Received: 7 May 2019 | Accepted: 3 June 2019

DOI: 10.1002/pamm.201900087

Uncertainty Quantification for Optimal Power Flow Problems

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The need to de-carbonize the current energy infrastructure, and the increasing integration of renewables pose a number of difficult control and optimization problems. Among those, the optimal power flow (OPF) problem—i.e., the task to minimize power system operation costs while maintaining technical and network limitations—is key for operational planning of power systems. The influx of inherently volatile renewable energy sources calls for methods that allow to consider stochasticity directly in the OPF problem. Here, we present recent results on uncertainty quantification for OPF problems. Modeling uncertainties as second-order continuous random variables, we will show that the OPF problem subject to stochastic uncertainties can be posed as an infinite-dimensional L2-problem. A tractable reformulation thereof can be obtained using polynomial chaos expansion (PCE), under mild assumptions. We will show advantageous features of PCE for OPF subject to stochastic uncertainties. For example, multivariate non-Gaussian uncertainties can be considered easily. Finally, we comment on recent progress on a Julia package for PCE.

1 Introduction

We are witnessing a paradigm shift in the production of electrical energy: the tremendous effort to generate electricity from renewable energy sources is unprecedented. This modification in energy generation brings about change in the way power systems are operated. The traditional modus operandi of power systems assumes and has been designed for an infrastructure consisting of a handful of large power plants that deliver energy to consumers. With the influx of renewable energy sources and production of electrical energy at lower-scale voltage levels the traditional modus calls for a critical assessment.

Mathematically, the task of delivering electrical energy to consumers in a cost-optimal way whilst respecting engineering limits such as generation limits and line limits is posed as a nonlinear optimization problem, the so-called optimal power flow problem (OPF) [1]. These problems are used for planning, dispatching, and operating the electric power system. It is a steady-state optimization problem that accounts for the so-called power flow equations, a set of nonlinear algebraic equations derived from Kirchhoff's laws that models the steady-state of an AC electrical network. Traditionally, the OPF problem is solved for fixed and presumably known values of uncontrollable active and reactive power demands. In other words, the demand is assumed to be deterministic. With the advent of renewable energy sources such as solar panels mounted in residential households, and so-called smart homes, the forecast power demand becomes more volatile and less predictable; a single point forecast may be insufficient. Rather, probabilistic forecasts are used that model the value to be forecast as a random variable [2]. This in turn affects the way the OPF problem is solved. Whereas traditionally OPF is solved for a single point forecast it must now be solved with probabilistic forecasts. The fact that the power demand is not known precisely must be accounted for. We show how the OPF problem can be formulated in the presence of uncertainties and how it can be solved in terms of a deterministic proxy problem.

2 Problem Formulation

We study an N-bus electrical network. Its set of bus indices is $\mathcal{N} = \{1, \dots, N\}$, and its set of line indices reads $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$. Each bus $i \in \mathcal{N}$ is described by its net active power p_i, net reactive power q_i, its real voltage v_i^{re}, and imaginary voltage v_i^{im}. We assume each bus $i \in \mathcal{N}$ connects to one controllable generation unit p_i^{g} and one uncontrollable power injection p_i^{u},

$$\forall i \in \mathcal{N}: \qquad p_i = p_i^{\mathrm{g}} + p_i^{\mathrm{u}}, \qquad q_i = q_i^{\mathrm{g}} + q_i^{\mathrm{u}}. \tag{1}$$

To model the effects of uncertainty we replace the deterministic description of (1) by random variables. To this end, let $L^2(\Omega, \mathbb{P}; \mathbb{R})$ be the space of all random variables of finite variance for the set of outcomes Ω and an associated probability measure $\mathbb{P}$.[1] This allows to introduce the N-valued random vectors $\mathsf{p}^{\mathrm{u}}, \mathsf{q}^{\mathrm{u}}$ where every element $\mathsf{p}_i^{\mathrm{u}}, \mathsf{q}_i^{\mathrm{u}} \in L^2(\Omega, \mathbb{P}; \mathbb{R})$ is a

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[1] Technically, the space $L^2(\Omega, \mathbb{P}; \mathbb{R})$ contains equivalence classes, but we stick to the convention and refer to the equivalence classes by their representatives.

real-valued random variable of finite variance. The OPF problem may then be formulated as [3, 4]

$$\min_{\mathsf{p}_i^{\mathrm{g}},\mathsf{q}_i^{\mathrm{g}},\mathsf{v}_i^{\mathrm{re}},\mathsf{v}_i^{\mathrm{im}}} \mathbb{E}\Big[\sum_{i\in\mathcal{N}} f_i(\mathsf{p}_i^{\mathrm{g}})\Big] \quad \text{subject to} \tag{2a}$$

$$\mathsf{p}_i = \mathsf{p}_i^{\mathrm{g}} + \mathsf{p}_i^{\mathrm{u}} = \sum_{j\in\mathcal{N}} G_{ij}(\mathsf{v}_i^{\mathrm{re}}\mathsf{v}_j^{\mathrm{re}} + \mathsf{v}_i^{\mathrm{im}}\mathsf{v}_j^{\mathrm{im}}) + B_{ij}(\mathsf{v}_i^{\mathrm{im}}\mathsf{v}_j^{\mathrm{im}} - \mathsf{v}_i^{\mathrm{re}}\mathsf{v}_j^{\mathrm{im}}), \tag{2b}$$

$$\mathsf{q}_i = \mathsf{q}_i^{\mathrm{g}} + \mathsf{q}_i^{\mathrm{u}} = \sum_{j\in\mathcal{N}} G_{ij}(\mathsf{v}_i^{\mathrm{im}}\mathsf{v}_j^{\mathrm{im}} - \mathsf{v}_i^{\mathrm{re}}\mathsf{v}_j^{\mathrm{im}}) - B_{ij}(\mathsf{v}_i^{\mathrm{re}}\mathsf{v}_j^{\mathrm{re}} + \mathsf{v}_i^{\mathrm{im}}\mathsf{v}_j^{\mathrm{im}}), \tag{2c}$$

$$\mathbb{P}\big(\mathsf{x} \geq x^{\min}\big) \geq 1-\varepsilon, \quad \mathbb{P}(\mathsf{x} \leq x^{\max}) \geq 1-\varepsilon, \quad \forall \mathsf{x} \in \{\mathsf{p}_i^{\mathrm{g}}, \mathsf{q}_i^{\mathrm{g}}, \mathsf{v}_i\}, \tag{2d}$$

$$\mathbb{P}(\mathsf{i}_{i\text{-}j} \leq i_{i\text{-}j}^{\max}) \geq 1-\varepsilon, \tag{2e}$$

$$\mathsf{v}_{i_{\theta V}}^{\mathrm{im}} = 0, \quad \forall i \in \mathcal{N},\, \forall ij \in \mathcal{L}, \tag{2f}$$

which minimizes the sum of the expected generation costs f_i in (2a), treats the power flow equations as a nonlinear system of algebraic equations in terms of random variables (2b), (2c), considers engineering limits with respect to generation limits and voltage magnitude limits (2d), and line flow limits (2e), each as chance constraints. The formulation as a chance constraint reduces conservatism as constraints are satisfied only up to a prescribed level of $1-\varepsilon \in (0,1)$. The matrix $Y = G + \mathrm{j}B \in \mathbb{C}^{N\times N}$ is the bus admittance matrix that collects physical parameters of the network. Problem (2) is an infinite-dimensional optimization problem. One possibility to render the problem finite-dimensional is to use polynomial chaos expansion (PCE), a Hilbert space technique for random variables of finite variance. Polynomial chaos has been applied to (optimal) power flow problems before [3, 5–8]. The advantages of PCE for Problem (2) are threefold: i) we are not restricted to a specific family of random variables such as Gaussians, ii) we can compute moments of random variables from the PCE coefficients alone, and iii) we can propagate uncertainties through the full power flow equations. If Problem (2) is solved with PCE, there is another post-processing advantage: knowing the realization of the uncertainty, the realization of the corresponding optimal generation is obtained instantaneously, simply by evaluating the PCE. To tame the computational burden associated with polynomial chaos there exist several software packages. Recently, an open source implementation in the Julia programming language [9] has been made available [10], which allows to construct orthogonal polynomials starting from the definition of the underlying probability density function.

3 Summary

Optimal power flow problems make up a cornerstone of the power system operations. In the presence of uncertainties (stemming for instance from renewable energy sources), the optimal power flow problem can be formulated as an infinite-dimensional optimization problem in terms of random variables. To render the problem finite-dimensional, polynomial chaos can be applied.

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