

# History relativism as extreme assessment relativism: A note on Prior's Ockhamism

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## Abstract

Since the early days of Ockhamist semantics, it has been recognized that the history-relative notion of truth which the theory postulates is problematic: it is unclear what it means that a sentence is true relative to a possible course of events; it is also unclear how such a notion of relative truth relates to the everyday notion of truth *simpliciter*. To rationalize the Ockhamist notion of truth I compare two relativistic theories: the assessment relativism of John MacFarlane and the history relativism of Belnap et al. In the end, I suggest that we may understand the history-relative notion of truth as the truth assessed relative to an end of time. On the formal level, I introduce a doomsday extension of a branching model and prove that history-relative truth in any given model is equivalent to doomsday-relative truth in the extended model. It turns out that this equivalence holds in general only if the end of time is also, in a sense, beyond time.

**Keywords:** Ockhamism, assessment relativism, branching time.

## 1 Introduction

With the publication of “Past, Present and Future” the branching-time model was incorporated into the mainstream of temporal logic. The

model introduced an important modal dimension to the discussion of time as it is based on the insight that the future, as opposed to the past, is open to multiple realizations. Nonetheless, Prior himself did not dwell on the metaphysical significance of the model because his main objective was to understand (and undermine) the main arguments for logical determinism. Thus, he immediately put the branching model to *semantic* work to show that some assumptions postulated by determinists are questionable and can be falsified in certain semantic theories. One of these theories is Ockhamism (this model helped Prior to show that not every sentence in the past tense truly concerns the settled past).

Ockhamist semantics is a simple and formally appealing theory that smoothly blends past, present, and future tenses with temporal possibility. However, there is certain controversy surrounding this semantics, as the basic Ockhamist notion of truth eludes clear comprehension [this concern was first clearly articulated in 11, pp. 270–1]. A few definitions are required to understand this problematic issue.

**Definition 1** (Branching Structure). Branching structure  $\mathfrak{B}$  is an ordered pair  $\langle M, < \rangle$ , where  $M \neq \emptyset$  and  $\leq$  is a partial order on  $M$  that satisfies the following conditions:

**Backward linearity**

$$\forall_{m_1, m_2, m_3} ((m_1 \leq m_3 \ \& \ m_2 \leq m_3) \Rightarrow (m_1 \leq m_2 \ \text{or} \ m_2 \leq m_1));$$

**Connectedness**  $\forall_{m_1, m_2} \exists_{m_3} (m_3 \leq m_1 \ \& \ m_3 \leq m_2)$ .

The structure represents all the possible ways the system (e.g. our world) might develop. Any one of the possible paths is called a *history* (it is a maximal, linearly ordered subset of  $M$ , sometimes also described as a “chronicle” or a “branch”).

To define an Ockhamist model based on a branching structure we use a very simple, sentential language containing operators of classical logic, two temporal operators, (‘it will be the case that’— $F$ —and ‘it was the case that’— $P$ ), and an operator of historical modality (‘It is settled that’— $\square$ )<sup>1</sup>.

Branching model  $\mathfrak{M}$ , based on a structure  $\mathfrak{B}$ , is a pair  $\mathfrak{M} := \langle \mathfrak{B}, V \rangle$ , where  $V$  is a valuation function which assigns a set of moments to every sentential constant,  $V: Atom \mapsto \wp(M)$ . In Ockhamism, sentences

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<sup>1</sup>The necessity operator is sometimes read as “it is inevitable that,” or “it is unpreventable that.”

are evaluated in a branching model at an *index* which contains two parameters: a *moment* parameter (shifted by temporal operators) and a *history* parameter (shifted by the modal operator). Consequently, sentences are evaluated at triples  $\langle \mathfrak{M}, m/h \rangle$ . The Ockhamist truth ( $\vDash$ ) of a sentence in a model at an index is inductively defined as follows:

**Definition 2** (Sentence  $\phi$  is true in model  $\mathfrak{M}$ , at index  $m/h$ ).

1. For  $p \in \text{Atom}$ ,  $\mathfrak{M}, m/h \vDash p$  iff  $m \in V(p)$ ;
2.  $\mathfrak{M}, m/h \vDash \neg\phi$  iff it is not the case that  $\mathfrak{M}, m/h \vDash \phi$ ;
3.  $\mathfrak{M}, m/h \vDash \phi \wedge \psi$  iff  $\mathfrak{M}, m/h \vDash \phi$  &  $\mathfrak{M}, m/h \vDash \psi$ ;
4.  $\mathfrak{M}, m/h \vDash P\phi$  iff  $\exists m' (m' < m \ \& \ \mathfrak{M}, m'/h \vDash \phi)$ ;
5.  $\mathfrak{M}, m/h \vDash F\phi$  iff  $\exists m' (m < m' \ \& \ m' \in h \ \& \ \mathfrak{M}, m'/h \vDash \phi)$ ;
6.  $\mathfrak{M}, m/h \vDash \Box\phi$  iff  $\forall h' (m \in h' \Rightarrow \mathfrak{M}, m/h' \vDash \phi)$ .

Importantly, in Ockhamist semantics the truth value of a sentence is relative to a modal parameter—a history. There is formally nothing wrong with such a relativization, but it creates an obstacle to the application of Ockhamism to its original purpose—analysis of future contingents. Let us take the sentence (S), “There will be a space battle in the 21st century,” ( $Fp$ ) as used during the NATO 2018 Summit in Brussels. How should we use Ockhamist semantics to evaluate this sentence? Well, we need to check if it is true at a moment/history pair. So far so good, but which exact moment and, more importantly, which history to use? After all, the Brussels Summit has many possible continuations. And so the trouble begins.

Our Ockhamist semantics gives us a definition of truth at a context and index (world/time pair) for arbitrary sentences in our language. But how can we move from this to the pragmatically relevant notion of truth at a context?

[6, pp. 207–208]

Thus, to apply Ockhamism, we need to somehow relate the pragmatically relevant notion of the truth of a sentence used at a particular

context to the technically relevant notion of the truth of a sentence evaluated at a semantic index. I use the symbol  $\Vdash$  to stand for the former notion of truth, and I use  $\models$  to stand for the latter. MacFarlane coined the term *postsemantics* to designate the theory whose job is to link the two notions of truth.

The easiest postsemantics simply identifies the truth at the context with the truth at the unique semantic index initialized by the context (we advocated this solution in Wawer and Malpass [12]; it is independently defended by Gallina [4]). Nonetheless, many branching theorists reject the easy solution. They argue that the context *does* designate a moment, but it *does not* designate a history [see especially 2, pp. 151–2, 231–3]. In their view, since the act of utterance is a part of many distinct histories, we cannot distinguish *the* history in which the utterance takes place and they conclude that the history parameter is not initialized by the context of use (see e.g. Belnap et al. [2, pp. 151–152, 163–164, 232–233]; John MacFarlane [5, p. 232]; [6, p. 208]; Tomasz Placek [9, p. 756]; or Thomas Müller [8, p. 350]). Therefore, they face what I call *an initialization failure*. Ockhamist semantics requires that the process of semantic evaluation begins at some specific index, but the context does not initialize the relevant index.<sup>2</sup>

Hence, the simple procedure does not work. According to many theorists the context of a sentence is not sufficient to designate the appropriate circumstance for the evaluation of the sentence. The content of the sentence does not seem to do the job either. The meaning of, “There will be a space battle in the 21st century,” does not indicate which history is being referred to. However, if neither the context nor the content initializes a history, then how shall one apply Ockhamist semantics?

Since the immediate route from the truth at a context to the truth at a semantic index is blocked, the authors need to find another, less direct way to relate the two notions of truth. Several postsemantic strategies have been proposed and I shall focus on two of them, both of whose distinctive feature is that they embrace a *relative* notion of truth. That is, I focus on the theories in which the meaning of the sentence supplemented by information provided by the context of use is not sufficient to assess the truth status of a sentence used at the context. According

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<sup>2</sup>I believe that the refusal to accept *the* history of the context partly results from specific metaphysical assumptions regarding the nature of the branching structure, but I set this issue aside (the issue is discussed in 12).

to these theories, the truth value of the sentence is *relative* to some extra factor.

## 2 Assessment relativism

MacFarlane thinks that the relevant additional factor is the context of assessment. He argues that the truth value of a sentence used at a given context can be determined only if we also take into account the context from which the truth value of the sentence is being assessed. A premonition of such an idea can be traced back to [11], in which it is suggested that:

[R]ather than making formulas true or false with respect only to the times at which they are true or false, we make their being true or false relative to subsequent times as well.  
[11, p. 268]

The idea was later revived by Nuel Belnap [1] under the name of “double-time reference.” (However, Belnap used the technique not to assess the truth value of a sentence, but to provide satisfaction conditions for assertions and other speech acts.) Finally, John MacFarlane used Belnap’s technical apparatus to formalize the double-relativized notion of truth in the form of “double-time reference postsemantics” [5, p. 331]. This postsemantics was later incorporated into a more general theory of assessment relativism [6].

The formal idea of assessment relativism, as applied to branching, is that when we assess the truth value of a sentence used at one context from the perspective of another context, we should check if the sentence assessed is true at the *context of use* with respect to the histories passing through *the context of assessment*. To state relativist postsemantics, we need an auxiliary notion of the set of histories passing through a moment:

**Definition 3.**  $H_m = \{h \mid m \in h\}$

and the set of histories passing through a pair of moments:

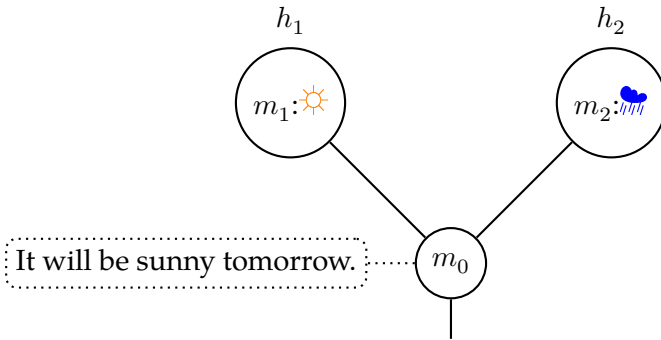
**Definition 4.**  $H_{m_1|m_2} = \begin{cases} H_{m_1} \cap H_{m_2}, & \text{if } m_1 \leq m_2, \\ H_{m_1}, & \text{otherwise.} \end{cases}$

We can now define assessment relativism as follows

**Definition 5.**  $m_u, m_a \Vdash^R \phi$  iff  $\mathfrak{M}, m_u/h \models \phi$  for every history  $h \in H_{m_u|m_a}$ .

A sentence is true at a pair of contexts  $m_u, m_a$  iff it is true at moment  $m_u$  in all histories passing through  $m_a$  (or all histories passing through  $m_u$  if  $m_u \not\leq m_a$ ). Assessment relativism truly deserves its name, since the very same sentence used in a single context can be true when assessed from one perspective, false when assessed from another perspective, and neither true nor false when assessed from still another perspective.

Let us study relativist postsemantics with a particular example:



In such a model:

- $m_0, m_0 \Vdash^R F_1(\text{sunny})$
- $m_0, m_1 \Vdash^R F_1(\text{sunny})$
- $m_0, m_0 \Vdash^R \neg F_1(\text{sunny})$
- $m_0, m_2 \Vdash^R \neg F_1(\text{sunny})$

So, at the pair of contexts  $m_0$  and  $m_0$ , the sentence “It will be sunny tomorrow” is neither true nor false: at  $m_0, m_1$  it is true, and at  $m_0, m_2$  it is false. MacFarlane is happy with these results as he believes that the relative truth fits well with our intuitive ascriptions of accuracy to assertions. He claims that each assertion of a future contingent should be judged to be inaccurate when assessed from the perspective of the context of use. Nonetheless, when the flow of time resolves the matter and confirms the previous prediction (i.e. the context of assessment changes), then the initial assertion should be judged accurate. Thus, MacFarlane finds intuitive support for his relative notion of truth.<sup>3</sup>

<sup>3</sup>I have my doubts with regard to MacFarlane’s postsemantic solution, but I set them aside as I am currently working on a paper focused uniquely on criticism of relativism.

### 3 History relativism

History relativism agrees with assessment relativism that the truth status of a sentence used in a context cannot be determined based solely on the meaning of the sentence and the features of the context of use. However, in history relativism we need to assume the point of view of the entire history to assess the truth status of a sentence used at a context. In Richmond Thomason's words, we are "adopting a whole possible future for  $\alpha$  as our perspective, rather than a single time in the future of  $\alpha$ " [11, p. 269].

Such an attitude is characteristic of Belnap et al. [2]. In their view, unless a specific possible history is specified, a future contingent cannot be evaluated at a given context.<sup>4</sup> The authors express their attitude in the following words:

Then the truth of that sentence (given indeterminism) depends not only on the moment at which the sentence is uttered. It depends in addition on which future course of events—which history—is being considered. [2, p. 225]

Nonetheless, we noticed that the authors ferociously argue against the idea that the context indicates which history *should* be considered (after all, an utterance is a part of many different courses of events). As a result, it is simply meaningless to call a future contingent true or false at the context of its use  $m_c$ . As the authors put it:

*" $\mathfrak{M}, m_c \models \text{Will: the coin lands heads}$ " does not make sense.*  
[2, p. 155]

Only when a continuation of the moment of utterance is independently specified can one ask about the truth value of the uttered sentence. Therefore, the history relativists' answer to the question whether the sentence "there is going to be a sea battle tomorrow" is true is somewhat evasive. They claim that the sentence is true relative to the continuation in which the sea battle takes place, but it is false relative to the alternative continuation. This is as much as can be said regarding the truth value of a sentence at a context.

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<sup>4</sup>Their terminology differs from mine. When I write about a sentence being true at a context, Belnap et al. [2] write about a stand-alone sentence being true at a context-initialized point of evaluation.

We may say that history relativists simply capitulate in face of the initialization failure. Given the evident indispensability of the history parameter in Ockhamist semantics, they simply duplicate the history parameter on the level of postsemantics. We end up with a theory according to which the truth value not only of a sentence-at-index but also of a sentence-at-context is relative to a history.

**Definition 6** (History relativism postsemantics).  $m/h \Vdash^H \phi$  iff  $m/h \vDash \phi$ .

When we apply history relativism to future contingents it becomes clear that their truth value (at context) is highly arbitrary—it depends on something as whimsical as an entirely unmotivated choice of a history parameter. This is a rather controversial consequence. One could argue that the truth value of a sentence used in a context should be grounded in something more solid than an *ad hoc* decision of a semanticist who needs one history or another to do her job.

It is also not entirely clear of what this decision should consist. When relativists talk in terms of abstract Ockhamist semantics, they say that a possible future needs to be “posited” [11, p. 271] or “supplied” [2, p. 156]. However, when they try to give a more down-to-earth description of the procedure, they often help themselves with intentional vocabulary. For example, Burgess writes that “The truth value of a future tense statement depends on which branch we *think of* as representing the course of events which is actually going to turn out to happen” [3, p. 575, emphasis mine] and Müller [8] echoes that “we normally need to specify which of the equally possible futures we *mean to refer to*” [8, p. 354, emphasis mine]. However, if all that is required to specify a possible history is an intention of a speaker (this procedure is called “inner baptism in [7]), then making predictions true or false would be all too easy. Such a procedure has very little in common with everyday usage. When Themistocles said to Eurybiades, “There will be a sea battle tomorrow,” no one could make this sentence true or false just by thinking of a specific possible future.<sup>5</sup>

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<sup>5</sup>Let me note that Belnap et al. [2] have their own ways of domesticating their proposal. Specifically, they argue that the “bare” truth value of a sentence at a context is *irrelevant* to the linguistic practice. What really matters in their view is the *settled* truth value. They notice that it is often settled that future contingents will have had a settled truth value and explain how this feature is sufficient to explain linguistic practice. A sketch of their proposal can be found in [2, sec. 6E]; the view is developed in [1].



In what follows, I will argue that there is another way to rationalize history relativism. In my proposal, the truth value of a sentence used at a context is still relative, but the choice of a specific history is neither arbitrary nor does it depend on a subject's intentions. I claim that the choice of the history becomes well motivated when we understand a history as a special context of assessment.

## 4 History relativism as extreme assessment relativism

In this section, I argue that history relativism can be understood as an extreme version of MacFarlane's assessment relativism; some inspiration for this approach can be found in a short passage by Prior:

[T]he Ockhamist seems to treat what is still future in a way in which it would only be proper to treat what *has been* future—he views it as it would be proper to view it from the end of time. [10, pp. 130–131]

This line of thinking about Ockhamism seems to have been discarded in the later development of the theory, but it might be used to establish a new understanding of the history-relative notion of truth. I intend to develop the idea that when history relativists relativize the truth value of a sentence at context to a history, they metaphorically situate themselves at the transcendent end of the history. Then, they indeed view a course of events “as it would be proper to view it from the end of time.”

In the spirit of Prior's philosophy, let me elucidate the insight above by giving it a formal reconstruction. I intend to apply MacFarlane's assessment relativism to demonstrate that the notion of truth at a context relative to a history can be understood as truth at a pair of contexts. We shall see that—in accordance with Prior's insight—the context of assessment needs to be situated at the end of time.

Let me begin the investigations with a simple example. Consider a branching model  $\mathfrak{M}$  in which there is a maximal element in every history—“the end of time”—in this history (there is at most one such element, given that histories are linearly ordered). If we symbolize the maximal moment in history  $h$  as  $m_h$ , it is easy to observe that:

**Lemma 1.** If  $\mathfrak{M}$  is a model in which there is a maximal element  $m_h$  in every history  $h$ , then  $\forall_h (H_{m_h} = \{h\})$

*Proof.* Take an arbitrary history  $h \in \mathfrak{M}$ ,  $m_h \in h$ , so  $h \in H_{m_h}$ , therefore,  $\{h\} \subseteq H_{m_h}$ . To prove that  $H_{m_h} \subseteq \{h\}$ , assume for reduction that  $\exists_{h'} h' \neq h \& m_h \in h'$ . Since  $h \neq h'$ ,  $\exists_{m' \in h'} m' \in h' \& m' \notin h$ . As  $m_h, m' \in h'$  and  $h'$  is linearly ordered, there are two options:

1. If  $m' < m_h$ , then (by no-backward-branching and maximality of  $h$ )  $m' \in h$ , which contradicts our assumption.
2. If  $m_h \leq m'$ , then (since  $m_h$  is the maximal element of  $h$  and  $h$  is a maximal, linearly ordered subset of  $M$ )  $m_h \not\leq m'$ . Thus,  $m' = m_h$ , but then  $m' \in h$ , which contradicts our assumption.

A simple proof is sufficient to establish that for any  $m$  in model  $\mathfrak{M}$  described above, a sentence is true relative to a history  $h$  iff it is assessed as true from the perspective of moment  $m_h$ , i.e. from the end of time in history  $h$ :

**Fact 1.**  $m/h \Vdash^H \phi$  iff  $m, m_h \Vdash^R \phi$

*Proof.* Since  $m, m_h \in h$  and  $m_h$  is the maximal element of  $h$ , then  $m \leq m_h$ . From this we can conclude that  $H_{m|m_h} = (H_m \cap H_{m_h}) = H_{m_h}$ . Hence,  $H_{m|m_h} = H_{m_h}$ . By lemma 1,  $H_{m_h} = \{h\}$ . Therefore,  $H_{m|m_h} = \{h\}$ .

1.  $m, m_h \Vdash^R \phi$  iff
2.  $m/h' \vDash \phi$ , for every  $h' \in H_{m|m_h}$  iff (since  $H_{m|m_h} = \{h\}$ )
3.  $m/h \vDash \phi$  iff (by def. 6)
4.  $m/h \Vdash^H \phi$ .

Thus, in the “upper-bounded” model it is easy to substantiate Prior’s claim; however, application of Ockhamism is not limited to such models. It might well be that some (or even all) of the histories in a model have no end. In such a case, what would it mean for an Ockhamist to view the future “as it would be proper to view it from the end of time?” I propose to read it along the following lines: an Ockhamist views the future as it would be proper to view it from a *transcendent* end of time.

To give formal meaning to the maxim, I construct what I call a *doomsday extension* of a branching model. Let  $\mathfrak{M} := \langle M, < \rangle$  be a branching model. We extend the model  $\mathfrak{M}$  with a set  $M_D$  such that:

- (i)  $\forall_h \exists!_{m_h \in M_D} \forall_{m \in h} m < m_h$
- (ii)  $\forall_{m_h \in M_D} \exists!_h \forall_{m \in h} (m < m_h \Leftrightarrow m \in h)$

This means that we attach a single extra moment on top of every history in the original model  $\mathfrak{M}$ . I will call such an extended structure  $\mathfrak{M}^D$ , and  $m_h$  is the moment which is attached on top of history  $h$ . Let me pause to show that model  $\mathfrak{M}^D$  is still a model of branching. Its ordering relation is evidently a partial order, so let me just check if it is a connected order without backward branching.

**Fact 2** ( $\mathfrak{M}^D$  is a *BT* model).

**Connectedness**  $\forall_{m,n \in M} \exists_{o \in M} o \leq m \ \& \ o \leq n$

*Proof.* The only interesting case is when we pick  $m_{h_1}, m_{h_2} \in M^D$ . In this case, we just need to choose any moment  $m \in h_1$  and  $n \in h_2$ . By definition of  $\mathfrak{M}^D$ ,  $m < m_{h_1}$  and  $n < m_{h_2}$ , and since  $m$  and  $n$  are connected and  $\leq$  is transitive,  $m_{h_1}$  and  $m_{h_2}$  are also connected.

### No-Backward-Branching

$\forall_{m_1, m_2, m_3} (m_1 \leq m_3 \ \& \ m_2 \leq m_3) \Rightarrow (m_1 \leq m_2 \vee m_2 \leq m_1)$

*Proof.* We just need to check if it is satisfied for every  $m_3 = m_h \in M_D$ . Take an arbitrary  $m_h \in M_D$ , then by condition (ii) we have that all the moments below  $m_h$  are in a single history. And since every history is linearly ordered, there is no danger of backward branching.

Before I proceed, let me observe that the construction of  $\mathfrak{M}^D$  guarantees that there is a maximal element in every history. Therefore, lemma 1 applies, and we have that in  $\mathfrak{M}^D$ ,  $\forall_h H_{m_h} = \{h\}$ .

Let us investigate the relations between history relativism and assessment relativism in the doomsday model.

Observe first that there is  $\mathfrak{M}$  and  $\phi$  such that:

$$\mathfrak{M}, m/h \Vdash^H \phi \ \& \ \mathfrak{M}^{\mathfrak{D}}, m, m_h \Vdash^R \phi$$

There are two kinds of reasons for the failure.

**“Material”** There may be  $\phi$  which is false everywhere in  $h$ , but true at  $m_h$ . (“Four horsemen of the Apocalypse are riding their horses” is a good candidate for  $\phi$ ). Then  $G\neg\phi$  is true at any moment in  $h$  in the base model, but false in the doomsday extension of the model.

**“Structural”** The addition of doomsday significantly modifies the structure of the histories. Most evidently, seriality no longer holds and thus  $G\phi \rightarrow F\phi$  is not valid in the extended model.

It is then a valid question whether we can give formal credit to Prior’s insight in the general case. I propose a relatively easy solution: limit the range of the future operator such that it does not reach all the way to doomsday. In this sense, doomsday is truly a transcendent end of time as it cannot be reached by the “mundane” future operator. The new definition of  $F$  in the doomsday model should be modified as follows:

**Definition 7.**

$$\mathfrak{M}^{\mathfrak{D}}, m/h \vDash F\phi \text{ iff } \exists_{m'} (m' \in h \ \& \ m' > m \ \& \ m' \neq m_h \ \& \ \mathfrak{M}^{\mathfrak{D}}, m'/h \vDash \phi).$$

By the duality of  $F$  and  $G$ , we obtain that

$$\mathfrak{M}^{\mathfrak{D}}, m/h \vDash G\phi \text{ iff } \forall_{m'} (m' \in h \ \& \ m' > m \ \& \ m' \neq m_h) \Rightarrow \mathfrak{M}^{\mathfrak{D}}, m'/h \vDash \phi.$$

Therefore, we end up with a model which has an extra element on top of every history, but the element is not attainable by the connective “it will be the case that.” So, doomsday is in one sense at the end of time, but in another it is outside of time. I am not sufficiently versed in theology to give a convincing account of this idea, but I am quite confident that it has been entertained at some point in the history of human thought. Importantly for us, this modification makes it possible to prove an analogue of fact 1 in full generality:

**Fact 3.** Let  $\mathfrak{M}$  be an arbitrary branching model and  $\mathfrak{M}^{\mathfrak{D}}$  its doomsday extension, and let  $m \in \mathfrak{M}$ , then:

$$\mathfrak{M}, m/h \Vdash^H \phi \text{ iff } \mathfrak{M}^{\mathfrak{D}}, m, m_h \Vdash^R \phi$$

*Proof.* By induction on complexity of  $\phi$ , in particular:

1.  $\mathfrak{M}, m/h \Vdash^H F\phi$  if (by def. 6)
2.  $\mathfrak{M}, m/h \vDash F\phi$  iff (by def. of  $F$ )
3.  $\exists_{m'|m' \in h \& m' > m} \mathfrak{M}, m'/h \vDash \phi$  iff (by def. 6)
4.  $\exists_{m'|m' \in h \& m' > m} \mathfrak{M}, m'/h \Vdash^H \phi$  iff (by inductive assumption)
5.  $\exists_{m'|m' \in h \& m' > m} \mathfrak{M}^{\mathfrak{D}}, m', m_h \Vdash^R \phi$  iff (by def. 5)
6.  $\exists_{m'|m' \in h \& m' > m} \forall_{h' \in H_{m'|m_h}} \mathfrak{M}^{\mathfrak{D}}, m'/h' \vDash \phi$  iff (since  $m' < m_h$ , by def. 4)
7.  $\exists_{m'|m' \in h \& m' > m} \forall_{h' \in H_{m_h}} \mathfrak{M}^{\mathfrak{D}}, m'/h' \vDash \phi$  iff (by Lemma 1)
8.  $\exists_{m'|m' \in h \& m' > m} \mathfrak{M}^{\mathfrak{D}}, m'/h \vDash \phi$  iff ( $m' \in h$ , so  $m' \neq m_h$ )
9.  $\exists_{m'} (m' \in h \& m' > m \& m' \neq m_h \& \mathfrak{M}^{\mathfrak{D}}, m'/h \vDash \phi)$  iff (by def. 7)
10.  $\mathfrak{M}^{\mathfrak{D}}, m/h \vDash F\phi$  iff (by Lemma 1)
11.  $\forall_{h \in H_{m_h}} \mathfrak{M}^{\mathfrak{D}}, m/h \vDash F\phi$  iff (since  $m < m_h$ , by def. 4)
12.  $\forall_{h \in H_{m|m_h}} \mathfrak{M}^{\mathfrak{D}}, m/h \vDash F\phi$  iff (be def. 5)
13.  $\mathfrak{M}^{\mathfrak{D}}, m, m_h \Vdash^R F\phi$

Thanks to the modification of the truth clause of  $F$  in the doomsday model, we can give full credit to Prior's insight. The Ockhamist looks at the future as if it has been future, that is, from the perspective of the end of time. A necessary addition to vindicate this insight is that in the models in which time has no end, the end of time is "beyond time."

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