Heavenly Order: Concept Dynamics, Mathematical Practices, and the Stability of the Solar System

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One of the key aspects of the evolution of knowledge is certainly the dynamics of concepts, that is how concepts are developed, disseminated, and appropriated over a long span of time. In this paper, I tackle this broad issue by adopting a comparative and deployment-oriented approach to concepts. By discussing the case of the stability proof of the solar system, I argue that the very idea of stability as a scientific problem emerged as a consequence of the transformation of the concept of natural order in astronomy from a metaphysical and theological notion into an effective tool for mathematical research. According to a deeply ingrained narrative, supported by astronomy textbooks, popular accounts, and some scholarly works, Newton's gravitation theory entailed very naturally the question whether the solar system would continue in its orderly periodic movements or would at some stage collapse. The narrative goes on that this fundamental question was eventually answered by Pierre Simon Laplace in the mid-1770 (or, alternatively, mid-1780) with the proof, now known to be partial, that the solar system is made up of strictly periodic motions. The aim of this paper is to present a totally different perspective. First, I argue that, in the original Newtonian framework, the stability of the solar system was not a subject of physical and mathematical inquiry. The cosmological question of stability only turned up within a larger theological and metaphysical discourse, which did not affect physical practice. Rather, Newton and his followers were much more concerned with the study of specific planetary inequalities, most notably the motion of the Lunar apogee and the great anomaly of Jupiter and Saturn.

After Euler's breakthrough in the late 1740s and the ensuing development of modern perturbation theory, these anomalies were attacked by new and more powerful analytical methods. The main protagonist of the vigorous grow of perturbation theory was Joseph Louis Lagrange. Throughout the 1770s, Lagrange refined, extended, and generalized the techniques used in physical astronomy to a level never seen before. Thus, my second claim is that, by introducing new formidable mathematical practices (changes of variables, integrals of motion, perturbation function), Lagrange realized that perturbation theory could aspire to more than just solving localized problems.

Eventually, it was in the early 1780s that Lagrange, for the first time, clearly posed the problem of stability of the solar system and provided a general solution. Laplace, who, admittedly, had also contributed to the development of perturbation theory in an important way, further improved Lagrange's methods and solved the long-standing mysteries of the Lunar motion and the anomalous motion of Jupiter and Saturn. In the first decades of the nineteenth century, Laplace's extremely influential Mécanique Céleste and the whole Laplacian school consolidated the narrative today still widely accepted.

This intricate episode, which, from Newton's Principia to Laplace's solution of the secular acceleration of the Moon, covers almost exactly one hundred years, will allow me to raise two important points. First, I will argue that, although mathematicians in the eighteenth century talk less about natural order than their predecessors in the seventeenth century, the concept of order does not disappear. Rather it is turned from a metaphysical notion into a tool of research. The necessity of solving by approximation Euler's equations of planetary motion led to the introduction of mathematical techniques (such as the method of variation of constants), which relied on the assumption that the heavenly motions were a complex composition of periodic trajectories. Only when perturbation theory reached a sufficient maturity, it became possible to reflect on the status of this assumption and to raise the question of stability. This deep connection between concept dynamics and mathematical practices suggests that one should look more carefully at how concepts are concretely deployed to produce knowledge rather than how they represent it.

Second, the development of perturbation theory provided, so to speak, the structural conditions for the solution of the problem of stability, but it did not, ultimately, determined the legitimacy of the question. To understand the dynamics of the concept of order, one needs to look at the modification of the general cultural and philosophical climate in mid-eighteenth century. There are two key factor, I argue, that facilitated a reflexive approach to order. The first factor was the development of epistemology as an autonomous field of philosophical research no longer related to the theological discourse. A de-theologized epistemology encouraged the search for the foundations of scientific knowledge within the knowledge process itself. The second factor was the appearance of the notion of self-organization. During the eighteenth century, scholars started to look at processes of self-organization in fields as diverse as life science, political economy, and social theory. The regular behavior of systems seemingly very complex such as the financial market suggested not only the idea that some laws were at work in there, but also that these systems had some sort of inherent order that can be made subject of scientific investigation. In order to enlarge the investigation to other discourses it is thus necessary to assume a comparative approach: one needs to include in the analysis the way in which concepts are deployed in the argumentative practices of neighbor fields of research and the dynamics that allows them to migrate from one field to another.