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Evaluating the Vulnerability of Time-Sensitive Transportation Networks: A Hub Center Interdiction Problem

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Abstract: Time-sensitive transportation systems have received increasing research attention recently. Examples of time-sensitive networks include those of perishable goods, high-value commodity, and express delivery. Much research has been devoted to optimally locating key facilities such as transportation hubs to minimize transit time. However, there is a lack of research attention to the reliability and vulnerability of time-sensitive transportation networks. Such issues cannot be ignored as facilities can be lost due to reasons such as extreme weather, equipment malfunction, and even intentional attacks. This paper proposes a hub interdiction center (HIC) model for evaluating the vulnerability of time-sensitive hub-and-spoke networks under disruptions. The model identifies the set of hub facilities whose loss will lead to the greatest increase in the worst-case transit time. From a planning perspective, such hubs are critical facilities that should be protected or enhanced by preventive measures. An efficient integer linear programming (ILP) formulation of the new model is developed. Computational experiments on a widely used US air passenger dataset show that losing a small number of hub facilities can double the maximum transit time.

Keywords: location–allocation model; time-sensitive transportation; system vulnerability

1. Introduction

A new trend of transportation since the turn of the century is the increasing emphasis on high value, time sensitive goods. The value of goods transported is projected to increase twice as fast as the tonnage from 2002 to 2035 [1]. Examples of high value/time-sensitive goods include perishable goods in the “cold chain” transportation, pharmaceutical supplies, electronics, and priority mails, among others. In order to reduce the costs associated with holding high-value goods in warehouses or in transit and to maximize responsiveness, many industries have adopted just-in-time delivery operations. Modern economy is becoming increasingly dependent on fast transportation systems.

Over the past three decades, researchers in transportation science [2,3], industrial engineering, operational research [4–6], and geography [7,8] have studied the transportation networks extensively using the hub-and-spoke models. Hub-and-spoke systems involve the movement of commodities (or people) from origins to destinations, routed through intermediate facilities called the hubs. Many time-sensitive transportation services, such as FedEx and air passenger transport, use global and regional hub facilities.

Multiple models of facility location have been proposed to optimize the operation of hub-and-spoke networks. From the works of O’Kelly [2] and Campbell [4,5], one of the most widely used models for hub location is the hub-median location problem. The goal of the model is to minimize the total transportation cost for moving commodities from origins to destinations. A discount factor is used to reflect the economy of scale associated with a larger or faster vehicle in inter-hub movement.

The hub median location problem, however, is not suitable for time-sensitive networks. Consider for example, time-sensitive networks such as FedEx. The maximum transportation time from origins to destinations represents the best time guarantee that a transport provider can offer to its customers [9], and therefore should be kept at the minimum. In cold-chain transportation for perishable food and horticultural products, long transit time from origins to destinations translates to costly reduction in shelf-time and deterioration of the quality of the commodity. In these cases, transit time for each customer is critical, and a hub median location problem is not suitable because it may prescribe a hub layout with excessively long transit times for certain individual customers to achieve lower average time. Traditionally, location in time-sensitive hub-and-spoke networks is analyzed using the hub center location problems [5], which are aimed at minimizing the greatest transportation time among all customers.

The difference between the hub median and hub center criteria can be illustrated in Figure 1 with the Civil Aeronautics Board (CAB) data of US air passenger transport widely used in the hub location literature. In the figure, 25 major US air passenger hubs are labeled with their three-letter IATA (International Air Transport Association) codes (see Table 1 for the names and IATA codes for the 25 cities). For comparison purposes, the demand of movement for each origin–destination pair is set to one in both the hub median and hub center models. It is assumed that the inter-hub transit time is discounted by a constant factor (0.6 in this example), as is with the majority of the hub location literature. Assuming that five hubs are to be established, the set of hubs that renders the smallest total transportation cost is [Phoenix—PHX, Pittsburgh—PIT, St. Louis—STL, Seattle—SEA, Tampa—TPA] with an average route cost of 948.6 miles (which can be easily translated into transit time based on speeds). The worst case route for this particular hub layout is the one from San Francisco (SFO) to Boston (BOS) via Phoenix (PHX) and Pittsburgh (PIT), with a combined length of 2244.8 miles. In contrast, for the p -hub center location problem, the optimal 5-hub configuration is [Atlanta—ATL, New York—JFK, PHX, SFO, SEA]. The worst-case route for this hub configuration is from Detroit (DTW) to Los Angeles (LAX) via a first hub at Atlanta (ATL) and a second hub at Phoenix (PHX), with a cost of 1916.2 miles. This cost is significantly (15%) lower than the maximum cost of routes for the hub median solution and it is the minimum possible for all 5-hub locations.

Table 1. City names and IATA (International Air Transport Association) codes for the Civil Aeronautics Board (CAB) dataset.

IATA Code	City	IATA Code	City	IATA Code	City
ATL	Atlanta	HOU	Houston	PHX	Phoenix
BWI	Baltimore	MCI	Kansas City	PIT	Pittsburgh
BOS	Boston	LAX	Los Angeles	STL	St. Louis
ORD	Chicago	MEM	Memphis	SFO	San Francisco
CVG	Cincinnati	MIA	Miami	SEA	Seattle
CLE	Cleveland	MSP	Minneapolis	TPA	Tampa
DFW	Dallas-Fort Worth	MSY	New Orleans	DCA	Washington DC
DEN	Denver	JFK	New York		
DTW	Detroit	PHL	Philadelphia		



Figure 1. Comparing hub median and center location models using the Civil Aeronautics Board (CAB) US Airport Dataset.

In time-sensitive transportation networks, the worst-case transit time is not only a function of hub locations, but also a function of service disruptions of the hub network. The hub facilities in such networks may be disrupted by extreme weather, equipment malfunctioning, congestion, and other unforeseeable factors. Such disruptions may render the promised longest transit time of the network operator infeasible, and should not be ignored in the design of the network. The goal of this paper is to analyze the vulnerability of time-sensitive networks. It is aimed at answering the following question: Given an existing time-sensitive hub-and-spoke network with p hub facilities, which hubs, if lost, will lead to the greatest maximum route time. The question is important in planning because knowledge about service disruption and critical facilities is crucial for developing contingency plans and promoting disaster preparedness [10]. The question also reflects the perspective of sustainable planning. As pointed out by Litman and Burwell [11], “sustainability planning is to development what preventive medicine is to health: It anticipates and manages problems rather than waiting for crises to develop”.

The rest of the paper is organized as follows. The next section provides a brief review of the related literature on hub location, system vulnerability and critical infrastructure. Section 3 presents an integer linear programming (ILP) formulation of the proposed hub interdiction center problem. Section 4 presents computational results based on the widely used CAB dataset (in Figure 1) as well as a larger 40 node dataset of US air passenger transport. Section 5 concludes with a summary of findings and points out possible future work.

2. Background

Even though early development in hub location research can be dated back to works of Goldman [12] and Marks [13], hub location did not become a distinct research area in location science until the works of O’Kelly [2,7]. Since then, numerous research articles about hub location problems appeared (see e.g., recent reviews by Alumur and Kara [6] and by Campbell and O’Kelly [3]). Two special journal issues in *Location Science* (October 1996) and in *Computers and Operations Research* (2009) are devoted to hub location research, respectively. The expansion of hub location research was motivated by a number of historical developments in the 1970s and 1980s [3]. The deregulation of transportation in the United States in the 1970s led to a focus by transportation firms in all modes (air passenger, air cargo, and trucking) on large-scale networks and optimization of their operations. The emergence of express delivery services like FedEx is another source of hub location research.

Despite the abundance of research papers, existing models in the hub location literature generally fall into several categories. An important distinction between hub location models is whether the “single-allocation” assumption is made. In a single-allocation hub location model, each origin (or destination resp.) node can be only connected to one intermediate hub regardless of the destination (or origin resp.) of the flow. First made in O’Kelly [2], this assumption is suitable when it is costly or difficult to establish multiple spoke links connecting an origin (or destination) to hub facilities. In contrast, in a multiple allocation model, flows from an origin can be routed through multiple hubs depending on the destination of a trip. O’Kelly and Miller [14] further proposed an eight-category classification of hub location models based on whether the single-allocation or multiple-allocation assumption is made; whether the network between hubs is a complete graph; and whether direct origin-to-destination movements are allowed.

From a different perspective, hub location problems can be classified according to the criteria used in choosing location patterns. This classification closely parallels a well-known classification of central facility location models into four basic models including the p -median problem, the p -center problem, coverage location models, and the simple plant location problem. Campbell [5] first explored the linkage between hub location and central facility location models, and developed integer programming formulations of the p -hub median problem, the hub center problems, hub covering problems, and the hub location problem with fixed costs. The p -hub median model discussed in Campbell [5] makes the multiple-allocation assumption, rather than the single-allocation assumption as in O’Kelly [2]. In the same article, Campbell also presented an integer programming formulation of three hub center location problems. As mentioned earlier, a p -hub center location problem differs from a p -hub median location problem in that rather than minimizing the average transportation cost, the p -hub center location problem minimizes the worst-case route cost among all origin–destination pairs. The p -hub center location problems are suitable for time-sensitive delivery networks where the best guarantee of delivery time is important. Alumur and Kara [6] provided a comprehensive review containing over 100 references on hub location research with a discussion of formulations and solution methods for hub location problems. As noted by Alumur and Kara, the majority of the hub location research deals with p -hub median location problems. Relative few papers have studied p -hub center location models. This is an area worthy of further investigation and will be reviewed briefly in the sequel.

Kara and Tansel [15] developed three different ILP formulations of the single-allocation p -hub center problem, and found computationally that one of the formulations to be consistently superior. Ernst et al. [16] proposed a new ILP formulation for the single-allocation p -hub center problem involving a new decision variable representing the maximum collection and distribution cost between a hub and the origin/destination nodes it is connected to. Through computational experiment using IBM ILOG CPLEX, Ernst et al. showed that their formulation took shorter computation time than that of Kara and Tansel [15]. Alumur, Kara, and Karasan [17] relaxed the assumption that the subgraph connecting the hub nodes is complete and proposed a single-allocation p -hub center problem with incomplete networks.

The time-sensitive aspect of hub center location problems is analyzed in detail in Kara and Tansel [18]. Kara and Tansel stated that the transient time spent at unloading, loading, and sorting operations at hub facilities may be significant, and proposed the latest arrival hub location problem to account for these elements. They focused on the single-allocation minimax version of the new problem. When the objective function only depends on maximum travel time, the latest arrival hub location problem was shown to be equivalent to the p -hub center location problem [19].

Campbell [5] presented a comprehensive set of integer programming formulations of the hub-center problems including a quadratic programming formulation of the multi-allocation p -hub center problem. Ernst et al. [16] studied the p -hub center location problems with either single-allocation or multiple allocation. Extending Campbell [5], they developed mixed integer linear programming (MILP) formulations of the multi-allocation p -hub center problem. Their first formulation uses path variables and similar constraints as in the ILP formulations of multi-allocation p -hub median problems (see e.g., [5,20]). With a slight change of variable names (to be consistent with Campbell [5]), their formulation is as follows:

$$\text{Minimize } z, \quad (1)$$

s.t.

$$z \geq \sum_{i \in I} \sum_{j \in J} x_{ijkm} (d_{ik} + \alpha \cdot d_{km} + d_{mj}) \quad \text{for each } i \in I, j \in J, \quad (2)$$

$$\sum_{k \in F} \sum_{m \in F} x_{ijkm} = 1 \quad \text{for each } i \in I, j \in J, \quad (3)$$

$$\sum_{k \in F} x_{ijkm} \leq y_m \quad \text{for each } i, j \in I, m \in H, \quad (4)$$

$$\sum_{m \in F} x_{ijkm} \leq y_k \quad \text{for each } i, j \in I, k \in H, \quad (5)$$

$$\sum_k y_k = p, \quad (6)$$

$$y_k \in \{0, 1\} \quad \text{for each } k \in H, \quad (7)$$

$$0 \leq x_{ijkm} \leq 1, \quad (8)$$

where I and i are the set and index of origins, respectively. J and j are the set and index of destinations, respectively. H is the set of candidate locations for hub facilities. d_{ij} is the cost of movement between locations i and j . α ($0 \leq \alpha < 1$) is a time discount factor that reflects higher speed of movement on links between hubs. For a route from i to j via hubs k and m , x_{ijkm} is a path variable, and $x_{ijkm} = 1$ if the flow from origin i to destination j is routed via hubs k and m , or 0 otherwise. y_k is the location variable, and $y_k = 1$, if a hub is located at site k , or 0 otherwise. Ernst et al. [16] also presented an alternative formulation of the multi-allocation p -hub center problem based on three-indexed variables, but they found it to be less effective computationally.

There are other types of models for analyzing hub locations. For example, Campbell et al. [9] (among others) have studied the allocation problem for the p -hub center problems in which the hub locations are given. They showed that certain special cases are polynomial-time solvable. Sasaki et al. [21] considered a restricted multi-allocation hub location problem in which each route can use at most one hub. They termed their model the one-stop multi-allocation median problem and presented an MILP formulation of the model. It is out of the scope of this paper to provide a comprehensive review of the hub location literature. The reader may refer to O'Kelly and Miller [14], Klincewicz [22], Alumur and Kara [6], and Campbell and O'Kelly [3] for a comprehensive review of the different types of hub location problems.

A related area of literature for this paper concerns the analysis of uncertainty and vulnerability of service and distribution systems. There are different kinds of uncertainties in hub-and-spoke systems and in general service systems. In the context of hub location, Alumur, Nickel, and

Saldanha-da-Gama [23] address the uncertainty of input data, including setup costs and demands to be transported. Sim, Lowe, and Thomas [24] detailed a hub covering model, which considers normally distributed random travel times along arcs.

While the uncertainty in data can cause inaccurate estimates of system performance, a more serious source of uncertainty pertains to system vulnerability and whether the hub facilities or the links connecting hubs and demands are available for service. In the context of the Internet hub network, O’Kelly et al. [25] proposed a framework to estimate the reliability of a hub-and-spoke network. The reliability of an O–D (origin–destination) path was shown to be the sum of the non-failure probabilities for each route from the origin to the destination in question. The authors computed the reliability by a combinatorial enumeration of all cases of link failures. Kim and O’Kelly [26] proposed the reliable p -hub location problem telecommunication networks that aims at maximizing O–D connectivity. Further, Kim [27] defines a p -hub protection problem for communication networks involving location of additional backup hubs to increase overall O–D reliability. He also developed an extended model, in which the total length of any O–D route is restricted.

Related to this work also is a class of network interdiction problems that analyze the vulnerability of a service system in terms of the impact of losing network nodes or links. These models often assume an (often imaginary) attacker, which tries to interdict network elements to maximally reduce system performance. This can involve removing network links to reduce the capacity of a flow network [28–31] or increasing the shortest path length from an origin to a destination [32,33]. A recent development in this line of research is the r -interdiction median (RIM) problem by Church, Scaparra, and Middleton [10]. The RIM model aims at identifying the set of critical facilities in an existing central facility system that, if lost, would lead to maximal total weighted transportation cost. Since the RIM model involves an objective function that maximizes total transportation cost, Church, Scaparra, and Middleton [10] found it necessary to employ a construct called the closest assignment (CA) constraint [34] to ensure that each demand node should be serviced by (or assigned to) its closest facility rather than the farthest. Extending the RIM model in the context of hub-and-spoke networks, Lei [35] developed a hub interdiction median (HIM) problem, which identifies critical hub facilities in terms of average transportation cost. The HIM model, however, cannot be applied to time sensitive networks due to its “maxisum” performance metric.

From the above review, we could observe that there is a lack of research in the existing literature regarding the vulnerability of time-sensitive hub-and-spoke networks. The hub interdiction center (HIC) problem proposed in this paper is inspired by the RIM model [10] and employs a hub center metric, which is naturally suitable for evaluating the vulnerability time-sensitive hub-and-spoke networks.

3. Model Formulation

Formally, the hub interdiction center (HIC) problem can be described as follows:

Find the set of r hubs in a time-sensitive hub-and-spoke network with p hubs that, if lost, results in the longest worst-case transit time.

To formulate the HIC problem, the following notation in addition to those in the previous section is required:

F is the set of existing hubs;

M is a sufficiently large constant;

$d_{ijkm} = d_{ik} + \alpha d_{km} + d_{mj}$ is the total cost for a route i – k – m – j . The two indices k and m may refer to the same hub, in which case $d_{km} = 0$.

$C_{ijkm} = \{(q, v) | d_{ijqv} < d_{ijkm} \text{ or } d_{ijqv} = d_{ijkm} \text{ and } (q < k \text{ or } q = k \text{ and } v < m)\}$.

For a given route i – k – m – j , the above set notation describes the relative order of costs among the routes originated from i and destined to j . It consists of all pairs of intermediate hubs that incur strictly lower route costs than a given route i – k – m – j . If there is a tie in costs between two routes, the route with lower indices for the hubs is considered better. The decision variables are:

$y_k = 1$, if hub k is not lost (or “interdicted”, from the perspective of an imaginary attacker) and 0 otherwise;

$x_{ijkm} = 1$, if the demand of movement from i to j is routed via hubs k and m , and 0 otherwise;

z_{ij} is the time cost for the route from origin i to destination j ;

u_{ij} is a binary choice variable, which is 1 if the route time from i to j is the maximum among all origin–destination pairs, and 0 otherwise.

The hub interdiction center (HIC) problem can be formulated now as follows:

$$\text{Maximize } \sum_{i \in I} \sum_{j \in J} z_{ij}, \quad (9)$$

subject to:

$$z_{ij} \leq \sum_{k \in F} \sum_{m \in F} d_{ijkm} x_{ijkm} \quad \text{for each } i \in I, j \in J, \quad (10)$$

$$z_{ij} \leq M \cdot u_{ij} \quad \text{for each } i \in I, j \in J, \quad (11)$$

$$\sum_{i \in I} \sum_{j \in J} u_{ij} = 1, \quad (12)$$

$$\sum_{k \in F} \sum_{m \in F} x_{ijkm} = 1 \quad \text{for each } i \in I, j \in J, \quad (13)$$

$$\sum_{k \in F} x_{ijkm} \leq y_m \quad \text{for each } i, j \in I, m \in F, \quad (14)$$

$$\sum_{m \in F} x_{ijkm} \leq y_k \quad \text{for each } i, j \in I, k \in F, \quad (15)$$

$$\sum_{k \in F} y_k = p - r, \quad (16)$$

$$\sum_{(q,v) \in C_{ijkm}} x_{ijqv} + x_{ijkm} \geq y_k + y_m - 1 \quad \text{for each } i \in I, j \in J, k, m \in F, \quad (17)$$

$$y_k \in \{0, 1\} \quad \text{for each } k \in F, \quad (18)$$

$$u_{ij} \in \{0, 1\} \quad \text{for each } i \in I, j \in J, \quad (19)$$

$$0 \leq x_{ijkm} \leq 1. \quad (20)$$

In the above formulation, z_{ij} represents the smallest transit time for routes from i to j if this transit time happens to be the largest among all pairs of origin–destinations; z_{ij} is zero otherwise. Objective function (9) maximizes the largest route time. Constraints (10) maintain that z_{ij} should be the shortest transit time among all routes originated from i and destined to j . Constraints (11) link z_{ij} to the “choice” variable u_{ij} defined above and maintain that z_{ij} must be zero if u_{ij} is zero. Constraints (10) and (11) together linearize the non-linear relation $z_{ij} = u_{ij} \cdot \sum_{k \in F} \sum_{m \in F} d_{ijkm} x_{ijkm}$. Equation (12) defines the choice variable u_{ij} and requires that exactly one O–D pair should be selected in computing the objective value. The maximization objective function (9) ensures that the maximum route time will be actually included in the optimal objective value. Constraints (13) are assignment constraints and maintain that the route from i to j can only be assigned to (or go through) one pair of (possibly identical) hubs. Constraints (14) and (15) link the assignment variables x_{ijkm} to the location variables y_k , and maintain that a route can only go through hubs that are not lost. Constraint (16) maintains that $p-r$ hubs are left after losing r of the p existing hubs. Constraints (17) maintain that x_{ijkm} equals one if and only if $i-k-m-j$ is the fastest route from i to j and hubs k and m are intact. More specifically, they state that if no route has a smaller time cost than route $i-k-m-j$ ($\sum_{(q,s) \in C_{ijkm}} x_{ijqs} = 0$) and if neither k nor m is lost ($y_k + y_m - 1 = 1$), then x_{ijkm} must be one. Constraints (17) are the so-called multi-stage closest

assignment (MSCA) constraints [35,36]. This constraint form is necessary in the above formulation because the hub center interdiction problem has a “maximax” objective function, which is the opposite of minimax objective function of the p -hub center problem. Without the closest assignment constraints, the hub center interdiction model will choose the assignment variable x_{ijkm} such that it is one for the longest-time route from i to j .

In the above formulation, it should be noted that the assignment variable x_{ijkm} is defined as a continuous decision variable (20) instead of a binary variable in the original problem description. This is possible because, as will be shown shortly, the MSCA constraints (17) and the assignment constraints (13) can force the assignment variables x_{ijkm} to integer values. Due to this important property, the number of binary variables is reduced from $O(n^2m^2)$ to $O(n^2)$, where $n = |I|, m = |F|$, yielding a compact formulation. Here, we provide a (simple) proof of this property.

Observation 1. *The MSCA constraints (17) and the assignment constraints (13), when combined, suffice to force assignment variables to integer values, if the location variables are binary.*

Proof: For any O–D pair (i, j) and the associated fastest route $i-j-a-b$ (via located hubs), we have $y_a = y_b = 1$, and $C_{ijab} = \varphi$.

Since no route has a lower time cost. Therefore (17) reduces to $x_{ijab} \geq y_a + y_b - 1 = 1$, or $x_{ijab} = 1$ considering the domain of assignment variables.

Now, by (13), for all *other* intermediate hubs $(k, m) \neq (a, b)$, we have:

$$x_{ijkm} = 0.$$

Therefore, all assignment variables are integers. \square

Of note, is that there are different ways of formulating closest assignment (CA) constraints. Certain forms may not enforce integrality of the assignment variable. Constraints (17) are based on an efficient formulation of CA due to Church and Cohon [37]. The interested reader is referred to Gerrard and Church [38] for a detailed review and comparison of single stage CA constraints and the properties of each CA constraint form.

4. Computational Experiments

The computational performance of the HIC model was tested on the widely used 25-node CAB dataset depicted in Figure 1. The CAB dataset was based on a Civil Aeronautics Board survey in the 1970s and contains air passenger travel demand data for 25 major US cities. Distance between each pair of cities was measured in miles. The computational tests were performed on a desktop computer with an Intel i7-2600 CPU at 3.4 GHz and 32 gigabyte of system memory. The ILP formulation for the HIC model in (9) through (20) was implemented in the OPL (Optimization Programming Language) algebraic modeling language in IBM ILOG CPLEX Studio 12.6.0. For comparison purposes, the parallel execution mode for CPLEX has been disabled and CPLEX is allowed to use one thread only in the solution process.

Computational results are presented in Table 2. In the experiment, we had tested five located hubs at [ATL, JFK, PHX, SFO, SEA] (i.e., $p = 5$). This configuration was an optimal solution to the multi-allocation p -hub center problem with $\alpha = 0.6$ (Figure 1). We tested the loss of $r = 0, 1, 2$, and 3 hubs. However, the HIC model could be easily applied to other values of p and r . The $r = 0$ case represents a base-case scenario, which is degenerate and represents the original p -hub center problem solution with no hub failures. Following prior work on p -hub center problem in the literature [15,16], we tested a range of α values, including $\alpha = 0.2, 0.4, 0.6, 0.8$, and 1.0. When $\alpha = 1.0$, the inter-hub movement speed is not discounted; otherwise, inter-hub movement is assumed to be faster than spoke movement. When divided by appropriate speeds, the objective function represents the worst-case maximum transit time after the loss of r hubs. Column 2 in Table 2 tabulates the optimal objective

function values for each value r of lost hubs in Column 1. Column 5 tabulates for each r , the critical set of hubs that if lost, lead to the worst-case scenario. For example, when $\alpha = 1.0$ and $r = 3$, the worst case route cost was 5229.62 miles, associated with the loss of hubs at JFK (New York), ATL (Atlanta), and PHX (Phoenix).

Table 2. Hub interdiction center (HIC) results on the CAB dataset, $p = 5$.

r	Critical Route Cost (mi)	Computation Time (s)	Nodes in B&B	Critical Hubs
$\alpha = 0.2$				
0	1820.24	0.06	0	[]
1	2515.35	0.84	0	[ATL]
2	4598.86	23.72	306	[ATL, JFK]
3	5229.62	17.6	78	[ATL, JFK, PHX]
$\alpha = 0.4$				
0	1874.16	0.05	0	[]
1	2549.13	0.79	0	[ATL]
2	4598.86	28.89	497	[ATL, JFK]
3	5229.62	18.79	217	[ATL, JFK, PHX]
$\alpha = 0.6$				
0	1916.16	0.062	0	[]
1	2583.17	0.671	0	[SEA]
2	4598.86	40.57	744	[ATL, JFK]
3	5229.62	29.11	340	[ATL, JFK, PHX]
$\alpha = 0.8$				
0	2340.09	0.047	0	[]
1	2781.78	0.702	0	[SEA]
2	4598.86	42.05	2622	[ATL, JFK]
3	5229.62	46.35	2171	[ATL, JFK, PHX]
$\alpha = 1.0$				
0	2725.79	0.046	0	[]
1	2781.78	0.374	0	[SEA]
2	4598.86	35.07	875	[ATL, JFK]
3	5229.62	69.12	4630	[ATL, JFK, PHX]

Note: The set of existing hub facilities is [ATL, JFK, PHX, SFO, SEA]. It is an optimal solution to the p -hub center location problem with $\alpha = 0.6$.

From Column 5, one could observe that the critical hub sets for different values of α are the same in cases where the number of interdicted hubs was relatively high (e.g., when $r = 3$), but varied in other cases (e.g., when $r = 1$). This is because when the number of interdicted hubs is high, only a small number of hubs remain intact and there are fewer ways to choose which hubs to leave intact. For example, when $r = 3$ ($p = 5$), only two hubs SFO (San Francisco) and SEA (Seattle) were left. The remaining sites were concentrated on the western seaboard of the US. This pattern makes sense because it makes routes from and to the eastern half of the country circuitous. For the $r = 3$ case, the worst-case (i.e., longest) route for all α values is from Boston (BOS) to a single hub at Seattle (SEA) and then to Miami (MIA). Since there is only one hub on the route, the discount factor α does not really apply. In general, when r value is higher, the interdiction results are less sensitive to α values.

For the $\alpha = 1.0$ case, the worst-case maximum transit time for losing two hubs is almost twice as large as the normal operations case when no hub is lost. This represents a serious degradation of service level, in terms of the best time guarantee that can be offered to customers in a time-sensitive network. However, when one hub is lost, the maximum transit time is essentially unchanged (increased by only about 2%). This reveals that in the original optimal 5-hub center configuration, the hub facility at SEA

(Seattle) is not cost-effective, if the inter-hub transportation speed is not discounted. When the discount factor α comes into play, the transit times between origin and destination change. Consequently, the impact of losing one hub facility on route cost is no longer negligible. For example, when $\alpha = 0.6$, route cost could increase by 35% from 1916.16 miles to 2583.17 when Seattle is lost.

Solution times are reported in Column 3 and range from a fraction of a second to about one minute. The number of nodes in the branch and bound process in CPLEX is tabulated in Column 4. They are generally low for $r = 1$ or 2 and higher for $r = 2$ or 3, indicating the efficiency and integer-friendliness of the proposed formulation.

To test the efficacy of the proposed model, we also tested the performance of the model on a larger 40-city US airport data [35], which is a superset of the CAB data. Distances between airports are great circle arc distances measured in kilometers. The set of existing hubs is [BDL, BOI, LAS, LIT, PIT, TPA], which is an optimal p -hub center problem solution on the 40-city dataset. As shown in Column 3 of Table 3, all problem instances can be solved within 24 min.

Table 3. Hub interdiction center (HIC) results on the 40-airport dataset, $p = 6$.

r	Critical Route Cost (km)	Computation Time (s)	Nodes in B&B	Critical Hubs
$\alpha = 0.2$				
0	2370	0.18	0	[]
1	3168	6.41	0	[LIT]
2	5730	1364.32	1347	[BOI, LAS]
3	7236	683.12	412	[BOI, LAS, LIT]
$\alpha = 0.4$				
0	2628.6	0.18	0	[]
1	3358.6	6.7	0	[BOI]
2	5730	1346.5	1553	[BOI, LAS]
3	7236	902.47	698	[BOI, LAS, LIT]
$\alpha = 0.6$				
0	3049	0.23	0	[]
1	3778.8	6.18	0	[BOI]
2	5730	892.93	3409	[BOI, LAS]
3	7236	1172.53	1092	[BOI, LAS, LIT]
$\alpha = 0.8$				
0	3761.4	0.26	0	[]
1	4143.6	5.45	0	[BOI]
2	5730	947.93	4012	[BOI, LAS]
3	7236	1435.69	1515	[BOI, LAS, LIT]
$\alpha = 1.0$				
0	4380	0.26	0	[]
1	4415	3.69	0	[BDL]
2	5730	994.92	3892	[BOI, LAS]
3	7236	1478.1	1640	[BOI, LAS, LIT]

* Note: The set of existing hub facilities is [BDL, BOI, LAS, LIT, PIT, TPA]. It is an optimal solution to the p -hub center location problem with $\alpha = 0.6$.

From Table 3, we can observe similar optimal interdiction patterns (i.e., worst case facility loss scenarios). When $r = 3$ ($p = 6$), the worst case scenario is the same for all tested α values. The remaining hubs are Pittsburgh (PIT), Bradley (BDL), and Tampa (TPA), which are all located near the east coast of the US, making routes from and to the western half of the country costly. As with the case of the CAB experiment, losing a small number of hubs can result in serious service degradation. For example, when $\alpha = 0.6$, losing two hubs can lead to a 88% increase of route cost; when $\alpha = 0.4$, losing the same

number of hubs can lead to a 118% increase of cost, which more than doubled the original cost under normal operations.

From the experiments, the proposed model is computationally effective and can be solved in a reasonable amount of time for all tested problem instances. In addition, the model is able to reveal the worst case network scenarios of facility loss that make sense in operations. In several occasions, the greatest disruption is associated with loss of hub facilities in a concentrated area forcing massive re-routing for origins and destinations. Overall, the results show that the loss of a small number of hub facilities can lead to significant increase in the maximum transit time, which in some cases more than doubles the designed level. Clearly, the effect of such events cannot be ignored given the possibility of service disruptions.

5. Conclusions

Time-sensitive hub networks are important in many transportation and logistics systems. Past research has primarily focused on the location of hub facilities to minimize the maximum transit time. The reliability of such networks has not received sufficient research attention. To address this shortcoming, this paper presents a new model for analyzing the vulnerability for time-sensitive distribution networks. The proposed hub interdiction center (HIC) model is based on the multi-allocation p -center hub location model, and can be used to uncover the worst-case scenarios for losing a certain number of hubs.

A preliminary analysis shows that identifying the worst case route for a fixed O–D pair is non-trivial technically. The multi-stage closest assignment (MSCA) constraints are required in developing a mixed integer linear programming (MILP) formulation of the HIC problem. Computational results showed that the formulation could be used to optimally solve the new model on the widely used CAB dataset as well as a larger dataset of US air passenger flows. Computational results showed that for a hub-and-spoke system with five hubs, the loss of two hubs could lead to an increase of more than 100% of maximum transit time, which is a serious degradation of service level for the hub-and-spoke network.

From a planning perspective, the hub interdiction center problem can help a transportation analyst identify the set of critical hub facilities in a time-sensitive delivery network to avoid such serious degradation of service. Planners may take preventive measures to keep critical facilities from simultaneously failing. This includes allocating additional personnel, adding equipment, and adding backup hub facilities near critical hubs.

Meanwhile, planners can also use the HIC model to test the cost-effectiveness of an existing time-sensitive network. If the loss of certain hub facilities in an existing system does not cause significant impact on the longest route time of the network, they may be re-allocated to other locations (or even removed).

Several areas are worthy of future investigation. First of all, the experiments in this paper have only considered the impact of facility loss on delivery time. For the transport of high-value and perishable goods, it is possible to consider the value of carried goods and model impact of lost goods due to delayed arrival. It is also possible to evaluate the impact of facility loss based on the monetary cost of late delivery and no-delivery instead of delivery time. This is left as future work.

Computationally, while the new model can be solved on medium-sized problems for all the tested cases, it is certainly worthwhile for future research to develop specialized algorithms or heuristics that can be used to solve problems faster in real time or near real time. Although the HIC model is posed for the basic multi-allocation p -center model, it is also worthwhile to investigate the vulnerability of other time-sensitive distribution systems such as single-allocation hub-and-spoke systems.

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