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Relativistic dissipative spin dynamics in the relaxation time approximation



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ABSTRACT

The concept of the Wigner function is used to construct a semi-classical kinetic theory describing the evolution of the axial current phase-space density of spin-1/2 particles in the relaxation time approximation. The resulting approach can be used to study spin polarization effects in relativistic matter, in particular, in heavy-ion collisions. An expression for the axial current based on the classical treatment of spin is also introduced and we show that it is consistent with earlier calculations using Wigner functions. Finally, we derive non-equilibrium corrections to the spin tensor, which are used to define, for the first time, the structure of spin transport coefficients in relativistic matter.

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1. Introduction

The spin polarization of various particles (Λ , K^* , ϕ) produced in relativistic heavy-ion collisions has been recently observed by the STAR experiment at the Brookhaven National Laboratory (BNL) [1–3]. In the case of the Λ hyperons, a quite substantial global polarization (of about 10%, due to the spin-orbit coupling) was theoretically foreseen in Refs. [4–6]. However, a smaller polarization (of about 1%, due to equilibration of spin degrees of freedom) was proposed later in Refs. [7–10] and such an effect was eventually observed by STAR [1,2]. Being the first experimental observation of a non-zero spin polarization in heavy-ion collisions, it has been commonly interpreted as one of the greatest discoveries in physics in 2017 [1].

While the global spin polarization of the Λ (and $\bar{\Lambda}$) hyperons can be explained by the assumption that the spin polarization is directly expressed by the so-called thermal vorticity [7–12], other features of the data lack convincing theoretical explanations [13, 14]. The interpretation problems appear also in the case of the K^* and ϕ mesons [15]. This situation has triggered many theoretical studies, for example, see [16–23].

The problems described above suggest that the spin effects in heavy-ion collisions can be independent of the thermal vorticity and governed by other type of dynamics. In Ref. [24], a hydrodynamic framework for particles with spin 1/2 was proposed, where the spin polarization is described by the spin polarization tensor $\omega_{\mu\nu}(x)$, whose dynamics follows from the conservation law for the angular momentum – similarly to the evolution of the local temperature $T(x)$ and the hydrodynamic flow vector $u^\mu(x)$ that follow from the conservation laws for the energy and linear momentum. Recently, this framework (with the updated forms of the energy-momentum and spin tensors, which have been related to the underlying Lagrangian of the Dirac field) has been derived from the kinetic theory [25] (for a recent review see [26]).

In this work, we go beyond the perfect-fluid setup constructed in [24]. We first introduce semi-classical kinetic equations describing the evolution of the Wigner function of massive spin-1/2 particles in the relaxation-time approximation (RTA) [27,28]. Then, we switch to the framework based on the classical description of spin. It has been shown in [26] that the classical approach is advantageous as it is not restricted to the case of small spin polarization. At the same time, it reduces to the Wigner-function approach if polarization is small. Moreover, it can be used to determine the structure of dissipative spin corrections in a completely analogous way to that known from the standard RTA.

Although the kinetic description of matter based on the RTA method may seem to be oversimplified, in the last years this

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method has turned out to be a very useful tool to address numerous physics problems, often of fundamental importance like testing applicability of hydrodynamics [29,30] or early thermalization and/or hydrodynamization puzzles [31–33]. Therefore, we propose here a version of the RTA, for particles with spin, which can play a very similar role in extended studies involving spin as an additional macroscopic degree of freedom.

In order to obtain a clear physics picture, our considerations are restricted to hydrodynamics and kinetic theory of spin-1/2, massive particles being on the mass shell. Their polarization is described by the axial current density that can be directly used to define the spin density matrix [34,35]. For the metric tensor, Levi-Civita symbol, and the scalar product, we use the following notation and conventions: $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$, $\epsilon^{0123} = -\epsilon_{0123} = 1$, $a \cdot b = g_{\mu\nu} a^\mu b^\nu = a^0 b^0 - \mathbf{a} \cdot \mathbf{b}$. Throughout the text we use natural units with $c = \hbar = k_B = 1$.

2. Kinetic equations for scalar and axial-vector components of the Wigner function

The studies of relativistic plasma for particles with spin 1/2 commonly use the Wigner function $\mathcal{W}(x, k)$ and its Clifford-algebra decomposition [36–41],

$$\mathcal{W}(x, k) = \frac{1}{4} \left[\mathcal{F}(x, k) + i\gamma_5 \mathcal{P}(x, k) + \gamma^\mu \mathcal{V}_\mu(x, k) + \gamma_5 \gamma^\mu \mathcal{A}_\mu(x, k) + \Sigma^{\mu\nu} \mathcal{S}_{\mu\nu}(x, k) \right]. \quad (1)$$

Here x is the space-time coordinate and $k^\mu = (k^0, \mathbf{k})$ denotes the particle momentum. The coefficient functions appearing on the right-hand side of Eq. (1) are sums of the particle and antiparticle contributions, for example, $\mathcal{F}(x, k) = \mathcal{F}^+(x, k) + \mathcal{F}^-(x, k)$ and $\mathcal{A}_\mu(x, k) = \mathcal{A}_\mu^+(x, k) + \mathcal{A}_\mu^-(x, k)$. We use the Dirac representation for gamma matrices with $\Sigma^{\mu\nu} = (i/4)[\gamma^\mu, \gamma^\nu]$ being the Dirac spin operator.

From the leading and next-to-leading orders of the semiclassical expansion of $\mathcal{W}(x, k)$ in powers of \hbar , one obtains two independent kinetic equations, for the scalar and axial-vector components [36–40],

$$k^\mu \partial_\mu \mathcal{F}(x, k) = C_{\mathcal{F}}, \quad (2)$$

$$k^\mu \partial_\mu \mathcal{A}^\nu(x, k) = C_{\mathcal{A}}^\nu, \quad k_\nu \mathcal{A}^\nu(x, k) = k_\nu C_{\mathcal{A}}^\nu = 0. \quad (3)$$

Here, we have neglected the effects of the mean fields (which are widely discussed in the literature) and schematically included complicated effects of collisions by adding collision terms on the right-hand sides of Eqs. (2) and (3).¹

If the collision terms vanish, Eqs. (2) and (3) describe free streaming of particles. There are also two other cases that can be analyzed with the help of those equations, namely, the global and local thermodynamic equilibrium. This requires, however, the knowledge of the Wigner function in a local thermodynamic equilibrium which can be generally expressed in terms of the scalar, \mathcal{F}_{eq} , and the axial-vector components, $\mathcal{A}_{\text{eq}}^\mu$ [25,26,42]. Recently, several works have used this concept to express the equilibrium Wigner function as a phase space integral [9,25,26,43]

$$\mathcal{F}_{\text{eq}}^\pm = 2m \int dP f_{\text{eq}}^\pm(x, p) \delta^{(4)}(k \mp p), \quad (4)$$

$$\mathcal{A}_{\text{eq}, \mu}^\pm = - \int dP \tilde{\omega}_{\mu\nu}(x) p^\nu f_{\text{eq}}^\pm(x, p) \delta^{(4)}(k \mp p), \quad (5)$$

¹ We stress that the coefficient functions \mathcal{F} , \mathcal{A}^ν , as well as the collision terms $C_{\mathcal{F}}$ and $C_{\mathcal{A}}^\nu$ are all of the same (leading) order in \hbar . The functions \mathcal{F} , \mathcal{A}^ν are also the only two independent coefficients of the Wigner function (1) within this approximation, hence, Eqs. (2) and (3) fully determine the evolution of the Wigner function provided the form of the collision terms is defined.

where the local equilibrium distributions in the classical (Boltzmann) approximation are given by the expression

$$f_{\text{eq}}^\pm(x, p) = \exp[-\beta(x) \cdot p \pm \xi(x)]. \quad (6)$$

Here $\beta^\mu = u^\mu/T$, where u^μ is the hydrodynamic flow vector and T is the local temperature, $\xi = \mu/T$ is the ratio of the chemical potential and temperature, $p^\mu = (E_p, \mathbf{p})$, and $dP = d^3p/E_p(2\pi)^3$, with $E_p = \sqrt{m^2 + \mathbf{p}^2}$ denoting the on-mass-shell particle energy. The quantity $\omega_{\mu\nu}$ ($\tilde{\omega}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\omega^{\alpha\beta}$) is the spin polarization tensor (dual spin polarization tensor). The former can be interpreted as the ratio of the spin chemical potential $\Omega_{\mu\nu}$ and the local temperature T , namely $\omega_{\mu\nu} = \Omega_{\mu\nu}/T$ [16]. We note that $\omega_{\mu\nu}$ as well as $\tilde{\omega}_{\mu\nu}$ are dimensionless quantities in our approach that play a similar role as ξ . The formalism presented here is valid for small values of $\omega_{\mu\nu}$ (i.e., for $\omega_{\mu\nu} < 1$), for a discussion of this point see Sec. 6.6 of Ref. [26].

3. RTA for the scalar component

We start our considerations with the scalar component that has been analyzed in many papers and its treatment is very well established now. The RTA collision term in the equation for the scalar coefficient takes the following form

$$C_{\mathcal{F}} = k \cdot u \frac{\mathcal{F}_{\text{eq}}(x, k) - \mathcal{F}(x, k)}{\tau_{\text{eq}}}, \quad (7)$$

where τ_{eq} is the relaxation time. With the definition

$$\mathcal{F}^\pm(x, k) = 2m \int dP f^\pm(x, p) \delta^{(4)}(k \mp p) \quad (8)$$

one finds

$$p^\mu \partial_\mu f^\pm(x, p) = p \cdot u \frac{f_{\text{eq}}^\pm(x, p) - f^\pm(x, p)}{\tau_{\text{eq}}}, \quad (9)$$

which is the Anderson-Witting model for the RTA [44].

In order to switch from the microscopic kinetic theory to the effective hydrodynamic description one takes the moments of Eq. (2). The zeroth and the first moments correspond then to the conservation laws for charge, energy, and linear momentum

$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0. \quad (10)$$

Here $N^\mu = \int dP p^\mu (f^+ - f^-)$ and, similarly, $T^{\mu\nu} = \int dP p^\mu p^\nu (f^+ + f^-)$. The necessary condition for the conservation laws (10) to be valid is that the appropriate moments of the collision term vanish. This leads to so-called Landau matching conditions:

$$u_\mu N_{\text{eq}}^\mu = u_\mu N^\mu, \quad u_\mu T_{\text{eq}}^{\mu\nu} = u_\mu T^{\mu\nu}. \quad (11)$$

These relations are used to determine the parameters appearing in the equilibrium distributions, namely T , u^μ , and μ .

4. RTA for the axial component

In this letter, we propose a natural generalization of the RTA approach that is applicable for the axial component,

$$C_{\mathcal{A}}^\nu = k \cdot u \frac{\mathcal{A}_{\text{eq}}^\nu(x, k) - \mathcal{A}^\nu(x, k)}{\tau_{\text{eq}}}. \quad (12)$$

With the definition

$$\mathcal{A}_{\pm}^\mu(x, k) = 2m \int dP a_{\pm}^\mu(x, p) \delta^{(4)}(k \mp p) \quad (13)$$

one finds

$$p^\mu \partial_\mu a_\pm^v(x, p) = p \cdot u \frac{a_{\pm \text{eq}}^v(x, p) - a_\pm^v(x, p)}{\tau_{\text{eq}}}, \quad (14)$$

where the local equilibrium distributions are defined as

$$a_{\pm \text{eq}}^v(x, p) = -\frac{1}{2m} \tilde{\omega}^{\nu\mu}(x) p_\mu f_{\text{eq}}^\pm(x, p). \quad (15)$$

The moments of the scalar equation (2) naturally lead to the conservation laws for charge, energy, and linear momentum. In the case of the axial equation (3), the situation is less obvious. Nevertheless, if the collisions are absent and no mean fields are present, then the axial equation describes free-streaming of spin degrees of freedom – the spin polarization tensor should be constant in this case.

This fact was known long time ago to be in contrast with the behavior of the canonical spin tensor [45–47]. The latter is obtained by the Noether theorem and is known to be not conserved even for a free Dirac field. This difficulty was overcome by switching from the canonical forms of the energy-momentum and spin tensors to the GLW expressions (GLW stands here for de Groot, van Leeuwen, and van Weert of Ref. [47]).

In our case, i.e., for particles being on the mass shell (note the Dirac delta functions in Eqs. (4) and (5)), the connection between the axial current and the GLW spin tensor takes a particularly simple form

$$S^{\lambda, \mu\nu}(x) = \frac{1}{2} \varepsilon^{\alpha\beta\mu\nu} \int d^4k \frac{k^\lambda k_\beta}{m^2} \mathcal{A}_\alpha(x, k). \quad (16)$$

Multiplying Eq. (3) by $\varepsilon^{\nu\beta\gamma\delta} k_\beta/m^2$ and integrating over k , we find

$$\partial_\mu S^{\mu, \gamma\delta} = u_\mu \frac{S_{\text{eq}}^{\mu, \gamma\delta} - S^{\mu, \gamma\delta}}{\tau_{\text{eq}}}. \quad (17)$$

Hence, the requirement of vanishing divergence of the GLW spin tensor leads to a constraint

$$u_\mu S_{\text{eq}}^{\mu, \gamma\delta} = u_\mu S^{\mu, \gamma\delta}. \quad (18)$$

This formula represents an additional, Landau-type, matching condition for the equilibrium distribution function. It allows to determine six independent components of the spin polarization tensor $\omega_{\mu\nu}$ appearing in Eq. (15).

In view of the above discussion it seems natural to consider the conservation of the GLW spin tensor as a parallel condition to the conservation of charge, energy, and linear momentum. The canonical energy-momentum tensor is asymmetric, and differs from the GLW form by quantum corrections. The latter give rise to mixing between orbital and spin components of the canonical angular momentum. This phenomenon, known as the spin-orbit coupling, is absent in the GLW pseudo gauge provided the collision term is local and the mean fields are neglected [48].

5. Approach with classical spin

Equations (9) and (14), together with the Landau matching conditions (11) and (18), form a consistent system of kinetic equations that allows for determination of the space-time evolution of the phase-space distribution as well as spin densities. They are, however, valid only in the case of small polarization tensor. In order to overcome this restriction, one can switch to a classical description of spin degrees of freedom. We have shown in [26] that the results obtained with the Wigner function for small ω can be exactly reproduced in the framework with the classical treatment of spin [49]. An advantage of the classical treatment is, however, that it can be applied to systems with arbitrary polarization.

In the classical approach, we introduce the internal angular momentum of a particle, $s^{\mu\nu}$, and the spin vector, s^μ , connected by

the relation $s^\alpha = (1/2m)\varepsilon^{\alpha\beta\gamma\delta} p_\beta s_\gamma \delta$. Accordingly, one introduces a classical distribution function $f_s^\pm(x, p, s)$ in an extended phase space that besides space-time coordinates and momenta includes the spin vector. The appropriate normalization is

$$\int dS f^\pm(x, p, s) = f^\pm(x, p), \quad (19)$$

$$\int dS s_\mu f^\pm(x, p, s) = a_\mu^\pm(x, p), \quad (20)$$

where $dS = (m/\pi s) d^4s \delta(s \cdot s + s^2) \delta(p \cdot s)$ with the length of the spin vector defined by the eigenvalue of the Casimir operator, $s^2 = \frac{1}{2}(1 + \frac{1}{2}) = \frac{3}{4}$. One can easily check that Eqs. (9) and (14) can be obtained as the zeroth and first moments in the spin space of the classical RTA equation

$$p^\mu \partial_\mu f^\pm(x, p, s) = p \cdot u \frac{f_{\text{eq}}^\pm(x, p, s) - f^\pm(x, p, s)}{\tau_{\text{eq}}}, \quad (21)$$

where the equilibrium, spin-dependent function reads

$$f_{\text{eq}}^\pm(x, p, s) = f_{\text{eq}}^\pm(x, p) \exp\left(\frac{1}{2}\omega_{\mu\nu} s^{\mu\nu}\right). \quad (22)$$

Moreover, one can check for small values of ω that the formula (22), when used in Eq. (20), yields Eq. (15). Consequently, one can use Eq. (22) in Eq. (21), together with the Landau matching conditions (11) and (18), where

$$N^\mu = \int dP dS p^\mu (f_s^+ - f_s^-), \quad (23)$$

$$T^{\mu\nu} = \int dP dS p^\mu p^\nu (f_s^+ + f_s^-), \quad (24)$$

$$S^{\lambda, \mu\nu} = \int dP dS p^\lambda s^{\mu\nu} (f_s^+ + f_s^-). \quad (25)$$

In the above equation $f_s^\pm = f^\pm(x, p, s)$.

6. Dissipative corrections

It is straightforward to search for solutions of the transport equation (21) in a series form $f^\pm(x, p, s) = f_{\text{eq}}^\pm(x, p, s) + \delta f^\pm(x, p, s) + \dots$, which yields

$$\delta f^\pm(x, p, s) = -\frac{\tau_{\text{eq}}}{p \cdot u} p^\mu \partial_\mu f_{\text{eq}}^\pm(x, p, s). \quad (26)$$

One can define dissipative corrections to the conserved current $\delta N^\mu \equiv N^\mu - N_{\text{eq}}^\mu$, energy-momentum tensor $\delta T^{\mu\nu} \equiv T^{\mu\nu} - T_{\text{eq}}^{\mu\nu}$, and the spin tensor $\delta S^{\lambda, \mu\nu} \equiv S^{\lambda, \mu\nu} - S_{\text{eq}}^{\lambda, \mu\nu}$, in terms of the moments of $\delta f_s^\pm(x, p, s)$ used in Eq. (25).

After straightforward but quite lengthy calculations, one obtains the following expressions valid in the case of small polarization:

$$\delta N^\mu = \tau_{\text{eq}} \beta_n (\nabla^\mu \xi), \quad (27)$$

$$\delta T^{\mu\nu} = \tau_{\text{eq}} (-\beta_\Pi \Delta^{\mu\nu} \theta + 2\beta_\pi \sigma^{\mu\nu}), \quad (28)$$

$$\delta S^{\lambda, \mu\nu} = \tau_{\text{eq}} \left[B_\Pi^{\lambda, \mu\nu} \theta + B_n^{\kappa\lambda, \mu\nu} (\nabla_\kappa \xi) + B_\pi^{\alpha\kappa\lambda, \mu\nu} \sigma_{\alpha\kappa} + B_\Sigma^{\kappa\beta\alpha\lambda, \mu\nu} (\nabla_\kappa \omega_{\beta\alpha}) \right]. \quad (29)$$

Here $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, $\nabla^\mu = \Delta^{\mu\nu} \partial_\nu$, θ is the expansion scalar, and $\sigma_{\mu\nu}$ is the shear flow tensor. Different coefficients appearing on the right-hand side of Eq. (29) have tensor structures expressed in terms of equilibrium tensor quantities u^μ , $\Delta^{\mu\nu}$ and $\omega^{\mu\nu}$. In particular, the structure of the new spin coefficients $B_X^{\mu_1\mu_2\dots}$ appearing in Eq. (29) is as follows:

$$\begin{aligned}
B_{\Pi}^{\lambda,\mu\nu} &= B_{\Pi}^{(1)} u^{[\mu} \omega^{\nu]\lambda} + B_{\Pi}^{(2)} u^{\lambda} u^{\alpha} u^{[\mu} \omega^{\nu]\alpha} + B_{\Pi}^{(3)} \Delta^{\lambda[\mu} u_{\alpha} \omega^{\nu]\alpha}, \\
B_{\pi}^{\alpha\kappa\lambda,\mu\nu} &= B_{\pi}^{(1)} \Delta^{[\mu\kappa} \Delta^{\lambda\alpha} u_{\gamma} \omega^{\nu]\gamma} + B_{\pi}^{(2)} \Delta^{\lambda\alpha} u^{[\mu} \omega^{\nu]\kappa} \\
&\quad + B_{\pi}^{(3)} u^{[\mu} \Delta^{\nu]\alpha} \Delta^{\lambda\gamma} \omega^{\gamma\kappa} + B_{\pi}^{(4)} \Delta^{\lambda[\mu} \omega^{\rho\kappa} u_{\rho} \Delta^{\nu]\alpha}, \\
B_n^{\kappa\lambda,\mu\nu} &= B_n^{(1)} \Delta^{\lambda\kappa} \omega^{\mu\nu} + B_n^{(2)} \Delta^{\lambda\kappa} u^{\alpha} u^{[\mu} \omega^{\nu]\alpha} \\
&\quad + B_n^{(3)} \Delta^{\lambda\alpha} \Delta^{[\mu\kappa} \omega^{\nu]\alpha} + B_n^{(4)} u^{[\mu} \Delta^{\nu]\kappa} u^{\rho} \omega^{\lambda\rho} \\
&\quad + B_n^{(5)} \Delta^{\lambda[\mu} \omega^{\nu]\kappa} + B_n^{(6)} \Delta^{\lambda[\mu} u^{\nu]} u_{\alpha} \omega^{\alpha\kappa}, \\
B_{\Sigma}^{\kappa\beta\alpha\lambda,\mu\nu} &= B_{\Sigma}^{(1)} \Delta^{\lambda\kappa} g^{[\mu\beta} g^{\nu]\alpha} + B_{\Sigma}^{(2)} u^{\alpha} \Delta^{\lambda\kappa} u^{[\mu} \Delta^{\nu]\beta} \\
&\quad + B_{\Sigma}^{(3)} (\Delta^{\lambda\kappa} \Delta^{\alpha[\mu} g^{\nu]\beta} + \Delta^{\lambda\alpha} \Delta^{[\mu\kappa} g^{\nu]\beta} \\
&\quad + \Delta^{\alpha\kappa} \Delta^{\lambda[\mu} g^{\nu]\beta}) + B_{\Sigma}^{(4)} \Delta^{\alpha\kappa} \Delta^{\lambda[\mu} \Delta^{\nu]\beta} \\
&\quad + B_{\Sigma}^{(5)} u^{\alpha} \Delta^{\lambda\beta} u^{[\mu} \Delta^{\nu]\kappa}. \tag{30}
\end{aligned}$$

The explicit forms of the scalar coefficients $B_X^{(i)}$ (which are expressed by very lengthy expressions involving complicated integrals) are provided in Ref. [50].

It is important to emphasize that the assumption of small polarization does not introduce any new dissipative corrections to $\delta T^{\mu\nu}$ and δN^{μ} in Eqs. (27) and (28) and their forms remain unchanged, compared to the results obtained in the usual analysis of spinless systems. Moreover, we see that the first three terms in the expression for $\delta S^{\lambda,\mu\nu}$ in Eq. (29) arise from the same ‘‘thermodynamic forces’’ as dissipation in conserved current and energy momentum tensor. Very interestingly, the last term in Eq. (29) leads to a new type of dissipation which is proportional to the gradient of the spin polarization tensor $\omega^{\mu\nu}$.

We note that all the kinetic coefficients obtained from Eq. (21) are proportional to the relaxation time that is common for all of them. This means that the equilibration times for momenta and spin degrees of freedom are the same. In phenomenological applications it is conceivable to vary the values of the relaxation times that appear in different kinetic coefficients, arguing that they describe independent physical phenomena. In any case, such modifications require further studies.

7. Summary and conclusions

In this work we have constructed kinetic theory describing evolution of the axial-current phase-space density of spin-1/2 particles in the relaxation time approximation. Our approach is based on the conservation laws for energy, linear momentum, and angular momentum. Choosing a special pseudo-gauge (by adopting the GLW version), one can split the conservation of total angular momentum into separate conservation of the orbital and spin parts. This procedure leads to the conserved spin tensor. After performing the calculations in the GLW pseudo gauge, one can switch to the canonical gauge where only the total angular momentum is conserved. We note that the pseudo-gauge transformations do not change the form of the conservation laws for the energy-momentum and angular momentum tensors. Moreover, the integrated values of energy, linear momentum and angular momentum also remain unchanged, however, they differently allocate energy density and spin density. The obtained framework has been used to derive non-equilibrium corrections to the spin tensor, which are then used to define, for the first time, the structure of spin transport coefficients in relativistic matter.

Approaches based on the relaxation-time approximation are attractive, since they allow for a rather simple treatment of dissipative processes and very often allow for finding exact solutions. From this point of view, we expect that our formalism will be useful in studies of dissipative effects connected with spin. Clearly, more microscopic approaches are required to construct more realistic collision terms. Such problems are currently investigated by several groups, for example, in Refs. [22,51–54].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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