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Quenching friction-induced oscillations in multibody-systems by the use of high-frequency excitation

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Dry friction can be a cause of undesired self-excited oscillations. One way to suppress this underlying mechanism is the superposition of high-frequency vibrations whereby the effective friction characteristics is changed and a quasi-equilibrium can be stabilized. This damping effect is analyzed in detail for single-degree-of-freedom systems [1] and experiments and simulations show a good accordance [2]. In this work, the analytical approach from [1] is used to analyze the stabilizing effect of superposed oscillations for a two-degree-of-freedom system subject to friction.

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1 Introduction

Friction-induced oscillations are often investigated by an elastically mounted mass on a moving belt. The discontinuous friction function and the decay of the friction force at small relative velocities can cause a destabilization of the equilibrium point and self-excited oscillations can occur. High-frequency excitation influences the system dynamics in such way, that the effective friction characteristic is no longer discontinuous and has a positive slope at small relative velocities [1]. This has a damping effect on the system, which is why a new, stable quasi-equilibrium can exist.

2 Investigated system

To investigate the suppression of friction-induced oscillations in multibody-systems, the system in figure 1 is considered. It consists of two masses, which are linked by linear springs. The first mass is additionally linked to the environment. Both masses lie on a revolving belt, which moves at a constant velocity v_0. The normal forces F_{N1} and F_{N2} act vertically on the masses and the friction coefficient between mass and belt is assumed as a function of the relative velocity in the contact, respectively. The first mass is excited by the force $F_1(t) = A\Omega^2 \sin \Omega t$.

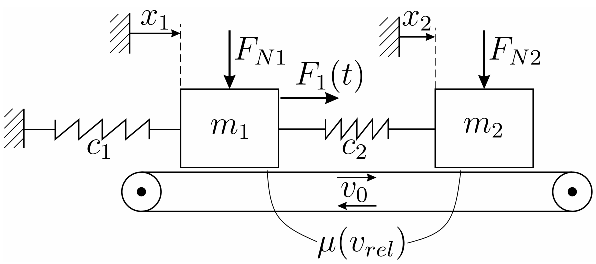

Fig. 1: Model of tow masses on a moving belt.

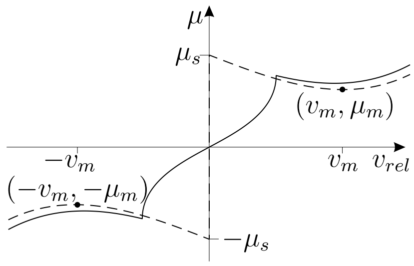

Fig. 2: Friction coefficient as a function of the relative velocity: discontinuous (- -) and averaged (–)

The dimensionless equations of motion in matrix form are given by

$$\underset{\sim}{x}'' + \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \underset{\sim}{x} = \underset{\sim}{a}\omega^2 \sin \omega\tau + \underset{\sim}{r}(\underset{\sim}{v}_{rel}) \,, \tag{1}$$

where $m_1 = m_2$, $c_1 = c_2$, $F_{N1} = F_{N2}$ for simplicity reasons. The dimensionless parameters are $\underset{\sim}{x} = (x_1,\, x_2)^\top$, $\eta^2 = \frac{c_1}{m_1}$, $\tau = \eta t$, $(\,)' = \frac{\mathrm{d}}{\mathrm{d}\tau}$, $\omega = \frac{\Omega}{\eta}$, $\underset{\sim}{a} = (a,\, 0)^\top$ with $a = \frac{A}{m_1}$, $\nu_0 = \frac{v_0}{\eta}$, $v_{rel,i} = x_i' - \nu_0$, $f_{Ni} = \frac{F_{Ni}}{c_1}$. It is $\omega \gg 1$, $a \ll 1$ and $a\omega = \mathcal{O}(1)$. The friction force is given by $r_i(v_{rel,i}) = f_{Ni}\mu(v_{rel,i})$, where the friction coefficient is given by the function

$$\mu(v_{rel}) = \mu_s \mathrm{sgn}(v_{rel}) - \frac{3}{2}\left(\mu_s - \mu_m\right)\left(\frac{v_{rel}}{v_m} - \frac{1}{3}\left(\frac{v_{rel}}{v_m}\right)^3\right) \,, \tag{2}$$

which is shown in figure 2. The function is discontinuous at $v_{rel} = 0$, has a negative slope at $|v_{rel}| < v_m$ where the absolute value has a local minimum at (v_m, μ_m). To investigate the system dynamics, the method of multiple scales is applied. The

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time is separated in a slow time τ and a fast time $\theta = \omega\tau$, the coordinates are separated in a big, slow motion and a fast, small motion: $\underset{\sim}{x}(\tau) = \underset{\sim}{z}(\tau) + \frac{1}{\omega}\underset{\sim}{\varphi}(\tau, \theta)$. Executing the derivatives and applying equation 1 yields

$$\left(\frac{\partial^2 \underset{\sim}{z}}{\partial\tau^2} + 2\frac{\partial^2 \underset{\sim}{\varphi}}{\partial\tau\partial\theta} + \frac{1}{\omega}\frac{\partial^2 \underset{\sim}{\varphi}}{\partial\tau^2} + \omega\frac{\partial^2 \underset{\sim}{\varphi}}{\partial\theta^2}\right) + \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\left(\underset{\sim}{z} + \frac{1}{\omega}\underset{\sim}{\varphi}\right) = \underset{\sim}{a}\omega^2 \sin\theta + \underset{\sim}{r}(\underset{\sim}{v}_{rel}) \,. \tag{3}$$

The order $\mathcal{O}(\omega^1)$ in equation 3 delivers the equation for the fast motion $\frac{\partial^2 \underset{\sim}{\varphi}}{\partial\theta^2} = \underset{\sim}{a}\omega\sin\theta$ which leads to $\underset{\sim}{\varphi}(\tau, \theta) = -\underset{\sim}{a}\omega\sin\theta + \underset{\sim}{h}_1(\tau)\theta + \underset{\sim}{h}_2(\tau)$, where $\underset{\sim}{h}_1(\tau)$ and $\underset{\sim}{h}_2(\tau)$ have to vanish, because it is claimed that the fast motion has zero mean and is bounded in the fast time. The order $\mathcal{O}(\omega^0)$ yields the equation for the slow motion, which is averaged over one period of the fast time θ

$$\frac{\partial^2 \underset{\sim}{z}}{\partial\tau^2} + \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\underset{\sim}{z} = \langle \underset{\sim}{r}(\underset{\sim}{v}_{rel})\rangle \,, \tag{4}$$

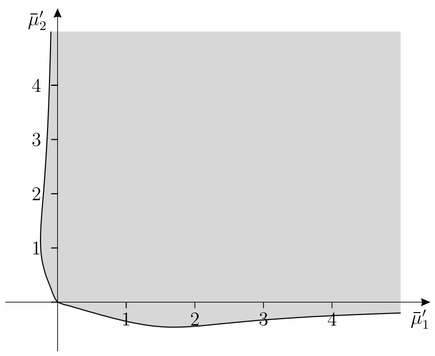
with $\langle \underset{\sim}{r}(\underset{\sim}{v}_{rel})\rangle = \frac{1}{2\pi}\int_0^{2\pi} \underset{\sim}{r}(\underset{\sim}{v}_{rel})\mathrm{d}\theta$. By applying equation 2, this integral can be solved analytically in dependence of the macroscopic relative velocity $V_{rel,i} = \frac{\partial z_i}{\partial\tau} - \nu_0$, which is shown in figure 2 . Following this, equation 4 can be linearized about its equilibrium point $\underset{\sim}{z}_0$, which yields

$$\frac{\partial^2 \Delta\underset{\sim}{z}}{\partial\tau^2} + \begin{pmatrix} \bar{\mu}_1' & 0 \\ 0 & \bar{\mu}_2' \end{pmatrix}\frac{\partial \Delta\underset{\sim}{z}}{\partial\tau} + \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\Delta\underset{\sim}{z} = \underset{\sim}{0} \,, \tag{5}$$

while $\bar{\mu}_i'$ describes the slope of the effective friction characteristics of mass i at the quasi-equilibrium. Using an exponential ansatz for equation 5 leads to the characteristic polynomial of order 4, which allows statements concerning the stability of $\underset{\sim}{z}_0$ using HURWITZ-criteria.

3 Results

In figure 2, the effective friction characteristics of excited mass 1 (–) and not excited mass 2 (- -) is displayed. For small relative velocities, the friction force at mass 1 has a positive slope, while mass 2 has a negative slope. Figure 3 shows the values of the slopes, for which the quasi-equilibrium $\underset{\sim}{z}_0$ is stable. In fact, the excitation of only mass 1 can suppress friction-induced vibrations in the whole system. Figure 4 shows the values of excitation frequency ω and belt velocity ν_0, for which the system is stable. For this map, the values of the discontinuous friction coefficient in equation 2 and its analytically averaged values for high-frequency excitation are inserted.



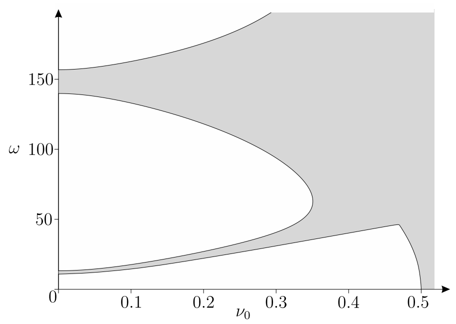


Fig. 4: Stable area (gray) using equation 2 in dependence of ω and ν_0.

Fig. 3: Stable area (gray) in dependence of friction coefficient slopes. Parameters: $\mu_s = 0.4$, $\mu_m = 0.25$, $v_m = 0.5$, $a = 0.01$.

4 Conclusions

The influence of high-frequency excitation on a simple two-body-system subjected to friction is analyzed using multiple scales. It is shown that friction-induced oscillations can be suppressed by exciting only one mass. Caused by the excitation, a quasi-equilibrium exists, where the system oscillates at a high frequency at small amplitudes, while the velocity of the slow, big coordinate is zero. This equilibrium can be stable, in dependence of the slopes of the effective friction characteristics.

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