Vibration sensor placement for delamination detection in a beam structure based on the vibration-based chaotic oscillator method

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Vibration Sensor Placement for Delamination Detection in a Beam Structure based on the Vibration-based Chaotic Oscillator Method

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ABSTRACT

This work aims to study the effect of sensor placement for delamination damage detection in a beam structure using a vibration-based chaotic oscillator method. A chaotic oscillator method is used due to its sensitivity to relatively small changes in measured vibration signal. The effect of vibration sensor placement to the delamination detection sensitivity for various delamination sizes is investigated. The Lyapunov Exponent (LE) is used in conjunction with the chaotic oscillator as a damage index to describe the extent of delamination damage in the laminated beam. The relationship between the damage index and sensor placement for different delamination size is studied to analyse the effect of sensor placement on detection performance. It is found that the sensor placement has a significant influence on the sensitivity of delamination detection with different delamination size.

1. INTRODUCTION

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Composite laminates are widely used in various engineering applications including in aerospace and other highly demanding applications because of its high modulus, low density and corrosion resistance. However, in engineering applications, damage in composite laminates can occur at times and it can be hard to be detected. Therefore, it is important to develop an effective method for damage detection, especially for the detection of delamination, which is an essential issue in composite laminate structure.

As delamination affects the vibration response of a laminate beam, its vibration characteristics can be used as an index for laminate beam's delamination detection [1]. Most of the previous research works focused on the detection and analysis of delamination based on the change of structural vibration response [2-3]. However, since the vibration response of a damaged laminate beam varies at different structural locations, the delamination detection sensitivity can also be affected by the locations of vibration sensors. Similar research works have focused on investigating the location of sensor for vibration control and measurement. Halim et al. [4] used the spatial observability measure to investigate the effect of sensor location in observing certain vibration modes. It is found that an optimized sensor location over the target structure can effectively enhance the sensing performance.

In addition, chaotic oscillators have attracted much research attention for structural damage detection. It has the advantages of being sensitive to the change of target structure's dynamic signal and being highly resistant to measurement noise if compared with other conventional methods. Nichols et al. [5] utilized chaotic input signal to excite target structures and detect the damages by analyzing the dynamic change of a chaotic attractor. Liu et al. [6] used a chaotic oscillator based on the observed weak Lamb wave and analyzed the attractor's property to detect micro-crack damage in an aluminum plate. Both of the research works demonstrated the feasibility and effectiveness of damage detection through a chaotic oscillator incorporated with the structure's dynamic response. However, as mentioned earlier, limited work has investigated the effect of sensor locations on the detection performance based on chaotic oscillators. Therefore, this work aims to study the effect of sensor location on the sensitivity of delamination damage detection when the vibration-based chaotic oscillator method is used.

The paper is structured as follows: Section 2 introduces the delaminated beam model and theory background of the chaotic oscillator. Section 3 describes the results and discussion based on the simulation model of a beam with delamination, while Section 4 provides the conclusions based on the obtained results.

2. THE MODELLING AND DELAMINATION DETECTION METHOD

2.1 The model for a delaminated beam

In this section, the configuration of a delaminated beam of length L is shown in Figure 1. There is a single delamination over the width of beam and the beam can be divided into four sub-beam as shown in the figure: sub-beams 1 and 4 are the respective left and right intact parts of the beam, while sub-beams 2 and 3 are the respective upper part and lower part of the delaminated region. Considering that the beam is split into 4 sub-beams, the governing equation based on the classical Euler-Bernoulli beam theory can be expressed as [7]:

$$EI_i \frac{\partial^4 u_i}{\partial x^4} + \rho A_i \frac{\partial^2 u_i}{\partial t^2} = 0, \ (i = 1,2,4) \tag{1}$$

where EI_i is the bending stiffness of the i-th sub-beam, ρ is density and A_i is the cross-sectional area.

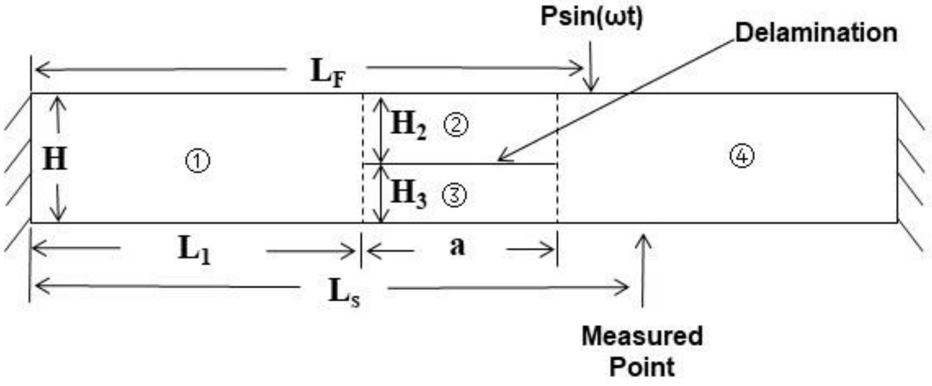


Figure 1 The model of a beam with delamination.

The solution for equation (1) can be described as:

$$u_i(x,t) = u_i(x) sin(\omega t). \tag{2}$$

where $u_i(x)$ is presented by:

$$u_i(x) = A_i cos(k_i x) + B_i sin(k_i x) + C_i cosh(k_i x) + D_i sinh(k_i x) \tag{3}$$

with $k_i^4 = \frac{\omega^2 \rho A_i}{EI_i}, (i = 1,4)$ and unknown coefficients A_i, B_i, C_i and D_i. Based on the method described in reference [7], the displacements of sub-beams 2 and 3 are assumed to be the same, which means $k_2^4 = \frac{\omega^2 \rho (A_2 + A_3)}{EI_2 + EI_3}$. Then the continuity conditions for the associated displacement $x = L_1$ can be described by:

$$u_1 = u_2, \dot{u}_1 = \dot{u}_2. \tag{4}$$

The continuity conditions for the shear force and bending moment at $x = L_1$ are as follows:

$$EI_1 \ddot{u}_1(L_1) = (EI_2 + EI_3) \ddot{u}_2(L_1) \tag{5}$$

$$EI_1\ddot{u}_1(L_1) + \frac{(H_2+H_3)^2 E_2A_2E_3A_3}{4a(E_2A_2+E_3A_3)} \times (\dot{u}_1(L_1) - \dot{u}_4(L_2)) = (EI_2 + EI_3)\ddot{u}_2(L_1). \quad (6)$$

Meanwhile at $x = L_2$, the continuity conditions are described as:

$$u_4 = u_2, \dot{u}_4 = \dot{u}_2 \quad (7)$$

$$EI_4\dddot{u}_4(L_2) = (EI_2 + EI_3)\dddot{u}_2(L_2) \quad (8)$$

$$EI_4\ddot{u}_4(L_2) + \frac{(H_2+H_3)^2 E_2A_2E_3A_3}{4a(E_2A_2+E_3A_3)} \times (\dot{u}_1(L_1) - \dot{u}_4(L_2)) = (EI_2 + EI_3)\ddot{u}_2(L_2). \quad (9)$$

In this work, the clamped boundary conditions are used for the beam structure of interest with:

$$u_1(0) = 0, \dot{u}_1(0) = 0 \quad (10)$$

$$u_4(L) = 0, \dot{u}_4(L) = 0. \quad (11)$$

Substituting equation (3) into equations (4)-(11), they can be in represented in a matrix form as:

$$T(\omega)X = 0 \quad (12)$$

where matrix $T(\omega)$ has the size of 12×12. To obtain the non-trivial solution for X, the determinant of matrix T must satisfy:

$$\det(T(\omega)) = 0. \quad (13)$$

The equation can then be solved to determine the natural frequencies and mode shapes.

2.2 The delamination detection method by using a chaotic oscillator

Chaotic oscillators have been widely used for damage detection and signal capture applications because of their high sensitivity in detecting dynamic changes of signals [5, 8], which allows the oscillators to be used for detecting certain types of damage in laminate structures [9]. In this work, a chaotic oscillator named the Duffing oscillator is used for detection, which is expressed as:

$$\ddot{x} + k \cdot \dot{x} - a \cdot x^3 + c \cdot x^5 = V\,cos(\,\omega \cdot t) + V_{in}\,cos(\,\omega_{in} \cdot t) \quad (14)$$

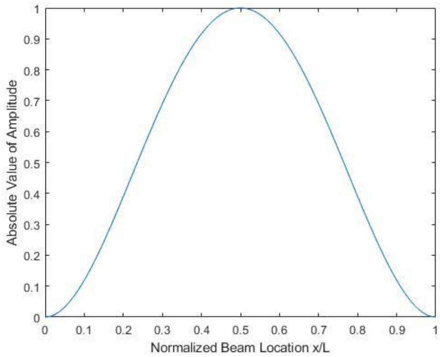
where k is the damping ratio; x is the associated displacement; a and c are coefficients of elasticity; $V_{in}\,cos(\,\omega_{in} \cdot t)$ is the vibration signal measured from a laminate beam with ω_{in} as the signal frequency; and $V\,cos(\,\omega \cdot t)$ is the harmonic driving force at excitation frequency ω.

The motion of the chaotic oscillator from equation (14) can then be analyzed based on the observed signal $V_{in}\,cos(\,\omega_{in} \cdot t)$ for different cases with varying delamination sizes. Equation (14) can be solved by using the Runge-Kutta method and the Lyapunov Exponent (LE) value can be used to quantify the chaotic feature of the nonlinear oscillator, which is influenced by $V_{in}\,cos(\,\omega_{in} \cdot t)$. The Lyapunov Exponent can be calculated using the method developed by Wolf [10].

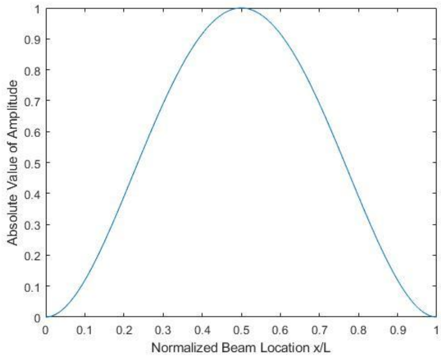
3. RESULTS AND DISCUSSION

3.1 Free vibration characteristics of laminated beam structures

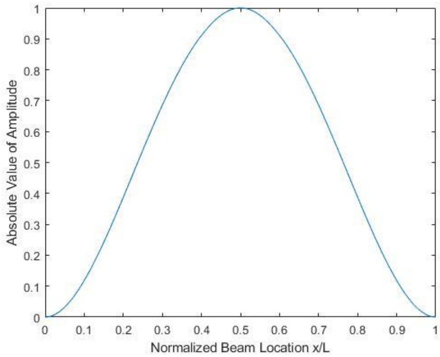
In this section, the vibration characteristics of delaminated beam structures with different delamination sizes are investigated. The delamination is assumed to be located in the middle-plane of beam ($H_2 = H/2$) and the length-wise location of delamination is at $L_1=(L-a)/2$ (i.e. the delamination is in the middle span of beam). The utilized delamination sizes are: *0* (undamaged), *0.1L*, *0.2L* and *0.3L*, respectively. The absolute value of the mode shape amplitude for the first three modes with different delamination sizes are shown in Figures 2-4.



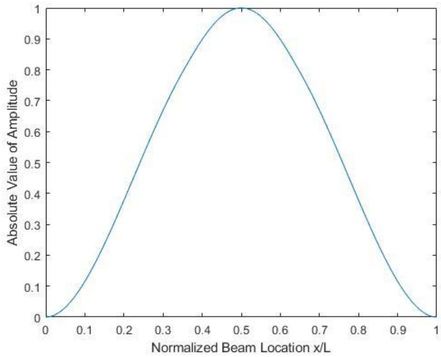
(a) Undamaged



(b) a=0.1L

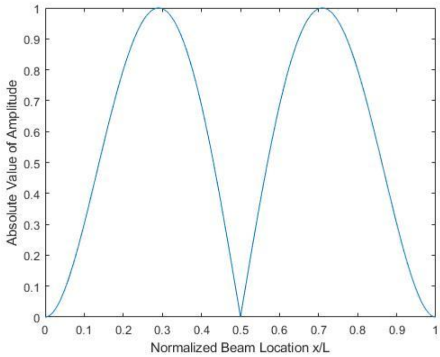


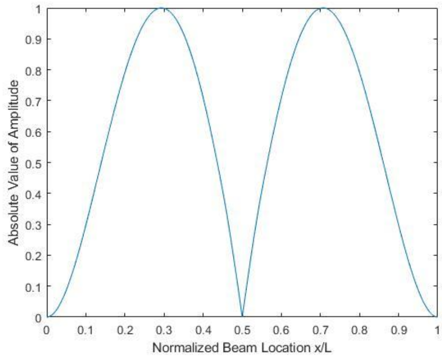
(c) a=0.2L



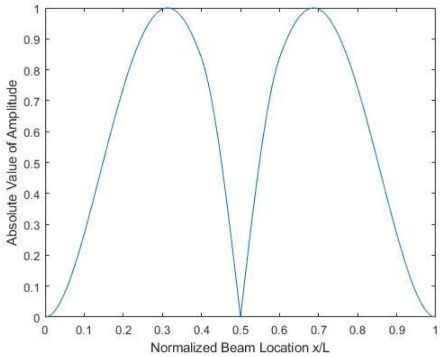
(d)a=0.3L

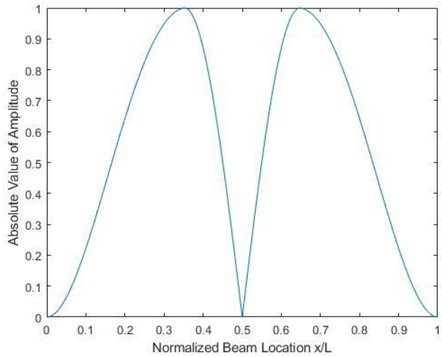
Figure 2 The absolute values of the first mode shape amplitude with different delamination sizes.





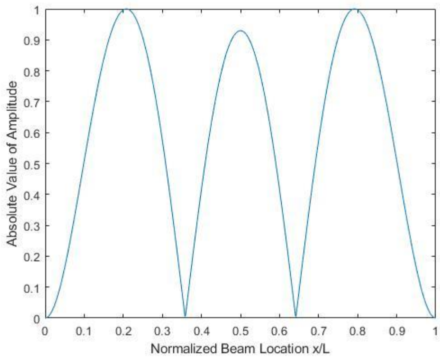
(a) Undamaged *(b) a=0.1L*

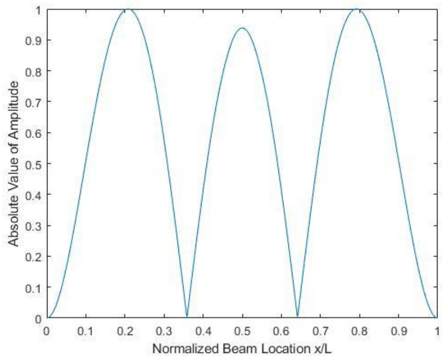




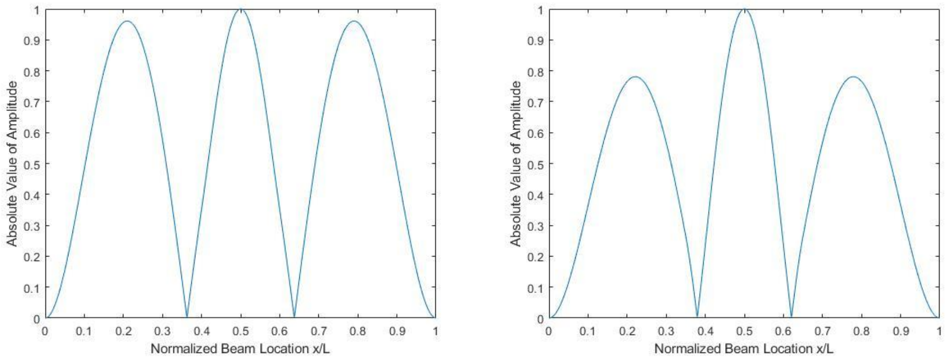
(c) a=0.2L *(d)a=0.3L*

Figure 3 The absolute values of the second mode shape amplitude with different delamination sizes.





(a) Undamaged *(b) a=0.1L*

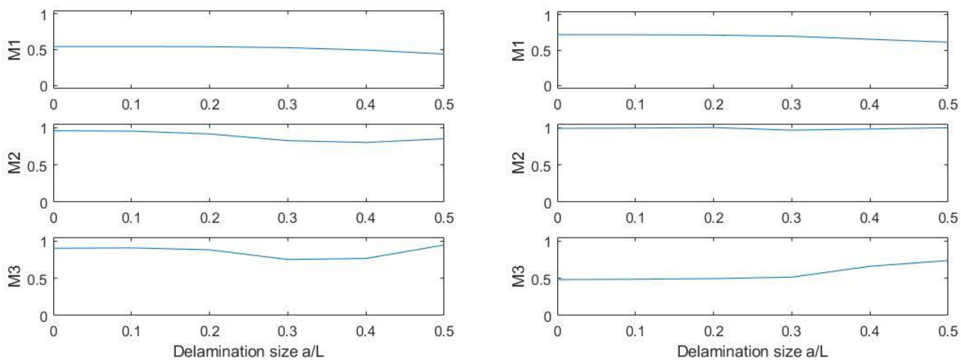


(c) a=0.2L *(d)a=0.3L*

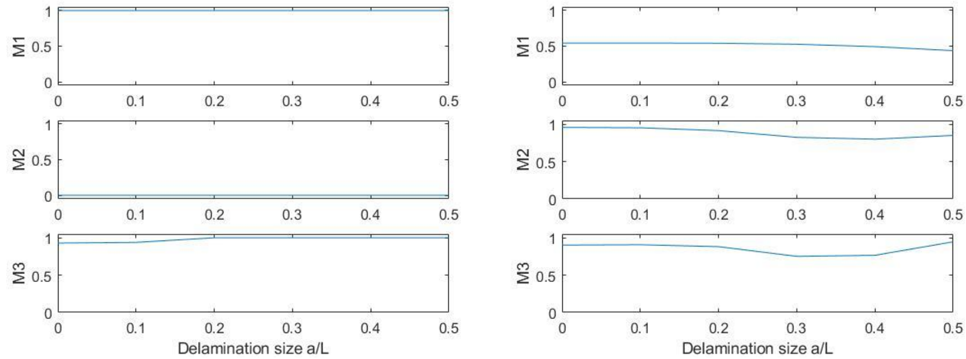
Figure 4 The absolute values of the third mode shape amplitude with different delamination sizes.

It can be observed from Figures 2-4 that the mode shape changes when there is a change in the delamination size as expected. The changes observed for the 3rd modes are more significant than those for the 1st and 2nd modes, indicating that higher frequency modes tend to be more sensitive to delamination of a particular size.

Figure 5 shows the absolute value of mode shape amplitude for *n*-th mode (Mn) when the delamination size is varied. Here, Mn is evaluated at 4 different locations (*Ls*) over the beam structure, representing 4 different vibration sensor locations. Considering accelerometer-type of sensors, it can be observed that for each sensor location, the sensitivity of sensor in observing each mode varies as reflected by the value of Mn, with the higher value indicating a higher sensitivity. The sensitivity in observing each mode also varies when the delamination size changes, which can be used to evaluate the effectiveness of a particular sensor location for observing certain modes.



(a) L_s=0.25L *(b) L_s=0.30L*

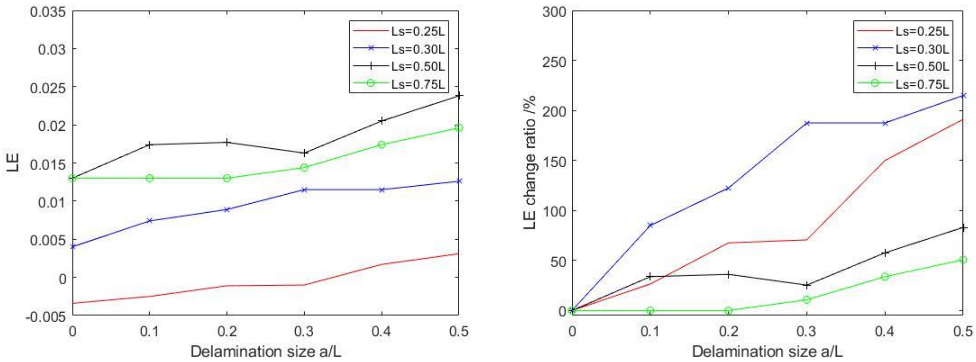


(c) L_s=0.50L (d) L_s=0.75L

Figure 5 The absolute value of mode shape amplitude for n-th mode (Mn) when the delamination size is varied, evaluated at 4 different sensor locations.

3.2 Lyapunov Exponent

Based on equation (14) and the Wolf's method [10], the LE value of different delamination cases can be determined. The input signal $V_{in}\,cos(\omega_{in} \cdot t)$ is generated by harmonic excitation with the angular frequency $\omega = 1\ rad/s$ and force amplitude $V = 1N$. The driving force location is located at L_F=0.8L. The LE value of the Duffing oscillator with the input vibration signal measured at different sensor locations is shown in Figure 6(a), with its LE change ratio shown in Figure 6(b).



(a) LE value (b) LE change ratio

Figure 6 The LE value and the change ratio of LE for different sensor locations (L_s) for varying delamination sizes.

As indicated in Figure 6, the LE value tends to increase as the delamination size increases for 4 different sensor locations. Figure 6(a) indicates that the LE value for sensor location *Ls=0.50L* is larger than those for other locations, because the driving frequency is closer to the first natural frequency. It can also be observed from Figure 5(c) that the value of M_1 for sensor location *Ls=0.50L* is at maximum for various delamination sizes.

However, it is clear that the LE change ratio can vary significantly for different sensor locations. Figure 6(b) shows that the change ratio of LE is the largest for sensor location at *Ls=0.30L,* which indicates that a vibration sensor location at this point can be more sensitive to detecting changes in delamination size compared to other 3 sensor locations. The results indicate that the LE change ratio can be used as a measure of the sensor sensitivity in detecting delamination in the beam structure, which can also be utilized for optimizing the sensor location to improve the delamination detection performance.

4. CONCLUSIONS

This work has investigated the effect of vibration sensor placement for delamination damage detection in a beam structure by using a vibration-based chaotic oscillator method. The chaotic oscillator can be used to detect delamination within the beam structure by using the Lyapunov Exponent as a damage index, with the LE value tending to increase as the size of delamination increases. It is also observed that the sensor location has a significant effect on the sensitivity in detecting delamination in a beam structure for different delamination sizes. It is found that the sensor sensitivity in detecting different delamination sizes can be indicated by observing the LE change ratio. The results demonstrate the potential of the proposed method to optimize the vibration sensor location for delamination detection in a beam structure, which can also be extended to other more general structures.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

1. Della, C.N. and D. Shu, Vibration of Delaminated Composite Laminates: A Review. *Applied Mechanics Reviews*, 2007. **60**(1): p. 1.
2. Ganesh S, Kumar K S, Mahato P K. Free Vibration Analysis of Delaminated Composite Plates Using Finite Element Method. *Procedia Engineering*, 2016. **144**: p. 1067-1075.
3. Ghaffari H, Zabihollah A, Saeedi E, et al, Vibration based damage detection in smart nonuniform thickness laminated composite beams, *in 2009 IEEE Toronto*

International Conference Science and Technology for Humanity (TIC-STH). 2009.

4. Halim D, Moheimani S O R. An optimization approach to optimal placement of collocated piezoelectric actuators and sensors on a thin plate. *Mechatronics*, 2003. **13**(1): p. 21.
5. Nichols J M, Virgin L N, Todd M D, et al. On the Use of Attractor Dimension as a Feature in Structural Health Monitoring. *Mechanical Systems and Signal Processing,* 2003. **17**(6): p. 1305-1320.
6. Liu X, Bo L, Yang K, et al. Locating and imaging contact delamination based on chaotic detection of nonlinear Lamb waves. *Mechanical Systems and Signal Processing*, 2018. **109**: p. 58-73.
7. Mujumdar P M, Suryanarayan S. Flexural vibrations of beams with delaminations. *Journal of Sound and Vibration*, 1988. **125**(3): p. 21.
8. Todd M D, Nichols J M, Pecora L M, et al., Vibration-based damage assessment utilizing state space geometry changes: local attractor variance ratio. *Smart Materials and Structures*, 2001. **10**(5): p. 1000-1008.
9. Li X, Halim D, Liu X, Rudd C, Delamination Detection in Composite Laminates using a Vibration-Based Chaotic Oscillator Method, *the 47th International Congress and Exposition on Noise Control Engineering*. 2018.
10. Wolf, A., Swift, J. B., Swinney, H. L., and Vastano, J. A, Determining Lyapunov exponents from a time series. *Physica D: Nonlinear Phenomena*, 1985. **16**(3): p. 33.