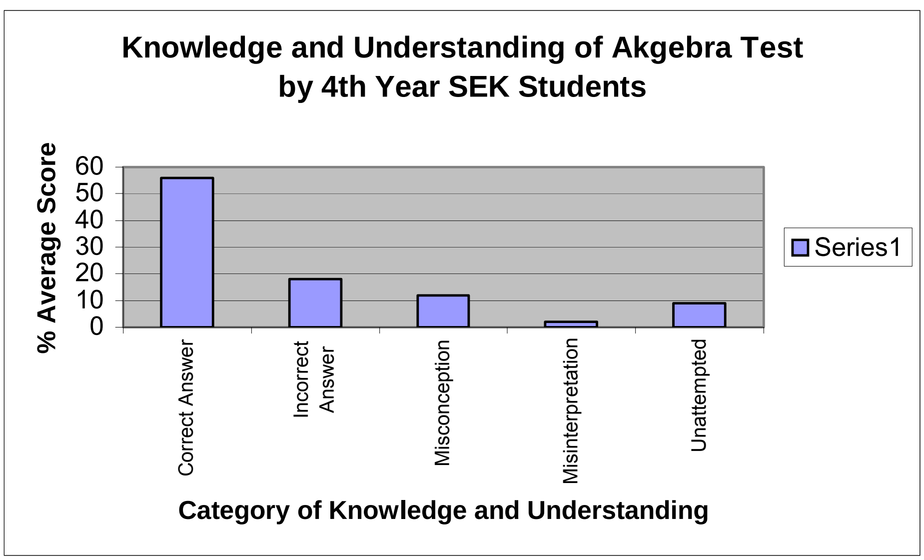
Item	Correct Answer	Incorrect Answer	Misconception	Misinterpretation	Unattempted	Percentage correct answers
1	14	0	0	0	0	100
2	12	2	0	0	0	86
3	10	2	0	0	2	71
4	0	6	8(57%)	0	0	0
5	4	6	0	0	4	29
6	10	4	0	0	0	71
7	10	2	0	0	2	71
8	10	0	2(14%)	0	2	71
9	10	0	2(14%)	0	2	71
10	8	0	0	6(42%)	0	57
11	12	2	0	0	0	86
12	12	2	0	0	0	86
13	6	6	0	0	2	43
14	12	2	0	0	0	86
15	4	4	6(42%)	0	0	29
16	6	2	4(29%)	0	2	43
17	2	2	8(57%)	0	2	14
18	12	0	0	0	2	86
19	2	2	8(57%)	0	2	14

20	2	6	4(29%)	0	2	14
Total No.	158	50	42	6	24	
Total %	56	18	12	2	9	56

Table 4.2: Results for 4th year SEK students.



Graph 4.3 Results of 4th year SEK students.



Graph 4.4 Knowledge and understanding of algebra by 4th year SEK students.

In this group the overall percentage of the total number of correct responses was 56%, two percent (2%) higher than the third year group. From Table 4.2 and graph 4.4 the misconceptions were 12% as compared to 20% for the third year group. Misinterpretations were 2 % compared to 3% for the third year group. Overall the fourth year group performed better than the third group in all aspects of the analyses. This should be expected as the fourth year group had spent more time at the college and the possibility that the best students are selected to pursue the fourth year programme is also there.

4.3.1. Category frequencies

Table 4.2 shows that out of the 20 items, 11 items (3, 5, 7-9, 13, 16-20) had students not attempting to answer, thus making a total percentage of 9% (See Graph 4.4). Under the incorrect answer category there were five items that could not be classified per the answers given by the participants, items 1,8,9,10,and 18. The percentage score for the incorrect category therefore contributed to 18% of the total score. Table 4.2 and Graph 4.3 show that 8 items had students scoring less than 50% (items 4, 5, 13, 15-17, 19 and 20). Graph 4.3 shows that six items had scores more than 80% (items 1, 2, 11, 12, 14 and 18)

4.3.2 Misconceptions

Overall 12% of the responses of the students from this group were classified under misconception (See Graph 4.4). Items 4, 8,9,15,16,17,19 and 20 were noted to have misconceptions. In item four 57% were found to have misconceptions. The most frequent answer to this question was c = 1,2,3,4 ignoring real numbers less than 5 as answers. In item eight, 14% were classified under misconception by the fact that they used the area formula for the perimeter of a rectangle. In item nine 14% were classified under misconception. These 14% of the students understood “older than” to mean “multiply by”, hence came up with an answer of 6y instead of 2y + 5. In item 15, 42% of the responses were categorised as misconception by multiplying both sides of the inequality and solving it as a normal equation. In item 16 the 29% who were classified as having misconceptions failed to take negative numbers as part of the solution to the answer. Item 17 was also poorly answered where 57% of the students were classified as having misconceptions. One of the students interviewed revealed how this topic is treated at the matric level (See I4SEK1 below I- Algebra test interview, 4- Fourth year, SEK- Name of college, and 1- student number).

I. Have you ever come across this type of a problem?
S. I cannot recall
I. What does that mean?
S. I mean I was not taught this type of a problem.
I. What level exactly were you not taught?
S. At the matric level as well as the college level.
I. Why not?

S. Because I think this topic is taught at the H.G. level only.

At the matric level this student said ‘this topic was not treated, it was for H.G students only’, he said only simple linear inequality was treated. Some failed to attempt this question probably because they might not have come across this type of a problem during the course of their study. This could lead to misconception, as students who did not study inequalities of this type were tempted to apply the method of linear equations they have learnt to inequalities as well. Item 19 followed a similar trend to that of item 17. In this item the responses of the majority of students (57%) were classified under misconception. In item 19 the problem some students had was in differentiating between equation and expression. These 57% of the students were able to tick the two statements in the question which make an expression but went further to tick the roots of an equation which come out of that expression. An interview with one of the students revealed that the moment you factorise an expression it means you will get the values of the unknown, which satisfy it. The excerpt (I4SEK1) below supports the belief this student had (I – Algebra test interview, 4 – 4th year, SEK- Name of College, 1 – students number).

I. Did you understand the question (item 20)?
S. Yes, I understood it.
I. What does it say?
S. It says tick all the statements, which describe an expression.
I. What is an expression?
S. I think it is two or more terms grouped together by any or all the four basic operations.
I. Is (x - 3)(x - 2) = 0 an expression or an equation?
S. Equation.
I. Why is it an equation?
S. Because the expression is equal to zero.
I. Is x = 2 and x = 3 an expression or an equation?
S. An expression because if you solve (x - 2)(x - 3) you get x = 2 and x = 3
I. How?
S. (She writes (x - 2) = 0 or (x-3) = 0 and comes out with x = 2 or x = 3)
I. But you said (x-2)(x-3) = 0 is an equation.
S. Yes, but if you want to get the values to (x - 2)(x - 3) you will have to equate to zero to give the answer x = 2 or x = 3.
I. Yes but the question did not ask for the roots of the expression.
S. Does it mean x = 2 or x = 3 is not an expression?
I. What do you think?
S. I think not because there is no equality sign in an expression.

This student feels the root of an equation is an expression but failed to realise that you cannot get the roots if the expression is not equated to zero.

4.3.3 **Misinterpretation**

It appears from Table 4.2 that this group could interpret the questions well. It was only in item10 that 42% of the answers were classified under misinterpretation. These students tried to solve for the value of x, which was not what the question required for an answer. i.e. solving for x to get x = 9 from the equation 3x + 4 = 41 (See Graph 4.4).

4.3.4 **Classroom observation**

Unfortunately the students who were observed during their teaching practice period did not treat specifically any of the topics in which they seemed to have misconceptions. The teaching however, was mostly teacher-centred (See 2.1.2). One of the participants who was observed during a grade 8 classroom teaching simplification of expressions started by trying to make learners know what an expression is. She first asked learners to give examples of monomials, binomials, and trinomials, which they did. They gave examples like: ax, $x^2 + 3$, $3 + 2x + 4x^2$. The teacher then led learners to know what an expression is by saying an expression is the addition and subtraction of terms. She did not explain that multiplication and division of terms could also be expressions. She wrote the expressions $(x - 3)(2x + 5)$ and $(2x^2 + 3x + 1)/(2(x + 1)$ on the chalkboard and asked the learners whether those were expressions. Some said they were not expressions because of the brackets. Here this teachers' knowledge of expression was confined to addition and subtraction whereas, multiplication and division also can be used in expressions. At the end of the lesson I requested the lesson plan for that lesson and read through it. A brief interview with this teacher on the lesson she presented to the class then followed. The questions presented to her and the responses were as follows (see post interview PI4SEK1, PI stands for post interview, 4 for fourth year, SEK the name of the college and 1 for the student number 1) below: (I stands for interviewer and S stands for student)

I: What were your areas of concern as you prepared this lesson?
S: To enable learners to know that you cannot add unlike terms.
I: What were your main goals as you prepared the lesson?
S: My main goals were to make
1) learners add unlike terms,
2) to simplify expressions
I: What method(s) did you plan to use to achieve these goals?
S: To use real-world problems and to make use of objects as examples.
I: How did your lesson go?
S: The lesson was fine
I: Why did you change the example of boys and girls to fruits? (see 4.3.5)
S: I realised that some of the learners were getting confused over the answers given by some learners about the number of boys and girls in the class.
I: Do you think the example was a wrong choice?

S: I do not think so, only some few of them wanted to confuse issues.
I: Is there no way you could have made those learners understand that example?
S: I was confused, so I had to resort to the fruit example, which I have seen in one textbook.

This student teacher was able to present the lesson using different examples as illustrated in the above excerpts, using number of boys and girls and changing to number of fruits as a way of using real world examples. This supports the notion that for conceptual understanding alternative ways of presenting the content plays an important role. However, the way she explained what an expression is can lead to learners having misconception since she attributed expression only to addition and subtraction of terms.

4.3.5 Pedagogical Content Knowledge

The teacher trying to introduce the addition of like and unlike terms started by writing “2 girls + 3 girls” on the chalkboard and asked the learners to respond. They answered “5 girls”. She went further to ask the same for 4 boys + 5 boys and got the answer 9 boys. She asked again 3 girls + 4 boys and the answer she got from one of the learners is 7 pupils. This teacher was surprised by this answer, so asked the question again: Can you add boys to girls? Again the response was yes. This generated into a long debate with another learner trying to support the friend said if you want the number of learners in this class you will have to add boys and girls to arrive at the expected answer. This teacher being confused over these answers decided to shift the example from human beings to fruits. She gave an example of a number of apples and bananas, she started by asking them to add some number of apples to apples and then bananas to bananas. Here, she was satisfied with the answers given to her. She then asked the learners to add 5 apples to 4 bananas. The learners this time said the answer remains 5 apples and 4 bananas and did not say the answer was 9 fruits.

The teacher then asked the learners to write down on papers the answer to 3a + 6b. Again after going around to see the answers, she was satisfied with what they had written down; the answer remained the same. The teacher on the contrary, concluded by saying "a" can represent apples and "b" can represent bananas in the above example. This is where this teacher is displaying a misconception, as objects "a" and "b" do not represent numbers as the term "5a" and "6b" denote. This establishes the fear Booth (1988) expressed that, one will be tempted not to multiply 5a and 4b, if they represent objects like apples and bananas. Multiplying 5a and 4b should give an answer of 20ab, where a and b represent the number of items or objects. In that case if a and b stand for apple and banana respectively then it would mean 20ab may imply 20 apple-banana. This misconception in algebra leads to the impression that variables are labels of objects, not number representatives reported by Driscoll (1982). This teacher’s pedagogical content knowledge by way of using alternative representations for a topic was displayed in this lesson. This teacher was quick enough to change the example from human beings to fruits to make the understanding of the learners possible. However, she failed to make learners know that "a" or "b" do not represent objects, but rather represent numbers. There are some difficulties related to this and other similar issues. A teacher trying to find ways of dealing with these issues, often uses concrete analogy. This strategy is part of PCK. However in doing so the concrete often introduces a misconception because it does not match the abstract sufficiently.

This student teacher lacked conceptual understanding of algebra. Her knowledge of simplification of expressions might have been good, only her choice of examples became controversial leaving learners the choice to answer in their own ways, but not the way the teacher expected (See LO4SEK2, LO-lesson observation, 4-fourth year student, SEK-name of college, and 2- student observed number 2). In this example, the teachers' knowledge and pedagogical understanding of algebra, if they were very good, should have been able to make the learners follow this example of boys and girls. The number of boys and girls will ever remain boys and girls, so the student teacher with a good knowledge of algebra should have been able to save the situation. The teacher’s explanation has contributed to learner’s errors and misconception as shown here by this teacher and this supports Kaur and Sharon’s (1994) view as reported in their study. They reported that the major cause of errors in mathematics is the misunderstanding of mathematical concepts. This misunderstanding, they stated, can lead to the formulation of misconceptions and false generalisations, which they reported could hinder the learning of mathematics. Notwithstanding, this student teacher was able to apply his pedagogical content knowledge of algebra to use an alternative representation to make the topic clearer to the learners. It is always said using concrete objects in the class puts learners at ease but sometimes these concrete objects make things more difficult or confuse issues as is seen from the example of boys and girls.

(S- Student teacher, L- Learner)
S: Can we do some addition
L: Yes
S: a + a; x + x + x
Ls: 2a, 3x (chorus)
S: 2 girls + 3 girls
Ls: 5 girls
S: 4 boys + 5 boys
Ls: 9 boys
S: 3 girls + 4 boys
Ls: Remains the same, 3 girls + 4 boys (but one learner said 7 pupils)
S: (Attention was directed to this learner who said 7 pupils). Why do you say 7 pupils? Can you add boys and girls to get one number answer?
L: Yes, you can add and get one number answer as 7 pupils (The whole class burst into

laughter and the class became noisy)

S: You cannot add number of boys to number of girls. Let us use another example. (Student teacher changed the example to apples and bananas)

This teacher's attention to individual difficulties was lacking, She did not probe further to make those learners understand that the answers they had given were wrong, she rather changed the example. The responses from learners were not probed further, the teacher was leaving individuals who gave wrong answers to search for correct answers from other learners. The constructive mode of instruction was not in use here, where the educator should be seen as a mediator. On the whole, the classroom arrangement depicted what was expected from this teacher as learners were seated in rows instead of in groups where learners are expected to discuss and share ideas among themselves in those groups. Under the groupings the learners are expected to construct their own knowledge of algebra from their experiential world and among themselves with the teacher serving as a mediator.

4.3.6 Comparison of 3rd year and 4th year College SEK algebra test results.

Tables 4.3.1, 4.3.2, 4.3.3 and Graph 4.5 show the comparisons between 3rd year SEK students as against the 4th year students in percentage scores according to individual items, the misinterpretations and misconceptions respectively.

Item Number	3rd Year % scores	4th Year % scores
1	89	100
2	89	86
3	84	71
4	26	0
5	84	29
6	74	71
7	84	71
8	74	71
9	80	71
10	53	57
11	74	86
12	84	86
13	42	43
14	52	86
15	26	29
16	52	43
17	0	14

18	11	86
19	11	14
20	0	14
Total %	54	56

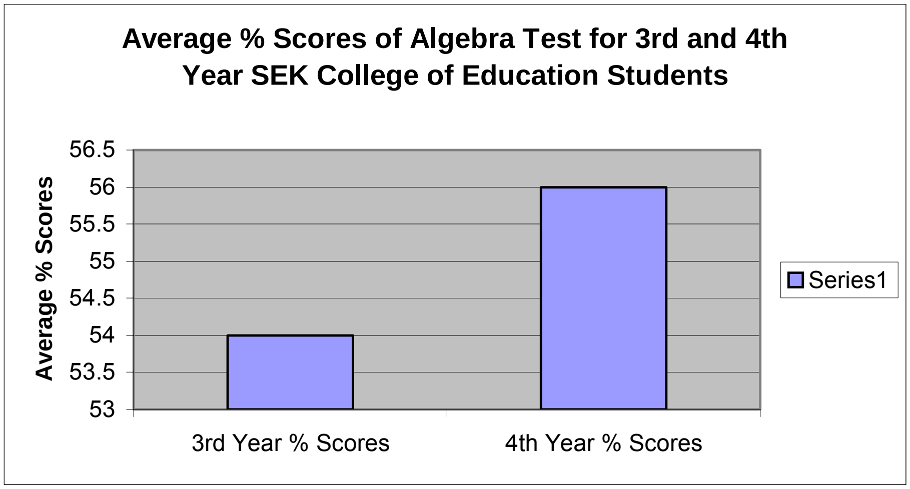
Table 4.3.1 Overall percentage scores of 3rd and 4th year SEK College students

Item No.	3rd Year % misinterpretation	4th Year % misinterpretation
9	11	0
10	37	42
15	5	0
Total %	3	2

Table 4.3.2 Overall percentage misinterpretation of 3rd and 4th year SEK College students.

Item No.	3rd Year % misconceptions	4th Year % misconceptions
4	52	57
7	5	0
8	11	14
9	11	14
10	11	0
15	26	42
16	26	29
17	84	57
18	84	0
19	42	57
20	47	29
Total %	20	12

Table 4.3.3 Percentage misconceptions of 3rd and 4th year College of Education students



Graph 4.5 Average % scores of algebra test for 3rd and 4th year SEK College of Education students

The results shown in the Tables 4.3.1, 4.3.2, and 4.3.3 showed that the percentages scores of the 4th year College students were better than those of the 3rd year College students: 56% as compared to 54% (Table 4.3.1 and Graph 4.5). In items 1-9, and 16 the third year students did better than the 4th year students. Table 4.3.2 and 4.3.3 show how the 4th year students had more misinterpretations and misconceptions than the third year students. In item 1, the students of the 4th year group showed 100% score better than the 3rd year group who scored 84%. Item 2 almost showed the same scoreline, 89% for the 3rd year group and 86% for the 4th year group. The score of 71% for the 4th year group was lower than that of the 3rd year group of 84%. Item 4 was badly answered by both groups none of the 4th year group got this item correct. The 3rd year group scored a low 26%. The most frequent answer from both groups was c = 1,2,3,4. This was classified under misconception as they did not include all real numbers less than 5. However, item 5 showed a low 29% correct as compared with 84% for the 3-year course students. The reason behind the overall increase in the percentage results is that this topic, “inequalities”, had just been taught by the lecturer to those students in the third year. Unfortunately the student who was interviewed here did not answer that item. He (See I3SEK1) stated that he got frustrated with the expansion of the second equation, which was noticed by the writings on his answer sheet. This is an indication of the mindset of many students who when confronted with this type of a problem, the only thing which comes to their mind is the expansion of the terms involved. After trying to expand in vain; the student decides to give up. This particular students' immediate reaction was typical. In item 6 the percentage score was fairly good, both groups scored 74% and 71% correct answers respectively. There was only one instance where a student wrote 4x cm, which has been classified as a misconception. This has been discussed earlier when some students seem to confuse “more than” to mean the same as “multiply by”. In item 8 two students each from the 3rd year and the 4th year groups had the misconception of taking the perimeter for the area of a rectangle. This contributed to 11% of the 3rd year group and 14% of the 4th year being classified under misconception. Item 9 followed the same pattern as item 8.

The misinterpretation of “older than” to mean the same as “multiply by”, was shown by both groups. Trying to look for a numerical answer was noticed from the two groups when some tried to solve for x and y from items 9 and 10. In item 11 the 3rd year group scored 74% correct as compared to 86% for the 4th year group, both group did not display any misconception or misinterpretations. In item 12 both the groups had scored 84% and 86% respectively as correct, however for the third year group 18% of the students had failed to attempt the question. In item 13 the responses were almost the same with the 3rd year group recording 42% while the 4th year group recorded 43%, some of the students from both groups had failed to attempt with a bigger percentage (47%) from the 4th year group. These two groups could not be classified under both misconception and misinterpretation. In item 14 the 3rd year group had scored lower percentage correct answer than the 4th year group. The 3rd year group had scored 52% while the 4th year group had scored a high 86%. Item 15 was however, poorly answered by both groups, the 3rd year group registered a 26% correct answer as to 29% for the 4th year group. The 3rd year group had displayed 26% misconceptions as compared to 42% for the 4th year group. The 3rd year group had done better in item 16 than the 4th year group, the percentages being 52% and 43% respectively. In item 17 the situation was more critical with the 3rd year group scoring 0% while the 4th year group scored only 14%. The 3rd year group showed 84% misconception as compared to 57% for the 4th year group. In item 18 the 4th year group had scored a high 86% as compared to a low 11% for the 3rd year students. Both had scored low correct answers for item 19 with the 3rd year group recording 11% while the 4th year group recorded 14%. Item 20 was no exception to the poor responses, the 3rd year group scored a 0% while the 4th year group scored 14%. The 3rd year group had 47% misconception as compared to 29% for the 4th year group.

4.3.7 General impression about teaching and learning in College SEK.

My interview questions (See appendix D) were intended to find out whether these students like algebra and mathematics in general, and also their mathematics teachers at school. Again to find out whether change of schools and mathematics teachers affected their interest in mathematics. At this particular College all seemed to like mathematics (when asked to express their interest in mathematics and algebra). They however, had different views about their various mathematics teachers along the way to matric class. Some of them said they had good mathematics teachers who motivated them towards offering mathematics at the tertiary level. Other responded by saying that they nearly gave up mathematics because of the behaviour of their teachers. They claimed, some of these teachers were not coming to class regularly and those who did come just gave them work without much assistance. This is what one of the students said in the interview, “ I nearly gave up studying mathematics if not because of one of my friends who help me and encouraged me because our teacher was most of the time absent from our mathematics class” (See I3SEK3) below. They recalled that some teachers became furious when asked questions, which the teachers were not sure how to respond. This type of behaviour can be attributed to lack of confidence and knowledge of the subject matter.

I. Do you like mathematics?
S. Yes, I do like mathematics that is why I am pursuing this course.
I. Were you motivated by any of your mathematics teachers to pursue mathematics course?
S. Not much.
I. What do you mean by that?
S. My teacher at grade 10 made me to dislike mathematics because he was most of the time absent from the class.
I. How did you the proceed with mathematics to this far?
S. I was interested in mathematics so with my friends we did study on our own.
I. But I am sure other mathematics teachers contributed to your efforts.
S. Yes, especially my grade 12 mathematics teacher, oh, he was good.
I. What do you mean by being good?
S. He knew his stuff and was always there to assist us.

These students responded that they preferred algebra to geometry, with the reason that in algebra one is given the rules and methods to follow and apply to arrive at answers to problems posed to them. They said in geometry one has to think and reason to draw conclusions. As far as explaining algebra to a layman, many talked of the branch of mathematics that deals with variables, unknowns and letters. Algebra some said is the continuation of arithmetic. They talked of algebra as the x and y mathematics. When it comes to teaching algebra at schools, the only problem some highlighted was that it is difficult to construct teaching aids in algebra (See I3SEK1). Most of the students preferred teaching algebra because they saw algebra as application of learnt formulas and rules, which suits the teacher-centred (See 2.1.3) approach, which was used by most of the student teachers and was also the way they were taught. The excerpt below (See I3SEK1) supports why learners prefer algebra to geometry.

I. Between algebra and geometry which one do you prefer teaching
S. I prefer teaching algebra.
I. Why?
S. Because the formulae and procedures in algebra are easy to understand and

follow. For example $(a + b)^2 = a^2 + 2ab + b^2$, can be easily applied to other expressions which need its application unlike in geometry where it becomes difficult to apply a theorem learnt to a given problem which needs its application.

I. So you mean geometry is difficult to teach?

S. Yes, because you need to give reasons for statements made in geometry while in algebra in most of the times you are not required to give reasons.

I. Do you construct teaching aids when teaching algebra?

S. Seldom

I. Why seldom?

S. Because it is difficult to construct teaching aids in algebra.

5.2 ANALYSIS OF COLLEGE STK RESULTS

Three groups of students were used from this institution. The first year group, the final third year group and the final fourth year group.

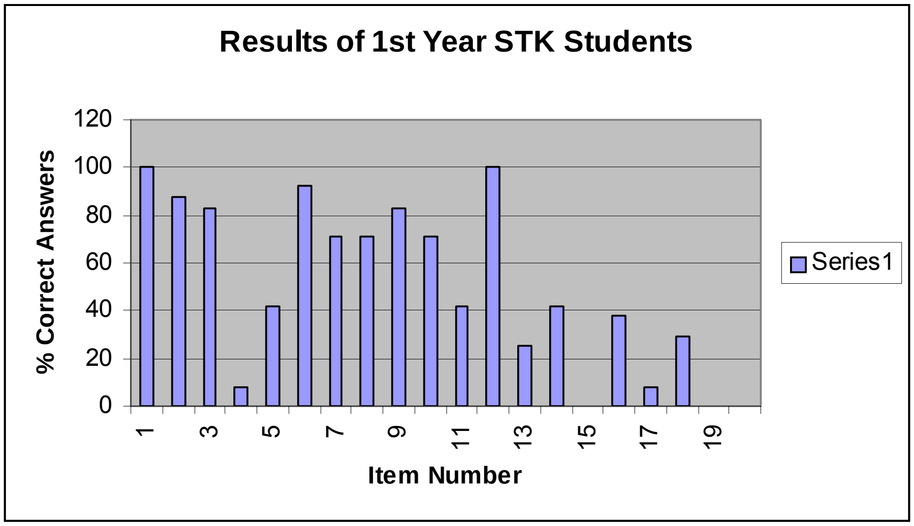
4.4.1 Category frequencies

This first year group had an overall 50% classified as correct (See Table 4.4 and Graph 4.6). 20% were classified under misconception and 4% of the rest classified under misinterpretation. Item 1 was very well answered leading to a 100% correct score by all the students. Items 2 and 3 were done well given percentage results of 88% and 83% respectively. Item 4 was poorly done with a low 8% correct response and a 38% classified under misconception. Item 5 with a 42% correct responses and a 21% of the students failing to attempt. Item 6 was well answered with 92% correct responses. Item 7 had 71% correct responses as compared to 8% misconceptions and 4% misinterpretation. Item 8 with 71% correct responses, 13% classified as misconception and 8% as misinterpreted. Item 9 was well answered with 83% classified as correct and 17% as misinterpreted. Item 10 with 71% classified as correct and 21% as misinterpreted. Item 11 showed a high 58% classified as misconceptions and 42% as correct. Item 12 gave 100% correct responses and item 13 however, gave a low 25% correct responses and 33% unattempted. Item 14 had 42% of the responses classified as correct while a high 54% were classified as incorrect. Item 15 had no correct response but a high 71% classified as misconceptions. Item 16 was also poorly done with 38% classified as correct and 63% as misconceptions. Item 17 was poorly answered with a low 8% classified as correct and 58% classified under misconception. Item 18 had 29% correct responses and 29% classified as incorrect. Items 19 and 20 could not be answered correctly resulting in 0% correct responses respectively. Item 19 had 50% classified as misconception and 50% classified as incorrect. Item 20 however had 38% classified as misinterpreted and 63% as incorrect.

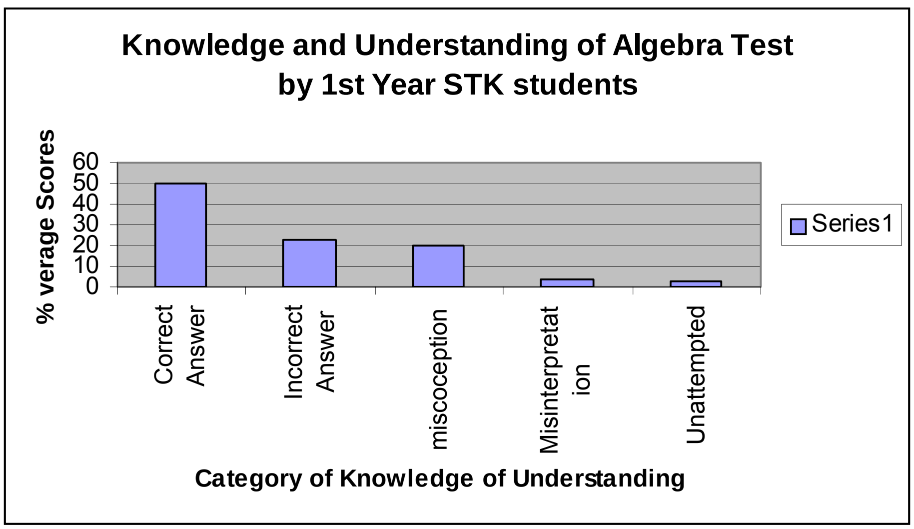
Table 4.4 shows results of 1st year students of college STK. Number of students = 24.

tem Number	Correct answer	Incorrect answer	Miscon ception	Misin terpretation	Un attempted	Percentage correct
1	24	0	0	0	0	100
2	21	3	0	0	0	88
3	20	4	0	0	0	83
4	2	13	9(38%)	0	0	8
5	10	9	0	0	5(21%)	42
6	22	2	0	0	0	92
7	17	4	2(8%)	1(4%)	0	71
8	17	2	3(13%)	2(8%)	0	71
9	20	0	0	4(17%)	0	83
10	17	2	0	5(21%)	0	71
11	10	0	14(58%)	0	0	42
12	24	0	0	0	0	100
13	6	11	0	0	8(33%)	25
14	10	13	0	0	1	42
15	0	5	17(71%)	0	2	0
16	9	0	15(63%)	0	0	38
17	2	6	14(58%)	0	2	8
18	7	7	10(42%)	0	0	29
19	0	12(50%)	12(50%)	0	0	0
20	0	15(63%)	0	9(38%)	0	0
TOTAL	238	108	96	21	18	1003
TOTAL%	50	23	20	4	3	50

Table 4.4 Results of 1st year STK students.



Graph 4.6 Algebra test results of 1st STK students



Graph 4.7 Knowledge and understanding of algebra by 1st year STK students

4.4.2 Misconceptions

An average 20% of the students' answers were classified under misconception (See Table 4.4 and Graph 4.7). These were attributed to items 4, 7, 8, 11, 15, 16, 17, 18 and 19. In item 4 the misconception was as a result of not including all real numbers less than 5. As a result of that, 38% of the students' answers were classified under this category. In item seven, the 8% of the students' answers which were classified under this category was as a result of the students writing 4*x* instead of 2*x* + 3. This misconception has been addressed in 4.2.2 when students tend to multiply instead of adding with the statement “more than”. In item 8, 13% of the answers were classified under misconception. This is when students factorised p = 2h + 18 to be p = h + 9. These students failed to divide both sides of the equation by 2. They rather divided one side of the equation leaving the other side not divided. These students are likely to be taking the factorisation of a formula as that of solving for an equation. For example when solving the equation 2h + 18 = 0, one can simplify to h + 9 = 0 but not when reducing p = 2h + 18 to the simple form.

Item 11 was poorly answered. 58% were classified under misconception. The total cost of fruits was given as a + p = 90c. In this case the student is seen to have failed to realise that **a** stands for the number of apples and **p** stands for the number of peaches. The answer gives an impression that **a** and **p** stand for one fruit each. Item 15 had been solved like an equation forgetting that they are dealing with inequalities. They had multiplied both sides of the inequality by 3 – x hence arriving at an answer of x > 5. This resulted in 71% of the answers classified under misconception. In item 16 a high percentage of 63% failed to realise that x = -y also satisfies the expression $x^2 = y^2$. The most frequent answers were x = y. In item 17, 58% of the students ticked (b), which is Y < ± 5. These students solved inequalities like solving an equation, with the value of y from $y^2 < 25$ being < ± 5. In item 18, 42% were classified under misconception since it was revealed from the answers given that there was no mistake found in the first statement (See appendix C) by these students, which depicts a misconception since there was the mistake of squaring a binomial expression by squaring individual terms in the binomial expression. In item 19, 5% of the answers were in the form of positive values ignoring negative values, which is a misconception. In item 20, there was nothing to classify under misconception.

4.4.3 **Misinterpretation**

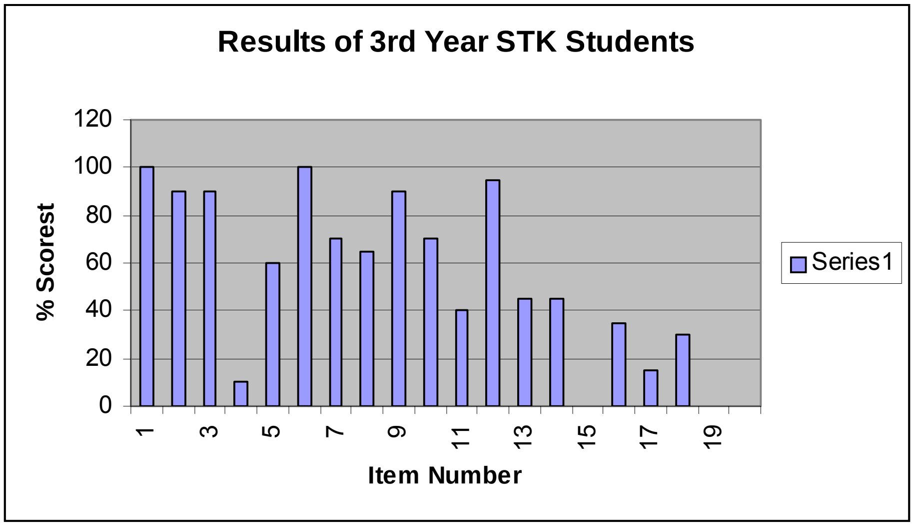
4% of the answers were classified as being misinterpreted. This occurred with some particular items, in item 7, 4% were attributed to misinterpretation as the students failed to add x to x + 3. This student simply wrote the answer as x + 3 given the impression that he had failed to interpret the question to be understood as the addition of AC and CB to give AB. In item 8, the 8% who misinterpreted were as a result of trying to find the numerical answer to h in that expression. In item 9, 17% of the answers were classified under misinterpretation because they had gone further to look for the value of y in that expression. In item 10, 21% had solved for x in the equation, which was derived. This was termed misinterpretation because the question only asked for the equation to be written not to be solved. In item 11, more than half the answers were in the form a + p = 90c. These students had misinterpreted the question by merely adding to get the cost of one apple and one peach. They failed to realise that a and p are not objects but rather represent the number of the objects. This can be linked to Stacey and Macgregor (1997) who stated that the use of misleading teaching materials intended to make the learning of algebra easy, could at times be a serious disadvantage to learners. In item 20, 38% of the answers have been classified under misinterpretation. These students had misinterpreted expression to mean equation, which resulted in looking for the roots of the expression.

4.5 ANALYSIS OF 3RD YEAR STK COLLEGE RESULTS

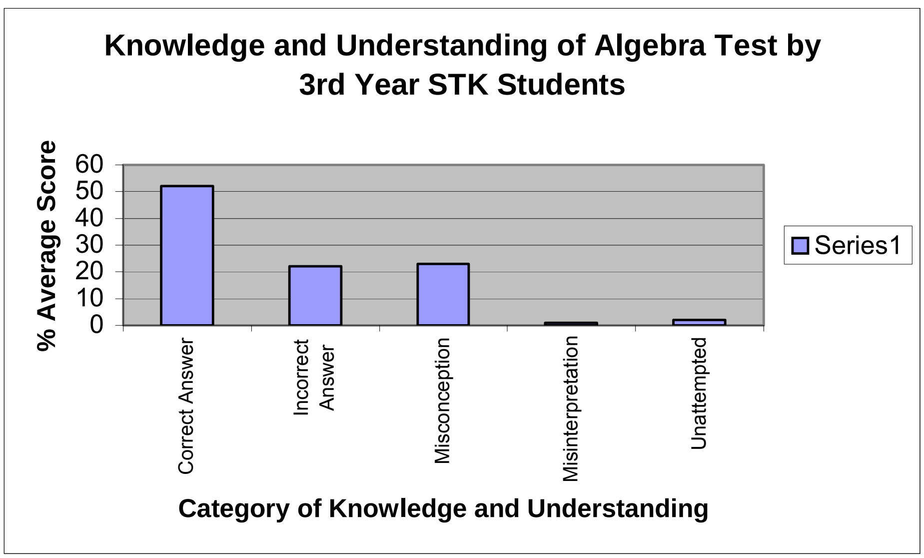
Table 4.5 shows the algebra test results of third year College STK students.
Number =20

Item Number	Correct answer	Incorrect answer	Miscon ception	Misin terpretation	Un attempted	Percentage correct
1	20	0	0	0	0	100
2	18	2	0	0	0	90
3	18	2	0	0	0	90
4	2	10	8(40%)	0	0	10
5	12	6	0	0	2	60
6	20	0	0	0	0	100
7	14	6	0	0	0	70
8	13	4	3(15%)	0	0	65
9	18	2	0	0	0	90
10	14	2	0	4(20%)	0	70
11	8	1	11(55%)	0	0	40
12	19	1	0	0	0	95
13	9	5	0	0	6	45
14	9	11	0	0	0	45
15	0	4	16(80%)	0	0	0
16	7	1	12(60%)	0	0	35
17	3	2	14(70%)	0	1	15
18	6	5	9(45%)	0	0	30
19	0	11	9(45%)	0	0	0
20	0	11	9(45%)	0	0	0
TOTAL	210	86	91	4	9	
TOTAL%	52	22	23	1	2	52

Table 4.5 Results of 3rd year STK students.



Graph 4.8 Algebra test results of 3rd year STK students



Graph 4.9 Knowledge and understanding of algebra by 3rd year STK students.

4.5.1 **Category frequencies**

From Table 4.5 the overall average percentage correct items were 52. Table 4.5 and Graph 4.9 show that 23% of the answers classified under misconception. A low 1% of the answers were classified under misinterpretation. Graph 4.6 shows that items 1,2,3, 6, 9 and 12 were well answered with the scores more than 80%. Items 15,19, and 20 were poorly done with 0% recorded respectively. In item 4 only 10% could answer correctly. In item 5, 40% of the respondents got the answer wrong with 10% failing to answer. Item 7 was fairly well done with 30% getting it wrong. Item 8 showed 65% correct answers with 15% classified as having misconceptions. In item 10, 70% had answered correctly with 20% having misinterpreted the question. Item 11 showed a 4% correct response with a high 55% having misconceptions. Item 13 showed a 45% correct answer, with 30% failing to attempt it. In item 14, 54% gave the answer as x/y, which could not be classified according to the classifications used in this study. Item 16 was poorly done with 35% classified as correct and 60% recorded as misconceptions. Item 17 was poorly done showing 15% correct answers with 70% showing misconceptions. Item 18 showed low 30% correct answers with 45% classified as having misconceptions.

4.5.2 Misconceptions

Items 4,8,11,15-20 make up the number of items for which misconceptions (See Table 4.5 and Graph 4.7) were detected from the scripts. In item 4, as has been discussed earlier in 4.3.2, students failed to write real numbers less than 5 as answers. 40% of the answers were classified under misconceptions, where only natural numbers were considered. In item 8, 15% of the answers were classified as having misconceptions as students simplified $p = 2h + 18$ to $p = h + 9$. In item 11 the 55% of the answers were classified as misconception as students wrote $a + p = 90c$, which was classified as misconception. In item 15, a high of 80% of the answers were given as $x > 5$, which is as a result of trying to solve inequality problem like an equation problem. In item 16, 60% of the answers were given as $x = y$, which means making use of only positive values to test whether they satisfy the conditions of the problem. Item 17 was also noted to have 70% misconceptions as students tried to solve for y like an equation. In items 18-20, Table 4.5 shows that 45% of the answers were classified as misconceptions. In item 18, the common answer was (c), implying that there was no mistake whereas there were mistakes in all the three statements. In item 19, the 45% of the students had ticked (a) as the answer. This means only positive values of y were considered instead of making use of both positive and negative numbers. In item 20, 45% the misconceptions were as a result of making roots of an equation as one of the characteristics of an expression by including (d).

4.5.3 Misinterpretation

A low percentage of 1% was shown in Table 4.5 to have misinterpreted the items. Item 10 was the only question where some students had gone further to solve for x, which means they misinterpreted the question to mean solve for (See Table 4.5 and Graph 4.7).

4.5.4 **Classroom observation**

It was observed from these student teachers from this College that the lesson had been planned for the whole class to work as individuals. Thus they applied the teacher centred approach instead of grouping the learners for a learner centred approach as has been suggested by the constructivists that grouping of learners provides an atmosphere of learner-centred teaching. The learners were seated in rows and columns with the teacher most of the time in front of the class. The student teachers were doing most of the talking. In one of the classes a student teacher was teaching addition of algebra to a grade 8 class. This teacher started the lesson by going through some worked examples. She started from the easier to the more complex ones. After going over the three different examples she asked the class what they had observed. The learners were able to tell the teacher that like terms could be added together whereas unlike terms could not be added. She went further to allow the learners to investigate two types of algebraic addition problems, which could likely create misconceptions. The learners were required to substitute numerical values to ascertain whether the additions of e.g.
(1). $3a^2 + 2a = 5a^3$ and
(2). $2b^2 + 3b = 5b^2$
are true or false. This teacher had designed a format to be followed by the learners in a form of a table. The teacher had gone around the classroom to assist those who were having problems or difficulties. At the end, the teacher asked them to come out with their own observations on the two investigated problems.

One of the learners said those statements were not true since they worked for only 0 and 1 but did not work for other values. The excerpt below (See LO3STK1, LO- Lesson observation, 3- Third year, STK- Name of College, 1- Student number) illustrates what happened in that classroom.
(S stands for student teacher and L stands for the learner)

S. Why do you say the statements are not correct?
L. Because they are unlike terms
S. Can you explain further?
L Because a^2 is different from a
S. What do you say about a^2 and 2a?
L. They are different
S. How are they different?
L. $a^2 = a \times a$ and $2a = a + a$
S. Good, can we conclude then that we cannot add unlike terms?
L. Yes, teacher

At the end of the post lesson interview I learnt that this teacher was trying a learner-centred (See 2.1.3) approach, which she had attended a course on. However at some stages she was forced to tell the learners because this approach was new to her. She was aware of some misconceptions some learners had over addition of unlike terms in algebra. She had therefore planned the lesson so that the learners could see the misconceptions, which are likely to appear in future (See excerpt PI3STK1, PI- Post Interview, 3- Third year, STK- Name of College, 1- Student number).

I. How did you like your lesson?
S. It was good I am satisfied with how it went.
I. What do you mean by it was good
S. I was able to apply what I learnt in a workshop I attended.
I. Tell me about it.
S. It was about learner-centred approach to teaching this particular topic, where learners were allowed to discover things for themselves.
I. What made it so successful to you?
S. The way the learners were able to discover the misconceptions with this topic.

4.5.5 Pedagogical content knowledge

This teacher had come up with the likely misconceptions in this topic and had tried to make the learners aware so that they did not have that type of misconception. She had used different examples in her lesson to illustrate the different types of additions in algebra. What she failed to do was only using the same type of variable. This teacher had allowed the learners to investigate some problems to see for themselves some of the misconceptions in this topic. It would have been better if this teacher had come out with an example like k + m not equal to km ($k + m \neq km$) and to allow the learners to state in words the meaning of such statements. This might help to overcome some of the misconceptions the statement “more than” is confused for “older than”, which had been interpreted in the algebra test to mean “times”.

In this lesson the teacher’s knowledge about the misconceptions about like and unlike terms had helped her to plan the lesson to include investigation of unlike terms. This teacher had used different examples to show why some terms could be added while others could not. This teacher’s lesson tells me how she had made use of the investigative approach (See 2.1.6) to make learners understand the concept of like and unlike terms. Most of the time she did not answer the questions posed by the learners but rather allowed them to debate around those questions. This teacher had used numerical values at some stage to let learners see why like terms can be added and unlike terms could not be added. This teacher had not only relied on the telling method but had tried to make learners draw their own conclusions. It was revealed from the observation that this teacher’s knowledge of addition in algebra was very good. She had incorporated the misconceptions some researchers have suggested, as far as this topic is concerned to help plan her lesson. This is what I think should prevail in an algebra classrooms if we would like to eradicate misconceptions in the teaching of algebra.

4.6 ANALYSIS OF 4TH YEAR STK COLLEGE RESULTS

Table 4.6 shows the results of 4 year STK College students, Number = 13.

Item Number	Correct answer	Incorrect answer	Miscon ception	Misin terpretation	Un attempted	Percentage correct
1	13	0	0	0	0	100
2	12	1	0	0	0	92
3	12	1	0	0	0	92
4	5	1	7(54%)	0	0	38
5	6	7	0	0	0	46
6	10	3	0	0	0	77
7	6	7	0	0	0	46
8	8	0	3(23%)	2(15%)	0	62
9	11	0	0	2(15%)	0	85
10	13	0	0	0	0	92
11	8	0	5(38%)	0	0	62
12	13	0	0	0	0	100
13	6	3	0	0	4	46
14	7	6	0	0	0	54
15	0	1	12(92%)	0	0	0
16	6	0	7(54%)	0	0	46
17	1	4	7(54%)	0	1	8
18	5	0	7(54%)	0	2	38
19	0	1	12(92%)	0	0	0
20	0	9	4(31%)	0	0	0
TOTAL	145	43	65	4	7	1084
TOTAL %	56	15	26	1	2	56

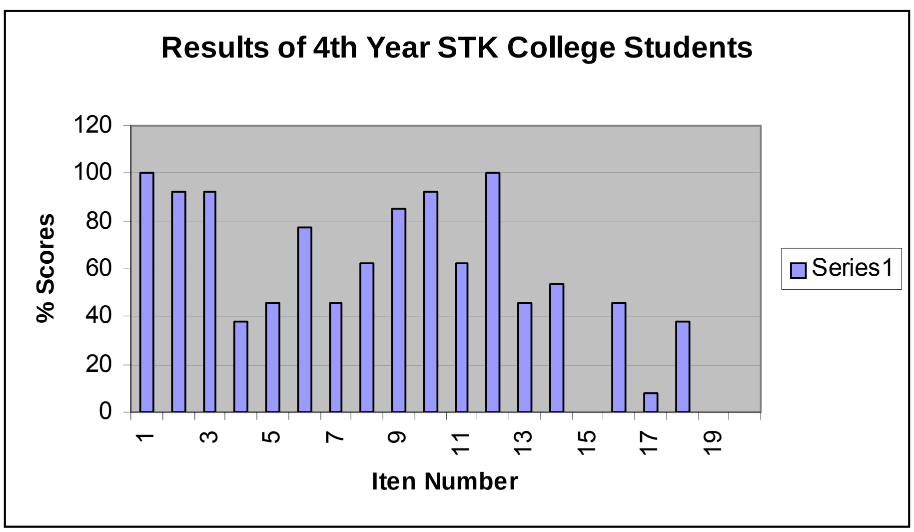
Table 4.6 Results of 4th year STK students

4.6.1 Category frequencies

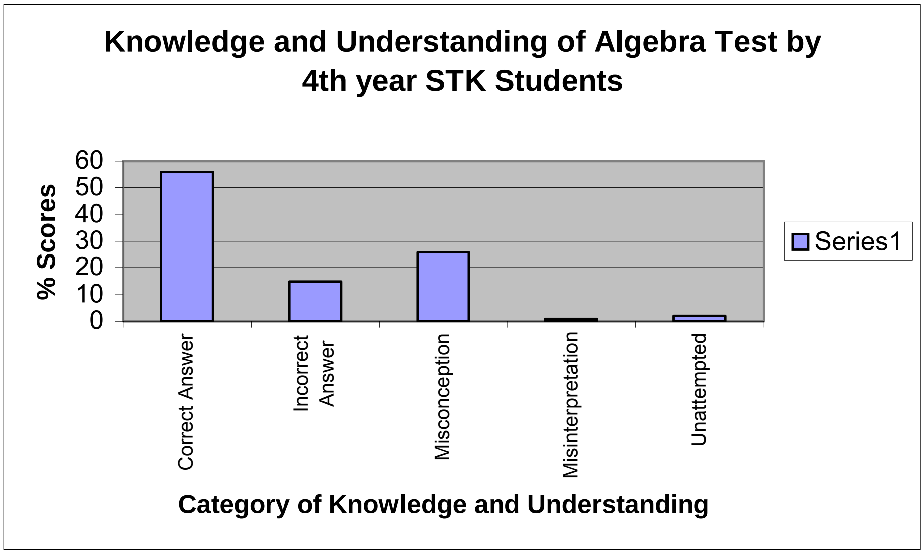
There were 13 participants from this group as shown by Table 4.6. Overall 56% were classified under correct answers. Table 4.6 and Graph 4.11 show that 14% were classified incorrect, 26% classified as having misconceptions, 4% as misinterpreting the items and 3% did not attempt some items. Graph 4.10 shows that items 1,2,3,6,9,10 and 12 were well answered with percentage score over 75%. Items 1 and 12 were perfectly answered with 100% score each. However, items 15,19, and 20 were poorly answered misconceptions. Item 5 showed 46% correct responses with 54% failing to answer the question. Item 7 showed that 46% got the answer correct as against 54% who got the answer wrong. Item 8 was satisfactorily answered with 62% getting the answer correct, 23% showing misconceptions and 15% misinterpreting the question. Item 11 showed 62% correct answers with 35% showing misconceptions. Item 13 showed that 46% got the answer correct and 31% failed to answer the question. Item 14 response was a little above average with 54% correct answers and 46% getting it wrong. Item 16 showed 46% correct answers and 54% classified as having misconceptions. Item 17 was poorly answered with low 8% classified as correct, 54% having misconceptions and 31% getting it wrong. Item 18 recorded 38% correct answers, 54% misconceptions and 15% getting the answer wrong.

4.6.2 **Misconceptions**

A total of 26% responses were classified under this heading (see Table 4.6 and Graph 4.11). These misconceptions were noticed in questions 4,8,11,15-20. In item 4 the misconception was a result of ignoring 0 as one of the answers to that item. 54% of the students failed to write $c < 5$, $c \in R$ but simply wrote the answer as $c = 1,2,3,4$ and therefore were classified as misconception as has been explained earlier in the discussion. In item 8 this is where 23% of the students had tried to simplify $P = 2h + 18$ to $P = h + 9$. In item 11, 38% of the students have answered this question by adding the cost of one apple to the cost of one peach to give the total cost of 90c. This type of misconception was peculiar to these college students. The assumption is that they used **a** and **b** to represent one apple and peach respectively instead of a number of unknown number of apples and peaches.



Graph 4.10 Algebra test results of 4th year STK students



Graph 4.11 Knowledge and understanding of algebra by 4th year STK students

4.6.3 Misinterpretation

Table 4.6 and Graph 4.11 show that 4% had been classified as having misinterpreted items 8 and 9. In item 8, 15% of the students have misinterpreted by trying to find the value of h, which was not asked for. The question demanded only the simplification of the expression arrived at for the perimeter of rectangular figure given. In item 9 some of the participants had gone further to solve for the value of y, which was not what was required to be done.

4.6.4 Classroom observation

The three final year students observed from this group had followed the teacher-centred approach. These student teachers first introduced the lessons by revising the previous knowledge with the class by asking leading questions (See LO4STK2, LO- lesson observation, 4- Fourth year, STK- Name of College, 2- Student number). The actual lessons then followed where the teachers went through examples and afterwards gave them exercises to do similar to the examples done on the chalkboard. These teachers had been asking questions as they developed the lessons and expected the learners to answer them. Where the learners failed to answer the teachers supplied them with the answers. In one of the lessons this teacher was going through the worked example from a textbook, which the learners also had copies of and at the end asked the learners to work through the exercises, which followed from the textbook. Below see what happened as I observed the lesson in the classroom (LO4STK2). This teacher asked all

the learners to close their textbooks (New Dimension in Mathematics Std 7) and look at the chalkboard as he went over a worked example from the textbook. He however checked their previous knowledge of the distributive law, addition of like and unlike terms, removal of brackets by asking them orally and getting chorus answers from the learners. The topic of the lesson was Identity $(a + b)^2 = a^2 + 2ab + b^2$. The teacher wrote $(a +b)^2$ in the form $(a + b)(a + b)$ and expanded in the form $a(a + b) + b(a + b)$ to arrive at $a^2 + 2ab + b^2$. The teacher then worked through two examples (1). $(x + 4)^2$ and (2). $(4x + 3y)^2$ and learners were asked to open their textbooks and do some class work exercises. Where a learner was faced with a problem this teacher had to stop the whole class and tried to explain or solve the problem on the chalkboard. I was expecting the teacher to have used another alternative method of expanding, for example the geometrical approach (scaffolding). This would have contributed to the conceptual understanding of this concept and would have demonstrated to the learners the alternative ways of solving this type of problem. Again from the examples I expected the teacher to ask the learners if they could deduce anything from them (i.e. to square a binomial expression you square the first term add two times the product of the two terms and add the square of the second term).

Excerpt LO4STK2

S: Do you know the meaning of $(a + b)^2$?
Ls: Yes
S: Can somebody write on the chalkboard another way of writing $(a + b)^2$.
L: (One learner stood up and wrote on the board $(a + b)^2 = (a + b)(a + b)$.
S: Good, now we are all going to remove the bracket using the distributive law.
L: Yes teacher
S: (Teacher wrote $(a + b)(a + b)$ and asked the class to use the distributive law to expand)
Ls: Learners in chorus said $a(a + b) + b(a + b)$
S: Let us remove the brackets, he began by writing a^2 and the learners continued telling the teacher the remaining terms
Ls: $a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.

4.6.5 Pedagogical content knowledge

These teachers pedagogical content knowledge did not include the command of the use of alternative representations (See 4.6.4). Student STK2 in excerpt LO4STK2 had only used the method found in the given textbook. The approach used by this teacher could not contribute to conceptual understanding of the topic by the learners. Learners were bound to follow the rules and procedures used by this teacher to solve problems on their own. The skills of this teacher to select and adapt alternative representation of the subject matter to meet the needs of the learners were lacking, in the sense that he could not relate the topic to the real world situations and also give different examples apart from the ones in the textbook. This teacher could not develop debates among his learners. In the context of presentation of the lesson this teacher could not introduce any new concepts. Most of the teaching was done through instrumental (Shulman, 1986) or procedural approach where the teacher just went through worked examples with the learners and in the end asked the learners to do similar problems. Alternative presentation of a solution to a problem is very important as far as knowledge and understanding of algebra is concerned. As was reported in Costello (1991) when pupils were asked to evaluate E + 12 when given E + 17 = 36 some pupils clearly evaluated E from E + 17 = 36 to get 19 and the substituted in E + 12. Others however, reason it out that the left hand side of E + 17 = 36 had been reduced by 5 hence the right hand side must be reduced by 5 to arrive at an answer of 31. Similar problem was addressed in the algebra test where these different approaches had been applied. In this example reported by Costello (1991) the second approach was easier and simpler than the first approach. It is therefore necessary for teachers of algebra to use alternative approaches to problems so that learners will decide and apply the more efficient and fast strategies when faced with a problem like one in item 5 where the students were asked to find the value of x in $(5x + 1)^3 + 5x = 349$ given $(x + 1)^3 + x = 349$. It was easier to replace x = 6 by 5x = 6 to arrive at the answer instead of expanding $(5x + 1)^3$ which almost all the students who had gone that route got the answer wrong.

4.6.6 **Comparison of the algebra test results for the three groups from College STK.**

4.6.6.1 **Overall percentage scores.**

Year of students	%Overall scores	% Misconceptions	% Misinterpretation
1^{st}	50	20	4
3^{rd}	52	23	1
4^{th}	56	26	1

Table 4.7 Overall percentage scores of students per course.

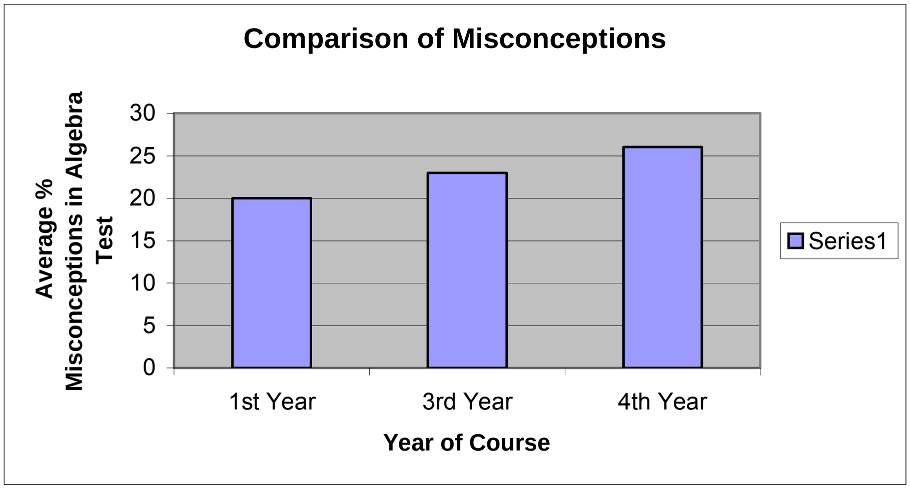
Table 4.7 shows that the percentage scores of the student teachers increased as the number of years at College increased. 50% correct response to the algebra test for the first year College students, 52% for the 3^{rd} year students and 56% for the 4^{th} year group. This should be expected since more teaching is supposed to have taken place according to the number of years at the College. However, the percentage difference between the first year group and the 3^{rd} year group is too small for the extra two years tuition by the 3^{rd} year students. Same comment can be given for the 4^{th} year group who had spent four years at the College. The percentage misconceptions were rather astonishing as the 4th year group showed a bigger percentage than the first and third year groups. The first year group showed 20% misconception, while the 3rd year group showed 23% and the fourth year 26%. This can be explained that new knowledge might have interfered with by the old knowledge retrieval. However the College of Education course is there to improve on the knowledge acquired from the matric education. It is hence surprising for the 4th year students to have had more misconceptions than the first year group, who had just entered the tertiary institution.

The percentage misinterpretation was however small with the first year group recording 4% and the other groups 1% each respectively. This is likely to happen as the more one studies and is exposed to instructions in English the better his or her chance of improving on understanding of some of the English words used in algebra. This assumption is made because the medium of instruction in the Colleges of Education is supposed to be English and most of the lecturers for mathematics are foreigners.

4.6.6.2 **Comparison of the misconceptions according to items.**

Item	1st Year (%)	3rd Year (%)	4th Year (%)
4	36	40	54
7	8	0	0
8	13	15	23
11	58	53	38
15	71	80	92
16	63	60	54
17	58	70	54
18	0	45	54
19	50	45	92
20	0	45	31
Overall Total %	20	23	26

Table 4.8 Comparison of misconceptions.



Graph 4.12 Comparison of Misconceptions from College STK

The overall misconceptions according to Table 4.8 and Graph 4.12 increased according to the number of years spent at the College, 20% for 1st year students, 23% for the 3rd year students and 26% for the 4th year students. Similarly in item 4 the percentage misconception increased as the number of years at College increased. The first year shows 36%, the third year 40% and the fourth year 54%. One can interpret this result to the fact that integers are treated well at the matric level so is likely to be fresh without much interference of any new learning from the College. As the year of College study increased it might mean that the learning of different concepts might have interfered with the old learning. This comparison is based on the test data alone without knowing how comparable the cohorts of the students were when entering the colleges. The 4th years might already have been better than the others when in their 1st year. In item 7 only the first year students had misconceptions and this might be attributed to the fact that English as a second language could not have been mastered well, thus confusing “more than” with “multiply by”. Item 11 showed that the fewer the years spent at the College the more misconceptions you have as is reflected here: that the first year showed 58% misconceptions, the third year 53% and the fourth year 38%. This is the misconception, which is attributed to letters used as objects instead of numbers. In item 15 the trend is opposite of item 11, the percentage misconceptions increased as the years at the College increased. The percentage misconceptions generally was very high; the first year students showed 71%, the third year 80% and the fourth year students as high as 92%. Inequalities of this form are not treated well at the College level as it is assumed that they might not teach it at the level they are supposed to teach after leaving the College. The first year students showed a smaller percentage than the other year groups, may be because this topic might have been fresh from the studies at the matric level. The overall analyses showed that this topic is not treated well both at the matric and the College levels.

Item 16 also showed very high percentages for all the groups. The first year showed a bigger percentage followed by the third and fourth years respectively. The first year showed 63%, the third year 60% and the fourth year 54%. Finding the square root of a square variable is treated at the matric level but most of the applications had been for numerical values; hence the first year students are more likely to have misconceptions than the fourth year students. In item 18, the first year students did not show any misconception, but the third and fourth year students showed misconceptions. The third year showed 45% as compared to 54% for the fourth year students. Finding the square of $\sqrt{(7 - m)} + m = 5$ is treated well at the matric level and therefore was likely to be fresh in the minds of first year students therefore do not have misconceptions. New learning at various levels might have interfered with the old learning therefore the misconceptions. In item 19 the percentage result is difficult to analyse since the trend did not follow any pattern. The first year students showed 50% misconceptions, the third year college students showed 45% and the fourth year students a high 92%. One can only infer that 3 years of new learning must have interfered with the 4th year group. In item 20 again the first year students did not show any misconceptions but the third year showed 45% and the fourth year 31%. This implies the first year students could differentiate between quadratic equation and quadratic expression better than the third and the fourth year students. This can be attributed to this topic being an examinable topic at matric level.

4.6.6.3 **Comparison of misinterpretation according to items**

Item	1st Year (%)	3rd Year (%)	4th Year (%)
7	4	0	0
8	8	0	15
9	17	0	15
10	21	20	0
14	0	0	4
17	0	1	0
20	38	0	0
Overall total %	4	1	1

Table 4.9 Comparison of misinterpretation.

From Table 4.9, in item 7 only the first year students showed misinterpretation, there was 4% misinterpretation where they had misinterpreted “more than” to mean “multiply by”. In item 8 however, 8% of the first year students had misinterpreted perimeter for the area of a rectangle. The same misinterpreted was noticed from the 4th year students. None of the students from the third year misinterpreted this item. Item 10 had been misinterpreted by some of the first and third year students to mean, “solve for”. The first year had 21% and the third years had 20% misinterpretations respectively. Item14 had been misinterpreted by only the 4th year students and the percentage was 4%. Similarly only 1% of the third year students had misinterpreted item 17. In item 20 as many as 38% of the first year students had misinterpreted it while the other groups showed no misinterpretation of the questions.

4.6.6.4 **Comparison of correctly answered items**

ITEM	1st year % score	3rd Year % score	4th Year % score
1	100	100	100
2	88	90	92
3	83	90	92
5	42	60	46
10	71	70	92
12	100	95	100
13	25	45	46
14	42	45	54

Table 4.10 Percentage scores of correctly answered items for College STK.

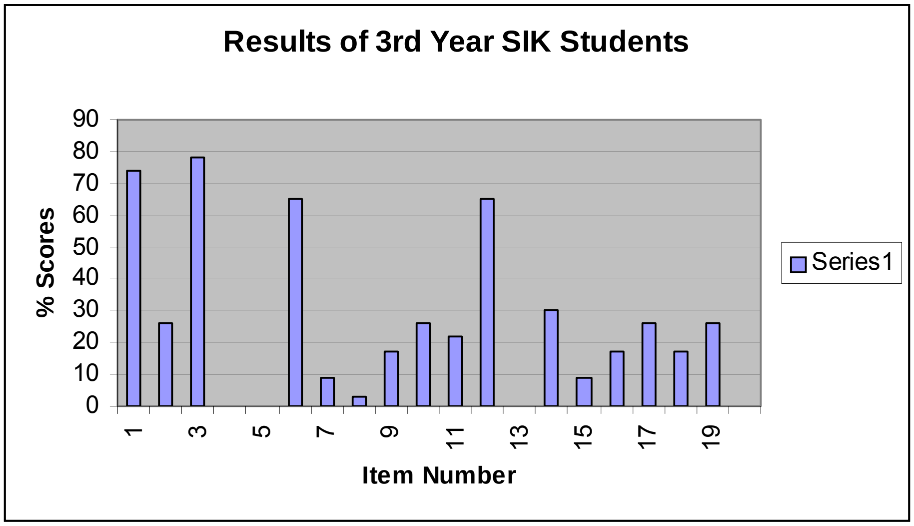
This comparison (See Table 4.10) is about items where there were no misconceptions or misinterpretations recorded. It is about either getting the question correct or getting the answer totally wrong. In item 1 as is shown in Table 4.10 all the participants scored 100%. This result conveys the message that all the participants had a sound knowledge and understanding of this concept. In item 2, the score was 88% high for the first year students with a 2% increase as the number of years for the students also increased. This high percentage scores again portray the knowledge and understanding of the concept of substitution. In item 5 the understanding of different ways of solving a problem was tested here and participants who had gone by the expansion method could not get the answer correct and really wasted time according to the scribbles seen on the answer sheets. On the whole the percentage score was satisfactory with a minimum percentage score of 42 and the maximum of 60%. In item 10 the percentage scores were high with the first years scoring 71%, the second years 70% and the 4th years 92% respectively. In this item the translation of a word problem using a letter was being tested. This can be interpreted to mean the number of years at the College might have contributed to improving their translation of word problems to symbolic form. Item 12 showed very high percentage scores by all the participants indicating that they had a sound knowledge and understanding of writing formula. In item 13 the definition of a function was what was required of them, however, the first year percentage score of 25% indicates that they could not define a function. More than half of the third and the fourth year students could also not define a function. The increase on the percentage scores over the first years by the 3rd and 4th year students show that there had been an improvement on the understanding of function at the College.

4.7 ANALYSIS OF 3RD YEAR SIK COLLEGE RESULTS

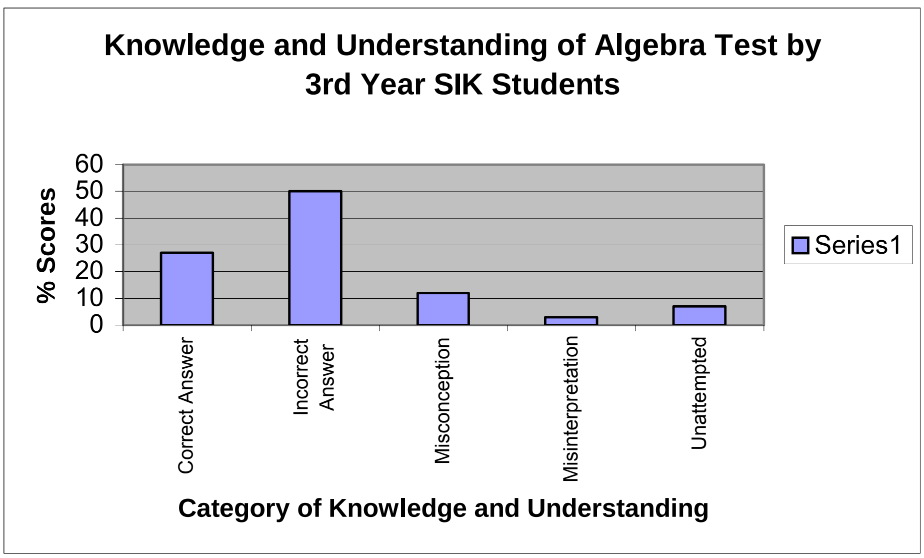
Table 4.11 shows the results of College SIK. Number of students = 23

Item Number	Correct answer	Incorrect answer	Misconception	Misin terpretation	Un attempted	Percentage correct
1	17	6	0	0	0	74
2	6	17	0	0	0	26
3	18	5	0	0	0	78
4	0	19	4(17%)	0	0	0
5	0	19	0	0	4	0
6	15	8	0	0	0	65
7	2	17	6(26%)	0	1	9
8	8	9	0	0	6	3
9	4	12	2(9%)	2(9%)	3	17
10	6	11	0	3(13%)	3	26
11	5	5	10(43%)	0	3	22
12	15	8	0	0	2	65
13	0	18	0	0	5	0
14	7	13	0	0	3	30
15	2	11	7(30%)	0	3	9
16	4	10	9(39%)	0	0	17
17	6	10	7(30%)	0	0	26
18	4	13	6(26%)	0	0	17
19	6	12	5(22%)	0	0	26
20	0	23	0	0	0	0
TOTAL	125	234	56	5	32	541
TOTAL %	27	50	12	3	7	27

Table 4.11 Results of 3rd year SIK students.



Graph 4.13 Algebra test results of 3rd year SIK students



Graph 4.14 Knowledge and understanding of algebra by 3rd year SIK students

4.7.1 Category frequencies

From Table 4.11 this institution performed badly with the test. The overall percentage is 27%. None of the items were correctly done by any of these students. Table 4.11 and Graph 4.14 show that 50% were classified under incorrect, which means the answers could not be placed under misconception or misinterpretation. 12% of them were classified under misconception and 3% under misinterpretation. Table 4.11 and Graph

4.14 show that 7% of the participants have failed to attempt some of the items like items 5, 7-15. In item 5 as many as 4 (17%) of the students left the item unanswered. In item 7 only one (4%) student failed to answer. In item 8, six (26%) of the students did not answer. Three (13%) each of the students failed to answer item 9-11 respectively. Two (9%) of the students failed to answer item 12 and five (22%) students also failed to answer item 13. Three (13%) students each failed to answer items 14 and 15 respectively. In items 4,5,13,20 the percentage scores were 0%. It is only in items 1,3,6,and 12 that students from this group could score more than 60%. In items 7,8, and 15 the percentage scores were below 10%. On the other hand items 16-20 were attempted by all may be because they had to tick the answers and therefore could have guessed. Items 1-15 were open statements and might not have given them the chance of guessing; therefore they left them blank.

4.7.2 **Misconceptions**

Table 4.11 and Graph 4.14 show that 12% overall had displayed some misconceptions from the answers given on their scripts. In item 4 none of these participants got the answer correct. 17% have shown some misconceptions. These participants had failed to write $c < 5$, $c \in R$ as answers to that item. The interview revealed that they only considered natural numbers. The excerpt (See I3SIK1, I-Algebra Test Interview, 3-Third year student, SIK- Name of college, 1- Student number) below will support this statement.

I: You wrote the answer as c =1,2,3,4
S: Yes
I: Why, can you explain?
S: I solved for the value of c from the two statements given.
I: What are the two statements?
S. She wrote c + d = 10 and c < d
I. Then what followed?
S. From c + d = 10, I could see that if c = d, then **c** will be equal to 5 as well as **d** too. But it is said **c** is less than **d** hence I wrote the answer like that.
I. Can you tell me why you started from 1?
S. Because 1 + 9 = 10.
I. Any other number you can think of?
S. No other number sir.
I. What name do you give to number 1,2,3,4?
S. Natural numbers
I. Why do you use natural numbers?
S. Do you mean I can use other numbers?

I. Yes that is exactly what I want you to come up with.

S. I see, (then she tried 0 and found it satisfied the conditions of the problem). I can add 0 to the values already put down.

I. Yes, you were not limited to only natural numbers, the answer includes all real numbers less than 5.

In item seven also 26% of these participants had arrived at AB = 4x cm. This has been termed misconception because they had added x to 3x to give 4x. The 3x had come out of "3 more than x" presumably. Interview excerpt (See I3SIK1, I- Algebra Test Interview, SIK- Name of College, 1- Student number) will justify that from below.

I. CB is 3cm more than AC, what is AC?
S. AC = x
I. What is CB?
S. CB = 3x
I. Why 3x?
S. Because CB is three more than AC
I. What is CB if CB is 3 times bigger than AC?
S. 3x
I. Are the two statements 3 more than and 3 times bigger the same?
S. Not exactly
I. What is the difference then?
S. Failed to respond
I. What is three times 4?
S. 12
I. What will 3 more than 4 be?
S.7
I. Can you see the difference between the two?
S. Yes
I. Then 3 more than x will be equal to what?
S. x + 3
I. Then AB will be equal to what?
S. She wrote x + x + 3 = 2x + 3.

In item 9, the 9% (two students) of the answers placed under misconception was also attributed to the statement 5 years older than. This student had a similar problem like "more than". In item 10 as many as 10 (43%) out of 23 students had written the answer as a + p = 90c. This answer has been discussed to be a misconception from the earlier analyses. This is where these students were assuming the cost of only one item instead of the cost of many items. In item 15, 30% of the answers were classified under misconception as a result of these students multiplying 3-x to both sides of the inequality just like an equation. In item 16, 39% of the students had ticked (a), that is x = y, which implies these students considered only the positive numbers. In item 17, 30%

had ticked (b) which go to support the notion that students solve inequality as a linear equation. In item 18, by failing not to include statement number one brings about misconception. Hence those students who had ticked (c) as the answer were classified as having misconceptions. In item 19, by ticking b and c as the answer implies those students had considered only positive numbers when verifying the statement to be true or false, these 22% were classified as having misconceptions.

4.7.3 **Misinterpretation**

Table 4.11 and Graph 4.14 show that 3% of the answers were classified as misinterpreted under items 9 and 10. In item 9, 9% of the answers had been classified as misinterpreted because the students had tried to solve for the value of y to arrive at an answer of y = -5/2. The question did not ask for the numerical value of y but these students had misinterpreted to solve for the value of y. This very answer should have been rejected if these students' knowledge and understanding of algebra were on a sound footing. They should have realised that the age of person could not be a negative number. In item 10 these students had tried to solve for the value of x, which the question did not ask for. This therefore was termed misinterpreted according to the classifications for this research.

4.7.4 **Classroom observation**

The student teachers had planned their lessons in different ways, but most were based on teacher centred approach. What I will discuss here is a particular lesson by one of the students. The topic of the lesson was expansion of binomial expressions. The teacher started with the four fundamental operations of multiplication. It was observed from the ways the lesson had been planned that the teacher was aware of some of the misconceptions pertaining to this topic. He had tried to involve the whole class through activities. He again tried to use real world example to clarify some concepts in the simplification of algebra. The teacher had used a worksheet where he asked the learners to complete a certain table involving multiplication of negative and positive numbers. The generalisation was written on the chalkboard.

He translated the multiplication of negative and positive numbers or terms into ordinary words incorporating realistic type of approach to the teaching of this concept e.g.

- x + = -
+ x - = -
+ x + = +
- x - = +

The teacher related these to the real world analogy by calling negative numbers, "enemy" and positive numbers "friend"

From the above the resulting analogy was (See LO3SIK3, LO- Lesson observation, 3-Third year, SIK- Name of College, 3- Student number),

the enemy(-) of my friend(+) is my enemy(-)
the friend(+) of my enemy(-) is my enemy(-)
the friend(+) of my friend(+) is my friend(+)
the enemy(-) of my enemy(-) is my friend(+)

This particular lesson at this stage related to the constructive principles. This teacher however, had approached the expansion of the square of binomial expression using the teacher-centred approach. The teacher showed on the chalkboard how to expand $(a + b)^2$ to get $a^2 + 2ab + b^2$. He wrote $(a + b)^2$ in the form $(a + b)(a + b)$ and arrived at the desired identity. Upon asking the learners to expand a similar expression of the form $(x + a)^2$ this teacher detected an error by one of the learners and hence wrote on the chalkboard what that learner had written.

'$(x + a)^2 = (x + a)(x + a)$………..(1)
$= x(x + a)a(x + a)$………(2)
$= x^2 + 2ax + b^2$ …………(3)'

The teacher then asked the learners whether the solution was correct. The reply from the majority of the learners was that the solution was correct. He then called on one of the learners who said the solution was not correct to explain why the solution was like that. This student said the solution was not correct because in statement (2) there was an omission of the '+' sign. One learner argued that it did not matter so far as the end result was correct. The teacher then intervened by saying the omission of the '+' sign matters a lot as the result would have been different if a ' – ' sign was the one which was omitted then it would have meant a wrong answer had been arrived at. The teacher drew their attention to the analogy of enemy and friend where '– x +' would have given a negative answer. The teacher then emphasised that the positive and the negative signs for the second term in the first bracket should not be omitted when used to multiply the second bracket in statement 2 above. This teacher used this particular error to draw the attention of all the learners to this type of error some learners make rather than overlooking it. At the end of the lesson this teacher tried to summarise by writing down on the chalkboard the expansion of three expressions dealt with in the class.

$(a + b)^2 = a^2 + 2ab + b^2$
$(a + x)^2 = a^2 + 2ax + x^2$ and
$(x + 3)^2 = x^2 + 6x + 9$

He then asked the learners to explain what they saw on the right-hand side of the identities in relationship to the left-hand side. The learners could not detect the relationship until the teacher led them to see this relationship (see the excerpt

LO3SIK3, LO- Lesson observation, 3- third year, SIK- Name of College, 3- Student number).

S. How many terms do you see on the right-hand side $(a + b)^2 = a^2 + 2ab + b^2$?
L. Three terms (in chorus)
S. On the right-hand side what happened to the first term on the left-hand side?
L. It is the squared of **a.**
S. What do you notice with the middle term on the right-hand side?
L. It is two times the product of the two terms **a** and **b** on the left-hand side.
S. What do you notice with the last term on the right-hand side?
L. The last term is the square of b.
S. Do you now see the relationship?
L. Yes sir (in chorus)
S. Can one of you express the relationship in words?
L. Quiet
S. The right-hand side is the square of the first term plus two times the product of the two terms plus the square of the last term.

The teacher asked the learners to repeat after him this relationship. At the end of the lesson he gave similar examples for the learners to do in their class work exercise books.

4.7.5 Pedagogical content knowledge

The teacher whose lesson is discussed above (See 4.7.4) had tried to use a real world analogy to explain why when you multiply terms with like signs you get a positive answer while multiplying terms with opposite signs yields a negative result. The teacher being aware of the error made by learners with the omission of positive or negative signs when multiplying terms in a bracket decided to draw the whole class attention to it. He had noticed that one of the learners was making that mistake. He referred them to the analogy of enemy and the friend he had talked of earlier.

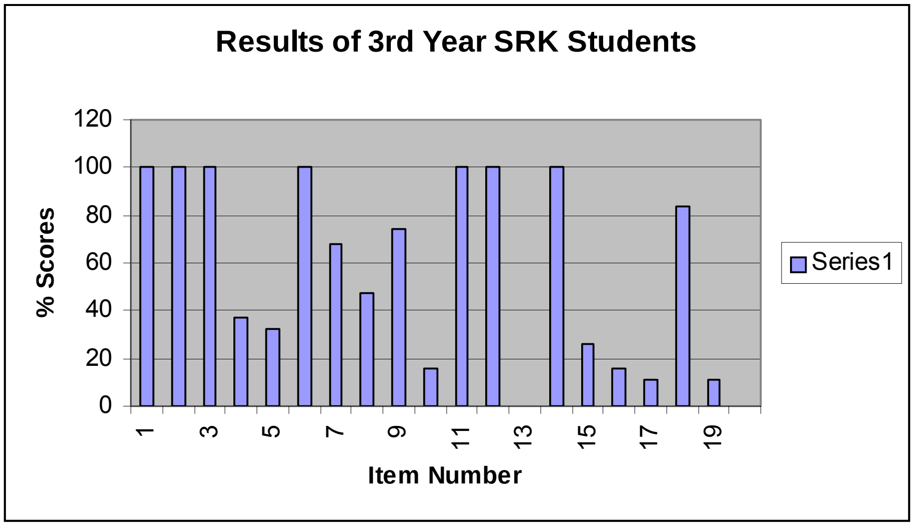
The approach by the teacher in LO3SIK3 relates to Vygotsky (1968) notion of scientific and spontaneous concepts. For Vygotsky the scientific and spontaneous concepts are related and contribute to the development of learning in schools. Vygotsky argues that true concepts are formulated as a result of combination of the individual experience together with generalised knowledge and these lead to the formation of meaningful, reflective and flexible thinking. He was able to use the analogy of the enemy and the friend to make the multiplication of operation signs clear. He further tried to make the learners deduce the expression for the expansion of a binomial expression by the geometric approach, which in essence is likely to bring about conceptual understanding of this topic.

4.8 ANALYSIS OF 3RD YEAR SRK COLLEGE RESULTS

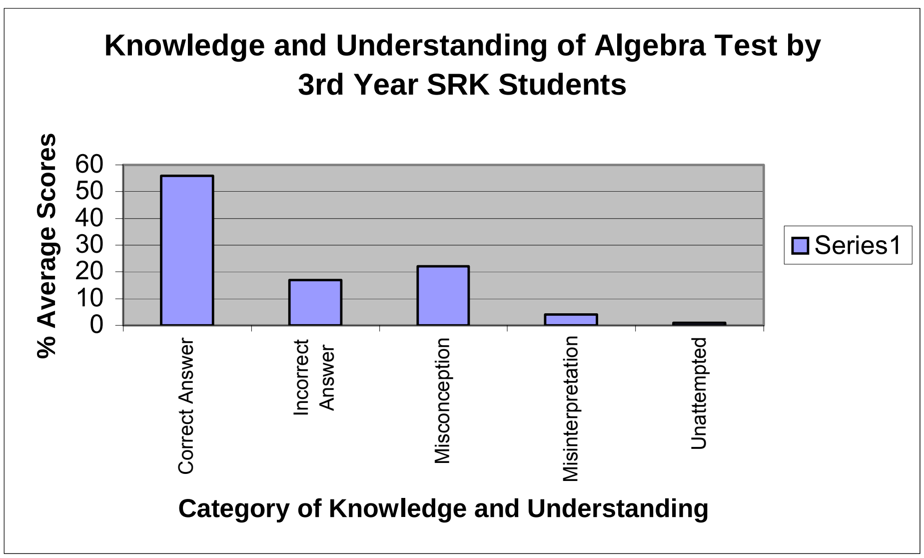
Table 4.12 shows the results of College SRK. Number of students = 19

Item Number	Correct answer	Incorrect answer	Misconception	Misin terpretation	Un attempted	Percentage correct
1	19	0	0	0	0	100
2	19	0	0	0	0	100
3	19	0	0	0	0	100
4	7	3	9(47%)	0	0	37
5	5	11	0	0	2(11%)	32
6	19	0	0	0	0	100
7	13	1	5(26%)	0	0	68
8	9	4	6(32%)	0	0	47
9	14	1	4(21%)	0	0	74
10	3	0	0	16(84%)	0	16
11	19	0	0	0	0	100
12	19	0	0	0	0	100
13	0	19	0	0	0	0
14	19	0	0	0	0	100
15	4	7	8(42%)	0	0	26
16	3	0	16(84%)	0	0	16
17	2	0	17(89%)	0	0	11
18	17	0	2(11%)	0	0	84
19	2	9	8(42%)	0	0	11
20	0	8	11(58%)	0	0	0
TOTAL	213	63	86	16	2	22
TOTAL %	56	17	22	4	1	56

Table 4.12 Results of 3rd year SRK students



Graph 4.15 Results of 3rd year SRK students



Graph 4.16 Knowledge and understanding of algebra by 3rd year SRK students

4.8.1 **Category frequencies**

Table 4.12 and Graph 4.16 show 56% of the items were responded to correctly. 17% were classified under the incorrect category. 22% seemed to have misconceptions, 4% misinterpreted some of the items while a low 1% failed to attempt some of the items in particular item 9 (See Graph 4.15). Table 4.12 and Graph 4.15 show that all the students answered items 1,2,3,6,11,12 and 14 correctly giving a 100% result. Items 13 and 20 on the other hand were not correctly answered by any of the students. Item 5 was poorly done but the responses could not be classified under either misconception or misinterpretation. Two (11%) of the students did not attempt this question either because they did not know what to do or because it was difficult for them. Item 7 was answered correctly by 68% of the participants from this college. Item 8 was however, not well answered, to the extent that, only 47% answered the question correctly. Item 10 was poorly done with the percentage score of only 16. Item 15 was also not well done with low 26% correct answers. Items 16 and 17 were poorly answered with the percentage score below 20. Item 18 was on the other hand well answered with 84% correct answers. Item 19 on the contrary to item 18 was poorly done with a percentage score of 11%.

4.8.2 Misconceptions

From Table 4.12 and Graph 4.16, in the overall 22% were classified to have had misconceptions. These were found in items 4,7,8,9,15,16,17,18,19 and 20. In item 4, 47% of the responses were classified as having misconceptions because these students failed to write $c < 5$, $c \in R$ as the answer. These students had written the answer as c = 1,2,3,4. In item 7 which had 26% misconception, the students had written 4x as the answer instead of 2x + 3. This misconception has been explained in the earlier analysis where these students interpreted “more than” to mean “multiply by”. Item 8 also show 32% of the responses to be under this category. This is where one (5%) student had used the area formula of the rectangle for the perimeter of a rectangle and the remaining 27% had arrived at the answer 2h + 14. This is where these students had failed to multiply the two items from the perimeter formula p = 2 (l + b) to arrive at 2h + 28. This student just multiplied the first term in the bracket by 2 forgetting about the second term. In item 9 some of the students had written the answer as 6y instead of 2y + 5. This again can be explained as in item 7 where the students failed to write “more than” in its correct perspective. This accounted for 21% of the responses classified under this category. In item 15 some of the students had answered this question by treating it as a linear equation. 42% of the responses had solved this question in the form of a linear equation, thus arriving at x > 5 as the answer. In item 16 as many as 84% had fallen into this category. These students had not recognised negative numbers as part of the answers; hence the misconception of using only positive numbers. Item 17 also showed 89% misconceptions as students treated the root of a square inequality like the root of a square linear equation. In item 18, 11% of the respondents had ticked (a) and (b) as having mistakes. If (a) was having a mistake then it implied that statement number 3 is correct where m = 32 came as a result of 7 + m = 32 which is not correct. There is a misconception here since instead of subtracting 7 from 25 the student rather added 7 to 25 to give 32. On the other hand the student who had ticked (b) implies that statement 2 was correct. This concurs with the misconception of adding

or subtracting unlike terms, – m is different from m^2 and therefore cannot be added together. 42% of the responses in item 19 were also classified to have had misconceptions. Answering the question under those statements based on x > y the students failed to use both the positive and negative numbers to check the truth of those statements. They relied only on positive numbers to check the validity of those statements, which is termed as misconception since negative numbers also had to be applied. In item 20, 58% could not differentiate between an expression and an equation. These students had included
x = -3/2 and x = 1 as part of the statements which denote an expression, which is not true.

4.8.3 **Misinterpretation**

Table 4.12 and Graph 4.16 show that 4% of the responses were classified under this category. These were found only in item 10 where the students tried to solved for the value of x in the equation 3x + 4 = 31. This question, as has been explained earlier did not require the value of x, it rather required the answer to be in the symbolic form
3x + 4 = 31.

4.8.4 **Classroom observation**

One of the teachers observed from this college SRK was teaching the topic "Introduction to algebra", to grade 8 learners in a class of 42 learners. It was a double period lesson, which took 70 minutes. The teacher started the lesson with a brief history of algebra, where he talked about people like the Chinese, Persians, and the Hindus who had used algebra thousands of years ago. This type of introduction really drew the attention of learners to the fact that algebra is not a Western type of mathematics and that many countries had used algebra before Western Europe.

This teacher had tried to use the geometrical approach as an example to introduce algebra. He used the example of finding the perimeter of a rectangle. He first started with the finding of the perimeter of a rectangle with numerical values, e.g. a rectangle with sides 3 and 5 units. He asked the learners the perimeter of the rectangle, which he had drawn on the chalkboard with sides 3 and 5. He wrote p =? and waited for the learners to supply the answer. One learner said 3 + 5 +3 + 5. The teacher completed by writing p = 3 + 5 + 3 + 5 = 16 (answer given by the learners).

The teacher then went further with an example of a rectangle with sides **a** and **b**, which was drawn on the chalkboard. The teacher together with the learners came to the conclusion that p was equal to 2(a + b). This teacher then tried to find the difference between the two sums discussed with the learners. The excerpt below depicts what happened in that classroom (See LO3SRK1, LO- Lesson observation, 3- Third year, SRK- Name of College, 1- Student number).

(S stands for the student teacher and L stands for a learner).
S: What is the difference between these two examples above?
L: The first example involves only numerical values while the second example involves numbers and letters.
S: Do you know the values of the letters?
L: Yes, they can be any number
S: So can the numbers –2, -3, -4, and 0, be some of the values of the letters?
L: No, because there is nothing like negative or zero length and breadth of a rectangle.
S: Yes, you are correct, do you all agree class?
Ls: Yes teacher (in chorus).
S: These examples show the difference between arithmetic and algebra (student teacher repeated that arithmetic involves numbers only while algebra involves numbers and variables or letters)

These two examples had been used to show the difference between arithmetic and algebra, however, the impression that was created was that variables could not be zero or negative numbers. This is a misconception, and as has been stated earlier in this chapter, trying to use concrete examples or models could lead to confusion and misconceptions. This teacher had gone further to use a different concept in his teaching resulting in compounding the confusion and the misconceptions. The excerpt below illustrates that (See LO3SRK1).

S: We have seen that we can use the formula A = l x b to obtain the perimeter of a rectangle. To determine the value of A we substitute values for l and b. (He wrote on the chalkboard, if l = 20 and b = 6, determine the value of A. He replaced l and b by 20 and 6 in the formula and wrote
A = l x b
= 20 x 6
= 120)

This teacher had confused the area formula with the perimeter formula of a rectangle. This can be traced back to the algebra test where some of the College of Education students answered item 7 in a similar manner. After the lesson I drew the attention of this teacher to the error made by using area formula for the perimeter of a rectangle. This teacher argued with me that it was not a mistake because he had copied that example from a textbook (he showed me that textbook with the same example written in it). The excerpt PI3SRK1 illustrates what transpired in the interview. (PI - Post interview, 3-Third year, SRK- Name of college and 1- Student number)

I. Did you notice anything in the use of that formula?
S. No
I. You said the perimeter of a rectangle is A = l x b

S. Yes, what is wrong with that?
I. There is something wrong, A = l x b is the formula for the area of a rectangle and not the perimeter.
S. But I got it from the textbook, which I can show you.
I. That means the textbook is also wrong, the perimeter is the addition of all the sides of any object (the teacher showed me this example from the textbook).
S. Yes
I. How will you then define a perimeter of a rectangle?
S Perimeter is two times the length plus the breadth.
I. How did you come with this definition?
S. (Quiet for sometime), but that is the formula for finding the perimeter of a rectangle.
I. I mean the definition not the formula
S. Quiet (could not give the definition)

This supports the notion that without the proper subject matter knowledge one is prone to having to accept this type of mistakes without challenging or altering such misconceptions. This again gives the impression that lack of proper subject matter knowledge is likely to make some teachers accept things from textbooks as irrefutable and cannot be challenged.

4.8.5 Pedagogical content knowledge

The students observed from this college had made use of the talk-and-do type of teaching to teach algebra. They by this type of approach gave examples, solved them on the chalkboard by asking leading questions from the learners to make it appear as if the learners were participating in the lesson. From the example of this teacher who was teaching introductory algebra, the teacher asked questions on arithmetic sums like, what is 3 + 5 + 3 + 5? which the learners answered as 16. Even with some examples the teacher led the way by answering some of the questions himself. For example when the teacher was finding the perimeter of a rectangle with sides a and b, the teacher had written (2a +..) expecting the learners to complete by saying 2b. The teacher asked again a leading question expecting the learners to respond with a yes answer. The excerpt below shows this (See LO3SRK1).

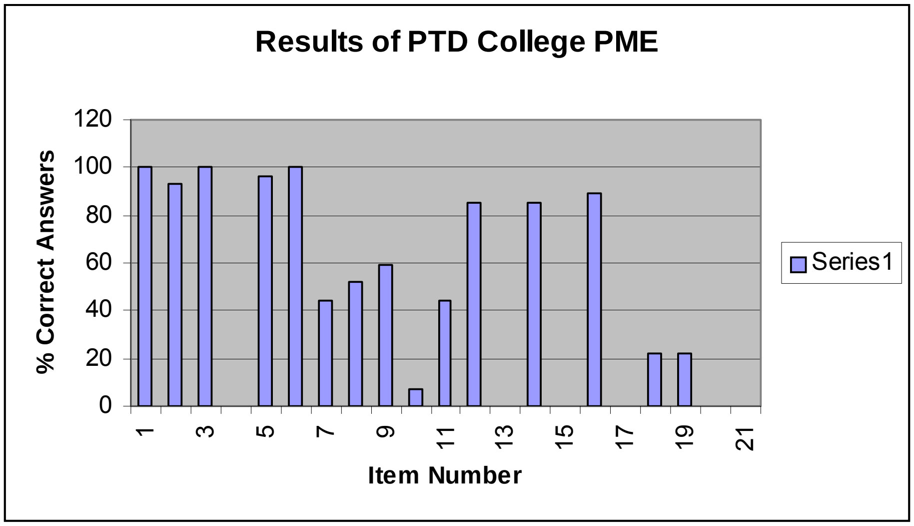
S: You see here 2 is common
L. Yes sir
S. Therefore we can take the common term out
L. Yes sir

The teacher completed the sum by writing 2(a + b). This teacher did not show the learners how 2a + 2b became 2(a + b). This was an introductory lesson to algebra so I expected every step of the lesson to be explained to the learners. On the other hand, I appreciated the way the teacher introduced algebra by bringing in the history of algebra and drawing their attention to the fact that algebra did not originate from Western Europe but that algebra had been used in some countries many years ago before the Western Europeans came to use it. The idea of using geometrical approach to introduce algebra was also there to show how algebra is integrated to other areas of mathematics. However, this approach was creating some misconception or confusion as the area formula was mixed up with the perimeter formula. This approach had made it easier for the teacher to show the difference between arithmetic and algebra by the learners drawing their own conclusion.

4.9 ANALYSIS OF 3rd YEAR PTD COLLEGE PME RESULTS

Item Number	Correct answer	Incorrect answer	Miscon ception	Misin terpretation	Un attempted	Percentage correct
1	27	0	0	0	0	100
2	25	2	0	0	0	93
3	27	0	0	0	0	100
4	0	19	8(30%)	0	0	0
5	25	2	0	0	0	96
6	27	0	0	0	0	100
7	12	4	2(7%)	7(26%)	2(7%)	44
8	14	2	9(33%)	2(7%)	0	52
9	16	8	0	0	3(11%)	59
10	2	6	0	19(70%)	0	7
11	12	5	10(37%)	0	0	44
12	23	3	0	0	1(4%)	85
13	0	24	0	0	0	0
14	23	4	0	0	0	85
15	0	4	23(85%)	0	0	0
16	24	3	0	0	0	89
17	0	19	8(30%)	0	0	0
18	6	10(37%)	11(41%)	0	0	22
19	6	19(70%)	0	0	2(7%)	22
20	0	3	24(89%)	0	0	0
TOTAL	269	137	95	28	11	998
TOTAL %	50	25	18	5	2	50

Table 4.13 Results of 3rd year PME students.



Graph 4.17 Algebra test results of final year College PME students

4.9.1 Category frequencies

Table 4.13 and Graph 4.17 show the summary results of third year final year students from College PME, it shows that 50% of the 27 students answered the algebra test correctly. Under incorrect answers, 2% of the responses fell into this category, 18% were classified as having misconceptions, 5% as misinterpreted and 25 failed to attempt some of the questions. Items 1, 3, and 6 were perfectly done recording 100% scores respectively. On the contrary in items 4, 13, 15, 17 and 20 none of the answers were classified as correct. In items 2, 5, 12, 14, and 16 the students scored more than 85% correct answers. In item 7 with the 44% correct responses, 7% were classified as having misconceptions, 26% as misinterpretation and 7% failed to attempt the question. In item 8 the responses were slightly above average, on the contrary, 33% were classified as having misconceptions and 7% had misinterpreted the question. In item 9, 59% were classified under category correct answer. The remaining percentage either got the question wrong or failed to attempt the question. The score for item 10 was very poor resulting in 7% correct answers and 70% having misinterpreted the question. Item 11 had a score of 44% correct and 37% showing misconceptions. Items 18 and 19 both show 22% correct answers with majority of the students getting the answers wrong. In item 18, 37% of the students got the answer wrong with 41% having misconceptions. In item 19 as high as 70% of the students' responses to the question were incorrect with 7% failing to attempt the question.

4.9.2 Misconceptions

Items 4, 7, 8, 11, 15,17, 18 and 20 contributed to the overall 18% misconceptions for the algebra test, which is shown by Table 4.13. In item 4 the misconception was noticed when the students failed to write $c < 5$, $c \in R$ as the set of answers required by the question. The excerpt below shows how a student had argued about the value zero during an interview. (See I3PME1, I- Algebra Test Interview, 3- Third year, PME- Name of College, 1- Student number)

(I- Interviewer, S- Student teacher)
I. How did you find the test?
S. It was tough
I. Yes I could see from your score, what was the problem?
S. I have never come across some of the questions before
I. Did you take mathematics to matric level?
S. Yes, but at the standard grade level
I. In item 4 you wrote the answer c = 1, 2, 3, 4
S. Yes and I checked them and they satisfied the conditions.
I. Did you check zero and any other numbers less than 5?
S. Why do I have to check zero?
I. I think zero could form part of the answers, can you check it?
S. Yes I think so (After she checked with zero), but that means d will be equal to 10.
I. Does it matter?
S. Yes because d takes all the values
I. Is zero a number?
S. Yes but not a natural number
I. Did the question ask for a counting number or a natural number or a real number?
S. Not specified
I. That means you are not restricted to any type of number
S. Oh I see, then, zero can be included in the set of values
I. Zero and all real numbers less than 5

The above excerpt shows how this student failed to accept zero as one of the answers. The score shows that many of the students failed to get this question correct and the few who got it correct failed to include all real numbers less than 5. In item 7 the misconception was identified when few students (7%) had written the answer as 4x instead of 2x + 3. This means that these students had implied ‘more than’ to mean ‘multiply by’. In item 8 some of the students had used area formula of rectangle for the perimeter of a rectangle. One student however, had written perimeter = (2h + 7) x 2 = 2h + 14. This student instead of writing 2 + h wrote 2h which can be classified as misconception. In item 11, 37% were classified under misconception where they had written a + p = 90c. This has been considered as misconception because a and p had been taken as objects and not as number of apples and peaches. In item 15, majority of the students (85%) had cross-multiplied both sides of the inequality that is applying the principles of an equation leading to a wrong answer or failing to answer it at all. Item 17, the 30% of the responses under misconception were as a result of the students solving the inequality problem like an equation hence ticking (b) as the correct answer, which implies that the solution was based on the principles of solving equations. In item 18, 41% of the students had ticked (c) as the answer meaning that statement (1) had been accepted, which means there was no mistake with it. Statement (1) had a mistake because of how the binomial expression had been done by squaring the individual terms, which is a misconception. This goes to confirm the misconception some learners of algebra have over the squaring of a binomial expression by squaring the individual terms in the binomial expression. Item 20 however, is of great interest as majority of the students were classified as having misconception because those 89% of the students had ticked x = -3/2 and x = 1 as part of the statements describing an expression, which should not be the case because they do not belong to statements representing expression.

4.9.3 Misinterpretation

It is only in three items that the learners were classified to have misinterpreted the questions. In item 7 some students (26%) had solved for the value of x giving an answer of x = -3/2. This question did not need the numerical value of x so whoever looked for the value of x is seen to have misinterpreted the question. Moreover, the negative answer is a misinterpretation for the length of a line, which depicts the poor conceptual understanding of algebra because length of a rectangle cannot be a negative number. In item 8 the students who had solved for the value of h had misinterpreted the question, because the question only asked for the perimeter of the rectangle and not the value of h, which was part of the dimensions. Only 7% of the students had fallen into this trap, which was classified as misinterpretation of the question. In item 10 as many as 70% of the students had solved for the value of x, which was termed as misinterpretation because the question required the students to just write down and simplify the equation, arrived at from the statement.

4.10 ANALYSIS OF 1ST YEAR PTD COLLEGE PUE RESULTS

4.10.1 Category frequencies

From Table 4.14 and Graph 4.18 the first year PTD students from this College of Education had done badly on this algebra test. According Table 4.14 only 31% of the responses were classified as correct. 46% had answered wrongly which could not be classified under misconception or misinterpretation. 11% were classified under misconception, 3% under misinterpretation and 9% failed to attempt some items. In items 4, 10, 13, 15, and 20 the responses were all wrong giving zero scores respectively. Item 1 was answered with 83% correct responses and item 2 had a score just above average with a 52% classified as correct. Item 3 was fairly done with 69% classified under category correct answers. Item 5 was poorly done, 14% were classified as correct, 34% failing to attempt the question and the rest answering wrongly which could not be classified as misconception or misinterpretation and therefore classified as incorrect answer. Item 6 was fairly well done with 66% correct responses while item 7 was poorly done with 24% of the students getting the answer correct and 66% getting the answer wrong. Item 8 was scored as 34% correct answers and 62% incorrect answers. Item 9 was also poorly done, 21% got the answer correct, 10% of the responses had misconceptions and another 10% were classified as having misinterpreted the question and a further 14% failed to attempt the question. In item 11, 48% had got the answer correct while 28% had misconceptions and the rest got the answer wrong. Item 12 was done well with 76% of the responses classified as correct, 10% as having misconceptions and only one student failed to answer the question. Item 14 was poorly done with 17% scored as correct, 7% failing to attempt the question and the rest getting the answer wrong which were classified under category incorrect answer. Item 16 was poorly done, 28% were classified as correct, 45% classified under misconception category and 7% failing to attempt any of the items. Item 17 had 17% correct responses, 31% classified under misconception and the rest falling under incorrect answers. Item 18 had responses under all the five categories, 31% correct answer, 34% incorrect answer, 21% misconception, 7% misinterpretation and 7% unattempted. Item 19 had a percentage score just below average with a percentage of 48 and the rest of 52% answering the question wrongly.

4.10.2 Misconceptions

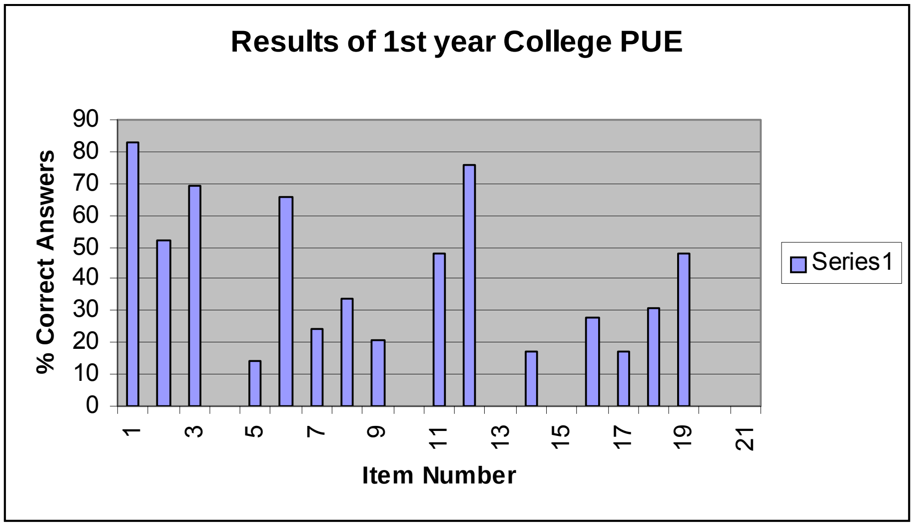
The overall misconception was 11% coming out of items 4, 9, 11, 12, 15, 16, 17 and 18 (See Table 4.14 and Graph 4.19). In item 4 as has been discussed before from earlier analyses, 17% of them had not written the answer as $c < 5$, $c \in R$ to the question.

In item 9, 10% of the students had written $y + 5$ as $5y$ resulting in getting the final answer as $6y$ for the total ages of Mpho and Sipho (See appendix C). In item 11, 28% had written the answer as $a + p = 90c$. This has been explained as students using a and p to represent objects instead of representing number of objects. Item 15 had 55% of the students cross-multiplying inequation like with the case of equation. This has been identified with almost all the participants and is said to be a misconception. Item 16 had 45% of the students ticking statement $x = y$ as the answer ignoring $x = -y$ (the negative answer) to be true to $x^2 = y^2$. This is a case where these students were not aware that two numbers whose squares are equal could either be positive or negative, it is therefore termed misconception. Item 17 was solved like in item 15, where inequation had been treated like equation. 31% of the students in this college had ticked (b) $y < \pm 5$ as the answer to $y^2 < 25$. Lastly, in item 18, 25% of them had used only positive numbers instead of both positive and negative numbers to check for the truth in the statement given (See appendix C). These students had ticked (b) as correct which, is however not the case as negative numbers would have given a false statement.

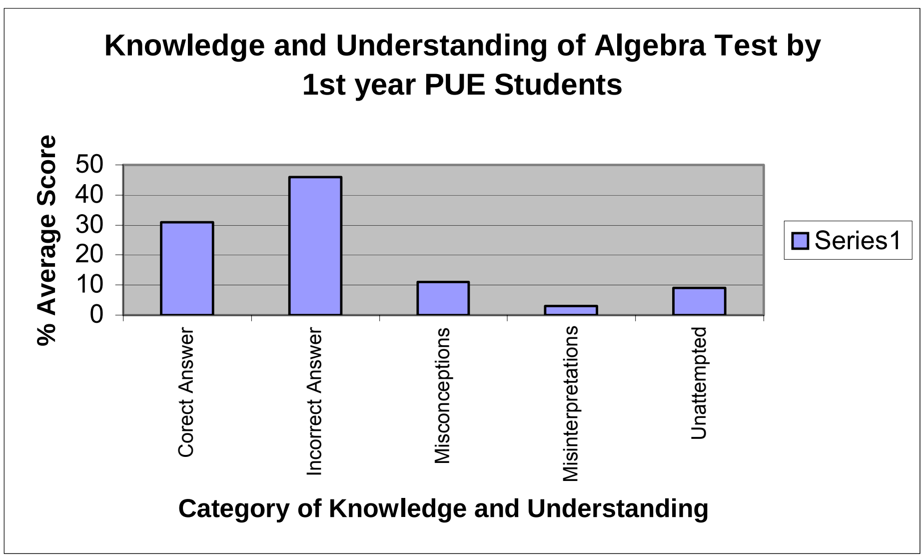
Table 4.14 shows the results of 1st year College PUE. Number of students = 29

Item Number	Correct answer	Incorrect answer	Miscon ception	Misin terpretation	Un attempted	Percentage correct
1	24	5	0	0	0	83
2	15	14(48%)	0	0	0	52
3	20	9	0	0	0	69
4	0	22	5(17%)	0	2	0
5	4	15	0	0	10	14
6	19	9	0	0	1	66
7	7	19(66%)	0	0	3	24
8	10	18(62%)	0	0	1	34
9	6	13(45%)	3(10%)	5(17%)	2	21
10	0	14(48%)	0	10(34%)	5	0
11	14	7	8(28%)	0	0	48
12	22	3	0	0	4	76
13	0	19	0	0	10	0
14	5	22	0	0	2	17
15	0	10(34%)	16(55%)	0	3	0
16	8	6	13(45%)	0	2	28
17	5	15	9(31%)	0	0	17
18	9	10(34%)	6(21%)	0	4	31
19	14	15(52%)	0	0	0	48
20	0	25	0	0	4	0
TOTAL	182	258	60	15	55	580
TOTAL %	31	46	11	3	9	31

Table 4.14 Results of 1st year PUE students.



Graph 4.18 Algebra test results of 1st year PUE students



Graph 4.19 Knowledge and understanding of algebra by 1st year PUE Students

4.10.3 Misinterpretation

Only 3% overall (See Table 4.14 and Graph 4.19) had misinterpreted items 9 and 10. In item 9, 17% of them had interpreted to question to mean solve for the value of y and therefore the ages of Mpho and Sipho. But this is not what the question had asked for, it had simply asked for a simplified equation for that statement. Solving for the value of y

therefore means misinterpretation of the instruction given. Similarly in item 10, they were
requested to write down an equation that could help find the value of the unknown number. Going further to solve for the value of x therefore meant they had misinterpreted the question.

4.11 ANALYSIS OF 3RD YEAR PTD COLLEGE PUE RESULTS

4.11.1 Category frequencies

The correct responses to this final year students were slightly higher than the first year counterparts. The percentage score was 36 (See Table 4.15 and Graph 4.20) as compared to 31% for the first year students. The correct answer responses from this College were generally poor. However, in item 1, the students had 100% implying a sound knowledge and understanding of this item. Items 2 and 3 had 61% correct responses each. There was no response from these two items, which could be classified under either misconception or misinterpretation. Items 4 and 5 recorded poor percentage scores of 9% each indicating that only a few had conceptual understanding of these items. Item 4 in particular, 13 % of the answers were classified as having misconception while 35% did not attempt to the question in item 5. Item 6 had a high correct responses of 87% thus conveying the message that most of the students appear
to have a sound knowledge of this item. Items 7 and 8 both recorded 35% correct responses, however, item 7 registered 22% misconceptions. Item 9 recorded 43% correct responses and had other responses categorised under the remaining four categories. Under the unattempted category three of the students 13% were registered. Item 10 registered 39% correct responses but surprisingly there were no misconceptions except for the misinterpretation of the question. In item 11 the correct response was low, the percentage correct response accounted to 30% of the categories with 41% falling under misconception. Item 12 was well done with correct responses as high as 83%. Items 13 and 14 recorded 22% correct answers each. Three students (13%) each failed to answer these questions. The score for item 15 was 0% indicating that these students had no knowledge and understanding of this question. 26% of these students were classified as having misconceptions. Item 16 also produced a low score of 17% correct answers with 61% classified as having misconceptions. Item 17 did not escape from these low scores, it recorded 13% correct answers, 30% had been classified under misconception category. Item 18 recorded 22% correct responses and 36% misconceptions with the rest falling under incorrect responses. Item 19 produced 43% correct responses, 22% misconceptions and the rest categorised under incorrect responses. Item 20 served to find out the difference between an expression and an equation with only 17% of them able to do that and the rest (83%) failing to distinguish between the two.

4.11.2 Misconceptions

Table 4.15 and Graph 4.21 show the average overall percentage of misconceptions as 14%, and these were found in items 4, 7, 9, 11, 15, 16, 17, 18 and19. In item 4 the failure to include 0 as one of the set of answers expected, contributed to having 13% of the responses classified under misconception. Item 7 was the case where the students interpreted "more than" to mean "multiply by", which resulted in having 22% classified under this category. Item 9 followed the same line of argument where the students had interpreted "older than" to mean "multiply" by instead of adding on for older than. Only 9% of the students had interpreted in this way. From item 11, 41% of the students had used letters to represent objects instead of number of objects. This had resulted in the students writing down the expression for the total cost of fruits to be that of one fruit each i.e. a + p = 90c. Item 15 had followed the same trend like other students from other colleges where they applied the principles of solving equation to that of solving for an unknown from an inequation. 26% of the students had made this mistake, which was termed misconception. Item 16 recorded the highest misconception among all the other items with a percentage of 61. This is where students had failed to used both positive and negative numbers to test for the truth for the statement $x^2 = y^2$. Item 17 was answered by 30% of the students like in item 15 where the principles of equation were applied for inequality. From item 18 those students who chose response (c) had a misconception since they implied that statement (1) was correct. This supports what Kaur and Sharon (1994) and many other researchers have observed which says that students had the tendency to square individual terms in a binomial expression if they are required to square. 35% of the students had ticked (c) as the answer to that item. From item 19 any of the students who ticked either (b) or (c) was termed to have misconception since they had only considered the positive numbers only to verify the statements ignoring the negative numbers; this is a misconception. 22% of the students had made this mistake of not checking with both positive and negative numbers.

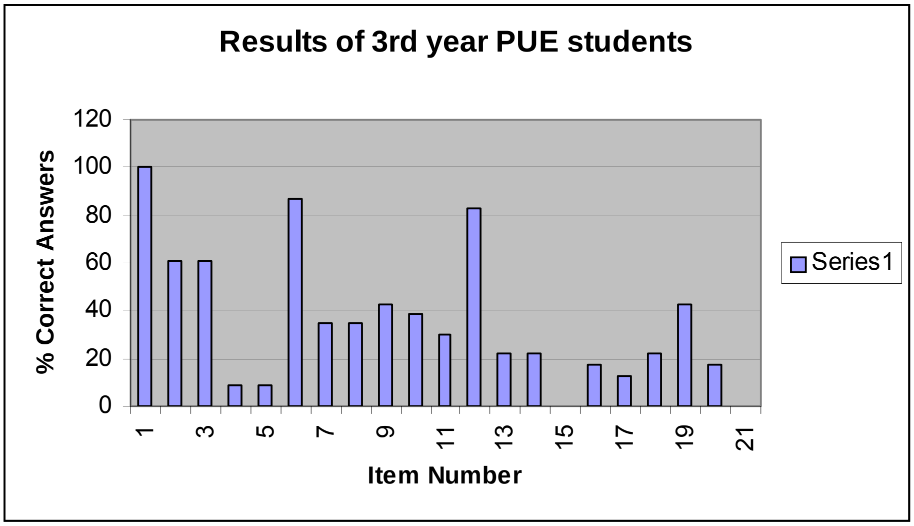
4.11.3 Misinterpretation

Table 4.15 show that misinterpretation was found in items 9 and 10, which resulted in an overall percentage of one (1). Students had tried to solve for the ages of Sipho and Mpho from item 9, which was not what was asked. This was therefore termed as misinterpretation of the question. Similarly in item 10 the students had been asked to write down an expression, which could help find the value of the unknown but 17% of them had actually solved for the value of x therefore termed as misinterpretation of the question.

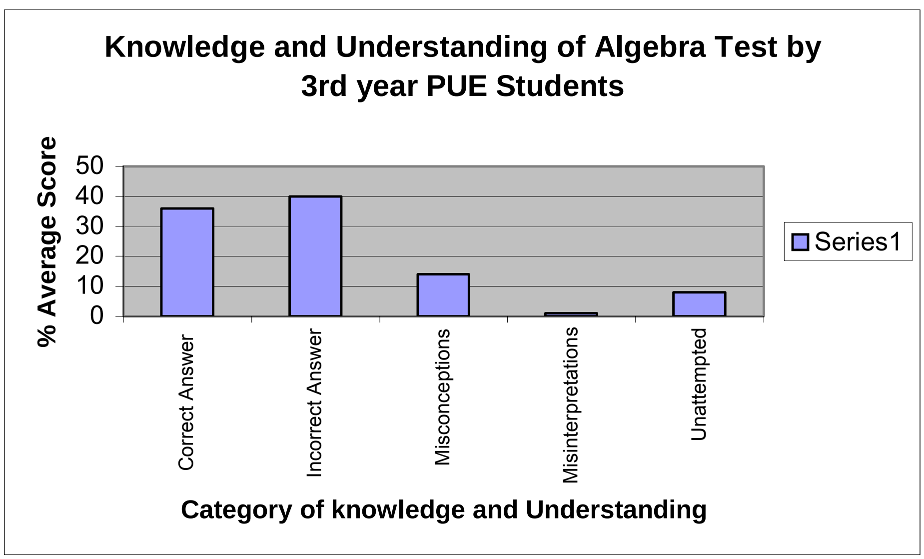
Table 4.15 shows the results of 3rd year College PUE. Number of students = 23

Item Number	Correct answer	Incorrect answer	Misconception	Misin terpretation	Un attempted	Percentage correct
1	23	0	0	0	0	100
2	14	9	0	0	0	61
3	14	7	0	0	2	61
4	2	18	3(13%)	0	0	9
5	2	13	0	0	8(35%)	9
6	20	3	0	0	0	87
7	8	8	5(22%)	0	2	35
8	8	14	0	0	1	35
9	10	6	2(9%)	2(9%)	3	43
10	9	6	0	4(17%)	4	39
11	7	4	12(41%)	0	0	30
12	19	2	0	0	2	83
13	5	15	0	0	3	22
14	5	15	0	0	3	22
15	0	14	6(26%)	0	3	0
16	4	3	14(61%)	0	2	17
17	3	13	7(30%)	0	0	13
18	5	10	8(35%)	0	0	22
19	10	8	5(22%)	0	0	43
20	4	19	0	0	0	17
TOTAL	172	188	61	6	33	460
TOTAL %	36	40	14	1	8	36

Table 4.15 Results of 3rd year PUE students



Graph 4.20 Algebra test results of 3rd year PUE students



Graph 4.21 Knowledge and understanding of algebra test by 3rd year PUE students

4.12 CONCLUSION

The above analyses show that chapter 4 has dealt mainly with the Colleges individually in terms of the algebra test, interviews based on the algebra test, and classroom observations. In chapter 5 the analyses will focus on all participants and Colleges as a group and where necessary compare and contrast the results of various Colleges.

The analyses so far have shown that there were no major changes in the knowledge and understanding of the individual Colleges as far as the misconceptions they carried from the 1st year college level is concerned. The PTD Colleges on the other hand scored a lower percentage than the STD College students. The reason might be the fact that most of the PTD College students did not have matric certificate and even those who had might not have passed it with good symbols. This is because the admission policy for PTD colleges differs from their STD counterparts. The STD students were supposed to have passed with at least a D symbol at standard grade or E at the higher grade in mathematics while the PTD students could be admitted with lower grade or might have attempted mathematics at the matric level. The data show that as the number of years increased at the Colleges the percentage scores of the algebra test also increased. As far as the Colleges of education syllabi (See appendix I) are concerned much emphases were placed on the pedagogy part of the content. This implies that these college students were assumed to have had enough content of mathematics based on admission policy, which should include algebra but little or no change is seen in their answering and teaching of the concept of algebra. The pedagogical content knowledge of these students is still based on the traditional method where the teacher becomes the sole authority in the classroom. Information is given to the learners instead of allowing them to discover things for themselves as has been proposed by Vygotsky and Piaget. These two constructivists focus on the learners constructing knowledge by themselves by interacting with the social and the physical world around them. The social aspect sees the role of the teacher as a mediator and not the transmitter of knowledge. Teaching should be crucial to learning but direct teaching is likely to inhibit learning since learners are not given the opportunity to discover things for themselves. The analyses so far point to the fact that most of the student teachers used the transmission mode of teaching (See 4.2.4, 4.3.4, 4.5.4, 4.6.4 and 4.8.4). The few students who had tried to use the method of facilitating or mediating had come across a lot of problems since this concept of teaching is said to be new to most of these students. This concept as has been discussed earlier is similar to the Outcomes-Based Education (OBE), which has been introduced to the South African educational system since 1998. This approach goes with the learner-centred mode of teaching where the learner is the centre of the learning process. These teachers had failed to produce atmosphere conducive to this type of teaching. Learners were seated in rolls and columns, not in small groups as is required by the constructivists. Another common feature among student teachers was the frequent use of the chalkboard, which allowed the teacher to work examples on it and provide enough space for demonstrations of worked examples prepared before the lesson was taught.

Generally, from the reflections, observations and interviews, it appears that student teachers drew on their previous knowledge and understanding of algebra. Consistent with the observations is that algebra or mathematics is taught in a particular manner using formulae or symbols to solve specific problems. For example, the perimeter of a rectangle is p = 2(l + b). Symbols have different meanings depending on the context in which they are used. For example most mathematicians would interpret f(x + 1) as meaning some function of x + 1 but would recognise m(x + 1) as m times x + 1 Therefore failure to know the meaning of these symbols in different contexts can lead to misconceptions or misinterpretation as was noticed from some of these students. Other comments from some student teachers indicated their heavy reliance on textbooks even if these carry wrong algebra problems and examples.

The observations have shown that most of the student teachers taught algebra by solving problems on the chalkboard by the chalk and talk method. They had demonstrated the skills of arriving at the answers, giving learners more similar examples to solve in order to master the techniques for arriving at the answers. This supports what they have argued earlier that algebra is difficult to do practically than geometry (See I3SEK1). The observation further unveiled a serious misconception, which passed unchecked by the student teachers because of the limited knowledge and understanding of certain concepts in algebra. The type of responses, which some student teachers gave to questions from learners did not stimulate class discussions and investigations that might have led the learners to adjust their understanding of algebra.

Shulman's concept of PCK requires of the teacher to translate their perceptions and understanding of algebra into the classroom activities to make algebra accessible to learners. This however, was not the case with most of the student teachers because they lacked these perceptions and understanding. The PCK of the student teachers were expected to go beyond the classroom to the wider social and cultural issues of the school in order to make meaning to the learners but these were not applied by most of these student teachers. The Procedural method of teaching algebra was what was observed in most of the classrooms. The practices described in this chapter do not lead to success in the knowledge and understanding of algebra. For successful teaching of algebra the teacher is required to explore practices and theories, raise questions, use communication and negotiation to open up new ways of seeing and understanding algebra.

Judging from the analyses so far in this chapter the student teachers' PCK had not helped in the teaching and learning of algebra. This may be as a result of the limited time to study in depth some of these concepts of algebra (for example, inequalities) which are not taught in some cases as reported by some student teachers (See I3SEK2). Classroom observation was limited to only STD Colleges since the research was on algebra, which is not taught at the senior primary level. The chapter which follows will look at the analyses of all the participants to find out if the above findings from the individual Colleges are applicable to them all.

CHAPTER 5

GENERAL ANALYSIS OF DATA FOR ALL COLLEGES OF EDUCATION IN THE STUDY

5.1 INTRODUCTION

This chapter will provide an overall analysis and interpretation across the various Colleges of Education in this study. The data in the study have been gathered through an algebra test, interviews and classroom observations. These methods were employed to obtain information relating to knowledge and understanding of final year College students in and around the Transkei, Eastern Cape province of South Africa. Within the study, there were noticeable similarities and differences among the college students. The chapter will also describe the main misconceptions and misinterpretations for each item and discuss some of the causes. Finally, it will answer the research questions in chapter 1. Table 5.1 shows the percentages of the responses to the correct answers to the various items in the algebra test.

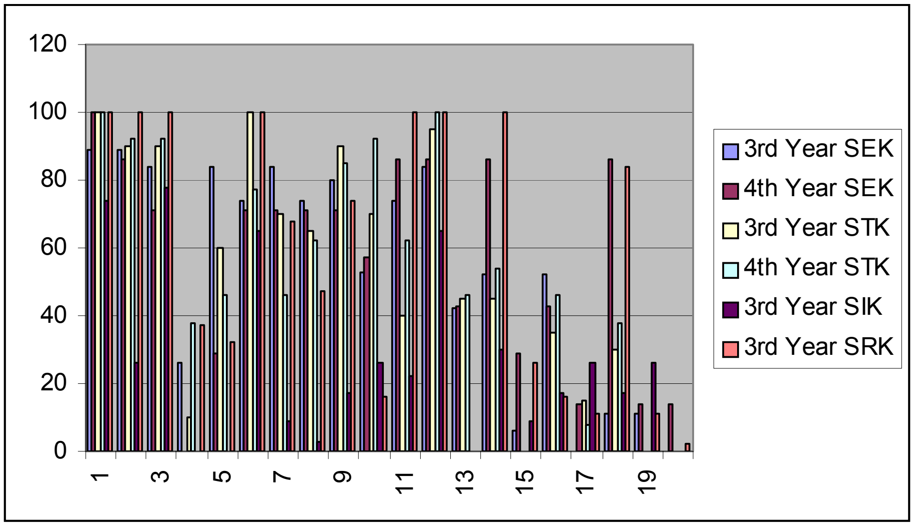
5.2. CATEGORY FREQUENCIES

The total overall average for the algebra test was not very good. As many as half of the participants failed to answer the question correctly (See Tables 5.1 and 5.2). This can be interpreted to mean that half of the participants knowledge and understanding was not sound enough to assist learners of these algebraic concepts after completing the teacher-training course.

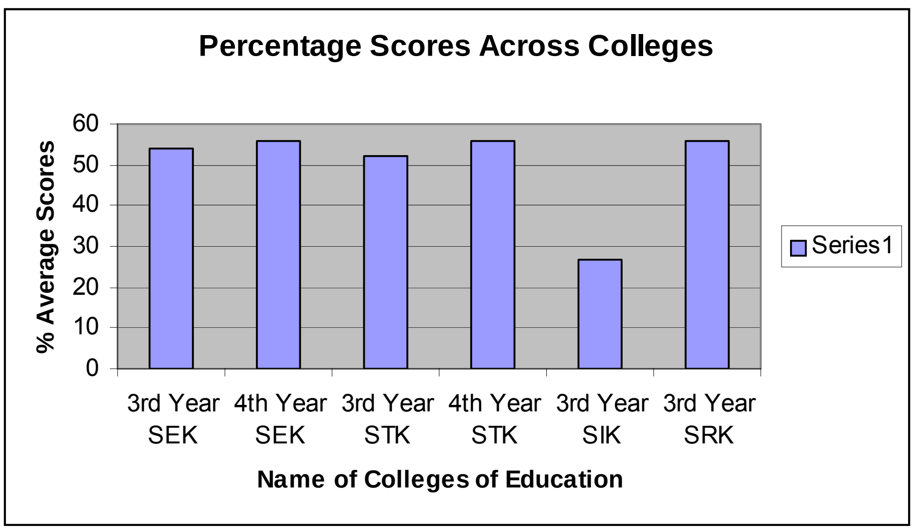
The percentage result per individual college shows that the fourth year students performed slightly better than the third year counterparts with the percentage score of 56. However, the percentage score for third year college students of SRK shows that they performed just as the 4th year students. The 3rd year students of college SIK percentage score of 27 indicates the least and hence the poor knowledge and understanding of the algebraic concept based on this algebra test.

ITEM	COLLEGE SEK	COLLEGE SEK	COLLEGE STK	COLLEGE STK	COLLEGE SIK	COLLEGE SRK	AVERAGE %
	YEAR 3	YEAR 4	YEAR 3	YEAR 4	YEAR 3	YEAR 3	
1	89	100	100	100	74	100	93.8
2	89	86	90	92	26	100	80.5
3	84	71	90	92	78	100	85.8
4	26	0	10	38	0	37	18.5
5	84	29	60	46	0	32	41.8
6	74	71	100	77	65	100	81.2
7	84	71	70	46	9	68	58.0
8	74	71	65	62	3	47	53.7
9	80	71	90	85	17	74	69.5
10	53	57	70	92	26	16	52.3
11	74	86	40	62	22	100	64.0
12	84	86	95	100	65	100	88.3
13	42	43	45	46	0	0	29.3
14	52	86	45	54	30	100	61.2
15	6	29	0	0	9	26	10.7
16	52	43	35	46	17	16	34.8
17	0	14	15	8	26	11	13.7
18	11	86	30	38	17	84	44.3
19	11	14	0	0	26	11	10.3
20	0	14	0	0	0	2.0	2.3
AVERAGE %	54	56	52	56	27	56	50

Table 5.1 Percentage scores across Colleges of Education



Graph 5.1 Percentage scores across Colleges per algebra test items



Graph 5.2 Percentage algebra test scores across Colleges

ITEM	AVERAGE %	IMPLICATIONS OF THE STATISTICS
1	93.8	Well answered across all Colleges
2	80.5	Well done across al Colleges
3	85.8	Well done across all Colleges
4	18.5	Poor grasp across all Colleges, 4th year STK best at 38%
5	41.8	Satisfactorily answered except for 3 groups with average score below 35%
6	81.2	Well answered across all Colleges
7	58	Well answered by almost all groups except poor 9% score by 3rd year SIK students
8	53.7	Well done by all participants except a very poor 3% score by 3rd year SIK students
9	69.5	Well answered by almost all groups except for the 17% score by 3rd year SIK students
10	52.3	Fairly well answered by all groups except for the 3rd year students of SIK and SRK with % scores of 26 and 16 respectively
11	64	Well answered across all Colleges except the poor 22% score from 3rd year SIK students
12	88.3	Well answered across all Colleges
13	29.3	Low scores across all Colleges with 0% scores from 3rd year SIK and SRK students
14	61.2	Satisfactorily answered by all colleges with 30% poor score from 3rd year SIK students
15	10.7	Very poor grasp of inequality concept across Colleges, best score of 29% by 4th year SEK students
16	34.8	Poor knowledge across all Colleges, 3rd year students from SIK and SRK scored below 20%
17	13.7	Very poor knowledge of the quadratic inequality concept, best score of 26% by 3rd year SIK
18	44.3	Satisfactorily answered across Colleges with poor score of 11% by 3rd year SEK students and best score of 86% by 4th year SIK students
19	10.3	Low scores across Colleges, 3rd and 4th year students from STK scored 0% respectively
20	2.3	Lowest % average score of all items. Knowledge of expression vrs equation was very poor. Far below expectations
TOTAL	50	Knowledge of algebra across Colleges not very good.

Table 5.2 Implications of average percentage scores.

5.3 ANALYSES OF COMMON MISINTERPRETATIONS ACROSS THE COLLEGES

Item No.	3rd Year College SEK	4th Year College SEK	3rd Year College STK	4th Year College STK	3rd Year College SIK	3rd Year College SRK	Average Percentage
10	37	42	20	0	13	84	32,7

Table 5.3 Common misinterpretations across Colleges of Education

Table 5.3 shows that it was only in one item that all the Colleges had misinterpreted the question, which registered an overall percentage of 32.7.The question for item 10 demanded only the writing down of the equation that could help find the value of the unknown number x. Although many of the students had written down the equation they went further to solve for the value of x, which I felt was a misinterpretation of the instruction. From College SEK the 3rd and 4th year students had fallen prey to this problem. The third years registered 37% as compared to 42% for the fourth year students. From College STK the situation is quite different as only the third year students had this problem, which registered 20% as compared to 0% for the fourth year students. The 3rd year students from college SIK registered 13% misinterpretation. Students from College SRK had registered the highest percentage of 84. This item serves to investigate students' interpretation of algebra word questions, the response is very disappointing for students from College SRK where almost all the students had made this mistake. The only reason for this response is the way they had misunderstood the question, as most of these students are English second language speakers.

5.4 ANALYSIS OF RESEARCH QUESTION 1

The first question was " To what extent do South African Colleges of Education students understand basic algebraic concepts?"

To answer this question I will refer to Table 5.1 and Table 5.2 and analyse each item of the algebra test. If the question was well answered, it means proper understanding of the concept being investigated. Where the question was not well answered it means poor understanding of that concept. A zero might be an indication of poor knowledge and understanding of that concept.

The percentage overall for the algebra test indicates that only half of the sample had scored correctly, which might indicate half of the sample had knowledge and conceptual understanding of algebra. As had been stated earlier, the algebra test comprised items from CDMTA and from Kaur and Sharon test on algebra. Items 1-12 were based on CDMTA and 13-20 on the Kaur and Sharon test. CDMTA was basically meant for children in the elementary schools; however I used it to test the understanding of the teachers who will teach these pupils. I now comment on the knowledge and understanding of aspects of algebra exhibited by each item separately.

Item 1 (If a + b = 34 then a + b + 3 = …).

The average score of 93,8 % implies that this sample of students were able to replace (a + b) by 34 in the expression a + b + 3 to arrive at 37.

Item 2 (If e + f = 8 then e + f + g =).

This might go to support the incomplete nature of algebraic expression because some of these students had failed to agree that 8+ g is the answer and therefore tried to find a numerical value for the answer. Some students (6%) had therefore equated 8 + g = 0 and hence arrived at an answer of g = -8 while others (20%) had got the answer 12 meaning they had made e = f = 4 and also g equal to 4. The response to this question has gone to support similar results reported by Hart et al.(1981) in the CSMS algebra test conducted on children in the elementary schools where 26% gave numerical answer. Similarly, the numerical answers given supports what Costello (1991) reported on 3rd year secondary school children where 12% also gave numerical answer of 12. This shows that the number of years of exposure and the age of these students might not have changed the conceptual thinking of some learners of algebra.

Item 3 the question stated: “what is the value of m if m = s + t and m + s + t = 24?”

Overall about 60% of the students had substituted m by s + t in the equation m + s + t = 24 arriving at 2(s + t) = 24 and implying s + t = 12, then concluding that m is also equal to 12. Others had replaced s + t by m and then getting 2m = 24, therefore m being equal to 12. Whichever approach to the question gave the correct answer but preferably the second approach should have been faster. The way students had approached the question brings to the fore the need to include the principle of alternative representation when teaching for conceptual understanding where the learners will approach the answering of such question with the method they prefer. This is an indication of the use of alternative representation as one of the characteristics of conceptual understanding.

Item 4 “What are the values of c if c + d = 10 and c is less than d?”

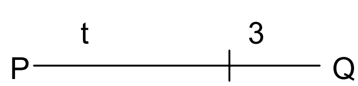
This question had two algebraic concepts, which could either help or lead to failure to answer the question; that is, ability to interpret ”less than” and solve simultaneous equation involving equal and unequal sign. Whereas some of the students (44.5%) could solve the problem they had failed to write $c < 5$, $c \in R$ as the set of answers. These students (44.5%) in this case did not have any problem with interpreting and answering the question. The problem is the misconception they had about numbers. The rest of the students however, lacked conceptual understanding of this type of problem. If they had followed Polya’s principle of problem solving they should have been able to interpret and check the solution to find out whether the answer was correct. Polya’s principle states that to solve a problem one has to follow four steps in order to arrive at the answer. i.e.

1.Understanding the problem
2. Devise a plan
3. Execute the plan
4. Look back

By following these four steps these students would have found out that they were required to solve a simultaneous equation involving a linear equation and linear inequality. Devising a strategy to solve it and checking the answers would have led them to see whether the solution set satisfied the conditions and that with proper interpretation they could have seen that conceptual understanding in this case should have included logical thinking, which was lacking.

Item 5 (If $x = 6$ is the solution to the equation $(x + 1)^3 + x = 349$, then what value of x will make the following equation true $(5x + 1)^3 + 5x = 349$? x = …).

In College SIK, none of them could get the answer correct. This is a question, which demanded logical thinking but some of the students had tried to expand the binomial expression and ended up not able to get the answer. This is a problem, which demanded an alternative way of solving it if even they understood the principles of expansion of a cubic expression. The conceptual understanding of this concept would have made the solution easier and faster to arrive at. The students needed to be able “*to judge whether the problem belongs to concept family by using an analytical judgement as opposed to a mere use of prototypical judgement*” Even (1990:523). To Even (1990) the analytical judgement is based on the concepts critical attributes like the concept’s mathematical definition. The prototype judgement is making use of visual or logical means. Logical deduction in this case made the solution easier than trying to expand the cubic binomial expression for the answer.

Item 6. 

Write down an expression for the length of PQ. PQ = …)

I expected some students to write t + 3 as 3t, that is conjoining to get a single answer like in arithmetic but this was not done in any of the scripts. This type of errors had been reported by Macgregor and Stacey (1997) where some learners of algebra write 10h to represent 10 plus h. Visualisation might have played an important role in answering this question, since they could see from the diagram that the total distance was that of adding 3 to t.

Item 7

Some students had arrived at 4x instead of 2x + 3 which is the correct answer. They interpreted “3 more than x” as 3x and therefore added x to get 4x. From Table 5.1 there is an indication that about half of the students did not have the conceptual understanding of this item and hence got the answer wrong.

Item 8 (See appendix C).

There were instances where students had used the area formula for the perimeter. Even after writing the formula they could not simplify. MacGregor and Stacy (1997) have reported this type of misconception due to interference from new learning. They reported that older students are prone to making mistakes because of interference from new schemas. Perimeter is taught before area and therefore could have interfered with the concept perimeter but at this stage of preparing to go out to teach such mistakes should be avoided. The way perimeter of a rectangle had been taught by some teachers could be a contributing factor as the wrong formula might have been used instead of the conventional way of adding the distance around the rectangle. Despite the gains from years in studying mathematics and therefore some algebra the success rate for this item did not match those years spent. In support of Blais (1988) who said students without a proper understanding of algebra fail to retain entire algorithm and association with the proper cue, these students who are misplacing the perimeter formula for the area of a rectangle had no proper understanding and therefore confused the correct formulae.

Item 9 (Sipho is y years old and Mpho is 5 years older than Sipho. Write down the sum of their ages in terms of y and simplify).

The phrase “older than’ just like “more than” had generated a lot of problems for a few of the students. Wheeler (1996) attributed some difficulties learners of algebra have with the use of language. Item 9, together with items 7 and 10 go to support this. The use of the phrases “more than” or “older than”, seems to confuse some of the students to the extent that they multiply instead of adding. It is therefore important for the teaching of these concepts to use similar phrases to bring out the actual meaning. For instance one could use these phrases “as many as”, “three times” etc, to bring out the meaning of “more than” or “older than”. The examples should begin from arithmetic then to algebra. It is therefore, necessary to show the difference between addition in arithmetic and in algebra. It has been suggested that it is necessary to show that in arithmetic 3 + 5 = 8 where 3 + 5 is the problem and 8 is the answer. On the other hand, x + 5 stands for the two operations in algebra, the problem and the answer. The misinterpretation of y + 5 to 6y supports what Robinson et al. (1994) state as "the incomplete nature of algebraic expression" i.e. 5m + 3 = 8m. Students at this level of studies were not expected to make such mistakes but as has been stated by Herscovics (1989) this type of problem is associated with learner's accommodation process. It is thus necessary to make the distinction between arithmetic operations as against algebraic operations. Human (1989) has expressed concern about what he referred to as conjoining of expressions as widespread occurrence, which takes time to remedy. He attributed this type of misconception to children's earlier studies of arithmetic, which resulted in a single number answer. Algebraic operations had been seen to follow the same line of thinking like arithmetic hence coming up with answers like $8x^3$ to the expression $3x^2 + 5x$. It is therefore not surprising to find final year College of Education students writing y + 5 as 6y, and similarly, arriving at 28h from this solution for item 8, which was worked out as follows:

P = 2(l x b)
  = 2(7 + 2h)
  = 2(14h)
  = 28h.

Obviously, these students wanted to arrive at a single answer and therefore got the wrong answers of 6y and 28h in items 9 and 8.

Tirosh et al. (1995) reported in their study that being aware of learners' tendency to conjoin or "finish" open expression did not go deep enough to explain to learners the source of such misconception which is attributed to previous experience of arithmetic or "*tension between the process and object facet of mathematical concept*" p61. It is therefore important for college of education students to go deeper to explain to their learners the source of such misconception, which is the result of previous learning of arithmetic, even if they are aware of such misconception in the teaching of algebra.

These errors are contrary to the philosophy of the Constructivists who claim that students will have to draw on previous knowledge to enhance understanding of new knowledge. In the above instances the previous knowledge of arithmetic seemed to have contributed to the errors. According to Macgregor and Stacey (1997) cognitive researchers believe that errors based on conjoining are particularly significant indicators of cognitive growth but the result here does not support this notion. College of Education students' responses to this item have proved that cognitive development does not change the already acquired misconception.

Item 10 (Four more than three times a number is 31.Write down an equation that can help find the number. Let the number be x).

Surprisingly only two students had written 12x = 31 as the answer where they might have interpreted more than to mean multiply. However it was easy for most of the students to write three times a number with the correct symbol 3x. This shows that the meaning became very clear when one had to write in symbol form “three times” a number. There was, however, a general misinterpretation of the question when most of the students tried to find out the value of x. The question was just to write down the formula but about 32% went further to solve for x. This could be termed misinterpretation of the question and could be linked to language understanding otherwise the question stated, “Write down an equation to help find the number” .The phrase “help to find” does not mean, “solve for”. The possible reason for such errors by the college students I believe is the way equation questions have been set. Most of the time questions have been phrased “solve for”, so the mere mention of an equation retrieves “solve” for the numerical value of the unknown. The possible suggestion for such errors being repeating is to apply Polya (1973) principle which may help to ascertain whether the final result complies with what the question had asked.

Item 11 (Apples cost 50 cents each and peaches 40 cents each. If “a” stands for number of apples and “p” stands for the number of peaches bought, write down an expression for the total cost of fruits bought.).

Some students had written a + p = 90c as the answer which was wrong. MacGregor and Stacey (1997) attributes the use of letters as abbreviated words and labels to the use of textbooks and interference from new learning that was applicable to beginners of algebra but one does not expect such misconception at this level where students could read and interpret what is in the textbooks. This can be attributed to poor conceptual understanding of algebra. With proper knowledge and understanding of algebra those students who wrote a + p = 90c could have detected that the answer was wrong if they had not interpreted a and p to be objects instead of a number of objects. Kuchemann (1981) also reports on a similar study on 2^{nd}, 3^{rd} and 4^{th} year students with ages from 13 to 15 which is termed as translation from algebra word problem to equation. The average percentage correct scores were 2%, 11%, and 13% respectively. He attributed 17% of the wrong answers to writing a + p = 90c among all the students who participated in the study. Similar percentage scores were recorded for this type of wrong answer in this study. According to the trend of the correct percentage scores stated by Kuchemann, College of Education students should have performed better than what was reported in this study.

Kuchemann attributes this kind of mistake to students viewing literal symbols as representing a set of objects. He further acknowledges the fact that the difficulty encountered in translating from algebra word problem to equation is not peculiar to only novices but that they persist into later years of studying algebra, citing some College students as culprits to this type of mistake. It is hence, not surprising to find some of these College of Education students making such mistakes. The percentage score for this study is, however, against the trend of improvement in making this mistake according to the data presented by Kuchemann. As the ages increase the percentage error decreases. It is hence surprising to find the College of Education students in this study performing against the trend given by Kuchemann. This cognitive problem should be addressed in the methodology class of these College students where students will be required to translate from word problem to equation and vice versa. It is again important to stress that when using letters for objects these letters should not be seen as static but varying. Letters are used to represent generalised numbers or specific unknowns. As Costello (1991) states letters are sometimes used to explain basic rules for algebraic manipulation. It is upon this that one can multiply 2a and 3b to get 6ab. However, if one sticks to a and b as objects then multiplication becomes impossible. It is hence necessary when teaching this concept to stress the statement "let **a** represent the number of apples" The writing of the product of say 3a as three times the number of apples will also help to do away with the misconceptions learners of algebra have about this concept.

Item 12 (A mathematical club has 15 members. Write a formula for finding the number of girls, "g", if you know the number of boys "b").

This item was well answered getting an answer of g = 15 – b but a few left the answer as g + b = 15, which was not the actual answer required by the question, however it had been accepted as correct for the analysis purpose.

Item 13 (What is a function?)

This was about the definition of a function. Function machines have been used as part of the introduction to algebra in some textbooks so failing to define it means these students had been solving problems involving functions without knowing what the definition is. It is very appropriate to define any concept used to bring about meaning so if students have been using functions without knowing the meaning then our approach to teaching has to be addressed to include the definition of concepts. Understanding of algebra should therefore include its definition and applications. The way most of the students had failed to define a function may imply that some teachers ignore the definition of concept when they have to teach it. This is similar to a case where during the interview (See PI3SRK1) a student who had used the formula of P = 2 (l + b) was asked to define a perimeter and was just interpreting the formula, which showed that the student actually did not understand the meaning of a perimeter. Failure to define a function can be seen as lack of understanding of the function concept.

Item 14

X	-1	0	1	2

Y	-3	0	3	6

(State the relation between x and y.)

The result (See Table 5.1) shows that these students could apply the knowledge of functions but could not define it in item 13. The failure of some students to answer the question means their knowledge and understanding were lacking. These students had studied functions and used them since they were at High school. Still, seeing a linear relation in this item did not immediately bring to mind the linear relationship of $y = mx + c$. A lack of rich relationships and connection between the two sets of numbers seemed to prevent them from answering the question correctly. Even (1990) further talks about Basic Repertoire, i.e. familiar examples closely connected to functions at High school level, which students should know, and which include linear and quadratic functions, polynomial functions, logarithmic and trigonometrical functions etc. Understanding the concept of linear function should include the structure of different number sets which serve as domain and range and that was what item 14 was investigating. Failure to get the correct answer could be as a result of Basic Repertoire (Even, 1990).

Item 15 (Find the solution set of the inequality $x-7/3-x > 1$)

The great interest with this item is the misconception shown by most of the students as the item serves to investigate students' knowledge and understanding of the inequality with unknown denominator. The response was very disappointing as a majority of the students (52%) cross-multiplied the inequation. This is an improvement on the same study by Kaur and Sharon (1994) where about 72% of the participants could not detect a mistake that one could not cross multiply an inequality with an unknown term. From their scripts one could detect that they knew the rules of linear inequality when they had to reverse the sign when multiplying by a negative number but failed to see that it was not always possible to cross-multiply both sides with an unknown quantity. The problem of non-linear inequalities arise in only the Higher Grade (H.G) stream at the High school and, as the result of the study shows, such brief experience is not enough for conceptual knowledge and understanding of this type of inequality. It should not surprise us that when solving non-linear inequalities many of the final year college students chose to use their knowledge with linear inequality.

Item 16 (If $x^2 = y^2$, then: (a) $x = y$
(b) $x = -y$
(c) $x = \pm y$
(d) $x = |y|$)

Most of these students had ticked (a) as the answer, which was wrong. The correct answer was (c) $x = \pm y$. Two reasons can be given for ticking (a) the wrong answer. Firstly because it appeared first which satisfies one of the conditions and therefore did not check with other alternatives. Secondly, it might be as a result of the tendency of many students checking answers with only positive numbers and forgetting about the negative numbers.

Item 17 (What can we conclude about y if $y^2 < 25$?:

a. $y < 5$
b. $y < \pm 5$
c. $y > 5$ or $y < 5$
d. $-5 < y < 5$)

Most of the students had ticked (b) $y < \pm 5$, a mistake which can occur with people without proper conceptual understanding of algebra; they normally apply their minds to their knowledge with linear quadratic equations. They tried to solve this quadratic inequality like quadratic equations. The solution to this item is almost similar to item 15 which means there is the likelihood that this topic was not treated well at the matric or the college level.

Item 18 (See appendix C)

There was lack of knowledge and understanding of this concept. Looking at the question there were mistakes with all the statements which were given. Anybody who failed to see these mistakes could be seen to have misconceptions with those algebraic concepts. Statement (1) investigated the misconception with squaring individual terms in a binomial. The same research studies have shown that if asked to square a binomial expression, some students end up writing $(a + b)^2 = a^2 + b^2$. This is however, not true so whoever gave statement (1) as correct has this misconception. Statement (2) again tested the knowledge of adding unlike terms, $7 - m + m^2 = 25$ should not give you $7 + m = 25$. By accepting statement (2) as having no mistake implies one had this misconception. Lastly, in statement (3) by answering $7 + m = 25$ as being equal to $m = 32$ means one had added 7 to both sides of the equation instead of subtracting 7 from both sides. The third statement is wrong therefore anybody who agreed to statement (3) as correct is also having misconception. The correct answer was therefore, (d) which stated that there were mistakes in all three statements.

Item 19 (If $x > y$, which of the following is true?

a. $\frac{1}{x} > \frac{1}{y}$
b. $\frac{1}{x} < \frac{1}{y}$
c. $\frac{1}{|x|} < \frac{1}{|y|}$
d. none of the above)

Anybody who said (a) was the correct answer did not check the answer by using both + and - numbers to verify. One could have seen that positive numbers do not satisfy while negative numbers were true. Hence the statement was not true for all integers. Likewise in (b) positive numbers hold while negative numbers do not hold. With (c) positive numbers would hold while negative numbers would not hold hence not true for all numbers. That left (d) to be the correct answer, but failure to take the pain to check with all the statements would have left students guessing and hence getting the wrong answer. This is a situation where the proper and sound knowledge of algebra would have prompted the students to check the answers with positive and negative numbers. The success rate for this item was almost the same as that reported by Kaur and Sharon (1994) where the success rate was also a low 11%. I agree with Kaur and Sharon who asserted that the failure to get the answer correct was due to the disregard students have over negative numbers in ascertaining the validity of statements and therefore the ineffective manipulation of counter-examples. From the scribbling on the scripts one could deduce that most of them did not have conceptual knowledge of absolute values. Most of them had failed to check the truth with statement (c). I presumed that those who chose (c) as the answer might have guessed. Lack of rich knowledge and understanding of absolute values might have prevented many of the students from checking with (c).

Item 20 (Check every answer you think is appropriate in describing $2x^2 + x – 3$:

a. It is a quadratic equation in x
b. It is a quadratic expression in x
c. It has factors (2x + 3) and (x – 1)
d. It has a solution x = -3/2 and x = 1)

This item serves to probe the students' understanding of the difference between equation and expression. The response is very disappointing seeing that these students have come across expressions and equations from the junior schools up to the college level. Failure not to know that (d) is not one of the characteristics of an expression is very demoralising. Another such widespread confusion is with the words "find the roots" and "solve for". Some students find it difficult to see that the two do mean the same. Some of the students had noticed that (b) and (c) were statements describing expression but had gone further to include (d) which is not a statement describing an expression. A lack of rich knowledge and understanding between equation and expression seemed to have prevented many of the students from including (d) as a statement depicting expression. Expressions and equations are the essence of algebra; students teachers should therefore have a good match between their characteristics.

5.4.1 Knowledge and Understanding of algebraic concepts

Even (1990) alludes to the fact that school mathematics tends to over-emphasise procedural knowledge without close relation to conceptual knowledge and meaning.

The classroom observations (See 4.2.4, 4.3.4, 4.5.4, 4.6.4) supported this conclusion since most of the students observed, had approached their lesson through this method. The South African school curriculum, until the time of the research, played a part in this type of teaching, (there is however, a curriculum change envisaged since 1998). The curriculum is mainly based on summative assessment where the teacher uses tests and examinations to promote or award certificates to learners. Teachers and textbooks therefore focus attention on activities that lead to procedural understanding. The classroom observations had confirmed this as rules and algorithms were what most of the student teachers used in their teaching. Learners were asked by some of the teacher to repeat after them some of the rules which they needed to use for some particular lessons. This led to the fact that the learners were meant to memorise facts and procedures without proper understanding, thus not paying much attention to conceptual understanding.

The response to item 12 bears testimony to the fact that these College students during their study of algebra had learnt functions without proper understanding. Function machines have been used as an introduction to the study of algebra by some textbooks and teachers alike. In the matric examination, questions have been asked based on functions in paper 1 but it is surprising to find out that many of the students could not define a function. This implies that these students had learnt this topic without proper understanding. Resnick and Ford (cited in Even, 1990) argue that procedural knowledge , which includes memorisation of certain facts and procedures is important since they serve to extend the capacity of the working memory. This they said could be done through development of automaticity of responding. They added that by automaticising facts and procedures more space becomes available in the working memory for other processes. They stated that both procedural and conceptual knowledge and understanding should be incorporated when solving problems in algebra or some nontrivial tasks. To gain mathematical/algebraic knowledge they agreed that both knowledge competencies of procedural and conceptual should be available to be successful.

For the final year college students their conceptual knowledge should rather be supplemented by the procedural knowledge since at this level of teaching, at the junior schools, they are mainly introducing new concepts in algebra to beginners of algebra. Conceptual knowledge and understanding was what was expected at this stage but what was noticed in most of the classrooms observed (See 4.2.4, 4.3.4, 4.5.4, 4.6.4) was the use of procedural knowledge approach. With reference to the teaching of simplification of algebraic expression earlier discussed the teacher gave the techniques of adding like and unlike terms by the use of - and + signs. That is, the number with the bigger sign was taken after subtracting the smaller number from the bigger one. This is testimony to what is termed procedural knowledge comprehension as against the conceptual understanding.

5.4.2. **Procedural and Conceptual understanding approach.**

It is important to consider concrete and visual representations to enhance an understanding of algebraic concepts. In addition to these it is the duty of the teacher to offer the appropriate explanations and introduce ideas that will assist in the comprehension of algebraic concepts. Accompanying the explanations with representations, models or practical application could lead to proper knowledge and understanding of algebraic concepts. If the teacher in the course of teaching the perimeter of rectangle, starts from the formula: Perimeter = 2(length + breadth) this will lead to procedural understanding as was noticed in the algebra test for item 8. Students were just recalling the formula for the perimeter of the rectangle and ended up confusing it with the area formula because they lacked conceptual understanding of this algebraic concept. For conceptual understanding the teacher should start by asking the learners to add the lengths of the sides of the rectangle to give the perimeter of the rectangle. It is after this that the learners are asked to derive their own formula and in this case there is the likelihood that they will remember and not confuse it with the area formula. In the same way the area of the rectangle should not start with the formula but should lead learners to derive it on their own. In items 6 and 8 the students could visualise from the diagram to enable them write down the correct lengths. This implies that for conceptual understanding there should be supportive materials like diagrams and concrete objects.

Procedural teaching was what was observed in most of the lessons observed. (See 4.24, 4.3.4, 4.5.5, 4.6.4). The teachers carefully demonstrated algorithms, explaining each step and then providing opportunities for the learners to practise the algorithms, which they had mastered. Activities were mostly drawn from textbooks. To assess the learners the student teachers sometimes went around the classroom as the learners were working and either ticked it correct or wrong without checking where the mistake had come from. These are some of the characteristics of procedural teaching. A few of the student teachers taught for conceptual understanding but struggled with conceptual explanations like what they do in case of procedural understanding. For example, one teacher (See PI3SRK1) had found it difficult to explain why the perimeter of a rectangle is written P = 2(l + b) during the post lesson interview. This student teacher could not explain that the perimeter of the rectangle is as a result of adding all the lengths of the sides of a rectangle. No wonder some of the student teachers had confused the perimeter formula with the area formula. This might be because they had learnt them without conceptual understanding and coupled with interference with current learning got confused with the formulae. As had been argued earlier, time constraints had also contributed to student teachers not teaching for conceptual understanding because the lesson had been planned for within a specified time so they tried as much as possible to finish and do assessment at the same time.

Most of the students teachers had applied procedural teaching of algebra just because of the way they themselves had learnt these concepts and their own limitations to conceptual understanding and knowledge of algebra. The importance of both procedural and conceptual knowledge and the relationships between them should be acknowledged by would-be teachers of algebra since they supplement each other, but conceptual understanding should come first.

In item 15 to solve the inequality (x-7)/(3-x) >1, lack of rich relationship and connectedness (Even,1990) between different representations of the same algebraic concept prevented most of the participants from solving the problem. Weak conceptual knowledge also did not help some of the student teachers when misusing procedural knowledge. Some got answers which did not make sense at all, such as x = 5, x = 3, x > 3, x < -3 etc. This is in accordance with Even (1990:539) who stated that "*prospective teachers lacked the connection which characterise conceptual knowledge to make this knowledge accessible*"

The number of learners in the classrooms might also be a contributing factor as some classes had more than 60 learners. Group activities and learner-centred teaching has been seen to be a means of teaching for conceptual understanding. Where the number is too big this type of teaching is practically impossible so in most cases teachers resorted to procedural teaching where they felt individual attention is not possible.

5.5 ANALYSIS OF RESEARCH QUESTION 2

Second question: What are the common misconceptions that South African COE students acquire in algebra and how do these misconceptions develop?

To answer second question 2, I will refer to Tables 5.4, 5.5 ,5.6 and Graph 5.3 which give the percentage misconceptions, common misconceptions across the Colleges, implications of the percentage scores and in addition the classroom observations from field notes and interviews. I will analyse each question with misconceptions and try to give an explanation as how they developed.

Table 5.4, 5.5 and Graph 5.3 show the percentage overall scores of the common misconceptions across the Colleges of Education. This overall percentage is calculated out of the raw data: (Number of students with misconceptions ÷ Total number of students*100%). Items 4,15,16,17, and 19 show the items, which had common misconceptions after comparing all the algebra results from all the groups. In item 4 the misconception was when they had failed to include all real numbers less than 5 as the set of answers required by that question, since 0 and other negative numbers fulfil the conditions of the question. The average misconceptions score was 44.5% with 3rd and 4th year College students from College SEK registering 52% and 57% respectively. With College STK the 3rd and 4th year students recorded 40% and 54%. For College SIK the third year students had low 17% misconceptions, while the third year College students of College SRK scored 47%. The students from 4th year College SEK had the highest percentage misconception score. This implies the conceptual misunderstanding of procedures of testing to be sure that all conditions and criteria had been met with this type of question is lacking. Number concepts play an important role in the understanding of this concept. It is therefore necessary to have a solid foundation in number concepts in order to solve this type of problem in algebra.

Item 15 exposed the misconception which appears when solving non-linear inequality problems with unknowns where the operations are supposed to be treated differently from those followed for an ordinary equation. It was seen here that 52% of the students had cross-multiplied like it is done for an equation. It is again apparent that most of the students do not verify their answers by checking with the original equation. The answer of $x > 5$ which most of the students got would have shown them that the answer was wrong if they had checked by substituting in the original inequation. This has confirmed Kaur and Sharons' findings where they found 72% of their participants had such a misconception. The misconception had also come out to be the failure of most of the students not solving this type of inequation at the matric levels as well as at the College of Education level. According to the syllabus of the Department of Education, this inequality topic is not treated by all students at the matric level. It is only treated by the Higher Grade students and even at the College of Education level, it is not surprising that only a few students and learners study this topic. Even at the College of Education level only those who at the end of their course will teach at the High school level do study this topic. It is no wonder that many of the students failed to solve it using the correct method by not relying on the principles of solving an equation. This can be attributed to the lack of interest in the topic or the conceptual understanding by most of teachers who try to avoid the teaching of this topic. This situation is understandable because inequalities had not been given a fair treatment as far as the syllabus is concerned. Within the syllabus emphasises had be laid on linear inequalities as against non-linear inequalities. While this could be acceptable with learners of algebra, it cannot be acceptable with College of Education students who are expected to teach this concept to learners of algebra. The incomplete conceptual knowledge of inequalities is a problem as it contributes to the cycle of misconceptions because with the poor knowledge from the students teachers the tendency is that they also transmit the same knowledge to learners of algebra, hence the cascading effect on learners of algebra.

Item 16 had an average misconception score of 48.7%. This means almost half of the final year College of Education students did not know that two numbers whose squares are equal could either be equal to or same magnitude but opposite sign. Kaur and Sharon (1994:46) had attributed such misconception to "*the uneasiness that some students face when working with letters and symbols, which are inherent in the language of algebra*". College SEK had registered 26% and 29% respectively for the 3rd year and the fourth year students as far as this misconception is concerned. College

STK recorded high 60% and 54% for the third and fourth year students. The third year students of College SIK had 39% of them having this misconception. College SRK recorded the highest misconception score of 84%. This can be seen as lack of knowledge and understanding for this concept in these Colleges.

Item 17 had the highest percentage misconception score among the items identified with misconception; it had registered 64% overall. The misconception was the ticking of (b) $y < \pm 5$ as the answer to the question: “What can you say about y if $y^2 < 25$?. 3rd year students of college SEK had 84% of them classified as having this misconception, the 4th year students from the same College had 57% of them having this misconception. College STK also had 70% misconception for the third year students and 54% for the fourth year students. From these two groups it was noticed that the misconception is higher for the third year students than with the fourth year students. This can be reasoned out to the fact that the fourth year students are meant to teach at the junior secondary school where such problems are met. The third year students might not solve such problems at the senior primary level at schools where they are supposed to teach at the end of their course, hence might not be taught well at the college level. At College SIK the third year students had 30% of the students having misconceptions. College SRK on the other hand, recorded a high 89% misconception for this item. This means an entire group of students from College SRK had poor knowledge and understanding of this concept. It is therefore, highly likely that in the years of teaching they may avoid this topic and hence deprive their learners from acquiring the knowledge and understanding of this concept.

Item 19 indicates that half of the participants showed misconceptions. This is a question, which demands the use of both positive and negative numbers to verify the statements given to check whether they fulfil the conditions. Half of these participants used only positive numbers to verify these statements hence were identified as having a misconception. These were the students who had ticked (b) $1/x < 1/y$ as the answer to the question asked in item 19: If $x > y$, which of the following is true? From College SEK the 3rd year students had 42% of the students having this misconception, 57% again for the fourth year students. The misconception for College STK was however strange with the 3rd year students registering 45% as against 92% for the fourth year counterparts from the same college. From these two Colleges SEK and STK the trend is that the fourth year students had higher misconception than the third year students. College SIK students recorded the lowest misconception of 22% for this question. The third year students from College SRK however registered 42% misconceptions for this item. These high misconceptions could be attributed to the interference from other learning, which could be the case with the 4th year students who by the nature of their course treat higher and more difficult topics than the 3rd year counterparts.

5.5.1 **Analysis and interpretations of misconceptions in the algebra test**

ITEM No	3rd Year College. SEK % SCORE	4th Year College SEK % SCORE	3rd Year College STK % SCORE	4th Year College STK % SCORE	3rd Year College SIK % SCORE	3rd Year College SRK % SCORE	AVERAGE %
1	0	0	0	0	0	0	0
2	0	0	0	0		0	0
3	0	0	0	0	0	0	0
4	52	57	40	54	17	47	44.5
5	0	0	0	0	0		0
6	0	0	0	0	0	0	0
7	11	0	0	0	26	26	10.5
8	11	14	15	23	0	32	15.8
9	11	14	0	0	9	21	9.2
10	11	0	0	0	0	0	1.8
11	0	0	55	38	43	0	22.7
12	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0
15	26	42	80	92	30	42	52.0
16	26	29	60	54	39	84	48.7
17	84	57	70	54	30	89	64.0
18	84	0	45	54	26	11	36.7
19	42	57	45	92	22	42	50.0
20	47	29	0	31	0	58	27.5
AVERAGE % OVERALL							19.2

Table 5.4 Percentage misconceptions for all items across Colleges

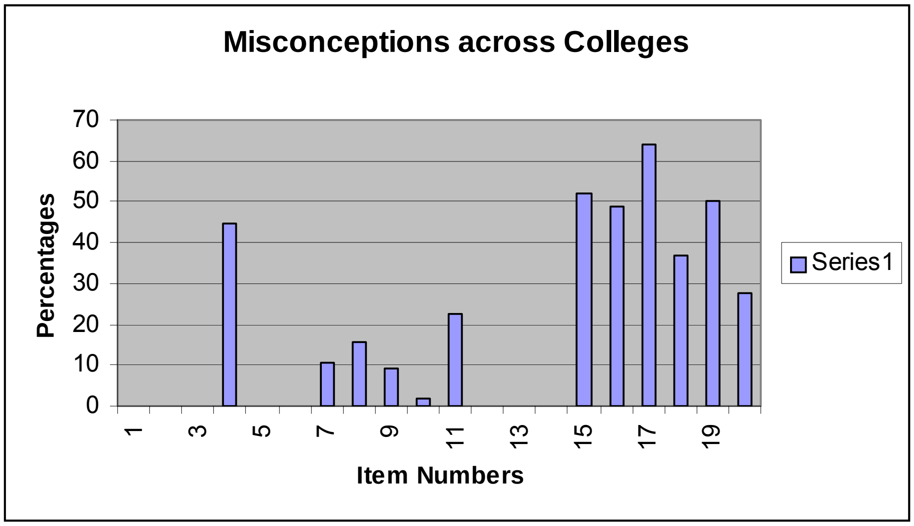
Item No.	3rd Year College SEK % SCORE	4th Year College SEK % SCORE	3rd Year College STK % SCORE	4th Year College STK % SCORE	3rd Year College SIK % SCORE	3rd Year College SRK % SCORE	Overall Average Percentage
4	52	57	40	54	17	47	44,5
15	26	42	80	92	30	42	52
16	26	29	60	54	39	84	48,7
17	84	57	70	54	30	89	64

19	42	57	45	92	22	42	50

Table 5.5 Common misconceptions across Colleges of Education

ITEM NO	AVERAGE %	IMPLICATION OF THE STATISTICS
1	0	No misconception, an indication of sound algebraic knowledge and understanding
2	0	No misconception, an indication of sound algebraic knowledge and understanding
3	0	No misconception, an indication of sound algebraic knowledge and understanding
4	44.5	High misconceptions an indication of weak algebraic knowledge and understanding
5	0	No misconception sound algebraic knowledge
6	0	No misconception, sound algebraic knowledge
7	10.5	Moderate misconceptions, an indication of slightly weak algebraic knowledge and understanding
8	15.8	Moderate misconceptions, an indication of slightly weak algebraic knowledge and understanding
9	9.2	Moderate misconceptions, an indication of slightly weak algebraic knowledge and understanding
10	1.8	Minor misconceptions, an indication of a small bit of lacking in algebraic knowledge and understanding
11	22.7	High misconceptions, an indication of not having sound algebraic knowledge and understanding
12	0	No misconception, an indication of sound algebraic knowledge and understanding
13	0	No misconception an indication of sound algebraic knowledge and understanding
14	0	No misconception an indication of sound algebraic knowledge and understanding
15	52.0	Very high misconceptions an indication of very weak algebraic knowledge and understanding
16	48.7	Very high misconceptions an indication of very weak algebraic knowledge and understanding
17	64.0	Extremely high misconceptions an indication of very weak algebraic knowledge and understanding
18	36.7	High misconceptions an indication of weak algebraic knowledge and understanding
19	50.0	Very high misconceptions an indication of very weak algebraic knowledge and understanding
20	27.5	High misconceptions an indication of not having sound knowledge and understanding of algebra
OVERALL AVE %	19.2	Moderate misconception an indication of slightly weak algebraic knowledge and understanding

Table 5.6 Implications of the percentage misconceptions scores



Graph 5.3 Misconceptions across Colleges

5.6. ANALYSIS OF RESEARCH QUESTION 3

Third question: How does subject matter knowledge of South African College of Education students affect their instructional practice?

5.6.1 Pedagogical Content Knowledge (PCK): Different representations

Concepts in mathematics do not always appear to affect behaviour in the same ways. From the literature (Even, 1990; Graeber, 1999) it is therefore advisable for the College students knowledge and conceptual understanding to include different representations in their teaching in the classrooms. The power of representation is the ability to make the content comprehensible and appropriate to the abilities and interest of learners. A good example is allowing children to solve problems on their own while the teacher moves around the classroom to offer suggestion and assist learners. When in a classroom the learner has written $2x^2 + x = 5x^2$ it is the duty of the teacher to make the learner know that the answer was wrong by offering advice on how to overcome such mistakes. The teacher at this stage has to act as a mediator where she or he targets the critical gap (Vygotsky, 1978) of development so that the child's understanding is shifted to a new level. The teacher at this stage presents clear concepts and facts that are not far beyond the learner's comprehension. The teacher has to adjust the presentation by using descriptions and examples to suit the level of comprehension of the learner.

The teacher does not have to dominate the social interaction or simply demonstrate a solution to a problem; he/she instead adjusts the problem to the level of the learner. The demand for a high level of competency on the part of the teacher in algebra is very important to assist the learner when he or she is stuck in the process of trying to solve an algebraic problem. In order to gain these competencies the teacher must have a mastery of that knowledge. For instance in this case of $2 \times 2 + 3x = 5x^2$ the teacher could ask the learner to substitute values in both the left hand side and the right hand side of the equation to see if they are equal to each other. This is what Vygotsky (1978) calls scaffolding (See 2.1.5.2) when the teacher provides support to the child in order to progress from the current position to the level of being able to solve the problem on his/her own. These seemed to be missing during the classroom observations (See 4.2.4, 4.3.4). The use of teaching aids had not been effective since most of the student teachers relied on working examples from textbooks. Where teaching aids had been used they had not been very effective like the bananas and apples episode discussed earlier (See 4.3.5). These student teachers were not self-confident enough about their algebraic knowledge and understanding to be able to create discussions and to be able to challenge learners' misconceptions with appropriate examples. This would have been in line with some of the principles of the Constructivists who believe the teacher should not transmit information to the learners but guide learners to construct their own knowledge through such discussions and probes. The use of classroom discussions and debates are also some of the principles of OBE the current curriculum is envisaging. This lack of self-confidence might have led to some of the student teachers not checking the reasonableness of their answers like when some arrived at an answer of $x > 5$ in item 15. They would have known that these values did not satisfy the original inequation.

Student teachers are likely to come out of Colleges with only one way of teaching a topic. The reason is not far fetched as the time constraints could be a contributing factor, where the student will be rushing to finish a lesson in order to give an exercise which he or she could assess. The textbooks also play a role as they normally dwell on only one method of solving a problem which leaves the prospective teacher with no other choice than the one in the textbook. The big numbers in most of the classrooms also play a part in making for student teachers use the procedural method of teaching. In some classrooms the number of learners were more than 60 while the teacher-pupil ratio stipulated by the DOE is at most 1:40. It became a big burden on the part of the student teachers to teach for conceptual understanding with these big numbers. Notwithstanding, there were classes where the ratio was less than the ratio stipulated by the DOE but teaching for conceptual understanding did not happen. The reason might be that they themselves had been taught these concepts through the procedural method during their time as learners of algebra and probably at the Colleges of Education.

It is however, of great importance according to (Even, 1990) if one could try to use different representation to bring about the proper and clear understanding of a concept. During the classroom observation a teacher who had tried to use number of boys and number of girls (See 4.3.5) to explain why you could not add unlike terms was challenged by a student who said you could add the number of girls to boys to get the total number of learners in the class. The teacher's PCK enabled him to use an alternative approach by using apples and bananas to explain the concept to the learners.

"Understanding a concept in one representation does not necessary mean that one understands it in another representation" (Even, 1990:523). Even (1990) further argues that different representations of a concept throw more light on the concept and gives more insight. This in effect allow for better, deeper, and more powerful and more complete knowledge and understanding of a concept. Approaching the teaching of algebraic concepts via different representations will go to augment the OBE approach to teaching. Grouping learners and giving them different tasks or activities, which could be done on their own, can lead to different approaches to the same concept. This is likely to add to acquiring meaningful knowledge and understanding of the concept. Doing things on your own by touching, feeling weighing, measuring etc can lead to conceptual understanding. From items 6 and 7 most of the students could write out the correct expression for the length of a line and the length of a rectangle which involved a letter and a number unlike in items 9 and 10 where they had to write down in symbolic terms "more than" and "older than". This might be because they could not visualise in items 9 and 10 but they could visualise from items 6 and 7 because of the diagrams.

With respect to PCK concrete and visual representations were important to drive the notion of conceptual understanding forward but few teaching aids were used in most of the observed lessons (See 4.3.5). Students had already admitted to the fact that it was difficult to construct teaching aids in the teaching of algebra. The only evidence of these visual aids was when one student teacher was teaching the addition of like and unlike terms. Bananas and apples had been used to confirm that unlike terms could not be added together. Likewise boys and girls in a class always remain the same even if you have to add them together. But jumping from concrete representation to formulae could lead to problems if explicit connection between the two is not made. As was discussed in 4.3.5 apples and bananas can also lead to misconception: x and y are symbols for numbers not objects. Without proper and sound knowledge and understanding of algebra some teachers end up confusing learners than helping them. If it is said one cannot add bananas to apples how then can you multiply 3a and 2b to get 6ab, if a stands for apples and b stands for bananas because multiplication is said to be repeated addition. This is where the PCK for the teacher comes into play, so without the proper PCK the teacher would be finding it difficult to clear the air in such a case. The PCK would help the teacher to diffuse the confusion which might arise from the use of such concrete example.

However, students' conceptions related to specific algebra topics may differ according to which curricula they study. It is important for student teachers to be acquainted with various teaching methods and to be aware of their pro and cons in different contexts with different teaching aims, and with different learners. As Tirosh et al.(1995:63) state

> " *teachers should consider possible short-and long-term implications of the use of each method on learner's knowledge and understanding of specific facts, procedures, concepts and ideas as well as on their knowledge about nature of mathematics".*

5.6.2 **Alternative ways of teaching algebra.**

Each of the alternative approaches to an algebra problem is different from the other and none of them is appropriate for all situations. In most instances, when more than one way can be used, some ways are more appropriate than others. Item 6 is an example of choosing between the different approaches to the same problem. There were two alternative ways of solving the problem in item 6. Firstly, one could either expand the cubic expression or use the replacement method. Interestingly, most of the student teachers who got the answer correct had used the replacement method and those who used the expansion method got the answer wrong. Similarly in item 15 all the students who attempted the question had used the algebraic method of cross-multiplying. An alternative method of using the graphical method could have yielded the same answer if students where however, aware of this method.

In order for student teachers to be able to assist their learners in their approach to algebra and make good choices between the different available approaches, the teachers themselves ought to have that knowledge and understanding. Knowing what algebraic concepts are and being able to work with them in different forms, representations and notations using appropriate ways, is important for student teachers to fulfil their duties in the classroom when they are obliged to teach for conceptual understanding.

Graeber (1999) has reported in her study that " *there is an obvious connection to preservice teachers pedagogical content knowledge related to alternative representations and success in helping students achieve conceptual understanding"* p203. It is therefore necessary and important for College of Education students to be armed within their methodology classes with variety of ways of solving algebraic problems in effect enhancing conceptual understanding. According to the Constructivists (Ernest, 1995 ; Von Glasersfeld, 1990) learners come to class with their own knowledge which helps them to construct new knowledge by either accommodating or assimilating the new information from the teacher. The result of this produces different ways of solving or understanding the same problem which might be different from the teachers' way of thinking or reasoning. These different ways or methods from the learners require that the student teacher or teacher in algebra should be able to recognise and understand these methods and solutions to be able to play the role of a mediator. Such situations demand that the alternative ways of solving algebraic problem are known by student teachers. Graeber (1999: 203) further acknowledges that

> " *if preservice teachers fail to provide alternative paths to understanding, they are apt to leave some students without understanding. If they fail to recognise and analyse alternative solutions to problems, students' reasoning may be undervalued or more seriously, be declared incorrect, if valid or correct when invalid"* .

College of Education students in order to help teach their learners for conceptual understanding of algebra demands that they experience the notion of multiple approaches in the method classroom and also be assisted to plan for multiple approaches to problems with their learners. The College of Education students should therefore be made to prepare different lessons on the same algebraic concept they want to teach to create confidence and to stimulate teaching for conceptual understanding instead of for procedural understanding as was the case with most of the lessons observed (See 4.2.4, 4.3.4, 4.5.4).

5.6.3. History and Language in algebra

During the classroom observation (4.2.4, 4.3.4, 4.5.4) it was rare to find the college students integrating the knowledge of the history of algebra (See 1.1.1) with the topics they were teaching. Student teachers had failed to bring in the teaching closer to the environment whereby they had to use examples from the real-world situations which history has shown has contributed to the development of some algebraic concepts and principles and how they were derived. Rather the examples and demonstrations were based on the textbooks, which in most cases were written from different backgrounds. This as a result does not create the interest in the topics being taught and in the end making algebraic concepts abstract and meaningless. The learners for the sake of interest should know why they are learning certain topics so failure to do so can end up hindering the learning of algebra (learners do not see any reason why they should learn certain algebraic concepts). This may lead to the development of learner apathy and negative attitude towards studying school algebra. Learning school algebra is therefore seen by certain learners as a way of meeting the requirements necessary to proceed to the next class, hence learning to pass test or examination becomes the ultimate goal for most of the learners. This in effect leads to instrumental (Skemp, 1976) or procedural learning where extrinsic instead of intrinsic motivation is involved.

It is important to see student teachers must ensure that the message they convey to the learners is properly communicated through the appropriate use of language. More often than not the way the learners interpret the instructions of the teacher differs from what the teacher wished to convey to them. There are instances where learners see multiplication and addition as synonymous. This dilemma may be caused by the manner in which we use language in general and in algebra in particular. “More than” and “older than” are examples of how some of the College of Education students had interpreted them different from what the question had asked for. During the classrooms observations I was expecting the student teachers to be code-switching (Setati, 1998) in order to clarify concepts being taught, but to my surprise this did not occur. My general observation was that most of the permanent teachers at these levels do code-switching, by using vernacular and English when teaching mathematics and some other subjects. My belief is that these student teachers realised that I did not understand the local language and hence tried as much as possible to stick to only English. There were instances where these learners would ask questions in vernacular but the teachers tried as much as possible to answer in English. For conceptual understanding I can support Setati (1998) when she states that teachers try as much as possible to make meaning of what they are teaching when they have to resort to code-switching. As part of PCK, student teachers were expected to make reference to already known misconceptions. All the student teachers observed failed to make reference to already noted misconceptions where they would try to clarify the misconception like, $(a + b)^2 = a^2 + b^2$, $3a^2 + b = 3a^2b$ etc.

5.6.4 Traditional and Learner-centred teaching

The classroom environment in almost all the classes I observed depicted that of the traditional way of teaching (See 2.1.2) where the teachers faced the whole class and seldom move around the class. The teachers did most of the talking while giving examples or demonstrated some ideas on the chalkboard. Learners were not grouped and therefore discussion among themselves were not much possible. Learners were attentive most of the time trying to follow what the teacher is doing or saying. Once in a while discussions among the whole class happened when one of the learners had asked a question. The answers to the questions were most of the time based on explanations and descriptions instead of asking probing questions to lead the learner to answer the question.

The learner-centred mode of teaching was not prominent in the classes observed. This may be because the activities which were planned by the student teacher did not allow for that. The whole class were taught together and when there was a misunderstanding by one of the students the teacher stopped the whole class and tried to explain using the chalkboard. The investigative type of teaching was not common. Investigative teaching happened on only one occasion when the teacher had asked the learners to investigate why $(3x + 2)^2$ is not equal to $9x^2 + 4$ but equal to $9x^2 + 12x + 4$ (See 4.7.4). This type of investigation goes a long way to dispel some of the known misconceptions which have been reported in the literature that some learners have.

5.7. CONCLUSION

The picture that has developed from the analysis shows a worrisome knowledge and understanding of algebra. The average overall score of 50% is not what is expected of would be teachers of the subject. The algebra test and the interview with some of the students revealed that they had some misunderstandings and misconceptions of some concepts that they had learnt earlier in the previous years. These include:

1. The use of number concepts, real, natural, counting etc. Some of them did not consider zero to be a real number. For example item 4 (See appendix C).

2. Expression and equation- Some students could not discriminate between expression and equation.

3. Solution of non-linear inequalities was a problem for most of the participants. Nonetheless, some of the students who were interviewed were able to make out the difference between linear equation and non-linear inequation and that the way to solve them were not the same. This supports the notion that discussions and appropriate use of questioning and probing techniques in the classroom could enhance student's participation and understanding and would facilitate learning.

4. Square root of an inequality. Some difficulties in the finding of a square root of an inequality. Items 16 and 17 are examples of such cases where some students simply used the knowledge of finding the root of an equation for an inequality.

5. Checking answers to quadratic equations and inequations with both positive and negative values was a problem with some of them. This was noticed in items 15, 16, 17, 18 and 19 of the algebra test.

Many of the student teachers do not explain what it is that they want their learners to learn from the concept they want to teach. This approach may contribute to making algebra look like an arbitrary collection of rules and algorithms, an approach which most of the lessons observed seemed to reflect. Not knowing why teaching a certain concept may influence pedagogical content choices may lead to teachers presenting to learners the easiest ways of solving algebraic problems, thus over-emphasising procedural knowledge without concern for meaning (Bromme and Steinbring,1994). This is what happened when some of the student teachers were teaching like and unlike terms (see 4.5.4). They gave the rule to be followed in order to get the correct answer without concern for understanding, i.e. subtract the smaller number from the bigger number and retain the sign of the bigger number when adding like and unlike terms.

According to Graeber (1999:194) understanding students' current understanding is important in giving direction to instruction. She states that

> "*if preservice teachers enter the classroom without valuing students' understanding, they are not apt to assess understanding or use knowledge of students' current understanding to make instructional decisions"*.

It is on the other hand very difficult for preservice teachers to understand their learners within a short space of time when they come in contact with the learners during teaching practice periods. It is however, important for College of Education students to know that their understanding of their learners understanding will help in their instructional practice.

The lessons observed (4.2.4, 4.3.4, 4.5.4) usually did not integrate with other subjects and seemed to have no useful purpose from the learners point of view. According to Macgregor and Stacey (1997:17), " *when algebraic concepts and methods are not used in other parts of the mathematics curriculum, students forget them and forget the notation for expressing them*". This is true to an extent, but the context and situation under which the concepts are used also can create confusion. For example in Chemistry, CO_2 is carbon plus two oxygen, but in algebra ab is a multiplied by b. Since learners build on their already acquired knowledge it would better for student teachers to be aware of the beliefs about meaning of letters and mathematics notations that learners of algebra bring with them to the class. The student teachers should take account of these beliefs in their teaching in order to assist learner build on their already acquired knowledge. For instance in Botswana Berry (1985), where it becomes a taboo to count in big numbers teachers will have to take cognisance of this fact so that cultural conflict does not happen. The study had demonstrated that in most parts the student teachers often had a weak conception of algebra and most of the times exhibit purely instrumental understanding and procedural teaching (Skemp,1976).

As the percentage score of the algebra test shows, 46% of the final year College students in this study did not have good conceptual understanding of algebra. Rather, their conceptions seemed similar to their High School days, since most of the algebra test had been solved using their prior knowledge and similar misconceptions and misinterpretations were found in both the PTD and the STD students answers to the algebra test. The algebra test interviews had also revealed that they were recalling from their high school experience (See excerpt I3SEK2) where the student said they had been taught inequalities at the high school but had forgotten. Knowledge is judged to be derived from experience; it is not passively received but rather actively built up Von Glasersfeld (1990) hence the College students knowledge of algebra had been seen to reflect what was learnt at High school instead of at the College level. One may not truly understand a concept until he/she is required to teach it. The data so far has not shown that student teachers or colleges who performed well in the algebra test also did well in teaching in the classroom. For example College SIK did not do well in the algebra test with a percentage score of 27 but in one of the lessons by student teacher LO3SIK3 (See 4.7.4) it was very interesting and the learners' participation had been excellent. The student teacher had related the lesson to the real-world situation and the examples had come from the environment. This student teacher had tried to make reference to misconceptions generally encountered in the learning of algebra like $(a + b)^2 = a^2 + b^2$. This student had scored less than 50% in the algebra test but demonstrated that she could teach for conceptual understanding by expanding of binomial expression and using the investigative approach (See 2.1.6). Similarly student of PI4SEK1 (See 4.5.4) had done well in the algebra test scoring more than 70%. In the classroom this student teacher had demonstrated that she was good in the simplification of like and unlike terms. This student had done well in her teaching practice in relation to the good score from the algebra test. These two students had both done well as far as instructional practice is concerned notwithstanding the poor and good scores in the algebra test. There is therefore in these instances no direct relationship between the algebra test scores and their instructional practices. Some of the fourth year students who performed better than their third year counterparts in the algebra test had similar misconceptions and even some worse than the 3rd year counterparts.

Misconceptions had been noted with the 4th year College students who according to the requirement for college entrance should do better than their 3rd year counterparts but some had performed badly, which could be attributed to poor conceptual understanding. While this study was limited in scope, several observations were made about the participants. I noted that some student teachers still exhibited gaps, sometimes very serious, in their conceptualisation of algebraic concepts. For example the use of area formula for the perimeter of a rectangle, identifying "more than" and "older than" with multiplication, sometimes relying on inappropriate verification of concepts, used equation principles for inequalities. Some of the students were very weak in translating algebraic word problems into symbolic forms. For example, some students got items 7and 9 wrong because they were required to translate the algebraic word problem to algebraic symbolic form before they could solve them. Some also lacked the knowledge that the value of "c" in item 4 should be more than one number because they could not translate more than to mean more than one answer as they were required to do. Indeed, language acts as a powerful tool for development of thought Ernest (1995). COE students may have difficulties in learning algebra if they have not understood the appropriate meaning of some of the statements and concepts like items 4, 7, and 9.

The message these analyses give is that we should not be disheartened with learners for their errors and misconceptions because these had been passed on to them by their teachers. If we make reference to the Constructivist principles, children do not make mistakes because they are stupid but because their teachers have not taught them well or because they receive from their teachers incomplete conceptual understanding of certain concepts. It is also important to bear in mind that the learning of mathematics/algebra is based on previous knowledge. If the foundation of the previous knowledge is weak it may affect the new learning as well and can result in errors and misconceptions. This goes with the analogy of a house which has got a weak foundation; the house may collapse during the building process or after completion. One may also agree that earlier concepts which were developed and learnt well stay longer and are resistant to change. It is therefore important and necessary to try and develop a good schema at the early stage of learning to assist learner in acquiring new knowledge. It is not enough for students to obtain correct answers; they should strive hard to explain their thinking and reasoning. Hence, they will experience the conceptual aspect of the concept beyond the computational algorithm. One would not understand a concept well until she/he is asked to explain it. So if a final year College of Education student lacks knowledge and understanding of algebraic concepts, there is the likelihood that he/she may not be able to share the knowledge with others.

However, from the Constructivist's perspective learners construct their own knowledge and do not depend on the knowledge transmitted by the teacher. This being the case, it is, according to Vygotsky (1978) the teachers duties to help the learners overcome the Zone of Proximal development (ZPD), so without proper and sound knowledge of algebra concepts these teachers may not be able to mediate at this stage of ZDP where learners of algebra will need help from the adult the teacher. Central to Vygotsky's theory is the potential for learners to gain from instruction. The theory of ZDP also suggests that learners' development to higher level of performance depends on the assistance from the adult. It is therefore, necessary for the student teacher to plan the activities of the lesson in a way that learners can be assisted. To do so means the student teacher should have the necessary knowledge and understanding of algebra to be able to help the learners gain conceptual understanding. Because teachers normally grade their work examples according to difficulty, scaffolding (See 2.1.5.2) techniques should be adopted to help learners understand better by providing the familiar and introducing the unfamiliar in a measured and calculated way.

Misconceptions have been attributed to the efforts made by learners trying to construct their own knowledge. Misconceptions are therefore bound to occur but if earlier concepts had been clearly taught to learners the tendency of learners having misconceptions might be reduced. The results so far do not exonerate the student teachers from having misconceptions in algebra and is therefore likely to cascade to new beginners of algebra when these student teachers do teach them in future. An attempt to minimise these misconceptions is by discussing them in their teaching. The lessons observed however, did not show this as the student teachers failed to make examples of misconceptions some researchers (Kaur and Sharon, 1994, Olivier, 1989, Kieran, 1984) have reported in many studies, to help alleviate the fear of having the same misconceptions, for example $(x + y)^2 = x^2 + y^2$ and as Kuchemann (1981) indicates $2a + 5b + a = 8ab$ or $3x^2 + 2 = 5x^2$.

These College students are mathematically advanced but it is interesting to note that the same problems and misconceptions experienced by beginners of algebra still present themselves. In my opinion, the poor nature of these students' algebraic concepts would extremely limit their ability to mediate in the learning of algebraic concepts by the children they are supposed to teach for conceptual understanding.

CHAPTER 6

SUMMARY AND FINDINGS

6.1 INTRODUCTION

In this chapter a summary is provided of the entire study in terms of aims of the investigation, literature review, methodology, data analyses and major findings. From this, conclusions are drawn and implications are discussed. Some suggestions and recommendations are made in an attempt to solve some of the problems College of Education students face as they undergo their studies for the teaching of algebra at the junior secondary schools in and around the Transkei region of the Eastern Cape of South Africa. Furthermore, recommendations for further research in this field of investigation are made. This chapter again serves two purposes, first it explores briefly what can be learned about this sample of COE students regarding their knowledge and understanding of algebra. The second aspect will look at issues raised and offer recommendations that may be useful towards conducting further research. The data in the study had been gathered through an algebra test, interviews and classroom observations implying triangulation methods (See 3.1.1.1). These methods have been employed to obtain information relating to the knowledge and understanding of final year College of Education students from in and around the Transkei region of the Eastern Cape province of South Africa as has been stated earlier in chapter 3. The study involved a total of 212 final year college students from six different colleges pursuing STD (3-year) and SSTD (4-year) courses. I believe that through these methods conclusions could be reached about the manner in which the knowledge and understanding of algebraic concepts of these participants affected their instructional practice.

6.2 CONCLUSIONS AND DISCUSSIONS

6.2.1 Conclusions to Research Questions

The study has endeavoured to identify some of the key conceptual understandings of algebra of the participants. The average score of the items from CDMTA in the Algebra Test in this study had been 79.3%. With the exception of items 4 and 5, which were below 50% the remaining items were above 50%. This can be said to be a good result and thus portray sound knowledge and understanding of these items. However, Table 3.2 shows that item 1 is a CDMTA level 1 question which was expected to be answered by every participant at a College of Education but 6.2% failed to answer it correctly. Further to the degree of difficulty item 6 was a CDMTA level 2 item and I expected all could answer the question with no difficulties but again 18.8% failed to answer. Item 4 was a CDMTA level 3 question but as high as 81.5% failed to answer it. This implies that the number of years at school or College of Education did not help. Items 7-12 were all CDMTA level 4 questions, the scores can be termed satisfactory as the scores were all above 70% judging from them being CDMTA level 4 questions and therefore of a higher degree of difficulty. On the other hand, item 10 was very disappointing as the score was only 53%.

On items 13-20 which was adapted from Kaur and Sharon (1994) which was originally meant for first year College students, the percentage overall was opposite to what items 1-12 indicated, resulting in a very poor score of 20.7%. With the exception of item 14 all other items had an average percentage correct score below 50%. In item 20 the average percentage correct score was 2.3% the lowest percentage of all the items. This question was meant to differentiate between an expression and an equation. Equations generally form the core of school algebra so failure to answer item 20 shows how little knowledge and understanding these subjects had of equations. It can be concluded from these percentage scores that the results were appalling, judging from the fact that these students were about to be certified to teach in schools. The knowledge and conceptual understanding of the concepts in items 1-12 were seen to be good but the 20.7% score with items 13-20 is very disappointing. Items 13-20 questions were more difficult than items 1-12 in that they required conceptual knowledge and understanding of algebra to answer them, so failure to answer them correctly means the participants were weak in these aspects. The overall mean percentage correct score of 50% (see Table 5.1) for the algebra test coupled with how badly they answered items13-20 helped to answer research question 1. This is an indication that the final year College of Education students in this study did not have proper knowledge and understanding of some basic algebraic concepts.

The second research question asks: ”What are the common misconceptions that a sample of College of Education students acquire in algebra and how do these misconceptions develop?” This question can be answered by using the analysis of the algebra test and the classroom observations. The overall quantified level of misconceptions from all the Colleges stand at 19.2% (See Table 5.4). Table 5.5 shows some of the misconceptions which were common across all the colleges; they include items 4, 15, 16, 17 and 19. The major misconceptions arose in items 15-19 where, the subjects failed to verify quadratic equations and inequations with both positive and negative numbers. Item 15 on the other hand had to do with solving an inequation. The misconception of solving an inequation using the method for equations was attributed to the way the concept had been taught previously (See 4.3.2) and the time, which had been allotted to the teaching of it.

The acquisition of this misconception is twofold. First it is the retrieving of previously incomplete knowledge (Macgregor and Stacey, 1997). The second reason is the fact that this concept is not examinable at Standard Grade (S.G) level in matric.

The majority of the College students are accepted with only S.G mathematics. The fact that they may not teach the algebraic concept in Junior schools where they intend to be employed also contributes to the poor test scores. Item 4 was however, about failure to write c as a set of real numbers less than 5 the solution to the problem. During classroom observations a student did have the misconception of using the area formula for the perimeter of a rectangle, which he attributed to the textbook he was using (See 4.8.4). Generally, not many misconceptions were detected during the teaching practice period possibly because of the short time I had with them. This study supports Robinson et al. (1994) who indicate that both precollege and college students have many misconceptions about algebraic concepts. Radatz (1979) classifies misconceptions under five categories (See 2.5) and the misconceptions found in this study fall under three of them:

1. Language difficulties
2. Deficient mastery of prerequisite facts and concepts
3. Incorrect associations or rigidity of thinking and application or irrelevant rules.

The above named factors have contributed to the misconceptions found in this study. Firstly, language difficulties contributed to the misconception where some subjects failed to interpret “older than” and “more than” correctly. As mentioned in section 3.1.1, majority of the subjects were second language English speakers. Some therefore had difficulty of interpreting the phrases like “older than” and “more than” in items 9 and 10 (See Appendix C). They had interpreted these phrases to mean, “multiply by”.

Secondly, deficient mastery of prerequisite facts and concepts contributed to the misconception where some subjects failed to retrieve the correct expansion of $(a + b)^2 = a^2 + 2ab + b^2$. Instead these subjects had applied the incorrect expansion of $(a + b)^2 = a^2 + b^2$ to item 18 (See appendix C). This led them to agree to statement 1 of item 18 as true.

Thirdly, incorrect associations or rigidity of thinking and application or irrelevant rules contributed to the misconception in the study where in item 5, some subjects could not recognise the simple substitution to arrive at the answer. Some instead applied the tedious application, in this case, the rule of the expansion of cubic binomial, that is, $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ to the expansion of $(5x + 1)^3$, which led to incorrect answer.

According to Blubaugh (1988) the influence of the teacher contributes to misconceptions. This study cannot conclude that lecturers are a contributing factor to the misconceptions student teachers had but it is likely to suggest so since the misconceptions differed from College to College. Lecturers were not included in the data collection; that is why I cannot emphatically say they were party to the misconceptions found. This can however, be verified in future research of this nature.

Finally, the third question, "How does College of Education students subject matter knowledge affects their instructional practice?", conclusions to this question can be drawn from the algebra test and the classroom observation. From the algebra test results and the practice teaching observations, I cannot directly relate the results to how they taught in the classroom. The reason being a student who performed badly in the algebra test taught well, making use of strategies which are meant to bring about conceptual understanding (See 4.5.4). This was possible because this student teacher had attended a course where they were shown how to teach for conceptual understanding on that particular topic. Similarly, another student who had performed very well in the algebra test also did well from my observation in the classroom (See 4.7.4). These two scenarios support what Lave (1990) states about apprenticeship learning. Teaching is a skill and can be learnt from experienced teachers in the field. Knowing the content alone does not make one a good teacher (Ball, 1990). The combination of the content knowledge and the skills learnt in the field can therefore enhance the conceptual understanding of algebra in College of Education students.

6.2.2 Discussions

It appears, though, that the COE students had the general knowledge needed to teach algebra but most of them were weak in conceptual knowledge and understanding. Furthermore, most of these students had not participated in social constructivist type of learning and teaching (See 2.1.5.1), which is centred on the learner and enhances the way algebra should be taught for conceptual understanding. Supporting evidence for these claims can be found in the student teachers judgement about the nature of mathematics and the emphasis placed on content knowledge to be taught which seemed to be absolute and therefore could not be changed. This could have led to student teachers accepting what is in the algebra textbooks wholeheartedly without doubting some of the information contained in them. For example, when one student supported a wrong answer, which was got from a textbook (See 4.8.4).

Teachers' knowledge and understanding play a major role in instruction and learning (Ball, 1990, Leinhardt and Smith, 1985, Even and Tirosh, 1995). The findings described by this study do have some support for the views expressed by these researchers. However, there were instances where students who did not perform well in the algebra test presented good lessons in the classroom when they were observed. It is a known fact that test results sometimes fail to reveal students' knowledge and understanding of a concept. The algebra test revealed some of the misconceptions and misinterpretations College of Education students had with inequalities, and misinterpretations of certain algebra words and statements like "more than:" and "older than". The findings seem to suggest that there is a high degree of similarity between 3rd year College students and their 4th year counterparts in the misconceptions of algebra they hold. The results of both of these samples on the Algebraic Test do not differ much from the first year students' results of the same test. One can infer from these results that much of their knowledge and understanding of algebra had come from the previous knowledge and understanding gained during high school. This means little change in knowledge and understanding of algebra happened during the years spent at the colleges. These conceptions and misconceptions were based on the traditional framework of knowledge acquisition rather than what curriculum 2005 and NCTM (1991) are trying to address. For instance in OBE the learner-centred approach, where the teacher places the learner at the centre stage of learning is advocated. Group work/teamwork is also recommended for conceptual understanding where peers will have the opportunity to assist each other to learn from each other and so develop conceptual knowledge and understanding.

This study suggests that 19.2% of the prospective teachers do not have a well developed concept of algebra (See Table 5. 4). For example, 19.2% viewed the concept of the perimeter of a rectangle solely in terms of the formula P= 2(l + b). The misconceptions, which have been revealed in this study, imply that there are certain sections in the high school syllabus which student teachers are inadequately equipped to teach and it would assist them greatly if they could be presented again to them. It will be of great assistance to the teaching and learning of algebra if some misconceptions in algebra are made known to student teachers. One of the ways in which awareness can come about will be through exposure to the literature on misconceptions. It will be particularly relevant and appropriate if the literature that is referred to deals mostly with misconceptions that are experienced by learners in schools because this is the target group for which student teachers are learning and preparing to teach. Another way in which awareness can be enhanced is to look at the misconceptions that teachers and student teachers have by conducting research. The present study shows that student teachers have misconceptions with inequalities. They should therefore be exposed to these misconceptions and be shown ways and strategies to address them.

The lack of conceptual understanding observed in the classrooms among some of the College of Education students probably reflects a more widespread problem pertaining to their future teaching after completion of the course at College, which is bound to have a cascading effect on the teaching and learning of algebra. I realised that in the schools where students were observed in the study, learning occurred according to the initiative of the College of Education students since most of the talking was done by these students while the learners most of the time remained passive (See 4.6.4). This could be due to my presence in the classroom since I thought they did not want to embarrass the teacher in my presence by asking questions, which might challenge the integrity of the student teacher. During the observed lessons student teachers most of the time failed to encourage learners to ask questions in the classroom (See 4.2.4, 4.3.4, 4.6.4). During the presentation of the lessons, student teachers provided most of the information and asked most of the questions. This could be as a result of the type of approach to teaching and learning which currently is prevalent in South Africa, the behaviouristic approach, leading to procedural teaching and learning. There was an indiscriminate use of questions from the student teachers, which learners most of the time answered in chorus (See 4.2.4) by giving one or two words. Most of the questions were intended to assess the level of understanding of the learners, the student teacher would ask, ”Do you understand”. In most cases the learners answered, “yes”, to such questions.

Invariably, questions requiring regurgitation dominated in the presentation of the lessons, with very few instances where learners posed questions. Teaching in the classroom was dominated and characterised by the lecture or telling method, the teacher-centred method which is a remnant of South African pedagogy based on fundamental pedagogics[3]. Classroom observation was a useful supplement to the interviews and the Algebra Test, since the opportunity of picking up misconceptions was created but at the same time reducing the chance of embarrassing the student teacher to a minimum. However, it is likely that teachers would show many more misconceptions in a formal interview situation, where probing is possible, than in a lesson. It was therefore appropriate that conceptual errors, which emerged during the lessons, were probed further in an interview with the student teachers concerned (See 3.1.3.3)

Leinhardt and Smith (1992) have shown that textbooks and teachers often provide incomplete descriptions of concepts. My study has shown that some of these participants had incomplete knowledge of some algebraic concepts and these did play a part in the students generating incorrect inferences, developing

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[3] Fundamental pedagogics, the study of science and basic principles of teaching where critical thought is not encouraged. Teaching methods revolve around “telling and informing” learners are expected to be passive and receptive.

misconceptions, and producing inaccurate answers to the algebraic test. As has been stated by Even and Tirosh (1995) and Shulman (1986), PCK includes several interrelated aspects “knowing what” and “knowing why” which have been seen to be important in the study of concepts in algebra (See 2.3). From my study it has come out that it is important for the student teachers to know about common misconceptions which have been identified by researchers such as, $(a + b)^2 = a^2 + b^2$ so that they do incorporate ways of avoiding these in their teaching. This will help student teachers to make sure that learners do not develop this misconception. Knowing why $(a + b)^2 = a^2 + 2ab + b^2$ is also important and by making learners know “why” leads to conceptual understanding of that concept. It is therefore necessary for a student teacher to arm himself/herself with this knowledge to help learners understand algebraic concepts well. The question that arises when one addresses misconceptions in algebra teaching to COE students is how pre-knowledge affects misconceptions.

Arithmetic has got a lot of influence on the learning of algebra. In most cases the misconceptions found in learners studying algebra are attributed to the early learning of arithmetic where some principles of arithmetic have great influence such as the incomplete nature (Robinson et al.,1994) of expression in algebra or conjoining (Human, 1989) of algebraic expression, which are linked. An example of such a misconception in the simplifying of expressions, which has a remarkable resistance to remediation, is learners writing $5x^2 + 3x = 8x^3$. My study has revealed that major misconceptions among College of Education students do exist and do hamper the knowledge and understanding of algebraic concepts. It is of great concern that even at the College level some final year students still hold erroneous views similar to those of the 1st year students.

Improving the teaching of algebra in schools could start with improvement in the algebra knowledge of student teachers. Without the knowledge and understanding of algebra student teachers would have great difficulty in using alternative representations and teaching for integration across topics, such as the use of geometrical representations in algebra. They would also be limited in the scope and depth of algebraic investigations of number patterns and sequences. Many student teachers failed to talk about or explain to their learners what it was that one could use of the concepts they were to learn. This approach to teaching might contribute to viewing algebra as arbitrary collections of rules and algorithms, an approach which most of the lessons observed (See 4.2.4, 4.3.4, 4.6.4) seemed to portray.

Teaching for conceptual understanding in algebra is a challenge, which is not an easy task. It requires a change in attitude among student teachers who still believe understanding a concept is being able to memorise and reproduce and who regard memorisation as a storehouse of rules and facts, held together by loose associative bonds which are either poorly retained or easily forgotten. My study has revealed that traditional methods of teaching are what prevailed in the classrooms observed. The classrooms were well organised with the student teacher most of the time in control of what happened in the class. Algebra was taught by making the learners learn formulae and tricks through memorisation in order to be able to solve certain algebraic problem. Failing to remember these formulae is likely to have resulted in misconceptions. I conclude with the statement by Vinner and Dreyfus (1989:365) "*One must remember that a concept is not acquired in one step. Several stages precede the complete acquisition and mastery of complex concept".* Observing lessons for only a day or two could not unearth all the strengths and weaknesses these students had. Macgregor and Stacey (1997), Eisenhart et al. (1993), Baturo and Nason (1996), and Shulman (1988) have all described and elaborated on what the learning and understanding of algebra means and how misconceptions could be explained. They propose that background knowledge of the learners' preconceptions is vital to assist in assimilation or accommodation in the process of conceptual change in learning and understanding new information. Brodie (1989), Costello (1991), Vygotsky (1962) and Ernest (1991) have investigated how conceptual and language difficulties affect algebra learning and understanding among learners. All these researchers have suggested some possible remedies to the difficulties investigated and have recommended how curricula could be developed to overcome the difficulties established by research and so lead to better instruction. In section 6.4, I also make my recommendations.

6.3 IMPLICATIONS OF THE STUDY

This research study has revealed that there is a great need for a review of College of Education method courses in algebra, which currently lead to procedural teaching. Such a method course should aim at assisting college students to learn for conceptual understanding to be able to teach for conceptual understanding. Moreover, student teachers should be subjected to continuous assessment (See 1.3.1) instead of gauging their knowledge and understanding only by written tests and examinations. Oral interviews should be part of the assessment in which student teachers will be probed further to correct some of the errors they make during the written examinations.

6.3.1 The Development of Conceptual understanding as opposed to Procedural Understanding

The frequent use of the traditional method of teaching whereby the teachers do most of the talking is reminiscent of how they themselves were taught and how as student teachers they were trained. All too often teachers have to go through this method of teaching where they are taught the techniques of arriving at answers instead of teaching learners the why of the methods to arrive at the answers. Von Glasersfeld (1990) discusses teaching and learning in terms of “radical constructivism” (See 2.1.5.1). He distinguishes between training and teaching. Training has to do with the trainee’s performance, while teaching which has to do with conceptual understanding. Conceptual understanding, he states, can only be inferred by the teacher based on how compatible the learner’s understanding is compared to the teacher’s understanding. So until the time when student teachers will be taught for conceptual understanding the vicious circle is bound to remain with teachers for decades as not much change is effected in the teaching of algebra as my study shows. Notwithstanding, the levels of accusations towards Interference-Response-Feedback (I-R-F) (Brodie, 1989), nothing so far has changed from this mode of teaching in the classrooms observed. The reason why this method is predominant in the classrooms is the assumption that a lot of information could be transmitted to learners within a short space of time. Coupled with big class sizes, the implication is that student teachers normally will adopt the behaviourist approach to help overcome classroom congestion and use the transmission method to attempt to teach a lot of content within a short space of time. With the behaviourist approach learners are not able to assimilate information properly and this leads to incomplete knowledge and understanding, which in turn leads to misconceptions. This traditional approach is evident with the chorus answers (See 4.2.4) given by learners and the emphasis placed on textbook-based knowledge.

The over reliance on textbook methods without knowing that there are alternative Methods and representations possible, which could be more viable in promoting pupil learning is a problem which could be addressed by exposing algebra teachers to different ways of teaching and making use of the learners environment and situations. I believe that algebra must be learnt with understanding. It will be important for teacher instruction in the classroom to be directed away from rote learning and routine manipulation towards conceptual understanding. Such understanding can be achieved in a variety of ways, being taught the algebraic concepts, using knowledge of instructional techniques, alternative representations, investigational approaches and applying constructivist principles in the classrooms (See 2.1.5). The student teachers’ PCK thus becomes very critical in order to fulfil these tasks of teaching for conceptual understanding of algebra. Most of the participants lacked these characteristics; the few who had sound knowledge were able to teach with confidence and for conceptual understanding. During the instructional process attention should be paid to social aspects of the prospective learners of algebra and to their inherent mathematical ability.

Ball’s (1990) investigation into preservice teachers knowledge and understanding of division in algebraic equations made the following assessment. She found that in most cases preservice teachers’ understanding tended to be rule-bound, fragmented and founded on memorisation rather than on conceptual understanding. My study shows a similar result. Ball (1990) further states that preservice teachers knowledge and understanding of mathematics is procedural rather than conceptual. With similar result for this study the implication is that the school system cannot rely on what COE students have learnt in algebra class before they enter the teaching field, since it is likely that the knowledge and understanding of algebra may be inadequate to teach for conceptual understanding. Intervention strategies are required to overcome or monitor these COE students. By so doing the effect of cascading to school learners of algebra might be minimised.

The teacher education course should aim at producing teachers as mediators (See 2.1.5.2) rather than as transmitters of knowledge. These College of Education students should be empowered with a thorough knowledge and insight into the teaching and learning of algebra to be able to face the daunting task and the challenges in the classrooms where algebra is taught for conceptual understanding instead of for procedural understanding, which is what is expected of teachers in Curriculum 2005. The psychological, the social and political views of what is necessary to acquire this knowledge in algebra could be attained through OBE and may lead to conceptual understanding instead of procedural understanding. The implications are that there will be hope that learners will experience the power of algebra through mediation and so correct construction of its basic concepts.

6.3.2 The Teaching of Methods in Colleges

The way in which some student teachers understand content influences classroom instruction to a certain extent as has been shown by my classroom observations (See 4.2.4, 4.3.4, 4.6.4). Some of the topics at the matric level were not treated during College of Education training courses because of a lack of time or due to specific Colleges or lecturers designing their own curriculum. (Some of the Colleges were semi-autonomous and had the opportunity to draw up their own syllabi). This therefore, might have led to the omission of the teaching of certain topics or concepts, which might have contributed to weak knowledge and understanding of the concepts involved. This supports what Nakahara et al. (2000:114) state " *instead of defining concepts, teachers tend to simply explain to the pupils how to manipulate the elements to which the definition refer*". This was evident in item 13 where only a few of the students could define a function but were able to answer the question in item 14 which is the application of the definition of a function this might have been the result of the omission of the definition of function from the curriculum. The mode of assessment which is mostly based on written examinations and tests has led to lecturers laying much emphasis on techniques and skills to arrive at answers instead of teaching for conceptual understanding.

As Brodie (1989:52) states “*meaningful mathematical learning must include explicit teaching of the language of mathematics with due emphasis on concept formation”*. Vygotsky (1962) emphasises that cognitive development is determined by language or linguistic tools of thought and the socio-cultural experiences of the learner. Vygotsky (1978) stresses also that the context in which teaching and learning occurs is important. This implies that language becomes a factor in such situations. The problem of inadequate vocabulary of the language of learning and teaching was evident during some lessons (See 4.2.2, 4.7.2). Learners, who could not express themselves well in English during discussions of algebra problems, resorted to their first language, Xhosa. As has been mentioned by Berry (1985) the learning of the mother tongue goes through various stages, as does the learning of an additional language. He further mentions that children who have not yet developed the basic vocabulary in their own language are likely to be unable to express their experiences in a second language. This study has shown that some of the College of Education students had problems interpreting “more than” and “older than”. In support of Brodie I maintain that student teachers should be made to include the teaching of such concepts coupled with their symbolic representations during their training years at the Colleges of Education. Inculcating the proper use of code-switching (Setati, 1998) during COE courses, in order to assist learners in such situations, is also important to consider.

6.3.3 Types of Assessment Practices

The findings of my study have implications for assessment practices. There is a need to revisit the basis on which judgement of COE students’ competencies are made. Awareness and reflections are dialectical concepts which student teachers have to develop in order to enrich their practices in general and their algebraic classroom assessment practices in particular. COE students need to be aware of the importance of evaluation of learner’s level of development as measurement of curriculum content and not as a means of grading learners. The use of tests and examinations as the most available means of collecting assessment data poses a challenge to teaching for conceptual understanding. There is therefore, the need for teachers to take risks and deviate in their ways of assessing so that the urgency, which normally accompanies teaching for conceptual understanding of algebra may be realised. Algebraic concepts should be examined in terms of problem solving. Often the problems have a “realistic” setting, but the real issue is meaningfulness. Even an abstract algebraic problem may be meaningful to a learner if the learner understands the problem and is engaged in determining the solution. No one way grouping should always be used: sometimes whole class, sometimes small cooperative groups, sometimes pairs, sometimes individual learners.

It can also be argued that the policies of the government towards teaching are mainly based on the so called “first world South Africans”. Efforts should be made so that the third world South Africans are also catered for in the policies. For the teaching of algebra for the third world South Africans a nomenclature for certain concepts should be developed in the other official languages since most of the student teachers and school learners are second language speakers of English who have to translate, reason and think in vernacular before it is translated back to English.

6.3.4 The Use of Networks of Teachers

It will be an advantage for student teachers to join some mathematical associations like the Association for Mathematics Education of South Africa (AMESA), a professional body for mathematics teachers, which attempts to address some of the problems and challenges teachers face in teaching mathematics.

6.3.5 The Situational Constraints

Situational constraints like large classes make it difficult for COE students to teach for conceptual understanding. These COE students should try to adopt different strategies and methods in the teaching of algebra to bring about conceptual understanding. They should play a role in facilitating learners of algebra to speak out so that some of the misconceptions that accompany their reasoning could be addressed through probing and reflection.

6.3.6 INSET

My findings also imply that In-service Education (INSET) through workshops for practising teachers are necessary supplement to the education provided by the Colleges of Education in algebra. There is need for the teachers of algebra to be aware of and reflect on the misconceptions and misinterpretations, which are likely to accompany the process of teaching algebraic concepts.

Secondly, INSET has a role in making teachers aware of the changes required to teach certain topics in algebra for conceptual understanding. The traditional mode of teaching is found to be based on behaviourist principles of teaching. There is therefore a need to change or include, more of the constructivist principles of teaching for conceptual understanding. The teachers beliefs in mathematics as absolute knowledge should be changed to view mathematical knowledge as fallible (See 2.1.4.4).

6.3.7 Summary

The student teachers involved in this study were from three different African background (rural, urban and city); however the sample was not fully representative of the South African population, as the learners and student teachers both belonged to the African population. The generalisation of the result of this study to all Colleges of Education students and learners in South Africa should be done with great care and caution. The implication for teaching from the study is that the approach to teaching algebra should be overhauled. In support of van Reeuwijk (1995) some algebraic topics should be revisited, and slowly but thoroughly develop problems that are "real" and make sense to learners. Teachers of algebra should make it their duty to allow learners to develop algebraic concepts at their own pace as suggested by Reeuwijk (1995). This will emphasis the student teachers' role as mediators who will guide the learning process in algebra. This role will put the teacher as the leader of class discussion, where he/she directs discussions, builds on prior knowledge, gives time for thinking, expects learners to explain or justify responses or thinking, knowing how to deal with different responses and so, puts himself/herself in the shoes of the learners. COE students should vary the mathematical difficulty or abstraction of the problem or to vary the size/type of the numbers in the problems, but not to vary the algebraic topic. They have to maintain whole class sharing, even if differing groups are solving different problems. This seems to be a hard task for College of Education students but with the belief and preparedness to change, this will not be an impossible task.

6.4 SUGGESTIONS AND RECOMMENDATIONS

The analyses have shown how important it is that we motivate our learners to actively struggle to make meaning of whatever they have to learn in algebra. This is because even at the College of Education level some students could not see that the length of a rectangle could not be a negative number or that the age of a person could not be negative. (Some student answers gave lengths and ages as negative, see 4.2.2). Improvement in the algebraic competencies in COE students would help to improve the reasoning and understanding of conceptual aspects as opposed to simple skill development. Components of competency should include multiple representations, understanding of basic principles such as: one cannot add unlike terms, checking solutions of equations and inequations to ascertain the correctness of the answers. For example, the teaching of algebra should include the use of geometrical problems like finding the area of rectangles and other geometrical shapes. These will enhance the conceptual understanding of algebra as it would then be seen from different perspectives and situations.

I think it is important to place College of Education students in a teaching situation where opportunity and support are provided to match the vision of Curriculum 2005 and the Standards of the NCTM (1991). They must be given the chance again to observe experienced teachers in the field who would model teaching strategies for the teaching of algebra which they might not have got at college. They must have time to do research to prepare lessons that focus on conceptual knowledge and understanding and should be given enough time to teach for conceptual understanding instead of for procedural knowledge. In this case it is the duty of the student teachers and their method lecturers to brainstorm around the factors, which lead to pressures in adopting procedural teaching instead of conceptual teaching and to help prevent similar situations in the classrooms from occurring. It is also important for a good teacher of algebra to teach with conceptual understanding, and to acknowledge what Graeber (1999) states that teachers should understand, support and acknowledge the learner’s reasoning as an important part of successful instruction. It is therefore necessary and important that classroom instruction is integrated with assessment as is suggested by Curriculum 2005 and the Standards of the NCTM (1991).

The COE course should be designed in such a way that the students, together with their lecturers will have time to explore and develop teaching strategies that will lead to conceptual understanding of algebra. Judging from the time constraints on the part of the lecturers I will suggest that method course lecturers meet with experienced teachers as well as examiners in algebra to share experiences and provide support for the College students. The Department of Education should allow for a apprenticeship period for the newly appointed teachers from the Colleges of Education to work with good experienced algebra teachers teach for some time before they are allowed to handle the classes alone. It should be the responsibility of the College students to confront their previous beliefs about the nature of mathematics teaching and about how to teach mathematics in order to keep abreast with the modern trends of teaching algebra.

The method course structure should be designed to include the use of algebraic language (See 2.1.5.4) to express real-world problems and work with different representation in order to solve the same problem. This should include the formulation and solving of equations, generalising, developing formulae from the learners’ environment. The beginning of each topic should start with a brief moment in the history of algebra (See 1.1.1); this can help create interest in algebra and bring about meaning to some of the concepts.

It can be presumed that some of the COE students who were subjects in this study had used the traditional method of teaching simply because they were ignorant or have poor grasp of changes to policies and practices envisaged by curriculum 2005. Teacher Education therefore, needs to partner the Department of Education (DOE) by alerting and instructing teachers enrolled in algebra courses of the changes in approach required. Teacher Education will need to identify sound programmes in algebra and support them in ways that will allow for effective changes in the classrooms.

How can knowledge of misconceptions that are encountered in the learning of algebra be used to improve classroom instruction? The answer is that teachers and student teachers should talk about these misconceptions in the classrooms and try to address them by allowing learners to investigate the truth and falsity of some of these statements or expressions. As has been reviewed in the literature (see 2.5) misconceptions are not due to students' lack of procedural knowledge, but are due to their lack of conceptual understanding of the concepts. Misconceptions in algebra have been attributed to the incomplete treatment of certain concepts or topics where learners try to fill in the gaps with false generalisation. It is therefore necessary for student teachers to come together and draw up programmes of action where particular misconceptions are addressed in workshops during which groups are tasked to come up with ways of addressing these misconceptions. This entails brainstorming teaching strategies that focus instruction on learners of algebra. Student teachers should then share teaching strategies that address specific misconceptions in algebra learning. This is important because the nature of the setting where various types of students, different levels of experience come together to share ideas makes the knowledge acquired richer and informative, stemming from experiences from different Colleges of Education. The brainstorming meeting should map out the objectives as well as activities, and resources that are needed to address each objective. Outcomes of such workshops should then be documented and circulated to schools and Colleges of Education.

In addition College of Education lecturers should visit and observe non-performing schools in mathematics and find out the causes. This can be done by both the lecturers and the student teachers in these non-performing schools sitting together to do item analyses of results of examinations or tests to identify problems and reasons for the low performance. The lecturers should then give student teachers projects to come up with better ways of teaching the concepts in algebra they identified as weak during the visit. These projects should then be presented to all students of algebra for modification or change where needed. This will then be instructional support, which will assist in the teaching of algebra at schools.

I believe that integrating teaching (See 2.7.2) with continuous assessment (See 1.3.1) suggested also by Van Reeuwijk (1995) should be recommended as is required by Curriculum 2005. The curriculum should be based on integrated learning and not single subject topic and learning such in algebra, geometry, trigonometry and calculus. Many of the topics dealt with in algebra can be connected to geometry. For example, the perimeter of a rectangle $P = 2(l + b)$ is both algebra and geometry. Assessment in algebra should be continuous to address misconceptions and misinterpretations so that they develop into accepted conceptions. Student teachers should also develop a reasonably rich relational understanding of algebra in order to help teach for conceptual understanding in the classrooms they will teach in future. Algebra courses should integrate with the overlapping subjects like geometry and calculus mentioned above. Materials development should also be integrated with assessment as suggested by Van Reeuwijk (1995). Materials however, do not teach so failure to incorporate them with proper instructional techniques and strategies will be of no advantage. For all these innovations to be successful the support of the DOE and Provincial Education Departments and the Teacher unions should be solicited. I am optimistic that all the aspects of change envisioned by Curriculum 2005 and the NCTM Standards for providing constructive learning and understanding of algebra will prevail with the support of those parties. This approach will end up producing student teachers who will teach algebraic concepts with meaning and sense.

I believe the formation of district and cluster mathematics networks will assist teachers in algebra to come together and share the problems and challenges in an effort to address them by way of model demonstration lessons. Model lessons can assist the College of Education students. One possibility is to develop a set of videotapes where exemplary teachers teach demonstration lessons and provide information concerning curriculum materials. At least one school in a cluster should be supplied with electricity and video and television sets to enable the use of these tapes, where electricity is not available in all the schools of the cluster. Likewise, an alternative model for conceptual teaching (See 1.4) of algebra should be developed where all College of Education students in a cluster are called to be part of the development. College of Education students could then be given the task of field testing these materials during their supervised practice teaching period. I believe that such a model will provide the COE students with supportive environment to implement the theoretical consideration discussed. Simultaneously, the COE students will receive additional training in both the content and method of teaching algebra.

Changing instruction in the classroom, however is difficult because it involves changing teachers' beliefs, attitudes, and habits. To encourage a greater degree of involvement of students teachers, the Department of Education must provide extra support to student teachers by providing them with extra money for resources. College lecturers, as instructional support personnel, can also provide student teachers with resources needed to use for instructional improvement. College lecturers usually have the trust of student teachers and are able to work with student teachers on a continual basis using a developmental approach, an essential practice for changing beliefs and attitudes. However, I believe that curriculum change in South Africa will have a major impact in the learning of algebra and mathematics as a whole. It is absolutely crucial that the Colleges of Education encourage and assist this change rather than impede it, for it will lead to meaningful learning and understanding of algebraic concepts. Different teaching methods and alternative ways of presenting a topic or concept should be acknowledged and accommodated in teaching and learning situations. As Hiebert

(1980b) and Lave (1990) suggest, apprenticeship learning leads to understanding.

Practice teaching should be videotaped so that lecturers can see and help College of Education students to reflect on what they see in the classroom and how learners understand certain algebraic concepts. This idea is what Van Reeuwijk (1995) discusses in his Realistic Mathematics Education (RME) strategy. He takes reflection and discussions as critical factors in the teaching and learning process. College of Education students should also try to include in their teaching practice a form of interviews with their learners to find out why they make certain conceptual mistakes. For practice teaching, College students should develop meaningful problems, which will require learners to explain their thinking and justify their procedures and answers. In so doing learners will be forced to talk and participate in the lesson.

I suggest (based on limited classroom observations) also that College of Education students should endeavour to use alternative representations in their teaching. As Graeber (1999) argues "*if preservice teachers fail to provide alternative paths to understanding they are apt to leave some students without understanding"* p203. Graeber further argues that alternative strategies are sometimes more efficient than the routine application of a well established method. Item 5 of the Algebra Test is testimony to such a situation where the established formula made the problem more difficult, while the intuitive application of replacing x by 5x gave the correct required answer. Shulman (1986) also shares the same sentiment with Graeber when he states that knowing alternative representations of regularly taught topics would provide teachers with a veritable armentarium of alternative forms of representation. This idea which should be adopted by College of Education students as a form of assistance to them as they perform their duties as mediators or facilitators of learning algebra.

My study shows that prospective teachers' knowledge and understanding of algebraic concepts is weak and fragile. This conclusion was also reached by Ball (1988, 1990) in relation to elementary and secondary mathematics where the knowledge and understanding of preservice teachers was found to be weak and fragile. If the knowledge and understanding of algebra by College of Education students is lacking in mathematical and pedagogical content knowledge, then the question I ask myself is, what kind of knowledge and understanding of algebra do these students need to have in order to teach for conceptual understanding? Some researchers have recommended some means to address this question. Even (1990) suggested that apart from the normal content courses an additional course should be added which will only be for preservice teachers to learn to teach. Additional algebra course should be designed to address the problem with these College of Education students. According to Even (1990) such courses should try to address issues like: attending to the essential features of a concept (algebraic in this case); understanding different representations of a concept; familiarity with alternative ways of approaching the same concept; having a basic repertoire of powerful examples that illustrate important principles and properties; and having mathematical knowledge which includes both procedural and conceptual knowledge and the relationship between the two.

It is therefore, important for the Department of Education and Teacher Education planners to implement changes in courses in algebra which will make student teachers come out of the colleges with a confidence and pride in teaching. These algebra courses should be designed and developed so as to create better conceptual knowledge and understanding in the algebra content they have to teach. These courses should be different from the way these College students have been previously taught.

In what follows the recommendations that arise out of this study are summarised under four different categories of people: the student teachers, College of Education lecturers, curriculum planners and researchers.

6.4.1. **Student teachers.**

- History of algebra should be included in their teaching.
- Integrated lesson should be encouraged and used in algebra classes.
- Different representations of the same topic should be used during instructional practice.
- Algebra teaching should be integrated with arithmetic, geometry and calculus since they have bearing on each other.
- Real life examples should be used to make meaning of concepts.
- Language and thought go together when teaching for conceptual understanding and hence student teachers should be assisted to explain algebraic words and symbols in algebra classrooms.
- The philosophies that mathematics is fallible should be exposed to student teachers.
- In teaching practice lessons the student teacher should be made to act as a facilitator and not a transmitter of knowledge and that the principles of constructivism should be used for developing conceptual understanding of algebra.

6.4.2. College of Education lecturers.

- In their courses emphasis needs to be placed on conceptual understanding and correct usage of algebraic notations and symbols.

- The meaning of phrases such as write, simplify, solve, more than, older than etc. need to be clarified to students of algebra.

- Steps should be taken by lecturers to identify College of Education students' difficulties/misconceptions and discuss them in an effort to make students aware of them.

- An evaluation by lecturers of College of Education students' relevant previous knowledge by means of diagnostic or baseline assessment strategies is necessary to help lecturers build on and introduce new topics.

- Lecturers should be made aware of the knowledge and skills required by Curriculum 2005. This will cater for the shift from a teacher-centred approach to a learner-centred approach.

- Lecturers should be encouraged to change their belief systems of the nature of mathematics as absolute knowledge to that of mathematics knowledge being fallible and that mathematics is a human activity which caters for all cultures.

- Lecturers should be encouraged to involve themselves in research studies to help improve teaching/learning of algebra with second and third language speakers of mathematics. The findings should be shared through mathematics associations e.g. AMESA, Journals, Symposiums etc. for onward dissemination to respective Colleges of Education.

- From the study, poor background content knowledge has been seen as one of the contributing factors towards poor performance. There is then the need to have extra mathematics courses where remedial teaching will be conducted by lecturers to lay the necessary foundation toward advanced knowledge in school algebra.

6.4.3. Curriculum planners

- The study revealed that some of the students have a poor conceptual understanding of algebraic concepts. Students are weak in conceptual knowledge and understanding probably because not much importance is given to conceptual understanding as most of the questions asked in the final examinations demand memorisation and reproduction. Curriculum developers should then guide examiners to include questions that demand conceptual understanding and not only procedural understanding.

- Curriculum developers should include history and relevance of certain algebraic topics in the prescribed curriculum in order to create interest and meaning for the concepts. However, relevance (appropriateness) should embrace students, lecturers, and governing bodies of schools. Without practising teachers being included in designing curricula, some resistance is bound to arise as teachers feel that their rights have been infringed on and thus see any change as an imposition.

- Instructional materials should be designed specifically for the local context. Such materials are more likely to succeed as vehicles of change than materials designed for foreign contexts which often include application to unfamiliar scenarios and make use of language unsuitable for second and third English language speakers. The most worthwhile concepts to target in such materials will be those which are both important to the topic but difficult to teach, and those which are most often subject to misunderstanding.

6.4.4. Researchers

- Similar research studies should be conducted involving Lecturers of Colleges of Education.

- There should be a longitudinal study in which the researcher follows the College students to their schools for at least a year.

- My study showed that most of the knowledge of algebra had been acquired at the high school level. It will thus be appropriate to investigate the cause of these misconceptions and misunderstandings at the high schools and try to address them before they are carried to the College of Education level. Poor performance in inequality problems was linked to poor pre-requisite knowledge. More efficient ways of incorporating previous knowledge in new lessons need to be developed and researched, taking into account large classes and lack of facilities such as videos and computers in most schools in rural parts of South Africa.

6.5 FUTURE RESEARCH

Perhaps future research needs to look more carefully at constructivists' principles as well as the investigative approach as to how algebra learning takes place. From the constructivist perspective, knowledge acquisition is as a result of construction by the learner, regardless of the context or nature of instruction. The evaluation of the constructivists' principles may provide a realistic basis on which to reconstruct current courses for Teacher Education. Secondly, the constructivist principles favour the learner-centred approach envisaged by Curriculum 2005, which is presently in its initial stage of implementation in South Africa. The constructivists raise serious questions from an epistemological position. According to Jarwoski (1994) the epistemologists main concern is with the status of knowledge. This study shows that student teachers not only lack deeper knowledge in school algebra but also that pedagogical content knowledge needs attention. The core principle of constructivism is that learners actively construct knowledge to make sense of the world, interpreting new information in terms of existing cognitive structures. To shift from procedural knowledge to conceptual knowledge and understanding of school algebra will mean adopting the constructivist's principles. Research however will be needed to show whether this is happening in South African classrooms.

My literature review has shown that there have been research studies into development of algebraic understanding and the acquisition of algebraic knowledge (See 2.1.2, 2.2.1, 2.3, and 2.9), but that little is known about the way one mathematical achievement relates to another. Is arithmetic the basic structure on which all algebraic concepts depend or can other aspects of mathematics, like geometry, form a foundation for algebra? Perhaps the answer is no, but arithmetic is still taken as the most basic means of teaching algebra. Arithmetic has been seen as one of the major causes of misconceptions and difficulties learners have in understanding the structure of algebra (See 2.6). There is a need for research into introducing algebra from a geometrical stance instead of from arithmetic, which will include the practicability of activities as has been noted as a concern by some of the COE students that they could not construct teaching aids in the teaching of algebra (See excerpt I3SEK1).

Research into ways of using the lecture method of teaching for the acquisition of knowledge and understanding of algebraic concepts should be looked into as most of the College of Education courses are based on the lecture method. The involvement of large samples from other Colleges of Education to accommodate other population groups in South Africa should be looked into as my study was on only a small section of the South African population.

Other methods of collecting data apart from the triangulation approach used in this study should be used. A longitudinal study should be undertaken where student teachers could be followed to the classrooms, as they become full time teachers of mathematics for more than one year. Inclusion of the College of Education lecturers in the study is also essential, as they also become a source of data. This is because these lecturers are part of sources of algebraic knowledge student teachers acquire. Little however, is known about the way student teachers' conceptions and misconceptions are affected by their lecturers at the Colleges of Education. Research can contribute significantly to improve instructional techniques and enrich knowledge and understanding of algebra if it is directed towards the goal of acquiring knowledge and understanding.

Research into ideas College of Education students hold about how algebraic concepts are learnt by secondary school learners of algebra would encourage them to concentrate on their methods of instruction and curriculum content at large. It is important not to allow the curriculum change which is currently taking place in South Africa to derail the teaching process where only one approach is used all the time. Strategies should be researched which can be used to enhance successful teaching for knowledge and understanding of algebra.

Research should also be conducted to find suitable forms of assessment which will demonstrate the knowledge and understanding of algebraic concepts by College of Education students. Future research should look into the constructivist mode of teaching, learning and assessment and so evaluate whether this approach to teaching, learning and assessment is realistic for developing the conceptual understanding of algebra by College of Education students.

Different types of research can contribute significantly to the improvement of instructional techniques and enrich algebraic knowledge and understanding of College of Education students. The aim should be to improve the pedagogical and mathematical development of would-be teachers.

6.6 CONCLUSION

This study is consistent with and adds to the research findings of Ball (1988, 1990) and Kaur and Sharon (1994). The findings support the original assumption that lead to this study; that is, College of Education students' knowledge and understanding of algebraic concepts are fragile and weak. In particular, the observed consistency between College of Education students' knowledge and understanding and the manner in which they approach the presentations of the lessons strongly suggests that their knowledge and understanding influence their instructional practice in one way or another. Although algebra is one of the most important branches of mathematics, it seems it is one about which students teachers have the most serious misconceptions. Intervention strategies are therefore needed to try to minimise these misconceptions, if not eradicate them.

The results of my research clearly indicate that the subjects knowledge and understanding of algebraic concepts were not good enough to assist learners as far as learning for conceptual understanding is concerned at schools. The ability to use different representations or strategies to make algebraic concepts clear were limited. This result is similar to that of Baturo and Nason (1996) who found first year teacher education students' understanding of area, as weak and lacking in conceptual understanding. They further state that the students' knowledge about the nature of and discourse of mathematics and about mathematics in culture and society was alarming. It was also found that my subjects' algebraic knowledge was mainly a collection of facts and rules. The situation in which these final year College of Education students have limited conceptual understanding of algebra is a problem which needs urgent attention. This problem is not going to support the constructivists approach to learning as Noddings (1990) and Confrey (1990) state that to establish a mathematical environment that encourages investigation, exploration and asking questions (NCTM, 1991), teachers are required to have strong knowledge and understanding of the topic to be taught. Final year College of Education students are therefore required to have the necessary pedagogical content knowledge (PCK) to face this task. As Even (1993) suggests a powerful content-specific pedagogical preparation in school algebra, based on meaningful and comprehensible subject-matter knowledge, is needed by the final year College of Education students to enable them enter the teaching field with confidence and enthusiasm to perform their duties as algebra educators.

It becomes the duty of South African mathematics experts and College of Education lecturers to work out an acceptable vernacular nomenclature of conceptual terms in algebra for use by teachers and students. Such nomenclature would help solve the problem of inconsistent definition of concepts which sometimes seem foreign to them because of the type of textbooks used. This will provide clear definitions and standardised translations. I wish to reiterate that some of the misconceptions that the study revealed were attributed to textbooks. As Laridon (1991) acknowledges, the mathematics conveyed by our textbooks in schools tends to give the impression that mathematics is the prerogative of Western culture and the European mind which leads to cultural conflict, thus hindering its usage by other cultures. The misconceptions displayed by some of the subjects revealed a number of conceptual and language difficulties, which had been identified by international researchers. Among my subjects the misconceptions were possibly worse than expected maybe because they were aggravated by language and cultural issues.

The findings, however, are specific to the situation in which the research was conducted. Hence the applicability may be limited to my sample. However, this study may resonate with similar situations in South Africa and internationally. The encouragement and reflection on ones' knowledge and understanding and misconceptions of algebra is done by introducing relevant materials which deal with the nature of algebra and the pedagogy related to the teaching and learning thereof. These will go a long way to help alleviate the problems and difficulties we envisage in the near future. To gain self-confidence to teach school algebra for conceptual understanding, I suggest that COE students should revisit the mathematical and pedagogical content knowledge of algebra. The constructivist mode of teaching and learning should form the basis for the revisit. This approach will address cultural, historical and language issues which are obstacles to learning for conceptual understanding as has been revealed in this study. There should be a paradigm shift toward a more interactive approach to teaching, where learners will be assisted to construct knowledge on their own.

The picture the study portrays here of final year College of Education students' knowledge and understanding of algebraic concepts can be seen as some of the reasons why the Department of Education in South Africa has closed down the Colleges of Education. Since year 2000 many of the Colleges of Education have closed down and some placed under the Higher Educational institutions like the Technikons and Universities. The training of teachers has since been under these institutions. The colleges especially, those that were admitting African students were seen to not have stringent admission policies. Most often students who did not obtain matric exemption to enter the Universities resorted to Colleges of Education as an alternative. My study can therefore be of assistance to the Universities and Technikons to address some of the problems and challenges they may face with these type of students.

Seemingly, financial problems might have been some of the reasons why Africans students failed to enter the universities. Financial support is hence needed to help students entering universities to pursue careers in the teaching of mathematics. The fact that Department of Education is faced with shortage of mathematics teachers will hence necessitate this suggestion. We must also not ignore the fact that some of the students had weak passes at matric , which could not allow them direct entry into the universities. A conscious effort should be made by the universities and Technikons to relate the teaching and learning of algebraic concepts to prior and existing knowledge structures.

One strategy to address issues raised in the preceding paragraphs is for the higher institutions and Department of Education to work together more closely to develop a set of learning to teach algebra opportunities for the would-be teachers. In particular, the collaboration between them can help to create contexts where University academicians will get to know what happens at the schools and be able to come out with measures to address them. What is certain is that curriculum planners and the University lecturers, etc do need to do something to acknowledge and help would-be teachers to overcome their conceptual difficulties in algebra. The would-be teachers need stronger algebraic knowledge and understanding, together with classroom confidence, if they are to teach algebra for conceptual understanding. It is unrealistic to expect that this will happen unless the would-be teachers have conceptual understanding of algebra.

According to Dewey (1933) "*it is the situatedness of the problem that causes the difficulty. Once we know what exactly the problem is, we are half way to the solution"* (p108). In fact, we know what the problems are; therefore the responsibility should now lie on curriculum planners, teacher education implementers, lecturers and College of Education students to remedy the situation.

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APPENDICES

APPENDIX A

ALGEBRAIC UNDERSTANDING TEST

Name................. Date....................
College.............. Course..................
Province................ Sex.................

Highest standard Passed in Mathematics................

INSTRUCTIONS

TIME : 1 Hour
ANSWER ALL QUESTIONS.
SHOW WORKING WHERE NECESSARY IN THE SPACES PROVIDED

1. If a + b = 34 then a + b + 3 =.........

2. If e + f = 8 then e + f + g =.........

3. What is the value of m if m = s + t and
   m + s + t = 24 ? ....

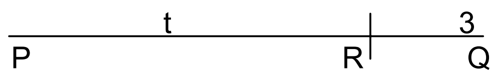
4. What are the values of c if c + d = 10 and c is less than d ? ....

5. If x = 6 is the solution to the equation
$(x + 1)^3 + x = 349$
then what value of x will make the following equation true

$$(5x + 1)^3 + 5x = 349$$

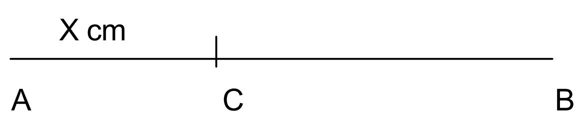
x = ...

6.



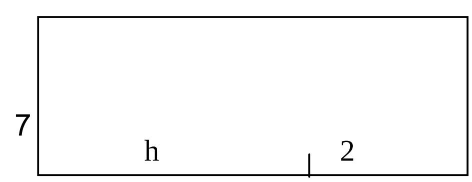
Write down an expression for the length PQ. PQ =......

7.



AC = x cm; CB is 3 cm more than AC. Write down the length of AB in terms of x and simplify. AB =..........

8. Write down the perimeter of this rectangle in terms of h, and simplify.
Perimeter =....



9. Sipho is y years old. Mpho is 5 years older than Sipho. Write down the sum of

their ages in terms of y and simplify.

10. Four more than three times a number is 31. Write down an equation that can help find the number. (Let the number be x)

11. Apples cost 50 cents each and peaches 40 cents each. If **"a"** stands for number of apples bought and **"p"** stands for number of peaches bought, write down an expression for the total cost of fruits bought.

12. A mathematical club has 15 members. Write a formula for finding the number of girls, "g", if you know the number of boys, "b".

14. Which of the following relations are functions?

(a) {(-10;10), (-5;10), (0;10), (5;0), 10;0)}
(b) {(-9;8), (-8;9), (-7;6), (-6;7), (7;-6), (-7;-6)}
(c) {(4x, 2x-3) | x an integer}
(d) {(-4;10), (-3;10), (-2; 10), (-1;10), (0;10)}
(e) {(|x|, x)| x an integer

14

x	-1	0	1	2
y	-3	0	3	6

State the relation between x and y.

15. Which is larger, 2n or n+2? Explain

APPENDIX B

PILOT INTERVIEW FORMAT

FILL IN THE INFORMATION BELOW

NAME................................
COLLEGE...............................
HIGHEST STANDARD PASSED IN MATHEMATICS................
NAME OF YOUR STD 5 SCHOOL............................
NAME OF YOUR STD 6 SCHOOL.............................
NAME OF YOUR STD 10 SCHOOL............................

1. Do you like mathematics ?
2. How did you like your std 5 teacher?
3. How did you like your std 10 teacher ?
4. Geometry and algebra which of the two do you like best ? Why ?
5. Which of the two, geometry and algebra did you score the highest marks ?
6. Can you tell a layman the meaning of algebra ? If yes what is the meaning of algebra ?
7. Name any four subtopics in algebra

8. Has algebra any intrinsic motivation for you ? If yes what is it ? If no why ?.
9. Can you relate algebra to arithmetic, How ?
10. What are the greatest problems you have to face with the teaching of algebra ?

Questions from the test results

APPENDIX C

ALGEBRAIC UNDERSTANDING TEST

Name................. Date....................
College.............. Course..................
Province................ Sex.................

Highest standard Passed in Mathematics................

INSTRUCTIONS

TIME : 1 Hour
ANSWER ALL QUESTIONS.
SHOW WORKING WHERE NECESSARY IN THE SPACES PROVIDED

1. If a + b = 34 then a + b + 3 =.........

2. If e + f = 8 then e + f + g =.........

3. What is the value of m if m = s + t and m + s + t = 24 ? ....

4. What are the values of c if c + d = 10 and c is less than d ? ....

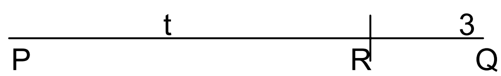
5. If x = 6 is the solution to the equation $(x + 1)^3 + x = 349$ then what value of x will make the following equation

true

$$(5x + 1)^3 + 5x = 349$$

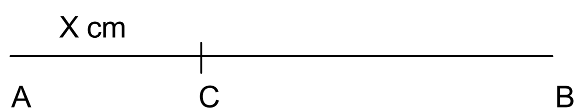
x = ...

6.



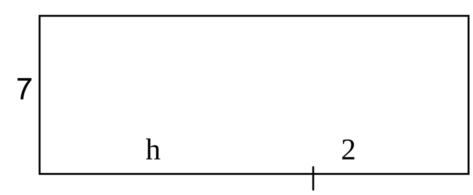
Write down an expression for the length PQ. PQ =......

7.



AC = x cm; CB is 3 cm more than AC. Write down the length of AB in terms of x and simplify. AB =..........

8. Write down the perimeter of this rectangle in terms of h, and simplify.
Perimeter =....



9. Sipho is y years old. Mpho is 5 years older than Sipho. Write down the sum of their ages in terms of y and simplify.

10. Four more than three times a number is 31. Write down an equation that can help find the number. (Let the number be x)

11. Apples cost 50 cents each and peaches 40 cents each. If **"a"** stands for number of apples bought and **"p"** stands for number of peaches bought, write down an expression for the total cost of fruits bought.

12. A mathematical club has 15 members. Write a formula for finding the number of girls, "g", if you know the number of boys, "b".

13. What is a function ?

....

14.

x	-1	0	1	2
y	-3	0	3	6

State the relation between x and y.

15. Find the solution set of the inequality

$$\frac{x - 7}{3 - x} > 1$$

From Nos 16 - 20 tick the correct answers only.

16.
If $x^2 = y^2$, then

(a) $x = y$
(b) $x = -y$
(c) $x = \pm y$
(d) $x = |y|$

17. What can we conclude about y if $y^2 < 25$?

(a) $y < 5$
(b) $y < \pm 5$
(c) $y > 5$ or $y < -5$
(d) $-5 < y < 5$

18 Consider the following calculation to find m from the equation

$\sqrt{(7 - m)} + m = 5$

$7 - m^2 + m = 25$ .... (1)

$7 + m = 25$ .... (2)

$m = 32$ .... (3)

In which lines do mistakes occur ?

(a) 1 and 2
(b) 1 and 3
(c) 2 and 3
(d) 1, 2 and 3
(e) There is no mistake

19 If x > y, which of the following is true ?

(a) $\frac{1}{x} > \frac{1}{y}$

(b) $\frac{1}{x} < \frac{1}{y}$

(c) $\frac{1}{|x|} < \frac{1}{y}$

(d) None of the above

20. Check every answer you think is appropriate in describing $2x^2 + x - 3$:

(a) It is a quadratic equation in x.
(b) It is a quadratic expression in x.
(c) It has factors (2x + 3) and (x-1).
(d) It has a solutions x = -3 and x = 1

APPENDIX D
MALUTI COLLEGE OBSERVATION FORMAT

APPENDIX E

THE BIOGRAPHICAL AND INTERVIEW QUESTIONAIRE

FILL IN THE INFORMATION BELOW
NAME................................
COLLEGE...............................
HIGHEST STANDARD PASSED IN MATHEMATICS................
NAME OF YOUR STD 5 SCHOOL TEACHER.....................
NAME OF YOUR STD 6 SCHOOL TEACHER ....
NAME OF YOUR STD 10 SCHOOL TEACHER ....

THE OPEN-ENDED QUESTIONS (Required verbal responses)

1. Do you like mathematics?
2. How did you like your std 5 teacher?
3. How did you like your std 10 teacher?
3. Geometry or algebra; which of the two do you like best? Why?
4. In which of the two, geometry or algebra, did you usually do best?
5. Explain in layman's terms the meaning of algebra.
7. Name any four subtopics in algebra. Explain them
8. How did your algebra studies at school differ from algebra studies at the college? – the way it is taught, and the way you learn it.
9. Has algebra any intrinsic motivation for you? If yes what is it? If no, why?
10. Relate algebra to arithmetic.
11. What are the greatest problems you have to face with the teaching of algebra?
12 How do you go about learning a new concept in algebra? Do graphs,

investigations, diagrams help?

13. Probing questions were asked from the test instrument, which required verbal and written responses, e.g. How? Why? what if? What do you mean by that, can you give me an example? etc.

APPENDIX F

Classroom Observation Schedule

Complete the information required for 1-5

1. Name of student teacher	
2. Topic of the algebra lesson	
3. Duration of lesson	
4. Number of learners	
5. Grade taught	
6. Type of teaching approach: (a) Teacher-centred (b) learner-centred	
7. Learner active involvement and participation: (a) active (b) passive (c) inactive (d) inattentive	

Provide comments for 8-19

8. Introduction of 'new' algebraic concept not previously known to learners:
   Comments:

9. Observing how algebraic concepts are defined and explained and developed over time:
   Comments:

10. Observing whether algebraic concepts are introduced informally.
    Comments:

11. Observing to what extent multiple representations were used so that the same problem may be seen from algebraic, arithmetical, and geometrical perspectives.
Comments:

12. Did the teacher have adequate knowledge of algebra?
Comments:

13 Did Student teachers questions elicit algebraic thinking in learners?
Comments:

14. What do the learners do in the lesson discussion? What does the communication with the teacher suggest about their algebraic understanding?
Comments:

APPENDIX H

Kaur and Sharon Instrument to Test Algebraic Misconceptions of First Year College Students

Instructions

Section A

Please not that correct responses are indicated by * and the number in [ ] indicates the number of students who selected that particular answer

APPENDIX I

REPUBLIC OF TRANSKEI TEACHER TRAINING COLLEGES AFFILIATED TO THE UNIVERSITY OF TRANSKEI SECONDARY TEACHERS DIPLOMA SYLLABUS FOR MATHEMATICS

APPENDIX J

EXCERPT I3SEK1

I Between algebra and geometry which one do you prefer teaching
S. I prefer teaching algebra.
I Why?
S. Because the formulae and procedures in algebra are easy to understand and follow. For example $(a + b)^2 = a^2 + 2ab + b^2$, can be easily applied to other expressions which need its application unlike in geometry where it becomes difficult to apply a theorem learnt to a given problem which needs its application.
I. So you mean geometry is difficult to teach?
S. Yes, because you need to give reasons for statements made in geometry while in algebra in most of the times you are not required to give reasons.
I. Do you construct teaching aids when teaching algebra?
S. Seldom
I. Why seldom?
S. Because it is difficult to construct teaching aids in algebra.
.
.
I. I saw from your script that you were trying to expand $(5x + 1)^3$
S. Yes, but I could not expand
I. What was the problem?
S. The expansion became too tedious and I got frustrated
I. Did you think there was an easier way to the problem?
S. I suspected so but could not reason it out.
I. Did you know how to solve it now?

S. Yes my friends showed me and it was a matter of replacing x by 5x in x = 6

EXCERPT I3SEK2

I. In item 9 you gave an answer of 6*y*.
S. Yes
I. What do you understand by 5 more than *y*?
S. 5*y*
I. If Sipho is 20 years old and Mpho is 5 years older than Sipho. What will be Mpho's age?
S. 20 + 5 = 25
I. What will be their total ages?
S. 45 years.
I. Why is Mpho's age not 5x20=100?
S. It cannot be, because Mpho is only five years older than Sipho
I. What do you understand by 6*y* then?
S. 6 multiply by *y.*
I. Then it means Mpho and Sipho's ages together should be 6x20 years if Sipho is 20 years old and Mpho is 5 years older.
S. No
I. Can you substitute 20 for *y* in 6*y* and tell me the answer according to the question on the test paper?
S. Yes, 6x20= 120 which is not the same as 45.
I. Can you now see your problem?
S. Yes, I see.
I. If Sipho is *y* years old, what will Mpho's age be, if Mpho is 5 years older than Sipho?
S. (Scratched the head) I think *y*+5 years.
I. Yes, *y*+5, then what should their total ages be?
S. He writes *y*+*y*+5= 2*y* +5
I. Good that is what was required from you.
.
.
I. You seemed to have difficulty with item 15
S. Yes
I. What was the problem?
S. I know how to solve inequality problems but this is not the type I am used to.
I. What type are you used to?
S. The linear inequality
I. Does it means you have never come across this type of inequality problem?
S. Yes
I. But this type is in the matric syllabus.
S. Yes, but it is for the Higher Grade students

I. Does the college syllabus not include this topic?
S. It is there but we have not treated it.
I. Why did you not leave the question altogether?
S. I thought I could use my previous knowledge of solving equations for this too.

EXCERPT I3SEK3

I. Do you like mathematics?
S. Yes, I do like mathematics that is why I am pursuing this course
I. Were you motivated by any of your mathematics teachers to pursue mathematics course?
S. Not much
I. What do you mean by that?
S. My teacher at grade 10 made me to dislike mathematics because he was most of the time absent from the class.
I. How did you proceed with mathematics to this far?
S. I was interested in mathematics so with my friends we did study on our own.
I. But I am sure other mathematics teachers contributed to your efforts.
S. Yes, especially my grade 12 mathematics teacher, oh, he was good.
I. What do you mean by being good?
S. He knew his stuff and was always there to assist us
.
.
I. You solved this problem and got an answer of $x > 5$?
S. Yes
I. Can you show me how you arrived at the answer?
S. Yes, wrote on a paper and multiply both sides of the equation by 3-x and arriving at an answer of $x>5$
I. Did you check your answer?
S. No
I. Check your answer.
S. How?
I. Substitute a number using your solution.
S. He puts 5 and arrived at $1>1$, oh there seems to be a problem
I. Where is the problem? (He was quiet, did not see the problem) I told him to substitute a number more than 5, which I asked him to suggest the number.
S. 6 he said
I. Can you do the substitution in the original inequality?
S. He puts 6 in place of x giving $(6 - 7)/(3-6) > 1$
(wrote on a piece of paper and got an answer of $1/3 > 1$)
I. Does your answer satisfy the conditions of the problem?
S. No but the solution is correct, he points to $x > 5$ on the paper.

I. I think you have a problem with solving inequality. You are solving it like using equal sign.

S. Oh, I remember now, we have solved this type of a problem before but I have forgotten how to solve it now, can you show me?

I. Yes, (the interviewer showed this student how this was solved to arrive at an answer of $3 < x < 5$. (We checked the answer with the value of 4 and it satisfied the inequation).

EXCERPT LO3SEK2

S: What is the common denominator of 3 and 4?

L: 12 (chorus)

S: 3 goes into 12?

L: 4 (chorus)

S: 4 multiply by 2x

L: 8x (chorus)

S: 4 goes into 12?

.

.

.

S: To collect like terms you add terms with the same sign and maintain the sign and you subtract in the case of unlike terms the smaller number from the bigger one keeping the sign of the bigger term. (The student teacher should have said “like” or “unlike” signs)

L: Yes teacher (chorus)

S: What is $-2x^2-x^2$ bearing in mind the rule that, you add like terms and you subtract unlike terms.

EXCERPT I4SEK1

I. Have you ever come across this type of a problem?

S. I cannot recall

I. What does that mean?

S. I mean I was not taught this type of a problem.

I. What level exactly were you not taught?

S. At the matric level as well as the college level.

I. Why not?

S. Because I think this topic is taught at the H.G. level only.

:

.

I. Did you understand the question (item 20)?

S. Yes, I understood it.

I. What does it say?

S. It says tick all the statements, which describe an expression.
I. What is an expression?
S. I think it is two or more terms grouped together by any or all the four basic operations.
I. Is (x-3)(x-2) = 0 an expression or an equation?
S. Equation.
I. Why is it an equation?
S. Because the expression is equal to zero.
I. Is x=2 and x=3 an expression or an equation?
S. An expression because if you solve (x-2)(x-3) you get x = 2 and x = 3
I. How?
S. (She writes (x-2) = 0 or (x-3) = 0 and comes out with x = 2 or x = 3)
I. But you said (x-2)(x-3) = 0 is an equation.
S. Yes, but if you want to get the values to (x-2)(x-3) you will have to equate to zero to give the answer x = 2 or x = 3.
I. Yes but the question did not ask for the roots of the expression.
S. Does it mean x = 2 or x = 3 is not an expression?
I. What do you think?
S. I think not because there is no equality sign in an expression.

EXCERPT PI4SEK1

I: What were your areas of concern as you prepared this lesson?
S: To enable learners to know that you cannot add unlike terms.
I: What were your main goals as you prepared the lesson?
S: My main goals were to make
1) learners add unlike terms,
2) to simplify expressions
I: What method(s) did you plan to use to achieve these goals?
S: To use real-world problems and to make use of objects as examples.
I: How did your lesson go?
S: The lesson was fine
I: Why did you change the example of boys and girls to fruits? (see 4.3.5)
S: I realised that some of the learners were getting confused over the answers given by some learners about the number of boys and girls in the class.
I: Do you think the example was a wrong choice?
S: I do not think so, only some few of them wanted to confuse issues.
I: Is there no way you could have made those learners understand that example?
S: I was confused, so I had to resort to the fruit example, which I have seen in one textbook.

EXCERPT LO4SEK2

(S- Student teacher, L- Learner)
S: Can we do some addition
L: Yes
S: a + a; x + x + x
Ls: 2a, 3x (chorus)
S: 2 girls + 3 girls
Ls: 5 girls
S: 4 boys + 5 boys
Ls: 9 boys
S: 3 girls + 4 boys
Ls: Remains the same, 3 girls + 4 boys (but one learner said 7 pupils)
S: (Attention was directed to this learner who said 7 pupils). Why do you say 7 pupils? Can you add boys and girls to get one number answer?
L: Yes, you can add and get one number answer as 7 pupils (The whole class burst into laughter and the class became noisy)
S: You cannot add number of boys to number of girls. Let us use another example. (Student teacher changed the example to apples and bananas)

EXCERPT LO3STK1

S. Why do you say the statements are not correct?
L. Because they are unlike terms
S. Can you explain further?
L Because a^2 is different from a
S. What do you say about a^2 and 2a?
L They are different
S. How are they different?
L a^2 = a x a and 2a =a + a
S. Good, can we conclude then that we cannot add unlike terms.
L Yes, teacher

EXCERPT PI3STK1

I. How did you like your lesson?
S. It was good I am satisfied with how it went.
I. What do you mean by it was good
S. I was able to apply what I learnt in a workshop I attended.
I. Tell me about it.
S. It was about learner-centred approach to teaching this particular topic, where learners were allowed to discover things for themselves.
I. What made it so successful to you?
S. The way the learners were able to discover the misconceptions with this topic.

EXERPT LO4STK2

S: Do you know the meaning of $(a + b)^2$?
Ls: Yes
S: Can somebody write on the chalkboard another way of writing $(a + b)^2$.
L: (One learner stood up and wrote on the board $(a + b)^2 = (a + b)(a + b)$.
S: Good, now we are all going to remove the bracket using the distributive law.
L: Yes teacher
S: (Teacher wrote (a + b)(a + b) and asked the class to use the distributive law to expand)
Ls: Learners in chorus said a(a + b) + b(a + b)
S: Let us remove the brackets, he began by writing a^2 and the learners continued telling the teacher the remaining terms
Ls: $a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.
EXCERPT I3SIK1

I: You wrote the answer as c =1,2,3,4
S: Yes
I: Why, can you explain?
S: I solved for the value of c from the two statements given.
I: What are the two statements?
S. She wrote c + d = 10 and c < d
I. Then what followed?
S. From c + d = 10 I could see that if c =d, then **c** will be equal to 5 as well as **d** too. But it is said **c** is less than **d** hence I wrote the answer like that.
I. Can you tell me why you started from 1?
S. Because 1 + 9 = 10.
I. Any other number you can think of?
S. No other number sir.
I. What name do you give to number 1,2,3,4, ….?
S. Natural numbers
I. Why do you use natural numbers?
S. Do you mean I can use other numbers?
I. Yes that is exactly what I want you to come out with.
S. I see, (then she tried 0 and found it satisfied the conditions of the problem). I can add 0 to the values already put down.
I. Yes, you were not limited to only natural numbers, the answer includes all real numbers less than 5 (i.e. $c < 5$, $c \in R$).
.

:
I. CB is 3cm more than AC, what is AC?
S. AC = x
I. What is CB?
S. CB = 3x
I. Why 3x?
S. Because CB is three more than AC
I. What is CB if CB is 3 times bigger than AC?
S. 3x
I. Are the two statements 3 more than and 3 times bigger the same?
S. Not exactly
I. What is the difference then?
S. Failed to respond
I. What is three times 4?
S. 12
I. What will 3 more than 4 be?
S.7
I. Can you see the difference between the two?
S. Yes
I. Then 3 more than x will be equal to what?
S. x + 3
I. Then AB will be equal to what?
S. She wrote x + x + 3 = 2x + 3.

EXCERPT LO3SIK3

S. How many terms do you see on the right-hand side $(a + b)^2 = a^2 + 2ab + b^2$?
L. Three terms (in chorus)
S. On the right-hand side what happened to the first term on the left-hand side?
L. It is the squared of **a.**
S. What do notice with the middle term on the right-hand side?
L. It is two times the product of the two terms **a** and **b** on the left-hand side.
S. What do you notice with the last term on the right-hand side?
L. The last term is the square of b.
S. Do you now see the relationship?
L. Yes sir (in chorus)
S. Can one of you express the relationship in words?
L. Quiet
S. The right-hand side is the square of the first term plus two times the product of the two terms plus the square of the last term.

EXCERPT LO3SRK1

(S stands for the student teacher and L stands for a learner).

S: What is the difference between these two examples above?

L: The first example involves only numerical values while the second example involves numbers and letters.

S: Do you know the values of the letters?

L: Yes, they can be any number

S: So can the numbers –2, -3, -4, and 0, be some of the values of the letters?

L: No, because there is nothing like negative or zero length and breadth of a rectangle.

S: Yes, you are correct, do you all agree class?

Ls: Yes teacher (in chorus).

S: These examples show the difference between arithmetic and algebra (student teacher repeated that arithmetic involves numbers only while algebra involves numbers and variables or letters)

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S: We have seen that we can use the formula A = l x b to obtain the perimeter of a rectangle. To determine the value of A we substitute values for l and b. (He wrote on the chalkboard, if l = 20 and b = 6, determine the value of A. He replaced l and b by 20 and 6 in the formula and wrote

A = l x b
  = 20 x 6
  = 120

:

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S: You see here 2 is common

L. Yes sir

S. Therefore we can take the common term out

L. Yes sir

EXCERPT PI3SRK1

I. Did you notice anything in the use of that formula?

S. No

I. You said the perimeter of a rectangle is A = lxb

S. Yes, what is wrong with that?

I There is something wrong, A = l x b is the formula for the area of a rectangle and not the perimeter.

S. But I got it from the textbook, which I can show you.

I That means the textbook is also wrong, the perimeter is the addition of all the sides of any object (the teacher showed me this example from the textbook).

S. Yes
I. How will you then define a perimeter of a rectangle?
S Perimeter is two times the length plus the breadth.
I. How did you come with this definition?
S. (Quiet for sometime), but that is the formula for finding the perimeter of a rectangle.
I. I mean the definition not the formula
S. Quiet (could not give the definition)

APPENDIX K

Letter to Colleges of Education