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QUASI-VISCO-ELASTO-PLASTIC CONSTITUTIVE MODEL OF CONCRETE FOR FATIGUE SIMULATION

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Abstract: A simple constitutive model, quasi-visco-elasto-plastic constitutive model, was proposed considering viscoelasticity and viscoplasticity for fatigue analysis. Applicability to the concrete and RC member was investigated by introducing into three-dimensional Rigid Body Spring Model. As a result, the proposed model could simulate fatigue behaviors of both concrete and RC members although fatigue strengths were underestimated. The effectiveness of the discrete analysis with proposed model was also shown by evaluating of the fatigue failure mechanisms of RC members.

1 INTRODUCTION

Fatigue is one of the severe problems for concrete structures. Many researchers have studied fatigue problems such as fatigue strength by experimental manner. Nowadays, influential factor on fatigue of concrete has been revealed from those studies. However, damage progress and mechanisms of fatigue failure are difficult to be understand from experiments because of the difficulty of measurement.

Recently, fatigue analysis considering damage accumulation into constitutive model of concrete has also been conducted by Finite Element analysis [1]. By this approach, fatigue strength not only for material level but also for RC member could be simulated. On the other hand, discrete modelling based on the mesoscopic features of concrete has also been developed for fatigue analysis [2]. This model considered viscoelasticity and viscoplasticity in constitutive model and damage progress in the specimen could be clarified. Analysis based on discrete model is effective for fatigue analysis because fatigue failure usually governed by accumulating microcracking.

In this study, a simplified constitutive model of concrete to simulate fatigue behaviour based on new concept, quasi-visco-elasto-plastic modelling was proposed. In this model, the features of both viscoelasticity and viscoplasticity are simply and indirectly modelled in static nonlinear constitutive model. Then, the proposed constitutive model introduced into Rigid Body Spring Model (RBSM), one of the discrete models, and fatigue analyses for concrete were conducted. Moreover, applicability of the model was investigated to simulate the RC beams failing in flexure and shear were conducted.

2 ANALYTICAL METHOD

2.1 3D-RBSM

Three-dimensional RBSM (3D-RBSM) by Yamamoto et al. was used in this study [3]. In this model, all element was represented as rigid body and 6 degrees of freedom, 3 translational and 3 rotational, were defined at the center of gravity of each element. Internal forces were calculated on the boundary of 2 elements, where stresses were obtained by setting the 1 normal spring and 2 shear springs at integration point. Note that the boundary surface was divided into several triangles and 1 integration point was set on each triangle in order to represent moment of force without any rotational spring.

Characteristics of constitutive model in 3D-RBSM is that fracture of concrete is expressed by combination of tensile failure in normal spring and shear failure in shear springs and never compressive failure in normal spring. That is, compressive failure such as in concrete cylinders at the macroscopic level results from combination failure of tension and shear. More details can be found in reference [3].

2.2 Influence of viscoelasticity and viscoplasticity on behavior in constitutive level

As described above, fracture of concrete in 3D-RBSM results from failure of tension and shear springs. Especially, softening behavior in shear spring had much effect on the damage progress and plastic deformation of concrete under compression from the results of preliminary analyses. Therefore, modification of constitutive model in shear spring is effective to simulate fatigue behavior of concrete.

Firstly, we tried to investigate the behavior in constitutive level when viscoelasticity and viscoplasticity were introduced into shear springs in series as shown in **Figure 1**. Here, elastoplasticity represents original nonlinear stress-strain relationship proposed by Yamamoto et al. [3].

Total strain in the shear spring consists of 3 components, elastoplastic strain, viscoelastic strain and viscoplastic strain. Viscoelasticity was assumed by Kelvin-Voigt material, in which a spring and a dashpot were set in parallel. While viscoplasticity was assumed by parallel model with a dashpot and a slider. Hooke's law was assumed in the spring and constitutive equations for the dashpot expressed as Newtonian fluid in both viscoelasticity and viscoplasticity were assumed that the stress depends on the strain rate. Rigid plasticity was assumed in the slider.

Cyclic loading was conducted by considering certain strain rate corresponding to one in the fatigue loading for cylindrical specimen. **Figure 2** shows one example of shear stress-strain relationship. As shown in **Figure 2**, a small difference could be distinguished in descending branch and reloading path. That is, both slopes obtained from the analyses modelling viscoelasticity and viscoplasticity became more gentle compare to the one of original constitutive model. According to these results, it is clearly thought that the difference comes from the influences of viscoelasticity and viscoplasticity.

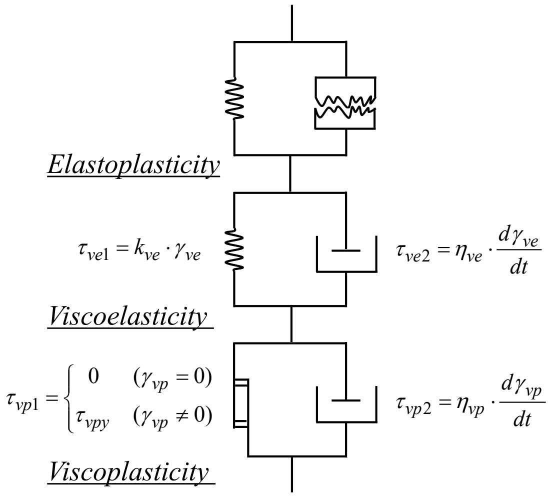

Figure 1: Series model of inelasticity, viscoelasticity and viscoplasticity in shear springs.

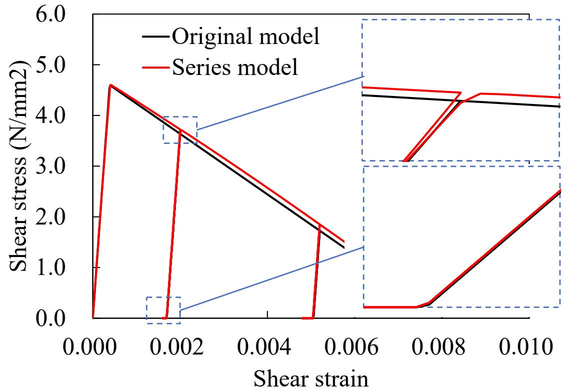

Figure 2: Comparison of shear stress-strain relationship.

2.3 Quasi-visco-elasto-plastic constitutive model

Generally, a huge time is necessary for the analysis considering both viscoelasticity and viscoplasticity because satisfying equilibrium becomes difficult and iteration process also becomes complex. The idea of quasi-visco-elasto-plastic constitutive model proposed in this study is quite simple. The macro response of shear spring, in which elastoplasticity, viscoelasticity and viscoplasticity are in series, is directly modelled as a static constitutive model. **Figure 3** shows the overview of the idea of proposed model. In this model, only reloading slope becomes small by decreasing rate α, which depends on the magnitude of maximum shear strain. This modelling also can be regarded as the deterioration of aggregate interlocking on the crack surface due to cyclic loading.

Decreasing rate α was assumed as the inverse trigonometric function shown in following equation and **Figure 4**.

$$\alpha = -\frac{1}{\pi}\arctan\left(\frac{\gamma_{\max} - 0.0029}{0.000035}\right) + 0.5038 \quad (1)$$

where, $\gamma_{\max}$ represents maximum shear strain in each shear spring.

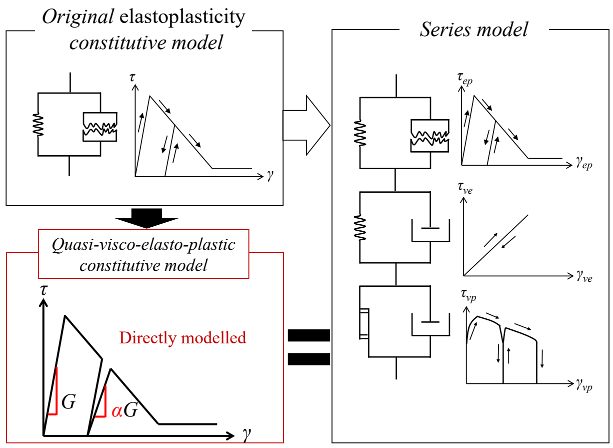

Figure 3: Overview of the idea of quasi-visco-elasto-plastic constitutive model.

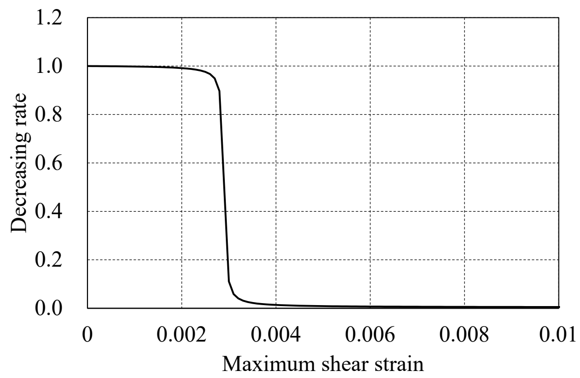

Figure 4: Relationship between decreasing rate and maximum shear strain.

3 FATIGUE SIMULATION OF CONCRETE UNDER COMPRESSION

3.1 Outline of analysis

Fatigue analysis for concrete subjected to cyclic compressive loading was conducted in order to evaluate the validity of proposed model. **Figure 4** shows the analytical model of the cylinder specimen of concrete with a diameter of 100mm and a height of 200mm. Average element size was about 10mm and total number of elements was 1215.

Material properties of concrete were set as shown in **Table 1**. Elastic modulus, E_c, compressive strength, f'_c, tensile strength, f_t, and fracture energy, G_F, were 24.4kN/mm^2, 32.8N/mm^2, 2.36N/mm^2 and 86.8N/m, respectively. Material parameters of constitutive model used in the analysis were set as the same with literature [3].

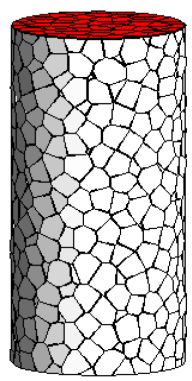

Figure 5: Cylinder specimen of concrete.

Table 1: Material properties of concrete

E_c (kN/mm^2)	f'_c (N/mm^2)	f_t (N/mm^2)	G_F (N/m)
24.4	32.8	2.36	86.8

Cyclic loading was applied by load control method. Upper load limit was set to the 90, 80, 70 and 60% of the maximum load in monotonic loading and unchanged during each analysis. Lower load limit was set to the 5% of the maximum load and the same in all cases. Modified Newton-Raphson method was adopted in iterative process. Force equilibrium was judged by an index which was the ratio of norm of residual force to norm of internal force, and its threshold was set to 0.01%. If the iterative number exceeded 50 times before satisfying equilibrium, residual force was carried over to next step. Fatigue failure was defined if the index value was larger than 2.0% at the upper load limit when the iterative number was 50 times.

3.2 Analytical results

Figure 6 shows stress-strain relationships of cyclic and monotonic loading. In this figure, dotted line represents result of monotonic loading and solid lines represent result of cyclic loading. We can see that stiffness decrease and residual strain increase as the cyclic number increase. The strain at the failure tend to be small when the upper load limit become large. These tendencies correspond with common experimental result.

Figure 7 shows S-N diagram obtained from cyclic loading analysis. In this figure, white circles represent the results of proposed model and black circles represent the results when the decreasing rate α kept constant value, 0.99. The solid line represents the fatigue strength curve proposed by Antrim and McLaughlin [4]. Fatigue strength obtained from both the analyses underestimate than fatigue strength curve especially in low upper load limit. However, proposed model can predict more long fatigue life compare to the case with constant decreasing rate in low upper load limit and slope is closer to the curve. The improvement of the model is necessary in order to evaluate the fatigue strength in low upper load limit more accurately.

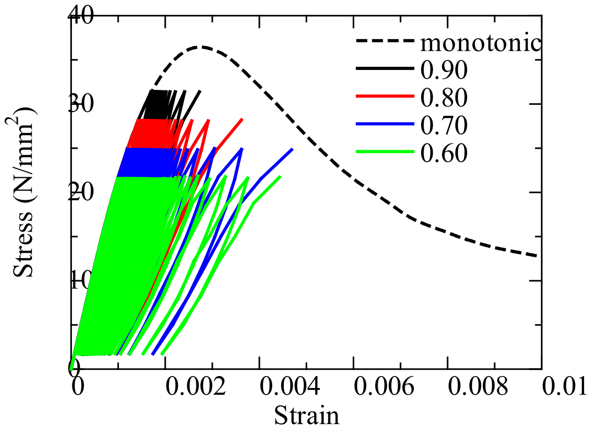

Figure 6: Stress-strain relationships of cylinder specimens.

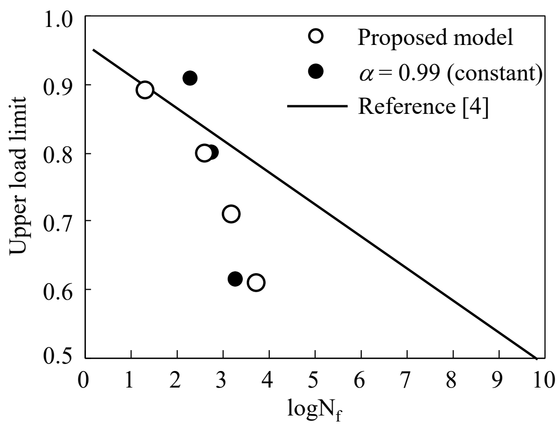

Figure 7: S-N diagram of cylinder specimens.

4 FATIGUE SIMULATION OF RC BEAM

4.1 Outline of analysis

Fatigue analysis for the RC beams tested by Inoue [5] was conducted. Two types of specimens with different failure modes, flexural failure and shear failure, were selected for the analysis. **Figure 8** shows outline of the specimens. Both specimens have same dimension, a width of 125mm and a height of 200mm. An effective depth of 165mm was also the same. Reinforcement ratio was 4.91% and shear span ratio was 3.03 for flexural specimen. This specimen designed to fail before yield of reinforcement. On the other hand, reinforcement ratio was 2.71% and shear span ratio was 2.0 for shear specimen. Stirrups were arranged in shear span only for flexural specimen, the ratio was 2.04%. Compressive strength of concrete for flexural and shear specimens were 54.5N/mm^2 and 56.7N/mm^2, respectively. **Figure 9** shows analytical mesh and rebar arrangement for flexural and shear specimens. Average element size was about 20mm and total number of elements was 4053.

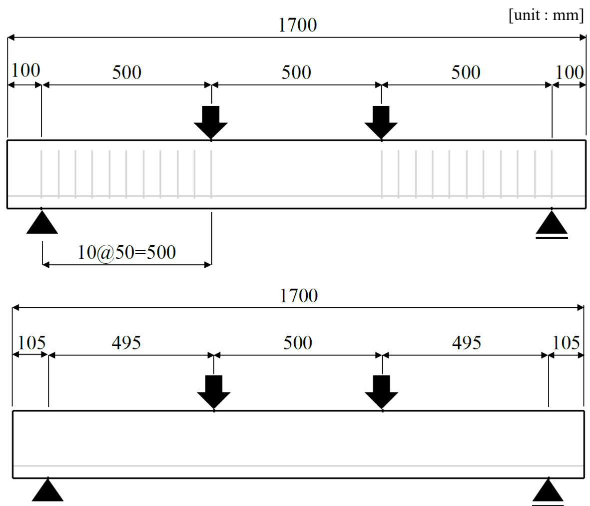

Figure 8: Outline of RC beams (upper: flexural specimen, lower: shear specimen).

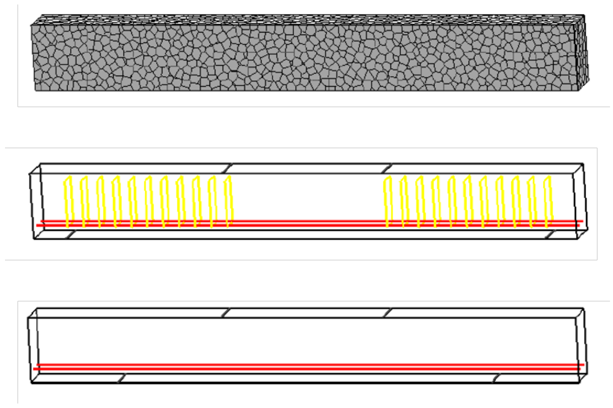

Figure 9: Outline of analytical mesh (upper) and rebar arrangements (middle: flexural specimen, lower: shear specimen).

Analytical conditions such as loading method, iterative algorithm, maximum iterative number, threshold of iteration and definition of failure, were set as the same with cyclic loading of concrete described in previous chapter. Upper load limit was set to the 90, 80 and 70% of the maximum load in monotonic loading. Lower load limit was set to the 10% of the maximum load and the same in all cases.

4.2 Fatigue lives of RC members

Figure 10 and **Figure 11** show S-N diagrams for flexural specimens and shear specimens, respectively. In these figures, analytical results are plotted as circle and experimental results are plotted as cross mark. For the flexural specimens, tendency of fatigue life with changing of upper limit load was different between the proposed model and the constant decreasing rate. This tendency is similar with the result of the analysis for cylinder specimen. It is thought that the reason why is that the failure of flexural specimen was governed by the compressive failure of concrete. The slope obtained from the analysis with proposed model is almost the same with the one of experiment although fatigue lives were small.

On the other hand, for the shear specimens, there were not notable difference between the proposed model and the constant decreasing rate. This is because the shear failure was governed by cracking. Fatigue life obtained by the experiment were quite scattered and it was difficult to evaluate the validity of the proposed model properly.

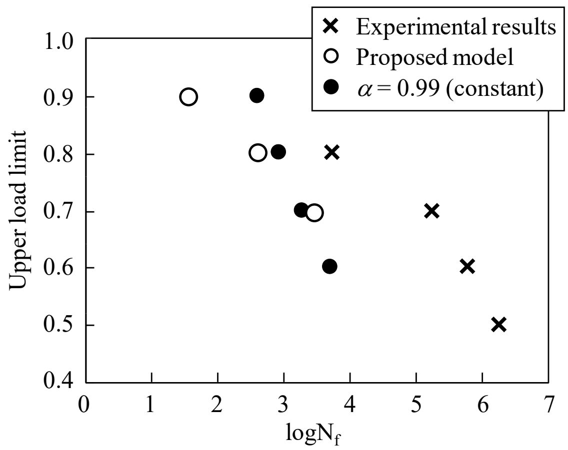

Figure 10: S-N diagram for flexural specimens.

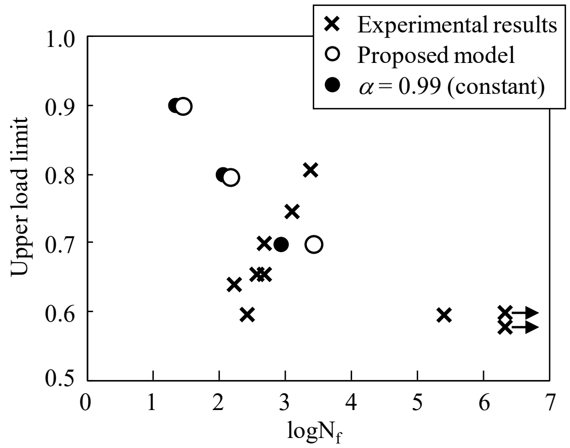

Figure 11: S-N diagram for shear specimens.

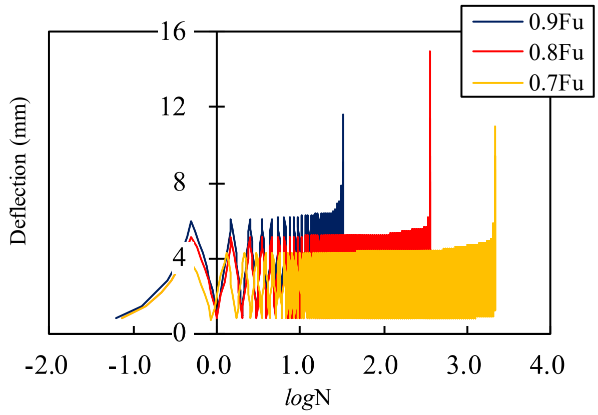

Figure 12: Deflection changes of flexural specimens under cyclic loading.

4.3 Deflection changes during cyclic loading

Figure 12 and **Figure 13** show relationships between cyclic number and deflection of RC beams for flexural specimens and shear specimens, respectively. For the flexural specimens, the amplitude of deflection kept the same during cyclic loading and dominantly become large just before failure. This tendency is obtained irrespective of magnitude of the upper load limit. On the other hand, for the shear specimens, different behaviors compared to the flexural specimens were obtained. That is, the deflection gradually became large by an increase of cyclic number and the amplitude also became large. The case of large upper load limit showed this behavior more apparently.

Figure 14 shows the crack patterns at the lower load limit in each cycle obtained from the case of upper load limit of 90%. The colors represent the crack width and it becomes large in the order blue, yellow and red. Diagonal cracks already appeared, and they became large due to the cyclic loading. That is, damage accumulated in the diagonal cracks. Finally, diagonal cracks opened, and fatigue failure occurred. This behavior suggests why the deflection became large during cyclic loading in the shear specimens.

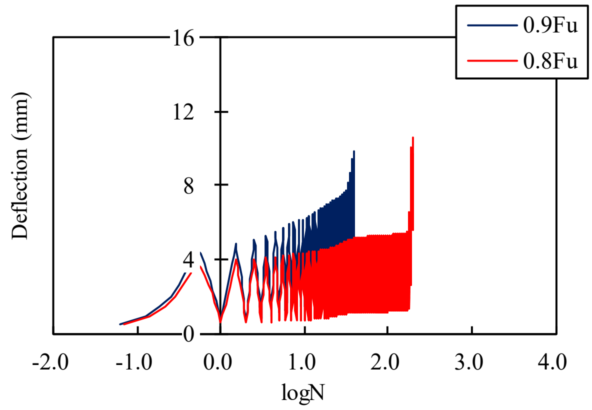

Figure 13: Deflection changes of shear specimens under cyclic loading.

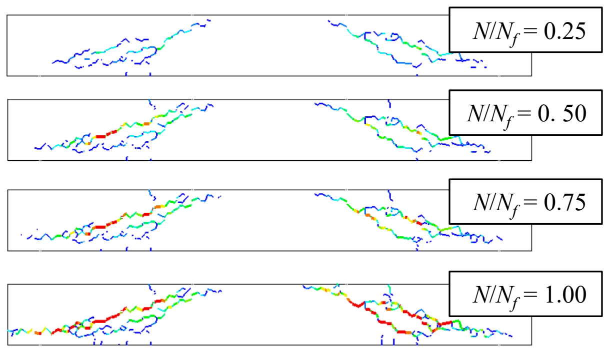

Figure 14: Crack patterns in shear specimen for the case of upper load limit of 90%.

5 CONCLUSIONS

The authors proposed a quasi-visco-elasto-plastic constitutive model, which was a static constitutive model based on the macroscopic behavior considering viscoelasticity and viscoplasticity. The fatigue simulations with the proposed model by 3D-RBSM were conducted and the validity of the model was investigated. Although fatigue strength of concrete and RC structures were underestimated, tendency of fatigue curves was similar with the experimental ones. The effectiveness of the model on the fatigue simulation of RC members was also shown. It was confirmed that crack progress and damage process obtained visibly is useful to understand the mechanisms of fatigue failure of RC members.

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