System of Equations for Hierarchical Decision Modeling

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Abstract

Decision-making is a cognitive process to select an alternative or a course of action in a specific context. In 1983, Dundar Kocaoglu proposed hierarchical decision modeling as a participative approach to program evaluation [1]. It has since been used to solve decision problems in many areas like the assessment of solar photovoltaic technologies [2] and enterprise data storage systems [3]. We present a simplified system of equations for hierarchical decision models with multiple perspectives, criteria, and alternatives.

Hierarchical Decision Modeling

In hierarchical decision modeling, the decision problem is modeled with perspectives and their related criteria in a hierarchy.

Perspective weights are calculated using pairwise comparison of all perspectives. Initial criteria weights are calculated by pairwise comparison of criteria within the same perspective. Final criteria weights are calculated by multiplying the initial criteria weights with related perspective weights.

The decision alternatives are compared with each other to find the initial alternative score for each criterion. A partial alternative score is obtained by multiplying the initial alternative score with criterion weight. The final score of a decision alternative is the sum of all partial alternative scores.

The decision alternative with the highest score is selected for the overall objective.

System of Equations:

Variables: Let, p represent perspectives, c represent criteria, and a represent decision alternatives in the hierarchical decision model. Also, let j be the index of perspectives ranging from 1 to J, i the index of criteria ranging from 1 to I, and k the index of decision alternatives ranging from 1 to K.

<u>The method of constant sum pairwise comparison ensures the following four conditions:</u>

Condition I: Sum of all perspective weights equal to one. Note: p_j is the weight of jth perspective.

$$\sum_{j=1}^{J} p_j = 1$$

Condition II: Sum of all initial criteria weights equal to one. Note: $c_{ji}|p_j$ is the initial weight of criterion i under perspective j. I_j is the maximum number of criteria in perspective j.

$$\sum_{i=1}^{I_j} c_{ji}|p_j = 1$$

Condition III: Sum of all final criteria weights equal to one. Note: C_{ji} represents the final weight of criterion i under perspective j.

$$\sum_{j=1}^{J} \sum_{i=1}^{I_j} C_{ji} = 1$$

Where $C_{ji} = p_j * c_{ji}|p_j$

Condition IV: Sum of all initial alternative scores for each criterion equal to one. Note: $a_k|c_{ji}$ represents the score of an alternative k for criterion i under perspective j for each fixed value of i and j.

$$\sum_{k=1}^{K} a_k|c_{ji} = 1$$

Score of a decision alternative:

Let A_k represent the score of the k^{th} decision alternative.

$$A_k = \sum_{j=1}^{J} \sum_{i=1}^{I_j} C_{ji} * a_k|c_{ji}$$

Selection of decision alternative:

The alternative with the largest score of all A_k is selected.

$$A^* = \max_{1 \le k \le K} A_k$$

Conclusion:

The method of constant sum pairwise comparison ensures the four conditions explained above. We presented the formula to calculate the score of a decision alternative. The alternative with the largest score has the highest value for the overall objective of the decision problem. The system of equations represents a hierarchical decision model with multiple perspectives, criteria, and alternatives.

Future Research:

In the future, we can expand the equations to include sub-criteria, factors, and desirability functions for more comprehensive decision models.

References:

[1] D. F. Kocaoglu, "A participative approach to program evaluation," in IEEE Transactions on Engineering Management, vol. EM-30, no. 3, pp. 112-118, Aug. 1983, doi: 10.1109/TEM.1983.6448602.

[2] Sheikh, Nasir Jamil, "Assessment of Solar Photovoltaic Technologies Using Multiple Perspectives and Hierarchical Decision Modeling" (2013). Dissertations and Theses. Paper 978. https://doi.org/10.15760/etd.978

[3] Shrestha, L., & Sheikh, N. J. (in press). Multiperspective Assessment of Enterprise Data Storage Systems: Literature Review. Portland International Conference on Management of Engineering and Technology (PICMET), (Accepted, May 2020).