Comfortable and Safe Decelerations for a Self-Driving Transit Bus

Alexis Mifsud, Matteo Ciocca, Pierre-Brice Wieber

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***Abstract*— We propose a combination of Model Predictive Control and Lexicographic Programming to address complex scenarios with conflicting goals related to various aspects of comfort and safety of passengers in a transit bus, generating different deceleration profiles depending on the speed of the bus and distance to obstacles, validated in experiments with a standard transit bus equipped with self-driving capabilities.**

I. Introduction

All self-driving vehicles have to solve a common set of problems [2], including perception and understanding of their surroundings, social interaction at a distance with other users of the road, complex and potentially conflicting sets of written and unwritten rules and laws [7]. Transit buses have to account for an additional problem: decelerations can present a risk of lethal impact for passengers *inside* the bus since seats are not equipped with airbags or seat belts and standing passengers may not be firmly grasping a handle [12] (falls are "the second leading cause of unintentional injury death, after road traffic injuries" in the world [23], causing even more deaths than traffic accidents in some countries [8]). We will see that for a bus with standing passengers to stop comfortably, the decision must be anticipated up to 12 seconds ahead of time. We implement therefore a Model Predictive Control (MPC) scheme where the motion of the bus is constantly re-planned for the next 12 seconds following a *receding horizon* scheme, a particularly efficient approach for dynamical systems with strong safety constraints [14].

Comfort and safety can be conflicting goals. Suppose for example that we want to keep two functions equal to zero, f_1 related to safety and f_2 related to comfort. If we can't keep both equal to zero, then it has to be f_2 (comfort) which is degraded, and not f_1 (safety) which has precedence. We approach this with a Lexicographic Program (LexP) [10]:

$$\text{lex minimize} \quad f_1^2,\ f_2^2 \tag{1}$$

which means that the value of f_1^2 is minimized first, down to zero if possible, and the value of f_2^2 is minimized afterwards, down to zero if possible, but without degrading the optimal value found beforehand for f_1^2. Combining MPC and LexP isn't new [13], [20], but has never been widely adopted despite its appeal for coherent performance degradation [4], [15]. Our first contribution is the incremental construction of an original MPC scheme that *optimally* handles *multiple conflicting goals* related to the comfort and safety of passengers in a self-driving transit bus using a fast online LexP solver [9]. The goals are to maintain a nominal speed *as long as possible*, drawing inspiration from [5], while maintaining comfort *as much as possible* and avoiding collisions *as much as possible* within safety limits. For clarity, we present this construction for a bus driving on a straight line, but steering and curves are also handled in our full implementation.

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Fig. 1. The STAR project (Système de Transport Autonome Rapide) aims at developing and integrating self-driving technology in a standard 12 m long, 19 t heavy transit bus reaching a maximum speed of 40 km/h.

This is part of the STAR project (Système de Transport Autonome Rapide[1]) which aims at developing and integrating self-driving technology in a 12 m long, 19 t heavy transit bus (Fig. 1) reaching a maximum speed of 40 km/h. Target use cases are public transport on reserved lanes and bringing airport passengers from terminals to planes. Very little academic research has been done on the specific issues of self-driving buses [3], [17] and apparently none at all on the specific deceleration issues mentioned above. Our second contribution is the experimental demonstration for the first time on a standard transit bus equipped with drive by wire braking and steering of an advanced LexP-MPC scheme.

We approach situations of increasing complexity: comfortable decelerations in Section II, uncomfortable but safe decelerations in Section III, and the choice that must be made occasionally between passenger safety and collisison avoidance in Section IV. We discuss approximate solutions in Section V and experimental results in Section VI before introducing defensive driving issues in Section VII.

II. Comfortable Decelerations

For passengers standing in the bus without grasping a handle, who are the most at risk of dangerous falls, maintaining balance depends on available contact forces between

[1]Partners are EasyMile, IVECO, Sector, Transpolis, ISAE-SUPAERO, Université Gustave Eiffel, Inria and Michelin.

feet and ground. These contact forces point from feet soles to Center of Mass (CoM) of the person in order to keep a low angular momentum and avoid unsafe arms and torso rotations [22]. Basic proportionality between sole half-length $s \approx 0.12$ m, height of the CoM $h \approx 1$ m and gravity acceleration $g \approx 9.8$ m.s^{-2} informs us that a horizontal deceleration beyond a limit $\frac{s}{h}g \approx 1.2$ m.s^{-2} would make the feet tilt, initiating a loss of balance. Stronger decelerations can be handled with postural adaptations, shifting the CoM with respect to feet by leaning or making a step, but this takes time to execute (around 1 second), so both amplitude and rate of change of deceleration affect the comfort and safety of passengers and must be kept low [18], [19].

As a result, we expect that whenever possible, the bus maintains a deceleration below a comfort limit $\ddot{x}_c = 1.23$ m.s^{-2}. And to limit its rate of change, we opt for a piecewise constant jerk

$$\dddot{x}_k = \frac{\ddot{x}_{k+1} - \ddot{x}_k}{T} \tag{2}$$

over time intervals of length $T = 1$ s, as this is the typical time constant for large balance recovery movements [19]. Under these conditions, for a bus at nominal speed (11.11 m.s^{-1}, 40 km/h), following the dynamics (2) means stopping in a minimum of 10 seconds: 1 s to ramp deceleration to $-\ddot{x}_c$, 8 s of full deceleration and 1 s to finally ramp deceleration back to 0, as shown in Fig. 2. In an MPC scheme, we need to anticipate at least 2 more seconds to account for possible delays in the perception and control pipeline and maintain the current desired speed for at least one sampling period before initiating this deceleration profile, resulting in a prediction horizon of at least 12 seconds.

In order to generate deceleration profiles automatically, let's start by introducing the difference between the speed of the bus $\dot{x}_k$ and a desired speed $\dot{x}_{des}$:

$$\Delta\dot{x}_k = \dot{x}_k - \dot{x}_{des}.$$

Maintaining the desired speed as long as possible can be approached as a LexP, following a similar idea as in [5]:

$$\text{lex minimize} \quad \Delta\dot{x}_1^2, \Delta\dot{x}_2^2, \ldots, \Delta\dot{x}_N^2 \tag{3}$$

$$\text{subject to} \quad \forall k \in \{0 \ldots N-1\},\ \text{dynamics (2)},$$

$$\forall k \in \{1 \ldots N-1\},\ |\ddot{x}_k| \leq \ddot{x}_c, \tag{4}$$

$$\forall k \in \{1 \ldots 2N-1\},\ 0 \leq \dot{x}_{\frac{k}{2}} \leq \dot{x}_{max}, \tag{5}$$

$$\dot{x}_N = 0,\ \ddot{x}_N = 0. \tag{6}$$

As discussed in [10], this corresponds to minimizing $\Delta\dot{x}_1^2$ first, then trying to minimize $\Delta\dot{x}_2^2$, then trying to minimize the difference in speed further into the future until finally trying to minimize $\Delta\dot{x}_N^2$. In constraint (5), we monitor the speed of the bus also in the middle of each k-th time interval in order to avoid undesirable speed oscillations. This speed can be obtained by direct integration of the jerk (2):

$$\dot{x}_{k+\frac{1}{2}} = \dot{x}_k + \frac{3}{8}T\ddot{x}_k + \frac{1}{8}T\ddot{x}_{k+1}.$$

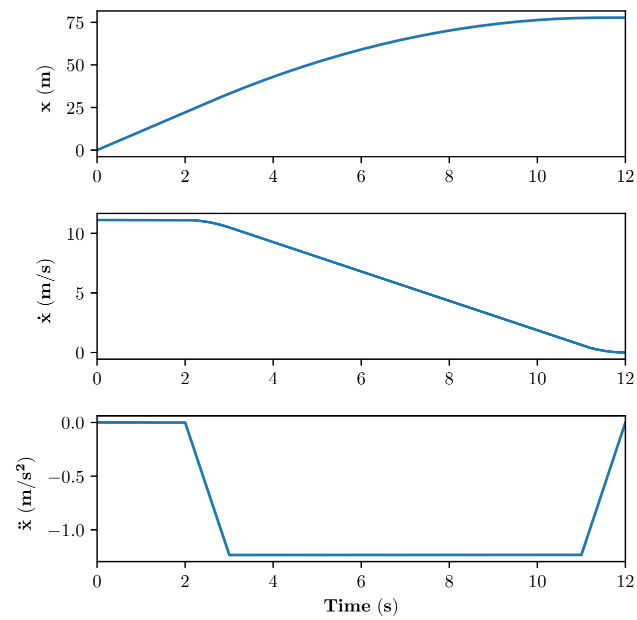

Fig. 2. For a bus driving at 11.11 m.s^{-1}, following the dynamics (2) while not exceeding a comfort limit $\ddot{x}_c = 1.23$ m.s^{-2} means stopping in a minimum of 10 seconds: 1 s to ramp deceleration to $\ddot{x}_c$, 8 s of full deceleration and 1 s to finally ramp deceleration back to 0.

The terminal constraint (6) provides a safety guarantee [6]: the bus must stop in the end of the prediction horizon, so that if a collision happens afterwards, the bus will be at rest, with zero kinetic energy. It also guarantees by induction that if all constraints can be satisfied at some time, they can all be satisfied at all future time as well [14].

Following the solution to this LexP with $\dot{x}_{max} = \dot{x}_{des} = 11.11$ m.s^{-1}, the bus maintains the desired speed as long as possible before decelerating comfortably and stop right in the end of the horizon, as shown in Fig. 2. When considering an obstacle, such as a red light $x_{obs} = 35$ m away, we also need that

$$\forall k \in \{1 \ldots N\},\ x_k \leq x_{obs}. \tag{7}$$

Following the solution to this LexP with (as an example) a lower speed $\dot{x}_{des} = 5.55$ m.s^{-1}, the bus maintains once again the desired speed as long as possible before decelerating comfortably right in front of the obstacle, as shown in Fig. 3.

III. Uncomfortable but Safe Decelerations

When an obstacle moves or appears suddenly in front of the bus, it may be necessary to exceed the comfortable deceleration limit $\ddot{x}_c$ to avoid collision. Consider for example the bus moving at 11.11 m.s^{-1} when an obstacle appears suddenly $x_{obs} = 30$ m away, less than 3 s away at full speed. In that case, the comfort constraint (4) on deceleration must be relaxed into

$$\forall k \in \{1 \ldots N-1\},\ |\ddot{x}_k| \leq \ddot{x}_c + s_k,$$

with some possible excess $s_k \geq 0$. We want, however, to exceed the comfort limit only when necessary to avoid

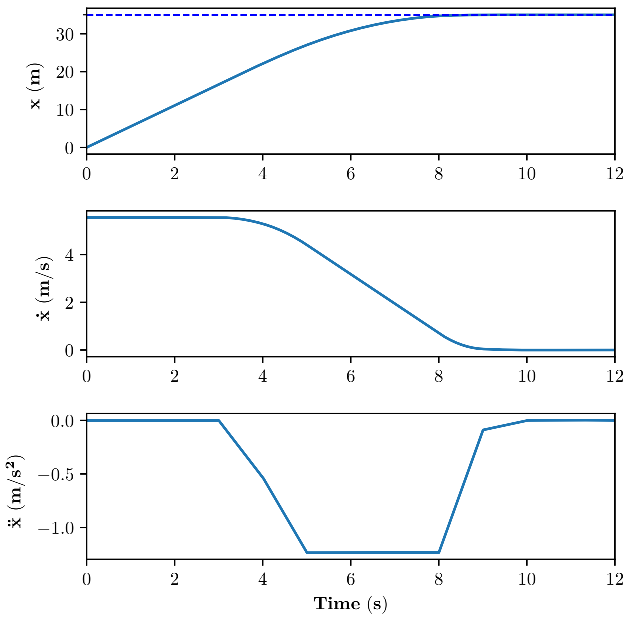

Fig. 3. Following the solution to the LexP (3)-(6), the bus maintains the existing speed as long as possible before decelerating comfortably right in front of the obstacle.

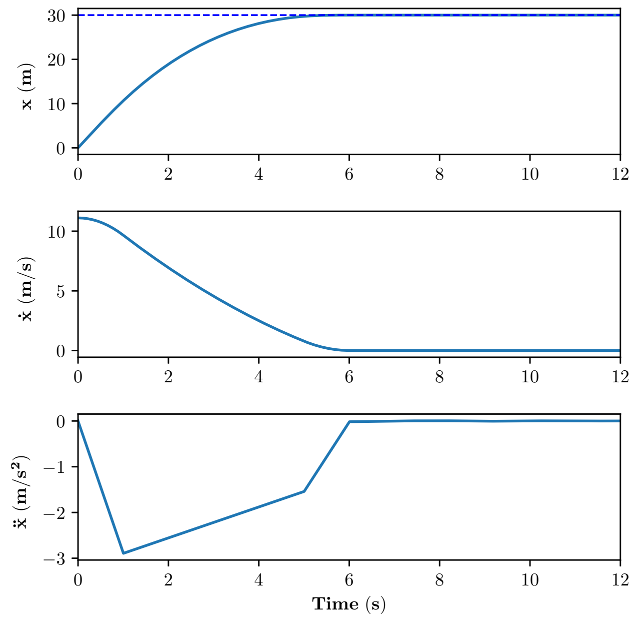

Fig. 4. Following the solution to the LexP (8), the bus starts decelerating immediately beyond the comfort limit in order to stop quickly enough to avoid collision. Even though we're minimizing an unweighted sum $\sum s_k^2$, the optimal solution displays a deceleration profile decreasing linearly with time, due to the fact that early decelerations are more efficient than late ones in actually stopping the bus before it reaches the obstacle.

collision, and maintain a comfortable deceleration otherwise. We can leverage the Lexicographic Programming approach to obtain such a behavior as follows:

$$\begin{aligned}
\text{lex minimize} \quad & \sum_{k=1}^{N} s_k^2,\ \Delta\dot{x}_1^2, \ldots, \Delta\dot{x}_N^2 && (8)\\
\text{subject to} \quad & \forall\, k \in \{0 \ldots N-1\},\ \text{dynamics (2)},\\
& \forall\, k \in \{1 \ldots N-1\},\ |\ddot{x}_k| \leq \ddot{x}_c + s_k,\\
& \forall\, k \in \{1 \ldots 2N-1\},\ 0 \leq \dot{x}_{\frac{k}{2}} \leq \dot{x}_{max},\\
& \forall\, k \in \{1 \ldots N\},\ x_k \leq x_{obs}, && (9)\\
& \dot{x}_N = 0,\ \ddot{x}_N = 0.
\end{aligned}$$

Here, the sum of squared excess $\sum s_k^2$ is minimized before considering the differences in speed $\Delta\dot{x}_1^2, \ldots, \Delta\dot{x}_N^2$. If this sum can be kept to zero, it means that the deceleration can be kept below the comfort limit, and as before, the existing speed of the bus can then be maintained as long as possible (as in Fig. 3). If it can't be kept to zero, it means some excess $s_k > 0$ must be introduced to immediately decelerate beyond the comfort limit in order to stop quickly enough to avoid collision, as shown in Fig. 4. Even though we're minimizing an unweighted sum $\sum s_k^2$, the optimal solution displays a deceleration profile decreasing linearly with time, due to the fact that early decelerations carry more weight than late ones in constraints (9) and are therefore more efficient to actually stop the bus before reaching the obstacle.

IV. Passenger Safety versus Collision Avoidance

Strong decelerations present a risk of lethal impact for passengers *inside* the bus since seats are not equipped with airbags or seat belts and standing passengers may not be firmly grasping a handle. As a result, decelerations beyond a safety limit $\ddot{x}_{max}$, which we consider here equal to 3.70 $m.s^{-2}$ [1], must be avoided. But when an obstacle moves or appears too close in front of the bus, avoiding collision may be possible only by exceeding this safety limit. Consider for example the bus moving at 11.11 $m.s^{-1}$ when an obstacle appears suddenly $x_{obs} = 20$ m away, less than 2 s away at full speed. A decision must be made in such a situation, whether the bus should avoid collision nevertheless, or not (collision should be mitigated anyway).

There could be justifications for maintaining a strict safety limit on deceleration, even if it means running into collision, as there are cases where collision means little consequence, if the obstacle is for example an inanimate object of little importance. If the obstacle is a person, this person might still evade collision in the time remaining before contact, whereas up to 100 passengers can't escape the bus and are at instant risk as soon as deceleration is initiated beyond the safety limit. We are certainly not in a position here to decide what should be the behavior of the bus in which situations, but we must propose solutions for both options: avoiding collision at all cost, or not.

If the bus avoids collision at all cost, *i.e.* exceeding if necessary the above safety limit on deceleration, then the LexP (8) remains adequate and needs not be changed. If the bus enforces instead a strict safety limit on decelerations,

$$\forall\, k \in \{1 \ldots N-1\},\ |\ddot{x}_k| \leq \ddot{x}_{max},$$

then the obstacle avoidance constraint (9) must be relaxed

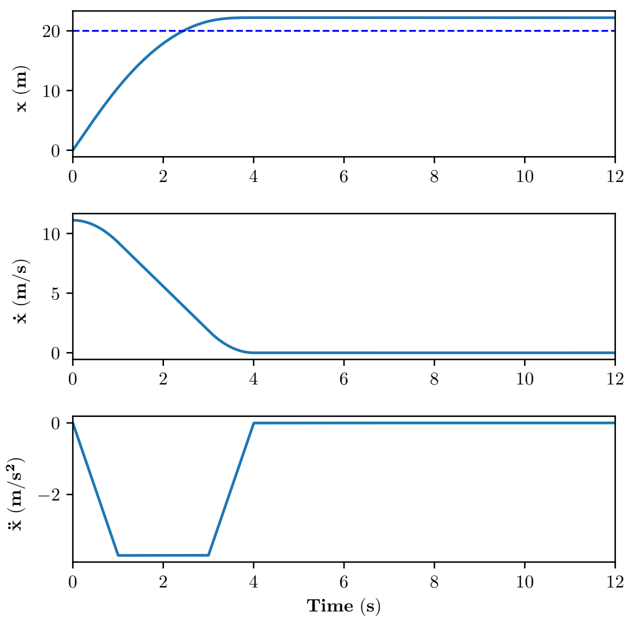

Fig. 5. Following the solution to the LexP (10), the bus starts immediately decelerating at the safety limit in order to stop as quickly as possible without putting its own passengers at risk. It is unable to avoid collision in this situation.

into

$$\forall\, k \in \{1 \dots N\},\ x_k \le x_{obs} + w_k,$$

with some possible violation $w_k \ge 0$. We want this violation minimal within the deceleration safety limit, to mitigate collision as much as possible, or even avoid it if we can keep $w_k = 0$. We can leverage once again the Lexicographic Programming approach as follows:

$$\begin{aligned} \text{lex minimize} \quad & \sum_{k=1}^{N} w_k^2,\ \sum_{k=1}^{N} s_k^2,\ \Delta\dot{x}_1^2, \dots, \Delta\dot{x}_N^2 \qquad (10) \\ \text{subject to} \quad & \forall\, k \in \{0 \dots N-1\},\ \text{dynamics (2)}, \\ & \forall\, k \in \{1 \dots N-1\},\ |\ddot{x}_k| \le \ddot{x}_{max}, \\ & \forall\, k \in \{1 \dots N-1\},\ |\ddot{x}_k| \le \ddot{x}_c + s_k, \\ & \forall\, k \in \{1 \dots 2N-1\},\ 0 \le \dot{x}_{\frac{k}{2}} \le \dot{x}_{max}, \\ & \forall\, k \in \{1 \dots N\},\ x_k \le x_{obs} + w_k, \\ & \dot{x}_N = 0,\ \ddot{x}_N = 0. \end{aligned}$$

Here, the sum of squared violation $\sum w_k^2$ is minimized before considering the sum of squared excess $\sum s_k^2$ and the differences in speed $\Delta\dot{x}_1^2, \dots, \Delta\dot{x}_N^2$. If this sum can be kept to zero, it means that the collision can be avoided within the safety limit and we can look to minimize then the excess deceleration $\sum s_k^2$ as before (see Fig. 4), and maintain the existing speed of the bus as long as possible, as before (see Fig. 3). If it can't be kept to zero, it means some violation $w_k > 0$ of the obstacle avoidance constraint must be introduced in order to maintain a deceleration within the safety limit, as shown in Fig. 5: in this situation and under this condition, the bus is unable to avoid collision.

V. Approximate Solutions

An interesting property of the LexP (10) is to generate qualitatively very different behaviors, depending on the situation the bus is facing. Lexicographic Programs, however, can be difficult to solve efficiently and reliably due to ill-conditioning [21]. As an alternative, the lexicographic multi-objective (10) can be approached as a single objective as follows:

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{N} (w_k + \alpha)^2 + (s_k + \beta)^2 + \\ & \gamma^{N-k+\frac{1}{2}} \Delta\dot{x}_{k-\frac{1}{2}}^2 + \gamma^{N-k} \Delta\dot{x}_k^2 \qquad (11) \\ \text{subject to} \quad & \forall\, k \in \{0 \dots N-1\},\ \text{dynamics (2)}, \\ & \forall\, k \in \{1 \dots N-1\},\ |\ddot{x}_k| \le \ddot{x}_{max}, \\ & \forall\, k \in \{1 \dots N-1\},\ |\ddot{x}_k| \le \ddot{x}_c + s_k, \\ & \forall\, k \in \{1 \dots 2N-1\},\ 0 \le \dot{x}_{\frac{k}{2}} \le \dot{x}_{max}, \\ & \forall\, k \in \{1 \dots N\},\ x_k \le x_{obs} + w_k, \\ & \forall\, k \in \{1 \dots N\},\ w_k \ge 0,\ s_k \ge 0, \\ & \dot{x}_N = 0,\ \ddot{x}_N = 0. \end{aligned}$$

The first two terms in this objective function implement a variant of exact ℓ_1 penalization [16] of $w_k \ge 0$ and $s_k \ge 0$, where a quadratic term is introduced (eventually resembling an Augmented Lagrangian approach) to keep the Hessian matrix of the QP positive definite. There are methods to find parameters α, β and γ such that solutions to this QP reproduce the same qualitative behaviors as solutions to the LexP (10) [20], but here we simply hand-tuned them to $\alpha = 5.0\text{e}7$, $\beta = 5.0\text{e}7$, $\gamma = 1.5$, being careful that the resulting QP is well conditioned numerically.

As an example, we can see in Fig. 6 that reproducing the situation from Fig. 4 with this choice of parameters, the bus starts decelerating immediately beyond the comfort limit in order to stop quickly enough to avoid collision, as desired. This solution, however, is slightly different from the LexP solution, with a shorter but stronger deceleration peak, getting back to the comfort limit more quickly. Which solution should be favored in this case is currently unclear: stronger deceleration can result in stronger loss of balance, but it can also trigger a stronger recovery behavior from passengers, resulting eventually in milder loss of balance due to its shorter duration. Controlled experiments with real passengers would be necessary to answer this question, but such experiments must be considered very carefully since loss of balance can be extremely dangerous. This QP takes approximately 150 ms to solve with qpOASES [11] instead of 200 ms to solve the LexP (10) with our solver [9] on an i5-6500 CPU at 3.2 GHz, but both computation times can be reduced if necessary with specific software optimizations.

VI. Experiments

Experiments are done with a standard 12 m long, 19 t heavy IVECO Urbanway transit bus (Fig. 1) equipped with drive by wire braking and steering, sensors and computers to provide self-driving capabilities. The control system

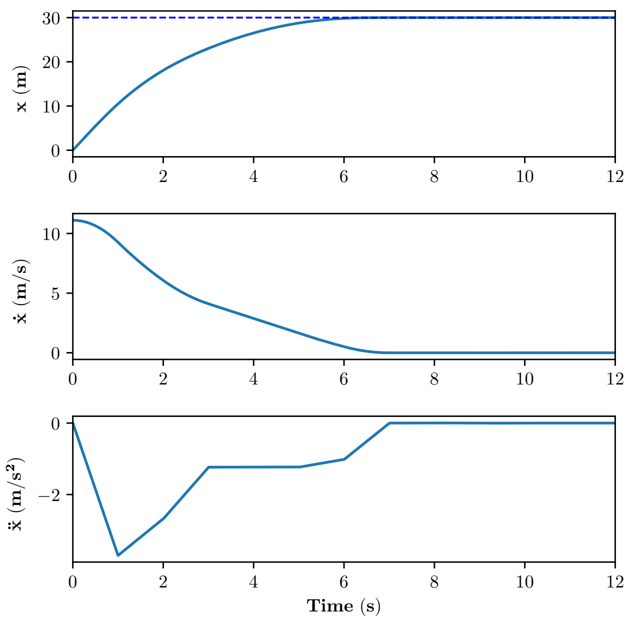

Fig. 6. Following an approximation of the solution to the LexP (8) provided by the QP (11), the bus starts decelerating immediately beyond the comfort limit in order to stop quickly enough to avoid collision, but differently from the LexP solution shown in Fig. 4, with a shorter but stronger deceleration peak, getting back to the comfort limit more quickly.

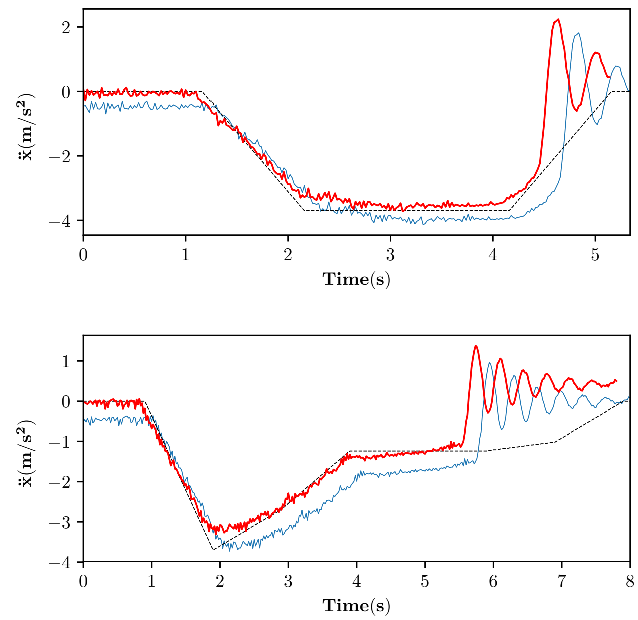

Fig. 7. Following the deceleration profiles of Figs. 5 (top) and 6 (bottom), the measured deceleration (in blue) can be observed to lag behind the desired one (in black dots), with a constant time delay of approximately 0.2 s. It is also constantly 0.42 m.s^{-2} stronger than desired. Correcting this constant offset and time delay (in red), we can see that deceleration profiles are very well tracked by the low-level control of the bus, at least until the bus stops, what triggers oscillations of its upper structure.

perceives the environment and sends steering and braking commands on a CAN bus every 20 ms. Current results are very preliminary as experiments with such massive hardware are difficult and costly to realize because of the extensive infrastructure involved and demanding safety measures enforced[2].

Typical deceleration results can be seen in Fig. 7, following the profiles of Figs. 5 and 6. The measured deceleration (in blue) can be observed to lag behind the desired one (in black dots), with a constant time delay of approximately 0.2 s. It is also constantly 0.42 m.s^{-2} stronger than desired, what corresponds to the constant deceleration that can be experienced when no control is applied, probably due to constant friction in the bus drivetrain. These two values are obtained by minimizing the sum of absolute difference between the two signals over the whole set of recorded experiments:

$$\underset{\ddot{x}_c,\,\delta}{\text{minimize}} \quad \sum_k |\ddot{x}_k^{des} + \ddot{x}_c - \ddot{x}_{k+\delta}^{meas}|, \tag{12}$$

with $\ddot{x}_c$ the constant deceleration offset, δ the constant time delay, $\ddot{x}^{des}$ and $\ddot{x}^{meas}$ the desired and measured decelerations. Correcting this constant offset and time delay (in red), we can see that deceleration profiles are very well tracked by the low-level control of the bus, at least until the bus stops.

When the bus stops, any remaining horizontal load on its suspension due to deceleration is suddenly removed, what triggers oscillations of its upper structure that can be clearly observed in Fig. 7. This effect can be reduced by considering deceleration profiles that gently move back to zero when stopping, as generated throughout this paper, but what happens here is that the bus stops earlier than planned, due to the constant deceleration offset, and it stops before the deceleration profile gets back to zero, resulting in strong oscillations. This constant deceleration offset (and time delay) should be easy to pre-compensate, so the bus would stop as planned, with little oscillations.

[2]Experiments were made possible by EasyMile, IVECO and Transpolis.

VII. Defensive Driving

Defensive driving aims at reducing the risk of collision by anticipating potentially dangerous actions from other drivers and pedestrians. As an example, consider a pedestrian on the sidewalk who might unwittingly set foot on the road. Depending on the time and distance where this pedestrian might step in, the bus could slow down and stop or continue forward and pass the area of potential collision before the risk of collision materializes. This is summarized in Fig. 8, where the black line represents the distance travelled by the bus at constant nominal speed (11.11 m.s^{-1}) while the blue curve represents the distance travelled when stopping as quickly as possible under the safety limit, as shown earlier in Fig. 5.

In regions I and II, the pedestrian would step on the road beyond the stopping distance of the bus, while in regions II, III and IV, the pedestrian would step on the road after the bus has passed the area of potential collision. In these regions, there is always at least one option to avoid collision if the pedestrian sets foot on the road. In region V, however, there

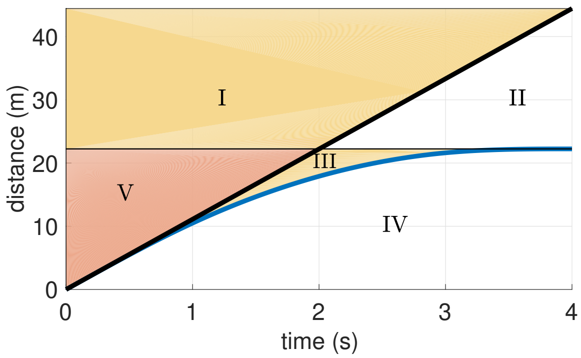

Fig. 8. The black line represents the distance travelled by the bus at constant nominal speed (11.11 m.s^{-1}) while the blue curve represents the distance travelled when stopping as quickly as possible under the safety limit. Depending on the time and distance where a pedestrian might step on the road, the bus could slow down and stop or continue forward and pass the area of potential collision before the risk of collision materializes.

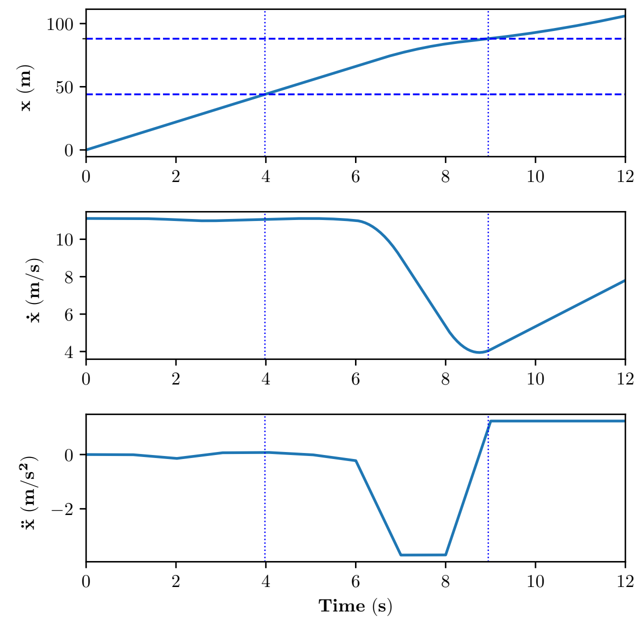

Fig. 9. Two pedestrians are waiting on the sidewalk. Neither steps on the road, but both are close enough that the bus needs to consider the risk that they could. The first pedestrian is passed by after 4 seconds and is far enough from the road that the bus doesn't need to slow down much to make sure that there is always one option available to avoid collision in case this pedestrian sets foot on the road. The second pedestrian is closer to the road, so the bus needs to slow down to 4 m.s^{-1} to make sure that there is always one option available to avoid collision in case this pedestrian sets foot on the road. This second pedestrian is passed by after 9 seconds.

is none: the pedestrian would step in too close for the bus to stop safely, too early for the bus to clear the area. Fortunately, this region shrinks with lower speeds, so an option for the bus to avoid it and make sure there is always at least one option available to avoid collision in case a pedestrian sets foot on the road, is to decelerate preventively.

A typical defensive driving behavior is presented in Fig. 9, where two pedestrians wait on the sidewalk. Neither steps on the road, but both are close enough that the bus needs to consider the risk that they could. The distance of the first pedestrian to the road is such that it would need 2 seconds to step in. In this case, the bus doesn't need to slow down to make sure that there is always one option available to avoid collision in case the pedestrian sets foot on the road. This first pedestrian is then passed by at nominal speed after 4 seconds. The second pedestrian is closer to the road and would need only 1 second to step in. In that case, the bus needs to slow down to 4 m.s^{-1} to make sure that there is always one option available to avoid collision in case this pedestrian sets foot on the road. This second pedestrian is then passed by after 9 seconds.

This defensive driving behavior can be obtained as follows. First, the MPC scheme presented earlier is computed with the optimistic assumption that the next pedestrian is not going to step on the road. If the solution obtained this way happens to pass the area of potential collision before the pedestrian can step in, we can proceed with this solution which appears to be safe, we are in regions II, III or IV of Fig. 8. Otherwise, we are in region I and the bus needs to prepare to stop safely if the pedestrian steps in. We compute therefore in parallel a similar MPC scheme, but with the pessimistic assumption that this pedestrian is going to step on the road. In this pessimistic view, we consider only the safety limit $\ddot{x}_{max}$ on deceleration, not the more constraining comfort limit $\ddot{x}_c$.

This results in the bus starting to decelerate after 6 seconds with maximal deceleration, making sure that it stays in regions I or II. After 8 seconds, the bus is closer to the pedestrian, but also slower, so the size and shape of the regions in Fig. 8 change and the bus ends up entering region II. It can continue decelerating and stop safely, but the optimistic MPC now provides another safe option, where the bus starts re-accelerating and passes the area of potential collision before the pedestrian can step in. This is what the bus elects to do, resuming nominal speed after a comfortable acceleration.

This dual optimistic/pessimistic motion planning provides a simple approach to defensive driving, but exploring optimistic and pessimistic assumptions separately for each pedestrian around the bus would lead to an unmanageable explosion of cases (this could be formulated as a mixed-integer program). The heuristic used above is more limited and explores both optimistic and pessimistic options only for the next pedestrian, based on the assumption that defensive driving with respect to pedestrians around the bus can be adressed sequentially.

VIII. Conclusion

We adopt a combination of Model Predictive Control and Lexicographic Programming to address complex scenarios with conflicting goals related to various aspects of comfort and safety of passengers in a transit bus, generating different deceleration profiles depending on the speed of the bus and distance to obstacles, validated in experiments with a standard transit bus equipped with self-driving capabilities.

Appendix

For simplicity, we consider here a bus driving on a straight line, with only one coordinate x. In order to have accelerations and decelerations change smoothly, we consider a triple integrator with piecewise constant jerk $\dddot{x}$ over time intervals of length T, with the following discrete-time formulation:

$$\begin{bmatrix} x_{k+1} \\ \dot{x}_{k+1} \\ \ddot{x}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix} + \begin{bmatrix} \frac{1}{6}T^3 \\ \frac{1}{2}T^2 \\ T \end{bmatrix} \dddot{x}_k. \quad (13)$$

Iterating this dynamics over a prediction horizon of N time intervals, we have

$$\begin{bmatrix} x_N \\ \dot{x}_N \\ \ddot{x}_N \end{bmatrix} = \begin{bmatrix} 1 & NT & \frac{1}{2}N^2T^2 \\ 0 & 1 & NT \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \end{bmatrix} + \sum_{k=1}^{N} \begin{bmatrix} \frac{3k^2-3k+1}{6}T^3 \\ \frac{2k-1}{2}T^2 \\ T \end{bmatrix} \dddot{x}_{N-k}. \quad (14)$$

We can parameterize the exact same motion with a sequence of accelerations $\ddot{x}_k$ instead of a sequence of jerks $\dddot{x}_k$, by applying the change of variables (2). This leads to a different discrete-time formulation of the exact same dynamics:

$$\begin{bmatrix} x_{k+1} \\ \dot{x}_{k+1} \\ \ddot{x}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{1}{3}T^2 \\ 0 & 1 & \frac{1}{2}T \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix} + \begin{bmatrix} \frac{1}{6}T^2 \\ \frac{1}{2}T \\ 1 \end{bmatrix} \ddot{x}_{k+1}. \quad (15)$$

Iterating this different formulation over the same prediction horizon of N time intervals, we have

$$\begin{bmatrix} x_N \\ \dot{x}_N \\ \ddot{x}_N \end{bmatrix} = \begin{bmatrix} 1 & NT & \frac{3N-1}{6}T^2 \\ 0 & 1 & \frac{1}{2}T \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \end{bmatrix} + \sum_{k=1}^{N-1} \begin{bmatrix} kT^2 \\ T \\ 0 \end{bmatrix} \ddot{x}_{N-k} + \begin{bmatrix} \frac{1}{6}T^2 \\ \frac{1}{2}T \\ 1 \end{bmatrix} \ddot{x}_N. \quad (16)$$

This alternative formulation doesn't involve k^2 and N^2 terms, unlike the first one. We have observed that this leads to much more reliable computations, probably due to a better conditioning of the underlying matrices.

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