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1 **Assessment of the Flexural Behavior of Reinforced Concrete Beams Strengthened with**
2 **Concrete Jackets**

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7 **Abstract:**

8 Analysis of continuous jacketed Reinforced Concrete (RC) beams requires accounting for
9 the nonlinear behavior of the interface and the materials as well as redistribution of moments. This
10 kind of analysis is complex and require an advanced level of knowledge and experience to perform.
11 Engineers need simplified yet robust tools to practically predict the actual behavior of jacketed RC
12 beams. In the current practice, slip is neglected in the analysis and monolithic behavior is assumed
13 for the jacketed section, which result in higher estimates of stiffness and/or capacity. This paper
14 provides a simplified method to analyze continuous jacketed RC beams taking into account the
15 interfacial slip distribution and the actual nonlinear behavior of both concrete and steel. An
16 iterative calculation algorithm is developed to determine the moment-curvature curves of a
17 jacketed beam at different sections. The developed method allows the evaluation of interfacial slip
18 and shear stress distributions in ductile reinforced concrete beams. The developed method is
19 utilized to conduct an extensive parametric study, which resulted into modification factors that can
20 be used to calculate the capacity and deformations of a strengthened beam considering the
21 interfacial slip.

22
23 **Keywords:** Reinforced Concrete; Continuous; Jacketing; Slip; Interface; Monolithic Factors;
24 Inelasticity; Flexure.

1 **1. Introduction**

2 The need to strengthen Reinforced Concrete (RC) structures emerges from various reasons,
3 such as new safety requirements, a change of structure occupancy, an incorrect design calculations
4 and/or degradation of materials with time. Jacketing is one of the widely spread procedures to
5 strengthen and repair RC beams. It comprises the addition of concrete layers that are usually
6 reinforced with longitudinal steel bars, stirrups, welded wire mesh or various kinds of fibrous
7 materials.

8 In the current practice, monolithic action is assumed between the original beam and the
9 attached jacket. This implies that the internal stresses developed in both substrates due to the
10 applied loads are distributed among them assuming infinite interfacial slip stiffness. This
11 assumption may result in higher estimates of stiffness and/or capacity depending on the
12 geometrical properties and interfacial surface treatment. The actual behavior of typical jacketed
13 beams is partially composite and depends on the frictional resistance between the surfaces and the
14 presence of steel anchors connecting the two substrates [1]. This implies that the analysis of
15 jacketed beams requires a knowledge of the nonlinear behavior of the interface as well as the
16 nonlinear properties of both concrete and the embedded steel bars at each loading step along the
17 beam.

18 Literature is ample with experimental programs and numerical investigations that have been
19 performed to address the influence of jacketing schemes, geometrical characteristics, mechanical
20 properties and interfacial treatment on the flexural behavior of determinate jacketed RC structural
21 members. For instance, Altun [2] and Bousias *et al.* [3] examined the effect of RC jacketing on the
22 mechanical performance of statically-determinate RC beams considering the load-displacement
23 behavior, ultimate load, ductility and toughness. Other researchers [1, 4] investigated the

1 significance of surface preparation of concrete members before applying the new concrete jacket.
2 The use of fiber reinforced cementitious composites as an alternative to adding steel reinforcement
3 within the jacket has been addressed by other studies [5 – 10]. In addition, the impact of using
4 shear studs to further attach the existing beam with the additional concrete layers has been
5 investigated by Shehata *et al.* [11]. Furthermore, the influence of varying the method of applying
6 the jacket on site, such as shotcrete or cast-in-place concrete, have been considered by many
7 researchers [12, 13, 14].

8 Experimental and numerical studies related to strengthening indeterminate RC beams using
9 concrete jackets is scarce in literature. At the time of writing, the only available relevant
10 experimental work was performed by Cheong and MacAlevey [15]. The rather extensive use of
11 indeterminate RC beams in building structures and bridges requires further research regarding the
12 influence of partial composite action their flexural performance.

13 This paper is a continuation of an ongoing research [16], which aims at proposing a simplified
14 method to capture the influence of interfacial slip on the moment-curvature ($M-\phi$) and load-
15 deflection ($P-\Delta$) relationships of jacketed continuous RC beams. This is achieved by performing
16 nonlinear analysis in view of the material properties and interfacial behavior. A calculation
17 algorithm is proposed to determine the slip distribution along the beam length and to obtain the
18 corresponding $M-\phi$ diagram at both the sagging and hogging moment regions. This analysis
19 procedure is sensitive to the bending moment distribution along the beam; therefore, the concept
20 of moment redistribution in indeterminate beams is illustrated and considered in the analysis. The
21 validated model is used to perform a parametric study aiming at examining the flexural behavior
22 of the strengthened beams. Finally, a regression analysis is performed to propose slip modification
23 factors that can be used to obtain the actual $M-\phi$ diagram of continuous RC beams considering

1 interfacial slip. The scope of the proposed work is limited to ductile RC beams by considering
2 sufficient reinforcement to prevent brittle modes of failure.

3

4 **2. Material Models**

5 The stress-strain relationship of concrete in compression is considered in view of Scott *et al.*'s
6 model [17] due to its simplicity and robustness. The tensile capacity of concrete is assumed to drop
7 after reaching the cracking point.

8 The constitutive relationship of the embedded steel bars is expressed according to the model
9 reported by Karthik and Mander [18] that was derived in view of the general formula proposed by
10 Ramberg and Osgood [19]. It conveniently combines the initial elastic response, yield plateau and
11 strain hardening stages in a single rigorous form to model the actual behavior of steel
12 reinforcement. The value of the strain hardening strain (ϵ_{sh}) is set equal to the yield strain (ϵ_y) and
13 the strain hardening modulus (E_{sh}) is taken as 1% of the Young's modulus of elasticity (E_s).

14

15 **3. Interfacial Shear Stress (τ) and Slip (S) Relationship**

16 The shear transfer mechanism is activated by the frictional resistance between the contact
17 surfaces and the axial forces developed in the anchors crossing the interface. The former
18 mechanism represents the concrete contribution; whereas the second case represents the influence
19 of dowel action. The concrete contribution (v_c) is determined in view of Tassios and Vintzeleou
20 [1] empirical model as a function of the lateral slip (S), ultimate slip value at the onset of frictional
21 mechanism failure (S_{cu}) and ultimate frictional capacity of the interface (v_{cu}). The overall
22 interfacial shear stress (τ) corresponding to any slip (S) value can be obtained as the summation of
23 concrete contribution and dowel action contribution for given material properties and interfacial

1 surface condition. A detailed description of this calculation procedure considering simply
2 supported beams is provided by Alhadid and Youssef [16].

3

4 **4. Assumptions**

5 Assumptions considered in the current study encompasses the following:

- 6 1) Plane sections remain plane after deformation, implying that shear deformations are small
7 relative to bending deformations.
- 8 2) Perfect bond exists between steel reinforcement and the surrounding concrete material. Thus,
9 strain in both concrete and steel bars at the same location is identical.
- 10 3) The failure criterion of the composite beam is defined by crushing of the extreme compression
11 fiber at a concrete ultimate strain (ϵ_{cu}) of 0.0035 [21].
- 12 4) The original RC beam and the added concrete layer are considered to deform by the same
13 curvature through the beam length [20, 23].
- 14 5) The interfacial shear stress distribution within each region is assumed to vary as a cubic
15 function with the distance from the zero moment section [22].

16

17 **5. Typical Jacketed Section**

18 The developed model is applicable to analyze symmetric continuous RC beams subjected to
19 either uniform or concentrate loads. Fig. 1 shows the geometry and reinforcement details of a
20 typical continuous beam that will be used for discussion throughout the chapter. The main steel
21 reinforcement in the positive and negative moment regions are assumed to be 20% and 40% of the
22 balanced steel reinforcement ratio, respectively. The compression steel reinforcement is 2-10M
23 bars. The amount of jacket reinforcement is assumed as 10M bars placed in one layer at the

1 maximum spacing provided by CSA 23.3-14 [21]. One half of the core and jacket steel bars from
2 the hogging moment region are assumed to extend throughout the beam.

3 Geometry and loading scheme of the continuous beam are assumed to be symmetric about the
4 intermediate support. Thus, one span of the beam can be modeled as a propped cantilever as shown
5 in Fig. 2(a). This span is assumed to be composed of several members rigidly connected at their
6 ends as illustrated in Fig. 2(b). Each segment has a defined length (L_i) and a distinct flexural
7 rigidity (EI_i). The segment length is set at about 50 mm, which was found to enhance the accuracy
8 based on a preliminary sensitivity analysis.

9 The expected trends of the moment-curvature diagrams in both the positive and negative
10 moment regions are shown in Fig. 3. The trend for the positive moment section is characterized by
11 three points; namely, the yielding of jacket reinforcement, yielding of the core reinforcement and
12 crushing of concrete. The trend of the negative moment section is defined by yielding of the core
13 reinforcement and crushing of concrete.

14

15 **6. Proposed Calculation Algorithm**

16 Slip, and consequently shear stress, reach their maximum value at the point of zero moment
17 and fade away as they approach the maximum bending moment section. In continuous RC beams,
18 each span can be divided into positive and negative moment zones as indicated in Fig. 4. To obtain
19 the complete slip distribution along the span, the analysis procedure is carried out individually for
20 each of the two zones. Assuming a propped cantilever model for each span, the analyzed segment
21 within the positive moment zone is taken from the pinned support to the point of maximum
22 bending; whereas, for the negative moment zone, this segment is taken from the point of
23 contraflexure to the point of maximum negative bending moment at the fixed end.

1 The proposed analysis method comprises two main stages. In the first one, an iterative sectional
2 analysis procedure is performed at different load levels only at the maximum sagging and hogging
3 moment sections. This results in determining the maximum slip strain ($\Delta\varepsilon_{max}$) at these locations
4 and the corresponding slip strain ($\Delta\varepsilon$) and slip (S) at the other segments along the span. In the
5 second stage, sectional analysis is conducted directly at the remaining segments taking into
6 account the $\Delta\varepsilon$ distribution evaluated from the first analysis step for each segment. The slip
7 distribution is obtained while satisfying the equilibrium and compatibility conditions at each
8 segment. Details about the mentioned steps are given below.

9 The primary challenges for the proposed calculation algorithm are prediction of the slip
10 distribution along the interface and determination of the moment-curvature relationships for the
11 beam segments shown in Fig. 2. Alhadid and Youssef [16] have proposed a calculation algorithm
12 to determine these relationships in jacketed RC simply supported beams considering slip effect. A
13 summary of the procedure is provided in section 6.1 showing the main changes used for analyzing
14 continuous beams. Sectional analysis procedure to determine the equilibrium conditions is
15 described in section 6.2. The influence of moment redistribution becomes substantial in the
16 prediction of slip distribution along continuous beams and is discussed in section 6.3. An
17 equivalent curvature distribution is then obtained based on the load-deflection relationship of the
18 actual curvature distribution considering slip effect as illustrated in section 6.4.

19 **6.1. Moment-Curvature at Maximum Moment Sections**

20 For each moment zone, the average value of interfacial shear stress (τ_{avg}) at any load level
21 can be calculated assuming a direct relationship with the maximum slip strain ($\Delta\varepsilon_{max}$) located at
22 the maximum moment section [22, 23, 24]. Therefore, the average shear stress can be given
23 according to the expression ($\tau_{avg} = \gamma_1 \gamma_2 k_s \Delta\varepsilon_{max} L'$) in terms of secant interfacial stiffness, k_s

1 (N/mm³); the ratios ($\gamma_1 = \tau_{avg} / \tau_{max}$) and ($\gamma_2 = \Delta\varepsilon_{avg} / \Delta\varepsilon_{max}$); the average slip strain ($\Delta\varepsilon_{avg}$) from point
 2 of zero moment to maximum positive or negative moment; and the corresponding length, L' (m).

3 For each of the two moment zones, the analysis procedure to determine interfacial slip
 4 distribution is carried out at each applied load level (i.e. assumed applied curvature value) until
 5 failure occurs. Firstly, initial values of the secant interfacial stiffness (k_s) and the shear stress
 6 distribution ratios (γ_1 and γ_2) are assumed. Then, for the total curvature (φ) value of the current
 7 load increment, two equilibrium conditions are applied at the maximum moment sections: (1)
 8 equilibrium between the internal forces; and (2) equilibrium between the resultant axial forces at
 9 one side of the interface and the resultant shear force acting along the interface. Hence, the moment
 10 (M) and maximum slip strain ($\Delta\varepsilon_{max}$) at the maximum moment sections corresponding to the
 11 current curvature value (φ) are obtained. After that, bending moment diagram is constructed along
 12 the span assuming uniform load and considering the obtained maximum moment values. Next, the
 13 slip strain ($\Delta\varepsilon$) distribution is determined along the span with respect to the location of each
 14 segment as shown in the proposed Equation 1.

$$\Delta\varepsilon_{(i,j)} = \Delta\varepsilon_{(m,1)} \left(\frac{x_j}{L'} \right) \quad (1)$$

15 Where i is the load step number, j is the segment number and m is the load step number that
 16 produces a bending moment in the mid-span segment equals to the moment applied at segment j .
 17 Once the slip strain ($\Delta\varepsilon$) distribution along the interface is established, both the slip (S) and the
 18 shear stress (τ) in each segment is obtained from Equations 2 and 3, respectively.

$$S_{(i,j)} = \sum_{n=1}^{n=j} [(\Delta\varepsilon_{(i,n)})(x_j)] \quad (2)$$

$$\tau_{(i,j)} = k_s S_{(i,j)} \quad (3)$$

1 Having obtained the slip distribution for both moment zones, continuity conditions is
2 checked at the point of contraflexure to ensure it is satisfied by calculating the error between the
3 obtained slip (S) from the sagging moment zone and the hogging moment zone. The procedure is
4 repeated if the error is more than 1% by adjusting the slip strain ($\Delta\varepsilon$) at all segments and repeating
5 the analysis to check equilibrium and compatibility conditions. Finally, based on the obtained slip
6 and shear stress distributions, the secant interfacial stiffness (k_s) and the shear stress distribution
7 ratios (γ_1 and γ_2) are calculated and compared to the initially assumed values. The analysis
8 continues if they are equal with a tolerance of 1%, otherwise the whole procedure is repeated with
9 the new calculated values.

10 If the beams are subjected to initial loading prior to jacketing, then a preliminary sectional
11 analysis on the unjacketed sections has to be carried out first to obtain the resulting moment-
12 curvature curve and strain profile at each beam segment. These diagrams are then included as an
13 input in the calculation algorithm of the jacketed beam to obtain its full behavior at different
14 loading stages before and after jacketing.

15 **6.2. Sectional Analysis in Jacketed Sections**

16 The sectional analysis procedure [25] is implemented to analyze the jacketed sections. The
17 upper limit for the thickness of each layer is taken as 0.5 mm as it results in a better accuracy. At
18 every loading step, an incremental curvature is applied and the strain at each strip in both the
19 concrete core and the jacket is calculated based on its location from the centroid (y_i) of the jacketed
20 section. The kinematic and compatibility conditions are considered in view of the corresponding
21 material stress-strain relationships and Equation 4, which relates the incremental applied moment
22 (ΔM) and axial load (ΔP) to the incremental curvature ($\Delta\varphi$) and axial strain ($\Delta\varepsilon_a$) by a defined
23 stiffness matrix. In this equation, n represents the number of discrete layers, E_i is the elastic

1 modulus of layer i , A_i is the area of layer i , subscript (c) represents concrete core and subscript (J)
 2 represents concrete jacket.

$$\begin{pmatrix} \Delta M \\ \Delta P \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n (E_{i,c}A_{i,c} + E_{i,J}A_{i,J})y_i^2 & -\sum_{i=1}^n (E_{i,c}A_{i,c} + E_{i,J}A_{i,J})y_i \\ -\sum_{i=1}^n (E_{i,c}A_{i,c} + E_{i,J}A_{i,J})y_i & \sum_{i=1}^n (E_{i,c}A_{i,c} + E_{i,J}A_{i,J}) \end{pmatrix} \begin{pmatrix} \Delta\varphi \\ \Delta\varepsilon_a \end{pmatrix} \quad (4)$$

3 6.3. Moment Redistribution in Continuous Beams

4 Matrix stiffness analysis is carried out to account for moment redistribution caused by the
 5 difference in stiffness between the hogging and sagging moment zones. Fig. 5 represents an
 6 arbitrary element of the propped cantilever model subjected to external static uniformly distributed
 7 load. The distorted shape of the element can be described in terms of a translational displacement
 8 (d_i) and in-plane rotation (θ_i) at its ends. The element stiffness is used in Equation 5 to express the
 9 joint internal forces (i.e. P_i and M_i) as functions of the corresponding displacements (i.e. d_i and θ_i)
 10 and fixed-end forces due to the applied loads (i.e. p_i and m_i) [26].

$$\begin{bmatrix} P_i - p_i \\ M_i - m_i \\ P_{i+1} - p_{i+1} \\ M_{i+1} - m_{i+1} \end{bmatrix} = (EI)_{i,j} \begin{bmatrix} \frac{12}{L_i^3} & \frac{6}{L_i^2} & \frac{-12}{L_i^3} & \frac{6}{L_i^2} \\ \frac{6}{L_i^2} & \frac{4}{L_i} & \frac{-6}{L_i^2} & \frac{2}{L_i} \\ \frac{-12}{L_i^3} & \frac{-6}{L_i^2} & \frac{12}{L_i^3} & \frac{-6}{L_i^2} \\ \frac{6}{L_i^2} & \frac{2}{L_i} & \frac{-6}{L_i^2} & \frac{4}{L_i} \end{bmatrix} \begin{bmatrix} d_i \\ \theta_i \\ d_{i+1} \\ \theta_{i+1} \end{bmatrix} \quad (5)$$

11 The proposed method modifies the matrix analysis procedure by incorporating the
 12 influence of slip. The $M-\varphi$ diagram for each section is first calculated while accounting for slip as
 13 explained in section 6.1. The secant stiffness is then evaluated for a given moment. For each
 14 loading step, the relationship in Equation 14 is carried out for each segment (i) considering the
 15 secant flexural stiffness (j) obtained from the corresponding $M-\varphi$ diagram at the specified load
 16 level. The equilibrium and compatibility conditions obtained from the matrix structural analysis

1 and the slip calculation algorithm must be verified simultaneously. Hence, nested iterations are
2 required for each load step to satisfy equilibrium and continuity for each segment along the beam.
3 The moment redistribution along the beam is dictated by the flexural stiffness ratio between the
4 hogging and sagging moment regions [27]. Fig. 3 shows the $M-\phi$ relationships for the positive and
5 negative moment sections of an arbitrary continuous beam. Because of the higher initial stiffness
6 of the negative moment section, the point of zero moment is shifted away from the intermediate
7 support towards the mid-span. A sketch of the bending moment diagram and the flexural rigidities
8 within the elastic loading stage for both the hogging and sagging regions are illustrated in Fig.
9 4(b). In this case, the flexural rigidity is constant within each region but vary between the positive
10 and negative zones. Bending moment diagram is obtained based on the stiffness distribution along
11 the span.

12 Once the negative moment section yields, its secant stiffness will decrease gradually with
13 the applied load until it equates the stiffness at the positive moment section. In this case, the
14 bending moment diagram obtained from stiffness analysis will be identical to that obtained from
15 elastic structural analysis. As the load keeps increasing, the hogging-to-sagging stiffness ratio
16 further decreases resulting in a shift of the point of zero moment towards the intermediate support
17 as more proportion of the additional load is carried by the sagging moment region. However, since
18 the length of each element, and consequently the reinforcement, is assumed to be fixed up to
19 failure, part of the assumed hogging moment region will start to resist small amount of positive
20 moment as shown in Fig. 4(c). The influence of this overlap is insignificant since the moment
21 values adjacent to the point of contraflexure are relatively low. Failure of the beam is activated by
22 crushing of the extreme concrete fibers at the intermediate support where the maximum moment
23 is anticipated. The expected load-deflection curve of the modeled propped cantilever is presented

1 in Fig. 4(d). It shows both the point of maximum deflection and the inflection point that is
2 determined at the initial loading steps and fixed throughout the analysis.

3 **6.4. Load-Deflection Relationship and Equivalent Curvature Distribution**

4 Once the slip effect is incorporated in a unique $M-\phi$ diagram for each segment, the area-
5 moment method is carried out to determine the deflection at distance of 0.4215 of the span away
6 from the edge support. This distance defines the location of maximum deflection for symmetric
7 typical continuous beam supporting a uniformly distributed load [27].

8 Having obtained the load-deflection curve of the jacketed beam including slip effect, the actual
9 curvature distribution of the propped cantilever is determined at different loading steps for each
10 segment. These values are obtained from the corresponding $M-\phi$ diagram taking into account the
11 partial composite action according to the jacketing scheme used. After that, positive (ϕ^+_{eq}) and
12 negative (ϕ^-_{eq}) equivalent curvatures are obtained by assuming that the curvature distribution along
13 the beam is similar to the monolithic behavior of jacketed beams. Therefore, at each loading value,
14 and consequently deflection, equivalent maximum positive and negative curvatures corresponding
15 to the applied moment can be obtained. Hence, an equivalent $M-\phi$ curve can be obtained for the
16 jacketed beam taking into consideration slip effect. The load-deflection curve can be determined
17 at any point using the moment-area theorem and the anticipated deflection shape.

18

19 **7. Validation**

20 The capability of the proposed model to capture the flexural behavior of simply supported
21 jacketed RC beams was previously validated [16]. For continuous jacketed beams, the
22 experimental program performed by Cheong and MacAlevey [15] is considered. Fig. 6 shows the
23 longitudinal and cross-sectional views of the jacketed continuous beam. Initially, the T-section

1 concrete core was cast according to the cross-sectional dimensions and reinforcement distribution
2 shown. After 28 days of curing, the contact surfaces were roughened prior to applying the jacket.
3 The concrete compressive strength was reported as 30 MPa for the core and 60 MPa for the
4 jacketing material. The tensile yield strength of bar size $\Phi 16$, $\Phi 25$, $\Phi 6$, $\Phi 10$ and $\Phi 8$ were 583
5 MPa, 567 MPa, 290 MPa, 321 MPa and 407 MPa, respectively. The tensile ultimate strength for
6 the same sequence of bars were 652 MPa, 670 MPa, 394 MPa, 424 MPa and 477 MPa,
7 respectively. The jacketed beam was subjected to two-point loading scheme at one span only as
8 shown in Fig. 6(a).

9 The proposed calculation method is carried out to determine the flexural behavior of the
10 jacketed continuous beam in terms of the load-deflection curve at the center of the loaded span.
11 The load-deflection curves assuming monolithic and partially composite behaviors are then plotted
12 and compared with the ones obtained experimentally by Cheong and MacAlevey [15]. Fig. 7
13 shows that the percent errors in initial stiffness between the experimental results and the proposed
14 analytical ones are 7.9% and 2.9% assuming both monolithic and partial interaction, respectively.

15 Cheong and MacAlevey [15] reported that a slip between the concrete core and the surrounding
16 jacket was detected without presenting any more data about the slip distribution along the interface.
17 The relatively close variations in the flexural stiffness in the elastic range indicates that the
18 proposed model is capable of predicting the load-deflection behavior prior to steel yielding.
19 Introducing the slip effect in the analysis further improves the predictions by lowering the stiffness
20 to approach the experimental trend. The value of friction coefficient chosen in the analysis is 0.8
21 to account for surface treatment using electric chisel used in the experiment (i.e. roughened
22 surface) [28]. Regarding the ultimate load, the percent error between the experimental and
23 proposed analytical results is 6.2% and 3.8% by ignoring and including the slip effect, respectively.

1 Cheong and MacAlevey [15] reported that the observed failure is brittle caused by the excessive
2 tensile stresses at the narrow bearing supports, which was not accounted for in the proposed model.

3 Due to the lack of experimental results, a finite element model is developed assuming
4 monolithic behavior using ANSYS Software for further validation. The connection between the
5 concrete core and the surrounding jacket is modeled assuming full bond between the adjacent
6 nodes. Geometrical details of both the concrete core and the jacket are modeled using SOLID65,
7 which is an 8–node solid element. This element is capable of cracking under tensile stresses and
8 crushing when subjected to excessive compressive stresses. The element is defined by 8 nodes
9 each having three translational degrees of freedom. The steel main and secondary steel bars in both
10 the core and the jacket are modeled using LINK180 element. This element is a uniaxial tension-
11 compression element with three degrees of freedom at each node corresponding to the X, Y and Z
12 coordinates. The supporting plates used at the location of applied concentrated loads or at the
13 supports are modeled using SOLID185 element assuming elastic behavior. This element is also
14 defined by 8 nodes each having 3 translational degrees of freedom in each direction. A sensitivity
15 analysis is performed to determine the optimum mesh size as shown in Fig. 8. The stiffness and
16 yield stress obtained from the finite element model are 8.7% and 10.2% higher than the
17 corresponding experimental values as shown in Fig. 7, respectively. This variation is attributed to
18 neglecting the influence of slip between the concrete core and the attached jacket. Fig. 7 also shows
19 that the load-deflection curve obtained from the proposed analytical model assuming monolithic
20 action follows the same trend as that obtained from the developed finite element model with a
21 percent difference of 5.2% and 6.4% for stiffness and yield stress, respectively.

22

23

1 **8. Parametric Study**

2 A parametric study is carried out to investigate the influence of different design parameters on
3 the performance of jacketed continuous RC beams. The concrete compressive strength is taken as
4 25 MPa, 30 MPa and 35 MPa; and the steel yield strength is taken as 300 MPa, 400 MPa and 500
5 MPa. For each of the analyzed sections, the mechanical properties are assumed to be the same for
6 the concrete core and its jacket. The coefficient of friction ranges according to ACI [28] between
7 0.4 for smooth concrete surface to 1.4 for intentionally highly roughened concrete in increments
8 of 0.2. The beam's cross-sectional dimensions are defined with reference to the existing beam
9 height (300 mm, 450 mm and 600 mm), jacket thickness (100 mm, 150 mm and 200 mm), existing
10 beam width (200 mm, 300 mm and 400 mm), and beam span (3 m, 4 m and 5 m). The steel
11 reinforcement distribution along the beam is shown in Fig. 1 in which the balanced steel
12 reinforcement ratio is determined with regard to CSA A23.3 [21]. Jacketing from one side at the
13 soffit of all beams is adopted in the analysis. Each section is analyzed 63 times to account for the
14 considered variables. Therefore, a total of 5103 different cases are considered in the current
15 parametric study.

16

17 **9. Moment-Curvature Behavior**

18 The following discussion refers to the beam sections whose geometrical and mechanical
19 properties are listed in Table 1. These sections are considered to examine the influence of slip on
20 flexural behavior of jacketed RC beams due to the variation of jacket thickness, beam width, beam
21 height, span, concrete compressive strength and steel grade. Figs. 9 and 10 show the initial stiffness
22 values for each section assuming full and partial composite actions (assuming a friction coefficient
23 of 0.4) under both sagging and hogging moments, respectively. The reduction in initial stiffness

1 caused by slip is indicated as a percentage in the corresponding figures. Fig. 11 describes the
2 variation of the reduction rate in stiffness (as percentage) due the variation of each of the
3 aforementioned parameters. Reference to Table 1 and Figs. 9, 10 and 11 is made throughout the
4 following discussion.

5 **9.1. Effect of Jacket Thickness (h_j)**

6 Beams B-1, B-2 and B-3 are considered for comparison. Figs. 12 and 13 illustrate the
7 influence of varying the jacket thickness on the flexural behavior of continuous beams in view of
8 the $M-\phi$ relationships along the sagging and hogging moment regions, respectively. The flexural
9 behavior in the sagging moment region is characterized by yielding of jacket reinforcement ensued
10 by yielding of core reinforcement and a yielding plateau until failure by concrete crushing.
11 Regarding the hogging moment region, the yielding plateau occurs immediately after yielding of
12 the tension steel bars located in the original beam. The same behavior is found for the remaining
13 parameters; therefore, only the stiffness values are included in the discussion.

14 The stiffness reduction rate in both the sagging and hogging moment zones slightly decreases
15 with increasing the jacket thickness. The ductility increase is insignificant when slip is considered
16 for the sagging moment region indicating that the compressive strains at the extreme compression
17 fibers reach the concrete crushing strain value at the same curvature. This happens since the axial
18 stress in the jacketing layer assuming both monolithic and partially composite actions become
19 identical beyond the yielding point of the jacket steel bars regardless of the slip strain. However,
20 in the hogging moment region, as the jacket thickness increases, the contribution of the concrete
21 material and the compression steel bars located in the jacket layer becomes more prevalent relative
22 to the entire section. Therefore, slip strain reduces the generated compressive stresses within the
23 jacket layer at the same curvature value. This results in delaying the concrete crushing and

1 consequently increasing the ductility as the jacket height increases relative to the monolithic
2 beams.

3 **9.2. Effect of Beam Width (b_c)**

4 Increasing the beam width results in a consequent increase in both the initial stiffness and
5 capacity with minor influence on the flexural ductility. Regarding the slip influence, increasing the
6 beam width results in decreasing the reduction rate of the initial stiffness in both sections. This is
7 justified by the larger contact area between the concrete core and the jacket that is provided by the
8 additional beam width. Two main differences arise from changing the location of the contact
9 surface with respect to the neutral axis. When the interface is located at the tension side (i.e.
10 sagging moment section), the reduction in the elastic stiffness is relatively smaller than the case of
11 hogging moment. This variation in stiffness reduction is attributed to the contribution of both
12 concrete and steel in determining the slip strain ($\Delta\varepsilon$) at each section. For the sagging moment
13 region, the bending stresses at the tension side are resisted by both the core steel bars and the jacket
14 steel bars especially after concrete cracking takes place. This means that the steel in the jacketing
15 layer sustains part of the generated tensile stresses and the remaining part is resisted by the steel
16 bars in the original beam. Thus, the slip strain required to achieve equilibrium at any section along
17 the jacketed beam is governed by a portion of the total tensile stress generated at a given applied
18 load. A different situation is observed along the hogging moment region where the jacketing layer
19 is at the compression side. In this case, the entire concrete material is utilized along with the jacket
20 steel bars to resist the same applied load. This indicates that a larger portion of bending is carried
21 by the jacket part causing an increase in the slip strain required to achieve equilibrium at any
22 segment along the hogging moment region.

1 The other difference that prevails from changing the location of the interface with respect to
2 the neutral axis is the point which the $M-\phi$ curves ignoring and including slip effects follow the
3 same path. For the sagging moment region, the major difference in the moment-curvature diagram
4 is within the elastic region before yielding of the jacket steel bars. This is justified by knowing that
5 the axial force at any section is determined by the jacket steel bars. So, once these bars yield, the
6 tensile forces in the jacket steel bars becomes almost constant and any increase depends on the
7 strain hardening modulus. Thus, after jacket yielding is reached, the influence of slip strain
8 becomes negligible in changing the behavior of the $M-\phi$ diagram compared to its monolithic
9 counterpart. Regarding the hogging moment region, yielding point is dictated by the tensile steel
10 reinforcement in the concrete core. Therefore, the yielding point considering slip occurs at a larger
11 curvature value compared to the monolithic case. Since the tensile stresses in the jacketing layer
12 are governed by the compression behavior of both the concrete material and the embedded jacket
13 steel bars, the influence of slip strain remains considerable in reducing the moment carrying
14 capacity at a given curvature. As the load increases, the effect of slip strain diminishes until the
15 moment-curvature behavior of the partially composite section becomes identical to the monolithic
16 one.

17 **9.3. Effect of Existing Beam Depth (h_c)**

18 In both the sagging and hogging moment cases, increasing the existing section height
19 increases both the elastic stiffness and capacity of the jacketed beams. This is justified by the larger
20 concrete material available in the compression side and the longer lever arm the tension steel bars
21 have. The ductility, on the other hand, decreases as the section height increases and becomes even
22 more pronounced if the interface is at the compression side. This drop in ductility is related to the
23 higher stresses developed in the tension steel bars as the original beam height rises at any curvature

1 level. Therefore, at the same applied bending moment, this higher stress at the tension steel bars is
2 translated into higher compressive stresses at the compression face of the jacketed beam causing
3 the concrete to reach its crushing strain at lower curvature values. Regarding the slip influence on
4 the flexural behavior of these beams, the initial stiffness reduction rate decreases as the original
5 section height increases for both the sagging and hogging moment cases. This decrease is a result
6 of the higher slip strain required to equilibrate the axial force within the jacket with the horizontal
7 shear force along the interface.

8 **9.4. Effect of Beam Span (L)**

9 The $M-\phi$ curve assuming monolithic interaction between the core and the jacket are identical
10 regardless of the span as they depend merely on the cross-sectional properties. However, including
11 the slip effect activates the partially composite action and consequently the horizontal shear
12 distribution along the interface becomes a major player in determining the flexural behavior of any
13 section along the beam. In both the sagging and hogging moment cases, as the span increases, the
14 elastic stiffness reduction rate decreases proportionally. This observation is justified by the higher
15 contact area provided by the larger span and consequently the increased frictional forces along the
16 jacketed beam. For the positive moment section, the partially composite flexural behavior becomes
17 identical to the monolithic counterpart once jacket steel bars yield. This happens due to the small
18 variation in the axial stresses governed by the strain hardening modulus of jacket steel bars after
19 yielding occurs. Thus, at the same curvature value, the stress in these steel bars is almost identical
20 to the ones in the monolithic case. Although it still exists, the influence of slip strain diminishes
21 even more at higher loading values due to the higher contribution of compression concrete and
22 tension core steel bars while the stresses in the jacket steel bars remain almost constant. Regarding
23 the hogging moment region, the variation between the partially composite scenario and monolithic

1 behaviors persists within a portion of the inelastic region. This occurs since the jacketing layer is
2 governed by the compressive stresses developed in concrete and the embedded steel bars rather
3 than the tensile stresses generated merely in the steel bars. Thus, even after yielding of the section
4 takes place, the axial force within the jacket at any section remains different from the monolithic
5 case due to the influence of slip strain which decreases the jacket stresses at any curvature value.
6 At higher loading values, the slip strain becomes less pronounced relative to the higher curvature
7 values and consequently its influence becomes less substantial.

8 **9.5. Effect of Concrete Compressive Strength (f'_c)**

9 Increasing the concrete compressive strength results in a consequent increase in the beam
10 capacity as it resists higher stresses for the same peak strain value. Also, increasing the concrete
11 grade rises the concrete modulus resulting in a higher elastic stiffness value. Regarding the slip
12 effect, increasing the concrete compressive strength decreases the stiffness reduction rate
13 indirectly through increasing the friction between the two surfaces. This is inferred by examining
14 the change in flexural behavior when slip is considered in both the sagging and hogging moment
15 regions.

16 **9.6. Effect of Steel Grade (f_y)**

17 Increasing the steel yield strength has a negligible influence on the initial stiffness of the
18 jacketed beams but a substantial enhancement to its capacity. The main reduction in stiffness will
19 be in the elastic zone in which the steel elastic modulus plays the major role. Considering slip in
20 the analysis shows that as the steel grade increases, the drop in flexural stiffness also increases for
21 both the sagging and hogging moment sections. This happens since the steel bars with higher grade
22 within the jacket resist larger axial forces before yielding and consequently result in higher shear

1 stresses to achieve equilibrium. These higher stresses result in larger slip and consequently larger
2 slip strain that reduces the flexural stiffness of the jacketed beams.

3

4 **10. Interfacial Slip Behavior**

5 The influence of interfacial slip between the concrete core and the underlying jacket layer is
6 investigated in view of the slip strain ($\Delta\varepsilon$), slip (S) and interfacial shear stress (τ) distribution along
7 the continuous beams under different loading values. Beam B-14 in Table 1 is considered for the
8 following discussion. The coefficient of friction between the two surfaces is taken as 0.4 and 1.0
9 which represent untreated surfaces and intentionally roughened surfaces, respectively. Figs. 14
10 through 18 represent the distribution along one span only of the continuous beam.

11 **10.1. Slip Strain ($\Delta\varepsilon$) Distribution**

12 Figs. 14 and 15 illustrate the slip strain distribution from the edge support towards the
13 intermediate support for coefficient of friction of 0.4 and 1.0, respectively. Four loading values
14 representing the elastic range, onset of jacket yielding, onset of core yielding and maximum
15 capacity of the section at the intermediate support at the onset of concrete crushing. Both figures
16 show the same trend in which the slip strain at any section increases with the applied load except
17 at the points of zero moment (i.e. the edge support and the point of contraflexure). This increase
18 corresponds to the rise in the axial stresses within the jacket layer to maintain the equilibrium
19 condition with the interfacial shear along the contact plane.

20 The maximum slip strain reaches the peak value at two points corresponding to the maximum
21 positive bending moment and the maximum negative bending moment sections. The slip strain is
22 always higher at the intermediate support than the maximum positive moment for two main
23 reasons. The first one is that the sagging moment region extends along a larger distance than the

1 hogging moment zone resulting in a larger contact area and friction resistance and consequently
2 less slip strain in the former case. Also, the slip strain is proportional to the bending moment that
3 develop axial stresses within the jacket. Since the negative moment at the middle support is always
4 larger than the maximum positive moment at any loading value, the slip strain follows the same
5 trend and becomes higher at the intermediate support. By comparing the curves in Fig. 14 for $\mu =$
6 0.4 to their counterparts in Fig. 15 for $\mu = 1.0$, higher slip strain values at any given load are
7 observed in the former case. This difference occurs due to the lower interfacial stiffness as the
8 friction coefficient decreases. Thus, for the same axial stresses in the jacket, higher slip strain is
9 required to achieve equilibrium with the interfacial shear stress. By roughening the concrete
10 surface prior to jacketing, the slip strain at the maximum positive moment section drops from about
11 0.39 to 0.16 indicating a ratio of 58.9%. This drop at the maximum negative moment section is
12 shown to be from 1.25 to 0.66 with a ratio of 47.2%. The slip strain increasing rate rises at higher
13 loading values compared to the elastic region in both the hogging and sagging moment regions.
14 For the maximum positive moment section, increasing the uniformly applied load from 30 kN/m
15 to 90 kN/m along the beam results in a consequent increase of the slip strain by just 0.07×10^{-3} for
16 the untreated surface and by just 0.03×10^{-3} for the roughened surface. After yielding occurs,
17 increasing the load by about 10 kN/m results in an increase of 0.16×10^{-3} and 0.08×10^{-3} for the
18 smooth and rough surfaces, respectively. The same observation applies for the maximum negative
19 moment section but with different increasing rate. This is explained by the larger curvature the
20 beam undergoes within the yielding plateau corresponding to any variation in the applied load
21 relative to the elastic range.

1 **10.2. Slip (S) Distribution**

2 The slip distribution along the interface considering both smooth and rough surfaces are
3 shown in Figs. 16 and 17, respectively. As shown in these figures, the maximum slip values are
4 obtained at the edge support and the point of contraflexure that both correspond to the sections of
5 zero moments. On the contrary, relative slip between the two surfaces becomes negligible at the
6 locations of maximum positive and negative moments. The same figures also show that slip values
7 at sections closer to the edge support are slightly less than those near the middle support. This
8 reduction is due to concrete cracking in the sagging moment region that takes place during the
9 initial loading stage. When the concrete jacket is cracked, only the jacket reinforcement contributes
10 in resisting the generated axial force resulting in lower level of shear stresses transferred along the
11 interface.

12 In the hogging moment region, both concrete and the jacket steel bars are active and resist
13 the bending moment in terms of compressive stresses. This results in larger contribution of the
14 jacket and consequently higher shear stress to be transferred along the interface as translated by
15 the higher slip values. The slip increasing rate after the first yielding point is higher than the rate
16 before yielding for both kinds of surface treatment. For example, increasing the applied load within
17 the elastic region from 30 kN/m to 90 kN/m results in a consequent increase in the maximum slip
18 at the edge support from 0.04 mm to just over 0.12 mm for the untreated surface case. However,
19 after the yielding point is reached, increasing the load by just 10 kN/m results in extra relative
20 sliding between the two surfaces of about 0.18 mm. The same observations are shown for the
21 roughened surface case. This higher rate of slip rise is justified by the higher curvature the beam
22 exhibits after reaching the yielding point for the same amount of load compared to the elastic
23 range. Consequently, the slip strain ($\Delta\varepsilon$) at the yielded segments increases resulting in a larger

1 increase in slip. By examining both figures, the slip values considering smooth surfaces are higher
2 than the ones obtained assuming roughened surfaces at any section for the same loading level. For
3 example, the slip at the edge support for the former case is 138.9% higher than the second case at
4 the ultimate loading value. This is justified by the higher frictional resistance and consequently the
5 higher interfacial stiffness as the original beam surface is roughened. It is worth mentioning that
6 at the ultimate load, the slip value at any section is less than the failure value defined in the slip
7 model of about 2 mm.

8 **10.3. Interfacial Shear Stress (τ) Distribution**

9 Figs. 18 and 19 detail the horizontal shear stress distribution along the interface between the
10 concrete core and the attached jacket layer considering untreated and roughened surfaces,
11 respectively. As shown in both figures, the distribution follows a third order parabolic function as
12 initially assumed. The shear values are then determined by carrying out both sectional and
13 longitudinal analyses to satisfy the equilibrium, compatibility and constitutive conditions. Also,
14 the figures demonstrate the direct relationship that relate the interfacial shear stress (τ) to the slip
15 (S) at any section through the interfacial stiffness (k_s). Since the shear-slip model at the interface
16 is non-linear, the secant interfacial stiffness varies depending on the slip value. For the smooth
17 connection, the interfacial shear to slip ratio at the edge support is obtained as 3.4 N/mm³ for all
18 distributions up to the yield point and 2.9 N/mm³ at ultimate. The same conclusion is drawn by
19 comparing the curves in the second figure but with the secant interfacial stiffness of 9.1 N/mm³ up
20 to the yield point and 7.7 N/mm³ at ultimate at the same section. As expected, the interfacial
21 stiffness at any given load is higher when the original beam surface is roughened compared to the
22 untreated case. Although the slip distribution along the interface is different for both cases, the
23 interfacial shear stress distribution is almost identical. This is justified by the variation of the

1 interfacial stiffness between both cases that result in equilibrium between the axial force in the
2 jacketing layer and the horizontal shear force at any segment along the interface. The same
3 observations are shown in the hogging moment region.

4

5 **11. Proposed Expressions for the Effective Stiffness**

6 Having developed and verified an analytical procedure to analyze jacketed continuous RC
7 beams considering the influence of interfacial slip, a parametric study including 5103 specimens
8 is carried out to determine the contribution of various parameters on the flexural behavior of such
9 beams. These parameters encompass the beams' geometrical properties, mechanical properties and
10 interfacial behavior between the core and the RC jacket. The outcomes show that ignoring the
11 relative slip between the two substrates may overestimate the flexural stiffness causing
12 serviceability issues such as larger deflections and unexpected cracking. Therefore, the influence
13 of slip should be considered when designing such jacketed beams. Including the influence of slip
14 in the analysis is tedious and requires a sequence of nested iterations that may not be convenient
15 for design engineers. Here comes the importance of providing the engineers with expressions that
16 improves the accuracy of their designs with less time and effort. The extent of flexural stiffness
17 reduction as well as the point at which both the monolithic and partially composite curves becomes
18 almost identical differ between the sagging and hogging moment regions. Therefore, different
19 expressions are provided to adjust the monolithic $M-\phi$ diagram of each region by considering the
20 slip effect. Equations 6 through 9 provide the expressions for α_y and α_u that represent the yield
21 monolithic factor and ultimate monolithic factor for the hogging moment section, respectively.
22 Equations 10 and 11 presents the yield monolithic factor (α_y) for the sagging moment section.

23

$$\alpha_y^{-ve} = (\xi_y^{-ve}) + [22.6645 (\xi_y^{-ve})^2 - 46.3178 (\xi_y^{-ve}) + 23.6573] \quad (6)$$

$$\begin{aligned} \xi_y^{-ve} = & 1.15545 - 2.661 \times 10^{-4} f'_c + 3.229 \times 10^{-5} f_y - 1.266 \times 10^{-5} L \\ & + 3.30 \times 10^{-7} b_c^2 - 2.811 \times 10^{-4} b_c - 1.704 \times 10^{-5} h_c \\ & + 5.22 \times 10^{-6} \frac{h_j^2}{b_c} - 1.57 \times 10^{-5} h_j - 0.037306 \mu \geq 1.0 \end{aligned} \quad (7)$$

$$\alpha_u^{-ve} = (\xi_u^{-ve}) + [1.4756 \exp(138.9291 \xi_u^{-ve})] \quad (8)$$

$$\begin{aligned} \xi_u^{-ve} = & 1.11070 - 1.108 \times 10^{-4} f'_c + 3.459 \times 10^{-5} f_y - 1.018 \times 10^{-5} L \\ & + 1.90 \times 10^{-7} b_c^2 - 1.784 \times 10^{-4} b_c - 8.39 \times 10^{-6} h_c \\ & + 2.857 \times 10^{-5} \frac{h_j^2}{b_c} - 9.06 \times 10^{-6} h_j - 0.033465 \mu \geq 1.0 \end{aligned} \quad (9)$$

$$\alpha_y^{+ve} = (\xi_y^{+ve}) + [20.3463 (\xi_y^{+ve})^2 - 41.0203 (\xi_y^{+ve}) + 20.6732] \quad (10)$$

$$\begin{aligned} \xi_y^{+ve} = & 1.11354 - 1.108 \times 10^{-4} f'_c + 3.459 \times 10^{-5} f_y - 1.018 \times 10^{-5} L \\ & + 2.20 \times 10^{-7} b_c^2 - 2.043 \times 10^{-4} b_c - 8.39 \times 10^{-6} h_c \\ & - 2.190 \times 10^{-5} h_j - 0.033465 \mu \geq 1.0 \end{aligned} \quad (11)$$

- 1 Where f'_c is the concrete compressive strength in MPa; f_y is the steel yield strength in MPa; L is
- 2 the beam span in mm; b_c is the section width in mm; h_c is the section height in mm; h_j is the jacket
- 3 thickness in mm and μ is the coefficient of friction between the original beam and the attached
- 4 jacket. If the beams were subjected to initial loading before jacketing, then the monolithic factors
- 5 should be reduced according to the expressions given in Equations 12, 13 and 14 for hogging
- 6 ultimate monolithic factor, hogging yield monolithic factor and sagging yield monolithic factor,
- 7 respectively.

$$(\alpha_y^{-ve})_{initial} = \alpha_y^{-ve} - \left(\frac{M_{initial}}{M_{u,unjacketed}} \right)^{1.327} (\alpha_y^{-ve} - 1.0) \geq 1.0 \quad (12)$$

$$(\alpha_u^{-ve})_{initial} = \alpha_u^{-ve} - \left(\frac{M_{initial}}{M_{u,unjacketed}} \right)^{0.849} (\alpha_u^{-ve} - 1.0) \geq 1.0 \quad (13)$$

$$(\alpha_y^{+ve})_{initial} = \alpha_y^{+ve} - \left(\frac{M_{initial}}{M_{u,unjacketed}} \right)^{1.113} (\alpha_y^{+ve} - 1.0) \geq 1.0 \quad (14)$$

1 Where $M_{initial}$ is the maximum applied moment during jacketing and $M_{u,unjacketed}$ is the flexural
 2 capacity of the unjacketed section. In these expressions, the section subjected to maximum
 3 negative moment is considered to determine the hogging moment, while the section subjected to
 4 maximum positive moment is used in evaluating the sagging moment. Figs. 20 (a) and 20(b) detail
 5 the variation in a typical equivalent moment-curvature diagrams assuming monolithic and partially
 6 composite sections without and with initially applied load, respectively.

7 The main parameters defining the curves in Fig. 20 are the yield moment (M_y) and the
 8 corresponding equivalent curvature assuming monolithic (ϕ_y) and partially composite (ϕ_y^*)
 9 actions; and ultimate moment (M_u) and the corresponding equivalent curvature assuming
 10 monolithic (ϕ_u) and partially composite (ϕ_u^*) actions. For the initially loaded beams, two additional
 11 terms are introduced that define the both the moment ($M_{initial}$) and the equivalent curvature ($\phi_{initial}$)
 12 corresponding to the initial loading value at the onset of jacketing as indicated in Fig. 20(b).

13 The proposed design procedure is summarized in the following three steps to obtain the actual
 14 load-deflection curve considering the sliding between the two surfaces:

- 15 1) Plot the $M-\phi$ diagram for the sections representing both the sagging and hogging moment
 16 regions assuming monolithic interaction between the original beam and the attached jacket. The
 17 hogging $M-\phi$ diagram is assumed bilinear and can be plotted by evaluating the yield and
 18 ultimate points. Regarding the sagging moment section, only the yield point is required since
 19 concrete crushing usually occurs at the negative moment section in continuous beams subjected
 20 to static loads.

1 2) Modify these $M-\phi$ diagrams in view of Fig. 10(a) and Equations 6 through 11 for beams not
2 subjected to initial load during jacketing. If the beam was subjected to initial loading prior to
3 jacketing, then modify the $M-\phi$ diagrams in view of Fig. 20(b) and Equations 12 through 14
4 taking into account the initial applied load level and the monolithic factors for unloaded beams
5 obtained from Equations 6 through 11.

6 3) Use the equivalent $M-\phi$ diagrams obtained at the sagging and hogging moment regions along
7 with the moment-area theorem to obtain the load-deflection diagram at any point along the
8 beam.

9 The expectation function of the proposed monolithic factors is determined considering
10 nonlinear regression analysis of the data. Figs. 21(a), 21(b) and 21(c) present the line of equality
11 corresponding to α_y^{-ve} , α_u^{-ve} and α_y^{+v} without initial loading, respectively.

12 The line of equality plots for all factors reveal that the model provides a very good prediction
13 of the actual behavior. Residual analysis for the three factors clearly shows a normally distributed
14 pattern of the residuals about the mean. The small positive value of mean indicates that the
15 proposed expressions tend to slightly round up the actual factor resulting in higher stiffness
16 reduction and therefore more conservative estimates. Similar statistical analysis is carried out for
17 the factors when initial load level is considered and a very good agreement is also found.

18

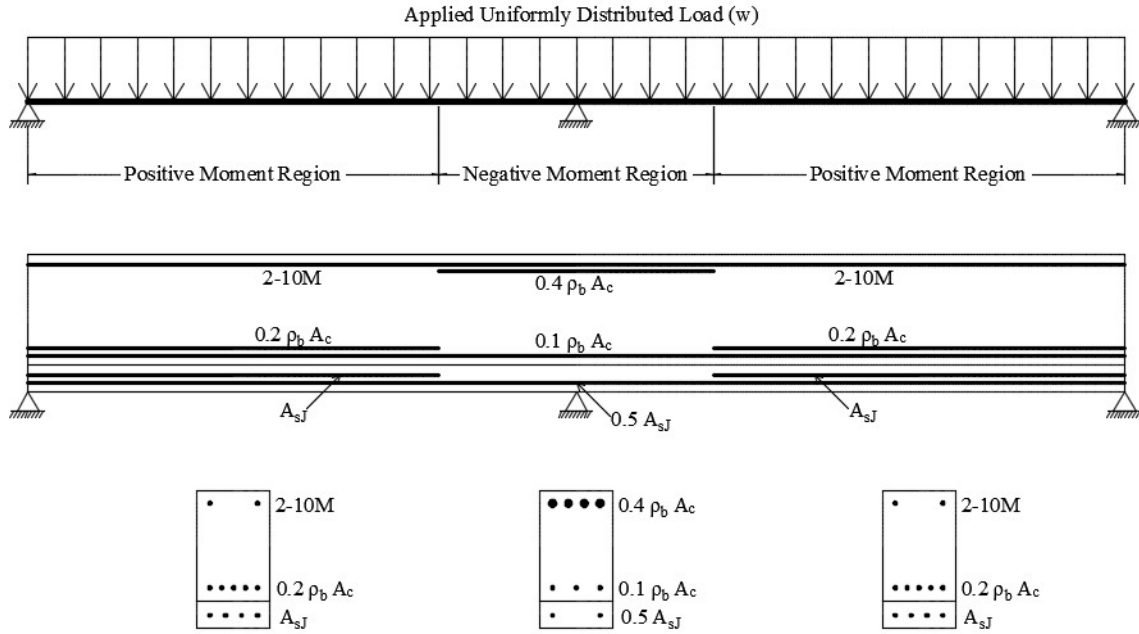
19 **12. Summary and Conclusions**

20 An investigation of the influence of RC jackets on the flexural behavior of continuous RC
21 beams was presented. A parametric study including 5103 symmetric continuous beams subjected
22 to uniformly distributed loads is carried out. The jacket is applied from one side at the soffit of all
23 beams. Different parameters including the geometrical properties (i.e. original beam width,

1 original beam depth, jacket thickness and beam span); mechanical properties (i.e. concrete
2 compressive strength and steel yield strength); and surface treatment (i.e. interfacial friction
3 coefficient) are investigated. An analytical model encompassing sectional and interfacial analyses
4 were developed taking into account that constitutive, compatibility and equilibrium conditions are
5 satisfied. The accuracy of the proposed model was validated in view of relevant experimental
6 results found in literature. The parametric study revealed that including the slip influence in the
7 analysis results in a reduction of stiffness that should be considered when designing jacketed
8 sections. For the beams considered in the analysis, ductile failure mode characterized by yielding
9 of tension steel bars followed by concrete crushing at the extreme compression fiber was detected.
10 The effect of each of the studied parameters on the $M-\phi$ relationship is similar for both the hogging
11 and sagging moment regions but shown to be more pronounced in the former zone. A design
12 procedure and stiffness monolithic factors are introduced in terms of the studied parameters to
13 obtain the flexural behavior of the continuous RC beams.

Table 1: Geometry of the discussed jacketed beams

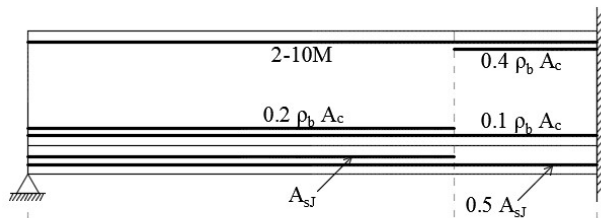
Section	Studied Parameters	Span (m)	b_c (mm)	h_c (mm)	h_J (mm)	f'_c (MPa)	f_y (MPa)
B-1	h_J, f'_c, f_y	3	200	300	100	30	400
B-2	h_J	3	200	300	150	30	400
B-3	$h_J, b_c, h_c, \text{Span}$	3	200	300	200	30	400
B-4	h_c	3	200	450	200	30	400
B-5	h_c	3	200	600	200	30	400
B-6	b_c	3	300	300	200	30	400
B-7	b_c	3	400	300	200	30	400
B-8	Span	4	200	300	200	30	400
B-9	Span	5	200	300	200	30	400
B-10	f'_c	3	200	300	100	25	400
B-11	f'_c	3	200	300	100	35	400
B-12	f_y	3	200	300	100	30	300
B-13	f_y	3	200	300	100	30	500
B-14	$\Delta\varepsilon, S, \tau$	3	200	450	150	30	300



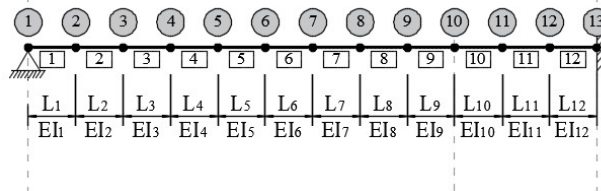
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Figure 1: Continuous beam loading scheme and reinforcement configuration

(a) Actual beam

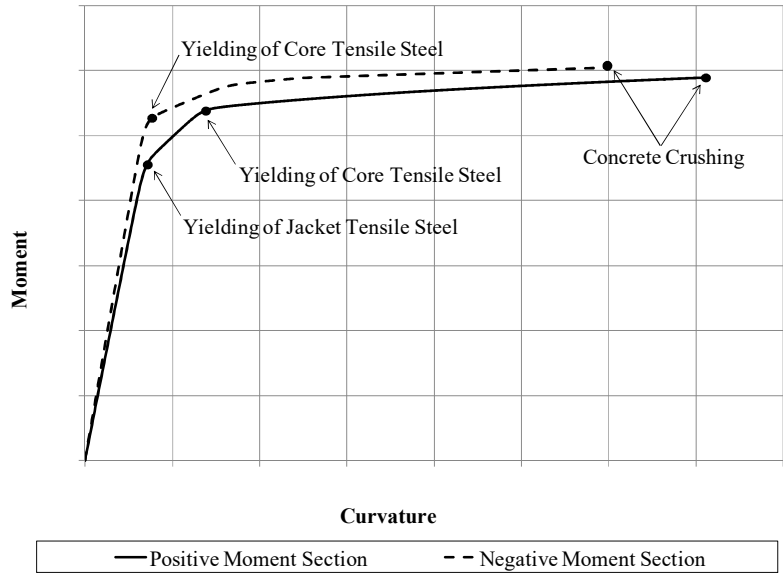


(b) Structure coordinate numbers



5

Figure 2: Propped cantilever analytical model



1

2

Figure 3: Moment-curvature diagrams for positive and negative moment sections

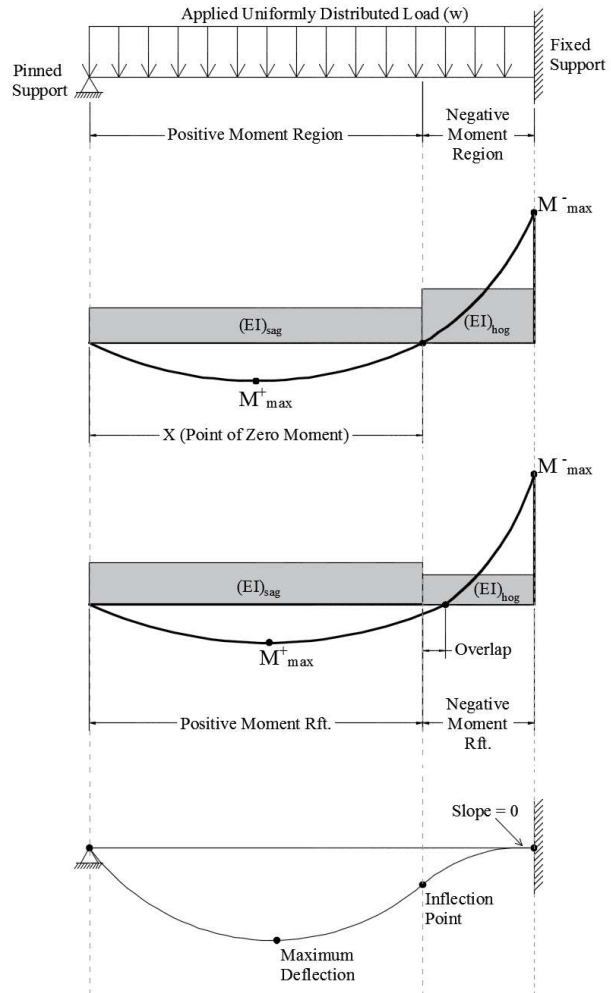
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(a) propped cantilever idealization

(b) bending moment diagram showing the point of zero moment

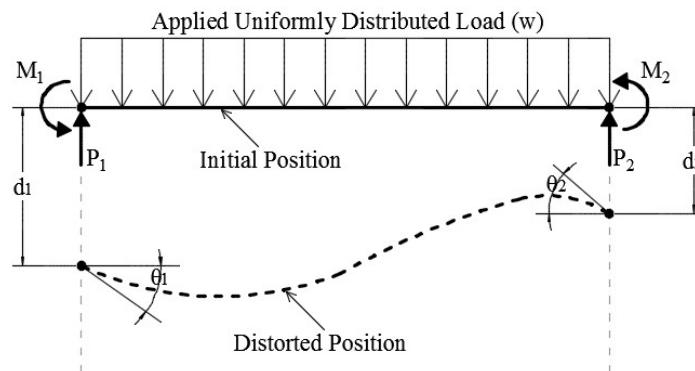
(c) bending moment diagram showing the point of zero moment

(d) anticipated deflection shape of the propped cantilever



1 **Figure 4: Bending moment and deflection profile of the propped cantilever model**

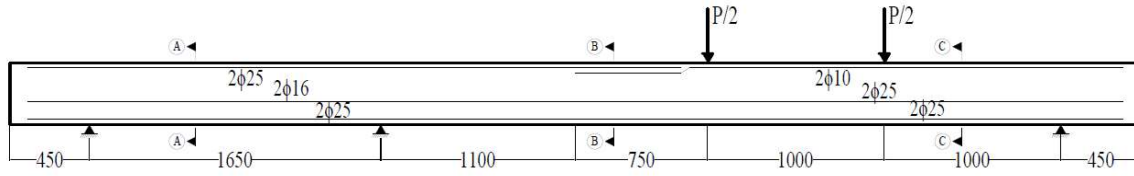
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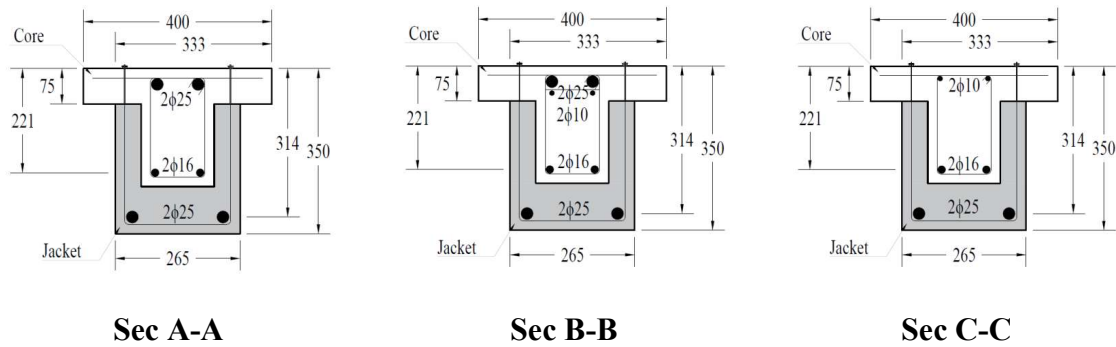
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Figure 5: Element forces and displacements



Note: Stirrups are ϕ 6 spaced at 115 mm along the beam

(a) jacketed beam longitudinal view and location of the applied loads



(b) cross-sectional views.

Figure 6: Longitudinal and cross-sectional views of the beams tested experimentally [15]

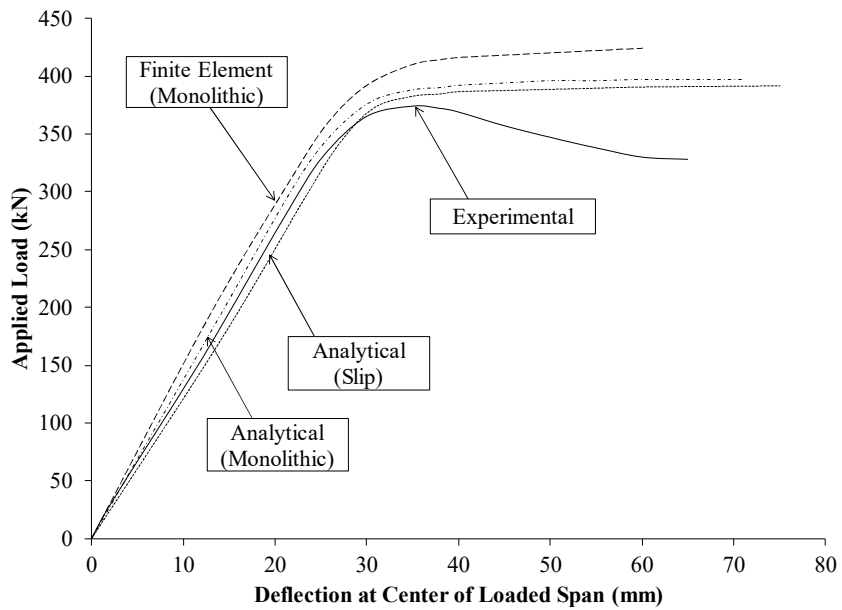
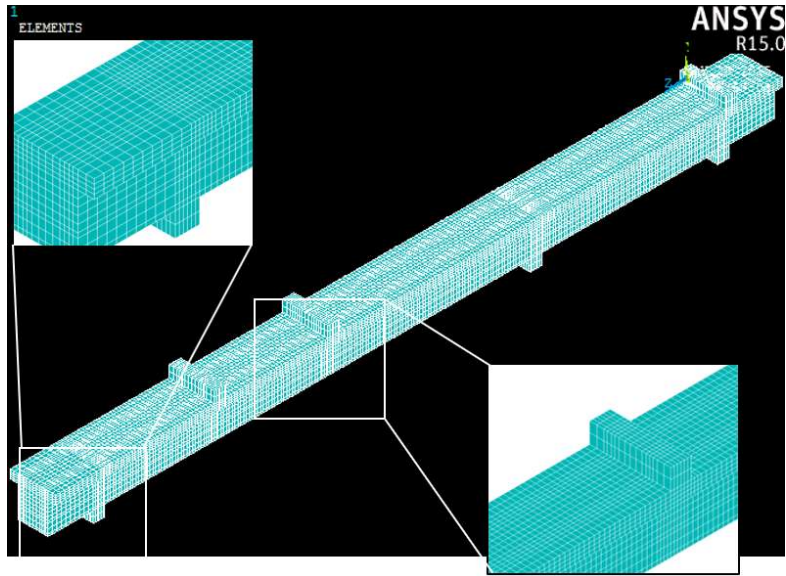


Figure 7: Validation of the proposed model



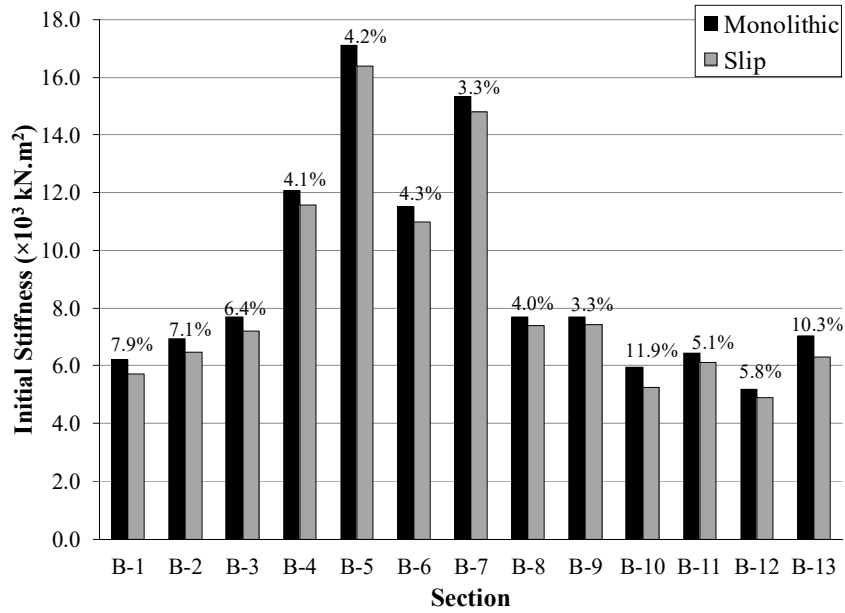
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Figure 8: Meshing of the jacketed beam in ANSYS

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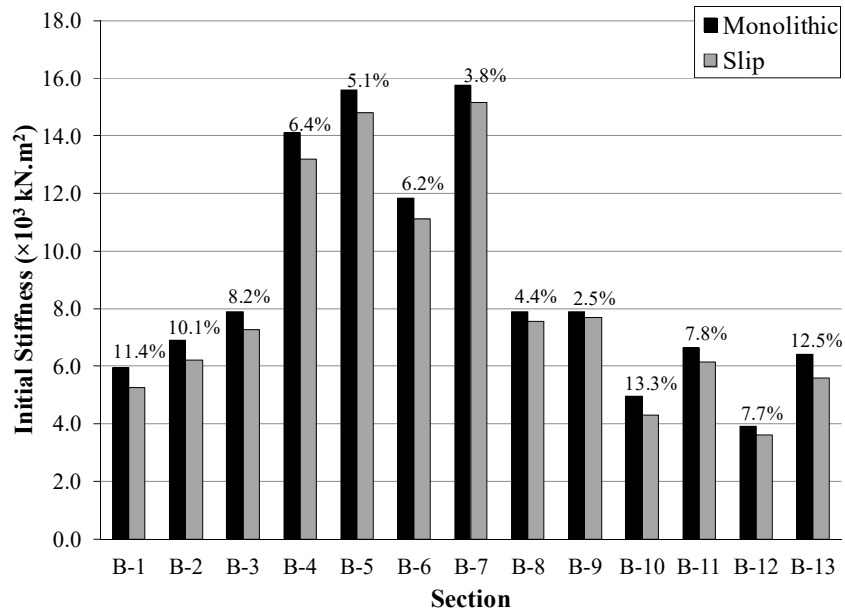


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Figure 9: Percent difference of initial stiffness with and without slip effect (sagging)

4

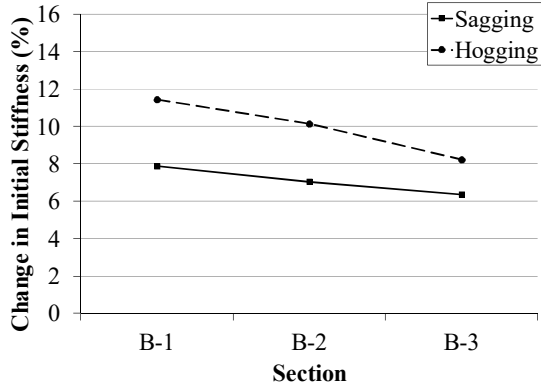


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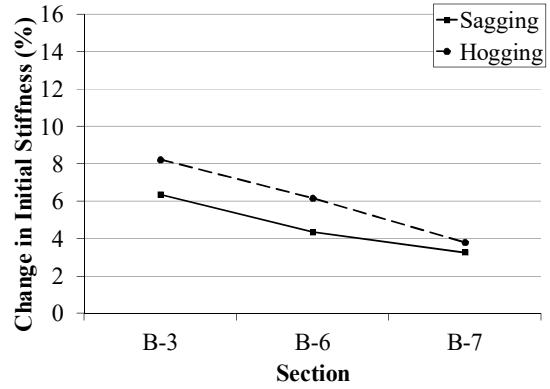
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Figure 10: Percent difference of initial stiffness with and without slip effect (hogging)

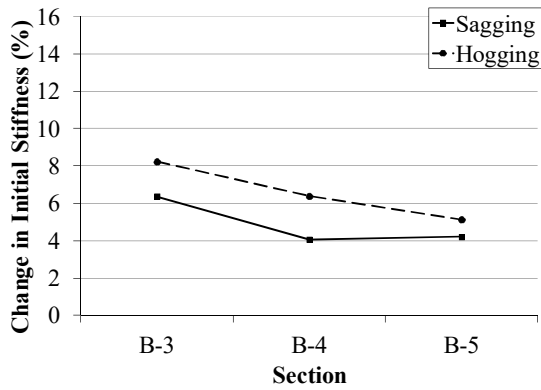
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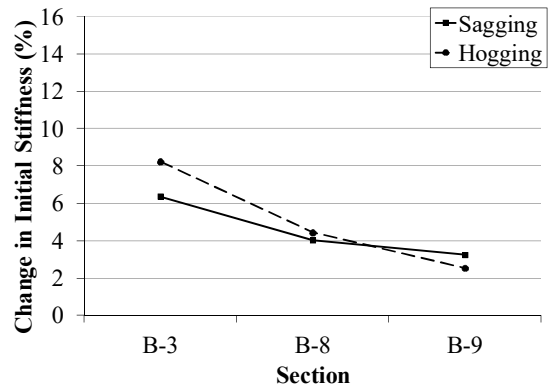
(a) jacket thickness



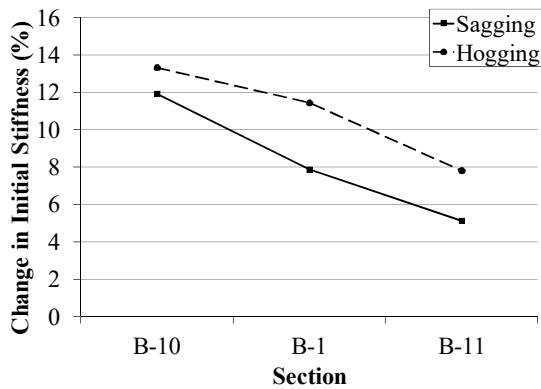
(b) section width



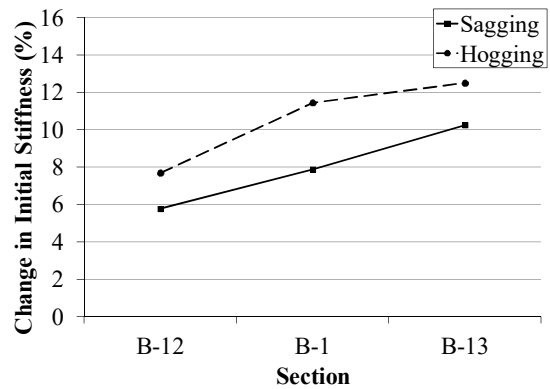
(c) section height



(d) span



(e) concrete compressive strength



(f) steel grade

Figure 11: Variation of sagging and hogging initial stiffness with various parameters

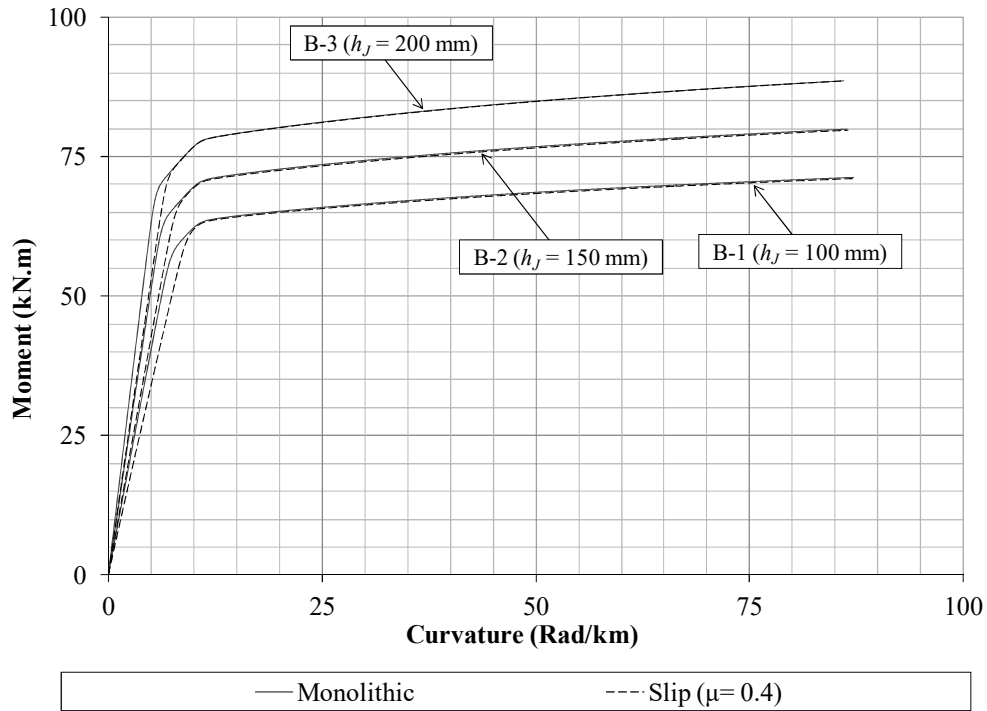


Figure 12: Effect of varying h_J on the $M-\phi$ relationship (sagging)

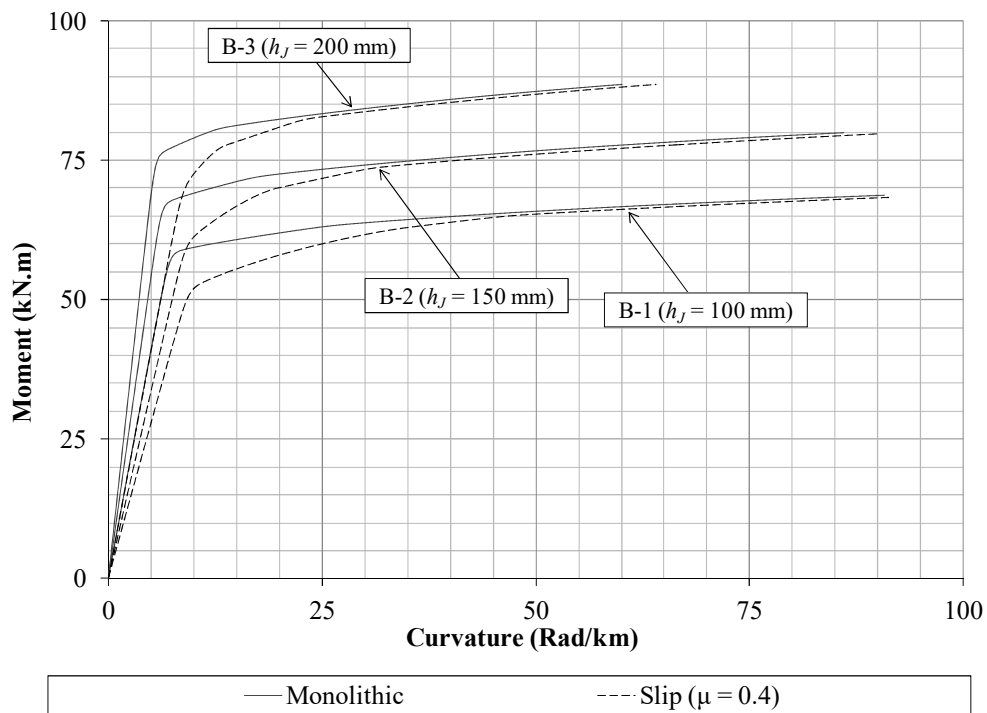


Figure 13: Effect of varying h_J on the $M-\phi$ relationship (hogging)

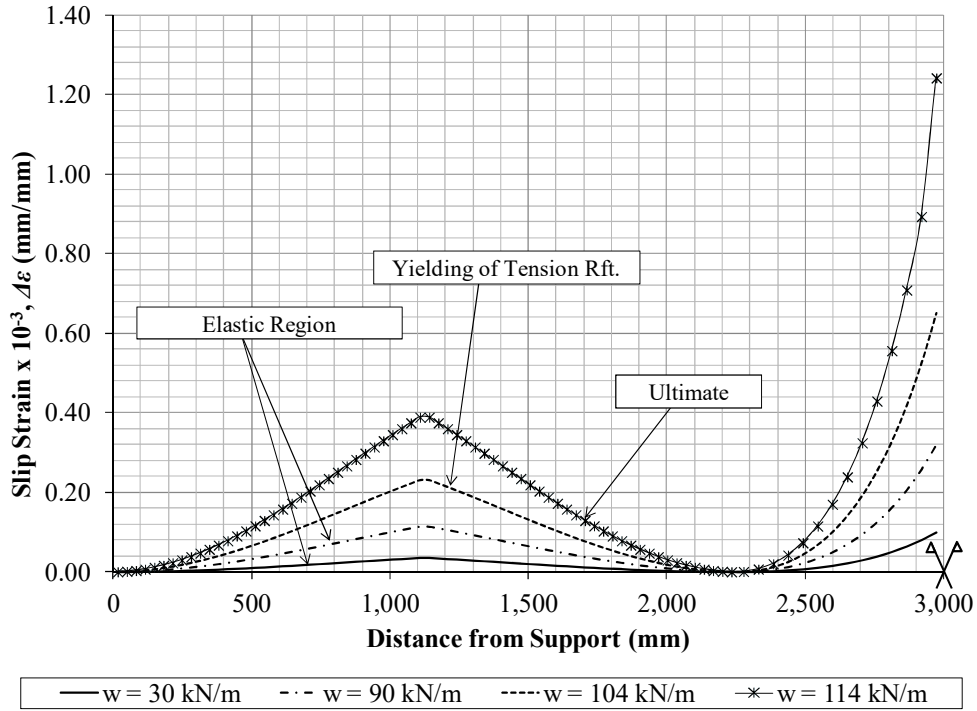


Figure 14: Slip strain ($\Delta\varepsilon$) distribution along beam B-5 ($\mu = 0.4$)

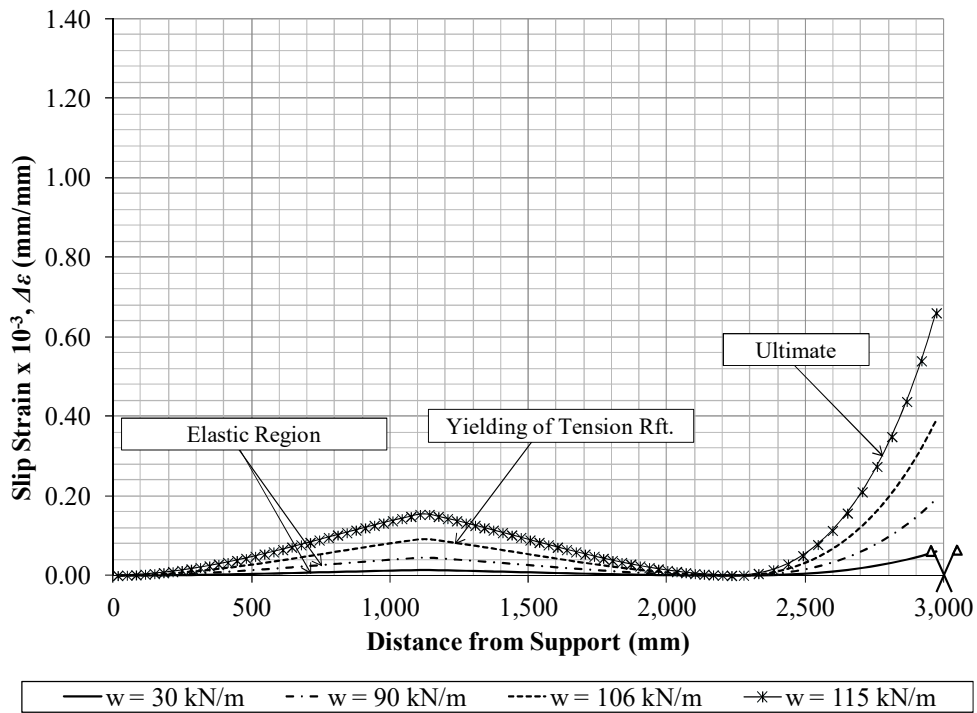


Figure 15: Slip strain ($\Delta\varepsilon$) distribution along beam B-5 ($\mu = 1.0$)

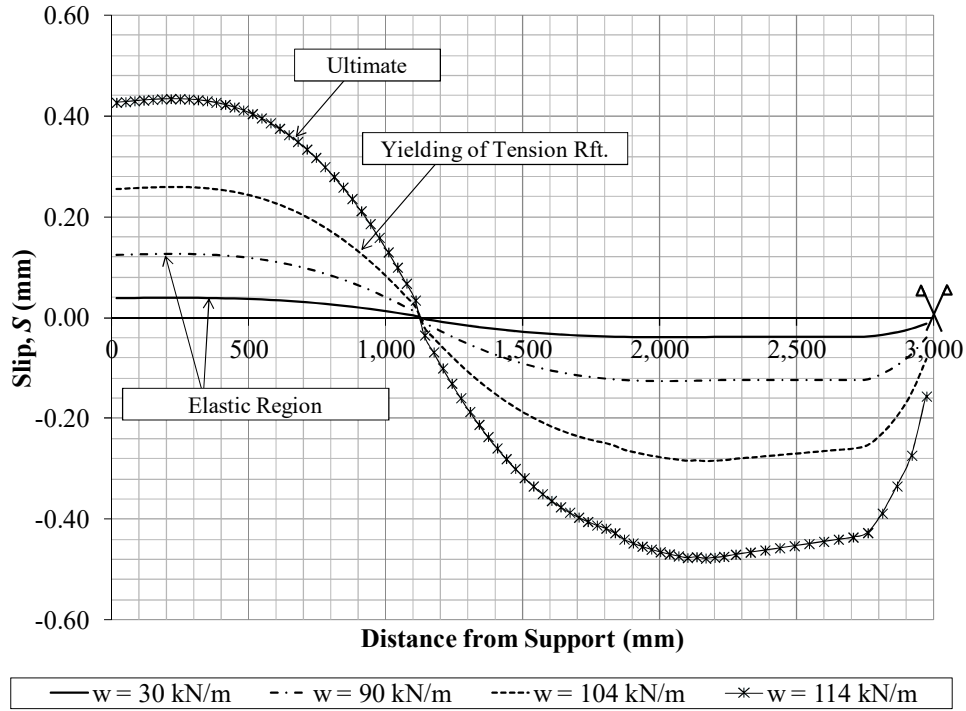


Figure 16: Slip distribution along beam B-5 ($\mu = 0.4$)

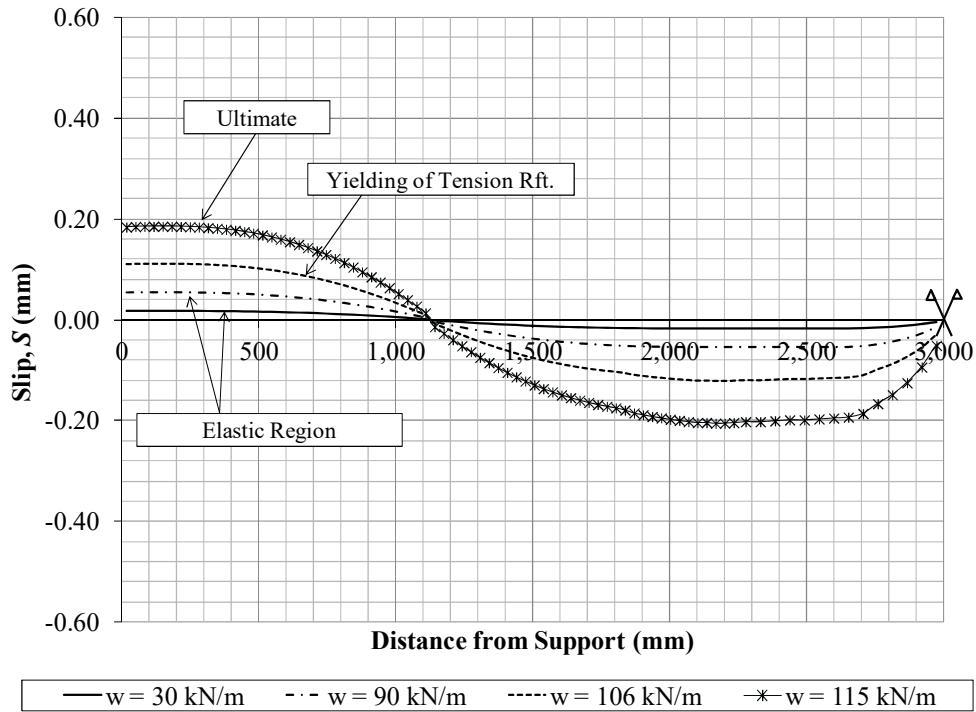


Figure 17: Slip distribution along beam B-5 ($\mu = 1.0$)

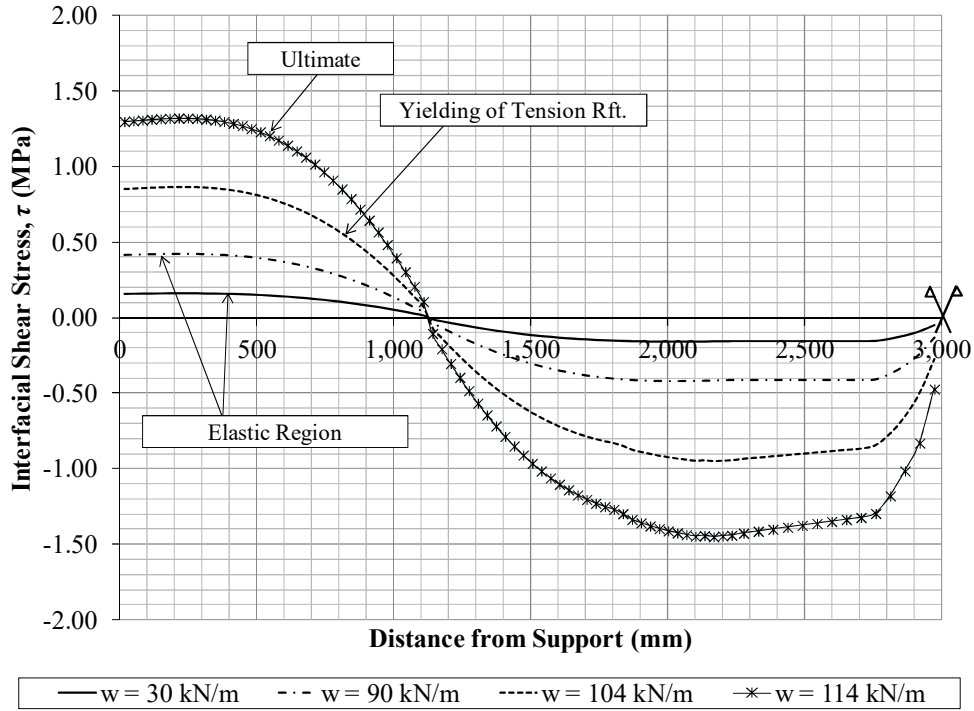


Figure 18: Interfacial shear stress distribution along beam B-5 ($\mu = 0.4$)

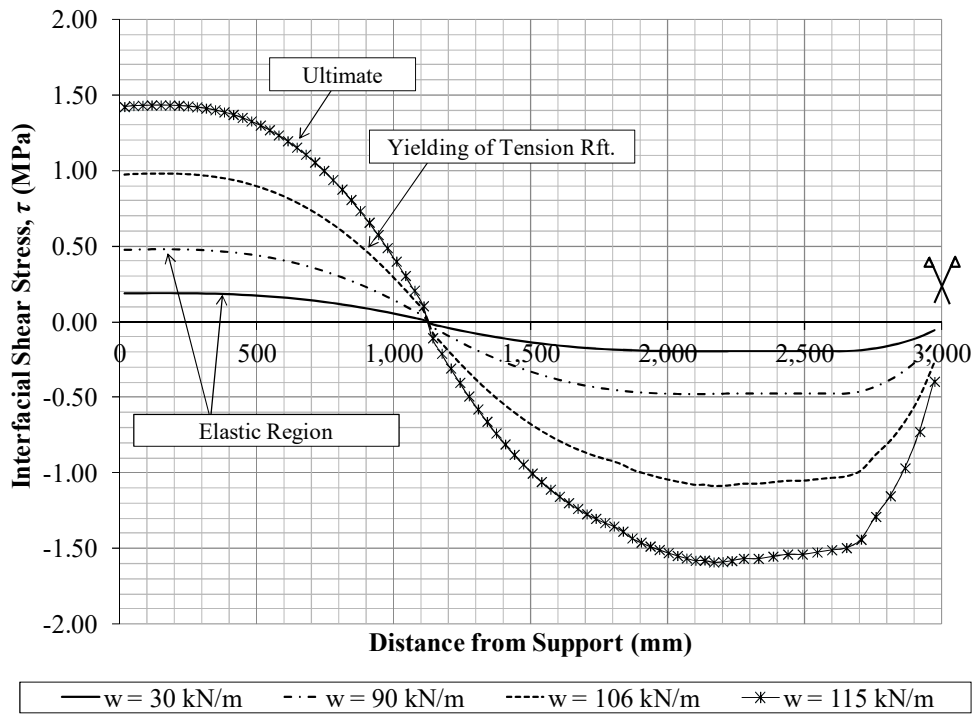


Figure 19: Interfacial shear stress distribution along beam B-5 ($\mu = 1.0$)

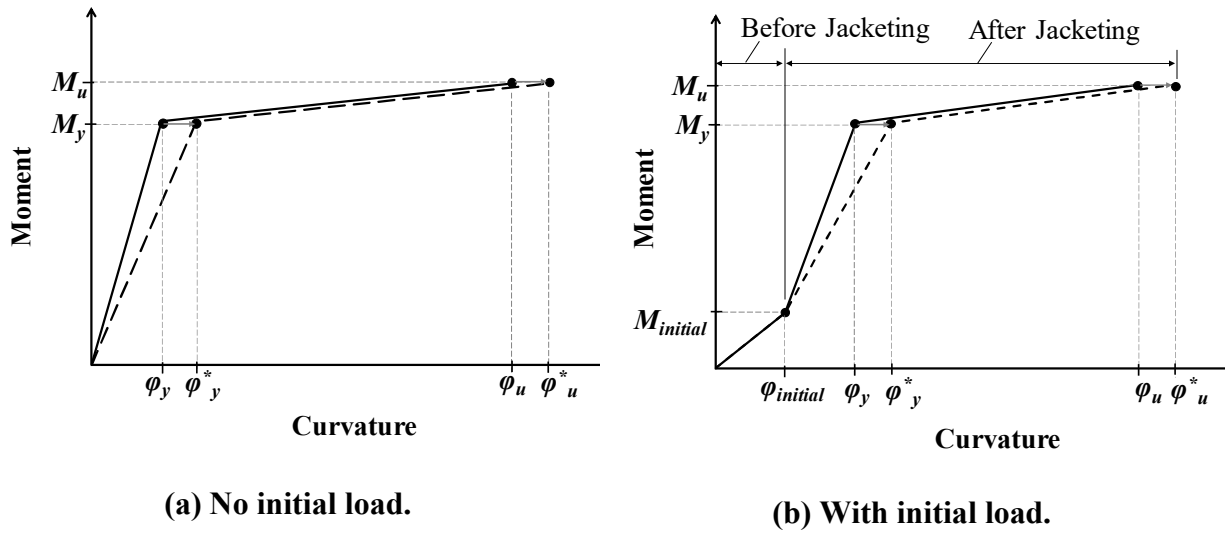


Figure 20: Typical moment-curvature diagram for jacketed beams.

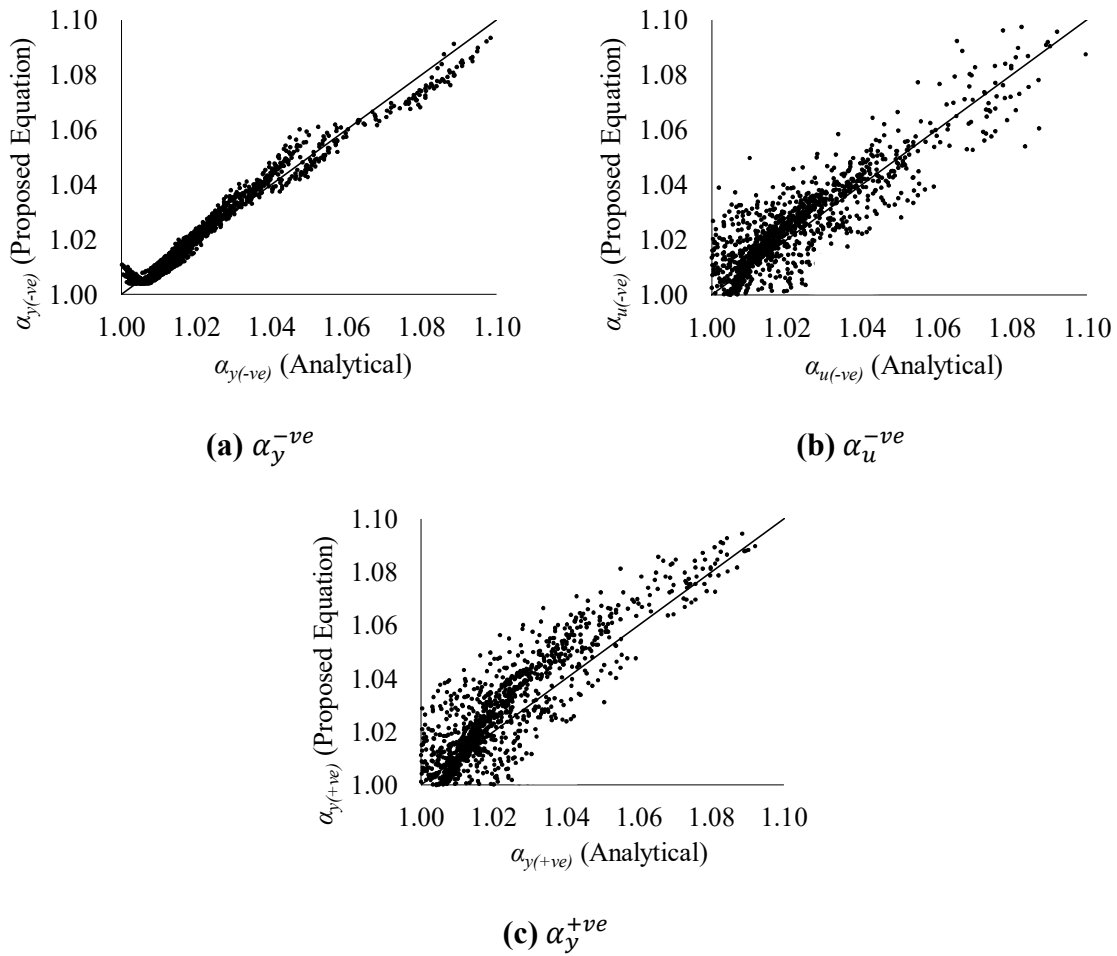


Figure 21: Statistical analysis for the proposed expressions

1 **13. References**

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3 [1] Tassios, T., Vintzeleou, E. "Concrete-to-Concrete Friction," Journal of Structural Engineering,
4 ASCE, 1987, Vol. 113, No. 4, pp. 832-849.

5 [2] Altun, F. "An experimental study of the jacketed reinforced-concrete beams under bending,"
6 Construction and Building Materials, 2004, Vol. 18, No. 8, pp. 611-618.

7 [3] Bousias, S., Spathis, A. and Fardis, M. "Seismic retrofitting of columns with lap-spliced
8 smooth bars through FRP or concrete jackets," Journal of Earthquake Engineering, 2007, Vol. 11,
9 pp. 653-674.

10 [4] Santos, P. and Júlio, E. "A State-of-the-Art Review on Shear-Friction," Engineering Structures,
11 2012, Vol. 45, pp. 435-448.

12 [5] Martinola, G., Meda, A., Plizzari, G.A. and Zinaldi, Z. "An Application of High Performance
13 Fiber Reinforced Cementitious Composites for RC Beam Strengthening," Fracture Mechanics of
14 Concrete and Structures - High Performance Concrete, Taylor and Francis Group, 2007, pp. 1541-
15 1548.

16 [6] Shimoyama, Y., Uzawa, M. "Properties and application of ductal," Journal of the Taiheiyo
17 Cement Corporation, 2002, Vol. 142, pp. 55-62.

18 [7] Vicenzino, E., Culhman, G., Perry, V., Zakariasen, D. and Chow, T. "The first use of UHPFRC
19 in thin precast roof shell for LRT Canadian station," PCI Journal, September- October, 2005.

20 [8] Li, V.C. "From Micromechanics to Structural Engineering - the design of cementitious
21 composites for civil engineering applications," Doboku Gakkai Rombun-Hokokushu/Proceedings
22 of the Japan Society of Civil Engineers, 1993, Vol. 471, No. 1-24, pp. 1-12.

- 1 [9] Rossi, P. "High Performance multimodal fiber reinforced fiber reinforced cement composite
2 (HPMFRCC): the LPC experience," *ACI Materials Journal*, 1997, Vol. 94, No. 6, pp. 478-483.
- 3 [10] Meda, A., Minelli, F., Plizzari, G.A. and Riva, P. "Shear behavior of steel fiber reinforced
4 concrete beams," *Materials and Structures*, 2005, Vol. 38, No. 3, pp. 343-351.
- 5 [11] Shehata, I., Shehata, L., Santos, E. and Simoes, M. "Strengthening of Reinforced Concrete
6 Beams in Flexure by Partial Jacketing," *Materials and Structures*, 2009, Vol. 42, No. 4, pp. 495-
7 504.
- 8 [12] Tsonos, A.D.G. "Performance enhancement of R/C building columns and beam-column joints
9 through shotcrete jacketing," *Engineering Structures*, 2010, Vol. 32, pp. 726-740.
- 10 [13] Hamilton, C., Pardoen, G., Navalpakkam, S. and Kanzasjy, R. "Reinforced concrete bridge
11 column performance enhancement through shotcrete jacketing," *ACI Structural Journal*, 2004,
12 Vol. 101, No. 3, pp. 332-340.
- 13 [14] Souza, R.H.F. and Appleton, J. "Flexural Behavior of Strengthened Reinforced Concrete
14 Beams," *Materials and Structures*, 1997, Vol. 30, pp. 154-159.
- 15 [15] Cheong, H.K. and MacAlevey, N. "Experimental Behavior of Jacketed Reinforced Concrete
16 Beams," *Journal of Structural Engineering – ASCE*, 2000, Vol. 126, No. 6, pp. 692-699.
- 17 [16] Alhadid, M.M. and Youssef, M.A. "Analysis of reinforced concrete beams strengthened using
18 concrete jackets," *Engineering Structures*, Elsevier, 2017, Vol. 132, pp. 172-187.
- 19 [17] Scott, B.D., Park, R. and Priestley, M.J.N. "Stress-Strain Behavior of Concrete Confined by
20 Overlapping Hoops at Low and High Strain Rates," *Journal of the American Concrete Institute*,
21 1982, Vol. 79, No. 1, pp. 13-27.

- 1 [18] Karthik, M.M and Mander, J.B. “Stress-block parameters for unconfined and confined
2 concrete based on a unified stress-strain model,” Journal of Structural Engineering – ASCE, 2011,
3 Vol. 137, No. 2, pp. 270-273.
- 4 [19] Ramberg, W. and Osgood, W.R. “Description of stress–strain curves by three
5 parameters,” Technical Note No. 902, National Advisory Committee for Aeronautics, 1943,
6 Washington DC.
- 7 [20] Thermou, G.E., Pantazopoulou, S.J. and Elnashai, A.S. “Flexural behavior of brittle RC
8 members rehabilitated with concrete jacketing,” Journal of Structural Engineering, ASCE, 2007,
9 Vol. 133, No. 10, pp. 1373-1384.
- 10 [21] CSA. “Design of concrete structures (CAN/CSA A23.3-14),” Cement Association of Canada,
11 2014, Ottawa, ON.
- 12 [22] Tsioulou, O.T. and Dritsos, S.E. “A theoretical model to predict interface slip due to bending,”
13 Materials and Structures, 2011, Vol. 44, pp. 825-843.
- 14 [23] Kotsira, E., Dritsos, S., Pilakoutas, K. “Effectiveness of techniques for flexural repair and
15 strengthening of RC members,” Proceedings of the 5th international conference on Structural faults
16 and repair, 1993, Edinburgh, UK, pp. 235-243.
- 17 [24] Saiidi, M., Vrontinos, S. and Douglas, B. “Model for the response of reinforced concrete
18 beams strengthened by concrete overlays,” ACI Structural Journal, 1990, Vol. 87, No. 6, pp. 687-
19 695.
- 20 [25] Youssef, M.A. and Rahman, M. “Simplified seismic modeling of reinforced concrete flexural
21 members,” Magazine of Concrete Research, 2007, Vol. 59, No. 9, pp. 639-649.
- 22 [26] Kassimali, A. “Matrix Analysis of Structures,” Thomson-Engineering, 2nd ed., 2011, 640 pp.

- 1 [27] Oehlers, D.J., Haskett, M., Mohamed, M.S. and Griffith, M.C. “Moment Redistribution in
- 2 Reinforced Concrete Beams,” Proceedings of the Institution of Civil Engineers, Structures and
- 3 Buildings 163, June 2010, pp. 165-176.
- 4 [28] ACI Committee 318. “Building code requirements for structural concrete and commentary
- 5 (ACI 318-14),” American Concrete Institute, 2014, Farmington Hills, MI.