

## Retreading the Path of Science: the case of independent motions

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The transformation of the subject of motion from pre-classical mechanics to the Newtonian world is an important part of the scholarship that integrates history and philosophy of science. In this paper, I bring to the fore the topic of conceptualizing the independence of motions and its empirical grounds. First, while Galileo and Descartes worked with rather different underlying conceptual assumptions of what makes two motions independent, in their common examples they rely on a similar inference-guiding rule of discerning between motions. Secondly, I show that several assumptions that they used are unmotivated by the conceptual tools that are available to both of them. Finally, I show how some of Newton's own struggles in trying to come up with a robust mathematical rule for composing motions were a reply to challenge of identifying a reliable notion of independence for motions.

There are at least two different ways of thinking of independence, depending on how motions are identified in the first place: the kinematical approach (dealing mainly with velocities) and the dynamical one (regarding forces). As I show, from a philosophical point of view, the role of diagrams and the geometrization of motion played a crucial role in how non-interference of motions is represented. This paper argues that Galileo's conceptualization of independent motions comes from his experiments and is represented in his diagrams, where the latter are constructed with the purpose of being accurate representations of natural motions (motions that would be empirically observable). On the other hand, Descartes's conception of independent motions is connected to the geometrical descriptions of curves and now the difficult problem is to find the corresponding motions within an orthogonal system of coordinates.

The structure of my paper is the following.

- 1) I begin by presenting how motions are identified and composed in pedagogical examples: a boat crossing a river, the motion of a projectile, etc. Quite often, students find the consequences of the mathematical (vectorial) counterintuitive. On the other hand, they find the mathematical (vectorial) representation almost trivial. I argue that the same reactions are recognizable, albeit in a rather different form, in Galileo's treatment of projectile motion in his *Dialogue on Two New Sciences*.
- 2) In the second part of my paper I develop the Galilean answer to the question: What are the component motions and what makes them independent?. I show that the mathematical representation by means of a diagram is necessarily part of the answer. In the case of Galileo's projectile's path, the motions that are independent are also on perpendicular directions. While this happens to be accurate for the particular motions that Galileo looked at, we could ask whether velocities that are perpendicular are also the representation of independent motions in general. Galileo understands this question in a particular way: Is this an empirically adequate representation of all projectile motions? For this reason his answer in the *Dialogue on Two New Sciences* only addresses the constraints under which his diagram is useful, but does not aim at a deeper understanding.
- 3) In his *Principles*, Descartes gives several examples where motions are decomposed on orthogonal directions. As I show, in some cases we seem to be forced in identifying one component (when the stone in a sling is released), in others the choice of motions seems arbitrary (the point on a circumference on a wheel). On the other hand, Descartes' geometrical work reveals a different

conceptualization of independence of motions under which certain curves are not considered geometrical. While Descartes recognizes the need to use independent motions in generations of some curves, he doesn't have the tools to give a determinate definition of this geometrical independence. Moreover, this failure forces him in part to exclude mechanical curves from the subject of geometry.

I argue that the Galilean and the Cartesian ways of understanding independence are in tension with each other and the paper concludes that a way of reading some of Newton's works is to follow his attempt at reconciliation. My answer delineates the philosophical challenges we meet when trying to find a sharper definition of independence of motions. If motions are described by velocities, then the velocities that are represented as orthogonal are independent in the following sense:

Adding one motion to the other does not affect the first.

I would note in passing that this formulation also includes a symmetry condition (i.e. it doesn't matter which motion is added) which the empirical Galilean and the Cartesian examples do not satisfy. However, the empirical problem is that in most cases we cannot separate/add de facto one motion from/to the other. Then, the empirical rule used to identify and describe independent motions is the following principle:

Any change that is designed to affect only one of the motions will not change the other.

My paper shows that, while they used the second rule, Galileo and Descartes have insufficient conceptual resources to theoretically motivate and isolate such changes in a predictive and consistent fashion. Why are independent motions represented by an orthogonal system? On their own, the rules above give no justification for why certain identified motions also happen to satisfy the perpendicularity condition. In the case of Galileo and Descartes, their diagrams are not mathematical explanations, but are generalizations from observed cases. Once imported in representation, the perpendicularity became a useful and fruitful assumption that is not justified by or inferred in any way from their respective conceptual frameworks. Finally, I show how this conclusion was part of the conceptual development of Newton's own works.

Read as a contribution to the history and philosophy of science, my paper shows that scientific understanding is achieved by retreading the path of certain problems while emphasizing the historical constraints- both conceptual and practical- under which the answers are sought for.